

Formal Derivation: Knot Harmonics as Mediators for Prime Gaps and Zeta Zeros

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Abstract

This document derives a universal relation connecting prime gaps, zeta zero spacings, and knot harmonics. We propose that periodic knots, particularly (p, q) -torus knots, act as harmonic templates, mediating the relationship between the chaotic distribution of prime gaps and the orderly pattern of zeta zeros.

1. Framework Setup

We hypothesize that prime gaps (Δp_n) , zeta zero spacings (Δt_n) , and knot harmonics $(K(p, q))$ are related by:

$$\Delta p_n = \Delta t_n + H(p, q), \tag{1}$$

where $H(p, q)$ is the harmonic correction provided by a specific knot.

1.1 Prime Gaps and Zeta Zeros

- **Prime gaps:** $\Delta p_n = p_{n+1} - p_n$, where p_n is the n th prime.
- **Zeta zero spacings:** $\Delta t_n = t_{n+1} - t_n$, where t_n is the n th imaginary part of the non-trivial zeros of $\zeta(s)$.

Both distributions exhibit periodic structures that can be analyzed via their Fourier transforms.

1.2 Knot Harmonics

Periodic knots, such as (p, q) -torus knots, have harmonics defined by their winding numbers p and q . The Fourier representation of a knot is:

$$H(p, q) = a \cdot \cos(p \cdot k \cdot t) + b \cdot \sin(q \cdot k \cdot t), \quad (2)$$

where k is the frequency index, and a, b are scaling factors.

2. Alignment Metric

To quantify the alignment between prime gaps, zeta zeros, and knots, we define the misalignment metric:

$$\mathcal{C}(p, q) = \int (F_p(k) - F_z(k) - F_k(k; p, q))^2 dk, \quad (3)$$

where:

- $F_p(k)$: Spectrum of prime gaps.
- $F_z(k)$: Spectrum of zeta zero spacings.
- $F_k(k; p, q)$: Spectrum of the (p, q) -torus knot.

The optimal knot parameters (p^*, q^*) minimize $\mathcal{C}(p, q)$.

3. Derivation of the Relation

3.1 Fourier Spectra

The Fourier transforms of the distributions are:

$$F_p(k) = \int e^{-ikt} \Delta p_n dt, \quad (4)$$

$$F_z(k) = \int e^{-ikt} \Delta t_n dt, \quad (5)$$

$$F_k(k; p, q) = \int e^{-ikt} H(p, q) dt. \quad (6)$$

3.2 Universal Relation

Substituting $H(p, q)$ into the alignment metric:

$$\Delta p_n = \Delta t_n + a \cdot \cos(p \cdot k \cdot t) + b \cdot \sin(q \cdot k \cdot t). \quad (7)$$

Minimizing $\mathcal{C}(p, q)$ aligns the chaotic prime gaps with the stable zeta zero spacings.

4. Implications

- **Predictive Framework:** Given a range of primes or zeta zeros, this formula predicts the optimal knot harmonic required to stabilize their relationship.
- **Universal Scaling:** If the same (p, q) -torus knot minimizes $\mathcal{C}(p, q)$ across multiple ranges, this establishes a universal harmonic framework.

5. Conclusion

This derivation formalizes the role of knot harmonics as mediators between prime gaps and zeta zeros. The proposed framework bridges chaotic and harmonic distributions, advancing our understanding of their deep connections.

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