

# Intro to Classification

## Question:

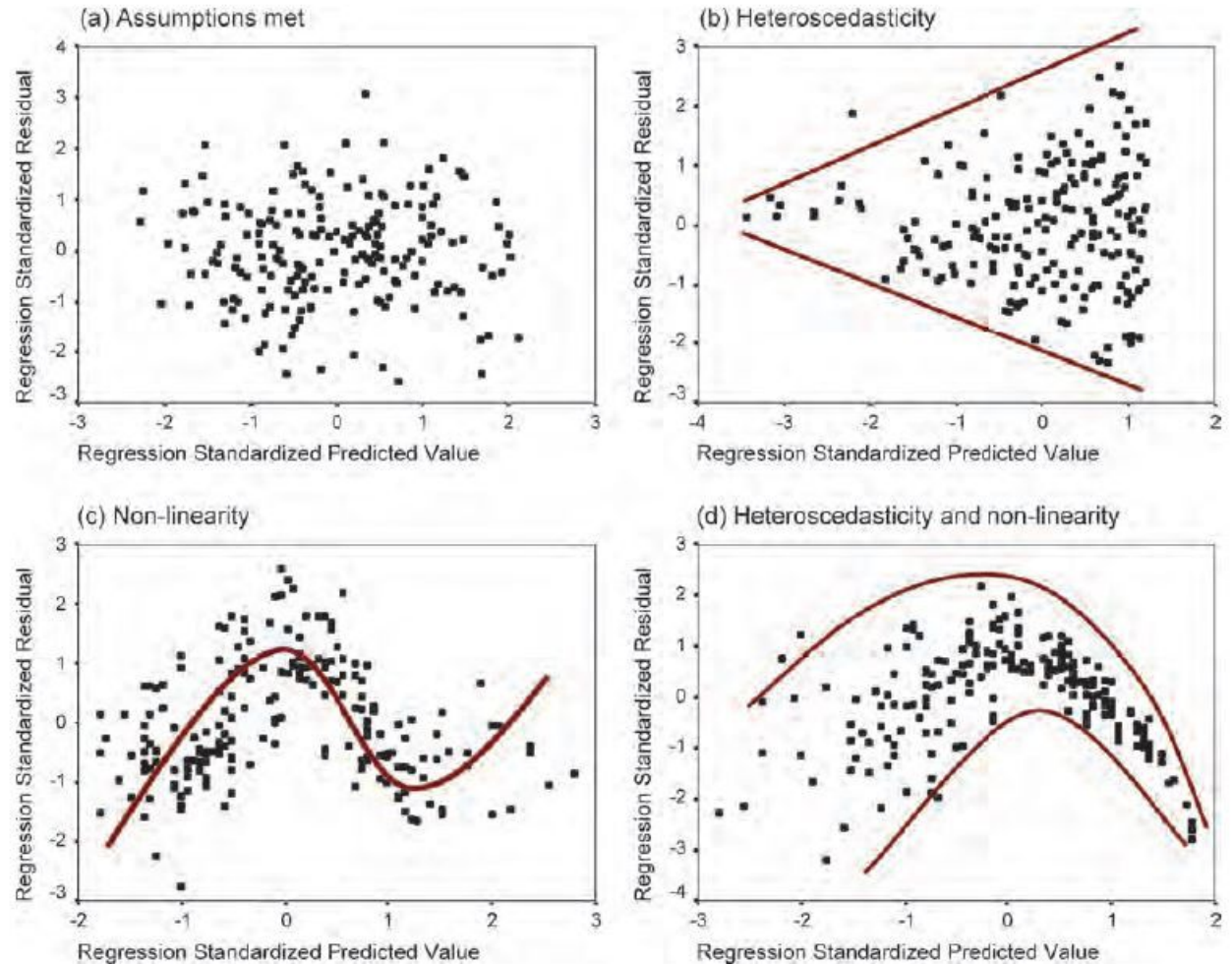
Last week we talked about regression. What is supervised learning? What is regression?



# Heteroscedasticity

- Variability of the data
- Check whether assumption of linearity is valid

[Source](#)



# Review: Least Squares Error

We define our error as follows:

$$\sum_{i=0}^n (y_i - (B_0 + B_1x_1 + \dots + B_px_p))^2$$

theoretical

observed

We call this **Least Squares Error**. Sum of squared *vertical* distance between observed and theoretical values.



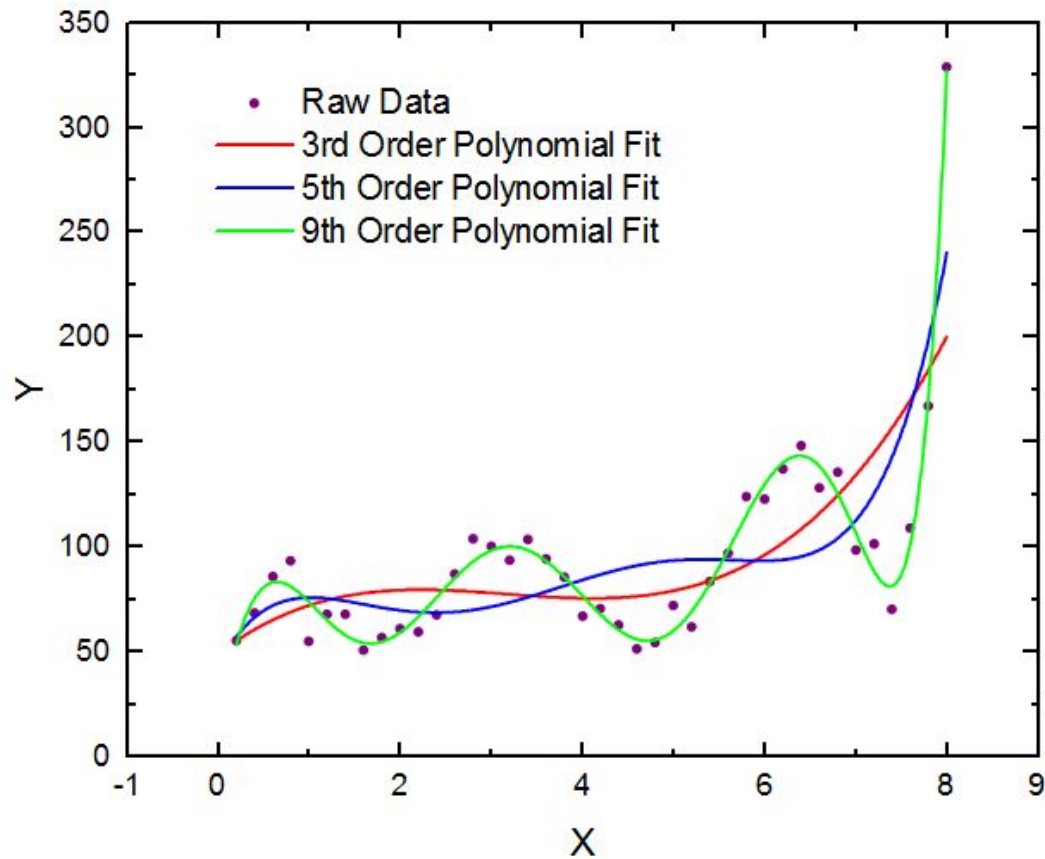
# Model “Goodness of Fit”

Common metric is called  $R^2$ .

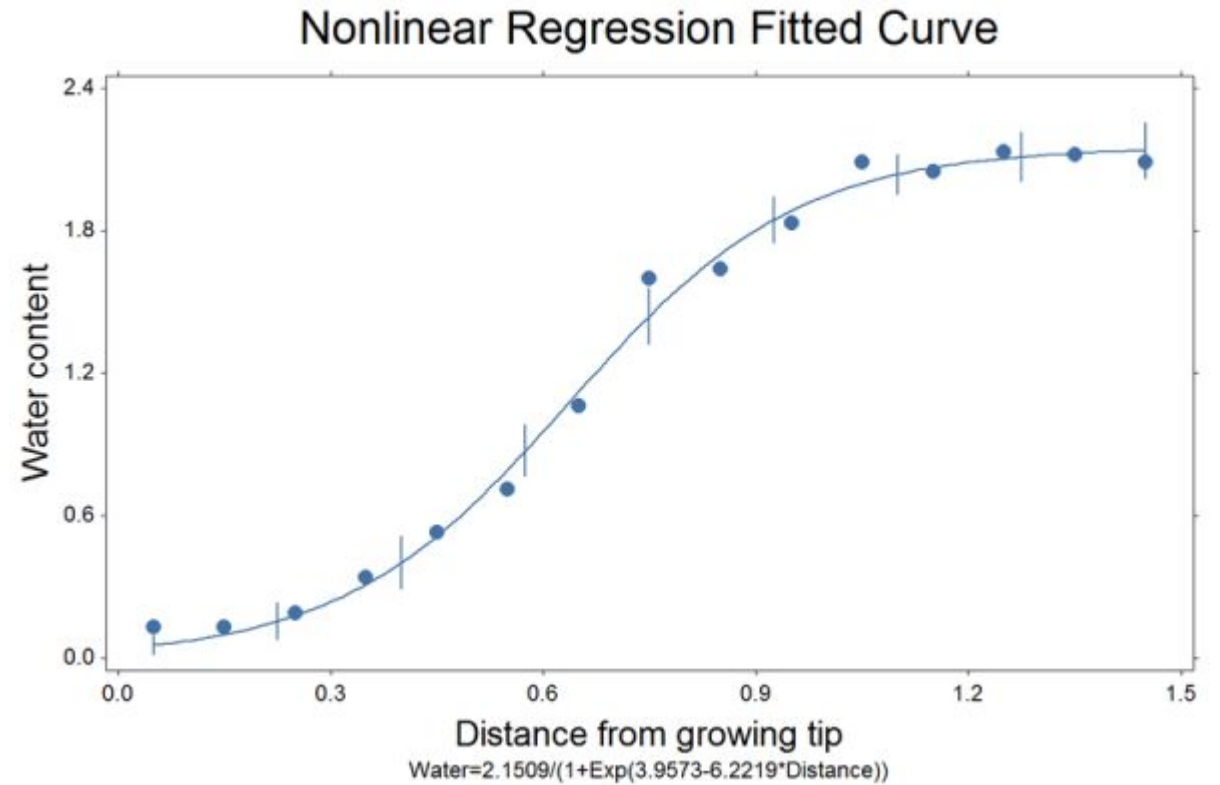
- We compare our model to a **benchmark model**
  - Predict the mean  $y$  value, no matter what the  $x_i$ 's are
- $SST$  = least-squares for benchmark
- $SSE$  = least-squares error for our model
- $R^2 = 1 - SSE/SST$



# Non-linear Regression



[Source](#)



[Source](#)

# Intro to Classification

- “What species is this?”
- “How would consumers rate this restaurant?”
- “Which Hogwarts House do I belong to?”
- “Am I going to pass this class?”



[Source](#)

# The Bayesian Classifier

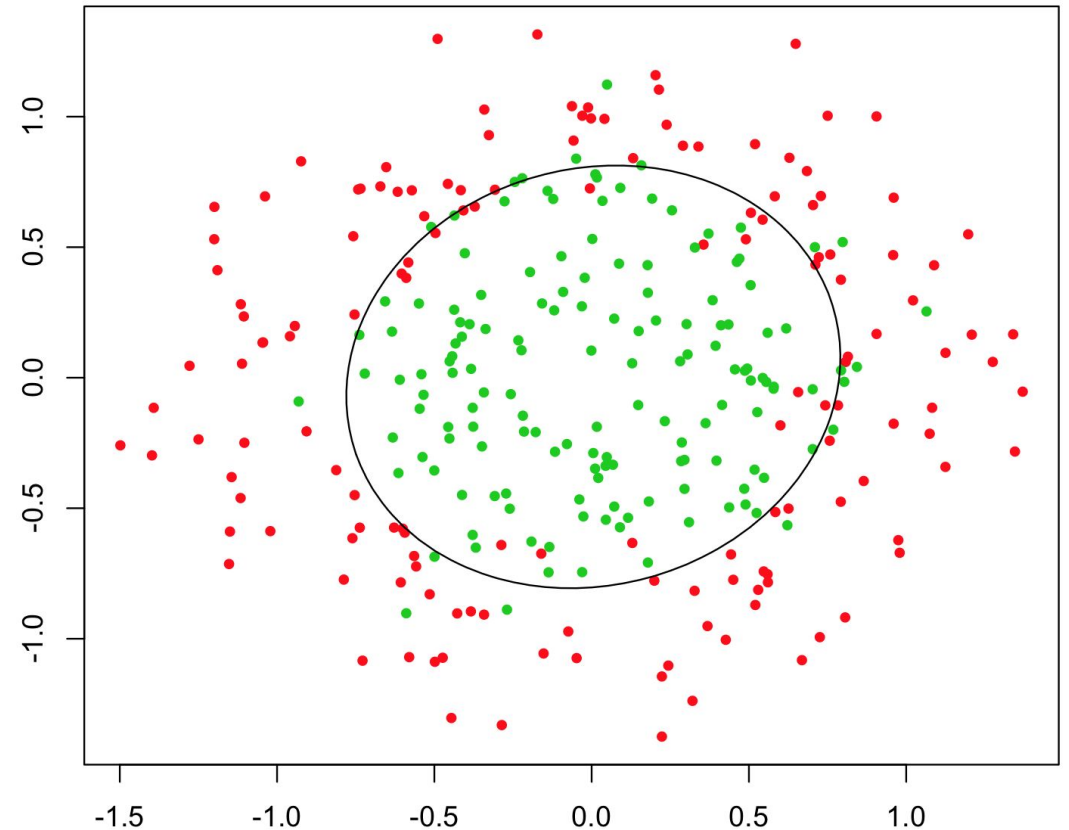
- The ideal classifier: a theoretical classifier with the highest accuracy
- Picks the class with the highest conditional probability for each point
- Assumes conditional distribution is known
- Exists only in theory!
- A conceptual **Golden Standard**





# Decision Boundary

- The **decision boundary** partitions the outcome space
- Classification algorithm you should use differs depending on whether the data is or is not linearly separable



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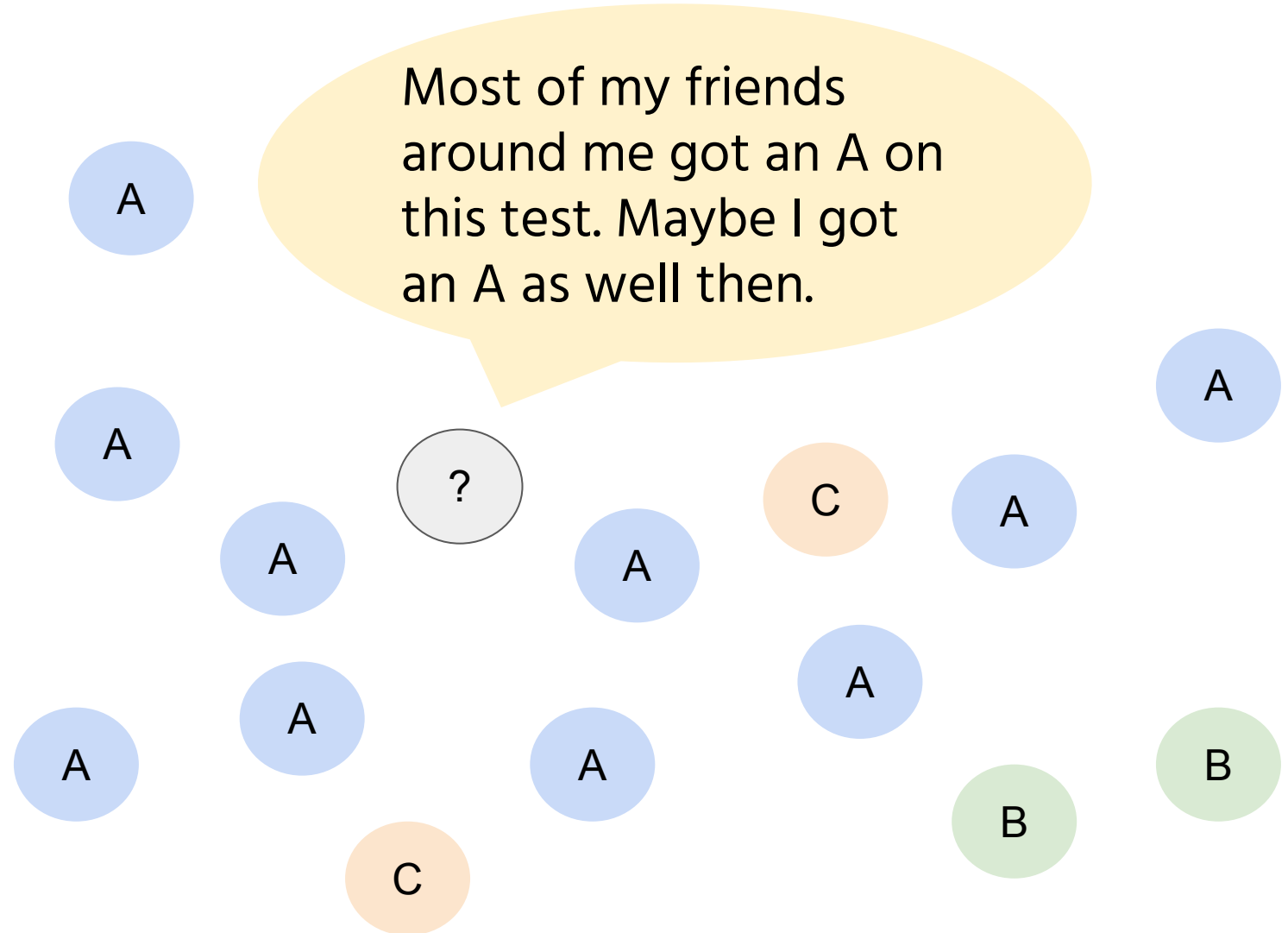
# k-Nearest Neighbors (KNN)

Easy to interpret

Fast calculation

No prior assumptions

Good for coarse analysis



# KNN

How does it work?

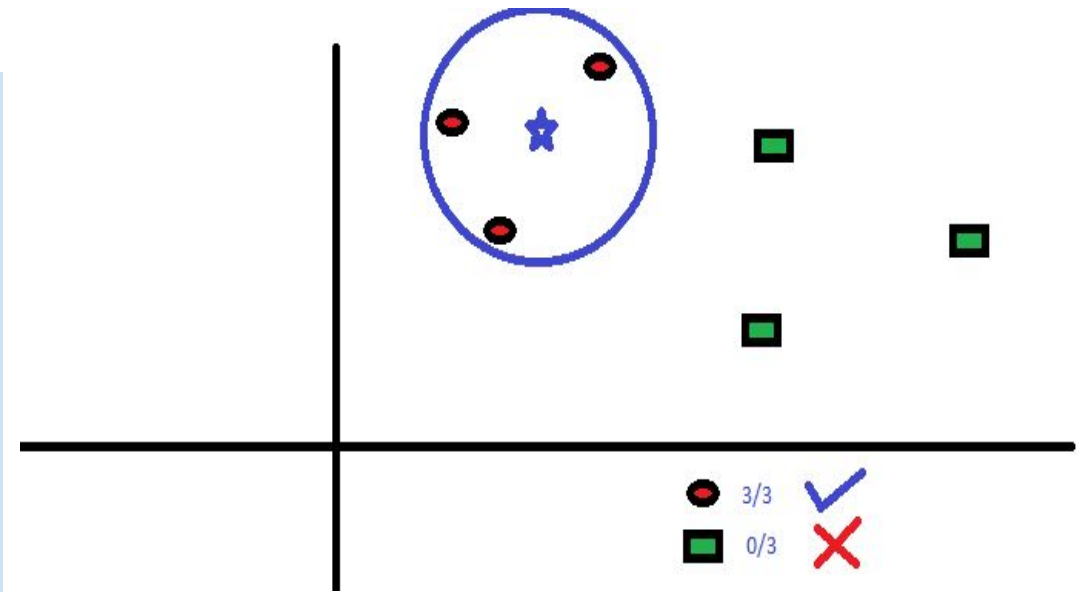
**Define** a  $k$  value (in this case  $k = 3$ )

**Pick** a point to predict (blue star)

**Count** the number of closest points

**Increase** the radius until the number of points within the radius adds up to 3

**Predict** the blue star to be a red circle!



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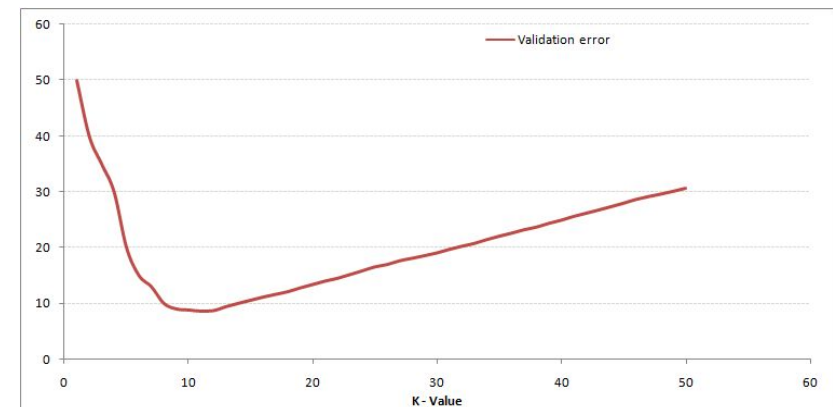
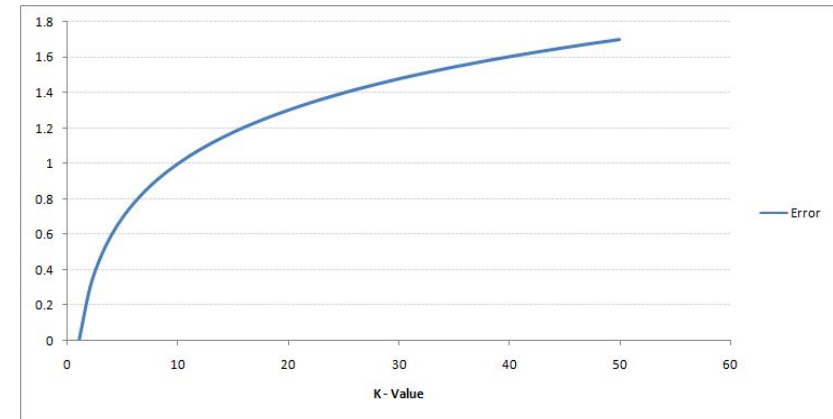
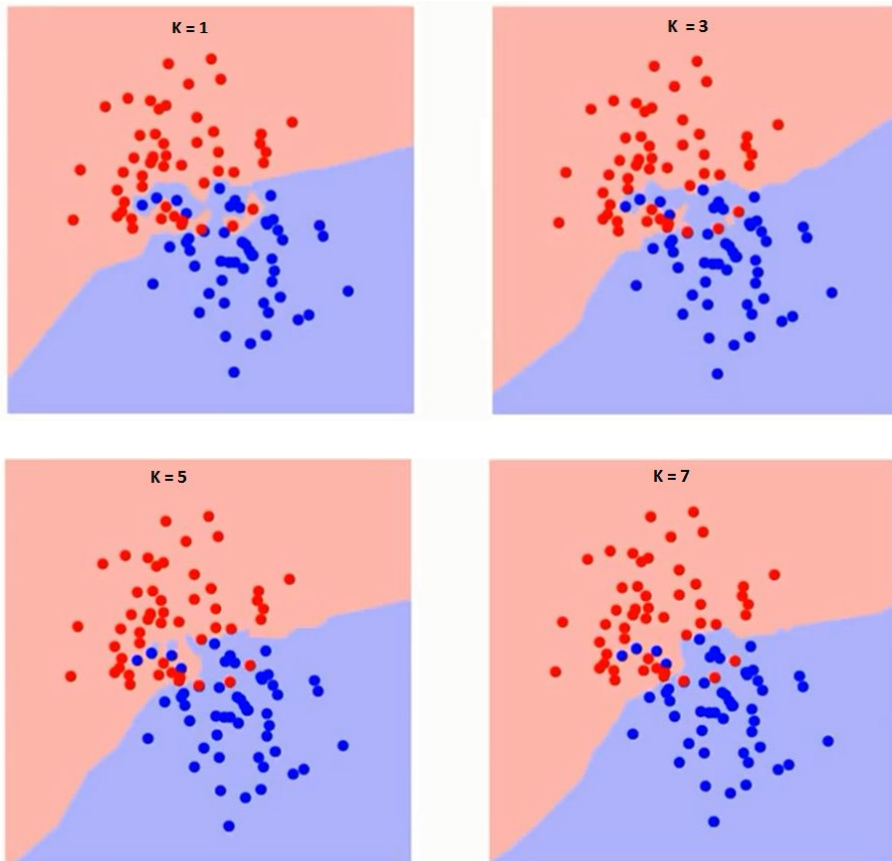


**Question:**  
What defines a good  $k$  value?

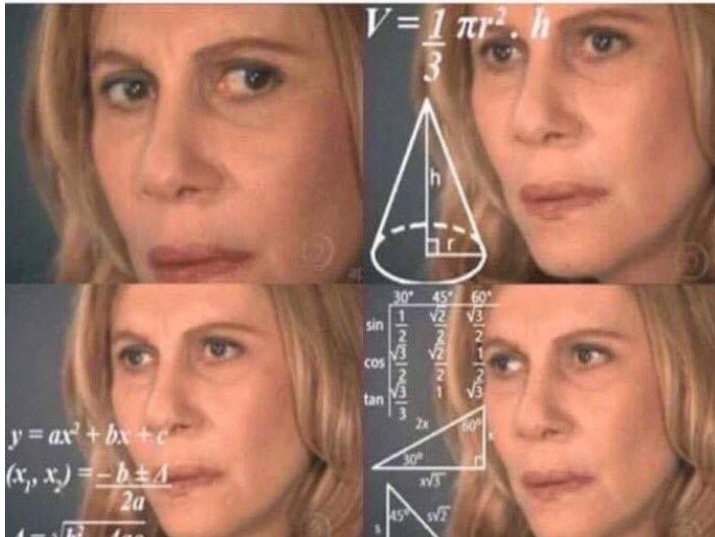


# KNN

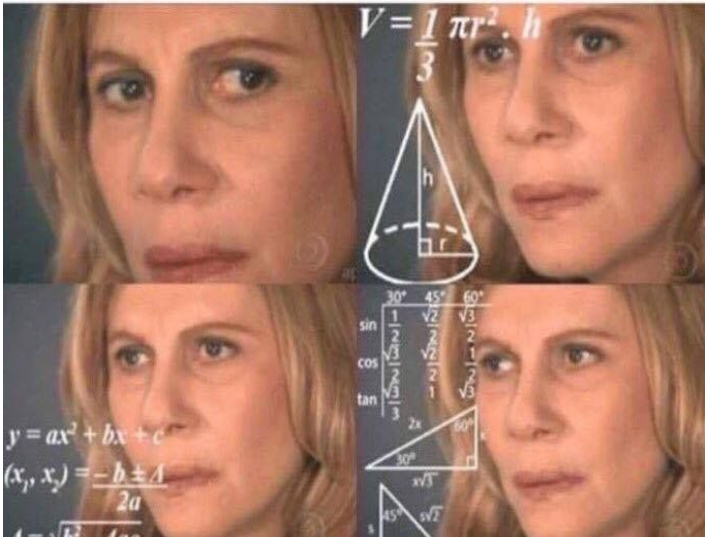
The  $k$  value you use has a relationship to the fit of the model.



# Confusion Matrix



	p' (Predicted)	n' (Predicted)
p (Actual)	True Positive	False Negative
n (Actual)	False Positive	True Negative



# Sensitivity

Also called **True Positive Rate**.

How many positives are correctly identified as positives?

Useful for:

- Airport security
- Initial diagnosis of fatal disease

$$\text{Sensitivity} = \frac{\text{True Positive}}{(\text{True Positive} + \text{False Negative})}$$





# Specificity

Also called **True Negative Rate**.

How many negatives are correctly identified as negative?

$$\text{Specificity} = \frac{\text{True Negative}}{(\text{True Negative} + \text{False Positive})}$$





## **Question:**

Name some examples of situations where you'd want to have a high specificity.



# Other Important measures

- **Overall accuracy** - proportion of correct predictions
- **Overall error rate** - proportion of incorrect predictions
- **Precision** - proportion of correct positive predictions among all positive predictions
- **Recall** = sensitivity



$$\begin{aligned}\textbf{Accuracy} &= \\ &(\text{True Positive} + \text{True Negative}) / \text{Total} \\ \textbf{Error Rate} &= \\ &(\text{False Positive} + \text{False Negative}) \\ &\quad / \text{Total}\end{aligned}$$

$$\begin{aligned}\textbf{Precision} &= \\ &\frac{\text{True Positive}}{(\text{True Positive} + \text{False Positive})} \\ \textbf{Recall} &= \\ &\frac{\text{True Positive}}{(\text{True Positive} + \text{False Negative})}\end{aligned}$$

# Example

Given this confusion matrix, what is the:

- Specificity?
- Sensitivity?
- Overall error rate?
- Overall accuracy?
- Precision?

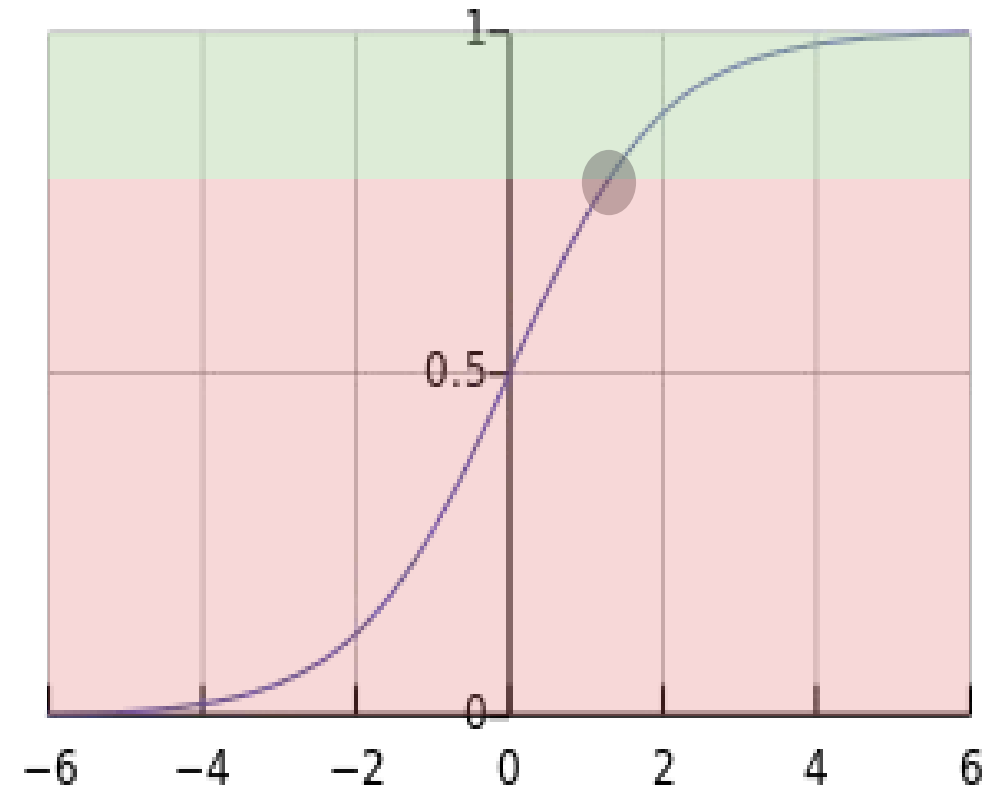
	$p'$ (Predicted)	$n'$ (Predicted)
$p$ (Actual)	146	32
$n$ (Actual)	21	590



# Threshold

Where between 0 and 1 do we draw the line?

- $P(x)$  below threshold:  
predict 0
- $P(x)$  above threshold:  
predict 1



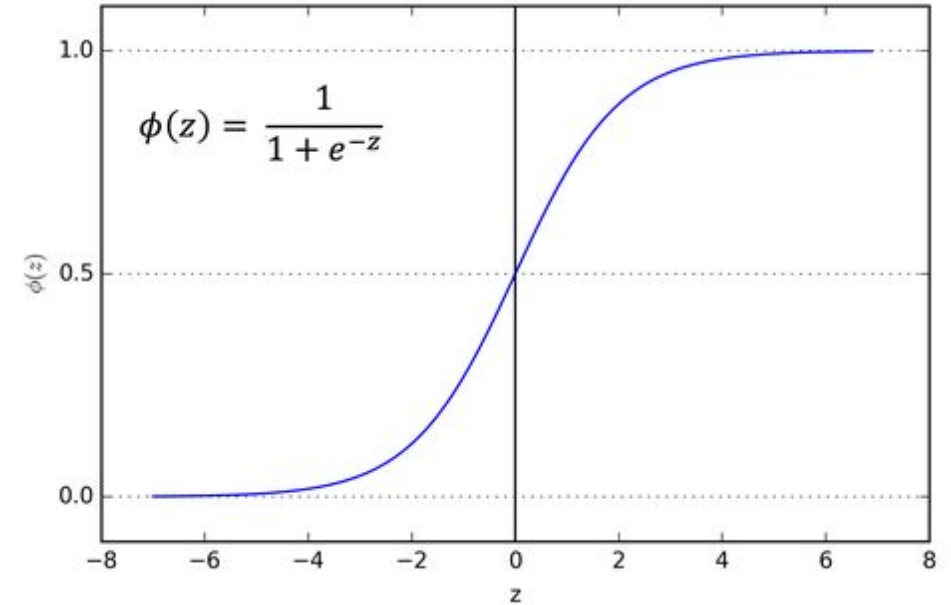
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# Thresholds matter (a lot!)

What happens to the specificity when you have a

- Low threshold?
- High threshold?



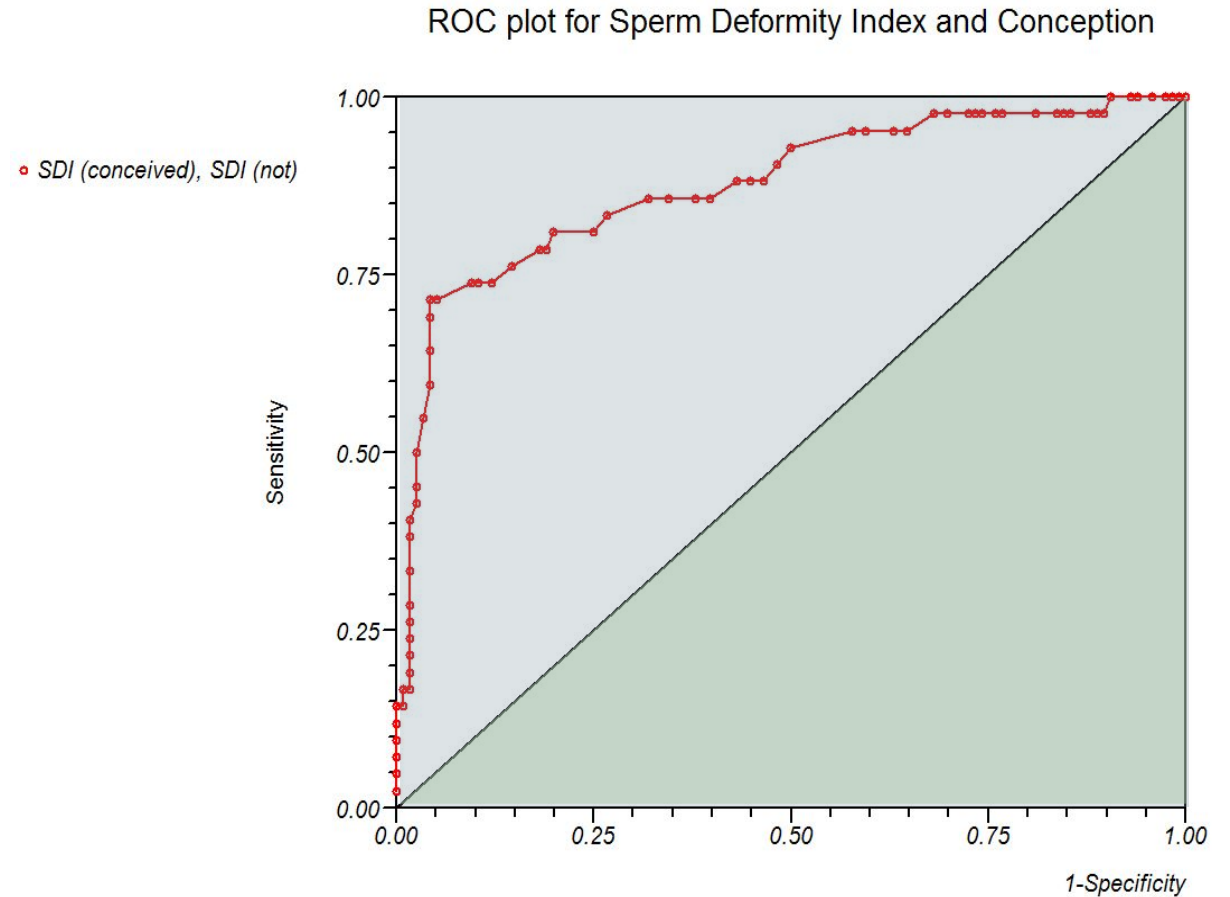
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# ROC Curve

## Receiver Operating Characteristic

- Visualization of trade-off
- Each point corresponds to a specific threshold value



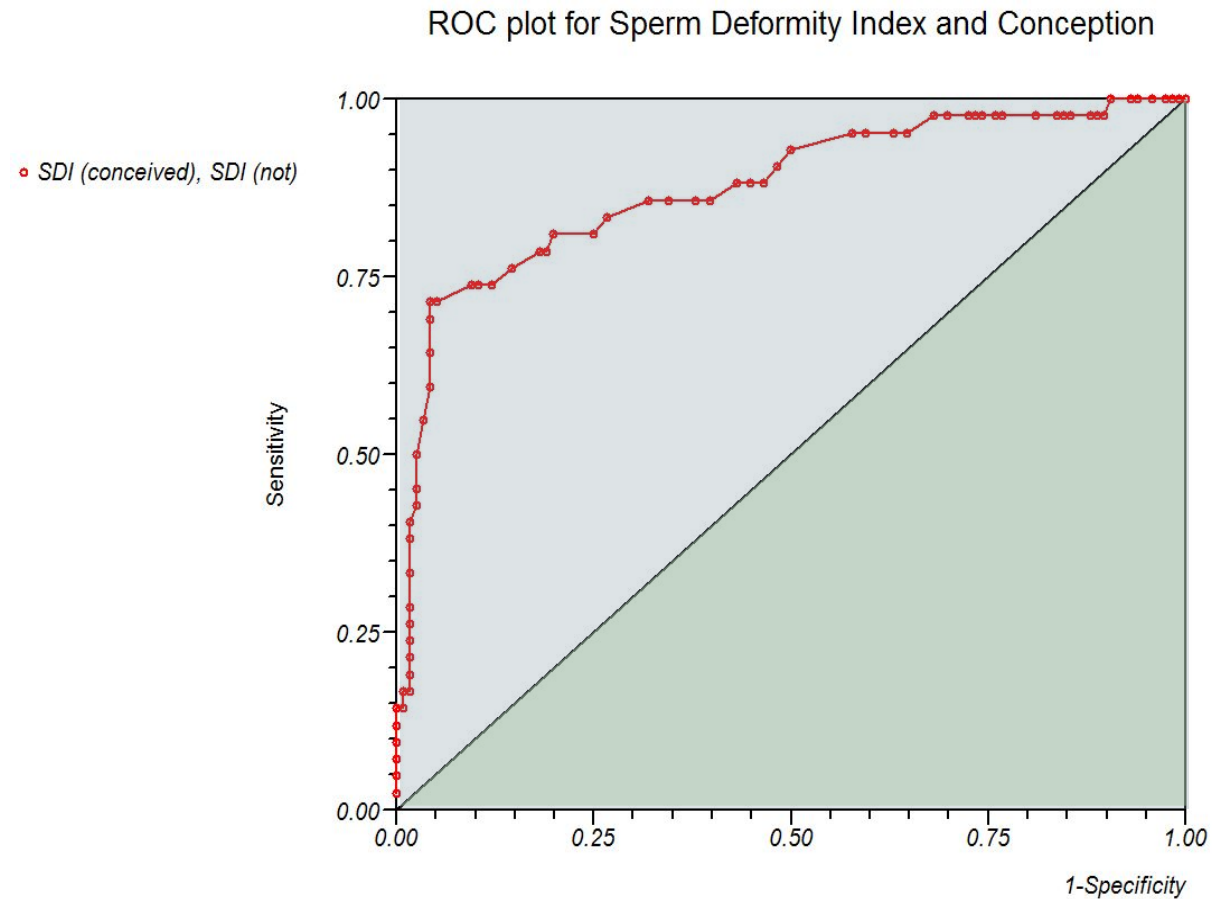
# Area Under Curve

$$AUC = \int ROC\text{-}curve$$

Always between 0.5 and 1.

Interpretation:

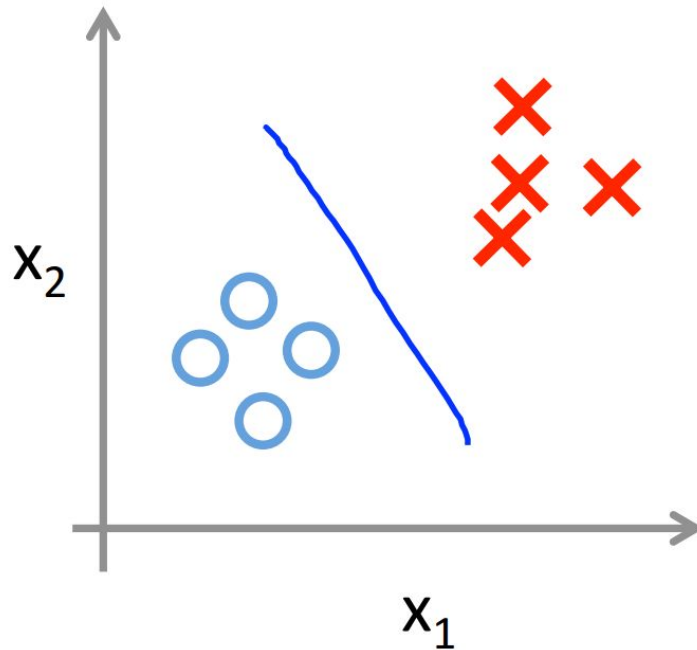
- 0.5: Worst possible model
- 1: Perfect model



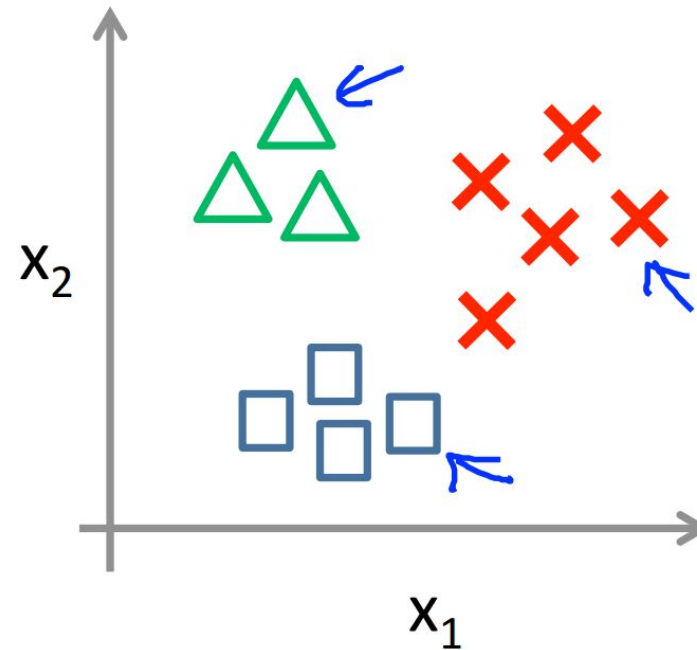
# Multi-class Classification

Classifying instances into three classes or more

Binary classification:



Multi-class classification:

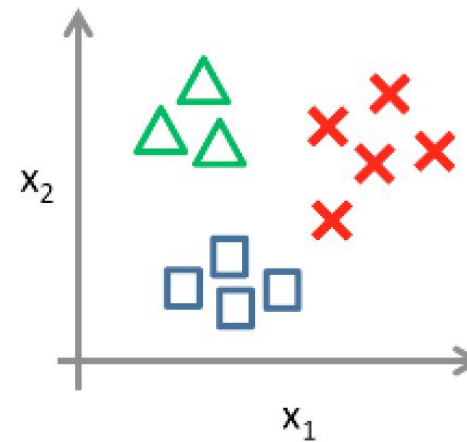







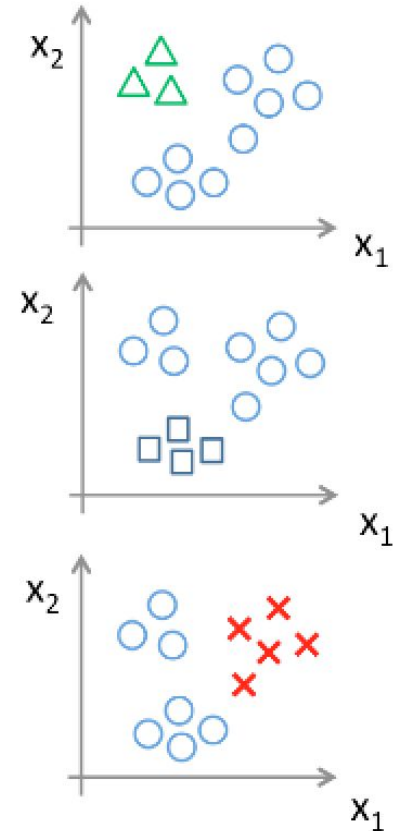
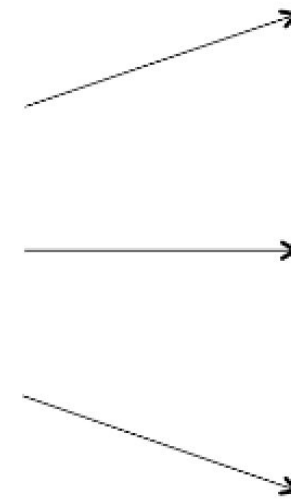
# One-vs-All

- Train a single classifier per class
- All samples of that class classified as positive, all other samples as negative

One-vs-all (one-vs-rest):



Class 1:   
Class 2:   
Class 3: 



# Coming Up

**Your problem set:** Start working on Project Part B

**Next week:** More classifiers

See you then!

