# Exercise: Maximum Cut

# Session Outline

- Introduce the maximum cut example
- Exercise: develop a QUBO for the maximum cut problem
- Review the solution

**Session Goals**

1. Practice formulating a QUBO

# Problem: Maximum Cut

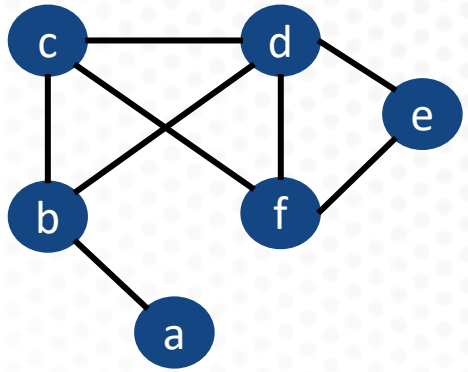
# Example: Maximum Cut Problem

**Problem**

The maximum cut problem seeks to cut through the maximum amount of edges in a graph.

Another way of saying this is:

A maximum cut is a subset of a graph's vertices such that the number of edges between this subset and the remaining vertices is as large as possible



# Example: Maximum Cut Problem

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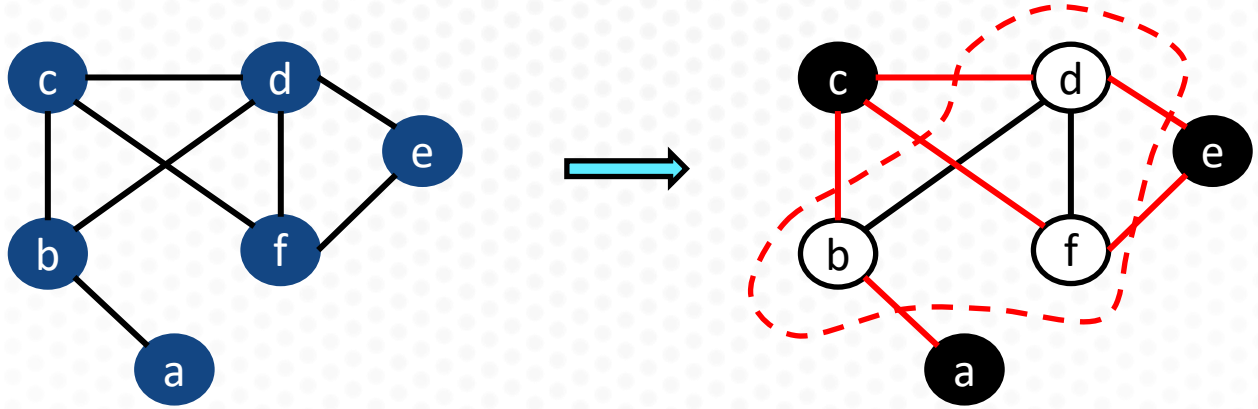


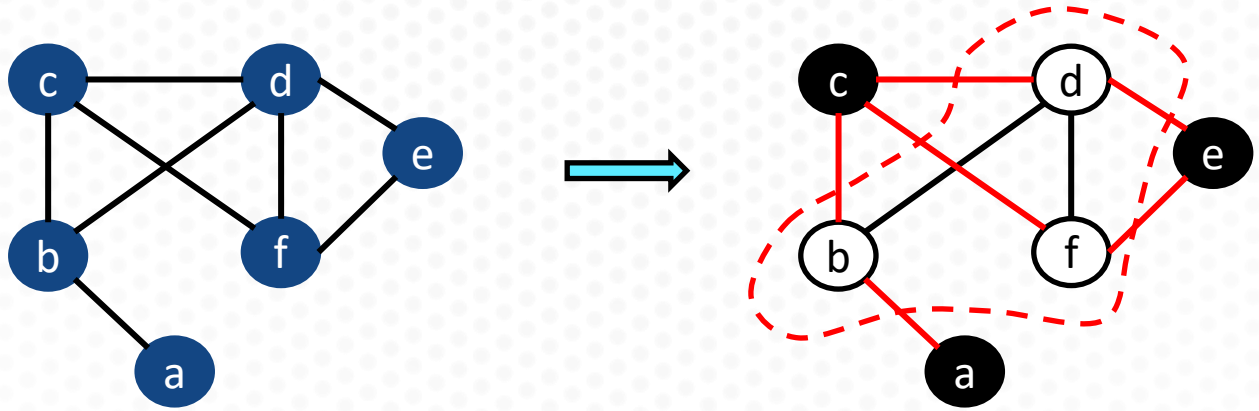
# Example: Maximum Cut Problem

**Problem**

The maximum cut problem seeks to cut through the maximum amount of edges in a graph.

**Exercise**

Follow the QUBO formulation steps to write a QUBO that finds the subset of the graph below that cuts through a maximum amount of edges.



# Example: Maximum Cut Problem

**Problem**

Partition the set so that the partition cuts through a maximum number of edges

**QUBO Writing Process**

1. Write out the objective and constraints in your problem domain
2. Define the binary variables
3. Write out objective in QUBO form
4. Write out constraints in QUBO form
5. Combine objectives and constraints
6. Solve and interpret results
7. Tune your QUBO to get better results

# Example: Maximum Cut Problem

**Problem**

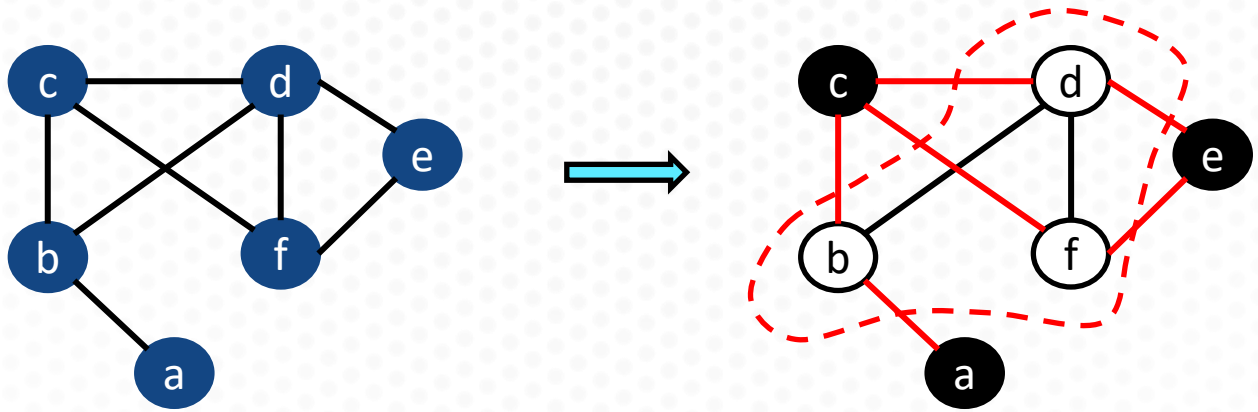
The maximum cut problem seeks to cut through the maximum amount of edges in a graph.

**Exercise**

Follow the QUBO formulation steps to write a QUBO that finds the subset of the graph below that cuts through a maximum amount of edges.

**Hint**

In this domain you're working with the graph's edges (whereas in the set partitioning problem you were thinking about the sums of numbers). You want edges in the same set to increase the QUBO's energy.



# Exercise: Maximum Cut

# Building blocks of QUBOs

To construct a QUBO for a particular problem you need to define a few things about that problem

**Binary Variables**
Each state of the binary variables must be assigned a meaning

**Objective**
The overall goal of the problem – what we're trying to minimize or maximize

**Constraints**
Rules that define what solutions are acceptable and which are not

# Example: Maximum Cut Problem

**Problem**

Partition the set so that the partition cuts through a maximum number of edges

**Step 1.** Write out the objective and constraints in your problem domain

**Objective**:

**Constraints:**

# Example: Maximum Cut Problem

**Problem**

Partition the set so that the partition cuts through a maximum number of edges

**Step 2.** Define the binary values

$$E_{qubo} = \sum_i a_i x_i + \sum_i b_{i,j} x_i x_j$$

We're working in QUBO so our binary variables are $x_i \in \{0, 1\}$

Let's define them as

$$x_i = \begin{cases} 1 & \_\_\_\_\_ \\ 0 & \_\_\_\_\_ \end{cases}$$

Assign meaning to binary values

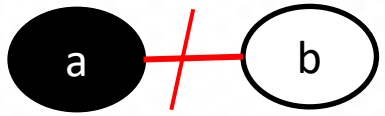
# Example: Maximum Cut Problem

**Problem**

Partition the set so that the partition cuts through a maximum number of edges

**Step 3.** Write out the objective in QUBO form

Hint:
Think about cutting through one edge first. How would you write this out mathematically?



# Example: Maximum Cut Problem

**Problem**

Partition the set so that the partition cuts through a maximum number of edges

**Step 3.** Write out the objective in QUBO form

| $x_i$ | $x_j$ | edge(i, j) |
|---|---|---|
| | | |
| | | |
| | | |
| | | |

Hint:
Can a truth table help?

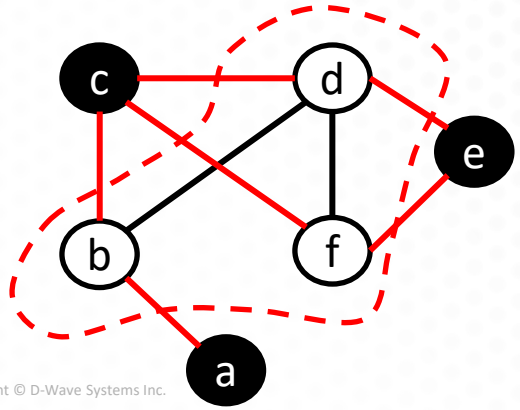
# Example: Maximum Cut Problem

**Problem**

Partition the set so that the partition cuts through a maximum number of edges

**Step 3.** Write out the objective in QUBO form

Hint:
How can you apply an expression for one edge to an entire graph?



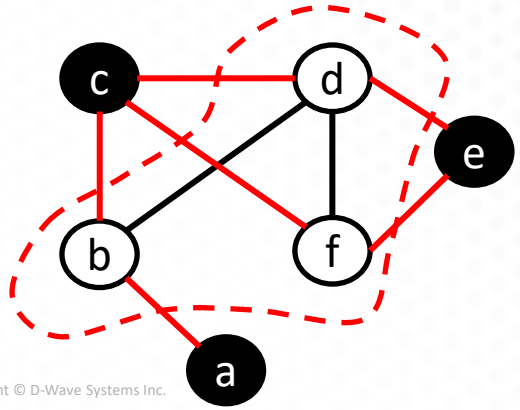
# Example: Maximum Cut Problem

**Problem**

Partition the set so that the partition cuts through a maximum number of edges

**Step 3.** Write out the objective in QUBO form

Hint: $E_{qubo} = \sum_i a_i x_i + \sum_i b_{i,j} x_i x_j$



# Example: Maximum Cut Problem

**Problem**

Partition the set so that the partition cuts through a maximum number of edges

**Step 4.** Write out the constraints in QUBO form

Hint:
Remember that constraints are rules about which solutions are feasible and which aren't.

# Example: Maximum Cut Problem

**Problem**

Partition the set so that the partition cuts through a maximum number of edges

**Step 5.** Combine objectives and constraints

$$E_{qubo} = \min(objective) + \gamma(constraints)$$