Programming D-Wave's Quantum Computer

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Session Outline

- **Review**
  - Problem formulation concepts
- **Example: Formulation to Programming**
  - Antennas

Session Goals

1. Work through one more problem formulation example
2. Create a general program to solve the problem

Review

Process for Constructing a QUBO

1. Write out the objective and constraints in your problem domain
2. Define the binary variables
3. Write out objective in QUBO form
4. Write out constraints in QUBO form
5. Combine objectives and constraints
6. Solve and interpret results
7. Tune your QUBO to get better results

Example: Antenna Placement

Antenna Placement Example

Given: A set of viable locations,

Determine where to build antennas so that coverage is maximized and there is no interference.

Nodes

Viable locations for antennas

Edges

Indicate interference between signals from two antennas

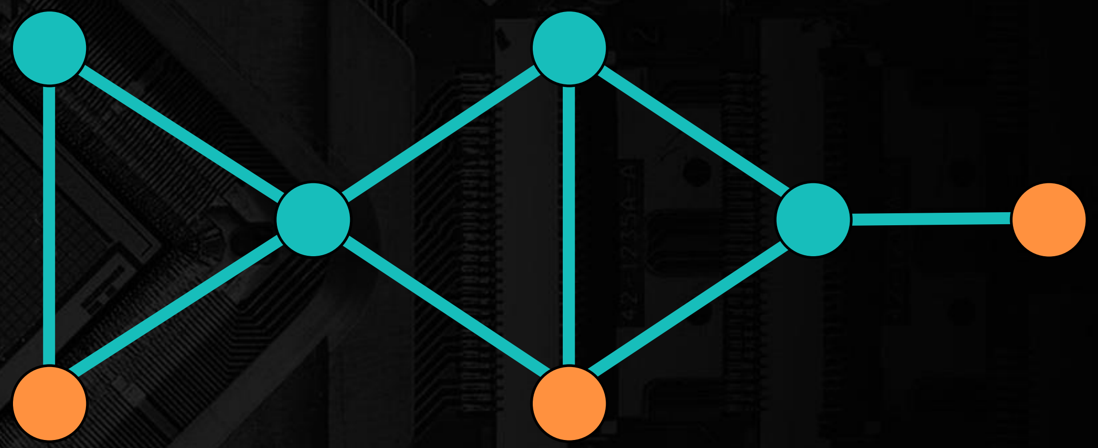
Antenna Placement Example

Given: A set of viable locations,

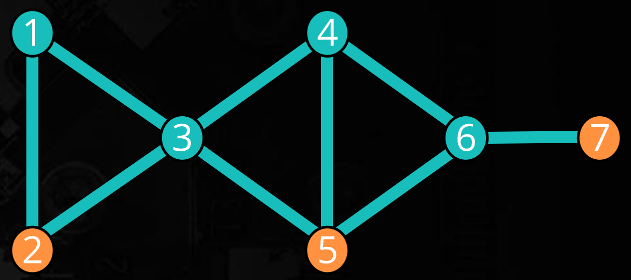
Determine where to build antennas so that coverage is maximized and there is no interference

A Maximum Independent Set

Find the maximum set of nodes such that there are no edges in the maximum set.



Antenna Placement Example



Given: A set of viable locations,

Determine where to build antennas so that coverage is maximized and there is no interference

1. Write out the objective and constraints in the problem domain

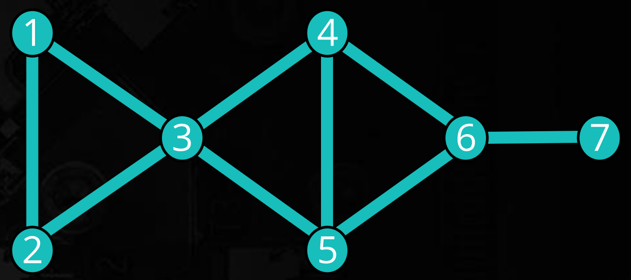
Objective

Maximize the number of antenna locations selected

Constraints

No interference between antennas

Antenna Placement Example



Given: A set of viable locations,

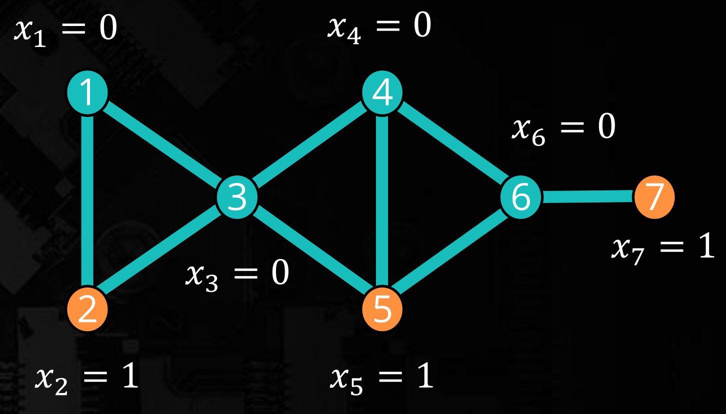
Determine where to build antennas so that coverage is maximized and there is no interference

2. Define binary variables

$$x_i = \begin{cases} 1 \;\; if\ node\ i\ is\ in\ our\ subset \\ 0 \;\; if\ node\ i\ is\ not\ in\ our\ subset \end{cases}$$

Binary variables for this problem: $x_1, x_2, x_3, x_4, x_5, x_6, x_7$

Antenna Placement Example



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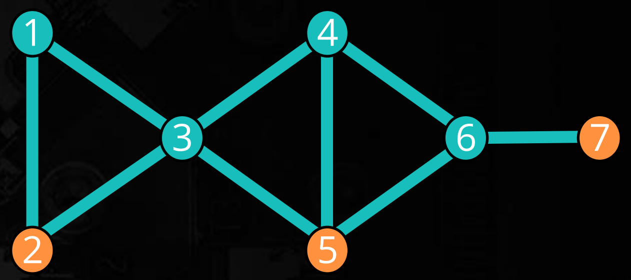


Given: A set of viable locations,

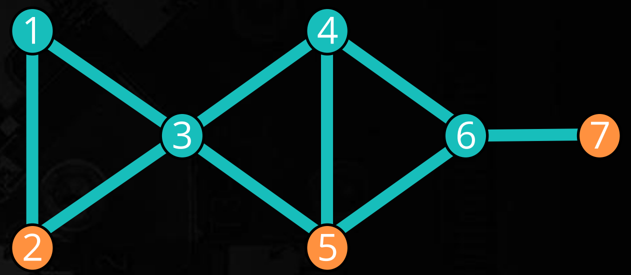
Determine where to build antennas so that coverage is maximized and there is no interference

3. Write out the objective in QUBO form

Maximize the number of antenna locations selected

$$max \sum_i x_i$$

Antenna Placement Example



Given: A set of viable locations,

Determine where to build antennas so that coverage is maximized and there is no interference

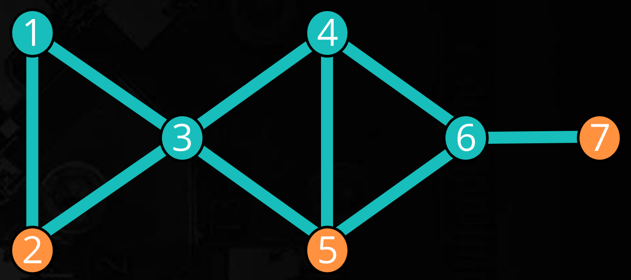
3. Write out the objective in QUBO form

Maximize the number of antenna locations selected

$$-\sum_i x_i$$

Trick:
Turn a maximization function into a minimization function by negating it

Antenna Placement Example



Given: A set of viable locations,

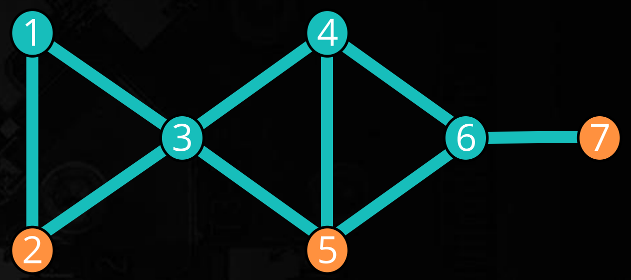
Determine where to build antennas so that coverage is maximized and there is no interference

4. Write out the constraints in QUBO form

No interference between antennas (no edges in the subset)

x_i	x_j	Edge?
0	0	Y
0	1	N
1	0	N
1	1	Y

Antenna Placement Example



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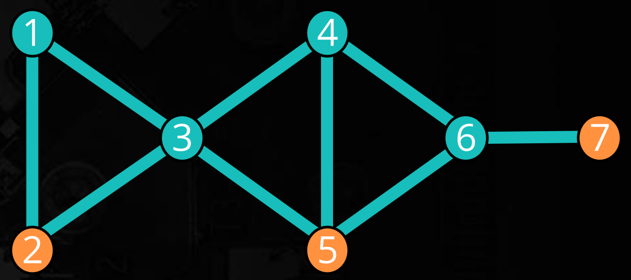

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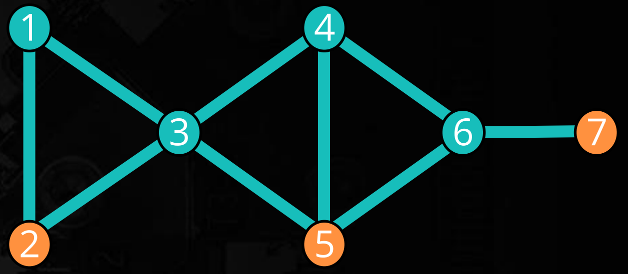
Solve system of equations:
ax_i + bx_j + c$x_i x_j$ + d

Antenna Placement Example



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Solve system of equations:
$ax_i + bx_j + cx_ix_j + d$

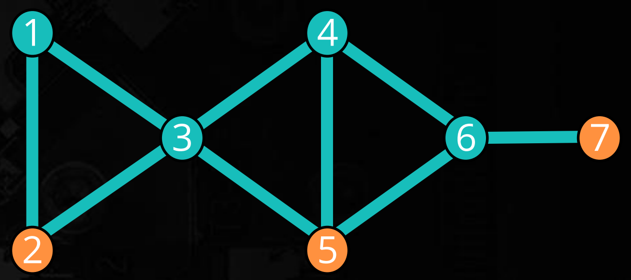
$0 + 0 + 0 + d = 0 \rightarrow d = 0$

$0 + b + 0 + d = 0 \rightarrow b = 0$

$a + 0 + 0 + d = 0 \rightarrow a = 0$

$a + b + c + d = 1 \rightarrow c = 1$

Antenna Placement Example



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Solve system of equations:
ax_i + bx_j + c$x_i x_j$ + d

0 + 0 + 0 + d = 0 → d = 0

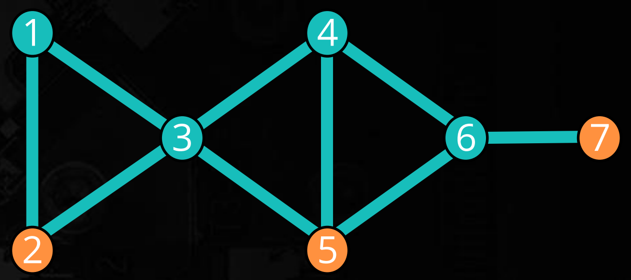
0 + b + 0 + d = 0 → b = 0

a + 0 + 0 + d = 0 → a = 0

a + b + c + d = 1 → c = 1

Penalty Model:
$x_i x_j$

Antenna Placement Example



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1	0	N	0
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To get an equation for the constraint, sum up over all edges:

$$\sum_{(i,j) in\ E} x_i x_j$$

Antenna Placement Example

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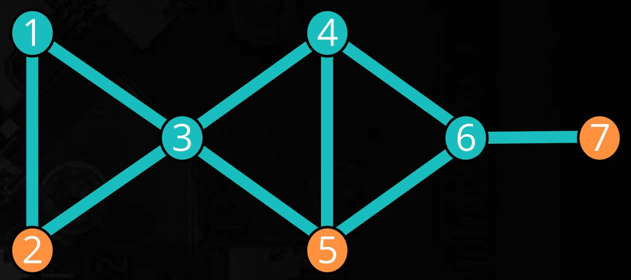
Determine where to build antennas so that coverage is maximized and there is no interference



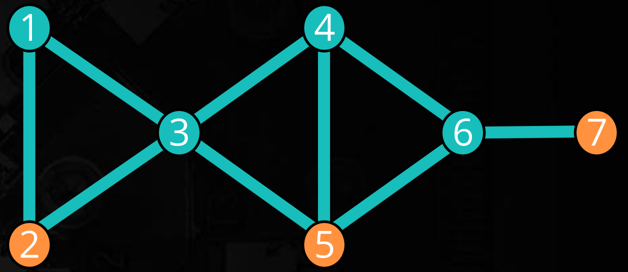
E = {(1,2), (1,3), (2,3), (3,4), (3,5), (4,5), (4,6), (5,6), (6,7)}

5. Combine the objective and constraints

$$E_{QUBO} = \min(objective) + \gamma(constraints)$$

$$E_{QUBO} = -\sum_i x_i + \gamma \sum_{(i,j)\in E} x_i x_j$$

Antenna Placement Example



Given: A set of viable locations,

Determine where to build antennas so that coverage is maximized and there is no interference

6. Solve and interpret the results

Objective

$$-\sum_i x_i$$

Constraint

$$\gamma \sum_{(i,j)\in E} x_i x_j$$

```
# Create a BQM object
bqm = dimod.BQM('BINARY')

# Add linear biases
bqm.add_variables_from({node: -1 for node in G.nodes})

```

```
# Add quadratic biases
bqm.add_interactions_from({(u,v): gamma for u,v in G.edges})

```

Antenna Placement Example

6. Solve and interpret the results

```python
# Import networkx for graph tools
import networkx as nx

# Import matplotlib.pyplot to draw graphs on screen
import matplotlib
matplotlib.use("agg")
import matplotlib.pyplot as plt

# Import the Ocean tools we're going to use
import dimod
from dwave.system import DWaveSampler, EmbeddingComposite
import dwave.inspector as inspector
```

```python
# 1. Define sampler
sampler = EmbeddingComposite(DWaveSampler())

# 2. Define problem
gamma = 2

# Create empty graph
G = nx.Graph()

# Add edges to graph - this also adds the nodes
G.add_edges_from([(1, 2), (1, 3), (2, 3), (3, 4), (3, 5), (4, 5), (4, 6), (5, 6), (6, 7)])

# Create a BQM object
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# Add quadratic biases
bqm.add_interactions_from({(u,v): gamma for u,v in G.edges})

# 3. Submit problem and parameters to the solver
sampleset = sampler.sample(bqm, num_reads=50)

# 4. Evaluate the solution
sample = next(iter(sampleset))
subset = [node for node in sample if sample[node] > 0]

print(sampleset)
print('Maximum independent set size found is', len(subset))
visualize_results(subset)
```

Antenna Placement Example

6. Solve and interpret the results

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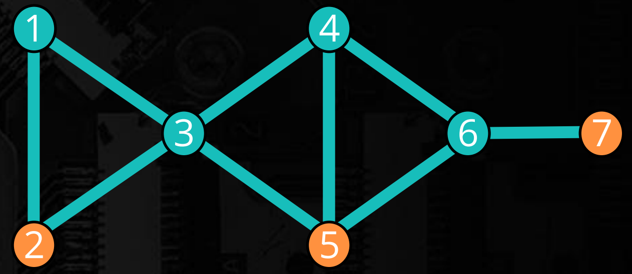

	1	2	3	4	5	6	7	energy	num_oc.	chain_.
0	0	1	0	0	1	0	1	-3.0	18	0.0
1	1	0	0	0	1	0	1	-3.0	18	0.0
2	1	0	0	1	0	0	1	-3.0	6	0.0
3	0	1	0	1	0	0	1	-3.0	8	0.0

Recap

Session Outline

- Formulated the maximum independent set problem
- Learned about the Lagrange parameter (gamma)
- Saw a general way to program a problem in Ocean
- Ran a fully formulated and implemented problem

Session Goals

1. Work through one more problem formulation example
2. Create a general program to solve the problem

Next Steps

Resources

Reading/Reference Material

- Learn more about the QPU
  https://docs.dwavesys.com/docs/latest/c_gs_1.html
- Ocean documentation
  - DWaveSampler
    https://docs.ocean.dwavesys.com/en/latest/docs_system/reference/samplers.html#dwavesampler
  - EmbeddingComposite
    https://docs.ocean.dwavesys.com/en/latest/docs_system/reference/composites.html#embeddingcomposite
  - ExactSolver
    https://docs.ocean.dwavesys.com/en/latest/docs_dimod/reference/sampler_composites/samplers.html#exact-solver
  - SimulatedAnnealingSampler
    https://docs.ocean.dwavesys.com/en/latest/docs_neal/reference/sampler.html
- Customer applications
  https://www.dwavesys.com/learn/featured-applications/

Practice and Exercises

- Collection of code examples
  https://cloud.dwavesys.com/leap/examples
- Jupyter notebook exercise
  - Example
    https://github.com/dwave-training/cdl_2021_jupyter/blob/master/part02_SolversI.ipynb
  - Exercise
    https://github.com/dwave-training/cdl_2021_jupyter/blob/master/part02_SolversI_practice1.ipynb