CV 2

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Universal CV quantum computation

As mentioned before, we can encode quantum information as CV wavefunction. The next task is to process the information. It is natural to think the most powerful type of CV quantum computer can perform the task

$$|\psi_{int}\rangle \equiv \int \psi_{int}(\vec{q})|\vec{q}\rangle d\vec{q} \longrightarrow \hat{U}|\psi_{init}\rangle = |\psi_{final}\rangle = \int \psi_{final}(\vec{q})|\vec{q}\rangle d\vec{q} \quad -(1)$$

multiple mode

i.e. it can transform any ψ_{int} to any ψ_{final}. This type of machine is called Universal CV quantum computer.

Like the qubit counterpart, ψ_{int} is usually the state that is easy to prepare, and we need to read out the computed information by measuring the final state ψ_{final}. Also like qubit QC, unfortunately, it is generally difficult to generate an arbitrary $\hat{U}$ in one step, so we have to decouple $\hat{U}$ into a sequence of elementary processes, also known as gates.

In the following, I will provide a brief overview of the types of state that can be typically prepared, types of gates that can be implemented easily, and the usual type of measurement that can be realized.

Gaussian state

A very important class of state in CVQC is Gaussian state. As its name suggests, the common feature of this class is that its wavefunction is a Gaussian function.

Ground state

The most important example of Gaussian state is the ground / vacuum state.

$$|0\rangle = \int \psi_0(q)|q\rangle dq, \text{ where } \psi_0(q) = \frac{1}{\sqrt[4]{\pi}} e^{-\frac{q^2}{2}} \quad -(2)$$

It is the most common initial state of each mode, because one can get this state without doing anything in optics, or cool the oscillator to ground state in other platform.

Coherent state

The next most common state is coherent state, usually denoted as $|\alpha\rangle$.

$$|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle = \int \psi_\alpha(q)|q\rangle dq, \text{ where } \psi_\alpha(q) = \frac{1}{\sqrt[4]{\pi}} e^{-\frac{(q-q_0)^2}{2}} e^{ip_0 q} \quad -(3)$$

Coherent state is an eigenstate of the annihilation operator

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle, \text{ where } \alpha \equiv \frac{q_0+ip_0}{\sqrt{2}} \quad -(4)$$

Generally, α is a complex number, its real part is the mean position of the state, $\langle\alpha|\hat{q}|\alpha\rangle = q_0$, and its imaginary part is the mean momentum, $\langle\alpha|\hat{p}|\alpha\rangle = p_0$. (Hw)

Coherent state is sometimes regarded as "classical state" for many reasons. One of which is that if we have only coherent state in every mode, no entanglement can be generated by linear optics. (Hw)

As such, a CV QC with only coherent input state and linear optics cannot provide any advantage over classical computer, because we can predict the output state and all its measurement outcome efficiently with classical computer.

The coherent state is also known as a state that has minimum uncertainty. This can be seen from the uncertainty of position and **momentum**, i.e.

$\Delta q \equiv \sqrt{\langle q^2\rangle - \langle q\rangle^2}$ and $\Delta p \equiv \sqrt{\langle p^2\rangle - \langle p\rangle^2}$. For coherent state, we can show

$$\Delta q^2 = \tfrac{1}{2}, \quad \Delta p^2 = \tfrac{1}{2}, \quad \Delta p \Delta q = \tfrac{1}{2} \quad \text{(Hw)}$$

The last relation is the minimum as guaranteed by uncertainty principle.

Note that the ground state is a coherent state with $\alpha = 0$.

Also note that the output of laser, as well as most natural light source, is coherent state.

Squeezed state

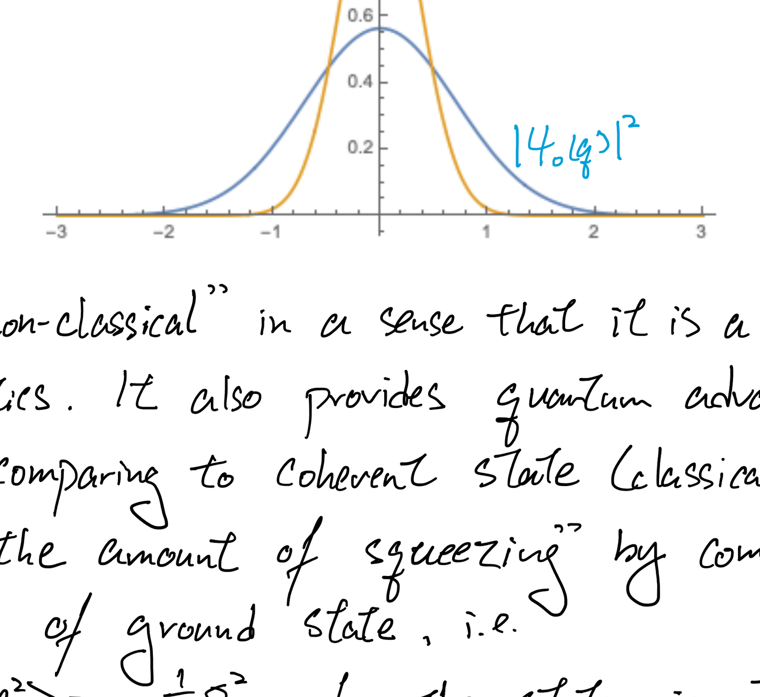
Because we cannot perform quantum computation with linear optics and coherent state, a CV QC thus needs something "non-classical". One simple but important example is squeezed vacuum state, or sometimes simply called squeezed state.

The wavefunction is given by

$$\psi_s(q) = \frac{1}{\sqrt{\sigma\sqrt{\pi}}} \exp\left\{\frac{-q^2}{2\sigma^2}\right\} \quad -(5)$$

This state has zero mean position and momentum, i.e. $\langle q\rangle = \langle p\rangle = 0$.

When comparing to the ground state, we can see their wave functions look very similar, except the "width" is changed by a factor of σ. That is probably the origin of the state, "squeezed" state.



Squeezed state is "non-classical" in a sense that it is a resource to generate entanglement together with linear optics. It also provides quantum advantage in different applications (e.g. sensing) when comparing to coherent state (classical).

We can quantify "the amount of squeezing" by comparing its position variance with that of ground state, i.e.

$$\Delta q^2 = \langle q^2\rangle = \tfrac{1}{2}\sigma^2 \quad \text{for the state in Eq. (5)}$$

So here the state is squeezed by σ^2.

Note that due to uncertainty principle $\Delta q \Delta p \geq \frac{1}{2}$, squeezing position variance must be accompanied by an anti-squeezing of momentum. It is easy to check that $\Delta p^2 = \langle p^2\rangle = \frac{1}{2\sigma^2}$ for the state in Eq. (5).

Note that a position eigenstate $|q=0\rangle$ can be treated as a squeezed state with $\sigma \to 0$, i.e. infinitely squeezed. And other position eigenstate is displaced infinitely squeezed state.

Two-mode squeezed state

The last Gaussian state we want to introduce is a two-mode entangled state

$$|\psi_{TMS}\rangle = \int \psi_{TMS}(q_1,q_2)|q_1\rangle|q_2\rangle dq_1 dq_2, \text{ where } \psi_{TMS}(q_1,q_2) = \frac{1}{\sqrt{\pi}} \exp\left\{\frac{(q_1-q_2)^2}{4\sigma^2}\right\} \exp\left\{\frac{(q_1+q_2)^2}{4}\sigma^2\right\}$$

It is easy to see that when $\sigma^2 \neq 1$, there will be cross terms in the exponent of the wavefunction. Therefore $\psi_{TMS}(q_1,q_2) \neq \phi(q_1)\phi(q_2)$, i.e. we cannot write the wavefunction as a product of mode 1 and mode 2 wave function, thus the state is entangled.

This state is called a two-mode squeezed (TMS) state because the correlation is squeezed. We can check

$$\left\langle \frac{(q_1-q_2)^2}{2} \right\rangle = \left\langle \frac{(p_1+p_2)^2}{2} \right\rangle = \tfrac{1}{2}\sigma^2 \leq \tfrac{1}{2} \quad \text{for } \sigma^2 \leq 1$$

$$\left\langle \frac{(q_1+q_2)^2}{2} \right\rangle = \left\langle \frac{(p_1-p_2)^2}{2} \right\rangle = \frac{1}{2\sigma^2} \leq \frac{1}{2} \quad \text{for } \sigma^2 \geq 1$$

TMS state is an important class of entangled state generated in bosonic system. In fact, it is the state generated by SPDC photon source, which is used in the majority of optical quantum experiment.

Furthermore, it can be the resource for many applications, e.g. sensing, teleportation, quantum key distribution. When $\sigma^2 \to 0$, TMS state will become the EPR state, the state that is employed by Einstein, Podolsky, Rosen to illstrate their famous EPR paradox.

By combining multiple EPR states with beam splitters, we can generate entanglement involving over million modes, which is the largest entangled state ever created (when comparing to ~20 qubit entanglement in DV approach.)

General Gaussian state

The definition of Gaussian state can be extended to multiple mode. It can also be extended to mixed state; Gaussian state generally includes the state that the Wigner function is Gaussian.

A crucial property of Gaussian state is that its Wigner function, and hence all its physical properties, is determined by two set of parameters:

1. The mean of quadratures, i.e. $\langle q_i\rangle$ and $\langle p_i\rangle$, where i is the mode index.
2. Variance and correlations, which is characterized by the covariance matrix, which contains the entries $\langle \Delta q_i \Delta q_j\rangle$, $\langle \Delta p_i \Delta p_j\rangle$, $\left\langle \frac{\Delta q_i \Delta p_j + \Delta p_j \Delta q_i}{2} \right\rangle$, where $\Delta q_i \equiv q_i - \langle q_i\rangle$, $\Delta p_i \equiv p_i - \langle p_i\rangle$