

Exercises for Image Processing 1

Problem Sheet 3

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1 Perspective Transforms

a) Straight Line Projection

A straight line in a 3D scene can be described by the following formula:

$$\vec{t} = \vec{p} + r \cdot \vec{d}, \quad (1)$$

where \vec{p} is a position vector and \vec{d} is a direction vector. Every possible point on the line \vec{t} can be reached by finding the appropriate r .

Furthermore, we can split the equation 1 into its components:

$$t_x = p_x + r \cdot d_x \quad (2)$$

$$t_y = p_y + r \cdot d_y \quad (3)$$

$$t_z = p_z + r \cdot d_z. \quad (4)$$

If we now perform the projection, we obtain the following formulas:

$$t'_x = t_x \cdot \frac{f}{t_z} \quad (5)$$

$$t'_y = t_y \cdot \frac{f}{t_z} \quad (6)$$

$$t'_z = f. \quad (7)$$

From equations 5 and 6 we can conclude a straight line vector specification for a 2D scene:

$$\vec{s} = \vec{0} + \frac{f}{t_z} \cdot \begin{pmatrix} t_x \\ t_y \end{pmatrix}. \quad (8)$$

Because we were able to construct a 2D straight line formula from any 3D one, we have shown that the obtained 2D projection stays a straight line. \square

b) Parallel Line Projection

Regarding perspective projection, parallel lines meet each other in the vanishing point, which concludes in the loss of parallelism of the former. If you use an **Orthographic projection** instead, the Z-axis of a Cartesian coordinate system is just pointing in some diagonal direction, no matter which X or Y position is applied. So, in that projection, all lines which are parallel in the real world will also be parallel in the orthographic projected world.

c) Sphere Projection

Imagining some points lying on the face of a circle in 3D space. If we apply a perspective projection on these, the points are shifted on the X- and Y-axis depending on their Z-value and the focal length. Because a sphere can be imagined as a stack of many circles with different Z-values, we can imagine that these circles are slightly shifting their positions because of their changing Z-values. This concludes in the form an ellipse, as you can see in the following figure.

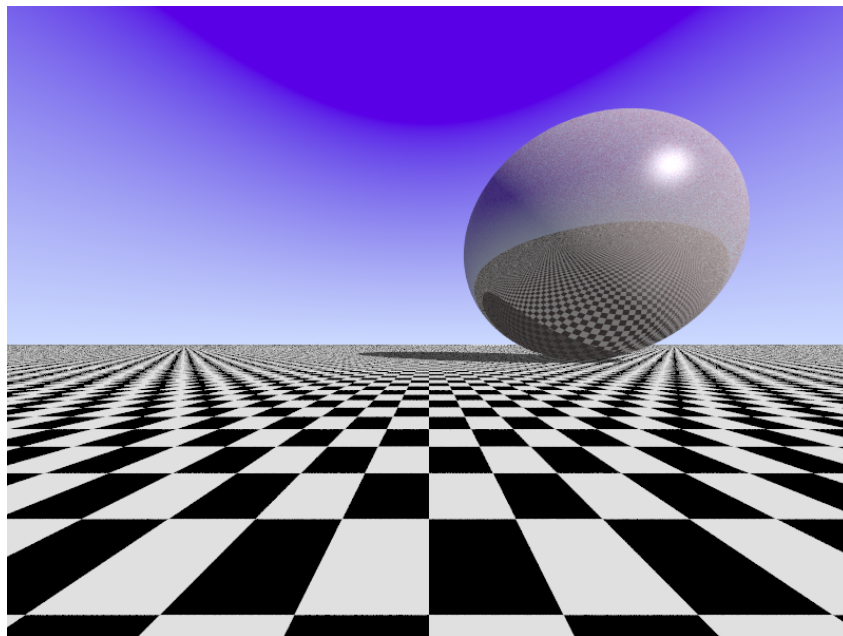


Figure 1: Sphere rendered with perspective projection in POV-Ray.