Exercises for Image Processing 1
Problem Sheet 3

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1 Perspective Transforms

a) Straight Line Projection

A straight line in a 3D scene can be described by the following formula:

$$\vec{t} = \vec{p} + r \cdot \vec{d}, \tag{1}$$

where $\vec{p}$ is a position vector and $\vec{d}$ is a direction vector. Every possible point on the line $\vec{t}$ can be reached by finding the appropriate r.

Furthermore, we can split the equation 1 into its components:

$$t_x = p_x + r \cdot d_x \tag{2}$$
$$t_y = p_y + r \cdot d_y \tag{3}$$
$$t_z = p_z + r \cdot d_z. \tag{4}$$

If we now perform the projection, we obtain the following formulas:

$$t'_x = t_x \cdot \frac{f}{t_z} \tag{5}$$
$$t'_y = t_y \cdot \frac{f}{t_z} \tag{6}$$
$$t'_z = f. \tag{7}$$

From equations 5 and 6 we can conclude a straight line vector specification for a 2D scene:

$$\vec{s} = \vec{0} + \frac{f}{t_z} \cdot \begin{pmatrix} t_x \\ t_y \end{pmatrix}. \tag{8}$$

Because we were able to construct a 2D straight line formula from any 3D one, we have shown that the obtained 2D projection stays a straight line. □

b) Parallel Line Projection

In the case of **Orthographic projection**. Then all lines which are parallel in the real world are also parallel in the projected world.

c) Sphere Projection

Spheres are looking like ellipses in a 3D projection:

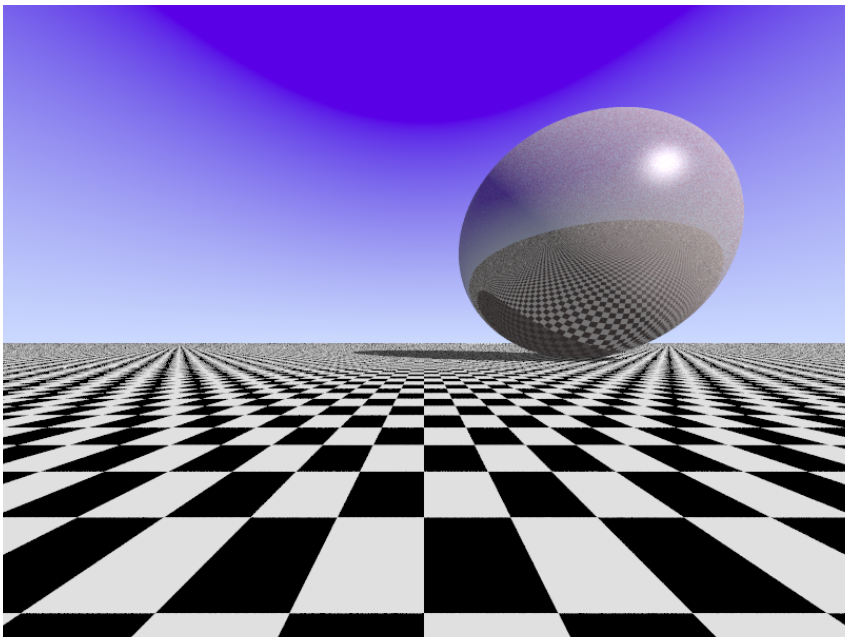

Figure 1: Sphere rendered with perspective projection in POV-Ray.