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The use of Game Theory in Small Business

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I dedicate my sisters: Caroline, Madeleine and Ursula

"Everything should be made as simple as possible, but not simpler."

– Albert Einstein

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Abstract

When in 1950 John Nash was getting his PhD degree on theory of non-cooperative games, no one could foresee that he was more than several decades ahead of his times. One could also not predict that almost half a century later his equilibria will be celebrated and Nash himself, along with other notable economists, will be given the Nobel Prize. Nowadays, no one undermines the relevance of Game Theory as science which influences other sciences, starting with mathematics and economy, and ending with philosophy and biology. The forefathers of GT are generally agreed to be John von Neumann and Oskar Morgenstern and their Theory of Games and Economic Behavior'. Since then, GT has developed into an extensive science branch that attempts to uniformly and unambiguously explain the behavior of humans, social groups, corporations, and governments, as well as animals and other living creatures, collectively called players. A player has their own interest in mind when behaving in a particular way. GT tries to explain that behavior and anticipates the best possible solutions.

The thesis focuses on the vital aspects of GT and the behavior of players in (non) coalition and (non) cooperative circumstances. In the following part, the art of implementing game theories in reality, particularly in the small business milieu- which is the focus of this paper. Finally, the expectations based on the knowledge presented in the previous chapters are juxtaposed with real life cases. For that, an interview with small business representatives has been done and the results are presented in form of reflections and tips of how a small business should act.

Chapter 1
Introduction

Game theory is a study with the main purpose of finding an answer to the question: how to react in both conflict and cooperation situations, as well as combined ones. This idea pre-defines a condition that there must me at least two sides in a relation towards each other to talk about conflict/cooperation. In the game theory nomenclature, these are players; in reality these can vary depending on the situation. Starting with the basic association of two players leaning over a board or holding a deck of cards, through people going about their daily activities (regardless of how common/uncommon these are), until entire nature's living, selfish organisms and social groups that, nonetheless, influence other "players". Selfish is the keyword here – each player's priority is their own interest.

The game theory assumes a rational behavior, i.e. one that will make the players situation better. They will design their tactics to earn as much as possible, therefore their motivation will be purely egotistical. The paradox is that even the altruistic players fit a certain model and can be classified as getting satisfaction from their own activities[1] [7]. Two terms that will need further explanation are: modeling and winning.

A game, by definition, should guarantee a result (a payoff). The result is strictly connected with the level of player's satisfaction from participating in a game. The payoffs have a numerical value and are an example of reality modeling. Unfortunately, the complexity of payoff theory does not allow an in-depth analysis of all its aspects. If, for example, we were to portray a conflict situation between two entities, the number of variables (non-linear, in most cases), would obstruct the legibility of the model [32].

The problem becomes more complex when we realize that even a poorly modeled reality is difficult to analyze in terms of a game. For instance, a game of chess, where all the rules are clearly stated and known to both players (at least they should be if they are to be of any interest for a game analysis). Each player knows the position the other player is in and can estimate a move. Moreover, the order of moves is

[1] It is not the author's intention to offend any charitable entities. What the author analyzes it the reasoning behind such actions – such as the feeling of fulfillment and satisfactions such actions bring.

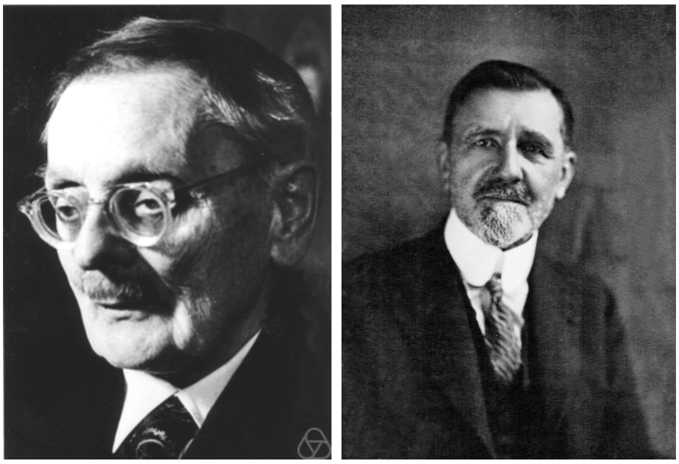
strictly regulated – the players cannot move simultaneously or many times at one go, but each move of player A must be followed by the move of player B. The 'surprise' element does not have its rights here, as opposed to, for example, a dice roll where the outcome is coincidental. There are also no surprises like in a game of cards, where the opponent does not reveal their cards. Also, the outcome is that the win of one player is a loss for the second one. The only other possibility is a tie. Therefore, the game represents a pure "conflict of interest" situation. In other words, we can't talk of cooperation between the players. The entire game takes place in a limited space of the chessboard, where a pre-defined number of figures can be used. However, the "limited" reality, completely immune to any external factors, does not make it easier to find the best possible strategy. It is not uncommon for pro payers to dedicate their lives to finding that strategy.

This introduces a new, highly important for the game theory term that needs defining – a strategy. A strategy is a set of rules, that if the point of reference for every possible situation during a game. In theory, each player's strategy is known prior to the game and the crossing of these yields a pre-defined result. In reality, the number of possible strategies, even for a simple game of chess, can be so big that finding the "optimal" one will not be possible anytime soon (even taking into consideration the advanced mathematical calculations and today's technology). We can risk a statement that even if we were presented with such a strategy (for example, by some intelligent life force not know to us), our memory capacity would make it impossible to store is[1], let alone use it without any aids. The example is to point out to the weight of the problem a gamer has to deal with.

First games that took place in reality can go back to the beginning of times – which with todays level of science – may be interpreted as the beginning of the universe. However, the games between matter and antimatter in baryogenesis[2] were, in fact, games of nature vs. nature, where neither of the players was rational. Winning was not the anticipated result on any end. The 'real' game, as is the authors opinion, appeared with the formation of life, and later on-consciousness. It is the consciousness that allows the feeling of satisfaction from winning, thus serving as

[1] This statement is based on simple estimates – namely the Checkers game whose state space is $5 * 10^{20}$ the size of the proof tree containing the optimal strategy is 10^7 [34]. Assuming that a similar relationship exists in the case of Chess, (this is a careful assumption because the average number of possible moves in this game is much greater than in Checkers), where the state space size is estimated at 10^{46} [8] resulting in the size of the optimal strategy amounting 10^{32}.

[2] We suspect that the matter had to win, because until now we see a small part of it that survived after this game.



(a) Ernst Zermelo (b) Émile Borel

FIGURE 1.1. Game theory pioneers

a drive for organisms (including humans) to seek fulfillment via smaller or bigger strategies. Actually, games were being played long before there were human beings, especially thinking human being, who start looking at games and begin formulating the theory and its rules.

The history of game theory is a long one and the thesis will not scrutinize it. The thesis will focus on certain subjectively chosen elements and people. For a more detailed account of the history of game theory, the reader is invited to Harold Kuhn's "Classics in game theory" [20].

One of the first people to break the grounds of game theory was Ernst Zermelo 1.1(a) who presented in 1913 the first theory [37]. He proved that in social games, such as chess, the final outcome is a tie for one or both players. The forefather title is also attributed to Émile Borel 1.1(b) – his major achievement being the use of mathematical apparatus in formalizing game theory [6]. The undisputed pioneer status belongs, however, to John von Neumann 1.2(a) and Oscar Morgenstern 1.2(b), and their work "Theory of Games and Economic Behavior" [24]. It would be difficult not to mention the colorful John Nash Jr 1.3(a), whose biography (although not without inconsistencies with the reality) served as inspiration for the Russel Crowe – starring movie "A Beautiful Mind". In 1950 he presented the famous Nash equilibria [25, 27], but unfortunately his health began to decline 10 years later due to paranoid schizophrenia (which he appears to have mostly combated by himself). A real breakthrough came in 1994 when Nash and two other mathematicians got the Nobel prize in economics. These were: Reinhard Selten 1.3(b) and John Harsanyi

(a) John von Neumann (b) Oskar Morgenstern

FIGURE 1.2. The authors of the "Theory of Games and Economic Behavior"

1.3(c),whose common denominator was the contribution to the Games Theory.

Nowadays, the game theory is no longer limited by the mathematical perception, but is studied in a wider spectrum: economical, physical, IT, anthropological, sociological, philosophical, polythological, legislative and biological. This helps systematize the perception of strategic situations and analyze the dependence of strategies and the assumptions connected with them. It helps verifying whether the assumptions there were on how the players will act match the actual behavior [9, 10, 16]. In case of dissonance it tries to correct the strategy, which more often than not results in change of rules set and behavior of the player [15]. To estimate whether the Game Theory gives a true account of the behavior, as well as correctly estimates the modeled players, two approaches are used [42]. It is possible to observe people in real situations and construct theories basing on these observation. These models can be used for statistical studies. The second approach tries to juxtapose the real-life situations with the simplified models. One example would be laboratory experiments. That type of research is called behavioral game theory. The drawback are that these have little reflection of the real-life situation, where the players motivation to win is the driving force.

Although the application of Game Theory in economics is obvious, one needs to think on the benefits this may brings to the world of business' representatives. For example, one can analyze the strategy of the competition and prepare the best counter – strategy. The questions is: is such analysis (consciously or unconsciously) done by the company management, and if so, would the knowledge of Game Theory

(a) John Nash Jr (b) Reinhard Selten (c) John Harsanyi

FIGURE 1.3. Winners of the Nobel Prize in Economics in 1994

help? Would it systematize the actions, or speed up the decision making process? A satisfactory answer would be: the Game Theory overlaps with the real behavior for small businesses, or can be used to correct the mistakes these businesses make and make their behavior more optimal.

The thesis focuses on using Game Theory in small businesses. The first chapter defines the mathematical formalism as basis for the remaining chapters, but also introduces types of games, the knowledge of which can be useful for small businesses. The second chapter presents free and widely available software that help find a strategic solutions and perform lab-like tests. The next chapter depicts models of example business situations an their analysis with respect to small companies. The last chapter focuses on confronting the the theory with actual entrepreneur experience – their reflections and comments on Game Theory.

It is worth to mention that Game Theory is a mathematical concept and the outcome should be optimal and unambiguous. Unfortunately, the sentence is not always true. It turns out that in case of cooperation, the outcome is not always straightforward. This proves how complex the issue is, in fact. This should force a reflection and respect on the decision making process. Otherwise, one may reach of point of no return, a completely new game.

Chapter 2

Short introduction to the Game Theory

The volume of literature on GT is overwhelming and presenting all the aspects of it will not be possible due to space limitations of the thesis. For that reason this chapter will focus on the basic aspects of GT and subjects, the knowledge of which will be required in the following chapters. The purpose of this chapter is also the systemization and detailing of GT formalism, which will be used in the chapters to come. One needs to start with some assumptions on GT:

- The rules of the game are precise, comprehensible and known to all players[1].
- The players adhere to these rules[2].
- There are between two and an infinite number of players[3].
- The game consist of moves performed at the same time (simultaneous game), or one at a time (sequential game); the number of moves is finite[4].
- Each player has a set of strategies (finite or infinite). In theory, a game is restricted to choosing between strategies by each player. A choice of strategy

[1]This assumption is true for simple games. The reality is, however, so complex that foreseeing all extraordinary situations that will require additional conditions seems difficult, if not impossible.

[2]In reality the players often can break the rules. The reasons for this are a few: it may be more beneficial that way, or the rules could have been misunderstood or misinterpreted (e.g. as a result of cultural differences, miscommunication or the unintelligibility of the rules). It seems that the possibility of rule-breaking forces a formulation of a new game, with a wider spectrum of rules. With a greater number of such exceptions, the model would become too complex to comprehend, as the same time making it impossible to study.

[3]To talk about a game, we need at least two players; otherwise we are dealing with optimizing a strategy towards and outcome, not a game per se. A player can be one person, a group of people (e.g. a pair in a game of bridge), a social group or a continuum of players (so called 'population games'), or nature itself.

[4]In theory, an infinite number of moves is possible, but not applicable. The game usually ends after a (pre-defined) number of moves in a set time frame, or after reaching a certain state. In the latter case, there's an additional rule preventing the players from making an infinite number of moves, such as the three-position repetition rule in chess.

can be a probability of selection among the subsets of strategies. What is means is that the player and their opponents do not know which strategy they will choose – this will be dictated by the chance.

- After the game ends, each of the players gets s certain result, its value being numerical and described by means of *payoff functions*[1].

Games can be differentiated depending on:

- Random situations – if there's at least one random situation that influences the outcome, we can talk of nondeterministic games[2], as opposed to deterministic games; deterministic games are the only ones that it is possible to foresee the outcome of, provided that we know the strategies of all players.
- Number of moves – in one-step (simultaneous) games[3], but also in multi-step (a collection of sequential and simultaneous[4].
- Completeness of information – the minimal information a player should have is the knowledge of their own payoff function. This is called playing a game with **hidden information**. When a player knows the entire set of strategies (their own and other players'), as well as the position they occupy in a game, one can talk of a game with **complete information**. If the players additionally knows the payoff functions of other players, the history of random moves and their outcome (both their ant the other players'), one can talk of having **complete information** [30].
- Cooperation elements – competition games are those where the win of one player has to be proportional to the loss of the other player(s). These games are zero-sum or constant-sum games[5]. Cooperation games are those, in which

[1]The sum of payoffs should reflect the usefulness of the player's win, i.e. the player should prefer a higher reward (preferably if with the n-times higher value comes n-times higher usefulness). That condition is very hard to fulfill, mainly because the player may not always realize which is more useful, but also because what may be valuable for one player, may not necessarily be valuable for another. A sub domain of GT devoted to this is called Usefulness Theory.

[2]The random situation may be a dice roll or natural occurrences. The randomness factor may, however, be also deliberately used by a player when resorting to a mixed strategy.

[3]We often deal with a repetition of identical one-step game (a repeated game), which in turn yields a multi-step game, because the history of the previous moves may influence the player's behavior in the future.

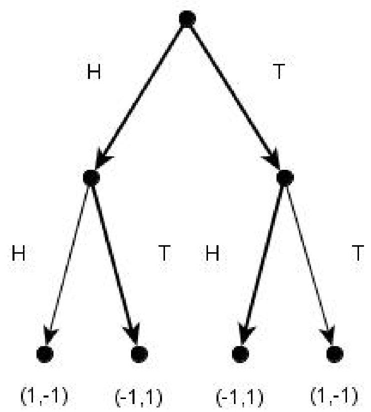
[4]In particular, there can be games with an infinite number of moves, but these usually are useful for topological and multitude theory-related studies.

[5]Each game can be, with the help of certain linear transformations, turned into a zero-sum game.

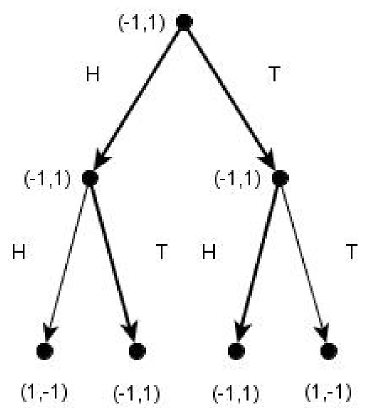
players use their strategies to gain as much as possible, without worsening the condition of other players.

2.1. Extensive games

The so called game tree is an ideal representation of sequential games. This structure consists of arches and vertices [19]. The vertices represent a position in a game, while the arc stands for the possible move of the player, which determines the next position in a game (that is why the arc join 2 vertices). The uppermost vertex – the root(rooted tree) represents the base position. We assume that within a given vertex, only one player can make a move. The vertices that do not generate vertices are terminals (leaves) and they represent the end of the round. Each such verticle contains a numerical vector with all players' payoffs. When a game allows that two players move at the same time, we are dealing with a simultaneous game and the outcome is that one player does not know which vertices he's reached (incomplete information). To report that we need so called *information sets* – represented by a dotted line joining verticles in one information set.



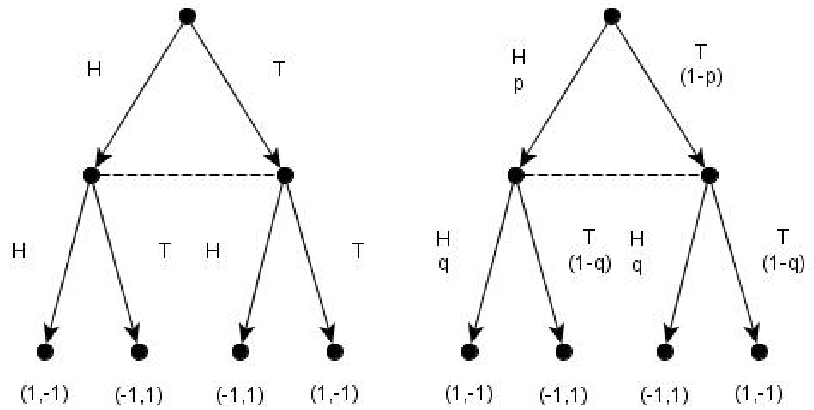
(a) Optimal strategy with final payoffs



(b) Wedial values counted via backwards induction

FIGURE 2.1. The extensive form of the modified Matching Pennies ($\Gamma_{H\to T}$)

To better show the structure of the game tree, we will use Matching Pennies game described in A.6. At first, let us analyze a situation without information sets ($\Gamma_{H\to T}$). Figure 2.1(a) reflects that structure. Player 1 makes a move (chooses the Head or the Tail), giving Player 2 the information on the position in the game and allowing P2 to make such a move, as to guarantee a win. The win is represented by 1, while the loss by -1. Therefore, an identical choice grants P1 a win or P2 a win. The conflict of interests is visible, as the win of P1 is equal to the loss of P2. This game belongs to the sum-zero game group – the sum of both players' payoffs is 0. In this case it's the 'other' player that has the winning strategy. That strategy is choosing the other side of the coin.



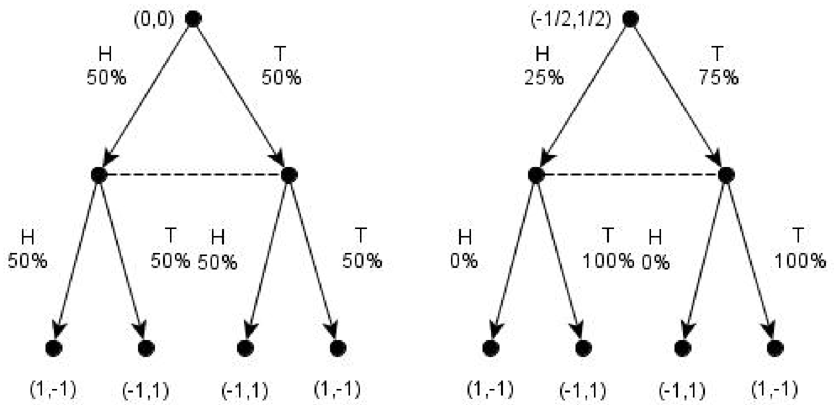
(a) Game tree (b) Player's decision probability

FIGURE 2.2. The extensive form of the original Matching Pennies (Γ_{HT})

Please note that a game modeled in that way can be represented with a reversed sequence by using backward induction. The last move belongs to the 'other' player, which means that his rational decision will be the best possible option [28]. These values can be assigned to the vertices one level up in the game tree, and will represent the player's move (pic. 2.1(b)). The 'next' move (going backwards), is P1's move, who chooses the best possible options from the set of new values. The alternative left by P2 is not beneficial for P1 (each move results in a loss), which proves the previous statement.

If players simultaneously reveal their choices, the original Matching Pennies (Γ_{HT}), then the players don't know the other player's choice until the game is over and the decision cannot be altered. This is depicted by the merge of the game's vertices, with the player is part of one information set. The picture 2.2(a) one can see that having made a choice P1 does not know which vertex he occupies. He knows, however, that he is one of the two joint dotted lines, which together form an information set . As the game is symmetrical, the P2's situation is identical, it does not matter which player is P1 or P2[1].

In that case backward induction will not work, because P2 does not know which vertex he occupies, and as a result cannot make an unambiguous decision. It is worth mentioning that in this situation P1 should benefit most from P2's lack of information. Intuition suggests that there are equal chances of choosing either side of the coin. We conclude that the chance of taking a Head is $p = 50\%$ and a Tail is $(p-1) = 50\%$[2].



(a) optimal strategies in equilibrium (b) non-optimal strategy of „first player"

FIGURE 2.3. The expected value of the Matching Pennies (Γ_{HT})

To count the win players can anticipate let us use. *expected value*. The expected value is the arithmetic mean of all payoffs, determined by the probability value of their realization. Suppose the probability of choosing heads by P1 is p, and by P2 is q (pic. 2.2(b)) the expected value for P1 is:

$$pq * 1 + p(1-q) * (-1) + (1-p)q * (-1) + (1-p)(1-q) * 1 = \mathbf{4pq\text{-}2p\text{-}2q\text{+}1}$$

.

[1]All information set verticles have to be identical/undistinguishable. This means that both the number and the probability layout of the given strategy have to be identical, otherwise the player might predict which vertex he occupies

[2]The probability sum must always yield a unified outcome. Especially in pure strategies, which the players do not use; such strategies give a probability value 0. In other words, the probability of realizing one and only one strategy by one player must be certain.

This is a two-variable equation, in which we substitute p for 50% and expect the sum to be zero. What is interesting is that the outcome is completely independent of P2's choice, i.e. q. This is an example of *Nash equilibria* (pic. 2.3(a)). It is, at the same time an optimal strategy, that helps achieve the best score (provided that P2 is also using their optimal strategy) [21].

If P1 makes a non-optimal choice of Head with a 25% probability, then its expected value would be $\frac{1}{2} - q$ and dependant on q. Therefore P2 would choose the minimal value $q = 0\%$ which means picking a Tail, to minimize P1's win. The expected value of P1 would be, in the presented situation $-\frac{1}{2}$, and since we are dealing with a zero sum game, P2 would be $\frac{1}{2}$ (pic. 2.3(b)).

To determine whether it is in fact so, the game needs to be played multiple times and the payoffs mean of both players is determined. The outcome will depend in the strategy they choose. Notice that a trivial strategy of choosing the same side of the coin or using schemata are easy to decipher by the opponent after a certain amount of tries. The best strategy (as has been argued before) is to make unexpected (random) choices of 50% – Heads and 50% – Tails. It's called a *mixed strategy*. Using a different probability formula might leads to disclosure of that strategy by P2, thus allowing him to choose a better one, i.e. choosing one side of the coin only.

2.2. Strategic game

Game Γ in a strategic (normal) form is a trinity [14,31] $\Gamma = (\mathbf{I}, \mathbf{S}, u)$ where:

- $\mathbf{I}$ – set of all players – a finite set[1] $\mathbf{I} = \{1, 2, \ldots, I\}$ with force I of 2 or more.
- $\mathbf{S}$ – set of strategies of all players – a finite set[2] $\mathbf{S} = \{S_1, \ldots, S_i, \ldots, S_I\}$ with force I where $S_i = s_{i_1}, s_{i_2}, \ldots, s_{i_{n_i}}$ for $i \in \mathbf{I}$ is a non-empty set of strategies for i-player. The number of n_i strategies can be different for each player[3].
- u – payoff function $u : S_1 \times S_2 \times \ldots \times S_I \to \mathbb{R}$, which transforms each strategy profile $s = (s_{1_{j_1}}, s_{2_{j_2}}, \ldots, s_{I_{j_I}})$ where $s_{i_{j_i}} \in S_i$ is a choice of a given j-strategy of i-player in payoff profile $u = (u_1, u_2, \ldots, u_i, \ldots, u_I)$ where $u_i \in \mathbb{R}$.

[1] There are games with an infinite, or even a continuum of players (population games), where the relationship between the statistical and dynamic properties of the games is analyzed. This thesis will be limited to analyzing situations with a finite number of players, where each can represent a certain group (finite or infinite).

[2] In literature, the game definition often has a second element, so called set of states $\mathbf{S} = S_1 \times S_2 \times \ldots \times S_I$ where S_i for $i \in \mathbf{I}$ is a non-empty strategy set of i-player.

[3] A player in particular can have an infinite or even an uncountable umber of strategies.

To represent a two-player strategic game, one uses matrix representation 2.4.

Γ	s_{1_1}	s_{1_2}	$\dots$	$s_{1_{n_1}}$
s_{2_1}	$u_{1,1}$	$u_{2,1}$	$\dots$	$u_{n_1,1}$
s_{2_2}	$u_{1,2}$	$u_{2,2}$	$\dots$	$u_{n_1,2}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$s_{2_{n_2}}$	u_{1,n_2}	u_{2,n_2}	$\dots$	u_{n_1,n_2}

FIGURE 2.4. The matrix representation of the game in the strategic form

Each extensive type of game can be brought down to an equivalent strategic form (while the conversion in the other direction is not as obvious). As an example, let us use the aforementioned Matching Pennies game. This is a two-player game $\mathbf{I} = \{1, 2\}$. In its modified version of the game ($\Gamma_{H\to T}$), the players are distinguished as P1 and P2 (determined by the order of moves)[1]. The strategy set for P1 is: $S_1 = \{H, T\}$ and determines the choice as Head or Tail. P2's set is richer, as it can be altered depending on P1's movement and looks as follows: $S_2 = \{(H \to H, T \to H), (H \to T, T \to T), (H \to H, T \to T), (H \to T, T \to H)\}$. The set elements can be interpreted so: always choose Head, always Choose Tail, always make and identical choice as the opponent, always make an opposite choice as the opponent. As the game is a two-player one, the payoff function is best presented by matrix 2.5. The two columns symbolize P1's strategies, while the 4 rows represent P2's strategies. The payoff value for both players can be read from the intersection of strategies.

$\Gamma_{H\to T}$	$Head$	$Tail$
$H \to H, T \to H$	$1, -1$	$-1, 1$
$H \to T, T \to T$	$-1, 1$	$1, -1$
$H \to H, T \to T$	$1, -1$	$1, -1$
$H \to T, T \to H$	$-1, 1$	$-1, 1$

FIGURE 2.5. Strategic form of the modified Matching Pennies ($\Gamma_{H\to T}$)

[1] Red represents P1 and Blue represents P2.

In constant-sum, two-player games it is possible to limit the representation to one player's payoff only. The opponents win can be easily counted, and with zero-sum games it will have an identical value, but with an opposite sign. Matrix 2.6 shows the reduced payoff function values with only P1's results.

$\Gamma_{H\to T}$	*Head*	*Tail*
$H \to H, T \to H$	1	−1
$H \to T, T \to T$	−1	1
$H \to H, T \to T$	1	1
$H \to T, T \to H$	**−1**	**−1**

FIGURE 2.6. The Matching Pennies ($\Gamma_{H\to T}$) with one player's payoff

Please note that regardless of P1's choice (whether Head or Tail), the payoff is dependent on P2's choice. This is because P2's strategy guarantees a win. That strategy is choosing the other side of the coin. This is in compliance with the anticipated choice, as has been presupposed that P2 wins when he makes a choice opposite to P1. Each of the bold matrix 2.6 elements is the so called *saddle point*. It is a value that is at the same time the lowest[1] in a given column (P2 cannot improve the score by changing the strategy/row) and the highest in a given row (P1 also cannot improve the score by changing the strategy/column).

$\Gamma_{H\to T}$	*Head*	*Tail*	$\min(\max) = -\mathbf{1}$
$H \to H, T \to H$	1	−1	1
$H \to T, T \to T$	−1	1	1
$H \to H, T \to T$	1	1	1
$H \to T, T \to H$	**−1**	**−1**	**−1**
$\max(\min) = -\mathbf{1}$	**−1**	**−1**	$game\_value = -\mathbf{1}$

FIGURE 2.7. Min-max value and the theoretical value of the Matching Pennies ($\Gamma_{H\to T}$)

The value of the *saddle point* is at the same time the value of the game. To check whether a game has a *saddle point* one must count the *safe strategies* for both

[1] Once needs to remember that the game value for P2 is equal to the value of a given cell multiplied by -1, therefore the lower it is, the better.

players. The *safe strategy* (or *minimax strategy*) is the maximal value that comes after the lowest values from all the strategies. The *minimax strategy* is a *safe strategy*, because it slows to anticipate a certain payoff. The 2.7 matrix presents the minimal values of all strategies for both players (where, in case of P2, these are the maximal values) and the lows of these values. If these are equal, one can positively identify the game as having a *saddle point*. We can also state that each of the players has a pure optimal strategy. As one can see from the calculations, the anticipated value of the game will be -1 and is in accordance with both our intuition and the backward induction calculations presented earlier.

Notice that should the players change the order of moves, the optimal strategy for P1 would mean choosing the same side of the coin as the rival. Should the player have two strategies, with one producing greater results than the opponent's, one says that the superior strategy dominates over the less effective one. The dominated strategies can be removed from the set of all strategies, as no rational player would use them [3, 29, 39]. In the presented case the strategy $H \rightarrow T; T \rightarrow H$ dominated over all of P2's strategies. Therefore, these can be removed. The matrix representation of that is 2.8.

$\Gamma_{H\rightarrow T}$	*Head*	*Tail*
$H \rightarrow T, T \rightarrow H$	$\mathbf{-1}$	$\mathbf{-1}$

FIGURE 2.8. Representation of the Matching Pennies ($\Gamma_{H\rightarrow T}$) without a dominated strategy

Focusing on the original Matching Pennies (Γ_{HT}) game . The strategy frame is presented in matrix 2.9. Analogically to the previous examples, only P1's payoffs have been presented (in columns)

Γ_{HT}	*Head*	*Tail*	$\min(\max) = \mathbf{1}$
Head	1	-1	$\mathbf{1}$
Tail	-1	1	$\mathbf{1}$
$\max(\min) = \mathbf{-1}$	$\mathbf{-1}$	$\mathbf{-1}$	$\mathbf{-1} \leq game\_value \leq \mathbf{1}$

FIGURE 2.9. Strategic form of the original Matching Pennies (Γ_{HT})

This time both players have two identical strategies – they can choose either head or tail: $S_1 = S_2 = \{H, T\}$. Regardless of the choice, the solution will be the worst possible. The value of the game varies between -1 and 1 which is represented by minimax values for both players. There's in no *saddle point* in this case. It is easily noticeable that none of the values is dominated by another one. Therefore, the optimal behavior for P1 and P2 needs to be looked for in mixed strategies, which have already been presented. Assuming that P1 takes a mixed strategy of selecting a Head (with the probability p) and a Tail (with the probability $(1-p)$. The *expected value* for P2 in a pure game will be respectively:

- Selecting Head $\Longrightarrow p * 1 + (1-p)(-1) = 2p - 1$,
- Selecting Tail $\Longrightarrow p(-1) + (1-p) * 1 = -2p + 1$.

P1 can make his choice independent of P2's choice if the payoff value of the opponent is identical (the choice will not be dependent on the other player's choice). The probability p can be calculated by juxtaposing the expected values for P2. The solution for $2p - 1 = -2p + 1$ gives $p = \frac{1}{2}$. This is, again, in accordance with both our intuition and the calculations presented earlier. Knowing the strategies (P1 and P2 have an identical one), it is possible to calculate the expected value for the game, which is $\frac{1}{4}(1) + \frac{1}{4}(-1) + \frac{1}{4}(-1) + \frac{1}{4}(1)$ As expected, the value is zero.

2.3. Cooperation games

Cooperation games are those in which the other players are no longer treats as opponents, but also as partially becoming partners, cooperators, etc. These are players with similar goals. In case of such games, one is dealing with variable sum games – an economically interesting phenomenon, as they mirror natural market situations and are most suitable for analyzing business relations, including the small businesses (which are the subject of the thesis).

2.3.1. Strategic divergence

Allow us to modify the Matching Pennies game such that: if both players choose Head, they will receive one payoff unit; if they both choose Tails, neither gets a payoff (0 unit payoff); if P1 and P2 choose Head and Tail respectively, P1 gets two units, while P2 loses one (-1 payoff). This is represented by matrix 2.10.

Γ_{PD}	$Head$	$Tail$
$Head$	$1, 1$	$-1, 2$
$Tail$	$2, -1$	$0, 0$

FIGURE 2.10. The Prisoner's Dilemma

This will help analyze the behavior of both players. It is important to note that Tail is always a better choice, regardless of the opponent's choice. If the opponent selects Head, then the Tail-player will gain 2 units instead of one if the choice had been Head. If the opponent chooses Tail, the Head-player will gain 0 units, instead of -1. This means that both players have dominating strategies which they are forced to use (as motivated by their rationality). Their behavior leads to the lack of payoff (both choose Tail), as either of them assumes the other player will try deception and protects himself from that. Choosing Head by both players would be a better solution, but neither has any interest in changing their strategy. A game where an alternative strategy may result in a gain for at least one, while guaranteeing no loss for others, is called *Pareto optimal*.
This case frequently occurs in reality and has been originally presented as *Prisoner's Dilemma* [22] a game described in A.3. In matrix 2.11 *generalized form of Prisoner's Dilemma* is presented:

Γ_{PD}	$Cooperate$	$Defect$
$Cooperate$	R, R	T, S
$Defect$	S, T	U, U

FIGURE 2.11. The generalized Prisoner's Dilemma

where:

- R – reward for cooperation,
- S – sucker's payoff,
- T – temptation payoff,
- U – uncooperative payoff.

The following condition must be obeyed: $T > R > U > S$. A situation where the individual interest counters the general interest is called the *strategic divergence of the first kind* [45]. The situation itself can have both positive and negative outcome for the society. For example: mass fishing – the number of fish caught exceeds the market demand, however it is in everybody's interest not to reduce the catch. A preferred situation is when others limit their catch. A positive example of *Prisoner's Dilemma* is competition – the superfluous catch causes price reduction.
There is a variety of methods that aim at Pareto-optimal outcome by influencing the players' behavior within the *Prisoner's Dilemma*. One such method is to ensure that one player is certain the other player will not betray then, and the other way around. Unfortunately, such situations are a rarity (unless the players will get a chance to win back the loss). Another method is reaching a conclusion on other players by observing previous games. This is called *iterated prisoners' dilemma*, and it requires multiple games to create a behavioral history. What is interesting is that one of the best strategies in that case is tit for tat one. This strategy assumes cooperation, unless P1 betrays P2, thus triggering P2's revenge – a situation mimicking real life? The fact that both players act rationally means they are making an optimal choice. Very often the coordination element is necessary. An example game described in A.7 (matrix 2.12) will serve to analyze this. Each player has their own preference for spending the evening, but holding blindly to their choices results in the worst of payoffs. To reach balance, they need to coordinate to some extent. Again, one of the possible solutions is observing the history of players' previous moves. If previously they chose a strategy that was favorable for one player more frequently, the other player may use this fact as an argument to achieve a more satisfactory result.

Γ_{BS}	*theater*	*cinema*
theater	$2, 1$	$0, 0$
cinema	$0, 0$	$1, 2$

FIGURE 2.12. The Battle of the sexes

Sometimes the players' individual interests may be fully convergent, but the lack of communication between them may have a determining effect in reaching a common, strategic goal. One example is a 2.13 game, described in A.4. Here, both equilibria are equally satisfying for both players, however the possibility of choosing non-optimal strategies and going apart is high. This is because they have no possibility to communicate. With no additional information that would let the player's choice select the same 'direction', it is up to the fate and the mixed strategy they choose whether they will ever meet. There' a 50% change they will. The described situation is known in literature as *strategic divergence of the second kind* [45].

$\Gamma_{Coor.}$	$point_A$	$point_B$
$point_A$	$1, 1$	$0, 0$
$point_B$	$0, 0$	$1, 1$

FIGURE 2.13. The Coordination

Sometimes the socially/globally non-optimal result for players is caused by the various coordination methods the players use. Once such instance is game 2.14 described in A.5, where the players can reach only two equilibria, one of which being Pareto-optimal. In this situation, realizing a non-optimal tactic by P1 leaves no choice but to fall in the line.

$\Gamma_{Coor.p}$	$standard_A$	$standard_B$
$standard_A$	$2, 2$	$0, 0$
$standard_B$	$0, 0$	$1, 1$

FIGURE 2.14. The Pareto-coordination

Surprisingly, such behavior is frequently observed in the market environment: for example the QWERTY standard (where a better solution is, say, DVORAK) [11], the standardized car plugs based on the lighter plug shape (where better ways of joining are available), etc. These non-optimal examples, as well as many others, show the lack of discrepancy between personal and global interests, but also the fact that there's no coordination missing. It is the difference in implementation that yields a non-optimal result for all, known as *strategic divergence of the third kind* [45].

2.3.2. Strategic moves

As has been noticed before, communication is one of the most vital elements – it allows the use of various strategic moves, which are: *promise*, *obligation* i *threat* [11, 35, 36].

In the previously mentioned *Prisoner's Dilemma* for example, P1 can *promise*, co-operation, provided that P2 does not betray them. The problem is that a player cannot always fulfill the *obligation*[1] and looking rationally at this – they shouldn't. A given *promise* can be made more credible with a preliminary move that will reduce the win (Reward – R) in case of treason to the level which does not generate temptation ($T < R$). In practice this could be a written contract, which starts a new game 2.15. If Red player makes the first move, the Blue player has no temptation to cheat.

Γ_{PD}	$Cooperate(Head)$	$Defect(Tail)$
$Cooperate(Head)$	$1, 1$	$2-(1+\epsilon), -1$
$Defect(Tail)$	$-1, 2$	$0, 0$

FIGURE 2.15. The promise in the Prisoner's Dilemma

As has been pointed out in Matching Pennies game, P1 always incurred a loss – a situation typical for zero-sum games, where the player that makes a move first can only lose. A different scenario happens in case of inconstant sum games.
Let us focus on the game of Chicken, described in A.2 and matrix 2.16. There are three equilibria in this game. The first is realized in mixed strategies, when the players have no additional information about the opponent. If there's no possibility to communicate between players (i.e. playing a simultaneous game), the players all select the strategy that is the best for them. This will be either driving or stopping, with a 50% chance each[2]. In that version, the payoff is identical for all, i.e: $-\frac{1}{2}$. The other version (one with communication between players), each player has the additional strategy of making the move first, hence choosing 'Straight' and leaving P2 with 'Swerve'. P1's payoff is 1, while P2's is -1, which proves that making the move first is beneficial for all players. This is called the *shut door strategy*. as P1 breaks off the communication with P2, having revealed their strategy. This leaves P2 no choice but to conform to P1's choice.

[1]As opposed to, say, a car crash, where both drivers suffer similar damages, P2 in a game can suffer a greater loss and is more prone to risk – a fact which may be used by P1 to gain more.
[2]We assume that the damages both players endure will be similar; otherwise P1 might use the knowledge that P2 will suffer greater damages against them and P1 will be more willing to risk.

$\Gamma_{chicken}$	*Swerve*	*Straight*
Swerve	$0, 0$	$-1, 1$
Straight	$1, -1$	$-2, -2$

FIGURE 2.16. The Chicken

There are types of games, however, where is it more beneficial to be P2, not P1. One such game is described in 2.17.

$\Gamma_{obligation}$	*A*	*B*
A	$1, 2$	$0, 1$
B	$3, 0$	$2, 3$

FIGURE 2.17. Game susceptible for *oblige*

Here, the Red player has the dominating strategy A. His opponent, knowing about this, will choose his strategy A and equilibrium AA, is formed (unfortunately, not a Pareto-optimal one). A much better solution for both players is BB. In this instance, both players want the Red to make the move first. Red's best interest is strategy B, and his partner will choose B. Please note that should Blue choose B first, the other player would be tempted to select A, which stems from the desirable domination (the highest payoff equals 3). At the same time, the payoff for Blue player would be the lowest, i.e. 0. Assuming the communication between players, Red can *oblige* himself to choose B, but with the temptation not to fulfill the *obligation*, the need for credibility arises. In practice, such credibility can be determined by behavioral history in a repeated game, or by drawing a contract. This boils down to creating a situation, where treason is too big a burden (in this case $1 + \epsilon$), that the payoff is reduced and yields a different game (2.18).

$\Gamma_{obligation}$	*A*	*B*
A	$1, 2$	$0, 1$
B	$3 - (1 + \epsilon), 0$	$2, 3$

FIGURE 2.18. Filing commitment in the game

Sometimes, instead of *Promise* one must use – *Threat*. In 2.19, regardless of who starts the game the result will be identical, i.e. 1 unit for Red player (who chooses the dominating A strategy), and 2 for Blue player (who also chooses a dominating strategy B). Here, the Red player cannot be obliged to choose B as it would negatively influence the situation of both players. He can threat to play B , if the other player does not choose A first. The idea of *threat* is that players declare to choose strategy that is both undesirable for him and the opponent.

Γ_{threat}	A	B
A	$2, 1$	$0, -1$
B	$1, 2$	$-1, 0$

FIGURE 2.19. Game susceptible for *threat*

The problem with *threat* is that is the addressee does not take it seriously, it will be in the interest of the person making the *threat* fulfill it. To make it credible, the threatening player must make a move he cannot withdraw from. Looking at the matrix, this means lowering the payoff to such a level where fulfilling the *threat* is worthwhile. This, in return, starts a new game 2.20. In practice, one trigger for *threat* is an unfavorable history of a player, who has previously made choices harmful for both players.

Γ_{threat}	A	B
A	$2, 1$	$0, -1$
B	$1-(2+\epsilon), 2$	$-1, 0$

FIGURE 2.20. Use of threats in the game

2.4. Coalition games

Until now, all analyzed games had two players, but the reality is more complex and it is not uncommon to have many more. Most problems connected with multi-player games can be analyzed on example of three-player games. As an example one can use a non-zero sum game introduced in matrix 2.21.

Γ_{I,II,III_A}	A	B
AA	$1, 2, -3$	$2, -1, -1$
BA	$-3, 3, 0$	$-4, -2, 6$

Γ_{I,II,III_B}	A	B
AB	$4, -2, -2$	$2, 1, -3$
BB	$-4, -3, 7$	$-2, 2, 0$

FIGURE 2.21. An example three-person, zero-sum game

Each of the players has two strategies, neither of which dominates the other, as can be seen in the *move diagram* (rys. 2.22). The verticles symbolize the preferred strategy of a player (where other players have revealed theirs). The vertex, which is formed by the conjunction of the three verticles is the so called *game balance* determined in pure strategies. It is in none of the players' interest to change the strategies, if others don't do it themselves. Unfortunately with more than two balances (in three or up player games), these need not be identical. In the analyzed game, we have two such balance points, which are not identical. The balance AAB is preferred by the Red player, while the other two players favor BAA. Moreover, if they all choose their preferred strategies, the outcome will be AAA – a non-optimal result for all the players. In the predetermined outcome (1,2,-3) each player would prefer to change their strategy to the opposite one to gain one payoff unit.

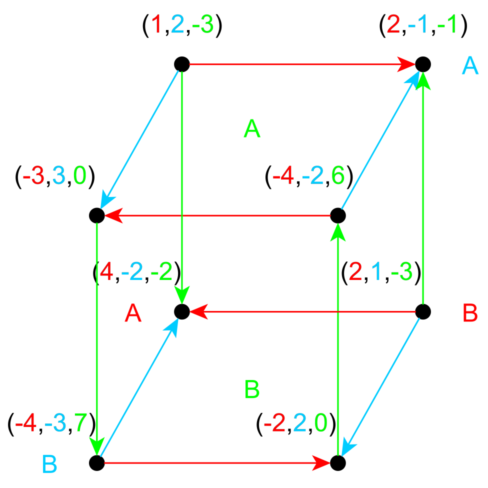


FIGURE 2.22. Move diagram of a three-person game

If communication between players and payoff transfer is allowed, they can form *coalitions*. It's a very strong presupposition, requiring that values within the function be identically useful for both coalition parties, between which the value transfer occurs. The condition is very hard to fulfill, as with the proverb 'Do not unto others as you would have others do unto you' – what is valuable for one, may not be as valuable to others. Even money – the most universal and measurable value determiner – can have different importance for a beggar and a rich man.

$\Gamma_{I \rightleftharpoons II,III}$	A	B
AA	1	2
BA	-3	-4
AB	4	2
BB	-4	-2

$\Gamma_{I,III \rightleftharpoons II}$	A	B
AA	2	3
BA	-1	-2
AB	-2	-3
BB	1	2

$\Gamma_{I,II \rightleftharpoons III}$	A	B
AA	-3	-2
BA	-1	-3
AB	0	7
BB	6	0

FIGURE 2.23. All possible two-player coalitions

There are all three possible coalitions between each pair of players (each pair with three strategies shows as rows) against a third player (in columns) presented in matrix 2.23. Note that these are zero-sum games of two players, therefore the matrix represents only the single player's payoffs. In 2.24 all dominating strategies have been removed. Two of these (both Blue player's) have two non-dominated strategies each, therefore one must look for the optimal strategy in a mixed strategy set. Blue and Green players' coalition has the strategies that dominate over the remaining ones. The optimal strategy for Blue and Green players is $\frac{2}{3}BA$, $\frac{1}{3}BB$ and the payoff value to be shared between the two is $3\frac{1}{3}$. In case of coalition of Red and Green players, the optimal strategy is AB which yields payoff value 2, whereas the Blue and Red coalition optimal strategy is $\frac{2}{3}AA$, $\frac{1}{3}BA$ and the payoff sum equal to $2\frac{1}{3}$. It is easy to notice that all players will benefit from a coalition, with the exception of a three-player coalition (where the payoff sum is 0).
Once can try to change the coalition, especially when other player offers a more beneficial payoff split (which is the lone player's priority). The fact is that the way in which the members of a coalition get together is not pre-defined and may depend on negotiator skills.

$\Gamma_{I \rightleftharpoons II,III}$	A	B
BA	-3	-4
BB	-4	-2

$\Gamma_{I,III \rightleftharpoons II}$	A
AB	-2

$\Gamma_{I,II \rightleftharpoons III}$	A	B
AA	-3	-2
BA	-1	-3

FIGURE 2.24. Two-player coalitions after removing the dominated strategies

Each game can be represented using characteristics functions. A characteristic function $v(C)$ where $C \subseteq I$ is the payoff value which the coalition guarantees. In case of zero-sum games, the condition $\forall_{C \subseteq I} v(C) - v(I - C) = 0$ has to be met, in particular $v(\emptyset) = 0$ and $v(I) = 0$. In the analyzed game, the possible coalition payoffs are as follows:

- $v(\emptyset) = 0$
- $v(i) = -3\frac{1}{3}$, $v(ii) = -2$, $v(iii) = -2\frac{1}{3}$
- $v(i, ii) = 2\frac{1}{3}$, $v(i, iii) = 2$, $v(ii, iii) = 3\frac{1}{3}$
- $v(I) = 0$

As the game is a zero-sum payoff game, the individual payoffs are identical with the coalition payoffs against the player, but with the opposite sign. Note that each of the players has the alternative of starting a coalition with any of the other two players. The assumption is that each player will choose the partner judging by the payoff size (more payoff units to split). While the red player will want a coalition with the blue on – thus guaranteeing a payoff of $2\frac{1}{3}$, the blue one will want a coalition with the green one, with the payoff of $3\frac{1}{3}$. It is important to remember that the red player may tempt the blue one with an additional transfer of $\frac{2}{3} + \epsilon$ (assuming that the payoff is split in half). Yet, in such a situation, the green player may also bribe the blue one, or threaten with a punishment by offering the red player a coalition, etc.

$\Gamma_{PD_{III}}$	$Cooperate$	$Defeat$
$Cooperate, Cooperate$	$1, 1, 1$	$3, 0, 0$
$Defeat, Cooperate$	$0, 3, 0$	$2, 2, -2$
$Cooperate, Defeat$	$0, 0, 3$	$2, -2, 2$
$Defeat, Defeat$	$-2, 2, 2$	$-1, -1, -1$

FIGURE 2.25. A three-person Prisoner's Dilemma diagram

The situation gets even more complicated in multi-player, non-constant sum games. The complexity will be shown on example of the aforementioned Prisoner's Dilemma in its three-player version, as is presented in matrix. 2.20.

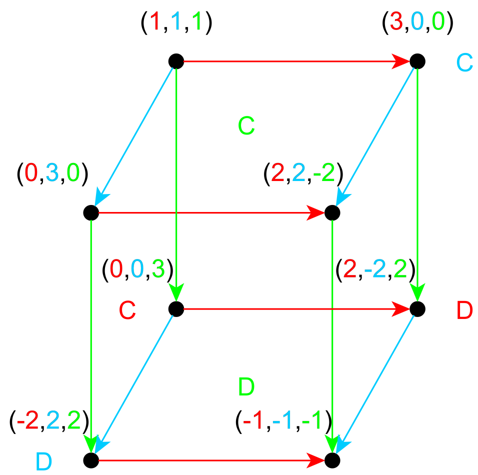


FIGURE 2.26. A three-person Prisoner's Dilemma move diagram

The *move diagram* is presented in 2.26 It is clear from this representation that each of the players has a dominating strategy, the use of which yields a non-Pareto optimal score (-1,-1,-1). In this game it is also possible to form coalitions, with the most fair of them all being the grand coalition of all players, yielding the Pareto optimal score (1,1,1). Let us analyze what will happen in case of a two-player coalition. This has been shown in matrix 2.27.

$\Gamma_{PD_{I \rightleftharpoons II,III}}$	$Cooperate$	$Defeat$
$Cooperate, Cooperate$	$1, 2$	$3, 0$
$Defeat, Cooperate$	$0, 3$	$2, 0$
$Cooperate, Defeat$	$0, 3$	$2, 0$
$Defeat, Defeat$	$-2, 4$	$-1, -2$

FIGURE 2.27. Three-player Prisoner's Dilemma – coalition of two players

The red player using Defeat has a guaranteed safety level -1 while the other players can guarantee an outcome 0 playing (Cooperate,Cooperate), (Defeat,Cooperate) or (Cooperate,Defeat). Thus, the characteristic function of the game is:

- $v(\emptyset) = 0$
- $v(i) = v(ii) = v(iii) = -1$
- $v(i, ii) = v(i, iii) = v(ii, iii) = 0$
- $v(I) = 3$

It would seem that entering a coalition will be beneficial for the player. In case of non-constant games, the characteristic function may mask some of the vital rules of the game. The fact is, it is more beneficent to abandon a coalition aspiration and leave it to the other players to form one. This way the coalition can guarantee a total payoff of 0 units (while with no coalition whatsoever this is -1). At the same time the player outside coalition has a guaranteed payoff of at least 2 units.
Finally, it is worth to mention the game proposed by an economist, Martin Shubik [40] in which a game theory based on a rational choice can lead to a ridiculous behavior of players. In the dollar auction presented in A.1 the players start with one cent bid. When the price reaches the one dollar it is not in their interest to stop the bid, as they would still have to pay the declared amount, so they prefer to raise the stakes with one cent, knowing they will win one dollar. This situation can be repeated into infinity, leading all players to bankruptcy.

Chapter 3

Game theory analysis programming tools

In the market there are currently at least two free IT projects helpful in modeling and analyzing business situations based on game theory.

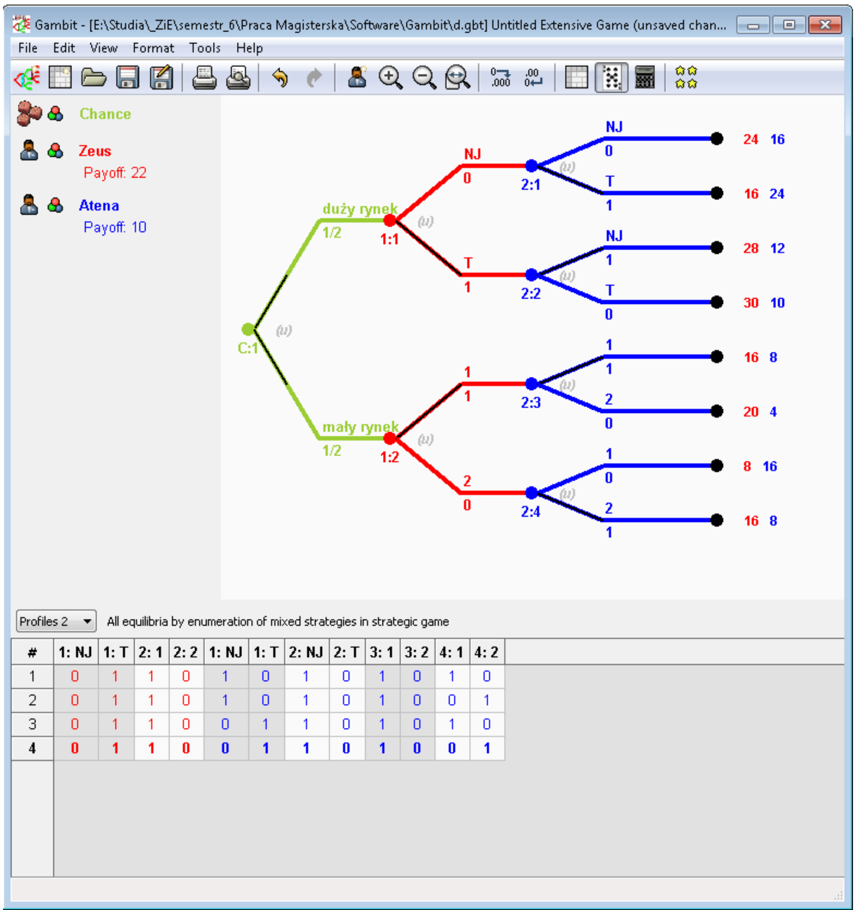


FIGURE 3.1. Gambit graphical interface

The first, most advanced position in **Gambit**[1] realized as part of the grant by California Institute of Technology (commonly referred to as Caltech). The project is still being developed by numerous institutions. The program allows modeling of

[1] http://www.gambit-project.org

finite non-cooperative, constant and non-constant games. It has a helpful, intuitive, graphical interface that can shows representations for both extensive and strategic games. The game models are saved as text files and can be transformed by using various algorithms, which are part of the project. The interface layout is presented in 3.1. The project is publicized via public license GNU, which allows modifying and free usage of the software.

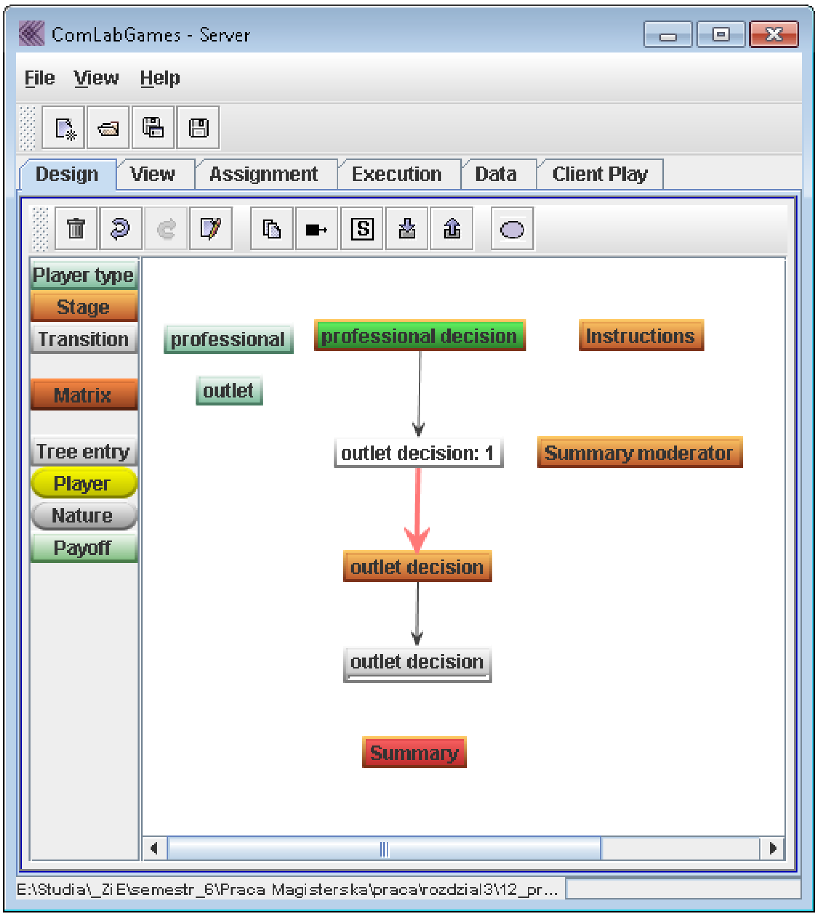


FIGURE 3.2. ComLabGames graphical interface – freestyle game „outsorcing" model

The second free software is ComLabGames. Its main functionality is the possibility to run computer experiments that allow testing in practice some theoretical economical models. The software is made of three modules, which help: model games, run tournaments and analyze data. Each game is saved in *.xml format. The modeled game can be later on played in distributed environment, on client-server architecture, using TCP/IP protocol. The launch environment can work in two modes: sequential (strategic, extensive, or freestyle games), or asynchronous games in the crash-test more (market games). Both modes, however, can be used simultaneously. One must define a freestyle game. It is comprised of stages, each with a formatted text that contains information of input data and variables. Each stage is a synchronization point – at the end of the move, the variables are updated.

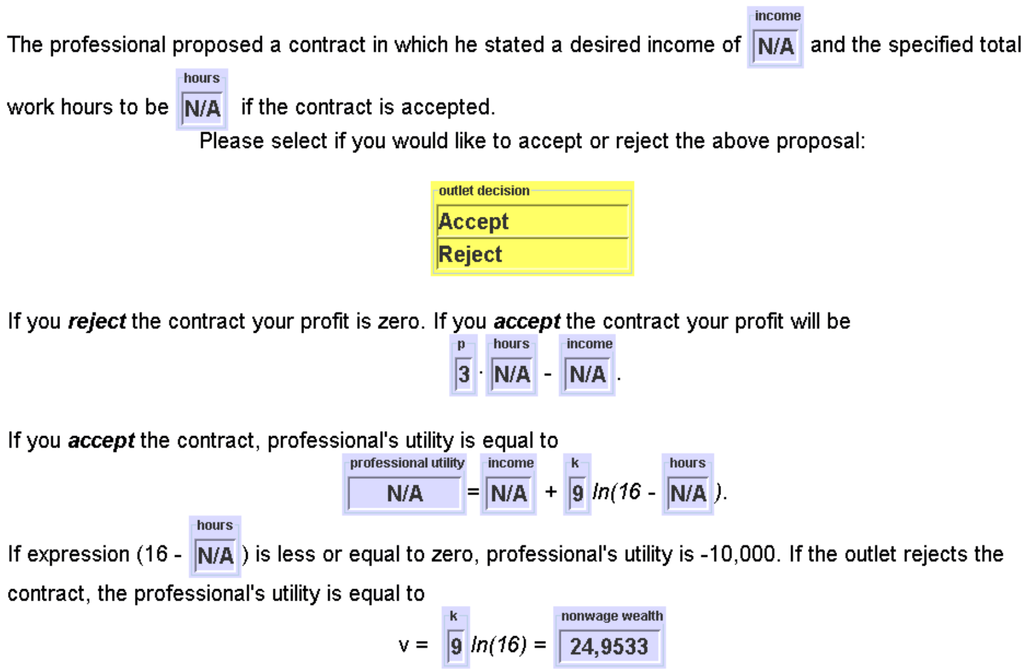


FIGURE 3.3. Stage of the freestyle form game modeling outsourcing process

Generally, the only step in a given freestyle type game can be a game modeled as strategic or extensive one. A freestyle type game has been shown in 3.2, Picture 3.3 shows one stage of such a game.
There's also a number Internet sites that help model and solve simple games:

- `http://people.hofstra.edu/Stefan_Waner/RealWorld/gametheory/games.html` – allows two-player game modeling. Each of the players can have up to five strategies. The software can remove dominated strategies, search saddle points, solve the game and run a human-computer tournament (with the computer using optimal mixed strategy). Picture 3.4 shows the screenshot whilst playing paper-scissors-rock.

- `http://gametheoryexplorer.appspot.com/builder` – it allows modeling, simulating and solving two-player zero sum games. The editor can build an extensive game and transform it into a strategic one. The modeled games can be saved in *.xml, *.fig, or *.png format. The current BETA version permits

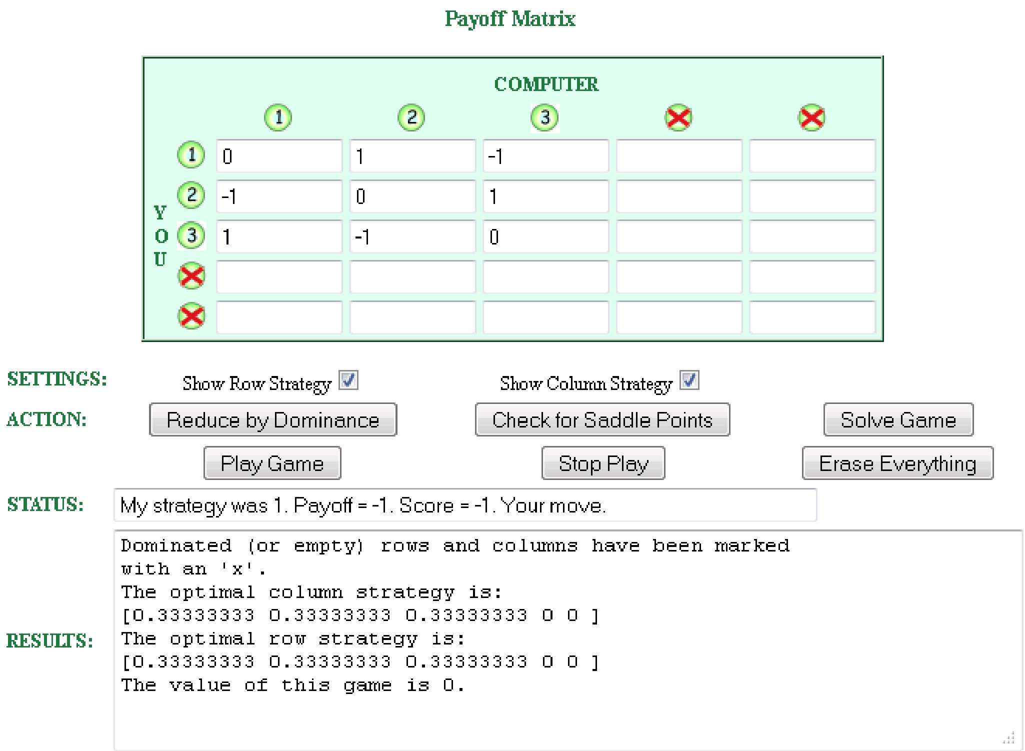


FIGURE 3.4. The gameplay in the paper-scissors-roc

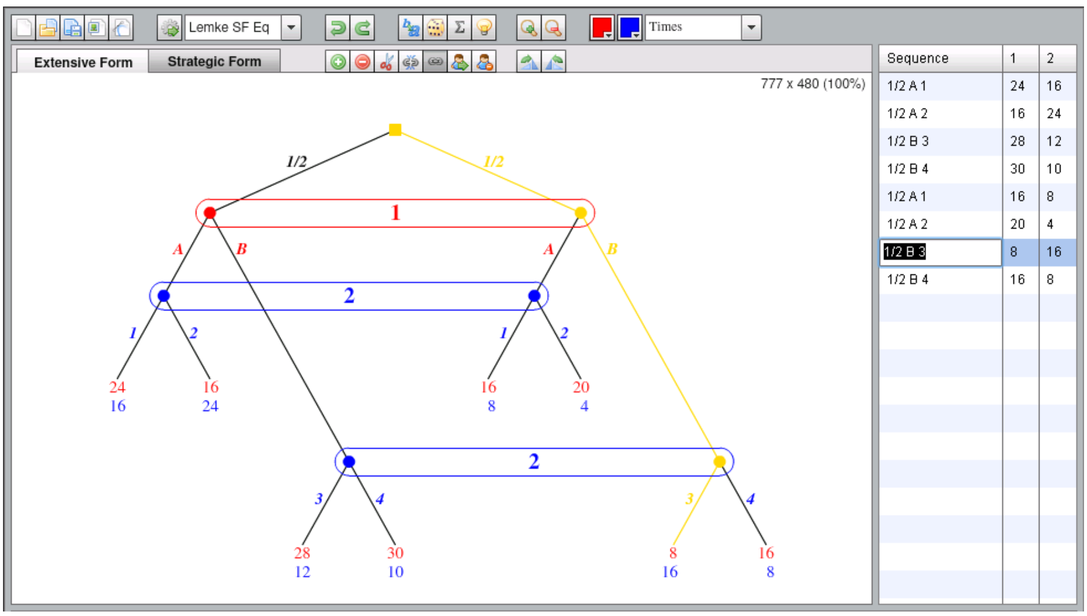


FIGURE 3.5. Game Theory Explorer in edit mode

the use of one algorithm only. The picture 3.5 shows the screen shot of an edited game.

Chapter 4

Modeling business situations with the use of Game Theory

While GT is undoubtedly science, its use gives it almost an art-like status. There is no simple and automated way to make practical use of it. Three key factors in doing so are: experience, intuition and intention of the players. In this chapter several examples of how business situations can be presented as games have been shown. It has been analyzed whether predictions on players' behavior can be made and whether the intuitive outcome is identical to the yielded by the simulation. First, let us determine who the 'players' in the small business sector are. Picture 4.1 shows potential candidates and one/two-sided relations between them. The players can be divided into:

- Internal – the boss, coworkers, employees
- External (micro) – clients, local community, competition, delivery team, subcontractors, investors, public organizations,
- External (macro) — government institutions, local government organizations.

The division is based on the strength of influence of the company towards workers. The internal interaction is very strong, as it is mainly on the experience, available resources, motivation, capabilities, knowledge, relations and character that the company's results depend. This does not suffice, however. A company does not function in a void, and external players (micro) also influence the outcome. The company and the external players' interests are usually mutual. A healthy relationship with the clients, delivery, and even with the competition bares the mark of cooperation. The third type of players is institutions on which a single company has no, or little influence on and cannot change their behavior. Such players include local and general governments. One possibility of forcing an interaction so as to influence the behavior of governments is forming an alliance (coalitions) of several companies – in accordance with the rule that a bigger entity can achieve more in such an interaction. What Game Theory tries to achieve is an understanding of the strategic meaning of market elements, such as: its structure, competition company organization, bidding,

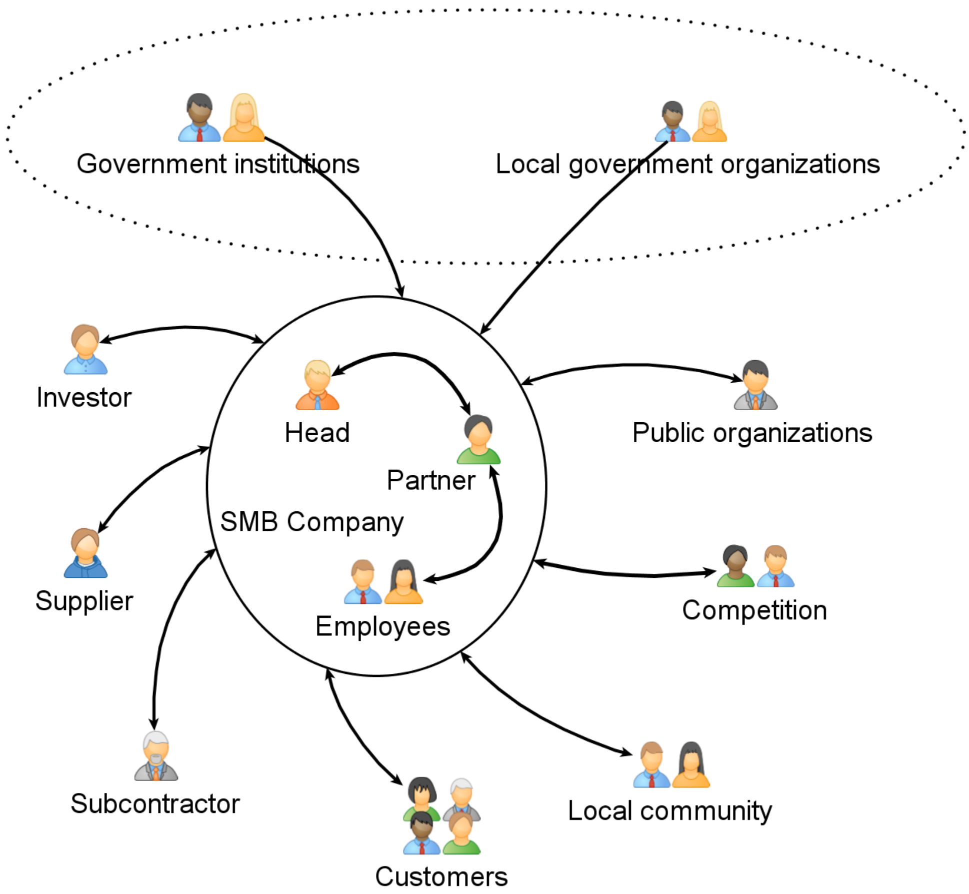


FIGURE 4.1. Players in small business market

advertising, the influence on the environment, company's policy, and interpersonal relationships – both inside and outside of the company. To show all these inter-player relations by means on one, big game would miss the point. The model itself would become too complicated or simply impossible to build. It would also be impractical, as no one, unified optimal strategy for the company to follow would emerge. As was shown in previous chapters, for a comprehensible analysis to be created, the number of players must be limited (which can be done by generalizing, i.e. combining a group of player into one player representative). Additionally, one must focus on the vital relations which help shape a payoff function that would match all players' preferences. A selection of business situation models and solution suggestions will be presented in the following sections.

4.1. Competition

When analyzing the competition on small business market, the simplest situation, i.e. Cournot duopoly [2,23], will be taken into consideration. Cournot duopoly shows how two companies can function on a local market. Both companies $\mathbf{I} = \{1, 2\}$ determine the merchandise volume q_i for $i \in \mathbf{I}$ delivered to the market with the price p that is determined by the given equation:

$$p = x - q_1 - q_2$$

With a stable merchandise unit production cost equal c, the combined income π for both companies:

$$q = q_1 + q_2$$

is as follows:

$$\pi = (x - q)q - cq$$

To calculate the revenue-maximizing production size, the first payoff differential:

$$\frac{d}{dq}(x - q)q - cq$$

must be equated to zero:

$$x - 2q - c = 0$$

which yields:

$$q = \frac{1}{2}(x - c)$$

and the combined outcome is:

$$\pi = (x - \frac{1}{2}(x - c))(\frac{1}{2}(x - c)) - \frac{1}{2}(x - c)c$$

Let us suppose that $x = 1000$ and $c = 100$, which set the π at 202 500.
In a situation where the companies split the market, to maximize the income, each company should deliver merchandise with the value on $\frac{1}{2}$ of the total cost, equaling:

$$q_i = \frac{1}{4}(x - c)$$

In a situation where one of the companies is tempted to breach the contract and delivers more than the estimated amount of merchandise to the market. That company's revenue will be:

$$\pi_1 = (x - q_1 - \frac{1}{4}(x - c))q_1 - cq_1$$

Once the differential:

$$\frac{d}{dq_1}(x - q_1 - \frac{1}{4}(x - c))q_1 - cq_1$$

is equated with to zero:

$$\frac{3}{4}x - \frac{3}{4}c - 2q_1 = 0$$

the outcome is the production size:

$$q_1 = \frac{3}{8}(x - c)$$

and revenue increase for $x = 1000$ and $c = 100$ to $\pi_1 = 113\,910$. Unfortunately, P1's behavior influences the competing player, and P2's revenue falls down to $\pi_2 = 75\,938$[1].

The analyzed situation resembles Prisoner's Dilemma, where each player is suggested to cheat on the opponent, while at the same time counting on players' loyalty. Let us look at the size of the production that would force equilibrium between the players, i.e. a situation where it is in neither of the players' interest to cheat by increasing/decreasing the amount of delivered merchandise. P1's payoff is:

$$\pi_1 = (x - q_1 - q_2)q_1 - cq_1$$

which when the partial differentia:

$$\frac{\partial}{\partial q_1}(x - q_1 - q_2)q_1 - cq_1$$

is equated to zero, it yields:

$$x - c - 2q_1 - q_2 = 0$$

Hence:

$$q_1 = \frac{x - c - q_2}{2}$$

and since this is a symmetrical game, P2's production will also be:

$$q_2 = \frac{x - c - q_1}{2}$$

Now, all that is left to be done is to find the value q_1 and q_2, that would be the best possible outcomes. These are:

$$q_1 = q_2 = \frac{x - c}{3}$$

[1] P2 payoff has been calculated with the formula: $\pi_2 = (x - q_2 - q_1)q_2 - cq_2 = (x - \frac{1}{4}(x - c) - \frac{3}{8}(x - c))\frac{1}{4}(x - c) - c\frac{1}{4}(x - c)$ which for the set values $x = 1000$ and $c = 100$ gives: $(1000 - \frac{1}{4}(1000 - 100) - \frac{3}{8}(1000 - 100))\frac{1}{4}(1000 - 100) - 100 * \frac{1}{4}(1000 - 100) = 75\,938$

Therefore, the combined market supply is:

$$q = \frac{2}{3}(x - c)$$

This is not an optimal outcome, given that with the previous assumptions that $x = 1000$ and $c = 100$, the combined revenues value is:

$$\pi = (x - \frac{2}{3}(x - c))\frac{2}{3}(x - c) - \frac{2}{3}(x - c)c =$$

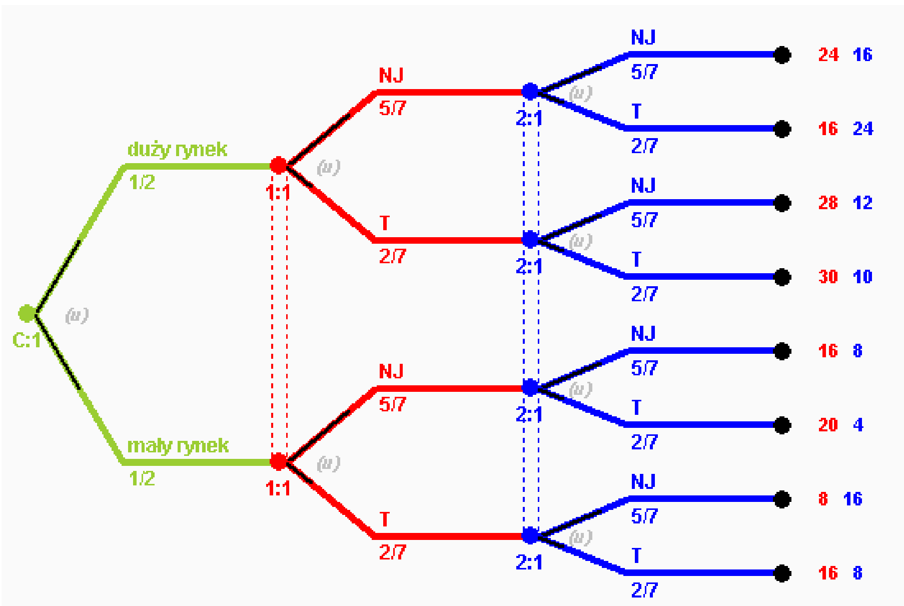
$$= (1000 - \frac{2}{3}(1000 - 100))\frac{2}{3}(1000 - 100)) - \frac{2}{3}(1000 - 100))100 = 180\,000$$

.

Zeus
Payoff: 136/7

Atena
Payoff: 88/7

#	1: NJ	1: T	1: NJ	1: T
1	$\frac{5}{7}$	$\frac{2}{7}$	$\frac{5}{7}$	$\frac{2}{7}$



	1		2	
1	20	12	18	14
2	18	14	23	9

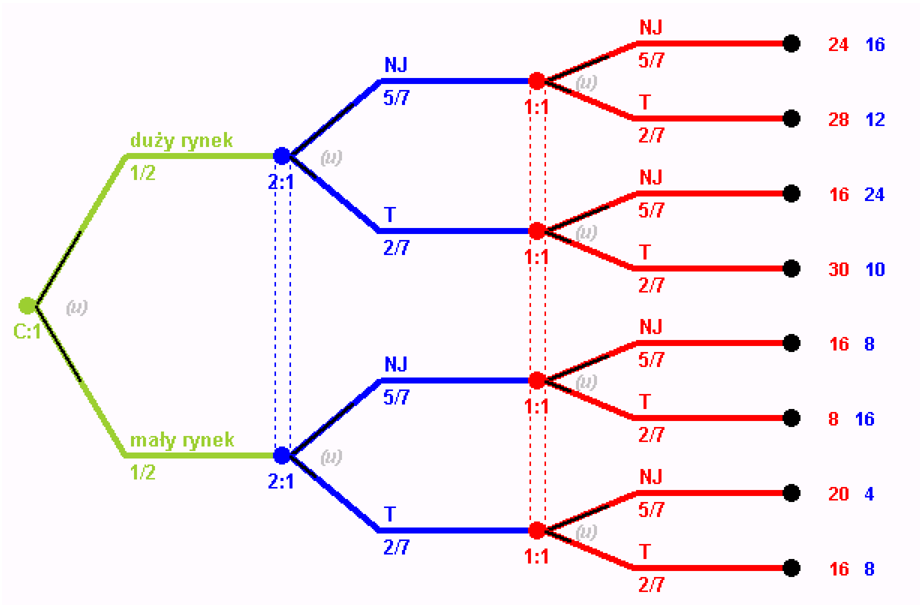
FIGURE 4.2. A case of Zeus and Athena making decisions simultaneously

It is unrealistic to assume that the companies will be the ones influencing the supply demand. They more often than not have influence over the price of the goods. This is reflected in Bertrand‘s duopoly [5], which leads to even more paradoxical conclusions, i.e. the competition leads the companies to produce the goods in a volume exceeding the optimal one, which in turn leads to price decrease and a formation of cross-companies balance in a zero-revenue situation. While being beneficial from the customers' point of view, the situation is best avoided by the companies, as it may lead to the destructive *price wars*.

Zeus
Payoff: 136/7

Atena
Payoff: 88/7

#	1: NJ	1: T	1: NJ	1: T
1	$\frac{5}{7}$	$\frac{2}{7}$	$\frac{5}{7}$	$\frac{2}{7}$



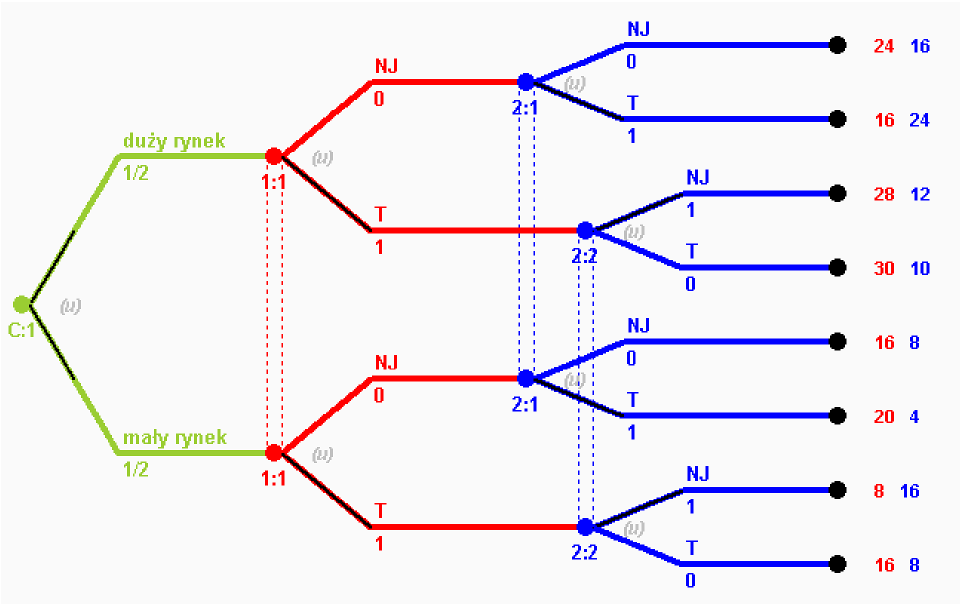
	1		2	
1	20	12	18	14
2	18	14	23	9

FIGURE 4.3. A case of Zeus and Athena making decisions (the tree shows Athena's information set first)

Game Theory will help in decision making competition-wise. The case-study example from Harvard Business School, analyzed with *Gambit*, the tool described in chapter 3 In an assumed market environment, there are two companies. One of them, Zeus, is a big company, adapted for mass-production on cheap merchandise, while the other one, Athena produces high-quality commodity in lower volumes. Both companies want to introduce new goods. The market estimates are: 50% – high demand (40 million $\frac{PLN}{year}$), 50% – low demand (24 million $\frac{PLN}{year}$).

Zeus
Payoff: 18
Atena
Payoff: 14

#	1: 1	1: 2	2: 11	2: 12	2: 21	2: 22
1	1	0	0	0	1	0
2	**0**	**1**	**0**	**0**	**1**	**0**



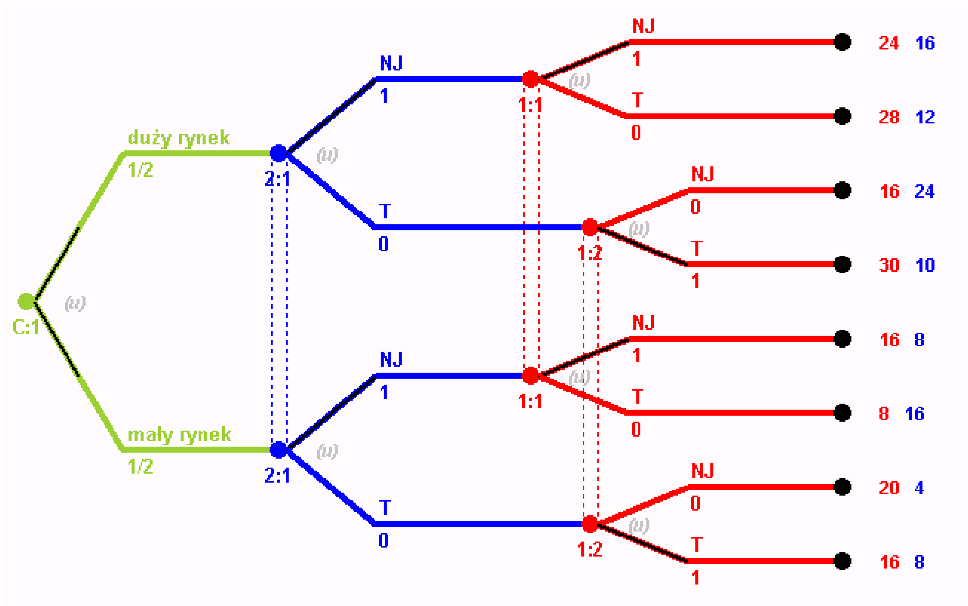
	11		12		21		22	
1	20	12	20	12	18	14	18	14
2	18	14	23	9	18	14	23	9

FIGURE 4.4. A case when Zeus makes the first decision

The commodity can be in luxury or regular versions. With lower demand, luxury version will be more beneficial, otherwise the regular-quality, mass-produced version will gross more income. Picture 4.2 depicts a game modeling the given example. When both companies make the decisions on the production size at the same time, this is a simultaneous game. This is a constant-sum game, i.e. purely competitive. As can be seen in the payoff function value, both companies will push towards expanding the market. In such a case, each company will want to produce the cheaper commodity, while the competition takes the luxury market. In an opposite situation, i.e. with a small market, each company will benefit more from producing luxury commodity (while the competition takes the mass-production of lower-quality merchandise). The solution to this game is a mixed strategy, where $\frac{5}{7} = 71,4\%$of commodity is top quality, while $\frac{2}{7} = 28,6\%$ of commodity is mass production. When both companies use this strategy, Zeus' income amounts to $\frac{136}{7} = 19,43$ million $\frac{PLN}{year}$), and Athena's to $\frac{88}{7} = 12,57$ million $\frac{PLN}{year}$. Note that in that scenario, the representation order is not important (as in picture 4.3), since it will influence neither the strategies, nor the outcome of either player.

Zeus
Payoff: 20
Atena
Payoff: 12

#	1: NJ	1: T	2: NJ	2: T	1: NJ	1: T
1	1	0	$\frac{3}{5}$	$\frac{2}{5}$	1	0
2	1	0	0	1	1	0



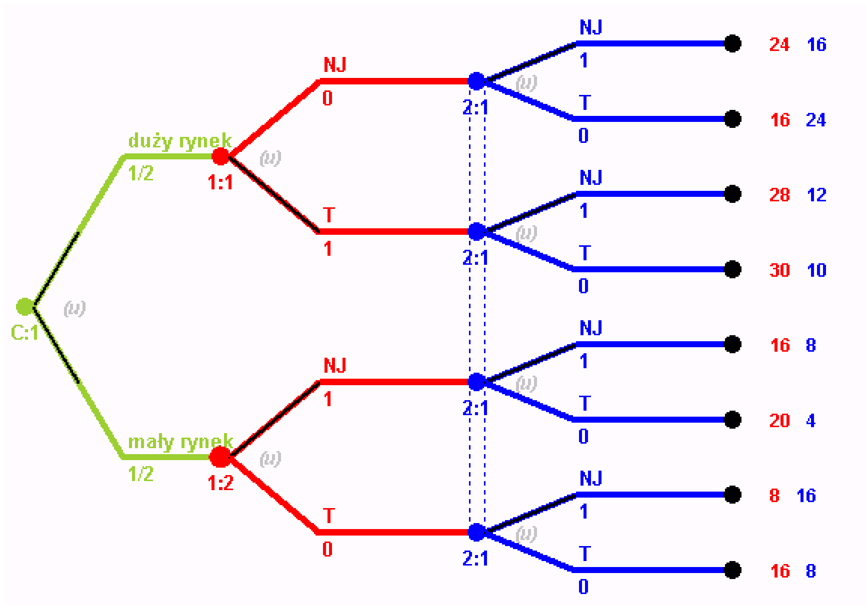
	1		2	
11	20	12	18	14
12	20	12	23	9
21	18	14	18	14
22	18	14	23	9

FIGURE 4.5. A case of Athena making the first move

Let us analyze a situation where Athena, as the smaller and more flexible company, can withhold the decision making until it knows Zeus' strategy. This has been presented in 4.4. Owing to this additional parameter, the single information set Athena was in could have been divided into two smaller ones. This forced a change in the strategy for both companies. The game now has two saddle points and regardless of Zeus' choice, Athena will always make a counter-decision. The payoffs are now: Zeus 18 million $\frac{PLN}{year}$, and Athena 14 million $\frac{PLN}{year}$. It is clear that Athena gained $1,43$ million $\frac{PLN}{year}$ of revenue. This is in compliance with the examples presented in chapter 2, where the analyzed cases proved that constant-sum games the position of P1 is less preferable.
In the opposite scenario, when Zeus takes the role of P2 (picture 4.5), the game

Zeus
Payoff: 22
Atena
Payoff: 10

#	1: NJ	1: T	2: 1	2: 2	1: NJ	1: T
1	0	1	1	0	1	0



	1		2	
11	20	12	18	14
12	16	16	16	16
21	22	10	25	7
22	18	14	23	9

FIGURE 4.6. A case of both Athena and Zeus making simultaneous decisions, but Zeus has done market research

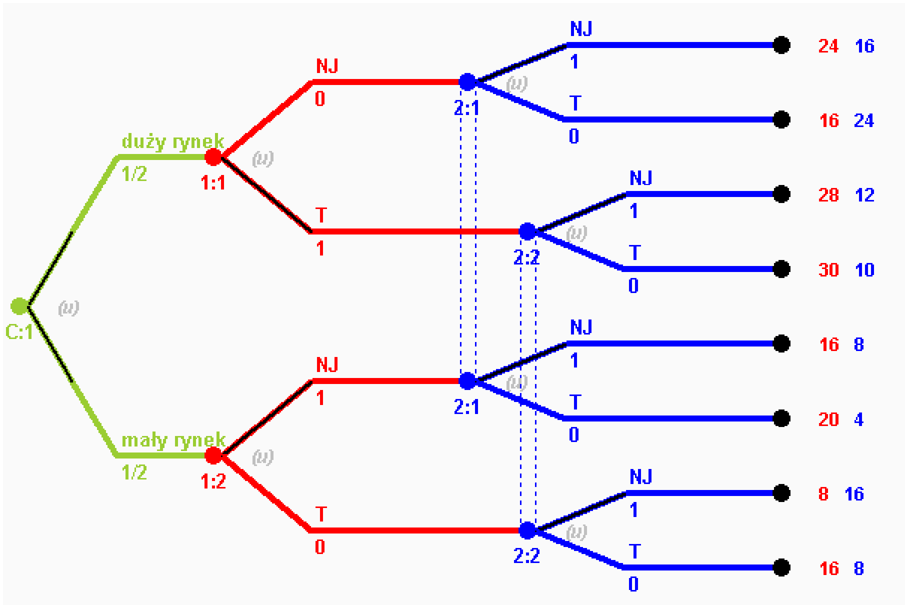
has two solutions. Athena always takes the 'luxury' approach. Zeus, on the other hand can stick to producing luxury commodity, or use mixed strategy, with 60% of top-quality stock, and 40% of mass-produced stock. Both solution (as has been presupposed) are beneficial for Zeus and increase the payoff to 20 million $\frac{PLN}{year}$, while at the same time reducing Athena's win to 12 million $\frac{PLN}{year}$.

Yet another case focuses on a situation where Zeus has carefully analyzed market trends and can estimate the size of the demand. Athena does not know the results, but is aware of the fact that such analysis has taken place. This is presented in picture 4.7 This time intuition suggests that Zeus will choose mass-production when the estimated market size is big; otherwise, in case of a small market it will produce fewer, better-quality goods. At the same time, Athena will choose top-quality production. This is because of a simple calculation Athena makes: either the market is

Zeus
Payoff: 22

Atena
Payoff: 10

#	1: NJ	1: T	2: NJ	2: T	1: NJ	1: T	2: NJ	2: T
1	0	1	1	0	1	0	1	0



	11		12		21		22	
11	20	12	20	12	18	14	18	14
12	16	16	20	12	12	20	16	16
21	22	10	23	9	24	8	25	7
22	18	14	23	9	18	14	23	9

FIGURE 4.7. A case of Zeus making the first move, having analyzed the market before doing so

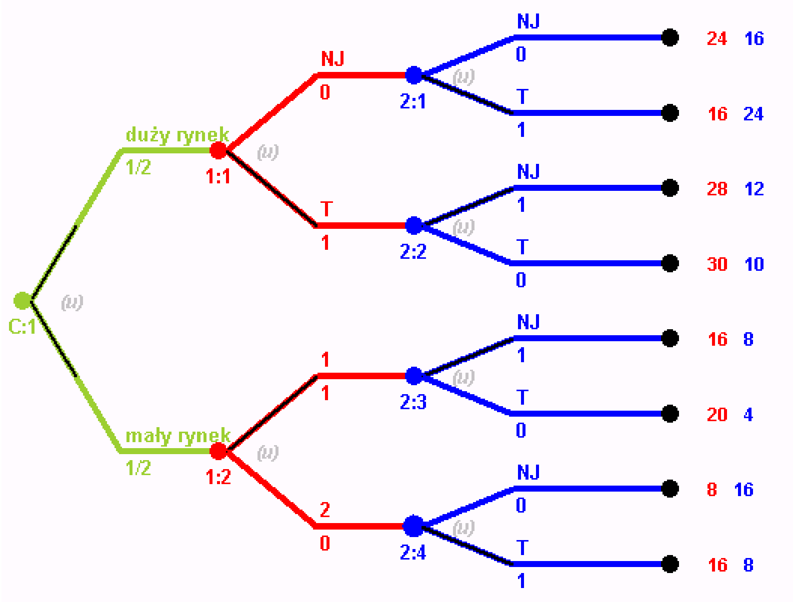
going to be small, or the competition is going to be big and better-adjusted. Athena loses and has 10 million $\frac{PLN}{year}$ revenue left, while Zeus reaps the benefits of having made the market analysis with a whooping 22 million $\frac{PLN}{year}$ income. The analysis cost (up to 4 million $\frac{PLN}{year}$) is, hence, justified.

Picture 4.7 shows Athena waiting for Zeus to make the first move. It also shows that such a strategy will not help improve the score, as it did then there was no prior market analysis.

An interesting case is made in a situation here Zeus hides the fact of having made such market analysis. Zeus would be playing 4.7 game, while Athena would perceive it as 4.4. Zeus knows that Athena will choose it best strategy: make the exact opposite choice Zeus would make. Zeus will use that knowledge to select mass-production for big market, and top-quality production for small market, which will

Zeus
Payoff: 22
Atena
Payoff: 10

#	1: NJ	1: T	2: 1	2: 2	1: NJ	1: T	2: NJ	2: T	3: 1	3: 2	4: 1	4: 2
1	0	1	1	0	1	0	1	0	1	0	1	0
2	0	1	1	0	1	0	1	0	1	0	0	1
3	0	1	1	0	0	1	1	0	1	0	1	0
4	**0**	**1**	**1**	**0**	**0**	**1**	**1**	**0**	**1**	**0**	**0**	**1**

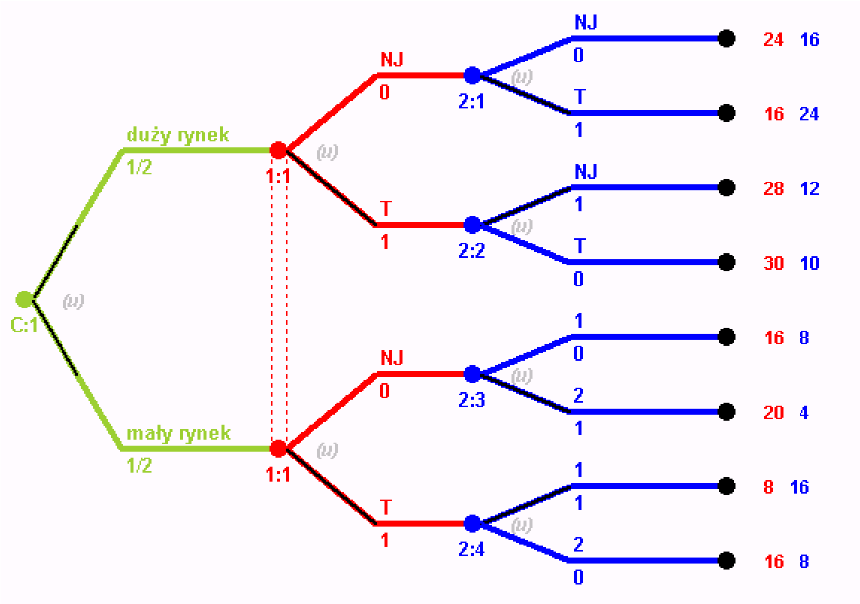


	1111		1112		1121		1122		1211		1212		1221		1222		2111		2112		2121		2122		2211		2212		2221		2222	
11	20	12	20	12	22	10	22	10	20	12	20	12	22	10	22	10	16	16	16	16	18	14	18	14	16	16	16	16	18	14	18	14
12	16	16	20	12	16	16	20	12	16	16	20	12	16	16	20	12	12	20	16	16	12	20	16	16	12	20	16	16	12	20	16	16
21	22	10	22	10	24	8	24	8	23	9	23	9	25	7	25	7	22	10	22	10	24	8	24	8	23	9	23	9	25	7	25	7
22	18	14	22	10	18	14	22	10	19	13	23	9	19	13	23	9	18	14	22	10	18	14	22	10	19	13	23	9	19	13	23	9

FIGURE 4.8. A case of Zeus and Athena making a careful market analysis

yield a mean income of 24 million $\frac{PLN}{year}$. By not giving out the information on having performed market analysis, the company earns additional 2 million $\frac{PLN}{year}$.
Let us analyze a case when Athena conducts a market analysis as well. The case will include both companies making such analysis and making a simultaneous decision. This has been presented in 4.8. Zeus's strategy has not changed, while Athena's analysis cannot be used to improve the outcome (the payoff is identical to the ones in games 4.8 and 4.7).
The last simulation will be a case of Athena making the analysis, while Zeus makes the first move (without such analysis). This is depicted in picture 4.9, where four optimal strategies exist. The payoffs, however, do not differ from the ones in the game 4.4, where Zeus is P1 (made analysis) and Athena is P2 (without analysis).
To sum up, a case study of business environment modeling the introduction of new

	#	1: NJ	1: T	1: NJ	1: T	2: NJ	2: T	3: 1	3: 2	4: 1	4: 2
Zeus Payoff: 18	1	0	1	0	1	1	0	1	0	1	0
	2	0	1	$\frac{1}{3}$	$\frac{2}{3}$	1	0	$\frac{2}{3}$	$\frac{1}{3}$	1	0
Atena Payoff: 14	3	0	1	$\frac{1}{2}$	$\frac{1}{2}$	1	0	1	0	1	0
	4	0	1	0	1	1	0	0	1	1	0



	1111	1112	1121	1122	1211	1212	1221	1222	2111	2112	2121	2122	2211	2212	2221	2222
1	20 12	20 12	22 10	22 10	20 12	20 12	22 10	22 10	16 16	16 16	18 14	18 14	16 16	16 16	18 14	18 14
2	18 14	22 10	18 14	22 10	19 13	23 9	19 13	23 9	18 14	22 10	18 14	22 10	19 13	23 9	19 13	23 9

FIGURE 4.9. A case of Zeus making the first move, while Athena makes a thorough market analysis

commodity into the market leads to an intuitive conclusion that small companies should wait for the big ones to make their move. By doing this, they can make the best out of their production flexibility and fill in the market niche. For big companies, however, it is more beneficial to run the market analysis (costly though it may be) to help decide on the best strategy.

4.2. The information management and manipulation

Information management is a vital element of optimal decision making. It is important for the company to have access to the stored crucial data from its past. This aids decision making and promotes the foreseeing of optimal moves that will bring about a satisfactory result. For example, if one of the supplies has a history or cyclical (though sporadic) delivery problems, the company can go one step ahead and

order a bigger deliver, or change the supplier. In a case where the delivery issues cause significant revenue loss, the company may even decide to switch to a more expensive supplier. If a company does not have such records, it may wish to obtain it from external sources. It must, however, understand that each external source will benefit only from positives reviews and the data may not be reliable.
To make it more reliable, the information must be *authenticated* which is a difficult task [12]. The negative and harmful news are easily authenticated. For example, is a company is declaring bankruptcy, it is probably so that the company has, in fact, gone bust. When the company produces documents showing a favorable status quo, we may suspect that demeaning data has been concealed. Once way to authenticate such revelations is by showing the accountancy records. Such sensitive data are (wisely) protected by the companies. One important aspect the companies must be aware of is that actions speak louder than words. It is best then to observe the competition, rather than believe their declarations. Knowing that opponents are aware of this, the company must carefully plan actions. Going further, the company can take conscious decisions that will have the power to *signalize* [41] certain pieces of information in a certain, satisfactory way. One may try *distorting signals*, when the one wants to hinder the reception. On the other hand, when it is in the company's interest to receive the signal, it may arrange a situation in such a way as to force the *vetted* [33] company to let it out. One signalization method may be warranty he company gives for a given product. This forms an idea in the customer's mind that the product must be high quality, as no company would give a warranty on a easily damaged commodity, knowing they would need to reimburse in case of a complaint. One *vetting* instance mechanism is asking the sales person for a warranty. A typical market where this applies is insurances, as it is the client who determines the risk factors. Those leading risky lifestyles will be interested in transferring greater responsibility to the insurance company, while those leading peaceful lives, will be opt for the cheaper insurances. Another example is price discrimination, which means offering two variants of the same product – regular and luxury ones. Each offer is meant for a different class of customers, serving the purpose of maximizing the revenue. If 'regular' and 'luxury' versions had the same standard, it would only be natural for the richer clients to choose the cheaper version. At the same time, if both versions have a similar price range, the regular client would not afford it or the selling volume would be too low to generate income. In each case, the revenue is insufficient. It may actually be in the company's interest to produce 'regular' commodities but conscious lowering of the 'luxury' stock quality (which, paradoxically, costs more to produce).

Very often it is so that such signaling is intentionally muffled. The rich do not flaunt their fortune, as opposed to, say, the nouveau-riche. The talented try not to boast when in company, etc. This is called *contra-signalizing* [13] This tactic is noteworthy, as it might do more good than overbearing *signalizing* (which, in turn, may help in *vetting* and *signal obstructing*). At last, it's worth to mention *shields of lies*, This leads to a ironic situation where the company purposefully reveals true data, hoping the competition will not believe it. Such cases serve as another reminder of the fact that, aside from official statements, it's better to focus on actions to get a clear picture.

4.3. Motivational system

One of the conditions for survival of a company is generating enough revenue to make the business profitable. While it is in the owner's and board's interest to generate as much revenue as possible, this is not always the case when for the workers. The workers require a proper motivational system, which is correlated with the company's situation and their engagement, as well as the amount of work put into it. Such a system forms a kind of a employer-employee game. The conditions it should meet are:

- Simplicity – the motivational system should be clear, unified and not too complex, so as to make it legible and calculable for everyone,
- The rule of participation – the offered pay should be higher or equal to the minimal one acceptable by the workers,
- The rule of sufficient incentive – the bonus needs to be adequate with respect to the effort that needs to be made to achieve the goal; the goal itself has to achievable – otherwise is will not be motivating.
- The rule of incorporation – the pay should be comprised of a set amount of a income and a bonus for the achieved results.

The basic problem is the calculation of how much of the outcome depended on the employee's effort, and how much is chance. The more depends on chance, the lesser influence should the outcome have on the pay. To estimate the chance factor, one can use the rivalry rule. The tendency is for the employee to boast of one's worthiness, while hiding one's imperfections. In such a case, a third party should serve as the assessor. There should be no personal (neither positive, nor negative) relations between the sides. The perfect situation would be to employ a different office or a

separate company to ensure the parties do not know another. The alternative solution is to compare the employee with the other ones, if multiple employees perform the same tasks. In that instance, the outcome of one employee can be compared against the average.
If we know the probability layout of the 'unlucky' situations and the trial number is high enough, then according to the law of large numbers it is possible to count the pay mean. This is because the chances or one employee having bad luck all the time are nil. An employee is motivated not only by their pay, but also by other factors, like: internal motivation, the interest in the work, the relations with the co-workers and the management, the promotion opportunity, etc. If the job requires devoting full attention, it is logical to employ a *pro quality job* that will reward the outcome, while at the same time include a strict penalty for unproductiveness.
In case of using a ladder motivation system, one must carefully set the next steps of the bonus levels. If the realization of the net step on the ladder proves an impossible task for the employee, it may serve as a de-motivator. One solution is a linear bonus system, where the value of the extras reflects the outcome. The flaw of this solution is that it cannot be applied to complex task. Until now, we assumed that the employee performs one sort of a task. In multilayer tasks a question arises – which assignments are complimentary and which are substitutive? In the first case, the more motivation there is for task A, the better the results are for task B. In the latter, the more emphasis is put on task A, the less attention task B gets, yielding worse results. The work division should consider the synergy of complementary tasks.

4.4. Negotiations

When preparing for the negotiations, the board should determine the best possible alternative in case of a failure. Such alternatives are collected under the acronym BATNA (Best Alternative to Negotiated Agreement). One basic requirement is that the negotiated goal be attractive for all sides (i.e. its value cannot be lower than that of BATNA). BATNA aids in establishing a starting point for negotiations. It is worth trying to estimate the starting point of the rivaling company, as the de facto negotiated value is the difference between BATNA and the possible outcome of such negotiations. The reason why BATNA is so important is because it helps negotiate a bigger part of the difference. To have a better starting point, the strategy of escalating ones BATNA, while decreasing the opponent's in employed. To accomplish this, one may resort to a declaration or a threat. Since it is the relative difference that counts, the player may put the threat into action is the results are harmful to a greater extent for the opponent, than for themselves.

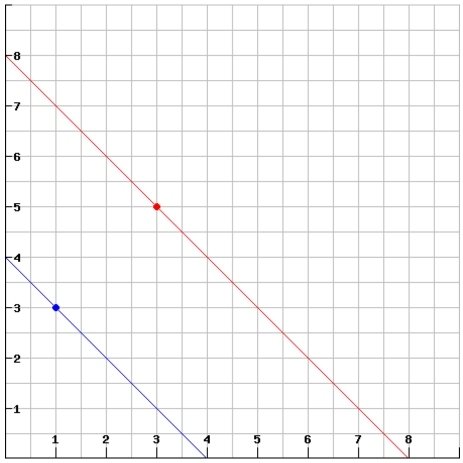


FIGURE 4.10. Negotiation set

Picture 4.10 shows an abstract bidding situation. The blue point with the coordinates $(1, 3)$ represents the values that would result in case of no negotiations (Status Quo point). The red point with the coordinates $(3, 5)$ represents the values that would result in case of a negotiation. The bidding value set is made of two straight lines (blue and red[1]), provided that a) there's a possibility of a money transfer, and b) money present equal utility for all players and has an additive character. An effective solution is one that does not have a point belonging to the negotiation set (to the right and up from it) [26]. As is noticeable, any t money transfer will be an ineffective solution (for any point set on the blue line there is an infinite number of red points with both coordinates of a non-lower value). For example, for $t = 2$, the coordinates of such a point will be $(3, 1)$. Each solution placed on the red line from $(3, 5)$ to $(7, 1)$, which represent a transfer deal $t \in < 0, 1 >$, is the more effective one.

The question remains: which of the infinite number of solutions is fair? As has been pointed out, Status Quo point denoting BATNA marks a section of the straight line, on which one can find a solution. Neither of the rational players will accept a solution of a lower payoff than the one they can achieve without a contract. Thus, the only real, negotiable value is the different between the payoff offer by a lack of contract, and that which the contract promises. In a situation presented in 4.10 the negotiated value is $5+3-(3+1) = 4$. The bidding force of the sides determines how it will be divided. The force π_i for $i \in \mathbf{I}$, I must meet the requirement $\sum_{i \in \mathbf{I}} \pi_i = 1$. One of the bidding forces might be, for example, the patience of both sides. In case of prolonged negotiations, the side with lower negotiation patience will have more costs to shoulder. Therefore, a vital element of a successful negotiation is to determine the

[1]To simplify matters, we shall assume that money can be divided into units of any given size.

bidding force of the opponent. This helps in getting higher takings without risking the breaking of negotiations.

4.5. Contracts

A contract is a two-stage process. Step 1 includes discussions on the terms, while step 2 is the implementation of these terms [44]. The first stage is a result of negotiations, which, in case of a success, lead to cooperation. Between two or more business entities a contract relation is set. The contract negotiating phase has been presented in the previous section. If the terms of the contract suffice to motivate all parties to comply with them, the contract is said to be *internally realized*. More often than not, the sheer fact of having signed a contract is not enough to guarantee its enforcing. Very often this requires a third party intervention. If such an arbiter is required to secure the contract, it is called *externally realized*. A third type of contract is one in which step 1 and 2 are simultaneously realized. Such a contract is said to be *automatically realized*. In reality, most contracts signed by business entities are a mixture of all three .

Γ_{GS}	W	N
W	z_1, z_2	x_1, y_2
N	y_1, x_2	$0, 0$

FIGURE 4.11. The associated game

The relationship of subjects called a *contract relation* can be presented via so called *associated game* pic. 4.11. Two players set a contract, which will require a lot of effort (W-strategy) or very little of it (N-strategy). The game is a simultaneous one, as all players make independent and decisions at the same time. An analysis of such case will be conducted $z_1 + z_2 > y_1 + x_2$, $z_1 + z_2 > x_1 + y_2$ and $z_1 + z_2 > 0$. If the possibility of *money transfer* is accepted, the only optimal strategy profile will be WW – a combination securing the biggest payoff to split. The contract is easily fulfilled when $z_1 > x_1$ and $z_2 > x_2$ when it is *internally realized* and Nash equilibria occurs. If at least one of these inequalities does not occur, a preferable WW profile will require a third party player. One instance of such third party is a court. When both players realize W strategy, the court does not mediate the payoff value. In case of one player engaging in N strategy, the transfer value is determined by the court. Adding money transfers α, β, γ to an *associated game* 4.11 gives an *inducted game*

4.12.

Γ_{GI}	W	N
W	z_1, z_2	$x_1 + \alpha, y_2 - \alpha$
N	$y_1 + \beta, x_2 - \beta$	$\gamma, -\gamma$

FIGURE 4.12. The inducted game

It is possible to formulate an equlibria situation in a WW profile if the court allows a *complete contract* to be penned. A complete contract is one that determines the exact transfer values α, β, γ, and meets the following condition: $z_1 \geq x_1 + \alpha$ and $z_2 \geq x_2 - \beta$. If the court can unequivocally determine which player realized which strategy, one may speak of *full verifiability.* Full verifiability allows penning a contract that will help reach any result an associate game can offer. This, unfortunately, rarely is the case. Usually, what the court can accomplish is to verify whether the optimal outcome has been achieved. This is a case of *limited verifiability.* As the court cannot determine which of the players took N strategy, it cannot conclude which transfer α, β or γ. The only solution is to order α, transfer in each of the three cases, which turns the game into and inducted one, picture 4.13.

Γ_{GI_n}	W	N
W	z_1, z_2	$x_1 + \alpha, y_2 - \alpha$
N	$y_1 + \alpha, x_2 - \alpha$	$\alpha, -\alpha$

FIGURE 4.13. The inducted game in the case of limited verifiability

In that case, an optimal-outcome-guaranteeing contract can be drawn only if the following inequality takes place $z_1 \geq x_1 + \alpha$ and $z_2 \geq x_2 + \alpha$ which gives, after conversion, the following $x_2 - z_2 \leq \alpha \leq z_1 - x_1$. Hence, the only guarantee of a successfully α transfer is a situation $z_1 + z_2 \geq x_1 + x_2$. Picture 4.14 shows an associated game, where an optimal outcome is impossible to reach in limited verifiability environment.

Γ_{GI_n}	W	N
W	$5,5$	$-2,6$
N	$6,-2$	$0,0$

FIGURE 4.14. Example associate game in which to obtain an effective result in conditions of limited verifiability is impossible

Limited verifiability is one of the reasons for *divergence of the third kind* (described in section 2.3.1) arising.
When transfer α, β, γ have not been set by the players, or have been ignored by the court, the contract is *incomplete*. Moreover, if one of the players breaches the contract, the partner can sue them. When dealing with an *incomplete* contract, the court can employ the *damages reimbursement policy*. The policy means ordering a transfer to the complainant that would be enough to guarantee an outcome identical to that a fulfilled contract would bring. That is depicted in picture 4.15 by an induced game.

$\Gamma_{GI_{nws}}$	W	N
W	z_1, z_2	$x_1 + y_2 - z_2, z_2$
N	$z_1, x_2 + y_1 - z_1$	$0,0$

FIGURE 4.15. The inducted game by the principle of *damages reimbursement policy*

In that game, the transfer values are as follows: $\alpha = y_2 - z_2, \beta = y_1 - z_1, \gamma = 0$. As a result, WW form a Nash equilibrium then and only then when $z_1 \geq x_1 + y_2 - z_2$ and $z_2 \geq x_2 + y_1 - z_1$ which, upon conversion yields two inequalities: $z_1 + z_2 \geq x_1 + y_2$ and $z_1 + z_2 \geq x_2 + y_1$. When both inequalities are fulfilled, the outcome will be effective in the sense of maximizing the sum of all players' payoffs[1]. The *damages reimbursement policy* is an excellent contract effectiveness guarantying method, as it eliminates the discrepancy between the individual and collective interests of contract-bound

[1]The effectiveness in the sense of maximizing the sum of all players' payoffs is a narrower idea then Pareto effectiveness. An optimal outcome may cause the lessening of the payoffs for one of the players. If player-player transfer is permitted, these can be used to yield a Pareto-effective outcome.

parties. Even when the values of α, β, γ transfers are undetermined when penning a contract, the sheer probability of a court ruling forces a globally effective behavior[1]. Unfortunately, for the court to rule *damages reimbursement*, it has to have full knowledge of the payoff function, as well as the information on the player who did not abide by the rules of the contract. In other words, it requires *full verifiability.* While the values of y_i for $i \in \mathbf{I}$ can be determined from invoices of the respondent, determining the values of z_i for $i \in \mathbf{I}$ may prove a task far more difficult. For instance, if the purpose of a contract is to diversify the market, it will be difficult to establish the revenue of the company in case of its full implementation.
When determining the value of z_i for $i \in \mathbf{I}$ is impossible, the court may order *restoring to the previous state.* This means restoring one of the players to the initial state in case of contract fulfillment failure. Such is the case in picture 4.16. The outcome ZZ will form Nash equilibrium then and only then when $z_1 \geq x_1 + y_2$ and $z_2 \geq x_2 + y_1$ which means the reimbursement cannot be too small.

$\Gamma_{GI_{psp}}$	W	N
W	z_1, z_2	$x_1 + y_2, 0$
N	$0, x_2 + y_1$	$0, 0$

FIGURE 4.16. The inducted game by the principle of *restoring to the previous state*

A third was of enforcing the contact obligations is *the return of undue payments.* This means stripping the dishonest contractor off all unwarranted income that the breach of contract brought. This has been presented in picture 4.17

$\Gamma_{GI_{zn}}$	W	N
W	z_1, z_2	$0, x_1 + y_2$
N	$x_2 + y_1, 0$	$0, 0$

FIGURE 4.17. The inducted game by the principle of *the return of undue payments*

[1]One exception would be a possibility of too low a value of court rules α, β, γ transfers. This means that the induced game will remain ineffective in terms of maximizing the payoff sum of all players.

The outcome ZZ will form Nash then when z_1 and z_2 are greater than 0 and form the give inequality $z_1 \geq x_2 + y_1$ and $z_2 \geq x_1 + y_2$.
All presented cases have been the subject of S. Shavell's publication [38]. In reality, the court's decision on the implementation of the contract depends on several factors. These are: the determinability of the payoff function and the values of the transfers, as well as the verification of each player's strategy. It is particularity difficult to determine the value of lost alternatives, the cost of eliminating the damages, etc.
The complex negotiation process can be represented by a common vertex [44]], which has 'mutual decisions' of more than one player. This helps simply the model and makes the analysis easier. The peak is additionally circled, to differentiate it from other peaks.

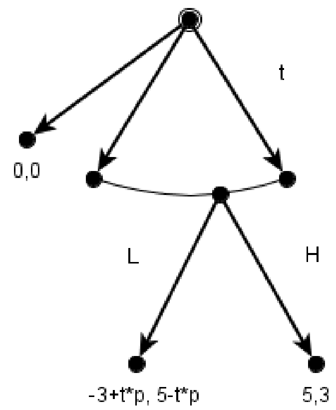


FIGURE 4.18. Extensive form of the game with the common vertex

Picture 4.18 presents a simple game reflecting the drawing of a contract for material delivery. In step one, the players establish the t transfer value, which stands for a compensation paid by the delivery company when the receiver proves that materials are of too low a quality. The chances of determining the stock's flaws are p. Once the negotiations are done, the supplier chooses the delivery strategies for high quality stock H or low quality stock L. In case of H, the supplier payoff is 3 and the receiver payoff is 5. However, with low p probability and t transfer, the supplier may find it more beneficial to look for a receiver with a payoff 5. A rational player will not accept a value lower than that a no-contract situations promises.

4.6. Reputation

The company's reputation is tightly related to its behavior in the previous years, or – using Game Theory jargon – with its early tournament [18]. The used strategies help in drawing conclusions regarding company's future behavior. If the past behavior shows a pattern of making *threats* and not fulfilling them, the reputation is unreliable and the success of such strategy is limited. Moreover, putting only private interests upfront in cases of strategy divergence of the first kind will result in co-players anticipating that in the future the company will also place personal interest over global one. Generally, to earn a good reputation, a company should employ a policy of maximizing the common wealth, as well as consistently fulfilling the declarations. One exception is when the company is under no risk of bearing the consequences of not complying with the declarations. In practice, this often happens when the company is about to declare bankruptcy. For that reason it is worth to observe the contractors' financial situation, to ensure that they are not tempted to abandon their trustworthiness.

$\Gamma_{GI_{zn}}$	X	Y
A	$4,3$	$0,0$
B	$0,0$	$2,1$
C	$1,4$	$0,0$

FIGURE 4.19. Two-stage game

Theoretically, the usefulness of a good reputation can be proven on a game played twice [17, 43]. For example, let us analyze the game presented in 4.19.

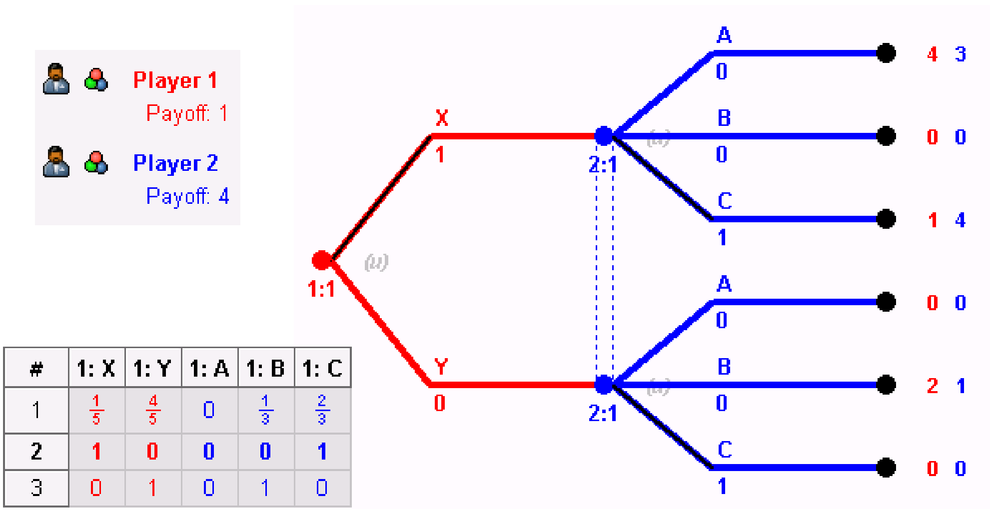


FIGURE 4.20. Extensive form of one step in the two step game

The calculations for 4.20 prove the game has two Nash equilibria in clean strategies X, C, and Y, B and one in mixed strategy $\frac{1}{5}X\frac{4}{5}Y, \frac{1}{3}B\frac{2}{3}C$. The chart in 4.21 shows all the possible outcomes for players taking two turns. The results from have been summed up. The values marked in green represent the possible results when using every available clean optimal strategies combination. P1 can offer P2 a deal, where in round 1 they choose a strategy profile X, A. If P2 agrees, then they will select X, C in round 2. This will yield a favorable result $(5, 7)$ (in red). If P2 fails to fulfill the promise and selects C in round 1, they will be 'punished' by P2 selecting Y in round 2. This will cause the strategy profile to be Y, B and the total outcome $(3, 5)$ – a ineffective result for both players.

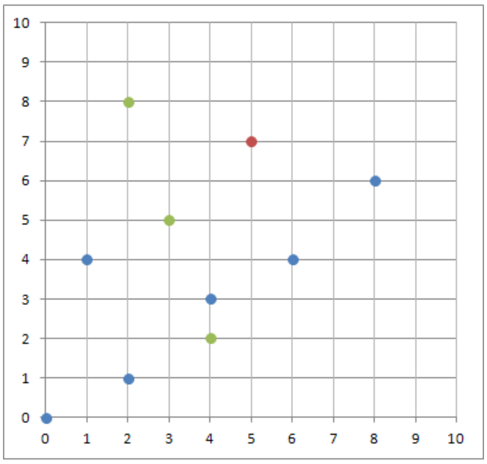


FIGURE 4.21. Payoff of the two-stage game

As has been mentioned, the common issue in building a reputation is that of Prisoner's Dilemma – type of strategy discrepancy 4.22. This creates a situation where player may choose an instant benefit over a long-term one. One of the prevention methods is building a reputation, at the same time being aware of the possible punishment. This simple, yet effective method is called tit for tat. To portray such a situation, we will use an indefinitely repeated game sample. At first glance, the situation seems unnatural, as nothing is indefinite, but this helps keep the sample simple and let's one focus on the normal phase (while removing the anomalies, as in the start and the end of the game) [4].

Γ_{PD}	$Cooperate$	$Defect$
$Cooperate$	R, R	T, S
$Defect$	S, T	U, U

FIGURE 4.22. The generalized Prisoner's Dilemma

In such a game two profiles can be distinguished: *corporate* profile and *punishment* profile. Player, whose best interest lays in cooperation and long-term benefits maximization, should choose *corporate* profile each time, which is not Nash equilibrium. If one of the players does otherwise, they may be sure the opponent will select *punishment* profile in the following round and will continue to do so until the 'traitor' starts using the corporate profile. We shall introduce *future payoff discounting factors* δ [1], in lieu with the rule that money owned in the present is worth more than money that will be owned in the future. This will help make the representation more real. The player that abides by the rules will receive a payoff:

$$player_{cooperative} = R + R\delta + R\delta^2 + R\delta^3 + \ldots = \frac{R}{1-\delta}$$

The player that decides to forgo the rules, and will continue to do so in future rounds, will get a payoff:

$$player_{not\_ooperative} = T+U\delta+U\delta^2+U\delta^3+\ldots = T+\delta(U+U\delta+U\delta^2+\ldots) = T+\delta\frac{U}{1-\delta}$$

Hence, the players will find it more beneficial to cooperate when payoff discounting factors meet the following:

$$\delta \geq \frac{R-T}{U-T}$$

For example, with the given values $R = 2, T = 3, U = 1$, it will be more beneficial for the players to cooperate when the payoff value in every round is not lower than: $\delta \geq \frac{1}{2}$, for $R = 4, T = 6, U = 0$ would be $\delta \geq \frac{1}{3}$.
The analysis of infinitely repeated games leads to a conclusion that with a higher payoff discounting factors ('patient' players) are more willing to build a reputation. Impatient players may try to cheat, which proves that making high bids with new, reputation-less partners is risky. One novelty is that by determining the player patience and their will to maintain a good reputation, we need not know the history

of behavioral patterns. While these factors are hard to determine (words), such behavioral history speaks for itself (actions).

Chapter 5

The confrontation of Game Theory apparatus with the small business milieu

To test the validity and uniformity of presuppositions made in earlier parts of the paper, these assumptions have been confronted with the representatives of this environment.
The interviewed entrepreneurs are owners of companies that manufacture, sell and offer their services with respect to packaging and related trade. They have over 10 years of experience in running a small company in Poland. During that period they evolved, changing their nature and headquarters. They merged with other companies, or divided into branches. They also declared bankruptcy. In the following sections reflections and suggestions regarding the practical aspects of GT, described in 4.

5.1. Competition

When competing against or cooperating with big providers, one must thoroughly analyze the nature of their position and behavior on the market. Big companies have proportionally big productions capabilities, so the production must reflect the company's scale to be profitable. With orders of a lower volume, the smaller suppliers will potentially be of interest to the company or to the buyers directly. Cultural differences also may give the market a boost. One such example is the Italian work culture. For an average Italian, working is a necessity to sustain themselves, not their purpose. This results in periodical idle times, which, in return, is a chance for the local entrepreneur to earn.
In chapter 4 section 4.1 it has been shown that competition is most beneficial for the end customer, which means it is mostly desirable form the social point of view. In practice, this may lead to seemingly unnoticeable quality degradation. For example, the producers offer a stock in an identical price, but they give the price without tax. The sticky tape is produced in shorter rolls and the metrics is changes from

meter to yard[1]. That tape, which was originally sold in 66 meter rolls, was later 66 yards, 60 meters, 60 yards, etc. Such practices result in an increased indirect costs (such as the cost of packaging), which, in the end, is incurred by the client.
Another entrepreneur issue is the degradation of the environment through aggressive competition (price wars). A newcomer company, more often than not established by former employees of a competing company, starts selling the stock at very attractive prices. After some time, the company is forced to declare bankruptcy and leaves a ‘spoilt market’ – the veterans find it hard to explain to the customer they will not be able to sell the stock at a reduced price.

5.2. The management and the handling of data

Every company is obliged to keep an archive of documents on previous activities. The company is not only legally bound to do so (to store the invoices and other documents) – it is also in their best interest, as this will help in selecting the best possible strategic solutions. The archive should be: safely stored (to prevent loss and theft), structured, easy to manage and presented in bulks (tables, charts, etc.). Such accessible data will help the company to look back at past solutions and draw conclusions in order to decide optimally.
Nowadays, running a company without internal data managing software seems unrealistic. The market offers numerous types of software dedicated to small and medium-sized companies. It is important that such software be customized and managed by both internal workers, as well as external providers. In a exemplary sales company it is viral to build a good rapport with the clients, as well as incorporating a sales, accountancy and HR-managing software. Such aid will be provided by CRM (Customer Relationship Management) program, which helps to not only in methodical building consumer – company relationship, but also to improve customer life cycle.
One important aspect it the signals a company sends to the surrounding environment. The company’s image, such as logo, business cars, the image of the representatives, the structure of an invoice and other documents, are examples of such *signaling*. An interesting instance is the company’s WWW site – both the layout and the content. In extreme cases, the virtual image of the company in no way reflects the real image. Moreover, this practice is often used and it significantly reduces the costs when launching a business. Equipped with that knowledge, the customer should take the online image the company creates with a pinch of salt.

[1]Yard to anglosaska jednostka długości, której długość wynosi 0,9144 metra – yard międzynarodowy.

The financial situation is also a crucial element of *signalizing*, as it creates a credible position in the eyes of suppliers, banks (when applying for a loan), etc. Frequently this is achieved via creative bookkeeping, which helps create desirable financial indicators.

5.3. Motivational system

A separate pay standards are applied to workers who do not directly bring revenue, but are indispensible (administration, accountancy, shipping, etc.). In their case, the most popular solution is not to offer incentives, but rather reduce costs. This is the case when two companies merge to form a bigger unit, and lessen the administrative costs by staff reductions.
The situation of workers who directly bring income is radically different. The company resorts to a mixed pay system. Each marketing employee (the seller, the driver, for instance) receives the base pay, which is sort of a guarantee of the financial minimum the employee can expect. This base, however, cannot be too high as these workers' pay should reflect the sales results. For that two mechanisms are utilized. The first mechanism is the commission calculated by taking the profit margin the customer pays. The company sets a progressive pricing. The other mechanism is to set targets. When these are exceeded in a billing month, the income percentage for that months closed sales is increased accordingly.
Its purpose, although not a sole one, is to fish out those employees who show no marketing potential. The main job of the second mechanism, however, is to serve as a deterrent to those who lack marketing skills. As has been shown in 5.2 the IT system will help in monitoring the seller's results on daily basis.
One drawback of the target mechanism is a premeditated practice of closing sales on the following month, once the current's month target has been reached. This has actually happened in the companies that took part in this interview.
The company made the target system more elastic by estimating the target on the basis of the previous season's results and anticipating the increase or decrease of closed sales. all this was done as preventive measures.
The punishment system is sporadically used and does not include the reduction on the pay (since they are guaranteed by the contract). The only punishment is to give two reprimands, and if that does not help – firing the worker.

5.4. Negotiations

When a local company wants to get a company renowned nationally or globally as a customer, it must realize that their negotiating force is very low. Big companies receive numerous offers of that sort, and may freely accept or reject them, which causes a stalling situation for small businesses. This means that the small company manufactures and sells a lot of stock at minimal profit (or no profit at all) to keep the sales and (potentially) get discounts from suppliers. The limited budget of the company usually prevents such solutions, and it is better to have smaller clients, but with a fair share of income. It is also a safer situation, as the risk of all companies not being able to pay or will postpone the payment is much lower than in case of a single, but big contractor.

By the end of the months, the sales people wanting to reach the set target offer a much lower price to companies, who often pretend not to need the stock. It is quite probable that should they withhold their impatience, the same stock would have been sold for a higher price.

A radically different situation occurs, when a random client needs the stock in short deadline. The company is in a position to earn more than usually, sometimes at the cost of 'offending' the customer (if they, for example, know the old pricing, which usually the companies explain as the increase of the material price – hiding the information).

An even more dramatic case is that of a typical player negotiating with a macro player. A solution to this could be forming an organization, such as the Polish Confederation of Private Entrepreneurs 'Lewiatan' (pol. Polska Konfederacja Pracodawców Prywatnych „Lewiatan"). However, a subjective opinion on that matter of one of the respondents was that the small business market is very disorganized.

5.5. Contracts

The majority of entrepreneurs admit the Polish court to be slow and incapable. Court vindication is often time consuming.

A possible example scenario: a client is starting to pay irregularly. Step 1 includes sending a reminder with at least 14-day waiting period. Step 2 includes sending another reminder, this time with at least a month-ahead deadline. Step 3 is sending the pre-court prompt with at least a 14-day notice. If is the holiday season, one must take that into consideration, as most court workers will be off and the court itself will not be 100% functioning. When the debtor receives such a notice, the first reaction is to declare in writing that they will settle the debt. A few months pass and the court decides to call for action. Another step may include appealing to second instance court by the defendant. The ‘procedure; can take up to a year, during which time the complainant must pay PIT or CIT, as well as VAT[1]. When finally we have the entire legal basis for vindication, the indebted company may no longer exist, or the complainant may have not enough of a budget to finance this. Moreover, the debtor may actually threaten to declare bankruptcy should the complainant decide on a court case. This all is aimed at slowing down the proceedings. It is up to the company to decide whether to take these threats as legitimate.

The arbitral court (amicable settlement) is a hope for a cheaper and faster solution. Nothing, however, surpasses simply avoiding a situation when the debt has to be settled in court. A few tips to prevent that are: not giving an unverified customer credit, not fulfilling contracts that exceed our financial capabilities, and not taking further orders for overdue clients. We may also monitor our contractors’ financial situation, so as to react quickly and withhold further orders. As always happens in life, this sixth sense for marketing comes with experience. It is vital that our actions do not result in bankruptcy. Ironically, the sooner an entrepreneur encounters such collection difficulties, the better will be his results in the future, as he will have experience to draw from.

5.6. Reputation

The company’s reputation proves most useful when reaching out to potential regular customers. If we meet the supplier’s reliability criteria, the chances of getting promotional offers and discounts increase. A one-time client will, on the other hand, get the less attractive price, as has been discussed in 5.4.

The company’s reputation is tightly connected with its financial condition. As was explained in 5.5, a company with a history of debts is potentially threatening the financial stability. In Poland, this may not necessarily be the case. In an overwhelming amount of cases this turns out to be the cheapest way of financing the company. What is more, the settling of the debt may actually helps improve the supplier-ordered relations.

[1]One light at the end of the tunnel may be the ‘bad taxes’ bill which forced the defendant to regulate these taxes. In case of defendants win, they can ask for the reimbursement after one year.

Chapter 6
Summary

Running a small business can be compared with playing a complex, multiplayer game. Whether the company will survive and thrive financially will depend on the strategy that is chosen. Our behavior will also influence the relations with the partners, workers, clients, suppliers, local community and other (directly or indirectly influenced) players. To participate is such a game is to realize the complexity of the game. It is also to accept that the game cannot be stopped and re-set as the players wish. Each game, whether well-played or disastrous, will lead to a change of state, whether we like it, or not. Often the decision or partners and co-workers make prove unexpected. It is important to store the data in a manageable database, where it may be used to plan strategies. In case of a failure, one should not give but, but rather draw conclusions and minimize the risk of re-occurrence. The participation can be a beneficial one. Those, who stay on the market, succeed. One must remember about the possibility of a loss, approach the ideas carefully and humbly, at the same time searching for the optimal strategy. This helps not only to grant financial success, but also a social respect, the feeling of self-fulfillment, and the emotions that the game brings.

In this paper it has been shown how Game Theory helps in reaching set goals and how to behave in a world with infinite number of strategies. It does not, however, explain every aspect. What is more, is shows that the complex reality prevents finding the one, optimal solution to fit all. What it attempts to do is to explain in a clear and coherent way what occurs in the small business environment. It helps estimate, when to go with the flow, hide information or send a signal. It tries to show why, in certain situations, people behave unexpectedly. It also prepares one for negotiations, managing risks and other aspects, which could not have been described in the paper due to space limitations.

An interesting side note is that the thesis has been written with the help of GT mechanisms, such as foreseeing and looking back. At the beginning of the writing process, a general subject has emerged. This, naturally, is put down in form of an introduction (which may cause some problems due to lack of conclusions at this point). Next, comes a theoretical chapter. The question is: which of the complex aspects of GT should be focused on? Which pieces of information should be extensively described, and which omitted and deemed less relevant? The next step is to consider the theory and find its practical use- this largely based on assumptions, as the interviews and observations come later. One is then considering postponing the writing of this chapter until they have more data. Going further, the presented software should be mastered by the author to help predict business situations. These would, too, have to be confronted with the reality. Only when all the factors have been met, should a concept the final confrontation of the mathematical apparatus (shaped as GT) with the experience of the interviewees be done. The experience and the conclusions helped focus on the vital elements and by using backward analysis- write the following chapters of the thesis. The presented outcome, hence, is consistent with the first claims made in the introduction.

Appendix A
The rules of games described in the thesis

A.1. Dollar auction

- multi-player, non-zero sum game,
- auction game, where the dollar is the subject of the auction,
- the auction is won by a person making the highest bid,
- each person must pay the declared amount, even if they didn't win it.

A.2. Chicken

- two-player, non-zero sum game,
- two players drive a speeding car in the other player's direction,
- the driver can drive forward or take a turn,
- if both players take a turn, nothing happens (a tie),
- if one turns, he loses (chicken), while the other wins (daredevil),
- if neither turns, there's a crash and both suffer damages.

A.3. Prisoner's Dilemma

- two-player, non-zero sum game,
- two players (prisoners) are suspected of committing a crime,
- the prisoners cannot communicate,
- each can declare their guilt or remain silent,
- if they both keep silent, they get the minimal penalty (6 months),

- if one keeps silent, he will get 10 years, while the partner will walk free,
- if they both testify, they will get up to 5 years each (1/2 of the maximal punishment).

A.4. Coordination

- two-player game,
- two friends get lost while seeing sights in a town with two characteristic points,
- the players can communicate,
- each must reach an individual decision on where to meet,
- both will get a payoff only when they make an identical decision.

A.5. Pareto-coordination

- two-player game,
- the market welcomes two new standards,
- two companies must choose one of these,
- both companies gain only if their choice is identical.

A.6. Matching Pennies

- two-player game,
- each player chooses a side of the coin in secrecy,
- it is determined whether both players choose one or two sides,
- the players revel their choice.

A.7. Battle of the sexes

- two-player game,
- one player decides to go to the theater,
- the partner wants to go to the movies,
- if one of the partners compromises, the other gets the highest payoff,
- they may argue and decide to stay home, thus both receiving the lowest payoff.

Appendix B
Content of the DVD-ROM

The enclosed DVD contains:

- digital version of the thesis adapted to print one-sided (file – *zie_MSc-one-sided.pdf*),
- digital version of the thesis adapted to print double-sided (file – *zie_MSc-two-sided.pdf*),
- source code of master's thesis in LaTeXwith attachments,
- digital version of the thesis to the test in the PLAGIAT.PL (file – *zie_MSc-two-sided.doc*),
- full version of thesis presentation (file – *MSc-presentation.pdf*),
- trimmed version of thesis presentation (file – *MSc-presentation-trimed.pdf*),
- source code of presentation in LaTeXwith attachments.

Description of the disc contains:

- thesis topic,
- the name of the promoter.

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