Linear Regression - MSIB AiCI

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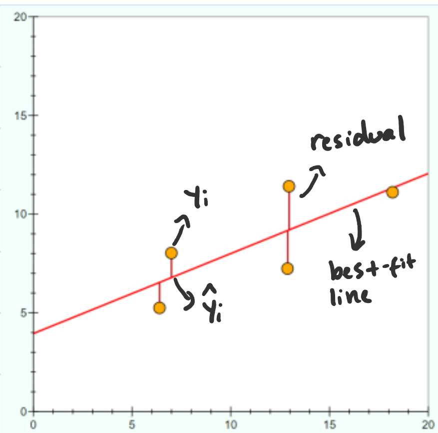
Finding m and c equation in least squared method to get the best fit line

$$y = \underline{m}x + \underline{c} \;\longrightarrow\; m = \sum_{i=1}^{n} \frac{(x_i - \bar{x})(y_i - \bar{y})}{(x_i - \bar{x})^2}$$

$$\longrightarrow\; c = \bar{y} - m\bar{x}$$



$\rightarrow y = mx + c$

$$\text{residual} = y_i - \hat{y}_i \longrightarrow \text{sum of squared residuals}$$

$$= \sum (y_i - \hat{y}_i)^2$$

$$= \underbrace{\sum (y_i - (c + mx_i))^2}_{\text{minimize!}}$$

Steps to get the equation for m and c:

1. Find the partial derivatives of sum of squared equation with respect to m and c
2. Set the partial derivatives equal to 0
3. Solve for m and c

1. Find the equation
    a. c
        i. Partial derivative

$$\frac{\partial}{\partial c} \sum (y_i - (c + mx_i))^2 = \sum \frac{\partial}{\partial c} (y_i - (c + mx_i))^2$$

$$= \sum 2 (y_i - (c + mx_i))^2 (-1)$$

$$= -2 \sum (y_i - (c + mx_i))$$

        ii. Set to 0

$$-2 \sum (y_i - (c + mx_i)) = 0$$

$$\sum (y_i - (c + mx_i)) = 0$$

        iii. Solve for c

$$\sum (y_i - (c + mx_i)) = 0$$

$$\sum y_i - \sum c - \sum m x_i = 0$$

sumasi seluruh data : 1 - n atau $\sum_{i=1}^{n}$

$$\sum c = \sum y_i - \sum m x_i$$

$$nc = \sum y_i - m \sum x_i \rightarrow c = \frac{\sum y_i}{n} - m \frac{\sum x_i}{n}$$

$$c = \bar{y} - m\bar{x} \quad //$$

a. m

i. Partial derivative

$$\frac{\partial}{\partial m} \sum (y_i - (c + m x_i))^2 = \frac{\partial}{\partial m} \sum \left(y_i - (m x_i + (\bar{y} - m\bar{x}))\right)^2$$

$$= \frac{\partial}{\partial m} \sum \left(y_i - (\bar{y} + m(x_i - \bar{x}))\right)^2$$

$$= \frac{\partial}{\partial m} \sum \left(y_i - \bar{y} - m(x_i - \bar{x})\right)^2$$

$$= 2 \sum (y_i - \bar{y} - m(x_i - \bar{x})) \cdot -(x_i - \bar{x})$$

$$= -2 \sum (y_i - \bar{y} - m(x_i - \bar{x})) \cdot (x_i - \bar{x})$$

i. Set to 0

$$-2 \sum (y_i - \bar{y} - m(x_i - \bar{x}))(x_i - \bar{x}) = 0$$

$$\sum (y_i - \bar{y} - m(x_i - \bar{x}))(x_i - \bar{x}) = 0$$

i. Solve for m

$$\sum (y_i - \bar{y} - m(x_i - \bar{x}))(x_i - \bar{x}) = 0$$

$$\sum y_i (x_i - \bar{x}) - \bar{y}(x_i - \bar{x}) - m(x_i - \bar{x})^2 = 0$$

$$\sum y_i (x_i - \bar{x}) - \bar{y}(x_i - \bar{x}) = \sum m (x_i - \bar{x})^2$$

$$\sum (y_i - \bar{y})(x_i - \bar{x}) = m \sum (x_i - \bar{x})^2$$

$$\boxed{m = \sum \frac{(y_i - \bar{y})(x_i - \bar{x})}{(x_i - \bar{x})^2}}$$

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