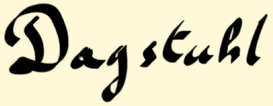
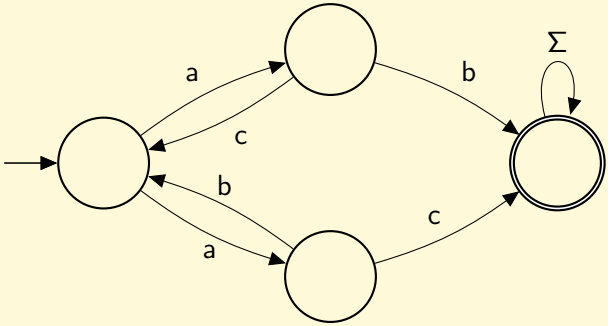
Strategy shapes for population games

Ongoing work with Hugo Gimbert and Patrick Totzke



Population games

$\mathcal{A}$ an NFA



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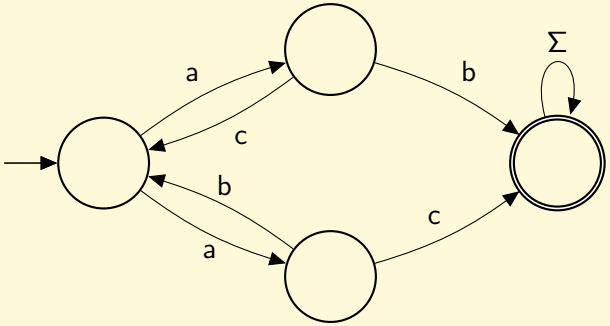

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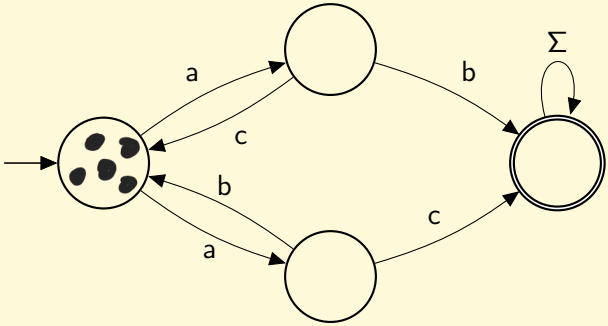

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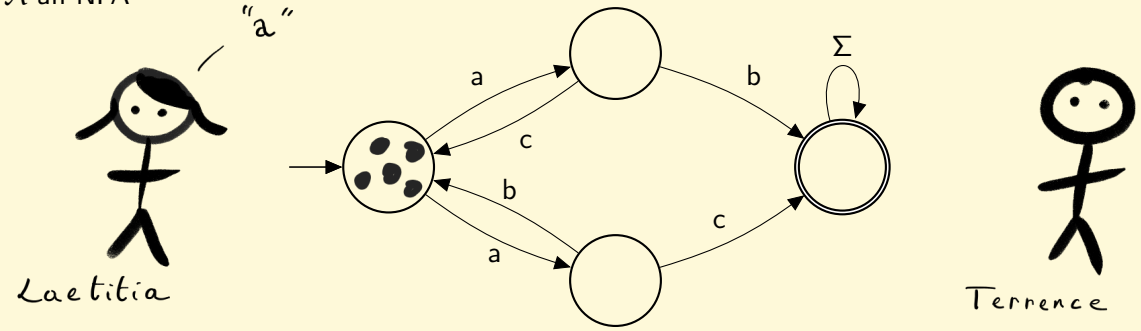

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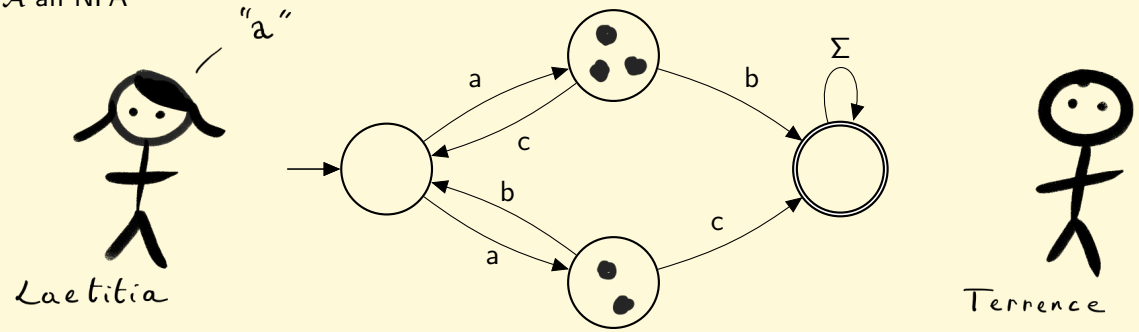

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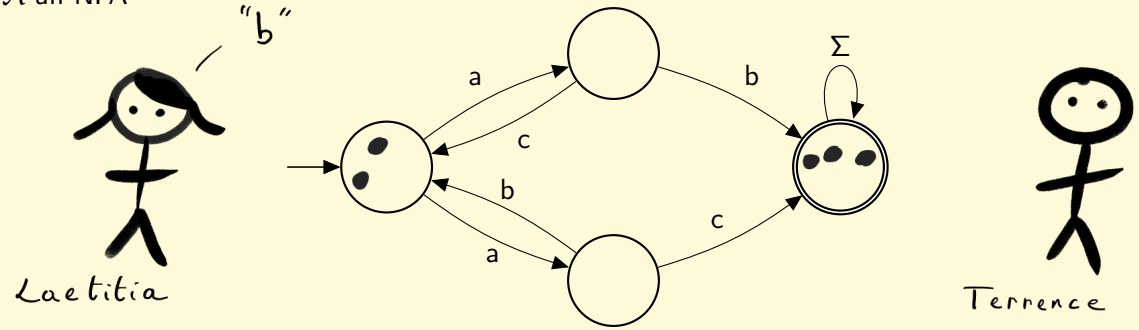

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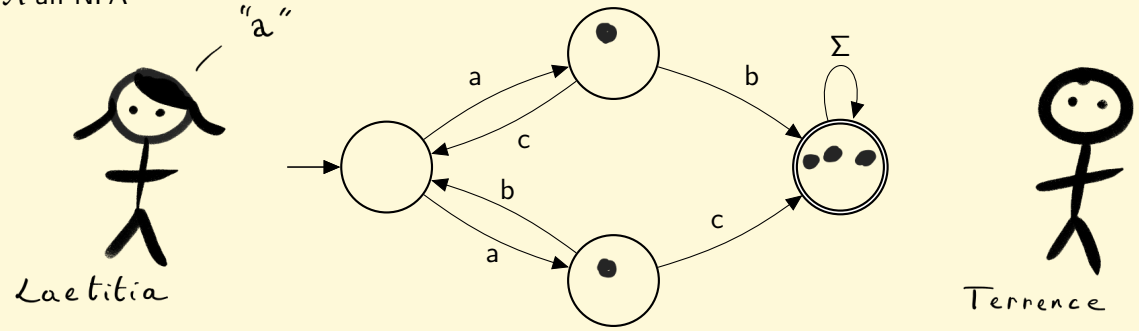

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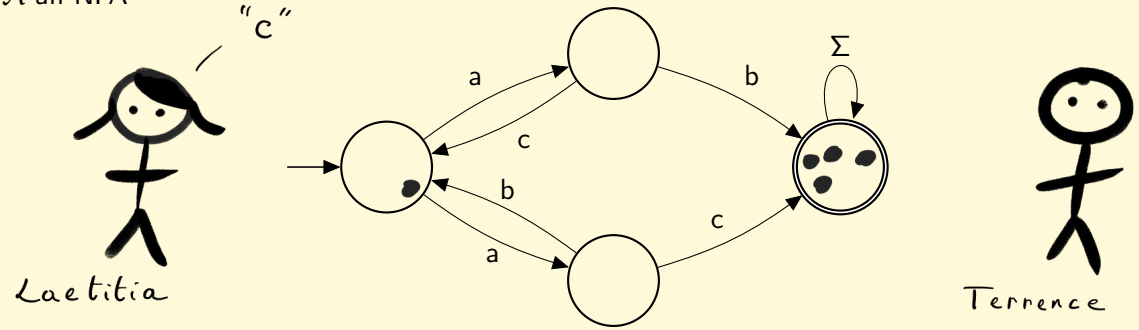

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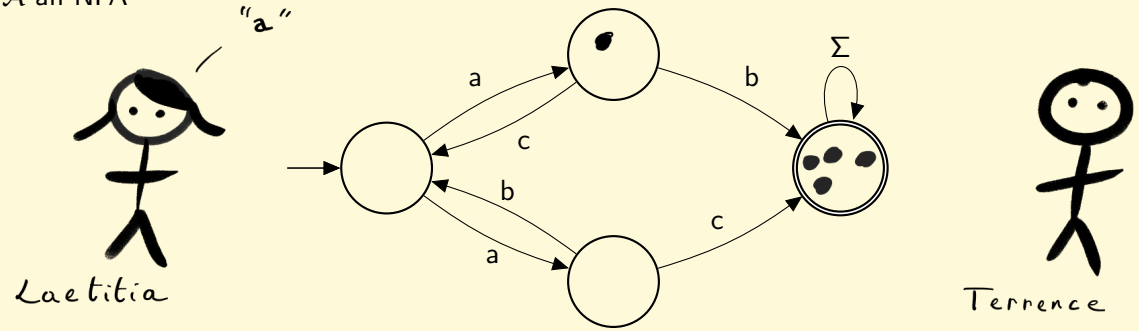

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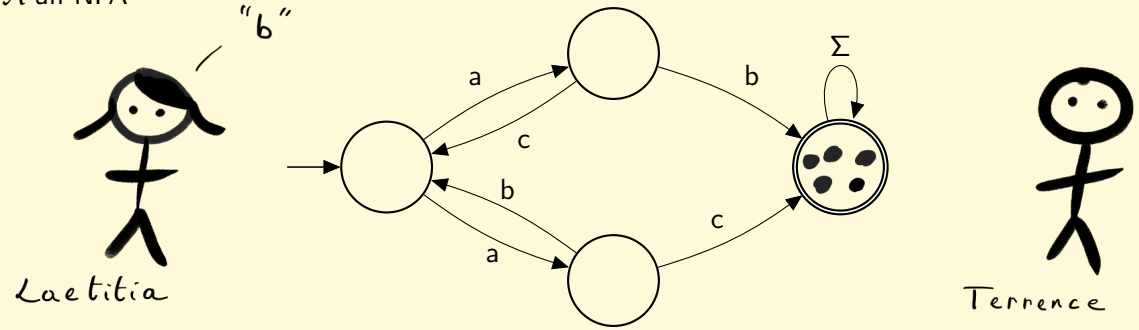

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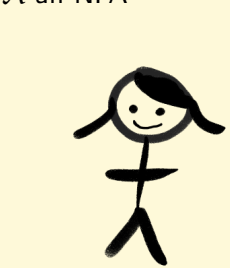

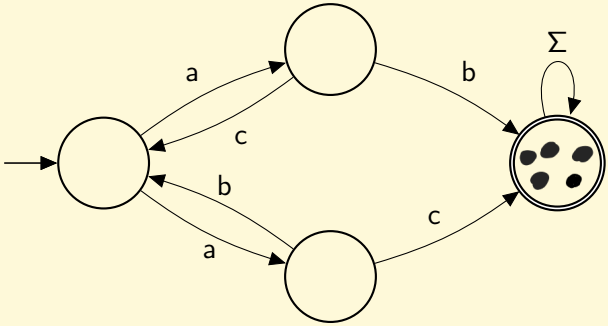

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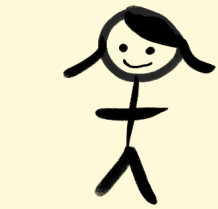

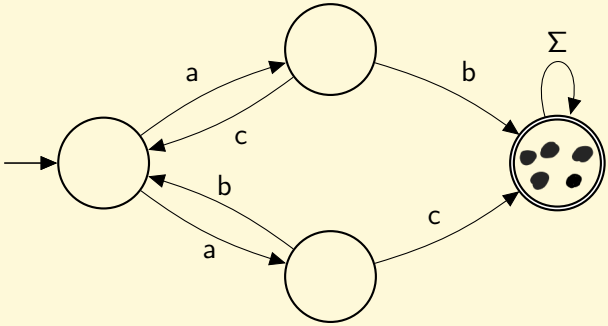

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Population control problem

I: an NFA $\mathcal{A}$

O: Can Laetitia win over $\mathcal{A}$ for all N?

[Bertrand, Dewaskar, Genest, Gimbert 2017]

The Population Control Problem is EXPTIME-complete.

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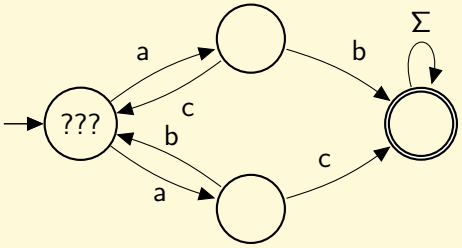
Proof:

Play a different game!



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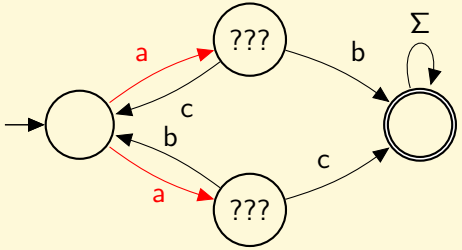

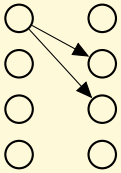

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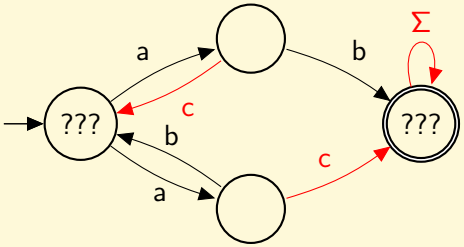

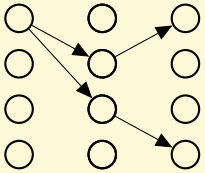

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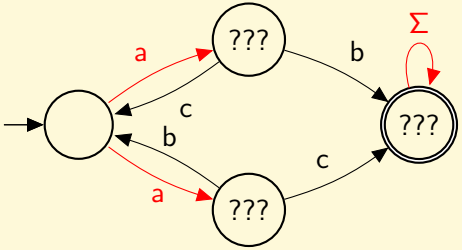

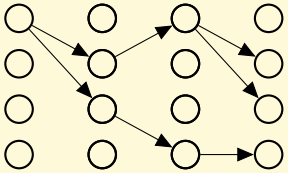

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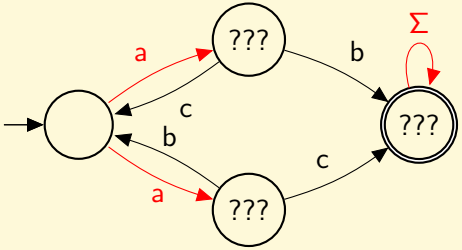

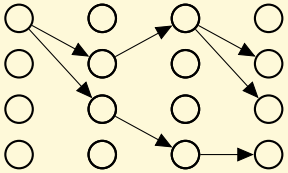

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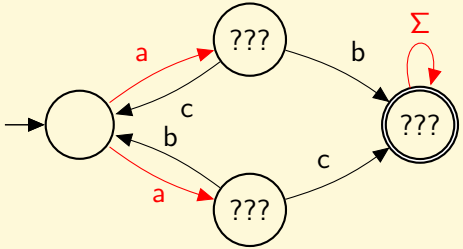

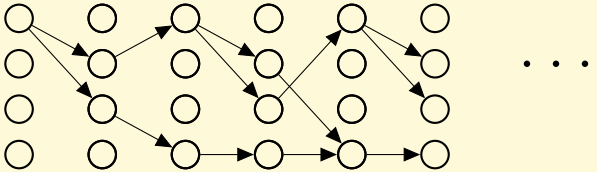

Randomised Population Control Problem

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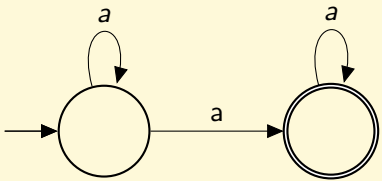

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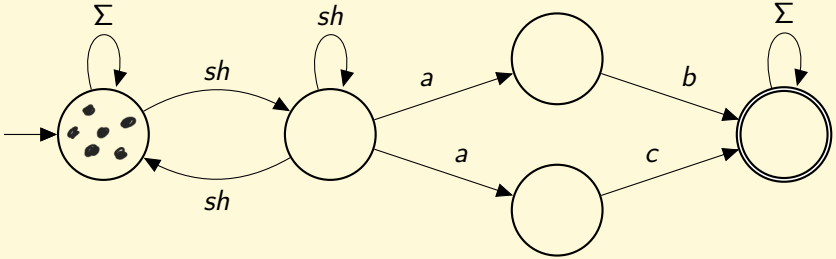

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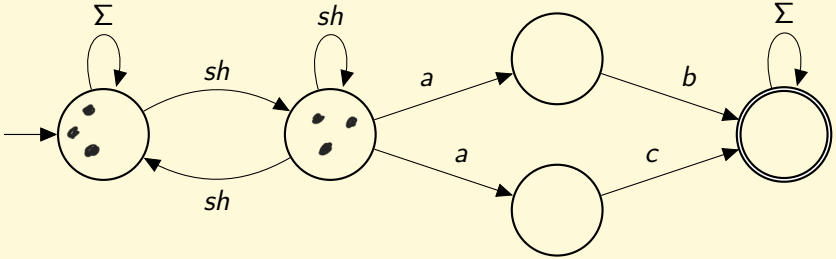

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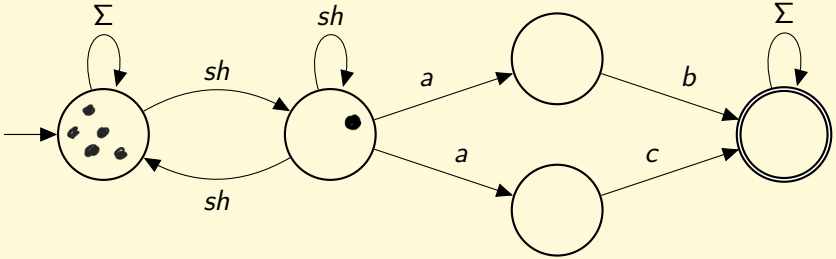

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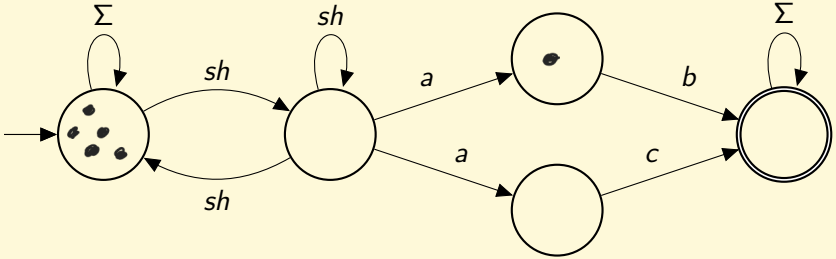

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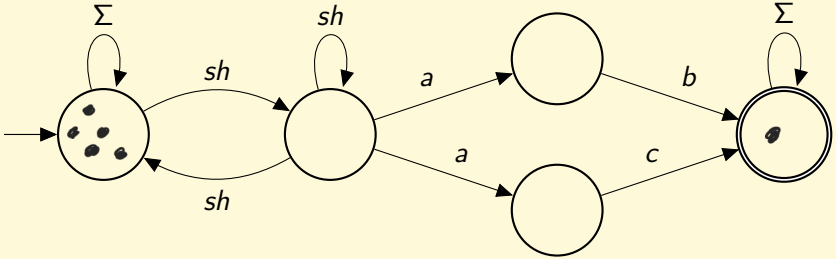

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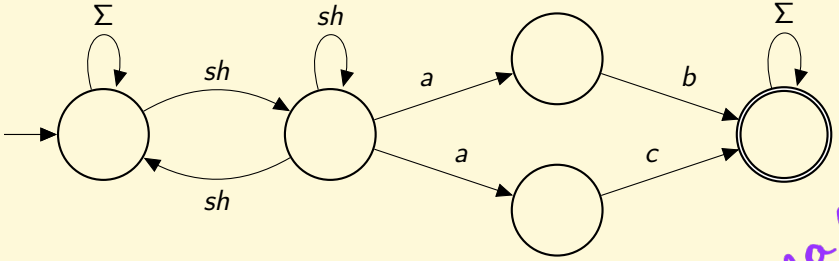

Strategies need to count!

Randomised Population Control Problem

[Colcombet, Fijalkow, Ohlmann 2020]

The Randomised Population Control Problem is decidable.

Proof: transform the game.

- Compute the winning region as a fixpoint.
- Iteration uses cost functions
- Terminates thanks to well quasi-orders ⇝ Ackermannian complexity

[M., Shirmohammadi, Totzke 2019]

The Randomised Population Control Problem is EXPTIME-hard.

Strategy for RPCP

[Gimbert, M., Totzke]

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→ We can make sure that tokens with $\neq$ colours never meet

↳ Constants are $\leq |Q|$

Open problems: Speed of victory

expected

Fact

If Laetitia wins the randomised game then she can win in exponential time

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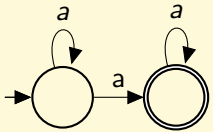

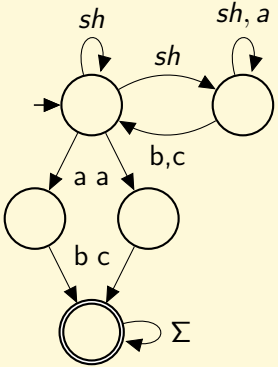

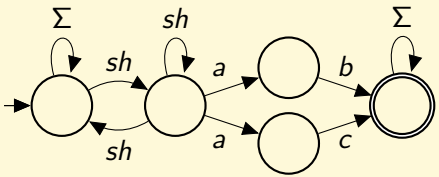

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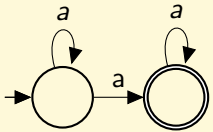
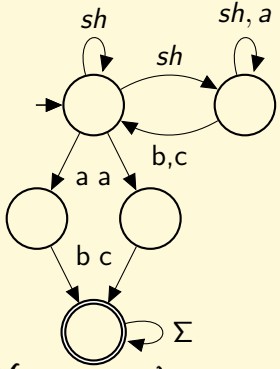


$O(\log(n))$



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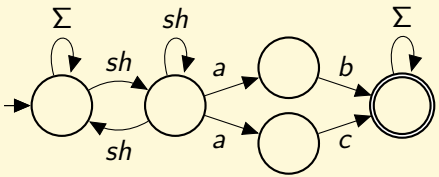

$O(n2^n)$

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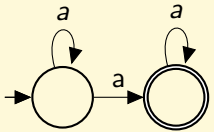
polylog



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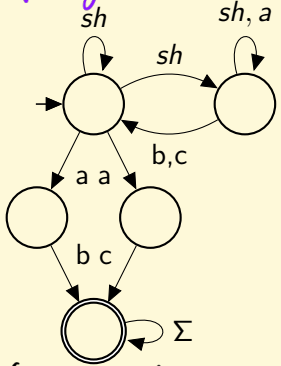
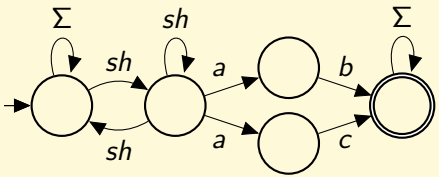


$O(n \log(n))$

exp.

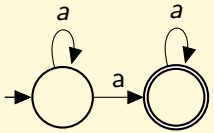


$O(n2^n)$

Open problems: Speed of victory

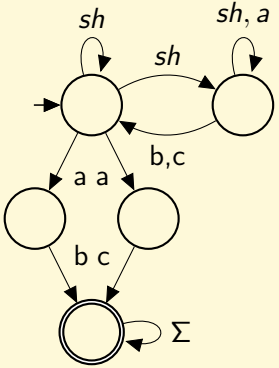
Fact

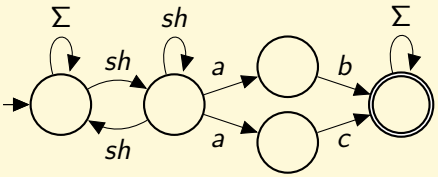
If Laetitia wins the randomised game then she can win in exponential time.



Conjecture:

Characterized by the Mankov monoid $(2^{Q \times Q}, \cdot, \#)$





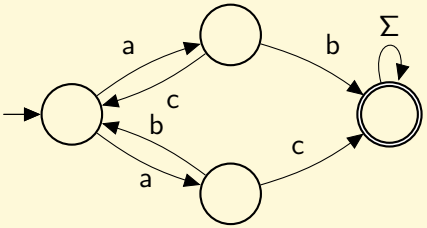
Open problems: Speed of victory

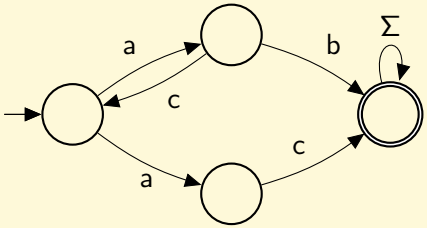
[Bertrand, Dewaskar, Genest, Gimbert, Godbole 2019]

If Laetitia wins the adversarial game then she can win in polynomial time.

What about polylogarithmic time?

in the number of tokens.





Open problems: Explorable automata [Hazard, Kuperberg 2023]

$\mathcal{A}$ a non-deterministic parity automaton

- Put N tokens on the initial state
- Laetitia picks letters, Terrence moves tokens

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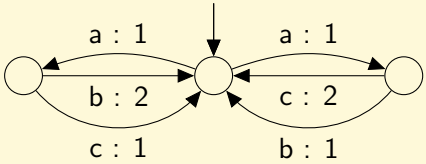

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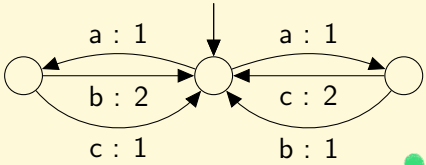

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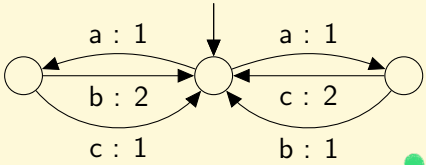

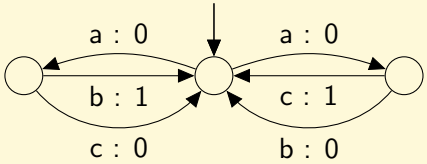

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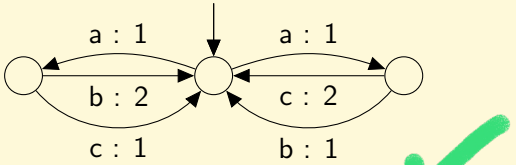

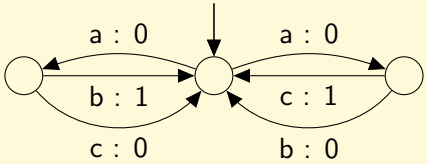

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[Hazard, Idir, Kuperberg]

The explorability problem is decidable on parity automata iff it is decidable on $[1, 3]$-automata

Future work

- Speed of victory
- Deciding (ω-)explorability for parity automata
- Threshold problems: Can Laetitia:
  - gather some proportion r of the tokens with probability 1?
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Thanks!