Verification and Synthesis of Distributed Systems with Weak Synchronisation

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- A system interacts with an environment
- A controller has to choose actions at runtime to achieve a given specification

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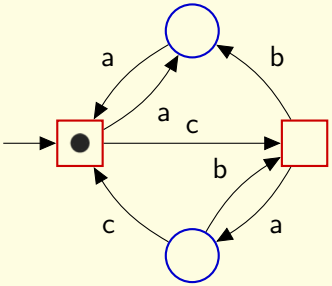

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◯ = Controller chooses the next action

□ = Environment chooses

Specification = constraint on the sequence of nodes visited

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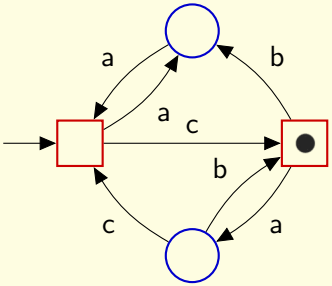

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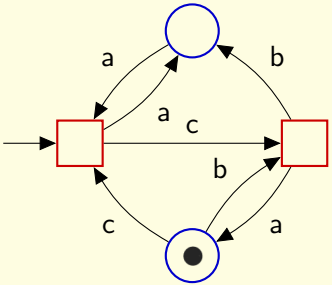

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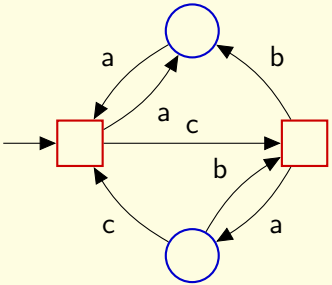

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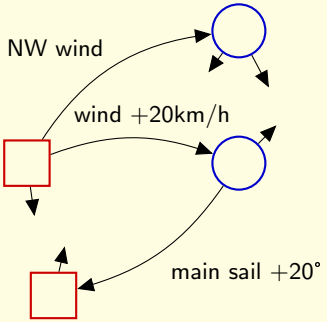

Distributed controller synthesis

- Several components interact with an environment
- Each component has a controller
- Each controller only has a partial view of the system

$\rightsquigarrow$ Multiplayer games with partial information (e.g.,[Mohalik, Walukiewicz '03])

Reduction to two-player games does not apply!

Distributed synthesis

- Fixed number of processes

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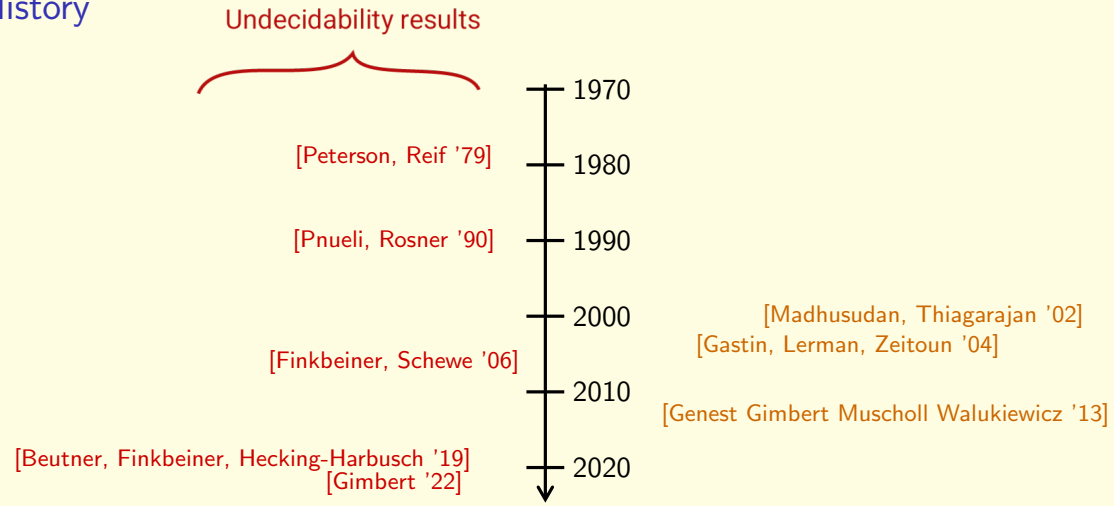
Parameterised distributed synthesis

(e.g. [Bertrand, Fournier, Sangnier '15], [Stan '17])

- Unknown number of identical processes
- Global objective
- Is there a local strategy (the same for all processes) guaranteeing the objective?

History


Undecidability results
1970
[Peterson, Reif '79]
1980
[Pnueli, Rosner '90]
1990
2000
[Madhusudan, Thiagarajan '02]
[Gastin, Lerman, Zeitoun '04]
[Finkbeiner, Schewe '06]
2010
[Genest Gimbert Muscholl Walukiewicz '13]
[Beutner, Finkbeiner, Hecking-Harbusch '19]
[Gimbert '22]
2020

What if...

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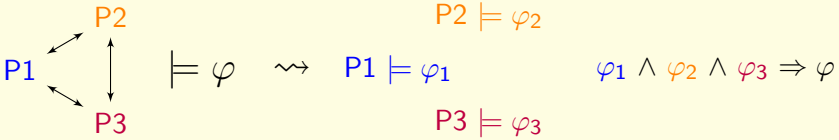
- Decompose the global specification into local objectives for each component
- Solve local control problems



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Consider **asynchronous** models of distributed systems, with weak synchronisation.

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Broadcast Networks of Register Automata

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Dynamic lock-sharing systems

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A model with fixed processes

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Let's apply it!



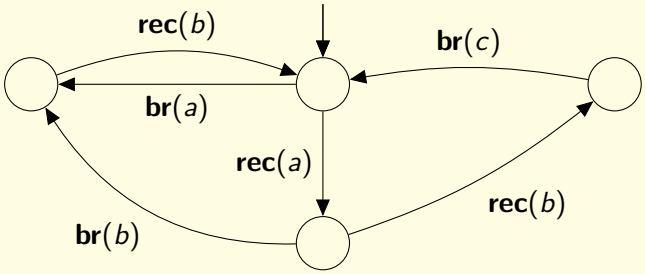

Part I.A

Broadcast Networks

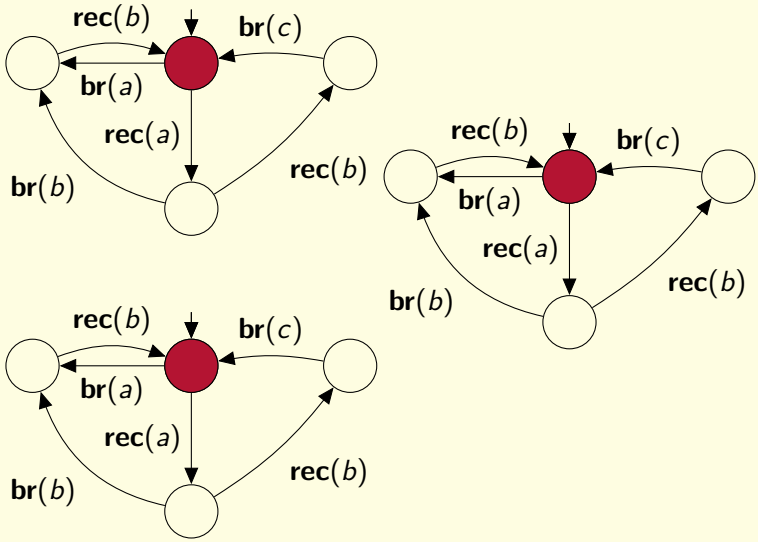




Reconfigurable Broadcast Networks [Delzanno, Sangnier, Traverso '10]



Reconfigurable Broadcast Networks



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- Lossy broadcast
- "Equivalent" to systems with a shared variable
  [Balasubramanian, Weil-Kennedy '21]

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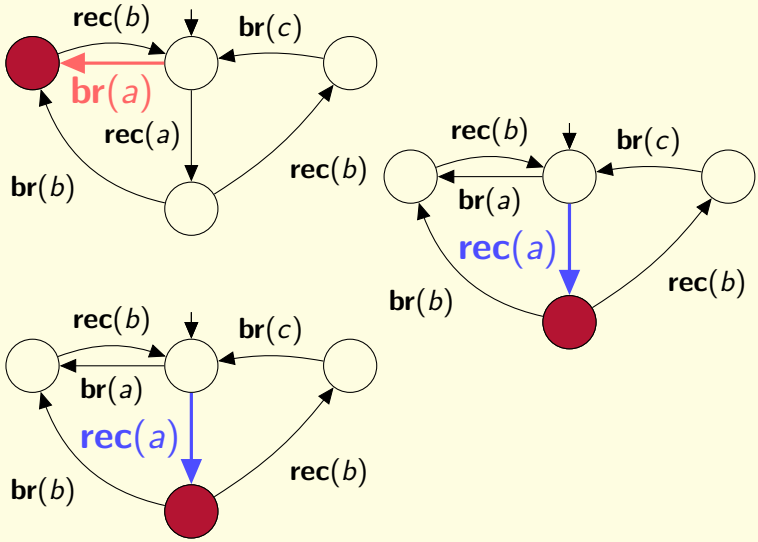

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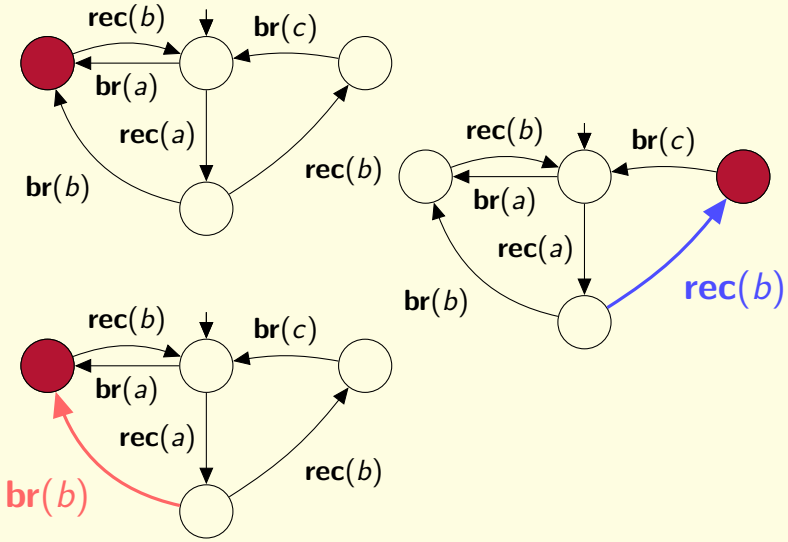

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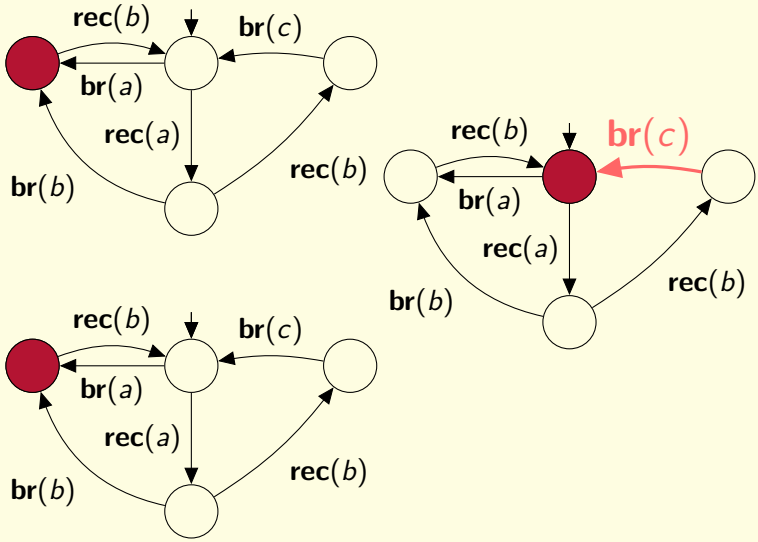

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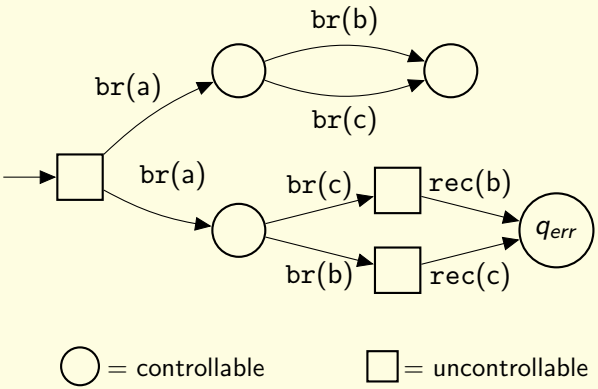

Δ = set of transitions

Strategy = function
$\sigma : \Delta^* \to \Delta$.

σ-run = run where every process chooses transitions from controllable states according to σ

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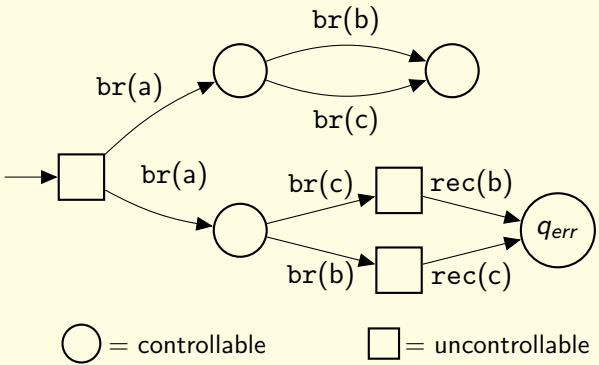

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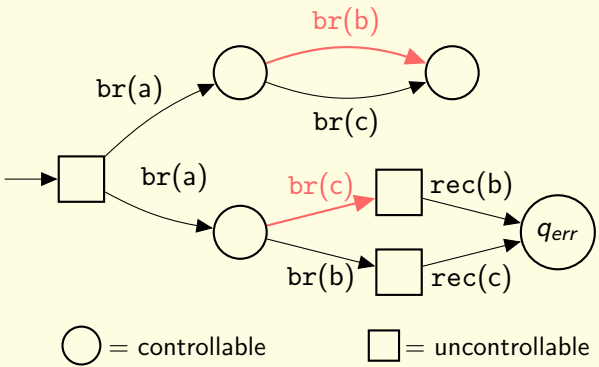

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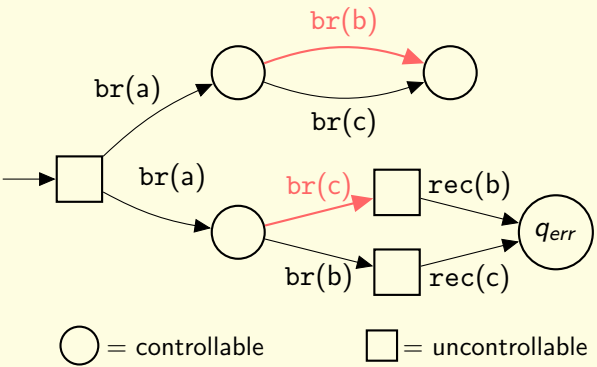

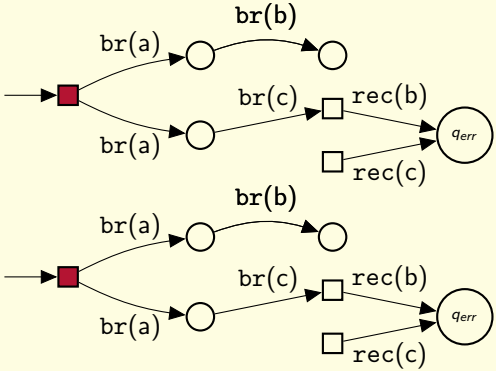

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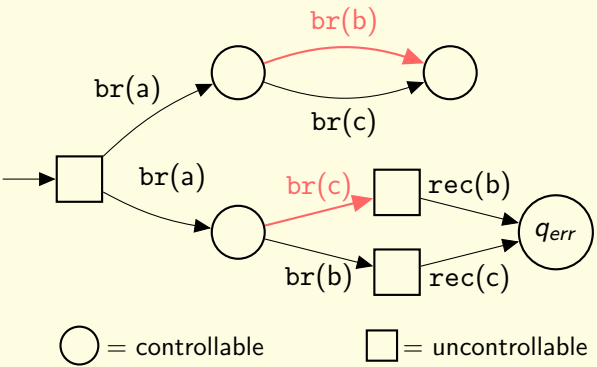

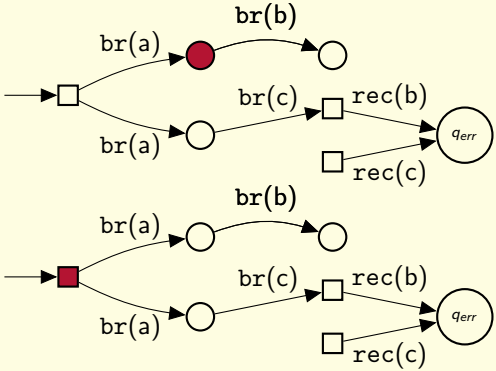

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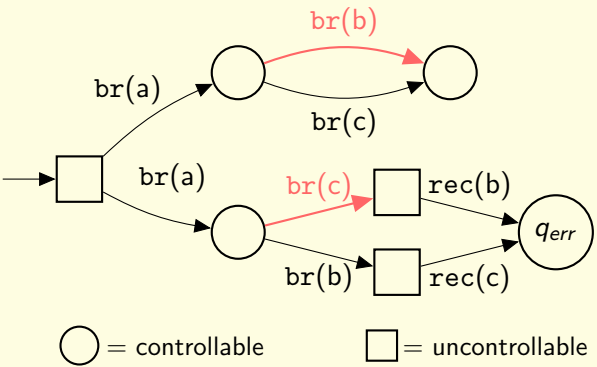

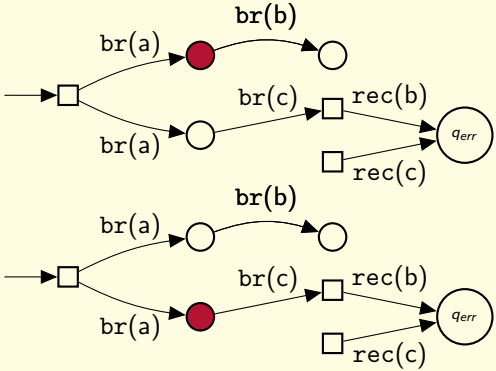

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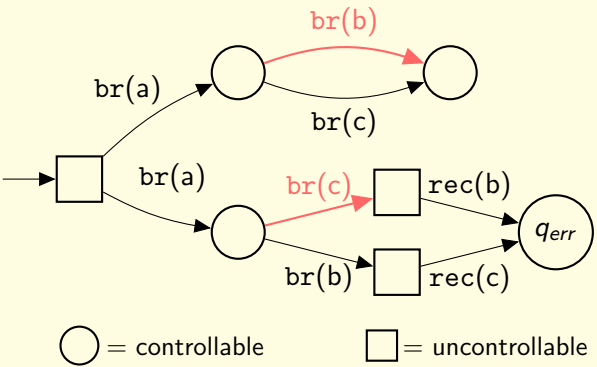

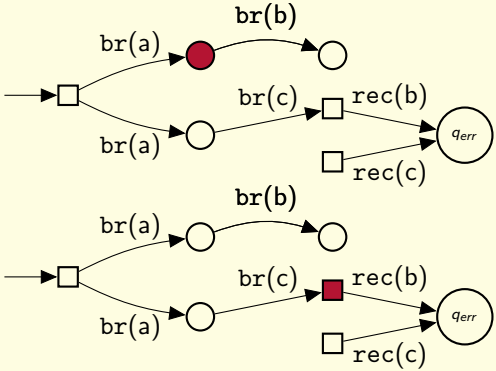

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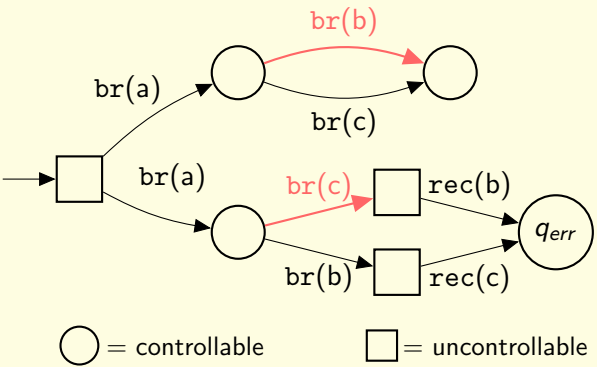

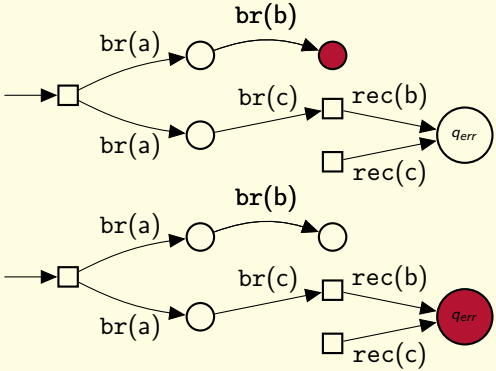

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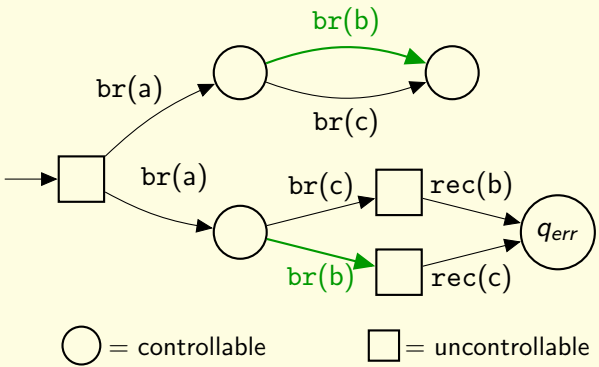

Decidability

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There is no run covering q_{err} if and only if there exists an **invariant** $\mathcal{I} \subseteq \Sigma$ *such that*

1. *Every local run that only receives messages of $\mathcal{I}$ broadcasts only messages of $\mathcal{I}$*
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  $\rightarrow$ Regular 2-player game.

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Theorem

The safe strategy synthesis problem for RBG is NP-complete.




Part I.B

Broadcast Networks with Registers




Broadcast Networks of Register Automata (BNRA)[1]

Each process has *registers* $\square_1, \ldots, \square_r$, containing data in an infinite dataset $\mathbb{D}$.

[1] Delzanno, Sangnier, Traverso, RP'13

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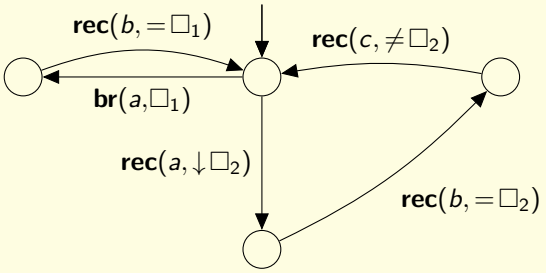
Message = letter + datum: $(m, d) \in \Sigma \times \mathbb{D}$.
A process can:



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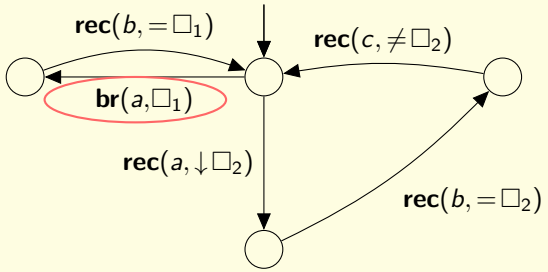

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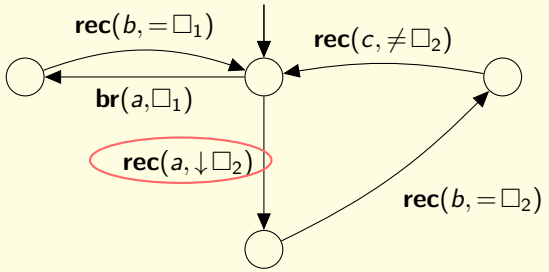

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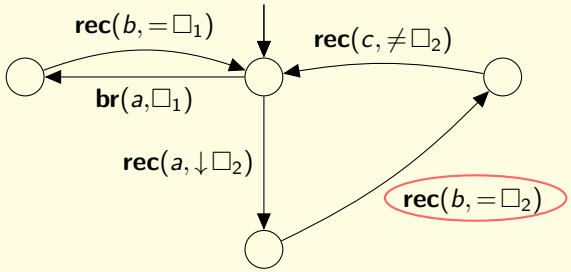

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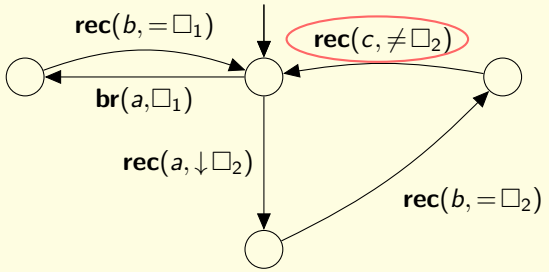

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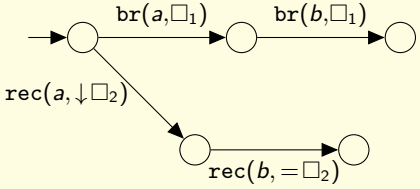
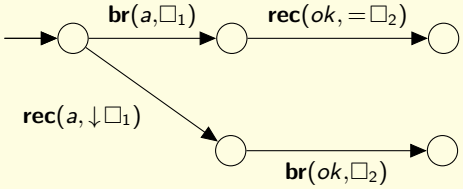
Things we can do

Check that a sequence of messages all come from the same process.



Check that a sequence of messages we sent was received.



Invariants (in a subcase)

A word u is a *subword* of v if it can be obtained by removing letters from v.
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Lemma

There is no run covering q_{err} if and only if there exists an **invariant** *$\mathcal{I} \subseteq \Sigma^*$ such that*

1. *Every local run that only receives words of $\mathcal{I}$ broadcasts only words of $\mathcal{I}$*
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Broadcast / receive $w \in \Sigma^*$ = broadcast / receive each letter of w with the same datum

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Theorem

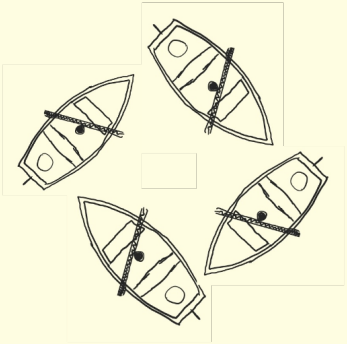
Safe strategy synthesis is decidable for broadcast networks with registers.

	0 registers	**1 registers**	**$\geq$ 2 registers**
Coverability	PTIME*	NP	$\mathbf{F}_{\omega^\omega}$
Safe strat.	NP	NEXPTIME	$\mathbf{F}_{\omega^\omega}$

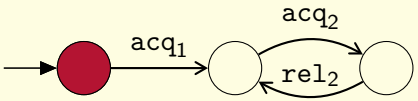
} [Guillou, M., Waldburger '23]

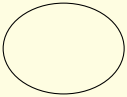
Part II

Locks



Lock-sharing systems[2]





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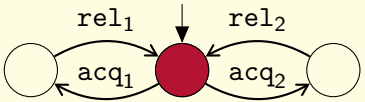

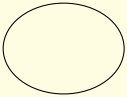

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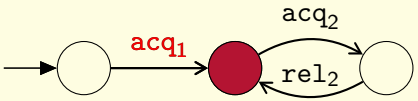

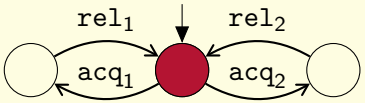

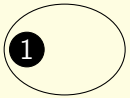

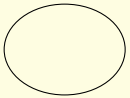

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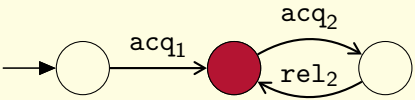

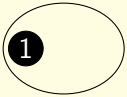

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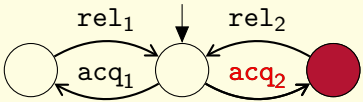

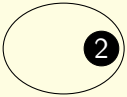

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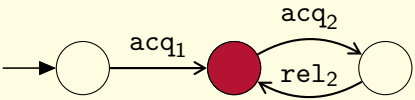

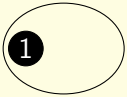

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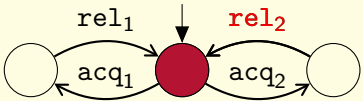

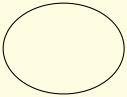

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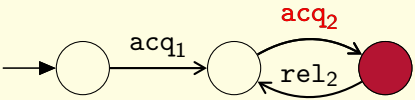

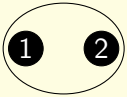

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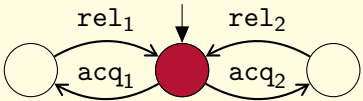

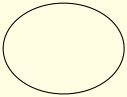

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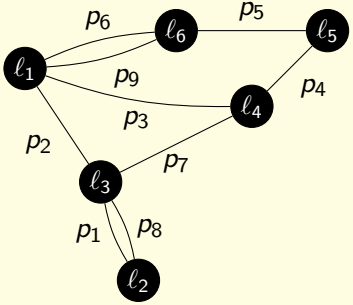
Restrictions

Two locks/process

Each process uses at most two locks.
A system defines a *locking graph*.



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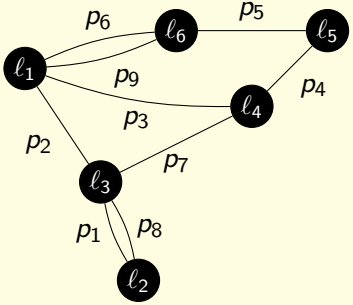

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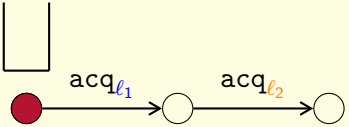

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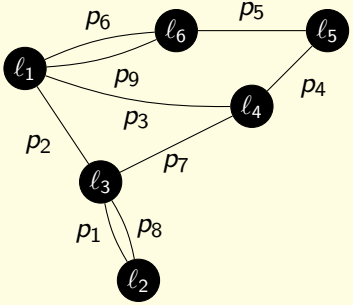

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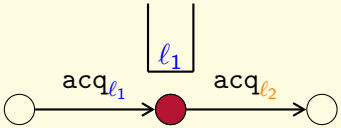

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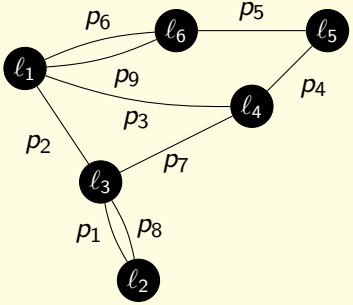

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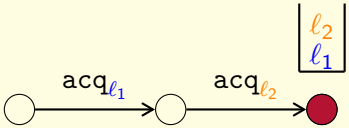

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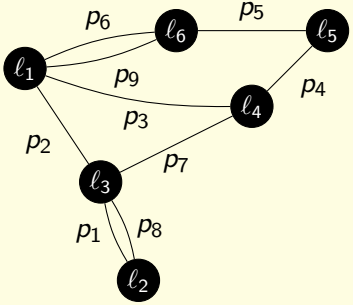

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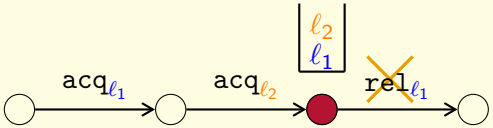

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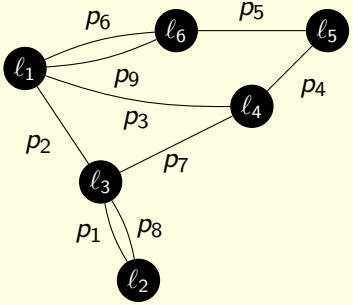

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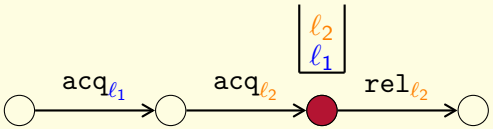

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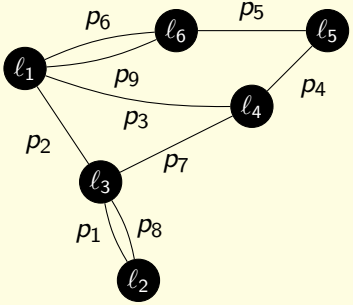

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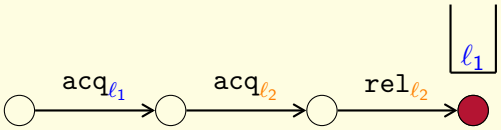

Patterns

Each local run has a type, called a *pattern*.

Schedulability lemma

Given a family of local runs $(u_p)_{p \in Proc}$, whether we can form a valid global run by interleaving them only depends on their *patterns*.

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Two locks/process

We colour the graph with the pattern of each run and analyse cycles.

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Patterns $\sim$ partial orders on locks

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Nested locking

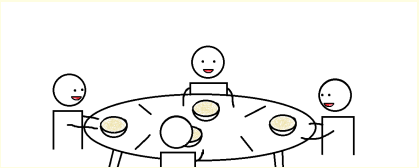
Patterns $\sim$ partial orders on locks

Schedulability $\sim$ finding a common order

Synthesis $\sim$ find sets of patterns $(\Pi_p)_{p \in Proc}$ such that:

- No bad run can be obtained from local runs with those patterns
- For all p there is a local strategy σ_p such that all local runs of p are in Π_p

Example



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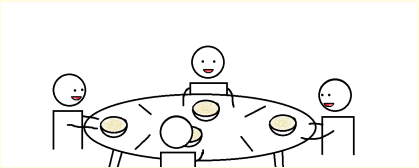

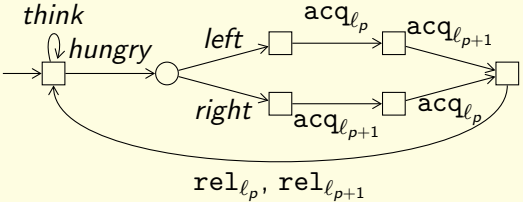
think
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$\texttt{acq}_{\ell_p}$
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$\texttt{rel}_{\ell_p}, \texttt{rel}_{\ell_{p+1}}$

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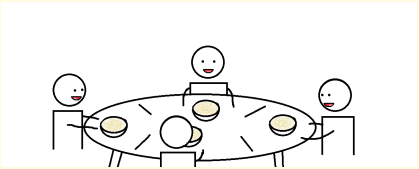

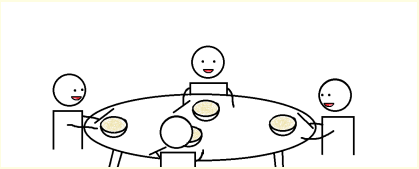

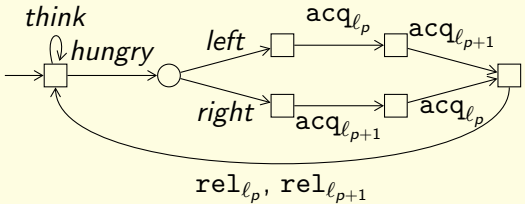
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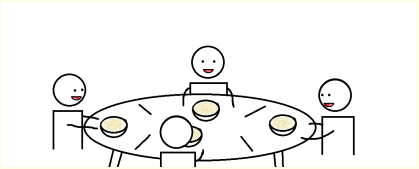

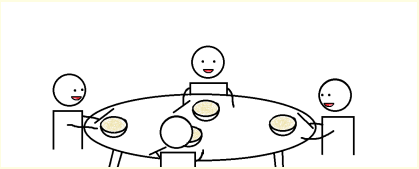

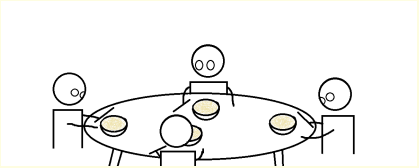

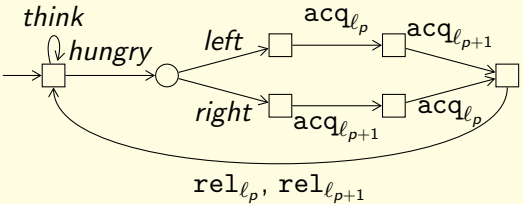
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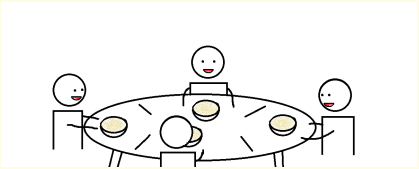

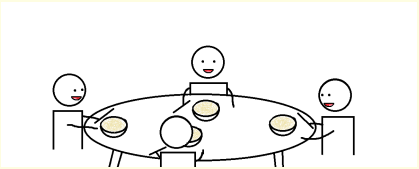

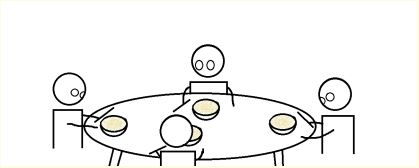

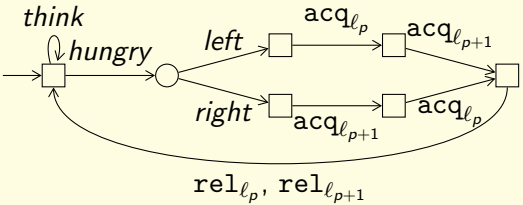
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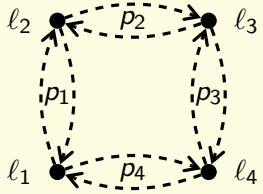
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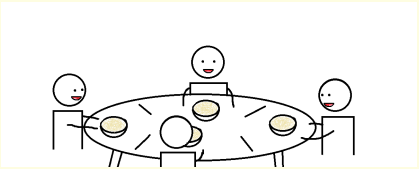

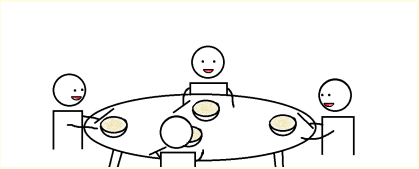

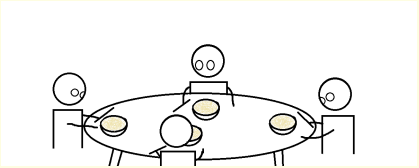

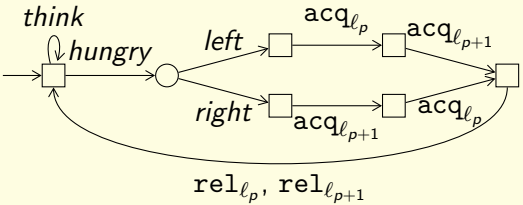
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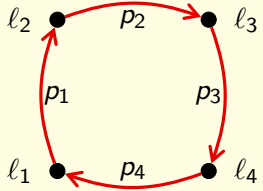
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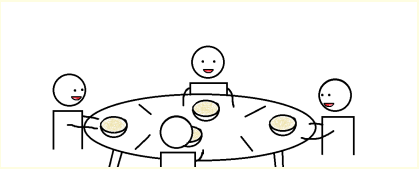
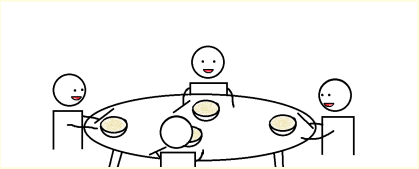
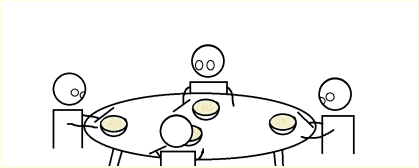
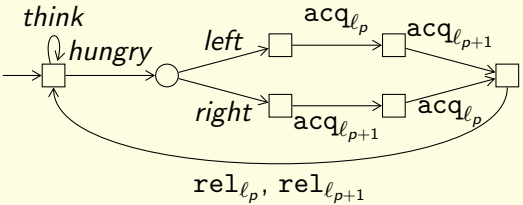
Example





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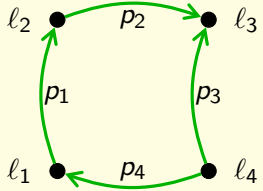
ℓ_2
p_2
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Results

Objectives

Regular objective = Boolean combination of local regular conditions

Global deadlock = Every process is blocked

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Verification	**Reg. obj.**	**Global**	**Process**
LSS	PSPACE	PSPACE	PSPACE
Nested	NP	NP	NP
2LSS	NP	NP	NP
Locally live 2LSS	NP	PTIME	NP
Exclusive 2LSS	NP	NP	PTIME

Synthesis	**Reg. obj.**	**Global**	**Process**
LSS	Undec.	Undec.	Undec.
Nested	NEXP	Σ_2^P	Σ_2^P
2LSS	NEXP	Σ_2^P	Σ_2^P
Locally live 2LSS	NEXP	NP	Σ_2^P
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Synthesis	**Reg. obj.**	**Global**	**Process**
LSS	Undec.	Undec.	Undec.
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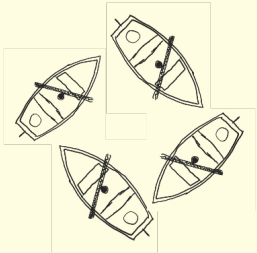
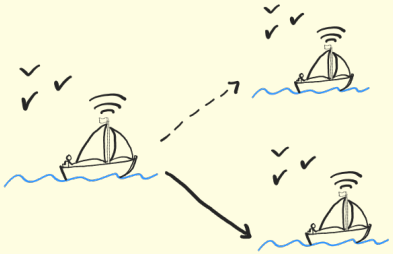


Part III

Locks and process spawning



Dynamic LSS

- We want to allow an unbounded number of processes and locks.

Dynamic LSS

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- A process can spawn other processes

Dynamic LSS

▷ We want to allow an unbounded number of processes and locks.

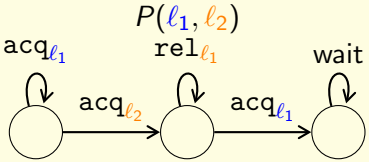
▷ A process can spawn other processes

▷ A process takes parameters, represented by *lock variables*

$Proc = \{P(\ell_1, \ell_2), Q(\ell_1, \ell_2, \ell_3), R(), ...\}$

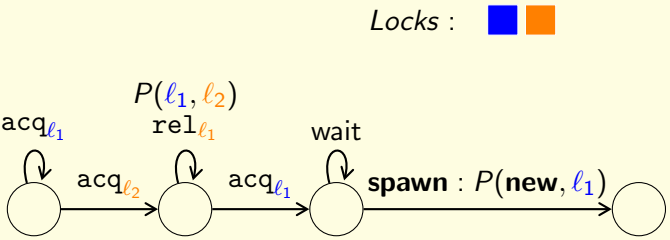
Dynamic LSS[3]

Locks : 



[3] Bouajjani, Müller-Olm, Touili, CONCUR 2005 + Kenter's thesis 2022

Dynamic LSS[3]



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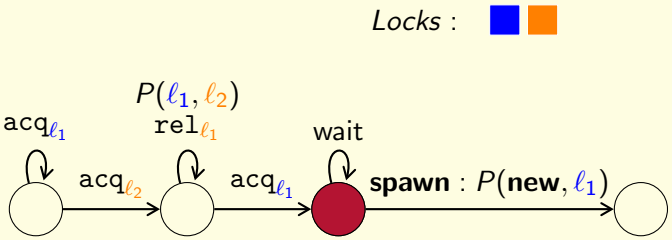
Dynamic LSS[3]



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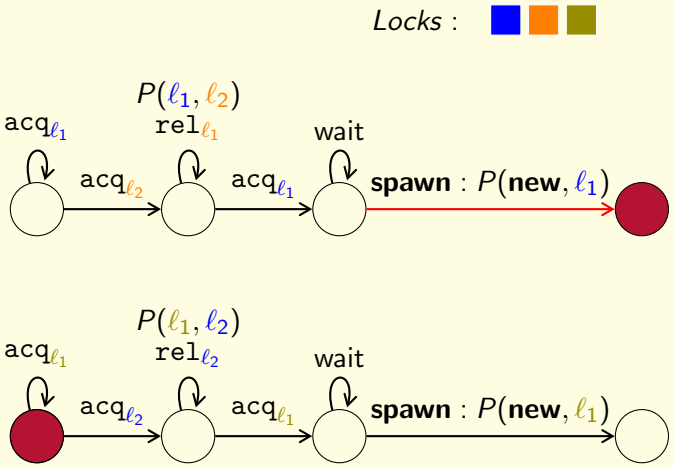
Dynamic LSS[3]



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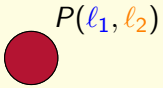
Tree representation



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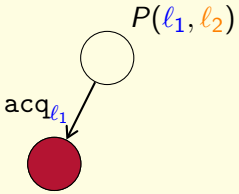

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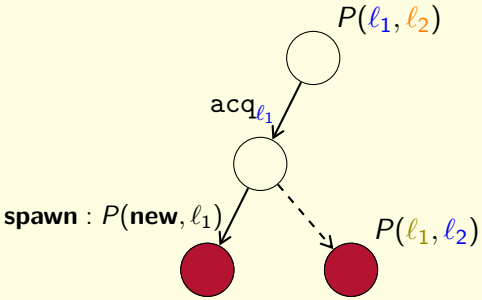

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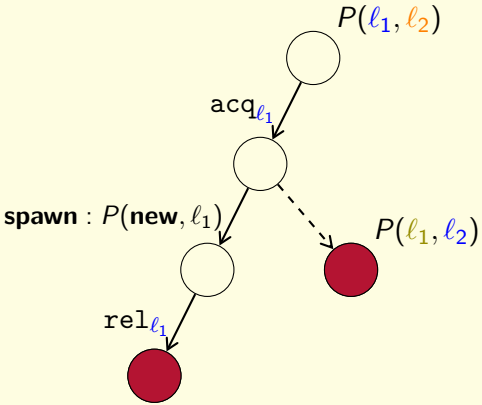

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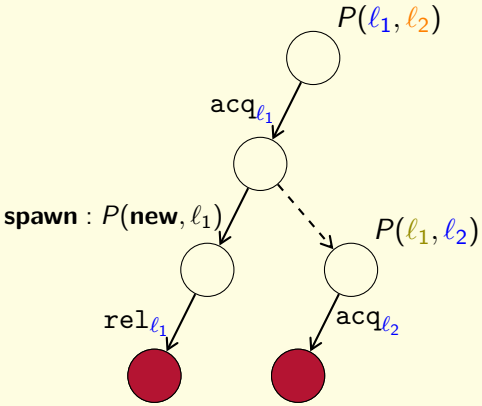

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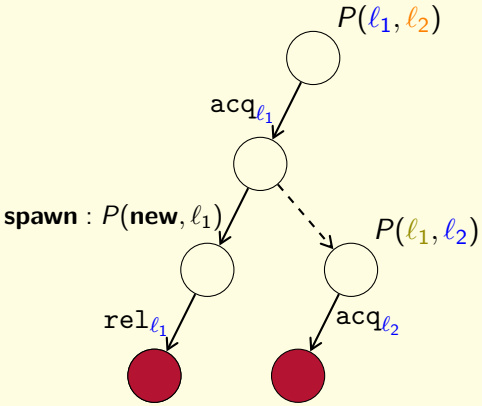

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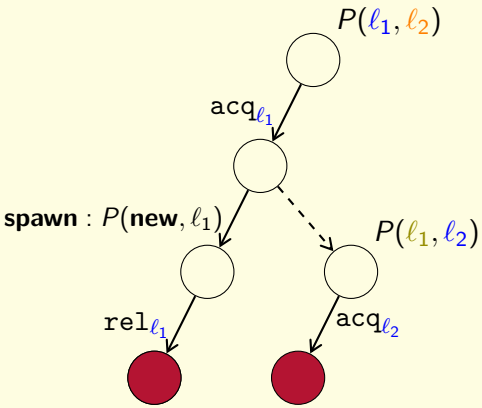

Specifications are ω-regular tree languages.

"Every process is blocked after some point"

"Finitely many processes are spawned"

"Infinitely many processes reach an error state q_{err}"

Deadlocks

Regular model-checking problem

Input: A DLSS $\mathcal{D}$ and a parity tree automaton $\mathcal{A}$.
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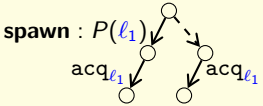

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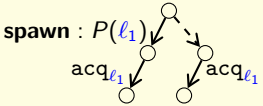

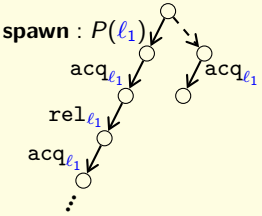

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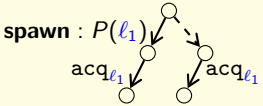

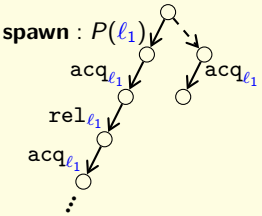

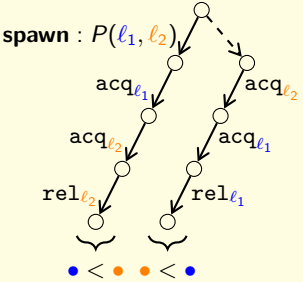

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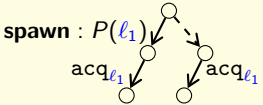

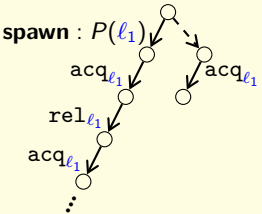

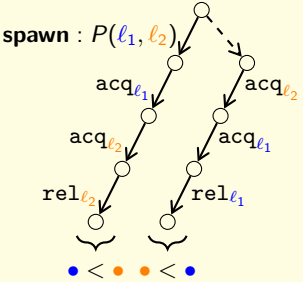

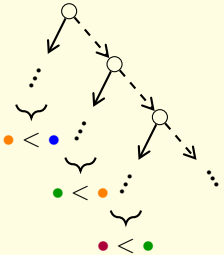

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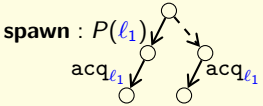

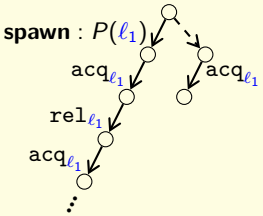

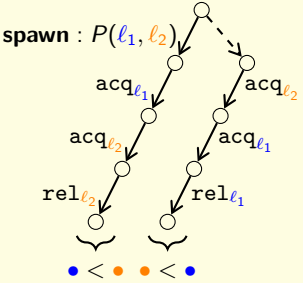

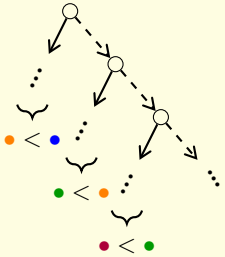

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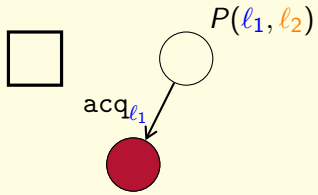
What about shared variables?

DLSS with variables

We add a variable and operations `wr` and `rd` writing and reading letters from a finite alphabet in the register.

DLSS with variables



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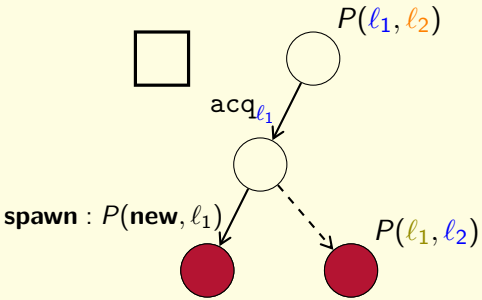

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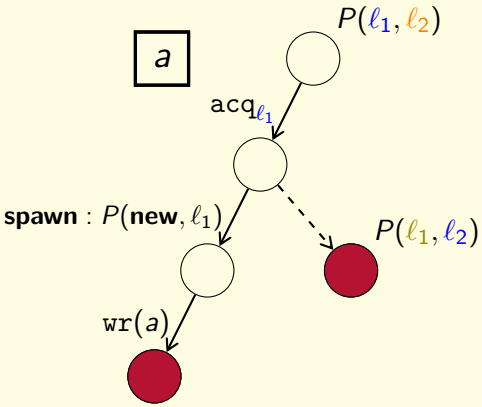

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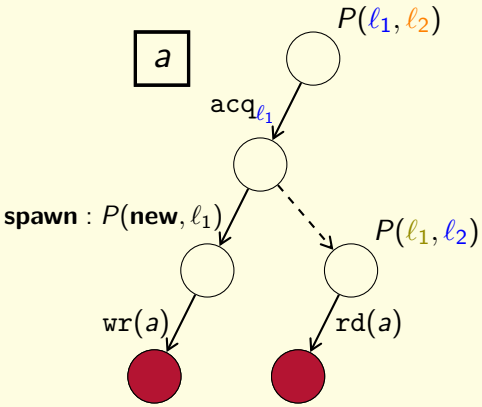

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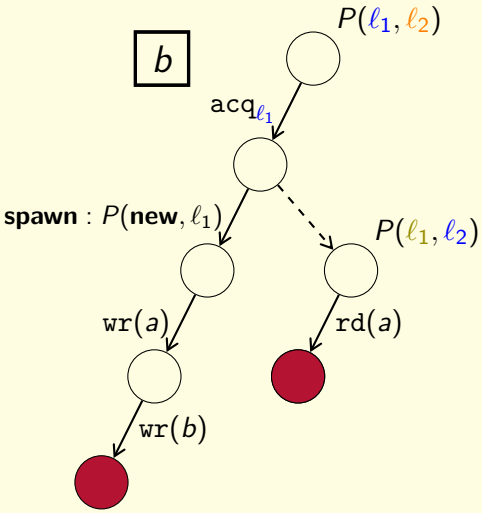

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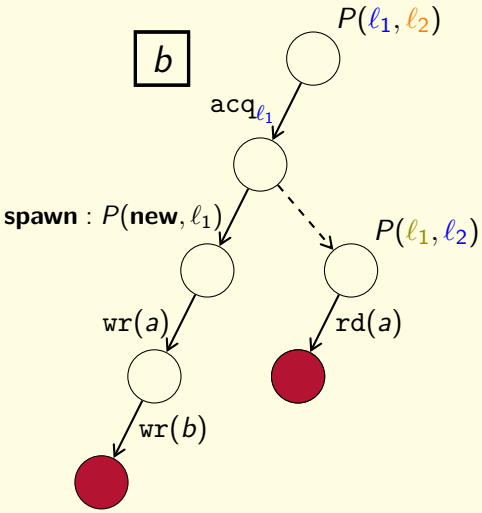

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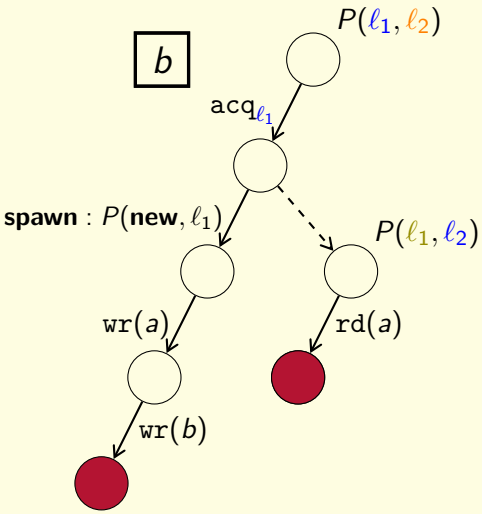

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State reachability is undecidable for DLSS with variables.

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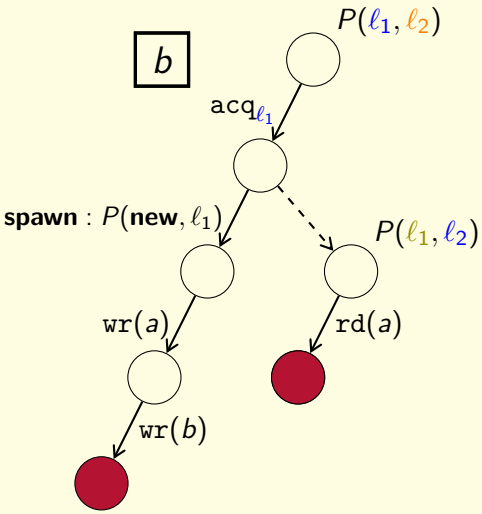

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Conjecture

Verification is decidable when the number of time the writing process switches is bounded.

Synthesis

- Processes have controllable and uncontrollable states
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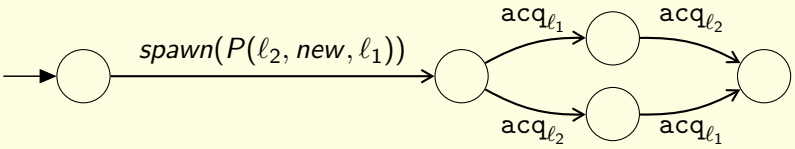
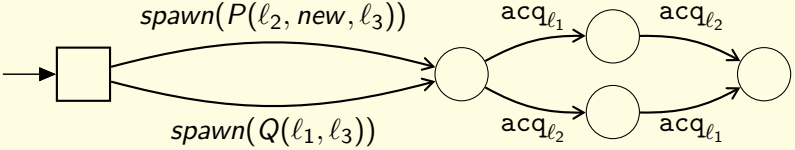
Is there a strategy $\sigma = (\sigma_p)_{p \in Proc}$ ensuring that there is no execution accepted by $\mathcal{A}$?

Example: Arbitrarily many dining philosophers

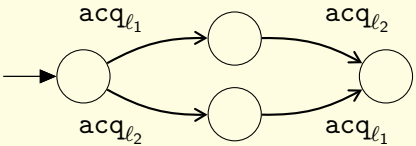
$P_{init}(\ell_1, \ell_2)$:

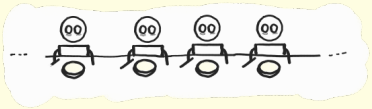


$P(\ell_1, \ell_2, \ell_3)$:



$Q(\ell_1, \ell_2)$:





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Lemma

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- Characterise sets of local runs with which we can form an execution of $\mathcal{D}$ accepted by $\mathcal{A}$.
- Check the existence of a strategy σ such that σ-local runs cannot form a tree accepted by $\mathcal{T}$, using a regular 2-player game.

Synthesis

Theorem

The controller synthesis problem is decidable in 2EXPTIME over DLSS.

Conclusion

Past

- New approach to distributed synthesis, applicable to models with
  - Fixed architectures
  - Parameterised number of processes
  - Dynamic set of processes
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Thank you for your attention!