Responsibility in verification

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- $\rightarrow$ An agent is considered a cause of an event if, had that agent acted differently, this event would not have happened.
- $\rightarrow$ In order to gauge the importance of a part of a system, we check whether the system still works when that part malfunctions.

Shapley values

We fix a set of players $\{1, \ldots, N\}$, and we note $\mathcal{P}(N)$ the set of partitions of $\{1, \ldots, N\}$.

Definition

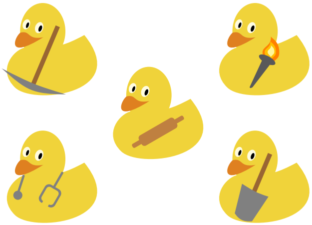
A *coalition* is a pair (T, P) with $P \in \mathcal{P}(N)$ and $T \in P$. The set of coalitions is denoted $\mathcal{C}(N)$.

Definition

A *coalitional game* is a function $\nu : \mathcal{C}(N) \to \mathbb{R}$.
We focus on games such that $\nu(T, P)$ only depends on T.

Process

- $\rightarrow$ We start with a system with agents $\{1, \ldots, n\}$.
- $\rightarrow$ Agents stop cooperating one by one in a random order.



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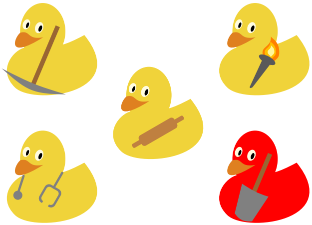

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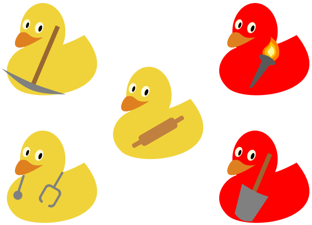

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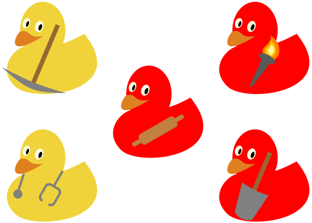

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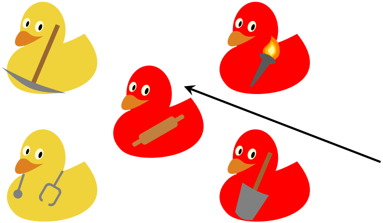

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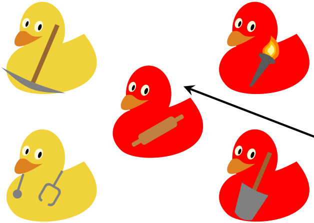

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The importance of an agent is its probability to be decisive.

Shapley values

The Shapley value function is the only one satisfying the following conditions.

1. **Efficiency** $\sum_{i=0}^{N} \varphi_i(\nu) = \nu(\{1,\ldots,N\},\{\{1,\ldots,N\}\})$
2. **Symmetry** Renaming players does not affect their rewards.
3. **Additivity** For all games μ, ν, and $C \in \mathbb{R}$, $\varphi_i(C\mu + \nu) = C\varphi_i(\mu) + \varphi_i(\nu)$, i.e., φ is a linear function.
4. **Null-Player Axiom** If for all $(T, P) \in \mathcal{C}(N)$, $\nu(T \cup \{i\}, P_{T \leftarrow i}) = \nu(T, P)$ then $\varphi_i(\nu) = 0$.

Cooperative games and the Shapley value

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The **Shapley value** is defined as:

$$\mathrm{Sh}(a) = \frac{1}{n!} \sum_{\pi \in \Pi_n} \nu(\pi_{\geq a}) - \nu(\pi_{\geq a} \setminus \{a\})$$

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→ The Shapley value of agent a is its probability to be decisive.

LTL model checking and importance of states

LTL is a logics designed to express properties of infinite words.

$$\varphi ::= a \mid \varphi \wedge \varphi \mid \neg\varphi \mid X\varphi \mid \varphi U \varphi \mid G\varphi \mid F\varphi$$

For instance, GFa expresses that at all positions of the word there is a further position at which a is true.

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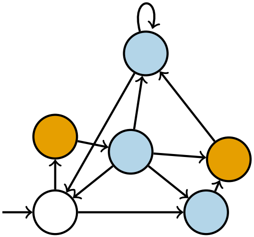

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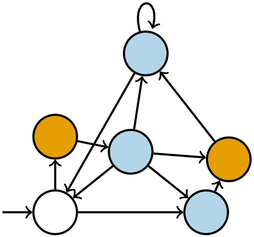

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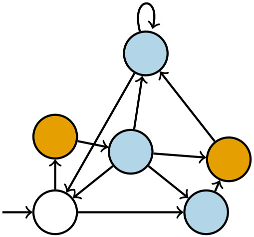

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Which states of K are important for φ?

A state of K is (more) important for φ if **its nondeterminism matters (more)**.

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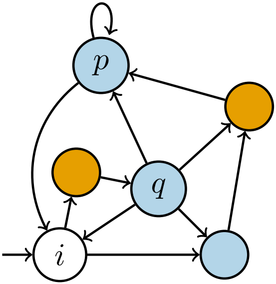
$$\mathcal{I}(q) = \frac{1}{|S|!} |\{\pi \in \Pi_S \mid q \text{ is decisive for } \pi\}|$$

Example



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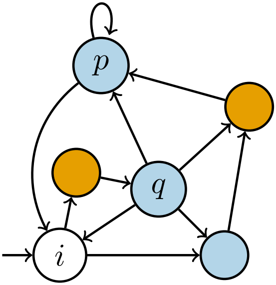

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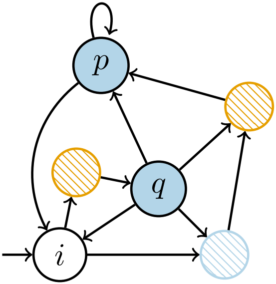

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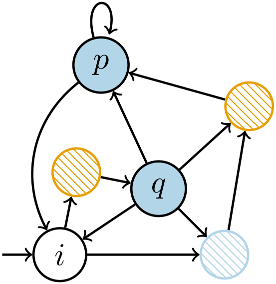

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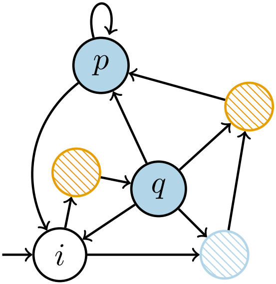

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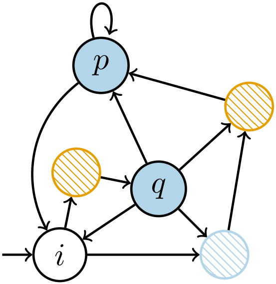

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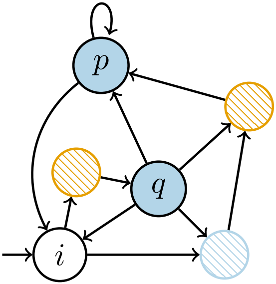

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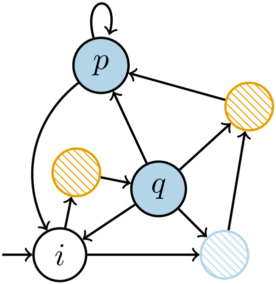

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$$\mathcal{I}(i) = \mathcal{I}(q) = 1/6$$

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Is C enough to make the system work?

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Importance computation problem. Given $q \in S$:

compute $|S|! \cdot \mathcal{I}(q)$

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TABLE I
A SUMMARY OF THE RESULTS ON THE COMPLEXITY OF THE VALUE, USEFULNESS AND IMPORTANCE PROBLEMS FOR VARIOUS TYPES OF SPECIFICATIONS.

	Büchi	**Rabin**	**Streett**	**Parity**	**Explicit Muller**
Value	P	NP	coNP	$\in$ NP $\cap$ coNP	P
Usefulness	NP	Σ_2^P	Σ_2^P	NP	NP
Importance	#P	$\#P^{NP}$	$\#P^{NP}$	#P	#P

	Emerson-Lei	**LTL**	**2-turn CTL**	**Concurrent CTL**
Value	PSPACE	2EXPTIME	Σ_2^P	$\in$ EXPTIME
Usefulness	PSPACE	2EXPTIME	Σ_3^P	$\in$ EXPTIME
Importance	PSPACE	2EXPTIME	$\#P^{\Sigma_2^P}$	$\in$ EXPTIME

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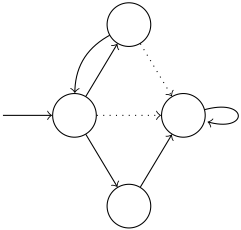

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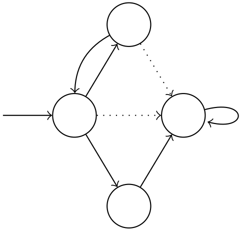

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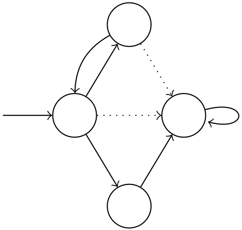


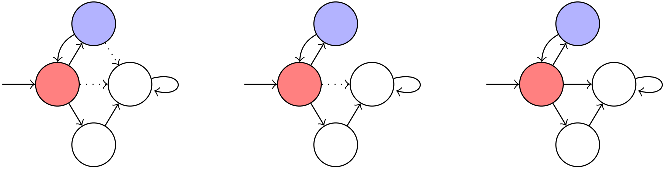
It is not clear how to design a **turn-based game**.

We considered **one-shot** games instead.

CTL complexity

We consider two interpretations for CTL:

1) One player chooses all its transitions, then the other one does.



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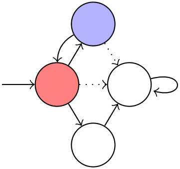

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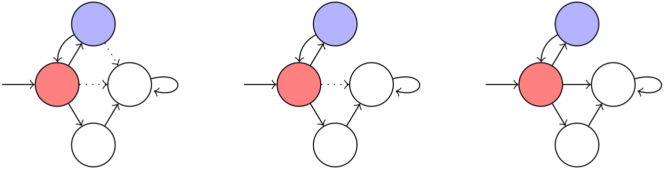

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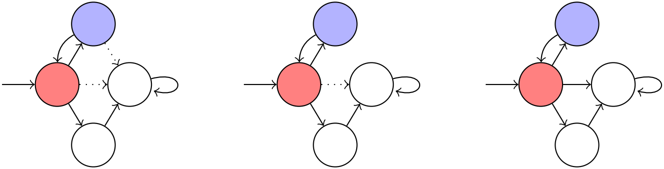

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- Players may use randomized strategies to choose their transitions.
- Computing the value of a set of states comes down to solving a linear optimization problem with exponential input.

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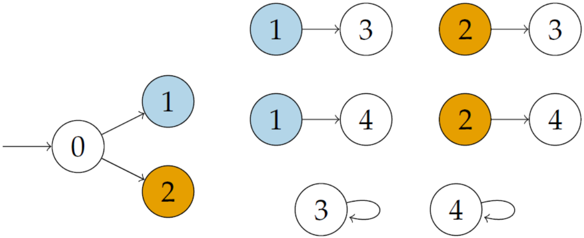
Definition

A *linear strategy* is a function $\sigma : E^* \to E$.

Sat (resp. Unsat) wins if they have a strategy to guarantee that the resulting tree (un)satisfies the objective. The game is sometimes undetermined.

Example of undeterminacy



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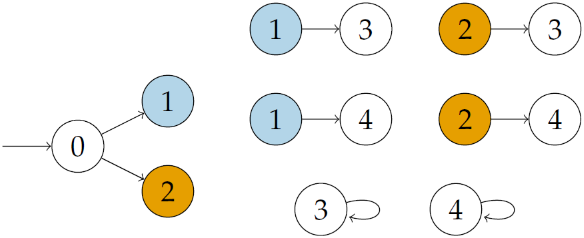

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Question

Which tree objectives allow tree games to be determined?

Game automata

Definition

A *game automaton* is an alternating tree automaton in which a pair (q, i) appears at most once in each transition.

Proposition

Languages expressed by game automata yield determined tree games.

However there exist languages non expressible by game automata which yield determined tree games.
It is the case with the language of trees having countably many branches fully labelled with a.

Continuation

$\rightarrow$ Fragment of CTL for which tree games are determined.

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- $\rightarrow$ Probabilistic approximation methods for Shapley values in this framework.

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- $\rightarrow$ Extension to probabilistic systems.
- $\rightarrow$ Control point of view (make the adversary all-knowing)

Thank you for your attention!

Axiomatic characterization of the Shapley value

Let $A = \{1, \ldots, n\}$ be **a set of agents**. A **value function** φ is a function that assigns to a cooperative game ν and an agent $a \in A$ a real number $\varphi(\nu, a)$. The **Shapley value** is the unique value function satisfying:

- **Efficiency.**

$$\sum_{a \in A} \varphi(\nu, a) = \nu(A) \quad \text{for all } \nu$$

- **Symmetry.** If $\nu(S \cup \{a_1\}) = \nu(S \cup \{a_2\})$ for all $S \subseteq A \setminus \{a_1, a_2\}$:

$$\varphi(\nu, a_1) = \varphi(\nu, a_2) \quad \text{for all } \nu$$

- **Dummy players**. If $\nu(S \cup \{a\}) = \nu(S)$ for all $S \subseteq A \setminus \{a\}$:

$$\varphi(\nu, a) = 0 \quad \text{for all } \nu$$

- **Additivity**. Let $(\nu \oplus \omega)(S) = \nu(S) + \omega(S)$. Then:

$$\varphi(\nu, a) + \varphi(\omega, a) = \varphi(\nu \oplus \omega, a) \quad \text{for all } \nu, \omega$$