

# Decidability of TeamLTL model-checking

## Context

TeamLTL is a temporal hyperlogic designed to express temporal properties of sets of traces (while logics like LTL only speak about a single trace). The most popular logic of that kind is HyperLTL, which offers a high expressivity but for which basic problems like satisfiability or model-checking are untractable or even undecidable.

The syntax is the one of LTL without negation.

$$\varphi ::= p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid X\varphi \mid \varphi U \varphi \mid \varphi R \varphi$$

The (synchronous) semantics is defined as follows (asynchronous semantics also exist, but will not be considered here). Most operators are naturally extended to sets of traces:

$$\begin{array}{ll} T \models p & \text{if } \forall t \in T, p \in t[0] \\ T \models \varphi \wedge \psi & \text{if } T \models \varphi \text{ and } T \models \psi \\ T \models X\varphi & \text{if } T[1 \dots] \models \varphi \\ T \models \varphi U \psi & \text{if } \exists k, T[k \dots] \models \psi \text{ and } \forall j < k, T[j \dots] \models \varphi \\ T \models \varphi R \psi & \text{if } \forall k, T[k \dots] \models \psi \text{ or } \exists j < k, T[j \dots] \models \varphi \end{array}$$

The twist is in the  $\vee$ : it is not defined as a boolean disjunction, but with *\*splits\**:

$$T \models \varphi \vee \psi \text{ if } \exists T_1, T_2 \text{ such that } T_1 \cup T_2 = T \text{ and } T_1 \models \varphi \text{ and } T_2 \models \psi$$

We define  $G\varphi$  and  $F\varphi$  as usual, as  $\perp R\varphi$  and  $\top U\varphi$ . The difficulty arises with formulas like  $G(F p \vee F q)$ , which expresses that “at all times,  $T$  can be split in two teams, with all traces of the first one synchronously satisfying  $p$  at some point and all the other ones synchronously satisfying  $q$  at some other point.”

While satisfiability comes down to LTL satisfiability (if a TeamLTL formula is satisfiable then it is satisfiable with a single trace), the decidability of model-checking (given a finite Kripke structure and a TeamLTL formula, does the set of traces of the former satisfy the latter?) is still an open problem.

## Credits and related work

This problem was presented to me by Martin Zimmermann. TeamLTL was introduced in this paper [1], with (amongst other results) a PSPACE-completeness proof for model-checking when the Kripke structure is a union of "lassos".

It was later shown that model-checking and satisfiability were undecidable for TeamLTL with the boolean negation  $\sim$ , as well as a "flattening" negation  $\neg\varphi$ , which says that  $\varphi$  should not be satisfied by any trace (seen as a singleton) in the set [2]. To the best of my knowledge, decidability of model-checking when only one of those negations is added is open.

In [3] decidability of model-checking is established for two fragments of TeamLTL (called "left-flat" and "k-coherent").

## References

### References

- [1] Andreas Krebs et al. "Team Semantics for the Specification and Verification of Hyperproperties". In: *43rd International Symposium on Mathematical Foundations of Computer Science, MFCS 2018, August 27-31, 2018, Liverpool, UK*. Ed. by Igor Potapov, Paul G. Spirakis, and James Worrell. Vol. 117. LIPIcs. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2018, 10:1–10:16. DOI: 10.4230/LIPIcs.MFCS.2018.10. URL: <https://doi.org/10.4230/LIPIcs.MFCS.2018.10>.
- [2] Martin Lück. "On the complexity of linear temporal logic with team semantics". In: *Theor. Comput. Sci.* 837 (2020), pp. 1–25. DOI: 10.1016/j.tcs.2020.04.019. URL: <https://doi.org/10.1016/j.tcs.2020.04.019>.
- [3] Jonni Virtema et al. "Linear-Time Temporal Logic with Team Semantics: Expressivity and Complexity". In: *41st IARCS Annual Conference on Foundations of Software Technology and Theoretical Computer Science, FSTTCS 2021, December 15-17, 2021, Virtual Conference*. Ed. by Mikolaj Bojanczyk and Chandra Chekuri. Vol. 213. LIPIcs. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2021, 52:1–52:17. DOI: 10.4230/LIPIcs.FSTTCS.2021.52. URL: <https://doi.org/10.4230/LIPIcs.FSTTCS.2021.52>.