Adjusted Count Quantification Learning on Graphs
Rebuttal Supplement

A ADDITIONAL RANDOM WALK DISTRIBUTION SHIFT

In the main paper, we evaluated the performance of different quantifiers using no shift, *prior probability shift* (PPS) and a BFS-induced covariate shift. Since the focus of our paper is on tackling covariate shift, we decided to extend the evaluation by a second type of covariate shift induced by random walks, instead of *breadth-first search* (BFS).

As described in the main paper, the BFS-based covariate shift is induced as follows: We select a random start node v and then perform a BFS traversal starting at v to obtain 100 direct or indirect neighbors. The resulting neighborhood is then used as the test set $\mathcal{X}_U$.

Analogously, for the RW-based covariate shift, we select a random start node v and perform 100 random walks of length 10 starting at v with a probability of 10% of not moving in a given step. The 100 end nodes of the random walks are then used as the test set $\mathcal{X}_U$.

While both types of covariate shift are structure-based, the BFS-based shift always produces connected test sets, whereas the RW-based shift can produce disconnected test sets. In all following tables and figures (Tables 1 and 2 and Figs. 1 and 2), we include quantification results for PPS, BFS-based covariate shift and RW-based covariate shift.

B COMPARISON WITH PRIOR GRAPH QUANTIFICATION METHODS (EGONET)

As suggested by reviwer LVAN, we compared our proposed *structural importance sampling* (SIS) and *Neighborhood-aware ACC* (NACC) methods with the previously proposed *Ego-network Quantification* (ENQ) method by Milli et al. [2015]. We do not compare against the link-based quantification method proposed by Tang et al. [2010], since it only applicable to binary quantification while our experiments consider multiclass quantification problems.

ENQ works by first assigning each node the majority class among its labeled (indirect) neighbors, i.e., its *ego-network*. Vertices without labeled nodes in their ego-network are assigned the class distribution of the entire labeled dataset $\mathcal{D}_L$, i.e., $\hat{\mathcal{P}}(Y)$. This classification is then combined with standard *Adjusted Classify & Count* (ACC) to obtain the final quantification.

Note that, since ENQ uses standard ACC to produce the final quantification, SIS and NACC can be used with ENQ as well. Table 1 compares the quantification results for 1-hop ego-network-based classification with the *graph neural network* (GNN) variants evaluated in the main paper. The ENQ method, as proposed by Milli et al. [2015], corresponds to the *EgoNet Adjusted* rows in the table. The more modern GNN-based variants clearly outperform the ENQ method on all datasets and quantification metrics. Nonetheless, SIS (using the PPR kernel) and NACC still significantly improve the quantification performance of the EgoNet classifier, compared to standard ACC.

Last, note that, in addition to adding the EgoNet quantifier and adding results for random walk-based covariate shift (see Appendix A), we report slightly different results for the PPR kernel with an interpolation parameter $\lambda = 0.9$, while in the main paper we use $\lambda = 1$ (see Appendix D.2).

Table 1: Quantification using probabilistic classifiers (absolute error, relative absolute error and KL divergence).

Model & Shift	Quantifier	CoraML AE	CoraML RAE	CoraML KLD	CiteSeer AE	CiteSeer RAE	CiteSeer KLD	Amazon Photos AE	Amazon Photos RAE	Amazon Photos KLD	Amazon Computers AE	Amazon Computers RAE	Amazon Computers KLD	PubMed AE	PubMed RAE	PubMed KLD	Avg. Rank AE	Avg. Rank RAE	Avg. Rank KLD
PPS	MLPE	.0903	.5692	.1605	.0469	.2623	.0396	.0924	.6660	.2107	.0770	.5358	.5358	.1268	.3948	.0834	-	-	-
EgoNet PPS	Unadjusted	.0750	.8692	.1960	**.0429**	**.3543**	**.0547**	.0480	1.319	.1199	.0453	.5297	.5297	.1159	.4764	.0931	3.8	4.2	3.8
	Adjusted	.0848	.6863	.4721	.0927	.5928	.3884	.0314	.3881	.0707	.0334	.3437	.3437	.0833	.3222	.0823	4.4	4.0	4.4
	NEIGH	**.0519**	**.4397**	**.1509**	.0594	**.3791**	.1305	**.0227**	**.3265**	**.0355**	**.0258**	**.2693**	**.2693**	**.0373**	**.1376**	**.0143**	2.2	2.0	2.2
	PPR	.0837	.6832	.4571	.0916	.5871	.3761	.0313	.3925	.0703	.0333	.3433	.3433	.0817	.3183	.0769	3.4	3.4	3.4
	PPR NEIGH	**.0514**	**.4369**	**.1465**	.0591	**.3775**	.1290	**.0227**	**.3330**	**.0354**	**.0258**	**.2691**	**.2691**	**.0371**	**.1374**	**.0141**	**1.2**	**1.4**	**1.2**
MLP PPS	Unadjusted	.0827	.8565	.2522	.0361	.2782	.0372	.0497	1.105	.1316	.0533	.6342	.6342	.0470	.1870	.0161	7.0	7.0	6.6
	Adjusted	.0481	.4186	.1404	.0336	.2271	.0384	.0191	.3036	**.0255**	.0334	.3690	.3690	**.0181**	.0649	**.0027**	4.2	4.2	4.8
	NEIGH	**.0326**	**.2865**	**.0493**	.0288	.1908	.0244	**.0163**	.3595	**.0249**	.0265	.2936	.2936	.0187	.0649	.0028	2.8	3.4	2.8
	SP	.0490	.4385	.1502	.0343	.2355	.0378	.0195	**.3003**	**.0261**	.0333	.3644	.3644	.0190	.0695	.0029	5.4	4.6	5.2
	PPR	.0486	.4237	.1403	.0327	.2218	.0366	.0192	**.2881**	**.0261**	.0338	.3708	.3708	**.0179**	**.0641**	**.0026**	4.4	3.6	4.4
	SP NEIGH	**.0330**	.2993	**.0511**	**.0271**	**.1825**	**.0215**	**.0165**	.3469	**.0249**	**.0255**	**.2844**	**.2844**	.0193	.0718	.0029	3.0	3.4	2.8
	PPR NEIGH	**.0320**	**.2847**	**.0469**	**.0271**	**.1799**	**.0216**	**.0162**	.3345	**.0248**	.0263	.2913	.2913	**.0178**	**.0616**	**.0026**	**1.2**	**1.8**	**1.4**
GCN PPS	Unadjusted	.0438	.4697	.0809	.0221	.1574	.0145	.0315	.8508	.0639	.0391	.4667	.4667	.0405	.1665	.0125	7.0	7.0	7.0
	Adjusted	.0246	.2216	.0359	.0190	.1259	.0109	**.0122**	**.2056**	**.0129**	.0228	.2411	.2411	.0161	**.0591**	**.0023**	4.4	4.4	4.6
	NEIGH	.0239	**.2073**	.0291	.0188	.1253	.0108	.0134	.2920	.0177	.0191	.2054	.2054	.0181	.0659	.0027	4.4	3.6	4.2
	SP	**.0237**	.2175	.0324	**.0181**	.1232	**.0100**	.0126	.2393	**.0130**	.0218	.2337	.2337	**.0153**	**.0588**	**.0021**	3.0	3.6	3.2
	PPR	**.0234**	.2163	.0300	**.0178**	**.1186**	**.0095**	**.0124**	.2295	**.0127**	.0223	.2386	.2386	**.0160**	**.0584**	**.0023**	**2.6**	**2.8**	2.8
	SP NEIGH	.0240	**.2116**	**.0279**	**.0180**	.1225	**.0100**	.0138	.3500	.0182	**.0183**	**.1975**	**.1975**	.0174	.0682	.0026	3.8	3.8	3.6
	PPR NEIGH	**.0232**	**.2073**	**.0258**	**.0176**	**.1177**	**.0094**	.0135	.3329	.0180	**.0188**	**.2029**	**.2029**	.0179	.0650	.0026	2.8	**2.8**	**2.6**
APPNP PPS	Unadjusted	.0374	.4124	.0598	.0214	.1509	.0133	.0318	.9795	.0682	.0390	.4657	.4657	.0398	.1664	.0125	7.0	7.0	7.0
	Adjusted	.0217	.1986	.0251	.0184	.1211	.0100	**.0124**	**.2442**	**.0136**	.0256	.2638	.2638	.0165	**.0597**	**.0023**	3.8	3.8	4.2
	NEIGH	.0224	**.1943**	.0245	.0184	.1222	.0103	.0139	.3133	.0205	.0231	.2471	.2471	.0187	.0676	.0028	4.8	4.2	4.8
	SP	.0211	.1994	.0236	**.0173**	.1170	.0093	**.0127**	.2901	.0144	.0240	.2513	.2513	**.0156**	**.0596**	**.0021**	**2.4**	3.4	2.6
	PPR	**.0203**	**.1926**	**.0205**	**.0171**	**.1132**	**.0087**	.0130	.3058	.0172	.0249	.2566	.2566	.0163	**.0589**	**.0022**	**2.4**	**2.2**	**2.4**
	SP NEIGH	.0224	**.1968**	.0244	.0178	.1211	.0100	.0144	.3782	.0215	**.0215**	**.2315**	**.2315**	.0180	.0703	.0027	4.2	4.4	3.8
	PPR NEIGH	.0214	**.1939**	**.0212**	**.0172**	**.1149**	**.0089**	.0143	.3780	.0222	.0226	.2424	.2424	.0185	.0668	.0027	3.4	3.0	3.2
BFS	MLPE	.1839	8.185	1.123	.2676	15.22	1.475	.1622	9.313	1.168	.1180	5.666	5.666	.3034	5.067	.5129	-	-	-
EgoNet BFS	Unadjusted	.1440	8.886	.7681	.2374	22.27	1.324	.0566	3.686	**.2027**	.0455	2.274	2.274	.2313	26.15	**.4463**	5.0	5.0	2.8
	Adjusted	.1005	**5.140**	.7202	.1908	17.87	1.086	**.0388**	**1.112**	**.2270**	.0419	**1.190**	**1.190**	.2108	24.61	.7576	3.6	2.6	4.4
	NEIGH	**.0825**	**4.300**	**.5547**	**.1552**	**14.32**	**.9307**	**.0359**	**1.151**	**.2050**	**.0384**	**1.156**	**1.156**	**.1605**	**18.38**	.6257	2.0	2.0	2.6
	PPR	.1003	**5.196**	.7114	.1909	17.93	1.085	**.0385**	**1.125**	**.2230**	.0413	**1.192**	**1.192**	.2119	24.96	.7615	3.4	3.6	3.8
	PPR NEIGH	**.0813**	**4.244**	**.5295**	**.1531**	**14.12**	**.9037**	**.0357**	**1.165**	**.2009**	**.0379**	**1.162**	**1.162**	**.1603**	**18.37**	.6237	**1.0**	**1.8**	**1.4**
MLP BFS	Unadjusted	.1243	7.212	.5915	.1588	14.84	.8652	.0668	4.028	.2597	.0662	3.635	3.635	.0800	10.44	**.1152**	6.0	6.6	5.0
	Adjusted	.0645	3.508	.5452	.1158	10.63	1.035	.0237	.9928	**.0963**	.0392	1.608	1.608	.0816	7.663	.2417	3.8	3.0	5.2
	NEIGH	**.0577**	**3.008**	**.3500**	**.0984**	**8.845**	**.5715**	.0290	1.237	.1148	.0400	1.817	1.817	.0878	**6.787**	.2365	4.4	3.0	3.8
	SP	.0709	4.268	.5405	.2096	19.96	1.300	.0259	1.312	.1032	**.0378**	1.720	1.720	**.0753**	7.759	.2151	4.0	5.4	4.4
	PPR	.0637	3.461	.4981	.1162	10.74	1.033	**.0222**	**.9079**	**.0882**	**.0370**	**1.509**	**1.509**	.0786	7.827	.2295	**2.2**	3.0	3.8
	SP NEIGH	.0671	4.082	.4421	.2059	19.63	1.212	.0308	1.496	.1218	**.0378**	1.830	1.830	.0791	**6.658**	.1985	4.8	4.8	3.6
	PPR NEIGH	**.0560**	**2.972**	**.3473**	**.0964**	**8.699**	**.5614**	.0266	1.097	.1014	**.0375**	1.694	1.694	.0840	**6.833**	.2231	2.8	**2.2**	**2.2**
GCN BFS	Unadjusted	.0539	3.489	**.2180**	.0783	7.060	.4402	.0256	1.513	**.0799**	.0418	2.255	2.255	**.0573**	9.553	**.1022**	4.6	6.6	**1.4**
	Adjusted	.0488	2.093	.2866	.0637	5.267	.5169	.0241	**.5966**	.1147	.0401	**.9320**	**.9320**	.0888	**6.713**	.2894	4.0	2.4	5.4
	NEIGH	.0474	2.020	.2646	.0653	5.428	.4571	.0261	.6773	.1212	.0379	**.9846**	**.9846**	.0977	7.994	.3400	5.0	3.8	5.4
	SP	.0567	3.216	.3403	.1986	18.85	1.190	.0252	.8467	.1094	.0378	1.045	1.045	.0821	**6.756**	.2556	4.6	4.8	4.4
	PPR	**.0415**	**1.943**	**.2334**	**.0618**	**5.132**	.4975	**.0207**	**.5932**	.0912	**.0358**	**.9569**	**.9569**	.0881	**6.706**	.2856	**2.0**	**1.6**	3.4
	SP NEIGH	.0572	3.269	.3406	.1984	18.89	1.190	.0271	.9204	.1142	.0361	1.096	1.096	.0896	7.947	.2902	5.6	5.8	5.2
	PPR NEIGH	**.0402**	**1.786**	**.2124**	**.0595**	**4.910**	**.4115**	.0240	.7070	.1043	**.0351**	1.004	1.004	.0968	7.980	.3343	2.2	3.0	2.8
APPNP BFS	Unadjusted	.0469	3.074	**.1871**	.0737	6.609	**.4078**	.0271	1.492	**.0829**	.0468	2.339	2.339	**.0569**	9.867	**.1055**	4.8	6.4	**1.4**
	Adjusted	.0457	1.881	.2682	.0603	4.944	.4750	.0225	**.5731**	.1114	.0430	**.9227**	**.9227**	.0927	**7.449**	.3091	3.4	2.2	4.8
	NEIGH	.0459	1.910	.2408	.0633	5.243	.4365	.0260	.6585	.1260	.0435	**.9606**	**.9606**	.1017	8.673	.3684	5.2	4.0	5.6
	SP	.0539	3.050	.3166	.1965	18.68	1.162	.0240	.8538	.1113	**.0401**	1.040	1.040	.0854	**7.344**	.2665	3.8	4.4	3.8
	PPR	**.0380**	**1.729**	.2126	**.0574**	**4.705**	.4466	**.0213**	**.5823**	.0968	**.0395**	**.9331**	**.9331**	.0874	**7.372**	.2726	**1.8**	**2.0**	3.2
	SP NEIGH	.0552	3.129	.3209	.1966	18.74	1.167	.0276	.9425	.1236	.0413	1.120	1.120	.0939	8.671	.3121	6.0	6.2	5.6
	PPR NEIGH	**.0378**	**1.615**	**.1853**	**.0562**	**4.600**	**.3866**	.0256	.6993	.1153	.0406	**.9747**	**.9747**	.0962	8.440	.3323	3.0	2.8	3.6
RW	MLPE	.1832	6.430	1.079	.2651	15.07	1.450	.1594	8.278	1.169	.1158	4.466	4.466	.3025	3.073	.4840	-	-	-
EgoNet RW	Unadjusted	.1355	6.275	.6304	.2237	21.00	1.151	.0538	2.895	**.1910**	**.0449**	1.703	1.703	.2171	4.937	**.3181**	4.6	5.0	2.2
	Adjusted	.0973	**2.916**	.6851	**.1477**	**13.48**	**.8540**	.0476	**1.061**	.2953	.0481	**1.052**	**1.052**	**.1506**	**1.716**	.6573	3.8	**1.2**	4.8
	NEIGH	**.0900**	**3.201**	**.5400**	**.1476**	**13.49**	**.8257**	**.0418**	**1.165**	.2398	**.0427**	**1.091**	**1.091**	**.1419**	**1.996**	.5182	2.0	3.2	2.6
	PPR	.0971	**2.939**	.6713	**.1484**	**13.55**	**.8433**	.0474	**1.076**	.2916	.0478	**1.058**	**1.058**	**.1508**	**1.756**	.6371	3.6	2.4	3.8
	PPR NEIGH	**.0887**	**3.101**	**.5200**	**.1455**	**13.29**	**.7978**	**.0416**	**1.173**	.2366	**.0424**	**1.100**	**1.100**	**.1416**	**2.003**	.5147	**1.0**	3.2	**1.6**
MLP RW	Unadjusted	.1263	5.275	.5574	.1494	13.84	.6561	.0727	3.820	.2918	.0718	3.224	3.224	.0913	1.376	**.0820**	6.4	6.6	5.4
	Adjusted	.0733	2.347	.4164	.0869	7.425	.5857	.0332	1.251	.1277	.0471	1.837	1.837	**.0882**	**.7452**	.2360	3.8	3.2	4.2
	NEIGH	.0644	2.153	.3042	**.0824**	**7.105**	.4413	.0326	1.369	.1330	.0434	1.859	1.859	.0915	**.7066**	.2614	3.4	2.8	3.6
	SP	.0824	3.250	.4786	.1993	18.85	1.152	.0360	1.566	.1414	.0473	1.979	1.979	**.0868**	.7628	.2168	5.4	6.0	5.0
	PPR	.0743	2.417	.4213	.0899	7.766	.5873	.0315	**1.144**	**.1194**	.0450	**1.765**	**1.765**	**.0864**	**.7580**	.2194	3.0	3.2	3.8
	SP NEIGH	.0748	3.213	.3957	.1979	18.82	1.124	.0344	1.631	.1398	.0424	1.898	1.898	**.0887**	**.7132**	.2289	4.6	5.0	3.8
	PPR NEIGH	**.0620**	**2.040**	**.2848**	**.0798**	**6.894**	**.4153**	**.0303**	**1.201**	.1179	**.0406**	**1.735**	**1.735**	**.0880**	**.6960**	.2378	**1.4**	**1.2**	**2.2**
GCN RW	Unadjusted	.0539	2.085	**.1841**	.0694	5.990	.3489	.0276	1.247	**.0859**	.0451	1.961	1.961	**.0566**	**.4972**	**.0475**	4.0	5.0	**1.2**
	Adjusted	.0571	1.267	.3115	.0554	**4.204**	.4110	.0298	**.6101**	.1476	.0428	**.8952**	**.8952**	.0956	.5915	.2990	5.0	2.6	6.0
	NEIGH	.0541	1.311	.2608	.0588	4.637	.3691	.0276	**.6622**	.1319	.0403	.9491	.9491	.0964	.6260	.3050	4.4	4.0	4.8
	SP	.0659	2.401	.3591	.1901	17.87	1.123	.0312	.8624	.1424	.0417	1.013	1.013	.0931	.5808	.2727	5.4	4.8	5.4
	PPR	.0494	**1.089**	.2408	**.0527**	**4.001**	.3840	.0266	**.5993**	.1189	.0392	**.9029**	**.9029**	.0952	.5897	.2940	2.4	**1.8**	3.6
	SP NEIGH	.0646	2.582	.3470	.1911	18.12	1.136	.0291	.9191	.1295	.0393	1.078	1.078	.0937	.6329	.2838	4.8	6.6	4.4
	PPR NEIGH	**.0461**	**1.034**	**.1960**	**.0524**	**4.049**	**.3198**	**.0256**	**.6682**	.1129	**.0375**	.9611	.9611	.0956	.6225	.2980	**2.0**	3.2	2.6
APPNP RW	Unadjusted	.0465	1.750	**.1505**	.0659	5.638	**.3308**	.0293	1.197	**.0924**	.0504	2.016	2.016	**.0546**	**.4160**	**.0469**	4.2	5.0	**1.2**
	Adjusted	.0527	1.121	.2848	.0541	4.060	.3897	.0282	**.5726**	.1500	.0452	**.8693**	**.8693**	.0979	.5958	.3099	4.6	2.6	5.6
	NEIGH	.0512	1.229	.2319	.0583	4.580	.3633	.0276	.6387	.1347	.0456	.9643	.9643	.0999	.6302	.3230	4.8	4.0	5.2
	SP	.0617	2.301	.3308	.1885	17.81	1.111	.0303	.8591	.1515	.0436	1.004	1.004	.0952	.5857	.2935	5.2	5.0	5.0
	PPR	.0448	**.9489**	.2138	**.0501**	**3.735**	.3589	**.0255**	**.5650**	.1269	**.0418**	**.8731**	**.8731**	.0944	.5821	.2816	**1.4**	**1.6**	2.8
	SP NEIGH	.0617	2.484	.3198	.1902	18.09	1.126	.0297	.9365	.1391	.0439	1.135	1.135	.0964	.6327	.3067	5.4	6.6	5.4
	PPR NEIGH	**.0425**	**.8951**	.1641	**.0508**	**3.873**	**.3140**	**.0261**	.6387	.1193	**.0424**	.9700	.9700	.0965	.6157	.2963	2.4	3.2	2.8

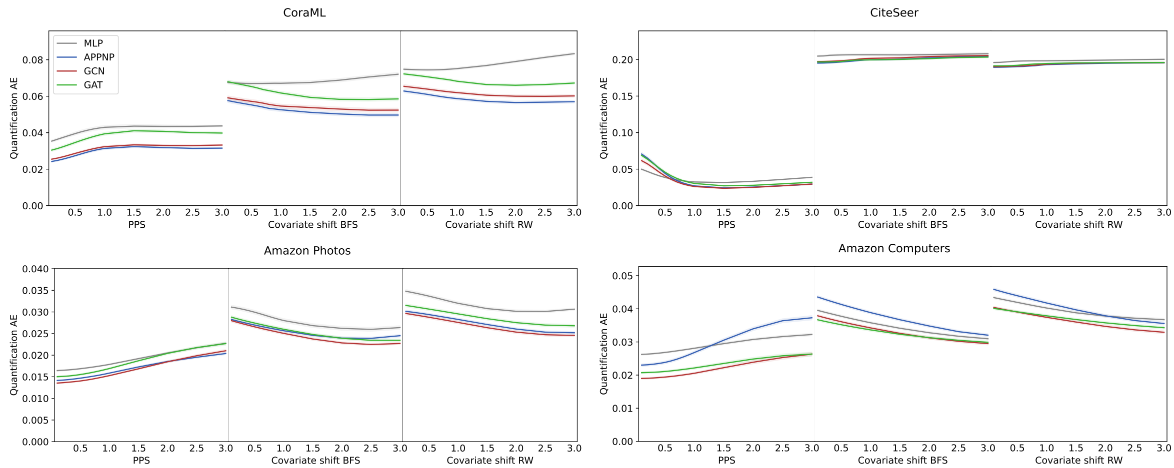

Figure 1: Quantification performance of SIS (with NACC) with the shortest-path kernel for different values of λ.

C STRUCTURAL IMPORTANCE SAMPLING FOR DISTRIBUTION MATCHING

In the conclusion of the main paper, we mentioned that the idea behind SIS could, in principle, also be applied to distribution matching approaches, such as KDEy [Moreo et al., 2025]. KDEy is a distribution matching approach that uses a kernel density estimator to estimate the distribution of class-conditional predictive distributions. Like ACC, it assumes PPS.

As pointed out by reviewers VuwU and U6qJ, an extension of SIS to distribution matching would be desirable. Since the submission, we have extended SIS accordingly and currently have a companion paper under review which describes this extension.

Here, we have included some of the results from the companion paper, showcasing the general effectiveness of SIS. More specifically, Table 2 compares *Probabilistic Adjusted Classify & Count* (PACC) and KDEy with and without SIS. Overall, the table shows that SIS is also effective in the distribution matching setting. KDEy without SIS tends out outperform PACC with SIS, while KDEy with SIS performs best overall. This further supports the claims made in the main paper.

D EVALUATION OF KERNEL HYPERPARAMETERS

In the main paper, we describe two possible choices for the vertex kernel k, the shortest-path and the PPR kernel. Both approaches can be seen as families of kernels, where many variations are possible. In this section we evaluate the impact of those hyperparameters on the quantification performance.

D.1 IMPACT OF λ ON THE SHORTEST-PATH KERNEL

We define the shortest-path kernel as

$$k_{\mathrm{SP}}(v, v') = \exp(-\lambda \cdot d_{\mathrm{SP}}(v, v')) \,, \tag{1}$$

where $d_{\mathrm{SP}}(v, v')$ is the length of the shortest path length between v and v' and $\lambda > 0$ a tunable hyperparameter.

Figure 1 shows the quantification performance of SIS with the shortest-path kernel for different values of λ. Under PPS, increasing λ leads to a decrease in performance on all datasets except CiteSeer. Under covariate shift (both, BFS and RW), increasing λ generally improves the quantification performance.

This is plausible, since under PPS, all training vertices are equally important, while under covariate shift, the training vertices that are close to the test vertices are more important than those that are far away. By increasing λ, the kernel becomes more peaked around close vertices. Under covariate shift, where far-away vertices are less important, this leads to better quantification performance, while under PPS more aggressive reweighting decreases performance.

Table 2: Comparison of PACC and KDEy with and without SIS (using the PPR kernel).

Model & Shift	Quantifier	CoraML			CiteSeer			Amazon Photos			Amazon Computers			PubMed			Avg. Rank		
		AE	RAE	KLD	AE	RAE	KLD	AE	RAE	KLD	AE	RAE	KLD	AE	RAE	KLD	AE	RAE	KLD
PPS	MLPE	.0903	.5692	.1605	.0469	.2623	.0396	.0924	.6660	.2107	.0770	.5358	.1952	.1268	.3948	.0834	-	-	-
EgoNet PPS	PCC	**.0750**	.8692	**.1960**	**.0429**	**.3543**	**.0547**	.0480	1.319	.1199	.0453	.5297	.1464	.1159	.4764	**.0931**	3.4	4.2	3.4
	PACC	.0848	**.6863**	.4721	.0927	.5928	.3884	**.0314**	**.3881**	**.0707**	.0334	.3437	.1199	**.0833**	**.3222**	**.0823**	3.6	3.2	4.0
	PPR PACC	.0837	**.6832**	.4571	.0916	.5871	.3761	**.0313**	**.3925**	**.0703**	.0333	.3433	.1190	**.0817**	**.3183**	**.0769**	**2.6**	2.6	2.6
	KDEy	**.0808**	**.6770**	.4343	.0970	.6279	.5134	**.0290**	**.3756**	**.0566**	**.0312**	**.3227**	**.0958**	**.0838**	**.3317**	**.0788**	**2.6**	**2.2**	2.8
	KDEy PPR	**.0805**	**.6768**	.4294	.0971	.6294	.5138	**.0290**	**.3772**	**.0564**	**.0312**	**.3229**	**.0957**	**.0837**	**.3324**	**.0778**	2.8	2.8	**2.2**
MLP PPS	PCC	.0827	.8565	.2522	.0361	.2782	**.0372**	.0497	1.105	.1316	.0533	.6342	.2015	.0470	.1870	.0161	5.0	5.0	4.0
	PACC	**.0481**	**.4186**	**.1404**	**.0336**	**.2271**	**.0384**	.0191	.3036	**.0255**	**.0334**	**.3690**	**.1337**	**.0181**	**.0649**	**.0027**	2.6	3.0	2.6
	PPR PACC	.0486	.4237	**.1403**	**.0327**	**.2218**	**.0366**	.0192	.2881	**.0261**	**.0338**	**.3708**	**.1372**	**.0179**	**.0641**	**.0026**	2.8	2.6	**1.6**
	KDEy	**.0469**	**.4076**	**.1383**	.0345	**.2289**	**.0391**	**.0178**	**.2642**	.0305	.0389	.4072	.2098	**.0178**	**.0623**	**.0026**	**2.2**	2.6	3.2
	KDEy PPR	**.0471**	**.4095**	**.1406**	**.0339**	**.2266**	**.0381**	**.0180**	**.2633**	.0309	.0388	.4055	.2094	**.0178**	**.0622**	**.0026**	2.4	**1.8**	3.6
GAT PPS	PCC	.0479	.5323	.0933	.0219	.1573	.0143	.0314	.9570	.0627	.0398	.4674	.1266	.0463	.1911	.0163	5.0	5.0	5.0
	PACC	.0297	.2660	.0485	.0192	.1262	.0111	.0147	.2776	.0190	**.0217**	**.2326**	**.0571**	.0176	.0635	.0026	3.6	3.8	3.4
	PPR PACC	.0290	.2635	.0453	**.0181**	.1200	**.0099**	.0148	.2901	.0190	**.0214**	**.2299**	**.0544**	.0174	.0630	.0025	2.6	2.6	2.6
	KDEy	.0254	**.2296**	.0355	.0185	.1208	.0102	**.0132**	**.2055**	**.0158**	**.0217**	**.2311**	.0664	**.0166**	**.0593**	**.0022**	2.4	2.2	2.6
	KDEy PPR	**.0246**	**.2267**	**.0317**	**.0178**	**.1168**	**.0094**	**.0132**	**.2078**	**.0154**	**.0216**	**.2302**	.0660	**.0164**	**.0585**	**.0021**	**1.4**	**1.4**	**1.4**
GCN PPS	PCC	.0438	.4697	.0809	.0221	.1574	.0145	.0315	.8508	.0639	.0391	.4667	.1074	.0405	.1665	.0125	5.0	5.0	5.0
	PACC	.0246	.2216	.0359	.0190	.1259	.0109	.0122	.2056	.0129	**.0228**	.2411	.0581	.0161	.0591	.0023	3.8	3.8	3.6
	PPR PACC	.0234	.2163	.0300	**.0178**	**.1186**	**.0095**	.0124	.2295	.0127	**.0223**	**.2386**	**.0542**	.0160	.0584	.0023	3.0	3.0	2.4
	KDEy	**.0212**	**.1971**	**.0236**	.0181	**.1197**	.0100	**.0102**	**.1430**	**.0095**	**.0223**	**.2325**	.0930	**.0152**	**.0553**	**.0020**	2.0	1.8	2.4
	KDEy PPR	**.0210**	**.1984**	**.0230**	**.0174**	**.1159**	**.0093**	**.0103**	**.1435**	**.0094**	**.0222**	**.2323**	.0933	**.0149**	**.0545**	**.0019**	**1.2**	**1.4**	**1.6**
APPNP PPS	PCC	.0374	.4124	.0598	.0214	.1509	.0133	.0318	.9795	.0682	.0390	.4657	.1072	.0398	.1664	.0125	5.0	5.0	4.6
	PACC	.0217	.1986	.0251	.0184	.1211	.0100	.0124	.2442	.0136	**.0256**	**.2638**	**.0986**	.0165	.0597	.0023	3.4	3.4	3.2
	PPR PACC	.0203	.1926	**.0205**	**.0171**	**.1132**	**.0087**	.0130	.3058	.0172	**.0249**	**.2566**	**.0929**	.0163	.0589	.0022	2.4	2.4	**2.4**
	KDEy	**.0193**	**.1783**	**.0200**	.0181	.1194	.0101	**.0102**	**.1535**	**.0105**	.0299	.3088	.1857	**.0154**	**.0550**	**.0019**	**2.0**	**2.0**	**2.4**
	KDEy PPR	**.0194**	**.1812**	**.0200**	**.0173**	**.1145**	.0092	**.0105**	**.1612**	.0123	.0300	.3089	.1894	**.0153**	**.0548**	**.0018**	2.2	2.2	**2.4**
BFS	MLPE	.1839	8.185	1.123	.2676	15.22	1.475	.1622	9.313	1.168	.1180	5.666	1.159	.3034	5.067	.5129	-	-	-
EgoNet BFS	PCC	.1440	8.886	.7681	.2374	22.27	1.324	.0566	3.686	**.2027**	.0455	2.274	**.2330**	.2313	**26.15**	**.4463**	5.0	4.8	**2.0**
	PACC	.1005	**5.140**	**.7202**	.1908	17.87	**1.086**	.0388	**1.112**	**.2270**	.0419	**1.190**	.3405	.2108	**24.61**	.7576	3.4	2.6	3.0
	PPR PACC	.1009	**5.351**	**.6656**	.1909	17.93	**1.085**	.0385	**1.125**	**.2230**	.0413	**1.192**	.3326	.2119	**24.96**	.7615	3.6	3.6	2.4
	KDEy	**.0847**	**4.742**	.8998	**.1786**	**16.73**	2.435	**.0280**	**1.052**	**.2283**	**.0316**	**1.182**	.3070	**.1667**	**25.75**	1.020	1.8	2.2	4.4
	KDEy PPR	**.0840**	**4.399**	**.7595**	**.1747**	**16.33**	2.363	**.0267**	**.9768**	**.2028**	**.0313**	**1.165**	**.3007**	**.1836**	**30.41**	1.165	**1.2**	**1.8**	3.2
MLP BFS	PCC	.1243	7.212	.5915	.1588	14.84	**.8652**	.0668	4.028	.2597	.0662	3.635	.4111	.0800	10.44	**.1152**	4.8	5.0	3.4
	PACC	.0645	3.508	.5452	.1158	10.63	1.035	.0237	.9928	**.0963**	.0392	1.608	.3212	.0816	**7.663**	.2417	3.8	3.6	2.8
	PPR PACC	.0637	3.461	**.4981**	.1162	10.74	1.033	.0222	.9079	**.0882**	**.0370**	1.509	**.2932**	.0786	**7.827**	.2295	2.8	3.4	**1.4**
	KDEy	**.0547**	**2.883**	**.5332**	**.1015**	**9.315**	1.360	**.0191**	**.6689**	.1041	.0394	**1.069**	.3890	**.0772**	**7.218**	.3295	2.0	1.6	3.8
	KDEy PPR	**.0552**	**2.972**	.5385	**.1039**	**9.621**	1.382	**.0187**	**.6592**	**.1001**	**.0374**	**1.023**	.3647	**.0743**	**7.146**	.3210	**1.6**	**1.4**	3.6
GAT BFS	PCC	.0741	4.840	**.3079**	.0820	7.349	**.4548**	.0291	1.757	**.0979**	.0455	2.415	.2347	**.0650**	9.922	**.1113**	4.2	5.0	**1.6**
	PACC	.0561	2.533	.3454	.0656	5.347	.5106	.0243	.7255	.1269	.0331	.9463	.2291	.0930	6.906	.3168	4.2	4.0	3.0
	PPR PACC	.0500	2.265	**.3003**	.0616	5.015	.4840	.0226	.7147	.1134	.0312	.9381	**.2102**	.0922	6.898	.3110	3.2	3.0	**1.6**
	KDEy	.0449	**1.735**	.3791	.0520	4.118	.7208	.0212	**.6491**	.1416	.0305	**.8432**	.2823	.0855	**5.659**	.3514	2.2	1.8	4.8
	KDEy PPR	**.0405**	**1.568**	.3280	**.0465**	**3.665**	.6381	**.0200**	**.6183**	.1305	**.0295**	**.8084**	.2697	.0850	**6.669**	.3679	**1.2**	**1.2**	4.0
GCN BFS	PCC	.0539	3.489	**.2180**	.0783	7.060	**.4402**	.0256	1.513	**.0799**	.0418	2.255	**.2021**	**.0573**	9.553	**.1022**	4.2	5.0	**1.0**
	PACC	.0488	2.093	.2866	.0637	5.267	.5169	.0241	.5966	.1147	.0401	.9320	.3295	.0888	6.713	.2894	4.2	3.8	4.0
	PPR PACC	.0415	1.943	**.2334**	.0618	5.132	.4975	.0207	.5932	.0912	.0358	.9569	.2763	.0881	6.706	.2856	3.2	3.2	2.2
	KDEy	.0355	**1.340**	.2797	.0555	**4.533**	.7772	**.0174**	**.5114**	.1049	.0326	**.7999**	.3485	.0716	**4.876**	.2716	2.2	1.6	3.8
	KDEy PPR	**.0340**	**1.376**	.2803	**.0513**	**4.211**	.7114	**.0167**	**.4799**	.0972	**.0315**	**.7807**	.3320	.0716	**5.732**	.2903	**1.2**	**1.4**	4.0
APPNP BFS	PCC	.0469	3.074	**.1871**	.0737	6.609	**.4078**	.0271	1.492	**.0829**	.0468	2.339	**.2448**	**.0569**	9.867	**.1055**	4.2	5.0	**1.0**
	PACC	.0457	1.881	.2682	.0603	4.944	.4750	.0225	.5731	.1114	.0430	.9227	.3620	.0927	7.449	.3091	4.2	3.6	4.0
	PPR PACC	.0380	1.729	.2126	.0574	4.705	.4466	.0213	.5823	.0968	.0395	.9331	.3235	.0874	7.372	.2726	3.2	3.4	2.2
	KDEy	.0334	**1.143**	.2452	.0506	4.023	.7147	**.0168**	**.4527**	.0964	.0362	**.7739**	.3782	.0735	**5.278**	.2874	1.8	1.6	3.8
	KDEy PPR	**.0304**	**1.071**	.2197	**.0457**	**3.671**	.6344	**.0176**	**.4705**	.0997	**.0351**	**.7556**	.3677	.0746	6.468	.3162	**1.6**	**1.4**	4.0
RW	MLPE	.1832	6.430	1.079	.2651	15.07	1.450	.1594	8.278	1.169	.1158	4.466	1.168	.3025	3.073	.4840	-	-	-
EgoNet RW	PCC	.1355	6.275	**.6304**	.2237	21.00	1.151	.0538	2.895	**.1910**	.0449	1.703	**.2140**	.2171	4.937	**.3181**	4.6	5.0	**1.6**
	PACC	.0973	2.916	.6851	**.1477**	**13.48**	**.8540**	.0476	**1.061**	.2953	.0481	**1.052**	.3802	.1506	**1.716**	.6573	3.8	3.0	4.4
	PPR PACC	.0971	2.939	**.6713**	**.1484**	**13.55**	**.8433**	.0474	**1.076**	.2916	.0478	**1.058**	.3734	.1508	**1.756**	.6371	3.6	4.0	3.4
	KDEy	**.0835**	**2.542**	**.6692**	**.1401**	**12.53**	1.885	**.0316**	**.8485**	**.2150**	**.0354**	**.9660**	.2977	**.1330**	**1.370**	.4807	1.6	1.6	3.0
	KDEy PPR	**.0801**	**2.234**	**.5922**	**.1367**	**12.15**	1.815	**.0314**	**.8211**	**.2062**	**.0354**	**.9730**	.3001	**.1340**	**1.386**	.4824	**1.4**	**1.4**	2.6
MLP RW	PCC	.1263	5.275	.5574	.1494	13.84	.6561	.0727	3.820	.2918	.0718	3.224	.4531	.0913	1.376	**.0820**	5.0	5.0	3.8
	PACC	.0733	2.347	.4164	.0869	7.425	**.5857**	.0332	1.251	**.1277**	.0471	1.837	.3737	.0882	.7452	.2360	3.6	3.4	3.0
	PPR PACC	.0743	2.417	.4213	.0899	7.766	**.5873**	.0315	1.144	**.1194**	.0450	1.765	**.3512**	**.0864**	.7580	.2194	3.2	3.6	2.6
	KDEy	**.0639**	**1.667**	**.3454**	**.0799**	**6.857**	.7411	.0261	**.7495**	**.1267**	.0457	**1.235**	.4296	**.0863**	**.6486**	.2171	1.8	**1.4**	3.4
	KDEy PPR	**.0653**	**1.713**	**.3387**	**.0824**	7.225	.7388	**.0245**	**.6922**	**.1180**	**.0438**	**1.171**	.4025	**.0845**	**.6638**	.2034	**1.4**	1.6	**2.2**
GAT RW	PCC	.0799	3.555	.2837	.0766	6.693	**.3655**	.0340	1.689	**.1125**	.0500	2.195	**.2548**	**.0691**	.7488	**.0575**	4.2	5.0	**1.8**
	PACC	.0610	1.648	.3237	.0594	4.563	.4000	.0293	.7767	.1479	.0382	.9639	.2656	.0952	.6084	.2958	3.8	3.8	4.0
	PPR PACC	.0550	1.427	.2744	.0554	4.203	**.3710**	.0277	.7664	.1326	.0366	.9656	**.2462**	.0946	**.6062**	.2913	2.8	3.0	**1.8**
	KDEy	.0490	.8774	.2828	.0475	3.353	.5716	.0239	**.6194**	.1439	.0330	**.7808**	.2862	.1035	**.6065**	.3141	2.6	2.0	4.4
	KDEy PPR	**.0443**	**.7559**	**.2314**	**.0425**	**2.938**	.4984	**.0226**	**.5964**	.1332	**.0321**	**.7815**	.2768	.0999	**.5951**	.2936	**1.6**	**1.2**	3.0
GCN RW	PCC	.0539	2.085	.1841	.0694	5.990	**.3489**	.0276	1.247	**.0859**	.0451	1.961	**.2124**	**.0566**	**.4972**	**.0475**	3.8	4.2	**1.2**
	PACC	.0571	1.267	.3115	.0554	4.204	.4110	.0298	.6101	.1476	.0428	.8952	.3455	.0956	.5915	.2990	4.6	4.0	4.6
	PPR PACC	.0494	1.089	.2408	.0527	4.001	.3840	.0266	.5993	.1189	.0392	.9029	.2972	.0952	.5897	.2940	3.2	3.4	3.2
	KDEy	.0412	.6563	.2056	.0489	3.550	.5825	.0190	**.4836**	.1080	.0325	**.7496**	.3288	.0904	.5459	.2344	2.2	2.2	3.6
	KDEy PPR	**.0373**	**.5533**	**.1702**	**.0432**	**3.092**	.5025	**.0182**	**.4516**	**.0972**	**.0314**	**.7375**	.3132	.0870	.5252	.2117	**1.2**	**1.2**	2.4
APPNP RW	PCC	.0465	1.750	**.1505**	.0659	5.638	**.3308**	.0293	1.197	**.0924**	.0504	2.016	**.2626**	**.0546**	**.4160**	**.0469**	4.0	4.2	**1.2**
	PACC	.0527	1.121	.2848	.0541	4.060	.3897	.0282	.5726	.1500	.0452	.8693	.3579	.0979	.5958	.3099	4.4	4.0	4.2
	PPR PACC	.0448	.9489	.2138	.0501	3.735	.3589	.0255	.5650	.1269	.0418	.8731	.3181	.0944	.5821	.2816	3.2	3.4	3.2
	KDEy	.0388	.5171	.1843	.0468	3.225	.5719	**.0197**	**.4548**	.1144	.0378	**.7761**	.3819	.0941	.5520	.2483	2.2	2.2	3.8
	KDEy PPR	**.0340**	**.4052**	**.1455**	**.0418**	**2.879**	.5018	**.0193**	**.4339**	.1075	**.0362**	**.7676**	.3665	.0902	.5327	.2279	**1.2**	**1.2**	2.6

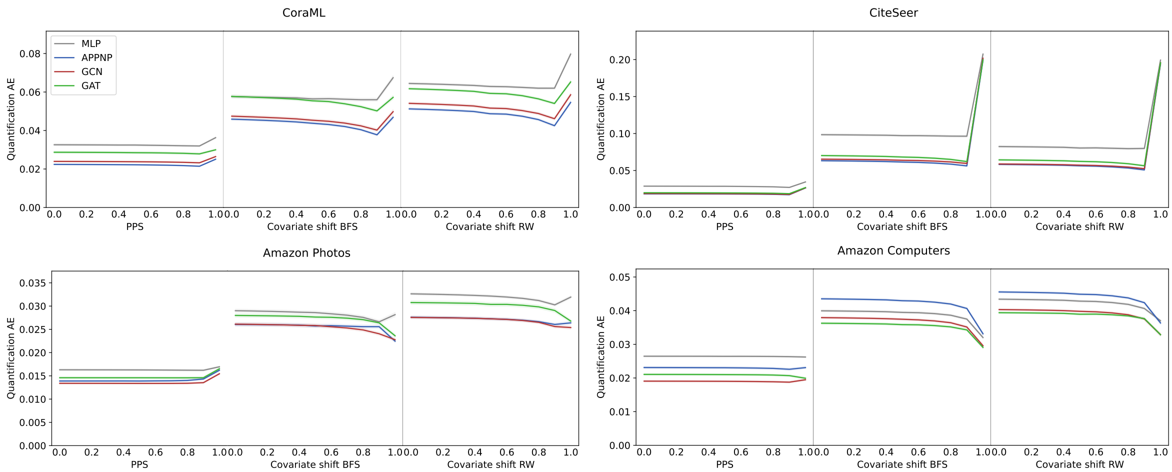

Figure 2: Quantification performance of SIS (with NACC) with the PPR kernel for different values of λ.

D.2 IMPACT OF LINEAR INTERPOLATION ON THE PERSONALIZED PAGERANK KERNEL

We define the interpolated PPR kernel as

$$k_{\text{PPR}}(v, v') = \lambda \Pi^L_{v',v} + (1 - \lambda) \;, \tag{2}$$

where $\Pi^L_{v',v}$ is the PPR kernel between v and v' and $\lambda \in [0, 1]$ a tunable hyperparameter. In the main paper, we did not define the kernel with the interpolation parameter λ, i.e., we chose $\lambda = 1$ as the default value.

Since the original submission we found, that this choice is not always optimal. Intuitively, λ controls the influence of training vertices $\mathcal{D}_L$ that are far-away from the test vertices $\mathcal{X}_U$. If $\lambda = 1$, far-away vertices are effectively ignored. If $\lambda < 1$, all vertices are considered at least to some degree, but the influence of far-away vertices is reduced. A large λ can have the advantage of reducing the influence of irrelevant or misleading vertices from different regions of the graph. However, if too many vertices are excluded, the effective sample size for the confusion matrix estimate is reduced, making it more noisy, which can, in turn, degrade performance.

Figure 2 shows the quantification performance of SIS with the PPR kernel for different values of λ. For CoraML and CiteSeer, $\lambda < 1$ clearly outperforms $\lambda = 1$, which $\lambda \approx 0.9$ performing very well. For the Amazon Photos and Computers datasets, $\lambda = 1$ performs best.

Since the main paper reports results for $\lambda = 1$, we show results for $\lambda = 0.9$ in Table 1 and Table 2. We plan to unify the results for different parameterizations, to simplify comparison.

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