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### **Master Thesis**

### **Learning to Aggregate on Structured Data**

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# **Abstract**

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Introduction

### 1.1 Motivation

The field of *machine learning* (ML) on graph-structured data has applications in many domains due to the general expressive power of graphs. Three common types of graph ML problems are

- **1. Link prediction:** A graph with an incomplete edge set is given and the missing edges have to be predicted. The generation of friendship suggestions in a social network is a typical example for this.
- **2. Vertex classification & regression:** Here a class or a score has to be predicted for each vertex of a graph. In social graphs this corresponds to the prediction of properties of individuals. Another example is the prediction of the amount of traffic at the intersections of a street network.
- **3. Graph classification & regression:** In this final problem type a single global class or continuous value has to be predicted for an input graph. The canonical example for this is the prediction of properties of molecule graphs, e.g. the toxicity or solubility of a chemical.

In this thesis we will focus on the last problem type, graph classification and regression. A ML method for this problem has to accept graphs of varying size and should be permutation invariant wrt. the vertices. Those requirements are not met by most of the commonly used learners that only accept fixed-size feature vectors as their input, e.g. logistic models, support vector machines or multilayer perceptrons.

1.2 Goals

1.3 Structure

**Chapter 2: Related Work** 

**Chapter 3: Learning to Aggregate on Graphs** 

**Chapter 4: Evaluation** 

### **Chapter 5: Conclusion**

Related Work

- 2.1 Learning to Aggregate
- 2.2 Graph Kernels
- 2.3 Graph Neural Networks
- 2.3.1 Spatial GNNs
- 2.3.2 Spectral GNNs

Learning to Aggregate on Graphs

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- 3.1 Formalization of LTA Methods
- 3.2 An LTA Interpretation of Graph Methods
- 3.3 LTA on Dynamically Decomposed Graphs

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Conclusion

- 5.1 Review
- 5.2 Future Directions

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### Erklärung zur Masterarbeit

Ich, Clemens Damke (Matrikel-Nr. 7011488), versichere, dass ich die Masterarbeit mit dem Thema *Learning to Aggregate on Structured Data* selbstständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe. Die Stellen der Arbeit, die ich anderen Werken dem Wortlaut oder dem Sinn nach entnommen habe, wurden in jedem Fall unter Angabe der Quellen der Entlehnung kenntlich gemacht. Das Gleiche gilt auch für Tabellen, Skizzen, Zeichnungen, bildliche Darstellungen usw. Die Masterarbeit habe ich nicht, auch nicht auszugsweise, für eine andere abgeschlossene Prüfung angefertigt. Auf § 63 Abs. 5 HZG wird hingewiesen.

Paderborn, 7. November 2019	
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