Linear-Time Suffix-Sorting

Proseminar Datenkompression

bei Prof. Böttcher – WS 16/17 – Clemens Damke

Konstruktion eines **Suffix Arrays** mit einem **rekursionsfreien Linearzeit-Algorithmus**.

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Was ist ein Suffix Array?

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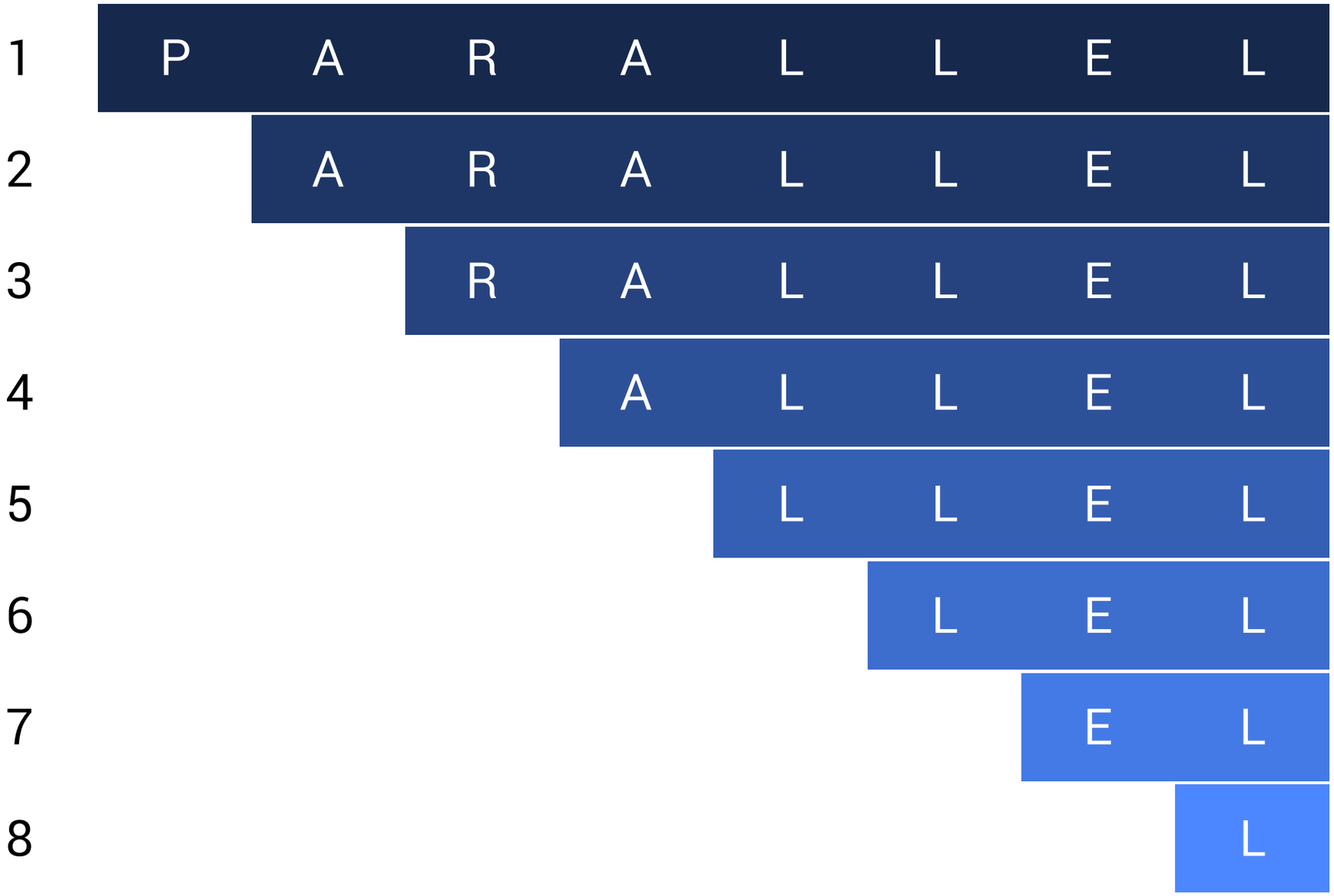

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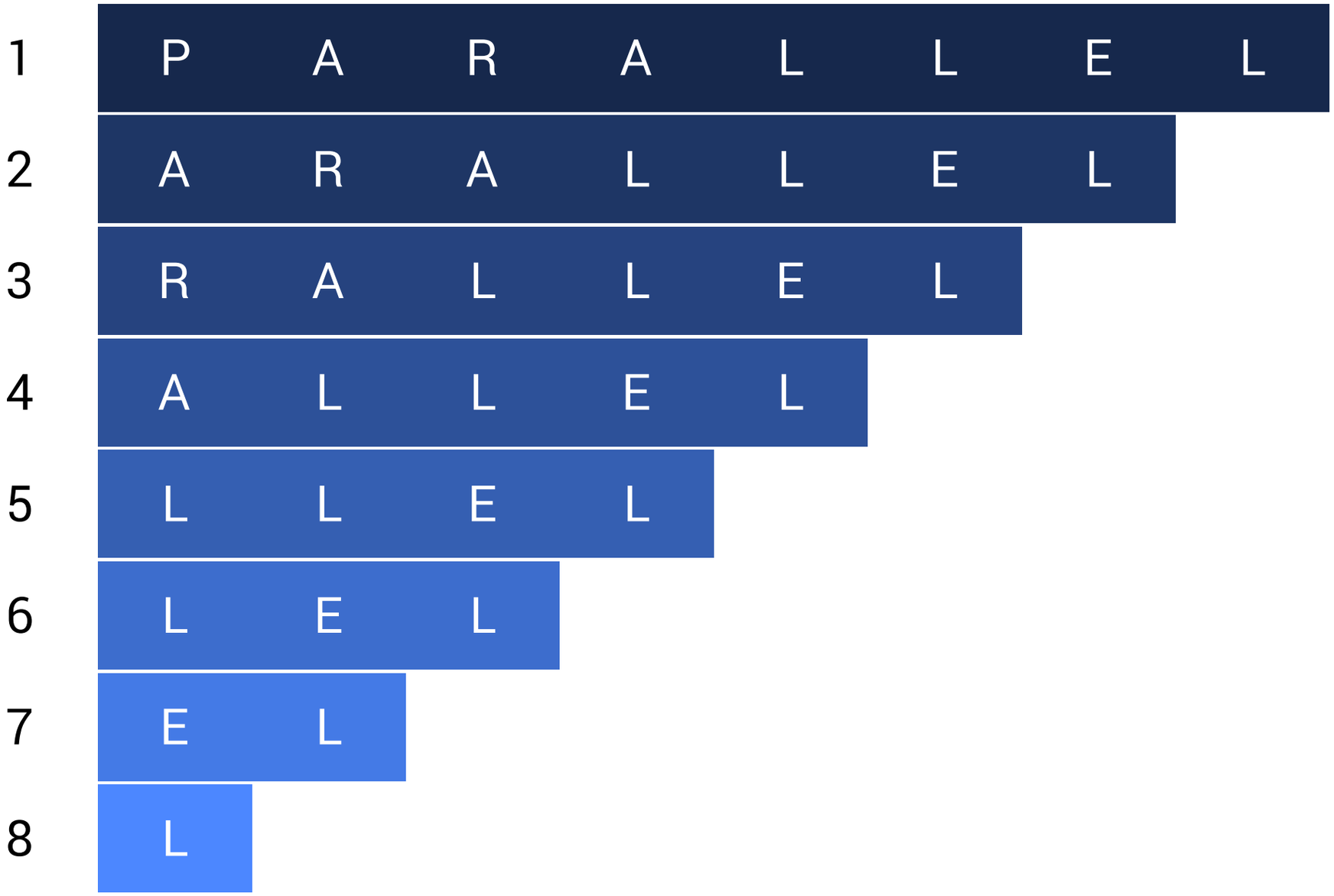

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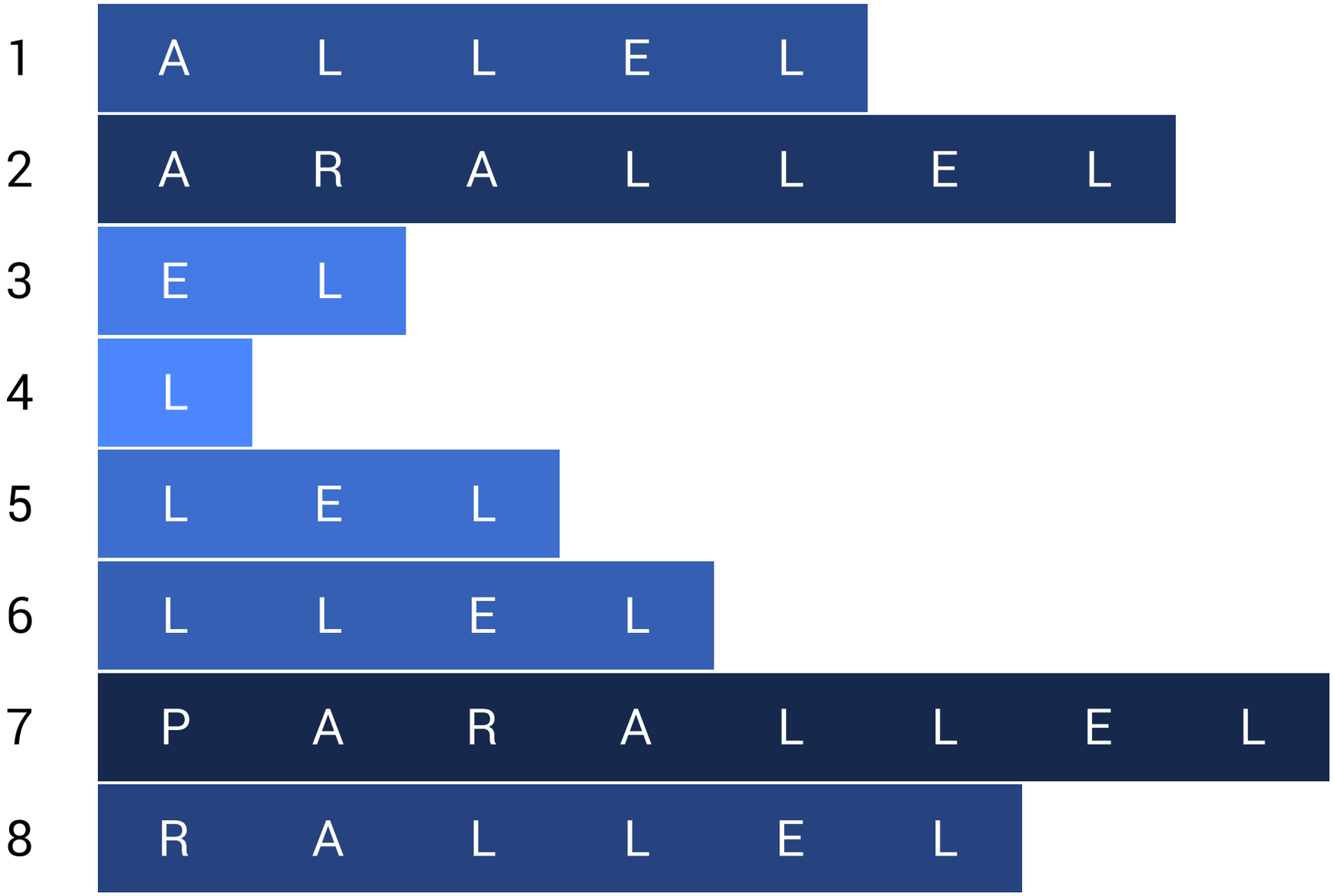

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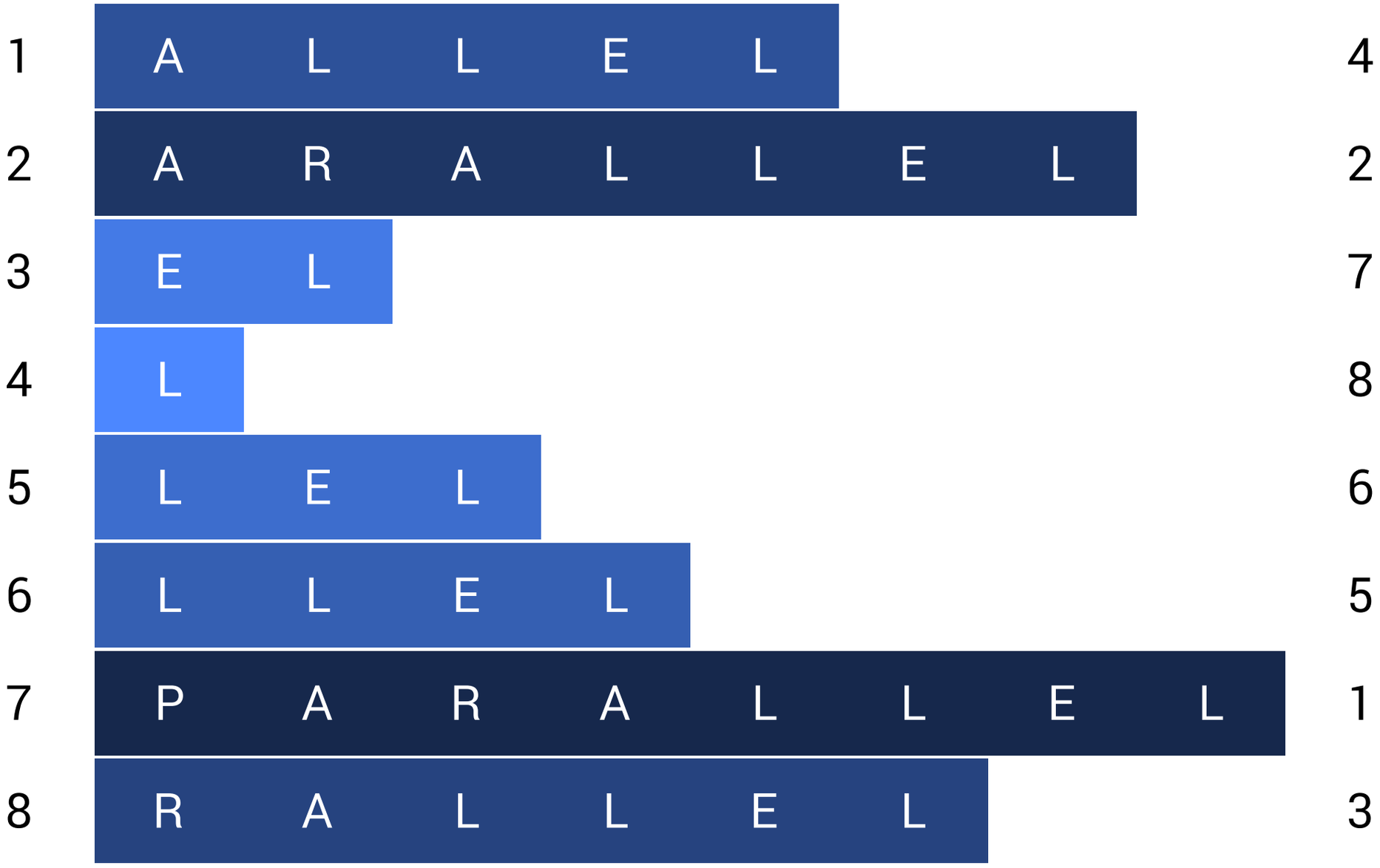
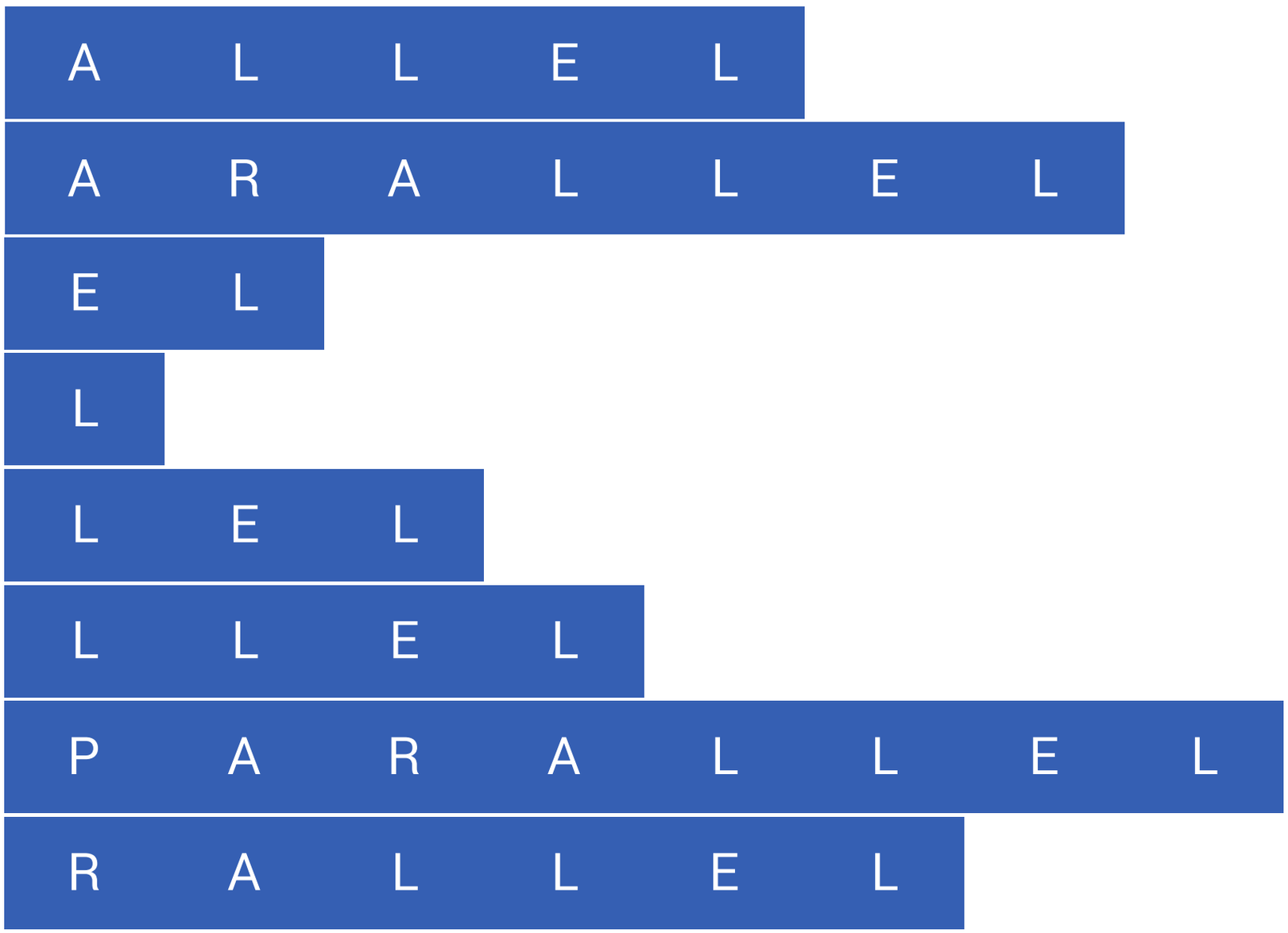


Einsatzgebiete

Substringsuche

Ist *alle* in *parallel* enthalten?



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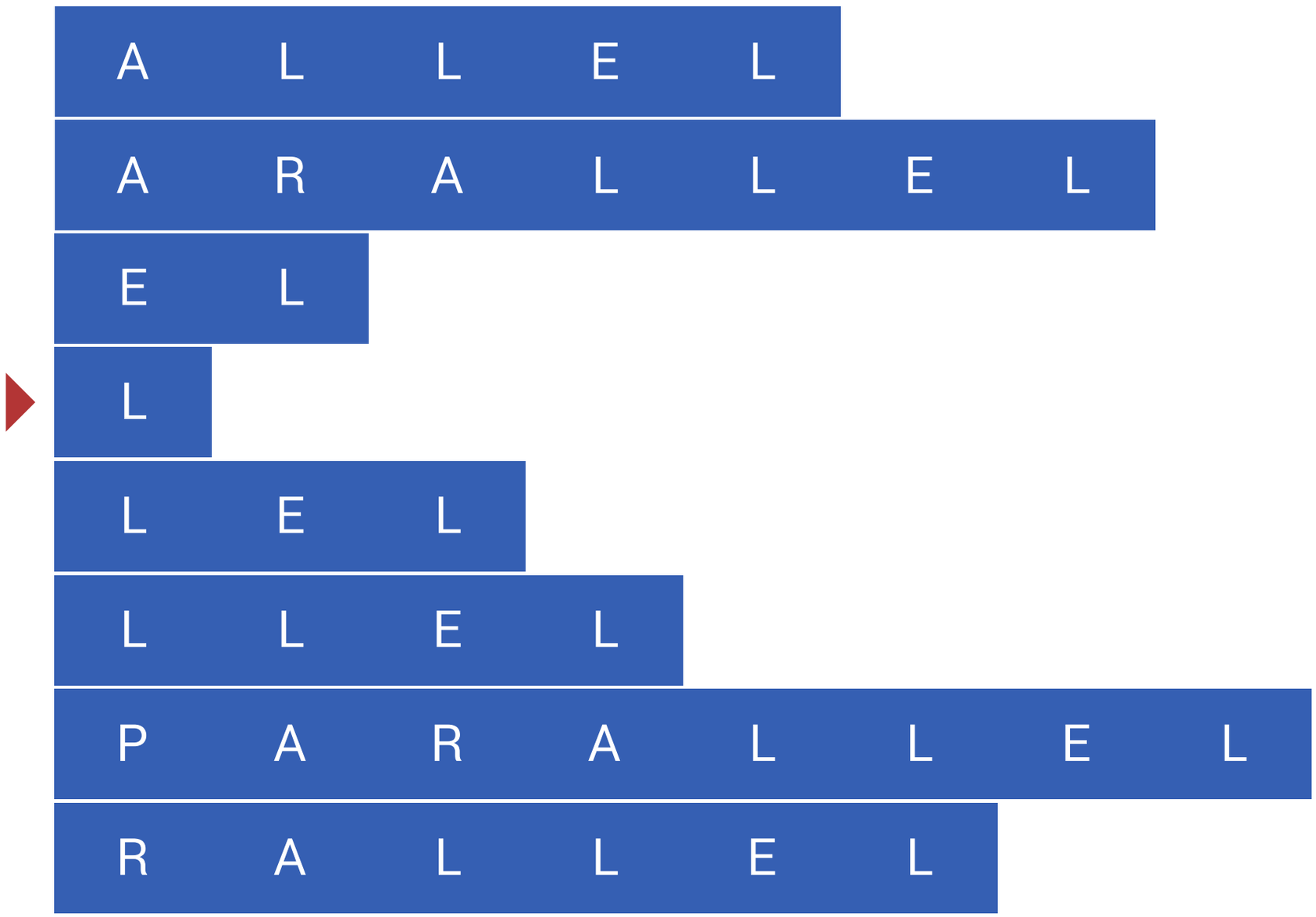

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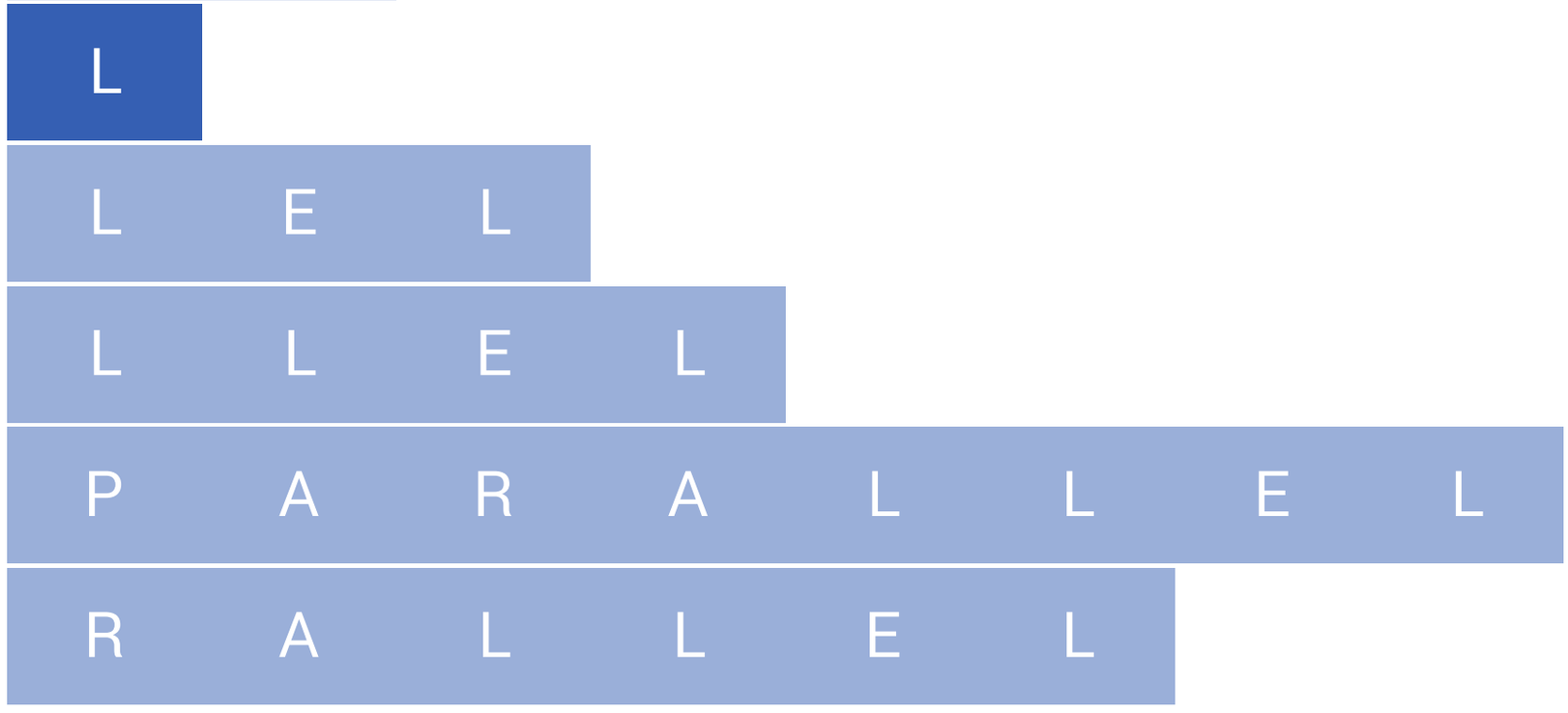

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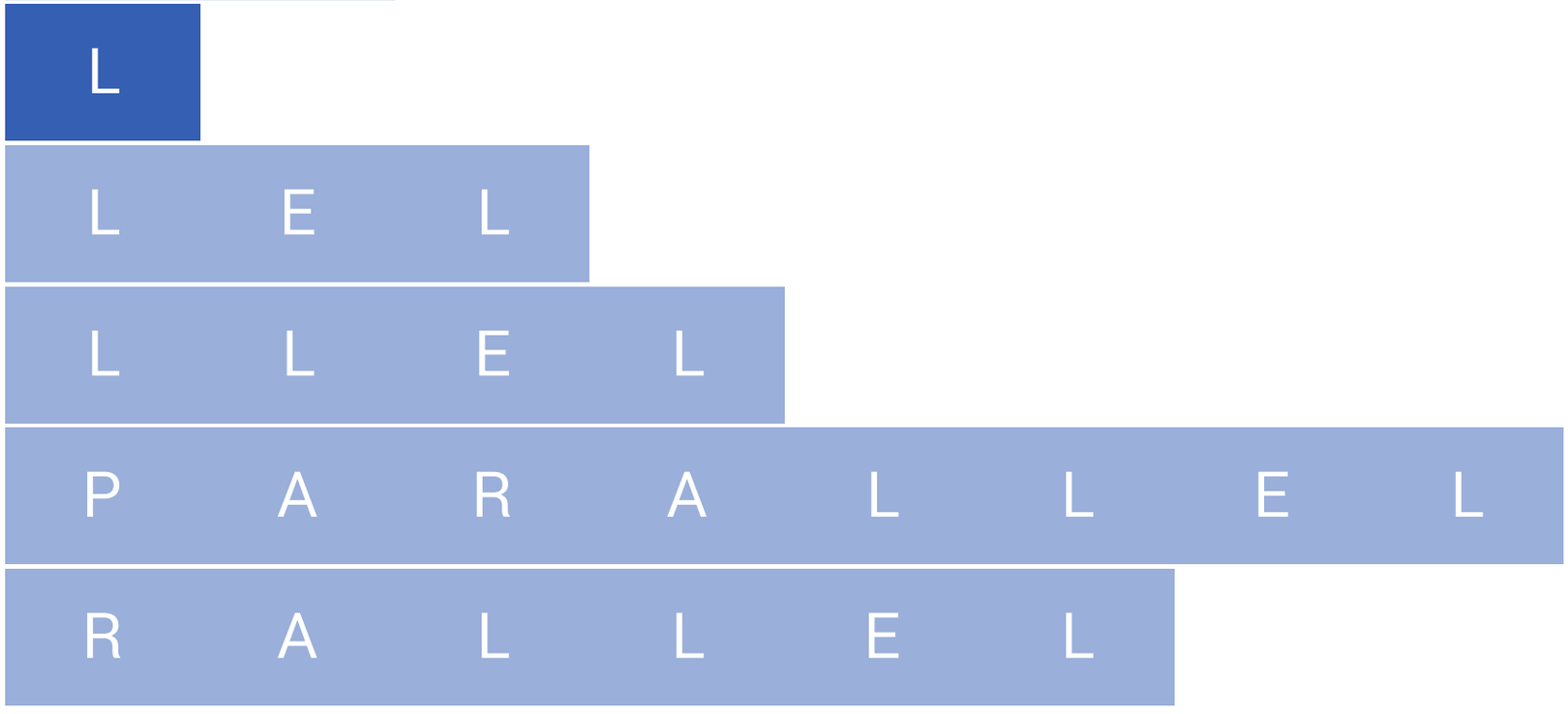

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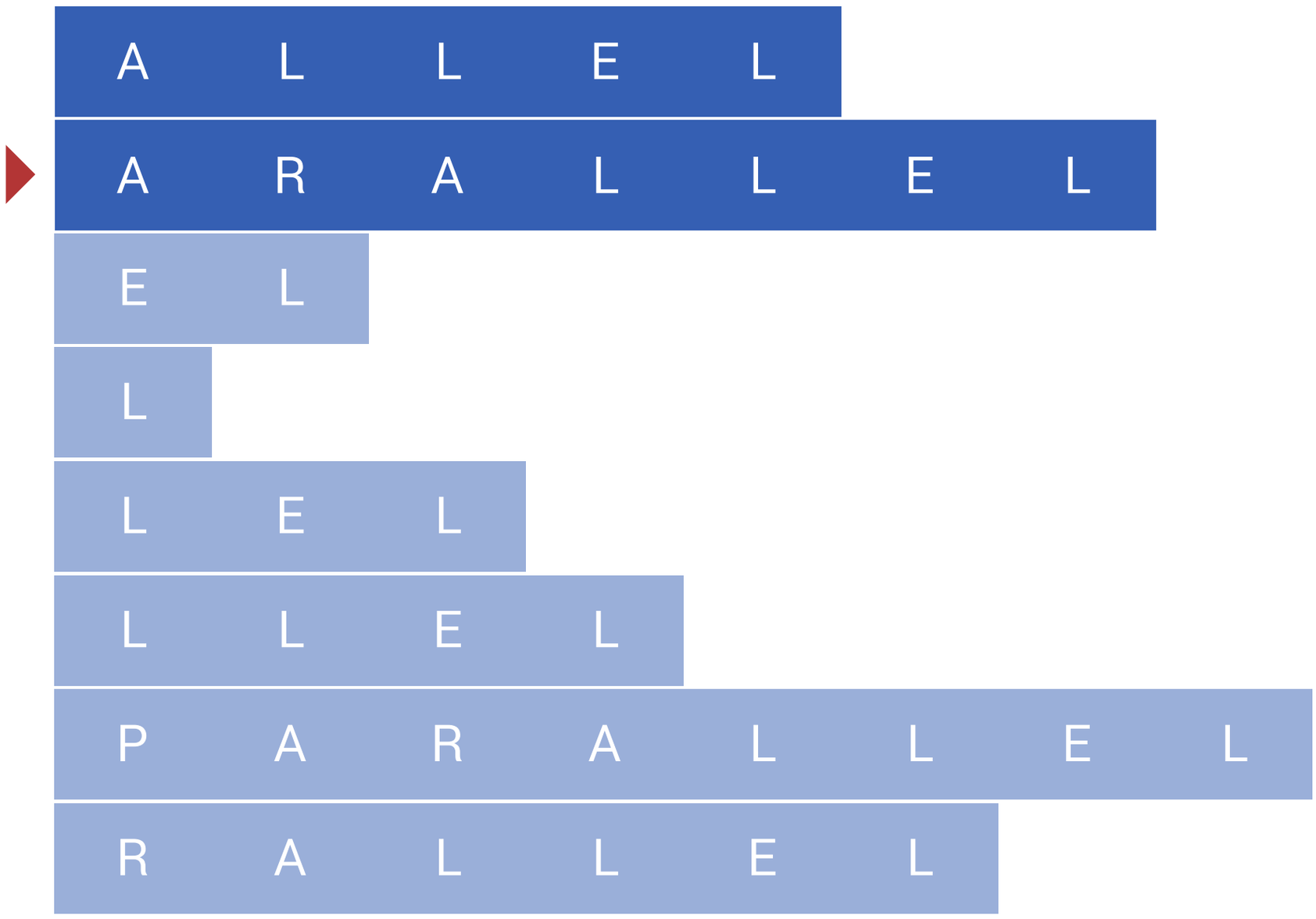

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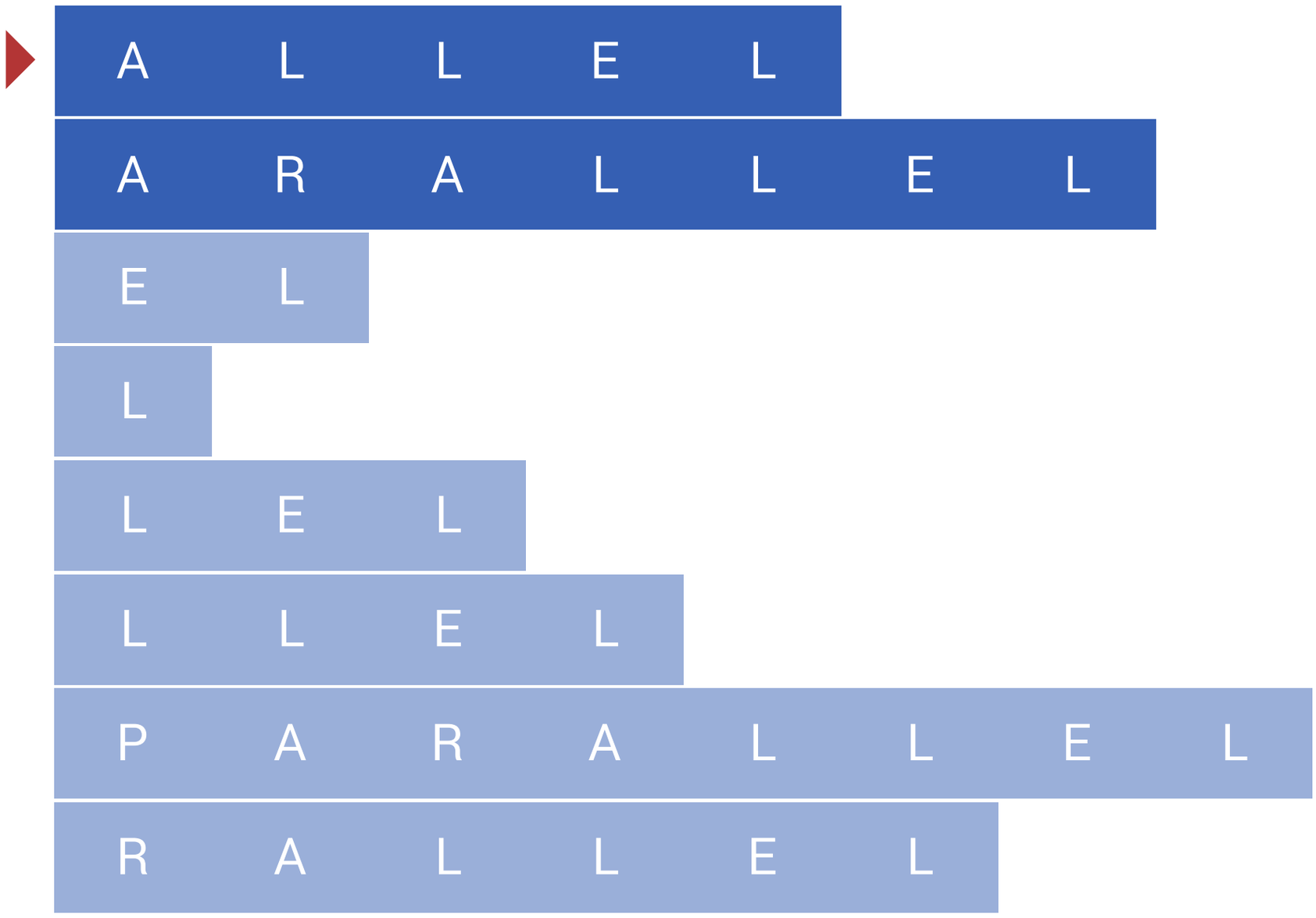

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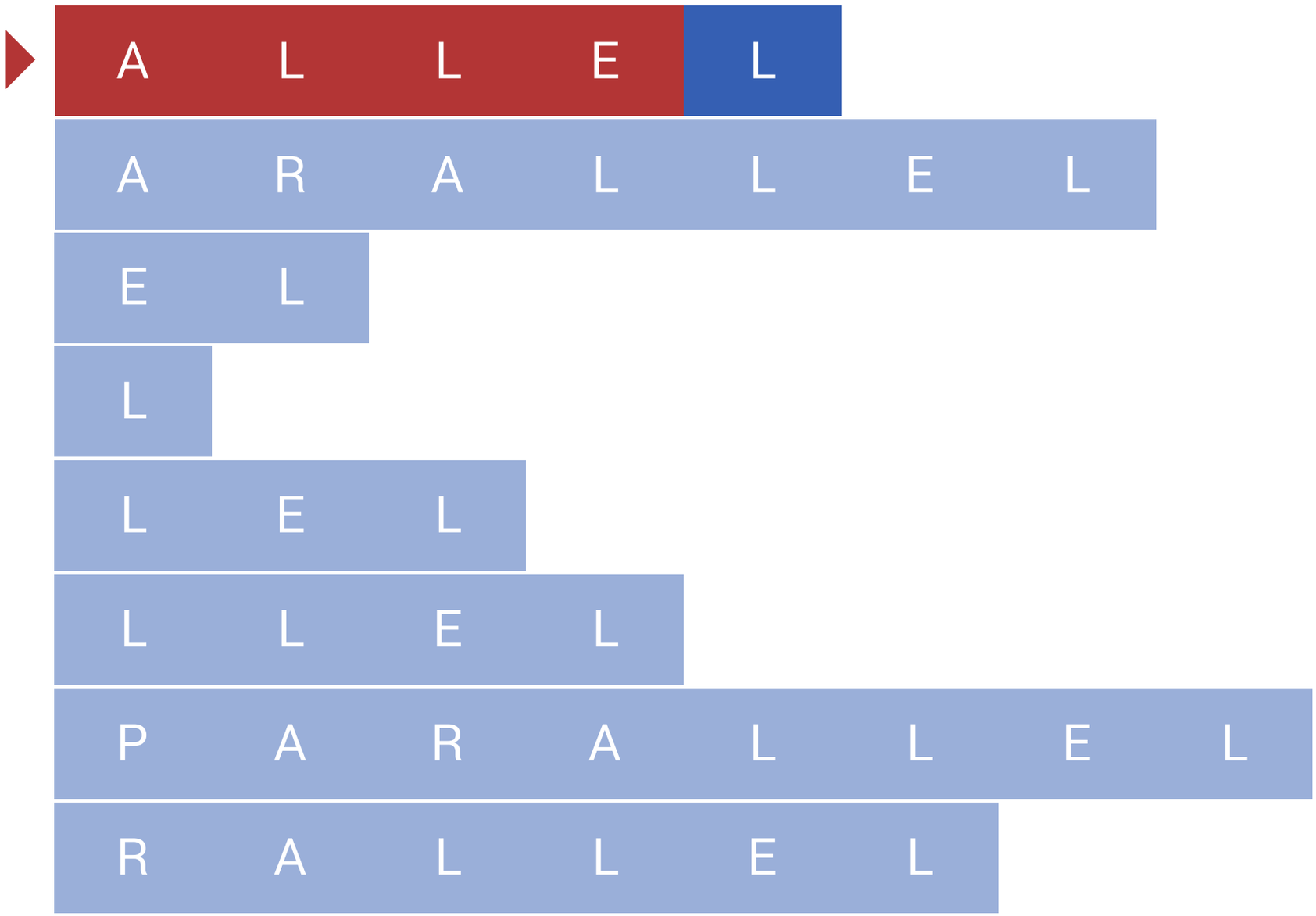
Substringsuche

Ist *alle* in *parallel* enthalten?

Ja, an Stelle 4.



Einsatzgebiete

Verwendet in Implementationen

des LZ77-Kompressionsalgorithmus

Konstruktion eines **Suffix Arrays** mit einem **rekursionsfreien Linearzeit-Algorithmus**.

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Übersicht

Problemstellung

Lösungsansätze

GSACA

Performance

Rückblick

Übersicht

Problemstellung ✓

Lösungsansätze

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Performance

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Lösungsansätze

Naiver Ansatz

Verwendung eines allgemeinen Sortierverfahrens

(z. B. Quicksort)

$$O(n \log n) \cdot O(n) = O(n^2 \log n)$$

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$$O(n \log n) \cdot O(n) = O(n^2 \log n) \neq O(n)$$

Linearzeit Ansätze

	Skew	SA-IS
Art	rekursiv	rekursiv
Zeit	$O(n)$	$O(n)$
Speicher	$O(\log n) + \max 24n$	$O(\log n) + \max 2n$

Linearzeit Ansätze

	Skew	SA-IS	?
Art	rekursiv	rekursiv	iterativ
Zeit	$O(n)$	$O(n)$	$O(n)$
Speicher	$O(\log n) + \max 24n$	$O(\log n) + \max 2n$	$O(1) + ?$

?

iterativ

$O(n)$

$O(1) + ?$

GSACA

iterativ

$O(n)$

$O(1) + ?$

GSACA

Greedy **S**uffix **A**rray **C**onstruction **A**lgorithm

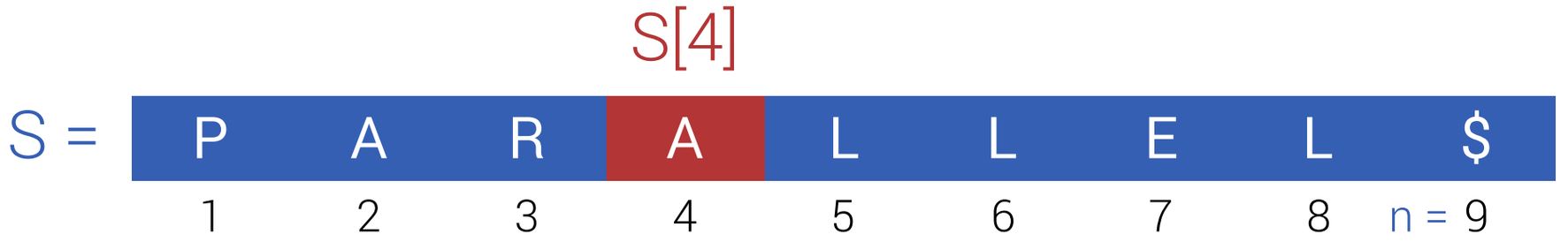
Definitionen

P	A	R	A	L	L	E	L	$
1	2	3	4	5	6	7	8	9

Definitionen

S := Eingabe, eine mit **$** terminierte Zeichenkette der Länge **n**

Definitionen



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S[**i**] := **i**-tes Zeichen von **S**

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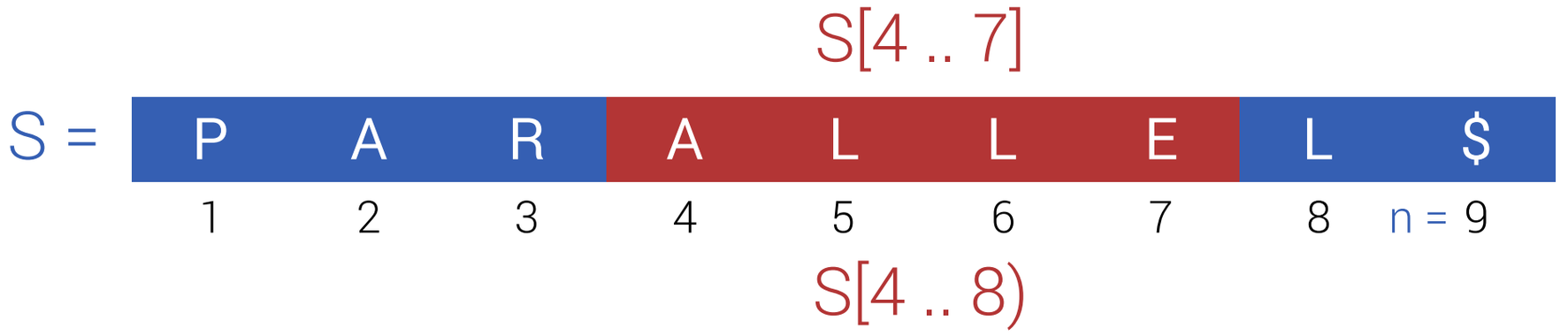

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S_i := **S[i .. n]**

Definitionen

$$\hat{\imath} := \mathit{min}\ \{\ j \in [i\ ..\ n]\colon S_j <_{lex} S_i\ \}$$

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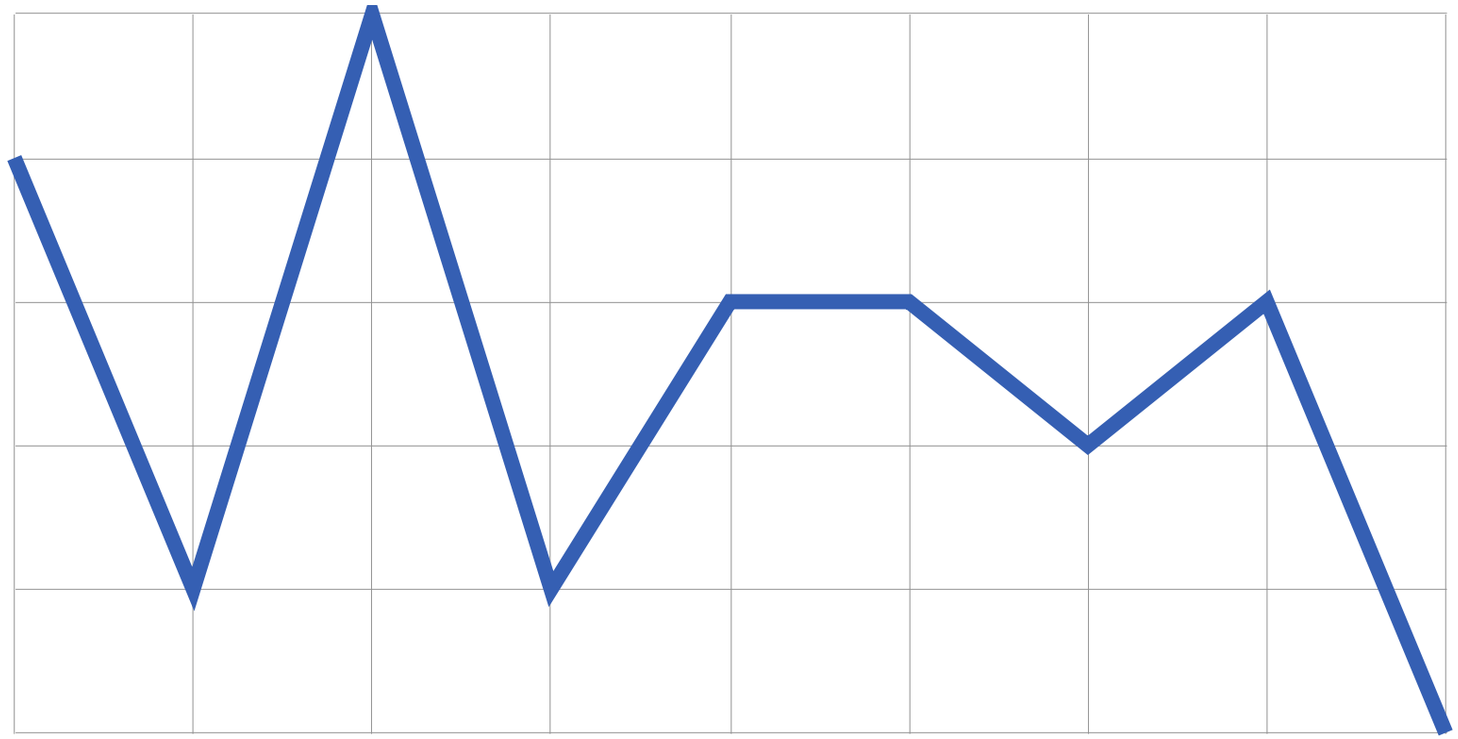

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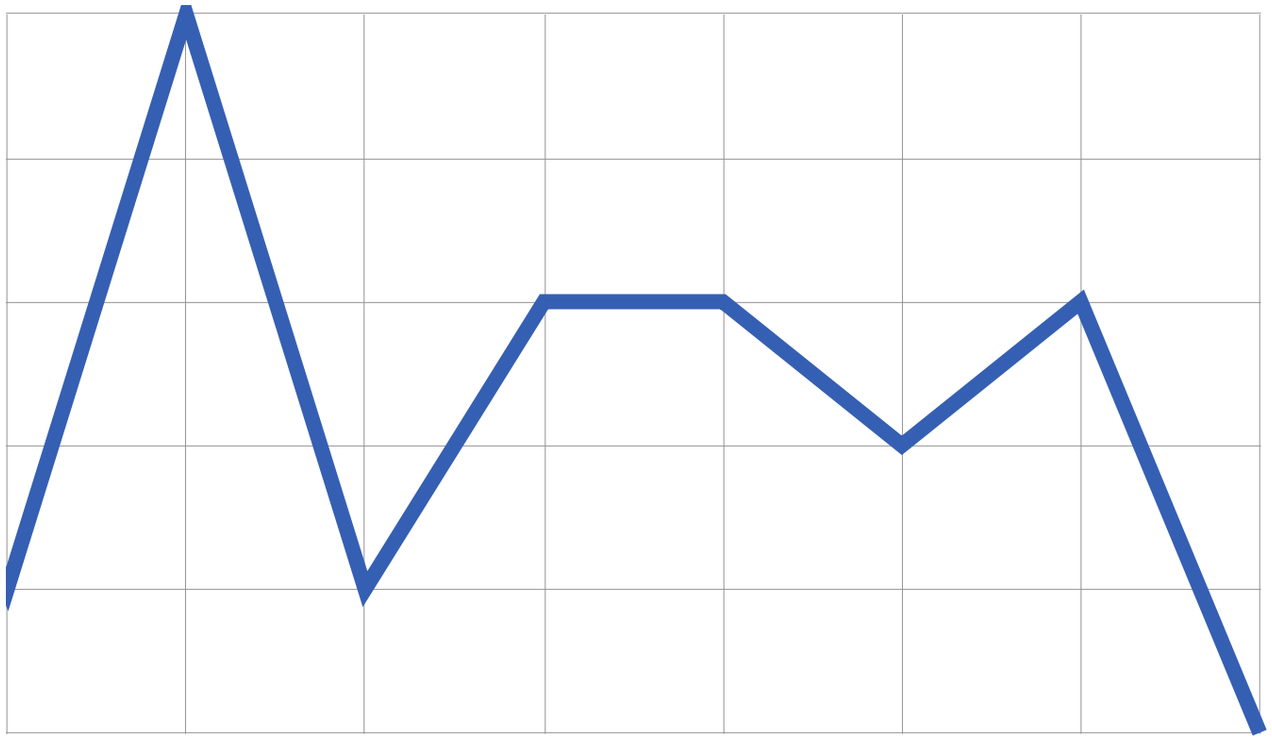

S_i =	A	R	A	L	L	E	L	$
i =	2	3	4	5	6	7	8	9

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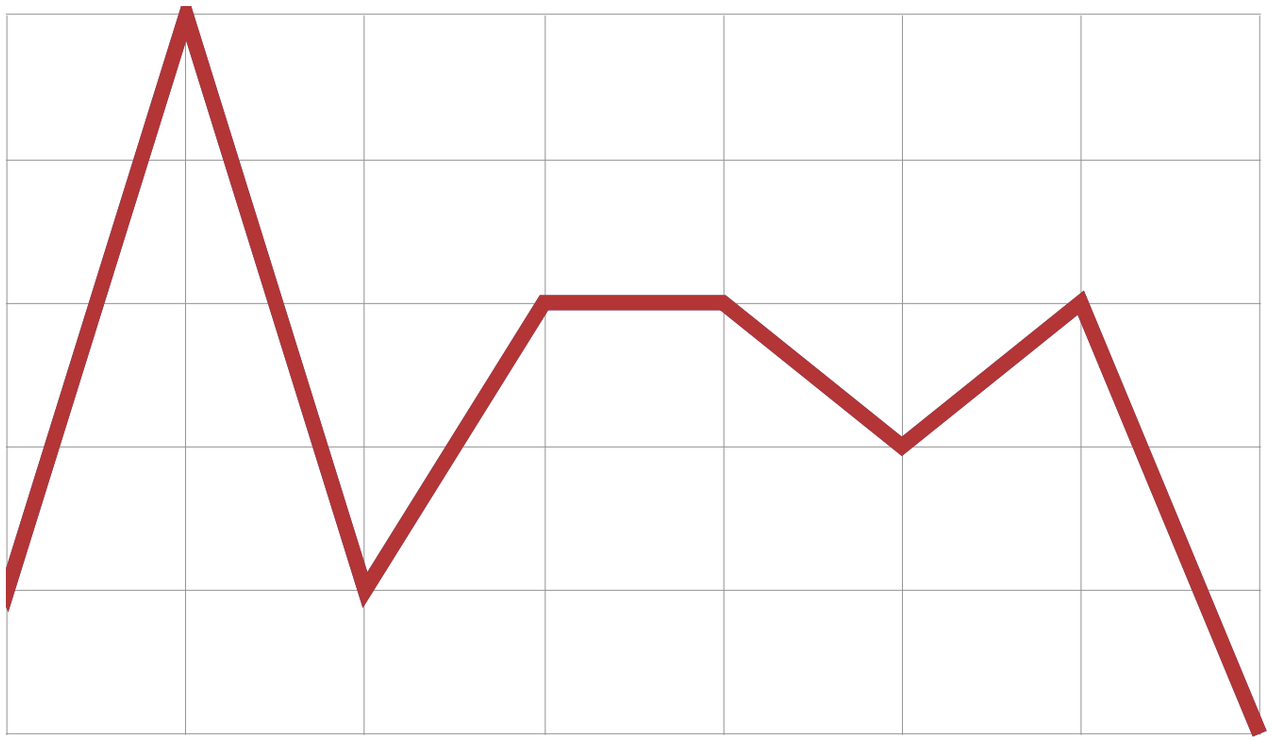

S_i =	A	R	A	L	L	E	L	$
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S_j =	A	R	A	L	L	E	L	$
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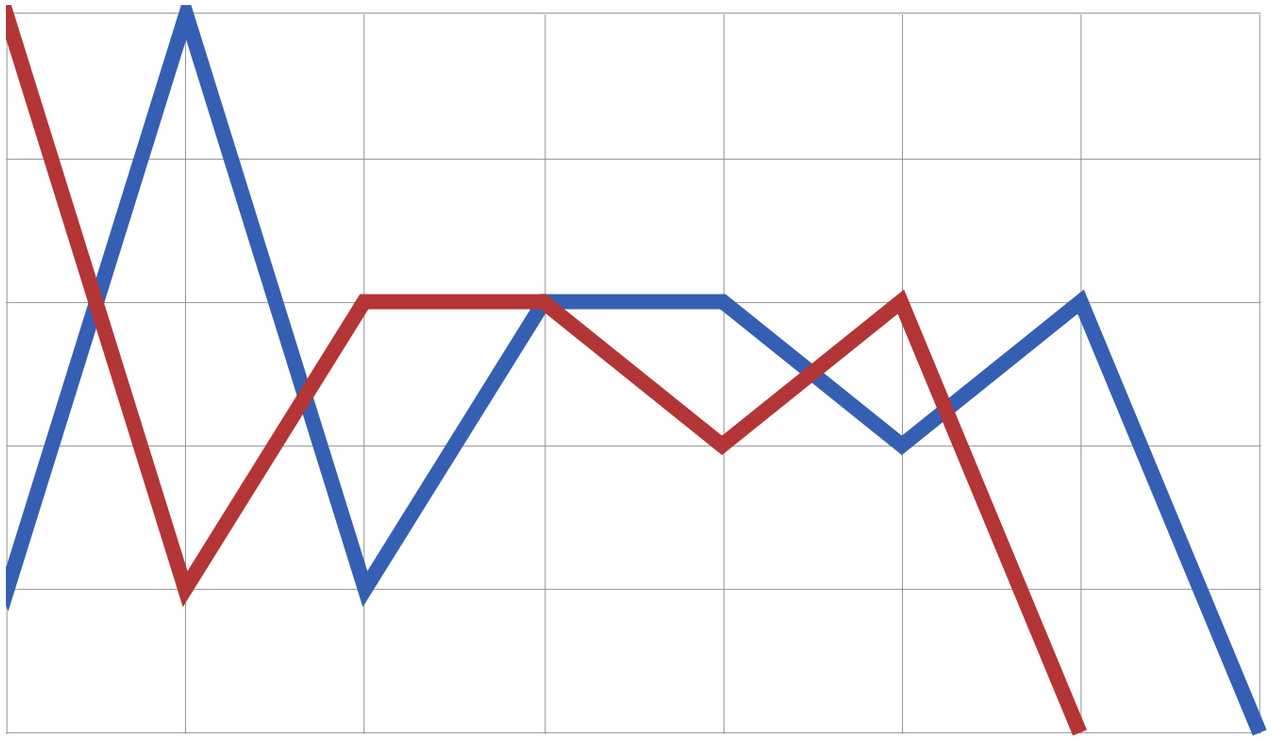

S_i =	A	R	A	L	L	E	L	$
i =	2	3	4	5	6	7	8	9
S_j =	R	A	L	L	E	L	$	
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Definitionen

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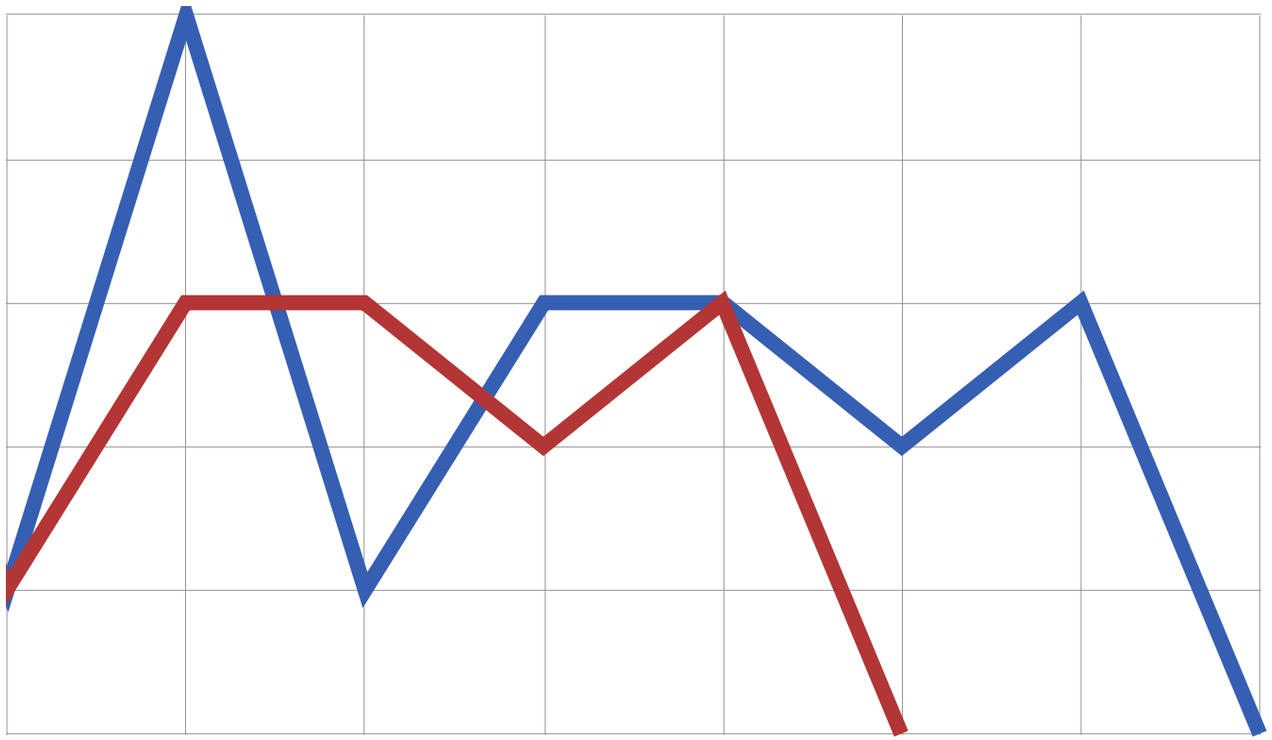

S_i =	A	R	A	L	L	E	L	$
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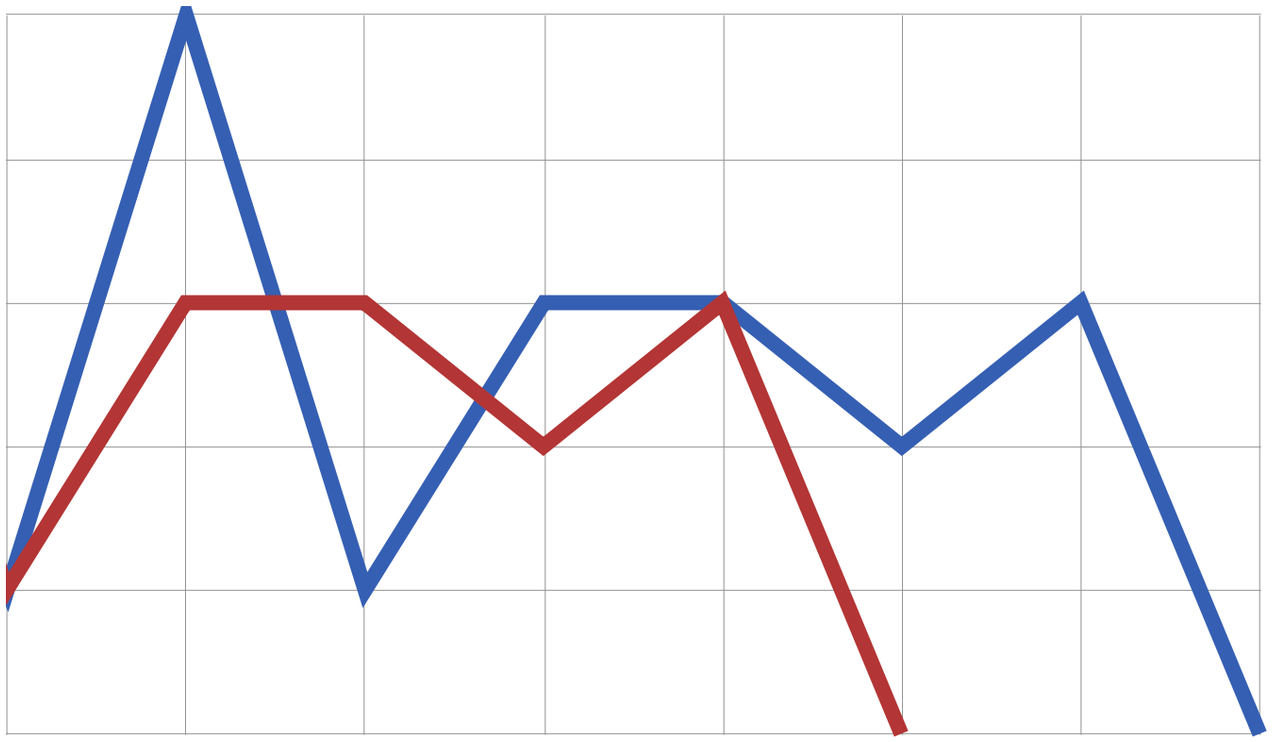

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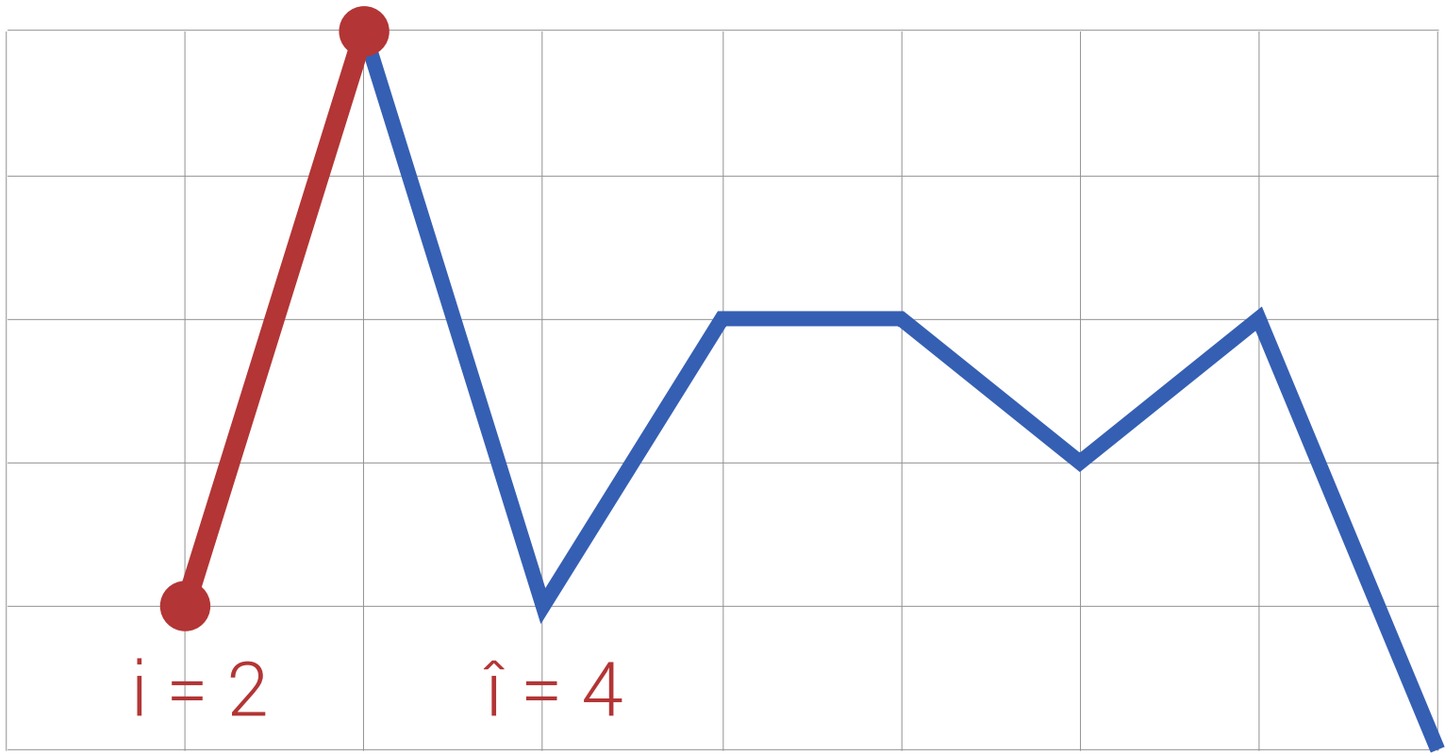
$\hat{\imath} = 4$

Definitionen

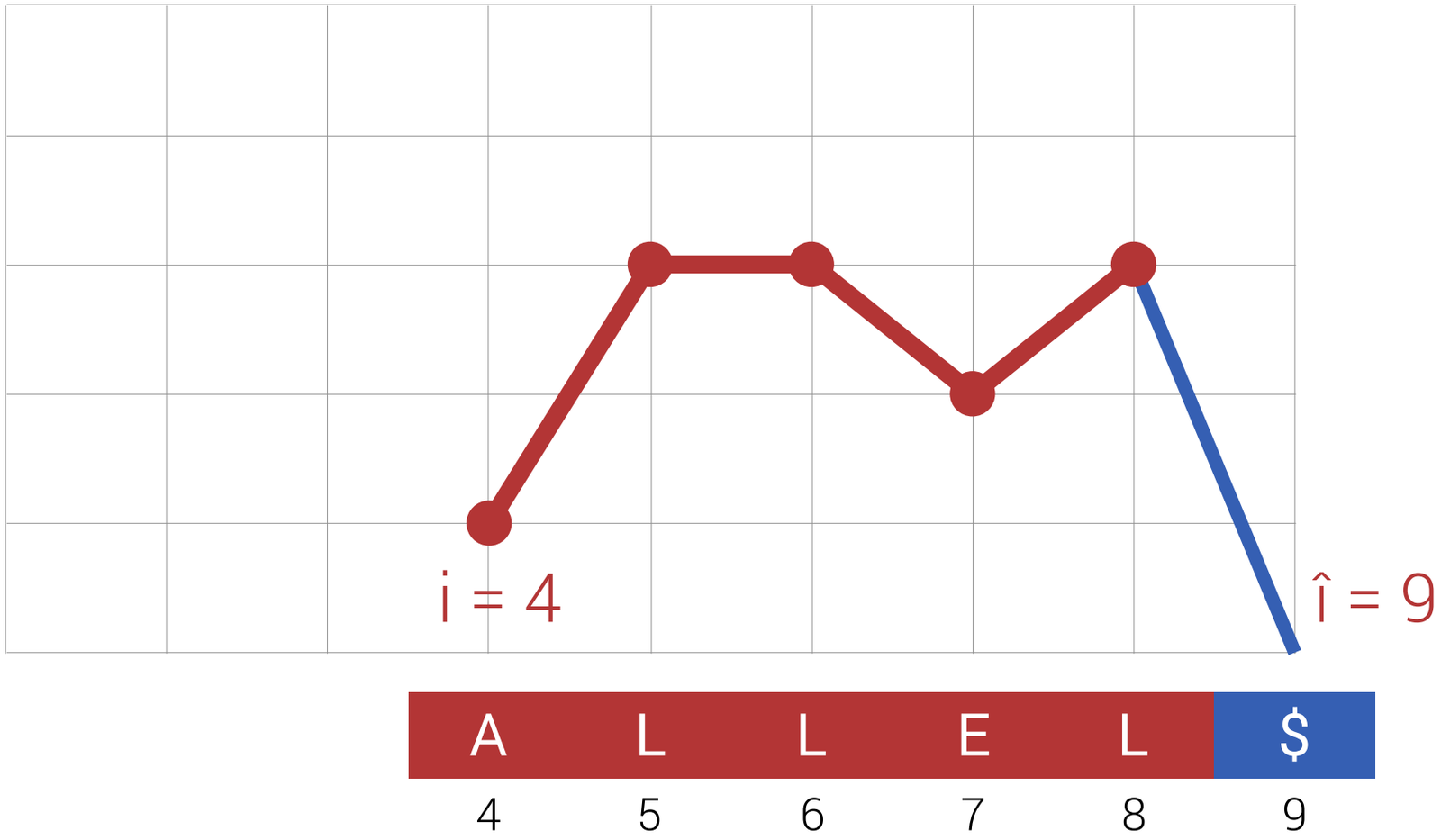
Gruppenkontext von S_i := S[i .. î)



A	R	A	L	L	E	L	$
2	3	4	5	6	7	8	9

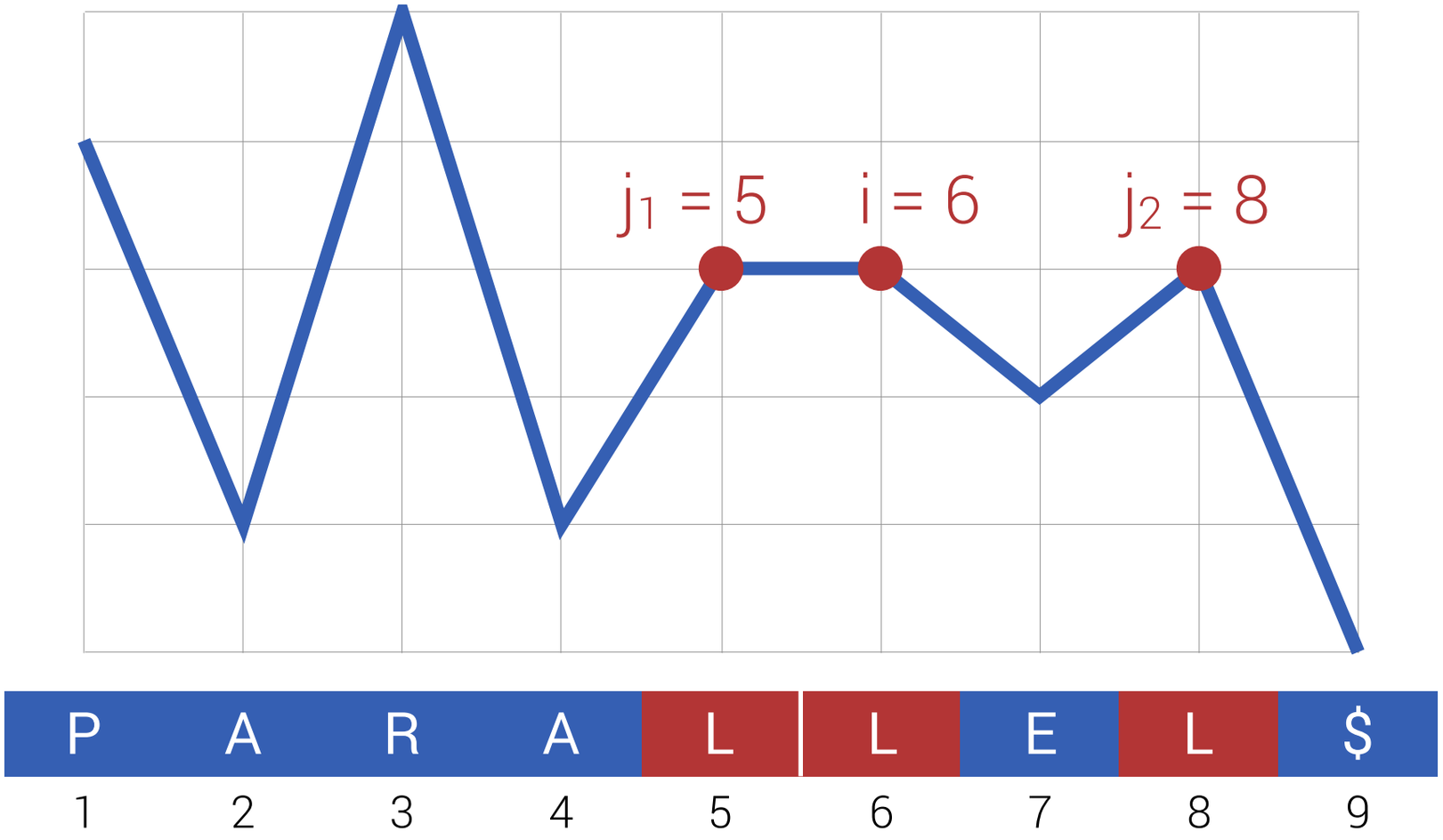
Definitionen

Gruppenkontext von S_i := $S[i \mathinner{..} \hat{\imath})$



Definitionen

Gruppe von S_i := { S_j : Gr.kontext S_j = Gr.kontext S_i }



Grundprinzip

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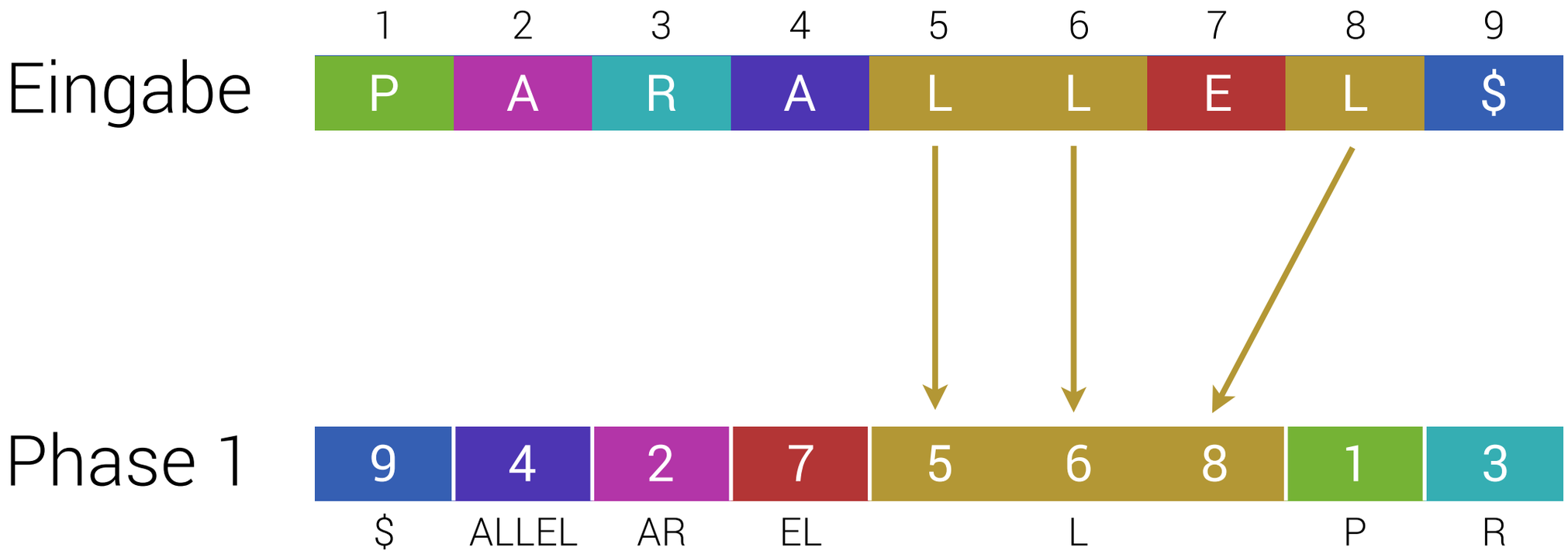

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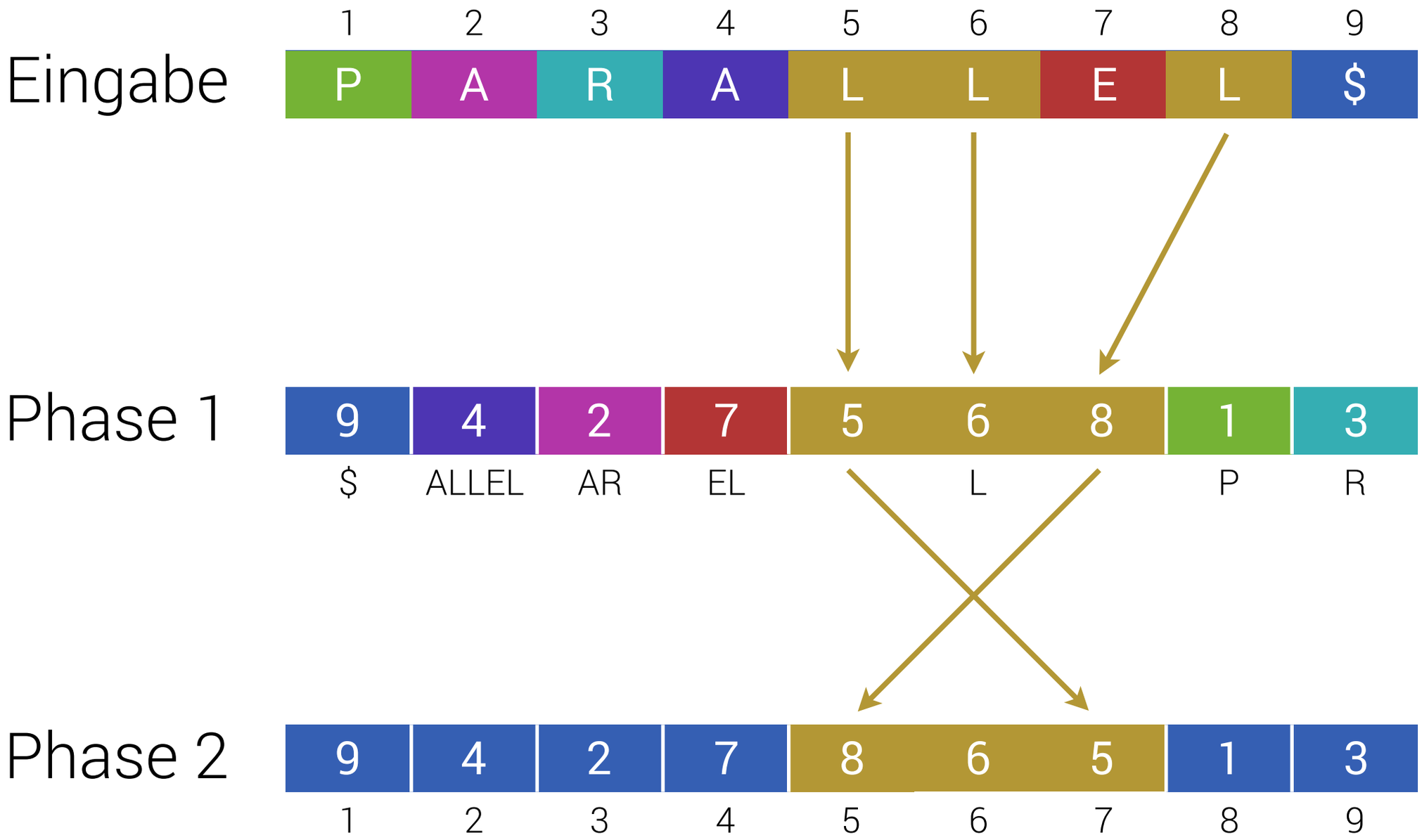

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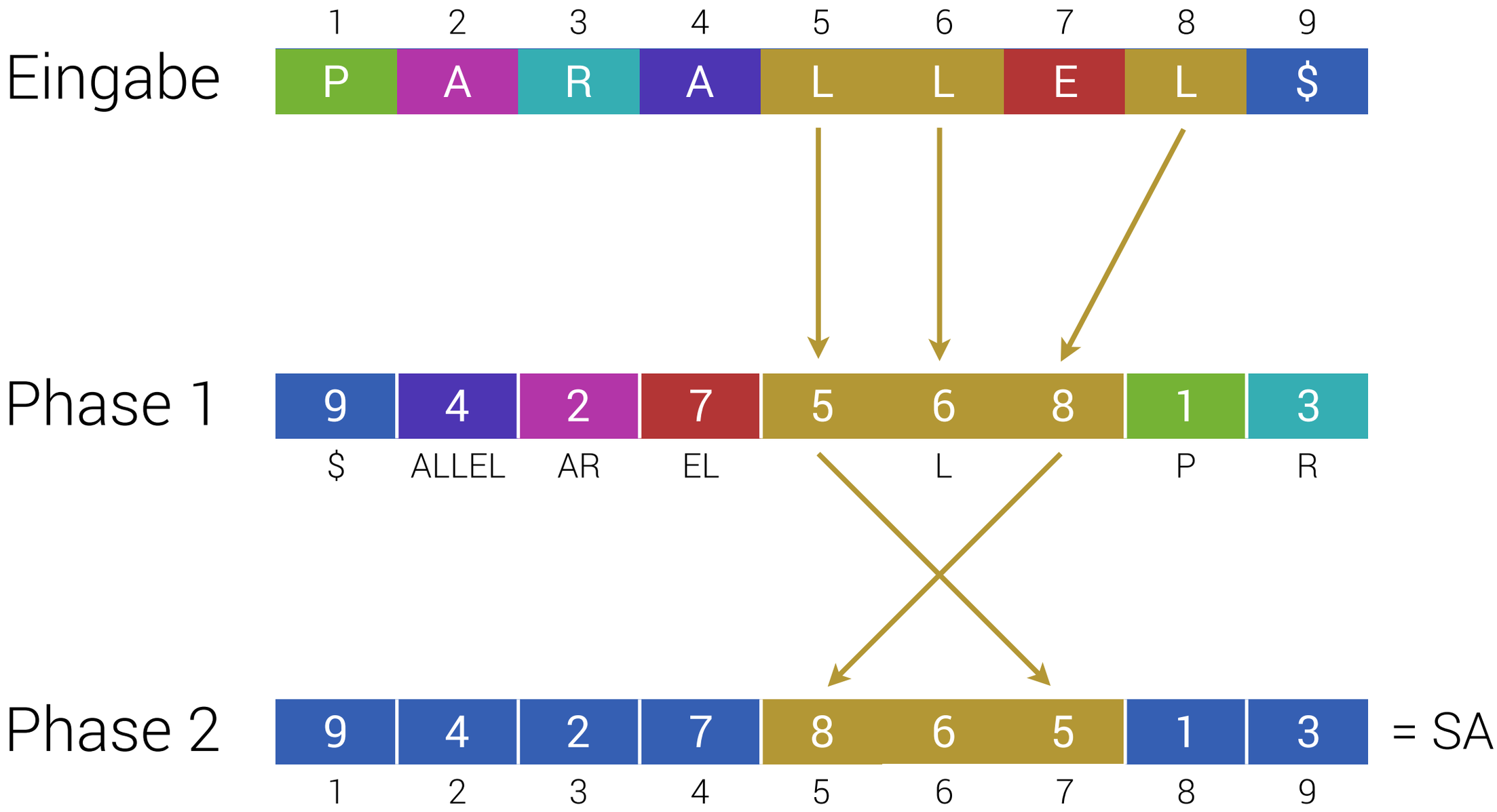

Grundprinzip

Eingabe

Phase 1 Sortierte Folge von Gruppen berechnen

Phase 2 Suffixe innerhalb der Gruppen sortieren

Grundprinzip

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Phase 1 Sortierte Folge von Gruppen berechnen $O(n)$

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Phase 1

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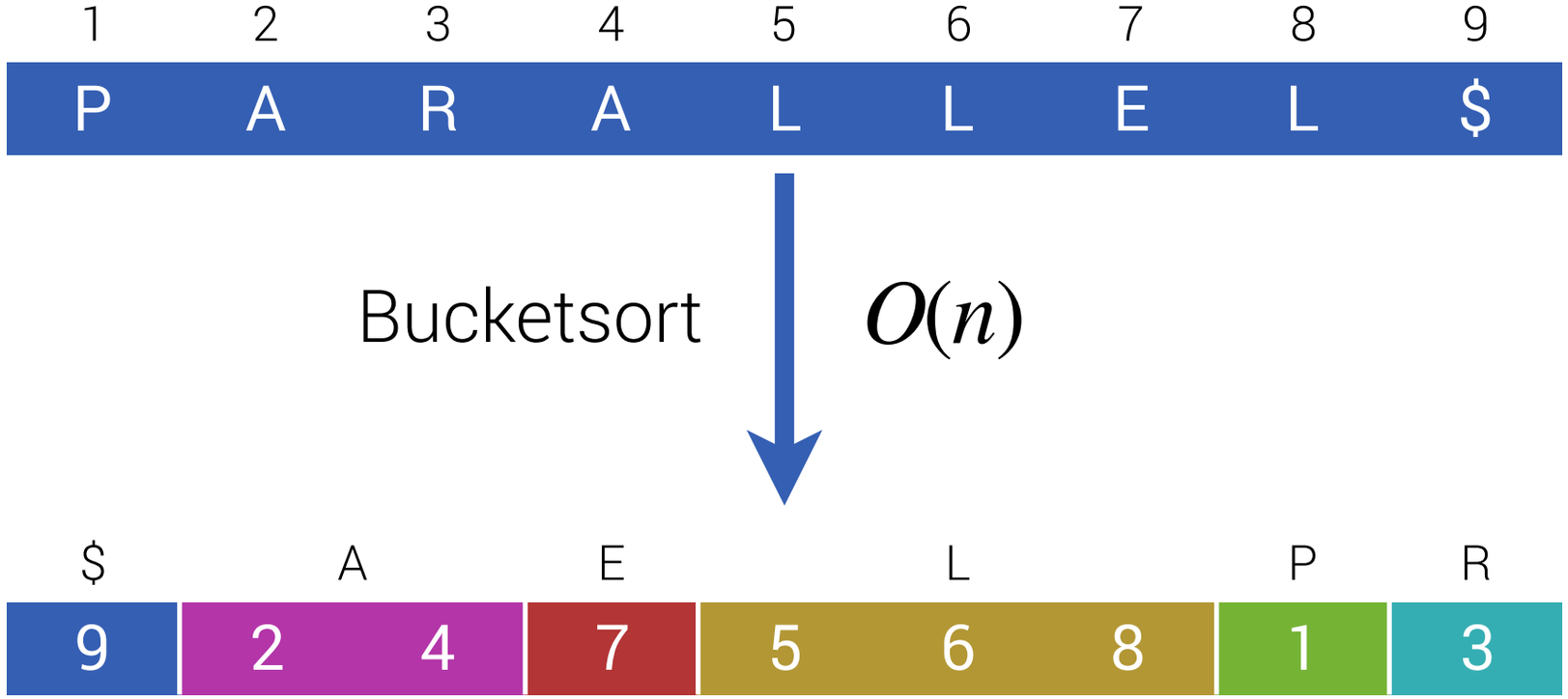
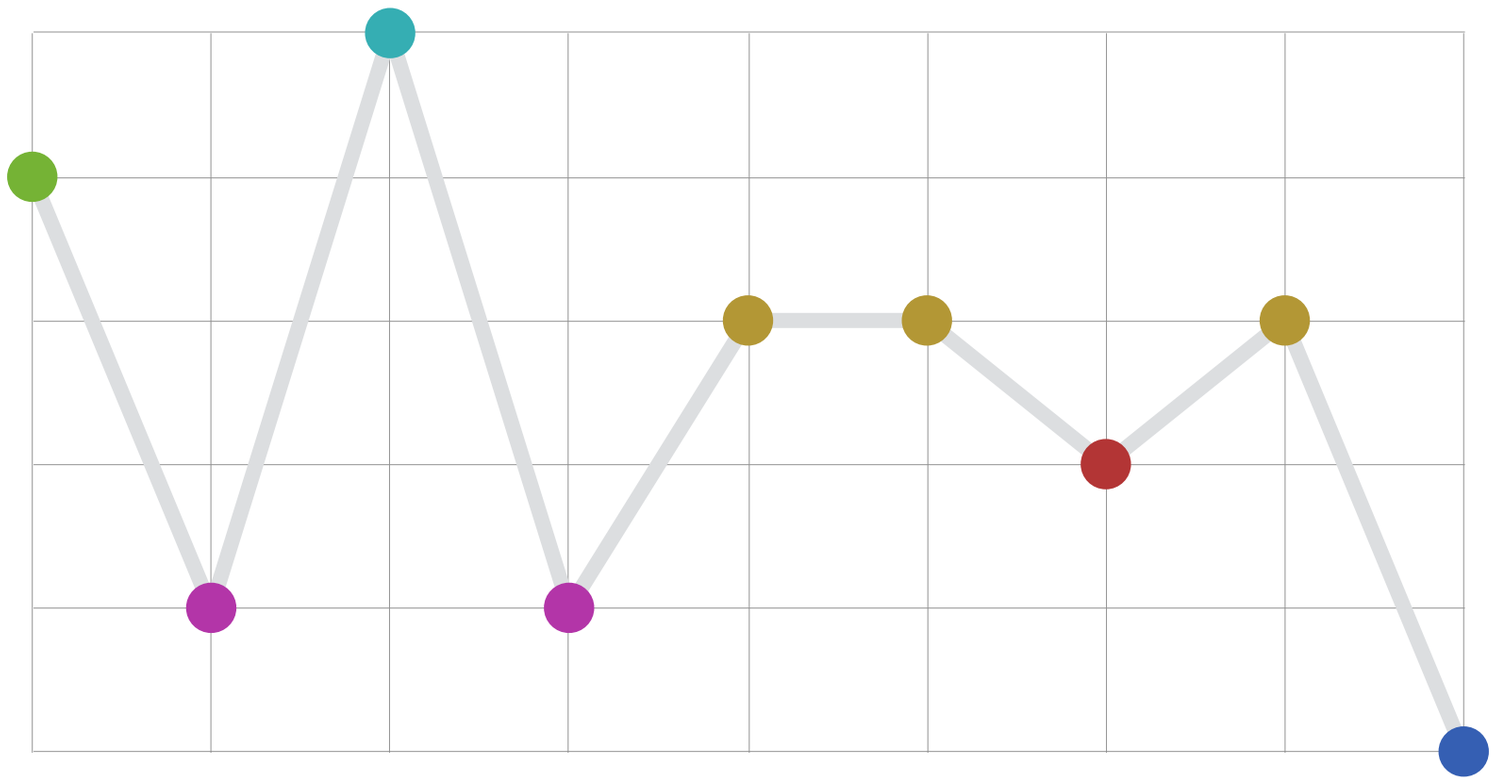


Phase 1

$	A		E	L			P	R
9	2	4	7	5	6	8	1	3



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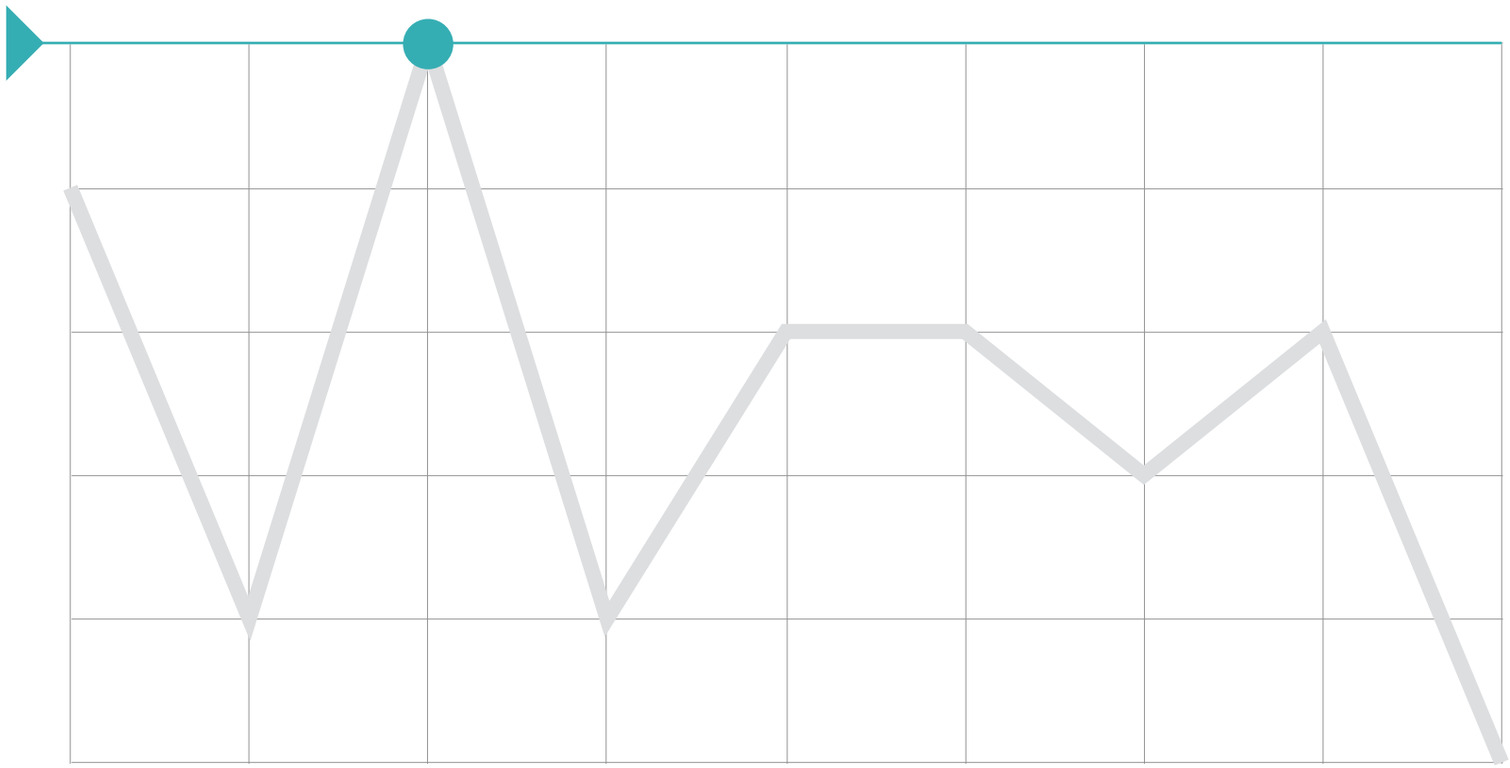

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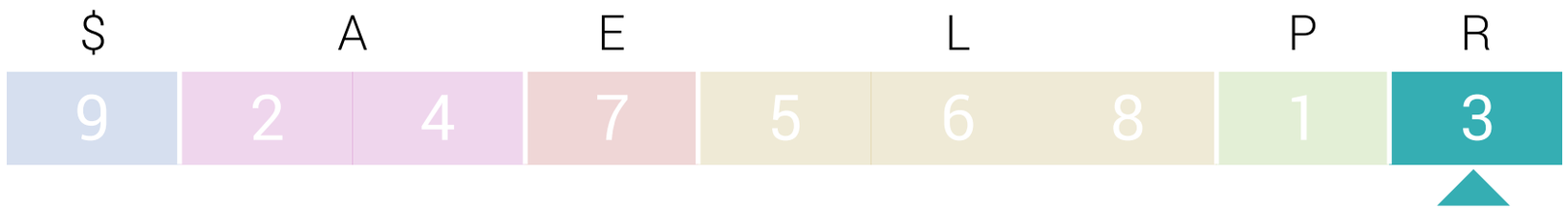
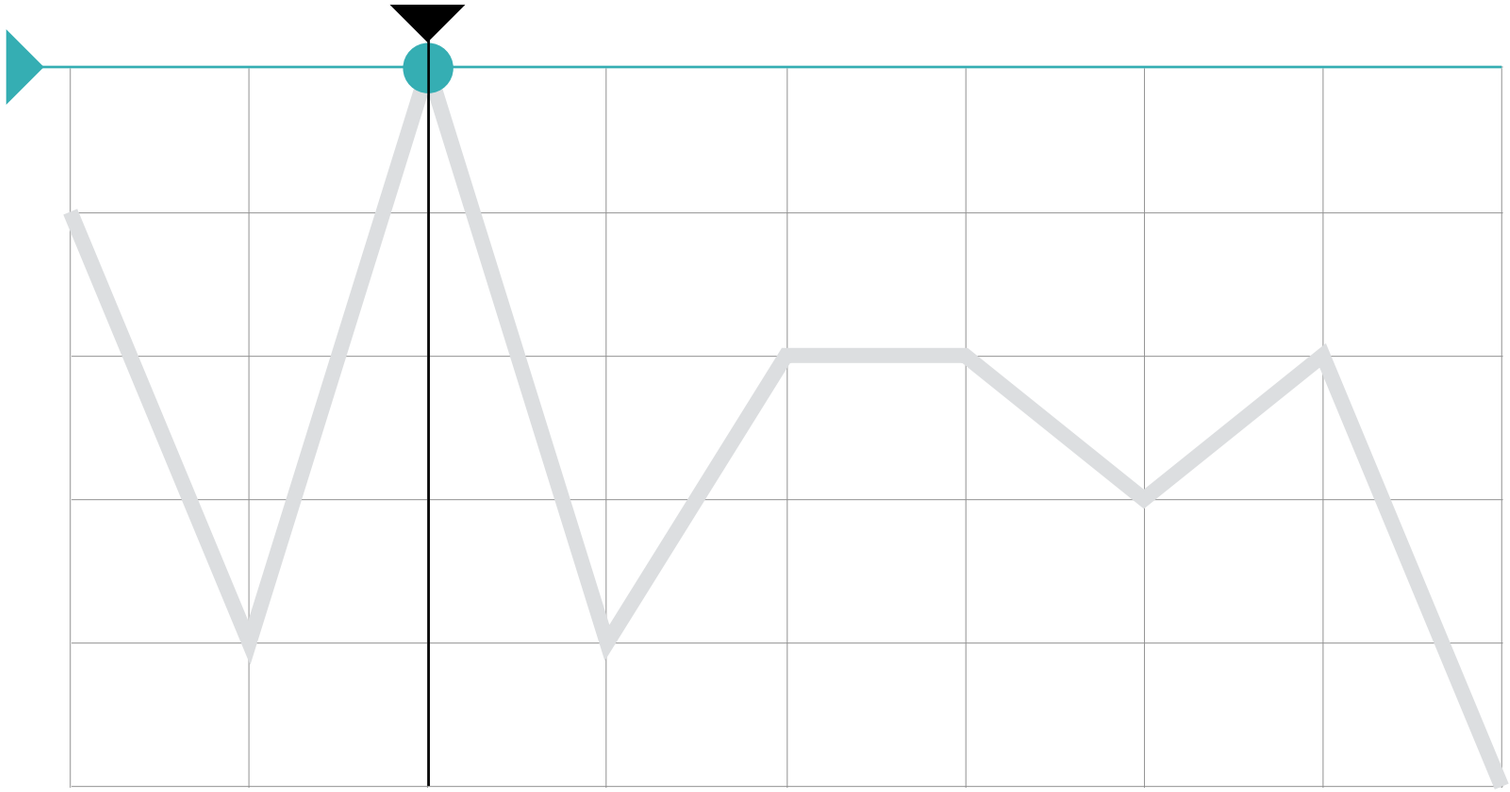


prev(**i**) := *max* { **j** ∈ [1 .. **i**]: Gr.kontext S_j <$_{lex}$ Gr.kontext S_i }



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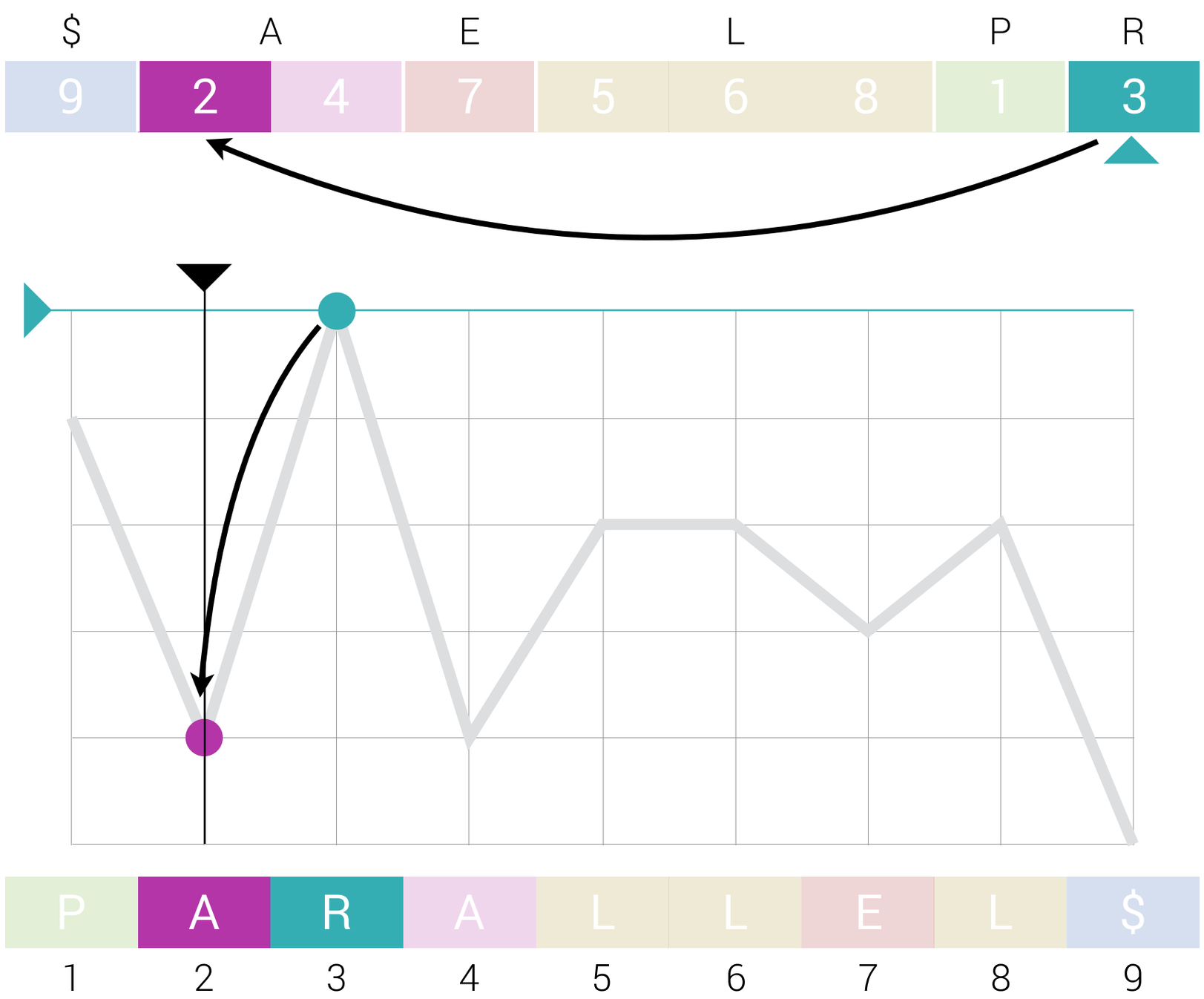

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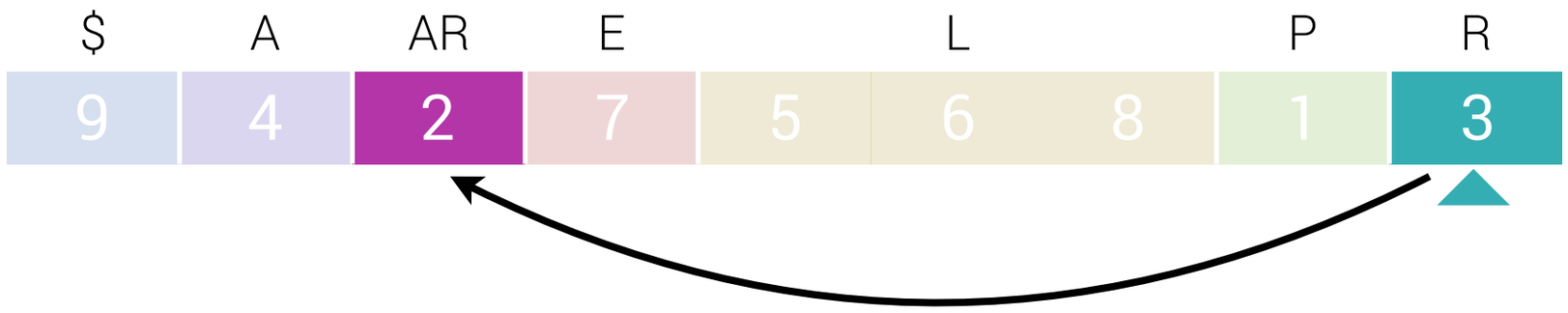

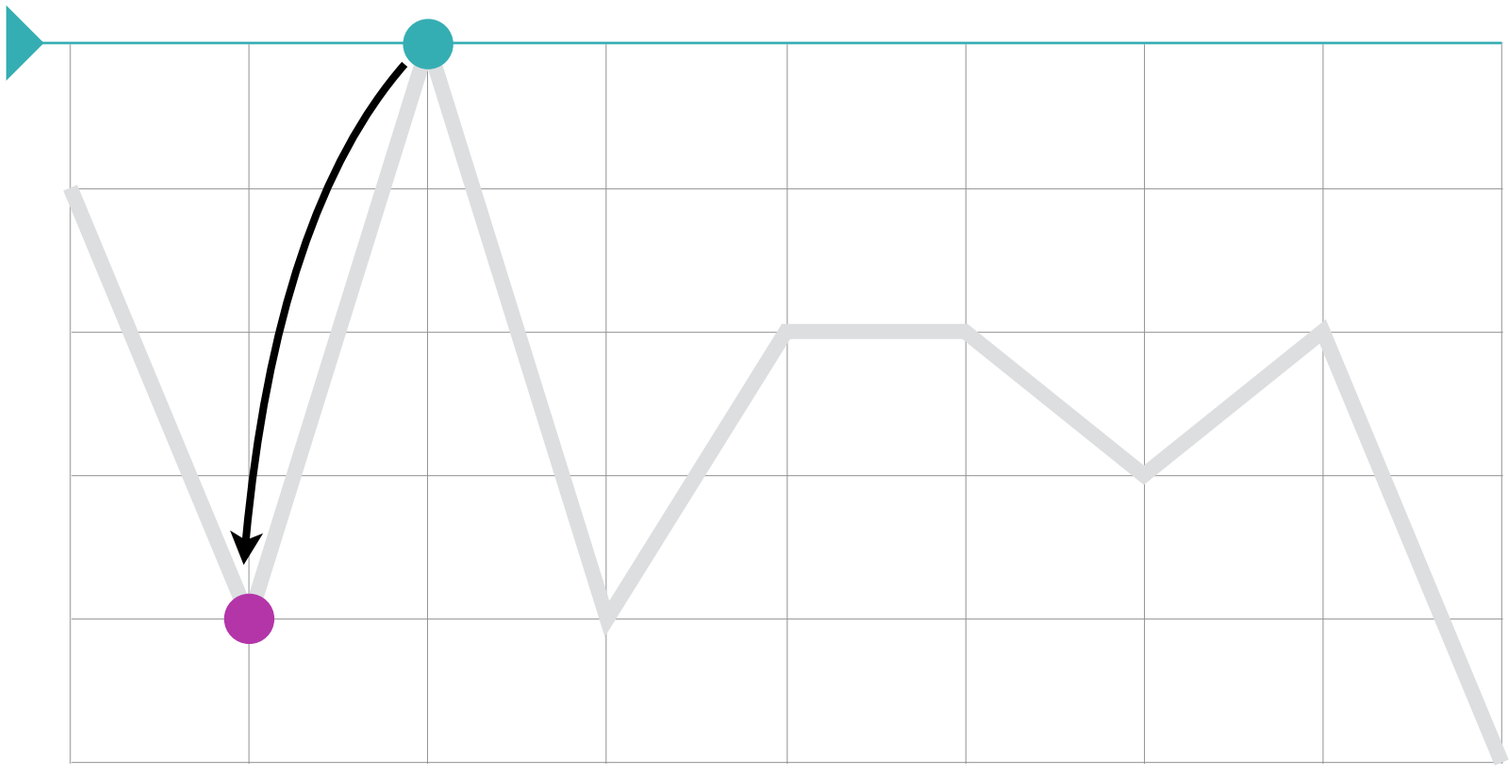

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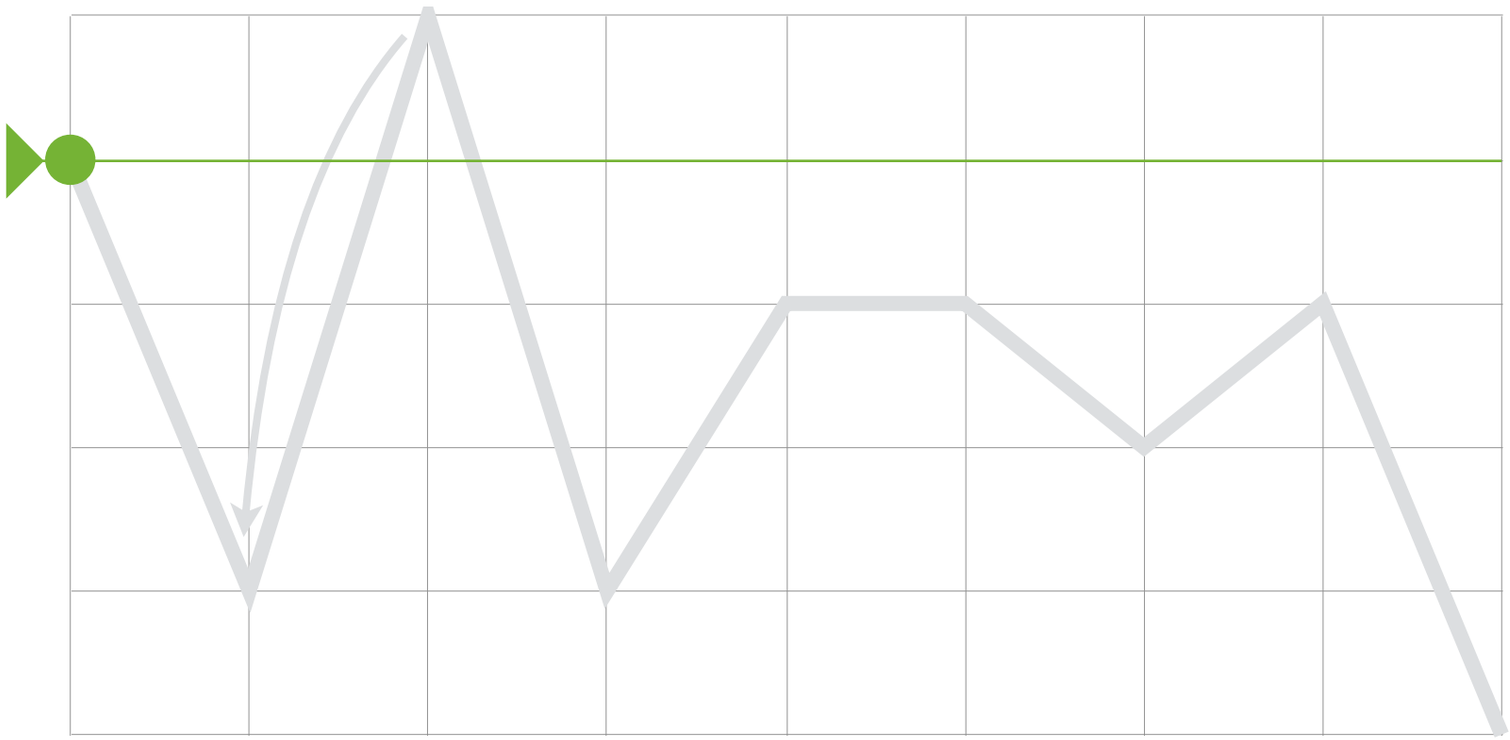

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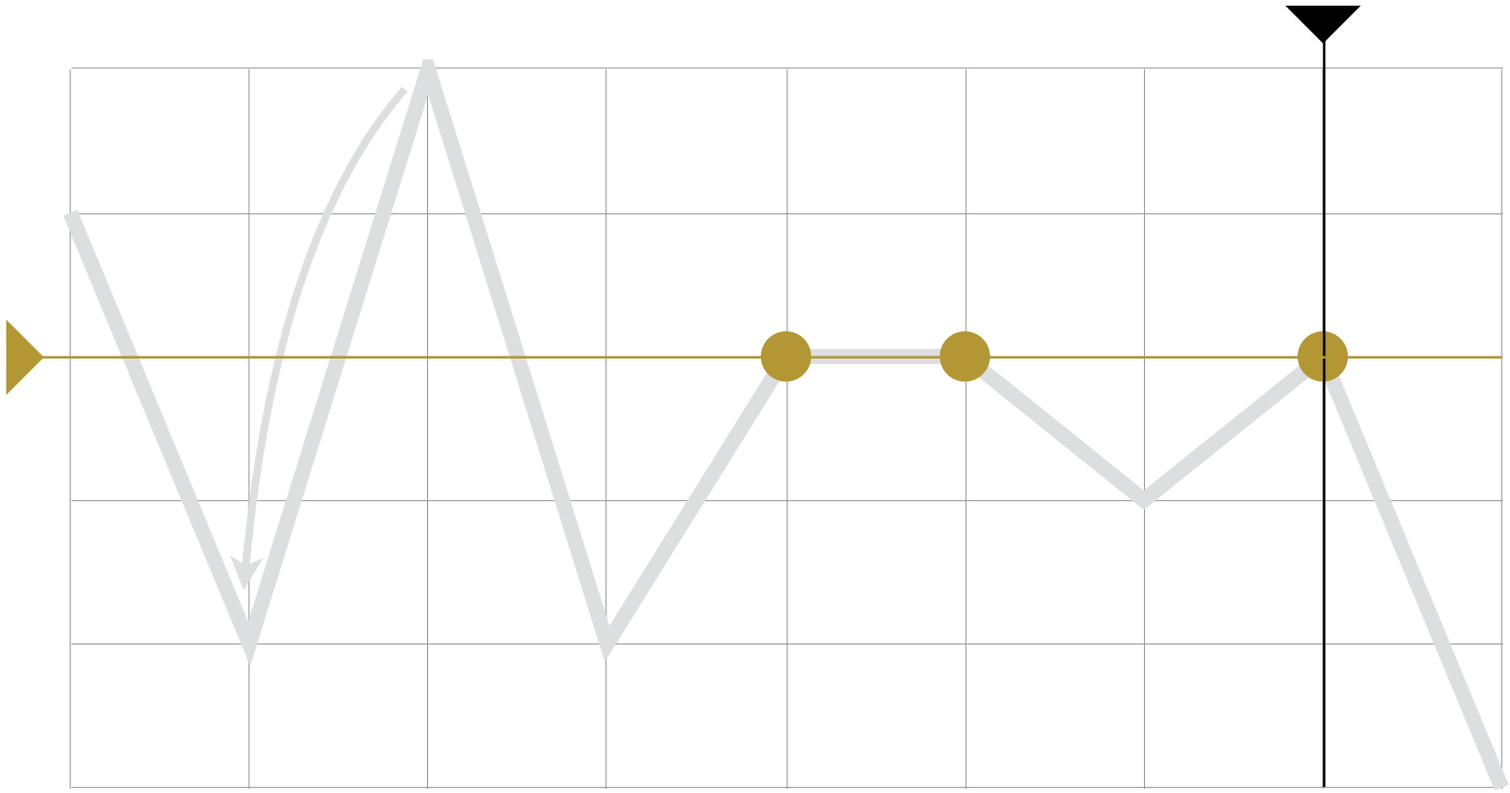

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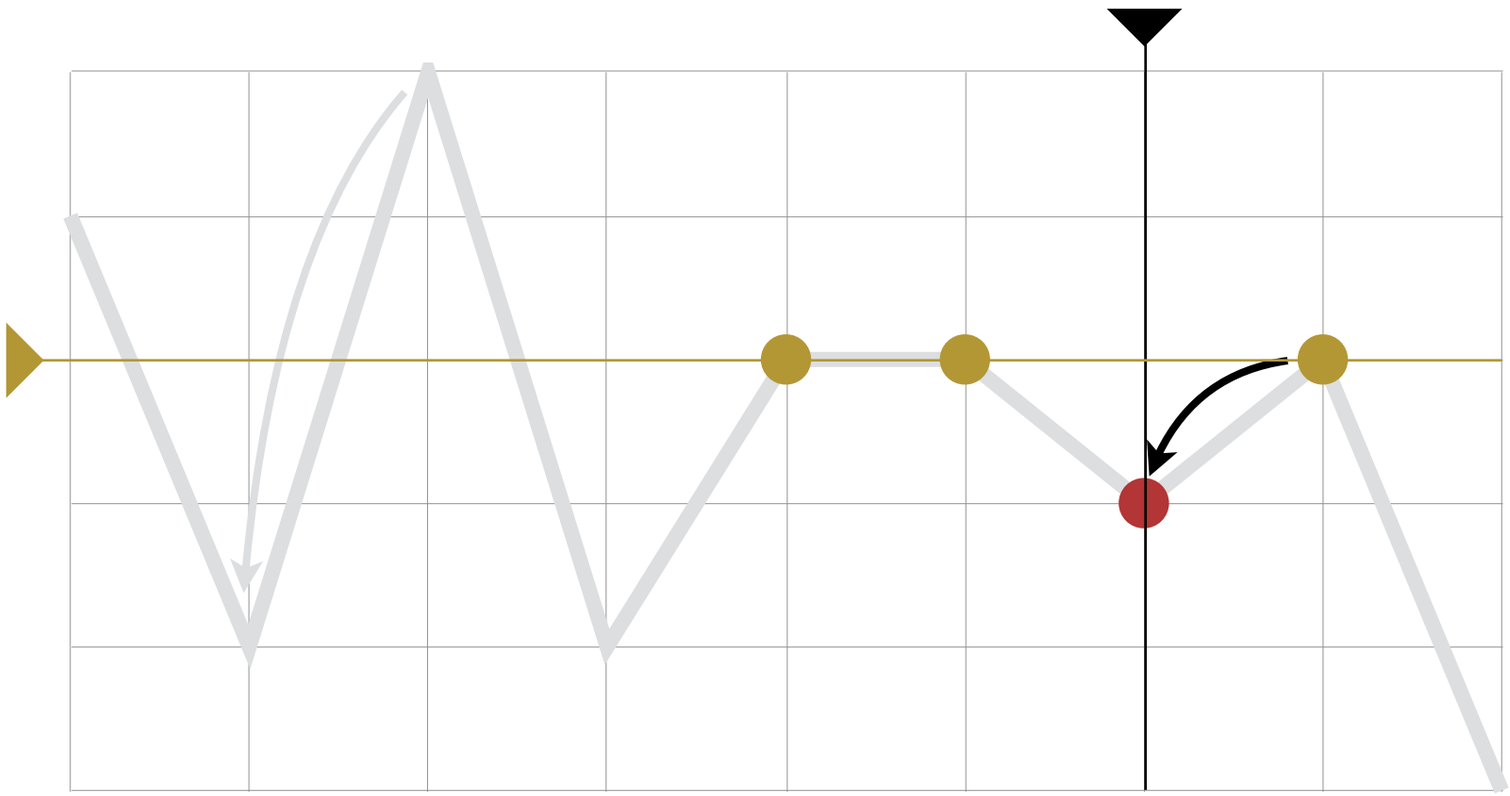

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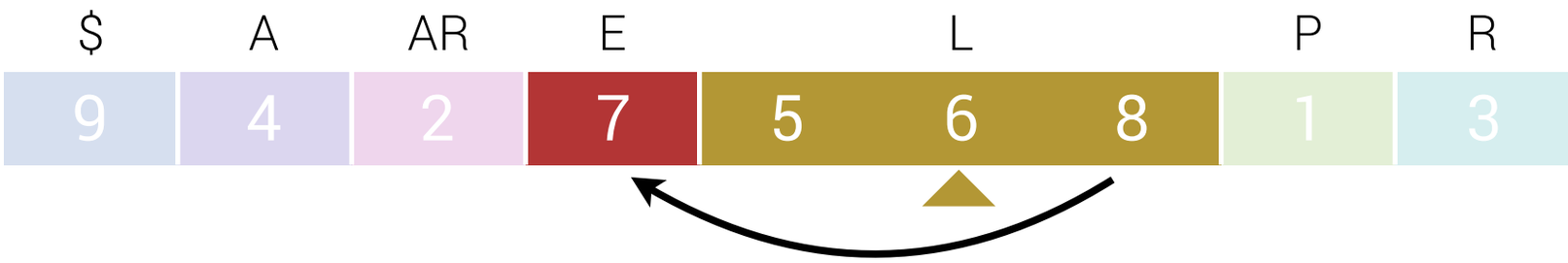

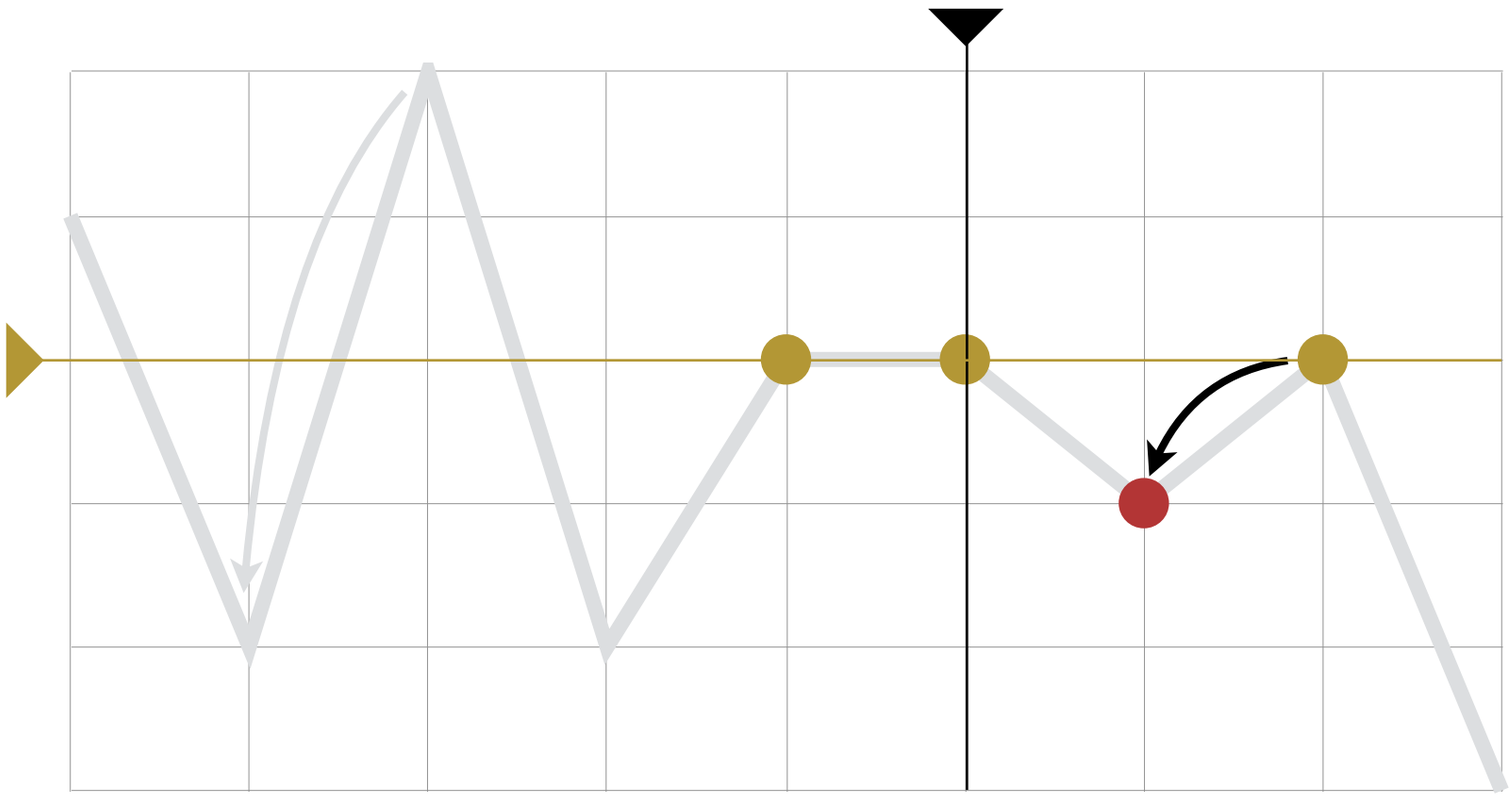

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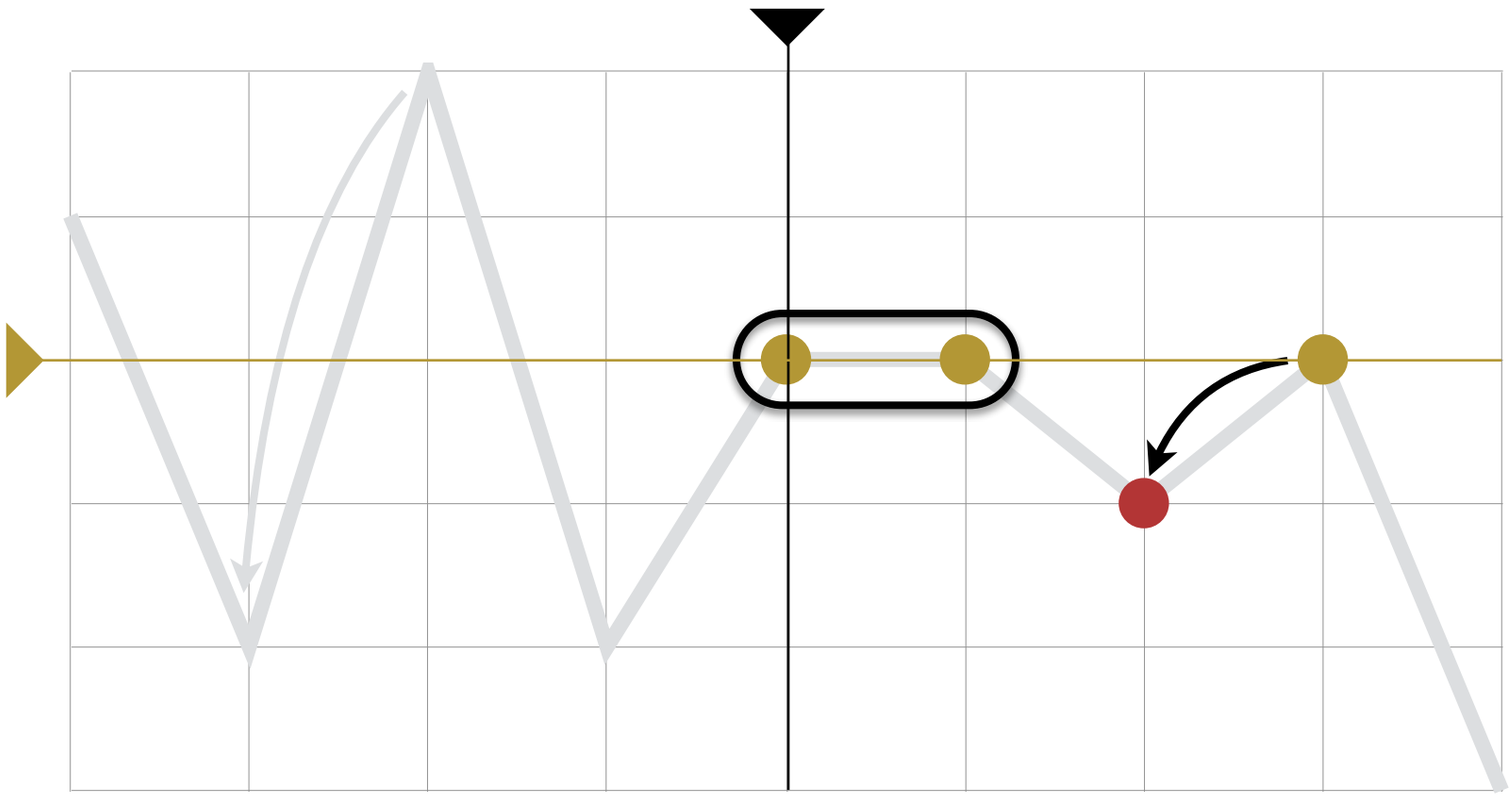

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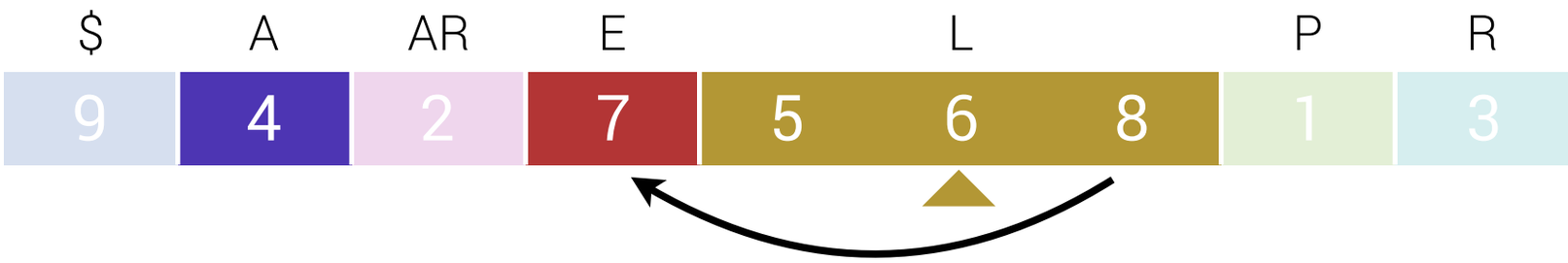
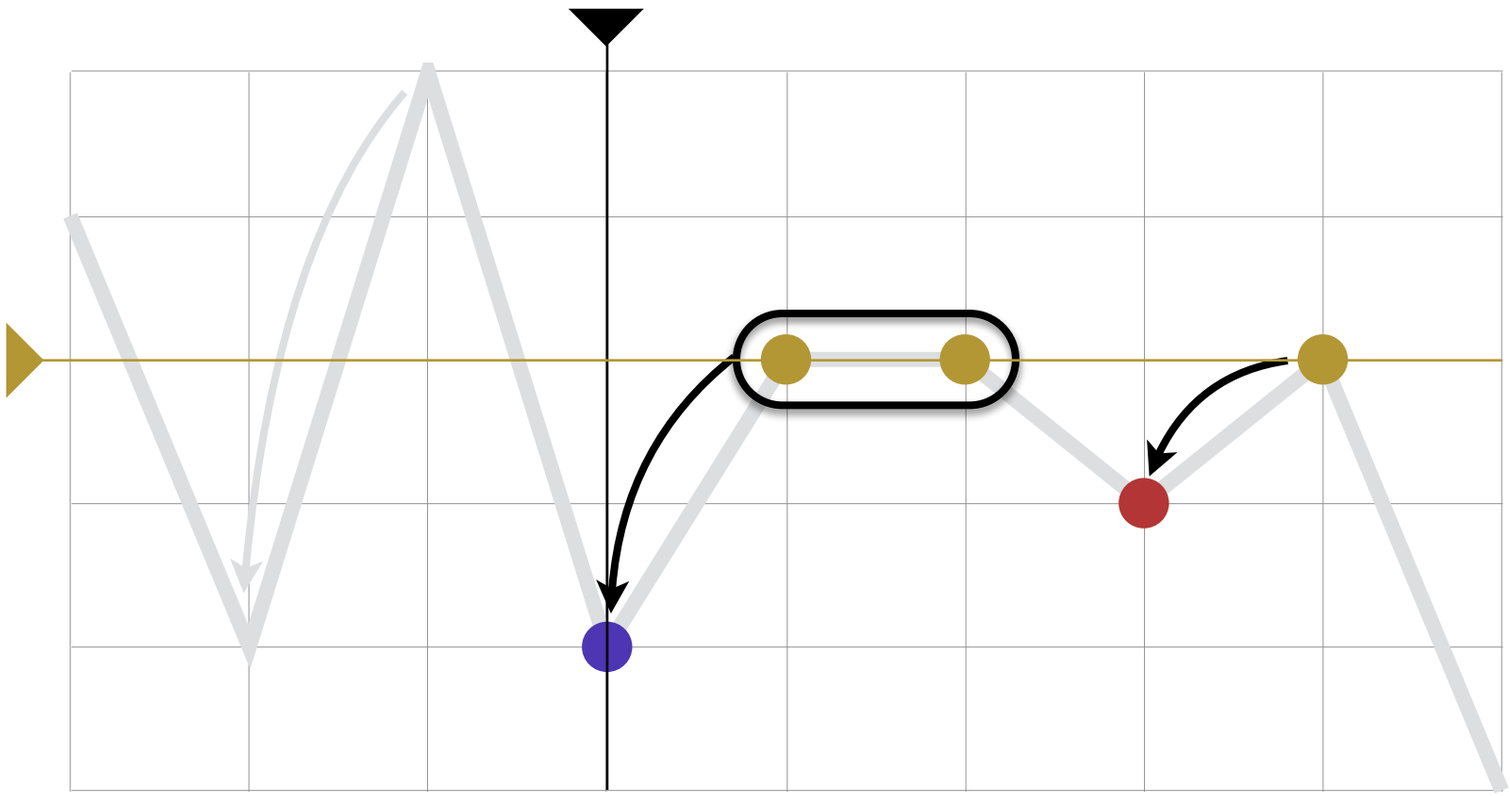
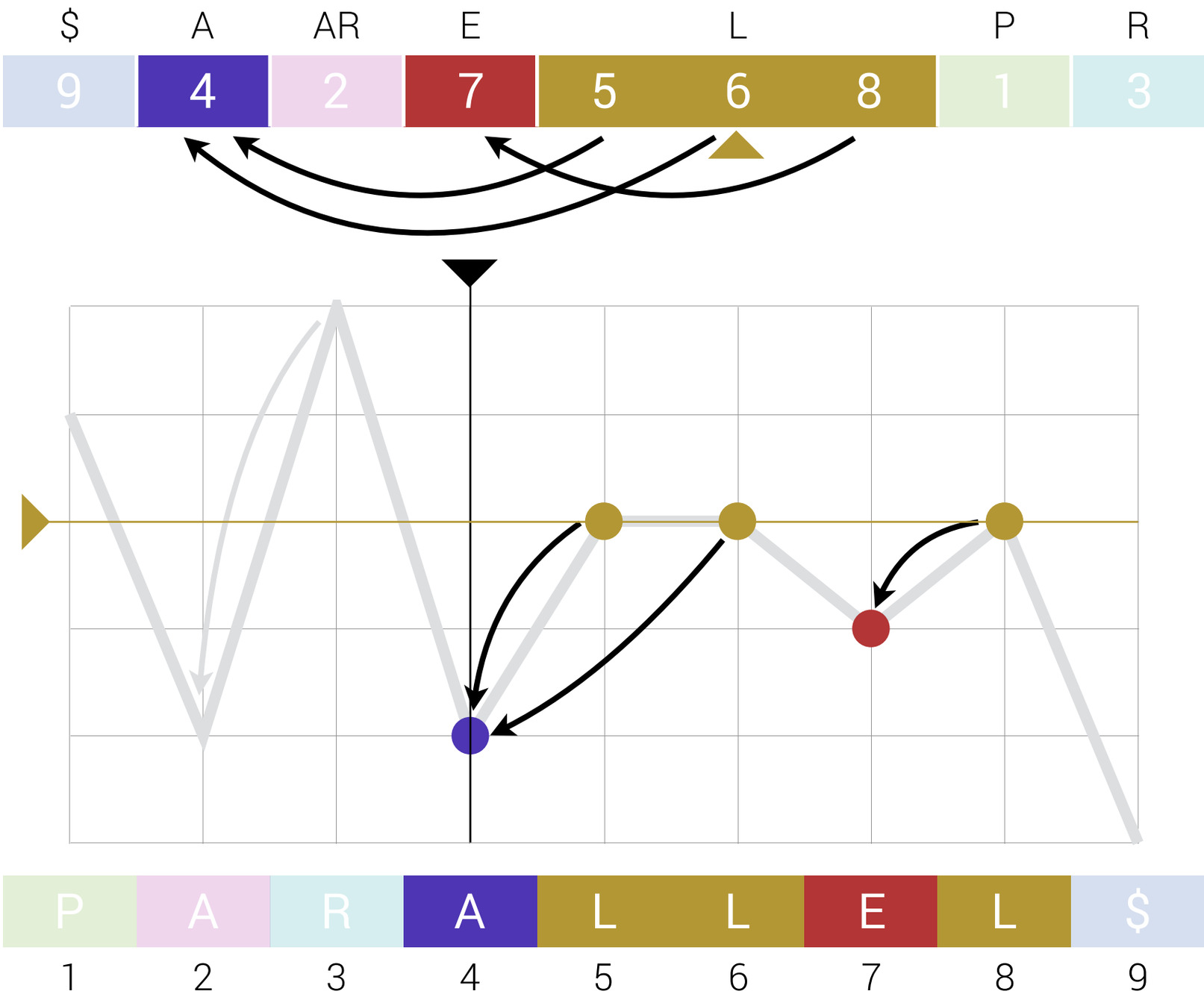
Phase 1

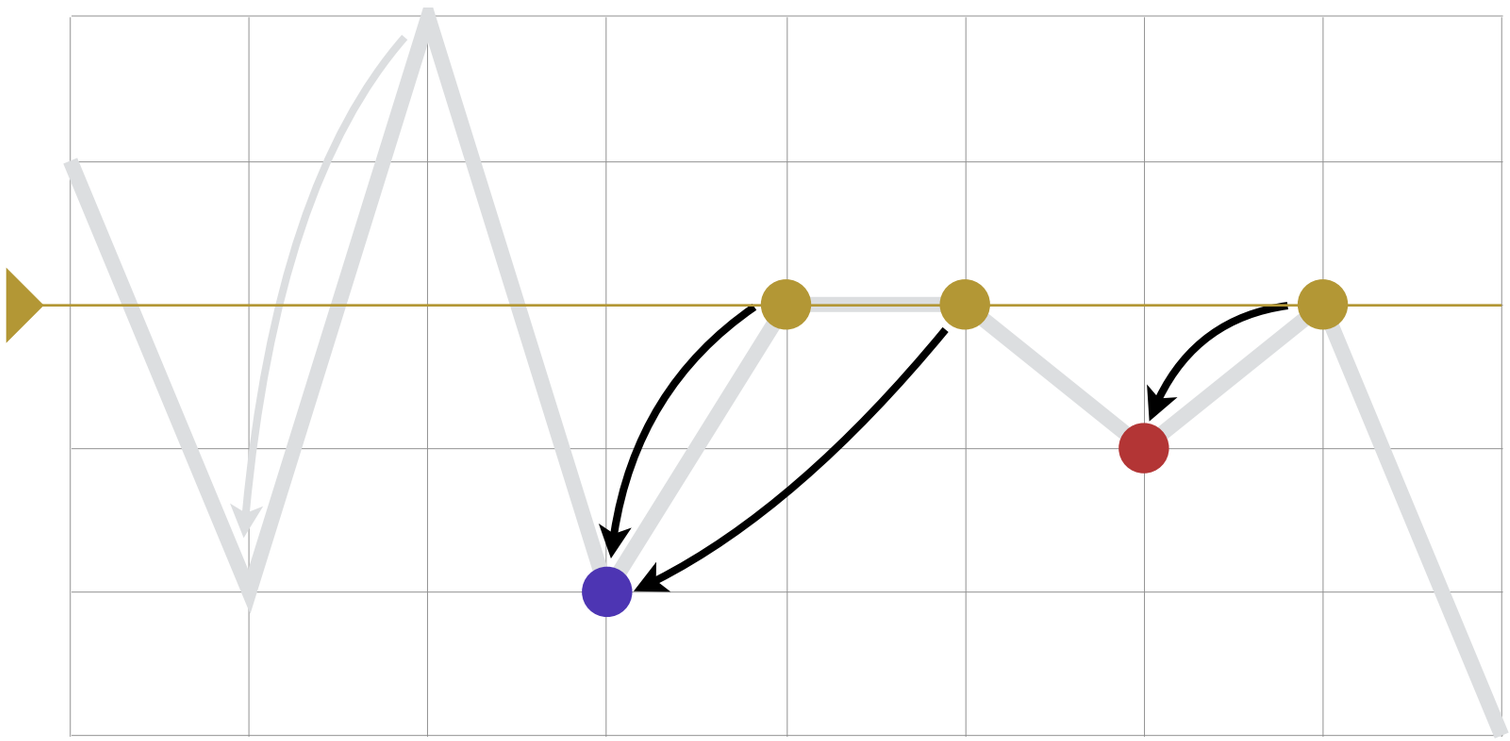


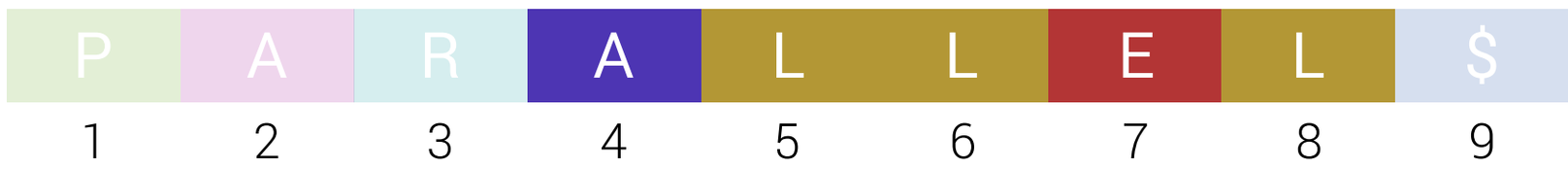


Phase 1

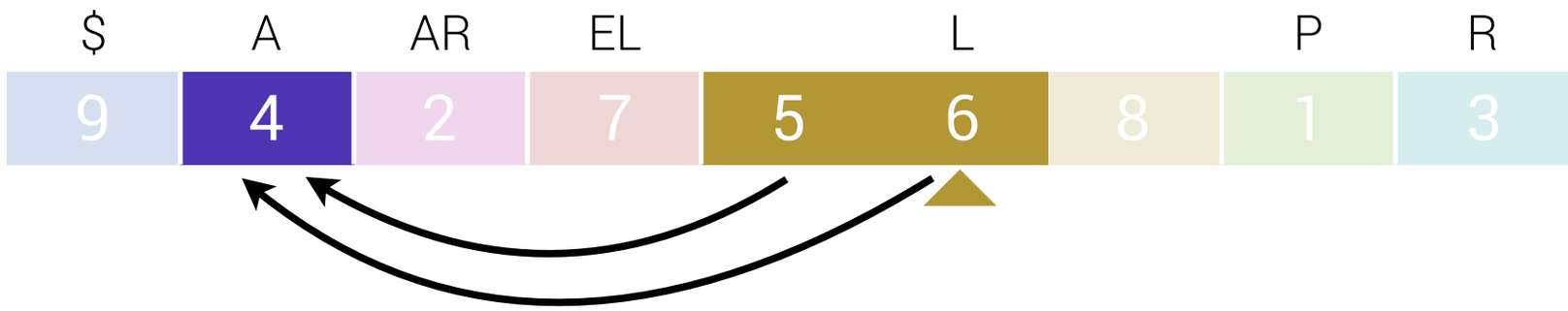


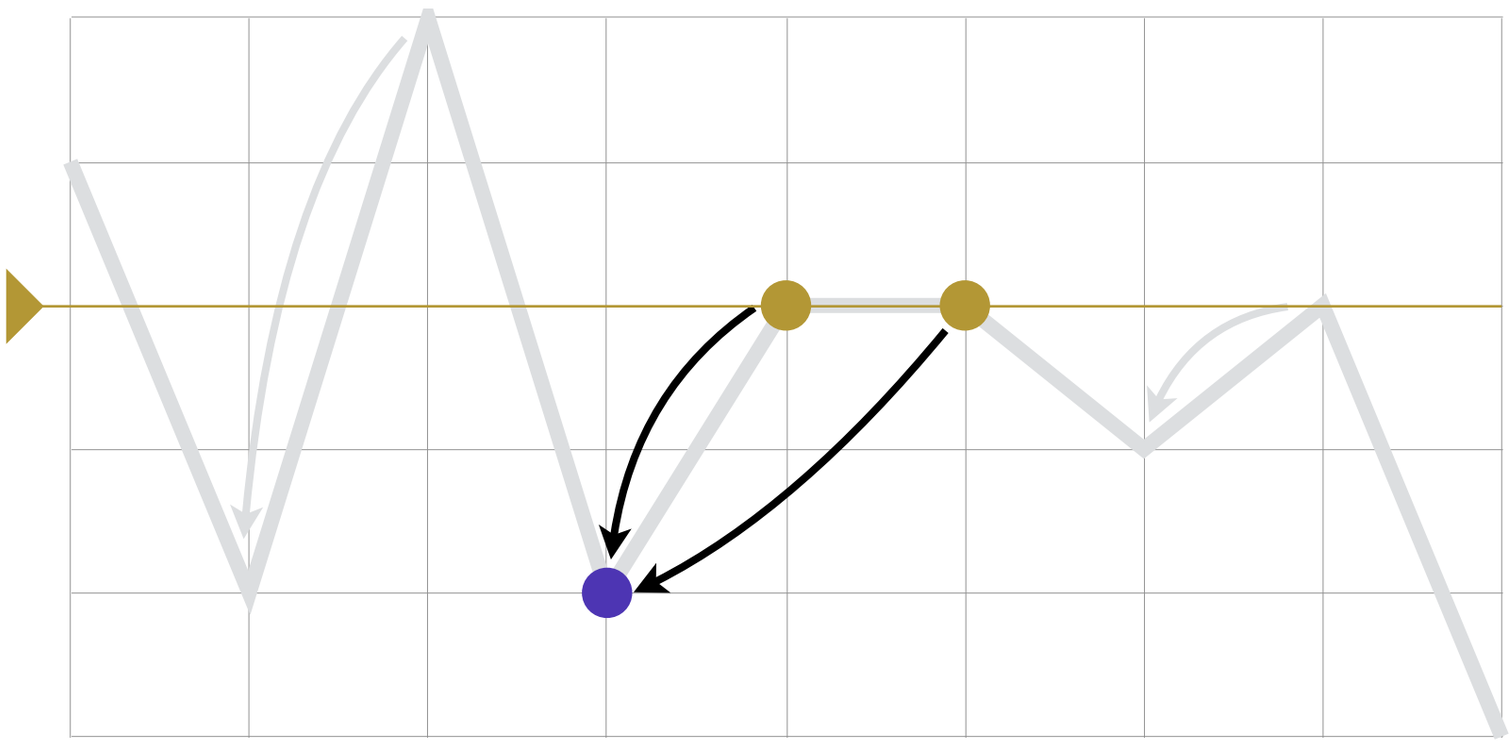
Phase 1

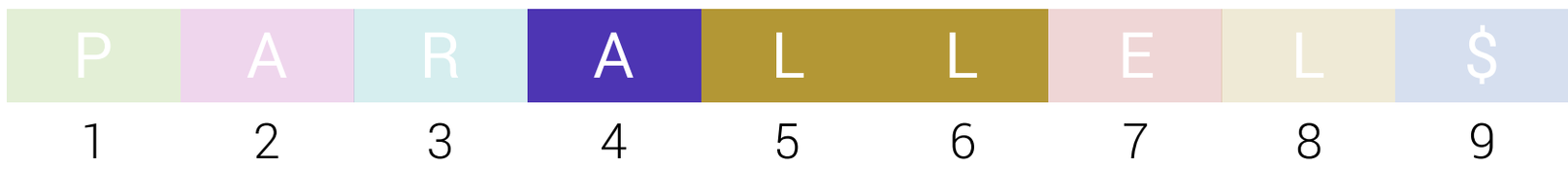




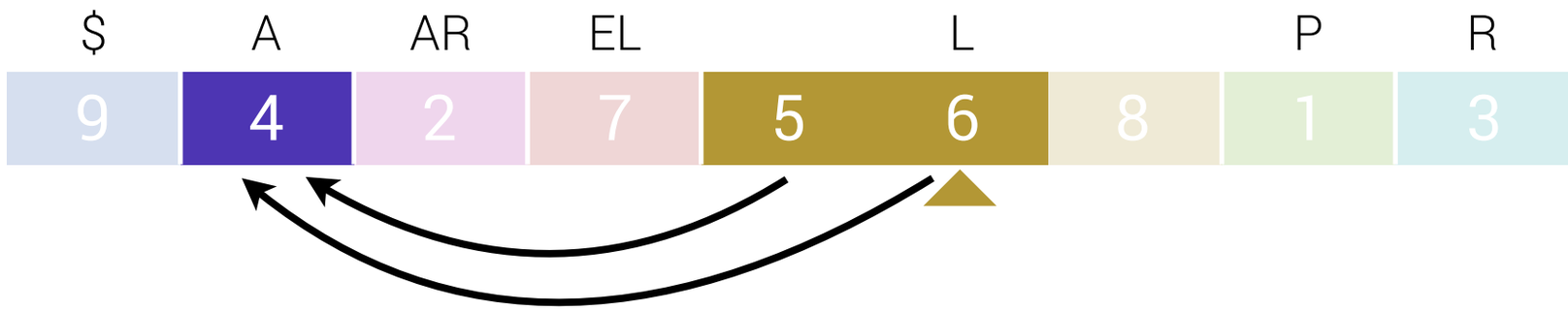
Phase 1

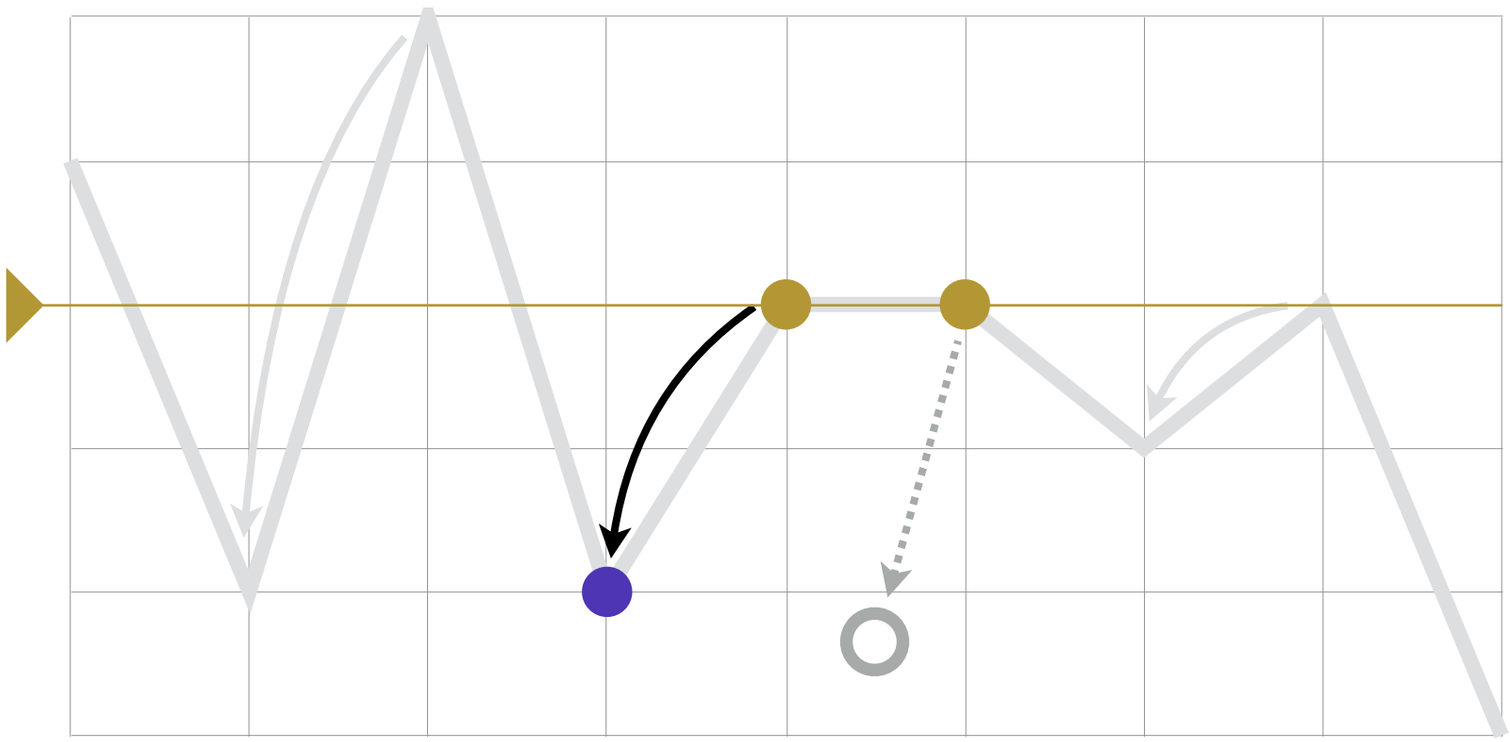


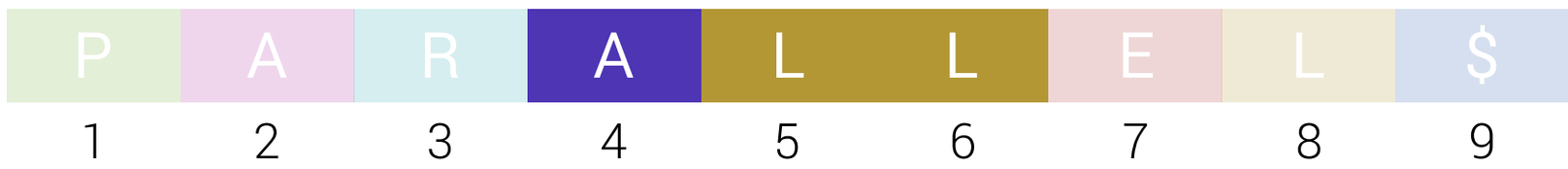




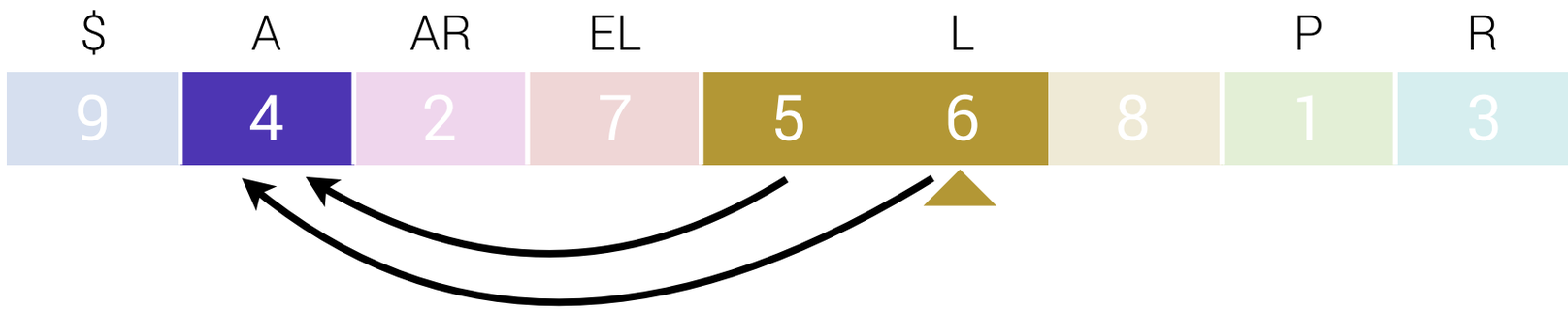
Phase 1

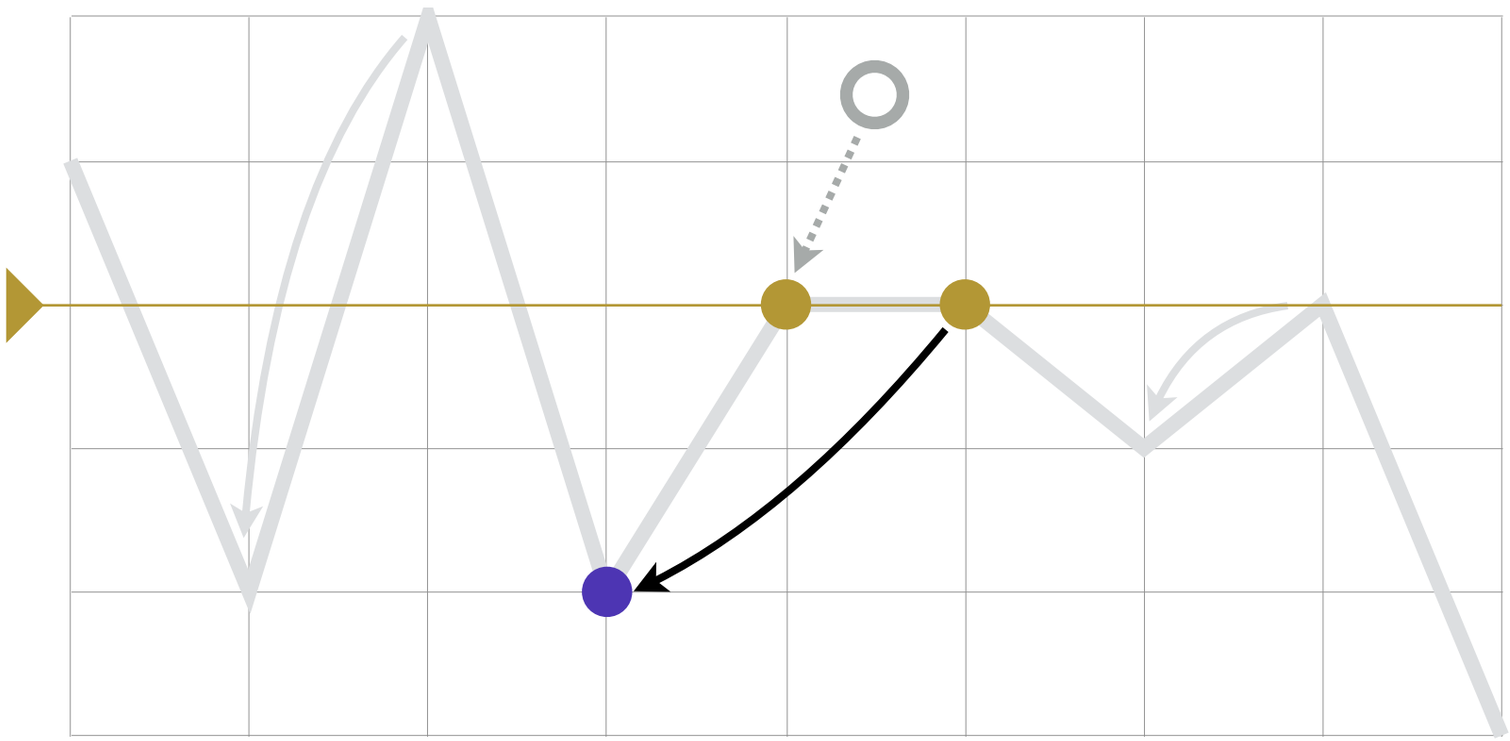


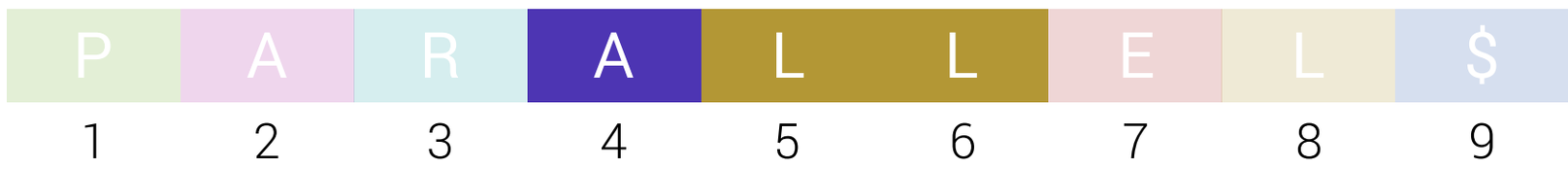




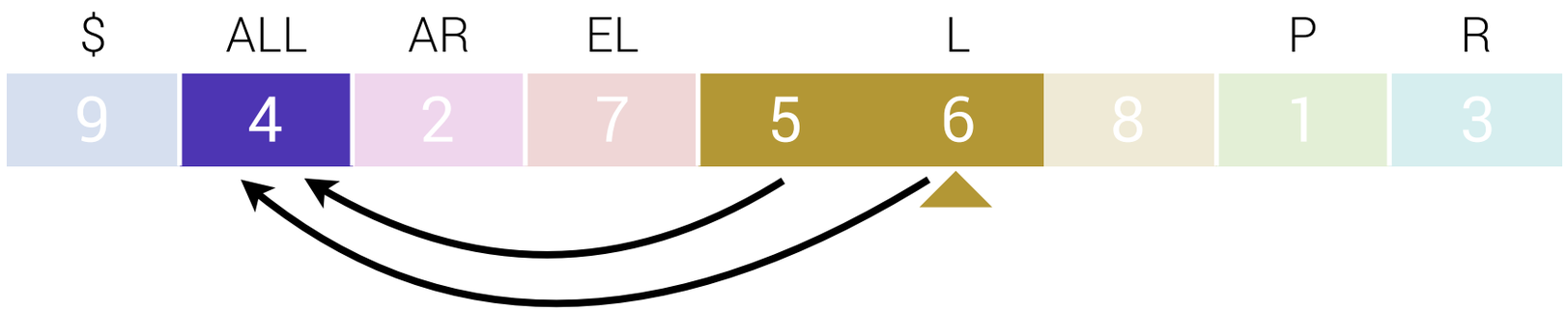
Phase 1

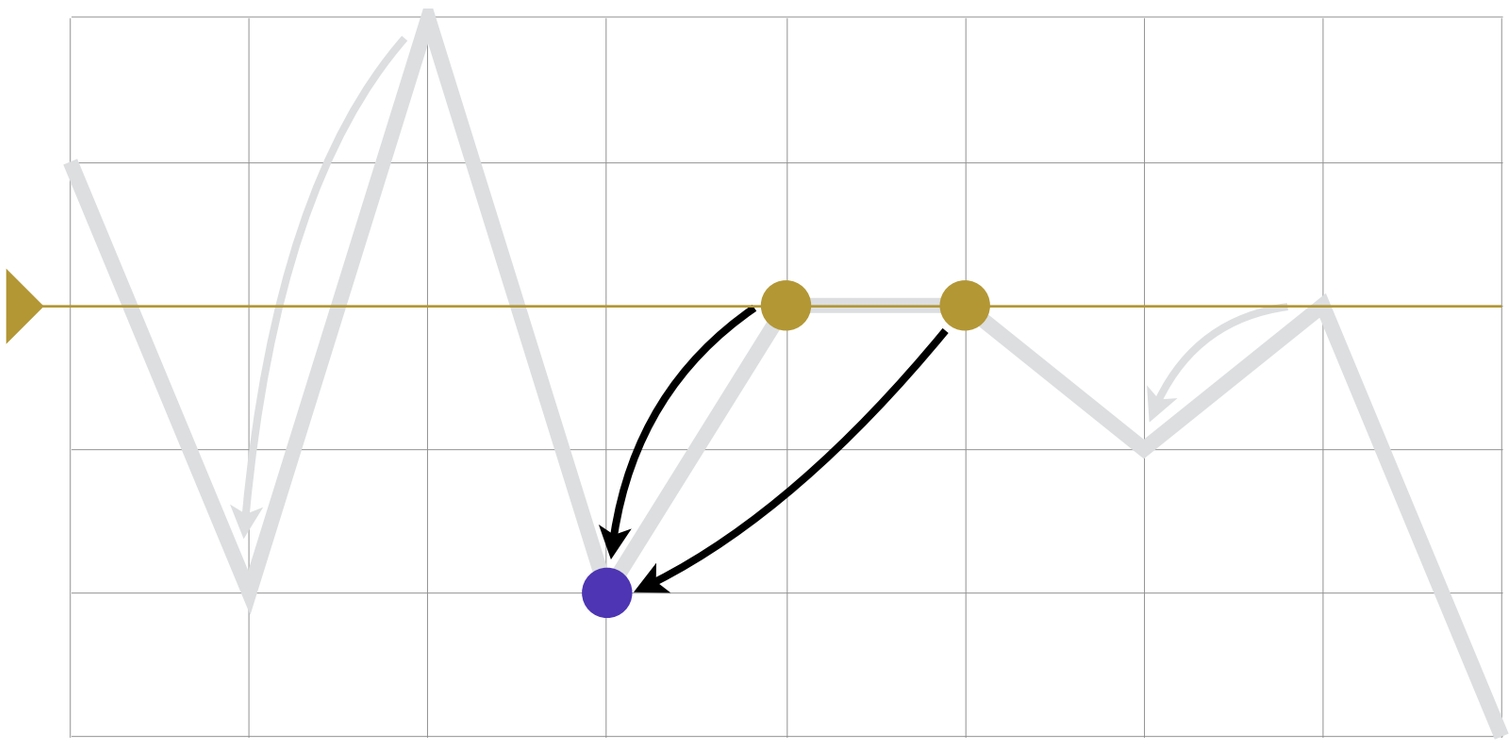






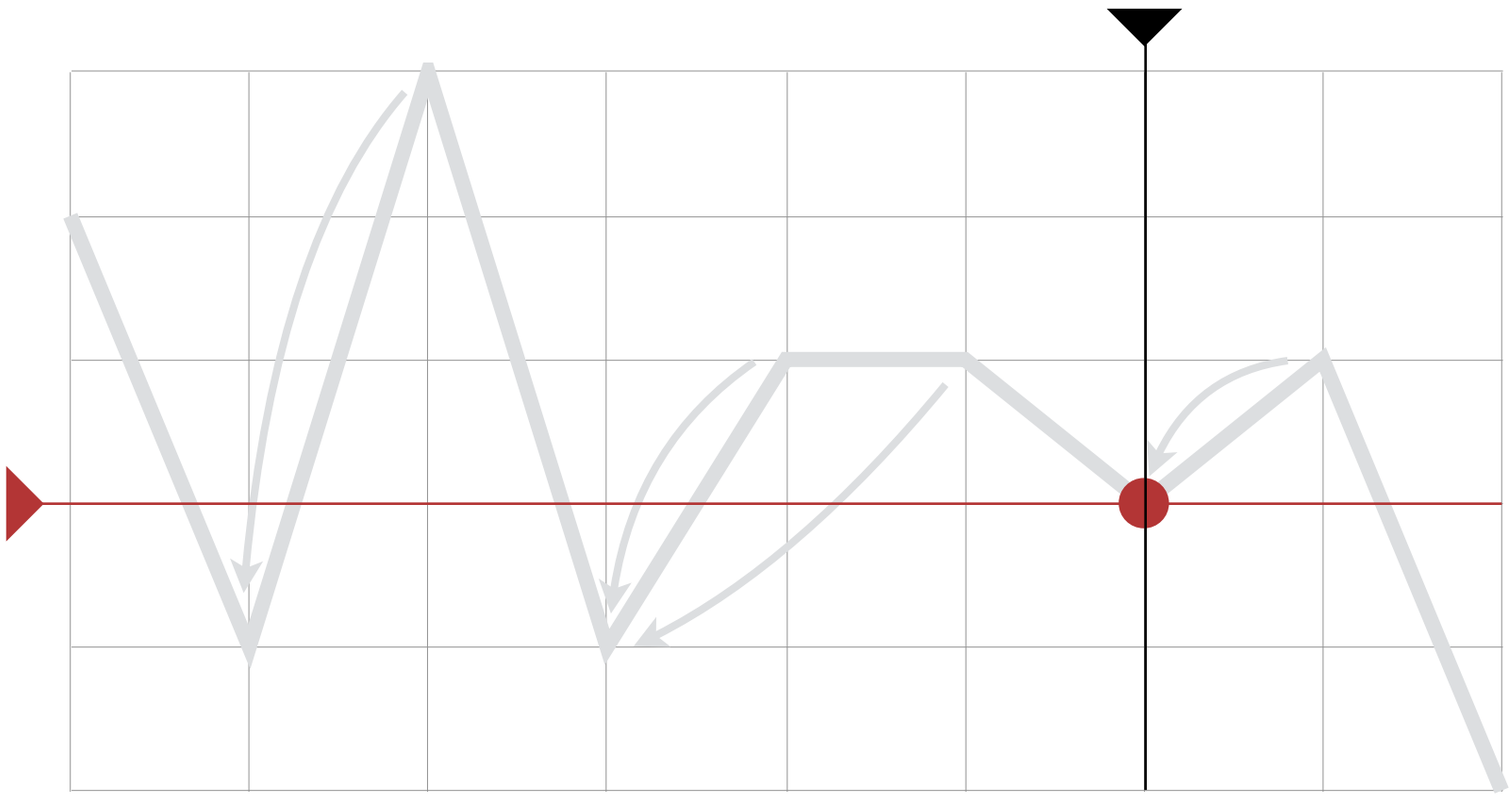
Phase 1





Phase 1

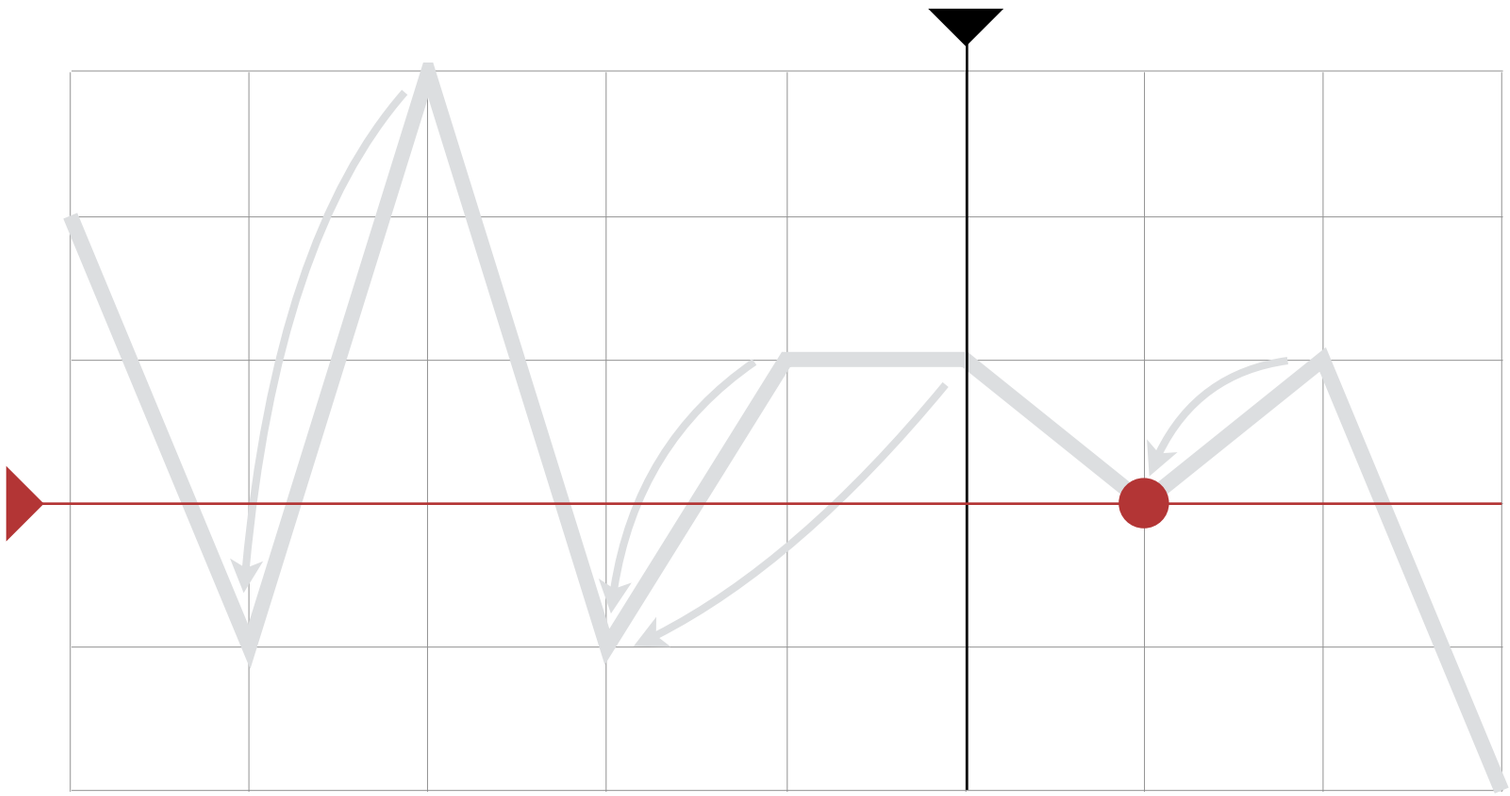
$	ALL	AR	EL		L		P	R
9	4	2	7	5	6	8	1	3



P	A	R	A	L	L	E	L	$
1	2	3	4	5	6	7	8	9

Phase 1

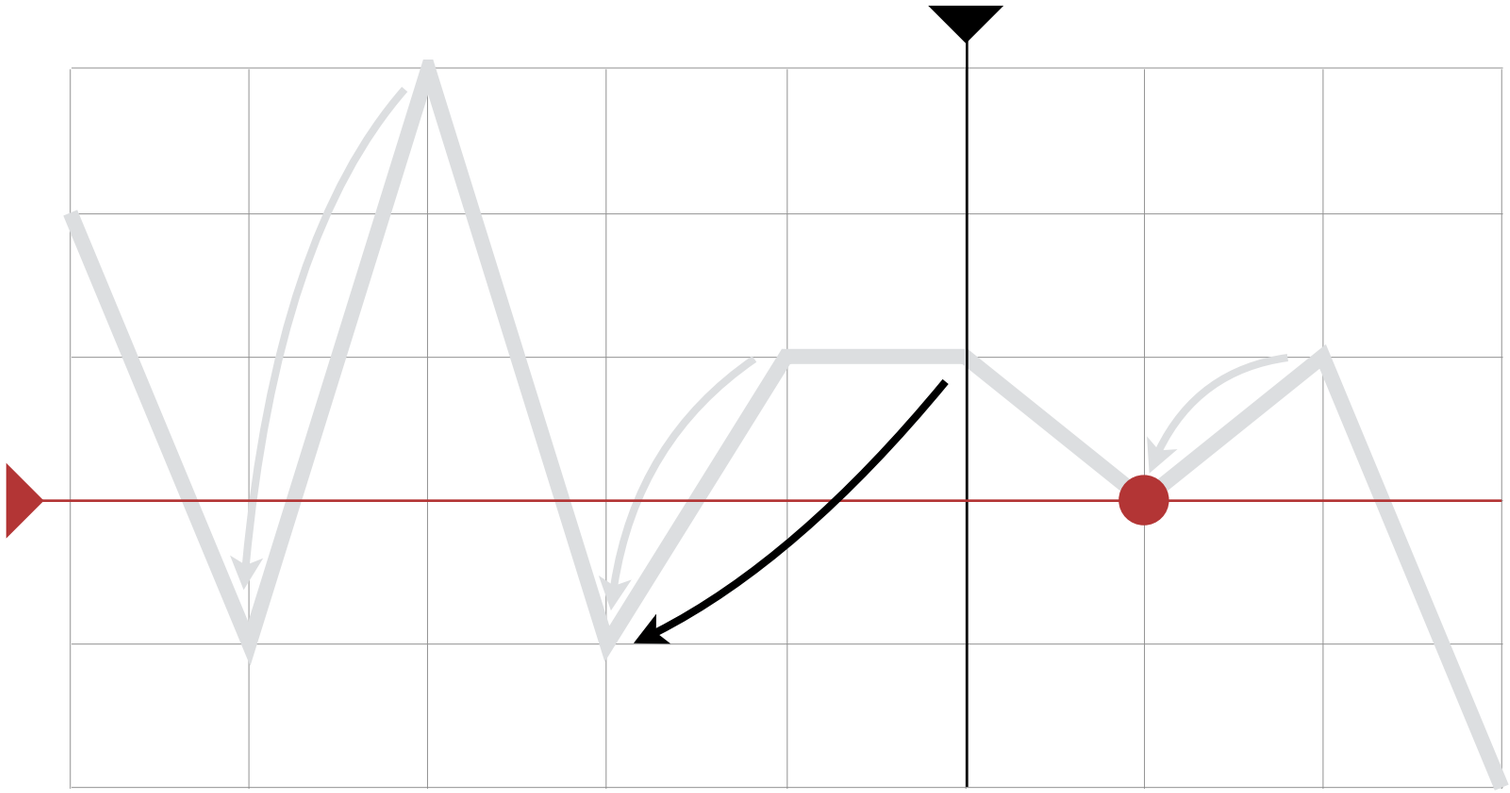
$	ALL	AR	EL		L		P	R
9	4	2	7	5	6	8	1	3



P	A	R	A	L	L	E	L	$
1	2	3	4	5	6	7	8	9

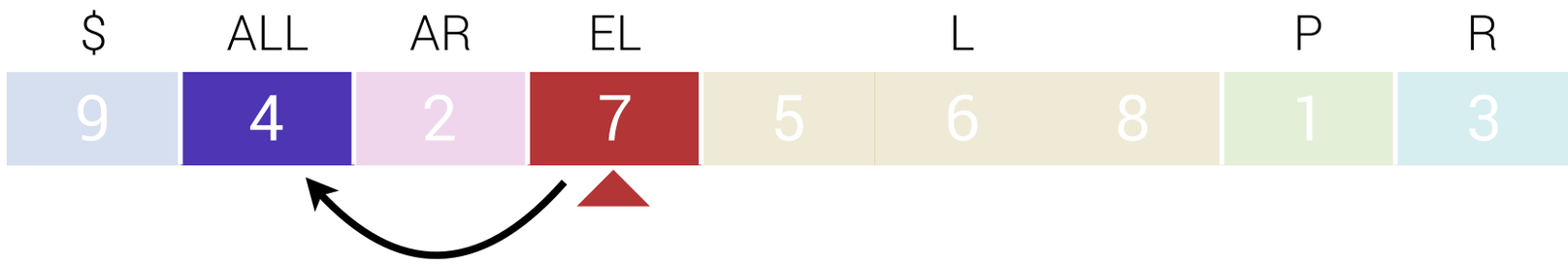
Phase 1

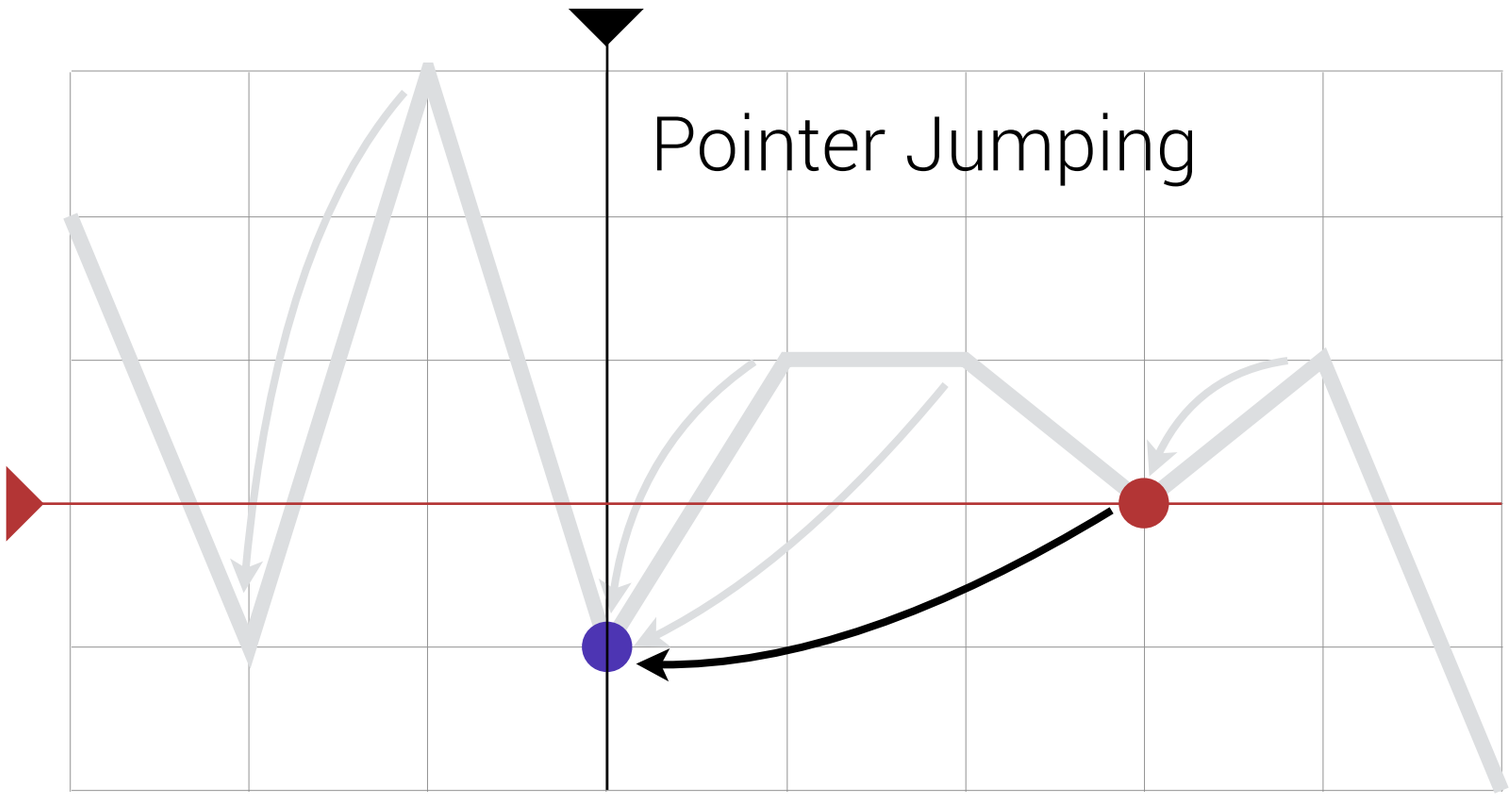
$	ALL	AR	EL		L		P	R
9	4	2	7	5	6	8	1	3

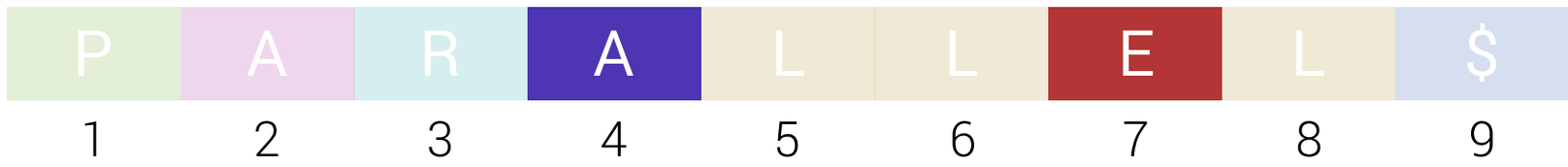


P	A	R	A	L	L	E	L	$
1	2	3	4	5	6	7	8	9

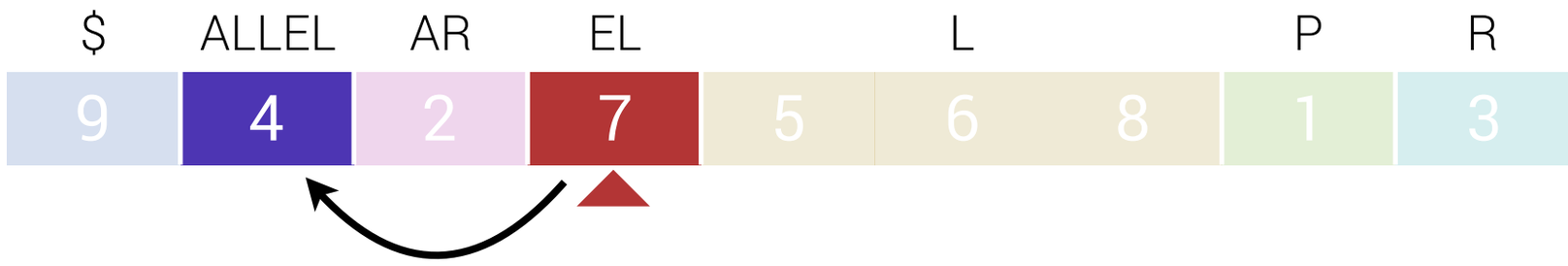
Phase 1

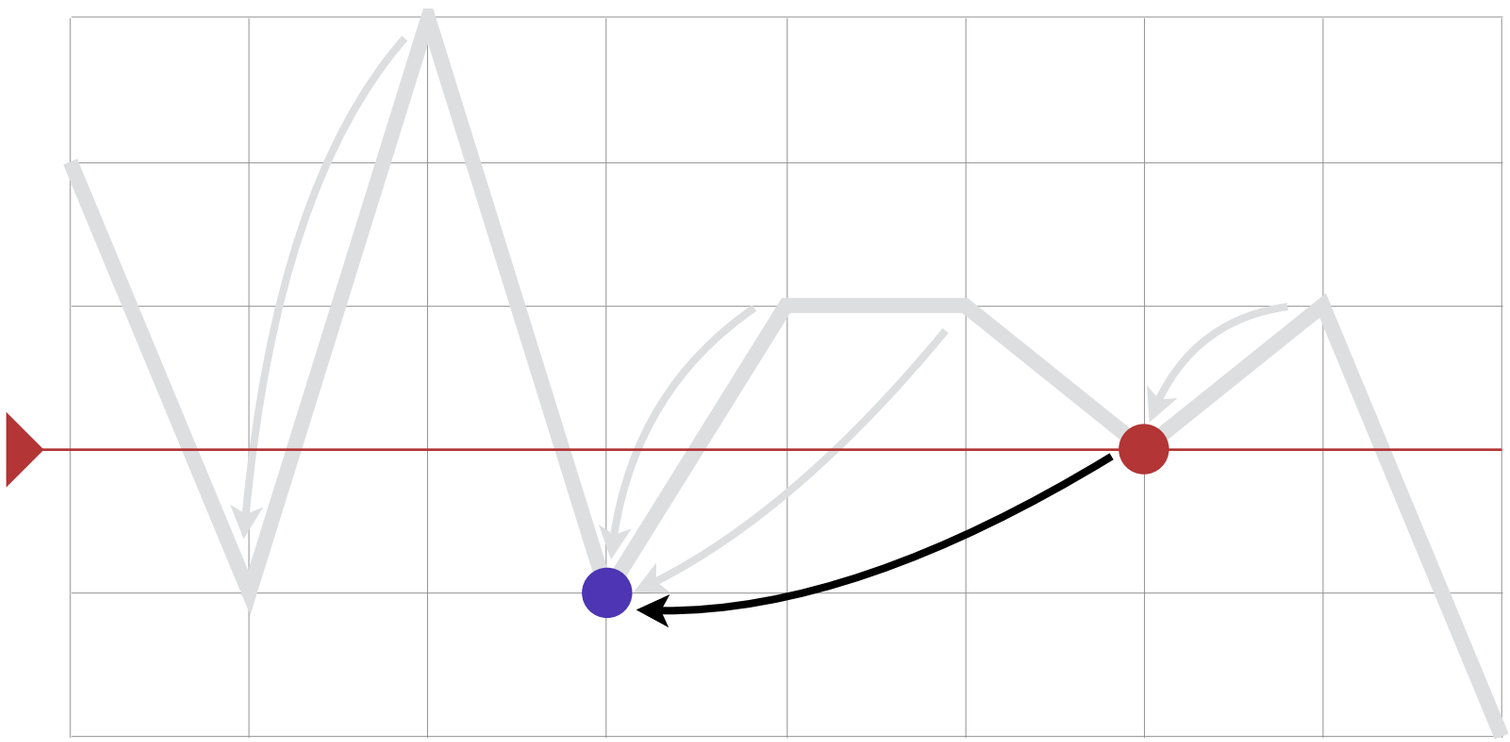

$ ALL AR EL L P R
9 4 2 7 5 6 8 1 3


Pointer Jumping


P A R A L L E L $
1 2 3 4 5 6 7 8 9

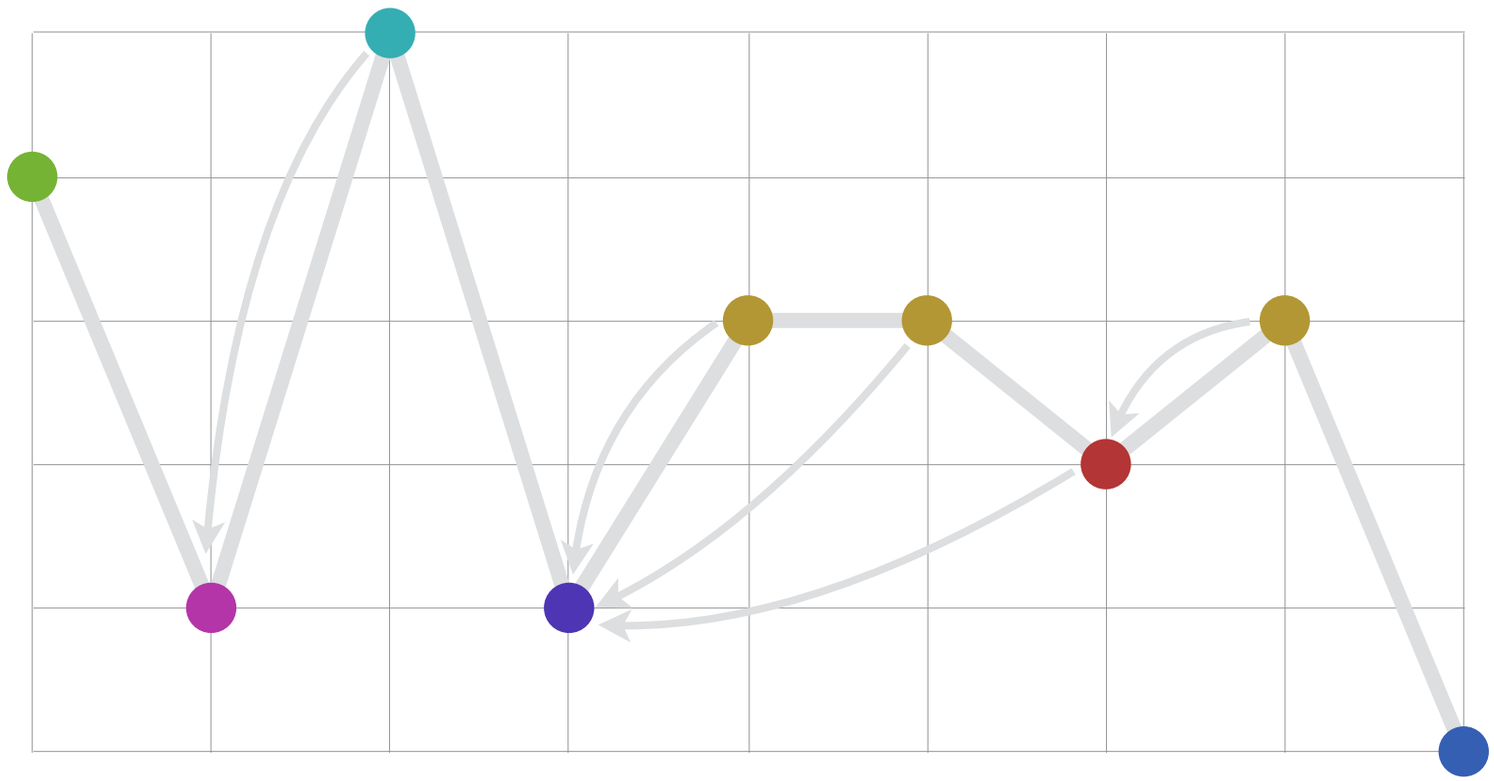
Phase 1





Phase 1

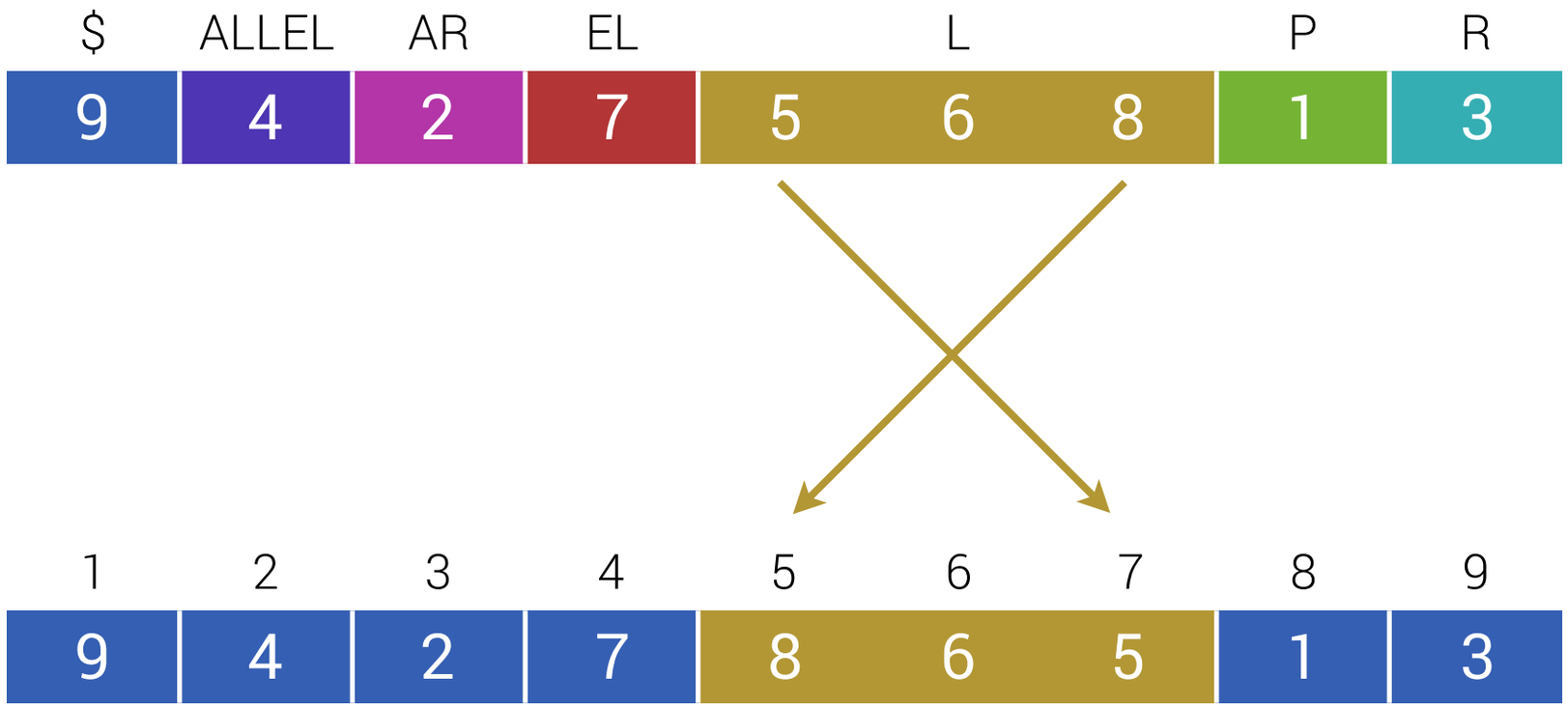
$	ALLEL	AR	EL		L		P	R
9	4	2	7	5	6	8	1	3



P	A	R	A	L	L	E	L	$
1	2	3	4	5	6	7	8	9

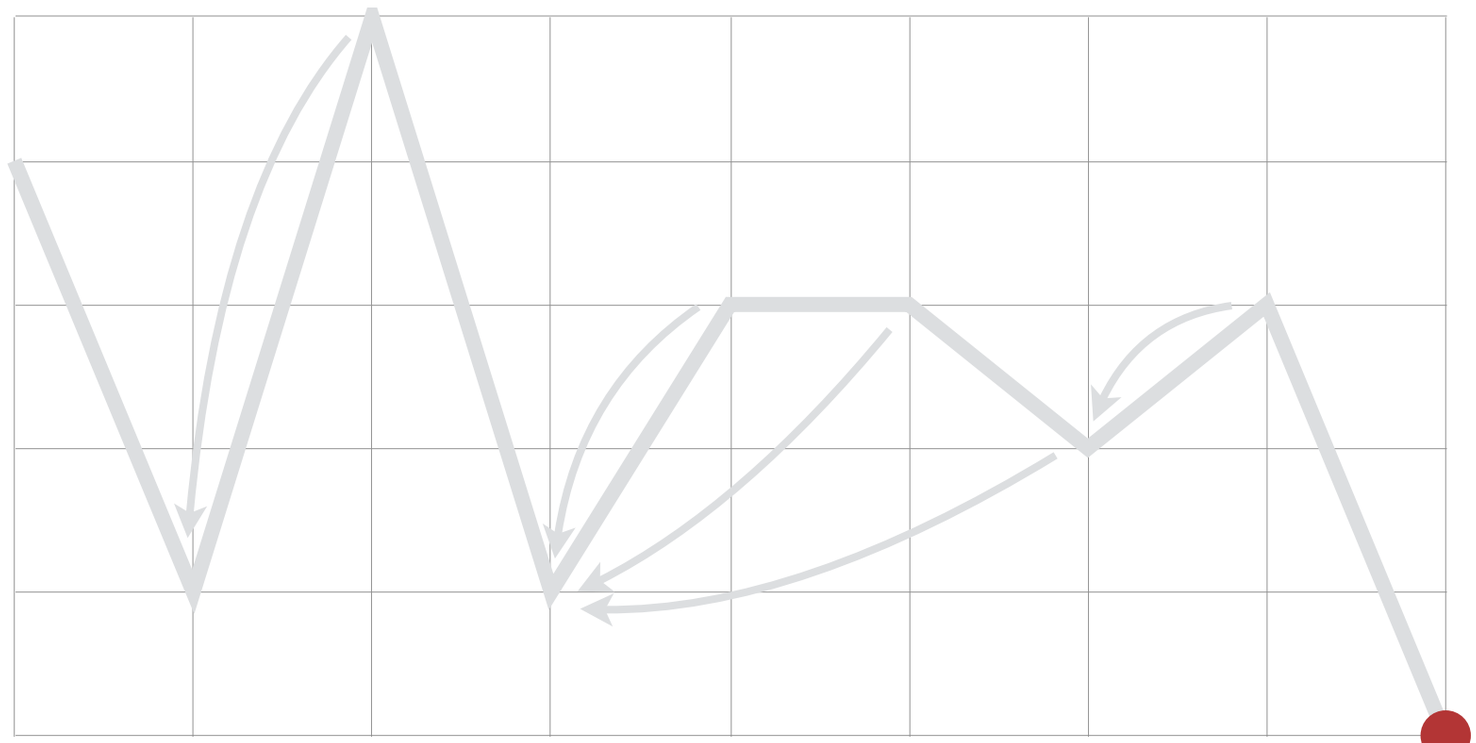
Phase 2

Suffixe innerhalb der Gruppen sortieren



Phase 2

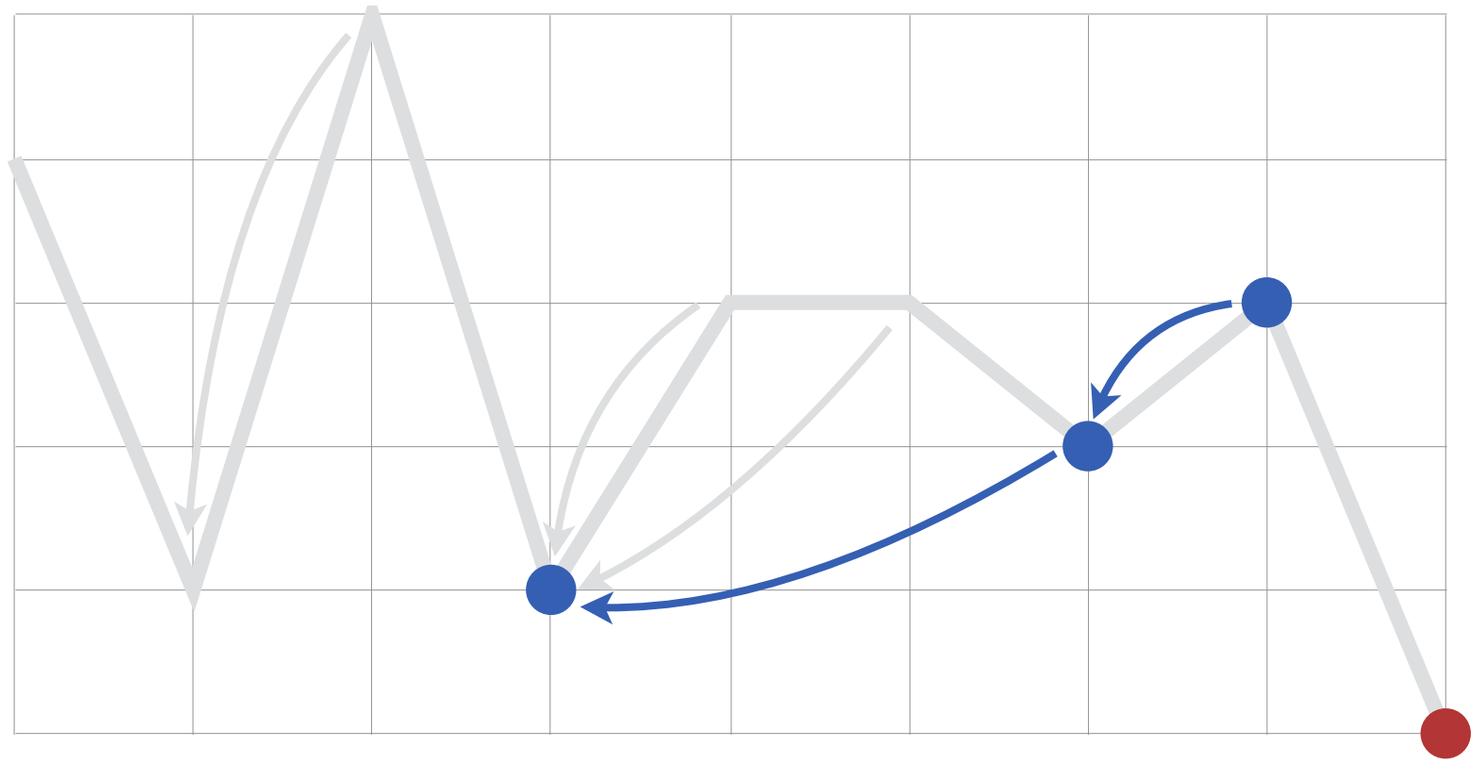
$	ALLEL	AR	EL		L		P	R
9	4	2	7	5	6	8	1	3



P	A	R	A	L	L	E	L	$
1	2	3	4	5	6	7	8	9

Phase 2

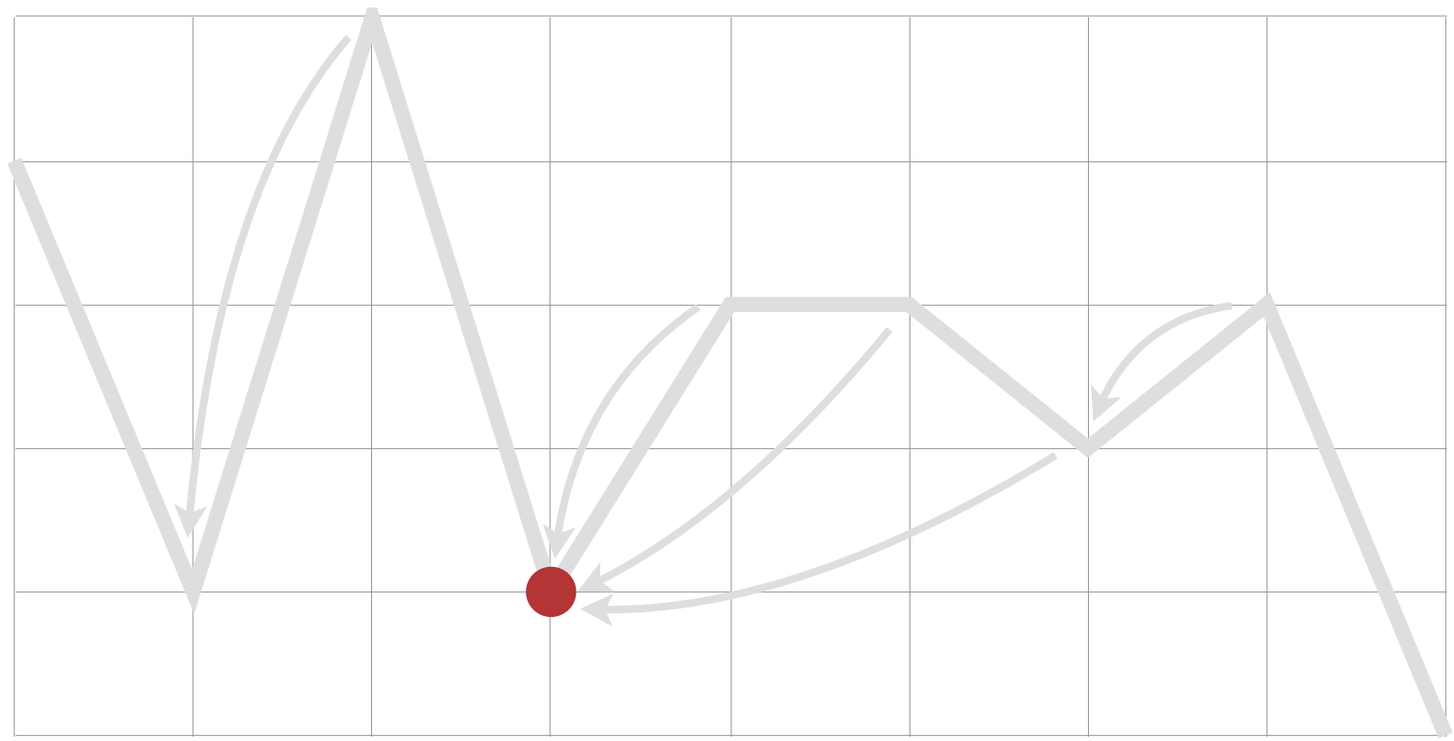
$	ALLEL$	AR	EL$	L$	L		P	R
9	4	2	7	8	5	6	1	3



P	A	R	A	L	L	E	L	$
1	2	3	4	5	6	7	8	9

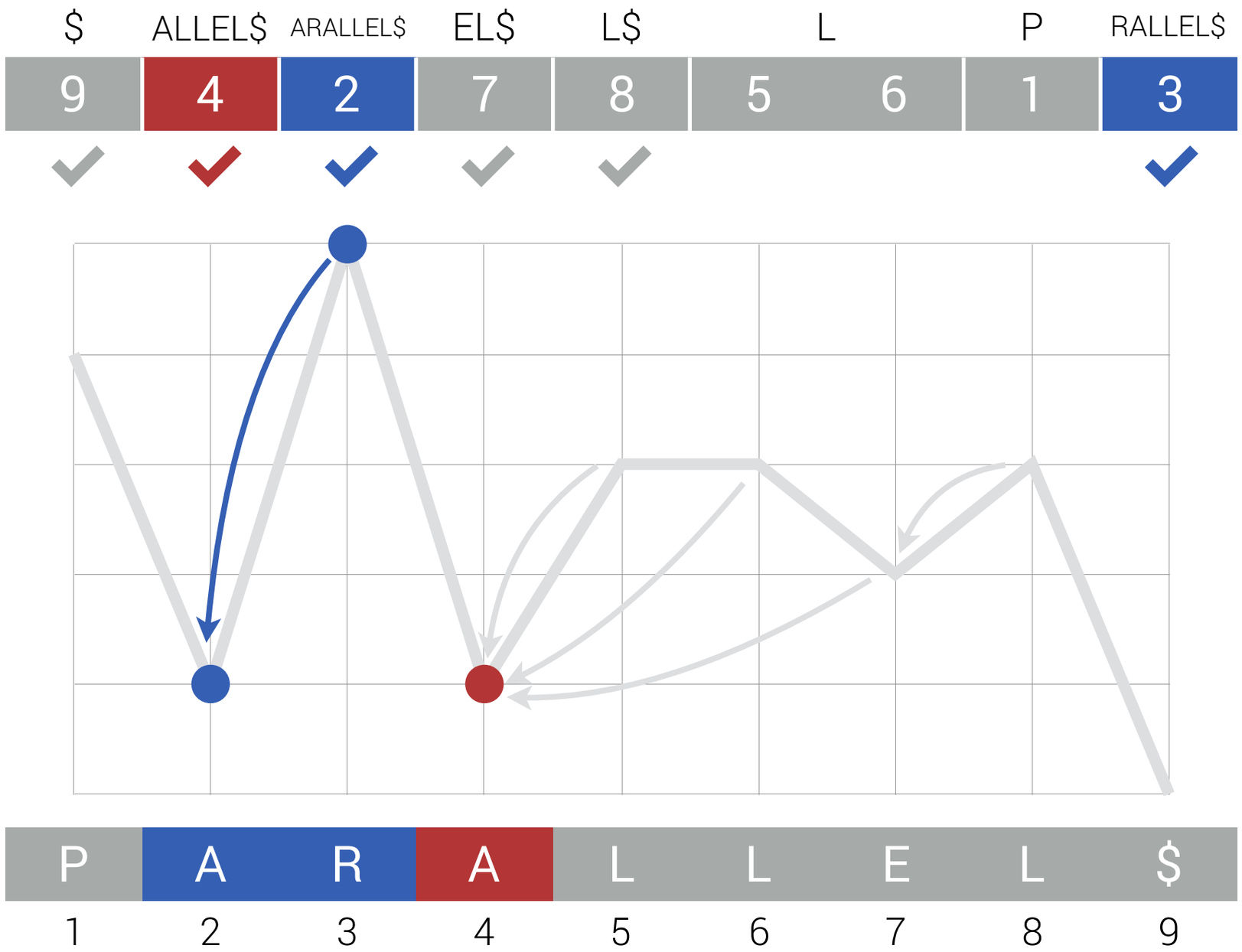
Phase 2

$	ALLEL$	AR	EL$	L$	L		P	R
9	4	2	7	8	5	6	1	3



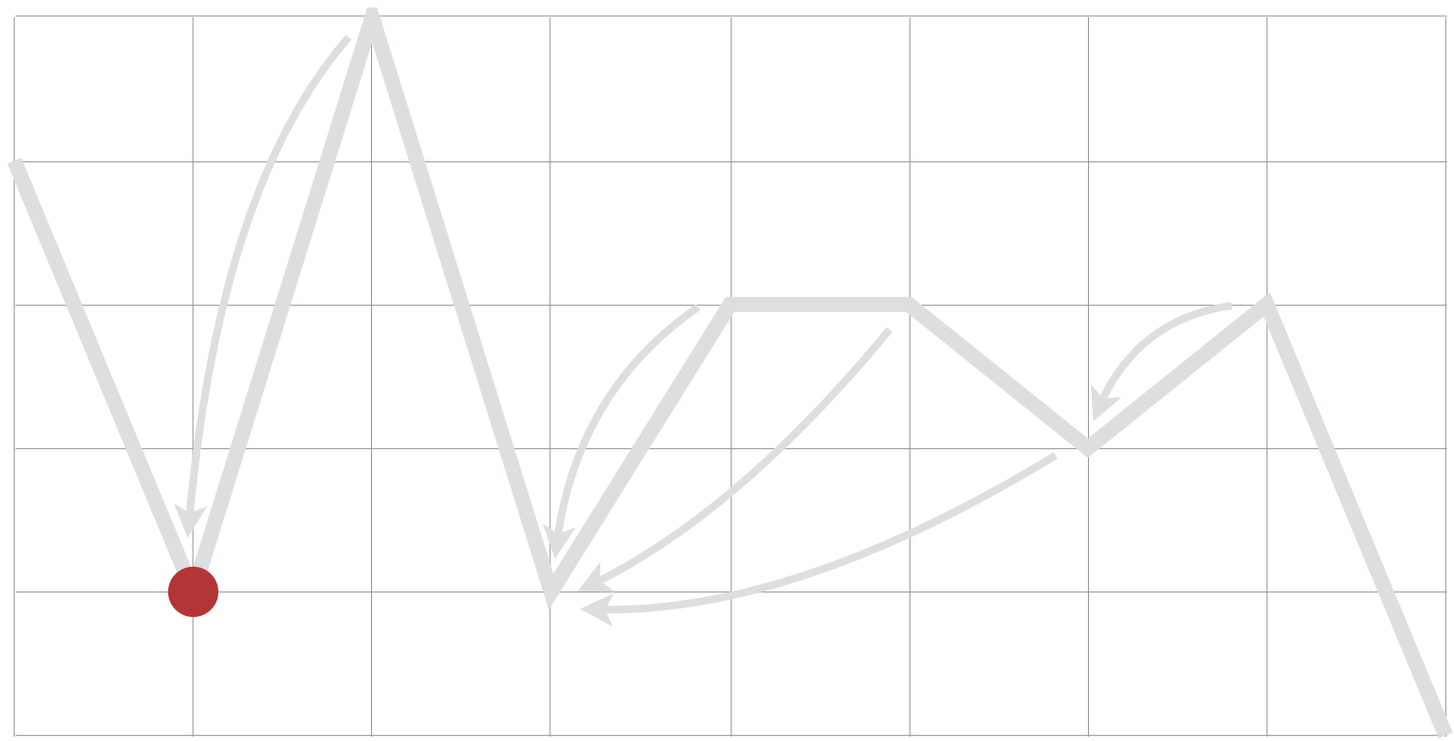
P	A	R	A	L	L	E	L	$
1	2	3	4	5	6	7	8	9

Phase 2



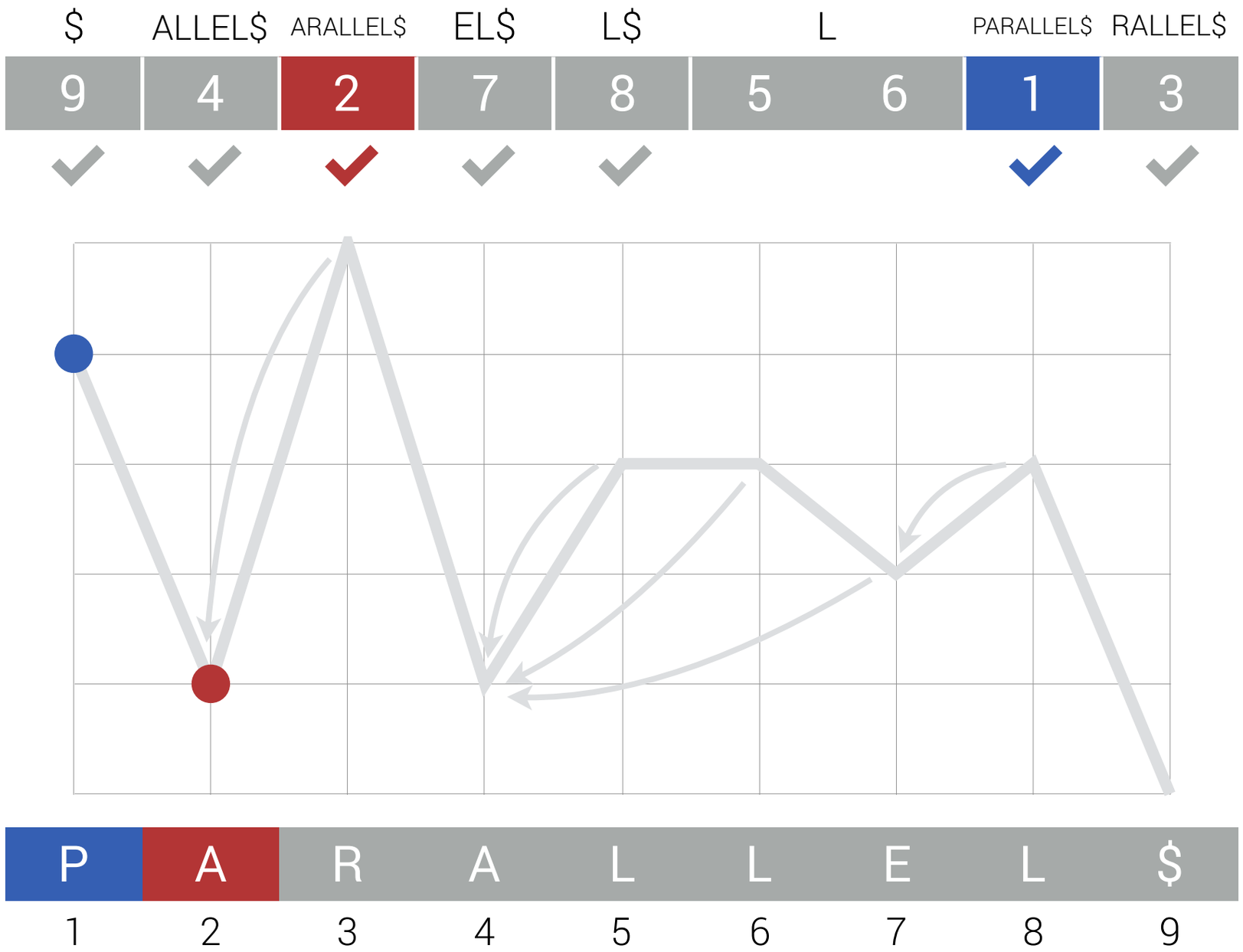
Phase 2

$	ALLEL$	ARALLEL$	EL$	L$	L		P	RALLEL$
9	4	2	7	8	5	6	1	3
✓	✓	✓	✓	✓				✓



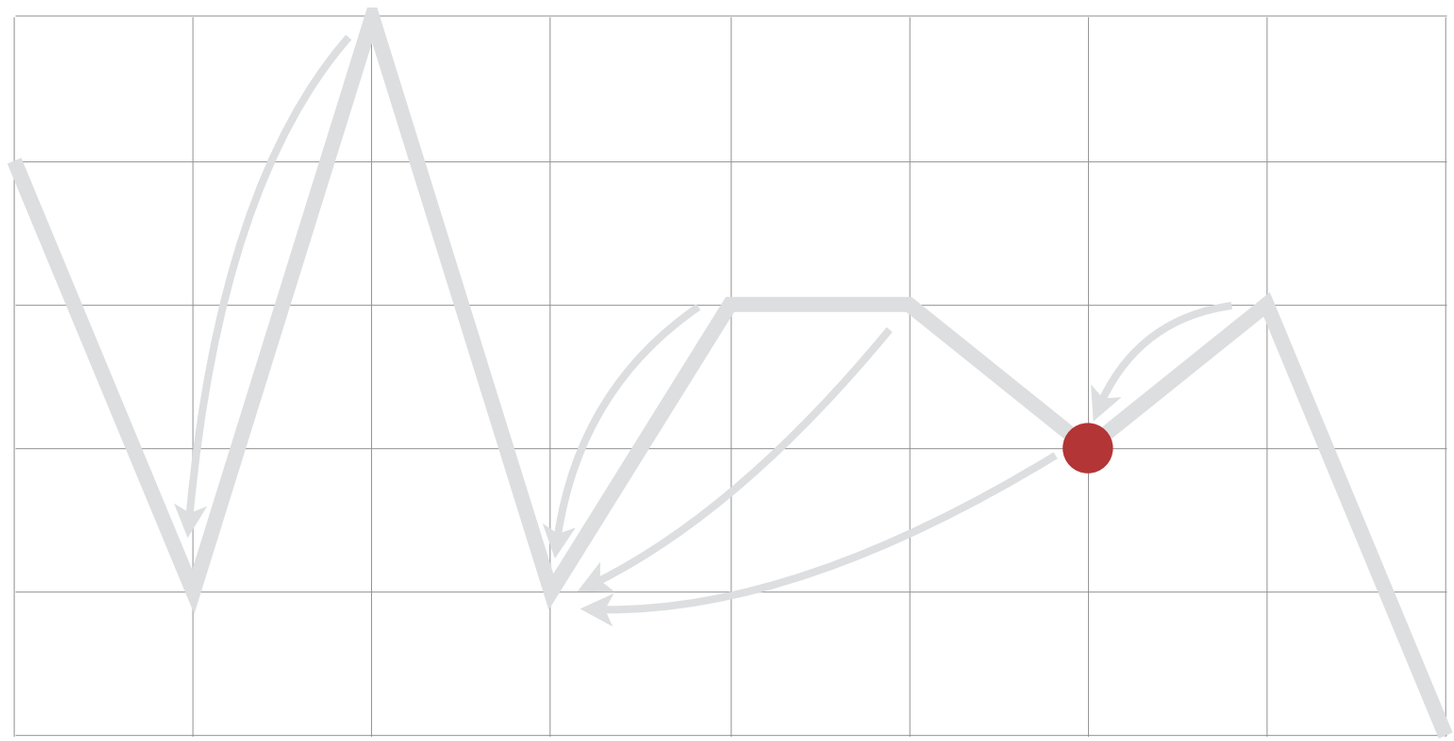
P	A	R	A	L	L	E	L	$
1	2	3	4	5	6	7	8	9

Phase 2


$ ALLEL$ ARALLEL$ EL$ L$ L PARALLEL$ RALLEL$
9 4 2 7 8 5 6 1 3
P A R A L L E L $
1 2 3 4 5 6 7 8 9

Phase 2

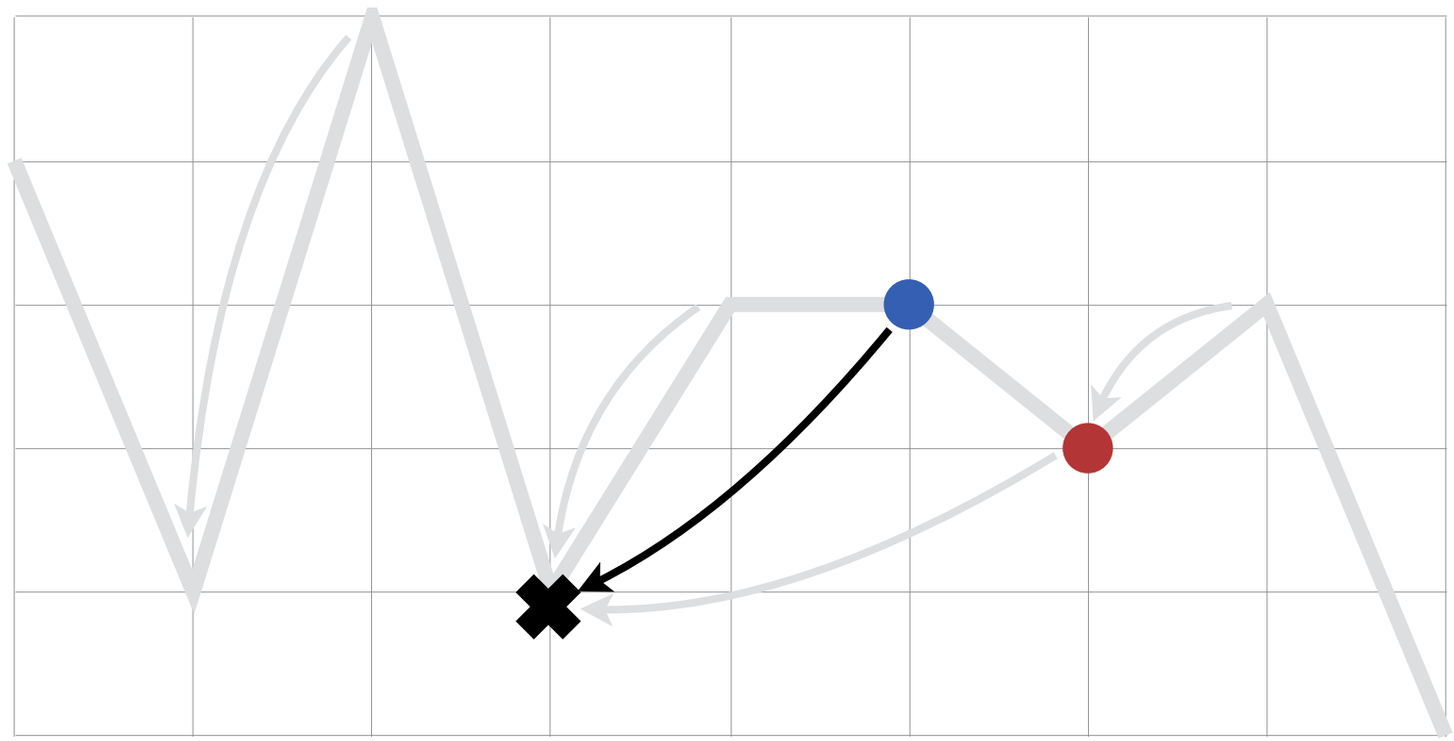
$	ALLEL$	ARALLEL$	EL$	L$	L		PARALLEL$	RALLEL$
9	4	2	7	8	5	6	1	3
✓	✓	✓	✓	✓			✓	✓



P	A	R	A	L	L	E	L	$
1	2	3	4	5	6	7	8	9

Phase 2

$	ALLEL$	ARALLEL$	EL$	L$	LEL$	L	PARALLEL$	RALLEL$
9	4	2	7	8	6	5	1	3



P	A	R	A	L	L	E	L	$
1	2	3	4	5	6	7	8	9

Phase 2

	$	ALLEL$	ARALLEL$	EL$	L$	LEL$	LLEL$	PARALLEL$	RALLEL$
SA =	9	4	2	7	8	6	5	1	3

Performance

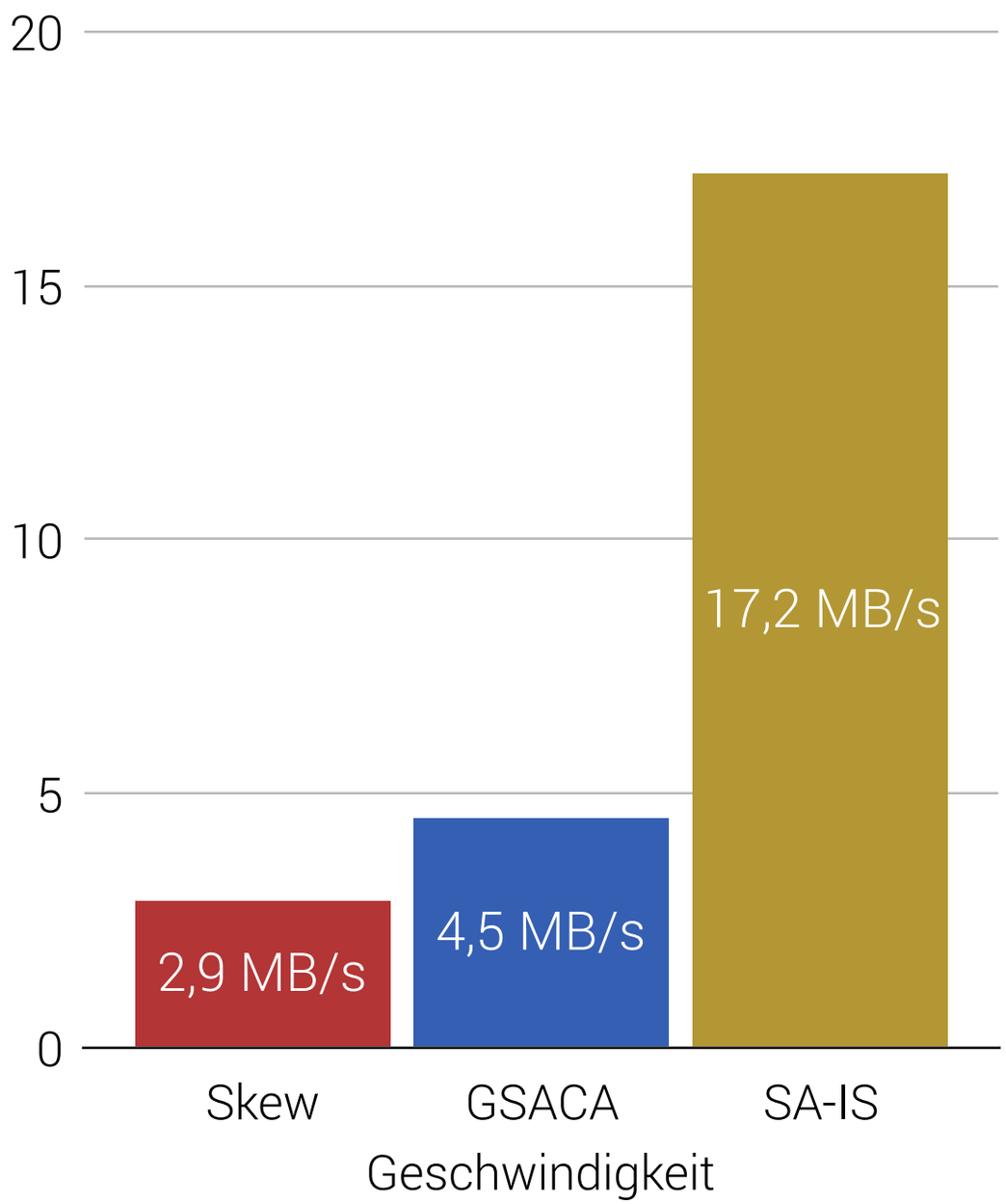
Linearzeit Ansätze

	Skew	SA-IS	GSACA
Art	rekursiv	rekursiv	iterativ
Zeit	$O(n)$	$O(n)$	$O(n)$
Speicher	$O(\log n) + \max 24n$	$O(\log n) + \max 2n$	$O(1) + ?$

Linearzeit Ansätze

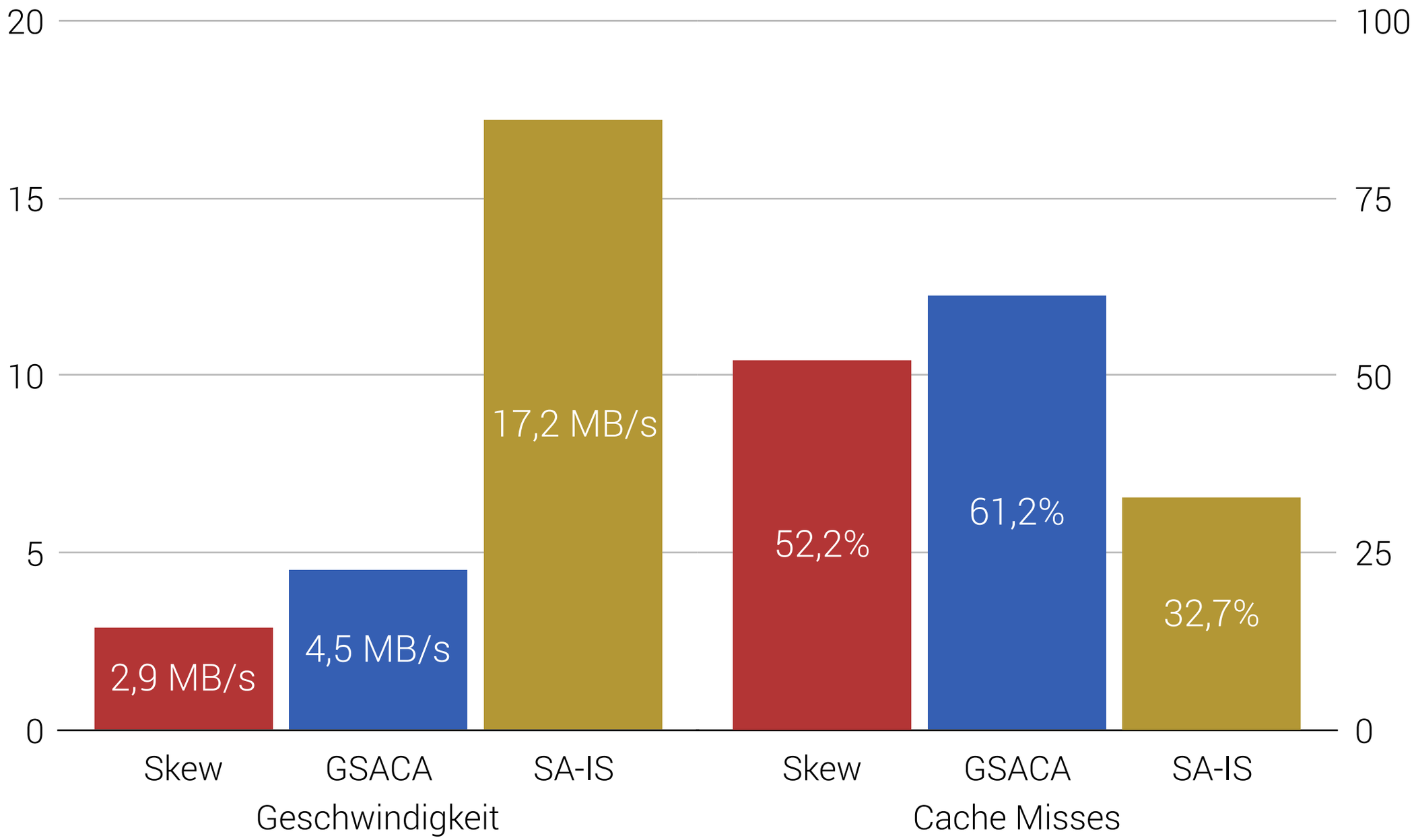
	Skew	SA-IS	GSACA
Art	rekursiv	rekursiv	iterativ
Zeit	$O(n)$	$O(n)$	$O(n)$
Speicher	$O(\log n) + \max 24n$	$O(\log n) + \max 2n$	$O(1) + 12n$

GSACA im Vergleich



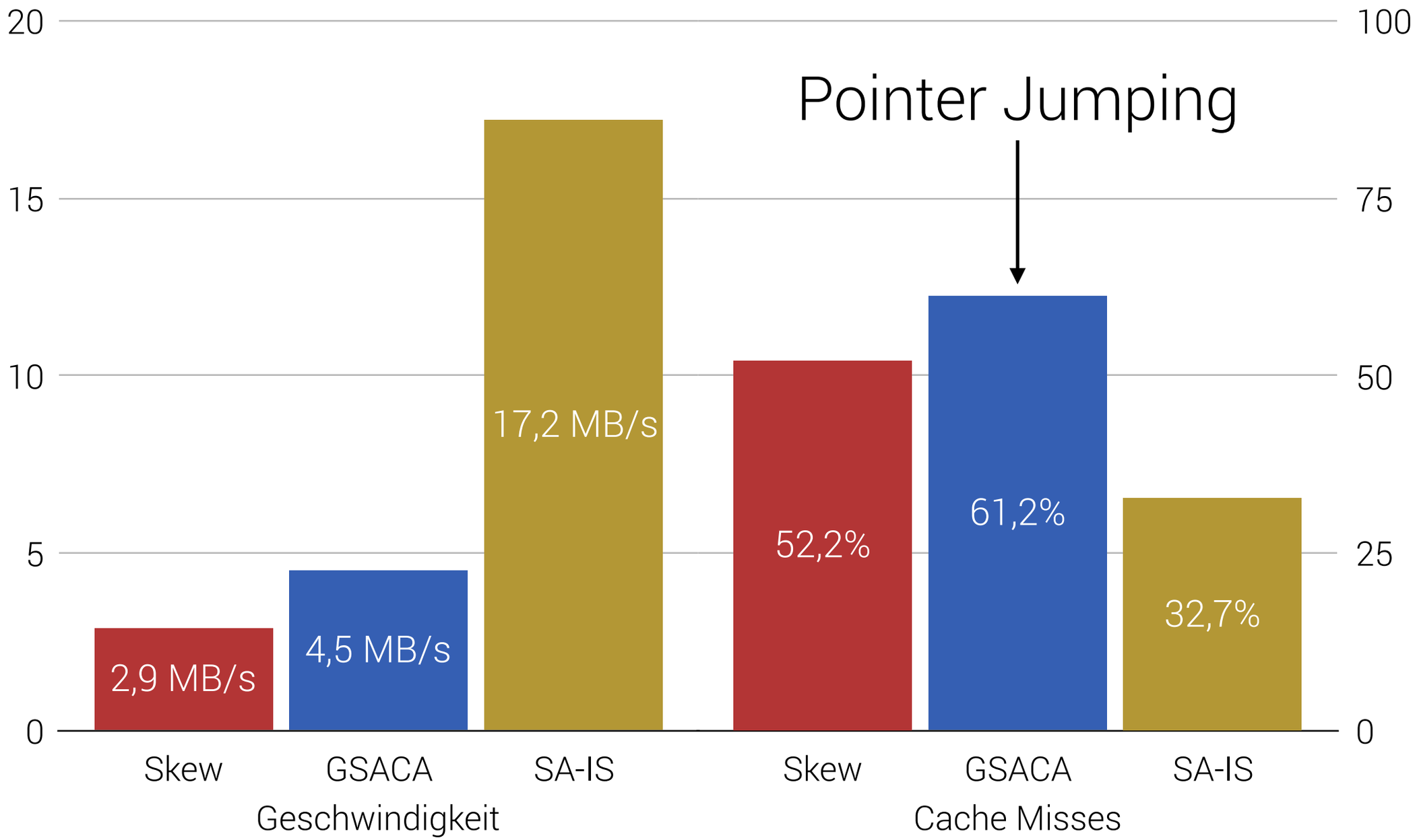
Testdaten: Silesia Corpus

GSACA im Vergleich



Testdaten: Silesia Corpus

GSACA im Vergleich

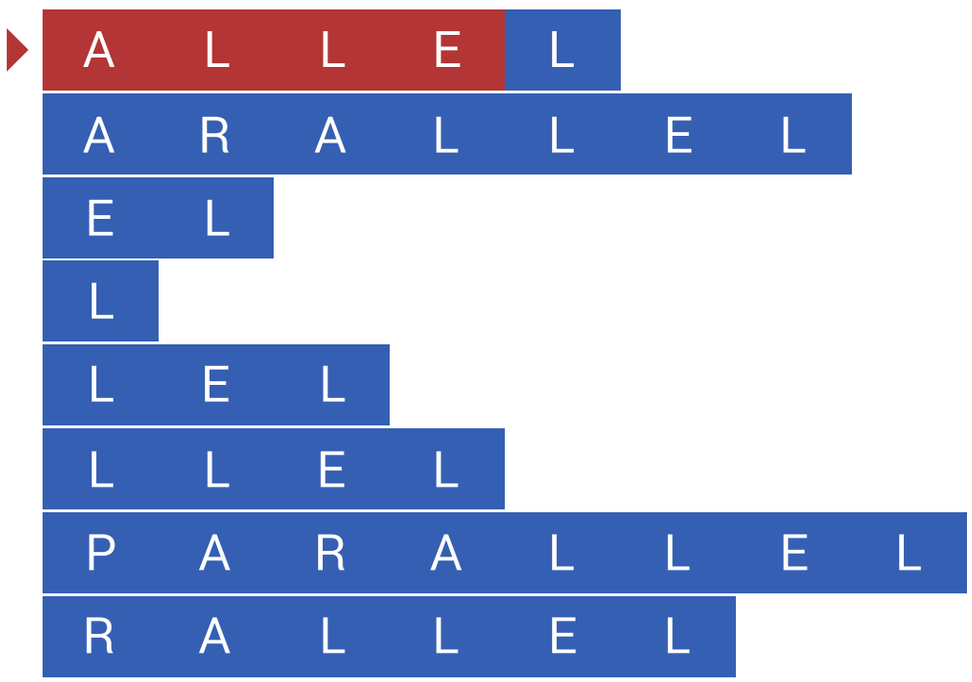


Testdaten: Silesia Corpus

Rückblick

Einsatzgebiete

Substringsuche



LZ77 Kompression

GSACA

Eingabe

Phase 1 Sortierte Folge von Gruppen berechnen $O(n)$

Phase 2 Suffixe innerhalb der Gruppen sortieren $O(n)$

Performance

Performance

Noch nicht praxistauglich.

Performance

~~Noch nicht praxistauglich.~~

Performance

~~Noch nicht praxistauglich.~~

Neuartiges Konzept mit vielen spannenden noch zu lösenden Problemen…

Danke!

Ergänzungen

Phase 1

1: order all suffixes of S into groups according to their first character:
Let S_i and S_j be two suffixes. Then, $\text{group}(i) = \text{group}(j) \Leftrightarrow S[i] = S[j]$.
2: order the suffix groups: Let $\mathcal{G}_1$ be a suffix group with group context character u,
$\mathcal{G}_2$ be a suffix group with group context character v. Then, $\mathcal{G}_1 < \mathcal{G}_2$ if $u < v$.
3: **for each** group $\mathcal{G}$ in descending group order **do**
4: **for each** $i \in \mathcal{G}$ **do**
5: $\text{prev}(i) \leftarrow \max(\{\ j \in [1 \ldots i] \mid \text{group}(j) < \text{group}(i)\ \} \cup \{0\})$
6: let $\mathcal{P}$ be the set of previous suffixes from $\mathcal{G}$,
$\mathcal{P} \coloneqq \{\ j \in [1 \ldots n] \mid \text{prev}(i) = j \text{ for any } i \in \mathcal{G}\ \}$.
7: split $\mathcal{P}$ into k subsets $\mathcal{P}_1, \ldots, \mathcal{P}_k$ such that a subset $\mathcal{P}_l$ contains
suffixes whose number of prev pointers from $\mathcal{G}$ pointing to them
is equal to l, i.e. $i \in \mathcal{P}_l \Leftrightarrow |\{\ j \in \mathcal{G} \mid \text{prev}(j) = i\ \}| = l$.
8: **for** $l = k$ **down to** 1 **do**
9: split $\mathcal{P}_l$ into m subsets $\mathcal{P}_{l_1}, \ldots, \mathcal{P}_{l_m}$ such that suffixes
of same group are gathered in the same subset.
10: **for** $q = 1$ **up to** m **do**
11: remove suffixes of $\mathcal{P}_{l_q}$ from their group and put them into a new
group placed as immediate successor of their old group.

Phase 2

12: $\mathsf{SA}[1] \leftarrow n$
13: **for** $i = 1$ **up to** n **do**
14: $\quad j \leftarrow \mathsf{SA}[i] - 1$
15: $\quad$ **while** $j \neq 0$ **do**
16: $\quad\quad$ let sr be the number of suffixes placed in lower groups,
i.e. $sr := |\{\ s \in [1 \ldots n] \mid \text{group}(s) < \text{group}(j)\ \}|$.
17: $\quad\quad$ **if** $\mathsf{SA}[sr + 1] \neq$ `nil` **then**
18: $\quad\quad\quad$ **break**
19: $\quad\quad \mathsf{SA}[sr + 1] \leftarrow j$
20: $\quad\quad$ remove j from its current group and put it in a new group
placed as immediate predecessor of j's old group.
21: $\quad\quad j \leftarrow \text{prev}(j)$

Performance

Text Corpus		divsufsort	SA-IS	Ko-Aluru	Skew	GSACA
Silesia Dateien < 40 MB	speed	15,9 MB/s	17,2 MB/s	8,1 MB/s	2,9 MB/s	4,5 MB/s
	cache misses	26,5 %	32,7 %	24,2 %	52,0 %	61,2 %
Pizza & Chilli Dateien ~200 MB	speed	9,2 MB/s	8,1 MB/s	3,5 MB/s	1,1 MB/s	3,0 MB/s
	cache misses	49,5 %	74,8 %	55,2 %	86,1 %	79,0 %
Repetitive Dateien > 45 MB	speed	12,5 MB/s	14,2 MB/s	5,3 MB/s	1,7 MB/s	3,5 MB/s
	cache misses	41,9 %	68,6 %	49,7 %	78,0 %	76,9 %