

$$k > 0 \quad e \quad \alpha > 0$$

1) $G(s) = \frac{1}{(s+1)^2}$

$$1 + k \cdot \frac{s + \alpha}{s(s+1)^2} = 0$$

Nyquist

$$\text{Im} \frac{\alpha + j\omega}{j\omega(1 - \omega^2 + 2j\omega)} = 0$$

$$\text{Re} \frac{\alpha + j\omega}{1 - \omega^2 + 2j\omega} = 0$$

$$\text{Re} (\alpha + j\omega)(1 - \omega^2 - 2j\omega) = 0$$

$$\alpha(1 - \omega^2) + 2\omega^2 = 0$$

$$(2 - \alpha)\omega^2 + \alpha = 0$$

$$\omega = \pm \sqrt{\frac{\alpha}{\alpha - 2}}$$

→ estabilidade

$$\boxed{0 < \alpha < 2}$$

Routh

| | | |
|-------|--------------------|------------|
| s^3 | 1 | $1 + k$ |
| s^2 | 2 | αk |
| s | $2 + 2 - \alpha k$ | |
| 1 | αk | |

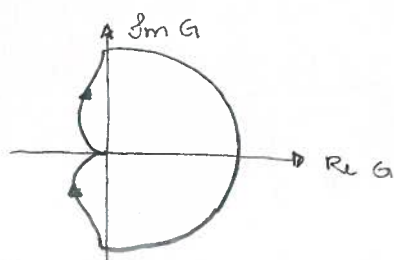
estabilidade

$$2 + 2k - \alpha k > 0$$

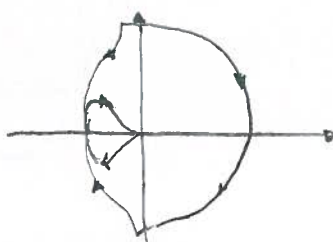
$$\alpha < 2 + \frac{2}{k}$$

$$\therefore \boxed{0 < \alpha < 2}$$

b) $\alpha < 2$



$\alpha > 2$



c) $\alpha = 9/4$

$$\omega = \sqrt{\frac{9/4}{9/4 - 2}} = \sqrt{9} = 3 \text{ rad/s}$$

$$G(s) = \frac{s + 9/4}{s(s+1)^2} = \frac{\frac{s}{9/4} + 1}{s(s+1)^2} \cdot \frac{9}{4}$$

$$|G(j\omega)| = \frac{3/9/4}{3 \cdot 3^2} \cdot \frac{9}{4} = \frac{1}{9}$$

$$-\frac{1}{k} < -\frac{1}{9} \rightarrow \boxed{k < 9}$$

→ Aproximação assintótica do Bode.

Routh

$$\frac{9}{4} < 2 + \frac{2}{k}$$

$$\frac{2}{k} > \frac{1}{4} \rightarrow \boxed{k < 8}$$

② $G(s) = \frac{1}{(s+1)(s+2)}$

a) $1 + k \cdot \frac{s+\alpha}{s(s+1)(s+2)} = 0$

$$s^3 + 3s^2 + (2+k)s + \alpha k = 0$$

Tabela de Routh:

| | | |
|-------|-----------------|------------|
| s^3 | 1 | $2+k$ |
| s^2 | 3 | αk |
| s | $6+3k-\alpha k$ | |
| 1 | αk | |

Estabilidade

$$\boxed{6 + k(3-\alpha) > 0}$$

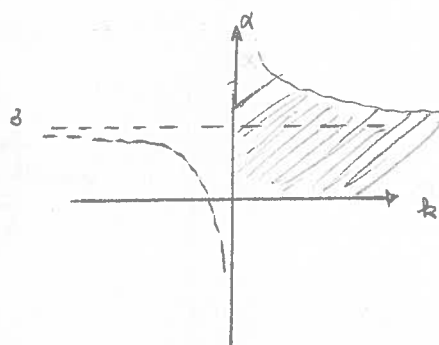
b) $6 > k(\alpha-3)$

$$\alpha-3 < \frac{6}{k} \rightarrow \boxed{\alpha < 3 + \frac{6}{k}}$$

$$k(3-\alpha) > -6$$

(1) $\alpha > 3 \rightarrow k < \frac{6}{\alpha-3}$

(2) $\alpha < 3 \rightarrow k > \frac{6}{\alpha-3}$



③

$$G(s) = \frac{2}{(s+2)^2} \quad \alpha = 5$$

$$1 + k \frac{2(s+5)}{s(s+2)^2} = 0$$

$$1 + \mu \frac{(s+5)}{s(s+2)^2} = 0 \quad \text{if } \mu = 2k$$

$$k = \frac{\mu}{2}$$

$$N'(s) \cdot D(s) - D'(s) N(s) = 0$$

$$2s^3 + 19s^2 + 40s + 20 = 0$$

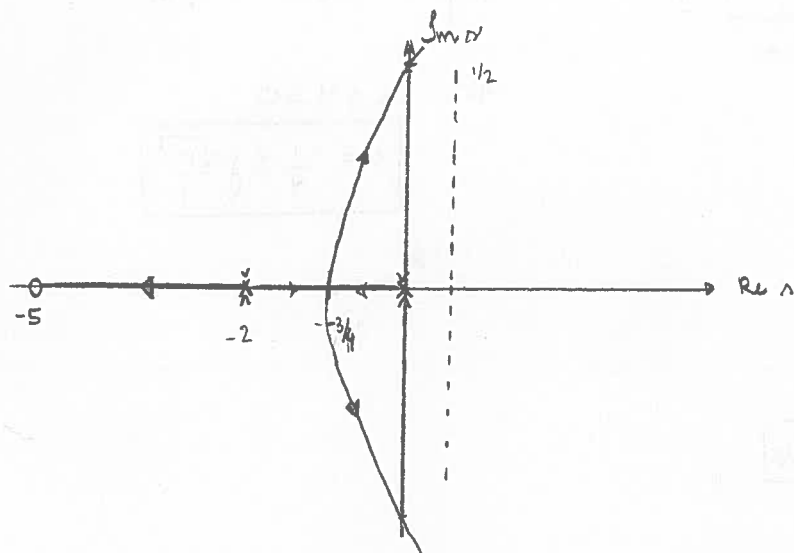
| | | | | |
|----|---|----|----|----|
| | 2 | 19 | 40 | 20 |
| -2 | 2 | 15 | 10 | 0 |

$$2s^3 + 15s + 10 = 0$$

$$s = \frac{-15 \pm \sqrt{225 - 80}}{4} = \frac{-15 \pm \sqrt{145}}{4}$$

$$s \approx -\frac{3}{4} \quad s \approx -\frac{27}{4}$$

b)



$$\sigma = \frac{-4 - (-5)}{2} = \frac{1}{2}$$

$$1 + \mu \frac{\lambda + 5}{\lambda(\lambda + 2)^2} = 0$$

$$\lambda^3 + 4\lambda^2 + (4 + \mu)\lambda + 5\mu = 0$$

| | | | |
|-------------|--------|-----------|--|
| λ^3 | 1 | $4 + \mu$ | |
| λ^2 | 4 | 5μ | |
| λ | 0 | | $\rightarrow 16 + 4\mu - 5\mu \rightarrow \mu_{sc} = 16$ |
| 1 | 5μ | | $\mu_{sc} = 8$ |

$$4\lambda^2 + 80 = 0$$

$$\lambda^2 + 20 = 0$$

$$\lambda = \pm j\sqrt{20} \approx \pm j4.5$$

$$c) \mu_{min} = \frac{3}{4} \cdot \left(\frac{5}{4}\right)^2 \cdot \frac{4}{17} \approx \frac{1}{3}$$

$$\mu_{min} \approx \frac{1}{6}$$

$$\mu_{opt} \approx \frac{1}{4}$$

$$d) \alpha = 2 \quad k = \frac{3}{4} \quad G(s) = \frac{1}{(s+1)^2}$$

$$F(s) = \frac{C(s)G(s)}{1+C(s)G(s)}$$

$$C(s) = \frac{3}{4} \left(1 + \frac{2}{s}\right)$$

$$F(s) = \frac{3s+6}{4s^3+8s^2+7s+6}$$

| | | | | |
|--------|---|---|---|---|
| | 4 | 8 | 7 | 6 |
| $-3/2$ | 4 | 2 | 4 | 0 |

$$4s^2 + 2s + 4 = 0$$

$$s = -\frac{1}{4} \pm j\frac{\sqrt{15}}{4}$$

$$F_{ap}(s) = \frac{\beta s + \alpha}{2s^2 + s + 2}$$

$$F_{ap}(0) = F(0)$$

$$\alpha = 2$$

$$F'_{ap}(0) = \frac{\beta \cdot 2 - \alpha}{4}$$

$$F'(0) = \frac{3 \cdot 6 - 7 \cdot 6}{36} = -\frac{2}{3} \rightarrow \beta = -1/3$$

$$F_{ap}(s) = \frac{-\frac{1}{3}s + 2}{2s^2 + s + 2}$$