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Galarito

(1)- a) $C(s)P(s) = \frac{-10\cancel{s}}{\cancel{s}(s+5)} = \frac{-10}{s+5}$. Malha fechada $\frac{C(s)P(s)}{1+C(s)P(s)} = \frac{-10}{(s-5)}$ ⇒ Instável

b) - Malha fechada: $\frac{C(z)P(z)}{1+C(z)P(z)F(z)} = \frac{\frac{0.5}{(z+0.5)(z-0.1)}}{1+\frac{2.0.5}{(z+0.5)(z-0.1)}} = \frac{0.5}{z^2+0.4z+0.95}$

$\Rightarrow z_{1,2} = -0.2 \pm j0.95 \Rightarrow |z_{1,2}| = 0.97 < 1 \Rightarrow$ Estável

Real. Unitária $\tilde{C}(z)\tilde{P}(z) = \frac{C(z)P(z)}{1+C(z)P(z)\underbrace{(F(z)-1)}_{1}} = \frac{C(z)P(z)}{1+C(z)P(z)} = \frac{0.5}{z^2+0.4z+0.45}$ ⇒ Tipo 0

$K_p = \lim_{z\to 1} \tilde{C}(z)\tilde{P}(z) = \frac{0.5}{1+0.4+0.45} = 0.27 \Rightarrow e_d = \frac{1}{1+K_p} = 0.787$

2)-

$A = \begin{bmatrix} 1 & -3 \\ 2 & 2 \end{bmatrix} \Rightarrow \lambda_{1,2} = 1.5 \pm j2.39$ ⇒ não possui pólo na origem ⇒ projeto proporcional não é suficiente para gerar erro nulo.

Condições para projeto integral

$\mathbb{C} = [B \;\; AB] = \begin{bmatrix} 1 & -2 \\ 1 & -4 \end{bmatrix} \Rightarrow \operatorname{rank}(\mathbb{C}) = 2$ ⇒ É controlável

$\operatorname{rank}\left(\begin{bmatrix} A & B \\ -C & 0 \end{bmatrix}\right) = 3$ ⇒ OK, é possível alocar arbitrariamente.

Projeto: $M_p = 10\%$, $t_s = 2s$

$\xi = \frac{-\ln(M_P)}{\sqrt{\pi^2+\ln(M_P)^2}} = 0.59 \approx 0.6$ $\quad \frac{4}{\xi\omega_n} = 2 \Rightarrow \omega_n = \frac{2}{0.6} = \frac{10}{3}$ ⇒ polinômio associado: $s^2 + 2\xi\omega_n s + \omega_n^2 = s^2 + 4s + \frac{100}{9}$

$\tau = \frac{1}{\xi\omega_n} = \frac{1}{2}$

Pólo adicional: $10\tau = 5$

$p_d(s) = (s^2+4s+\frac{100}{9})(s+5) = s^3 + 9s^2 + \frac{280}{9}s + \frac{500}{9}$

Alocação $K = [K_1 \;\; K_2]$ e K_I

$\det\left(sI - \begin{bmatrix} A-BK & BK_I \\ -C & 0 \end{bmatrix}\right) = s^3 + s^2(K_1+K_2-3) + s(K_2+K_2-5K_1+8) - 5K_I$

$$\boxed{K = \left[-\frac{200}{54} \;\; \frac{848}{54}\right] \quad K_I = -\frac{100}{9}}$$

$-5K_I = \frac{500}{9} = -\frac{100}{9}$

$K_1 + K_2 = 9+3$
$-5K_1 + K_2 - \frac{100}{9} = \frac{280}{9} - 8$
$\Rightarrow K_1 + K_2 = 12 \Rightarrow K_2 = 12 - K_1$
$-5K_1 + K_2 = \frac{308}{9} \Rightarrow -5K_1 + 12 - K_1 = \frac{308}{9} \Rightarrow -6K_1 = \frac{308}{9} - 12 \Rightarrow -6K_1 = \frac{200}{9} \Rightarrow K_1 = -\frac{200}{54}$

$\Rightarrow K_2 = 12 + \frac{200}{54} = \frac{848}{54}$

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3) $\dot{x} = Ax + Bu$, $u = [K \;\; 1 \;\; 3]x$ $\Rightarrow \dot{x} = (A+BK)x \Rightarrow \det(sI - A - BK) = \det\left(\begin{bmatrix} s & & \\ & s & \\ & & s\end{bmatrix} - \begin{bmatrix} 2 & 1 & 3 \\ 0 & 0 & 4 \\ -2 & -1 & -1 \end{bmatrix} - \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}[K \;\; 1 \;\; 3]\right) =$

$= s^3 - 3s^2 + (6-2K)s + (12-6K)$

negativo $\Rightarrow$ sempre instável

4) a) Tustin: $C(z) = C(s)\Big|_{s = \frac{2}{T}\frac{(z-1)}{(z+1)}}$ $\Rightarrow T=2$ $C(z) = \dfrac{\frac{z-1}{z+1} + 1}{\frac{z-1}{z+1} + 2} = \boxed{\dfrac{2z}{3z+1}}$

b) Emulação: zero em $-1 \Rightarrow z = (z - e^{-2})$

pólo em $-2 \Rightarrow p = (z - e^{-4})$

$C(z) = K\dfrac{(z-e^{-2})}{(z-e^{-4})} \Rightarrow C(z)\Big|_{z=1} = C(s)\Big|_{s=0} \Rightarrow K\dfrac{(1-e^{-2})}{(1-e^{-4})} = \dfrac{1}{2} \Rightarrow K = 0.5\dfrac{(1-e^{-4})}{(1-e^{-2})} \Rightarrow \boxed{C(z) = 0.5\dfrac{(1-e^{-4})}{(1-e^{-2})}\cdot\dfrac{(z-e^{-2})}{(z-e^{-4})}}$

c) $K_v = \lim_{z\to 1} \dfrac{(z-1)C(z)P(z)}{T} = \dfrac{\cancel{(z-1)}\frac{2z}{3z+1}\cdot\frac{(z+0.5)}{(z+0.25)^2\cancel{(z-1)}}}{2} = \dfrac{\frac{2 \cdot 1.5}{4 \cdot (1.25)^2}}{2} = 0.24$

$\boxed{e_r = \dfrac{1}{K_v} = 4.166}$

5) $x_1 = y \quad x_2 = \dot{y}$

$\dot{x}_1 = x_2$

$\dot{x}_2 = \ddot{y} = u - \alpha x_2 - \sqrt{2}\,\mathrm{sen}(x_1)$ $\Rightarrow$ $\dot{x}_1 = x_2$, $\dot{x}_2 = -\alpha x_2 - \sqrt{2}\,\mathrm{sen}(x_1) + u$

P.E $\Rightarrow x_2 = 0$

$-\alpha x_2 + \sqrt{2}\,\mathrm{sen}(x_1) + \underbrace{u}_{=1} = 0 \Rightarrow \mathrm{sen}(x_1) = \dfrac{1}{\sqrt{2}} \Rightarrow \mathrm{sen}(x_1) = \dfrac{\sqrt{2}}{2} \Rightarrow x_1 = \mathrm{asen}\left(\dfrac{\sqrt{2}}{2}\right) = \boxed{\pi/4}$ P.E $= (\pi/4, 0)$

)

$A = \begin{bmatrix} 0 & 1 \\ -\sqrt{2}\cos(x_1) & -\alpha \end{bmatrix}\Bigg|_{x = (\pi/4, 0)} = \begin{bmatrix} 0 & 1 \\ -\sqrt{2}\frac{\sqrt{2}}{2} & -\alpha \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -\alpha \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\Rightarrow \dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & -\alpha \end{bmatrix}x + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u$

$y = [1 \;\; 0]x$

) $\mathcal{C} = [B \;\; AB] = \begin{bmatrix} 0 & 1 \\ 1 & -\alpha \end{bmatrix} \rightarrow \det(\mathcal{C}) = -1 \Rightarrow$ É controlável $\forall\alpha$

$\mathcal{O} = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \det(\mathcal{O}) = 1 \Rightarrow$ É observável $\forall\alpha$

$p(\lambda) = \lambda^2 + \alpha\lambda + 1 \Rightarrow$ É assintoticamente estável $\alpha > 0$

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(6) $P(s) = \frac{10}{s+1} \Rightarrow \frac{P(s)}{s} = \frac{10}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1} = \frac{10}{s} - \frac{10}{s+1}$

$\mathcal{L}^{-1}\left\{\frac{P(s)}{s}\right\} = 10u(t) - 10e^{-t}u(t)$

Amostragem: $10u(kT) - 10e^{-Tk}u(kT)$

Transformada Z

$$\mathcal{Z}\{\cdot\} = \frac{10z}{z-1} - \frac{10z}{z-e^{-T}}$$

$$P(z) = \frac{(z-1)}{z}\mathcal{Z}\{\cdot\} = \frac{\cancel{(z-1)}}{\cancel{z}} \cdot \cancel{z}\left(\frac{10z - 10e^{-T} - 10z + 10}{\cancel{(z-1)}(z-e^{-T})}\right) = \frac{10(1-e^{-T})}{(z-e^{-T})}$$

$$G(z) = \frac{P(z)}{1+P(z)} = \frac{10(1-e^{-T})}{z-e^{-T}+10-10e^{-T}} = \frac{10(1-e^{-T})}{z-(11e^{-T}-10)}$$

Estabilidade

$$|11e^{-T}-10| < 1 \Rightarrow -1 < 11e^{-T} - 10 < 1 \Rightarrow$$
$$9 < 11e^{-T} < 11 \Rightarrow$$
$$\frac{9}{11} < e^{-T} < 1 \Rightarrow$$
$$-0.2 < -T < 0 \Rightarrow$$
$$\boxed{0 < T < 0.2}$$