Resolução da Prova

1) $P(-1,-1,3)$, $Q(1,-1,2)$ e $R(-1,2,1)$

a.

$\vec{a} = \overrightarrow{PQ} = (2,0,-1)$

$\vec{b} = \overrightarrow{PR} = (0,3,-2)$

$\vec{a}+\vec{b} = (2,3,-3)$

$\vec{a}\cdot\vec{b} = 2$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 2 & 0 & -1 \\ 0 & 3 & -2 \end{vmatrix} = 6\,\hat{a}_z + 3\,\hat{a}_x + 4\,\hat{a}_y$$

$\vec{a}\times\vec{b} = (3,4,6)$

$\vec{a}\cdot(\vec{a}\times\vec{b}) = (2,0,-1)\cdot(3,4,6) = 6-6 = 0$

b. plano de $\vec{a}$ e $\vec{b}$ com $P(-1,-1,3)$

$\vec{n} = (3,4,6)$ $\vec{a} = (2,0,-1)$ $\vec{b} = (0,3,-2)$

$\vec{n}\cdot P = d$

$(3,4,6)\cdot(-1,-1,3) = d$

$d = 11$

$\pi: 3x+4y+6z = 11$

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2) $Q = \dfrac{1}{\sqrt{x^2+y^2}} + \dfrac{y}{\sqrt{x^2+y^2}} + z$

$$\begin{cases} x = r.\cos\varphi \\ y = r.\operatorname{sen}\varphi \\ z = z \end{cases} \qquad r = \sqrt{x^2+y^2}$$

$Q = \dfrac{1}{r} + \dfrac{y}{r} + z$

$Q = \dfrac{1}{r} + \operatorname{sen}\varphi + z$

$\nabla.Q = \dfrac{1}{r} \cdot \dfrac{\partial\left(r.\frac{1}{r}\right)}{\partial r}\hat{a}_r + \dfrac{1}{r}\cos\varphi\,\hat{a}_\varphi + 1\,\hat{a}_z$

$\nabla.Q = 0 + \dfrac{1}{r}\cos\varphi\,\hat{a}_\varphi + 1\,\hat{a}_z$

$z = 0$

$r = \sqrt{4} = 2$

$\varphi = \operatorname{arc\,tg}\left(\dfrac{y}{x}\right) = \operatorname{arc\,tg}\left(\dfrac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}\right) = \dfrac{\pi}{4}$

$\nabla.Q = \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2}\,\hat{a}_\varphi + 1\,\hat{a}_z$

$\nabla.Q = \dfrac{\sqrt{2}}{4}\,\hat{a}_\varphi + 1\,\hat{a}_z$

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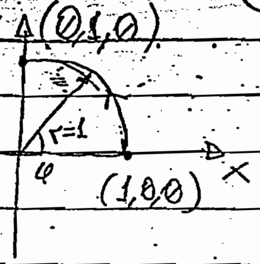
b) $C = \int_C \nabla Q \cdot d\vec{l}$



$C = \int_0^{\frac{\pi}{2}} \frac{1}{r} \cos\varphi \, \hat{a}_\varphi \, d\varphi$ $\qquad \varphi: 0 \rightarrow \frac{\pi}{2}$

$C = \frac{1}{r} \cdot -\operatorname{sen}\varphi \Big|_0^{\frac{\pi}{2}} = \frac{1}{r} \cdot \left(-\operatorname{sen}\frac{\pi}{2} + \cancel{\operatorname{sen} 0}\right)$ $\rightarrow 0$

$C = \frac{1}{r}(-1) = -\frac{1}{r} = -1$

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3) $f = y^2 z + z x^2$

a. $Dd = ?$ $P(-1,1,-1)$ $Q(3,1,2)$ $\vec{PQ} = ?$

$\vec{PQ} = (4,0,3)$

$\nabla f(x,y,z) = 2zx\,\hat{a}_x + 2zy\,\hat{a}_y + (x^2+y^2)\,\hat{a}_z$

$\nabla f(-1,1,-1) = (2\cdot -1\cdot -1)\hat{a}_x + (2\cdot -1\cdot 1)\hat{a}_y + (1+1)\hat{a}_z$

$\nabla f(-1,1,-1) = 2\hat{a}_x - 2\hat{a}_y + 2\hat{a}_z$

$Dd = \frac{1}{\sqrt{16+9}} \cdot (4,0,3) \cdot (2,-2,2) = \frac{8+6}{5} = \frac{14}{5}$

b. máx Dd nesse ponto

$\|\nabla f\| = \sqrt{4+4+4} = \sqrt{12}$

4) $\vec{A} = \left[\frac{1}{2}x^2\right]\hat{a}_x + \left[\frac{1}{2}y^2\right]\hat{a}_y + \left[\frac{1}{2}z^2\right]\hat{a}_z$

a.

$\nabla\cdot\vec{A}(x,y,z) = x\,\hat{a}_x + y\,\hat{a}_y + z\,\hat{a}_z$

$\nabla\cdot\vec{A}(1,1,1) = \hat{a}_x + \hat{a}_y + \hat{a}_z$

b. $F = \iint_S \vec{A} \cdot d\vec{S} = \iiint_V div \vec{A}\, dvol$

limites $x: 0 \to 1 \quad y: 0 \to 1 \quad z: 0 \to 1$

$F = \iiint_V (x\hat{a}_x + y\hat{a}_y + z\hat{a}_z) \cdot dvol$

$F = \frac{x^2}{2}\Big|_0^1 + \frac{y^2}{2}\Big|_0^1 + \frac{z^2}{2}\Big|_0^1$

$F = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$

5)

a) $\vec{G} = (4y + 13z)\hat{a}_x + (11x + 8z)\hat{a}_y + (18y + 3x)\hat{a}_z$
$P(-7, -31, 11)$

$$\nabla \times \vec{G} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \vec{G}_x & \vec{G}_y & \vec{G}_z \end{vmatrix}$$

$$= \left(\frac{\partial \vec{G}_z}{\partial y} - \frac{\partial \vec{G}_y}{\partial z}\right)\hat{a}_x + \left(\frac{\partial \vec{G}_x}{\partial z} - \frac{\partial \vec{G}_z}{\partial x}\right)\hat{a}_y + \left(\frac{\partial \vec{G}_y}{\partial x} + \frac{\partial \vec{G}_x}{\partial y}\right)\hat{a}_z$$

$= (18 - 8)\hat{a}_x + (13 - 3)\hat{a}_y + (11 - 4)\hat{a}_z$

$= 10\,\hat{a}_x + 10\,\hat{a}_y + 7\,\hat{a}_z$

$\nabla \times \vec{G} = 10\,\hat{a}_x + 10\,\hat{a}_y + 7\,\hat{a}_z$

2)

$$I = \oint_C \vec{G} \cdot d\vec{l} = \iint_S \nabla \times \vec{G} \cdot dS = \iiint_V \nabla \cdot (\nabla \times \vec{G})\, dV$$

$$\nabla \cdot (\nabla \times \vec{G}) = 0$$

$I = 0$

$I =$

b.