1) - $\ddot{x} = u$, $x(0) = 1$, $\dot{x}(0) = 0$, $x_1 = x$, $x_2 = \dot{x}$, $y = x_1$

$\dot{x}_1 = \dot{x} = x_2$

$\dot{x}_2 = \ddot{x} = u$ $\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$ $\Rightarrow x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} x(0) \\ \dot{x}(0) \end{bmatrix}$

$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$x(t) = e^{At} x(0) + \int_0^t e^{A(t-\tau)} B u(\tau) d\tau$, $t \geq 0$

$e^{At} = \mathcal{L}^{-1}\{(sI - A)^{-1}\}$

$(sI - A) = \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix} \Rightarrow (sI - A)^{-1} = \begin{bmatrix} 1/s & 1/s^2 \\ 0 & 1/s \end{bmatrix} \Rightarrow \mathcal{L}^{-1}\{(sI-A)^{-1}\} = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} u(t) = e^{At}$

$e^{At} x(0) = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$

$\int_0^t e^{A(t-\tau)} B u(\tau) d\tau = \int_0^t \begin{bmatrix} 1 & t-\tau \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} . u(t) d\tau = \int_0^t \begin{bmatrix} t-\tau \\ 1 \end{bmatrix} . 1 \, d\tau = \begin{bmatrix} \tau t - \tau^2/2 \\ \tau \end{bmatrix}_0^t = \begin{bmatrix} t^2 - t^2/2 \\ t \end{bmatrix} u(t)$

$x(t) = \begin{bmatrix} 1 + t^2/2 \\ 0 + t \end{bmatrix} u(t)$ $\quad y(t) = x_1(t) = (1 + t^2/2) u(t)$

2) -

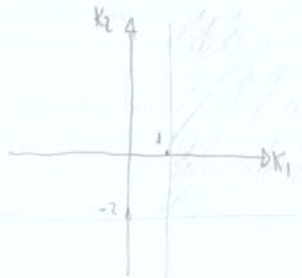
$\det(sI - A + BK) = \det\left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & -2 \\ 10 & 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} [K_1 \quad K_2] \right) = s^2 + (K_1 - 1)s + 10K_2 + 20$

Routh

s^2	1	$10(K_2+2)$
s^1	$K_1 - 1$	0
s^0	$\frac{(K_1-1)(10(K_2+2))}{(K_1-1)}$	

$\Rightarrow K_1 - 1 > 0 \Rightarrow K_1 > 1$

$10 K_2 > -20 \Rightarrow K_2 > -2$



3) - $M_p \leq 5\%$ $\quad t_s \leq 1s$

$\zeta = \frac{-\ln(M_P)}{\sqrt{\pi^2 + \ln^2(M_P)}} \Rightarrow \zeta = \frac{-\ln(0.05)}{\sqrt{\pi^2 + \ln^2(0.05)}} = 0.69 \approx 0.7$ $\quad t_s = \frac{4}{\zeta \omega_n} \leq 1 \Rightarrow \omega_n \geq 4/\zeta \Rightarrow \omega_n \geq 5.71$

$P_c(s) = (s^2 + 2.0.7.5.71s + 5.71^2)(s + 3.0.7.\omega_n) = s^3 + 19.98s^2 + 128,46s + 390.95$

$\det(sI - A + BK) = s^3 + (5+K_3)s^2 + (3+K_2)s + 2 + K_1 \Rightarrow$

$5 + K_3 = 19.98$

$3 + K_2 = 128.46$

$2 + K_1 = 390.95$

$\Rightarrow K = [14.98 \quad 125.46 \quad 388.95]$

(4) $\dddot{y}+6\ddot{y}+11\dot{y}=6u$, $x_1=y$ $x_2=\dot{y}$. $x_3=\ddot{y}$

$\dot{x}_1=\dot{y}=x_2$

$\dot{x}_2=\ddot{y}=x_3$

$\dot{x}_3=\dddot{y}=6u-6\ddot{y}-11\dot{y}=6u-6x_3-11x_2$

$$\Rightarrow \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -11 & -6\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\\0\\1\end{bmatrix}u$$

$$y=\begin{bmatrix}1 & 0 & 0\end{bmatrix}x$$

$$\mathcal{C}=\begin{bmatrix}B & AB & A^2B\end{bmatrix}=\begin{bmatrix}0 & 0 & 1\\ 0 & 1 & -6\\ 1 & -6 & 25\end{bmatrix} \Rightarrow \text{rank}(\mathcal{C})=3 \Rightarrow \text{É controlável}$$

$$\mathcal{O}=\begin{bmatrix}C\\ CA\\ CA^2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \Rightarrow \text{É observável}$$

$p(\lambda)=\det(\lambda I-A)=\lambda(\lambda^2+6\lambda+11)$ $\Rightarrow$ não é assintoticamente estável pois tem pólo na origem.

(5) $$\mathcal{O}=\begin{bmatrix}C\\ CA\\ CA^2\end{bmatrix}=\begin{bmatrix}0 & 0 & 1\\ 0 & 1 & -5\\ 1 & -5 & 22\end{bmatrix} \Rightarrow \text{rank}(\mathcal{O})=3 \Rightarrow \text{É observável}$$

$p_o(s)=(s+3)^3=s^3+9s^2+27s+27$

$$L=\begin{bmatrix}l_1\\ l_2\\ l_3\end{bmatrix}$$

$\det(sI-A+LC)=s^3+(5+l_3)s^2+(3+l_2)s+2+l_1$

$$\begin{aligned}5+l_3&=9\\ 3+l_2&=27\\ 2+l_1&=27\end{aligned} \Rightarrow L=\begin{bmatrix}25\\24\\4\end{bmatrix}$$