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1) 3
2) 4
3) 3
10

1) a) $Z_0 = \sqrt{\frac{l}{c}} = 250\Omega$ $\quad \bar{\theta} = j\omega\sqrt{lC} = j\omega\, 8 \cdot 10^{-6}$

$$\rho_r = \frac{Z_2^{\infty} - Z_0}{Z_2^{\infty} + Z_0} = 1$$

$$\rho_t = \frac{Z_1 - Z_0}{Z_1 + Z_0} = \frac{100-250}{100+250} = \frac{-150}{350} = -3/7$$

$$E_x = \frac{Z_0}{Z_1+Z_0} E \left[\frac{\exp(-\bar{\theta}(d-x)) + \rho_r \exp(-\bar{\theta}(d+x))}{1 - \rho_r\rho_t \exp(-2\bar{\theta} d)} \right]$$

$$= \frac{5}{7} E \left(e^{-\bar{\theta}} + e^{-3\bar{\theta}} \right) \cdot \frac{1}{1 - \left(-\frac{3}{7} e^{-4\bar{\theta}} \right)}$$

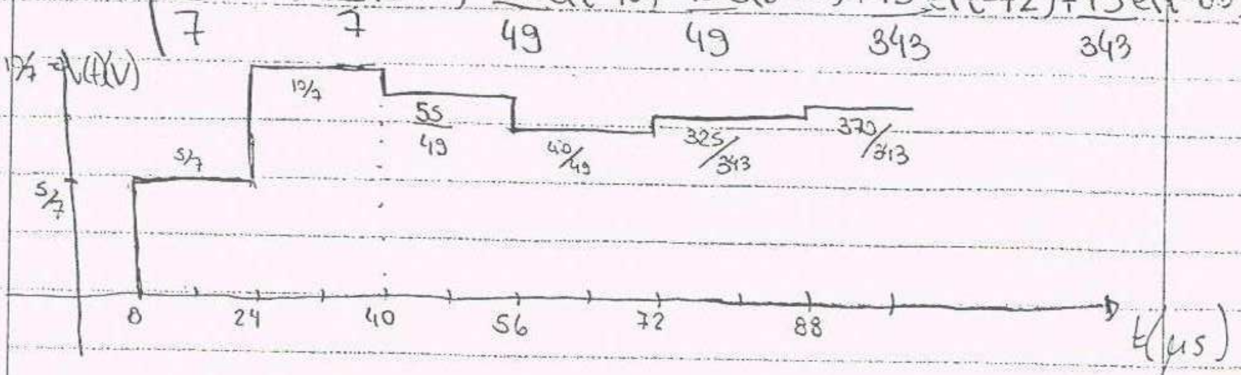
$$= \frac{5}{7} \left(e^{-\bar{\theta}} + e^{-3\bar{\theta}} \right) \left(1 - \frac{3}{7} e^{-4\bar{\theta}} + \left(\frac{3}{7}\right)^2 e^{-8\bar{\theta}} - \left(\frac{3}{7}\right)^3 e^{-12\bar{\theta}} + \left(\frac{3}{7}\right)^4 e^{-16\bar{\theta}} \dots \right)$$

$$= \frac{5E}{7} \left(e^{-\bar{\theta}} - \frac{3}{7} e^{-5\bar{\theta}} + \left(\frac{3}{7}\right)^2 e^{-9\bar{\theta}} - \left(\frac{3}{7}\right)^3 e^{-13\bar{\theta}} + \dots + e^{-3\bar{\theta}} - \frac{3}{7} e^{-7\bar{\theta}} + \left(\frac{3}{7}\right)^2 e^{-11\bar{\theta}} - \dots \right)$$

tempo em μs

$$v(t) = \frac{5E}{7} \left(e(t-8) + e(t-24) - \frac{3}{7} e(t-40) - \frac{3}{7} e(t-56) + \frac{9}{49} e(t-72) + \frac{9}{49} (t-88) \dots \right)$$

$$= E \left(\frac{5}{7} e(t-8) + \frac{5}{7} e(t-24) - \frac{15}{49} e(t-40) - \frac{15}{49} e(t-56) + \frac{45}{343} e(t-72) + \frac{45}{343} e(t-88) \dots \right)$$



b) Quando $t \to \infty$ temos que:

$$v(t) = \frac{5}{7}\left(2\sum_{n=0}^{\infty}\left(\frac{3}{7}\right)^{2n} - 2\sum_{n=0}^{\infty}\left(\frac{3}{7}\right)^{2n+1}\right)$$

$$= \frac{10}{7}\left(\sum_{n=0}^{\infty}\left(\frac{9}{49}\right)^{n} - \frac{3}{7}\sum_{n=0}^{\infty}\left(\frac{9}{49}\right)^{n}\right)$$

$$= \frac{10}{7} \cdot \frac{4}{7} \cdot \sum_{n=0}^{\infty}\left(\frac{9}{49}\right)^{n}$$

$$= \frac{40}{49} \cdot \frac{1}{1 - 9/49} = 1$$

$$\lim_{t\to\infty} v(t) = 1\,V$$

(2) Trecho 3:

$$\bar{\Theta} = \alpha + j\beta = j3 \qquad d = 0,4\,km$$

$$Z_0 = 100\,\Omega$$

$$\Theta = \bar{\Theta} d = j1,2$$

$$A = D = \cosh\Theta = 0,36236$$

$$B = Z_0 \,\text{senh}\,\Theta = j93,2039$$

$$C = \frac{\text{senh}\,\Theta}{Z_0} = j9,32039 \cdot 10^{-3}$$

$$Z_3 = \frac{A\cancel{Z}^{\infty} + B}{C\cancel{Z}^{\infty} + D} = \frac{A}{C} = -j38,878\,\Omega$$

Trecho 2:

$$\bar{\Theta} = j2,5 \qquad \bar{\Theta} d = \Theta = j2,25$$

$$A = D = \cosh\Theta = -0,62817$$

$$B = Z_0 \,\text{senh}\,\Theta = j77,8073$$

$$C = \frac{\text{senh}\,\Theta}{Z_0} = j7,78073 \cdot 10^{-3}$$

$$Z_2 = \frac{AZ + B}{CZ + D} = \frac{A(100 + j100) + B}{C(100 + j100) + D} = 38,72 + j10,76\,\Omega$$

Trecho 1:

$$\bar{\theta} = 0,1 + j2 \qquad \theta = \bar{\theta} d = 0,08 + j1,6$$

$$A = D = \cosh\theta = -2,9293\cdot10^{-2} + j0,08005$$

$$B = Z_0 \operatorname{senh}\theta = -0,23385 + j100,2774$$

$$C = \frac{\operatorname{senh}\theta}{Z_0} = -2,3385\cdot10^{-5} + j1,002774\cdot10^{-2}$$

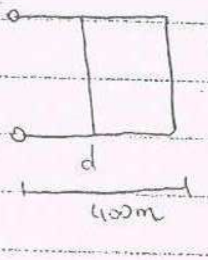
$$Z_1 = \frac{AZ + B}{CZ + D} = \frac{A(Z_2 // Z_3) + B}{C(Z_2 // Z_3) + D}$$

$$Z_2 // Z_3 = \frac{Z_2 Z_3}{Z_2 + Z_3} = 25,5584 - j20,3178\ \Omega$$

$$Z_1 = 242,059 + j123,03\ \Omega$$

(3) $$\frac{1}{\sqrt{lc}} = 200\cdot10^{6} \Rightarrow \sqrt{lc} = 5\cdot10^{-9}$$

$$\bar{\theta} = j\omega\sqrt{lc} = j2\pi(100\cdot10^{3})5\cdot10^{-9} = j\pi\cdot10^{-3}$$



$$Z_1 = Z_0 \frac{\cancel{Z_2}^{\,0}\cosh(\bar{\theta}d) + Z_0 \operatorname{senh}(\bar{\theta}d)}{\cancel{Z_2}^{\,0}\operatorname{senh}(\bar{\theta}d) + Z_0\cosh(\bar{\theta}d)}$$

$$Z_1 = Z_0 \cdot tgh(\bar{\theta}d) \Rightarrow |Z_1| = |Z_0| \cdot tg(\pi\cdot10^{-3} d)$$

$$d = \frac{tg^{-1}(2)}{\pi\cdot10^{-3}} = 352,42m$$