(1) $u = -\underbrace{[K_1 \quad K_2]}_{K} x + r$

$$\begin{cases} \dot{x} = Ax + B(-Kx + r) \\ y = Cx \end{cases} \Rightarrow \begin{cases} \dot{x} = (A - BK)x + Br \\ y = Cx \end{cases}$$

$$(A - BK) = \begin{bmatrix} 1 & 1 \\ 0 & -25 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} [K_1 \quad K_2] = \begin{bmatrix} 1 & 1 \\ -K_1 & -25-K_2 \end{bmatrix}$$

Autovalores $\Rightarrow$ raízes de $p(\lambda) = \det(\lambda I - (A - BK)) = 0 \Rightarrow$

$p(\lambda) = s^2 + (24 + K_2)s - 25 + K_1 - K_2$

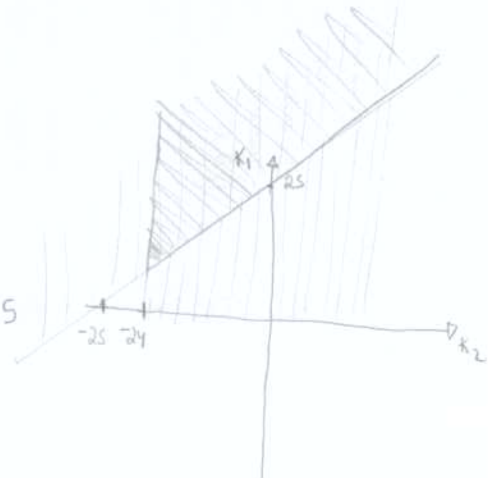
raízes $= \dfrac{-(24+K_2) \pm \sqrt{(24+K_2)^2 - 4(K_1 - K_2 - 25)}}{2}$

estabilidade $\Rightarrow$ Routh

s^2	1	$K_1 - K_2 - 25$
s^1	$24 + K_2$	0
s^0	$\dfrac{(24+K_2)(K_1-K_2-25)}{(24+K_2)}$	

$24 + K_2 > 0 \Rightarrow K_2 > -24$

$K_1 - K_2 - 25 > 0 \Rightarrow K_1 > K_2 + 25$



$$\frac{y}{R} = C(sI - A + BK)^{-1}B$$

$$= [2500 \quad 0] \cdot \frac{1}{s^2 + (24+K_2)s + K_1 - K_2 - 25} \begin{bmatrix} s + 25 + K_2 & 1 \\ -K_1 & s-1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{2500}{s^2 + (24+K_2)s + K_1 - K_2 - 25}$$

$$\Rightarrow Y(s) = \frac{R(s) \cdot 2500}{s^2 + (24+K_2)s + K_1 - K_2 - 25}$$

$R(s) = 1/s$

$$y(\infty) = \lim_{s \to 0} sY(s) = \frac{2500}{0 + 0 + K_1 - K_2 - 25} = 1 \Rightarrow K_1 - K_2 - 25 = 2500 \Rightarrow \boxed{K_1 = K_2 + 2525}$$

(2) $s = \dfrac{z+1}{z-1}$ $\qquad z = \dfrac{s+1}{s-1}$

Se $Re(s) < 0 \Rightarrow |z| = \dfrac{\sqrt{(1+Re(s))^2 + Im(s)^2}}{\sqrt{(Re\,s - 1)^2 + Im\,s^2}}$

Se $Re(s) < 0 \Rightarrow (Re\,s - 1)^2 > (1 + Re(s))^2 \Rightarrow$ portanto o módulo do denominador sempre será maior $\Rightarrow |z| < 1$

Se $s = 0 \Rightarrow z = -1 \Rightarrow |z| = 1$

Se $Re(s) > 0 \Rightarrow |z| = \dfrac{\sqrt{(1+Re(s))^2 + Im(s)^2}}{\sqrt{(Re\,s - 1)^2 + Im(s)^2}}$

Nesse caso o numerador sempre será maior e $|z| > 1$

② (cont)

$$z^2-2z-z = z^2-3z \Rightarrow \frac{(s+1)^2}{(s-1)^2} - \frac{3(s+1)}{(s-1)} = \frac{s^2+2s+1}{(s-1)^2} \quad \frac{-3s-3}{(s-1)} = \frac{s^2+2s+1-3(s+1)(s-1)}{(s-1)^2} = \frac{s^2+2s+1-3(s^2-1)}{(s-1)^2}$$

$$\Rightarrow \frac{s^2+2s+1-3s^2+3}{(s-1)^2} = \frac{-2s^2+2s+4}{(s-1)^2} = 0 \Rightarrow 2s^2-2s-4=0$$

$$s_{1,2} = \frac{2 \pm \sqrt{4-4.2.(-4)}}{4} \Rightarrow s_1 = 2 \quad s_2 = -1$$

$\boxed{\text{instável}}$

③ $\ddot{y}+3\dot{y}+2y = u$

$x_1 = y \Rightarrow \dot{x}_1 = \dot{y} = x_2$

$x_2 = \dot{y} \Rightarrow \dot{x}_2 = \ddot{y} = u - 3\dot{y} - 2y = u - 3x_2 - 2x_1$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$\zeta = 0.5 \quad w_n = 2 \text{ rad/s} \Rightarrow s^2 + 2\zeta w_n + w_n^2 = s^2 + 2s + 4$

$\det(sI - A + BK) = s^2 + (3+K_2)s + 2 + K_1 \Rightarrow \begin{matrix} 3+K_2 = 2 \Rightarrow K_2 = -1 \\ 2+K_1 = 4 \Rightarrow K_1 = 2 \end{matrix}$

$$\boxed{K = [2 \quad -1]}$$