EA721 - P2 - 1s2004

① - $G_1 = KP$ $K = 1440$

a) $K_v = ?$

$32.03 = 20\log K_v - 20\log\omega \Rightarrow 32.03 = 20\log K_v - 20\log 1 \Rightarrow \frac{32.03}{20} = \log K_v \Rightarrow K_v = 10^{\frac{32.03}{20}} = \boxed{39{,}94 \approx 40\ s^{-1}}$

b) $C(s) = K_p$, $MF = 40°$?

$MF = 40 \Rightarrow \angle P = -140 \Rightarrow \omega = 25.83 \Rightarrow |G_1|_{\omega=25.83} = 1.71\ dB$

$20\log K_c = -1.71 \Rightarrow \log K_c = \frac{-1.71}{20} \Rightarrow K_c = 10^{-1.71/20} = \boxed{0.82}$

c) $C(s)$ atraso, $MF = 45+10$

$\angle P = 55° \Rightarrow \omega = 16{,}95$ rad/s

$\frac{1}{T} = 1{,}695 \Rightarrow T = 0.589$

$|P|_{\omega=16.95} = 6.46\ dB$

$-20\log\beta = -6.46 \Rightarrow \log\beta = \frac{6.46}{20} \Rightarrow \beta = 10^{6.46/20} = 2.10$

$\frac{1}{\beta T} = \frac{1}{2.10 \cdot 0.589} = 0{,}807$

$K = K_c\beta \Rightarrow 1440 = K_c \cdot 2.10 \Rightarrow K_c = 684{,}73$

$C(s) = \frac{684.73(s+1.695)}{(s+0.807)}$

---

② - Tipo 2 $G(s) = C(s)P(s)$ $K_a = \lim_{s\to 0} s^2 G(s)$

Assíntota de baixa freq: $|C(j\omega)P(j\omega)| = 20\log\left|\frac{K_a}{(j\omega)^2}\right| = 20\log K_a - 20\log\omega^2 \Rightarrow$

$|C(j\omega)P(j\omega)| = 0 \Rightarrow \log K_a = \log\omega^2 \Rightarrow \sqrt{K_a} = \omega = \omega_c$

③ - $P(s) = \frac{10}{s^2}$ $MF = 45+5°$ $K=1$

freq. cruz: $\sqrt{10}$ rad/s

$\angle P(s) = 180° \Rightarrow MF = 0°$

$\phi_m = 50° \Rightarrow \alpha = \frac{1-\operatorname{sen}50}{1+\operatorname{sen}50} = 0.132$

log comp. $20\log\frac{1}{\sqrt{\alpha}} = 8.77\ dB$

$|G_1(j\omega)|_{dB} = -8.77$ ou $\left|\frac{10}{(j\omega)^2}\right| = \sqrt{\alpha} \Rightarrow \frac{10}{\omega^2} = \sqrt{\alpha} \Rightarrow \omega = \frac{\sqrt{10}}{\sqrt{\sqrt{0.13}}} = 5.24$ rad/s

$\omega_m = \omega_c = 5.24$ rad/s $= \frac{1}{T\sqrt{\alpha}} \Rightarrow T = 0.524$

zero $\frac{1}{T} = 1.90$

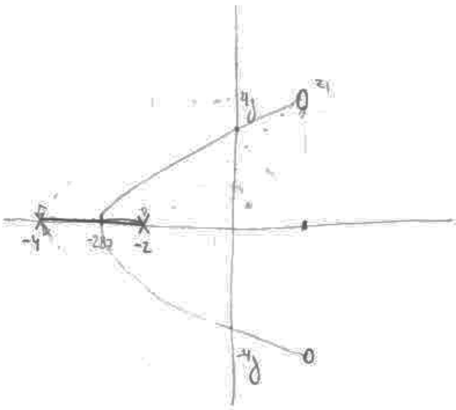
polo $\frac{1}{\alpha T} = 14.4$

$K = 1 = K_c\alpha \Rightarrow K_c = 1/\alpha = 7.54$

$C(s) = \frac{7.54(s+1.90)}{(s+14.4)}$

(4) $1 + K\left(\frac{s^2-4s+20}{(s+2)(s+4)}\right) = 0$

$z_1 = 2+4j$
$z_2 = 2-4j$
$p_1 = -2$
$p_2 = -4$



$D = s^2+6s+8$, $D' = 2s+6$
$N = s^2-4s+20$, $N' = 2s-4$
$m=2$, $n=2$, $n-m=0$

$D'N - DN' = 0 \Rightarrow$
$(2s+6)(s^2-4s+20) - (s^2+6s+8)(2s-4) = 0 \Rightarrow$
$-10s^2+24s+152 = 0 \Rightarrow$
$s_1 = 5.27$
$s_2 = -2.87$

$\phi_{p_1} = \phi_{z_1} + \phi_{z_2} - \phi_{p_2} + 180n$
$= -135 + 135 + 0 + 180n$
$= 180$

$\phi_{z_1} = \phi_{p_1} + \phi_{p_2} - \phi_{z_2} + 180$
$= 45 + 33.69 - 90 + 180$
$= 168.9°$

$\phi_{p_2} = \phi_{z_1} + \phi_{z_2} - \phi_{p_1} + 180n$
$= -146 + 146 - 180 + 180n = 0$

$D(j\omega) + KN(j\omega) = -\omega^2 + 6j\omega + 8 - K\omega^2 - 4Kj\omega + 20K = 0$

$6\omega = 4K\omega \Rightarrow K = 6/4$

$\omega^2 = \frac{20K+8}{1+K} \Rightarrow \omega = 3.89$ $s = \pm j3.89$

(5)- $1 + \frac{10}{(s+2)(s+\alpha)} = 0$ $0 < \alpha < \infty$ α : valores reais positivos

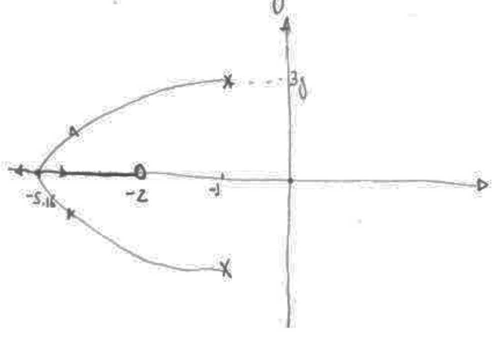
$s^2 + 2s + \alpha s + 2\alpha + 10 = 0 \Rightarrow 1 + \frac{\alpha(s+2)}{s^2+2s+10} \Rightarrow D = s^2+2s+10$ $D' = 2s+2$
$N = s+2$ $N' = 1$

$s_1 = -1+3j$ $z_1 = -2$
$s_2 = -1-3j$

$n=2$, $m=1$ $n-m=1 \Rightarrow \theta_\infty = 180°$

$D'N - DN' = s^2+4s-6$ $\boxed{s_1 = -5.16}$
$s_2 = 1.16$

$K\left|\frac{(s+2)}{(s^2+2s+10)}\right|_{s=-5.16} = 1 \quad 8.32$



(6)-

7. $P(s) = \dfrac{K}{s(s+7)}$ $\quad M_p = 20\%$

$G(s) = \dfrac{K}{s^2+7s+K}$

$\omega_n^2 = K \Rightarrow K = 49$

$2\zeta\omega_n = 7 \Rightarrow 2 \cdot 0.5\, \omega_n = 7 \Rightarrow \omega_n = 7$

$M = e^{-(\zeta/\sqrt{1-\zeta^2})\pi} \Rightarrow \ln(0.2) = \dfrac{-\zeta \cdot \pi}{\sqrt{1-\zeta^2}} = \dfrac{-\zeta\pi}{\sqrt{1-\zeta^2}} = -1.60 \Rightarrow \zeta\pi = 1.60\sqrt{1-\zeta^2} \Rightarrow \zeta^2\pi^2 = 2.59(1-\zeta^2) \Rightarrow \zeta = 0.45 \approx 0.5$

$K_v = \lim_{s\to 0} s\, G(s) = \dfrac{s \cdot 49}{s(s+7)} = 7$

$e_r = \dfrac{1}{7} = 0.11$

$\zeta = \dfrac{-\ln(\%OV/100)}{\sqrt{\pi^2 + \ln^2(\%OV/100)}}$

b) Para $e_r = \dfrac{1}{20}\dfrac{1}{7} = \dfrac{1}{140} \Rightarrow K_v = 140$

$140 = \lim_{s\to 0} s\, C(s) P(s) = \dfrac{\cancel{s} K_c\beta(Ts+1)}{(T\beta s+1)} \cdot \dfrac{49}{s(s+7)} \Rightarrow 140 = K_c\beta\, 7 \Rightarrow K_c\beta = 20$

$K_c = 1 \quad , \beta = 20$ $\qquad C(s) = \dfrac{(s+0.01)}{(s+0.0005)}$