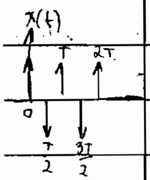
1-3,00
2-3,00
~~3-3,00~~
9,00

Tiago Rodarte Ricciardi 036305 24/05/2006

P2 - EA614

$x(t)$ é periódico com período fundamental T



① (a) $X(\omega) = \mathcal{F}\{x(t)\} = \mathcal{F}\left\{ \sum_{n=-\infty}^{\infty} \delta(t-nT) - \delta(t-nT-T/2) \right\} =$

$= \mathcal{F}\left\{ \sum_{n=-\infty}^{\infty} \delta(t-nT) \right\} - \mathcal{F}\left\{ \sum_{n=-\infty}^{\infty} \delta(t-T/2-nT) \right\} =$

$= \omega_0 \cdot \sum_{n=-\infty}^{\infty} \delta(\omega-n\omega_0) - \left[\omega_0 \cdot \sum_{n=-\infty}^{\infty} \delta(\omega-n\omega_0) \right] \cdot e^{-j\frac{\omega T}{2}}$ $\quad \omega_0 = \frac{2\pi}{T}$

$= \omega_0 \cdot \sum_{n=-\infty}^{\infty} \delta(\omega-n\omega_0) \left[1 - e^{-j\omega T/2} \right] =$

$= \omega_0 \sum_{n=-\infty}^{\infty} \left[1 - e^{-j\frac{\omega T}{2}} \right] \cdot \delta(\omega-n\omega_0) \quad ; \quad \omega_0 = \frac{2\pi}{T}$

$$\boxed{\text{Resp: } \omega_0 \sum_{n=-\infty}^{\infty} [\underbrace{1-e^{-j\frac{\omega T}{2}}}_{=?}] \cdot \delta(\omega-n\omega_0) \quad ; \quad \omega_0 = 2\pi/T}$$

1,5

(b) $X(\omega) = 2\pi \sum_{n=-\infty}^{\infty} \underbrace{\frac{\omega_0}{2\pi} \cdot \left[1 - e^{-j\frac{\omega T}{2}} \right]}_{c_k} \delta(\omega-n\omega_0)$

$c_k = \frac{\omega_0}{2\pi} \cdot \left[1 - e^{-j\frac{k\omega_0 T}{2}} \right] = \frac{1}{T} \cdot \left[1 - e^{-j\frac{k \cdot 2\pi}{2}} \right] = \frac{1}{T} \cdot \left[1 - e^{-jk\pi} \right]$

$c_k = \frac{1}{T} \left[1 - \cos(k\pi) + j\underbrace{\sin(k\pi)}_{=0} \right] \Rightarrow \begin{cases} a_k = \frac{1-\cos(k\pi)}{T} \\ b_k = \frac{\sin(k\pi)}{T} = 0 \end{cases}$

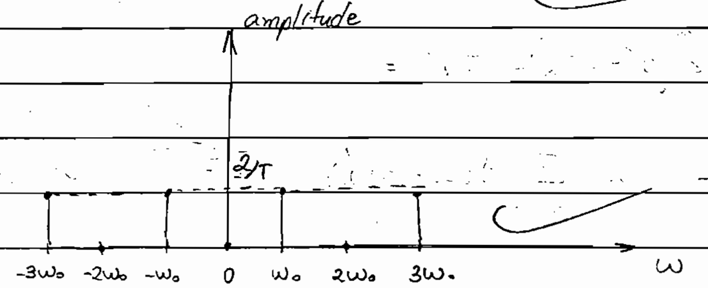
$c_0 = \frac{1}{T} \left[1 - \cos(0) + j\sin(0) \right]$

$c_0 = 0$

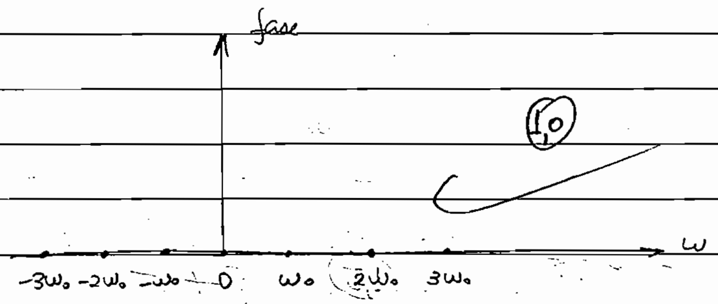
$$x(t) = 0 + \sum_{k=1}^{\infty} 2 \cdot \left(\frac{1 - \cos(k\pi)}{T} \right) \cdot \cos(k\omega_0 t) - \underbrace{2 \cdot \frac{\operatorname{sen}(k\pi)}{T}}_{=0!} \cdot \operatorname{sen}(k\omega_0 t)$$

$$x(t) = \sum_{k=1}^{\infty} 2 \cdot \left(\frac{1 - \cos(k\pi)}{T} \right) \cdot \cos\left(\frac{2\pi k t}{T} \right) - \underbrace{2 \cdot \left(\frac{\operatorname{sen}(k\pi)}{T} \right)}_{=0!} \cdot \operatorname{sen}\left(\frac{2\pi k t}{T} \right)$$

3



Para $|k|$ = impar, (harmônicos ímpares) a amplitude de C_k vale $2/T$, para $|k|$ = para, a amplitude de C_k é nula.



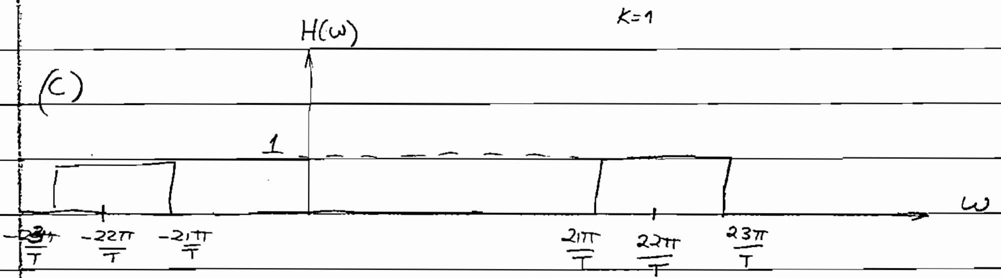
1,0

A fase de C_k é nula para todos os k

$\omega_0 = 2\pi/T$

$$x(t) = \sum_{k=1}^{\infty} 2 \cdot |C_k| \cdot \cos(k\omega_0 t) \qquad |C_k| = \sqrt{\left(\frac{\operatorname{sen}(k\pi)}{T}\right)^2 + \left(\frac{1-\cos k\pi}{T}\right)^2}$$

(c)



$$Y(\omega) = X(\omega) \cdot H(\omega)$$

O filtro irá selecionar apenas a frequencia correspondente à $22\pi/T$, é a $\frac{22\pi}{T}$, $n = 11, -11$

$$X(\omega = \pm 22\pi/T) = \omega_0 \cdot \sum_{-\infty}^{\infty} \left[1 - e^{-j\frac{22\pi}{T}\cdot\frac{T}{2}}\right] \cdot \delta\left(\frac{22\pi}{T} - n\omega_0\right) +$$

$$\omega_0 \cdot \sum_{-\infty}^{\infty} \left[1 - e^{+j\frac{22\pi}{T}\cdot\frac{T}{2}}\right] \cdot \delta\left(-\frac{22\pi}{T} - n\omega_0\right) = 2\omega_0 \sum_{-\infty}^{\infty} \cdot \delta\left(\frac{22\pi}{T} - n\omega_0\right) + \delta\left(-\frac{22\pi}{T} - n\omega_0\right)$$

$$= 2\omega_0 \sum_{-\infty}^{\infty} \delta\left(\frac{22\pi}{T} - n\omega_0\right) + \delta\left(\frac{22\pi}{T} + n\omega_0\right)$$

$$Y(\omega) \overset{0,5}{=} \frac{8\pi}{T} \sum \delta\left(\frac{22\pi}{T} + n\cdot\frac{2\pi}{T}\right) = \frac{8\pi}{T} \underbrace{\sum \delta\left(\frac{2\pi}{T}(11+n)\right)}_{0,5}$$

$$\cancel{y(t) = \frac{1}{4}\sum_{-\infty}^{\infty}\delta(t + KT)} \;??$$

$$(2) \quad X(\omega) = R_2(\omega+\omega_0)\cdot\cos(2\pi(\omega+\omega_0)) + R_2[\omega-\omega_0]\cdot\cos(2\pi(\omega-\omega_0))$$

Propriedade da Dualidade:

$$x(t) \longleftrightarrow X(\omega)$$

$$X(t) \longleftrightarrow 2\pi\, x(-\omega)$$

$$X(t) = R_2(t+t_0)\cdot\cos(2\pi(t+t_0)) + R_2(t-t_0)\cdot\cos(2\pi(t-t_0))$$

$$\mathcal{F}\{X(t)\} = \frac{1}{\cancel{2\pi}}\cdot\cancel{2}\cdot Sa(\omega)\, e^{j\omega t_0} * \cancel{\pi}(\delta(\omega-2\pi)+\delta(\omega+2\pi))\cdot e^{j\omega t_0} +$$

$$+ \frac{1}{\cancel{2\pi}}\,\cancel{2}\cdot Sa(\omega)\, e^{-j\omega t_0} * \cancel{\pi}(\delta(\omega-2\pi)+\delta(\omega+2\pi))\, e^{-j\omega t_0} =$$

$$= \left[Sa(\omega-2\pi) + Sa(\omega+2\pi)\right]\cdot 2\cdot\cos(t_0\omega) =$$

$$= 2\cdot\cos(t_0\omega)\cdot[Sa(\omega-2\pi)+Sa(\omega+2\pi)]$$

$$2\pi\, X(-\omega) = 2\cdot\cos(t_0\omega)\cdot[Sa(\omega-2\pi)+Sa(\omega+2\pi)]$$

$$\omega_0 = \frac{2\pi}{t_0}$$

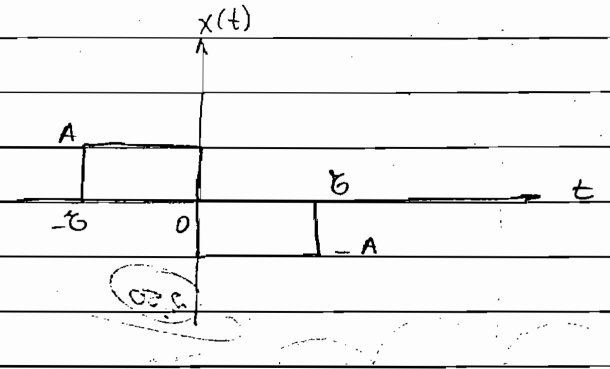
$$X(\omega) = \frac{1}{\pi} \cdot \cos(t_0 \omega) \cdot [Sa(\omega - 2\pi) + Sa(\omega + 2\pi)]$$

$$x(t) = \frac{1}{\pi} \cdot \cos(\omega_0 t) \cdot [Sa(t - t_0) + Sa(t + t_0)]$$ (3,0)

(2,0)

(b) (0,0)

(3)



$$x(t) = A \cdot r_\tau(t + \tau/2) - A \cdot r_\tau(t - \tau/2)$$

$$X(\omega) = \mathcal{F}\{x(t)\} = A \cdot \tau \cdot Sa\left(\frac{\omega\tau}{2}\right) e^{\frac{j\omega\tau}{2}} - A \cdot \tau \cdot Sa\left(\frac{\omega\tau}{2}\right) e^{-\frac{j\omega\tau}{2}} =$$

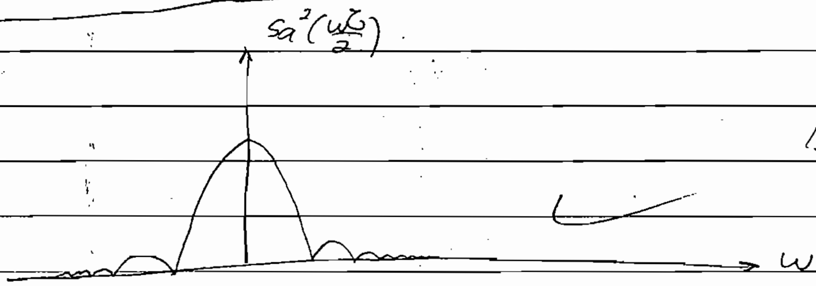
$$= A\tau\, Sa\left(\frac{\omega\tau}{2}\right) \left[e^{\frac{j\omega\tau}{2}} - e^{-\frac{j\omega\tau}{2}} \right] = A\tau\, Sa\left(\frac{\omega\tau}{2}\right) \cdot 2 \cdot j \sin\left(\frac{\omega\tau}{2}\right) =$$

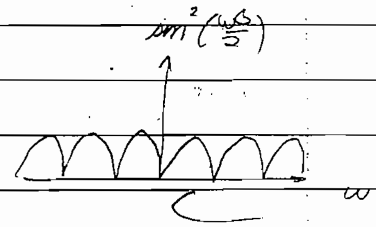
$$= j2A\tau \cdot Sa\left(\frac{\omega\tau}{2}\right) \cdot \sin\left(\frac{\omega\tau}{2}\right) = j2A\tau \cdot Sa\left(\frac{\omega\tau}{2}\right) \cdot \sin\left(\frac{\omega\tau}{2}\right)$$

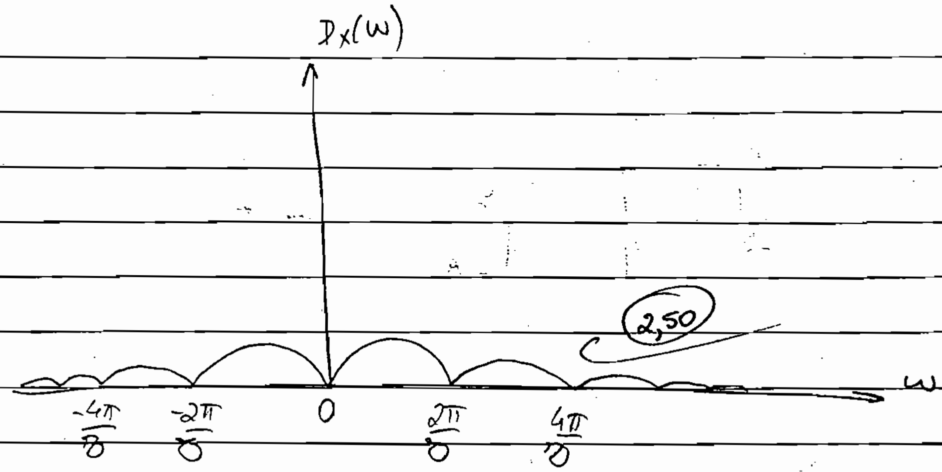
$$D_x(\omega) = \frac{|X(\omega)|^2}{2\pi}$$

$$D_x(\omega) = \frac{1}{2\pi} \cdot 4 \cdot A^2 \cdot \tau^2 \cdot Sa^2\left(\frac{\omega\tau}{2}\right) \cdot \sin^2\left(\frac{\omega\tau}{2}\right)$$

$$\boxed{D_x(\omega) = \left(\frac{2A^2\tau^2}{\pi}\right) \cdot Sa^2\left(\frac{\omega\tau}{2}\right) \cdot \sin^2\left(\frac{\omega\tau}{2}\right)}$$



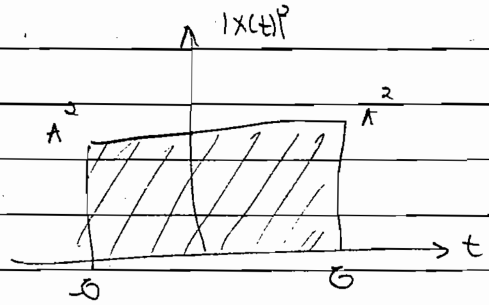




(2,50)

b) $\int_{-\infty}^{\infty} D_x(\omega)\, d\omega = \int_{-\infty}^{\infty} \frac{|X(\omega)|^2}{2\pi} d\omega = \int_{-\infty}^{\infty} |x(t)|^2 dt =$

$= 2 \cdot A^2 \tau$



$\boxed{\text{Resp: } 2A^2\tau}$ (0,5)