Gabarito P3

1.- $\frac{\partial f}{\partial x}(x,y) = e^x \cos y + y$

$f(x,y) = e^x \cos y + xy + C(y)$

$\frac{\partial f}{\partial y}(x,y) = -e^x \operatorname{sen} y + x + C'(y) = x - e^x \operatorname{sen} y$

Assim $C'(y) = 0 \Rightarrow C(y) = D$

$f(x,y) = e^x \cos y + xy + D$ → 1,0

$\vec{F}$ é um campo vetorial conservativo

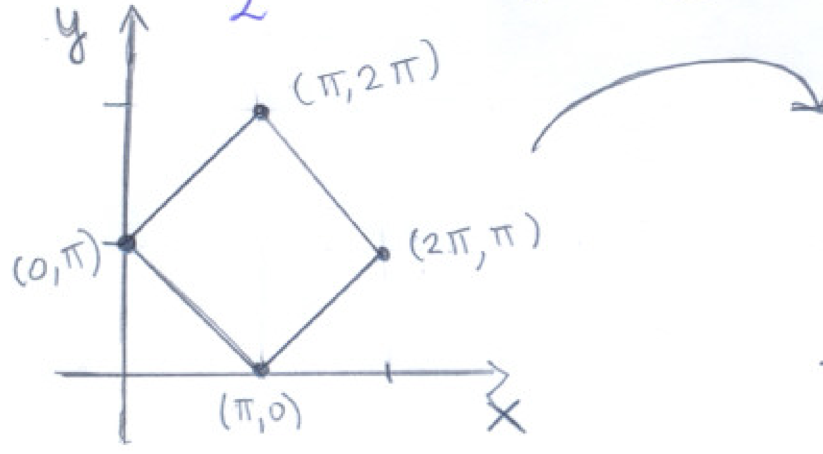
$$\int_C \vec{F} \cdot d\vec{r} = \int_C \nabla f \cdot d\vec{r} \overset{(*)}{=} f(1,0) - f\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$ → 1,0

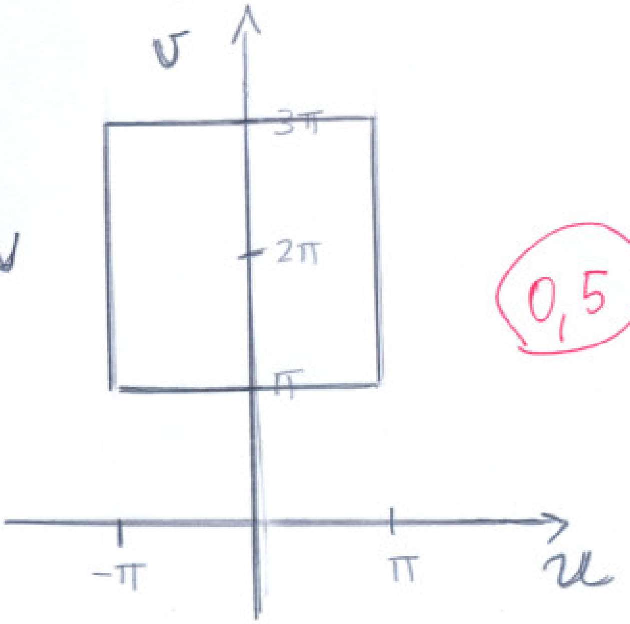
$$= e - e^{-\frac{\sqrt{2}}{2}} \cos\left(\frac{\sqrt{2}}{2}\right) + \frac{1}{2}$$ → 0,5

(*) Teorema Fundamental do Cálculo para integrais de linha.

2.- Sejam: $u = x - y$, $v = x + y$

$x = \frac{u+v}{2}$, $y = \frac{v-u}{2}$





0,5

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{2} \longrightarrow 0,5$$

$$\iint_D (x-y)^2 \operatorname{sen}^2(x+y)\,dx\,dy = \frac{1}{2}\int_{-\pi}^{\pi}\int_{\pi}^{3\pi} u^2 \operatorname{sen}^2 v \, dv\, du \longrightarrow 0,5$$

$$= \frac{1}{2}\int_{-\pi}^{\pi} \frac{u^2}{2}\left(\int_{\pi}^{3\pi}(1-\cos(2v))\,dv\right) du$$

$$= \frac{1}{4}\int_{-\pi}^{\pi} u^2 \left(\int_{\pi}^{3\pi} dv - \int_{\pi}^{3\pi} \cos(2v)\,dv\right) du$$

$$= \frac{1}{4}\int_{-\pi}^{\pi} 2\pi u^2\, du = \frac{\pi}{2}\left(\frac{u^3}{3}\Big|_{-\pi}^{\pi}\right) = \frac{\pi^4}{3} \longrightarrow 1,0$$

3 – Segue do Teorema de Green com $P=0$ e $Q=x$ que

$$\oint_C x\,dy = \iint_D dA = \text{área de } D$$

falta de orientação
−0,2

onde D é a região limitada por C. Então $\longrightarrow$ 0,7

$$\text{Área} = \int_0^{2\pi} \cos t\,(3\operatorname{sen}^2 t)\cos t\,dt = 3\int_0^{2\pi} \operatorname{sen}^2 t \cos^2 t\,dt \longrightarrow 0,8$$

$$= \frac{3}{4}\int_0^{2\pi} (\operatorname{sen}(2t))^2 = \frac{3}{4}\int_0\left(\frac{1}{2} - \frac{\cos 4t}{2}\right) dt$$

$$= \frac{3}{4}\left[\frac{1}{2}t - \frac{\operatorname{sen}(4t)}{8}\right]_0^{2\pi} = \frac{3}{4}\,\frac{1}{2}\cdot 2\pi = \frac{3\pi}{4}.$$

$\longrightarrow$ 1,0

(4)

$$I = \iiint_E x\, e^{\sqrt{x^2+y^2+z^2}}\, dV \qquad \begin{cases} x = \rho \,\text{sen}\,\phi \cos\theta \\ y = \rho \,\text{sen}\,\phi \,\text{sen}\,\theta \\ z = \rho \cos\phi \end{cases}$$

$$\rightarrow \quad I = \int_0^{\pi/2}\int_0^{\pi/2}\int_1^2 \rho \,\text{sen}\,\phi \cos\theta \, e^{\rho} \rho^2 \,\text{sen}\,\phi \; d\rho \, d\phi \, d\theta \quad 1{,}0$$

$$= \underbrace{\int_0^{\pi/2} \text{sen}^2\phi \, d\phi}_{0{,}5} \; \underbrace{\int_0^{\pi/2} \cos\theta \, d\theta}_{0{,}2} \; \underbrace{\int_1^2 \rho^3 e^{\rho} \, d\rho}_{0{,}8}$$

$$= \frac{\pi}{4} \cdot 1 \cdot \int_1^2 \rho^3 e^{\rho} d\rho$$

Integrando por partes :

$$\int_1^2 \rho^3 e^{\rho} d\rho = \left[e^{\rho} \left(\rho^3 - 3\rho^2 + 6\rho - 6 \right) \right]_1^2$$

$$= 2e(1+e)$$

Logo: $$I = \frac{\pi}{2} e(1+e)$$