5,5

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2,6 1 a-) Deve-se resolver a equação de Helmholtz

$$\frac{\partial^2 H_z^0}{\partial x^2} + \frac{\partial^2 H_z^0}{\partial y^2} + h^2 H_z^0 = 0$$

H_z^0 deve seguir as seguintes condições de contorno:

Para $y=0$ e $y=b \Rightarrow E_x^0 = 0 \Rightarrow \frac{\partial H_z^0}{\partial y} = 0$

0,42

Para $x=0$ e $x=a \Rightarrow E_y^0 = 0 \Rightarrow \frac{\partial H_z^0}{\partial x} = 0$

A resolução da equação de Helmholtz que obedece a essas condições de contorno é:

$$H_z^0(x,y) = H_0 \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

$$H_x^0 = \frac{\gamma}{h_{mn}^2}\left(\frac{m\pi}{a}\right) H_0 \,\mathrm{sen}\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right), \quad h_{mn}^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$H_y^0 = +\frac{\gamma}{h_{mn}^2}\left(\frac{n\pi}{b}\right) H_0 \cos\left(\frac{m\pi x}{a}\right)\mathrm{sen}\left(\frac{n\pi y}{b}\right)$$

$$E_x^0 = \frac{j\omega\mu}{h_{mn}^2}\left(\frac{n\pi}{b}\right) H_0 \cos\left(\frac{m\pi x}{a}\right)\mathrm{sen}\left(\frac{n\pi y}{b}\right)$$

$$E_y^0 = -\frac{j\omega\mu}{h_{mn}^2}\left(\frac{m\pi}{a}\right) H_0 \,\mathrm{sen}\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

b) Para o modo TE_{10}:

Não é correto dizer $H_y^0 = E_x^0$ | $H_y^0 = 0$ e $E_x^0 = 0$

$H_y^0 = E_x^0 = 0 \qquad H_x^0 = \frac{\gamma\pi}{h^2 a} H_0 \operatorname{sen}\left(\frac{\pi}{a}x\right) \qquad E_y^0 = -\frac{\omega\mu\pi}{j h^2 a} H_0 \operatorname{sen}\left(\frac{\pi}{a}x\right)$

Como há componentes não nulas, existe o modo

0,4/2

Para o modo TE_{01}:

$H_x^0 = 0$ e $E_y^0 = 0 \qquad H_y^0 = \frac{\gamma\pi}{h^2 b} H_0 \operatorname{sen}\left(\frac{\pi}{b}y\right) \qquad E_x^0 = \frac{j\omega\mu\pi}{j h^2 b} H_0 \operatorname{sen}\left(\frac{\pi}{b}y\right)$

Como há componentes não nulas, existe o modo

c) $\nu_{c_{mn}} = \frac{c}{2}\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$ (precisa demonstrar ..)

0,3

d) $\beta_{mn} = k\sqrt{1 - \left(\frac{\nu_{c_{mn}}}{\nu}\right)^2}$ 0,4/2

Demonstração:

$\gamma = j\beta$ (a onda é transmitida sem ser atenuada) $\Rightarrow \beta^2 = \left(\frac{2\pi}{c}\right)^2(\nu^2 - \nu_c^2)$

$h^2 = \gamma^2 + k^2$

$\beta^2 = k^2 - h^2$

$\beta^2 = \left(\frac{\omega}{c}\right)^2 - \pi^2\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)$

$= \left(\frac{2\pi\nu}{c}\right)^2 - \frac{4\pi^2}{c^2}\nu_c^2$

$\frac{\beta^2}{\nu_c^2} = \left(\frac{2\pi}{c}\right)^2\left(1 - \frac{\nu_c^2}{\nu^2}\right)$

$\beta^2 = k^2\left(1 - \frac{\nu_c^2}{\nu^2}\right)$

$\beta = k\sqrt{1 - \left(\frac{\nu_c}{\nu}\right)^2}$

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1,6 3

a) Com $kr \ll 1 \Rightarrow e^{-jkr} \rightarrow 1$

Com $kr \ll 1 \Rightarrow \frac{1}{(jkr)^3} \gg \frac{1}{(jkr)^2} \gg \frac{1}{(jkr)}$

Nas expressões de campo elétrico as parcelas $\frac{1}{(jkr)^3}$ dominam sobre as outras, e na expressão do campo magnético a parcela $\frac{1}{(jkr)^2}$ domina sobre $\frac{1}{jkr}$

0,45

$$E_r = -\frac{2 I dl}{4\pi} \eta_0 k^2 \cos\theta \cdot j \cdot \frac{1}{k^3 r^3} = -j \frac{2 I dl}{4\pi} \eta_0 \cos\theta \cdot \frac{1}{kr^3}$$

$$E_\theta = -\frac{I dl}{4\pi} \eta_0 k^2 \sin\theta \cdot j \frac{1}{k^3 r^3} = -j \frac{I dl}{4\pi} \eta_0 \sin\theta \cdot \frac{1}{kr^3}$$

$$H_\phi = -\frac{I dl}{4\pi} k^2 \sin\theta \cdot \frac{-1}{k^2 r^2} = \frac{I dl}{4\pi} \sin\theta \cdot \frac{1}{r^2}$$

(Interpretação física)

b) Com $kr >> 1$, $\frac{1}{jkr} >> \frac{1}{(jkr)^2} >> \frac{1}{(jkr)^3}$

$$E_r = -\frac{2Idl}{4\pi}\eta_0 k^2\cos\theta\left(\frac{-1}{k^2r^2}\right)e^{-jkr} = \frac{2Idl}{4\pi}\eta_0\cos\theta.\frac{1}{r^2}\cancel{e^{-jkr}} \approx 0$$

$$E_\theta = -\frac{Idl}{4\pi}\eta_0 k^2\sin\theta\left(\frac{-j}{kr}\right)e^{-jkr} = j\frac{Idl}{4\pi}\eta_0\sin\theta\frac{k}{r}e^{-jkr}$$

0,45

$$H_\phi = -\frac{Idl}{4\pi}k^2\sin\theta\left(\frac{-j}{kr}\right)e^{-jkr} = j\frac{Idl}{4\pi}\sin\theta\frac{k}{r}e^{-jkr}$$

(Interpretação física)

c) $\langle \vec{S}_{med}\rangle = \frac{1}{2}Re\{\vec{E}\times\vec{H}^*\} = \frac{1}{2}Re\{(E_r\hat{r} + E_\theta\hat{\theta})\times(H_\phi^*\hat{\phi})\}$

$= \frac{1}{2}Re\{(E_\theta H_\phi^*\hat{r} - E_r H_\phi^*\hat{\theta})\}$ 0,1

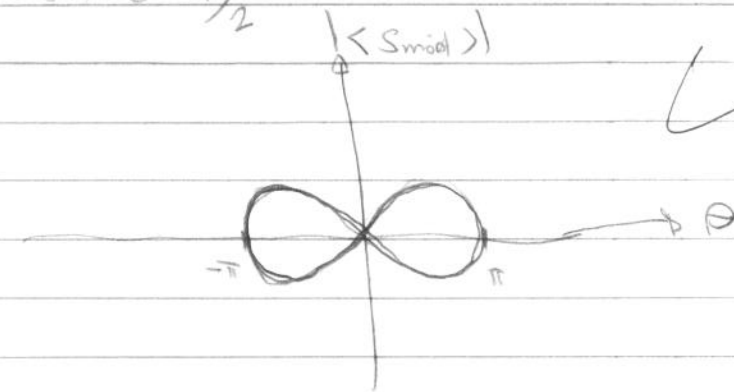
$$E_\theta H_\phi^* = \left(\frac{Idl}{4\pi}\right)^2\eta_0 k^4\sin^2\theta\left(-\frac{1}{k^2r^2} - \frac{1}{jk^3r^3} - \frac{1}{jk^3r^3} + \frac{1}{k^4r^4} + \frac{1}{k^4r^4} + \frac{1}{jk^5r^5}\right)$$

$$E_rH_\phi^* = 2\left(\frac{Idl}{4\pi}\right)^2\eta_0 k^4\sin\theta\cos\theta\left(\frac{1}{-jk^3r^3} - \frac{1}{k^4r^4} + \frac{1}{k^4r^4} - \frac{1}{jk^5r^5}\right)$$

$$\langle \vec{S}_{méd} \rangle = \frac{1}{2}\left(\left(\frac{Idl}{4\pi}\right)^2 \eta_0 k^4 \sin\theta^2 \left(\frac{-1}{k^2r^2} + \frac{2}{k^4r^4}\right)\hat{r} - 2\left(\frac{Idl}{4\pi}\right)^2 \eta_0 k^4 \sin\theta\cos\theta \left(\frac{2}{k^4r^4}\right)\hat{\theta}\right)$$

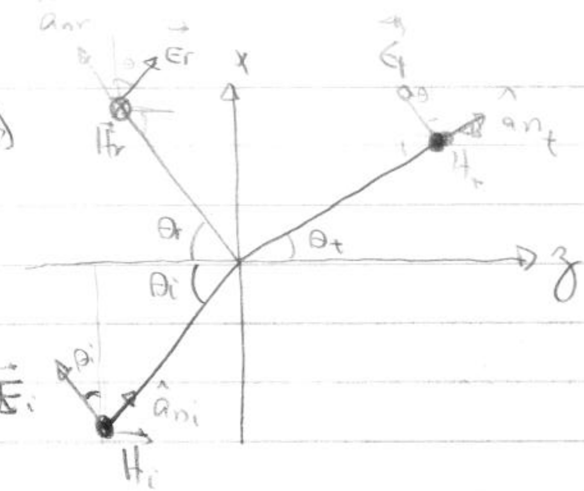
$$= \frac{\left(\frac{Idl}{4\pi}\right)^2 \eta_0 k^4 \sin\theta}{2} \left(\cancel{\left(\frac{2}{k^4r^4} - \frac{1}{k^2r^2}\right)} \sin\theta\, \hat{r} - \frac{4}{k^4r^4}\cos\theta\,\hat{\theta}\right)$$

d) Devido ao termo $\sin^2\theta$ que há em $\langle \vec{S}_{méd} \rangle$, o fluxo é mais intenso em $\theta = \pi/2$.



0,6

1,33 (2) a)



$\theta_i = \theta_r$

0,4

$E_{1T} = E_{2T} \Rightarrow E_{i0}\cos\theta_i + E_{r0}\cos\theta_i = E_{t0}\cos\theta_t$

$$\therefore (1+\Gamma)\cos\theta_i = \tau\cos\theta_t$$

$H_{1N} = H_{2N} \Rightarrow \frac{E_{i0}}{\eta_1} - \frac{E_{r0}}{\eta_1} = \frac{E_{t0}}{\eta_2}$

$$\therefore \frac{1-\Gamma}{\eta_1} = \frac{\tau}{\eta_2}$$

b) $\vec{E}_t(x,z) = \tau E_{i0}(\hat{x}\cos\theta_t - \hat{z}\sin\theta_t)e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$

$\vec{E}_t(x,z,t) = \tau E_{i0}(\hat{x}\cos\theta_t - \hat{z}\sin\theta_t)\cos[\omega t - \beta_2(x\sin\theta_t + z\cos\theta_t)]$

Onde $\tau = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$, com $\theta_i < \theta_c \Rightarrow \tau$ real

0,83

$\vec{H}_t(x,z) = \hat{y}\frac{\tau E_{i0}}{\eta_2}e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$

$\vec{H}_t(x,z,t) = \hat{y}\frac{\tau E_{i0}}{\eta_2}\cos(\omega t - \beta_2(x\sin\theta_t + z\cos\theta_t))$