EA721 - PRIMEIRA LISTA DE EXERCÍCIOS

1) a) $\ddot{y} + 2\dot{y} + 5y = t \quad \dot{y}(0) = 0 \quad y(0) = 0$

$(p^2 + 2p + 5)y = t$ → MODO FORÇADO te^{0t} $\quad \Delta = b^2 - 4ac = 4 - 4 \cdot 5 = -16$

$\lambda_1, \lambda_2 = \frac{-2 \pm j4}{2} = -1 \pm j2$

$\bar{D}(p) = (p)^2 \Rightarrow y_f = b_1 t + b_2$; e^{0t} e te^{0t} → SÃO MODOS FORÇADOS

→ SUBSTITUINDO NA EQUAÇÃO:

$(p^2 + 2p + 5)(b_1 t + b_2) = t \Rightarrow 2b_1 + 5b_1 t + 5b_2 = t$

SE $t = 0 \Rightarrow 2b_1 + 5b_2 = 0$

$t = 1 \Rightarrow 2b_1 + 5b_1 + 5b_2 = 1$

$\left.\begin{array}{l} 2b_1 + 5b_2 = 0 \ (1) \\ 7b_1 + 5b_2 = 1 \ (2) \end{array}\right\} \Rightarrow (2) - (1) \Rightarrow 5b_1 = 1 \Rightarrow b_1 = \frac{1}{5}$

$2 \cdot \frac{1}{5} + 5b_2 = 0 \Rightarrow b_2 = -\frac{2}{25}$

$y_f = \frac{1}{5}t - \frac{2}{25}$ • TESTANDO: $(p^2 + 2p + 5)\left(\frac{1}{5}t - \frac{2}{25}\right) = 2\frac{1}{5} + \frac{5}{5}t - \frac{5 \cdot 2}{25} = t \quad$ (OK!)

$y_h = a_1 e^{-(1+j2)t} + a_2 e^{-(1-j2)t} = c_1 e^{-t}\cos 2t + c_2 e^{-t}\sin 2t$

$y = c_1 e^{-t}\cos 2t + c_2 e^{-t}\sin 2t + \frac{1}{5}t - \frac{2}{25}$

$y(0) = 0 = c_1 - \frac{2}{25} \Rightarrow c_1 = \frac{2}{25}$

$\dot{y}(t) = c_1(-e^{-t}\cos 2t - 2\sin 2t\, e^{-t}) + c_2(-e^{-t}\sin 2t + 2\cos 2t\, e^{-t}) + \frac{1}{5}$

$\Rightarrow \dot{y}(0) = 0 = c_1(-1) + c_2(2) + \frac{1}{5} = 0 \Rightarrow 2c_2 = -\frac{1}{5} + c_1 = \frac{2}{25} - \frac{1}{5} \Rightarrow 2c_2 = \frac{2}{25} - \frac{5}{25} = \frac{-3}{50}$

$c_2 = \frac{-3}{50}$

b) $\ddot{y} + 8\dot{y} + 15y = 2e^{-3t} + 5$, $\dot{y}(0) = 0$, $y(0) = 1$

$(p^2 + 8p + 15)y = 2e^{-3t} + 5$; te^{-3t} → JÁ É MODO PRÓPRIO e $5e^{0t}$ SÃO OS MODOS FORÇADOS

$\begin{array}{l} 8 \\ 15 \end{array} < \begin{array}{l} -3 \\ -5 \end{array}$ → $(p+3)(p+5)y = 2e^{-3t} + 5$

→ JÁ É MODO PRÓPRIO

$\bar{D}(p) = (p+3) \cdot p \Rightarrow \bar{D}(p)x = (p^2 + 3p)(2e^{-3t} + 5) = (+18e^{-3t}) - 3 \cdot 6e^{-3t} = 0$

$y_f = b_1 t e^{-3t} + b_2$

SUBSTITUINDO NA EQUAÇÃO

$(p^2 + 8p + 15)(b_1 t e^{-3t} + b_2) = 2e^{-3t} + 5 \Rightarrow p\left(b_1(e^{-3t} - 3te^{-3t})\right) + 8(b_1 e^{-3t} - 3b_1 t e^{-3t}) + 15b_1 t e^{-3t} + 15b_2$

$= 2e^{-3t} + 5 \Rightarrow -3b_1 e^{-3t} - 3b_1(\cancel{e^{-3t}} - 3te^{-3t}) + 8b_1 e^{-3t} - 24 b_1 t e^{-3t} + 15b_1 t e^{-3t} + 15b_2 = 2e^{-3t} + 5$

$\Rightarrow -3b_1 e^{-3t} - 3b_1 e^{-3t} + 8b_1 e^{-3t} + 9b_1 t e^{-3t} - 24b_1 t e^{-3t} + 15b_1 t e^{-3t} + 15b_2 = 2e^{-3t} + 5$

$2b_1 e^{-3t} + 15b_2 = 2e^{-3t} + 5$
SOL FORÇADA

$t = 0 \Rightarrow 2b_1 + 15b_2 = 2 + 5 \Rightarrow b_1 = +1$

$t = 1000 \Rightarrow 15b_2 = 5 \Rightarrow b_2 = \frac{5}{15} = \frac{1}{3}$

$y = a_1 e^{-3t} + a_2 e^{-5t} + te^{-3t} + \frac{1}{3} =$

$y(0) = a_1 + a_2 + \frac{1}{3} = 0$

$\dot{y}(t) = -3a_1 e^{-3t} - 5a_2 e^{-5t} + (e^{-3t} - 3te^{-3t}) \Rightarrow \dot{y}(0) = -3a_1 - 5a_2 + 1 = 1$

$\Rightarrow a_1 + a_2 = -\frac{1}{3} \Rightarrow 3a_1 + 3a_2 = -1 \ (1)$

$-3a_1 - 5a_2 = 0 \ (2)$

$(1) + (2) \Rightarrow -2a_2 = -1 \Rightarrow a_2 = \frac{1}{2}$

$a_1 = -\frac{1}{3} - a_2 = -\frac{1}{3} - \frac{1}{2} = \frac{-2 - 3}{6} = \frac{-5}{6}$

c) $\ddot{y} + 3\dot{y} + 2y = 8e^{-t}\operatorname{sen} t$ $\qquad \frac{\Delta}{2} = \sqrt{4 - 4 \cdot 1 \cdot c} = \sqrt{-4}$

$r = \left< \begin{matrix} -1 \\ -2 \end{matrix} \right.$

$\bar{D}(p) = (p^2 + 2p + 2) \Rightarrow \bar{D}(p)x = 0 \Rightarrow (p^2$

$\hookrightarrow \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -1 \pm \frac{\sqrt{-4}}{2} = -1 \pm \sqrt{-1} = -1 \pm j$

$y_f = b_1 e^{-t}\operatorname{sen} t + b_2 e^{-t}\cos t$ TESTAR A SOLUÇÃO

$(p^2 + 3p + 2)y = 8e^{-t}\operatorname{sen} t \Rightarrow$

$\dot{y}_f = b_1(-e^{-t}\operatorname{sen} t + e^{-t}\cos t) + b_2(-e^{-t}\cos t - e^{-t}\operatorname{sen} t)$

$\ddot{y}_f = b_1\left(-(-e^{-t}\operatorname{sen} t + e^{-t}\cos t) + (-e^{-t}\cos t - e^{-t}\operatorname{sen} t)\right) + b_2\left(-(-e^{-t}\cos t - e^{-t}\operatorname{sen} t) - (-e^{-t}\operatorname{sen} t + e^{-t}\cos t)\right)$

$= b_1(\cancel{e^{-t}\operatorname{sen} t} - e^{-t}\cos t - e^{-t}\cos t - \cancel{e^{-t}\operatorname{sen} t}) + b_2(\cancel{e^{-t}\cos t} + e^{-t}\operatorname{sen} t + e^{-t}\operatorname{sen} t - \cancel{e^{-t}\cos t})$

$= 2b_2 e^{-t}\operatorname{sen} t - 2b_1 e^{-t}\cos t$

$2b_2 e^{-t}\operatorname{sen} t + 3b_1 e^{-t}\cos t - 3b_1 e^{-t}\operatorname{sen} t - 3b_2 e^{-t}\cos t - 2b_2 e^{-t}\operatorname{sen} t + 2b_1 e^{-t}\operatorname{sen} t + 2b_2 e^{-t}\cos t = 8e^{-t}\operatorname{sen} t$

$\Rightarrow -b_1 e^{-t}\operatorname{sen} t - b_1 e^{-t}\operatorname{sen} t - b_2 e^{-t}\cos t + b_1 e^{-t}\cos t = 8e^{-t}\operatorname{sen} t$

$e^{-t}\operatorname{sen} t(-b_2 - b_1) + e^{-t}\cos t(-b_2 + b_1) = 8e^{-t}\operatorname{sen} t$

$-b_2 - b_1 = 8 \qquad -2b_1 = 8 \Rightarrow b_1 = -4$

$-b_2 + b_1 = 0 \Rightarrow b_1 = b_2$

$y = a_1 e^{-t} + a_2 e^{-2t} - 4e^{-t}\operatorname{sen} t - 4e^{-t}\cos t$

$y(0) = a_1 + a_2 - 4 = 0 \quad (1)$

$\dot{y}(0) = -a_1 - 2a_2 \cancel{-4} - 4(-1) = 1$

$= -a_1 - 2a_2 = 1 \quad (2)$

$(1) + (2)$

$-a_2 = 5 \Rightarrow a_2 = -5$

$a_1 = 4 + 5 = 9$

d) $\ddot{y} + 2\dot{y} + y = \delta(t) + 2u(t)$, $y(0) = 0$, $\dot{y}(0) = 0$ $\qquad$ PERGUNTAR $y(0) \neq 0$

$s^2\hat{Y} - \cancel{sy(0)} - \cancel{\dot{y}(0)} + 2s\hat{Y} - \cancel{2y(0)} + \hat{Y} = 1 + 2\frac{1}{s}$ $\qquad 2e^{-t} - e^{-t} + te^{-t}$

$\Rightarrow (s^2 + 2s + 1)\hat{Y} = 1 + 2\frac{1}{s} + \cancel{(s+2)}y_0 + \cancel{\dot{y}(0)} = \frac{s+2}{s}$

$s = -2 \quad -1$

$P = 1 \quad -1$

$\hat{Y} = \frac{s+2}{s(s+1)^2} = \frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{(s+1)^2} = \frac{A(s+1)^2 + Bs(s+1) + Cs}{s(s+1)^2}$ $\qquad s + 2$

$s = 0 \Rightarrow 2 = A \Rightarrow A = 2$ $\qquad \Rightarrow \hat{Y} = \frac{2}{s} - \frac{2}{s+1} - \frac{1}{(s+1)^2}$ $\qquad 2s^2 + 4s + 2$

$s = -1 \Rightarrow C(-1) = 1 \Rightarrow C = -1$

$1 = A(2s + 2) + B(2s + 1) + C$ $\qquad \hookrightarrow y(t) = (2 - 2e^{-t} - te^{-t})u(t)$

$s = -1 \Rightarrow 1 + 1 = B(-1)$

$B = -2$

2) b) $\ddot{y} + 5\dot{y} + 4y = te^{-2t}$, $\dot{y}(0) = 1$, $y(0) = 1$

$$s^2\hat{Y} - sy(0) - \dot{y}(0) + 5(s\hat{Y} - y(0)) + 4\hat{Y} = \frac{1}{(s+2)^2}$$

$$\Rightarrow (s^2 + 5s + 4)\hat{Y} = \frac{1}{(s+2)^2} + s + 1 + 5 = \frac{1}{(s+2)^2} - (s+6)$$

$S = -5$, $P = 4$ $\begin{cases} -1 \\ -4 \end{cases}$

$(s+1)(s+4) = s^2 + 5s + 4$

$(s+2)(s^2 + 5s + 4) =$
$s^3 + 5s^2 + 4s + 2s^2 + 10s + 8$

$$\hat{Y} = \underbrace{\frac{1}{(s+2)^2(s+1)(s+4)}}_{①} \; \boxed{-} \; \frac{(s+6)}{(s+1)(s+4)} \; ② =$$

$(s+2)^2 = s^2 + 4s + 4$
$(s+4)(s^2+4s+4) = s^3 + \cancel{4s^2} + 4s + \cancel{4s^2} + 16s + 16$
$(s+1)(s^2+4s+4) = s^3 + 4s^2 + 4s + s^2 + 4s + 4$

①

$$1 = \frac{A}{(s+2)^2} + \frac{B}{(s+2)} + \frac{C}{(s+1)} + \frac{D}{(s+4)} = \frac{A(s+1)(s+4) + B(s+2)(s+1)(s+4) + C(s+2)^2(s+4) + D(s+2)^2(s+1)}{(s+2)^2(s+1)(s+4)}$$

$s = -1$: $1 = C(1)^2(3) \Rightarrow C = \frac{1}{3}$

$s = -4$ $\quad 1 = D(-2)^2(-3) \Rightarrow D = \frac{-1}{12}$

$s = -2$ $\quad 1 = A(-1)(2) \Rightarrow A = \frac{-1}{2}$

DERIVANDO:

$$0 = A(2s+5) + B(3s^2 + 14s + 14) + C(3s^2 + 16s + 20) + D(3s^2 + 10s + 8)$$

$s = 0 \Rightarrow 0 = 5A + B14 + C20 + D8 \Rightarrow 14B = -5A - 20C - 8D$

$$B = \frac{-5A - 20C - 8D}{14} = -\frac{1}{4}$$

②

$$\frac{s+6}{(s+1)(s+4)} = \frac{A}{(s+1)} + \frac{B}{(s+4)} = \frac{A(s+4) + B(s+1)}{(s+1)(s+4)}$$

$s = -4 \Rightarrow 2 = B(-3) \Rightarrow B = -\frac{2}{3}$

$s = -1 \Rightarrow 5 = A\,3 \Rightarrow A = \frac{5}{3}$

$$\hat{Y} = -\frac{1}{2}\left(\frac{1}{(s+2)^2}\right) - \frac{1}{4}\left(\frac{1}{(s+2)}\right) + \frac{1}{3}\left(\frac{1}{s+1}\right) - \frac{1}{12}\left(\frac{1}{s+4}\right) + \frac{5}{3}\left(\frac{1}{s+1}\right) - \frac{2}{3}\left(\frac{1}{s+4}\right)$$

$$= \frac{-1}{2}\left(\frac{1}{(s+2)^2}\right) - \frac{1}{4}\left(\frac{1}{s+2}\right) + 2\left(\frac{1}{s+1}\right) - \frac{9}{12}\left(\frac{1}{s+4}\right) =$$

$$y = -\frac{1}{2}te^{-2t} - \frac{1}{4}e^{-2t} + 2e^{-t} - \frac{3}{4}e^{-4t}$$

OU $\hat{Y} = \frac{1 + (s+6)(s^2+4s+4)}{(s+2)^2(s+1)(s+4)} = \frac{A(s+1)(s+4) + B(s+2)(s+1)(s+4) + C(s+2)^2(s+4) + D(s+2)^2(s+1)}{(s+2)^2(s+1)(s+4)}$

$\cancel{s^2}+3s+25$ $\cancel{s^2}+\cancel{6s}+4$ 134 4 136 36

4

2) c-) $\ddot{y}+6\dot{y}+34y = e^{-3t}\operatorname{sen}(5t+\pi/2)$, $y(0)=0$, $\dot{y}(0)=1$

$\ddot{y}+6\dot{y}+34y = e^{-3t}\cos 5t$

$\frac{-6 \pm \sqrt{36-4\cdot 34}}{2} = -3 \pm j5$

$s^2\hat{Y} - \cancel{sy(0)} - \dot{y}(0) + 6(s\hat{Y} - \cancel{y(0)}) + 34\hat{Y} = \frac{s+3}{(s+3)^2+5^2}$

$s^2+6s+9+25 = s^2+6s+34$

$\Rightarrow s^2\hat{Y} - 1 + 6s\hat{Y} + 34\hat{Y} = \frac{s+3}{(s+3)^2+25} \Rightarrow (s^2+6s+34)\hat{Y} = \frac{s+3}{(s+3)^2+25} + 1$ (1)

$S=-6$

$P=34$

$\Rightarrow \hat{Y} = \frac{s+3 \quad + (s+3)^2+25}{((s+3)^2+25)(s+3-j5)(s+3+j5)} = \frac{s+3 \quad + (s+3)^2+25}{((s+3)^2+25)((s+3)^2+25)} = \frac{s^2+7s+37}{((s+3)^2+25)^2}$

$\to s^2+6s+9+25$

$= \frac{A}{((s+3)^2+25)^2} + \frac{B}{((s+3)^2+25)} = \frac{A \quad + \quad B((s+3)^2+25)}{((s+3)^2+25)}$

$2s+7 = B(2s+6)$

$s=0 \Rightarrow 7 = B6 \Rightarrow B = \frac{7}{6}$

$\Rightarrow s^2+7s+37 = A + \frac{7}{6}(s^2+6s+34)$

$s^2 + \cancel{7s} + 37 - \frac{7}{6}s^2 - \cancel{\frac{7}{6}6s} - \frac{7}{6}34 = A$

$s=0 \quad 37 - \frac{7}{6}\cdot 34 = A \Rightarrow A = \frac{3\cancel{37}}{3} - \frac{7\cdot 17}{3} = \frac{-8}{3}$

$\Rightarrow \mathcal{L}\{e^{-at}f(t)\} = \int_0^\infty e^{-at}e^{-st}f(t)\,dt = \hat{f}(s+a)$

$\mathcal{L}\{e^{-3t}\cos 5t\} = \frac{s+3}{(s+3)^2+5^2}$

OU

e (1): $\hat{Y} = \frac{s+3}{((s+3)^2+5^2)^2} + \underbrace{\frac{1}{(s+3)^2+5^2}}$ $\Rightarrow$

$\mathcal{L}^{-1}\left\{\frac{1}{(s+3)^2+5^2}\right\} = \frac{1}{5}e^{-3t}\operatorname{sen} 5t$

$\mathcal{L}\{t\,f(t)\} = -\frac{d}{ds}\mathcal{L}\{f(t)\} = -\frac{d}{ds}\cdot\underbrace{\frac{5}{(s+3)^2+5^2}}_{\mathcal{L}\{e^{-3t}\operatorname{sen}5t\}} = \frac{10(s+3)}{[(s+3)^2+5^2]^2}$

$y(t) = \frac{1}{5}e^{-3t}\operatorname{sen}(5t) + \frac{1}{10}t\,e^{-3t}\operatorname{sen}5t, \quad t \geq 0$

$\to$ s=0 PERTENCE AO DOMÍNIO OK

↳ Se é pedido $I = \int_0^\infty y(t)\,dt \Rightarrow I = \hat{y}(0)$.

4) a) $p^2y[k] + 9py[k] + 14y[k] = 5^k$ $\quad x[k] = 5^k$ $\quad$ (RESOLVENDO POR COEFICIENTES A DETERMINAR)

$\bar{D}[p] = p - 5$ $\quad \to D[p] = p^2 + 9p + 14 \; \begin{smallmatrix} S: -9 \\ P: 14 \end{smallmatrix} \; \begin{smallmatrix} -2 \\ -7 \end{smallmatrix}$

$\bar{D}[p]\,x[k] = (p-5).5^k = 5^{k+1} - 5\,5^k = 5^{k+1} - 5^{k+1} = 0$

SOL FORÇADO $y_f[k] = a5^k$

$\to$ TESTANDO NA EQ:

$p^2[a5^k] + 9pa5^k + 14a5^k = 5^k \Rightarrow a.5^{k+2} + 9a5^{k+1} + 14a5^k = 5^k$

$\Rightarrow 25a5^k + 45a5^k + 14a5^k = 5^k \Rightarrow 25a + 45a + 14a = 1 \Rightarrow a = \frac{1}{84}$

$(p+2)(p+7)y[k] = 0$

$y_h = a_1(-2)^k + a_2(-7)^k$

$y = a_1(-2)^k + a_2(-7)^k + \frac{5^k}{84} \Rightarrow$ APLICANDO NA EQ:

$a_1(-2)^{k+2} + a_2(-7)^{k+2} + \frac{5^{k+2}}{84} + 9\left(a_1(-2)^{k+1} + a_2(-7)^{k+1} + \frac{5^{k+1}}{84}\right) + 14\left(a_1(-2)^k + a_2(-7)^k + \frac{5^k}{84}\right) = 5^k$

$\cancel{4a_1(-2)^k} + \cancel{49a_2(-7)^k} + 25.\frac{5^k}{84} \cancel{-18a_1(-2)^k} \cancel{-63a_2(-7)^k} + \frac{45.5^k}{84} + \cancel{14a_1(-2)^k} + \cancel{14a_2(-7)^k} + \frac{14}{84}5^k = 5^k$ OK!

$y[0] = 2 \Rightarrow$ $2 = a_1 + a_2 + \frac{1}{84} \times 2 \Rightarrow 4 = 2a_1 + 2a_2 + \frac{2}{84}$ (1)

$y[1] = 1 \Rightarrow$ $1 = -2a_1 - 7a_2 + \frac{5}{84}$ (2)

(1) + (2) $\Rightarrow 5 = -5a_2 + \frac{7}{84}$

$5a_2 = \frac{7}{84} - \frac{5\,84}{84} \Rightarrow a_2 = \frac{7 - 5\,84}{84.5}$

$a_2 = -\frac{59}{60}$ $\quad a_1 = \frac{104}{35}$

4) b) MULTIPLICANDO POR U[k] E APLICANDO-SE A TRANSFORMADA Z:

$z(z\hat{Y} - zy[0]) + 6z\hat{Y} - zy[0] + 5\hat{Y} = 32\,\frac{.3z}{(z-3)^2}$ $\quad \Rightarrow Z\{kf(k)\} = \sum_{k=0}^{\infty} kf(k)z^{-k} \Rightarrow$

$\Rightarrow z^2\hat{Y} - z^2y[0] - zy[1] + 6z\hat{Y} - 6zy[0] + 5\hat{Y} = \frac{3z}{(z-3)^2}.32$ $\quad \frac{d}{dz}Z\{f(k)\} = \frac{d}{dz}\sum_{k=0}^{\infty} f(k)z^{-k}$

$= -\sum kf(k)z^{-k-1}$

$\Rightarrow Z\{kf(k)\} = -z\frac{d}{dz}\hat{f}(z)$

$\begin{smallmatrix} S: -6 \\ P: 5 \end{smallmatrix} \; \begin{smallmatrix} -1 \\ -5 \end{smallmatrix}$

$\Rightarrow (z^2 + 6z + 5)\hat{Y} = \frac{32.3z}{(z-3)^2} + 8z + z^2 \Rightarrow \hat{Y} = \frac{96z + 8z(z^2 - 6z + 9) + z^2(z^2 - 6z + 9)}{(z-3^2)(z+1)(z+5)}$

$\hat{Y} = \frac{96z + 8z^3 - 48z^2 + 72z + z^4 - 6z^3 + 9z^2}{(z-3)^2(z+1)(z+5)} = \frac{z^4 + 2z^3 - 39z^2 + 168z}{(z-3)^2(z+1)(z+5)}$ $\quad$ 96 + 72 = 168

$\frac{Az}{(z-3)^2} + \frac{Bz}{(z-3)} + \frac{Cz}{(z+1)} + \frac{Dz}{(z+5)} = A\,3z(z+1)(z+5) + Bz(z-3)(z+1)(z+5) + Cz(z-3)^2(z+5) + Dz(z-3)^2(z+1)$

$z = -1 \Rightarrow 1 - 2 - 39 - 168 = -C.16.4$ $\quad B = -\frac{9}{8}$

$z = 3 \Rightarrow 288 = A3.3.4.8 \Rightarrow A = 1$

$z = -5 \Rightarrow -1440 = D(-5).64.(-4)$

4) c) $z(z\hat{Y} - zy(0)) - z\hat{Y} + zy(0) - 2\hat{Y} = \frac{z}{z-2}$ $\quad y(0) = 0$, $y(1) = 1$

$\Rightarrow z^2\hat{Y} - \cancel{z^2 y(0)} - zy(1) - z\hat{Y} - \cancel{zy(0)} - 2\hat{Y} = \frac{z}{z-2} \Rightarrow z^2\hat{Y} - zY - 2Y - z = \frac{z}{z-2}$

$S = 1$, $P = -2$ $\langle {}^{-1}_{2}$

$\Rightarrow (z^2 - z - 2) = \frac{z}{z-2} + z \Rightarrow (z^2 - z - 2)\hat{Y} = \frac{z + z(z-2)}{(z-2)}$

$\Rightarrow \hat{Y} = \frac{z + z^2 - 2z}{(z-2)(z+1)(z-2)} = \frac{z^2 - z}{(z-2)^2(z+1)} = \frac{z(z-1)}{(z-2)^2(z+1)} = \frac{A.2z}{(z-2)^2} + \frac{Bz}{(z-2)} + \frac{Cz}{(z+1)} = \frac{2A(z+1)z + B(z-2)(z+1)z + C(z-2)}{(z-2)^2(z+1)}$

$z^2 + z$ $\quad z(z^2 - z - 2)$ $\quad z^2 + 4z + c$

$z = -1 \Rightarrow 2 = C.(9).-1 \Rightarrow C = -\frac{2}{9}$

$z = +2 \Rightarrow \cancel{2} = A.(3).\cancel{2}.2 \Rightarrow A = \frac{1}{6}$

$2z - 1 = A.2.(2z+1) + B(3z^2 - 2z - 2) + C(3z^2 - 8z + 4)$ $\quad B = -\frac{2}{9}$

3) a) $y(k+2) + 3y(k+1) + 2y(k) = 11(-2)^k$, $y(0) = 1$, $y(1) = 1$

$S = -3$, $P = 2$ $\langle {}^{-1}_{-2}$ $(p^2 + 3p + 2)y = 11(-2)^k$ $\quad \bar{D}(p) = (p+2)$, MAS $\lambda = -2$ JÁ É UM MODO PRÓPRIO

$(p+2)(p+1)$

$y_f = bk(-2)^k \Rightarrow$ SUBSTITUINDO NA EQUAÇÃO

$b(k+2)\cancel{(-2)^k}.4 + 3b(k+1)\cancel{(-2)^k}(-2) + 2bk\cancel{(-2)^k} = 11\cancel{(-2)^k}$

$4b(k+2) - 6b(k+1) + 2bk = 11 \overset{k=0}{\Rightarrow} 8b - 6b = 11 \Rightarrow b = \frac{11}{2}$

$y = a_1(-2)^k + a_2(-1)^k + \frac{11}{2}k(-2)^k$

$y[0] = a_1 + a_2 = 1 \Rightarrow a_2 = 1 + 13 = 14$

$-a_1 - 11 = 2 \Rightarrow a_1 = -13$

$y[1] = -2a_1 - a_2 - \frac{11}{\cancel{2}}\cancel{2} = 1$

3 c) $y(k+2) + 5y(k+1) + 6y(k) = 10\cos\left(\frac{\pi}{2}k\right)$, $y(0) = 1$, $y(1) = 0$

$\lambda_1 = jak$, $\lambda_2 = -jak$

$2\cos(ak) = e^{jak} + e^{-jak}$

$(p^2 + 5p + 6)y(k) = 10\cos\left(\frac{\pi}{2}k\right)$ $\quad \bar{D}(p) = p^2 + (ak)^2$

$S = -5$, $P = 6$ $\langle {}^{-2}_{-3}$

$(p - jak)(p + jak) = (p^2 + (ak)^2$

$a = \frac{\pi}{2}$

y_f É DO TIPO $b_1(ja)^k + b_2(-ja)^k \Rightarrow$ SUBSTITUINDO NA EQ:

$b_1(ja)^{k+2} + b_2(-ja)^{k+2} + 5b_1(ja)^{k+1} + 5b_2(-ja)^{k+1} + 6b_1(ja)^k + 6b_2(-ja)^k = 5(ja)^k + 5(-ja)^k$

$-b_1a^2(ja)^k - b_2a^2(-ja)^k + 5b_1ja(ja)^k - 5b_2ja(-ja)^k + 6b_1(ja)^k + 6b_2(-ja)^k = 5(ja)^k + 5(-ja)^k$

$-b_1a^2\cancel{(jak)} + 5jb_1a\cancel{(ja)^k} + 6b_1\cancel{(ja)^k} = 5\cancel{(ja)^k}$

EA721 - Primeira lista de exercícios (7)

5) $y(t) = 20t - 30 + 40e^{-t} - 10e^{-2t} \quad \forall t \geq 0$

a) $Y(s) = \frac{20}{s^2} - \frac{30}{s} + \frac{40}{s+1} - \frac{10}{s+2} = \frac{20(s+1)(s+2) - 30s(s+1)(s+2) + 40s^2(s+2) - 10(s^2)(s+1)}{s^2(s+1)(s+2)}$

$Y(s) = H(s) \cdot X(s) \Rightarrow Y(s) = H(s) \cdot \frac{1}{s^2} \Rightarrow H(s) = \frac{20(s+1)(s+2) - 30s(s+1)(s+2) + 40s^2(s+2) - 10s^2(s+1)}{(s+1)(s+2)}$

b) $x(t) = \delta(t) \Rightarrow$

$Y(s) = H(s) \cdot 1 \Rightarrow \quad H(s) = \frac{20(s^2+3s+2) - 30(s^3+3s^2+2s) + 40(s^3+2s^2) - 10(s^3+s^2)}{(s+1)(s+2)}$

$30s^3 - 90s^2 + 80s^2 - 10s^2 \quad 60s - 60s$

$= \frac{40}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2} = \frac{A(s+2) + B(s+1)}{(s+1)(s+2)} = \frac{40}{s+1} - \frac{40}{s+2}$ $\quad B = -40,\ A = 40$

$\Rightarrow h(t) = \mathcal{L}^{-1}\{H(s)\} = 40e^{-t} - 40e^{-2t}$

c) Para $x(t) = g(t) = 30\,\text{sen}(20t) \Rightarrow X(s) = 30 \cdot \frac{20}{s^2+400}$

$Y[s] = H(s) \cdot X(s) = \frac{40}{(s+1)(s+2)} \cdot 30 \cdot \frac{20}{(s^2+400)} = \frac{24000}{(s^2+3s+2)(s^2+400)}$

$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s\,Y[s] = \frac{24000\,s}{s^4 + 400s^2 + 3s^3 + 1200s + 2s^2 + 800}$

e então $\lim_{s\to 0} sY[s] = 0$ (só que o resultado não vale, já que $s = 0$ não pertence ao domínio)

$\text{sen}(\theta t) = \frac{e^{j\theta t} - e^{-j\theta t}}{2j}$

$\cos\theta t + j\,\text{sen}\,\theta t$

$-\cos\theta t + j\,\text{sen}\,\theta t \quad 2j$

$\text{sen}\,\theta t = \frac{1}{2j}(e^{j\theta t} - e^{-j\theta t})$

$\mathcal{L}\{\text{sen}\,\theta t\} = \frac{1}{2j}\left(\frac{1}{s-j\theta} - \frac{1}{s+j\theta}\right)$

$= \frac{1}{2j}\left(\frac{s+j\theta - s + j\theta}{(s-j\theta)(s+j\theta)}\right) = \frac{1}{2j}\left(\frac{2j\theta}{s^2+\theta^2}\right)$

$= \frac{\theta}{s^2+\theta^2}$

6) $y[k] = 10(1)^k - (0,4)^k + (-0,5)^k \qquad x[k] = u[k]$

$X[z] = \frac{z}{z-1}$

$Y[z] = \frac{10 \cdot z}{z-1} - \frac{z}{z-0,4} + \frac{z}{z+0,5} = \frac{10z}{z-1} - \frac{5z}{5z-2} + \frac{2z}{2z+1} =$

$Y[z] = H[z] \cdot X[z] \Rightarrow H[z] = \frac{Y[z]}{X[z]} = 10 - \frac{5\cancel{z}}{5z-2}\frac{(z-1)}{\cancel{z}} + \frac{2\cancel{z}}{2z+1}\frac{(z-1)}{\cancel{z}} =$

$\Rightarrow H[z] = \frac{10(5z-2)(2z+1) - 5(z-1)(2z+1) + 2(z-1)(5z-2)}{(5z-2)(2z+1)} =$

Dividir polinômio

$100z^2 + z - 11 \;\; | \; 10z^2 + z - 2$

$-100z^2 - 10z + 20 \quad 10$

$-9z + 9$

$= \frac{10(10z^2 + z - 2) - 5(2z^2 - z - 1) + 2(5z^2 - 7z + 2)}{(5z-2)(2z+1)} = \frac{100z^2 + z - 11}{(5z-2)(2z+1)} = \frac{10(5z-2)(2z+1) + 9(1-z)}{(5z-2)(2z+1)}$

$H_1[z] \quad H_2[z]$

$= 10 + \frac{9(1-z)}{(5z-2)(2z+1)} \Rightarrow H_2[z] = \frac{9(1-z)}{z(5z-2)(2z+1)} = \frac{A}{z} + \frac{B}{(5z-2)} + \frac{C}{(2z+1)} = \frac{A(5z-2)(2z+1) + Bz(2z+1) + Cz(5z-2)}{(5z-2)(2z+1)z}$

$z = 0 \Rightarrow 9 = A(-2)(1) \Rightarrow A = -\frac{9}{2}$

$z = \frac{2}{5} \Rightarrow 9\left(1 - \frac{2}{5}\right) = B\frac{2}{5}\left(\frac{4}{5} + 1\right) \Rightarrow 9 \cdot \frac{3}{5} = B\,\frac{2}{5}\,\frac{9}{5} \Rightarrow B = \frac{15}{2}$

$z = -\frac{1}{2} \Rightarrow 9\left(1 + \frac{1}{2}\right) = C\left(-\frac{1}{2}\right)\left(-\frac{5}{2} - 2\right) \Rightarrow C = 6$

$H(z) = \underbrace{10}_{H_1[z]} - \frac{9}{2} + \underbrace{\frac{15(z)}{2(z-4)}}_{H_2[z]} + \frac{6z}{(2z+1)} = \frac{11}{2} + \frac{6z}{(\ldots)} + \frac{15}{\ldots} \ldots \frac{\ldots}{z+0{,}5} + \frac{3}{2}\frac{z}{(z-\ldots)}$

$h[n] = \frac{11}{2}\delta[n] + 3(-0{,}5)^n + \frac{3}{2}(\ldots)^n$

$H(s) = \frac{K}{(s+\ldots)(s+16)(\ldots)} = (\ldots) \cdot 40^2 \quad => \quad 8 = \frac{K_2\, 16}{16}$

$H(s) = \frac{K_1 \cdot s\, 4\left(\frac{s}{4}+1\right) 40^2}{2\left(\frac{s}{2}+1\right)16\left(\frac{s}{16}+1\right)\left(s^2+40s+40^2\right)}$

$\delta = \frac{K_1\, 16 \cdot 4 \cdot 4}{2 \cdot 4 \cdot 16}\left(\ldots\right)$

$H(s) = K\, s \cdot \frac{1}{\left(\frac{s}{2}+1\right)} \cdot \left(\frac{s}{4}+1\right) \cdot \frac{1}{\left(\frac{s}{16}+1\right)} \cdot \frac{40^2}{\left(s^2+40s+40^2\right)} = \frac{1 \cdot s(s+4) \cdot 1 \cdot 40^2}{(s+2)(s+16)(s^2+40s+40^2)} \cdot \frac{1}{4} \cdot 16^{-1}$

$H_{dB}(s) = 20\log K + 20\log s + \underbrace{20\log\left(\frac{1}{\frac{s}{2}+1}\right)}_{-20\log\left(\frac{s}{2}+1\right)} + 20\log\left(\frac{s}{4}+1\right) + \underbrace{20\log\left(\frac{1}{\frac{s}{16}+1}\right)}_{-20\log\left(\frac{s}{16}+1\right)} + 20\log\left(\frac{40^2}{s^2+40s+40^2}\right)$

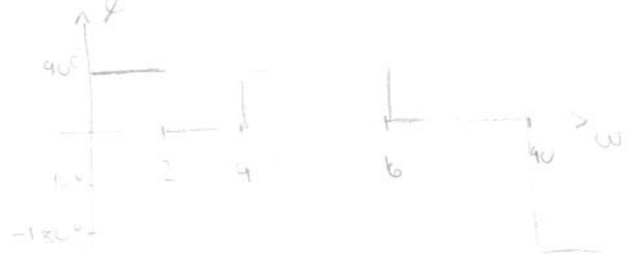
$\ldots => $

$\ldots = 20\log K + 20\log 1 - 20\log 2 + 20\log 4 - 20\log 1 \ldots$

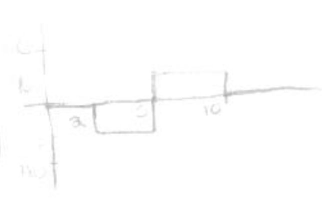
$\ldots K \cdot \frac{4}{2} \ldots => K = \ldots = \frac{1}{16}$

1/ $x(t) = \sin 10t$ (PELO DIAGRAMA ASSINTÓTICO) $\quad Y_{\ldots} = \ldots \log 4 \ldots \log 16$

$Y_{\ldots} = 20\log K + 20\log\frac{1}{16} = 20\log\frac{K\, 16}{16 \ldots} \ldots \quad |G(\ldots)| = \ldots$



8) $G(s) = K \cdot \frac{1}{\left(\frac{s}{2}+1\right)} \ldots \frac{1}{\left(\frac{s}{16}+1\right)} \ldots \quad 20\log|G(\ldots)| = 12 = 20\log\left(\frac{K}{5}(2)^2 \frac{1}{\ldots}\right)$



$=> 20\log 2^2 + 20\log\left(\frac{K}{5} \cdot 4\right) => \frac{K4}{5} = 4 => K = 5$

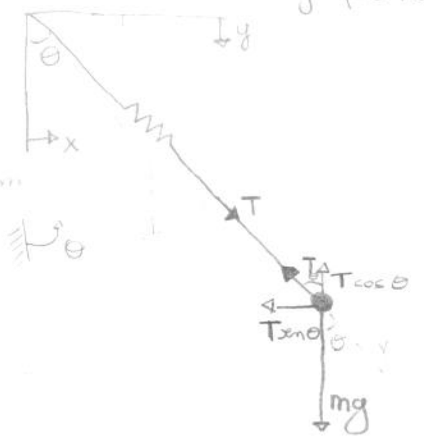
$x(t) = \sin(4t)$

$|G(j\omega)|$ em $s = j2 = |G(\ldots)| = 14\text{dB} \quad Y_{dB} - 14 = -20(\log 4 - \log 2) => Y_{dB} = 20\log 5 - 20\log\left(\frac{1}{4}\right) = 20\log\frac{5}{4} \ldots 2 \quad |G(j4)| = \frac{10}{4}$

$T = K(l - l_0)$

10) 

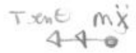
→ l_0 é considerado sem mais da massa m



$y = (l_0 + l)\cos\theta$

VERTICAL

HORIZONTAL



$\frac{mg}{x} = \cos\theta$

$x = l\,\text{sen}\,\theta$

$y = l\cos\theta$

$$\Rightarrow \left.\begin{aligned} m\ddot{y} &= mg - T\cos\theta \\ m\ddot{x} &= -T\,\text{sen}\,\theta \end{aligned}\right\} \quad \begin{aligned} m\ddot{y} &= mg - K(l-l_0)\cos\theta \\ m\ddot{x} &= -K(l-l_0)\,\text{sen}\,\theta \end{aligned}$$

$\dot{x} = \dot{l}\,\text{sen}\,\theta + l\cos\theta\,\dot{\theta} \Rightarrow \ddot{x} = \ddot{l}\,\text{sen}\,\theta + \dot{l}\cos\theta\,\dot{\theta} + \frac{d}{dt}(l\cos\theta).\dot{\theta} + l\cos\theta.\ddot{\theta} =$

$\ddot{x} = \ddot{l}\,\text{sen}\,\theta + \dot{l}\cos\theta\,\dot{\theta} + (\dot{l}\cos\theta - l\,\text{sen}\,\theta.\dot{\theta}).\dot{\theta} + l\cos\theta.\ddot{\theta}$

$= \ddot{l}\,\text{sen}\,\theta + 2.\dot{l}\cos\theta\,\dot{\theta} - l\,\text{sen}\,\theta.\dot{\theta}^2 + l\cos\theta.\ddot{\theta}$

$\dot{y} = \dot{l}\cos\theta - l\,\text{sen}\,\theta.\dot{\theta} \Rightarrow \ddot{y} = \ddot{l}\cos\theta - \dot{l}\,\text{sen}\,\theta.\dot{\theta} - \frac{d}{dt}(l\,\text{sen}\,\theta).\dot{\theta} - l\,\text{sen}\,\theta.\ddot{\theta}$

$\ddot{y} = \ddot{l}\cos\theta - \dot{l}\,\text{sen}\,\theta\,\dot{\theta} - (\dot{l}\,\text{sen}\,\theta + l\cos\theta.\dot{\theta})\dot{\theta} - l\,\text{sen}\,\theta.\ddot{\theta}$

$= \ddot{l}\cos\theta - 2\dot{l}\,\text{sen}\,\theta\,\dot{\theta} - l\cos\theta\,\dot{\theta}^2 - l\,\text{sen}\,\theta.\ddot{\theta}$

Igualando-se ...

$m(\ddot{l}\cos\theta - 2.\dot{l}\,\text{sen}\,\theta.\dot{\theta} - l\cos\theta\,\dot{\theta}^2 - l\,\text{sen}\,\theta.\ddot{\theta} - g) + K(l-l_0)\cos\theta = 0$ $\quad \times(-\text{sen}\,\theta)$ (3) $\times$ ...

$m(\ddot{l}\,\text{sen}\,\theta + 2\dot{l}\cos\theta.\dot{\theta} - l\,\text{sen}\,\theta.\dot{\theta}^2 + l\cos\theta.\ddot{\theta}) + K(l-l_0)\,\text{sen}\,\theta = 0$ $\quad \times \cos\theta$ (4) $\times$ ...

$m(-\ddot{l}\,\text{sen}\,\theta\cos\theta + 2.\dot{l}\,\text{sen}^2\theta.\dot{\theta} + l\,\text{sen}\,\theta\cos\theta.\dot{\theta}^2 + l\,\text{sen}^2\theta.\ddot{\theta} - g\,\text{sen}\,\theta) - K(l-l_0)\,\text{sen}\,\theta\cos\theta = 0$

$m(\ddot{l}\,\text{sen}\,\theta\cos\theta + 2\dot{l}\cos^2\theta.\dot{\theta} - l\,\text{sen}\,\theta\cos\theta.\dot{\theta}^2 + l\cos^2\theta.\ddot{\theta}) + K(l-l_0)\,\text{sen}\,\theta\cos\theta = 0$

$m(2\dot{l}\dot{\theta} + l\ddot{\theta} - g\,\text{sen}\,\theta) = 0$

$m(\ddot{l} - l\dot{\theta}^2 - g\cos\theta) + K(l-l_0) = 0$

b) Um ponto de equilíbrio é quando $\dot{l} = 0$ e $\dot{\theta} = 0$ e $\ddot{l} = 0$ e $\ddot{\theta} = 0$

$\text{sen}\,\theta = 0 \Rightarrow \theta = K\pi, \; K = 0, 1, 2$

$-mg\cos\theta + K(l-l_0) \Rightarrow l = \frac{mg}{K} + l_0$

c) $\text{sen}\,\theta = f(0).1 + f'(0)\theta = 0 + \theta$

$\cos\theta = 1$

(1) $m(2\dot{l}\dot{\theta} + l\ddot{\theta} - g\theta) = 0$

(2) $m(\ddot{l} - g) + K(l-l_0) = 0$

a) $3\dot{V}_2 + V_2 = V_1$ $\quad V_1 = t\,u(t) - (t-20)\,u(t-20)$ → PERGUNTAR MATHEMATICA

$$(3s+1)\hat{V}_2 = \frac{1}{s^2} - \frac{e^{-20s}}{s^2} \Rightarrow \hat{V}_2 = \frac{1 - e^{-20s}}{s^2}\frac{1}{(3s+1)}$$

$$\hat{V}_2 = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{(3s+1)} = \frac{A(3s+1) + Bs(3s+1) + Cs^2}{(3s+1)s^2} = \frac{1 - e^{-20s}}{s^2(3s+1)} = \frac{1}{s^2(3s+1)} - \frac{e^{-20s}}{s^2(3s+1)}$$

$$\frac{A}{s^2} + \frac{B}{s} + \frac{C}{(3s+1)} = \frac{A(3s+1) + Bs(3s+1) + Cs^2}{s^2(3s+1)} = \frac{1}{s^2(3s+1)} \Rightarrow \frac{1}{s^2} - \frac{3}{s} + \frac{9}{(3s+1)}$$

$s = -\frac{1}{3} \Rightarrow C\frac{1}{9} = 1 \Rightarrow C = 9$

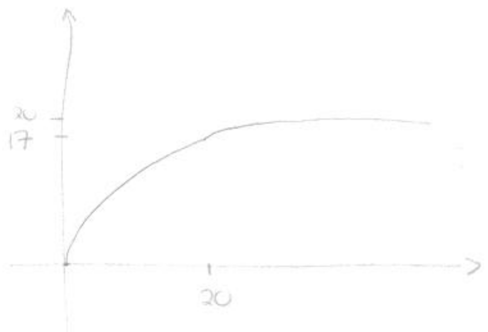
$s = 0 \Rightarrow A = 1$

DERIVANDO $\quad A\,3 + B(6s+1) + \cancel{2Cs}^{\,0} = 0$

$\hookrightarrow s = 0 \quad B = -3A = -3$

$$V_2(t) = t - 3 + 3e^{-t/3} + \left(-t + 20 + 3 - 3e^{-\frac{(t-20)}{3}}\right).u(t-20)$$

20 - 3



DEPOIS DE 20s:

$$20 + 3e^{-t/3} - 3e^{-t/3} \cdot e^{20/3} = 20.3e^{t/3}(1 - 1e^{20/3})$$

P/ $t = 15s \Rightarrow V_2(t) \cong 15 - 3 = 12\,m/s$
$\quad V_1(t) = 15\,m/s$ $\Big\}$ $V_1 - V_2 = 3\,m/s$

$t = 50s \Rightarrow V_2(t) \simeq 20\,m/s$
$\quad V_1(t) \simeq 20\,m/s$ $\Big\}$ $V_1 - V_2 = 0$

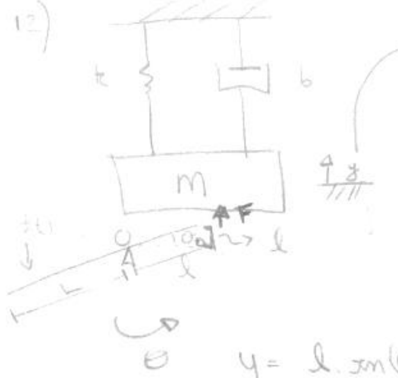
$$\Delta s_1 = \int_0^{20} V_1(t)dt + C = \frac{t^2}{2}\Big]_0^{20} + \overset{=40m}{C} = \frac{20^{\,10}}{2}20 + 40 = 1020 + 40 = 240\,m$$

$$\Delta s_2 = \int_0^{20} V_2(t)dt = \frac{t^2}{2}\Big]_0^{20} - 3t\Big]_0^{20} - 3.3e^{-t/3}\Big]_0^{20} = 200 - 3.20 + 3.3 = 200 - 60 + 9$$

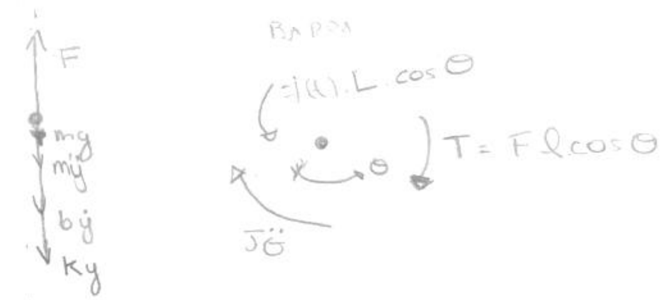
$\Delta s_1 - \Delta s_2 = 40 + 60 - 9 = 91\,m$

CA721 - PRIMEIRA LISTA DE EXERCÍCIOS

12)



COMPARADO COM A INCLUSÃO ...



$y = l.\text{sen}(\theta + \theta_0)$

$\dot y = l\cos(\theta+\theta_0).\dot\theta$

MASSA $\ddot y = l\cos(\theta+\theta_0).\ddot\theta - l\,\text{sen}(\theta+\theta_0)\dot\theta^2$

$m\ddot y + mg + b\dot y + ky = F$

BARRA

$f(t)L\cos\theta - J\ddot\theta = F\,l\cos\theta$

$J\ddot\theta + l\cos\theta(m\ddot y + mg + b\dot y + ky) = f(t)L\cos\theta$

$\Rightarrow J\ddot\theta + l\cos\theta\left(m(l\cos\theta.\ddot\theta - l\,\text{sen}\theta\,\dot\theta^2 + g) + b(l\cos\theta\,\dot\theta) + Kl\,\text{sen}\theta\right) = f(t)L\cos\theta$

$\frac{J\ddot\theta}{\cos\theta} + l(ml\cos\theta\,\ddot\theta - ml\,\text{sen}\theta\,\dot\theta^2 + ml + bl\cos\theta\,\dot\theta + Kl\,\text{sen}\theta) = f(t)L$

b) $\theta = 0$
$\cos\theta = 1$
$\text{sen}\theta \approx \theta$
$tg\theta \approx \theta$

$J\ddot\theta + ml^2\ddot\theta - ml^2\theta.\dot\theta^2 + ml^2 + bl^2\dot\theta + Kl^2\theta = f(t)L$

$(J + ml^2)\ddot\theta + bl^2\dot\theta + Kl^2\theta = f(t).L - ml^2$

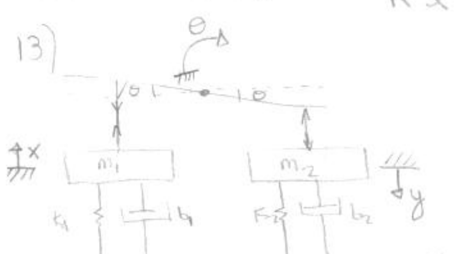
$\hookrightarrow \ddot\theta + \frac{bl^2}{(J+ml^2)}\dot\theta + \frac{Kl^2\theta}{(J+ml^2)} = \frac{f(t)L - ml^2}{(J+ml^2)}$

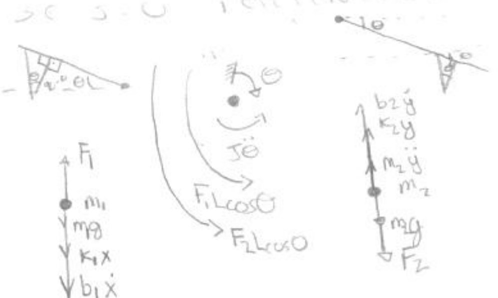
$\omega_N = \sqrt{\frac{Kl}{J+ml^2}}$

c) $\left(s^2 + \frac{bl^2}{J+ml^2}s + \frac{Kl^2}{J+ml^2}\right)\hat\theta = \frac{f_0 L - ml^2}{J+ml^2} \Rightarrow \hat\theta(s) = \frac{\frac{1}{s}(f_0.L - ml^2 s)}{(J+ml^2)s^2 + bl^2 s + Kl^2}$

$\lim_{t\to\infty}\theta = \lim_{s\to 0} s\hat\theta(s) = \frac{f_0 L}{Kl^2}$, SE $s \to 0$ ... AO TÉRMINO DE $\hat\theta(s)$

13)





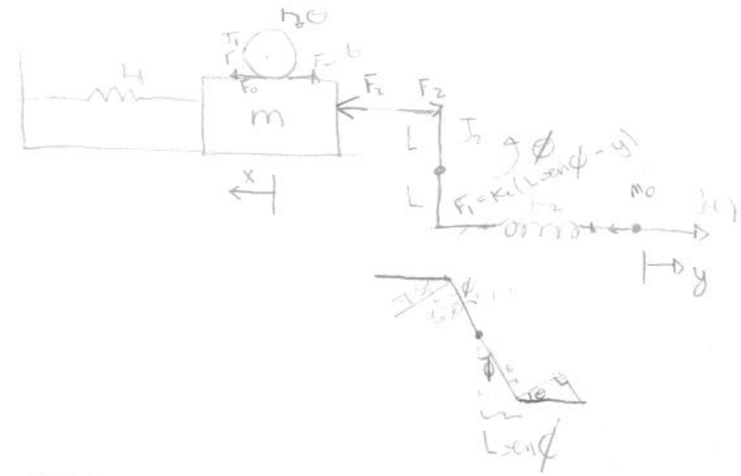
$$\begin{cases} m_1 g + k_1 x + b_1\dot x + m_1\ddot x = F_1 \Rightarrow m_1 g + k_1 L\theta + b_1 L\dot\theta + m_1 L\ddot\theta = F_1 \\ -m_2 g + k_2 y + b_2\dot y + m_2\ddot y = F_2 \Rightarrow -m_2 g + k_2 L\theta + b_2 L\dot\theta + m_2 L\ddot\theta = F_2 \\ J\ddot\theta = -L\cos(F_1+F_2) \Rightarrow J\ddot\theta = -L(F_1+F_2) \end{cases}$$

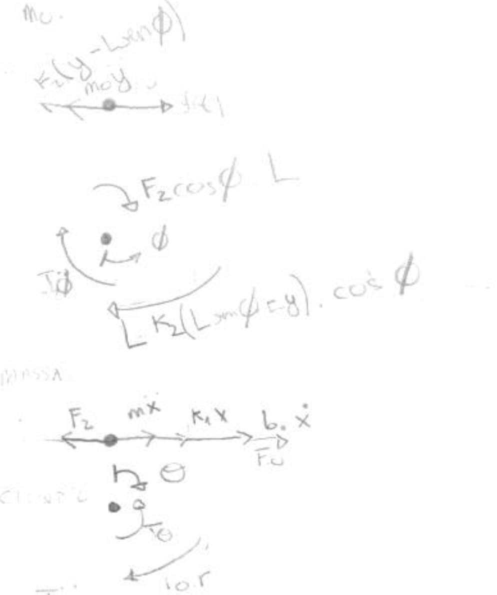
$\Rightarrow F_1 + F_2 = g(m_1 - m_2) + (k_1+k_2)L\theta + (b_1+b_2)L\dot\theta + (m_1+m_2)L\ddot\theta$

SÓ QUE (P/ PEQUENAS PERTURBAÇÕES E L GRANDE) $\underbrace{x = L\theta \;\; e \;\; y = L\theta}_{x = y}$, E $\cos\theta = 1$

$J\ddot\theta = L\left(g(m_2 - m_1) - (k_1+k_2)L\theta - (b_1+b_2)L\dot\theta - (m_1+m_2)L\ddot\theta\right) \Rightarrow (J + L^2(m_1+m_2))\ddot\theta + (b_1+b_2)L^2\dot\theta + (k_1+k_2)L^2\theta = Lg(m_2 - m_1)$

14)





· CILINDRO:

$J_1\ddot{\theta} - F_0 r = 0 \Rightarrow J_1\ddot{\theta} = +F_0 r \Rightarrow \frac{J_1\ddot{\theta}}{r} = F_0$ (1)

MASSA:

$m\ddot{x} + K_1 x + b\dot{x} = -F_0 + F_2 \Rightarrow F_2 = m\ddot{x} + K_1 x + b\dot{x} + \frac{J_1\ddot{\theta}}{r}$ (2)

- PARTE

$J_2\ddot{\phi} + L K(L\,sen\phi - y)\cdot\cos\phi + L F_2\cos\phi = 0$

CONDIÇÕES DE CONTORNO

$\begin{cases} x = \theta r \\ x = L\,sen\phi \end{cases} \Rightarrow x - \theta r = L\,sen\phi$

$\dot{\theta} r = L\cos\phi\cdot\dot{\phi}$

$\ddot{\theta} r = L(\cos\phi\cdot\ddot{\phi} - sen\phi\cdot\dot{\phi}^2)$

m_0

$K_2(y - L\,sen\phi) + \cancel{m_0\ddot{y}}^{\,0} - f(t) = 0$ (4)

$\Rightarrow -f(t) = K_2(L\,sen\phi - y) = K_2(\theta r - y) = -f(t)$ (6)

(2) em (3) $\Rightarrow J_2\ddot{\phi} + L K_1(L\,sen\phi - y)\cos\phi + L(m\ddot{x} + K_1 x + b\dot{x} + \frac{J_1}{r}\ddot{\theta})\cos\phi = 0$

$\Rightarrow J_2\ddot{\phi} + L K_2(\theta r - y)\cos\phi + L\cos\phi(m r\ddot{\theta} + b r\dot{\theta} + K_1 r\theta + \frac{J_1}{r}\ddot{\theta}) = 0$ (5)

Equilíbrio: $\dot{y} = y = 0$, $\dot{x} = x = 0$, $\dot{\phi} = \phi = 0$

(6) em (5) $\Rightarrow J_2\ddot{\phi} - L f(t) + L\cos\phi(m r\ddot{\theta} + b r\dot{\theta} + K_1 r\theta + \frac{J_1}{r}\ddot{\theta}) = 0$

TEMOS DE CG P/ PEQUENAS PERTURBAÇÕES

$L\cos\phi(K_1 r\theta) = 0 \quad \therefore \theta = 0, \cos\phi = 1$ (JÁ QUE $0 = L\,sen\phi$)

$\phi = 0$

$K_2(y - L\phi) = f(t)$

$K_2 y - K_2\theta r = f(t)$

$y = \frac{\cancel{K_2} r\theta}{\cancel{K_2}} + \frac{f(t)}{K_2}$

OUTRA CONDIÇÃO DE CONTORNO:

$\ddot{\theta} r + L\,sen\phi\cdot\dot{\phi}^2 = L\cos\phi\cdot\ddot{\phi} \Rightarrow \ddot{\phi} = \frac{\ddot{\theta} r}{L\cos\phi} + tg\phi\,\dot{\phi}^2$ (7) em (5) $\Rightarrow$

$\Rightarrow J_2\frac{\ddot{\theta} r}{L\cos\phi} + J_2\,tg\phi\cdot\dot{\phi}^2 + L\cos\phi(m r\ddot{\theta} + b r\dot{\theta} + K_1 r\theta + \frac{J_1}{r}\ddot{\theta}) = L f(t)$

↳ LINEARIZANDO $\frac{J_2 r}{L}\ddot{\theta} + \frac{L J_1}{r}\ddot{\theta} + L m r\ddot{\theta} + L b r\dot{\theta} + L K_1 r\theta = L f(t)$

$\Rightarrow \underbrace{\left(\frac{J_2 r}{L} + \frac{J_1 L}{r} + L m r\right)}_{a}\ddot{\theta} + L b r\dot{\theta} + L K_1 r\theta = L f(t) \Rightarrow \ddot{\theta} + \frac{L b r}{a}\dot{\theta} + \frac{L K_1 r}{a}\theta = \frac{L}{a} f(t)$

$\left(p^2 + \frac{L b r}{a}p + \frac{L K_1 r}{a}\right)\theta = \frac{L}{a} f(t)$

$\theta = v_1$

$\dot{\theta} = \dot{v}_1 = v_2$

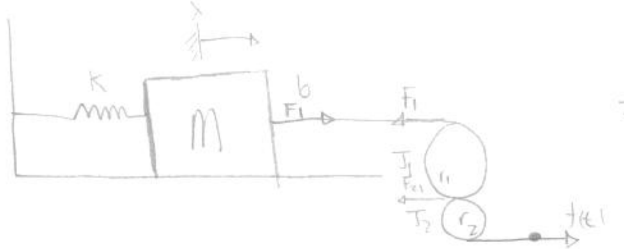
$\ddot{\theta} = \ddot{v}_1 = \dot{v}_2 = v_3$

$$\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{L K_1 r}{a} & -\frac{L b r}{a} \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{L}{a} \end{bmatrix} f(t)$$

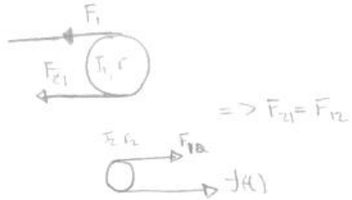
$$Z = \begin{bmatrix} \theta \\ y \end{bmatrix} \Rightarrow Z = \begin{bmatrix} 1 & 0 \\ r & 0 \end{bmatrix}\begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/K_2 \end{bmatrix} f(t)$$

$y = a_{21}\theta + a_{22}\dot{\theta}$

13)

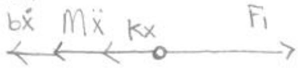


$\Rightarrow$



$\Rightarrow F_{21} = F_{12}$

Massa



$\Rightarrow M\ddot{x} + b\dot{x} + kx = F_1$ (1)

$x = \theta . r_1$ C. Contorno

$\theta r_1 = \phi r_2$

$x = \theta r_1 = \phi r_2$

$F_{21} = F_{12}$

Cilindro I:

$\Rightarrow J_1\ddot{\theta} + F_1 r_1 = F_{21} r_1$ (2)

Cilindro II:

$\Rightarrow J_2\ddot{\phi} + F_{12} r_2 = f(t) r_2$ (3) $\Rightarrow F_{12} = f(t) - \frac{J_2}{r_2}\ddot{\phi}$

(1) em (2) $\Rightarrow J_1\ddot{\theta} + (M\ddot{x} + b\dot{x} + kx) r_1 = F_{12} r_1 \Rightarrow \frac{J_1}{r_1^2}\ddot{x} + (M\ddot{x} + b\dot{x} + kx) = F_{12}$ (4)

$\Rightarrow \frac{J_1\ddot{x}}{r_1^2} + M\ddot{x} + b\dot{x} + kx = f(t) - \frac{J_2}{r_2^2}\ddot{x} \Rightarrow \left(\frac{J_1}{r_1^2} + \frac{J_2}{r_2^2} + M\right)\ddot{x} + b\dot{x} + kx = f(t)$

b)

$$\left(\left(\frac{J_1}{r_1^2} + \frac{J_2}{r_2^2} + M\right) s^2 + bs + k\right)\hat{X} = \hat{f}(s) \Rightarrow$$

$$\hat{X} = \frac{f(s)}{\left(s^2 + \frac{b}{\left(\frac{J_1}{r_1^2} + \frac{J_2}{r_2^2} + M\right)} s + \frac{k}{\left(\frac{J_1}{r_1^2} + \frac{J_2}{r_2^2} + M\right)}\right)\left(\frac{J_1}{r_1^2} + \frac{J_2}{r_2^2} + M\right)}$$

$$\omega_N = \sqrt{\frac{k}{\left(\frac{J_1}{r_1^2} + \frac{J_2}{r_2^2} + M\right)}}$$

c) $$\hat{X} = \frac{1}{s} \cdot \frac{1}{\left(s^2 + \frac{b}{\left(\frac{J_1}{r_1^2} + \frac{J_2}{r_2^2} + M\right)} s + \frac{k}{\left(\frac{J_1}{r_1^2} + \frac{J_2}{r_2^2} + M\right)}\right)\left(\frac{J_1}{r_1^2} + \frac{J_2}{r_2^2} + M\right)}$$

$$\lim_{t \to \infty} x(t) = \lim_{s \to 0} s\hat{X}(s) = \frac{1}{k}$$

d) $$H(s) = \frac{1}{\left(\left(\frac{J_1}{r_1^2} + \frac{J_2}{r_2^2} + M\right)s^2 + bs + k\right)} \Rightarrow H(j2) = \frac{1}{2(j2)^2 + 1,5j2 + 0,25} = -0,11 - 0,043j = 0,12e^{-}$$

$y(t) = 0,12 \operatorname{sen}(2t - 2,77)$

15) e)

$X = F.G$

$F = C(X_{ref} - X)$

$X = C(X_{ref} - X).G \Rightarrow (1 + CG)X = CGX_{ref}$

f) $\Rightarrow X = \dfrac{CG}{1+CG} X_{ref}$, p/ $G(s) = K_p \Rightarrow X = \dfrac{K_p G}{1 + K_p G}$

SE $\hat{X}_{ref} = \dfrac{x_{ref}}{s} \Rightarrow X = \dfrac{K_p G}{1 + K_p G} \cdot \dfrac{x_{ref}}{s}$

EM REGIME: $X = \lim_{t \to \infty} x(t) = \lim_{s \to 0} sX = \lim_{s \to 0} \dfrac{K_p G}{1 + K_p G} X_{ref} = \dfrac{K_p \frac{1}{0,25}}{1 + K_p \frac{1}{0,25}} X_{ref} = \dfrac{4K_p}{1 + 4K_p} \cdot X_{ref}$

g) P/ $C(s) = \dfrac{K_1}{s} \Rightarrow \hat{X} = \dfrac{\frac{K_1}{s}.G}{1 + \frac{K_1}{s} G} \hat{x}_{ref} = \dfrac{\frac{K_1}{s} \cdot \frac{1}{(2s^2 + 1,5s + 0,25)}}{1 + \frac{K_1}{s}\left(\frac{1}{2s^2 + 1,5s + 0,25}\right)} =$

$= \dfrac{K_1}{\cancel{s(2s^2+1,5s+0,25)}} \cdot \dfrac{\cancel{s(2s^2+1,5s+0,25)}}{s(2s^2+1,5s+0,25) + K_1} = \dfrac{K_1}{2s^3 + 1,5s^2 + 0,25s + K_1} \cdot X_{ref}$

$\lim_{t \to \infty} x = x_{final} = \lim_{s \to 0} s\hat{X} = \dfrac{K_1}{K_1} . x_{ref} = x_{ref}$

NÃO É POSSÍVEL UTILIZARMOS QUALQUER VALOR DE GANHO [illegible] O DENOMINADOR DE H(s) DEVE TER RAÍZES < 0 P/ O SISTEMA SER ESTÁVEL

INVESTIGANDO O VALOR DE K_1 [illegible] ROUTH-HURWITZ

s^3	2	0,25
s^2	1,5	K_1
s	( )	
s^0	( )	

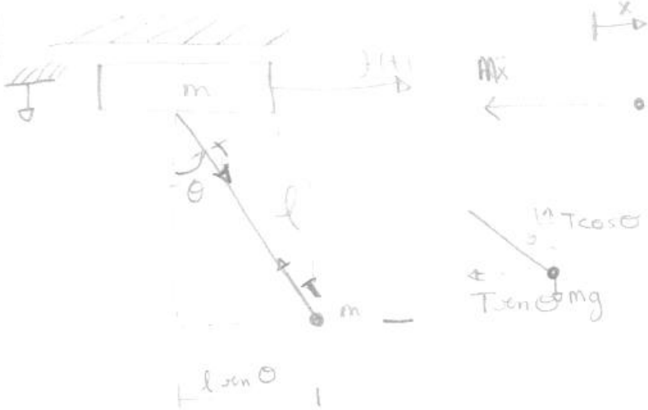
TODOS OS ELEMENTOS DA PRIMEIRA COLUNA DEVEM [illegible] P/ QUE O SISTEMA APRESENTE POLOS NO [illegible]

$\dfrac{1,5 \cdot 0,25 - 2K_1}{0,25} > 0 \Rightarrow \dfrac{1,5 \cdot 0,25}{2} > K_1$

$\dfrac{\frac{(1,5 \cdot 0,25 - 2K_1)}{0,25} \cdot K_1}{\frac{(1,5 \cdot 0,25 - 2K_1)}{0,25}} = K_1 > 0$

$\boxed{0 < K_1 < \dfrac{1,5 \cdot 0,25}{2}}$

10)



$\frac{d}{dt}(\ell\cos\theta) = -\ell\,\mathrm{sen}\theta\,\dot\theta \Rightarrow \frac{d^2}{dt^2} = -\ell\cos\theta\,\dot\theta^2 - \ell\,\mathrm{sen}\theta\,\ddot\theta$

$\frac{d}{dt}(\ell\,\mathrm{sen}\theta) = \ell\cos\theta\,\dot\theta \Rightarrow \frac{d^2}{dt^2} = -\ell\,\mathrm{sen}\theta\,\dot\theta^2 + \ell\cos\theta\,\ddot\theta$

$$\begin{cases} M\ddot{x} = T\,\mathrm{sen}\theta + f(t) \\ m\frac{d^2}{dt^2}(x + \ell\,\mathrm{sen}\theta) = -T\,\mathrm{sen}\theta \Rightarrow m\ddot{x} + m\ell\cos\theta\,\ddot\theta - m\ell\,\mathrm{sen}\theta\,\dot\theta^2 = -T\,\mathrm{sen}\theta \quad (\cos\theta) \\ m\frac{d^2}{dt^2}(\ell\cos\theta) - mg = -T\cos\theta \Rightarrow -m\ell\,\mathrm{sen}\theta\,\ddot\theta - m\ell\cos\theta\,\dot\theta^2 - mg = -T\cos\theta \quad (-\mathrm{sen}\theta) \end{cases}$$

$$\begin{cases} m\ddot{x}\cos\theta + m\ell\ddot\theta + mg\,\mathrm{sen}\theta = 0 \\ M\ddot{x} + m\ddot{x} + m\ell\cos\theta\,\ddot\theta - m\ell\,\mathrm{sen}\theta\,\dot\theta^2 = f(t) \Rightarrow (M+m)\ddot{x} + m\ell(\cos\theta\,\ddot\theta - \mathrm{sen}\theta\,\dot\theta^2) \end{cases}$$

b) PONTO DE EQUILÍBRIO $\dot\theta = \ddot\theta = 0$ e $\dot{x} = \ddot{x} = 0$

$\mathrm{sen}\theta = 0 \Rightarrow \theta = 0$

ENTÃO $\mathrm{sen}\theta \approx \theta$ e $\cos\theta \approx 1$ $\quad \dot\theta^2 \approx 0$

$$\begin{cases} \ddot{x} + \ell\ddot\theta + g\theta = 0 \\ (M+m)\ddot{x} + m\ell\ddot\theta = f(t) \Rightarrow -(M+m)(\ell\ddot\theta + g\theta) + m\ell\ddot\theta = f(t) \end{cases}$$

c) P/ $f(t) = 0 \Rightarrow (m\ell - (M+m)\ell)\ddot\theta - (M+m)g\theta = 0$

$\Rightarrow (m\ell - (M+m)\ell)\lambda^2 = (M+m)g$

$$\lambda = \sqrt{\frac{(M+m)g}{m\ell - (M+m)\ell}} = \sqrt{\frac{(M+m)g}{-ML}} = \pm j\underbrace{\sqrt{\frac{(M+m)g}{ML}}}_{\omega_N}$$

e) $(M+m)\ddot{x} - m(\ddot{x} - g\theta) = f(t)$

P/ θ pequeno

$M\ddot{x} = f(t) \Rightarrow Ms^2\hat{X} = \hat{f}(s) \Rightarrow \hat{X} = \frac{\hat{f}(s)}{Ms^2}$ $\quad H(s) = \frac{1}{Ms^2}$

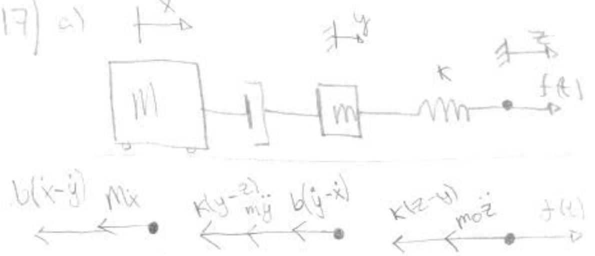
d) $(-M\ell)s^2\hat\theta - (M+m)g\hat\theta = \hat{f}(s) \Rightarrow \hat\theta = \frac{\hat{f}(s)}{-M\ell s^2 - (M+m)g} \Rightarrow H(s) = -\frac{1}{M\ell s^2 + (M+m)g}$

f) $\hat\theta = \frac{1/s}{s(-M\ell s^2 - (M+m)g)}$ [illegible] $\theta_{reg} = \lim_{s\to 0} s\hat\theta = \frac{1}{(-M-m)g}$

16) h) $C(s) = K_p + K_d s$

$$X = \frac{CG}{1+CG} r(\,) \Rightarrow X = \frac{(K_p + K_d s)\cdot \frac{1}{Ms^2}}{1 + \frac{(K_p+K_d s)}{Ms^2}} = \frac{K_p + K_d s}{\cancel{Ms^2}} \cdot \frac{\cancel{Ms^2}}{(Ms^2 + K_p + K_d s)} = \frac{K_p + K_d s}{Ms^2 + K_d s + K_p}$$

17) a)



$$\hat{v} = \dot{x} \Rightarrow$$

3) $M\ddot{x} + b(\dot{x}-\dot{y}) = 0 \Rightarrow Ms v + bv = bs\hat{y} \rightsquigarrow \hat{y} = \frac{Msv + bv}{bs}$ (1)

4) $m\ddot{y} + b(\dot{y}-\dot{x}) + k(y-z) = 0 \Rightarrow ms^2\hat{y} + b(s\hat{y} - \hat{v}) = \hat{f}(s)$ (2)

5) $\cancel{m\ddot{z}} \to 0 \quad + k(z-y) = f(t) \Rightarrow k(y-z) = -f(t) \Rightarrow$

b) (1) em (2) $\quad ms^2\left(\frac{Msv + bv}{b\cancel{s}}\right) + bs\left(\frac{Msv + bv}{bs}\right) - bv = \hat{f}(s)$

$$\Rightarrow ms\frac{(Msv + bv)}{b} + (Msv + \cancel{bv}) \cancel{- bv} = \hat{f}(s)$$

$$\Rightarrow \frac{ms}{b}(Ms + b)v + Msv = \hat{f}(s) \Rightarrow \left(\frac{Mms^2}{b} + Ms + ms\right)v = \hat{f}(s)$$

$$H(s) = \frac{1}{\frac{Mms^2}{b} + (M+m)s}$$

c) $m\ddot{y} + b\dot{y} + k(y-z) = b\dot{x} \Rightarrow \dot{x} = \frac{m}{b}\dddot{y} + \frac{b}{b}\dot{y} + \frac{k}{b}(\dot{y}-\dot{z})$ (6)

(6) em (3)

$$M\frac{m}{b}\dddot{y} + M\ddot{y} + M\frac{k}{b}(\dot{y}-\dot{z}) + m\ddot{y} + \cancel{b\dot{y}} + k(y-z) - \cancel{b\dot{y}} = 0$$

$$\frac{Mm}{b}\dddot{y} + (M+m)\ddot{y} + \frac{Mk}{b}(\dot{y}-\dot{z}) + k(y-z) = 0 \quad (8)$$

5 $\Rightarrow y = z - \frac{f(t)}{k}$ (7) $\Rightarrow$ (7) em (8) $\Rightarrow \frac{Mm}{b}\left(\dddot{z} - \frac{\dddot{f}(t)}{k}\right) + (M+m)\left(\ddot{z} - \frac{\ddot{f}(t)}{k}\right) + \frac{Mk}{b}\left(\cancel{\dot{z}} - \frac{\dot{f}(t)}{k} - \cancel{\dot{z}}\right)$

$$+ k\left(\cancel{z} - \frac{f(t)}{k} - \cancel{z}\right) = 0$$

$$\Rightarrow \frac{Mm}{b}\dddot{z} + (M+m)(\ddot{z}) = \frac{Mm}{bk}\dddot{f}(t) + \frac{(M+m)}{k}\ddot{f}(t) + \frac{M}{b}\dot{f}(t) + f(t)$$

em Laplace

$$\left(\frac{Mm}{b}s^3 + (M+m)s^2\right)\hat{z} = \left(\frac{Mm}{bk}s^3 + \frac{(M+m)}{k}s^2 + \frac{M}{b}s + 1\right)\hat{f}(s) \Rightarrow \hat{z} = \underbrace{\frac{\left(\frac{Mm}{bk}s^3 + \frac{(M+m)}{k}s^2 + \frac{M}{b}s + 1\right)}{\left(\frac{Mm}{b}s^3 + (M+m)s^2\right)}}_{H(s)}\hat{f}(s)$$

$$\Rightarrow \hat{z} = \left(\frac{1}{k} + \frac{\frac{Ms}{b} + 1}{\left(\frac{Mm}{b}s^3 + (M+m)s^2\right)}\right)\hat{f}(s) = \left(\frac{1}{k} + \frac{\frac{M}{b}\frac{b}{Mm}s + \frac{b}{Mm}}{s^3 + \frac{(M+m)b}{Mm}s^2}\right)\hat{f}(s)$$

$\dot{v} = Av + Bx$ $\qquad$ $z = Cv + Dx$

17) d)

$z = \beta_0 v_1 + \beta_1 v_2 + \beta_2 v_3 + \beta_m x$

$\dot{v}_1 = v_2$

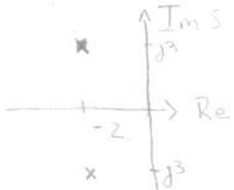
$\dot{v}_2 = v_3$

$$\Rightarrow \dot{v} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{(m+m)b}{m_m} \end{bmatrix} v + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} f(t); \quad z = \begin{bmatrix} \frac{b}{m_m} & \frac{1}{m} & 0 \end{bmatrix} v + \left[\frac{1}{k}\right] f(t)$$

$D(p)y = N(p)x \Rightarrow D(p)y = (\beta_m D(p) + \bar{N}(p))x$

$$z = \left(\frac{1}{K} + \frac{\frac{1}{m}p + \frac{b}{m_m}}{p^3 + \frac{b(m+m)}{m_m}p^2} \right) f(t)$$

23) a) $H(s) = \dfrac{13}{s^2+4s+13}$ $\qquad$ Polos em $\dfrac{-4 \pm \sqrt{16-4.13}}{2} = -2 \pm \dfrac{\sqrt{-36}}{2} = -2 \pm j3$



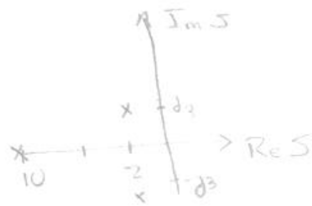
$\hookrightarrow 2\xi\sqrt{13} = 4 \Rightarrow$

$t_e = \dfrac{4}{2} = 2s$ $\qquad$ $-\ln(0{,}02)$

Sim:

$\hookrightarrow$ p/ $\varepsilon = 0{,}02 \rightarrow t_\varepsilon = 1{,}62$

b) $\dfrac{10 \cdot 13}{(s+10)(s^2+4s+13)}$ $\qquad$ Polos em $s = -10, \; -2 \pm j3$



Aproximação por polos dominantes $-2 \pm j3$

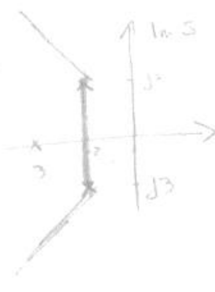
$\dfrac{13}{s^2+4s+13}$ $\qquad$ $t_e = 2s$

Sim $\rightarrow t_e = 1{,}73s$

c) $\dfrac{3 \cdot 13}{(s+3)(s^2+4s+13)}$ $\qquad$ Polos em $-3, -2 \pm j3$

$2\xi\sqrt{13} = 4$

$\xi = \dfrac{2}{\sqrt{13}}$



$\Rightarrow$ Por polos dominantes: $-2 \pm j3$

$t_e = \dfrac{4}{2} = \dfrac{4}{2} = 2s$

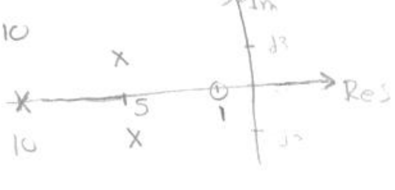
sim $\rightarrow t_e = 1{,}83s$

d) $\dfrac{10(s+1) \cdot 34}{(s+10)(s^2+10s+34)}$ $\qquad$ Polos em:

$\circ\; -5 \pm \dfrac{1}{2}\sqrt{100-4.34} = -5 \pm j3$

$\circ\; -10$

$s = -10$
$p = 34$



Por polos dominantes

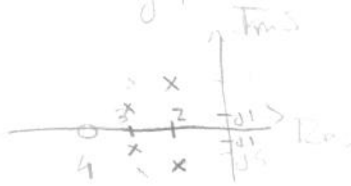
$H_{APROX}(s) = \dfrac{34}{s^2+10s+34}$ $\qquad$ $-5 + j3$, $-5 - j3$

$t_e = \dfrac{4}{5} = 0{,}8s$

Sim: $t_e = 1{,}1s$

23) e) $(s^2+4s+20) \; \dfrac{-4 \pm \sqrt{16-80}}{2} = -2 \pm \dfrac{j}{2}\sqrt{64} = -2 \pm j4$

$(s^2+6s+10) \; -3 \pm \dfrac{1}{2}\sqrt{36-40} = -3 \pm j1$



APROXIMAÇÃO POR POLOS DOMINANTES

$H_{APRO}(s) = \dfrac{20}{s^2+4s+20}$ $\quad te = \dfrac{4}{2} = 2s \quad$ sim: 1,49s - te

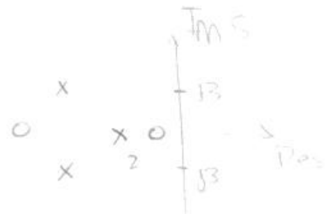
f) $H(s) = \dfrac{25 \cdot 2\,(s+1)(s+5)}{(s+2)(s^2+8s+25)}$; POLOS EM

$-2 \quad e \quad \dfrac{-4 \pm \sqrt{64-100}}{2} = -4 \pm j3$

ZEROS EM -1 E -5

POLO DOMINANTE $s=-2$

$te = \dfrac{4}{2} = 2s \quad$ POR SIM: $te = 2s$



g) $\dfrac{(163{,}2/25)(s^2+4s+20)}{(s+3)(s+6)(s^2+12s+136)}$ $\Rightarrow$ POLO DOMINANTE $s=-3$

$te = \dfrac{4}{3} = 1{,}33s$

te -> SIMULAÇÃO -> te = 1,43s

h) $H(s) = \dfrac{-13/4\,(s-1)(s+4)}{(s+1)(s^2+6s+13)}$ POLOS EM (-1) -> POLO DOMINANTE

$\dfrac{-3 \pm \sqrt{36-52}}{2} \Rightarrow -3 \pm j2$

$te = \dfrac{4}{1} = 4s$

POR SIM: te = 4,41s

28) iii) $C(s) = \dfrac{s+4}{s+2}$ (SOZ)

$$C_D(z) = \mathcal{Z}\left\{ \mathcal{L}^{-1}\left\{ \frac{1-e^{-Ts}}{s} C(s) \right\} \Big|_{T=kT} \right\}$$

$$= \mathcal{Z}\left\{ \mathcal{L}^{-1}\left\{ \frac{C(s)}{s} \right\}\Big|_{t=kT} - \mathcal{L}^{-1}\left\{ \frac{C(s)}{s} \right\}\Big|_{t=(k-1)T} \right\}$$

$$\Rightarrow C_D(z) = (1-z^{-1})\,\mathcal{Z}\left\{ \mathcal{L}^{-1}\left\{ \frac{C(s)}{s} \right\} \Big|_{t=kT} \right\}$$

$$\frac{C(s)}{s} = \frac{s+4}{(s)(s+2)} = \frac{2}{s} + \frac{-1}{s+2} \Rightarrow \mathcal{L}^{-1}\left\{ \frac{C(s)}{s} \right\} = 2 - e^{-2t}$$

$$C_D(z) = (1 - z^{-1})\,\mathcal{Z}\left\{ 2 - (e^{-2T})^k \right. = \frac{z-1}{z}\left[2 \cdot \frac{z}{z-1} - \frac{z}{z-e^{-2T}} \right]$$