(1)

$$\iint_S f(x,y,z)\,dS = \iint_A f(r(u,v))\,|r_u \times r_v|\,dA \quad (0,5)$$

- $f(r(u,v)) = f(u\cos v,\, u\,\mathrm{sen}\, v,\, v) =$
  $= \sqrt{u^2\cos^2 v + u^2\,\mathrm{sen}^2 v} = u$

- $|r_u \times r_v| = \sqrt{1+u^2}$. $\quad (1,5)$

$$\iint_S f\,dS = \iint_A u\sqrt{1+u^2}\,dA = \int_0^{\pi}\int_0^1 u\sqrt{1+u^2}\,du\,dv$$

$$= \pi\int_0^1 u\sqrt{1+u^2}\,du \qquad (x = 1+u^2)$$

$$= \pi\int_1^2 \frac{\sqrt{x}}{2}\,dx = \frac{\pi}{2}\int_1^2 \sqrt{x}\,dx$$

$$= \frac{\pi}{2}\left(\frac{2}{3}x^{\frac{3}{2}}\Big|_1^2\right) = \frac{\pi}{3}\left(2\sqrt{2}-1\right).$$

$(2,5)$

② Gráfico: $\cdot\ r(x,y) = (x, y, y^2 - x^2)$; $1 \leq x^2 + y^2 \leq 4$.

$$A(S) = \iint_S dS = \iint_D |r_x \times r_y|\, dA \quad 0{,}5$$

$$\cdot\ |r_x \times r_y| = \sqrt{(-2x)^2 + (+2y)^2 + 1} = \sqrt{4(x^2+y^2)+1}.$$

$$A(S) = \iint_D \sqrt{4(x^2+y^2)+1}\, dA \quad + 0{,}5 \quad \left(\text{até aqui } 1{,}0\right)$$

$$= \int_0^{2\pi}\int_1^2 \sqrt{4r^2+1}\, r\, dr\, d\theta$$ mudança p/ coord. polares $+ 0{,}5$ $(1{,}5)$

$$= 2\pi \int_1^2 \sqrt{4r^2+1}\, r\, dr$$ $\left(u = 4r^2+1,\ du = 8r\,dr \right)$

$$= 2\pi \int_5^{17} \frac{\sqrt{u}}{8}\, du = \frac{\pi}{4}\int_5^{17} \sqrt{u}\, du$$

$$= \frac{\pi}{4}\left(\frac{2}{3} u^{\frac{3}{2}} \Big|_5^{17}\right) = \frac{\pi}{6}\left(17\sqrt{17} - 5\sqrt{5}\right).$$

2,5 a conta correta

(Se errou os números tira 0,5)

③ Seja S a parte da esfera de raio 2 ~~unitária~~ no 1º octante. Então

$$\underbrace{W = \oint_C F \cdot d\vec{r}}_{0,5} = \iint_S \text{rot}\, F \cdot d\vec{S} \quad (\text{Teorema de Stokes}).$$

(+ 0,5 aplicar Stokes)

$$= \iint_S \text{rot}\, F \cdot N \, dS = \iint_D \text{rot}\, F(r(\phi,\theta)) \cdot (r_\phi \times r_\theta)\, dA.$$

Temos: • $\text{rot}\, F = (2y, 2z, 2x)$.

• $S: \ r(\phi,\theta) = (2 \,\text{sen}\,\phi \cos\theta, 2\,\text{sen}\,\phi\,\text{sen}\,\theta, 2\cos\phi)$

(+ 0,5 pela parametrização) $0 \le \phi \le \frac{\pi}{2}$, $0 \le \theta \le \frac{\pi}{2}$.

• $r_\phi \times r_\theta = (4\,\text{sen}^2\phi\cos\theta, 4\,\text{sen}^2\phi\,\text{sen}\,\theta, 4\,\text{sen}\,\phi\cos\phi)$

• $\text{rot}\, F(r(\phi,\theta)) = (4\,\text{sen}\,\phi\,\text{sen}\,\theta, 4\cos\phi, 4\,\text{sen}\,\phi\cos\theta)$

• $\text{rot}\, F(r(\phi,\theta)) \cdot r_\phi \times r_\theta = 16(\text{sen}^3\phi\,\text{sen}\,\theta\cos\theta + \cos\phi\,\text{sen}^2\phi\,\text{sen}\,\theta$

(+0,5 se calculou o integrando corretamente) $+ \text{sen}^2\phi\cos\phi\cos\theta)$

• $\iint_S \text{rot}\, F \cdot d\vec{S} = \iint_D \text{rot}\, F(r(\phi,\theta)) \cdot r_u \times r$

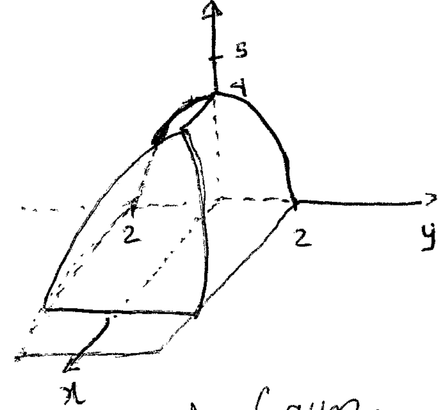
$$= 16 \int_0^{\pi/2}\int_0^{\pi/2} \text{sen}^3\phi\,\text{sen}\,\theta\cos\theta \, d\phi\, d\theta + 16 \int_0^{\pi/2}\int_0^{\pi/2} \cos\phi\,\text{sen}^2\phi\,\text{sen}\,\theta$$

$$+ 16 \int_0^{\pi/2}\int_0^{\pi/2} \text{sen}^2\phi\cos\phi\cos\theta \, d\phi\, d\theta$$

$$= 16.$$

+0,5 contas finais

(4) turma W : $z = 4 - y^2 - x^2$



• Seja E o sólido limitado por S. Orientando S positivamente, temos, pelo Teorema de Gauss,

$$\iint_S F\cdot dS = \iiint_E \operatorname{div} F \, dV.$$

0,5 ponto pelo teo de Gauss.

Note que:

• $E = \{(x,y,z);\ -2 \le y \le 2,\ 0 \le z \le 4-y^2$ e $0 \le x \le 5-z\}$.

• $\operatorname{div} F = 3x^2 + x^2 = 4x^2$. (Calc. do div e do sólido +1,0)

Assim:

$$\iint_S F\cdot dS = \iiint_E \operatorname{div} F \, dV = \iiint_E 4x^2 \, dV$$

$$= \int_{-2}^{2}\int_0^{4-y^2}\int_0^{5-z} 4x^2 \, dx \, dz \, dy$$

$$= \int_{-2}^{2}\int_0^{4-y^2} \frac{4}{3}(5-z)^3 \, dz \, dy$$

$$= \int_{-2}^{2}\left(-\frac{1}{3}(5-z)^4 \Big|_0^{4-y^2}\right) dy$$

$$= \int_{-2}^{2}\left(\frac{5^4}{3} - \frac{1}{3}(1+y^2)^4\right) dy = 8\,\frac{5^4}{3} - \frac{1}{3}\int_{-2}^{2}(1+y^2)^4 \, dy$$

até aqui 2,3 pts.

cálculo da integral +1,0

$$= 8\,\frac{5^4}{3} - \frac{1}{3}\int_{-2}^{2}(1 + 4y^2 + 6y^4 + 4y^6 + y^8) \, dy$$

$$= 8\,\frac{5^4}{3} - \frac{2}{3}\int_{0}^{2}(1 + 4y^2 + 6y^4 + 4y^6 + y^8) \, dy$$

$$= \left(\frac{8}{3}5^4 - \frac{2}{3}\left(1 + \frac{4\,2^2}{3} + \frac{6\,2^4}{5} + \frac{4\,2^6}{7} + \frac{1\,2^8}{9}\right)\right).$$