EE400A - Métodos da Engenharia Elétrica 2º Sem. 2007

Prova P1 duração: 2 horas 29/08/2007

1(2,0) Dado o campo escalar $f(x,y,z) = x+y$

a- Calcule e esboce as superfícies de nível

b - Calcule $\frac{df}{ds}$ em (1,1,0) nas direções: i) $\vec{n} = \vec{a}_x$; ii) $\vec{n} = \frac{1}{\sqrt{2}}\vec{a}_x + \frac{1}{\sqrt{2}}\vec{a}_y$; iii) $\vec{n} = \frac{1}{\sqrt{2}}\vec{a}_x - \frac{1}{\sqrt{2}}\vec{a}_y$ e justifique os resultados obtidos baseado nas superfícies de nível

2(1,5) Considere um fluido incompressível girando num vaso cilíndrico com velocidade $\vec{v}(r,\phi,z) = \vec{w}\times\vec{r}$, $\vec{w} = w_0\,\vec{a}_z$. Calcule $\nabla\cdot\vec{v}$ e justifique o resultado obtido.

3(1,5) Dado $\vec{v}(r,\theta,\phi) = \frac{K}{r^2}\vec{a}_r + r.\text{sen}\theta.3\phi\,\vec{a}_\phi$, $r>0$, calcule $\oint_S \vec{v}\cdot\vec{dS}$ para S um icosaedro de volume unitário centrado em $r=10$, $\theta=\theta_0$, $\phi=\phi_0$. O resultado depende de θ_0 e ϕ_0? Justifique.

4(1,5) Seja $\vec{v}(x,y,z) = -3y\,\vec{a}_x + 3xz\,\vec{a}_y + yz\,\vec{a}_z$, γ uma curva fechada $x^2+y^2=4$, $z=1$, sentido anti-horário de quem olha de $z>1$. Calcule $\oint_\gamma \vec{v}\cdot\vec{dl}$ usando o Teorema de Stokes.

5(2,0) Deduza a componente $\vec{a}_z$ do rotacional de um campo vetorial em coordenadas cilíndricas.

6(1,5) Dado $\vec{E}(x,y,z) = -\nabla U(x,y,z)$, obtenha o potencial U do campo $\vec{E} = x\,\vec{a}_x + y\,\vec{a}_y - 1.\vec{a}_z$

Formulário

$$\frac{\partial f}{\partial s} = \nabla f \cdot \vec{u}$$

$$\nabla f(x,y,z) = \frac{\partial f}{\partial x}\vec{a}_x + \frac{\partial f}{\partial y}\vec{a}_y + \frac{\partial f}{\partial z}\vec{a}_z$$

$$\nabla f(r,\phi,z) = \frac{\partial f}{\partial r}\vec{a}_r + \frac{1}{r}\frac{\partial f}{\partial \phi}\vec{a}_\phi + \frac{\partial f}{\partial z}\vec{a}_z$$

$$\nabla f(r,\theta,\phi) = \frac{\partial f}{\partial r}\vec{a}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\vec{a}_\theta + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial f}{\partial \phi}\vec{a}_\phi$$

$$\nabla\cdot\vec{v}(x,y,z) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$\nabla\cdot\vec{v}(r,\phi,z) = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\nabla\cdot\vec{v}(r,\theta,\phi) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial}{\partial \theta}(\mathrm{sen}\,\theta\, v_\theta) + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial v_\phi}{\partial \phi}$$

$$\nabla\times\vec{v}(x,y,z) = \left[\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right]\vec{a}_x + \left[\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right]\vec{a}_y + \left[\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right]\vec{a}_z$$

$$\nabla\times\vec{v}(r,\phi,z) = \left[\frac{1}{r}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right]\vec{a}_r + \left[\frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r}\right]\vec{a}_\phi + \frac{1}{r}\left[\frac{\partial (r v_\phi)}{\partial r} - \frac{\partial v_r}{\partial \phi}\right]\vec{a}_z$$

$$\nabla\times\vec{v}(r,\theta,\phi) = \frac{1}{r\,\mathrm{sen}\,\theta}\left[\frac{\partial (v_\phi\,\mathrm{sen}\,\theta)}{\partial \theta} - \frac{\partial v_\theta}{\partial \phi}\right]\vec{a}_r + \frac{1}{r}\left[\frac{1}{\mathrm{sen}\,\theta}\frac{\partial v_r}{\partial \phi} - \frac{\partial (r v_\phi)}{\partial r}\right]\vec{a}_\theta +$$

$$+ \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}\right]\vec{a}_\phi$$

T. de Gauss $\displaystyle\oint_S \vec{v}\cdot d\vec{S} = \int_V (\nabla\cdot\vec{v})\, dV$

T. de Stokes $\displaystyle\oint_\gamma \vec{v}\cdot d\vec{l} = \int_S (\nabla\times\vec{v})\cdot d\vec{S}$

T. de Green $\displaystyle\int_S \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dx\,dy = \oint_\gamma f\,dx + g\,dy$