1. 1,0+1,0
2. 1,0+1,0
3. 1,0+0,5+0
4. 0,5+1,0
5. 0,5

7,5

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1) a) $\ddot{y} + 5\dot{y} + 6y = e^{-3t}$ (1)

$\Delta(\lambda) = \lambda^2 + 5\lambda + 6$ $\quad S = -5,\ P = 6 \Rightarrow \Delta(\lambda) = (\lambda-2)(\lambda-3)$

-> PERCEBE-SE QUE e^{-3t} JÁ É MODO PRÓPRIO, ENTÃO,

$y_h = a_1 e^{-2t} + a_2 e^{-3t}, \quad y_f = b t e^{-3t}$

=> SUBSTITUINDO y_f NA EQUAÇÃO (1):

$f(be^{-3t} - 3bte^{-3t}) + 5be^{-3t} - 15bte^{-3t} + 6bte^{-3t} = e^{-3t}$

$\Rightarrow -3be^{-3t} - 3b(e^{-3t} - \cancel{3te^{-3t}}) + 5be^{-3t} - 15bte^{-3t} + 6bte^{-3t} = e^{-3t}$

$-1b\cancel{e^{-3t}} + \cancel{9bte^{-3t}} + \cancel{6bte^{-3t}} - \cancel{15bte^{-3t}} = \cancel{e^{-3t}}$

$b = -1$

$y = y_h + y_f = a_1 e^{-2t} + a_2 e^{-3t} - te^{-3t}$

DAS CONDIÇÕES INICIAIS: $\quad \Rightarrow \dot{y}(t) = -2a_1e^{-2t} - 3a_2e^{-3t} - e^{-3t} + 3te^{-3t}$

$y(0) = 1 \Rightarrow a_1 + a_2 = 1$
$\dot{y}(0) = -1 \Rightarrow -2a_1 - 3a_2 - 1 = -1$
$\left.\begin{matrix} a_1 + a_2 = 1 \\ +2a_1 + 3a_2 = 0 \end{matrix}\right\}$ $\quad a_1 = 1 - a_2$

$\Rightarrow 2(1 - a_2) + 3a_2 = 0 \Rightarrow 3a_2 - 2a_2 = -2 \Rightarrow a_2 = -2$

$a_1 = 3$

ENTÃO,

$\boxed{y = 3e^{-2t} - 2e^{-3t} - te^{-3t}} \quad t \geq 0$

b) $I = \int_0^\infty t\, y(t)\, dt$, PELA DEFINIÇÃO DA TRANSFORMADA DE LAPLACE,

TEMOS QUE $\mathcal{L}\{f(t)\} = \int_0^\infty f(t) e^{-st} dt$ E $\mathcal{L}\{t f(t)\} = \int_0^\infty \underbrace{t f(t) e^{-st}}_{-\frac{d}{ds} f(t) e^{-st}} dt$

$\Rightarrow \mathcal{L}\{t f(t)\} = -\frac{d}{ds}\int_0^\infty f(t) e^{-st} dt = -\frac{d}{ds}\mathcal{L}\{f(t)\}$, ISTO É,

$\mathcal{L}\{t f(t)\} = -\frac{d}{ds}\mathcal{L}\{f(t)\}$. COMPARANDO-SE I COM $\mathcal{L}\{t f(t)\}$

$I = \int_0^\infty t\, y(t) e^{-0t} dt = \mathcal{L}\{t y(t)\}\Big|_{s=0}$, ENCONTREMOS AGORA $\mathcal{L}\{t f(t)\}$

• $\mathcal{L}\{y(t)\} = \frac{3}{s+2} - \frac{2}{(s+3)} - \frac{1}{(s+3)^2}$, $\quad \mathcal{L}\{t y(t)\} = -\frac{d}{ds}\mathcal{L}\{y(t)\}$

$Re\{s\} > -2$

1) b) (continuação)

$$\mathcal{L}\{t\,f(t)\} = \frac{3}{(s+2)^2} - \frac{2}{(s+3)^2} - \frac{2}{(s+3)^3}$$

E, pela discussão anterior, sabemos que

$$I = \mathcal{L}\{t\,f(t)\}\Big|_{s=0} = \frac{3}{4} - \frac{2}{9} - \frac{2}{27} = \frac{3.27}{4.27} - \frac{2.3.4}{9.3.4} - \frac{2.4}{4.27}$$

$$I = \frac{81 - 24 - 8}{108} = \frac{49}{108}$$

2) $y[k+2] + y(k+1) - 6y[k] = -4(1)^k$, $y[0] = 1$

Por transformada Z: $y[1] = 0$

$Z\{y[k+1]\} = z\hat{y}[k] - zy[0]$

$$z^2\hat{y} - z^2y[0] - \cancel{zy[1]} + z\hat{y} - zy[0] - 6\hat{y} = -4\frac{z}{z-1}$$

$$(z^2 + z - 6)\hat{y} = -4\frac{z}{z-1} + z^2 + z$$

$$\Rightarrow (z+3)(z-2)\hat{y} = \frac{-4z}{z-1} + z^2 + z$$

$$\frac{\hat{y}}{z} = \underbrace{-4\frac{z}{(z-1)(z+3)(z-2)}}_{\hat{y}_1(z)} + \underbrace{\frac{z}{(z+3)(z-2)}}_{\hat{y}_2(z)} + \underbrace{\frac{1}{(z+3)(z-2)}}_{\hat{y}_3(z)}$$

$$\hat{y}_1(z) = \frac{-4}{(z-1)(z+3)(z-2)} = \frac{A(z+3)(z-2) + B(z-1)(z-2) + C(z-1)(z+3)}{(z-1)(z+3)(z-2)}$$

$z = 1 \Rightarrow -\cancel{4} = A(\cancel{4})(-1) \Rightarrow A = 1 = 5/5$

$z = -3 \Rightarrow \cancel{-4} = B(\cancel{-4})(-5) \Rightarrow B = -1/5$

$z = 2 \Rightarrow -4 = C.5 \Rightarrow C = -4/5$

$$\hat{y}_2(z) = \frac{z}{(z+3)(z-2)} = \frac{A(z-2) + B(z+3)}{(z+3)(z-2)}$$

$z = 2 \Rightarrow 2 = B.5 \Rightarrow B = 2/5$

$z = -3 \Rightarrow -3 = A(-5) \Rightarrow A = 3/5$

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2) a) (continuação)

$$y_3(z) = \frac{A(z-2)+B(z+3)}{(z+3)(z-2)} = \frac{1}{(z+3)(z-2)}$$

$z=-3 \Rightarrow A(-5)=1 \Rightarrow A=-1/5$

$z=2 \Rightarrow B(5)=1 \Rightarrow B=1/5$

Então

$$\frac{\hat{y}}{z} = \frac{5}{5}\frac{1}{(z-1)} - \frac{1}{5}\frac{1}{(z+3)} - \frac{4}{5}\frac{1}{(z-2)} + \frac{3}{5}\frac{1}{(z+3)} + \frac{2}{5}\frac{1}{(z-2)} - \frac{1}{5}\frac{1}{(z+3)} + \frac{1}{5}\frac{1}{(z-2)}$$

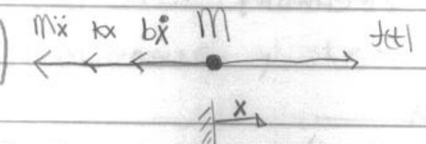
$$\hat{y} = \frac{z}{z-1} + \frac{1}{5}\frac{z}{(z+3)} - \frac{1}{5}\frac{z}{z-2} = \frac{1}{1-z^{-1}} + \frac{1}{5}\frac{1}{(1-(-3)z^{-1})} - \frac{1}{5}\frac{1}{1-2z^{-1}} \qquad |z|<|2|$$

Finalmente, $y[k] = Z^{-1}\{\hat{y}\} = 1 + \frac{1}{5}(-3)^k - \frac{1}{5}(2)^k$, $k \in \mathbb{N}$

b) Da definição de transformada Z, temos que

$Z\{y[k]\} = \sum_{k=0}^{\infty} y[k]\, z^{-k}$, por inspeção, observa-se que:

$$S = \sum_{k=0}^{\infty} y[k] \underbrace{(1/4)^k}_{4^{-k}} = Z\{y[k]\}\Big|_{z=4} = \frac{4}{3} + \frac{4}{5\cdot(7)} - \frac{4}{5(2)}$$

3) 

$\Rightarrow \quad m\ddot{x} + b\dot{x} + kx = f(t)$

a) $\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = \frac{f(t)}{m}$

$$\Rightarrow \left(s^2 + \frac{b}{m}s + \frac{k}{m}\right)\hat{x} = \frac{\hat{f}(s)}{m} \Rightarrow \frac{\hat{x}}{\hat{f}(s)} = \frac{1}{m\left(s^2+\frac{b}{m}s+\frac{k}{m}\right)} = \frac{\frac{k}{m}}{m\frac{k}{m}\left(s^2+\frac{b}{m}s+\frac{k}{m}\right)}$$

Pela forma padrão $K\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$, $\omega_n^2 = \frac{k}{m} \Rightarrow \omega_n = \sqrt{\frac{k}{m}}$

e $2\xi\underbrace{\sqrt{\frac{k}{m}}}_{\omega_n} = \frac{b}{m} \Rightarrow \xi = \frac{1}{2}\frac{b}{m}\sqrt{\frac{m}{k}} = \frac{1}{2}\frac{b}{\sqrt{km}}$

3) b) P/ $M = 1$ e $\zeta = 0{,}25$, $\omega_N = 10$

-> Se um pombo de massa m pousa sobre M, (segundo caso)

$$\frac{\hat{X}_2}{\hat{f}(s)} = \frac{1}{s^2 + \frac{b}{M+m}s + \frac{k}{M+m}} = \frac{\frac{k}{M+m}}{\frac{k}{M+m}\left(s^2 + \frac{b}{M+m}s + \frac{k}{M+m}\right)} \quad \text{e} \quad \omega_{N_2} = \sqrt{\frac{k}{M+m}}$$

$$\text{e} \quad \zeta = \frac{\sqrt{M+m}}{2\sqrt{k}} \cdot \frac{b}{(M+m)} = \frac{1}{2} \frac{b}{\sqrt{k(M+m)}}, \quad \text{onde } \omega_{N2} = 5 \text{ (pelo diagrama)}$$

Primeiro caso:

$$\omega_N^2 = 100 = \frac{k}{M} \Rightarrow \boxed{k = 100}$$

$$\zeta = 0{,}25 = \frac{1}{2} \frac{b}{\sqrt{k \cdot M}}$$

$$\Rightarrow 0{,}5\sqrt{100} = b \Rightarrow b = 0{,}5 \cdot 10 = 5$$

Segundo caso

$$\omega_{N_2}^2 = \frac{k}{M+m} = 25 \Rightarrow$$

$$25(1+m) = 100 \Rightarrow \boxed{m = 100 - 25 = 75\,\text{kg}}$$

4) a) $C(s) = \mu \frac{s+\lambda}{s+\sigma}$, $G(s) = \frac{1}{s-1}$

$$\hat{y} = GC(\hat{r} - \hat{y}) \Rightarrow \hat{y} = CG\hat{r} - CG(\hat{y}) \Rightarrow (1 + CG)\hat{y} = CG\hat{r}$$

$$\Rightarrow \hat{y} = \frac{CG}{1+CG}\hat{r} \quad \text{(em malha fechada)}$$

$$\hat{y} = \frac{\frac{s+\lambda}{s+\sigma} \cdot \frac{1}{s-1}}{\frac{(s+\sigma)(s-1) + (s+\lambda)}{(s+\sigma)(s-1)}} = \frac{s+\lambda}{(s+\sigma)(s-1) + (s+\lambda)}$$

-> queremos que os polos em malha fechada sejam -1 e -2, então,

$$(s+\sigma)(s-1) + (s+\lambda) = (s+1)(s+2)$$

$$\Rightarrow s^2 + (\sigma - 1)s - \sigma + s + \lambda = s^2 + 3s + 2$$

$$\Rightarrow s^2 + \sigma s + (\lambda - \sigma) = s^2 + 3s + 2$$ e, por inspeção,

$\sigma = 3 \Rightarrow (\lambda - 3) = 2 \Rightarrow \lambda = 5$

O controlador é do tipo atraso, pois desloca o polo

PARA TRÁS NO EIXO REAL

4) b) $C(s) = \frac{s+5}{s+3}$

$$C_D(z) = (1 - z^{-1})\, \mathcal{Z}\left\{ \mathcal{L}^{-1}\left\{ \frac{C(s)}{s} \right\}\Big|_{t=kT} \right\}$$

$$\frac{s+5}{s(s+3)} = \frac{A(s+3) + Bs}{s \cdot (s+3)} \Rightarrow A = \frac{5}{3} \quad e \quad B = -\frac{2}{3}$$

$$\mathcal{L}^{-1}\left\{ \frac{5}{3}\frac{1}{s} - \frac{2}{3}\frac{1}{(s+3)} \right\}\Big|_{t=kT} = \frac{5}{3} - \frac{2}{3} e^{-3Tk} = \frac{5}{3} - \frac{2}{3} (e^{-3T})^{k}$$

$$C_D(z) = (1 - z^{-1}) \cdot \left(\frac{5}{3}\frac{z}{z-1} - \frac{2}{3}\frac{z}{z-e^{-3T}} \right)$$

$$C_D(z) = \frac{z-1}{z}\left(\frac{5}{3}\frac{z}{(z-1)} - \frac{2}{3}\frac{z}{(z-e^{-3T})} \right) = \frac{5}{3} - \frac{2}{3}\frac{(z-1)}{(z-e^{-3T})}$$

5) $$F(s) = \frac{\dfrac{10 \cdot 13}{s(s^2+14s+53)}}{\dfrac{s(s^2+14s+53) + 10 \cdot 13}{s(s^2+14s+53)}} = \frac{10 \cdot 13}{s^3+14s^2+53s+130}$$

PARA UMA ENTRADA $r(t) = 1$, CONSIDERANDO $s = -10$ PÓLO DOMINANTE, $t_e = -\frac{\ln(\varepsilon)}{\sigma} = \frac{4}{10} \cong 0,4\,s$