Gabarito

1.a) $\frac{\partial f}{\partial x}(x,y) = 2k(x-y)$, $\frac{\partial f}{\partial y}(x,y) = -2k(x-y)+y^3-y$

$\frac{\partial f}{\partial x}(x,y) = 0 \Rightarrow 2k(x-y)=0$

$\frac{\partial f}{\partial y}(x,y) = 0 \Rightarrow -2k(x-y)+y^3-y=0 \longrightarrow 0{,}5$

Já que $k \neq 0$ temos que $x=y$ e portanto

$y^3-y=0 \Rightarrow y(y^2-1)=0 \Rightarrow y=0, 1$ ou -1.

Assim os pontos críticos são $(0,0)$, $(1,1)$ e $(-1,-1)$. $\longrightarrow 0{,}5$

b)

$f_{xx}(x,y) = 2k$, $f_{yy}(x,y) = 2k+3y^2-1$, $f_{xy}(x,y) = -2k$

$D = D(a,b) = f_{xx}(a,b)\, f_{yy}(a,b) - [f_{xy}(a,b)]^2 \longrightarrow 0{,}75$

$D(0,0) = f_{xx}(0,0)\, f_{yy}(0,0) - [f_{xy}(0,0)]^2$

$= 2k(2k-1) - 4k^2 = 4k^2 - 2k - 4k^2 = -2k < 0$

pois $k>0$, então $(0,0)$ é um ponto de sela

$- D(1,1) = f_{xx}(1,1)\, f_{yy}(1,1) - [f_{xy}(1,1)]^2$

$= 2k(2k+2) - 4k^2 = 4k^2 + 4k - 4k^2 = 4k > 0$

$f_{xx}(1,1) = 2k > 0$, pois $k>0$ então

$f(1,1) = -\frac{1}{4}$ é um valor mínimo local

$- D(-1,-1) = f_{xx}(-1,-1)\, f_{yy}(-1,-1) - [f_{xy}(-1,-1)]^2$

$= 2k(2k+2) - 4k^2 = 4k^2 + 4k - 4k^2 = 4k > 0$

$f_{xx}(-1,-1) = 2k > 0$, pois $k>0$ então

$f(-1,-1) = -\frac{1}{4}$ é um valor mínimo local. $\longrightarrow$ 0,75

2 Sejam $f(x,y) = x^2 + (y-1)^2$, $g(x,y) = y - x^2$

$$\begin{cases} \nabla f(x,y) = \lambda \nabla g(x,y) \\ g(x,y) = 0 \end{cases}$$

Assim $(2x, 2(y-1)) = \lambda(-2x, 1)$

$2x = -2\lambda x \Rightarrow x = -\lambda x \Rightarrow x(1+\lambda) = 0$

$2(y-1) = \lambda$

$y = x^2 \longrightarrow$ 1,0

- Se $x = 0$ então $y = 0$
- Se $\lambda = -1$ então $y - 1 = -\frac{1}{2}$

$\Rightarrow y = \frac{1}{2} \Rightarrow x = \pm\frac{1}{\sqrt{2}} \longrightarrow$ 1,0

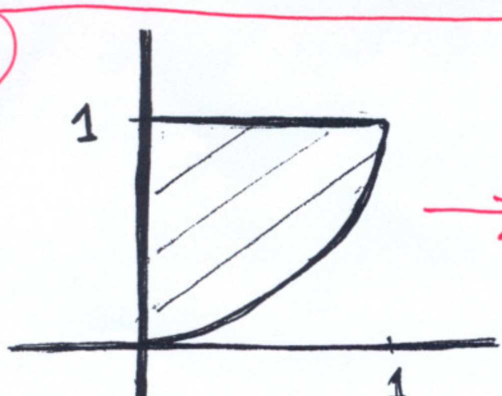
$f(0,0) = 1$

$f\left(\frac{1}{\sqrt{2}}, \frac{1}{2}\right) = f\left(-\frac{1}{\sqrt{2}}, \frac{1}{2}\right) = \frac{3}{4}$

Portanto os pontos mais próximos são

$\left(\frac{1}{\sqrt{2}}, \frac{1}{2}\right)$ e $\left(-\frac{1}{\sqrt{2}}, \frac{1}{2}\right) \longrightarrow$ 0,5

3.a) 3



$\longrightarrow$ 1,0

$$\int_0^1 \left(\int_{x^2}^1 x^3 sen(y^3)\, dy \right) dx =$$

$$\int_0^1 \left(\int_0^{\sqrt{y}} x^3 sen(y^3)\, dx \right) dy \longrightarrow 0,5$$

$$= \int_0^1 sen(y^3) \left(\int_0^{\sqrt{y}} x^3\, dx \right) dy$$

$$= \int_0^1 sen(y^3) \left(\frac{x^4}{4} \Big|_0^{\sqrt{y}} \right) dy \longrightarrow 0,5$$

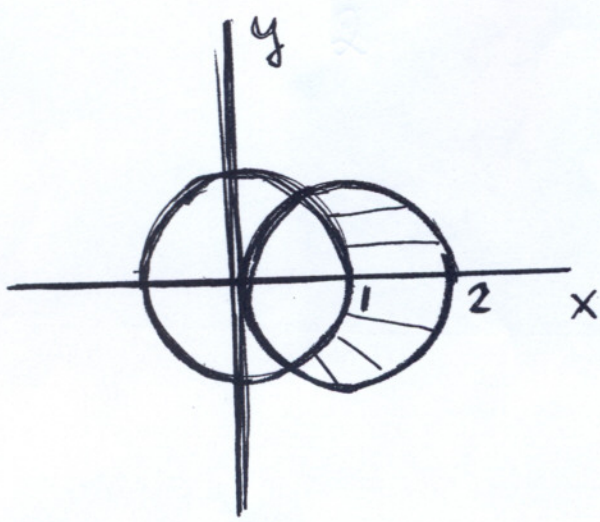
$$= \frac{1}{4} \int_0^1 y^2 sen(y^3)\, dy$$

Seja $u = y^3$
$du = 3y^2 dy$

$$= \frac{1}{12} \int_0^1 sen(u)\, du \quad 0,5$$

$$= \frac{1}{12} \left(-\cos(u) \Big|_0^1 \right) = \frac{1}{12} (1 - \cos(1))$$

4.- 4



$$x^2 - 2x = x^2 - 1 \Rightarrow x = \frac{1}{2}$$

$$\cos\theta = \frac{1}{2} \Rightarrow \theta = \pm \pi/3 \longrightarrow 0{,}5$$

$$r^2\cos^2\theta + r^2 \operatorname{sen}^2\theta = 2r\cos\theta \Rightarrow r^2 = 2r\cos\theta$$

$$\Rightarrow r = 2\cos\theta \longrightarrow 0{,}5$$

$$V = \int_{-\pi/3}^{\pi/3} \left(\int_1^{2\cos\theta} r^3\,dr \right) d\theta \longrightarrow 0{,}5$$

$$= \int_{-\pi/3}^{\pi/3} \left(\frac{r^4}{4} \Big|_1^{2\cos\theta} \right) d\theta$$

$$= \frac{1}{4} \int_{-\pi/3}^{\pi/3} (2^4 \cos^4\theta - 1)\, d\theta \longrightarrow 1{,}0$$

$$= 4 \int_{\pi/3}^{\pi/3} \cos^4\theta\, d\theta - \frac{1}{4} \int_{-\pi/3}^{\pi/3} d\theta$$

$$= \frac{5}{6}\pi + \frac{7}{8}\sqrt{3}$$

Obs: Os valores assinalados nesse gabarito correspondem às partes dos exercícios feitos corretamente. O erro de um dos itens compromete os valores dos demais ítens da questão.