T_4 ( ) F520 ( ) MS550 · Nome: ______________________ RA: __________

Use as definições e propriedades das funções gama e beta para mostrar que

$$\text{(i)} \int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta = \frac{\pi}{\sqrt{2}}, \qquad \text{(ii)} \int_{-1}^{1} \left(\frac{1+x}{1-x}\right)^{1/2} dx = \pi.$$

$$\text{(i)} \int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta = \int_0^{\pi/2} \sin^{1/2}\theta \cos^{-\frac{1}{2}}\theta\, d\theta = \int_0^{\pi/2} \sin^{2(\frac{3}{4})-1}\theta \cos^{2(\frac{1}{4})-1}\theta\, d\theta$$

$$\overset{⑥}{=} \frac{1}{2} B\left(\frac{3}{4}, \frac{1}{4}\right) \overset{⑤}{=} \frac{1}{2} \frac{\Gamma(\frac{3}{4})\Gamma(\frac{1}{4})}{\underbrace{\Gamma(1)}_{1}} = \frac{1}{2}\Gamma\left(\frac{1}{4}\right)\Gamma\left(1-\frac{1}{4}\right)$$

+2,0

$$\overset{③}{=} \frac{\pi}{2\sin\frac{\pi}{4}} = \frac{\pi}{2\frac{\sqrt{2}}{2}} = \frac{\pi}{\sqrt{2}}$$

+3,0

$$\text{(ii)} \int_{-1}^{+1} \left(\frac{1+x}{1-x}\right)^{1/2} dx \overset{x=2t-1}{=} \int_0^1 \left(\frac{1+2t-1}{1-2t+1}\right)^{1/2} 2\,dt = 2\int_0^1 t^{1/2}(1-t)^{-1/2}\,dt$$

$$= 2\int_0^{1} t^{\frac{3}{2}-1}(1-t)^{\frac{1}{2}-1}\,dt \overset{⑦}{=} 2B\left(\frac{3}{2}, \frac{1}{2}\right) \overset{⑤}{=} 2 \cdot \frac{\Gamma(\frac{3}{2})\Gamma(\frac{1}{2})}{\Gamma(2)}$$

+2,0

$$\overset{②}{=} \frac{2\Gamma(\frac{3}{2})\Gamma(\frac{1}{2})}{1 \cdot \underbrace{\Gamma(1)}_{1}} \overset{②}{=} 2 \cdot \frac{1}{2}\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right) = \Gamma\left(\frac{1}{2}\right)\Gamma\left(1-\frac{1}{2}\right)$$

$$\overset{③}{=} \frac{\pi}{\sin\frac{\pi}{2}} = \pi$$

+3,0

FORMULÁRIO EVENTUALMENTE ÚTIL

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1}\, dt, \;① \quad \Gamma(z+1) = z\Gamma(z), \;② \quad \Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin \pi z}, \;③ \quad 2^{2z-1}\Gamma(z)\Gamma(z+1/2) = \sqrt{\pi}\,\Gamma(2z), \;④$$

$$B(z,w) = \frac{\Gamma(z)\Gamma(w)}{\Gamma(z+w)}, \;⑤ \quad B(z,w) = 2\int_0^{\pi/2} \cos^{2z-1}\theta \sin^{2w-1}\theta\, d\theta, \;⑥ \quad B(z,w) = \int_0^1 t^{z-1}(1-t)^{w-1}\, dt. \;⑦$$