T_1 ( ) F520 ( ) MS550 · Nome: ______________________ **RA:** __________

Sejam as coordenadas esféricas (r, θ, ϕ) dadas por

$$x = r\sin\theta\cos\phi, \quad y = r\sin\theta\sin\phi, \quad z = r\cos\theta,$$

onde $0 \le r < \infty$, $0 \le \theta \le \pi$ e $0 \le \phi < 2\pi$.
(i) Mostre que os fatores de escala são dados por

$$h_r = 1, \quad h_\theta = r, \quad h_\phi = r\sin\theta.$$

(ii) Seja $\mathbf{A}$ o campo vetorial dado por

$$\mathbf{A} = \frac{r}{3}\sin\theta\mathbf{e}_\phi.$$

Usando coordenadas esféricas, calcule $\mathbf{B} = \nabla\times\mathbf{A}$ e $\rho = \nabla\cdot\mathbf{B}$.

(i) $h_i = \left|\dfrac{\partial\vec{r}}{\partial q_i}\right| = \sqrt{\left(\dfrac{\partial x}{\partial q_i}\right)^2 + \left(\dfrac{\partial y}{\partial q_i}\right)^2 + \left(\dfrac{\partial z}{\partial q_i}\right)^2}$

$h_r = \sqrt{(\sin\theta\cos\phi)^2 + (\sin\theta\sin\phi)^2 + \cos^2\theta} = \sqrt{\sin^2\theta + \cos^2\theta} = 1$

$h_\theta = \sqrt{(r\cos\theta\cos\phi)^2 + (r\cos\theta\sin\phi)^2 + (-r\sin\theta)^2} = \sqrt{r^2\cos^2\theta + r^2\sin^2\theta} = r$

$h_\phi = \sqrt{(-r\sin\theta\sin\phi)^2 + (r\sin\theta\cos\phi)^2} = \sqrt{r^2\sin^2\theta} = r\sin\theta$

+0,4

(ii) $\vec{A} = \frac{r}{3}\sin\theta\,\vec{e}_\phi \Rightarrow \begin{cases} A_r = 0, \ A_\theta = 0 \\ A_\phi = \frac{r}{3}\sin\theta \end{cases}$

$$\nabla\times\vec{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{e}_r & r\vec{e}_\theta & r\sin\theta\,\vec{e}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ 0 & 0 & \frac{r^2\sin^2\theta}{3} \end{vmatrix}$$

$$= \frac{1}{r^2\sin\theta}\left(\vec{e}_r\,\frac{2r^2}{3}\sin\theta\cos\theta - r\vec{e}_\theta\,\frac{2r}{3}\sin^2\theta\right)$$

$$= \frac{2}{3}\cos\theta\,\vec{e}_r - \frac{2}{3}\sin\theta\,\vec{e}_\theta$$

+0,8

FORMULÁRIO

$$\nabla\cdot\mathbf{V} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial q_1}(h_2h_3V_1) + \frac{\partial}{\partial q_2}(h_3h_1V_2) + \frac{\partial}{\partial q_3}(h_1h_2V_3)\right], \qquad \nabla\times\mathbf{V} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\mathbf{e}_{q_1} & h_2\mathbf{e}_{q_2} & h_3\mathbf{e}_{q_3} \\ \frac{\partial}{\partial q_1} & \frac{\partial}{\partial q_2} & \frac{\partial}{\partial q_3} \\ h_1V_1 & h_2V_2 & h_3V_3 \end{vmatrix}.$$

$$\vec{B} = \frac{2}{3}\cos\theta\,\vec{e_r} - \frac{2}{3}\sin\theta\,\vec{e_\theta} \Rightarrow \begin{cases} B_r = \frac{2}{3}\cos\theta \\ B_\theta = -\frac{2}{3}\sin\theta \\ B_\phi = 0 \end{cases}$$

$$\nabla\cdot\vec{B} = \frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}\left(r^2\sin\theta\,\frac{2}{3}\cos\theta\right) + \frac{\partial}{\partial\theta}\left(r\sin\theta\left(-\frac{2}{3}\sin\theta\right)\right)\right]$$

$$= \frac{1}{r^2\sin\theta}\left[\frac{4}{3}r\sin\theta\cos\theta - \frac{4}{3}r\sin\theta\cos\theta\right] = 0 \quad // \quad +0{,}8$$