EA721 - QUARTA LISTA DE EXERCÍCIOS

$20\log 2 = 6\,dB$

$20\log 10 = 20$

1) $G(s) = K\frac{1}{(s+1)^2} \Rightarrow 20 = 20\log K$

$K = 10$

$G(s) = \frac{10}{(s+1)^2} = \frac{10}{s^2+2s+1} = \frac{\hat{y}}{\hat{u}}$ $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ \dot{y}_1 \end{bmatrix}$

$\hat{y}(s) = G(s)\,\hat{u}(s)$

$$\left.\begin{array}{l} \dot{x} = Ax + Bu \\ y = Cx + Du \end{array}\right\} \quad \dot{x} = \overbrace{\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}}^{A} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 10 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \end{bmatrix} u$$

$\ddot{y}_1 = -y_1 - 2\dot{y}_1 + u$

$y = 10y_1 \quad y_1 = \frac{y}{10}$

$\frac{\ddot{y}}{10} + 2\frac{\dot{y}}{10} + 1\frac{y}{10} = u$

b) $\hat{u}(s) = \hat{r}(s) - K\hat{x}(s)$

$\cancel{\dot{x} = Ax + B(\hat{r}(s) - Kx)} \Rightarrow$ $\left.\begin{array}{l}\cancel{\dot{x} = Ax + B(r - Kx)} \\ \cancel{\hat{y} = C\hat{x}}\end{array}\right\}$ X

$y = Cx$

$\hat{u}(s) = r(s) - K\hat{x}(s) \Rightarrow u(t) = r(t) - Kx(t)$

$$\Rightarrow \left.\begin{array}{l} \dot{x} = Ax + B(r - Kx) \\ y = Cx + D(r - Kx) \\ \cancel{y = Cx} \end{array}\right\} \Rightarrow \begin{array}{l} \dot{x} = (A - BK)x + Br \\ \cancel{y = Cx} \quad y = (C - DK)x + Dr \end{array}$$

, impondo que $r(t) = 0 \Rightarrow \lim_{t\to\infty} x(t) = 0$, ou seja, queremos que

$\hat{y}(s) = \det(sI - (A - BK))$ = eq característica do sistema em malha fechada

$= P(s) = (s+2)(s+3) = s^2 + \underset{p_1}{5}s + \underset{p_0}{6}$ → dos polos que queremos

$\begin{bmatrix} 0 & 0 \\ 5 & 3 \end{bmatrix}$

por inspeção

$K = [p_0 - a_0 \quad p_1 - a_1] = [6 - 1 \quad 5 - 2] = [5 \quad 3]$

$A - BK = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix}[5 \quad 3]$

c) $F(s) = (C - DK)(sI - (A - BK))^{-1} B + \cancel{D}^{=0}$

$$= [10 \quad 0]\left(\begin{bmatrix} s & -1 \\ +6 & s+5 \end{bmatrix}\right)^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = [10 \quad 0]\frac{1}{s^2+5s+6}\begin{bmatrix} s+5 & 1 \\ -6 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$$

$$= \frac{1}{s^2+5s+6}[10 \quad 0]\begin{bmatrix} 1 \\ s \end{bmatrix} = \frac{10}{s^2+5s+6} \Rightarrow$$

$\lim_{s\to 0} s\,\frac{1}{s}\,\frac{10}{s^2+5s+6}$

$F(0) = \frac{10}{6} \Rightarrow E_{perm} = 1 - \frac{10}{6} = \frac{-4}{6} =$

d) ~~Temos que a eq característica é do tipo~~

$A - BK = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ K_1 & K_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1-K_1 & -2-K_2 \end{bmatrix}$

$$F(s) = [10 \quad 0]\left(\begin{bmatrix} s & -1 \\ 1+K_1 & s+2+K_2 \end{bmatrix}\right)^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{s^2 + (2+K_2)s + (1+K_1)} \cdot [10 \quad 0]\begin{bmatrix} s+2+K_2 & 1 \\ -1-K_1 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^2+(2+K_2)s+(1+K_1)} \cdot 10 = \frac{10}{(s+2)(s+p)} \rightarrow$$ para erro nulo, $F(0) = 1 \Rightarrow 1 + K_1 = 10$

$K_1 = 9$

$s^2 + (2+K_2)s + 10 = s^2 + (2+p)K_2 + 2p \quad 2p = 10,\ p = 5$

$2 + K_2 = 2 + p = 5$

$\Rightarrow K = [9 \quad 5]$

1) $(s^2+3s+2)\hat{Y} = (s+3)\hat{U}$

$\Rightarrow G(s) = \dfrac{\hat{Y}}{\hat{U}} = \dfrac{s+3}{s^2+3s+2} \Rightarrow \begin{array}{l}\dot{x} = Ax + Bu \\ y = Cx + Du\end{array}$, $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$C = \begin{bmatrix} 3 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 0 \end{bmatrix}$.

$\hat{Y} = G(s)\hat{U}$

b) $u(t) = 1 \Rightarrow \hat{U}(s) = \dfrac{1}{s}$, $\lim\limits_{t\to\infty} y(t) = \lim\limits_{s\to 0} s \cdot \dfrac{1}{s}\dfrac{s+3}{s^2+3s+2} = \dfrac{3}{2} = 1{,}5$

c) $u(t) = r(t) - Kx(t)$

$\Rightarrow \dot{x} = Ax + Bu = Ax + B(r-Kx) = (A-BK)x + Br$

$y = Cx + Du = Cx + D(r-Kx) = (C-DK)x + Dr$

$\xi = \sqrt{2}/2$ $\quad e_{perm} = 1 - \dfrac{3}{4} \rightarrow F(0)$

$(A-BK) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} K_1 & K_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2-K_1 & -3-K_2 \end{bmatrix}$

SENDO $F(s) = (C-DK)(sI-(A-BK))^{-1}(B) + D$

$= \begin{bmatrix} 3 & 1 \end{bmatrix}\left(s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2-K_1 & -3-K_2 \end{bmatrix}\right)^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 3 & 1 \end{bmatrix}\left(\begin{bmatrix} s & -1 \\ 2+K_1 & s+3+K_2 \end{bmatrix}\right)^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \dfrac{1}{s^2+(3+K_2)s+2+K_1}\begin{bmatrix} 3 & 1 \end{bmatrix}\begin{bmatrix} s+3+K_2 & 1 \\ -2-K_1 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$= \dfrac{1}{s^2+(3+K_2)s+(2+K_1)}\begin{bmatrix} 3 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ s \end{bmatrix} = \dfrac{s+3}{s^2+(3+K_2)s+(2+K_1)}$

QUEREMOS $\xi = \sqrt{2}/2$ e $F(0) = \dfrac{\cancel{3}}{4} = \dfrac{\cancel{3}}{2+K_1} \Rightarrow 4 = 2+K_1 \Rightarrow K_1 = 2$

$\omega_N^2 = 4 \Rightarrow \omega_N = 2$

$(3+K_2) = 2\dfrac{\sqrt{2}}{\cancel{2}}\cancel{2} \Rightarrow K_2 = 2\sqrt{2} - 3 \approx 0$

3) a) $G(s) = K \cdot s\,\dfrac{1}{s+1}\,\dfrac{1}{\left(\frac{s}{2}+1\right)} \Rightarrow$ em $j\omega \Rightarrow \overset{\rightarrow 20\,dB}{10} = K$, $K = 10$

$G(s) = \dfrac{20s}{(s+1)(s+2)} = \dfrac{20s + 0}{s^2+3s+2}$

$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}x + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u$

$y = \begin{bmatrix} 0 & 20 \end{bmatrix}x + \begin{bmatrix} 0 \end{bmatrix}u$

b) $u = r - Kx \Rightarrow \begin{array}{l}\dot{x} = Ax + B(r-Kx) \\ y = Cx + D(r-Kx)\end{array} \Rightarrow \begin{array}{l}\dot{x} = (A-BK)x + Br \\ y = (C-DK)x + Dr\end{array}$

SENDO $F(s) = (C-DK)(sI-(A-BK))^{-1}B + D = \begin{bmatrix} 0 & 20 \end{bmatrix}\dfrac{1}{s^2+(3+K_2)s+(2+K_1)}\begin{bmatrix} s+3+K_2 & 1 \\ -2-K_1 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$= \dfrac{1}{s^2+(3+K_2)s+(2+K_1)}\begin{bmatrix} 0 & 20 \end{bmatrix}\begin{bmatrix} 1 \\ s \end{bmatrix} = \dfrac{20s}{s^2+(3+K_2)s+(2+K_1)} = \dfrac{20s}{(s+3)(s+3)} = \dfrac{20s}{s^2+6s+9}$

$\tau_0 = \dfrac{1}{3}$, $\Rightarrow$ DOIS PÓLOS EM $s = -3$

$2+K_1 = 9 \Rightarrow K_1 = 7$

$3+K_2 = 6 \Rightarrow K_2 = 3$ $\quad K = \begin{bmatrix} 7 & 3 \end{bmatrix}$

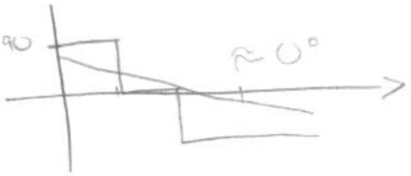
OU $K = \begin{bmatrix} p_0 - a_0 & p_1 - a_1 \end{bmatrix} = \begin{bmatrix} 7 & 3 \end{bmatrix}$

c) em j3 -> CALCULA-SE A RESPOSTA EM FREQUÊNCIA P/ FASE E MÓDU

$$F(s) = \frac{20 \cdot s}{s^2+6s+9} = \frac{20\, s}{(s+3)(s+3)} = \frac{\frac{20}{9}\, s}{\left(\frac{s}{3}+1\right)\left(\frac{s}{3}+1\right)} = \frac{20}{9}\frac{s}{\left(\frac{s}{3}+1\right)^2}$$

$$|F(j3)| = \frac{20}{\cancel{9}} \cdot \cancel{3} = \frac{20}{3}$$

$$\rightarrow y = |F(j3)|\, 6\, \text{sen}(3t+\phi) = \frac{20}{\cancel{3}}\, \cancel{6}^{2}\, \text{sen}(3t) \approx 40\,\text{sen}\,3t.$$



4) a)
$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = [1 \quad 0]\, x + [0]\, u \quad \to \text{CI nulas}$$

$$x = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$$

$$s^2 Y - \cancel{s y(0)}^{0} - \dot{y}(0) - 2sY - 2y(0) + Y$$

$$(s^2 - 2s + 1)\, Y = \hat{U} + 1$$

$$Y = \frac{\hat{U}}{s^2 - 2s + 1} + 1$$

$$A - \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} [1 \quad 0] = \begin{bmatrix} -L_1 & 1 \\ -1-L_2 & 2 \end{bmatrix}$$

$$(sI - (A-LC)) = \begin{bmatrix} s+L_1 & -1 \\ 1+L_2 & s-2 \end{bmatrix}$$

$$\cancel{\det(A - LC)} = P(s) = (s+2)^2 \Rightarrow$$

$$\det(sI - (A-LC)) = P(s) = (s+L_1)(s-2) + (1+L_2)$$

$$= s^2 + (L_1 - 2)s + L_2 - 2L_1 + 1 = s^2 + 4s + 4$$

$$4 = L_1 - 2 \qquad 4 = L_2 - 2 \cdot 6 + 1$$

$$L_1 = 6 \qquad L_2 = 4 + 11 = 15$$

b) $\dot{e}_0 = (A - LC) \cdot e_0 \overset{\mathcal{L}}{\Rightarrow} s\hat{e}_0 - e_0(0) = (A - LC)\hat{e}_0 \Rightarrow (sI - (A-LC))\hat{e}_0 = e_0(0)$

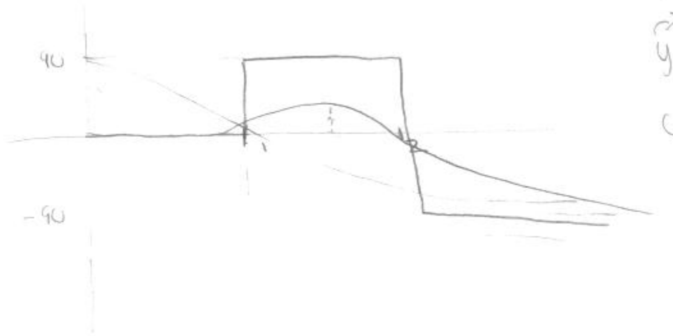
$$\hat{e}_0 = (sI - (A-LC))^{-1} e_0(0)$$

$$e_0(t) = x(t) - \hat{x}(t)$$

5) $G(s) = K(s+1)\dfrac{1}{\left(\frac{s}{2}+1\right)^2} = \dfrac{\frac{10^{7/10}}{2}(s+1)\cdot 4}{(s^2+4s+4)} = \dfrac{2\,10^{7/10}(s+1)}{s^2+4s+4}$ $\quad 14 = 20\log c$, $c = 10^{\frac{14}{20}}$

$20\log K + 20\log 2 = 14 \Rightarrow 20\log K = -20\log 2 + 20\log 10^{14/20}$

$20\log K = 20\log\dfrac{10^{7/10}}{2} \Rightarrow K = \dfrac{10^{7/10}}{2}$



$|G(s)| = 1 \Rightarrow$ (PELO BODE)

$\overset{0\,dB}{y} - y_0 = m(x - x_0)$

$0 - 14 = -20(\log\omega - \log 2)$

$\Rightarrow 20\log\omega = 14 + 20\log 2$

$\log\omega = \frac{14}{20} + \log 2 \qquad \log 10^{\frac{14}{20}}$

$\log\omega = \log 10^{14/20}\cdot 2$

$\omega = 2\cdot 10^{7/10}$ RAD/S $\gg 2$ RAD/S

$\Rightarrow$ FASE em $\omega = 2\,10^{7/10}$ RAD/S $\to -90° \Rightarrow MF \simeq 90°$

b) $\dot{X} = \begin{bmatrix} 0 & 1 \\ -4 & -4 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

$y = \begin{bmatrix} 2\,10^{7/10} & 2\,10^{7/10} \end{bmatrix} X + \begin{bmatrix} 0 \end{bmatrix} u$

c) $\dot{x}_o = Ax_o + Bu + L(y - y_o)$

$y_o = Cx_o + Du$

$\dot{x}_o = Ax_o + Bu + L(y - Cx_o - Du) \Rightarrow \dot{x}_o = Ax_o + Bu + L(Cx + \cancel{Du} - Cx_o - \cancel{Du})$

$\left.\begin{array}{l} \Rightarrow \dot{x}_o = (A - LC)x_o + Bu + LCx \\ \dot{x} = Ax + Bu \end{array}\right\}$ $\dot{x} - \dot{x}_o = \dot{e}_o = Ax + \cancel{Bu} - (A - LC)x_o - \cancel{Bu} - LCx$

$= (A - LC)x - (A - LC)x_o$

$= (A - LC)(x - x_o) \Rightarrow \dot{e}_o = (A - LC)e_o$

$\Rightarrow$ PARA O OBSERVADOR, PRECISAMOS ALOCAR OS POLOS DE $\det(sI - (A - LC))$ EM $\tau_0 = 0{,}2$ E $\tau_1 = 0{,}25\,s$

$A - LC = \begin{bmatrix} 0 & 1 \\ -4 & -4 \end{bmatrix} - \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}\begin{bmatrix} 2\cdot 10^{7/10} & 2\,10^{7/10} \end{bmatrix} = \begin{bmatrix} -L_1\,2\,10^{7/10} & 1 - L_1\cdot 2\,10^{7/10} \\ -4 - L_2\,2\,10^{7/10} & -4 - L_2\cdot 2\,10^{7/10} \end{bmatrix}$.

$\det(sI - (A - LC)) = 0 \Rightarrow$ EQ CARACTERÍSTICA $\Rightarrow \det\begin{bmatrix} s + L_1\cdot 2\,10^{7/10} & -1 + L_1\,2\,10^{7/10} \\ 4 + L_2\,2\,10^{7/10} & s + 4 + L_2\,2\,10^{7/10} \end{bmatrix} = 0$

$s^2 + (4 + L_2\,2\,10^{7/10} + L_1\,2\,10^{7/10})s + (L_1\,2\,10^{7/10})(\cancel{4} + L_2\,2\,10^{7/10}) + (+1 \cancel{- L_1\,2\,10^{7/10}})(4 + L_2\,2\,10^{7/10}) = 0$

$s^2 + (4 + L_2\,2\,10^{7/10} + L_1\,2\,10^{7/10})s + (4 + L_2\,2\,10^{7/10}) = P(s)$

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5) c) (CONTINUAÇÃO)

$$P(s) = \left(\frac{1}{4}s + 1\right)\left(\frac{1}{5}s + 1\right) = \frac{1}{4}\,\frac{1}{5}(s+4)(s+5) = P(s) = 0 = s^2 + 9s + 20$$

$$^{2} + L_2\, 2\, 10^{7/10} = 20^{10} \Rightarrow L_2 = \frac{10}{10^{7/10}} - \frac{2}{10^{7/10}} \cong 1,59 \ldots$$

$$4 + L_2\, 2\, 10^{7/10} + L_1\, 2\, 10^{7/10} = 9 \Rightarrow L_1 = -1,09$$

$$L = \begin{bmatrix} -1,09 \\ 1,59 \end{bmatrix}$$

d) $u(t) = 1 \Rightarrow U(s) = \frac{1}{s}$ $\quad X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$\dot{e}_o = (A - LC)e_o \overset{\mathcal{L}}{\Rightarrow} se_o - e_o(0) - (A-LC)e_o = 0 \Rightarrow (sI - (A-LC))e_o = e_o(0)$$

$$\hat{e}_o = (sI - (A-LC))^{-1} e_o(0)$$

$$X_o(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$e_o(0) = X(0) - X_o(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$\rightarrow$ ADOTANDO $2\, 10^{7/10} \cong 10$

$$(sI - (A-LC))^{-1} = \frac{1}{(s+4)(s+5)} \begin{bmatrix} s+20 & 11 \\ -20 & s-11 \end{bmatrix}$$

$$\hat{e}_o = \frac{1}{s^2 + 9s + 20} \begin{bmatrix} s+20 & 11 \\ -20 & s-11 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\hat{e}_o = \frac{1}{(s+4)(s+5)} \begin{bmatrix} s+20 \\ -20 \end{bmatrix} \Rightarrow \hat{e}_o \begin{bmatrix} \hat{e}_{x_1} \\ \hat{e}_{x_2} \end{bmatrix} = \frac{1}{(s+4)(s+5)} \begin{bmatrix} s+20 \\ -20 \end{bmatrix}$$

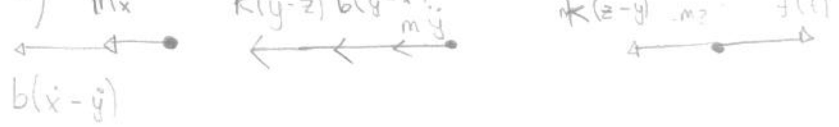
$$\hat{e}_{x_2} = \frac{-20}{(s+4)(s+5)} = \frac{A}{s+4} + \frac{B}{s+5} \Rightarrow A(1) = -20$$

$$B(-1) = -20 \Rightarrow B = 20$$

$$= \frac{-20}{s+4} + \frac{20}{s+5}$$

$$\mathcal{L}^{-1}\{\hat{e}_{x_2}(s)\} = -20e^{-4t} + 20e^{-5t}$$

6)



$$X = \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix}$$

$M\ddot{x} + b(\dot{x} - \dot{y}) = 0 \quad b(\dot{y} - \dot{x}) = M\ddot{x}$

$K(y-z) + b(\dot{y} - \dot{x}) + m\ddot{y} = 0 \Leftarrow$

$K(z-y) = f(t) \Rightarrow K(y-z) = -f(t)$

$Kz - Ky = f(t) \Rightarrow z = y + \frac{1}{K} f(t)$

$M\ddot{x} + b\dot{x} - b\dot{y} = 0 \Rightarrow \ddot{x} = \frac{-b}{M}\dot{x} + \frac{b}{M}\dot{y}$

$m\ddot{y} + b\dot{y} - b\dot{x} = f(t) \quad \ddot{y} = \frac{-b}{m}\dot{y} + \frac{b}{m}\dot{x} + \frac{f(t)}{m}$

6) CONT

$$\dot{X} = \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-b}{M} & 0 & \frac{b}{M} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{b}{m} & 0 & \frac{-b}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/m \end{bmatrix} f(t)$$

$$\overset{\to z}{y} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} X + \left[\frac{1}{k} f(t)\right]$$

a) NÃO É POSSÍVEL ATENDER A ESPECIFICAÇÃO POIS O POLO EM MALHA FECHADA MAIS RÁPIDO SERIA AINDA MAIOR QUE 2s ⇝ NÃO CONTROLÁVEL

b) NÃO É POSSÍVEL POIS NÃO CONSEGUIMOS ALOCAR OS POLOS $A-LC$ DE MODO QUE ~~A DINÂMICA DO SISTEMA SEJA MAIS~~ ~~OS POLOS DE $(A-LC)$~~ FIQUE MAIS RÁPIDA QUE A DINÂMICA DO SISTEMA EM MALHA ABERTA

↳ NÃO OBSERVÁVEL ⇒ $C_b = 0$

7) $G(s) = \dfrac{2(s+4)}{(s+1)(s+3)(s+6)} = \dfrac{2s+8}{(s+1)(s^2+9s+18)} = \dfrac{2s+8}{s^3+10s^2+27s+18}$

a)

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -18 & -27 & -10 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 8 & 2 & 0 \end{bmatrix} X + [0] u$$

P/ $u = K(M\hat{r} - x)$,

$\dot{x} = Ax + Bu \Rightarrow \dot{x} = (A - BK)x + BKM\hat{r}$

$y = Cx + Du \Rightarrow y = (C - DK)x + DKM\hat{r}$

$F(s) = (C - \cancel{DK})(sI - (A-BK))^{-1} BKM + \cancel{DKM}$

$= \underbrace{C(sI - (A-BK))^{-1} B}_{S(s)} \cdot KM$

b) $\varphi(\xi) = e^{-\pi\xi/\sqrt{1-\xi^2}} \Rightarrow \xi \geq 0,4$

$\xi w_N \geq 4$

$w_N \leq 2\pi \cdot 50$

$F(0) = 1$

SENDO A EQ CARACT. $\Rightarrow \det(sI - (A-BK)) = P(s)$ → ONDE IREMOS ALOCAR OS POLOS

EM $s=-4$, POR EXEMPLO

$P(s) = (s+4)(s+4)(s+4) = s^3 + 12s^2 + 48s +$

$K = [p_0 - a_0 \quad p_1 - a_1 \quad p_2 - a_2]$

$= [64 - 18 \quad 48 - 27 \quad 12 - 10] = [46 \quad 21 \quad 2]$

$$A - BK = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -18 & -27 & -10 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 46 & 21 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -64 & -48 & -12 \end{bmatrix}$$

$$F(s) = \begin{bmatrix} 8 & 2 & 0 \end{bmatrix} \frac{1}{s^3+12s^2+48s+64} \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 64 & 48 & s+12 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} K \cdot M$$

7) CONTINUAÇÃO

$$\begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} \\ \Delta_{21} & \Delta_{22} & \Delta_{23} \\ \Delta_{31} & \Delta_{32} & \Delta_{33} \end{bmatrix} = \begin{bmatrix} s^2+12s+48 & -64 & -64s \\ +s+12 & s^2+12s & -48s-64 \\ +1 & s & s^2 \end{bmatrix}$$

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}^t = \begin{bmatrix} s^2+12s+48 & s+12 & 1 \\ -64 & s^2+12s & s \\ -64s & -48s-64 & s^2 \end{bmatrix}$$

$$F(s) = [8 \quad 2 \quad 0] \frac{1}{s^3+12s^2+48s+64} \begin{bmatrix} 1 \\ s \\ s^2 \end{bmatrix} K\,M$$

$$= \underbrace{\frac{2s+8}{s^3+12s^2+48s+64}}_{S(s)} . K\,M$$

$$K\,K' = [46 \quad 21 \quad 2] \begin{bmatrix} 46 \\ 21 \\ 2 \end{bmatrix}$$

→ P/ ERRO NULO

$$F(0) = 1 = S(0)\,K\,M \Rightarrow 1 = \frac{8}{64} . K\,M$$

$$M = S(0)^{-1} . K'(KK')^{-1} = 8 \begin{bmatrix} 46 \\ 21 \\ 2 \end{bmatrix} \frac{1}{46^2+21^2+4} = \frac{8}{2561} \begin{bmatrix} 46 \\ 21 \\ 2 \end{bmatrix}$$

$$M = \begin{bmatrix} 0,1437 \\ 0,0656 \\ 0,0062 \end{bmatrix}$$

$$F(s) = \frac{2s+8}{s^3+12s^2+48s+64} \quad 8 = \frac{16s+64}{s^3+12s^2+48s+64}$$

c) SENDO ENTÃO $U = -Kx \Rightarrow U = -[46 \quad 21 \quad 2] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow$

SÓ QUE: $\left.\begin{array}{l} x_2 = \dot{x}_1 \\ x_3 = \dot{x}_2 \end{array}\right\} \overset{\mathcal{L}}{} \begin{array}{l} \hat{x}_2 = s\hat{x}_1 \\ \hat{x}_3 = s\hat{x}_2 \end{array}$

$$U = -[46 \quad 21 \quad 2] \begin{bmatrix} \frac{\hat{x}_2}{s} \\ \hat{x}_2 \\ s\hat{x}_2 \end{bmatrix} = -46\frac{\hat{x}_2}{s} - 21\hat{x}_2 - 2s\hat{x}_2$$

→ VARIÁVEL A SER MEDIDA

$$= -\hat{x}_2\left(\frac{46}{s} + 21 + 2s\right)$$

$$C(s) = \frac{46}{s} + 21 + 2s$$

7) d) $\mathring{x}_o = Ax_o + Bu + L(y - y_o)$ $\quad$ $\mathring{x}_o = Ax_o + Bu + L(Cx + \cancel{Du} - Cx_o - \cancel{Du})$ $\quad$ d

$y_o = Cx_o + Du$ $\quad$ $\mathring{x}_o = (A - LC)x_o + Bu + LCx$

$\dot{x} = Ax + Bu, \quad \dot{x} - \dot{x}_o = Ax + \cancel{Bu} - (A - LC)x_o - \cancel{Bu} - LCx$

$\Rightarrow \dot{x} - \dot{x}_o = (A - LC)x - (A - LC)x_o \Rightarrow \boxed{\mathring{e}_o = (A - LC)e_o}, \quad e_o = x - x_o$

OS POLOS DO OBSERVADOR DEVEM SER MAIS RÁPIDOS DO QUE A DINÂMICA O SISTEMA ORIGINAL!

→ PÓLO MAIS RÁPIDO DO SISTEMA ORIGINAL

OU SEJA, DEVE SER $|s| > 6$

$$A - \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} \begin{bmatrix} 8 & 2 & 0 \end{bmatrix} = \begin{bmatrix} -8L_1 & 1 - 2L_1 & 0 \\ -8L_2 & -2L_2 & 1 \\ -18 - 8L_3 & -27 - 2L_3 & -10 \end{bmatrix}$$

EQ CARACT $\Rightarrow \det(sI - (A - LC)) = 0 = P(s) = (s + 7)^3$

$$L = \begin{bmatrix} -1{,}75 \\ +12{,}5 \\ -45 \end{bmatrix}$$

f) $G(s) = \dfrac{1}{s(s+1)} = \dfrac{1}{s^2 + s}$

i) $\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x + [0] u$

ii) Se $u = r - Kx$,

$\mathring{x} = Ax + B(r - Kx)$ $\quad$ $\dot{x} = (A - BK)x + Br$ $\quad$ $\mathcal{L} \overset{\text{CI NULAS}}{=}$ $\quad$ $s\hat{x} - (A - BK)\hat{x} = B\hat{r}$

$y = Cx + D(r - Kx)$ $\quad$ $y = (C - DK)x + Dr$ $\quad$ $(sI - (A - BK))\hat{x} = B\hat{r}$

$\hat{x} = (sI - (A - BK))^{-1} B\hat{r}$

$\hat{y} = \{(C - DK)(sI - (A - BK))^{-1} B + D\} \hat{r}$

ONDE $F(s) = (C - DK)(sI - (A - BK))^{-1} B + D \Rightarrow$ A EQ CARACTERÍSTICA É

$\det(sI - (A - BK)) = 0$

$\tau_o s + 1 \rightsquigarrow$ PÓLOS EM $-\dfrac{1}{\tau_o} = -2$

$\left.\begin{matrix} \zeta\omega_N = 2 \\ \zeta = 0{,}5 \end{matrix}\right\}$ $\quad$ $\omega_N = \dfrac{2}{0{,}5} = 4$ $\rightsquigarrow$ $P(s) = s^2 + 4s + 16$

$K = [\alpha_1 - a_1 \quad \alpha_2 - a_2] = [16 \quad 3]$

8) c) CONSIDERANDO $u = -Kx \Rightarrow U = -\begin{bmatrix} 16 & 3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = -\begin{bmatrix} 16 & 3 \end{bmatrix} \begin{bmatrix} X_1 \\ sX_1 \end{bmatrix}$

$\dot{x}_1 = x_2 \xrightarrow{\mathcal{L}} sX_1 = X_2$

$$= -X_1 \underbrace{(16 + 3s)}_{C(s)}$$

d) SENDO (1) $\dot{x}_o = Ax_o + Bu + L(y - y_o)$

$y_o = Cx_o + Du$

(1) $\rightarrow \dot{x}_o = Ax_o + Bu + L(Cx + \cancel{Du} - Cx_o - \cancel{Du})$

$\dot{x}_o = (A - LC)x_o + Bu + LCx$

$\dot{x} - \dot{x}_o = Ax + \cancel{Bu} - (A - LC)x_o - \cancel{Bu} - LCx \Rightarrow \dot{e}_o = (A - LC)x - (A - LC)x_o$

$\Rightarrow \dot{e}_o = (A - LC)e_o \xrightarrow{\mathcal{L}} (sI - (A - LC))e_o = e_o(0) \Rightarrow \frac{e_o}{e_o(0)} = (sI - (A - LC))^{-1}$

ALOCANDO AS RAÍZES DE $\det(sI - (A - LC)) = 0$ EM $s = -5$, POR EXEMPLO

$A - LC = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} -L_1 & 1 \\ -L_2 & -1 \end{bmatrix}$

$\varphi(sI - (A - LC)) = \det\left(\begin{bmatrix} s + L_1 & -1 \\ L_2 & s + 1 \end{bmatrix}\right) = s^2 + (L_1 + 1)s + L_1 + L_2 = 0 = (s + 5)^2$

$s^2 + (L_1 + 1)s + (L_1 + L_2) = s^2 + 10s + 25 \Rightarrow L_1 = 9, \quad L_2 = 16 \quad L = \begin{bmatrix} 9 \\ 16 \end{bmatrix}$

$$\begin{array}{r} 0{,}49 \\ \underline{1{,}7} \\ 343 \\ \underline{49+} \\ 0{,}833 \\ \underline{0{,}3} \\ 0{,}2499 \end{array}$$

e) $\rho = 0{,}5 \quad V = \begin{bmatrix} 1 & 0 \end{bmatrix} = C = \begin{bmatrix} 1 & 0 \end{bmatrix}$

$\varphi(s) = V(sI - A)^{-1}B = C(sI - A)^{-1}B = G(s) = \frac{1}{s^2 + s}$

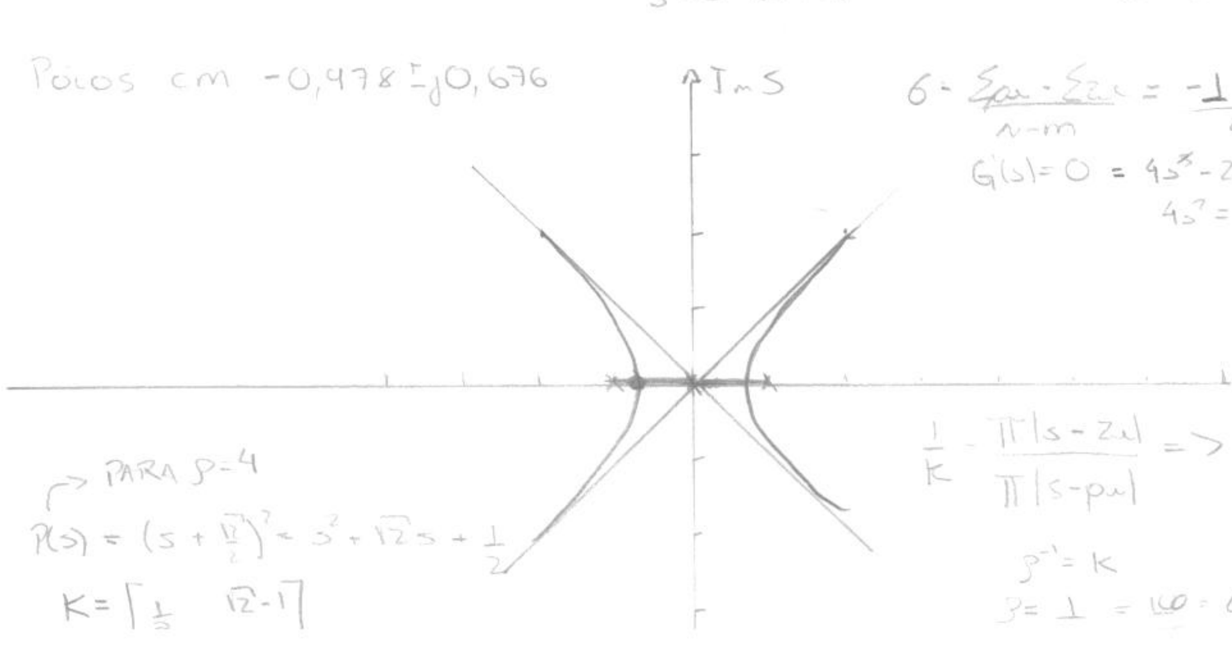
$1 + \rho^{-1}\varphi(s)\varphi(-s) = 0 \Rightarrow 1 + 2 \cdot \frac{1}{s^2 - s} \frac{1}{(s^2 + s)} = 1 + \rho^{-1} \frac{1}{s^2(s-1)(s+1)} = \frac{1}{s^4 - s^2}$

POLOS EM $-0{,}478 \pm j0{,}676$

$\sigma = \frac{\sum p_i - \sum z_i}{n - m} = \frac{-1 + 1}{4} = 0$

$G'(s) = 0 = 4s^3 - 2s = 0$

$4s^2 = 2 \quad s^2 = \frac{2}{4} \Rightarrow \pm\frac{1}{\sqrt{2}}$



$\frac{1}{K} = \frac{\prod|s - z_i|}{\prod|s - p_i|} \Rightarrow K = 0{,}3 \cdot 0{,}7 \cdot 0{,}7 \cdot 1{,}7$

$K \cong 0{,}25$

$\rho^{-1} = K$

$\rho = \frac{1}{} = 100 = 4$

$\rightarrow$ PARA $\rho = 4$

$P(s) = \left(s + \frac{\sqrt{2}}{2}\right)^2 = s^2 + \sqrt{2}s + \frac{1}{2}$

$K = \begin{bmatrix} \frac{1}{2} & \sqrt{2} - 1 \end{bmatrix}$