T_1 ( ) F520 ( ) MS550 · Nome: ______________________ RA: __________

Sejam as coordenadas cilíndricas parabólicas (u, v, z) dadas por

$$x = \frac{1}{2}(u^2 - v^2), \quad y = uv, \quad z = z,$$

onde $-\infty \leq u < \infty$, $v \geq 0$ e $-\infty < z < \infty$, e o campo vetorial $\mathbf{V}$ dado por

$$\mathbf{V} = \frac{-y\mathbf{i} + x\mathbf{j}}{\sqrt{x^2 + y^2}}.$$

Utilizando as coordenadas cilíndricas parabólicas, calcule o divergente e o rotacional de $\mathbf{V}$.

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k} = \frac{1}{2}(u^2 - v^2)\vec{i} + uv\vec{j} + z\vec{k}$$

$$\frac{\partial \vec{r}}{\partial u} = u\vec{i} + v\vec{j} \Rightarrow h_u = \sqrt{u^2+v^2} \quad \therefore \vec{e_u} = \frac{u\vec{i} + v\vec{j}}{\sqrt{u^2+v^2}}$$

$$\frac{\partial \vec{r}}{\partial v} = -v\vec{i} + u\vec{j} \Rightarrow h_v = \sqrt{u^2+v^2} \quad \therefore \vec{e_v} = \frac{-v\vec{i} + u\vec{j}}{\sqrt{u^2+v^2}}$$

$$\frac{\partial \vec{r}}{\partial z} = \vec{k} \Rightarrow h_z = 1 \quad \therefore \vec{e_z} = \vec{k}$$

$$\therefore \vec{i} = (\vec{i}\cdot\vec{e_u})\vec{e_u} + (\vec{i}\cdot\vec{e_v})\vec{e_v} + (\vec{i}\cdot\vec{e_z})\vec{e_z} = \frac{u\vec{e_u} - v\vec{e_v}}{\sqrt{u^2+v^2}}$$

$$\vec{j} = (\vec{j}\cdot\vec{e_u})\vec{e_u} + (\vec{j}\cdot\vec{e_v})\vec{e_v} + (\vec{j}\cdot\vec{e_z})\vec{e_z} = \frac{v\vec{e_u} + u\vec{e_v}}{\sqrt{u^2+v^2}}$$

$$\vec{k} = \vec{e_z}$$

$$x^2 + y^2 = \left[\frac{1}{2}(u^2-v^2)^2\right]^2 + (uv)^2 = \frac{1}{4}(u^4 - 2u^2v^2 + v^4 + 4u^2v^2) = \frac{1}{4}(u^2+v^2)^2$$

$$\therefore \vec{V} = \frac{2}{(u^2+v^2)}\left[(-uv)\frac{(u\vec{e_u} - v\vec{e_v})}{\sqrt{u^2+v^2}} + \frac{1}{2}(u^2-v^2)\frac{(v\vec{e_u} + u\vec{e_v})}{\sqrt{u^2+v^2}}\right]$$

$$= \frac{2}{(u^2+v^2)^{3/2}}\left(-\frac{1}{2}u^2v\vec{e_u} - \frac{1}{2}v^3\vec{e_u} + \frac{1}{2}uv^2\vec{e_v} + \frac{u^3}{2}\vec{e_v}\right)$$

$$\vec{V} = \frac{1}{\sqrt{u^2+v^2}}(-v\vec{e_u} + u\vec{e_v}) \qquad \therefore V_u = \frac{-v}{\sqrt{u^2+v^2}}$$

$$V_v = \frac{u}{\sqrt{u^2+v^2}}$$

$$V_z = 0$$

+1,0

FORMULÁRIO

$$\nabla \cdot \mathbf{V} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial q_1}(h_2h_3V_1) + \frac{\partial}{\partial q_2}(h_3h_1V_2) + \frac{\partial}{\partial q_3}(h_1h_2V_3)\right], \qquad \nabla \times \mathbf{V} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\mathbf{e}_{q_1} & h_2\mathbf{e}_{q_2} & h_3\mathbf{e}_{q_3} \\ \frac{\partial}{\partial q_1} & \frac{\partial}{\partial q_2} & \frac{\partial}{\partial q_3} \\ h_1V_1 & h_2V_2 & h_3V_3 \end{vmatrix}.$$

$$\therefore \operatorname{div} \vec{V} = \frac{1}{u^2+v^2}\left[\frac{\partial}{\partial u}(h_v h_z V_u) + \frac{\partial}{\partial v}(h_u h_z V_z) + \frac{\partial}{\partial z}(h_u h_v V_z)\right]$$

$$= \frac{1}{u^2+v^2}\left[\frac{\partial}{\partial u}(-v) + \frac{\partial}{\partial v}(u)\right] = 0$$

+0,5

$$\operatorname{rot} \vec{V} = \frac{1}{u^2+v^2}\begin{vmatrix} \sqrt{u^2+v^2}\,\vec{e_u} & \sqrt{u^2+v^2}\,\vec{e_v} & \vec{e_z} \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial z} \\ \cancel{\sqrt{u^2+v^2}}\frac{(-v)}{\cancel{\sqrt{u^2+v^2}}} & \cancel{\sqrt{u^2+v^2}}\frac{u}{\cancel{\sqrt{u^2+v^2}}} & 0 \end{vmatrix}$$

$$= \frac{1}{(u^2+v^2)}\left[\left(\frac{\partial}{\partial u}(u) - \frac{\partial}{\partial v}(-v)\right)\vec{e_z}\right]$$

+0,5

$$= \frac{2}{(u^2+v^2)}\,\vec{e_z}$$