T$_3$ ( ) F520 ( ) MS550 · Nome: ______________________ RA: __________

Mostre que nenhuma solução não trivial da equação

$$z^2y'' + zy' + y = 0$$

que é real no semieixo real positivo do plano complexo pode ser real no semieixo real negativo.

$$\left.\begin{matrix} z^2y''+zy'+y=0 \\ y=z^r \end{matrix}\right\} \Rightarrow r(r-1)+r+1=0 \Rightarrow r^2+1=0 \Rightarrow \begin{cases} r_1 = i \\ r_2=-i \end{cases}$$

$$\therefore y = c_1 z^{i} + c_2 z^{-i} = c_1 e^{i\ln z} + c_2 e^{-i\ln z}, \text{ onde } \underline{c_1, c_2 \in \mathbb{C}}$$

+0,5

Escrevendo $z = re^{i\theta}$, $c_1 = \alpha_1 + i\beta_1$, $c_2 = \alpha_2 + i\beta_2$, temos

$$y = (\alpha_1+i\beta_1)e^{i\ln r}e^{-\theta} + (\alpha_2+i\beta_2)e^{-i\ln r}e^{\theta}$$

$$= (\alpha_1+i\beta_1)e^{-\theta}(\cos(\ln r)+i\sin(\ln r)) + (\alpha_2+i\beta_2)e^{\theta}(\cos(\ln r)-i\sin(\ln r))$$

$$= [(\alpha_1 e^{-\theta}+\alpha_2 e^{\theta})\cos(\ln r) + (-\beta_1 e^{-\theta}+\beta_2 e^{\theta})\sin(\ln r)]$$

$$+ i[(\alpha_1 e^{-\theta}-\alpha_2 e^{\theta})\sin(\ln r) + (\beta_1 e^{-\theta}+\beta_2 e^{\theta})\cos(\ln r)] \quad \circledast$$

I $\boxed{x>0} \Rightarrow \underline{\theta = 0}$

$$\circledast \Rightarrow y_+ = (\alpha_1+\alpha_2)\cos(\ln r) + (\beta_2-\beta_1)\sin(\ln r)$$
$$+ i[(\alpha_1-\alpha_2)\sin(\ln r) + (\beta_1+\beta_2)\cos(\ln r)]$$

$$y_+ \in \mathbb{R} \Leftrightarrow \boxed{\alpha_1 = \alpha_2, \ \beta_1 = -\beta_2} \quad \star$$

II $\boxed{x<0} \Rightarrow \underline{\theta = \pi}$

$$\circledast \Rightarrow y_- = (\alpha_1 e^{-\pi}+\alpha_2 e^{\pi})\cos(\ln r) + (-\beta_1 e^{-\pi}+\beta_2 e^{\pi})\sin(\ln r)$$
$$+ i[(\alpha_1 e^{-\pi}-\alpha_2 e^{\pi})\sin(\ln r) + (\beta_1 e^{-\pi}+\beta_2 e^{\pi})\cos(\ln r)]$$

$$\therefore y_- \in \mathbb{R} \Leftrightarrow \boxed{\alpha_1 e^{-\pi} = \alpha_2 e^{\pi}, \ \beta_1 e^{-\pi} = -\beta_2 e^{\pi}} \quad \star\star$$

+1,5

$\star$ e $\star\star$ NÃO podem ser satisfeitas ao mesmo tempo!