GABARITO — TURMAS SEXTA

Q1

$$F = (2xyz^3,\ x^2z^3,\ 3x^2yz^2)$$

$$\text{rot}\, F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xyz^3 & x^2z^3 & 3x^2yz^2 \end{vmatrix} =$$

$$= (2xz^3 - 2xz^3,\ -6xyz^2 + 6xyz^2,\ 2xz^3 - 2xz^3) = 0$$

$\Rightarrow$ **F é conservativo** $\longrightarrow$ 0,5

$F = \nabla f$ onde $f = x^2yz^3$ $\longrightarrow$ 0,5

Ponto final e inicial da curva

$$r(0) = (0, 0, 1) = A$$

$$r\left(\frac{\pi}{2}\right) = (1, 1, 1) = B$$

Então pelo Teorema Fundamental

$$\int_C \mathbf{F} \cdot dr = f(B) - f(A) = 1 - 0 = 1. \longrightarrow 1{,}5$$

Q2) $r(u,v) = (uv,\ u+v,\ u-v)\ ,\quad u^2+v^2 \leq 1$

a) $(c,1,0) \in S \iff (c,1,0) = r(u,v)$ para algum par (u,v)

$$\iff \begin{cases} c = uv \implies c = \frac{1}{4} \\ u+v=1 \implies 2u=1 \implies u = 1/2 \\ u-v=0 \implies \boxed{u=v} \end{cases}$$

0,5

b) $r_u = (v, 1, 1) \qquad r_v = (u, 1, -1)$

$$r_u \times r_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ v & 1 & 1 \\ u & 1 & -1 \end{vmatrix} = (-1-1)\vec{i} + (u+v)\vec{j} + (v-u)\vec{k} \doteq (-2,\ u+v,\ v-u)$$

$$\|r_u \times r_v\|^2 = 4 + (u+v)^2 + (v-u)^2 = 4 + u^2 + \cancel{2uv} + v^2 + v^2 - \cancel{2uv} + u^2$$
$$= 2u^2 + 2v^2 + 4 = 2(u^2+v^2+2)$$

0,5

$$\begin{cases} u = r\cos\theta \\ v = r \operatorname{sen}\theta \\ r = \sqrt{u^2+v^2} \end{cases}$$

$$\text{Área} = \iint\limits_{u^2+v^2\leq 1} \|r_u \times r_v\|\, dA = \iint\limits_{u^2+v^2 \leq 1} \sqrt{2}\sqrt{u^2+v^2+2}\, dA =$$

0,5

$$\begin{cases} 0 \leq \theta \leq 2\pi \\ 0 \leq r \leq 1 \end{cases}$$

$$= \int_0^{2\pi}\int_0^1 \sqrt{2}\sqrt{r^2+2}\ r\, dr\, d\theta$$

0,5

$$\begin{cases} z = r^2+2 \\ dz = 2r\, dr \end{cases}$$

$$= 2\sqrt{2}\,\pi \int_0^1 \sqrt{r^2+2}\ r\, dr$$

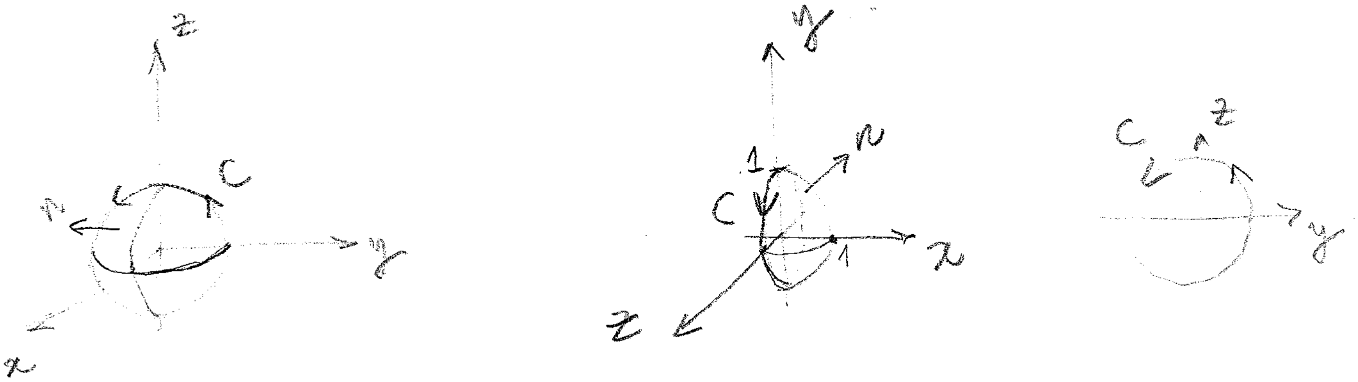
$$= \sqrt{2}\,\pi \int_2^3 z^{1/2}\, dz = \sqrt{2}\,\pi\ \frac{z^{3/2}}{3/2}\Bigg|_{z=2}^{z=3} = \frac{2\sqrt{2}\,\pi}{3}\left(3^{3/2} - 2^{3/2}\right)$$

0,5

$$= \frac{\pi}{3}\left(6^{3/2} - 4^{3/2}\right) = \frac{\pi}{3}\left(6^{3/2} - 8\right) = \pi\left(2\sqrt{6} - \frac{8}{3}\right)$$

OBS.: Descontar 0,2 para cada erro de conta

Q3



$$F = \left(e^{xy}\cos z,\ (x^2+1)z,\ -y\right)$$

$$\gamma(t) = (0,\ \cos t,\ \operatorname{sen} t)\ ,\quad 0 \le t \le 2\pi$$

$x = 0$
$y = \cos t$
$z = \operatorname{sen} t$

$$\gamma'(t) = (0,\ -\operatorname{sen} t,\ \cos t) \longrightarrow 1,0$$

Teo. de Stokes.

$$\iint_S \operatorname{rot} F \cdot n\, ds = \int_C F \cdot \gamma'(t)\, dt =$$

$$= \int_C \left(e^{xy}\cos z,\ (x^2+1)z,\ -y\right) \cdot (0,\ -\operatorname{sen} t,\ \cos t)\, dt \longrightarrow 0,7$$

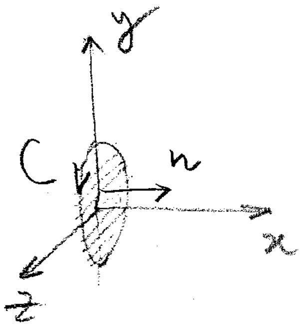
$$= \int_0^{2\pi} \left(-\operatorname{sen}^2 t - \cos^2 t\right) dt \longrightarrow 0,3 \quad = -\int_0^{2\pi} dt = -2\pi \longrightarrow 0,5$$

Outra solução

$$\operatorname{rot} F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^{xy}\cos z & (x^2+1)z & -y \end{vmatrix} =$$

$$= \left(-2 - x^2,\ -e^{xy}\operatorname{sen} z,\ 2xz - xe^{xy}\cos z\right) \longrightarrow 0,5$$

trocando a semi-esfera por um plano



$n = (1, 0, 0) \longrightarrow 0{,}5$

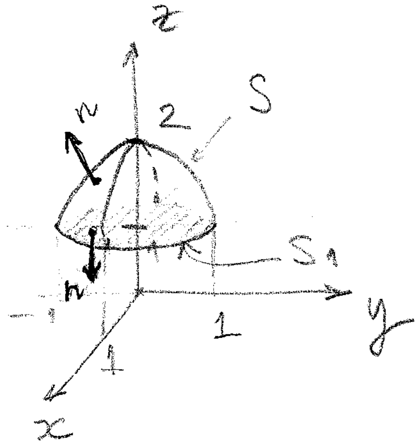
$$\iint_S \text{rot}\, F \cdot n \, dS = \iint_D -2 \, dy\, dz = \underline{-2\pi}.$$

1,0

0,5

<u>Obs.</u>: Inverteu a orientação -0,5

Q4



$$F = \left(z\,tg^{-1}(y^2),\ z^3 \ln(x^2+1),\ z\right)$$

$$\operatorname{div} F = 1 \quad \text{———} \quad \boxed{0.2}$$

S_1 = fundo = parte do plano $z = 1$ acima do disco $x^2+y^2 \leq 1$.

Teorema da Diverg.

$$\iint_S F\cdot n\, dS + \iint_{S_1} F\cdot n\, dS = \iiint_E \operatorname{div} F\, dV \quad \text{———} \quad \boxed{+0.8}$$

Tem

$$\iiint_E \operatorname{div} F\, dV = \iiint 1\, dV = \text{coord. cilíndricas}$$

$$\boxed{\begin{aligned} z &= 2 - x^2 - y^2 \\ &= 2 - r^2 \end{aligned}}$$

$$= \int_0^{2\pi}\int_0^1\int_1^{2-r^2} r\, dz\, dr\, d\theta =$$

$$= \int_0^{2\pi}\int_0^1 r(1-r^2)\, dr\, d\theta = \int_0^{2\pi}\left(\frac{r^2}{2} - \frac{r^4}{4}\right)\Big|_0^1 d\theta$$

$$= \left(\frac{1}{2} - \frac{1}{4}\right)\cdot 2\pi = \frac{1}{4}\, 2\pi = \frac{\pi}{2} \quad \text{———} \quad \boxed{+0.8}$$

$$\iint_{S_1} F\cdot n\, dS = \iint_{S_1} \left(z\,tg^{-1}(y^2),\ z^3\ln(x^2+1),\ z\right)\cdot(0,0,-1)\, dS$$

$$= \iint_{S_1} -1\, dx\,dy = -\text{área } S_1 = -\pi \quad \text{———} \quad \boxed{+0.7}$$

Portanto o fluxo de F através de S =

$$\iint_S F \cdot n \, dS = \frac{\pi}{2} + \pi = \frac{3\pi}{2}$$