Questão 1: Efetue uma mudança de variáveis conveniente e calcule a integral

$$I = \int_0^2 \int_0^{\sqrt{4-y^2}} \int_0^{\sqrt{4-x^2-y^2}} \frac{1}{x^2+y^2+z^2}\, dz\, dx\, dy.$$

$$I = \iiint_E \frac{1}{x^2+y^2+z^2}\, dz\, dx\, dy$$

onde

$$E = \{(x,y,z) : 0 \leq y \leq 2,\ 0 \leq x \leq \sqrt{4-y^2},\ 0 \leq z \leq \sqrt{4-x^2-y^2}\},$$

ou seja

$$E = \{(x,y,z) : x^2+y^2+z^2 \leq 4,\ x \geq 0,\ y \geq 0,\ z \geq 0\}$$

Usando coordenadas esféricas

$x = \rho \operatorname{sen}\varphi \cos\theta$, $y = \rho \operatorname{sen}\varphi \operatorname{sen}\theta$, $z = \rho \cos\varphi$

$$E = \{(\rho, \varphi, \theta) : 0 \leq \rho \leq 2,\ 0 \leq \varphi \leq \frac{\pi}{2},\ 0 \leq \theta \leq \frac{\pi}{2}\} \quad \boxed{1,3}$$

$$I = \int_0^2 \int_0^{\pi/2} \int_0^{\pi/2} \cancel{\frac{1}{\rho^2}}\, \cancel{\rho^2} \operatorname{sen}\varphi \, d\theta\, d\varphi\, d\rho \quad \boxed{0,7}$$

$$I = \int_0^2 1\, d\rho \int_0^{\pi/2} 1\, d\theta \int_0^{\pi/2} \operatorname{sen}\varphi\, d\varphi = 2 \cdot \frac{\pi}{2} \cdot (-\cos\varphi)\Big|_0^{\pi/2} = \pi \quad \boxed{0,5}$$

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1 erro nos limites de ρ, φ, θ : $-0,3$

2 erros nos limites de ρ, φ e θ : $-0,7$

QUESTÃO 2: $I = \iint_R \cos\left(\frac{y-x}{y+x}\right) dA$

R = região trapezoidal com vértices $(1,0)$, $(2,0)$, $(0,2)$ e $(0,1)$

$$\begin{cases} u = y - x \\ v = y + x \end{cases} \Rightarrow u + v = 2y \Rightarrow y = \frac{u+v}{2}, \quad x = y - u = \frac{u+v}{2} - \frac{2u}{2} = \frac{v-u}{2}$$

0,3

$$\begin{cases} x = \frac{v-u}{2} \\ y = \frac{u+v}{2} \end{cases} \qquad \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{vmatrix} = -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2}$$

0,2

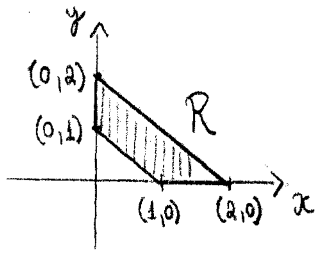
$(x,y) = (1,0) \Rightarrow (u,v) = (-1,1)$

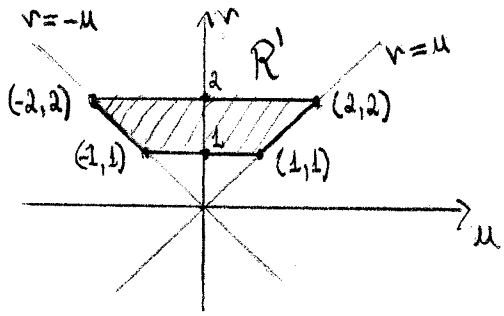
$(x,y) = (2,0) \Rightarrow (u,v) = (-2,2)$

$(x,y) = (0,1) \Rightarrow (u,v) = (1,1)$

$(x,y) = (0,2) \Rightarrow (u,v) = (2,2)$

$$R': \begin{cases} 1 \leq v \leq 2 \\ -v \leq u \leq v \end{cases}$$





1,0

$$I = \iint_{R'} \left(\cos \frac{u}{v}\right) \frac{1}{2} \, du \, dv$$

$$= \frac{1}{2} \int_1^2 \int_{-v}^{v} \cos \frac{u}{v} \, du \, dv =$$

0,5

$$= \frac{1}{2} \int_1^2 \left(v \cdot \operatorname{sen} \frac{u}{v} \Big|_{u=-v}^{u=v} \right) dv = \frac{1}{2} \int_1^2 \left(v \operatorname{sen} 1 - v \operatorname{sen}(-1) \right) dv$$

$$= \frac{1}{\cancel{2}} \cancel{2} \operatorname{sen} 1 \int_1^2 v \, dv = (\operatorname{sen} 1) \frac{v^2}{2} \Big|_1^2 = (\operatorname{sen} 1)\left(\frac{4}{2} - \frac{1}{2}\right) = \frac{3}{2} \operatorname{sen} 1$$

0,5

Esqueceu do Jacobiano: − 1,0

" o módulo " : − 0,2

Não justificou por que as curvas na região R' são retas: − 0,3

QUESTÃO 3 : $I = \int_C e^{2y}\,dx + (1+2xe^{2y})\,dy$,

C: $r(t) = \left(te^t,\ 1 + \operatorname{sen}\left(\frac{\pi}{2}t\right)\right)$, $0 \leq t \leq 1$.

$P(x,y) = e^{2y}$, $Q(x,y) = 1 + 2xe^{2y}$

$\frac{\partial P}{\partial y} = 2e^{2y}$, $\frac{\partial Q}{\partial x} = 2e^{2y}$

$D = \mathbb{R}^2$ domínio das funções P e Q é um aberto simplesmente conexo

$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} = 2e^{2y}$

$\Longrightarrow$ O campo $F = (P, Q)$ é conservativo

0,8

f = função potencial de F $\Rightarrow$ $f_x = e^{2y}$ e $f_y = 1 + 2xe^{2y}$

$f = \int e^{2y}\,dx = xe^{2y} + h(y)$

$1 + \cancel{2xe^{2y}} = f_y = \cancel{2xe^{2y}} + h'(y) \Rightarrow h'(y) = 1 \Rightarrow h(y) = y$

$\boxed{f(x,y) = xe^{2y} + y}$

1,0

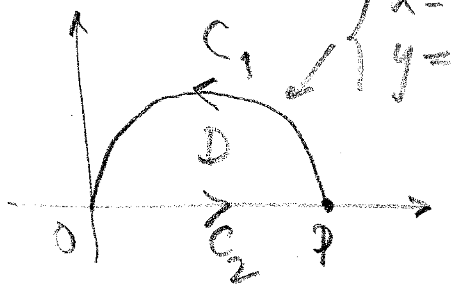
$I = \int_C F \cdot dr = f(r(1)) - f(r(0)) = f(e, 2) - f(0, 1)$

$= (e\,e^4 + 2) - (0 \cdot e^1 + 1) = \boxed{e^5 + 1}$

0,7

OBSERVAÇÃO. $-0,5$ se não observou que do domínio $D = \mathbb{R}^2$ do campo vetorial é um aberto simplesmente conexo.

QUESTÃO 4



C_1 $\swarrow$ $\begin{cases} x = t - \text{sen}\, t \\ y = 1 - \cos t \end{cases}$

No ponto P, $y = 0 = 1 - \cos t \Rightarrow t = 2\pi$ $\longrightarrow$ 0,5

Área de $D = \iint_D dx\,dy \underset{\substack{\text{Teo. de Green} \\ \text{com } P = -y \\ \text{e } Q = 0}}{=} \oint_C -y\,dx$

Parametrização da curva C_2 - segmento

$$C_2 = \begin{cases} x = t \\ y = 0 \end{cases} \longrightarrow 0{,}5$$

Então

$$\oint_C -y\,dx = \cancel{\int_{C_1} -y\,dx} + \int_{C_2} -y\,dx =$$

$= 0$ porque $y = 0$

$$= \int_{2\pi}^{0} -(1-\cos t)(1-\cos t)\,dt \longrightarrow 1{,}0$$

$$= \int_0^{2\pi} (1-\cos t)^2\,dt = \int_0^{2\pi} \left(1 - 2\cos t + \frac{1}{2} + \frac{\cos 2t}{2}\right) dt$$

$$= \int_0^{2\pi} \frac{3}{2}\,dt = \frac{3}{2}\,t\Big|_0^{2\pi} = 3\pi \longrightarrow 0{,}5$$