T_4 ( ) F520 ( ) MS550 · Nome: ____________________ RA: __________

Seja $E(x)$ a função definida como

$$E(x) = \frac{\pi}{2}\ {}_2F_1\left(1/2, -1/2, 1; x^2\right).$$

Mostre que $E(x)$ satisfaz a equação diferencial

$$\frac{d}{dx}\left(x\frac{dy}{dx}\right) + \frac{x}{1-x^2}y = 0.$$

Como $F(\alpha,\beta,\gamma;t)$ satisfaz

$$t(1-t)\frac{d^2y}{dt^2} + [\gamma - (\alpha+\beta+1)t]\frac{dy}{dt} - \alpha\beta y = 0$$

então $E(x) = \frac{\pi}{2}\,{}_2F_1(\frac{1}{2}, -\frac{1}{2}, 1; x^2)$ satisfaz

$$t(1-t)\frac{d^2y}{dt^2} + \left[1 - \left(\frac{1}{2} + \left(-\frac{1}{2}\right) + 1\right)t\right]\frac{dy}{dt} - \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)y = 0$$

com $t = x^2$. Passando para x:

$$\frac{d}{dt} = \frac{1}{2x}\frac{d}{dx} \quad ; \quad \frac{d^2}{dt^2} = \frac{1}{4x^2}\frac{d}{dx} - \frac{1}{4x^3}\frac{d}{dx}$$

$$\therefore\ x^2(1-x^2)\left[\frac{1}{4x^2}\frac{d^2y}{dx^2} - \frac{1}{4x^3}\frac{dy}{dx}\right] + (1-x^2)\frac{1}{2x}\frac{dy}{dx} + \frac{1}{4}y = 0$$

$$(1-x^2)\frac{d^2y}{dx^2} - \frac{(1-x^2)}{x}\frac{dy}{dx} + \frac{2(1-x^2)}{x}\frac{dy}{dx} + y = 0$$

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{x}{1-x^2}y = 0$$

$$\frac{d}{dx}\left(x\frac{dy}{dx}\right) + \frac{x}{1-x^2}y = 0$$

x2,0

FORMULÁRIO

$${}_2F_1(\alpha,\beta,\gamma;x) = \frac{1}{B(\beta,\gamma-\beta)}\int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-xt)^{-\alpha}\,dt, \quad {}_2F_1'(\alpha,\beta,\gamma;x) = \frac{\alpha\beta}{\gamma}\,{}_2F_1(\alpha+1,\beta+1,\gamma+1;x),$$

$$x(1-x)y'' + [\gamma - (\alpha+\beta+1)x)]y' - \alpha\beta y = 0, \quad {}_2F_1(\alpha,\beta,\gamma;x) = \sum_{n=0}^{\infty}\frac{(\alpha)_n(\beta)_n}{(\gamma)_n}\frac{x^n}{n!}, \quad (\alpha x^2+\beta)y'' + \alpha x y' + \gamma y = 0.$$