Gabarito Q1 - Sexta Feira:

a) (1,5 ponto): Teorema de Green: $\iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\, dA = \int_C P dx + Q dy$ (0,5 ponto)

aplicando corretamente, $\Rightarrow$ $Q = x$ $P = 0$ implica o resultado crédito completo.

b) (1,0 ponto): Achou a integral $3\int_0^{2\pi} \cos^2 x \,\text{sen}^2 x \, dx$ (0,5 ponto)

achou o resultado correto $\frac{3\pi}{4}$ (0,8 ponto)

+ verificou que C estava com orientação correta - pto integral

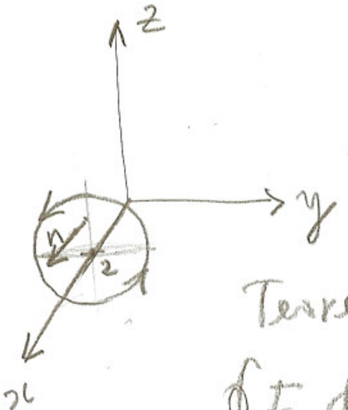
2ª Quartão — sexta feira

$$F = (2xyz - 2y,\ x^2z + 2x,\ x^2y + 2y)$$

$$\text{rot}(F) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xyz - 2y & x^2z + 2x & x^2y + 2y \end{vmatrix} =$$

$$= (x^2 + 2 - x^2,\ -2xy + 2xy,\ 2xz + 2 - 2xz + 2)$$

$$= (2, 0, 4)$$



Orientação figura

$$n = (1, 0, 0)$$

Teorema Stokes

$$\oint_C F \cdot dr = \iint_S \text{rot}(F) \cdot n \, dS =$$

$$= \iint_S (2,0,4) \cdot (1,0,0)\, dy\, dz = 2 \iint_S dy\, dz =$$

$$= 2\,(\text{área de } S) = 2\pi.$$

Parametrizando $\gamma(r, \theta) = (2,\ r\cos\theta,\ r\operatorname{sen}\theta)$

$$\gamma_r \times \gamma_\theta = (r, 0, 0)$$

$$\iint_S (2, 0, 4) \cdot (r, 0, 0)\, dr\, d\theta = 2 \iint_S r\, dr\, d\theta = 2\pi$$

VALORES: Teorema 0,5 pt
Orientação e o vetor n — 1,0 pt
rot(F) 0,5 pt
contas 0,5 pt

Sexta - feira

3) a) $\vec{F}$ definido em $\mathbb{R}^3$ simplesmente conexo, então $\vec{F}$ é conservativo $\Longleftrightarrow \nabla \times \vec{F} = \vec{0}$.

$$\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ e^z & 2yz & xe^z + y^2 \end{vmatrix} = (2y - 2y, e^z - e^z, 0 - 0) = \vec{0}$$

0,4 0,4 0,2

$\therefore \vec{F}$ é conservativo.

b) $f_x = e^z \Rightarrow f = xe^z + g(y,z) \Rightarrow f_y = g_y = 2yz \Rightarrow g = y^2 z + h(z)$

0,4 0,4

$\therefore f = xe^z + y^2 z + \underset{0}{h(z)}$, onde h = cte que pode ser tomada como zero.

0,2

c) $\int_C \vec{F} \cdot d\vec{r} = \int_C (\nabla f) \cdot d\vec{r} \overset{TF}{\underset{IL}{=}} f(\vec{r}(2\pi)) - f(\vec{r}(0)) = f(1,0,2\pi) - f(1,0,0) =$

$= 1 . e^{2\pi} + 0^2 . 2\pi - [1 . e^0 + 0^2 . 0] = e^{2\pi} - 1$

0,5

mas desconta 0,1 p/ cálculo errado

Outro modo: $\int_0^{2\pi} [-e^t . \text{sen}\, t + 2 . t . \text{sen}\, t . \cos t + (\cos t . e^t + \text{sen}^2 t)] \, dt =$

0,2

$= \int_0^{2\pi} [\underbrace{e^t . (\cos t - \text{sen}\, t)} + t . \text{sen}\, 2t + \frac{1 - \cos 2t}{2}] \, dt =$

$= \left[e^t . (\text{sen}\, t + \cos t) + t . \frac{-\cos 2t}{2}\right]_0^{2\pi} - \int_0^{2\pi} \left[e^t (\text{sen}\, t + \cos t) - \frac{\cos 2t}{2} + \frac{1 - \cos 2t}{2}\right] dt$

$= e^{2\pi} - e^0 + 0 - \left[\frac{\text{sen}\, 2t}{2} + \frac{t}{2}\right]_0^{2\pi} - \underbrace{\left[e^t (-\cos t + \text{sen}\, t)\right]_0^{2\pi}}_{-e^{2\pi} + e^0} - \underbrace{\int_0^{2\pi} e^t (\cos t - \text{sen}\, t) \, dt}$

Como $\int_0^{2\pi} e^t (\cos t - \text{sen}\, t) \, dt = \frac{2(e^{2\pi} - e^0)}{2}$, então $\boxed{\int_C \vec{F} \cdot d\vec{r} = e^{2\pi} - 1}$

0,3

④ $$\iint_S F\cdot dS \overset{②}{=} \iiint_E \overset{①}{\operatorname{div} F}\, dV$$

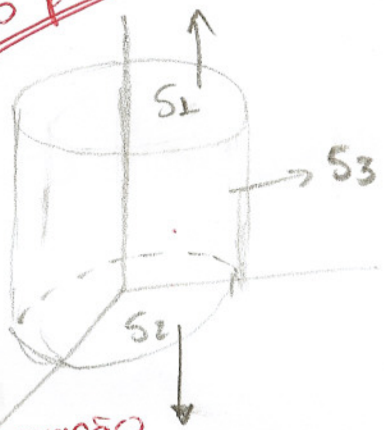
1,0 ponto

$$\operatorname{div} F = z + z + 0 = 2z$$

① $$\iiint_E 2z\, dV = \int_0^{2\pi}\int_0^1\int_0^1 2zr\,dz\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^1 r\,dr\,d\theta = \int_0^{2\pi} \frac{1}{2}\,d\theta = \frac{1}{2}\cdot 2\pi = \pi$$

1,5 pontos



② $$\iint_S F\cdot dS$$

$$\parallel$$

$$\iint_{S_1} F\cdot dS_1 + \iint_{S_2} F\cdot dS_2 + \iint_{S_3} F\cdot dS_3$$

Observação

$$\boxed{\text{Como } \iint_{S_1} F dS_1 = -\iint_{S_2} F dS_2}$$, então

$$\iint_S F\cdot dS = \iint_{S_3} F\cdot dS_3$$

Parametrização de S_3: $r(\theta, z) = \cos\theta\,\hat{i} + \operatorname{sen}\theta\,\hat{j} + z\,\hat{k}$

Calculamos a normal $\quad r_\theta = -\operatorname{sen}\theta\,\hat{i} + \cos\theta\,\hat{j} + 0\,\hat{k}$

$$r_z = 0\,\hat{i} + 0\,\hat{j} + 1\,\hat{k}$$

regrão D. $\begin{cases} 0 \le \theta \le 2\pi \\ 0 \le z \le 1 \end{cases}$

$$r_\theta \times r_z = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\operatorname{sen}\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = \cos\theta\,\hat{i} + \operatorname{sen}\theta\,\hat{j} + 0\,\hat{k}$$

Logo, a integral fica

$$\iint_D (z\,\mathrm{Cos}\,\theta, z\,\mathrm{Sen}\,\theta, 2)\cdot(\mathrm{Cos}\,\theta, \mathrm{Sen}\,\theta, 0)\,dA$$

$$= \iint_D z\,dA = \int_0^{2\pi}\int_0^1 z \cdot dz\,d\theta = \int_0^{2\pi} \frac{1}{2}\,d\theta = \frac{1}{2}\cdot 2\pi = \pi$$