1ª $f(x,y) = 4 + x + y - \ldots$

(1)

$f_x = 3x^2 - 3y = 0 \implies y = x^2$

$f_y = 3y^2 - 3x = 0$

$\implies x^4 - 3x = 0 \iff x(x^3 - 1) = 0$

$\implies x = 0$ ou $x = 1$

Pontos críticos $A = (0,0)$, $B = (1,1)$ → 1,0

Observação:
Note que são apenas 2 pontos críticos!
Caso tenha considerado mais pontos, essa parte vale 0,3

$f_{xx} = 6x$

$f_{yy} = 6y$

$f_{xy} = -3$

$f_{xx}(A) = 0$, $f_{yy}(A) = 0$ e $f_{xy}(A) = -3$

$\implies D = f_{xx} f_{yy} - (f_{xy})^2 = -9 < 0$

$\implies$ A é ponto de sela

0,7

$f_{xx}(B) = 6$, $f_{xy}(B) = 6$ e $f_{xy}(B) = -3$

$\implies D > 0$ e $f_{xx}(A) > 0 \implies$ B é ponto de mínimo

0,8

2ª Q

2

$$f(x,y,z) = x^2+y^2+z^2$$

$$g(x,y,z) = xyz = 1 \text{ — vínculo}$$

$$\nabla f = \lambda \nabla g \iff$$

$$(2x, 2y, 2z) = \lambda (yz, xz, xy)$$

0,5

$$\Rightarrow \begin{cases} 2x = \lambda yz \\ 2y = \lambda xz \\ 2z = \lambda xy \\ xyz = 1 \end{cases} \Rightarrow \begin{cases} 2x^2 = \lambda \\ 2y^2 = \lambda \\ 2z^2 = \lambda \\ xyz = 1 \end{cases}$$

1,0

$$\Rightarrow \begin{cases} x^2 = y^2 = z^2 \\ xyz = 1 \end{cases} \iff x = \pm y,\ y = \pm z \ \text{e}\ xyz = 1$$

Os pontos são: $(1, 1, 1)$

$(1, -1, -1)$

$(-1, 1, -1)$

$(-1, -1, 1)$

1,0

Outra maneira

$$z = \frac{1}{xy}$$

$$h(x,y) = f\left(x, y, \frac{1}{xy}\right) = x^2 + y^2 + \frac{1}{x^2y^2} \Rightarrow$$

3

$$\begin{cases} \dfrac{\partial h}{\partial x} = 2x - \dfrac{2}{x^3 y^2} = 0 & (1) \\ \dfrac{\partial f}{\partial y} = 2y - \dfrac{2}{x^2 y^3} = 0 & (2) \end{cases}$$

0,5

De (1) $\quad x = \dfrac{1}{x^3 y^2} \Longleftrightarrow \boxed{x^4 = \dfrac{1}{y^2}}$

De (2) $\quad y = \dfrac{1}{x^2 y^3} \Longleftrightarrow y^4 = \dfrac{1}{x^2} \Leftrightarrow \boxed{x^2 = \dfrac{1}{y^4}}$

1,0

Então

$$x^4 = \frac{1}{y^8} = \frac{1}{y^2} \Longrightarrow y^8 = y^2 \Rightarrow y^6 = 1$$

$$\Rightarrow \boxed{y = \pm 1} \Longrightarrow x = \pm 1$$

$x = 1,\ y = 1 \Rightarrow z = 1 \Longrightarrow (1, 1, 1)$

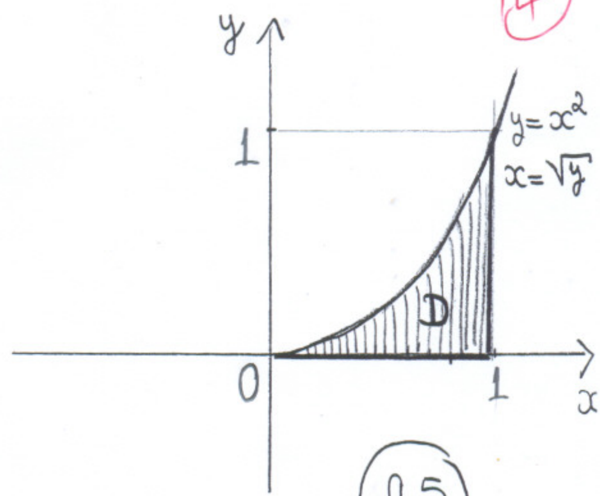
$x = 1,\ y = -1 \Rightarrow z = -1 \Longrightarrow (1, -1, -1)$

$x = -1,\ y = 1 \Rightarrow z = -1 \Longrightarrow (-1, 1, -1)$

$x = -1,\ y = -1 \Rightarrow z = 1 \Longrightarrow (-1, -1, 1)$

1,0

3) $$I = \int_0^1 \int_{\sqrt{y}}^1 \operatorname{sen} x^3 \, dx \, dy$$



$$D : \begin{cases} 0 \leqslant y \leqslant 1 \\ \sqrt{y} \leqslant x \leqslant 1 \end{cases} \iff \begin{cases} 0 \leqslant x \leqslant 1 \\ 0 \leqslant y \leqslant x^2 \end{cases}$$

0,5

0,5

$$I = \int_0^1 \int_0^{x^2} \operatorname{sen} x^3 \, dy \, dx$$

0,5

$$= \int_0^1 \left(y \operatorname{sen} x^3 \Big|_{y=0}^{y=x^2} \right) dx = \int_0^1 x^2 \operatorname{sen} x^3 \, dx$$

0,3

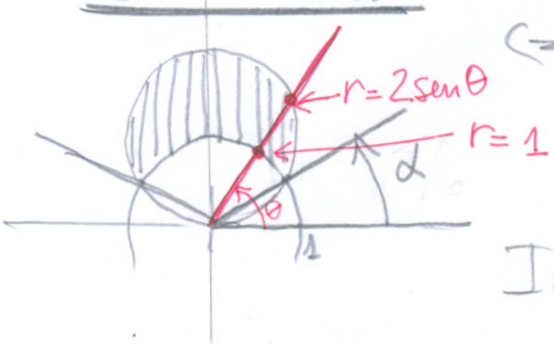
$$\begin{cases} u = x^3 \\ du = 3x^2 dx \end{cases}$$

$$= \frac{1}{3} \int_0^1 \operatorname{sen} u \, du =$$

$$= -\frac{1}{3} \cos u \Big|_0^1 = \frac{1}{3} \left(1 - \cos 1 \right)$$

0,7

4ª Questão



$$x^2+(y-1)^2=1$$

$$\iff \quad x^2+y^2=2y \quad (1)$$

$$x^2+y^2=1. \quad (2)$$

Interseção dos círculos $(y=1/2)$

$$\alpha=\pi/6=30^\circ.$$

0,5

$$x^2+y^2=2y$$

$$r^2=2r\cos\theta \implies r=2\,\text{sen}\,\theta$$

0,3

$$\text{Área}=\int_{\pi/6}^{\frac{5\pi}{6}}\int_{1}^{2\,\text{sen}\,\theta} r\,dr\,d\theta$$

0,7

$$=\int_{\pi/6}^{\frac{5\pi}{6}} \frac{r^2}{2}\Big|_{1}^{2\,\text{sen}\,\theta} d\theta=\int_{\pi/6}^{\frac{5\pi}{6}}\left(2\,\text{sen}^2\theta-\frac{1}{2}\right)d\theta=$$

$$=\int_{\pi/6}^{5\pi/6}\left(\cancel{2}\,\frac{1-\cos 2\theta}{\cancel{2}}-\frac{1}{2}\right)d\theta=\int_{\pi/6}^{5\pi/6}\left(\frac{1}{2}-\cos 2\theta\right)d\theta$$

$$=\left[\frac{\theta}{2}-\frac{\text{sen}\,2\theta}{2}\right]_{\pi/6}^{5\pi/6}=\left[\frac{\theta}{2}-\text{sen}\,\theta\cos\theta\right]_{\pi/6}^{5\pi/6}=$$

$$=\frac{5\pi}{12}+\frac{1}{2}\frac{\sqrt{3}}{2}-\frac{\pi}{12}+\frac{1}{2}\frac{\sqrt{3}}{2}=\boxed{\frac{\pi}{3}+\frac{\sqrt{3}}{2}}$$

Resolver $\int \text{sen}^2\theta\,d\theta$ — vale 0,8

Outra maneira de resolver

$$\int \operatorname{sen}^2\theta \, d\theta =$$

$$= -\int \operatorname{sen}\theta \, (\cos\theta)' \, d\theta \overset{\text{p. partes}}{=}$$

$$= -\operatorname{sen}\theta\cos\theta + \int \cos^2\theta \, d\theta =$$

$$= -\operatorname{sen}\theta\cos\theta + \int (1 - \operatorname{sen}^2\theta) \, d\theta$$

$$\Longrightarrow$$

$$2\int \operatorname{sen}^2\theta \, d\theta = -\operatorname{sen}\theta\cos\theta + \int d\theta$$

$$\Longrightarrow \int \operatorname{sen}^2\theta \, d\theta = \frac{\theta}{2} - \frac{\operatorname{sen}\theta\cos\theta}{2} + C.$$