Gabarito da Q1, prova de 6ª feira,
P3 de MA 211 - 2º sem de 2006

#1) A questão envolvia:

1) Determinar uma mudança de variáveis adequada (0 pts) $\begin{cases} u = x - y \\ v = x + 2y \end{cases} \rightsquigarrow \quad y = \frac{v-u}{3}, \quad x = \frac{2u+v}{3}$

2) Encontrar o domínio de integração adequado (0.6 pts) $0 \le u \le 1$, $0 \le v \le 2$

3) Calcular o Jacobiano (0.6 pts) $\det \begin{bmatrix} \partial x/\partial u & \partial x/\partial v \\ \partial y/\partial u & \partial y/\partial v \end{bmatrix} = \det \begin{bmatrix} 2/3 & 1/3 \\ -1/3 & 1/3 \end{bmatrix} = 1/3$

4) Escrever a integral iterada nas novas variáveis (0.6) pts : $\int_0^2 \int_0^1 \frac{v}{\cos u} \cdot |1/3| \, du \, dv$

5) Calcular a integral iterada (0.7 pts)

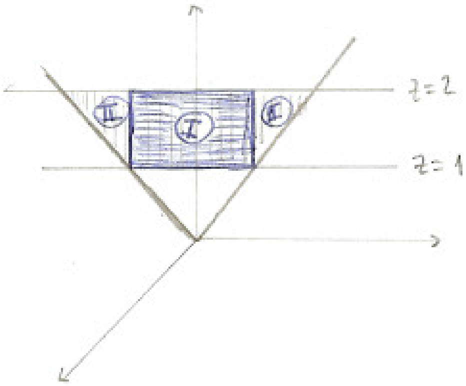
$$\int_0^2 \int_0^1 \frac{v}{\cos u} |1/3| \, du \, dv = 1/3 \int_0^2 v \, dv \int_0^1 \sec u \, du = \frac{2}{3} \log|\sec u + \operatorname{tg} u| \Big|_0^1 =$$

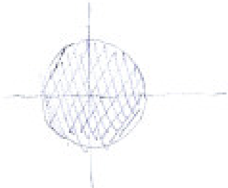
$$= \underline{\frac{2}{3} \log|\sec 1 + \operatorname{tg} 1|}$$

Alguns erros comuns:
+ Esqueceu Jacobiano mas calculou domínio e integral : 1.0
+ Esqueceu o valor absoluto no Jacobiano : 1.5
* Encontrou resposta com log 1, tan 0, sec 0 : 2,0

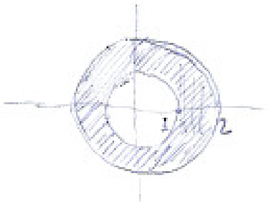
QUESTÃO 2:

(i) coordenadas cilíndricas (1 ponto)



Região I: 

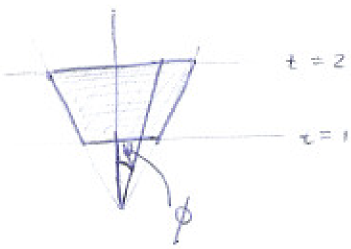
$$E_{r\theta z}: \begin{cases} 0 \le r \le 1 \\ 0 \le \theta \le 2\pi \\ 1 \le z \le 2 \end{cases}$$

Região II: 

$$\begin{cases} 1 \le r \le 2 \\ 0 \le \theta \le 2\pi \\ r \le z \le 2 \end{cases}$$

$$\therefore \left[vol(E) = \int_0^{2\pi}\int_0^1\int_1^2 r \; dz\,dr\,d\theta + \int_0^{2\pi}\int_1^2\int_r^2 r \; dz\,dr\,d\theta. \right]$$

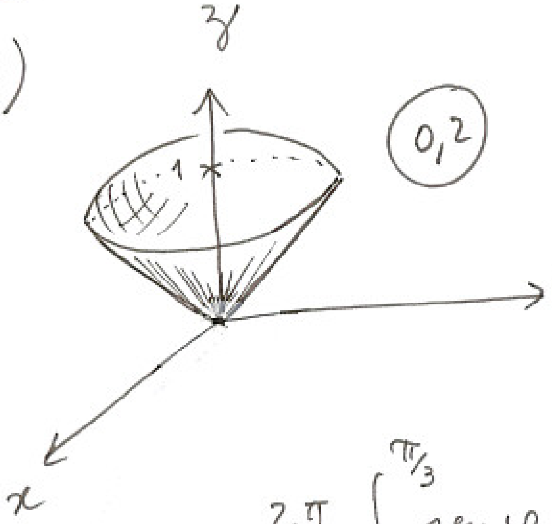
(ii) coordenadas esféricas (1 ponto)



$$E_{\theta\phi\rho}: \begin{cases} 0 \le \theta \le 2\pi \\ 0 \le \phi \le \frac{\pi}{4} \\ \frac{1}{\cos\phi} \le \rho \le \frac{2}{\cos\phi} \end{cases}$$

$$\left[vol(E) = \int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_{\frac{1}{\cos\phi}}^{\frac{2}{\cos\phi}} \rho^2 \cdot \operatorname{sen}\phi \; d\rho\,d\phi\,d\theta. \right]$$

3)



0,2

$$Vol(E)=\int_0^{2\pi}\int_0^{\pi/3}\int_0^{1} \rho^2 \operatorname{sen}\varphi \, d\rho \, d\varphi \, d\theta$$

0,3

$$= 2\pi \int_0^{\pi/3} \left[\frac{\rho^3}{3} \operatorname{sen}\varphi \right]_0^1 d\varphi$$

0,2

$$= \frac{2\pi}{3} \int_0^{\pi/3} \operatorname{sen}\varphi \, d\varphi = \frac{2\pi}{3} \left[-\cos\varphi \right]_0^{\pi/3} = \frac{2\pi}{3}\cdot\frac{1}{2} = \frac{\pi}{3}$$

0,2   0,1

$$\bar{z} = \frac{3}{\pi} \cdot \int_0^{2\pi}\int_0^{\pi/3}\int_0^{1} \rho\cos\varphi \; \rho^2 \operatorname{sen}\varphi \; d\rho \, d\varphi \, d\theta$$

0,5

$$= \frac{3}{\cancel{\pi}} \cdot 2\cancel{\pi} \cdot \int_0^{\pi/3} \left[\frac{\rho^4}{4} \cos\varphi \operatorname{sen}\varphi \right]_0^1 d\varphi$$

0,3

$$= \frac{6}{4} \int_0^{\pi/3} \operatorname{sen}\varphi \, d(\operatorname{sen}\varphi) = \frac{3}{2} \cdot \left[\frac{\operatorname{sen}^2\varphi}{2} \right]_0^{\pi/3}$$

0,5

$$= \frac{3}{4} \cdot \left[\frac{3}{4} - 0 \right] = \frac{9}{16}$$

0,2

4ª Questão — 6ª feira

(a) $x = t\cos t \implies x'(t) = \cos t - t\,\text{sen}\,t$

$y = t\,\text{sen}\,t \qquad y'(t) = \text{sen}\,t + t\cos t$

$$S = \int_0^{2\pi} ds = \int_0^{2\pi} \sqrt{x'(t)^2 + y'(t)^2}\,dt =$$

$$= \int_0^{2\pi} \sqrt{1+t^2}\,dt$$

Valores
derivadas 0,5
fórmula 0,5
contas 0,5
1.5

(b) $\int_C x\,dx + y\,dy =$

$$= \int_0^{2\pi} \Big(t\cos t\,(\cos t - t\,\text{sen}\,t) + t\,\text{sen}\,t\,(\text{sen}\,t + t\cos t) \Big)\,dt$$

$$= \int_0^{2\pi} t\,dt = \frac{t^2}{2}\Big|_0^{2\pi} = 2\pi^2$$

Valores
Montar a integral 1,0
Calcular 0,5
1.5