Métodos I - 1S11 - Lista 2

Resolva as equações diferenciais abaixo utilizando séries (use $x_0 = 0$ exceto quando indicado).

$$\text{(a)} \quad x^2 y'' + x y' + (x^2 - 1/4) y = 0,$$

$$\text{(b)} \quad x(1-x) y'' - 3y' + 2y = 0,$$

$$\text{(c)} \quad x y'' + y' = 0,$$

$$\text{(d)} \quad x^4 y'' + 2x^3 y' - \omega^2 y = 0, \qquad x_0 = +\infty,$$

$$\text{(e)} \quad y'' + x y' + y = 0,$$

$$\text{(f)} \quad y'' + 5x^3 y = 0,$$

$$\text{(g)} \quad 4x y'' + 2(1-x) y' - y = 0,$$

$$\text{(h)} \quad x^2 y'' + x y' + (x^3 - 2) y = 0,$$

$$\text{(i)} \quad 3(x^2 + x) y'' + (x+2) y' - y = 0,$$

$$\text{(j)} \quad 2x y'' + y' - y = 0,$$

$$\text{(k)} \quad 8x^2 y'' + 2x y' + (1-x) y = 0,$$

$$\text{(l)} \quad (x-1) y'' - x y' + y = 0, \qquad x_0 = 1,$$

$$\text{(m)} \quad x^2(1+x) y'' + x(1+x) y' - y = 0,$$

$$\text{(n)} \quad x y'' + (x-1) y' - y = 0.$$

Respostas: (a) $y_1 = x^{-1/2}\cos x$, $y_2 = x^{-1/2}\sin x$; (b) $y_1 = x^2 + 2x + 3$, $y_2 = x^4/(1-x)^2$; (c) $y_1 = 1$, $y_2 = \ln x$; (d) $y_1 = \sum_{k=0}^{\infty} (\omega x)^{-2k}/(2k)!$, $y_2 = \sum_{k=0}^{\infty} (\omega x)^{-2k-1}/(2k+1)!$; (e) a_0 e a_1 arbitrários, $a_k = -\frac{1}{k} a_{k-2}$, $k \geq 2$; (f) a_0 e a_1 arbitrários, $a_2 = a_3 = a_4 = 0$ e $a_k = -\frac{5}{k(k-1)} a_{k-5}$, $k \geq 5$; (g) $a_k = \frac{1}{2k} a_{k-1}$, $a_k = \frac{1}{2k+1} a_{k-1}$, $k \geq 1$; (h) $a_k = -\frac{1}{k(k+2\sqrt{2})} a_{k-3}$, $a_k = -\frac{1}{k(k-2\sqrt{2})} a_{k-3}$, $k \geq 3$, $a_1 = a_2 = 0$; (i) $a_{k+1} = -\frac{(3k-2)(3k+2)}{(3k+3)(3k+4)} a_k$, $k \geq 1$, $y_2 = 1 + x/2$; (j) $a_k = \frac{1}{k(2k-1)} a_{k-1}$, $a_k = \frac{k(2k+1)}{a}{}_{k-1}$, $k \geq 1$; (k) $a_k = a_{k-1}/[8(k+r)(k+r-1) + 2(k+r) + 1]$, $k \geq 1$, $r_1 = -1/2$, $r_2 = -1/4$; (l) $a_k = \frac{1}{k+2} a_{k-1}$, $k \geq 1$, $y_2 = x$; (m) $a_k = -\frac{k}{k+2} a_{k-1}$, $k \geq 1$, $y_2 = 1 + x^{-1}$; (n) $a_k = -\frac{1}{k+2}$, $k \geq 1$, $y_2 = 1 - x$.