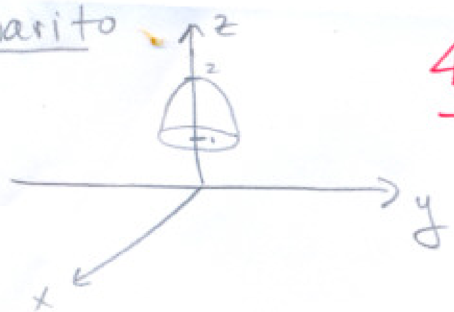
Gabarito

(1)



4ª PROVA

Considere o sólido E cujas fronteiras são S e T, onde T é dado pelo conjunto

$$\{(x,y,z) \in \mathbb{R}^3 : x^2+y^2 \leq 1,\ z=1\}$$

Verificar as hipoteses do teorema da Divergência

Logo

$$\iint_{S \cup T} F\, d\vec{S} = \iiint_E \operatorname{div} F\, dV = 0 \text{ , pois } \operatorname{div} F = 0 \longrightarrow 1 \text{ pt}$$

Assim

$$\iint_S F\, d\vec{S} = -\iint_T F\, d\vec{S}$$

Uma parametrização para T é $r(x,y)=(y,x,1)$

$$D = \{(x,y) : x^2+y^2 \leq 1\} \longrightarrow 0{,}5$$

$r_x = (0,1,0)$, $r_y = (1,0,0)$, $r_x \times r_y = -k$

$$F(r(x,y)) \cdot r_x \times r_y = \frac{1}{(x^2+y^2+1)^{3/2}}(y,x,1)\cdot(0,0,-1) = \frac{-1}{(x^2+y^2+1)^{3/2}}$$

$$\iint_S F\, d\vec{S} = -\iint_T F\, d\vec{S} = \iint_D \frac{1}{(x^2+y^2+1)^{3/2}}\, dA$$

$x = r\cos\theta$ $\quad 0 \leq r \leq 1$
$y = r\operatorname{sen}\theta$ $\quad 0 \leq \theta\ 2\pi$

$$= \int_0^1 \int_0^{2\pi} \frac{r}{(r^2+1)^{3/2}}\, d\theta\, dr = \pi(2-\sqrt{2})$$

$$\longrightarrow 1 \text{ pt}$$

2) - Sejam $x = r\cos t$ e $y = r\,\text{sen}\,t$, então

$$z = \sqrt{x^2 - y^2} = \sqrt{r^2\cos^2 t - r^2\text{sen}^2 t} = r\sqrt{\cos 2t}$$

onde $\cos(2t) \geq 0$, $\cos t \geq 0$ e $\text{sen}\,t \geq 0$ implicam que $0 \leq t \leq \pi/4$. Como o pedaço do cone está dentro do cilindro temos que $r^2\cos^2 t + r^2\text{sen}^2 t \leq a^2$, ou seja, $r \leq a$.

Parametrização para S:

$$S(r,t) = (r\cos t,\, r\,\text{sen}\,t,\, r\sqrt{\cos 2t}) \quad 0{,}5$$

$$D = \{(r,t) : 0 \leq r \leq a,\ 0 \leq t \leq \pi/4\} \quad 0{,}5$$

$$S_r = (\cos t,\, \text{sen}\,t,\, \sqrt{\cos 2t})$$

$$S_t = \left(-r\,\text{sen}\,t,\, \cos t,\, -\frac{r\,\text{sen}(2t)}{\sqrt{\cos(2t)}}\right)$$

$$S_r \times S_t = \begin{vmatrix} i & j & k \\ \cos t & \text{sen}\,t & \sqrt{\cos 2t} \\ -r\,\text{sen}\,t & r\cos t & -\frac{r\,\text{sen}(2t)}{\sqrt{\cos(2t)}} \end{vmatrix}$$

$$= \left(-\frac{r\cos t}{\sqrt{\cos(2t)}},\, \frac{r\,\text{sen}\,t}{\sqrt{\cos(2t)}},\, r\right)$$

$$|S_r \times S_t| = r\sqrt{\frac{1+\cos(2t)}{\cos(2t)}} \quad 0{,}5$$

A área da superfície S é

$$A(S) = \iint_D |S_r \times S_t|\, dA = \int_0^{\pi/4}\int_0^a r\sqrt{\frac{1+\cos(2t)}{\cos(2t)}}\, dr\, dt = \frac{\pi a^2}{4}$$

3. a) Uma representação paramétrica é:

$$r(x,y) = \left(x,\ y,\ \frac{x^2}{a^2} + \frac{y^2}{b^2}\right) \longrightarrow 0{,}5$$

$$D = \mathbb{R}^2 \longrightarrow 0{,}5$$

b) $r_x = \left(1,\ 0,\ \dfrac{2x}{a^2}\right) \qquad r_y = \left(0,\ 1,\ \dfrac{2y}{b^2}\right)$

$$r_x \times r_y = \begin{vmatrix} i & j & k \\ 1 & 0 & \frac{2x}{a^2} \\ 0 & 1 & \frac{2y}{b^2} \end{vmatrix} = \left(-\frac{2x}{a^2},\ -\frac{2y}{b^2},\ 1\right) \longrightarrow 0{,}75$$

Agora $r(x_0, y_0) = (-a\pi, 0, \pi^2) \Leftrightarrow x_0 = -a\pi,\ y_0 = 0$

Assim $(r_u \times r_v)_{(-a\pi,0)} = \left(\dfrac{2\pi}{a},\ 0,\ 1\right) \longrightarrow 0{,}25$

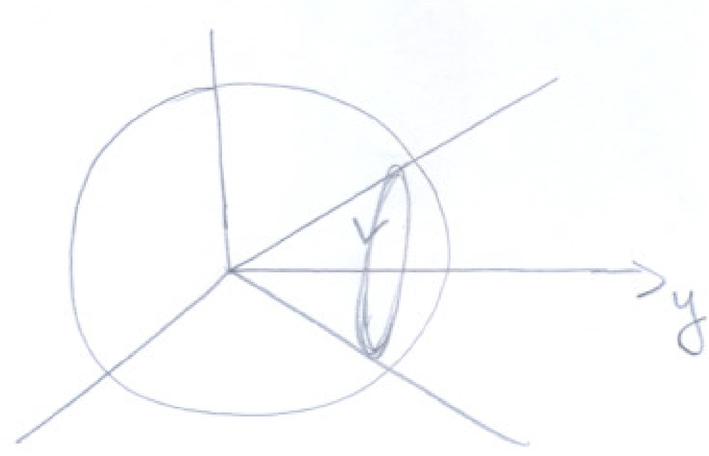
A equação do plano tangente é:

$$(r_x \times r_y)_{(-a\pi,0)} \cdot (x + a\pi,\ y,\ z - \pi^2) = 0$$

$$\left(\frac{2\pi}{a},\ 0,\ 1\right) \cdot (x + a\pi,\ y,\ z - \pi^2) = 0$$

$$2\pi(x + a\pi) + a(z - \pi^2) = 0 \longrightarrow 0{,}5$$

4. 1



Verificar as hipoteses do teorema de Stokes

Logo

0,5

4

$$\int_C F\, d\vec{r} = \iint_S rot(F)\, d\vec{S} = \iint_S rot(F) . n\, dS$$

$$rot\, F = (2y - 2z,\ 2z - 3x^2,\ 3x^2 - 2y)$$

$$n = (0,1,0)$$

1 pt

Portanto

$$\int_C F\, d\vec{r} = \iint_S (2z - 3x^2)\, dS$$

$x = r\cos\theta \quad 0 \leq r \leq 1$

$z = r\,sen\,\theta \quad 0 \leq \theta \leq 2\pi$

$$= 2\int_0^1 \int_0^{2\pi} r^2 sen\theta\, dr\, d\theta - 3\int_0^1 \int_0^{2\pi} r^3 \cos^2\theta\, d\theta\, dr$$

$$= -3\left(\frac{r^4}{4}\Big|_0^1\right) \int_0^{2\pi} \cos^2\theta\, d\theta = -\frac{3}{4}\int_0^{2\pi}\left(\frac{1+\cos 2\theta}{2}\right) d\theta$$

$$= -\frac{3}{4} \cdot 2\pi \cdot \frac{1}{2} = -\frac{3}{4}\pi$$

1 pt