EA044A – Planejamento e Análise de Sistemas de Produção

1o. Semestre de 2009 - Prova 2 - Prof. Vinícius A.Armentano

Questão 1

a)

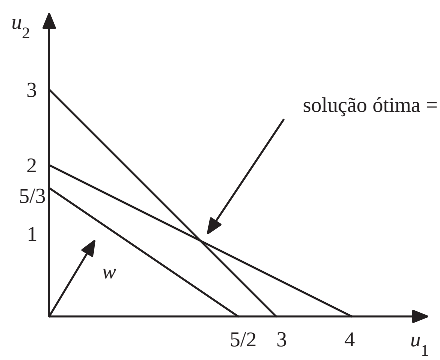
Problema primal

$$\begin{array}{rcccl} \max z = & 5x_1 & +3x_2 & +4x_3 & \\ & 2x_1 & +x_2 & +x_3 & \leq 18 \\ & 3x_1 & +x_2 & +2x_3 & \leq 30 \\ & x_1 \geq 0 & x_2 \geq 0 & x_3 \geq 0 & \end{array}$$

Problema dual

$$\begin{array}{rccl} \min w = & 18u_1 & +30u_2 & \\ & 2u_1 & +3u_2 & \geq 5 \\ & u_1 & +u_2 & \geq 3 \\ & u_1 & +2u_2 & \geq 4 \\ & u_1 \geq 0 & u_2 \geq 0 & \end{array}$$

b)



c) Solução do dual: $w = 66,\ u_1 = 2,\ u_2 = 1$. Como a primeira restrição do dual não está ativa, tem-se $x_1 = 0$. Portanto, a solução ótima do primal é dada pela solução do sistema.

$$\begin{array}{ccl} x_2 & +x_3 & = 18 \\ x_2 & +2x_3 & = 30 \end{array}$$

que fornece $x_2 = 6,\ x_3 = 12,\ z = 66.$

Questão 2

a)

Para que o lucro de x_3 seja competitivo, $\bar{c}_3^{'} = \bar{c}_3 - \delta \leq 0$, e portanto, $\delta \geq 2$.

Daí, $c_3^{'} = c_3 + \delta \geq c_3 + 2 = 15$.

b)

$$\mathbf{a}_1^{'} = \mathbf{B}^{-1}\mathbf{a}_1 = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$$\bar{c}_1^{'} = c_1^{'} - \mathbf{c_B}\mathbf{B}^{-1}\mathbf{a}_1 = 24 - \begin{bmatrix} 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix} = 14$$

z	x_1	x_2	x_3	s_1	s_2	LD	VB
1	-14		2	5		100	z
	2	1	3	1		20	x_2
	-3		-2	-4	1	10	s_2

x_1 entra na base e $\min\{20/2\} = 10 \rightarrow x_2$ sai da base.

z	x_1	x_2	x_3	s_1	s_2	LD	VB
1		7	24	12		240	z
	1	$1/2$	$3/2$	$1/2$		10	x_1
		$3/2$	$-5/2$	$-5/2$	1	40	s_2

c)

$$\bar{c}_4^{'} = c_4^{'} - \mathbf{c_B}\mathbf{B}^{-1}\mathbf{a}_4 = 20 - \begin{bmatrix} 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 5 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix} = -5$$

$$\mathbf{a}_4^{'} = \mathbf{B}^{-1}\mathbf{a}_4 = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 5 \end{bmatrix} = \begin{bmatrix} 5 \\ -15 \end{bmatrix}$$

z	x_1	x_2	x_3	x_4	s_1	s_2	LD	VB
1	10		2	5	5		100	z
	1	1	3	5	1		20	x_2
	8		-2	-15	-4	1	10	s_2

d)

$$\mathbf{B} = \begin{bmatrix} 1/2 & 0 \\ 1 & 1 \end{bmatrix} \qquad \mathbf{B}^{-1} = \begin{bmatrix} 2 & 0 \\ -2 & 1 \end{bmatrix}$$

$$\mathbf{c_B}\mathbf{B}^{-1} = \begin{bmatrix} 10 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 20 & 0 \end{bmatrix}$$

$$\bar{c}_1 = c_1 - \mathbf{c_B}\mathbf{B}^{-1}\mathbf{a}_1 = -5 - \begin{bmatrix} 20 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 12 \end{bmatrix} = -15$$

$$\bar{c}_2 = c_2 - \mathbf{c_B}\mathbf{B}^{-1}\mathbf{a}_3 = 10 - \begin{bmatrix} 20 & 0 \end{bmatrix} \begin{bmatrix} 1/2 \\ 1 \end{bmatrix} = 0$$

$$\bar{c}_3 = 13 - \begin{bmatrix} 10 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 10 \end{bmatrix} = -47$$

$$\bar{c}_{s_1} = 0 - \begin{bmatrix} 10 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = -20$$

$$\bar{c}_{s_2} = 0 - \begin{bmatrix} 10 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0$$

$$\mathbf{a}_1^{'} = \mathbf{B}^{-1}\mathbf{a}_1 = \begin{bmatrix} 2 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

$$\mathbf{a}_3' = \mathbf{B}^{-1}\mathbf{a}_3 = \begin{bmatrix} 2 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

$$\mathbf{a}_{s_1}' = \mathbf{B}^{-1}\mathbf{a}_{s_1} = \begin{bmatrix} 2 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

$$\mathbf{B}^{-1}\mathbf{b} = \begin{bmatrix} 2 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 90 \end{bmatrix} = \begin{bmatrix} 40 \\ 50 \end{bmatrix}$$

$$\mathbf{c_B}\mathbf{B}^{-1}\mathbf{b} = 400$$

z	x_1	x_2	x_3	s_1	s_2	LD	VB
1	15		47	20		400	z
	1	1	2	2		40	x_2
	8		6	6	1	50	s_2

e)

$2x_1 + 2x_2 + 5x_3 + s_3 = 30$

z	x_1	x_2	x_3	s_1	s_2	s_3	LD	VB
1	10	0	2	5	0	0	100	z
	1	1	3	1	0	0	20	x_2
	8	0	-2	-4	1	0	10	s_2
	0	2	5	0	0	1	30	s_3

z	x_1	x_2	x_3	s_1	s_2	s_3	LD	VB
1	10	0	2	5	0	0	100	z
	1	1	3	1	0	0	20	x_2
	8	0	-2	-4	1	0	10	s_2
	2	0	$\boxed{-1}$	-2	0	1	-10	s_3

s_3 sai da base. $\max\{-2/1, -5/2\} = -2 \rightarrow x_3$ entra na base.

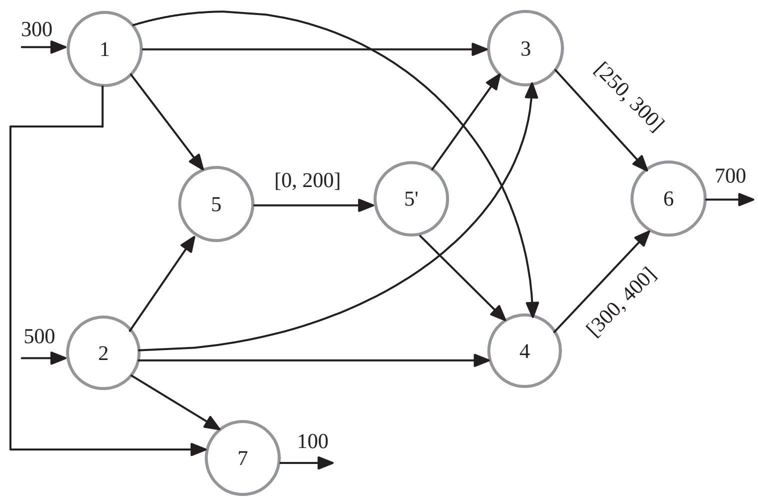
z	x_1	x_2	x_3	s_1	s_2	s_3	LD	VB
1	10			1		2	80	z
	1	1		-5		3	-10	x_2
	8			0	1	-2	30	s_2
	2		1	2		-1	10	x_3

x_2 sai da base. $\max\{-1/5\} = -1/5 \rightarrow s_1$ entra na base.

z	x_1	x_2	x_3	s_1	s_2	s_3	LD	VB
1	$51/5$	$1/5$				$13/5$	78	z
	$-1/5$	$-1/5$		1		$-3/5$	2	x_2
	8	0			1	-2	30	s_2
	$2/5$	$2/5$	1			$1/5$	6	x_3

Questão 3

a)



$$
\begin{array}{cccccccccccccc}
220x_{13} & +280x_{14} & +80x_{15} & & +140x_{23} & +170x_{24} & +100x_{25} & & & +40x_{5'3} & +50x_{5'4} & & & \\
x_{13} & +x_{14} & +x_{15} & +x_{17} & & & & & & & & & & = 300 \\
 & & & & x_{23} & +x_{24} & +x_{25} & +x_{27} & & & & & & = 500 \\
 & & -x_{15} & & & & -x_{25} & & +x_{55'} & & & & & = 0 \\
 & & & -x_{17} & & & & -x_{27} & & & & & & = -100 \\
 & & & & & & & & -x_{55'} & +x_{5'3} & +x_{5'4} & & & = 0 \\
-x_{13} & & & & -x_{23} & & & & & -x_{5'3} & & +x_{36} & & = 0 \\
 & -x_{14} & & & & -x_{24} & & & & & -x_{5'4} & & +x_{46} & = 0 \\
 & & & & & & & & & & & -x_{36} & -x_{46} & = -700
\end{array}
$$

Canalização das variáveis, por exemplo, $250 \leq x_{36} \leq 300$.

Solução ótima:

$$x_{15} = 200,\ x_{17} = 100,\ x_{23} = 300,\ x_{24} = 200,\ x_{55'} = 200,\ x_{5'4} = 200,\ x_{36} = 300,\ x_{46} = 400$$