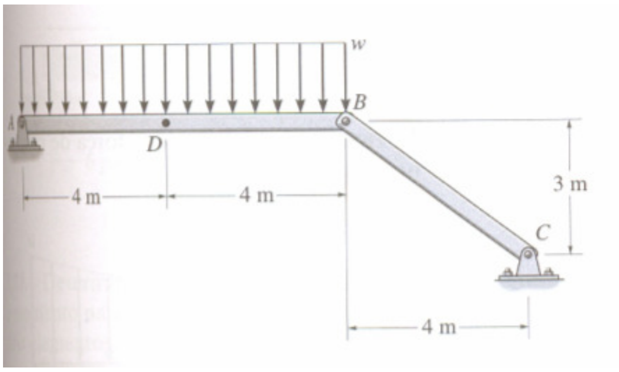
# EM 406 - RESISTÊNCIA DE MATERIAIS

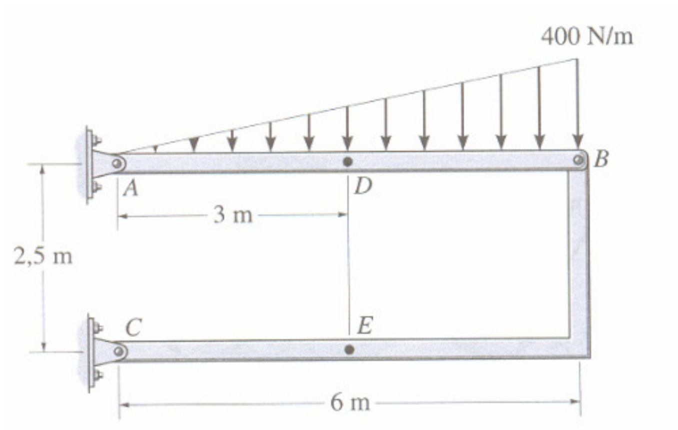
## LISTA DE EXERCICIOS 3

Resolva os exercícios 1 a 13 utilizando as técnicas de equações diferenciais e funções de singularidade, sempre que possível, e verifique pelo método das seções. Faça um esboço dos diagramas de V(x) e M(x) indicando os valores máximos e verifique as condições de contorno.

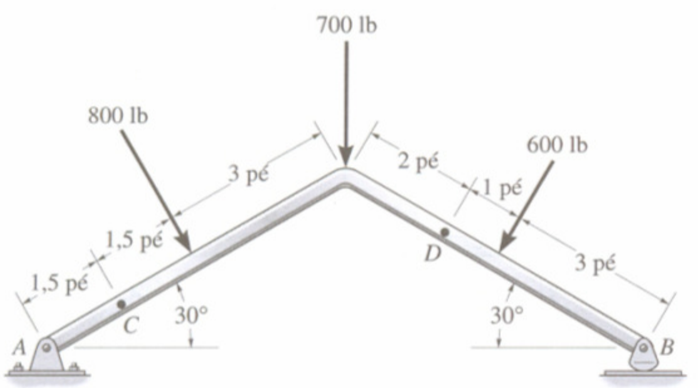
1- A viga AB cederá se o momento interno máximo em D atingir o valor de 800 Nm ou a força normal no elemento BC for de 1500 N. Determine a maior carga *w* que pode ser sustentada pela viga. (Resp.: $w$ = 100 N/m)



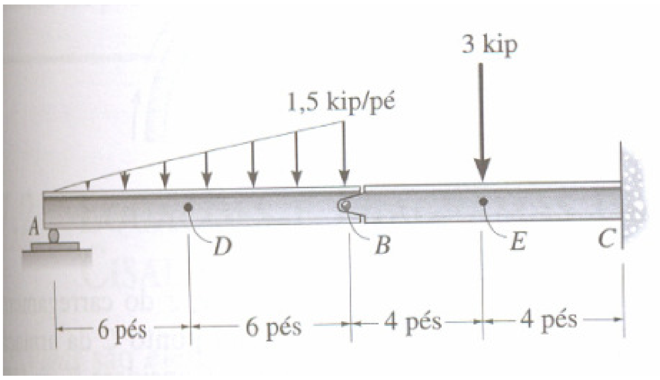

2- Determine a força normal, a força de cisalhamento (cortante) e o momento na seção transversal que passa pelo ponto D da estrutura de dois elementos. (Resp.: $N_D$ = 1.92 KN, $V_D$ = 100 N, $M_D$ = 900 Nm)



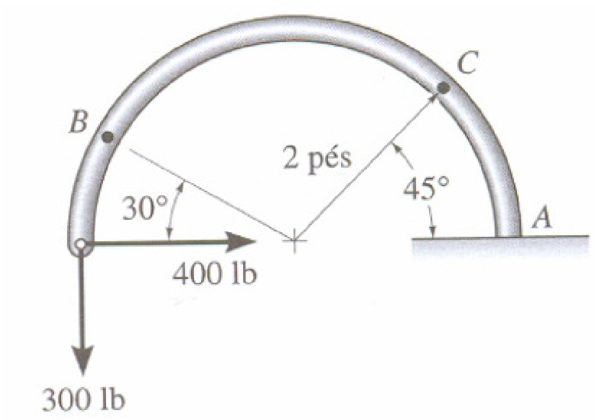
3- Determine a força normal, a força de cisalhamento e momento atuantes na seção que passa pelo ponto D. (Resp.: $N_D$ = -464 lb, $V_D$ = -203 lb, $M_D$ = 2.61 kip.pés)




4- Determine a força normal, cortante e momento sobre a viga nas seções que passam pelos pontos D e E. O ponto E está imediatamente à direita da carga de 3 kip. (Resp.: $N_D$ = 0, $V_D$ = 0.75 kip, $M_D$ = 13.5 kip.pés; $N_E$ = 0, $V_E$ = -9 kip, $M_E$ = -24 kip.pés)



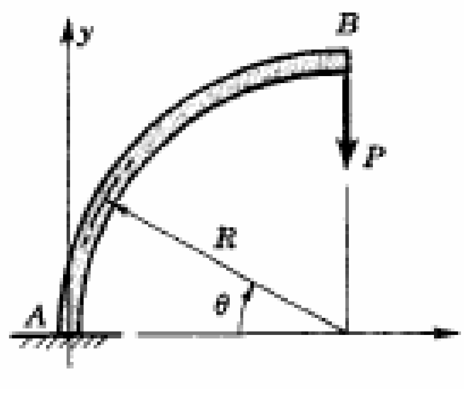
5- Determine a força normal, a força de cisalhamento e o momento atuantes nas seções que passam pelos pontos B e C na viga curva. (Resp: $N_B$ = 59.8 lb, $V_B$ = -469 lb, $M_D$ = -480 lb.pés; $N_C$ = -495 lb, $V_C$ = 70.7 lb, $M_C$ = -1.59 kip.pé)




6-

**3.9.** Determine the axial force, the shear force, and the bending moment acting at any section $\theta$ in the circular arc $AB$.

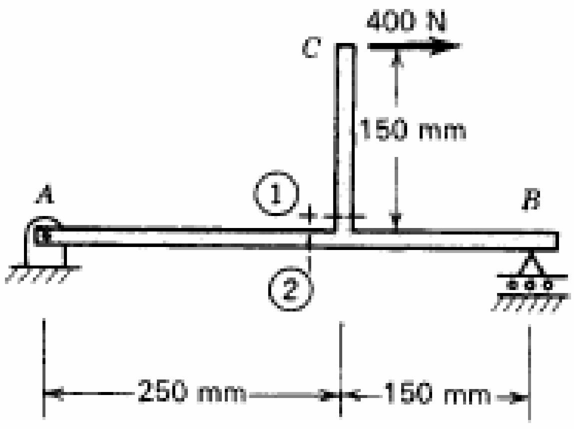
185




**Prob. 3.9**

7-

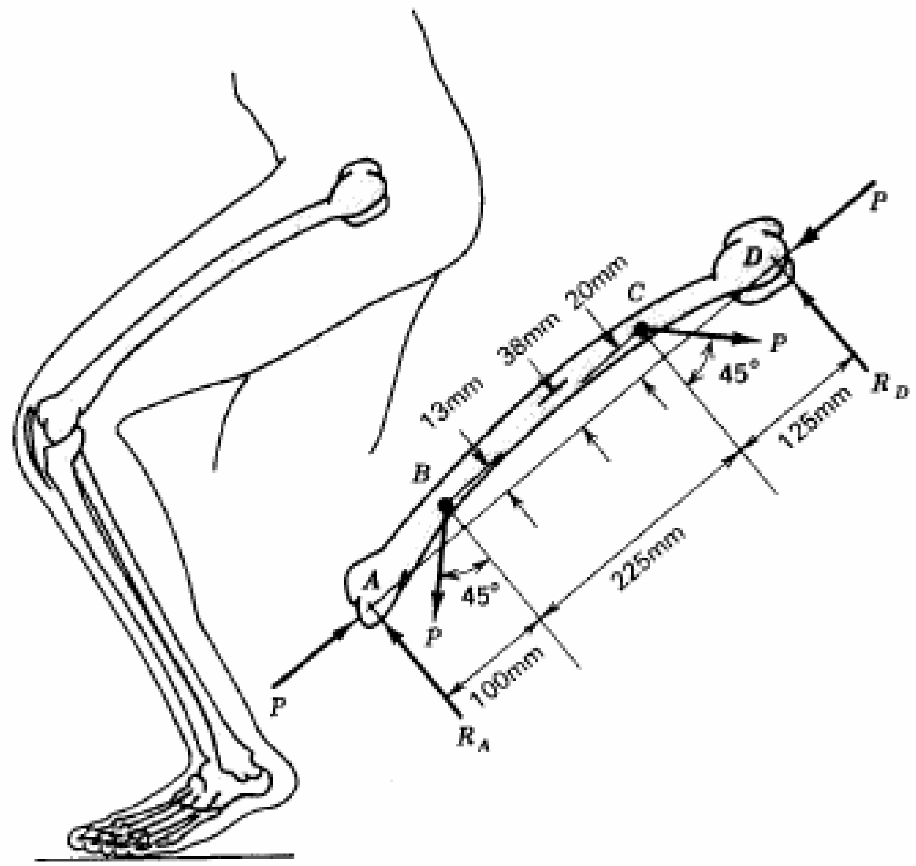
**3.14.** Calculate the internal forces and moments acting at sections 1 and 2 in the structure shown.




**Prob. 3.14**

8 –

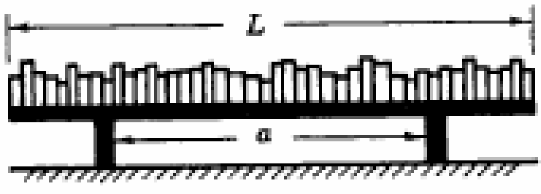
**3.16.** The sketch shows a possible set of muscle forces acting on the femur of a man who is running upstairs. Find the unknown reactions $R_A$ and $R_D$ in terms of $P$ and show how the transverse force varies along the femoral shaft. Show how the bending moment varies along the shaft, and comment on the compensating effect of the muscles attached at $B$ and $C$ in terms of reducing the bending moments in the shaft.




**Prob. 3.16**

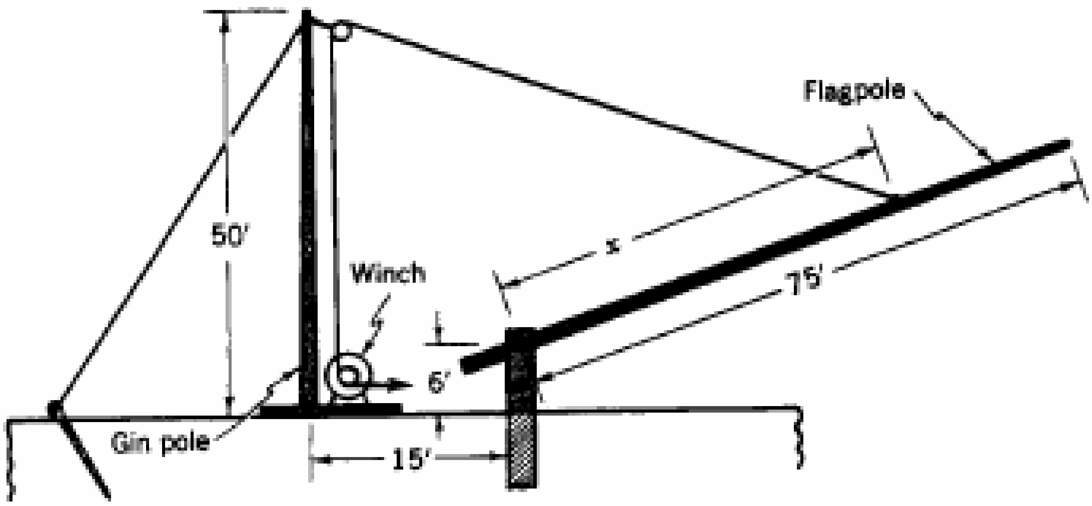
9 – e 10 –

**3.25.** A bookshelf is made by placing a wooden plank on two brick supports. Where should the bricks be placed so as to make the maximum bending moment as small as possible?




**Prob. 3.25**

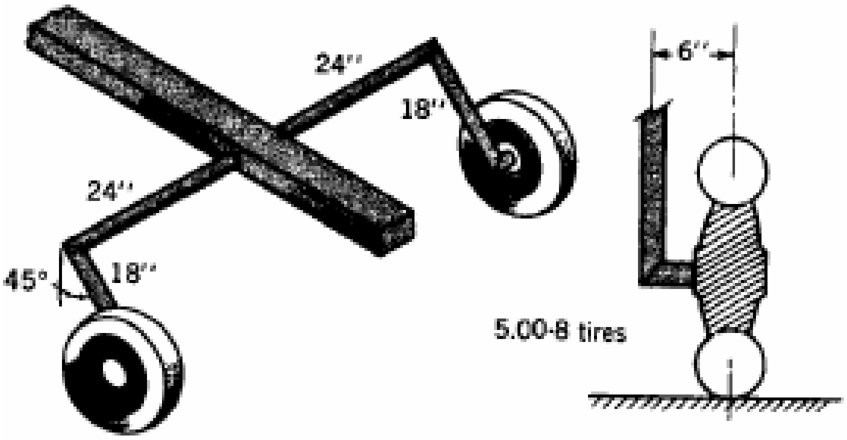
**3.26.** A pivoted flagpole is to be erected by using a gin pole and winch as shown. Where should the rope be attached to the flagpole so that during erection the maximum bending moment is as small as possible?




**Prob. 3.26**

11 –

3.30. A number of small, two-wheeled boat trailers have a suspension system similar to that shown. Estimate the maximum twisting and bending moments in the 18-in. bar and the 24-in. bar when the wheels are carrying loads of 500 lb each. A wheel with a 5.00–8 tire has an outside diameter of 5 in. + 8 in. + 5 in. = 18 in.

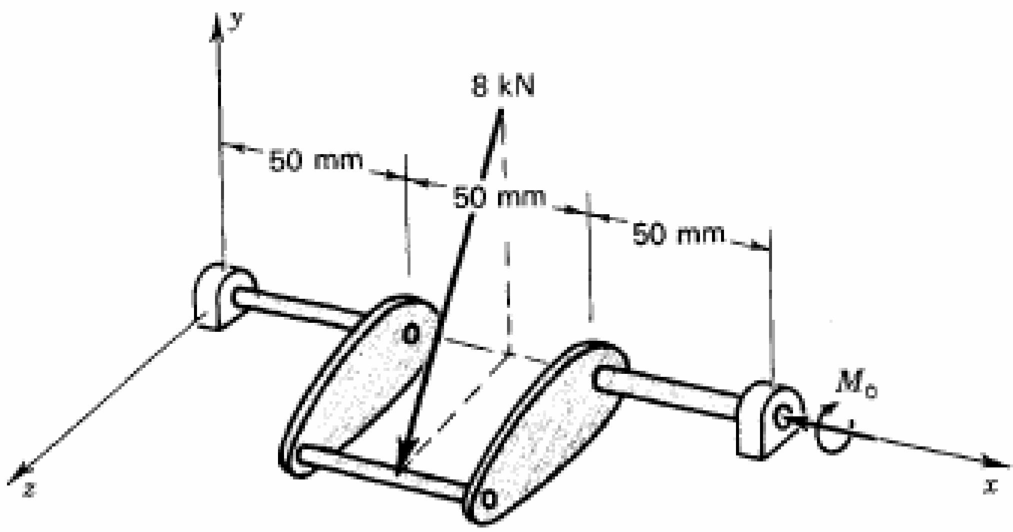



12 –

3.40. A crankshaft for a single-cylinder engine is shown mounted in bearings at each end. It is in equilibrium under the action of the connecting-rod force and the shaft torque $M_o$. The engine has:

Bore 64 mm
Stroke 75 mm
Connecting-rod length 125 mm

Show diagrams for shear, bending moment, and twisting moment for the two end sections of the crankshaft.



Respostas dos exercícios 6 a 12

**3.6.** $V_{max} = w_o L$, $(M_b)_{max} = \frac{1}{2} w_o L^2$, at wall

**3.9.** $F = -P\cos\theta$, $V = P\sin\theta$, $M_b = -PR\cos\theta$

**3.14.** At 1, $F_x = 400$ N, $M_b = 60$ N·m
At 2, $F_x = 400$ N, $F_a = 150$ N, $M_b = 37.5$ N·m

**3.25.** $a = 0.586L$

**3.26.** $x = 0.7L$

**3.28.** $T = 0.432D$

**3.29.** $M_t = 0.988PR$, $M_b = 0.157PR$

**3.30.** $M_b = 6{,}700$ in.-lb, $M_t = 2{,}120$ in.-lb in the 18-in. bar; $M_b = 15{,}000$ in.-lb, $M_t = 6{,}370$ in.-lb in the 24-in. bar

**3.37.** $V_{max} = 339$ lb ($A$ to $B$)
$(M_b)_{max} = 4{,}060$ in.-lb (at $B$)
$(M_t)_{max} = 900$ in.-lb ($B$ to $C$)

**3.39.** $V = 2.0$ kN, $(M_b)_{max} = 733$ N·m
$M_t = 115$ N·m

**3.40.** $V = 4.0$ kN, $(M_b)_{max} = 200$ N·m, $M_t = 282$ N·m

13 – (Exercício do Prof. Euclides)

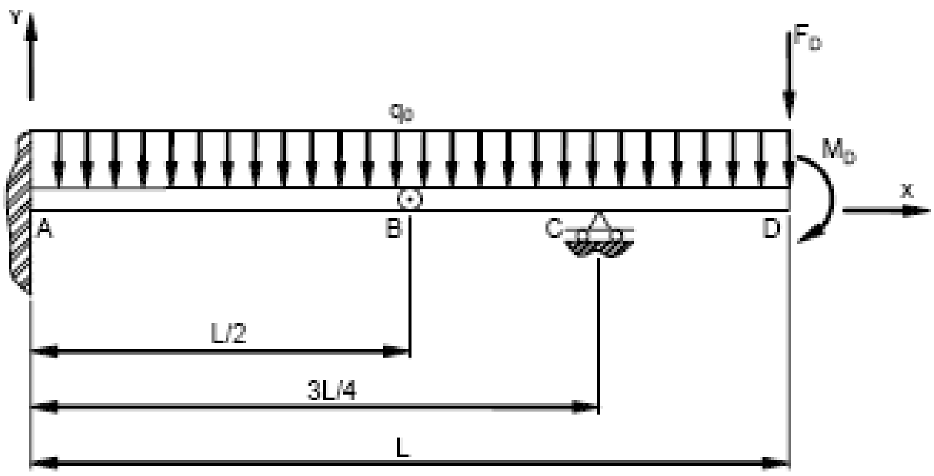
**EXERCÍCIO - EQ DIF VIGA 13:** Viga engastada com um apoio e rótula, carregamento uniforme, força e momento concentrados na extremidade, esforços resultantes

**Enunciado:**

A partir do modelo físico mostrado na Figura eq dif viga 13-1, abaixo, determine os esforços resultantes, esforço cortante $V_y(x)$ e momento fletor $M_z(x)$, atuando na seção transversal.

**Resolução:**

**I) Modelo físico**



14 -

(1) The components of stress at a point in Cartesian coordinates are given by $\sigma_{xx} = 500$; $\sigma_{xy} = \sigma_{yx} = 500$; $\sigma_{yy} = 1000$; $\sigma_{yz} = \sigma_{zy} = -750$, $\sigma_{zx} = 800$; $\sigma_{zz} = -300$.

(2) A plane is defined by the unit vector

$$\mathbf{n} = \frac{1}{2}\mathbf{e}_x + \frac{1}{2}\mathbf{e}_y + \frac{1}{\sqrt{2}}\mathbf{e}_z.$$

(3) It is desired to compute the traction and the normal and tangential components on the plane.

Resp: $T^n = 1104$ ; $T_N^n = 511$ ; $T_S^n = 979$

15 –

Verificar a condição para que o seguinte tensor represente um estado de tensões que satisfaça a equação de equilíbrio, na ausência de forças de corpo.

$$\sigma_{11} = x^2 + y + 3z^2; \quad \sigma_{22} = 2x + y^2 + 2z; \quad \sigma_{33} = -2x + y + z^2$$

$$\sigma_{12} = \sigma_{21} = -xy + z^3; \quad \sigma_{13} = \sigma_{31} = y^2 - xz; \quad \sigma_{23} = \sigma_{32} = x^2 - yz$$

Resp:

$$2x - x - x = 0$$

$$-y + 2y - y = 0$$

$$-z - z + 2z = 0.$$

16 – e 17 –

**2.1** Find the components of the traction on a plane defined by $n_1 = 1/\sqrt{2}$, $n_2 = 1/\sqrt{2}$, $n_3 = 0$ for the following states of stress:

(a) $\sigma_{11} = \sigma, \quad \sigma_{12} = 0, \quad \sigma_{13} = 0,$

$\sigma_{22} = \sigma, \quad \sigma_{23} = 0, \quad \sigma_{33} = \sigma,$

(b) $\sigma_{11} = \sigma, \quad \sigma_{12} = \sigma, \quad \sigma_{13} = 0,$

$\sigma_{22} = \sigma, \quad \sigma_{23} = 0, \quad \sigma_{33} = 0.$

**2.2** The state of stress at a point $P$ in a structure is given by

$$\sigma_{11} = 20{,}000, \quad \sigma_{22} = -15{,}000, \quad \sigma_{33} = 3000$$

$$\sigma_{12} = 2000, \quad \sigma_{23} = 2000, \quad \sigma_{31} = 1000.$$

(a) Compute the scalar components $T_1$, $T_2$, and $T_3$ of the traction $\mathbf{T}$ on the plane passing through $P$ whose outward normal vector $\mathbf{n}$ makes equal angles with the coordinate axes $x_1$, $x_2$, and $x_3$ (from [2.1]).

(b) Compute the normal and tangential components of stress on this plane.

18 – e 19 –

**2.4** Determine the body forces for which the following stress field describes a state of equilibrium (from [2.6]):

$$\sigma_{xx} = -2x^2 - 3y^2 - 5z, \quad \sigma_{xy} = z + 4xy - 6,$$
$$\sigma_{yy} = -2y^2 + 7, \quad \sigma_{xz} = -3x + 2y + 1,$$
$$\sigma_{zz} = 4x + y + 3z - 5 \quad \sigma_{yz} = 0.$$

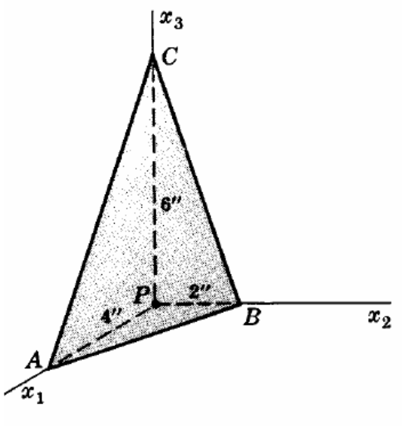
**2.5** Determine whether the following stress field is admissible in an elastic body when body forces are negligible.

$$\boldsymbol{\sigma} = \begin{bmatrix} yz + 4 & z^2 + 2x & 5y + z \\ \cdot & xz + 3y & 8x^3 \\ \cdot & \cdot & 2xyz \end{bmatrix}.$$

20 –

The stress tensor at point P is given by the matrix:

$$\boldsymbol{\sigma} = \begin{bmatrix} 7 & -5 & 0 \\ -5 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$




Determine the stress vector on the plane passing through P and parallel to the plane ABC shown in the figure.