(A)

$$\oint_C \frac{f(z)}{(z-z_0)^n} = \frac{2\pi i}{(n-1)!} f^{(n)}(z_0)$$

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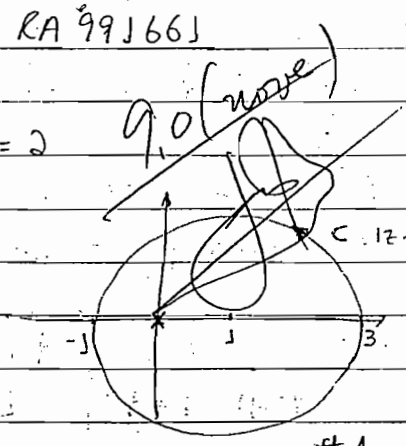
01) $f(z) = \frac{e^{-z}}{z^9}$ $\quad C: |z-1| = 2$



$f(z)$ é analítica p/ $z \neq 0$

$$\int_{C:|z-1|=2} f(z)\,dz = \int_{|z|=\frac{1}{2}} \frac{\overbrace{e^{-z}}^{g(z)}}{(z-\underbrace{0}_{z_0})^9}\,dz = \frac{2\pi i}{8!} g^{(8)}(z_0)\Big|_{z_0=0}$$

#1 – 2/2
#2 – 2/2
#3 – 2/2
#4 – 1,5/2
#5 – 1,5/2

$g(z) = e^{-z}$
$g'(z) = -e^{-z}$
$g''(z) = e^{-z}$
$g^{(3)}(z) = -e^{-z}$
$g^{(4)}(z) = e^{-z}$
$g^{(5)}(z) = -e^{-z}$
$g^{(6)}(z) = e^{-z}$
$g^{(7)}(z) = -e^{-z}$
$g^{(8)}(z) = e^{-z}$

$$\int_{|z-1|=2} f(z)\,dz = \frac{2\pi i}{8!}\left[e^{-z_0}\right]_{z_0=0} = \frac{2\pi i}{8!} e^{-0}$$

$$\int_{|z-1|=2} f(z)\,dz = \frac{2\pi i}{8!} = \frac{\pi i}{20160}$$

03) $f(z) = \frac{(z+3)^3}{z^4+5z^3+6z^2} = \frac{(z+3)^3}{z^2(z^2+5z+6)} = \frac{(z+3)^3}{z^2(z+3)(z+2)}$

$z^2+5z+6=0$

$\Delta = 25-24 = 1$

$z = \frac{-5 \pm 1}{2}$ $\quad z=-3$ $\quad z=-2$

$$\frac{(z+3)^3}{z^2(z+3)(z+2)} = \frac{A_2}{z^2} + \frac{A_1}{z} + \frac{B_1}{z+3} + \frac{C_1}{z+2}$$

$$A_2 = \lim_{z\to 0}\left[\frac{z^2\,(z+3)^3}{z^2(z+3)(z+2)}\right] = \frac{9}{2}$$

$$A_1 = \frac{1}{(2-1)!}\lim_{z\to 0}\left\{\frac{d}{d[z]}\left[\frac{(z+3)^2}{z+2}\right]\cdot\frac{1}{d[z]}\right\} = \frac{3}{4}$$

$$\frac{d}{d[z]}\left[\frac{(z+3)^2}{z+2}\right]\Bigg|_{z=0} = \left[\frac{2(z+3)(z+2)-(z+3)^2}{(z+2)^2}\right]\Bigg|_{z=0} = \frac{12-9}{4} = \frac{3}{4}$$

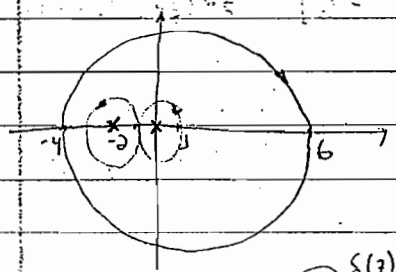
$$\frac{d[z]}{d[z]} = 1$$

$$B_1 = \lim_{z\to -3}\left[\frac{(z+3)\,(z+3)^3}{z^2(z+3)(z+2)}\right] = \lim_{z\to -3}\left[\frac{(z+3)^3}{z^2(z+2)}\right] = 0$$

$$C_1 = \lim_{z\to -2}\left[\frac{(z+2)\,(z+3)^3}{z^2(z+3)(z+2)}\right] = \lim_{z\to -2}\left[\frac{(z+3)^2}{z^2}\right] = \frac{1}{4}$$

$$f(z) = \frac{\frac{9}{2}}{z^2} + \frac{\frac{3}{4}}{z} + \frac{\frac{1}{4}}{z+2} = \frac{\frac{9}{2}z + 9 + \frac{3}{2}z + \frac{3}{4}z^2 + \frac{1}{4}z^2}{z^2(z+2)} = \frac{z^2+6z+9}{z^2(z+2)} = \frac{(z+3)^2}{z^2(z+2)}$$

C: $|z-1| = 5$ — $f(z)$ não é analítica p/ $z=0$ e $z=-2$



no sentido direto

$$\oint_{|z-1|=5} f(z)\,dz = \oint_{|z|=\frac{1}{2}} \frac{\overbrace{\frac{9}{2}}^{g(z)}}{(z-0)^2}\,dz + \oint_{|z|=\frac{1}{2}} \frac{\overbrace{\frac{3}{4}}^{g(z)}}{z}\,dz + \oint_{|z+2|=\frac{1}{2}} \frac{\frac{1}{4}}{[z-(-2)]}\,dz$$

z_0

no sentido direto

$$= \frac{2\pi j}{1!}\left[d\left(\frac{9}{2}\right)\right]\Big|_{z_0=0} + \frac{2\pi j}{0!}\left[\frac{3}{4}\right]\Big|_{z_0=0} + \frac{2\pi j}{0!}\left[\frac{1}{4}\right]_{z_0=-2} = \frac{3}{2}\pi j + \frac{\pi}{2}j = 2\pi j$$