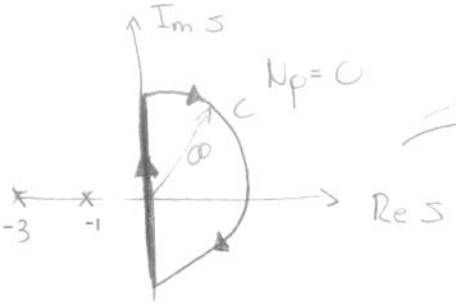
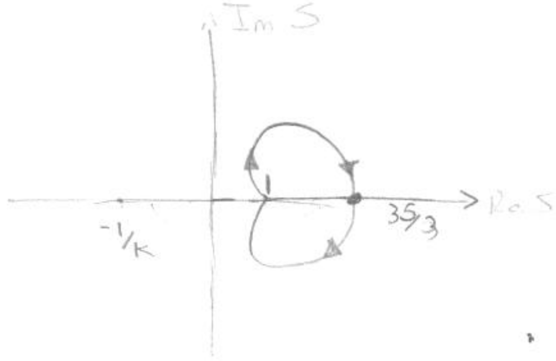
1) a) Eq. característica

$1 + CGH = 0 \Rightarrow 1 + K \dfrac{\overbrace{(s+5)(s+7)}^{N(s)}}{\underbrace{(s+1)(s+3)}_{D(s)}} = 0$





Por Routh

$s^2 + 4s + 3 + Ks^2 + K12s + 35K = (1+K)s^2 + (4+12K)s + 3 + 35K$

s^2	$1+K$	$3+35K$	$1+K>0$	$K>-1$
s^1	$4+12K$		$4+12K>0$	$K>-4/12$
s^0	$3+35K$		$3+35K>0$	$K>-3/35$

$\Rightarrow$ Perguntar qual escolher?

b) $1 + CGH = 0 \quad 1 + K\dfrac{s^2+6s+25}{s(s^2+2s+5)}$

$\hookrightarrow \dfrac{s(s^2+2s+5) + K(s^2+6s+25)}{s(s^2+2s+5)} = 0$

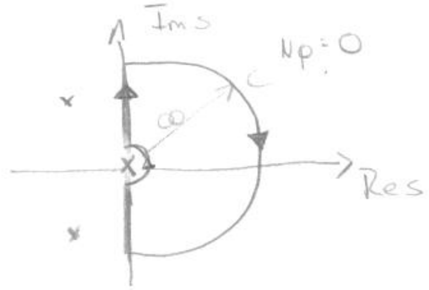
$s^3 + 2s^2 + 5s + Ks^2 + 6Ks + 25K = 0$

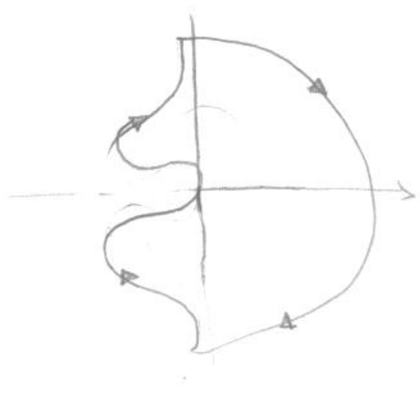
$s^3 + (2+K)s^2 + (5+6K)s + 25K = 0$

Routh

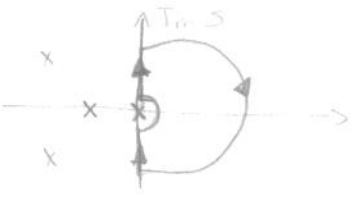
s^3	1	$5+6K$
s^2	$2+K$	$25K$
s^1	$\dfrac{(2+K)(5+6K)-25K}{2+K}$	
s^0	$25K$	$K>0$

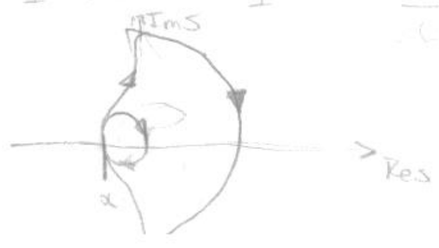
$6K^2 + 17K + 10 - 25K > 0 \Rightarrow$





c) $G(s) = \dfrac{s+1}{s(s^2+4s+?)}$ Eq. carac. $1 + CG(s) = 1 + K\dfrac{(s+1)}{s(s^2+4s+?)}$





CA721 - Exercícios Lista II

1) c) (continuação)

Routh.

Eq carac.

$$1 + \frac{K(s+3)}{s(s^2+4s+5)}\frac{1}{(s+1)} = 0 \Rightarrow (s^3+4s^2+5s)(s+1) + Ks + 3k = 0$$

$$\Rightarrow \cancel{s^4} + \cancel{4s^3} + \cancel{5s^2} + \cancel{s^3} + \cancel{4s^2} + \cancel{5s} + Ks + 3K = 0$$

$$\Rightarrow s^4 + (4+1)s^3 + (5+4)s^2 + (5+k)s + 3K = 0$$

s^4	1	9	3K
s^3	5	5+K	
s^2	(1)	(2)	
s^1	(3)		
s^0	(2)		

$(1) \to \frac{5.9 - (5+K)}{\cancel{5}} .5$

$(2) \to \frac{15K}{5} .5$

$(3) \to \frac{(5.9-(5+K)).(5+K) - 5.15K}{5.9-(5+K)}$

(2) $15K > 0 \Rightarrow K > 0$

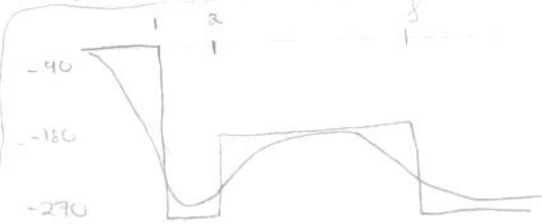
(1) $45 - 5 - K > 0 \Rightarrow K < 40$

(3) $(40-K)(5+K) - 5.15K > 0$ $K < 4.49$

$20 < K < 40$

$0 < K < 4.49$

$\alpha = \frac{-1}{-20+10\sqrt{6}}$ ~> Por Routh



d) $G(s) = \frac{1}{s(s^2+2s+10)}$ $H(s) = \frac{s+2}{s+8}$ ↳ Perguntar (fase)

Eq carac $1 + CGH = 0 \Rightarrow 1 + K\frac{1}{s(s^2+2s+10)}\frac{(s+2)}{(s+8)} = 0$

$\Rightarrow (s^3+2s^2+10s)(s+8) + K(s+2) = 0 \Rightarrow \cancel{s^4} + \cancel{2s^3} + \cancel{10s^2} + \cancel{8s^3} + \cancel{16s^2} + 80s + Ks + 2K = 0$

$\Rightarrow s^4 + 10s^3 + 26s^2 + (80+K)s + 2K = 0$

s^4	1	26	2K
s^3	10	80+K	
s^2	(1)	(2)	
s^1	(3)		
s^0	(2)		

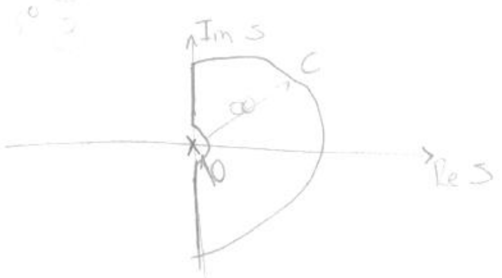
$(1) \to 260 - 80 - K = 180 - K > 0$ $K < 180$

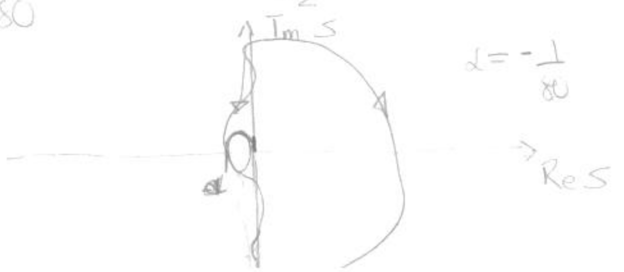
$(2) \to 20K - 20K > 0$ $K > 0$

$(3) \to (180-K)(80+K) - 10.20K > 0$

$-K^2 - 100K + 180.80$ $\frac{-100 \pm \sqrt{100.100 + 4.180.80}}{-2} \Rightarrow K < 80$

$0 < K < 80$



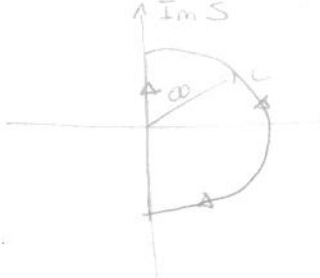


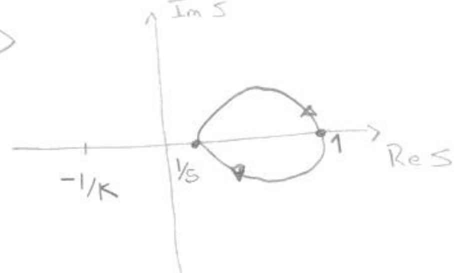
$\alpha = -\frac{1}{80}$

2) a) $G(s) = \frac{s+2}{s+10}$ $\quad 1 + K\frac{(s+2)}{s+10} = 0 \Rightarrow s+10+Ks+2K=0 \Rightarrow (1+K)s + 10+2K = 0$

$(1+K)>0 \Rightarrow \boxed{K>-1}$

$10+2K>0 \quad K>-\frac{10}{2} = -5$





b) $G(s) = \frac{1}{(s+10)(s+2)^2} \Rightarrow$ Eq carac $1 + K\frac{1}{(s+10)(s+2)^2} = 0$

$\Rightarrow (s+10)(s+2)^2 + K = 0 \Rightarrow (s+10)(s^2+4s+4) + K = 0 \Rightarrow$

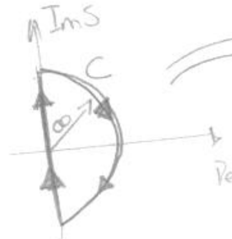
$s^3 + 14s^2 + 44s + 40 + K = 0$

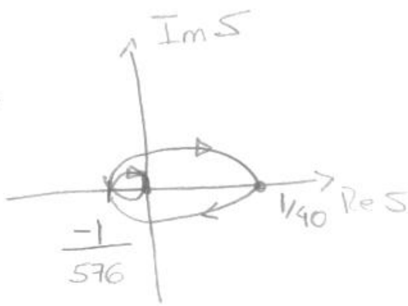
s^3	1	44
s^2	14	40+K
s^1	(1)	
s^0	40+K	

$(1) \to 14\cdot44 - 40 - K > 0$

$40+K>0 \Rightarrow K>-40$

$K<576$





c) $G(s) = \frac{(s+1)(s+10)}{(s+100)(s+2)^3} \overset{\text{Eq carac}}{\Rightarrow} 1 + K\frac{(s+1)(s+10)}{(s+100)(s+2)^3} = 0 \Rightarrow$

$s^4 + 106s^3 + (612+K)s^2 + (1208+11K)s + 800+10K = 0$

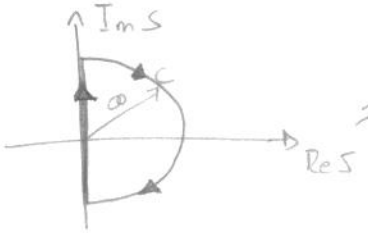
s^4	1	612+K	800+10K
s^3	106	1208+11K	
s^2	(1)	(2)	
s^1	(3)		
s^0	(2)		

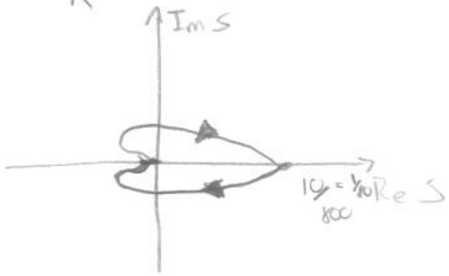
$(1) \to 106(612+K) - 1208 - 11K > 0 \quad K > -\frac{63664}{95}$

$(2) \to \frac{106(800+10K)}{(1)} > 0 \quad \boxed{K>-80}$

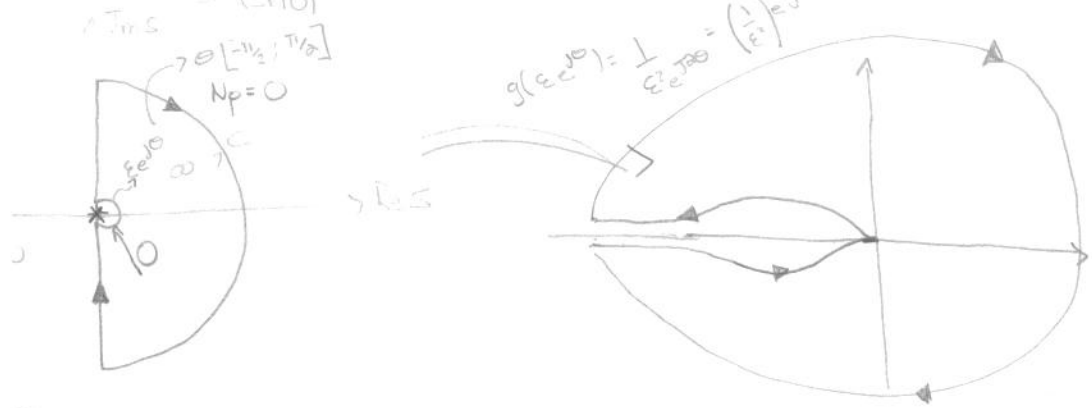
$(3) \to (106(612+K) - 1208 - 11K)(1208+11K) - 106(800+10K)(106) > 0$

↳ K





1) d) $G(s) = \dfrac{s+1}{s^2(s+10)}$ => Eq caract. $1 + CG = 1 + K\dfrac{(s+1)}{s^2(s+10)} = 0$



Por ROUTH:

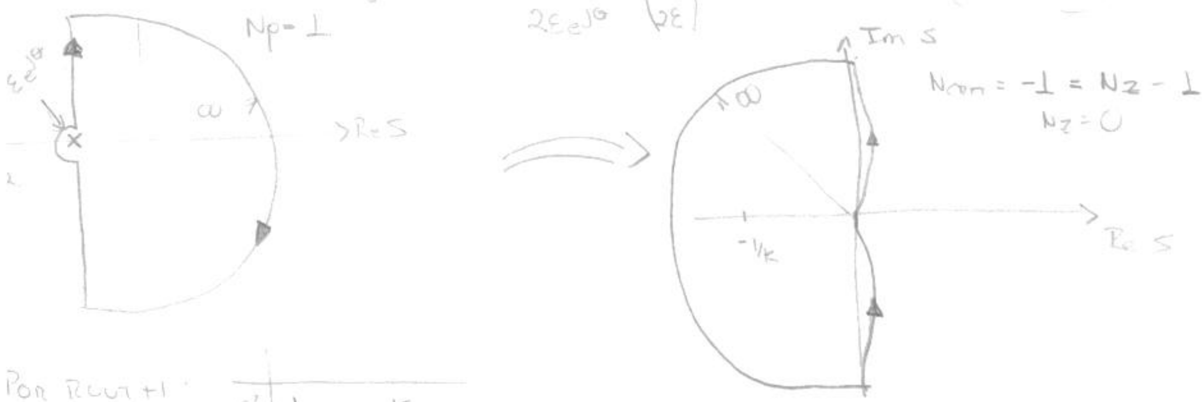
$1 + CG = 0 \Rightarrow s^3 + 10s^2 + ks + K = 0$

s^3	1	K
s^2	10	K
s^1	9K	
s^0	K	

$K > 0$

e) $G(s) = \dfrac{s+1}{s(s+2)}$ Eq. caract. $1 + CG = 0 \Rightarrow 1 + \dfrac{K(s+1)}{s(s+2)} = 0$

$g(\varepsilon e^{j\theta}) = \dfrac{1}{2\varepsilon e^{j\theta}} = \left(\dfrac{1}{2\varepsilon}\right)e^{-j\theta}$



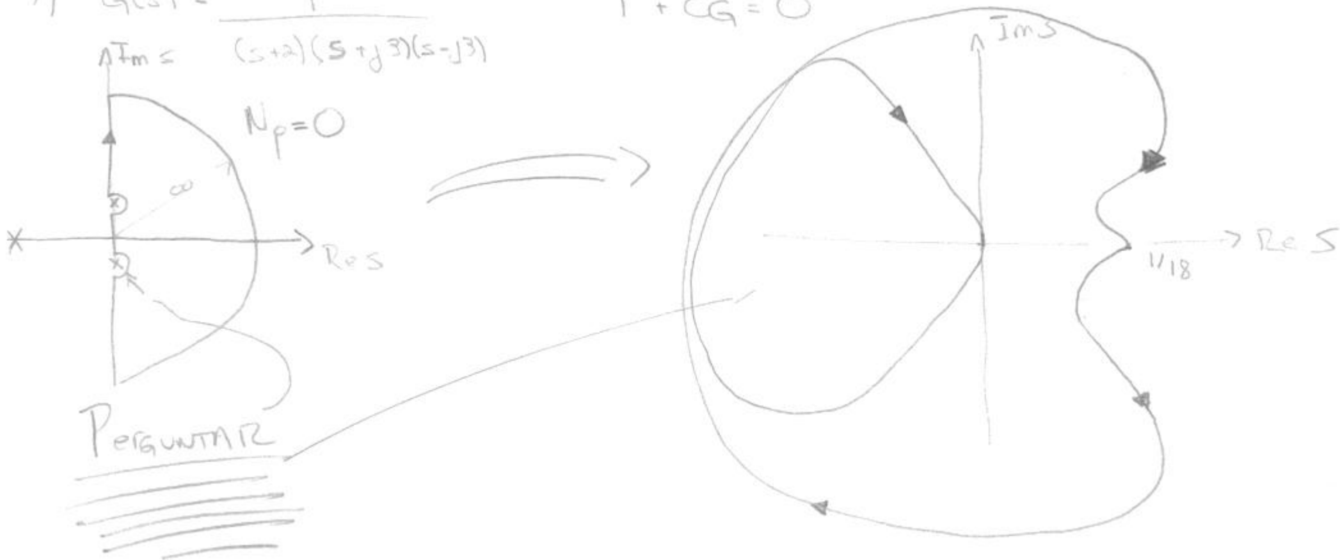
Por ROUTH:

s^2	1	K
s^1	2+K	
s^0	K	

$1 + K\dfrac{(s+1)}{s(s+2)} = 0 \Rightarrow s^2 + (2+K)s + K = 0$

$K > 0$

f) $G(s) = \dfrac{1}{(s+2)(s+j3)(s-j3)}$ $1 + CG = 0$



PERGUNTAR

2) f) (Routh)

$$(s+2)(s^2+9) + K = 0 \Rightarrow s^3 + 2s^2 + 9s + 18 + K = 0$$

s^3	1	9
s^2	2	18+K
s^1	(1)	
s^0	(2) 18+K	

(1) $\rightarrow 18 - 18 - K > 0 \Rightarrow K < 0$

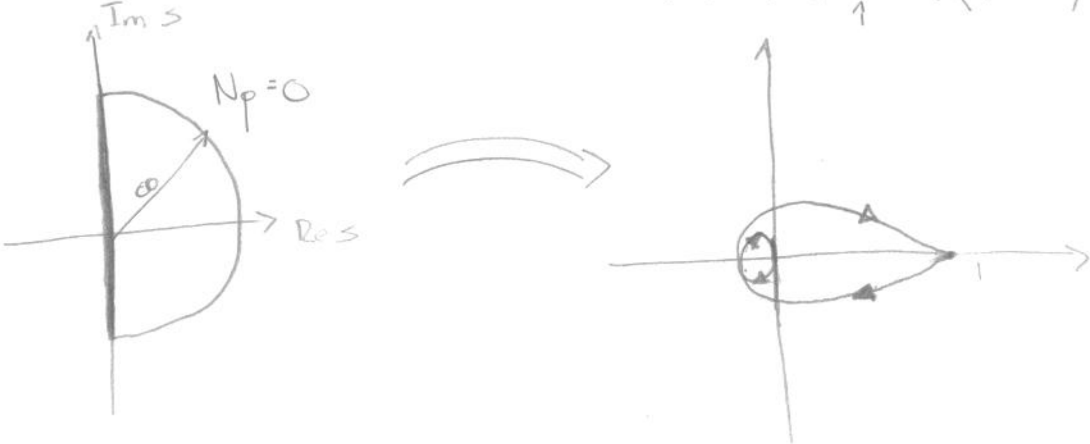
(2) $\rightarrow 18 + K > 0 \quad K > -18$

$$-18 < K < 0$$

g) $$G(s) = \frac{(1+10s)}{(1+20s)^2(1+5s)(1+s)} = \frac{\left(\frac{s}{1/10}+1\right)}{\left(\frac{s}{1/20}+1\right)^2\left(\frac{s}{1/5}+1\right)\left(\frac{s}{1}+1\right)}$$

Eq. característica

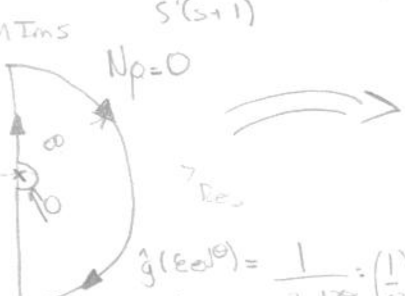
$1 + CG = 0$



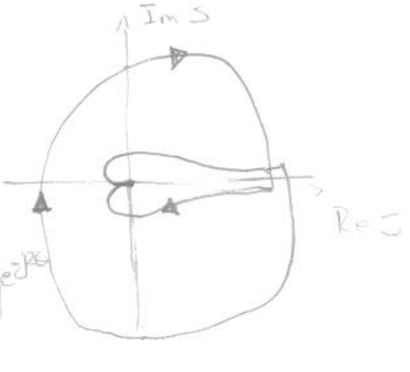
Routh $\quad 1 + KG = 0 \Rightarrow 2000s^4 + 2600s^3 + 645s^2 + (46 + 10K)s + 1 + K = 0$

s^4	2000	645	1+K
s^3	2600	46+10K	
s^2			
s^1			
s^0			

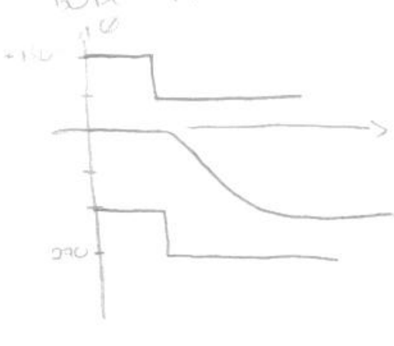
h) $G(s) = \dfrac{s-1}{s^2(s+1)}$ zero de fase não mínima $\quad$ Eq. característica: $1 + C\hat{g} = 0 \Rightarrow 1 + KG = 0$



$\hat{g}(\varepsilon e^{j\theta}) = \dfrac{1}{\varepsilon^2 e^{j2\theta}} = \left(\dfrac{1}{\varepsilon^2}\right)e^{-j2\theta}$



Bode fase



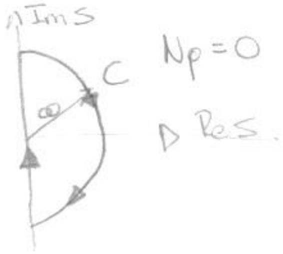
2) h) (CONTINUAÇÃO)

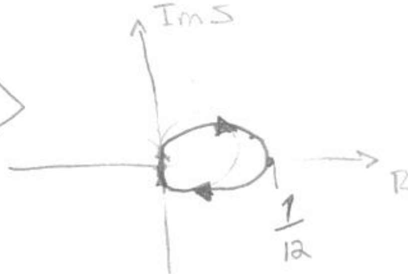
POR ROUTH: $Q(s) = s^3 + s^2 + Ks - K = 0$

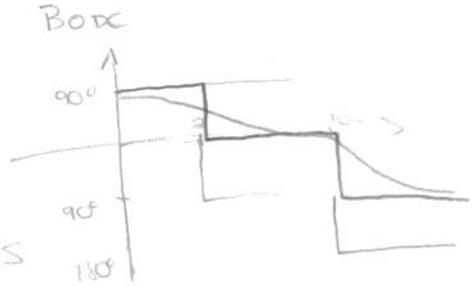
s^3	1	K
s^2	1	-K
s^1	2K	
s^0	-K	

$2K > 0 \Rightarrow K > 0$
$-K > 0 \Rightarrow K < 0$
NENHUMA REGIÃO FACTÍVEL P/ ESTABILIDADE

c) $G(s) = \dfrac{s}{(s+2)(s+10)}$







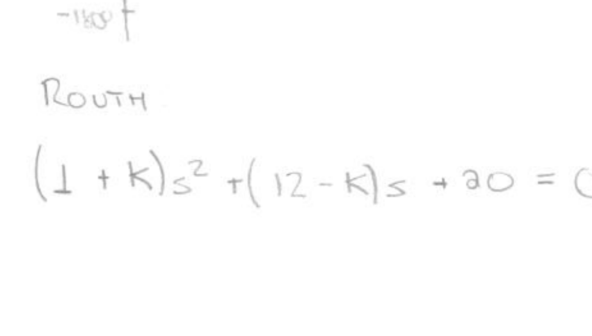
ROUTH

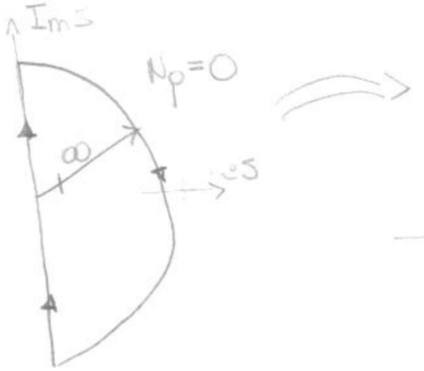
$s^2 + (12 + K)s + 20 = 0$

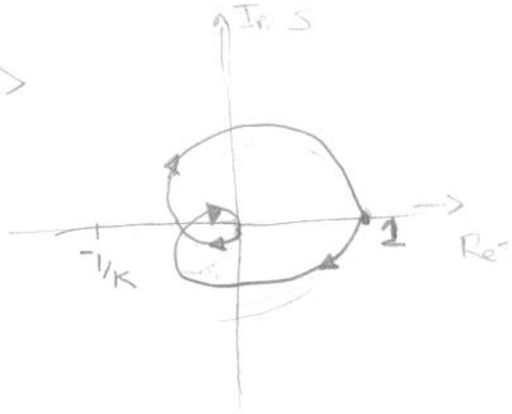
s^2	1	20
s^1	12+K	
s^0	20	

$12 + K > 0$
$K > -12$

d) $G(s) = \dfrac{s(s-1)}{(s+2)(s+10)}$







ROUTH

$(1 + K)s^2 + (12 - K)s + 20 = 0$

s^2	1+K	20
s^1	12-K	
s^0	20	

$12 - K > 0 \Rightarrow K < 12$
$1 + K > 0 \Rightarrow K > -1$

3) a) $s^2(s^2+1{,}5s+0{,}5)+Ks+K\lambda=0 \Rightarrow s^4+1{,}5s^3+0{,}5s^2+Ks+K\lambda=0$

s^4	1	0,5	$K\lambda$
s^3	1,5	K	
s^2	(1)	$1{,}5K\lambda$	
s^1	(2)		
s^0	$1{,}5K\lambda$		

(1) $\to 0{,}75-K$ $\quad K<0{,}75$

(2) $\to (0{,}75-K)K - 1{,}5\,1{,}5K\lambda$

$\hookrightarrow -K^2+0{,}75K-2{,}25\lambda K>0$

$K\lambda>0$ $\quad \hookrightarrow -K^2+(0{,}75-2{,}25\lambda)K>0$

$(0{,}75-2{,}25\lambda)K>K^2$

$(0{,}75-2{,}25\lambda)>K$

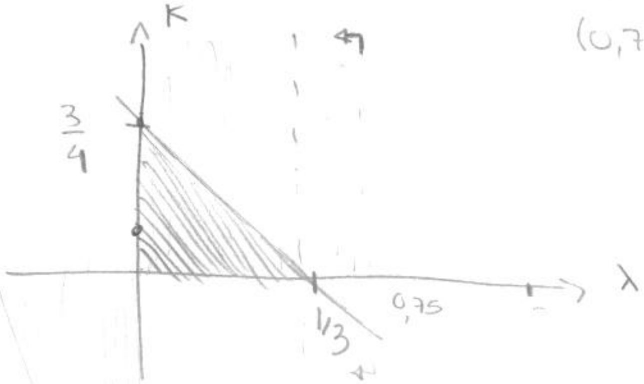
$0{,}75-K>2{,}25\lambda$

$\Rightarrow \frac{3}{4}-K>\frac{9}{4}\lambda$

$\Rightarrow 3-4K>9\lambda$

$3-9\lambda>4K$

$K<\frac{3}{4}-\frac{9}{4}\lambda$



$K=\frac{3}{4}-\frac{9}{4}\lambda$

b) $s^4+2s^3+Ks^2+\lambda s+K=0$

s^4	1	K	K
s^3	2	λ	
s^2	(1)	$2K$	
s^1	(2)		
s^0	$2K$		

(1) $\to 2K-\lambda>0$ $\quad 2K>\lambda$

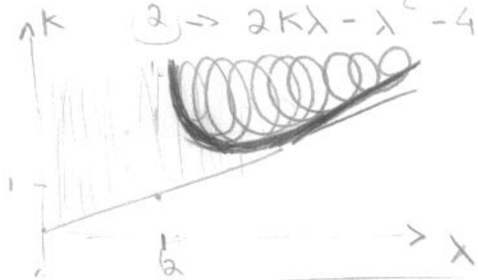
(2) $\to (2K-\lambda)\lambda-4K>0$

$2K>0$

(2) $\to 2K\lambda-\lambda^2-4K>0 \Rightarrow K(2\lambda-4)>\lambda^2$

$K>\frac{\lambda^2}{2\lambda-4}$

$K>\frac{\lambda^2}{2(\lambda-2)}$



i) a)

$F(s)=\frac{\hat{y}}{\hat{r}}=\frac{CG}{1+CG}$, Eq caract. $1+CG=0 \Rightarrow 1+\frac{K}{(s+\lambda)}\frac{1}{(s^2+2s+2)}=0$

$\Rightarrow s^3+(2+\lambda)s^2+(2+2\lambda)s+2\lambda+K$

s^3	1	$2+2\lambda$
s^2	$2+\lambda$	$2\lambda+K$
s^1	(1)	
s^0	$2\lambda+K$ (2)	

(1) $\to (2+\lambda)(2+2\lambda)-2\lambda-K>0$

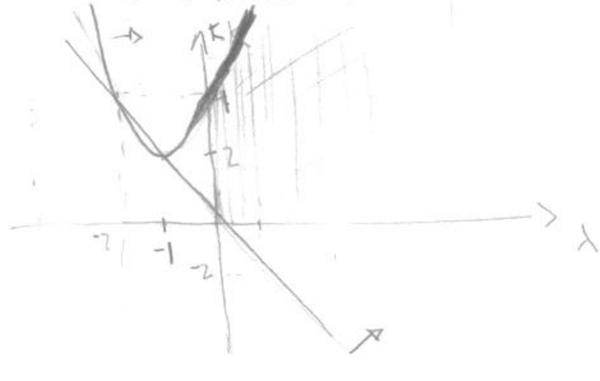
(2) $\to 2\lambda+K>0 \Rightarrow K>-2\lambda$

(1) $(2\lambda^2+6\lambda+4)-2\lambda-K>0$

$2\lambda^2+4\lambda+4>K$

$4\lambda+4=0$

$K=2\lambda$



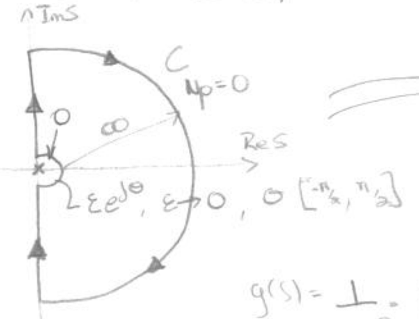
EA721 - Exercícios Lista II

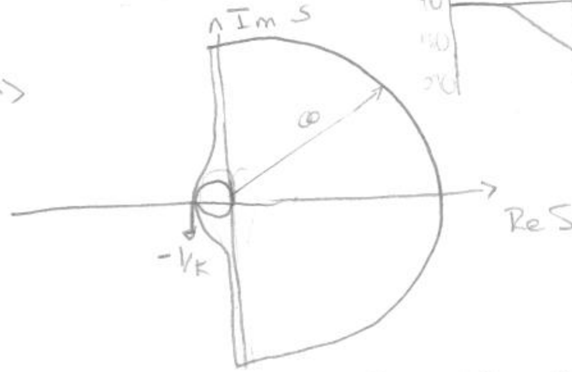
1 b) $\lambda = 0$ Eq característica: $s^3 + 2s^2 + 2s + K = 0$

$$s(s^2+2s+2) + K = 0 \Rightarrow 1 + K\frac{1}{s(s^2+2s+2)} = 0$$

Perguntar: método p/ encontrar K ($-1/K$) por Nyquist (método espartano)

$$1 + \frac{K}{s}\frac{1}{(s^2+2s+2)} = 1 + K\underbrace{\frac{1}{s(s^2+2s+2)}}_{g(s)} = 0$$





$$g(s) = \frac{1}{\varepsilon e^{j\theta}} = \left(\frac{1}{\varepsilon}\right)e^{-j\theta}$$

$$\frac{1}{j\omega(-\omega^2+j2\omega+2)} = \frac{1}{-2\omega^2+j(2\omega-\omega^3)}\frac{(-2\omega^2-j(2\omega-\omega^3))}{(-2\omega^2+j(2\omega-\omega^3))} = \frac{-2\omega^2-j(2\omega-\omega^3)}{D(j\omega)}$$

$2\omega - \omega^3 = 0$
$2\cancel{\omega} = \omega^{\cancel{3}2}$
$\omega = \pm\sqrt{2}$

$$\Rightarrow g(\sqrt{2}) = \frac{1}{j\sqrt{2}(\cancel{-2}+j2\sqrt{2}+\cancel{2})} = \frac{1}{-2\cdot 2} = -\frac{1}{4} \qquad K < 4$$

$\hookrightarrow \left(\frac{-1}{K} < \frac{-1}{4}\right)$

2) $s^4 + 6s^3 + 10s^2 - 2s - 15 = 0$, $\mathrm{Re}\{s\} < -1$

$\mathrm{Re}\{\underbrace{s+1}_{\gamma}\} < 0$ $\Rightarrow (\gamma-1)^4 + 6(\gamma-1)^3 + 10(\gamma-1)^2 - 2(\gamma-1) - 15 = 0$

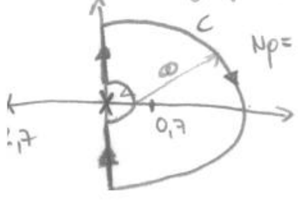
$\gamma \Rightarrow s = \gamma - 1$ $\quad \gamma^4 - \cancel{4\gamma^3} + \cancel{6\gamma^2} - \cancel{4\gamma} + 1 + 6(\gamma^3 - 3\gamma^2 + 3\gamma - \cancel{1}) + 10(\gamma^2 - \cancel{2\gamma} + 1) - \cancel{2\gamma} + 2 - 1\ldots$

$\Rightarrow \gamma^4 + 2\gamma^3 - 2\gamma^2 - 8\gamma - 8 = 0 \rightsquigarrow \gamma^2(\gamma^2 + 2\gamma - 2) - 8\gamma - 8 = 0$

$$\Rightarrow 1 + \frac{(-8\gamma-8)}{\gamma^2(\gamma^2+2\gamma-2)} = 0 \Rightarrow 1 + \underbrace{\frac{-8(\gamma+1)}{\gamma^2(\gamma^2+2\gamma-2)}}_{G(s)} = 0$$

Bode

$$\Rightarrow 1 + \frac{-8(\gamma+1)}{\gamma^2(\gamma+2{,}7)(\gamma-0{,}7)}$$

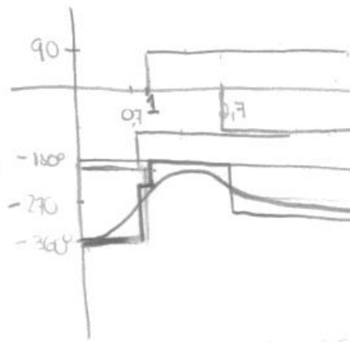


$N_{CRIT} = 0 - N_Z - N_P$

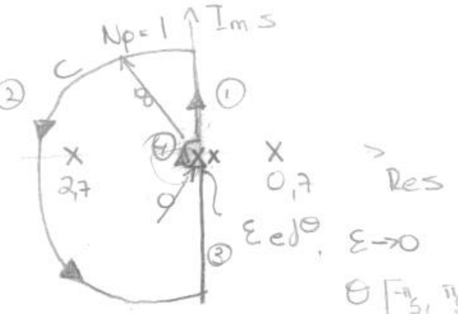
$N_Z = 1$

$\Downarrow$

N^o zeros da eq original no semiplano $\mathrm{Re} < -1$

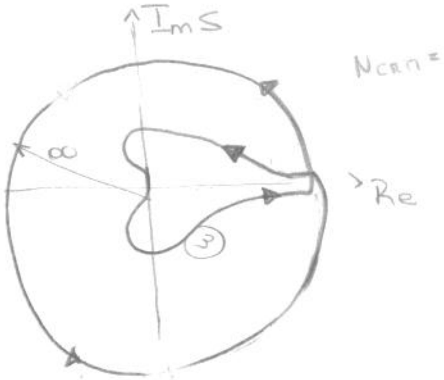


5) (Outra abordagem)



$\varepsilon e^{j\theta}$, $\varepsilon \to 0$
$\theta\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$g(\varepsilon e^{j\theta}) = \left(\frac{1}{\varepsilon^2}\right) e^{-j2\theta}$



$N_{CRN} = 2 = N_Z - 1$

$N_Z = 3$

Perguntar: duas abordagens dif. (dão círculos dif?)

6) Eq caract: $s^3 + 3s^2 + 2s + \left(\alpha + \frac{1}{\alpha}\right) = 0$

s^3	1	2
s^2	3	$\alpha + \frac{1}{\alpha}$
s^1	①	
s^0	$\alpha + \frac{1}{\alpha}$ ②	

① $\to$ $6 - \alpha - \frac{1}{\alpha} > 0 \Rightarrow \alpha^2 - 6\alpha + 1 < 0$

② $\to$ $\alpha + \frac{1}{\alpha} > 0 \Rightarrow \alpha^2 + 1 > 0$

① $\alpha^2 - 6\alpha + 1 = 0 < {0,17 \atop 5,8}$ $\quad \alpha < 0,17$ $\quad \alpha > 5,8$

② $\alpha^2 + 1 = 0 \Rightarrow$ OK!

$0,17 < \alpha < 5,8$

7) Sendo $C(s) = K$

$$C_D(z) = \mathcal{Z}\left\{\mathcal{L}^{-1}\left\{\frac{1-e^{-Ts}}{s}C(s)\right\}\Big|_{t=kT}\right\} = \mathcal{Z}\left\{\mathcal{L}^{-1}\left\{\frac{C(s)}{s}\right\}\Big|_{t=kT} - \mathcal{L}^{-1}\left\{\frac{C(s)}{s}\right\}\Big|_{t=(k-1)T}\right\}$$

$$= (1 - z^{-1})\,\mathcal{Z}\left\{\mathcal{L}^{-1}\left\{\frac{C(s)}{s}\right\}\Big|_{t=kT}\right\}$$

$$\mathcal{Z}\{a^k\} = \frac{1}{1-az^{-1}} = \frac{1}{1-\frac{a}{z}} = \frac{z}{z-a}$$

$$\mathcal{Z}\{k\,a^k\} = \frac{1}{(1-az^{-1})^2} = \frac{1}{\left(1-\frac{a}{z}\right)^2} = \frac{a \cdot 1}{\frac{(z-a)^2}{z}} = \frac{az}{(z-a)^2}$$

a) $F(s) = \dfrac{CG}{1+CG} = \dfrac{K\frac{1}{s(s+1)}}{\frac{s(s+1)+K}{s(s+1)}} = \dfrac{K}{s^2+s+K}$ $\quad -1 \pm \sqrt{1-4K}$

$$F_D(z) = \mathcal{Z}\left\{\mathcal{L}^{-1}\left\{\frac{K}{s(s^2+s+K)}\right\}\Big|_{t=kT}\right\}$$

Perg e revisar

8) a) $G(s) = \dfrac{10}{s^2 + 7s + 10} = \dfrac{Y(s)}{X(s)}$

$v_1 = y$
$\dot{v}_1 = \dot{y} = v_2$
$\dot{v}_2 = \ddot{y}$

$\dot{v} = \begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -10 & -7 \end{bmatrix} \overbrace{\begin{bmatrix} y \\ \dot{y} \end{bmatrix}}^{v} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} x$

$\dot{v} = \begin{bmatrix} 0 & 1 \\ -10 & -7 \end{bmatrix} v + \begin{bmatrix} 0 \\ 10 \end{bmatrix} x$

$y = \begin{bmatrix} 1 & 0 \end{bmatrix} v + [0] x$

OU

$\dot{v} = \begin{bmatrix} 0 & 1 \\ -10 & -7 \end{bmatrix} v + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x$

$y = \begin{bmatrix} 10 & 0 \end{bmatrix} v + [0] x$

=> MATLAB $P = \begin{bmatrix} 1{,}1357 & 0{,}5 \\ 0{,}5 & 0{,}0786 \end{bmatrix}$ $\lambda_1 = 0{,}07$, $\lambda_2 = 1{,}138$ isto é, $P > 0$ e o sistema é estável

b) $G(s) = \dfrac{10s + 1}{s^3 + 2s^2 + 3s + 7}$

$v_1 = y$

$\dot{v} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7 & -3 & -2 \end{bmatrix}}_{A} v + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} x$

$y = \begin{bmatrix} 1 & 10 & 0 \end{bmatrix} v + [0] x$

$A'P - PA + Q = 0$

$P = \begin{bmatrix} -33{,}3 & -14{,}85 & 0{,}0714 \\ -14{,}85 & -16{,}07 & -4{,}7857 \\ 0{,}0714 & -4{,}7857 & -2{,}1429 \end{bmatrix}$ $\lambda_1 = -41{,}9$, $\lambda_2 = -9{,}75$, $\lambda_3 = 0{,}24$ $P < 0$ (A instável

9) $A'PA - P + Q = 0$

a) $G(z) = \dfrac{z - 0{,}5}{z^2 - z + 0{,}26}$

$v = \begin{bmatrix} 0 & 1 \\ -0{,}26 & -1 \end{bmatrix} v + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x$

$y = \begin{bmatrix} -0{,}5 & 1 \end{bmatrix} v + [0] x$

$A'PA - P + Q = 0$

$P = \begin{bmatrix} 1{,}39 & 1{,}19 \\ 1{,}19 & 5{,}78 \end{bmatrix}$ $\lambda = 1{,}08$, $\lambda_2 = 6{,}1$ $P > 0$ => estável

b) $G(z) = \dfrac{z + 1}{z^2 - z + 1{,}06}$ => $\dot{v} = \overbrace{\begin{bmatrix} 0 & 1 \\ -1{,}06 & -1 \end{bmatrix}}^{A} v + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x$

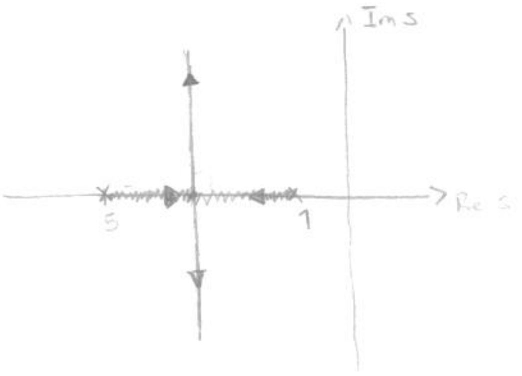
$y = \begin{bmatrix} 1 & 1 \end{bmatrix} v + [0] x$

$A'PA - P + Q = 0$

$P = \begin{bmatrix} -22{,}78 & -10{,}89 \\ -10{,}89 & -21{,}17 \end{bmatrix}$ $\lambda_1 = -32{,}9$, $\lambda_2 = -11{,}05$

$P < 0$, instável

10) a) $G(s) = \dfrac{1}{(s+1)(s+5)}$ Eq. $= 1 + K \dfrac{1}{(s+1)(s+5)} = 0$



$\sigma = \dfrac{\sum p_i - \sum z_i}{N - M} = \dfrac{-1-5}{2} = -3$

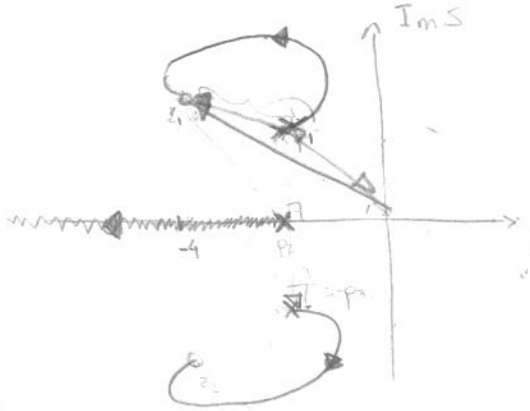
10) b) $G(s) = \dfrac{s^2+8s+20}{(s+2)(s^2+4s+7)}$ ① ② $\Rightarrow$ Eq caract. $1 + KG = 0 \Rightarrow 1 + \dfrac{K(s^2+8s+20)}{(s+2)(s^2+4s+7)}$

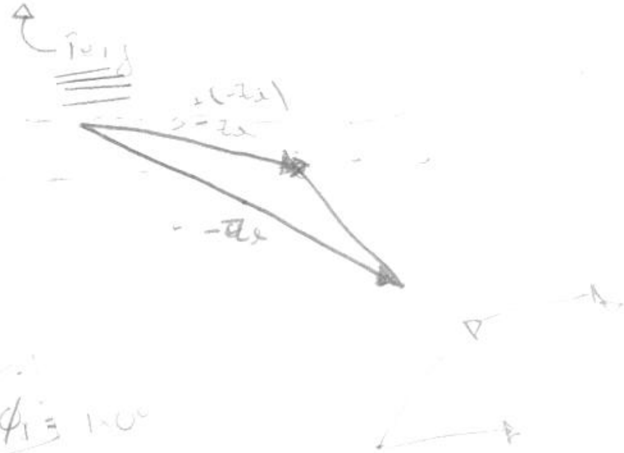
① $-4 \pm \frac{1}{2}\sqrt{64-80} \; < \begin{matrix} -4+j2 \\ -4-j2 \end{matrix}$ Eq caract $\Rightarrow 1 + \dfrac{K(s+4-j2)(s+4+j2)}{(s+2)(s+2-j\sqrt{3})(s+2+j\sqrt{3})} = 0$

② $-2 \pm \frac{1}{2}\sqrt{16-28} \; < \begin{matrix} -2+j\sqrt{3} \\ -2-j\sqrt{3} \end{matrix}$

$\sigma = \dfrac{\sum p_i - \sum z_i}{N-m} = \dfrac{-2 \cancel{-2} -2 \cancel{+4} + 4}{1} = 2$

$N-m$ → Im 1





Cálculo (p/ p_1)

$\angle s - z_i - \sum \angle s - p_i = 180° \Rightarrow 90° + 60° - 90° - 90° - \phi_1 = 180°$

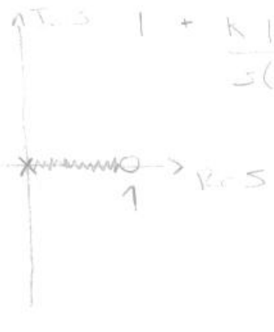
$\phi_{p1} = 30°$

C, p/ z_1

$\phi_{z1} + 90° - 135° - 160° - 110° = 180° \Rightarrow \phi_{z1} = 90 + 135 + 160 + 110$

c) $= \dfrac{10(s-1)}{s(s+1)} = G(s) \Rightarrow$ Eq caract: $1 + K\dfrac{10(s-1)}{s(s+1)}$ $\quad s = -2 \pm \frac{1}{2}\sqrt{16-16}$

$1 + \dfrac{K\,10(s-1)}{s(s+2)^2} = 0 \qquad \sigma = \dfrac{\sum p_i - \sum z_i}{N-m} = \dfrac{(-2)(-2)-(+1)}{2} = -2{,}5$

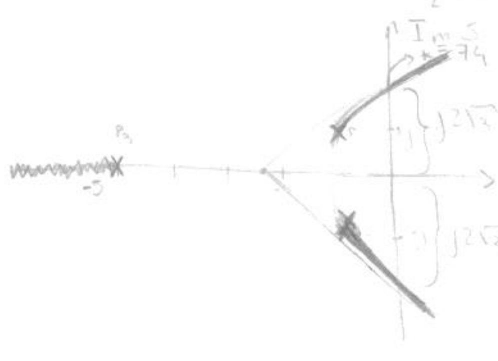


$(180°) - (\phi_{p1} + 90° + 180°) = 180°$

$-90° - 180° = \phi_{p1} \Rightarrow \phi_1 = -270° = 90°$

d) $G(s) = \dfrac{1}{(s+5)(s^2+2s+2)}$ ① $\Rightarrow$ Eq característica $1 + CG = 0 \Rightarrow 1 + K\dfrac{1}{(s+5)(s+1-j)(s+1+j)}$

① $< \begin{matrix} -1+\frac{1}{2}\sqrt{4-8} = -1+j \\ -1-\frac{1}{2}\sqrt{4-8} = -1-j \end{matrix}$



$\sigma = \dfrac{-5-1-1}{3} = \dfrac{-7}{3}$

$-(\phi_1 + 90° + 15°) = 180° \Rightarrow \phi_1 = -105 - 180° = -285° = 75°$

Eq caract: $(s+5)(s^2+2s+2) + K = 0$

$\Rightarrow s^3 + 7s^2 + 12s + K + 10 = 0$

10) c) (CONTINUAÇÃO)

s^3	1	12
s^2	7	$K+10$
s^1	①	
s^0	$K+10$	

$① \rightarrow 7.12 - K - 10 > 0 \;\; \Rightarrow K < 74$

$\Delta(s) = 7s^2 + K + 10 = 0$

$K = 74$

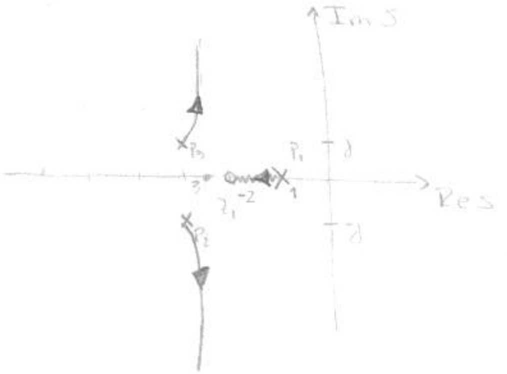
$\hookrightarrow \Delta(s) = 7s^2 + 84 = 0$

$\Rightarrow s = \pm\sqrt{-12} = \pm j2\sqrt{3}$

e) $G(s) = \dfrac{s+2}{(s+1)(s^2+6s+10)}$ $\Rightarrow$ Eq. caract. $1 + CG = 0 \Rightarrow 1 + \dfrac{K(s+2)}{(s+1)}$

$-3+j$, $-3-j$

$\Rightarrow 1 + \dfrac{K(s+2)}{(s+1)(s+3-j)(s+3+j)}$



$\sigma = \dfrac{\sum p_i - \sum z_i}{N-m} = \dfrac{-1-3-3-(-2)}{2} = -\dfrac{5}{2}$

$\theta_k = \dfrac{(2k-1)\pi}{N-m}$, $k = 1 \ldots (N-m)$

$z_1 \quad p_1 \quad$ cálculo

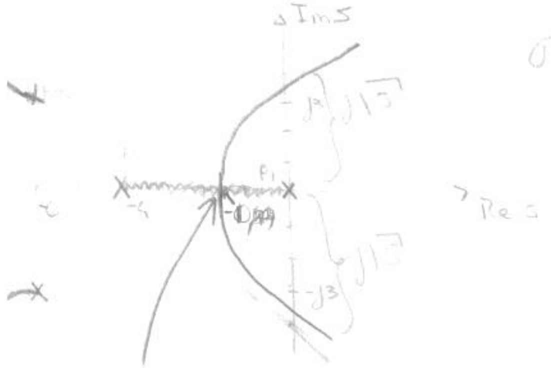
$(135°) - (\phi_{p_3} + 90° + 150°) = 180°$

$\Rightarrow 135 - 90 - 150 - 180 = \phi_{p_3} = 75°$

f) $G(s) = \dfrac{5}{s(s+4)(s^2+12s+45)} = \dfrac{5}{(s)(s+4)(s+6-j3)(s+6+j3)}$ $\Rightarrow$ Eq. caract. $1 + CG = 0 \Rightarrow$

$\hookrightarrow \Delta = 144 - 180$

$1 + K\dfrac{5}{s(s+4)(s+6-j3)(s+6+j3)}$



$\sigma = \dfrac{\sum p_i - \sum z_i}{N-m} = \dfrac{-4-6-6}{4} = -4$

$-(\phi_{p_3} + 90° + 110° + 135°) = 180 \Rightarrow \phi_{p_3} = -90 + 110 + \ldots$

$= 165°$

Por Routh (eq. caract. $s^4 + 16s^3 + 93s^2 + 180s + 5K$)

s^4	1	93	5K
s^3	16	180	
s^2	81,75	5K	
s^1	$81,75 \cdot 180 - 5 \cdot 16K > 0$		
s^0	5K		

$\rightarrow K < \dfrac{81,75 \cdot 180}{5 \cdot 16}$

$\Rightarrow K < 183,9375$

$\rightarrow \Delta(s) = 81,75s^2 + 919,6875 = 0$

$s = \pm j\sqrt{11,25} \approx \pm j3,3541$

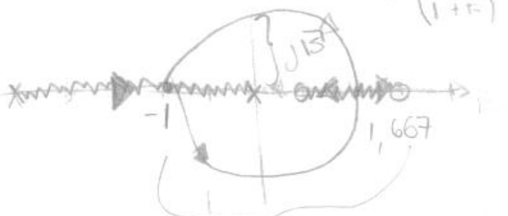
$G'(s) = 0 \Rightarrow 4s^3 + 48s^2 + 186s + 180 = 0$

$s_1 = -1,436$ (ÚNICA SOL REAL)

g) $G(s) = \dfrac{(s-1)(s-3)}{s(s+5)}$ $\Rightarrow$ Eq. caract. $1 + CG = 0 \Rightarrow 1 + K\dfrac{(s-1)(s-3)}{s(s+5)} = 0$

$\hookrightarrow s^2 + 5s + Ks^2 - 4Ks + 3K = 0$

$\Delta \; (1+K)s^2 + (5-4K)s + 3K = 0$



s^2	$1+K$	$3K$
s^1	$5-4K$	
s^0	$3K$	

$5 - 4K > 0 \quad K < \dfrac{5}{4}$

$\Delta(s) = \left(\dfrac{9}{4}\right)s^2 + \dfrac{15}{4} = 0$

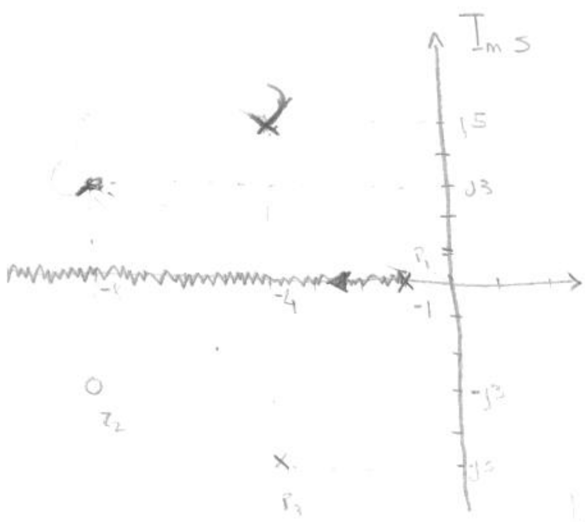
$s = \pm\sqrt{\dfrac{-15}{9}} = \pm j\sqrt{\dfrac{5}{3}}$

SOLUÇÃO DE $G'(s) = 0$, ONDE $G'(s) = \dfrac{DN' - ND'}{D^2}$

$\hookrightarrow$ Para $\rightarrow$ Qual polo ...

EA 721 - LISTA DE EXERCÍCIOS II $G'(s)=0$

h) $G(s) = \dfrac{s^2+16s+73}{(s+1)(s^2+8s+41)}$ → Eq. CARACT. $1 + KG = 0 \Rightarrow 1 + \dfrac{K(s+8-j3)(s+8+j3)}{(s+1)(s+4+j5)(s+4-j5)}$



$\sigma = \dfrac{\sum p_i - \sum z_i}{n-m} = \dfrac{-4-4-1-(-8)-(-8)}{1}$

Em rel. a P_2:

$(30° + 60°) - (\phi_{P2} + 140° + 90°) = 180° \Rightarrow \phi_{P2} \simeq 140°$

Em rel. a Z_1

$(\phi_{Z_1} + 90°) - (160° + 210° + 135°) = 180$

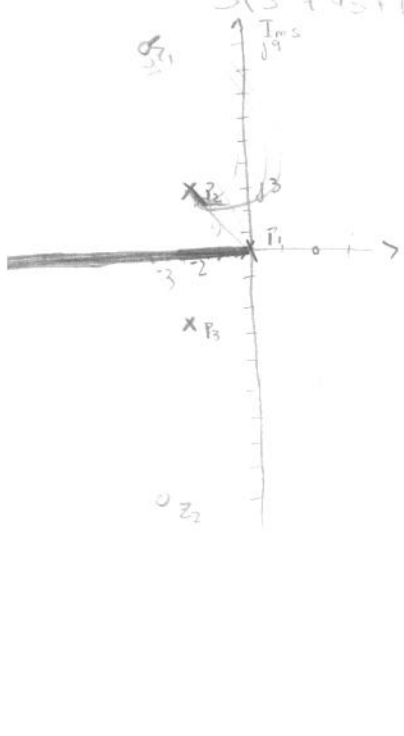
→ P/ cruzamento dos ramos

$G'(s) = \dfrac{DN' - ND'}{D^2} = 0 \Rightarrow DN' - ND' = 0$

$\Rightarrow (s^3 + 9s^2 + 49s + 41)(2s+16) - (s^2+16s+73)(3s^2+18s+49) = 0$

↳ Perg (quando calcula, qdo não mesmo existindo $G'(s) = 0$, como em b), tem a ver com o ângulo de chegada no zero? (olhar l)

e) $G(s) = \dfrac{2(s^2+6s+90)}{s(s^2+4s+13)}$ Eq. caract. $1 + KG = 0 \Rightarrow 1 + \dfrac{K2(s^2+6s+90)}{s(s^2+4s+13)}$



$\sigma = \dfrac{\sum p_i - \sum z_i}{N-M} = \dfrac{-2-2-(-3)-(-3)}{1} = 2$

P/ P_2

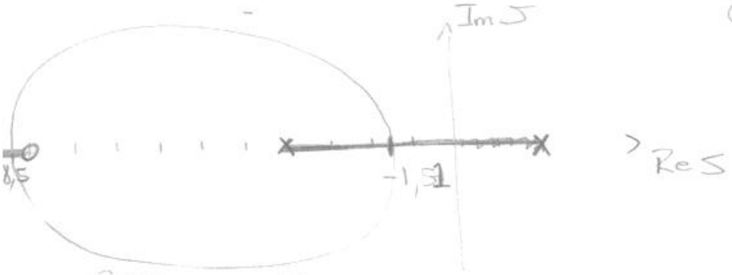
$(-60° + 70°) - (\phi_{P2} + 90 + 125°) = 180°$

P/ Z_1

$(90° + \phi_{Z_1}) - (100° + 110° + 95°) = 180°$

10) j) $G(s) = \dfrac{s+10}{(s-2)(s+4)}$

Eq caract: $1 + CG = 0 \Rightarrow 1 + K\dfrac{(s+10)}{(s-2)(s+4)} = 0$



$\sigma = \dfrac{\sum p_i - \sum z_i}{N-m} = \dfrac{2-4+10}{1} = 8$

$G'(s) = 0 \Rightarrow (s^2+2s-8)1 - (s+10)(2s+2) = 0$

$\Rightarrow s^2+2s-8 - 2s^2-22s-20 = 0$

$\Rightarrow -s^2-20s-28 = 0 \Rightarrow s^2+20s+28$

$-10 \pm \dfrac{1}{2}\sqrt{400-112}$

$\begin{array}{r} 28 \\ -4 \\ \hline 112 \end{array}$

$G(s) = \dfrac{s+10}{s^2+2s-8}$

Eq caract

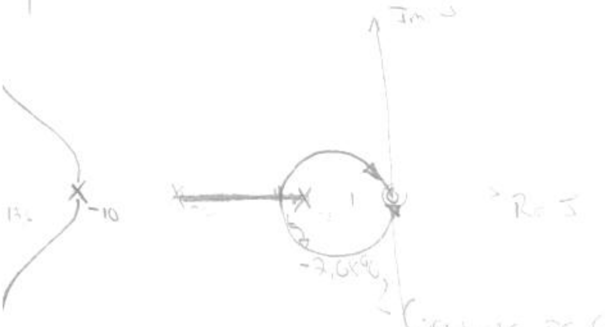
$s^2+2s-8+K(s+10) = 0$

$\Rightarrow s^2 + (2+K)s - 8 + 10K = 0$

s^2	1	$-8+10K$
s^1	$2+K$	
s^0	$-8+10K$	

$-8+10K \rightarrow -8+10K > 0 \Rightarrow K > \dfrac{8}{10}$

k) $G(s) = \dfrac{s^2}{(s+2)(s+5)(s+10)^2} \rightarrow$ Eq caract: $1 + CG = 0 \Rightarrow 1 + K\dfrac{s^2}{(s+2)(s+5)(s+10)^2}$



(cálculo de $G'(s) = 0$)

$\sigma = \dfrac{\sum p_i - \sum z_i}{N-m}$

$= \dfrac{-2-5-10-10}{2} = \dfrac{-27}{2}$

$G'(s) = 0$

Eq caract

$s^4 + 27s^3 + (250+K)s^2 + 900s + 1000$

s^4	1	$250+K$	1000
s^3	27	900	
s^2	(1)	27 1000	
s^1	(2)		
s^0	27 1000		

(1) $\rightarrow 27(250+K) - 900 > 0$

(2) $\rightarrow$ (1) $.900 - 27.27\ 1000 > 0$

(1) $\rightarrow 27K + 27\ 250 - 900 > 0$

(2) (por Wolfram)

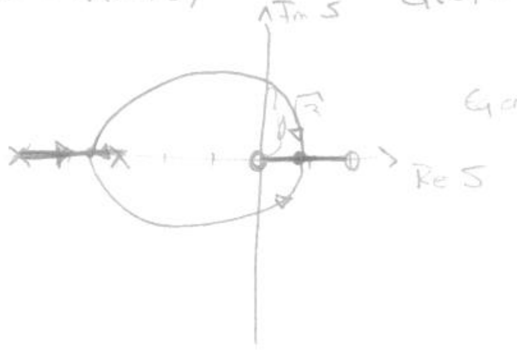
Ambas eq. p/ $K \rightarrow \infty$ (estável)

l) $G(s) = \dfrac{s(s-2)}{(s+3)(s+5)}$

$N - m = 0$

$G'(s) = (s^2+8s+15)(2s-2) - (s^2-2s)(2s+8) = 0$

$s \begin{cases} -3,8 \\ 0,79 \approx 0,8 \end{cases}$



Eq caract

$s^2+8s+15 + Ks^2 - 2Ks = 0$

$\Rightarrow (1+K)s^2 + (8-2K)s + 15 = 0$

s^2	$1+K$	15
s^1	$8-2K > 0 \Rightarrow K < 4$	
s^0	15	

$\Delta(s) = (1+4)s^2 + 15 = 0$

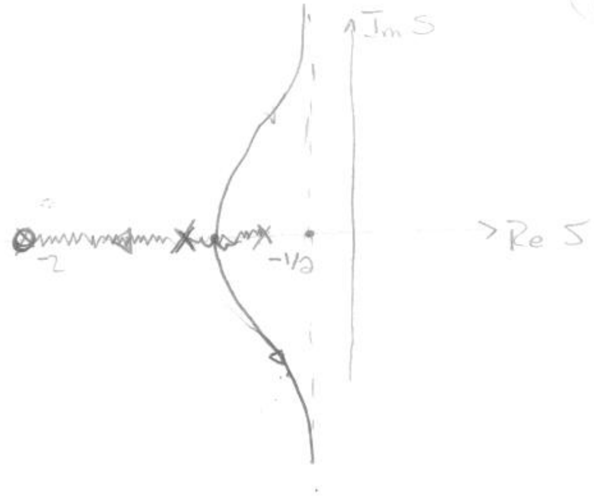
$\Rightarrow 5s^2 + 15 = 0$

$s = \pm j\sqrt{3}$

CA721 - Exercício Lista II

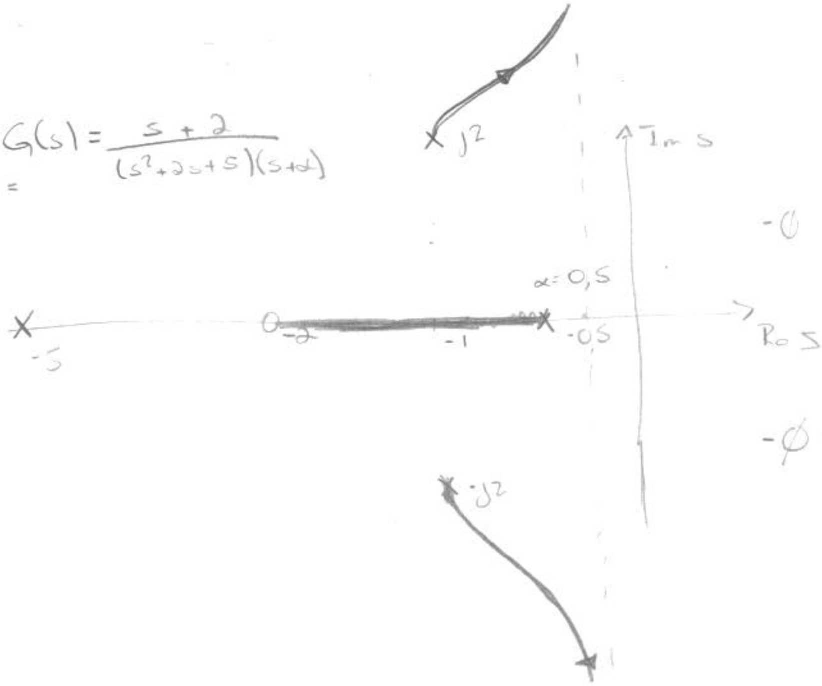
11) a) $G(s) = \dfrac{s+2}{(s+1)^2(s+\alpha)}$

$\alpha = 1/2 \Rightarrow \sigma = \dfrac{\sum p_i - \sum z_i}{N-m} = \dfrac{-1/2 - 1 - 1 - (-2)}{2}$



b) $G(s) = \dfrac{s+2}{(s^2+2s+5)(s+\alpha)}$

$\alpha =$

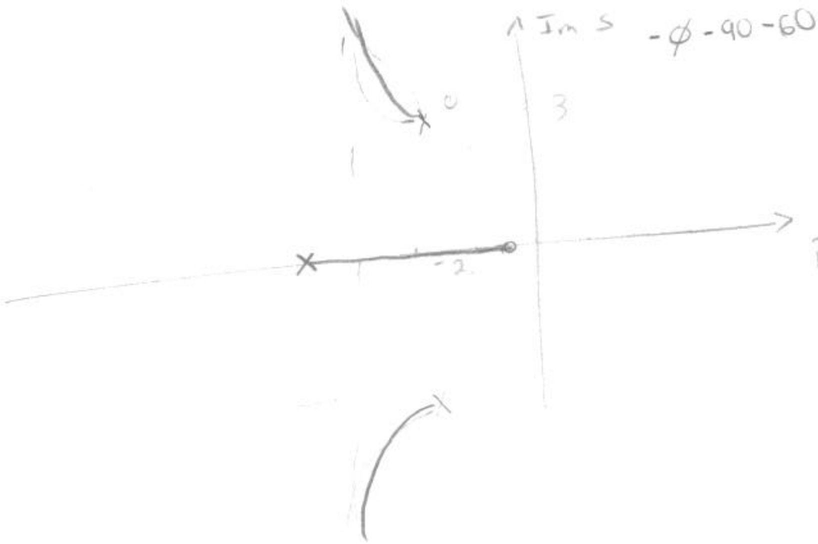


$-\phi - 110 + 60 - 90 = 180$

$-\phi - 90 + 60 + 30 = 180$

c) $G(s) = \dfrac{s+2}{(s+4)(s+3)^2}$

d) $G(s) = \dfrac{s+a}{(s+4)(s^2+4s+13)}$



$-\phi - 90 - 60 + 110 = 180$

13) a) $F(s) = K \dfrac{s \quad 8^2}{\left(\frac{s}{1}+1\right)\left(s^2+2\frac{1}{4}8s+8\right)\left(\frac{s}{16}+1\right)} = \dfrac{2\, s\, 8^2 \cdot 16}{(s+1)(s^2+4s+8^2)(s+16)}$

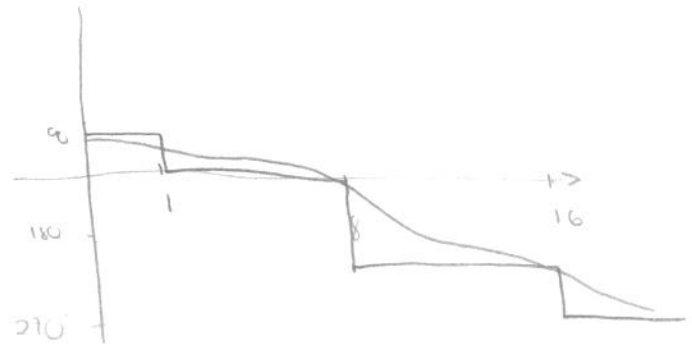
P/ DESCOBRIR O GANHO K, OLHAMOS PARA $s = j1$:

$|F(s)| = 2 = \dfrac{K \cdot 1 \cdot \cancel{8^2}}{1 \cdot \cancel{8^2} \cdot 1}$ $\quad K = 2$

b) $Y(s) = F(s) \cdot X(s) = \dfrac{32 \cdot \cancel{s} \cdot 8^2}{(s+1)(s^2+4s+8^2)(s+16)} \dfrac{1}{s^{\cancel{2}}}$

$\lim\limits_{t\to\infty} y(t) = \lim\limits_{s\to 0} s\,Y(s) = \dfrac{32 \cdot \cancel{8^2}}{\cancel{8^2} \cdot 16}$

c)



d) $y - y_0 = m(x - x_0)$ (BODE DE MODULO)

$Y_{dB} - 20\log 2 = -40(\log 12 - \log 8)$

$Y_{dB} = 20\log 2 + 40\log\frac{8}{12} = 20\log 2 + 40\log\frac{2}{3} = 20\log\frac{2\cdot 4}{9} = 20\log\frac{8}{9}$

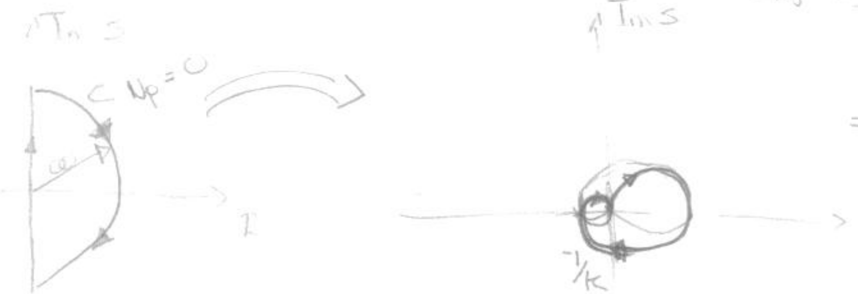
$|y| = \frac{8}{9}$

P/ $u(t) = \cos 12t \Rightarrow y(t) = \frac{8}{9}\cos(12t - \pi) = -\frac{8}{9}\cos(12t)$

e) $G(s) = \dfrac{32 \cdot s \cdot 64}{(s+1)(s^2+4s+8^2)(s+16)}$ $\quad$ Eq CARACT $1 + CG = 0 \Rightarrow 1 + K\dfrac{32 s\, 64}{(s+1)(s^2+4s+8^2)(s+16)} = 0$

$G(j\omega) = \dfrac{32 j\omega 64}{(j\omega+1)(-\omega^2+4j\omega+64)(j\omega+16)}$

$= \dfrac{2048 j\omega}{\omega^4 - 21j\omega^3 - 148\omega^2 + 1152 j\omega + 1024}$



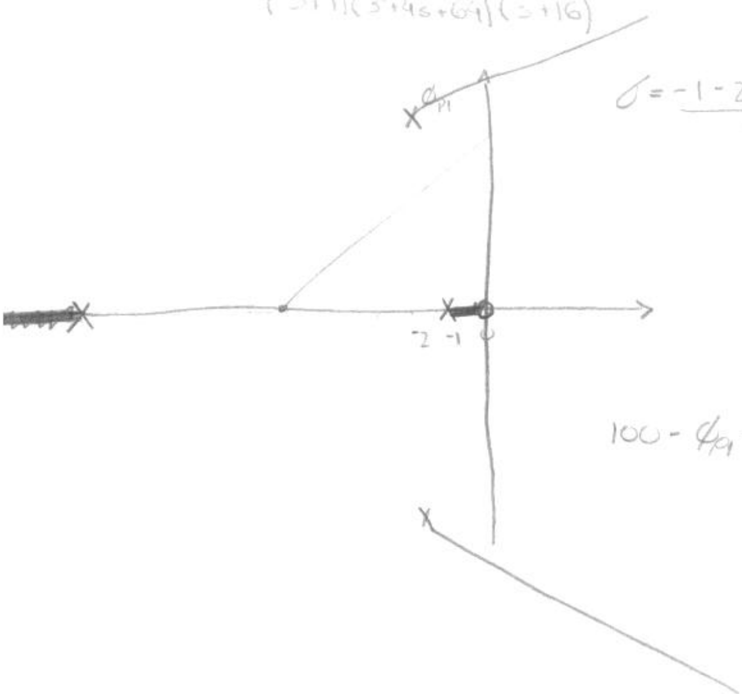
Re S $\quad \omega^4 - 148\omega^2 + 1024 = 0$

$\omega = 11{,}8627$

$g(j11{,}8627) = -1{,}135 \Rightarrow -1{,}135 > -\frac{1}{K}$

$k < 0{,}88$

13) f) $G(s) = \dfrac{32s\ 64}{(s+1)(s^2+4s+64)(s+16)}$



$\sigma = \dfrac{-1-2-2-16}{3} = \dfrac{-21}{3} = -7$

$s^2 + 2\xi\omega_N + \omega_N^2$

$-\xi\omega_N \pm \dfrac{1}{2}\sqrt{4\xi^2\omega_N^2 - 4\omega_N^2}$

$-\xi\omega_N \pm \dfrac{1}{2}\sqrt{4\omega_N^2(\xi^2-1)}$

$-\xi\omega_N \pm \omega_N\sqrt{\xi^2-1}$

$-\xi\omega_N \pm j\omega_N\sqrt{1-\xi^2}$

$100 - \phi_q - 95 - 90° - 30 = 180°$

ROUTH: $1 + KG = 0$

$s^4 + 21s^3 + 148s^2 + (1152 + 2048K)s + 1024$

s^4	1	148	1024
s^3	21	1152+2048K	
s^2	(1)	(2)	
s^1	(3)		
s^0	(2)		

(1) $21 \cdot 148 - 1152 - 2048K > 0$ $\quad K < 0{,}95$

(2) $21 \cdot 1024$ /

(3) $\to$ (1) $\cdot (1152 + 2048K) - 21$ (2) > 0

(4) $\to$ (2)

$-0{,}88 < K < 0{,}88$

$K < 0{,}88 \to$ P/ LUGAR DAS RAÍZES $K > 0$

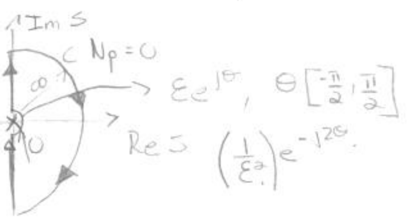
$\Delta(s) = (21 \cdot 148 - 1152 - 2048 \cdot 0{,}88)s^2 + 21 \cdot 1024 = 0$

$s = \pm j11{,}8627$

14) $G(s) = \dfrac{(s+5)(s+10)}{s^2(s+2)}$

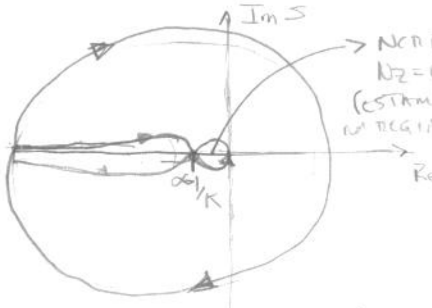
a) $C(s) = K$ $\quad$ Eq CARACTERÍSTICA: $1 + CG = 0 \Rightarrow 1 + K\dfrac{(s+5)(s+10)}{s^2(s+2)} = 0$ $\quad \hat{g}(s)$

$\Rightarrow$

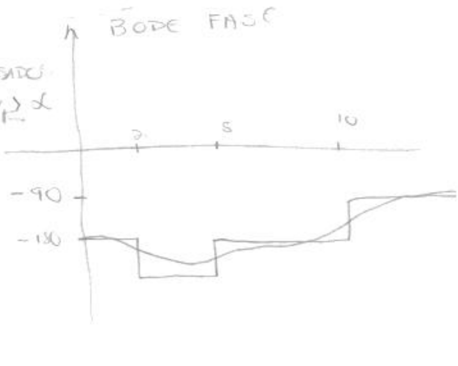


$\varepsilon e^{j\theta}$, $\theta\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\left(\frac{1}{\varepsilon^2}\right)e^{-j2\theta}$



(-1+i)
$\to N_{CRIT} = 0$
$N_Z = 0$
(ESTAMOS INTERESSADOS
NA REGIÃO ONDE $-1/K > \alpha$



$\hat{g}(j\omega) = \dfrac{(j\omega+5)(j\omega+10)}{-\omega^2(j\omega+2)} = \dfrac{-\omega^2 + j15\omega + 50}{-2\omega^2 - j\omega^3}\ \dfrac{(-2\omega^2 + j\omega^3)}{(-2\omega^2 + j\omega^3)}$

$= \dfrac{\cdots - j30\omega^3 - j\omega^5 + 50j\omega^3}{D(\omega)}$ $\to$ SABEMOS QUE A PARTE Im DEVE SER NULA

$\Rightarrow 20\omega^3 - \omega^5 = 0 \Rightarrow \omega^5 = 20\omega^3 \Rightarrow \omega^2 = 20 \Rightarrow \omega = \pm\sqrt{20}$

$\hat{g}(j\sqrt{20}) = -0{,}75 \Rightarrow \dfrac{-1}{K} > -0{,}75 \Rightarrow \boxed{K > \dfrac{4}{3}}$

14) b) $G(s) = \dfrac{\overset{s^2+15s+50}{(s+5)(s+10)}}{s^2(s+2)}$

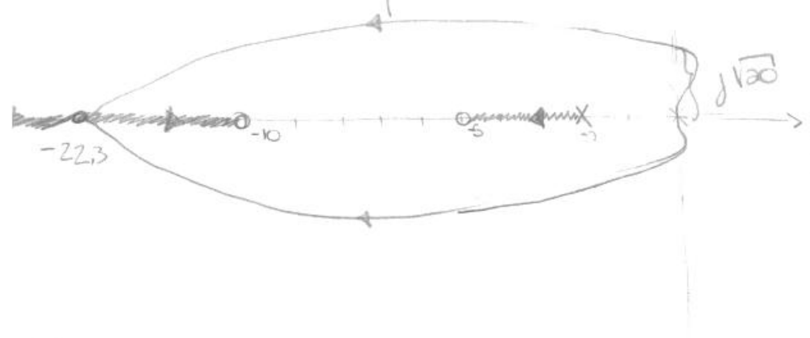
$\sigma = \dfrac{\overset{\Sigma p_i - \Sigma z_i}{-2-(-5)-(-10)}}{1} = +13$

$G'(s) = \dfrac{DN' - ND'}{D} = 0$

$(s^3+2s^2)(2s+15) - (s^2+15s+50)(3s^2+4s) = 0$

$\Rightarrow \cancel{2s^4} \cancel{+15s^3} \cancel{+4s^3} + \cancel{30s^2} \cancel{-3s^4} \cancel{-4s^3} \cancel{-45s^3} - 60s^2$
$\cancel{-150s^2} - 200s = 0$

$-s^4 - 30s^3 - 180s^2 - 200s = 0$



Por Routh $1 + KG = 0 \Rightarrow s^3 + (2+K)s^2 + 15Ks + 50K = 0$

s^3	1	15K
s^2	2+K	50K
s^1	①	
s^0	②	

① $\rightarrow (2+K)15K - 50K > 0$

② $50K$

① $\rightarrow 15K^2 - 20K > 0$

$K(15K - 20) = 0 \Rightarrow K = 0 \quad K < 0$ (NÃO)

$K = \frac{20}{15} = \frac{4}{3} \quad K > 4/3$

$\Delta(s) = (2+K)s^2 + 50K = 0$

Perg. dado ζ

$\Rightarrow \Delta(s) = \cancel{10}/\cancel{3}\, s^2 = -\cancel{50}/\cancel{3} \Rightarrow s = \pm j\sqrt{20}$

P/ $\zeta = \frac{1}{\sqrt{2}}$, $\Rightarrow \dfrac{1}{K} = \dfrac{\prod^m |s - z_i|}{\prod^n |s - p_i|} =$

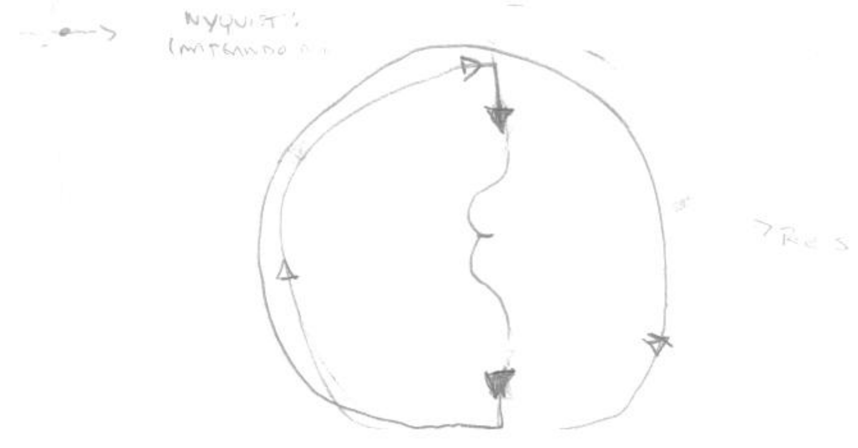
Se parte Real = Im

(Computacional, $\zeta = \frac{1}{\sqrt{2}} \Rightarrow K = 20,7$, polos em $-12 \pm j12$ e $-4,61$)

Não é possível aceitar uma aproximação de segunda ordem pois o polo dominante (mais lento) é $-4,61$ e há somente 3 polos, sendo os outros dois complexos conjugados.

c) Se $C(s) = K/s \Rightarrow$ Eq. caract. $\Rightarrow 1 + K\dfrac{(s+5)(s+10)}{s^3(s+2)}$

$\sigma = \dfrac{\Sigma p_i - \Sigma z_i}{2} = \dfrac{-2+5+10}{2} = \dfrac{13}{2} = 6,5$



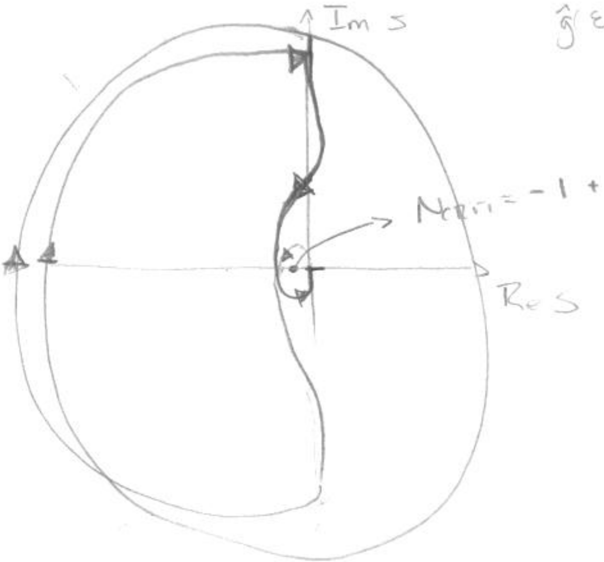
CA721 - SEGUNDA LISTA DE EXERCÍCIOS

$\pi - 180$

4) d) $C(s) = K\frac{(s+20)}{s}$

$\Rightarrow$ Eq: $1 + K\frac{(s+20)(s+5)(s+10)}{s^3(s+2)}$

Nyquist:



$\hat{g}(\varepsilon e^{j\theta}) = \left(\frac{1}{\varepsilon^3}\right)e^{-j3\theta}$

$-3\frac{\pi}{4}$

$G(s) = \frac{s^3 + 35s^2 + 350s + 1000}{s^4 + 2s^3} \rightarrow G(j\omega) = \frac{-j\omega^3 - 35\omega^2 + j350\omega + 1000}{\omega^4 - j2\omega^3} \frac{(\omega^4 - j2\omega^3)}{(\omega^4 + j2\omega^3)}$

Representando somente parte img

$= \dots \frac{-j\omega^7 + j(350)\omega^5 - j70\omega^5 + j2000\omega^3}{\dots} \rightarrow \omega = -16{,}94$

$\hat{g}(j16{,}94) = -0{,}1098 \qquad \frac{-1}{K} > -0{,}1098$

$K > 9{,}01$

Usando $K = 67.3$ (p/ $\xi = 0{,}707$), a função em malha fechada possui os seguintes polos:

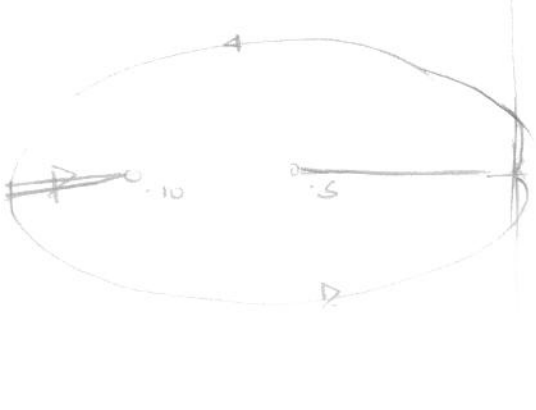
$-27{,}8 \pm 27{,}8$

$-6{,}5181$

$-5{,}0819$

$\rightarrow$ sendo possível uma aproximação de segunda ordem

e) $CG(s) = \frac{K(s+5)(s+10)}{s^3}$

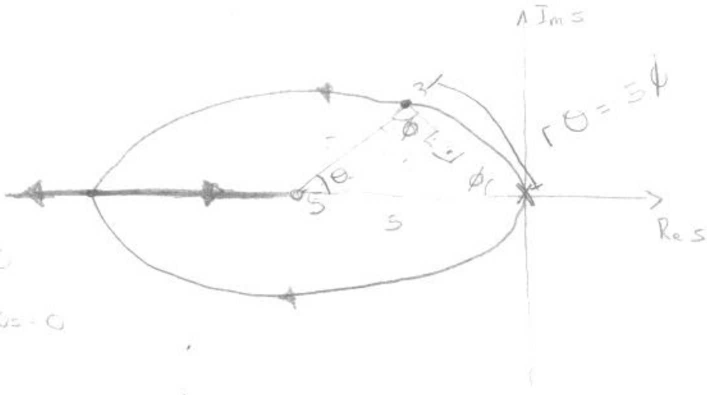


16) Eq caract $1 + CGH = 0$

$$\Rightarrow 1 + \frac{K\left(1+\frac{s}{5}\right)}{s^2\left(\frac{s}{12}+1\right)} \cdot \left(\frac{s}{12}+1\right) = 0 \Rightarrow 1 + \frac{K\left(1+\frac{s}{5}\right)}{s^2} = 0$$

$\hat{g}(s)$

$$1 + \frac{K \cdot 0{,}2(s+5)}{s^2} = 0$$



$\hat{g}'(s) = 0 \Rightarrow s^2(1) - 2s(s+5) = 0$

$\Rightarrow -s^2 - 10s = 0 \Rightarrow s^2 + 10s = 0$

$s((s+10)) = 0 < \begin{matrix} 0 \\ -10 \end{matrix}$

Por Routh: $s^2 + K \cdot 0{,}2s + 1 = 0$

s^2	1	1
s^1	$K0{,}2$	
s^0	1	

$K > 0$

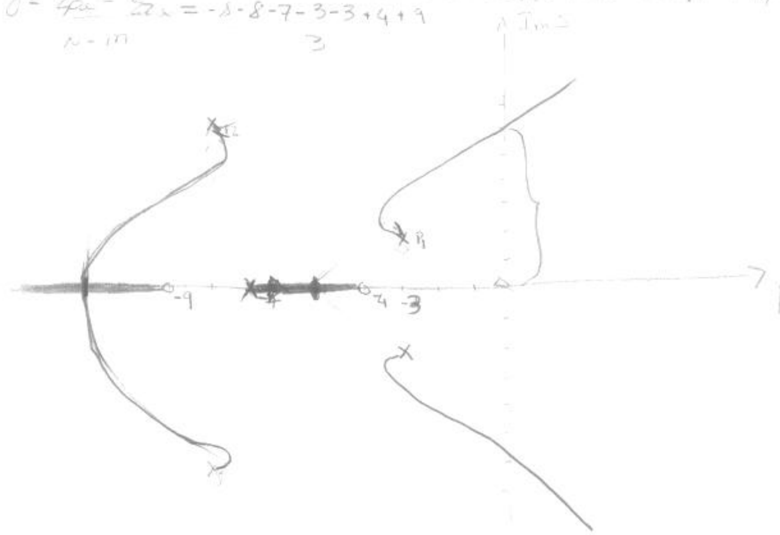
Queremos que os polos possuam parte real $= -3$ $\left(\frac{-1}{6}\right)$

cond. módulo

$$\frac{1}{K \cdot 0{,}2} = \frac{\prod^m |s - z_i|}{\prod^n |s - p_i|} \overset{5{,}47\ 5{,}47}{\Rightarrow} \quad K = 30$$

17) P/ malha) $G(s) = \dfrac{150\,(s+4)(s+9)}{(s^2+6s+13)(s^2+16s+100)(s+7)}$

$\sigma = \dfrac{\sum p_i - \sum z_i}{n-m} = \dfrac{-8-8-7-3-3+4+9}{3}$



Chamando p/ p_1

$45° + 20° - (\phi_{p_1} + 90° + 45 + 30 - 45°) = 180°$

p/ p_2

$145° + 30° - (\phi_{p_2} + 150° + 140° + 110°) = 180°$

$G_{ap}(0) = G(0) = \dfrac{891}{1801}$

$$\frac{\beta}{(s^{2} + 6{,}5\cancel{112}s + 43{,}0654)} = \frac{891}{1801}$$