T_5 ( ) F520 ( ) MS550 · Nome: ______________________ **RA:** __________

A função geratriz das funções de Bessel é

$$g(x,t) = \mathrm{e}^{x(t-t^{-1})/2}.$$

(i) Use-a para mostrar que

$$J_n(u+v) = \sum_{m=-\infty}^{\infty} J_m(u) J_{n-m}(v).$$

(ii) Use esse resultado para mostrar que

$$|J_0(x)| \leq 1, \qquad |J_n(x)| \leq 1/\sqrt{2} \quad (n = 1, 2, 3, \ldots)$$

(i) $$e^{(u+v)(t+t^{-1})/2} = e^{u(t+t^{-1})/2}\, e^{v(t+t^{-1})/2}$$

$$\sum_{n=-\infty}^{+\infty} J_n(u+v)\, t^n = \sum_{m=-\infty}^{+\infty} J_m(u)\, t^m \sum_{k=-\infty}^{+\infty} J_k(v)\, t^k$$

$$\sum_{n=-\infty}^{+\infty} J_n(u+v)\, t^n = \sum_{m=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} J_m(u) J_k(v)\, t^{m+k} \qquad (m+k=n)$$

$$= \sum_{n=-\infty}^{+\infty} \left(\sum_{m=-\infty}^{+\infty} J_m(u) J_{n-m}(v) \right) t^n$$

$$\therefore \quad J_n(u+v) = \sum_{m=-\infty}^{+\infty} J_m(u) J_{n-m}(v)$$

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(ii) Tomando $n=0$, $u=-v=x$

$$J_0(0) = \sum_{m=-\infty}^{+\infty} J_m(x)\, J_{-m}(-x) = \sum_{m=-\infty}^{+\infty} J_m(x)\, (-1)^m J_m(-x)$$

Mas, da def. de $J_m(x)$, segue que $J_m(-x) = (-1)^m J_m(x)$, e lembrando $J_0(0)=1$:

$$1 = \sum_{m=-\infty}^{+\infty} J_m^2(x) = J_0^2(x) + \sum_{m=-\infty}^{-1} J_m^2(x) + \sum_{m=1}^{\infty} J_m^2(x)$$

$$1 = J_0^2(x) + 2\sum_{m=1}^{\infty} J_m^2(x) \implies \begin{cases} J_0^2(x) \leq 1 \\ 2J_m^2(x) \leq 1 \quad (m=1,2,\ldots) \end{cases}$$

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