EA721 – P2 1s2011

1 - $C(s) = K_c \quad P(s) = \dfrac{2}{s(s+1)^2} \quad , F(s) = 1$

K_c tal que $MG \geq 10\,dB$

$$\angle C(s)P(s) = \angle K_c + \angle 2 - \angle j\omega - \angle j\omega+1 - \angle j\omega+1$$
$$= 0^\circ + 0^\circ - 90 - 2\,\mathrm{atan}\,\omega$$
$$= -90 - 2\,\mathrm{atan}\,\omega$$

$\angle C(s)P(s) = 180^\circ \Rightarrow -180 = -90 - 2\,\mathrm{atan}\,\omega \Rightarrow -2\,\mathrm{atan}\,\omega = -90 \Rightarrow \mathrm{atan}\,\omega = 45^\circ \Rightarrow \omega = 1$ rad/s

$$|C(j\omega)P(j\omega)|_{\omega=1} = K_c\left|\frac{2}{j\omega(j\omega+1)^2}\right| = K_c\left|\frac{2}{j(j+1)^2}\right| = K_c\left|\frac{2}{j(j^2+2j+1)}\right| = K_c\left|\frac{2}{j^3+2j^2+j}\right| = K_c\left|\frac{2}{\cancel{-j}+\cancel{j}-2}\right| = K_c \cdot 1$$

$|C(j\omega)P(j\omega)|_{dB} = 20\log K_c$

$MG \cdot |C(j\omega)P(j\omega)| = 1 \Rightarrow$

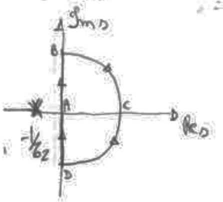
$MG_{dB} + 20\log K_c = 0 \Rightarrow MG = -20\log K_c$

$-20\log K_c \geq 10\,dB \Rightarrow$

$-20\log K_c = 10 \Rightarrow K_c = 10^{-1/2} = 0.3162$

$0 < K_c \leq 0.3162$

2 - $F(s) = 1, \quad C(s)P(s) = \dfrac{-K}{(\tau_1 s+1)(\tau_2 s+1)^2}, \quad K>0, \tau_1>0, \tau_2>0$



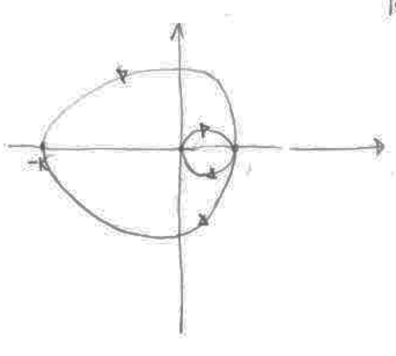
Trecho AB, $s = j\omega \quad 0^+ < \omega < \infty$

$\angle C(j\omega)P(j\omega) = 180 - \mathrm{atan}\,\tau_1\omega - 2\,\mathrm{atan}\,\tau_2\omega$

$|C(j\omega)P(j\omega)| : \dfrac{K}{\sqrt{\tau_1^2\omega^2+1}\,(\tau_2^2\omega^2+1)}$

$\omega \to 0 \Rightarrow \angle CP : 180^\circ$
$\quad |CP| : K$

$\omega \to +\infty \Rightarrow \angle CP : -90$
$\quad |CP| : 0$



Estavel $-K > -1 \Rightarrow K < 1$

Instavel $K > 1$

Marg Est $K = 1$

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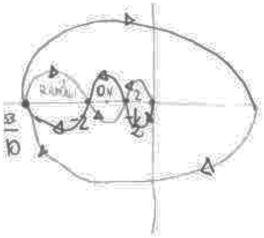
③ $P=0$, para ser estável $\Rightarrow N=0 \Rightarrow Z=0$



$-\frac{33}{10}K > -1 \Rightarrow \frac{33K}{10} < 1 \Rightarrow K < \frac{10}{33}$

$-2K < -1 < -\frac{1}{2}K$

$-2K < -1 \Rightarrow 2K > 1 \Rightarrow K > \frac{1}{2}$

$-1 < -\frac{1}{2}K$

$1 > \frac{1}{2}K \Rightarrow K < 2$

estável   estável

0   0.3 $\frac{1}{2}$ 2

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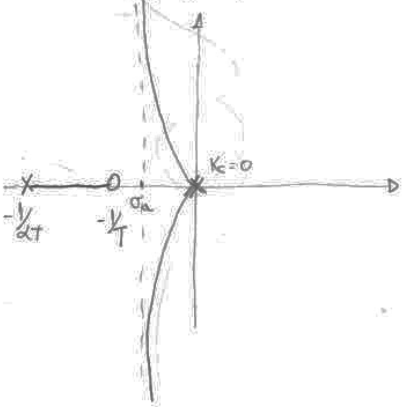
④ - $P(s) = \frac{K}{s^2}$, $K>0$ $F(s)=1$

a) $C(s) = \frac{K_c\alpha(Ts+1)}{\alpha Ts+1}$ $K_c>0$, $T>0$, $0<\alpha<1$

$C(s)P(s) = \frac{KK_c\alpha(Ts+1)}{s^2(\alpha Ts+1)}$ $n=3$, $m=1$



$n-m=2$

Assint: $-90, +90$

Int ass: $\sigma_a = \frac{0+0+\frac{-1}{\alpha T}+\frac{1}{T}}{2} = \frac{\frac{-1+\alpha}{\alpha T}}{\frac{2}{1}} = \frac{-1+\alpha}{2\alpha T} = \frac{\alpha-1}{2\alpha T} < 0$

$\Delta(s) = s^2(\alpha Ts+1) + KK_c\alpha(Ts+1) = 0$

$= \alpha Ts^3 + s^2 + KK_c\alpha Ts + KK_c\alpha$

s^3	αT	$KK_c\alpha T$
s^2	1	$KK_c\alpha$
s^1	$KK_c\alpha T(1-\alpha)$	
s^0	$KK_c\alpha$	

$KK_c\alpha T - \alpha TKK_c\alpha$

$KK_c\alpha T(1-\alpha)$

$\alpha T>0$

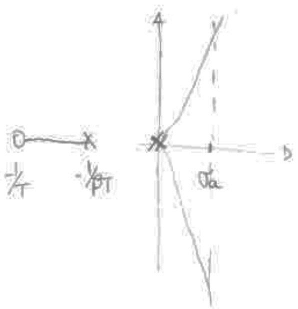
$1>0$

$KK_c\alpha T(1-\alpha)>0$

$KK_c\alpha>0$

sempre Hurwitz

b) $C(s) = \frac{K_c\beta(Ts+1)}{\beta Ts+1}$ $\beta>1$



$n-m=2$

Assint: $-90, +90$

$\sigma_a = \frac{-\frac{1}{\beta T}+\frac{1}{T}}{2} = \frac{\frac{-1+\beta}{\beta T}}{2} = \frac{\beta-1}{2\beta T} > 0$

$\Delta(s) = s^3 + \frac{1}{\beta T}s^2 + KK_cs + \frac{KK_c}{T}$

s^3	1	KK_c
s^2	$\frac{1}{\beta T}$	KK_c/T
s^1	$\left(\frac{KK_c}{\beta T}-\frac{KK_c}{T}\right)/\frac{1}{\beta T}$	
s^0	$\frac{KK_c}{T}$	

$= \frac{KK_c}{T}\underbrace{\left(\frac{1}{\beta}-1\right)}_{<0}$

sempre instável

2 trocas de sinal

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⑤ - $C(s) = K_P + K_D s$ $\quad P(s) = 50/(s(2s+1))$ $\quad F(s) = 1$

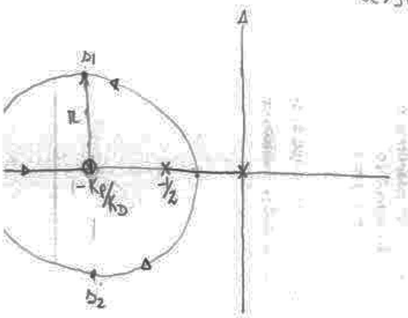
Sistema tipo 1

$t_s < 2s$, $t_s = \frac{4}{\zeta\omega_n}$

$2 = \frac{4}{\zeta\omega_n} \Rightarrow \zeta\omega_n = 2 \Rightarrow \zeta\omega_n \geq 2$

parte real das raízes ≤ -2

$$C(s)P(s) = \frac{50(K_P + K_D s)}{s(2s+1)} = \frac{50K_D(s + K_P/K_D)}{2s(s+1/2)} = \frac{25K_D(s + K_P/K_D)}{(s+0)(s+1/2)}$$



$r = \sqrt{(z-p_1)(z-p_2)}$

$= \sqrt{\left(-\frac{K_P}{K_D}\right)\left(-\frac{K_P}{K_D} + \frac{1}{2}\right)}$

$K_P/K_D = 2$

$r = \sqrt{(-2)(-2+1/2)} = \sqrt{3}$

$s_1 = -2 + j\sqrt{3}$

$s_2 = -2 - j\sqrt{3}$

$s^2 + 2\zeta\omega_n s + \omega_n^2$

$$s_{1,2} = \frac{-2\zeta\omega_n \pm \sqrt{2\zeta\omega_n^2 - 4\omega_n^2}}{2} = -\zeta\omega_n \pm \frac{\sqrt{2\zeta\omega_n^2 - 4\omega_n^2}}{2}$$

$$\left|\frac{K_D(s+2)\,25}{s(s+1/2)}\right|_{s=-2+j\sqrt{3}} = 1 \Rightarrow K_D = \frac{35}{250}$$

$K_P = \frac{70}{250}$