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6,2

1,0 (1)

a-) $\frac{\varepsilon''}{\varepsilon'} = 10^{-2}$ (tangente de perda)

$\gamma = j2\pi f \sqrt{\mu\varepsilon'} \left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{1/2}$

0,5

$\gamma = j2\pi f \sqrt{\mu_0 2{,}5\varepsilon_0} \left(1 - j10^{-2}\right)^{1/2} = 0{,}49 + j99{,}3$

b)

$\alpha = 0{,}49 \frac{Np}{m}$ $\beta = 99{,}3 \frac{rad}{m}$

$e^{-0{,}49 z} = 1/2 \Rightarrow z = 1{,}41\,m$

b) $\varepsilon_c = \varepsilon' - j\varepsilon'' \Rightarrow \frac{\varepsilon_c}{\varepsilon'} = 1 - j\frac{\varepsilon''}{\varepsilon'} = 1 - j10^{-2}$

$\varepsilon_c = 2{,}5\varepsilon_0(1 - j10^{-2})$

0,5

$\eta = \sqrt{\frac{\mu_0}{\varepsilon_c}} = 238 + j1{,}19\,\Omega$

$u_p = \frac{\omega}{\beta} = 1{,}89 \cdot 10^{8}\,m/s$

$\lambda = \frac{2\pi}{\beta} = 6{,}3 \cdot 10^{-2}\,m$

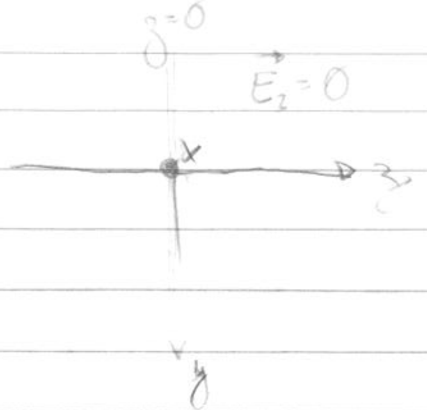
$u_g = ?$

c)

$$\vec{E} = E_0 e^{-j\gamma x}\,\hat{y}$$

$$\vec{E}(x,t) = E_0 e^{-\alpha x} \cos(\omega t - \beta x)\,\hat{y}$$

(1) (2) a)



$E_{1T} = E_{2T} = 0$

$E_{ix} + E_{rx} + E_{iy} + E_{ry} = 0$

$E_0 + jE_0 = -E_r - jE_r \Rightarrow E_r = -E_0$

$\vec{E}_r = E_r e^{+j\beta z}\hat{x} + jE_r e^{+j\beta z}\hat{y}$

0,8²³

$\vec{E}_r = E_0(-\hat{x} + j\hat{y})e^{+j\beta z}$

$-\hat{z}$

Polarização circular negativa.

b) $\vec{H}_i = \dfrac{E_0}{\eta_0}(\hat{y} + j\hat{x})e^{-j\beta z}$

$\vec{H}_r = \dfrac{E_0}{\eta_0}(\hat{y} + j\hat{x})e^{+j\beta z}$

0,3

$\vec{H}_{1total} = \dfrac{2E_0}{\eta_0}\cancel{(\hat{y} + j\hat{x})e^{j\beta z}}$?

$\vec{J}_s = -\hat{z} \times \vec{H}_{1total} = \dfrac{2E_0}{\eta_0}(\hat{x} - j\hat{y})$

está certo, mas ~~... ... ...~~

c) $\vec{E}_{total} = E_0\left[(-\hat{x} + j\hat{y})e^{j\beta z} + (\hat{x} - j\hat{y})e^{-j\beta z}\right]$

0,8⁵³

$\vec{E}_{total}(x,y,z,t) = E_0\left(-\cos(\omega t + \beta z)\hat{x} - \text{sen}(\omega t + \beta z)\hat{y} + \cos(\omega t - \beta z)\hat{x} + \text{sen}(\omega t - \beta z)\hat{y}\right)$

$= E_0\left[(\cos(\omega t - \beta z) - \cos(\omega t + \beta z))\hat{x} + (\text{sen}(\omega t - \beta z) - \text{sen}(\omega t + \beta z))\hat{y}\right]$

$[\cos\omega t \cos\beta z + \sin\omega t \sin\beta z - \cos\omega t \cos\beta z + \sin\omega t \sin\beta z]\hat{x}$

Jansel da Costa ...

④ b-) $\nu_{mn} = \dfrac{1}{2\sqrt{\mu\varepsilon}} \sqrt{\left(\dfrac{m}{a}\right)^2 + \left(\dfrac{n}{b}\right)^2}\ Hz$

$$\gamma = j\beta \Rightarrow j\beta = \sqrt{h^2 - k^2} = \sqrt{h^2 - \omega^2\mu\varepsilon}$$

Como $h^2 = \left(\dfrac{m\pi}{a}\right)^2 + \left(\dfrac{n\pi}{b}\right)^2$

0,625

$$\beta = -j\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon}$$

$$\beta_{mn} = \sqrt{\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

$$v_{f_{mn}} = \frac{\omega}{\beta_{mn}} = \omega\left(\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2\right)^{-1/2} m/s$$

$$v_{g_{mn}} = \frac{1}{d\beta/d\omega} \qquad \frac{d\beta}{d\omega} = \frac{1}{2}\left(\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2\right)^{-1/2} \cdot 2\omega\mu\varepsilon$$

$$v_{g_{mn}} = \frac{\left(\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2\right)^{1/2}}{\omega\mu\varepsilon}$$

c-) $\vec{E} \perp \vec{H} \Rightarrow \vec{E} \cdot \vec{H} = 0$

0,625

$$\vec{E} \cdot \vec{H} = \left(E_x^0 H_x^0 + E_y^0 H_y^0\right) e^{-\gamma z}$$

$$\vec{E} \cdot \vec{H} = \left(-j\frac{\gamma\omega\varepsilon}{h^4}\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right) E_0^2 \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) +\right.$$

$$\left. + j\frac{\gamma\omega\varepsilon}{h^4}\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right) E_0^2 \sin\left(\frac{m\pi}{a}x\right) \cdots \right) e^{-\gamma z} = 0 \quad c.q.d$$

d-) $\nu_{c_{11}} = \frac{c}{2}\sqrt{\left(\frac{1}{0,005}\right)^2 + \left(\frac{1}{0,004}\right)^2} = 48\ GHz$

$\nu_{c_{12}} = \frac{c}{2}\sqrt{\left(\frac{1}{0,005}\right)^2 + \left(\frac{2}{0,004}\right)^2} = 80,7\ GHz$

0,625

$\nu_{c_{21}} = \frac{c}{2}\sqrt{\left(\frac{2}{0,005}\right)^2 + \left(\frac{1}{0,004}\right)^2} = 70,7\ GHz$

OK!

Como os modos TM_{01} e TM_{10} não existem

$$48\ GHz \leq f < 70,7\ GHz$$

1,0 (3) a) $\vec{E_r} = \Gamma_{11} E_{i0} \left(\cos\theta_r \hat{x} + \operatorname{sen}\theta_r \hat{z}\right) e^{-j\beta_1 (x \operatorname{sen}\theta_r - z\cos\theta_r)}$

$\vec{H_r} = \frac{1}{\eta_1} \hat{a}_{nr} \times \vec{E_r} = -\frac{\Gamma_{11} E_{i0}}{\eta_1} e^{-j\beta_1 (x \operatorname{sen}\theta_r - z\cos\theta_r)} \hat{y}$

0,625

$\vec{E_t} = \tau_{11} E_{i0} \left(\cos\theta_t \hat{x} - \operatorname{sen}\theta_t \hat{z}\right) e^{-j\beta_2 (x \operatorname{sen}\theta_t + z\cos\theta_t)}$

$\vec{H_t} = \frac{1}{\eta_2} \hat{a}_{nt} \wedge \vec{E_t} = \frac{\tau_{11} E_{i0}}{\eta_2} e^{-j\beta_2 (x \operatorname{sen}\theta_t + z\cos\theta_t)} \hat{y}$

b) $\vec{E_t}(x, y, z, t$

c) Ângulo de Brewster é um ângulo no qual ocorre a transmissão total de uma parcela da onda (paralela ou perpendicular) e a reflexão total da outra parcela. Seja o ângulo $\theta_{B\parallel}$, no qual não há reflexão da parcela paralela:

$$\Gamma_{\parallel} = 0 \Rightarrow \eta_2 \cos\theta_t = \eta_1 \cos\theta_{B\parallel}$$

Da lei de Snell $\Rightarrow \cos\theta_t = \sqrt{1-\sin^2\theta_t} = \sqrt{1 - \frac{n_1^2}{n_2^2}\sin^2\theta_i}$

$$\eta_2 \sqrt{1 - \frac{n_1^2}{n_2^2}\sin^2\theta_{B\parallel}} = \eta_1 \cos\theta_{B\parallel}$$

0,4

$$1 - \frac{n_1^2}{n_2^2}\sin^2\theta_{B\parallel} = \frac{\eta_1^2}{\eta_2^2}\cos^2\theta_{B\parallel}$$

$$-\frac{\eta_1^2}{\eta_2^2} - \frac{n_1^2}{n_2^2}\sin\theta_{B\parallel} = \frac{\eta_1^2}{\eta_2^2}\cos^2\theta_{B\parallel} - \frac{\eta_1^2}{\eta_2^2} - 1 = -\frac{\eta_1^2}{\eta_2^2}\operatorname{sen}^2\theta_{B\parallel}$$

$$\frac{\eta_2^2}{\eta_1^2}\sin^2\theta_{B\parallel} = \frac{\eta_1^2}{\eta_2^2}\operatorname{sen}^2\theta_{B\parallel} + 1 - \frac{\eta_1^2}{\eta_2^2}$$

$$\left(\frac{\eta_2^4 - \eta_1^4}{\eta_1^2\eta_2^2}\right)\operatorname{sen}^2\theta_{B\parallel} = \frac{\eta_2^2 - \eta_1^2}{\eta_2^2}$$

$$\operatorname{sen}^2\theta_{B\parallel} = \frac{\eta_1^2}{\eta_1^2 + \eta_2^2}$$

2,3

4 a) Para modos TM, $H_z = 0$ e $E_z(x,y,z) = E_z^0(x,y)e^{-\gamma z}$

Para encontrar E_z devemos resolver a equação de Helmholtz

$$\nabla^2 E_z^0 + k^2 E_z^0 = 0$$

$$\frac{\partial^2 E_z^0}{\partial x^2} + \frac{\partial^2 E_z^0}{\partial y^2} + h^2 E_z^0(x,y) = 0$$

Condições de contorno ???

Usando o método de separação de variáveis tem-se:

$$E_z^0(x,y) = E_0 \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right) \ V/m$$

A partir de E_z^0 podemos encontrar os componentes H_x^0, H_y^0, E_x^0 e E_y^0 a partir das seguintes relações:

$$H_x^0 = +\frac{1}{h^2} j\omega\varepsilon \frac{\partial E_z^0}{\partial y} \qquad H_y^0 = -\frac{j\omega\varepsilon}{h^2}\frac{\partial E_z^0}{\partial x}$$

0,4

$$E_x^0 = \frac{-\gamma}{h^2}\frac{\partial E_z^0}{\partial x} \qquad E_y^0 = \frac{-\gamma}{h^2}\frac{\partial E_z^0}{\partial y}$$

Com isso temos:

$$H_x^0 = \frac{j\omega\varepsilon}{h^2}\left(\frac{n\pi}{b}\right) E_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$$

$$H_y^0 = -\frac{j\omega\varepsilon}{h^2}\left(\frac{m\pi}{a}\right) E_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$$

$$E_x^0 = \frac{-\gamma}{h^2}\left(\frac{m\pi}{a}\right) E_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$$

$$E_y^0 = \frac{-\gamma}{h^2}\left(\frac{n\pi}{b}\right) E_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$$