T_2 ( ) F520 ( ) MS550 · Nome: ______________________ RA: __________

Seja a equação diferencial

$$x(1-x)y'' + (1-5x)y' - 4y = 0. \qquad (*)$$

Ao utilizar o método de Frobenius para resolver essa equação diferencial encontramos que a equação indicial correspondente apresenta raízes reais iguais $r_1 = r_2 = 0$, e com isso obtemos que uma das soluções em forma de série dessa equação é

$$y_1(x) = \sum_{n=0}^{\infty} (n+1)^2 x^n.$$

Utilize o método de Frobenius para encontrar uma segunda solução $y_2(x)$ linearmente independente.

$$y_2(x) = y_1(x)\ln x + x^0 \sum_{n=1}^{\infty} a_n x^n = y_1(x)\ln x + \sum_{n=1}^{\infty} a_n x^n$$

$$y_2'(x) = y_1' \ln x + y_1 \frac{1}{x} + \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$y_2''(x) = y_1'' \ln x + 2y_1' \frac{1}{x} - y_1 \frac{1}{x^2} + \sum_{n=1}^{\infty} n(n-1) a_n x^{n-1}$$

+0,2

$$(*) \Rightarrow \underbrace{[x(1-x)y_1'' + (1-5x)y_1' - 4y_1]}_{=0}\ln x + 2y_1' - \cancel{y_1\frac{1}{x}} + \sum_{n=1}^{\infty} n(n-1)a_n x^{n-1}$$

$$-2xy_1' + y_1 - \sum_{n=1}^{\infty} n(n-1)a_n x^n + \cancel{y_1\frac{1}{x}} + \sum_{n=1}^{\infty} n a_n x^{n-1} - 5y_1 - 5\sum_{n=1}^{\infty} n a_n x^n - 4\sum_{n=1}^{\infty} a_n x^n = 0$$

usando a expressão de $y_1(x)$:

$$2\sum_{n=1}^{\infty} n(n+1)^2 x^{n-1} + \sum_{n=1}^{\infty} n(n-1)a_n x^{n-1} - 2\sum_{n=1}^{\infty} n(n+1)^2 x^n - \sum_{n=1}^{\infty} n(n-1)a_n x^n$$

$$+\sum_{n=1}^{\infty} n a_n x^{n-1} - 4\sum_{n=0}^{\infty} (n+1)^2 x^n - 5\sum_{n=1}^{\infty} n a_n x^n - 4\sum_{n=1}^{\infty} a_n x^n = 0$$

$$\therefore\ 2\cdot 1\cdot 2^2 x^0 + 2\sum_{n=1}^{\infty} (n+1)(n+2)^2 x^n + \sum_{n=1}^{\infty} (n+1)n a_{n+1} x^n - 2\sum_{n=1}^{\infty} n(n+1)^2 x^n$$

$$-\sum_{n=1}^{\infty} n(n-1)a_n x^n + 1\cdot a_1 + \sum_{n=1}^{\infty} (n+1)a_{n+1} x^n - 4 - 4\sum_{n=1}^{\infty} (n+1)^2 x^n$$

$$-\sum_{n=1}^{\infty} (5n+4)a_n x^n = 0$$

+0,8

$$\therefore \begin{cases} 8-4+a_1=0 \quad \Longrightarrow \quad \boxed{a_1=-4=-2\cdot 2} \\ 2(n+1)(n+2)^2+(n+1)n a_{n+1}-2n(n+1)^2-n(n-1)a_n+(n+1)a_{n+1} \\ \quad -4(n+1)^2-(5n+4)a_n=0 , \qquad n=1,2,3,\dots \quad \circledast \end{cases}$$

$$\circledast \quad \underbrace{(n+1)[2(n+2)^2\underbrace{-2n(n+1)-4(n+1)}_{-2(n+2)(n+1)}]}_{\underbrace{(n+2)[2(n+2)-2(n+1)]}_{(n+2)\cdot 2}}+(n+1)^2 a_{n+1}-\underbrace{\underbrace{[n(n-1)+5n+4]}_{n^2+4n+4}}_{(n+2)^2}a_n=0$$

$$\therefore 2(n+1)(n+2)+(n+1)^2a_{n+1}-(n+2)^2a_n=0$$

$$\therefore \boxed{a_{n+1}=\left(\frac{n+2}{n+1}\right)\left[\left(\frac{n+2}{n+1}\right)a_n-2\right]} \qquad n=1,2,3,\dots \qquad +0,5$$

$\underline{n=1}$

$$a_2=\frac{3}{2}\left[\frac{3}{2}\cdot a_1-2\right]=\frac{3}{2}(-6-2)=-3\cdot 4=-3\cdot 2\cdot 2$$

$\underline{n=3}$

$$a_3=\frac{4}{3}\left[\frac{4}{3}(-3\cdot 4)-2\right]=\frac{4}{3}(-16-2)=-4\cdot 3\cdot 2$$

$\underline{n=4}$

$$a_4=\frac{5}{4}\left[\frac{5}{4}(-4\cdot 6)-2\right]=\frac{5}{4}(-32)=-5\cdot 4\cdot 2$$

$$\therefore \boxed{a_n=-2n(n+1)} \qquad , n=1,2,3\dots$$

$$\therefore y_2(x)=y_1(x)\ln x-2\sum_{n=1}^{\infty}n(n+1)x^n \qquad +0,5$$

onde: $y_1(x)=\sum_{n=0}^{\infty}(n+1)^2x^n$