1. Seja $z = \ln(y) \approx a_1 + a_2 x$

$$\Leftrightarrow e^{z} = y \approx e^{a_1 + a_2 x} = \underbrace{e^{a_1}}_{\alpha_1} \cdot \underbrace{e^{a_2 x}}_{\alpha_2}$$

$\frac{1}{2}$

Ajuste de

x	1	1.25	1.5	1.75	2
$z = \ln y$	1.6292	1.7561	1.8764	2.0082	2,1353

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1.25 \\ 1 & 1.5 \\ 1 & 1.75 \\ 1 & 2 \end{pmatrix}, \quad A \cdot \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \approx z$$

$\frac{1}{2}$

$$A^{+}A \cdot \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = A^{+} z$$

$$A^{+}A = \begin{pmatrix} 5 & 7,5 \\ 7,5 & 11,875 \end{pmatrix}, \quad A^{t} \cdot z = \begin{pmatrix} 9.4053 \\ 14.4241 \end{pmatrix}$$

$\frac{1}{2}$

Resolve $\left(\begin{array}{cc|c} 5 & 7,5 & 9,4053 \\ 7,5 & 11,875 & 14.4241 \end{array}\right)$

$$\rightsquigarrow \left(\begin{array}{cc|c} 5 & 7,5 & 9.4053 \\ 0 & 0,6250 & 0.3161 \end{array}\right)$$

$a_2 = 0.3161 / 0.6250 = 0.5057$

$$\rightsquigarrow \left(\begin{array}{cc|c} 5 & 7.5 & 9.4053 \\ 0 & 1 & 0.5057 \end{array} \right)$$

$$\rightsquigarrow \left(\begin{array}{cc|c} 5 & 0 & 5.6124 \\ 0 & 1 & 0.5057 \end{array} \right)$$

$$\rightsquigarrow \left(\begin{array}{cc|c} 1 & 0 & 1.1225 \\ 0 & 1 & 0.5057 \end{array} \right)$$

$\frac{1}{2}$ $a_1 = 1{,}1225 \Rightarrow \alpha_1 = e^{a_1} = \underline{3{,}0725}$

$a_2 = 0{,}5057 \Rightarrow \alpha_2 = a_2 = \underline{0{,}5057}$ $\frac{1}{4}$

~~Um ajuste apropi à curva~~

A curva $3{,}0725 \cdot e^{0{,}5057 \cdot x}$

Ofrece um ajuste adequado aos dados

(x_i, y_i) $\frac{1}{4}$

2. $y'' + 2y' + 2y = 0$

$\Leftrightarrow y'' = -2y' - 2y = -2(y' + y)$

(a)

$y(0) = 2$

$y'(0) = -2$

$\frac{1}{2}$

Seja $\bar{Y} = \begin{pmatrix} y \\ y' \end{pmatrix} \Rightarrow \bar{Y}' = \begin{pmatrix} y' \\ y'' \end{pmatrix} = \begin{pmatrix} y' \\ -2(y'+y) \end{pmatrix}$

$Y(0) = \begin{pmatrix} y(0) \\ y'(0) \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

$\Rightarrow (\bar{y}'_{k+}$

x_k	$\bar{Y}_k = \begin{pmatrix} y_k \\ y_k' \end{pmatrix}$	$\bar{Y}_k' = \begin{pmatrix} y_k' \\ -2(y_k'+y_k) \end{pmatrix}$	$\tilde{Y}_{k+1} = Y_k + Y_k' \cdot h = \begin{pmatrix} \tilde{y}_{k+1} \\ \tilde{y}'_{k+1} \end{pmatrix}$	$\tilde{Y}'_{k+1} = \begin{pmatrix} y_{k+1} \\ -2(\tilde{y}'_{k+1}+\tilde{y}_{k+1}) \end{pmatrix}$	$\Delta Y_k = h \cdot \frac{\bar{Y}_k' + \tilde{Y}'_{k+1}}{2}$ $\frac{1}{2}$
0	$\begin{pmatrix} 2 \\ -2 \end{pmatrix}$	$\begin{pmatrix} -2 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1.8 \\ -2 \end{pmatrix}$ $\frac{1}{2}$	$\begin{pmatrix} -2 \\ +0.4 \end{pmatrix}$	$\begin{pmatrix} -0.2 \\ 0.02 \end{pmatrix}$ $\frac{1}{2}$
0.1	$\begin{pmatrix} 1.8 \\ -1.98 \end{pmatrix}$	$\begin{pmatrix} -1.98 \\ 0.36 \end{pmatrix}$	$\begin{pmatrix} 1.6020 \\ -1.9440 \end{pmatrix}$	$\begin{pmatrix} -1.9440 \\ 0.6840 \end{pmatrix}$	$\begin{pmatrix} -0.1962 \\ 0.0522 \end{pmatrix}$
0.2	$\begin{pmatrix} 1.6038 \\ -1.9278 \end{pmatrix}$ $\frac{1}{4}$				

Reposta $y(0.2) \approx y_2 = 1.6038$ $\frac{1}{4}$

3. (a)

Ordem

x	0	1	2	3
-.75	-.0718			
		0.1880		
-.5	-.0248		2.5016	
		1.4388		0.9995
-.25	.3349		3.2512	
		3.0644		
0	1.101			

$\frac{1}{2}$

Utilizamos os nós $(-.5, -0.0248)$
$(-.25, 0.3349)$ $\frac{1}{2}$
$(0, 1.101)$

porque $\left|-\frac{1}{3} - 0\right| < \left|-\frac{1}{3} - (-0.75)\right|$

$p_2(x) = -.0248 + 1.4388(x - x_0) + 3.2512(x-x_0)(x-x_1)$

$= -0.248 + 1.4388(x+0.5) + 3.2512(x+0.5)(x+0.25)$

$= 1.101 + 3.8772 \cdot x + 3.2512x^2$ $\frac{1}{2}$

$p_2\cancel{(-0.3333)} \approx f(-\frac{1}{3}) \approx p_2(-0.3333)$

$\simeq \cancel{-0.0534}$ $\underline{0.1698}$ $\frac{1}{2}$

(b) $E_2(x) \hat{\leq} (x-x_0)(x-x_1)(x-x_2)$

$\cdot \max|\text{dif. div. de ordem } 3|$

$= (x+0.5)(x+0.25)(x) \cdot \cancel{3.2512}$ $0{,}9995$ $\frac{1}{4}$

$E_2(-\frac{1}{3})$ é aproximadamente $\leq$

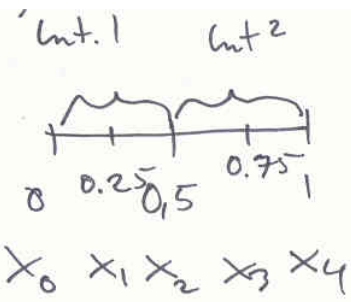
$(-.3333+0.5)(-.3333+0.25) -.3333$

$\cdot \cancel{3.2512}$ $\approx \cancel{0.0151}$ $\frac{1}{4}$

$0{,}9995$

0.0046

Ex. 4(e)



$$\int_0^1 f(x)\,dx =$$

$h = \frac{1}{4}$

$$= \frac{1}{12}\left[f(0) + 4f(0.25) + 2f(0.5) + 4f(0.75) + f(1)\right]$$

$$= \frac{1}{12}\left[\frac{1}{2} + 4\cdot\frac{1}{e^{0.25^2}+1} + 2\,\frac{1}{e^{0.5^2}+1} + 4\cdot\frac{1}{e^{0.75^2}+1} + \frac{1}{e^{1}+1}\right]$$

$\frac{1}{2}$

$$= \frac{1}{12}\left[0.5 + 1.9375 + 0.8756 + 1.4519 + 0.2689\right]$$

$$\approx \frac{1}{12}\cdot 5.0339 \approx 0.4195$$

$\frac{1}{2}$

4(b) $\displaystyle\int_{\varphi(-1)=0}^{\varphi(1)=1} \frac{1}{e^{x^2}+1}\,dx$

$$\begin{aligned} \alpha-\beta&=0\\ \alpha+\beta&=1 \end{aligned}$$

$\varphi(t)=\alpha+\beta t$

$2\beta=1 \Longleftrightarrow \beta=0{,}5$

$2\alpha=1 \Longleftrightarrow \alpha=0{,}5$

$\frac{1}{4}$

$\varphi(t)=0{,}5(1+t)$

$$\int_0^1 f(x)\,dx=\int_{-1}^{1} f(0{,}5+0{,}5t)\cdot 0{,}5\,dt \quad \frac{1}{4}$$

$$=\int_{-1}^{1}\frac{1}{e^{0{,}25(1+t)^2}+1}\cdot 0{,}5\,dt \quad \frac{1}{4}$$

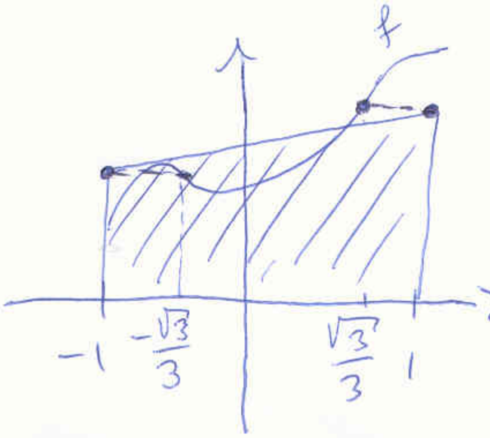
$$=\frac{1}{2}\left[\frac{1}{e^{0{,}25\left(1-\frac{\sqrt{3}}{3}\right)^2}+1}+\frac{1}{e^{0{,}25\left(1+\frac{\sqrt{3}}{3}\right)^2}+1}\right]$$

$$=\frac{1}{2}\left[0{,}4888+0.3493\right]$$

$$\approx 0.4191$$

$\frac{1}{4}$

(c)



$\frac{1}{2}$

$$f\left(-\frac{\sqrt{3}}{3}\right)+f\left(\frac{\sqrt{3}}{3}\right)=2\cdot\frac{f\left(-\frac{\sqrt{3}}{3}\right)+f\left(\frac{\sqrt{3}}{3}\right)}{2}$$

4. PVC

$$y''y - y'x = 2$$

$y(0) = 1$ $\qquad$ $y_0 \quad y_1 \quad y_2 \quad y_3 \quad y_4$

$y(1) = 2$ $\qquad$ $0 \quad 0.25 \quad 0.5 \quad 0.75 \quad 1$ $\qquad$ $\frac{1}{4}$

$h = 0.25$

$$y(x_k) \approx y_k$$

$$y'(x_k) \approx \frac{y_{k+1} - y_{k-1}}{2h}$$ $\qquad$ $\frac{1}{2}$

$$y''(x_k) \approx \frac{y_{k+1} - 2y_k + y_{k-1}}{h^2}$$

para $k = 1, 2, 3$ temos: $\qquad$ $\frac{1}{4}$

$$\frac{y_{k+1} - 2y_k + y_{k-1}}{h^2} \cdot y_k - \frac{y_{k+1} - y_{k-1}}{2h} \cdot x_k = 2$$

$\frac{1}{2}$

$$(y_{k+1} - 2y_k + y_{k-1})y_k - \frac{1}{2}(y_{k+1} - y_{k-1})h \cdot x_k = 2h^2$$

$$(2y_{k+1} - 4y_k + 2y_{k-1})y_k - (y_{k+1} - y_{k-1})h x_k = 4h^2$$