

So Ivan had predicted that we could extract  $\mathbf{Var}(\tilde{h}^{KPZ}(0, s))$  from the relation:

$$\mathbf{var}(Q_B(N, t)) = \mathbf{var}(\tilde{h}^{KPZ}(0, x_0^4)) \frac{t^{1/2}}{x_0^2} \quad (1)$$

where  $s = \frac{4\log(N)^2}{t}$ ,  $x_0^4 = s$  and  $Q_B(N, t)$  is the position of the Nth quantile at time  $t$ . A graph of  $\mathbf{Var}(\tilde{h}^{KPZ}(0, s))$  is shown below as the blue triangles. I think the key feature is that it's monotonically decreasing and as  $s \rightarrow \infty$  it approaches a value between 0.5 and 1. However, if Equation 1 is only valid for  $t > \log(N)^2$  then shouldn't we only care about the portion of the graph where  $s < 1$ .

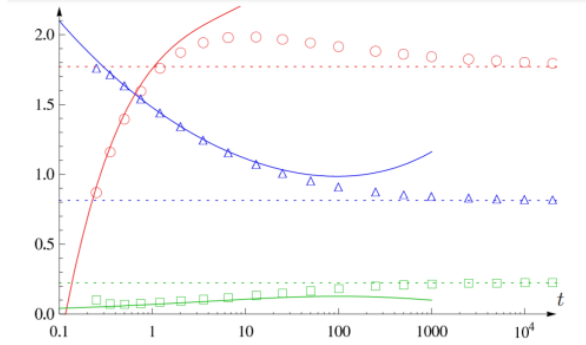


FIG. 3: (Color online) First three normalized cumulants of the distribution  $F_t$  as a function of  $t$ . Short time expansion of  $-C_1$  (solid red line),  $C_2$  (solid blue line), and  $C_3/C_2^{3/2}$  (solid green line) as in (24) and numerical evaluation of the cumulants  $-C_1$  (red circles),  $C_2$  (blue triangles),  $C_3/C_2^{3/2}$  (green squares) for the values of  $t$  as listed in (19). The dotted lines represent the asymptotic Tracy-Widom values (27) – (29).

Figure 1: Variance of  $\tilde{h}^{KPZ}(0, s)$  is in the blue triangles.

So now if we plug in  $s$  and  $x_0^2 = \frac{2\log(N)}{t^{1/2}}$  into Equation 1 we get

$$\mathbf{var}(Q_B(N, t)) = \mathbf{var}(\tilde{h}^{KPZ}(0, \frac{4\log N^2}{t})) \frac{t^{1/2}t^{1/2}}{2\log(N)} \quad (2)$$

Solving for  $\mathbf{var}(\tilde{h}^{KPZ}(0, \frac{\log N^2}{t}))$ ,

$$\mathbf{var}(\tilde{h}^{KPZ}(0, \frac{4\log N^2}{t})) = \frac{2\log(N)}{t} \mathbf{var}(Q_B(N, t)) \quad (3)$$

Then if we plot the RHS of Equation 3 on the y-axis and  $\frac{4\log(N)^2}{t}$  on the x-axis we should see the graph in Figure 1. Note that we aren't talking about

the same time that is on the x-axis in Figure 1 since we want the plot of  $\text{var}(\tilde{h}^{KPZ}(0, \frac{4 \log N^2}{t}))$ . I think this is most likely where the issue lies.

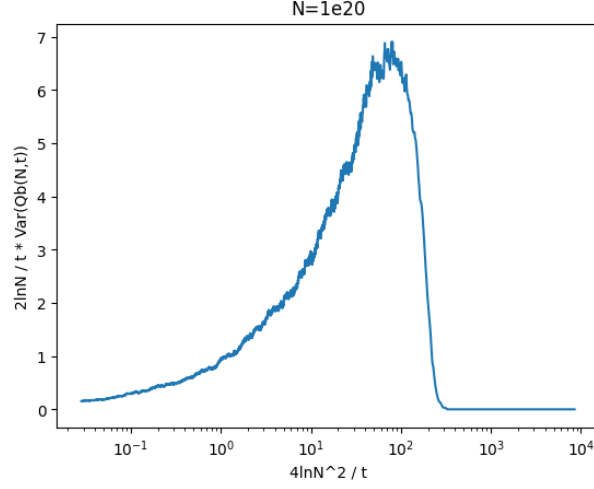


Figure 2: Calculated curve for  $\tilde{h}^{KPZ}(0, s)$

The figure was generated using the PDF for 1000 systems out to  $t=300,000$  and for the  $1e20$  quantile. An interesting thing to note is that the graphs are not the same for different quantiles. Additionally, the unscaled variance is plotted in the figure below.

If there aren't any issues above, could there be an assumption that was made to get the  $t^{1/2}$  scaling that we need to do here. Such as it only holds for  $t \gg \ln N^2$ .

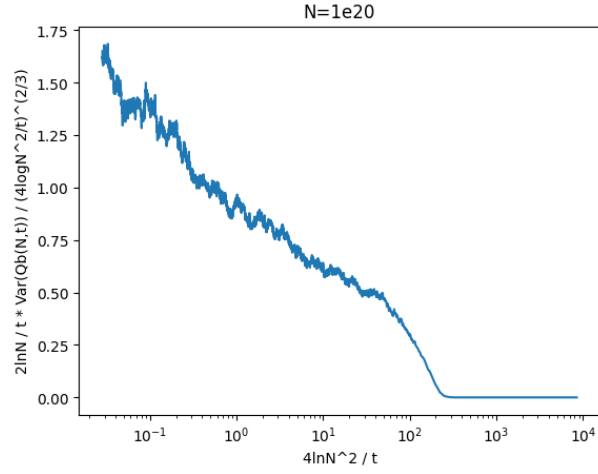


Figure 3: Calculated curve for  $\tilde{h}^{KPZ}(0, s)$  but y-axis is scaled by  $s^{(2/3)}$

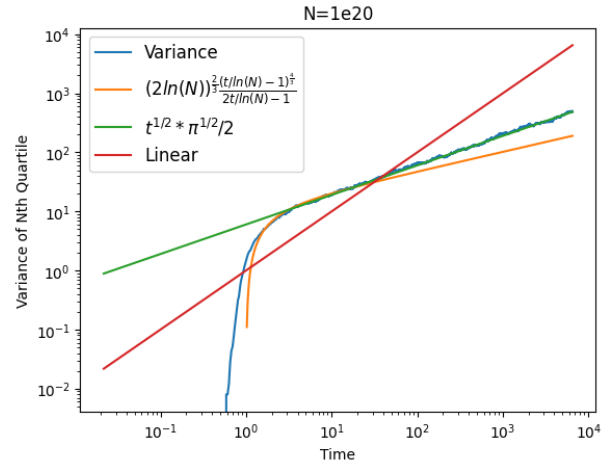


Figure 4: Variance of the 1e20 quantile.