Introduction to Spacecraft Navigation – MANE 6964

Spring Term 2024

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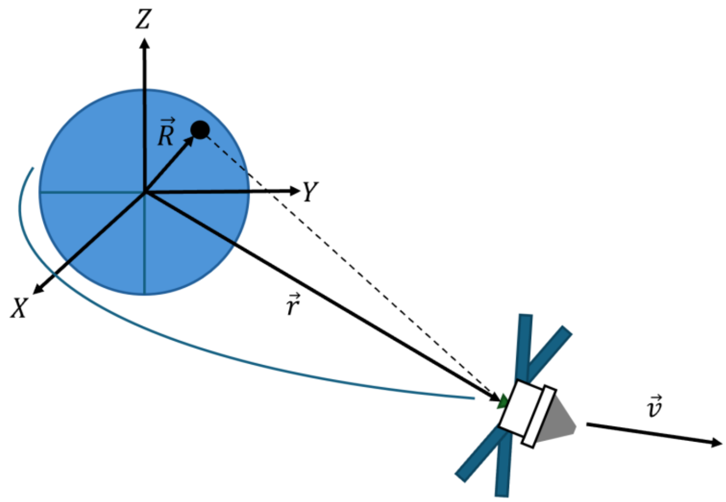
Homework 5: Kalman Filtering **Due: 04/05/24**

Consider a spacecraft on its way to the Moon. After its trans-lunar injection (TLI) burn, it is said to be on trans-lunar coast. During this period, it is being tracked by the Deep Space Network ground station at Goldstone, CA (35° 25′36″ N latitude, 116° 53′24″ W longitude).



The Goldstone station takes range, range-rate, and bearing measurements (often abbreviated as "R3B") of the spacecraft. These are tabulated in MANE6964_HW5_meas.csv. Note that not all types of measurements are available simultaneously. The true state of the spacecraft is tabulated every minute in MANE6964_HW5_traj.csv. All times are given in seconds since the start of the sidereal day (when the ECI frame is aligned with the ECEF frame).

You will be tasked with coding up an Extended Kalman Filter (EKF) to perform sequential estimation of the state of the spacecraft. The states you will track are the inertial position and velocity (with respect to the ECI frame). That is to say

$$\vec{x} = \begin{bmatrix} \vec{r} \\ \vec{v} \end{bmatrix}$$

You can initialize the filter using $\vec{x}_0^T$ = [500,6500,3500,-10,2,3] and a 6x6 covariance matrix describing 100km uncertainty in position and 0.1km/s uncertainty in velocity. Your dynamic model will be nonliner and will take the form

$$\dot{\vec{x}} = f(\vec{x}, t) + Gw$$

We will assume that acceleration has Gaussian random white noise. That is to say

$$G = \begin{bmatrix} O_{3\times3} \\ I_{3\times3} \end{bmatrix} \qquad Q = \sigma_w^2\, I_{3\times3}$$

You will need to derive equations describing the range, range-rate, and bearing measurements as observed at the Goldstone station as functions of the current spacecraft state. Then, you will need to take partial derivatives of these measurement models to build the measurement sensitivity matrix H for each measurement type. This may prove to be the most difficult part of the assignment for some.

We will say that the range uncertainty is $\sigma_\rho = 10km$ the range-rate uncertainty is $\sigma_{\dot{\rho}} = 100m/s$ and the bearing uncertainty can be measured using the QUEST model, with bearing error of $\sigma_{\hat{u}} = 1arcmin$.

Once you have coded up the filter (both propagate and update steps), store the estimated state and covariance at each time step. It is highly recommended that you propagate the state every minute so you can more easily check your state error against the trajectory file. Your final deliverable will be six plots (x,y,z position and velocity in the ECI frame) showing the error and three-sigma uncertainty in each state.