

Analysis of electrical power and energy systems

Practical session 1

21 September 2020

1 Exercises¹

- Express the following voltages as phasors:
(a) $v_1(t) = \sqrt{2} \times 100 \cos(\omega t - 30^\circ)$ V
(b) $v_2(t) = \sqrt{2} \times 100 \cos(\omega t + 30^\circ)$ V
- The following series R-L-C circuit (Figure 1) is in a sinusoidal steady state at a frequency of 60 Hz. $V = 120$ V, $R = 1.5\Omega$, $L = 20$ mH and $C = 100\mu$ F. Calculate $i(t)$ in this circuit by using the phasor-domain analysis.

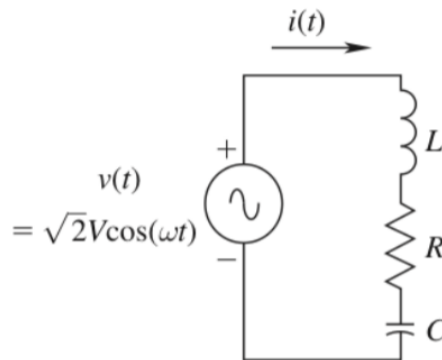


Figure 1: RLC series circuit.

- In a linear circuit in sinusoidal steady state with only one active source $\bar{V} = 90\angle 30^\circ$ V, the current in a branch is $\bar{I} = 5\angle 15^\circ$ A. Calculate the current in the same branch if the source voltage were to be $100\angle 0^\circ$ V.
- To the circuit of Figure 2, if a voltage of $100\angle 0^\circ$ V is applied, calculate P , Q and the power factor. Show that $Q = \sum_k I_k^2 X_k$.
- In the circuit (Figure 3) the complex power drawn by the load impedance was calculated as $P_L + jQ_L = (1858.4 + j1031.3)$ VA, calculate the capacitive reactance in parallel, necessary to make the overall power factor to 0.9 (leading) if the applied voltage has an rms value of 120 V.
- A positive sequence (a-b-c), balanced, wye-connected voltage source has the phase-a voltage given as $\bar{V}_a = \sqrt{2} \times 100\angle 30^\circ$ V. Obtain the time-domain voltages $v_a(t)$, $v_b(t)$, $v_c(t)$ and $v_{ab}(t)$, and show all of these as phasors.
- A balanced three-phase inductive load is supplied in steady-state by a balanced, wye-connected, three-phase voltage source with a phase voltage of 120V RMS. The load draws a total of 10kW at a power factor of 0.9. Calculate the RMS value of the phase currents

¹From Chapter 2 of Ned Mohan's book "Electric power systems, a first course"

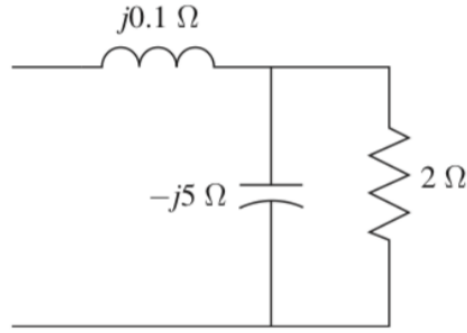


Figure 2: Ex 4 circuit.

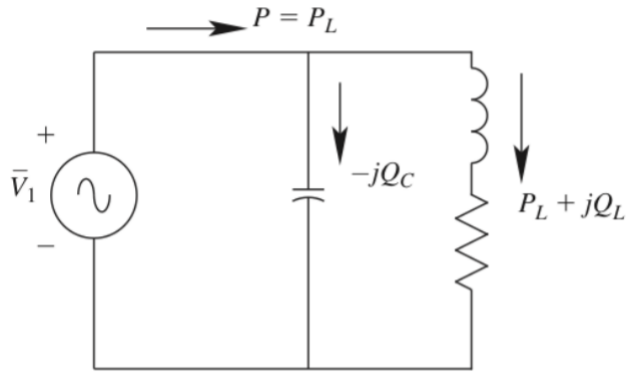


Figure 3: Power factor correction.

and the magnitude of the per-phase load impedance, assuming a wye connected load. Draw a phasor diagram showing all three voltages and currents.

8. The balanced circuit of Figure 4 shows the impedance of three-phase cables connecting the source terminals (a, b, c) to the load terminals (A, B, C), where $Z_{self} = (0.3 + j1.5)\Omega$ and $Z_{mutual} = j0.5\Omega$. Calculate \bar{V}_A if $\bar{V}_a = 1000\angle 0^\circ$ V and $\bar{I}_a = 10\angle -30^\circ$ A, where \bar{V}_A and \bar{V}_a are voltages with respect to a common neutral.

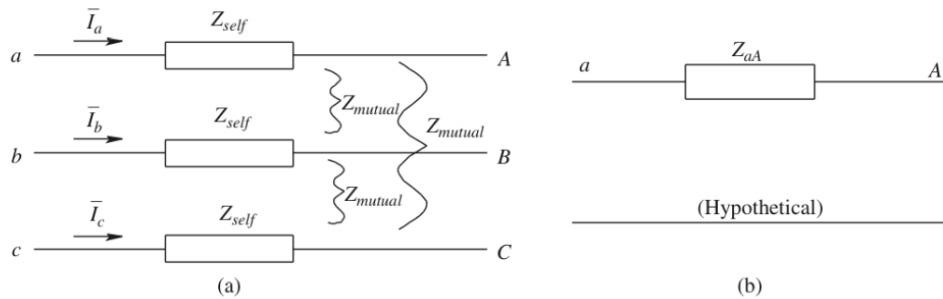


Figure 4: Balanced three-phase network with mutual coupling.

9. In the per-phase circuit of Figure 5, the power transfer per-phase is 1 kW from side 1 to 2. $V_S = 100$ V, $\bar{V}_R = 95\angle 0^\circ$ V, and $X = 1.5\Omega$. Calculate the current, the phase angle of \bar{V}_S , and the per-phase Q_R supplied to the receiving end.

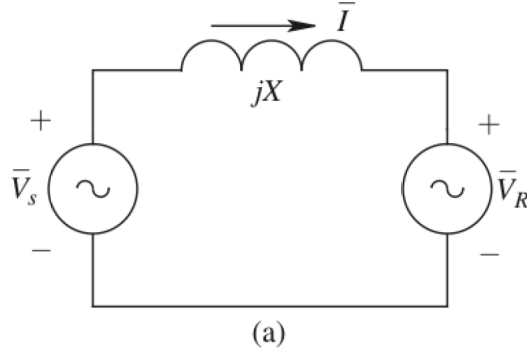


Figure 5: Power transfer between two AC systems.

10. In a radial system represented by the circuit of Figure 5, $X = 1.5 \, \Omega$. Consider the source voltage to be constant at $\bar{V}_S = 100\angle 0^\circ \text{ V}$. Calculate and plot V_S/V_R if the load varies in a range from 0 to 1 kW at the following three power factors: unity, 0.9 (lagging), 0.9 (leading).
11. In the three-phase circuit of Figure 6, $|Z_L| = 10 \, \Omega$, and the per-phase power factor is 0.8 (lagging). Calculate the per-unit values of the per-phase voltage, the load impedance, the load current, and the load real and reactive powers,
 - (a) if the line-to-line voltage base value is 208 V (RMS) and the base value of the three-phase power is 3.6 kW.
 - (b) if the line-to-line voltage base value is 240 V (RMS) and the base value of the three-phase power is 5.4 kW.
 - (c) if the line-to-line voltage base value is 240 V (RMS) and the base value of the three-phase power is 3.6 kW.

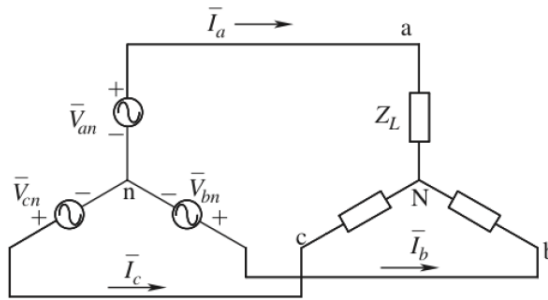


Figure 6: Balanced wye connected, three-phase circuit.