

# Analysis of electrical power and energy systems

## Practical session 2

30 September 2021

### 1 Exercises<sup>1</sup>

1. A positive sequence (a-b-c), balanced, wye-connected voltage source has the phase-a voltage given as  $\bar{V}_a = \sqrt{2} \times 100 \angle 30^\circ \text{ V}$ . Obtain the time-domain voltages  $v_a(t)$ ,  $v_b(t)$ ,  $v_c(t)$  and  $v_{ab}(t)$ , and show all of these as phasors.
2. A balanced three-phase inductive load is supplied in steady-state by a balanced, wye-connected, three-phase voltage source with a phase voltage of 120V RMS. The load draws a total of 10kW at a power factor of 0.9. Calculate the RMS value of the phase currents and the magnitude of the per-phase load impedance, assuming a wye connected load. Draw a phasor diagram showing all three voltages and currents.
3. In the per-phase circuit of Figure 1, the power transfer per-phase is 1 kW from side 1 to 2.  $V_S = 100 \text{ V}$ ,  $\bar{V}_R = 95 \angle 0^\circ \text{ V}$ , and  $X = 1.5 \Omega$ . Calculate the current, the phase angle of  $\bar{V}_S$ , and the per-phase  $Q_R$  supplied to the receiving end.

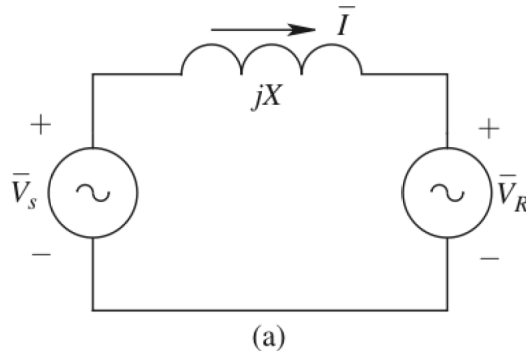


Figure 1: Power transfer between two AC systems.

4. In a radial system represented by the circuit of Figure 1,  $X = 1.5 \Omega$ . Consider the source voltage to be constant at  $\bar{V}_S = 100 \angle 0^\circ \text{ V}$ . Calculate and plot  $V_S/V_R$  if the load varies in a range from 0 to 1 kW at the following three power factors: unity, 0.9 (lagging), 0.9 (leading).
5. In the three-phase circuit of Figure 2,  $|Z_L| = 10 \Omega$ , and the per-phase power factor is 0.8 (lagging). Calculate the per-unit values of the per-phase voltage, the load impedance, the load current, and the load real and reactive powers,

(a) if the line-to-line voltage base value is 208 V (RMS) and the base value of the three-phase power is 3.6 kW.

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<sup>1</sup>Exercises 2.11, 2.12, 2.16, 2.17, 2.18, 2.19 and 2.20 from Ned Mohan's book "Electric power systems, a first course"

- (b) if the line-to-line voltage base value is 240 V (RMS) and the base value of the three-phase power is 5.4 kW.
- (c) if the line-to-line voltage base value is 240 V (RMS) and the base value of the three-phase power is 3.6 kW.

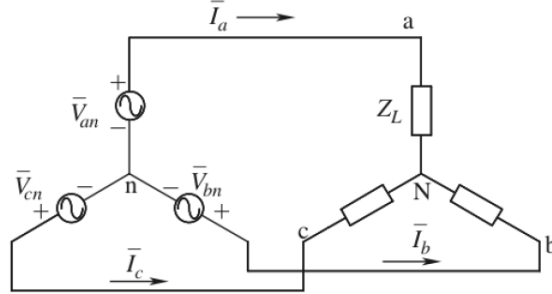


Figure 2: Balanced wye connected, three-phase circuit.

## 2 Solutions

- For a frequency of 50 Hz,  $v_a(t) = 200 \cos(100\pi t + \frac{\pi}{6})$  V,  $v_b(t) = 200 \cos(100\pi t + \frac{3\pi}{2})$  V,  $v_c(t) = 200 \cos(100\pi t - \frac{5\pi}{6})$  V,  $v_{ab}(t) = \sqrt{3} 200 \cos(100\pi t + \frac{\pi}{3})$  V
- $I = 30.864$  A,  $|Z_L| = 3.888 \Omega$
- $\bar{V}_A = 992.428 \angle -0.413^\circ$  V
- $\bar{I} = 10.82 \angle -13.35^\circ$  A,  $\delta = 9.08^\circ$ , per phase  $Q_R = 237.221$  var supplied to the receiving end
- The plots will be shown during the practical session.
- $\bar{V}_{ph} = 1 \angle 0^\circ$  pu,  $Z_L = 0.832 \angle -36.87^\circ$  pu,  $\bar{I}_L = 1.202 \angle -36.87^\circ$  pu,  $P_L = 0.961$  pu,  $Q_L = 0.721$  pu
  - $\bar{V}_{ph} = 1 \angle 0^\circ$  pu,  $Z_L = 0.937 \angle -36.87^\circ$  pu,  $\bar{I}_L = 1.067 \angle -36.87^\circ$  pu,  $P_L = 0.853$  pu,  $Q_L = 0.64$  pu
  - $\bar{V}_{ph} = 1 \angle 0^\circ$  pu,  $Z_L = 0.625 \angle -36.87^\circ$  pu,  $\bar{I}_L = 1.6 \angle -36.87^\circ$  pu,  $P_L = 1.28$  pu,  $Q_L = 0.96$  pu