Analysis of electrical power and energy systems

Assignment 1

Due date: 5 October 2020

Power flow analysis

In the system of Figure 1, the voltage base (3-phase) is 345 kV and the power base (3-phase) is 100 MVA. Bus-1 is a slack bus with $V_1 = 1.0$ pu and $\theta_1 = 0$. Bus-2 is a PV bus with $V_2 = 1.05$ pu and $P_2^0 = 2.0$ pu. Bus-3 is a PQ bus with injections of $P_3^0 = -5.0$ pu and $Q_3^0 = -1.0$ pu. Lines electrical resistance and reactance are provided by $r = 0.037\ \Omega$/km and $x = 0.376\ \Omega$/km. Shunt susceptances are ignored. Lines length are specified in Figure 1. Use the Newton-Raphson procedure to solve the power flow problem by following the steps below:

1. Compute the line impedances of the system $Z_{1,2}$, $Z_{1,3}$, $Z_{2,3}$ where $Z_{i,j}$ is the impedance of the line between bus-i and bus-j.

2. Calculate the base impedance Z_B and compute the per-unit line impedances.

3. Compute the per-unit admittance matrix of the system $\mathbf{Y}_{pu}$ where $Y_{i,i}$ is the sum of the admittances incident to bus i and $Y_{i,j} (i \neq j)$ is the opposite of the sum of the admittances connecting bus i to bus j. Calculate from the latter, the per-unit conductance and susceptance matrices, respectively $\mathbf{G}_{pu}$ and $\mathbf{B}_{pu}$.

4. Initialize the unknown voltages of the multidimensional Newton-Raphson procedure: there are two unknowns at the PQ bus (V_3, θ_3) and one unknown at the PV bus (θ_2). Initialize the unknown magnitudes at 1 pu and unknown phases at 0 rad (first arbitrary estimation).

$$\bar{\mathbf{V}}^{(0)} = \begin{pmatrix} \theta_2^{(0)} \\ V_3^{(0)} \\ \theta_3^{(0)} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tag{1}$$

5. Perform one iteration in the Newton-Raphson procedure.

   - Use the active and reactive power equations (2) in the nodes where active and reactive power are known and establish $P_2(\theta_2, V_3, \theta_3)$, $P_3(\theta_2, V_3, \theta_3)$ and $Q_3(\theta_2, V_3, \theta_3)$

$$\begin{cases} P_i = G_{ii} V_i^2 & + V_i \sum_{j \in \mathcal{N} \setminus i} V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) \\ Q_i = -B_{ii} V_i^2 & + V_i \sum_{j \in \mathcal{N} \setminus i} V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) \end{cases} \tag{2}$$

   - Compute the error of the initial step $\mathbf{F}^0 - \mathbf{f}(\bar{\mathbf{V}}^{(0)})$: calculate the P_2, P_3 and Q_3 values resulting from the initial estimate and compute the error with respect to P_2^0, P_3^0 and Q_3^0.

   - Compute the Jacobian matrix $\mathbf{J}$ of the system and evaluate it at $\bar{\mathbf{V}}^{(0)}$.

- Find the update step $\mathbf{\Delta\bar{V}}$ by solving the following linear system:

$$\underbrace{\mathbf{J}(\mathbf{\bar{V}}^{(0)})}_{\mathbf{A}}\underbrace{\Delta\mathbf{\bar{V}}^{(0)}}_{\mathbf{x}} = \underbrace{\mathbf{F}^0 - \mathbf{f}(\mathbf{\bar{V}}^{(0)})}_{\mathbf{b}} \tag{3}$$

- Update the estimated voltages magnitudes and phases.

$$\mathbf{\bar{V}}^{(1)} = \mathbf{\bar{V}}^{(0)} + \mathbf{\Delta\bar{V}}^{(\mathbf{0})} \tag{4}$$

6. Repeat the process until the error $\mathbf{F}^0 - \mathbf{f}(\mathbf{\bar{V}}^{(0)})$ is low enough.

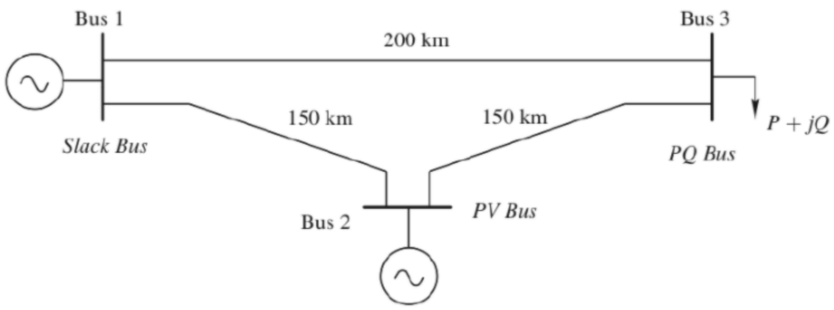

Figure 1: Three-bus power system.