Analysis of electrical power and energy systems

Practical sessions 1 & 2

21 & 28 September 2020

1 Exercises[1]

1. Express the following voltages as phasors:
(a) $v_1(t) = \sqrt{2} \times 100 \cos(\omega t - 30°)$ V
(b) $v_2(t) = \sqrt{2} \times 100 \cos(\omega t + 30°)$ V

2. The following series R-L-C circuit (Figure 1) is in a sinusoidal steady state at a frequency of 60 Hz. $V = 120$V, $R = 1.5\Omega, L = 20$mH and $C = 100\mu$F. Calculate $i(t)$ in this circuit by using the phasor-domain analysis.

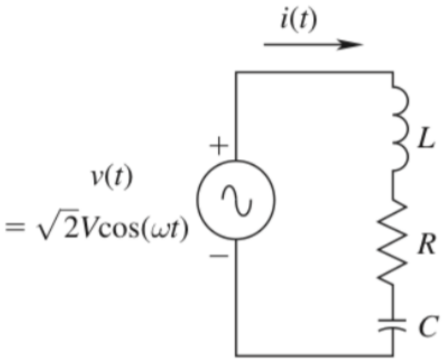

Figure 1: RLC series circuit.

3. In a linear circuit in sinusoidal steady state with only one active source $\overline{V} = 90\angle 30°$ V, the current in a branch is $\overline{I} = 5\angle 15°$ A. Calculate the current in the same branch if the source voltage were to be $100\angle 0°$V.

4. To the circuit of Figure 2, if a voltage of $100\angle 0°$ V is applied, calculate P, Q and the power factor. Show that $Q = \sum_k I_k^2 X_k$.

5. In the circuit (Figure 3) the complex power drawn by the load impedance was calculated as $P_L + jQ_L = (1858.4 + j1031.3)$ VA, calculate the capacitive reactance in parallel, necessary to make the overall power factor to 0.9 (leading) if the applied voltage has an rms value of 120 V.

6. A positive sequence (a-b-c), balanced, wye-connected voltage source has the phase-a voltage given as $\overline{V}_a = \sqrt{2} \times 100\angle 30°$V. Obtain the time-domain voltages $v_a(t)$, $v_b(t)$, $v_c(t)$ and $v_{ab}(t)$, and show all of these as phasors.

7. A balanced three-phase inductive load is supplied in steady-state by a balanced, wye-connected, three-phase voltage source with a phase voltage of 120V RMS. The load draws

[1] Exercises 2.1, 2.2, 2.4, 2.5, 2.9, 2.11, 2.12, 2.14, 2.16, 2.17, 2.18, 2.19 and 2.20 from Ned Mohan's book "Electric power systems, a first course"

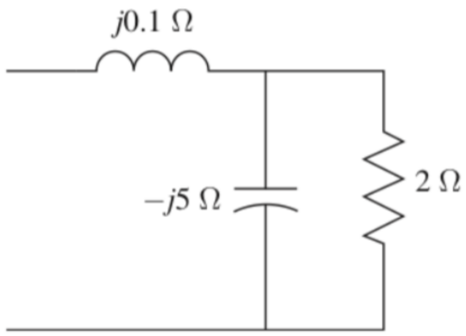

Figure 2: Ex 4 circuit.

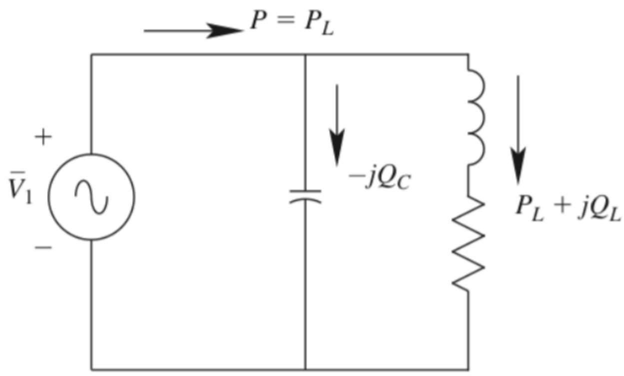

Figure 3: Power factor correction.

a total of 10kW at a power factor of 0.9. Calculate the RMS value of the phase currents and the magnitude of the per-phase load impedance, assuming a wye connected load. Draw a phasor diagram showing all three voltages and currents.

8. The balanced circuit of Figure 4 shows the impedance of three-phase cables connecting the source terminals (a, b, c) to the load terminals (A, B, C), where $Z_{self} = (0.3 + j1.5)\Omega$ and $Z_{mutual} = j0.5\ \Omega$. Calculate $\overline{V}_A$ if $\overline{V}_a = 1000\angle 0°$ V and $\overline{I}_a = 10\angle -30°$ A, where $\overline{V}_A$ and $\overline{V}_a$ are voltages with respect to a common neutral.

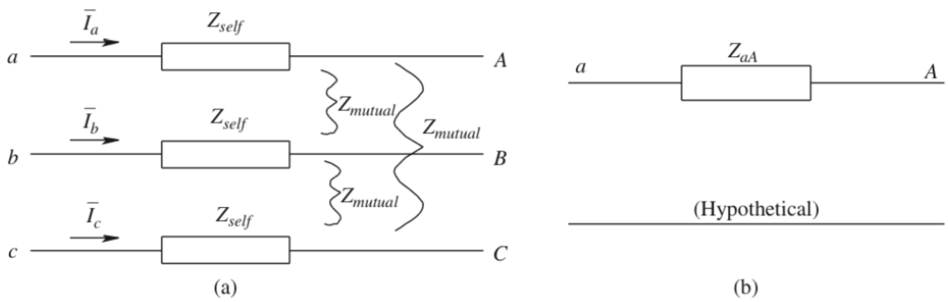

Figure 4: Balanced three-phase network with mutual coupling.

9. In the per-phase circuit of Figure 5, the power transfer per-phase is 1 kW from side 1 to 2. $V_S = 100$ V, $\overline{V}_R = 95\angle 0°$ V, and $X = 1.5\ \Omega$. Calculate the current, the phase angle of $\overline{V}_S$, and the per-phase Q_R supplied to the receiving end.

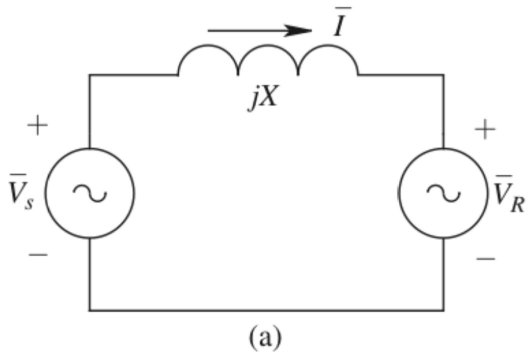

Figure 5: Power transfer between two AC systems.

10. In a radial system represented by the circuit of Figure 5, $X = 1.5\ \Omega$. Consider the source voltage to be constant at $\overline{V}_S = 100\angle 0°$ V. Calculate and plot V_S/V_R if the load varies in a range from 0 to 1 kW at the following three power factors: unity, 0.9 (lagging), 0.9 (leading).

11. In the three-phase circuit of Figure 6, $|Z_L| = 10\ \Omega$, and the per-phase power factor is 0.8 (lagging). Calculate the per-unit values of the per-phase voltage, the load impedance, the load current, and the load real and reactive powers,

    (a) if the line-to-line voltage base value is 208 V (RMS) and the base value of the three-phase power is 3.6 kW.

    (b) if the line-to-line voltage base value is 240 V (RMS) and the base value of the three-phase power is 5.4 kW.

    (c) if the line-to-line voltage base value is 240 V (RMS) and the base value of the three-phase power is 3.6 kW.

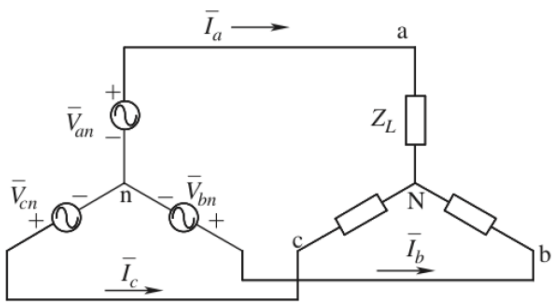

Figure 6: Balanced wye connected, three-phase circuit.

2 Solutions

1. (a) $\overline{V}_1 = 100\angle -30°$ V (b) $\overline{V}_2 = 100\angle 30°$ V

2. $i(t) = 4.65\sqrt{2}\cos(376.99t + 1.51)$ A

3. $\overline{I} = 5.556\angle - 0.262$ A

4. $P = 5192.64$ W, $Q = -1775.89$ var, $\cos\phi = 0.946$, $Q_R = 0$ var, $Q_L = 301.17$ var, $Q_C = -2077.06$ var

5. $X_C = -7.46\ \Omega$, $C = 0.356$ mF

6. For a frequency of 50 Hz, $v_a(t) = 200\cos(314.16t + 0.52)$ V, $v_b(t) = 200\cos(314.16t + 2.62)$ V, $v_a(t) = 200\cos(314.16t - 1.57)$ V, $v_{ab}(t) = 346.41\cos(314.16t)$ V

7. $I = 30.864$ A, $|Z_L| = 3.888\ \Omega$

8. $\overline{V}_A = 992.428\angle - 0.413$ V

9. $\overline{I} = 10.82\angle - 13.35°$ A, $\delta = 9.08°$, per phase $Q_R = 237.221$ var supplied to the receiving end

10. The plots will be shown during the practical session.

11. (a) $\overline{V}_{ph} = 1\angle 0°$ pu, $Z_L = 0.832\angle - 36.87°$ pu, $\overline{I}_L = 1.202\angle - 36.87°$ pu, $P_L = 0.961$ pu, $Q_L = 0.721$ pu
    (b) $\overline{V}_{ph} = 1\angle 0°$ pu, $Z_L = 0.937\angle - 36.87°$ pu, $\overline{I}_L = 1.067\angle - 36.87°$ pu, $P_L = 0.853$ pu, $Q_L = 0.64$ pu
    (c) $\overline{V}_{ph} = 1\angle 0°$ pu, $Z_L = 0.625\angle - 36.87°$ pu, $\overline{I}_L = 1.6\angle - 36.87°$ pu, $P_L = 1.28$ pu, $Q_L = 0.96$ pu