# Analysis of electric power and energy systems

Lecture 4: Transformers, power flow analysis part 2

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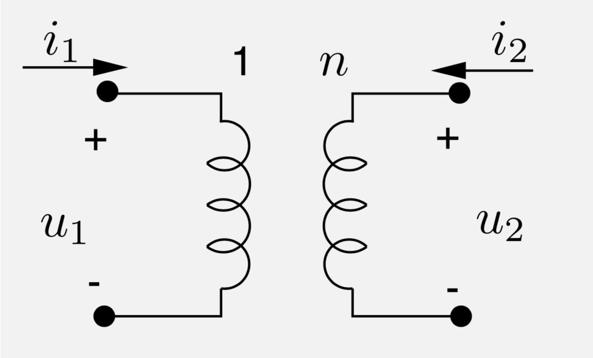
# What will we learn today?

- The power transformer
- The next part of power flow analysis: how to include transformers, and transformers with tap changers

You will be able to do exercises 6.2, 6.3, 6.4 from the Ned Mohan's book.

# The transformer

A (single phase) transformer is made of two magnetically coupled coils or windings. An ideal transformer is a two-port represented as




with

$$u_2 = n u_1$$

$$i_2 = -\frac{1}{n} i_1$$

In power systems, transformers are mainly used to transmit power over long distances by changing the voltage level, thus decreasing the current for a given power level. Voltage level of synchronous generator around 20kV.

Voltage is changed around 5 times between generation and load.

It is also used to measure currents and voltages, electrically isolate parts of a circuit (not the auto-transformer we will see), match impedances.

How does a Transformer work ?

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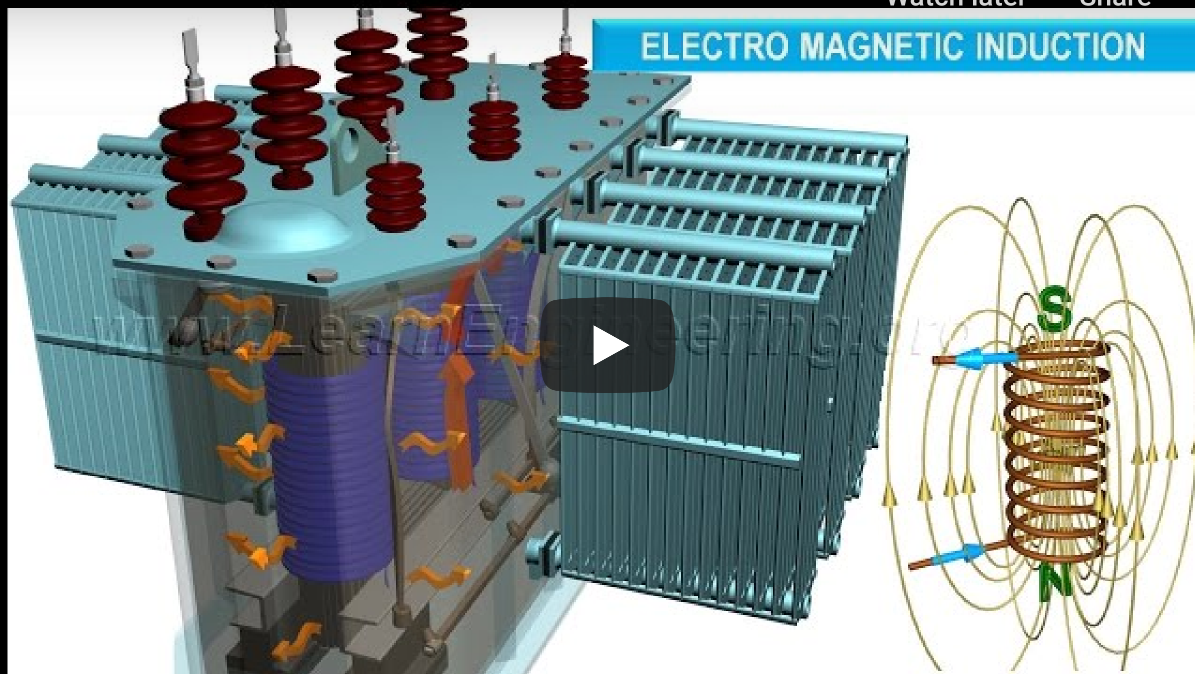
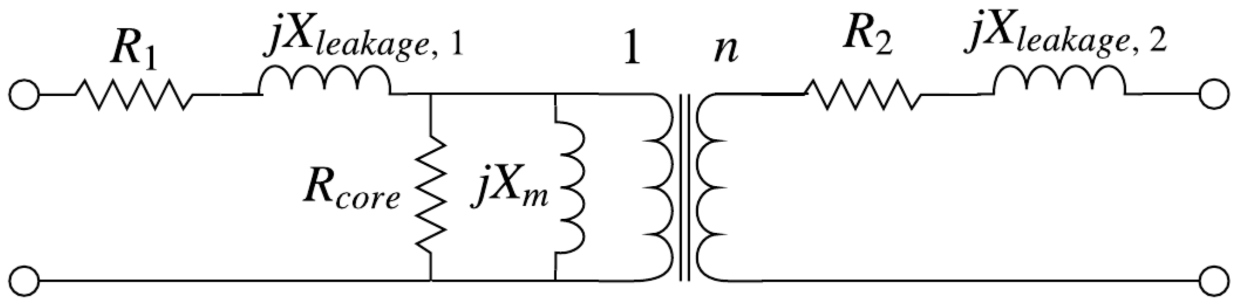


# Non-ideal model



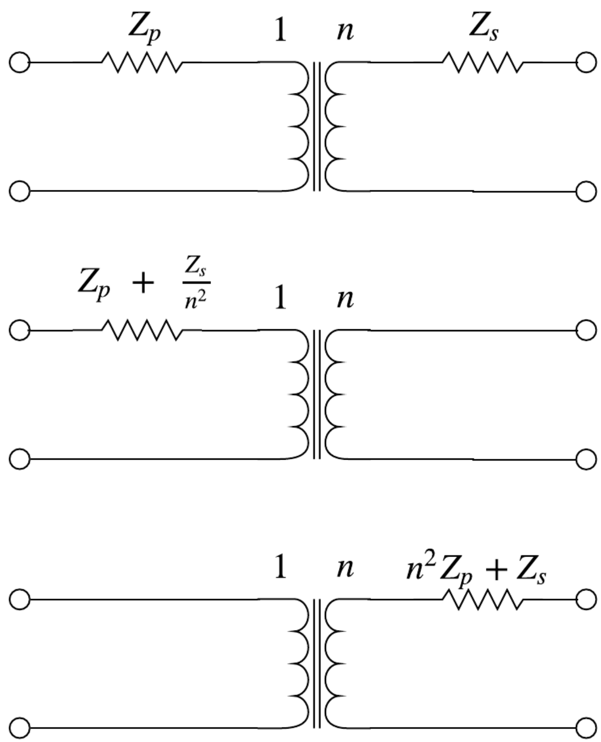

The ideal model is complemented by elements

- $X_m$ that models the magnetizing inductance
- $X_{leakage,i}$ that models the flux not captured by the core on side $i$
- $R_{core}$ that models eddy current and hysteresis losses, i.e. losses in the iron core
- $R_1$ and $R_2$ that model (coil) copper losses

Parameters are either given in the datasheet or obtained by open-circuit and short-cirtuit tests.

Laminated core to decrease losses.

The excitation current, sum of the currents in $R_{core}$ and $X_m$, is often neglected, leading to a simpler non-ideal model, and the series impedances can be transferred from one side to the other:




with

$$Z_p = R_1 + jX_{leakage,1}$$

and

$$Z_s = R_2 + jX_{leakage,2}$$

## Per unit representation

Let's consider the rated voltages and currents on both sides of the (ideal) transformer as base values. As

$$V_{s,base} = nV_{p,base}$$

and

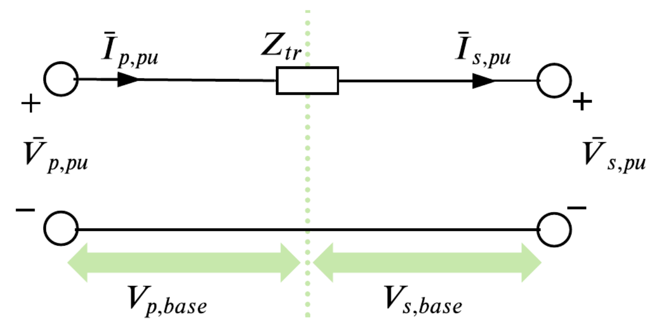
$$I_{p,base} = nI_{s,base} ,$$

the MVA base is the same on both sides, and thus

$$Z_{s,base} = n^2 Z_{p,base}$$

Hence, in per unit, the transformer can be replaced by a single impedance

$$Z_{tr} = \frac{Z_p}{Z_{p,base}} + \frac{Z_s}{Z_{s,base}}.$$



Thus we have also that

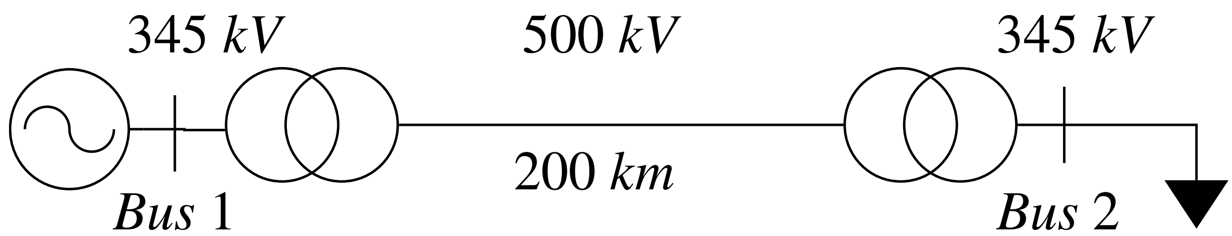
$$\begin{aligned} Z_{tr} &= \frac{Z_p + Z_s/n^2}{Z_{p,base}} \\ &= \frac{n^2 Z_p + Z_s}{Z_{s,base}} \end{aligned}$$

i.e. the impedance if the same whether we see it from the primary or the secondary side, although the voltage bases differ.

Also, if the three-phase transformer is wye-delta connected, a 30° phase shift must be applied (more on this later).

## Example 6.1

Consider the one-line diagram




with

- a 200 km line with $R = 0.029\Omega/km$, $X = 0.326\Omega/km$, neglected shunt impedances
- two ransformers with a leakage reactance of $0.2pu$ in the (500 kV, 1000 MVA) base, and losses neglected.

What is the equivalent model in a (345 kV, 100 MVA) base?

In the (500 kV, 1000 MVA) base:

- $$Z_{line,pu} = 200 \times (0.029 + j0.326)/(500^2/1000) = 0.0232 + j0.2608pu$$
- hence the total impedance between buses 1 and 2 is

$$Z_{12} = 0.0232 + j0.2608 + 2 * j0.2pu = 0.0232 + j0.6608pu$$

In the (345 kV, 100 MVA) base:

- the pu value of the impedance is the same in the (500 kV, 1000 MVA) and (345 kV, 1000 MVA) bases,
  - since we can transfer the impedance from one side of each transformer to the other, cf. a previous remark
- if we now change the MVA base to 100 MVA,

$$Z_{12} = 0.0232 + j0.6608 \times (100/1000)pu = 0.00232 + j0.06608pu$$

since the base impedance is proportional to the inverse of the MVA base.

## Efficiency

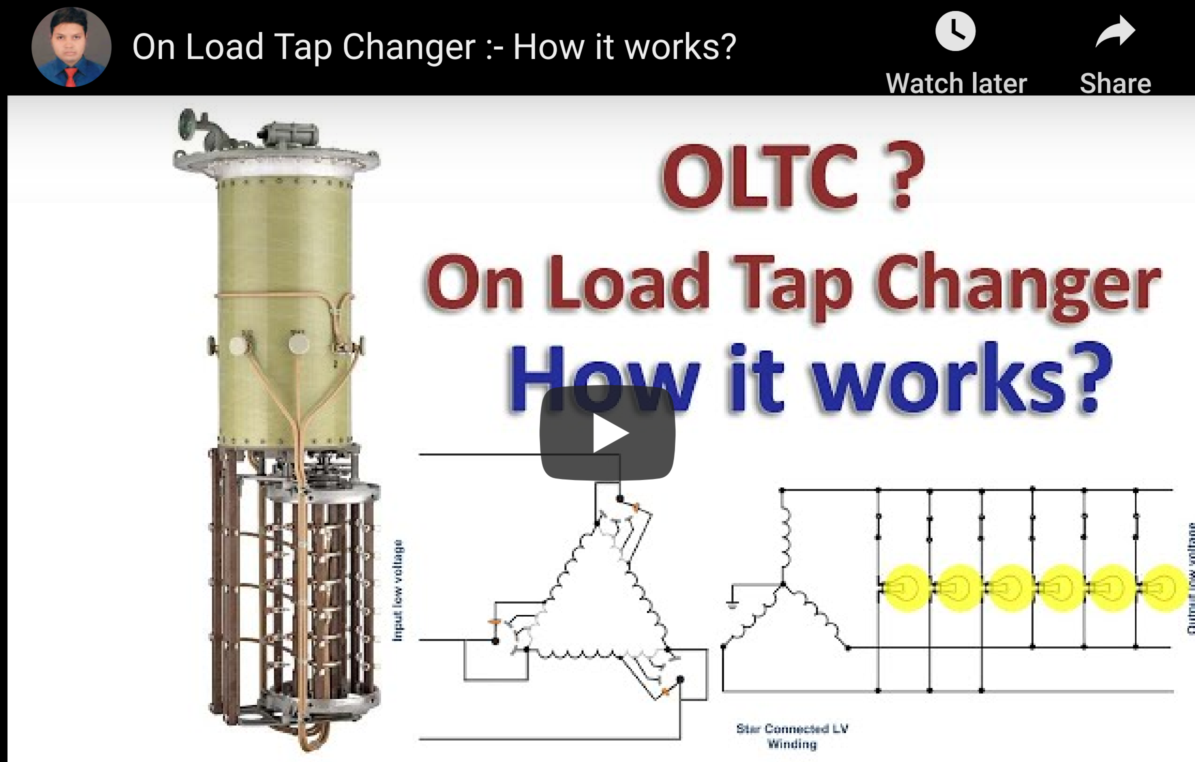
The efficiency expressed in % is

$$100 \times \frac{P_{output}}{P_{input}} = 100 \times \left(1 - \frac{P_{losses}}{P_{input}}\right)$$

- maximal when copper losses = iron losses (cancel derivative of efficiency w.r.t current)
- Around 99.5 % in large power transformers at full load.

# Tap changers

- Some transformers are equipped with a system allowing to change the $1 : n$ ratio
- The ability to change the tap under load is called load tap changer (LTC) or on-load tap changer (OLTC)
- This is mainly used for voltage control
- It is usually implemented using auto-transformers
- We will see later on how to include this in the power flow analysis



On Load Tap Changer :- How it works?
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OLTC ?
On Load Tap Changer
How it works?
Input low voltage
Output low voltage
Star Connected LV Winding

## Auto-transformers

The two windings (of the same phase) are connected in series, without galvanic insulation. They are commonly used when the ratio is limited.

Advantages:

- Physically smaller
- less costly (less copper)
- higher efficiency
- easy to implement tap changes
- "solid" earth grounding

Disadvantages:

- no electrical insulation
- higher short circuit current
- full voltage at secondary if it breaks (in case of a step down)


Auto Transformer working principle, how Var...
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Auto Transformer's
Concept & Use
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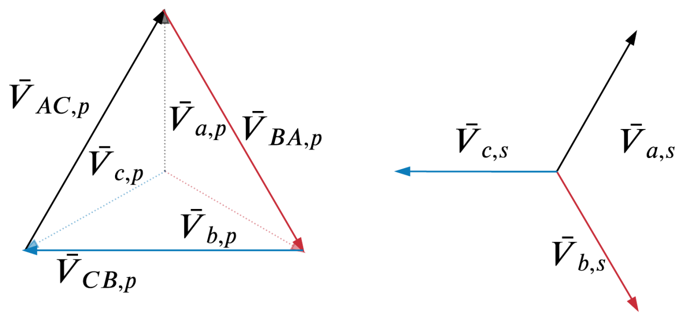
## Phase shift in delta-star transformers

The star part has $n$ times the number of turns of the delta part (primary side).

Let's reason on phase $a$,

- Voltage $\bar{V}_{a,s}$ is on the same core as $\bar{V}_{AC,p} = \sqrt{3}\bar{V}_{a,p}\angle{-30^\circ}$ where $\bar{V}_{a,p}$ is the (virtual) phase-neutral voltage on the primary side.
- Since $\bar{V}_{a,s} = n\bar{V}_{AC,p}$, $\bar{V}_{a,s} = n\sqrt{3}\bar{V}_{a,p}\angle{-30^\circ}$

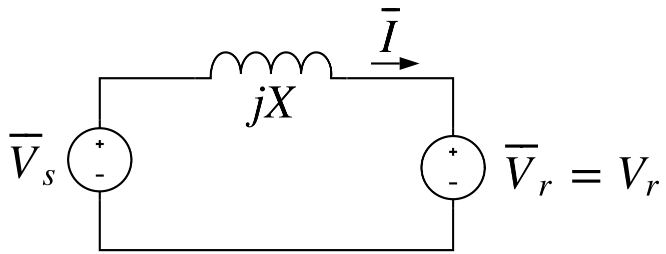



We gain a $\sqrt{3}$ factor in the amplification, but a lagging phase shift of 30°.

The same reasoning holds for phases $b$ and $c$.

## Power flow regulation by phase shifting

We have seen in lecture 2 that active power flows are dictated by the voltage magnitudes but also the sine of the angle difference between buses:




$$S_r = \bar{V}_r \bar{I}^* = V_r \left( \frac{V_s \angle -\delta - V_r}{-jX} \right)$$
$$= \frac{V_s V_r \sin\delta}{X} + j \frac{V_s V_r \cos\delta - V_r^2}{X}$$

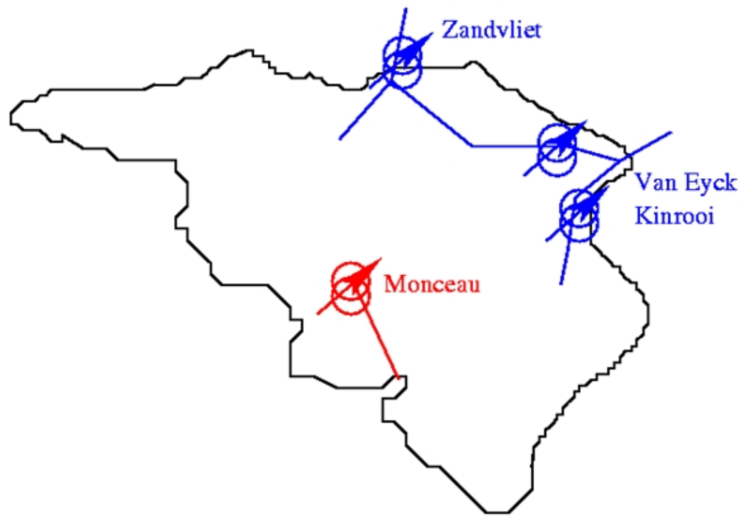
$\delta$ is the angle between $\bar{V}_r$ and $\bar{V}_s$

If we have a device that can generate an adjustable phase shift, we can control the power flows. This is the purpose of phase-shifting transformers.

In practice phase shifting is achieved by "combining the signal with a fraction of itself shifted by 90°". For the details of how it is implemented or modeled, see

- Wikipedia
- Section 5.7. of the Weedy or ELEC0014.
- ENTSO-E - Phase Shift Transformers Modelling, Version 1.0.0, May 2015

# Example: phase shifting transformers on the borders of Belgium




380/380 kV : in series with:

1. line Zandvliet (B) - Borssele (NL) and Zandvliet (B) - Geertruidenberg (NL)
2. line Meerhout (B) - Maasbracht (NL)
3. line Gramme (B) - Maasbracht (NL)
   - nominal power 3VN Imax = 1400 MVA
   - phase shift adjustment: 35 positions, +17/-17 × 1.5° (at no load)

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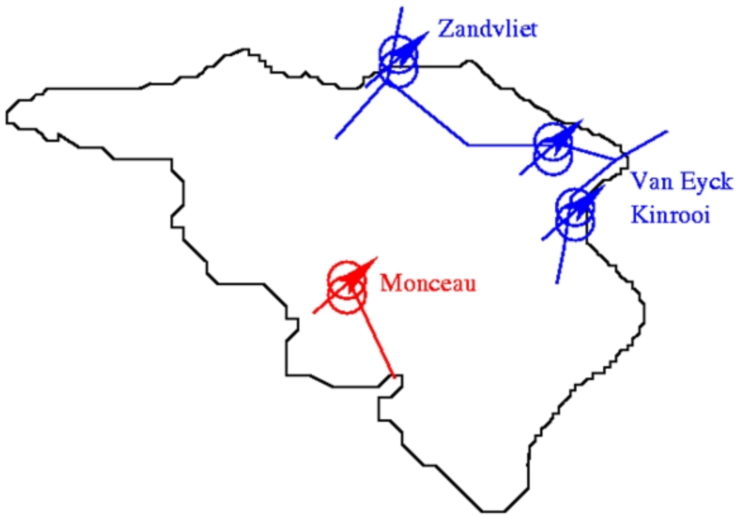
From ELEC0014.




220/150 kV :

- in series with the Chooz (F) - Monceau (B) line nominal power: 400 MVA
- in-phase adjustment : 21 positions, +10/-10 × 1.5 %
- quadrature adjustment: 21 positions, +10/-10 × 1.2°

## Remarks

In three-phase operation,

- either there are three separate single-phase transformers (easier to fix when there is a problem on a phase, more modular)
- or a three-phase transformer, that is a single core with three auto-transormers on it, cf. the video at the beginning of this presentation (cheaper, lighter core and less copper).

Some transformers called three-winding transformers are equiped with a third winding (not to be confused with a three-phase transformer) that is used for auxiliary purposes (feeding auxiliary devices e.g. fans, providing reactive power support, ...)

# Transformers in the power flow analysis

## Transformer without regulation

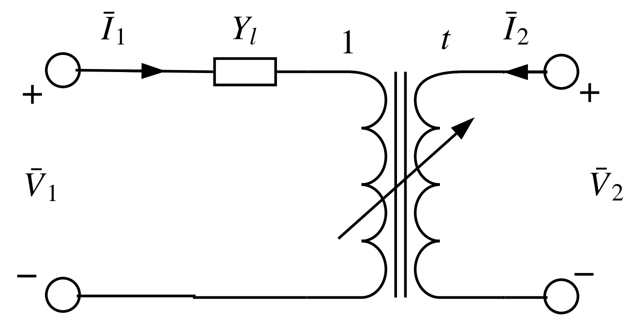
A transfomer, in the per unit representation, can thus be represented

- as a two-port if the shunt admittance are considered
- as a simple series leakge impedance if the shunt admittance is neglected

## Representing taps and phase shifts

Let $Y_l$ be the leakage admittance and $t$ be the off-nominal turns ratio:

- if $0 < t \leq 1$, this corresponds to a simple tap-changer
- if $0 < |t| \leq 1$ but is complex, then this is a phase-shifter ( $\angle t < \pi/2$)




We have

$$\bar{I}_1 = \left(\bar{V}_1 - \frac{\bar{V}_2}{t}\right) Y_l$$

and since $\frac{\bar{V}_2}{t} \bar{I}_1^\star = \bar{V}_2 \bar{I}_2^\star$ by energy conservation

$$\bar{I}_2 = -\frac{\bar{I}_1}{t^\star} = -\bar{V}_1 \frac{Y_l}{t^\star} + \bar{V}_2 \frac{Y_l}{|t|^2}$$

Thus tap and phase shift can be represented by the admittance matrix

$$\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \end{bmatrix} = \begin{bmatrix} Y_l & -\frac{Y_l}{t} \\ -\frac{Y_l}{t^\star} & \frac{Y_l}{|t|^2} \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \end{bmatrix}$$

- if $0 < t \leq 1$, this can be represented as a $\pi$ two-port
- if $0 < |t| \leq 1$ but is complex, this is not the case

In the power flow analysis, you must pay attention to this when contructing the system-wide admittance matrix.

## Pandapower example

See python notebook / video recording.

# References


- Mohan, Ned. Electric power systems: a first course. John Wiley & Sons, 2012. Chapter 6.
- Weedy, Birron Mathew, et al. Electric power systems. John Wiley & Sons, 2012. Section 3.8, Section 5.7.
- Course notes of ELEC0014 by Pr. Thierry Van Cutsem.

The end.