ELEC0447 — Analysis of electric power and energy systems

Sinusoïdal steady-state analysis

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What will we learn today?

- Recap of conventions
- Sinusoidal steady state analysis

You will be able to do exercises 2.1, 2.2, 2.4, 2.5, 2.9, 2.11, 2.12, 2.14, 2.16, 2.17, 2.18, 2.19 and 2.20 from the Ned Mohan's book.

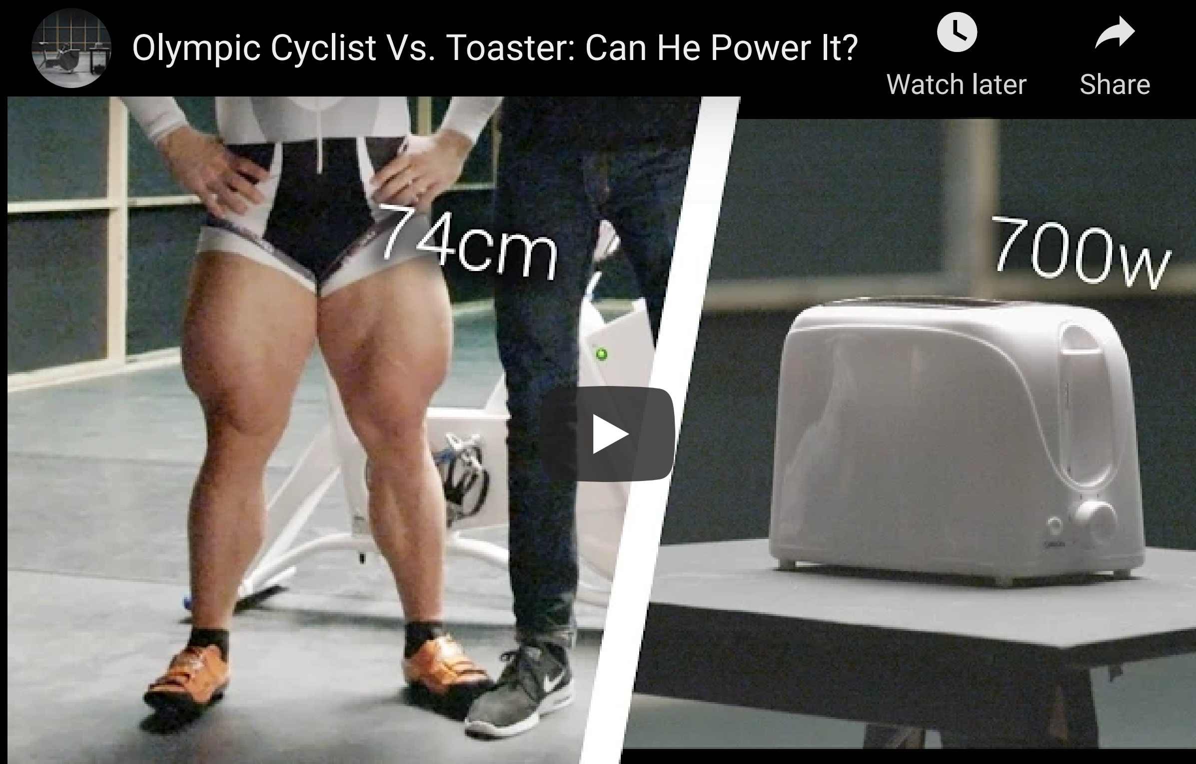
Basics and conventions

Power and energy

- Power measures the rate of use of energy
- It is expressed in Watt [W]: 1 W = 1 Joule/second
- In an electric system,

$$p(t) = u(t) \times i(t)$$

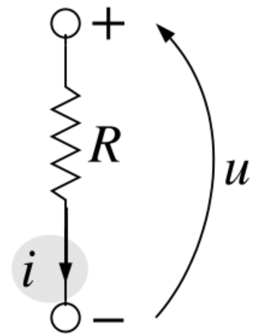
  - $u(t)$ is the voltage measured in volt [V], the line integral of the electric field between two points.
  - $i(t)$ is the current measured in amps [A]
  - t is the time
- To measure energy in power systems, we use units ranging from a kWh (a microgrid) to a TWh (a country)
- Devices have power ratings ranging from W to GW (although we generally speak in VA for ratings)



The cyclist vs the toaster.

Motor convention (or standard reference)

When using the motor convention to direct u w.r.t. i, $p(t)$ represents the power consumed by a device (here a resistor):



- The power consumed can be < 0, = 0, or > 0 depending on the device
- E.g. for a resistor we always have $p(t) \geq 0$
- The opposite convention is the generator convention
- We will sometimes use a mix of both conventions, based on intuition, so that in general we have few negative numbers: pay attention to the orientations!

Magnetic fields, etc.

Magnetic fields have a central role to model the behavior of equipment (lines, transformers, generators, etc.).

As this course will not be focused on modeling devices, but rather a system of devices, magnetic effects will often be highly abstracted (an inductance, or a turns ratio).

Let's just recall that

- a magnetic field is due to charges in movement or magnetized materials
- it is measured in amps/meter (for H) or in Tesla (for B) when we capture the effect of the material (the μ_0 coefficient in the air)
- the magnetic flux (ϕ in weber) measures the magnetic field crossing a surface
- a time varying magnetic flux induces a voltage (Lenz), this is the fundamental principle behind transformers.

Sinusoidal steady state analysis

Sinusoidal signals and phasor representation

Unless otherwise specified, we will always work with sinusoidal signals and in steady state:

$$y(t) = \sqrt{2}Y \cos(\omega t + \phi_y).$$

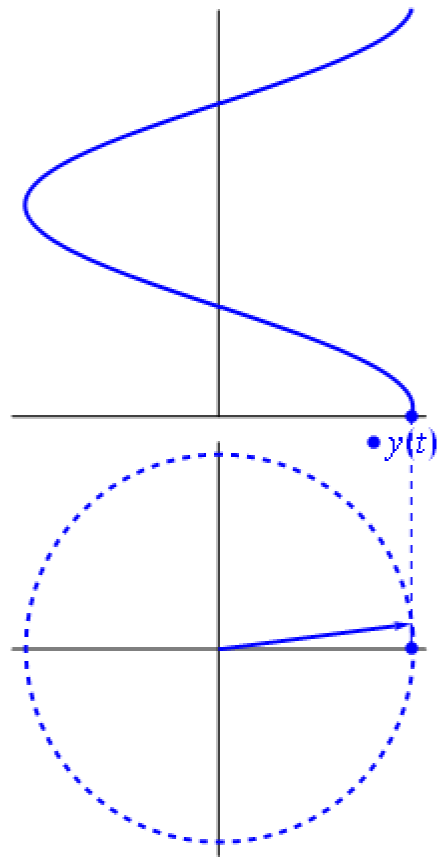
Y is the rms value of the signal, ϕ_y its phase and ω its angular frequency.

At a specific frequency $f = \frac{\omega}{2\pi}$, the signal can be represented as a phasor

$$\bar{Y} = Y \angle \phi_y = Y e^{j\phi_y}$$

Phasors allow to work in the frequency domain, which is much more handy for computations.

How do you get the time expression from the phasor?



https://en.wikipedia.org/wiki/Phasor

Impedance

Let $u(t)$ and $i(t)$ be the voltage and current across a one-port, respectively, in sinusoidal steady state and with the motor convention.

- For a resistor, $u(t) = Ri(t)$ hence $\bar{U} = R\,\bar{I}$
- For a self, $u(t) = L\frac{di(t)}{dt}$ hence $\bar{U} = j\omega L\,\bar{I}$
- For a capacitor, $i(t) = C\frac{du(t)}{dt}$ hence $\bar{I} = j\omega C\bar{U}$

The impedance, a complex number, generalizes this notion

$$Z = R + jX \quad [\Omega]$$

such that $\bar{U} = Z\,\bar{I}$ with

- for a resistor, $Z = R$
- for a self, $Z = jX = j\omega L$
- for a capacitor, $Z = jX = -j\frac{1}{\omega C}$

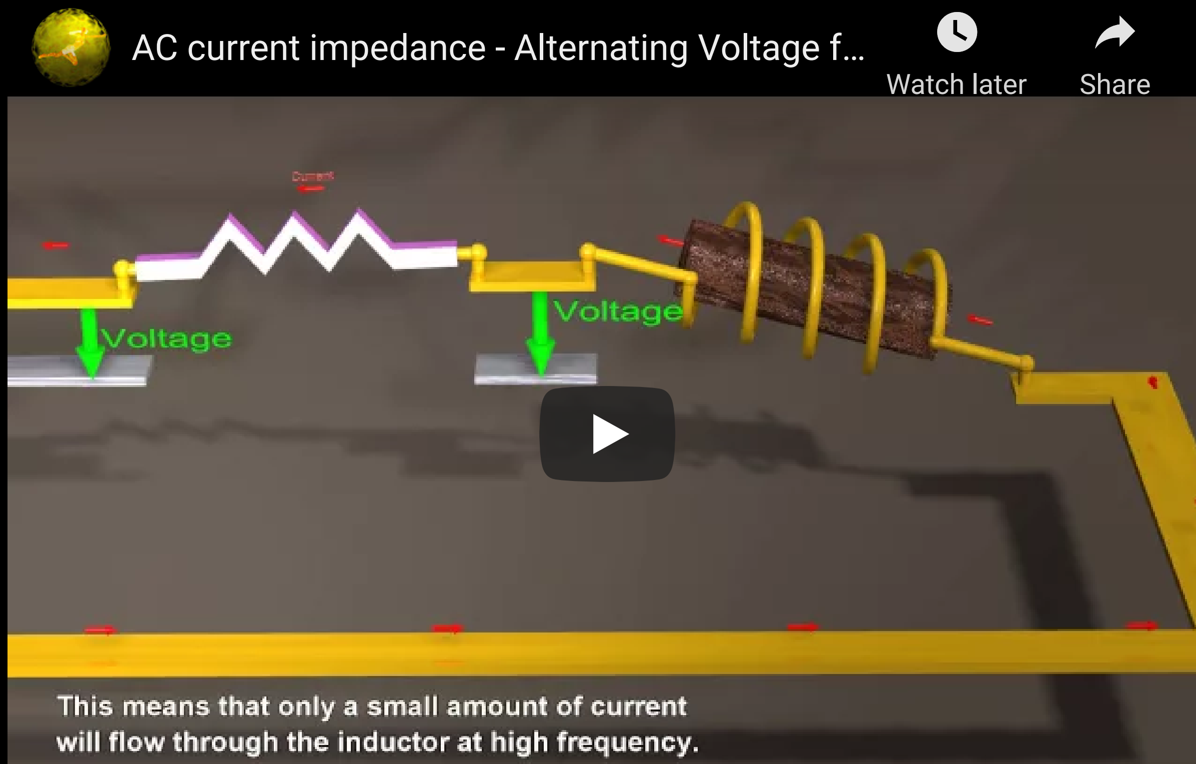
Impedance, admittance, etc.

The imaginary part of the impedance, X, is called reactance

The admittance Y is the inverse of the impedance, expressed in Siemens:

$$Y = G + jB$$

- G is the conductance
- B is the susceptance



A nice animation.

Complex calculus

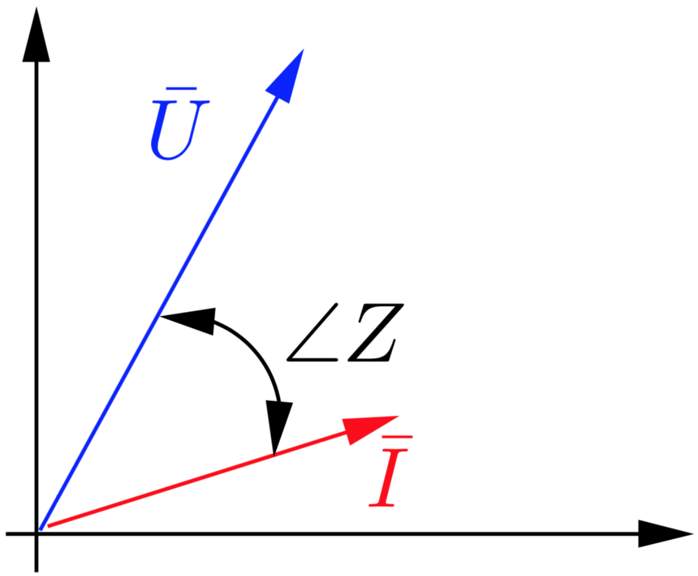
$$|Z| = \sqrt{R^2 + X^2}$$

$$\angle Z = \arctan \frac{X}{R}$$

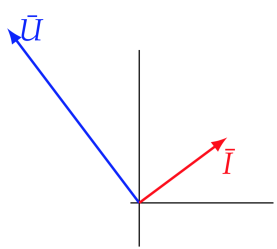
$$Z = \frac{\bar{U}}{\bar{I}} = \frac{U}{I} \angle(\phi_u - \phi_i)$$

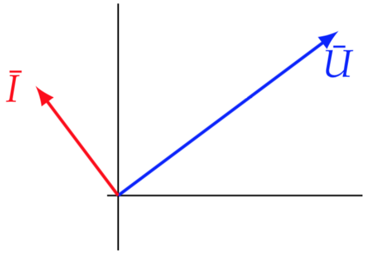
Phasor diagrams

Plot the phasors in the complex plane!



Inductive or capacitive? Which is which?





The notions of power

The complex power is defined as

$$S = \bar{U}\,\bar{I}^*$$

Let

$$\phi = \phi_u - \phi_i$$

then

$$S = UIe^{j\phi} = P + jQ$$

- $P = UI\cos\phi$ is the active power, measured in watt
- $Q = UI\sin\phi$ is the reactive power, measured in var
- $\cos\phi$ is the power factor

Reactive power is, in general, undesirable.

The apparent power is $|S| = UI$, measured in VA

Useful formulas

$$P = RI^2 = \frac{U^2}{R}$$

$$Q = XI^2 = \frac{U^2}{X}$$

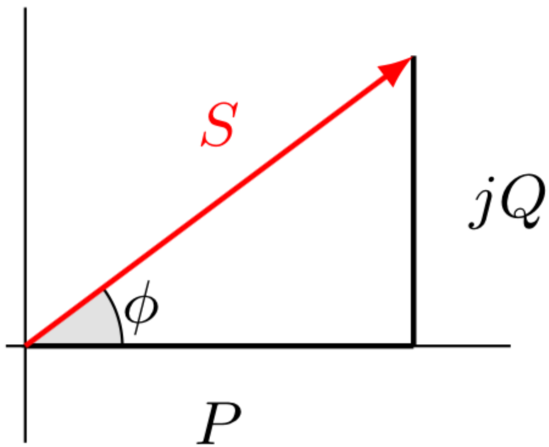
$$\tan\phi = \frac{Q}{P}$$

$$\cos\phi = \frac{P}{|S|}$$

The power factor does not tell you whether the system is leading or lagging

- in an inductive system, $u(t)$ precedes $i(t)$, $i(t)$ is lagging, thus $Q > 0$ (motor convention)
- in a capacitive system, this is the opposite (leading).

Power factor compensation



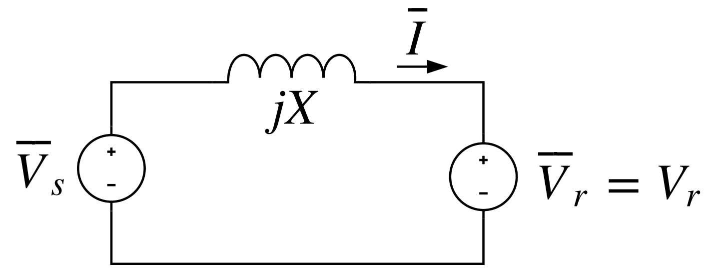
Produce some Q to cancel out ϕ.

Example:

A 120V voltage source at 60 Hz that feeds a R-L load $1858.4 + j1031.4\ VA$

Power transfer between AC systems

Consider the following simple system



We have $\bar{I} = \frac{\bar{V}_s - \bar{V}_r}{jX}$

Let δ be the angle between $\bar{V}_r$ and $\bar{V}_s$, then

$$S_r = \bar{V}_r \bar{I}^* = V_r \left(\frac{V_s \angle -\delta - V_r}{-jX} \right)$$
$$= \frac{V_s V_r \sin\delta}{X} + j \frac{V_s V_r \cos\delta - V_r^2}{X}$$

Let's remember two things:

- The active power is highly sensitive to δ
- The reactive power acts on the voltage magnitude (look at what happens for $\delta = 0$)

The end.