Evolutionary multi-objective optimization (EMO)

Cosijopii García García
Cosijopii@inaoep.mx
`https://cosijopiii.github.io/teaching/`

Computer Science Lab, INAOE

April 5, 2023

The following slides take figures and information from :

- Carlos A. Coello Coello, David A. Van Veldhuizen and Gary B. Lamont, **Evolutionary Algorithms for Solving Multi-Objective Problems**, Kluwer Academic Publishers, New York, March 2002, ISBN 0-3064-6762-3.
- Kalyanmoy Deb. **Multi-Objective Optimization using Evolutionary Algorithms**, John Wiley & Sons, Chichester, UK, 2001, ISBN 0-471-87339-X.
- `https://www.cs.cinvestav.mx/~emooworkgroup/tutorial-slides-coello.pdf`
- `https://www.cs.cinvestav.mx/~emooworkgroup/tutorial-slides-zitzler.pdf`
- `https://engineering.purdue.edu/~sudhoff/ee630/Lecture09.pdf`

Multiobjective Optimization

Why Multiobjective Optimization?

Most optimization problems naturally have several objectives to be achieved (normally conflicting with each other), but in order to simplify their solution, they are treated as if they had only one (the remaining objectives are normally handled as constraints)

Basic concepts

- **The Multiobjective Optimization Problem** can then be defined (in words) as the problem of finding:
- *"A.* ***vector of decision variables*** *which satisfies* ***constraints*** *and optimizes a* ***vector function*** *whose elements represent the objective functions. These functions form a mathematical description of performance criteria which are usually in* ***conflict*** *with each other. Hence, the term "optimize" means finding such a solution which would give the values of all the objective functions acceptable to the decision maker.*

Basic concepts

$$\left.\begin{aligned} \text{Minimize/Maximize } \mathbf{f}_m(\mathbf{x}), m = 1, 2, \ldots, k; \\ \text{subject to } g_j(\mathbf{x}) \geq 0, j = 1, 2, \ldots, m; \\ h_k(\mathbf{x}) = 0, k = 1, 2, \ldots, p; \\ x_i^{(L)} \leq x_i \leq x_i^{(U)}, i = 1, 2, \ldots, t; \end{aligned}\right\} \quad (1)$$

- with k objectives, m and p are the number of inequality and equality constraints. A solution $\mathbf{x} \in \mathbf{R}^n$ is a vector of n decision variables: $\mathbf{x} = [x_1, x_2, \ldots, x_n]$, which satisfy all constraints and variable bounds.

Basic concepts
Pareto

- Having several objective functions, the notion of "*optimum*" changes, because in MOPs, we are really trying to find good compromises (or "trade-offs") rather than a single solution as in global optimization. The notion of "*optimum*" that is most commonly adopted is that originally proposed by Francis Ysidro Edgeworth in 1881.
- This notion was later generalized by Vilfredo Pareto (in 1896). Although some authors call **Edgeworth-Pareto optimum** to this notion, we will use the most commonly accepted term: **Pareto optimum.**

Basic concepts

Pareto

We say that a vector of decision variables $x^* \in \mathcal{F}$ is Pareto optimal if there does not exist another $x \in \mathcal{F}$ such that $f_i(x) \leq f_i(x^*)$ for all $i = 1, \ldots, k$ and $f_j(x) < f_j(x^*)$ for at least one j.

Basic concepts

Non-dominated solutions

In words, this definition says that x^* is Pareto optimal if there exists no feasible vector of decision variables $x \in \mathcal{F}$ which would decrease some criterion without causing a simultaneous increase in at least one other criterion. Unfortunately, this concept almost always gives not a single solution, but rather a set of solutions called the **Pareto optimal set**. The vectors x^* corresponding to the solutions included in the Pareto optimal set are called **non-dominated**. The plot of the objective functions whose non-dominated vectors are in the Pareto optimal set is called the **Pareto front**.

Decision and Objective Space

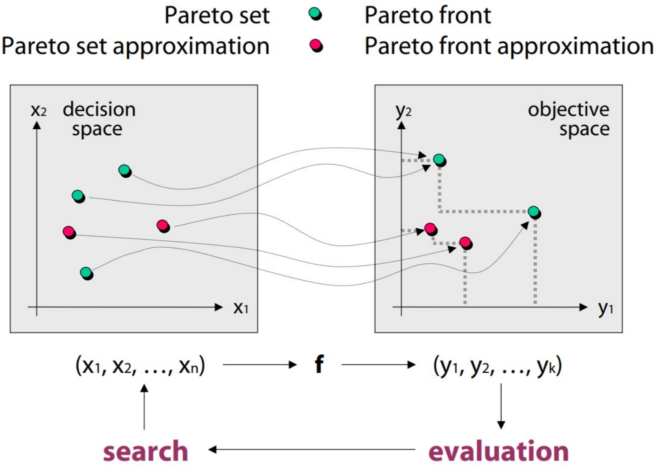

Figure: Decision variables space and objective function space.

Classifying Techniques

References I