Trigonometry

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January 7, 2020

Weeks: 2
Dates: Monday 6/1, Wednesday 8/1, Thursday 9/1, Tuesday 14/1, Thursday 16/1, Friday 17/1

Monday 6/1

1st half module:

- Exam - Anu

2nd half module:

- Math Activity

Wednesday 8/1

1st half module:
Trigonometrical functions

- Sine Ratio
- Cosine Ratio
- Tangent Ratio

2nd half module:
Choosing the right trig function?

- SohCahToa
- How to use the SohCahToa triangles.
- Questions

Thursday 9/1

1st half module:
Recap of the trigonometrical functions

Finding an unknown angle

- Inverse functions
- Questions

Angles of elevation and depression

- Elevation
- Depression
- Questions

2nd half module:
Multi-stage problems

- Triangles split into two right angle triangles.
- Pythagoras' Theorem
- Bearings
- Questions

Tuesday 14/1

1st half module:
Sine Rule

- Ambiguous case of the sine rule
- Questions

2nd half module:
Cosine Rule

- Questions

Thursday 16/1

1st half module:
Recap of Right Angles
Recap of Sine rule and Cosine rule

2nd half module:
Area of a Triangle

- Questions

Friday 17/1

Trigonometry in 3-D:

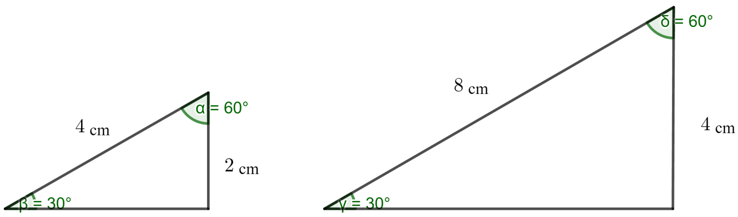
- Questions

1 Notes Monday 6/1

2 Notes Wednesday 8/1

The Sine Ratio:

Consider two triangles with similar angles. The ratio will be same regardless of the different lengths. The larger triangle has been enlarged by a scale factor.

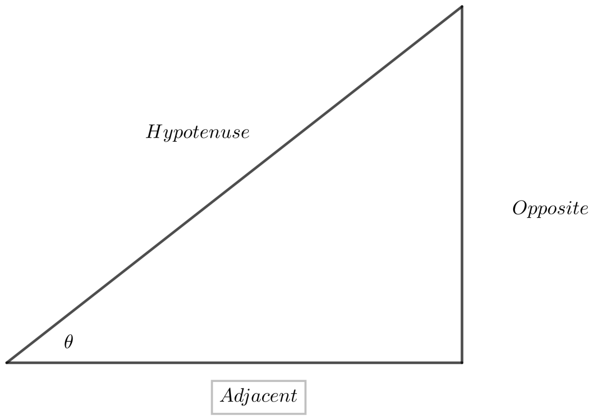


In right angle triangles, the answer obtained by dividing the length opposite the angle by the hypotenuse is called the sine of that angle.

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \tag{1}$$

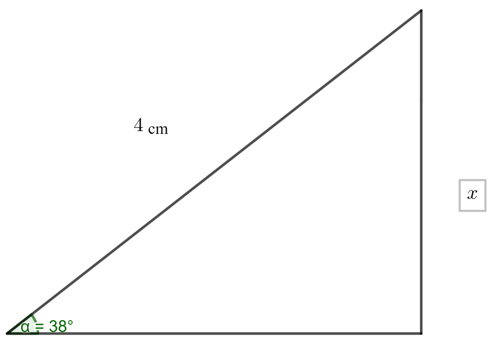
Remember - Calculators set to degrees (DEG)

To calculate a missing length, Isolate the missing length from the sine formula.



$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \tag{2}$$
$$\implies \text{opposite} = \sin(\theta) \times \text{hypotenuse} \tag{3}$$

Example:



$$\text{opposite} = \text{hypotenuse} \times \sin(\theta) \tag{4}$$
$$x = 4 \times \sin(38^\circ) \tag{5}$$
$$x = 2.462645901^\circ \approx 2.46^\circ \tag{6}$$

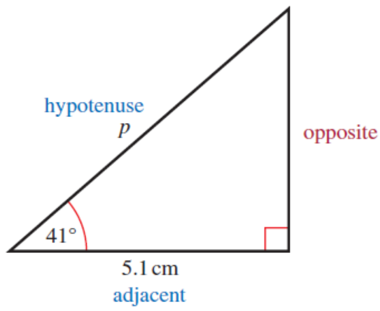
Show that the hypotenuse can also be isolated.

Cosine Ratio:

Instead of trying to find the opposite length, the adjacent length can be found using the cosine ratio.

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} \tag{7}$$
$$\implies \text{adjacent} = \text{hypotenuse} \times \cos(\theta) \tag{8}$$

Labelling the sides as seen from the 41° angle:



In this triangle, the side adjacent to 41° is 5.1 cm, and the hypotenuse is p cm.

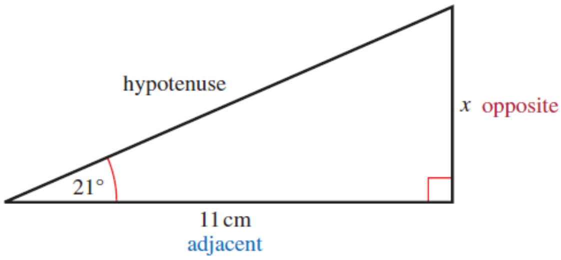
$$\begin{aligned} \text{adjacent} &= \text{hypotenuse} \times \cos\theta \\ 5.1 &= p \times \cos 41^\circ \\ p &= \frac{5.1}{\cos 41^\circ} \\ &= \underline{6.76\text{ cm}} \text{ (3 s.f.)} \end{aligned}$$

Tangent Ratio:

The last ratio is defined as:

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} \tag{9}$$
$$\implies \text{opposite} = \text{adjacent} \times \tan(\theta) \tag{10}$$

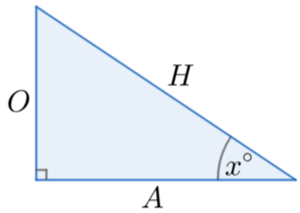
Labelling the sides as seen from the 21° angle:



In this triangle, the side opposite to 21° is x, and the adjacent is 11 cm.

$$\begin{aligned} \text{opposite} &= \text{adjacent} \times \tan\theta \\ x &= 11 \times \tan 21° \\ &= 4.222\,504\,385 \\ &= \underline{4.22\text{ cm}}\ (3\text{ s.f.}) \end{aligned}$$

SohCahToa When solving questions the trigonometrical ratio rhyme can be used and triangles can be drawn to help isolate the missing length or angle(but more on that later).



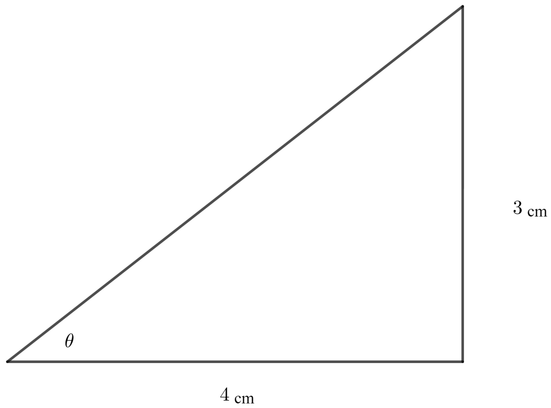
How to use: Cover the quantity you are looking for and the triangle will tell to either multiple or divide the quantities you were given in the question.
Questions:

- 1 through to 8 (Page 317)

3 Notes Thursday 9/1:

Finding an unknown angle:

- Inverse - "Reverse" function
- Inverse Angle - Calculators -> shift, sin/cos/tan.



-
- $\tan(\theta) = \frac{3}{4} \implies \theta = \arctan(\frac{3}{4}) = 36.86989765° \approx 36.9°$

Questions:

- 2,4,6,8,10,12 (Page 319)

Angles of elevation and depression:
Angle of elevation - Above the horizontal line
Angle of depression - Below the horizontal line.

These problems are a combination of pythagoras' theorem, trigonometrical equations and alternate angles.

Example: A climber is sitting on the summit S of a mountain. He looks down and sees his camp C in the valley below. The direct distance from the summit to the camp is 2200 metres. The summit is at an altitude 420 meters higher than the camp.
a: Calculate the angle of elevation of the summit as seen from the camp. Give your answer to the nearest 0.1°.

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \tag{11}$$

$$\sin(\theta) = \frac{420}{2200} \tag{12}$$

$$\theta = \arcsin(\frac{420}{2200}) = 11.005° \approx 11° \tag{13}$$

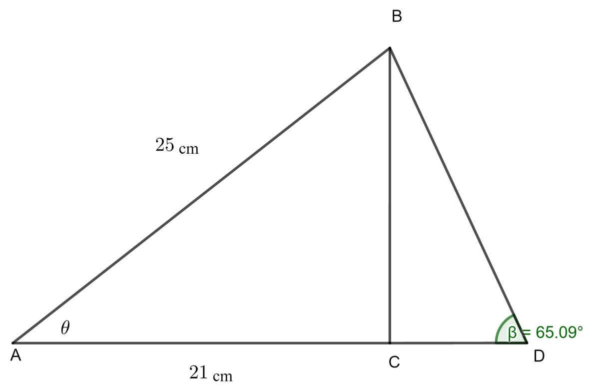
$$\tag{14}$$

b: Write down the angle of depression of the camp as seen from the summit. = 11°

Questions:

- 1,3,4 (Page. 325-326)

Multi-stage problems: Some problems will involve non-right angle triangles, such as:



Now you might be able to see, two right angle triangles!
a: Calculate the height BC, to three significant numbers.
b: Calculate length BD.
c: Calculate θ.

$$c^2 = a^2 + b^2 \tag{15}$$
$$\implies b^2 = c^2 - a^2 \tag{16}$$
$$BC^2 = 25^2 - 21^2 = 184 \tag{17}$$
$$\implies BC = \sqrt{184} = 13.5646 \approx 13.6 \tag{18}$$

Use Soh triangle!

$$\sin(65^\circ) = \frac{BC}{BD} = \frac{13.6}{BD} \tag{19}$$
$$BD = \frac{13.56465997}{\sin(65^\circ)} = 14.96694629 \approx 15.0 \tag{20}$$

$$\cos(\theta) = \frac{21}{25} \tag{21}$$
$$\theta = \arccos(\frac{21}{25}) = 32.85988038 \approx 32.9^\circ \tag{22}$$

Bearing: Absolute bearing refers to the angle between the magnetic North (magnetic bearing) or true North (true bearing) and an object.

Example: A ship leaves its harbour and sails due south for 10km. It then sails due East for 20km then stops. a:How far is the ship from its harbour?

b:The ship wishes to return directly to its habour. On what bearing must it sail?

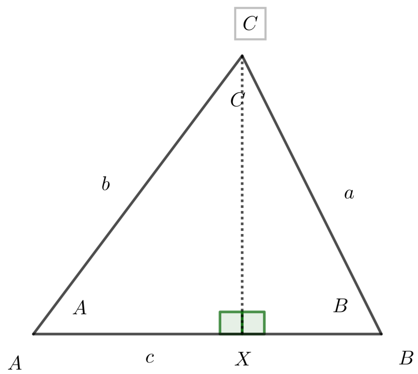
$$d^2 = 20^2 + 10^2 = 500 \tag{23}$$
$$d = \sqrt{500} = 22.36067978 \approx 22.4km \tag{24}$$

$$\tan(\theta) = \frac{10}{20} = 0.5 \tag{25}$$
$$\theta = \arctan(0.5) = 26.56505118 \approx 26.6^\circ \tag{26}$$
$$\text{Bearing} = 270^\circ + 26.6^\circ = 296.6^\circ \tag{27}$$

4 Notes Tuesday 14/1

The sine rule: Earlier we worked on right angle triangle, and looked at the trigonometrical formulae we can use to solve those problems. But what happens when the triangles are not right angle triangles? We can break them into two right angle triangles or we can use the Sine rule, and cosine rule.

Consider the following triangle:



The triangle ACX is right angled triangle, and the height h can be calculated using the sine ratio formula:

$$h = b \times \sin(A) \tag{28}$$

The triangle BXC is also a right angled triangle, height is calculated:

$$h = a \times \sin(B) \tag{29}$$

Both of the equations are equal to the height of the triangle:

$$a \times \sin(B) = b \times \sin(A) \tag{30}$$

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} \tag{31}$$

Drawing a perpendicular line from A to BC will obtain similar results using c and $\sin(C)$.

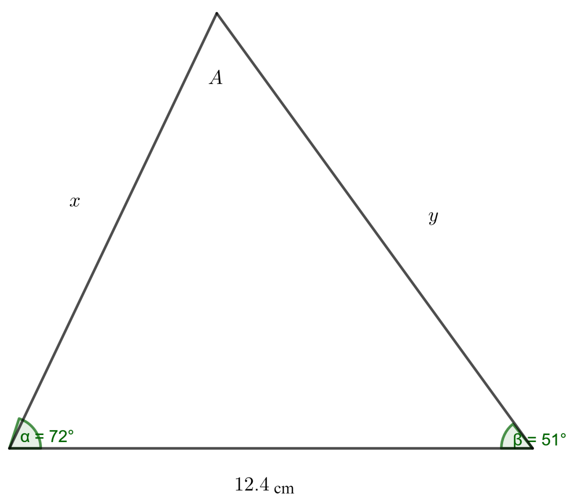
The Sine Rule

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} \tag{32}$$

This rule is used is you have two angles, one length and want to find a missing length, or you have two lengths, one angle and want to find a missing angle.

Example:

Consider the triangle:



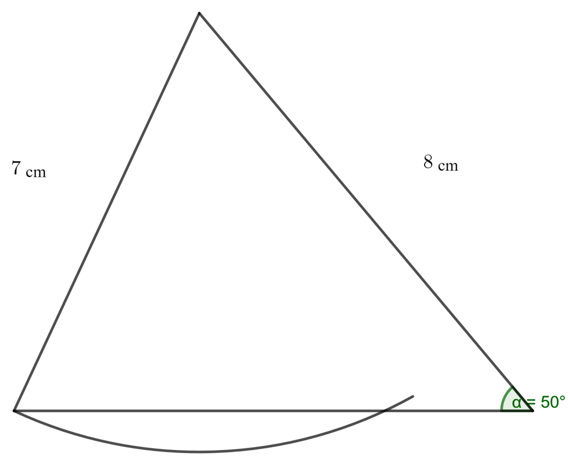
$$A = 180 - 72 - 51 = 57 \tag{33}$$
$$\frac{x}{\sin(51)} = \frac{12.4}{\sin(57)} \tag{34}$$
$$x = 11.49033994 \approx 11.5 \tag{35}$$
$$\frac{y}{\sin(72)} = \frac{12.4}{\sin(57)} \tag{36}$$
$$y = 14.06166051 \approx 14.1 \tag{37}$$

Questions:

- 28.1 - 2,4,6 (Page 511)
- 28.1 - 8,10,12 (Page 512)

The ambiguous case of the sine rule You have to be careful when using the sine rule! An acute angle and obtuse angle can give the same result.

Consider a triangle:



Using the sine rule, the acute and obtuse angle made the side rotated through the arc with radius 7 cm.

$$\frac{7}{\sin(50^\circ)} = \frac{8}{\sin(C)} \tag{38}$$
$$\sin(C) = 0.875479 \tag{39}$$
$$C = \arcsin(\frac{8 \times \sin(50)}{7}) = 61.10176^\circ \tag{40}$$
$$\sin(180 - 61.10176) = 0.875479372 \tag{41}$$

Questions:

- 2,4,6,8 Page(515.)

The cosine rule The cosine rule is a version of Pythagoras's theorem and is used when calculating either at missing length or angle. If you need to calculate an angle, you need three lengths. If you need to calculate a length, you need an angle and two lengths.

$$c^2 = a^2 + b^2 - 2ab\cos(C) \tag{42}$$
$$a^2 = b^2 + c^2 - 2ab\cos(A) \tag{43}$$
$$b^2 = a^2 + c^2 - 2ab\cos(B) \tag{44}$$

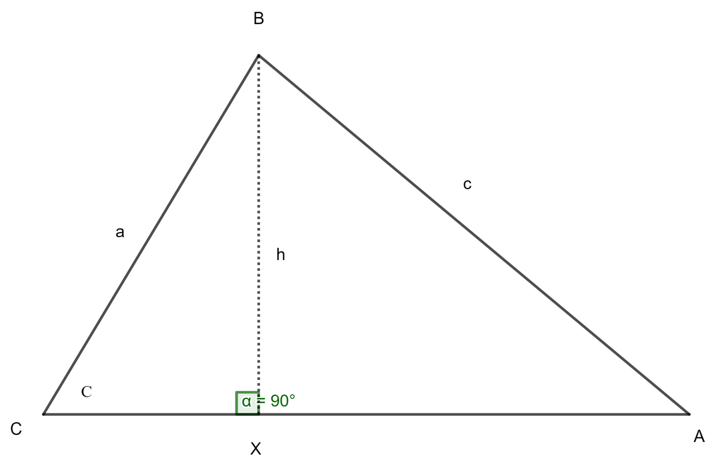
$$\cos(C) = \frac{a^2 + b^2 - c^2}{2ab} \tag{45}$$

The other angles can be isolated from the other cosine rules.
Questions:

- 3, 6, 8, 10 (Page 518)
- 2, 4, 8, 10 (Page 519)

5 Notes Thursday 16/1

Consider the following triangle:



The area of the triangle is:

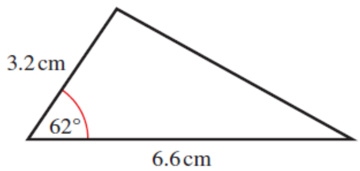
$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{Height} \tag{46}$$
$$= \frac{1}{2} \times AC \times BX \tag{47}$$
$$= \frac{1}{2} \times b \times (a \times \sin(C)) \tag{48}$$
$$\text{Area} = \frac{1}{2} ab \sin(C) \tag{49}$$

Learn the formula in the three versions:

$$\text{Area} = \frac{1}{2} ab \sin(C) = \frac{1}{2} bc \sin(A) = \frac{1}{2} ac \sin(B) \tag{50}$$

Example:

Find the area of this triangle, correct to 3 significant figures.



SOLUTION

Area of triangle $= \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 3.2 \times 6.6 \times \sin 62°$

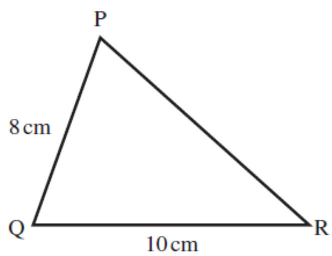
$= 9.323\,926\,581$

$= 9.32 \text{ cm}^2$ (3 s.f.)

The diagram shows a triangle PQR in which PQ = 8 cm and QR = 10 cm. The triangle has an area of 20 cm^2. Angle PQR is acute.

a) Calculate the size of angle PQR.

b) Calculate the length of PR. Give your answer to 3 significant figures.



a) Since the area of the triangle is 20 cm^2:

$$
\begin{aligned}
\text{`}\tfrac{1}{2}ab\sin C\text{'} &= \text{area of triangle}\\
\tfrac{1}{2}\times 8\times 10\times \sin Q &= 20\\
40\sin Q &= 20\\
\sin Q &= \frac{20}{40}\\
\sin Q &= 20\\
Q &= \sin^{-1}(0.5)\\
Q &= 30^\circ
\end{aligned}
$$

Thus angle PQR $= 30^\circ$.

Since the sine rule has been used, another possibility might be $Q = 180 - 30 = 150^\circ$, but you are told that angle PQR is acute, so 150° may be discounted.

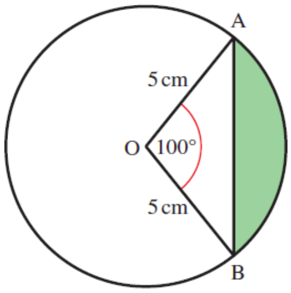
b) The length of PR may now be found, using the cosine rule:

$$
\begin{aligned}
PR^2 &= 8^2 + 10^2 - 2\times 8\times 10\times \cos 30\\
&= 64 + 100 - 160\cos 30^\circ\\
&= 25.435\,935\,39\\
PR &= \sqrt{25.435\,935\,39}\\
&= 5.04\text{ cm (3 s.f.)}
\end{aligned}
$$

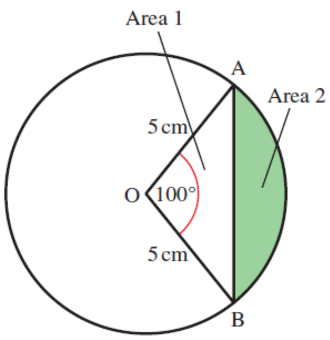
Link between area and sector

$$\text{Sector area} = \frac{\text{sector angle}}{360^\circ}\times \pi \times r^2 \tag{51}$$

Find the area of the segment shaded in the diagram below.
Give your answer to 3 significant figures.



SOLUTION



Area 1 + Area 2 form a sector of angle 100°.
The area of the sector is:

$$\frac{100}{360}\times \pi \times 5^2 = 21.816\,6\text{ cm}^2$$

Area 1 on its own forms a triangle.
Its area is:

$$\tfrac{1}{2}\times 5\times 5\times \sin 100^\circ = 12.310\,1\text{ cm}^2$$

Thus the area of the segment, Area 2, is:

$$
\begin{aligned}
21.816\,6 - 12.310\,1 &= 9.506\,5\text{ cm}^2\\
&= 9.51\text{ cm}^2\text{ (3 s.f.)}
\end{aligned}
$$

Questions:

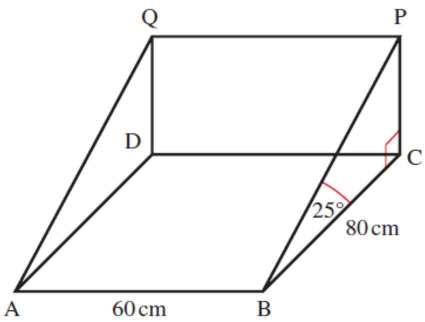
- 1, 2, 3, 4

6 Notes Friday 17/1

Important

- Break down the problem into two or more 2-D triangles.
- Draw a sketch, it will help you!

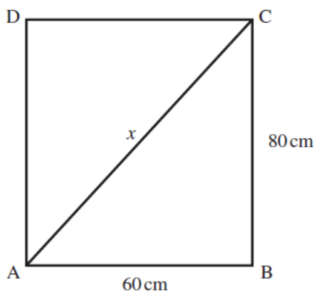
The diagram shows a wedge.
The base of the wedge is a horizontal rectangle measuring 60 cm by 80 cm.
The sloping face ABPQ is inclined at 25° to the horizontal.



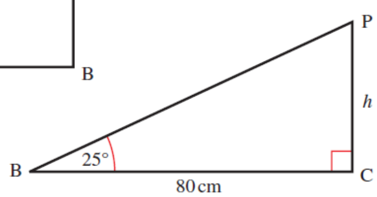
a) Calculate the lengths AC and PC.
b) Calculate the length AP.
c) Calculate the angle that AP makes with the horizontal plane ABCD.

a) By Pythagoras' theorem:

$x^2 = 60^2 + 80^2$
$= 3600 + 6400$
$= 10\,000$
$x = \sqrt{10\,000}$
$= 100$ cm

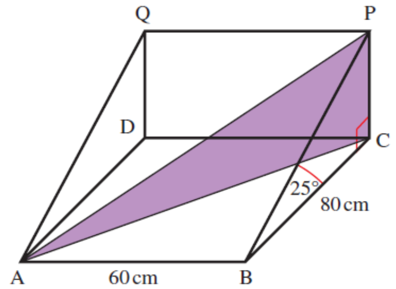


$h = 80 \times \tan 25°$
$= \underline{37.3 \text{ cm}}$ (3 s.f.)

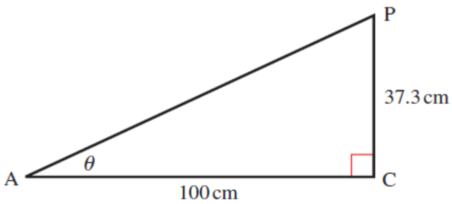


Thus AC = $\underline{100 \text{ cm}}$ and PC = $\underline{37.3 \text{ cm}}$ (3 s.f.)

b) From the prism, pick out triangle ACP.



This triangle may be turned flat:



By Pythagoras' theorem:

$$\begin{aligned} AP^2 &= 100^2 + 37.3^2 \\ &= 10\,000 + 1391.6 \\ &= 113\,91.6 \\ AP &= \sqrt{113\,91.6} \\ &= 106.731\,5\ldots \\ &= \underline{106.7\text{ cm}}\ (4\text{ s.f.}) \end{aligned}$$

c) Using the same triangle as above:

$$\begin{aligned} \tan\theta &= \frac{37.3}{100} \\ &= 0.373 \\ \theta &= \tan^{-1}(0.373) \\ &= \underline{20.5^\circ} \end{aligned}$$