Measurement

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January 20, 2020

Weeks: 1
Dates: Monday 20/1, Wednesday 22/1 and Thursday 23/1

Monday 20/1

First half module: Similar shapes and solids

- Recap of enlargement factor
- Enlargement of length, area and volume
- Questions

Second half module: Construction of 3d objects

- Making cubes and Pyramids
- Coordinates in 3-D and midpoint formula
- Questions

Wednesday 22/1

First half module: Volume and surface of pyramids, cones and spheres

- Description of pyramids and cones
- Formulas
- Questions

Second half module: Continuation

Thursday 23/1

First half module: Converting between units

- Examples
- Questions

Second half module: Continuation

1 Notes to Monday 20/1

Ask these questions:

- What is enlargement?
- What is the scale factor?
- In what intervals is there enlargement, reduction, inverted enlargement and reduction.

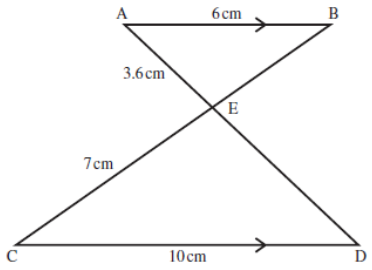
How do we use the scale factor to enlarge or reduce, length, area and volume?
Enlargement factor n

$$\text{Length1} : n \tag{1}$$
$$\text{Area1} : n^2 \tag{2}$$
$$\text{Volume1} : n^3 \tag{3}$$

EXAMPLE

In the diagram, AB and CD are parallel.
AB = 6 cm, CD = 10 cm, AE = 3.6 cm and CE = 7 cm.

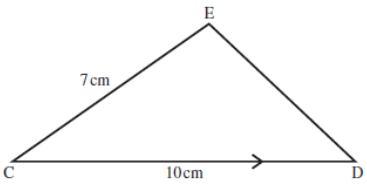
a) Explain carefully why triangles AEB and DEC are similar.
b) Calculate the length BE.
c) Work out the length DE.

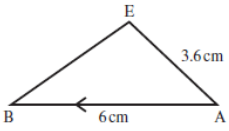


SOLUTION

a) Angles ABE and DCE are equal (alternate angles).
Angles BAE and CDE are equal (alternate angles).
Angles AEB and DEC are equal (vertically opposite).
Thus both triangles contain exactly the same angles, so they must be similar.

b) Redrawing the similar triangles so that they are the same way up:





Then, by comparing corresponding sides:

$$\frac{BE}{7} = \frac{6}{10}$$

Thus, cross-multiplying:

$$10 \times BE = 6 \times 7$$
$$10 \times BE = 42$$
$$BE = \frac{42}{10}$$
$$BE = 4.2 \text{ cm}$$

c) Likewise:

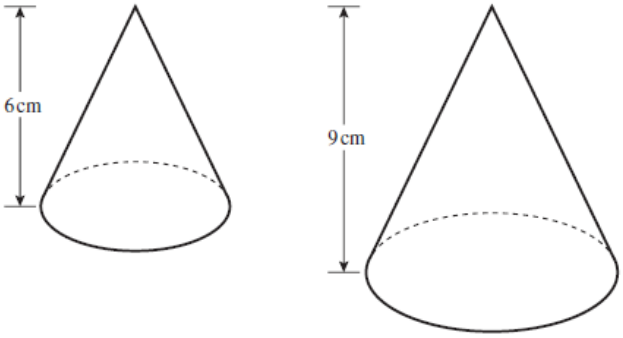
$$\frac{DE}{3.6} = \frac{10}{6}$$

Thus, cross-multiplying:

$$6 \times DE = 10 \times 3.6$$
$$6 \times DE = 36$$
$$DE = \frac{36}{6}$$
$$DE = 6 \text{ cm}$$

EXAMPLE

The diagram shows two solid cones. They are mathematically similar.



The smaller cone has a curved surface area of 64 cm^2.

a) Work out the curved surface area of the larger cone.

The two cones are made of the same material. The larger cone has a mass of 1080 grams.

b) Work out the mass of the smaller cone.

SOLUTION

The enlargement factor is $9 \div 6 = 1.5$

a) Area of larger cone $= 64 \times (1.5^2)$
$= 144\ cm^2$

b) Mass of smaller cone $= 1080 \div (1.5^3)$
$= 320$ grams

Questions:

- 4,5,9 Page(281-282)

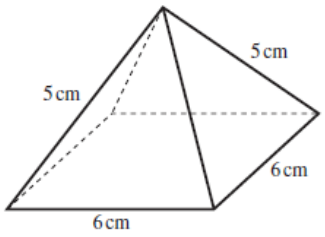
Activity: Constructing 3D shapes - Cubes and Triangles

A pyramid has a square base whose sides are 6 cm long. The triangular faces have sides 5 cm, 5 cm, 6 cm.

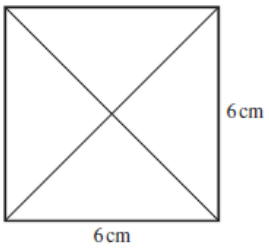
a) Draw a sketch of the pyramid.
b) Draw a plan view of the pyramid.
c) Draw a net for the pyramid.

SOLUTION

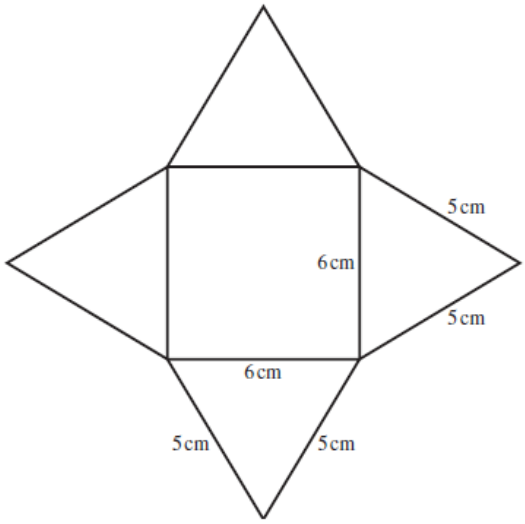
a)



b)

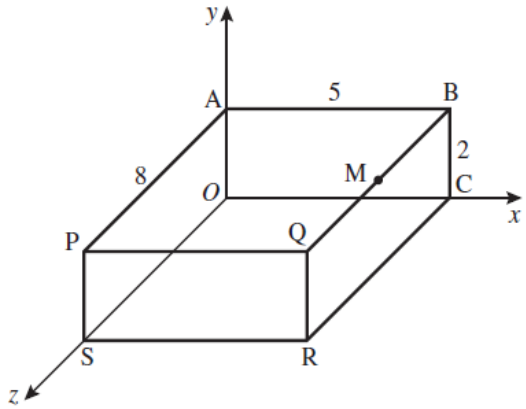


c)



Coordinates in 3D

The diagram shows a cuboid ABC*O*PQRS.
AB = 5 units, BC = 2 units, AP = 8 units.



a) Write down the coordinates of B, C, P and Q.
b) M is the midpoint of BQ. State the coordinates of M.

SOLUTION

a) B is at (5, 2, 0).
C is at (5, 0, 0).
P is at (0, 2, 8).
Q is at (5, 2, 8).
b) M is midway between B (5, 2, 0) and Q (5, 2, 8).
Thus M is at (5, 2, 4).

The midpoint of a line formula:

$$\left(\frac{a+p}{2}, \frac{b+q}{2}, \frac{c+r}{2}\right) \tag{4}$$

Questions:

- 18.2 - All questions Page(341-342)

2 Notes to Wednesday 22/1

A pyramid or a cone has a base and an apex. Rays are drawn from the edge of the base and converges at a point, the apex. Pyramids can have square or triangle bases, and cones have a circular base.

The volume of a pyramid is given:

$$\text{Volume} = \frac{1}{3} \times \text{area of the base} \times \text{Perpendicular height} \tag{5}$$

The volume of a cone is given:

$$\text{Volume} = \frac{1}{3} \times \pi r^2 h \tag{6}$$

where r is the radius of the base, and h is the perpendicular height. And the surface area:

$$A = \pi r l \tag{7}$$

where l is the slant length.

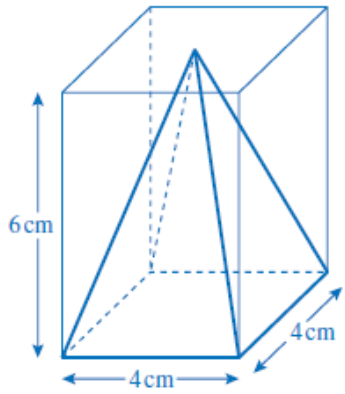
The volume for a sphere is given:

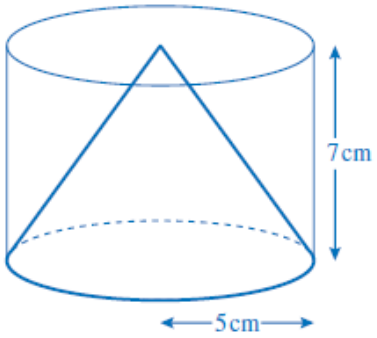
$$V = \frac{4}{3} \times \pi r^3 \tag{8}$$

and the surface area:

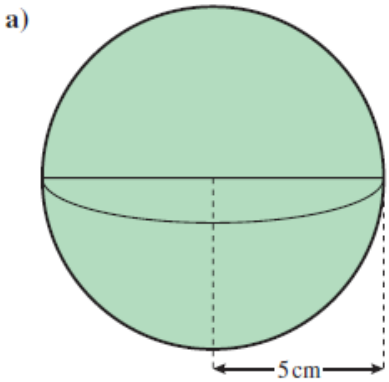
$$A = 4\pi r^2 \tag{9}$$

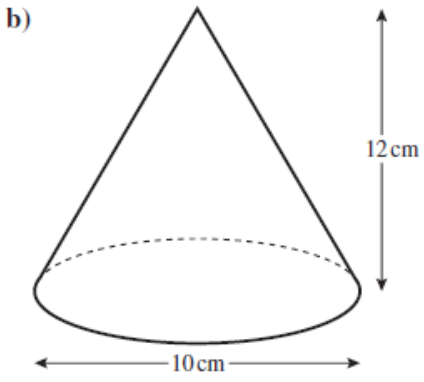
Examples:





Ex - A spherical ball bearing has a diameter of 6 mm. Find its volume?





Questions:

- 18.3 - All questions Page(345-346)

3 Notes to Thursday 23/1

Converting units:

$$1\text{cm} = 10\text{mm} \tag{10}$$
$$1\text{cm}^2 = 10 \times 10 = 100\text{mm}^2 \tag{11}$$
$$1\text{cm}^3 = 10 \times 10 \times 10 = 100\text{mm}^3 \tag{12}$$
$$1\text{km} = 1000\text{meters} \tag{13}$$

Questions:

- 18.4 1-10 Pages 347