Probability

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Weeks: 2
Dates: Monday 6/1, Tuesday 7/1, Monday 13/1 (Cancelled because of Ess Work), Thursday 16/1

Monday 6/1

1st half module

- What is probability?
  - Represented numerically
  - Determine probabilities
  - Where is probability used?
- Experimental Probability
  - Trials, Outcomes, Frequency, Relative frequency.
  - Relative frequency = Experimental probability(TOK 1)
  - Example : Dice and Two way table
  - Questions

2nd half module

- Sample Space
  - Sample space, Events
  - Set notation, subsets, Venn diagram
  - complementary events
  - Example: Venn diagram, 2D dimensional grid and Tree diagram
- Theoretical Probability
  - Equally likely definition
  - $P(A) = \frac{n(A)}{n(U)}$
  - Complementary Events $P(A) + P(A') = 1$
  - Questions

Tuesday 7/1

1st half module

- Addition law of probability
  - Work through Investigation 4
  - Definition $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
  - Mutually exculsive
  - Examples:
  - Questions

2nd half module

- Independent events
  - Work through Investigation 5
  - Definition $P(A \cap B) = P(A) \times P(B)$
  - Examples:
  - Questions

Thursday 16/1

1st half module

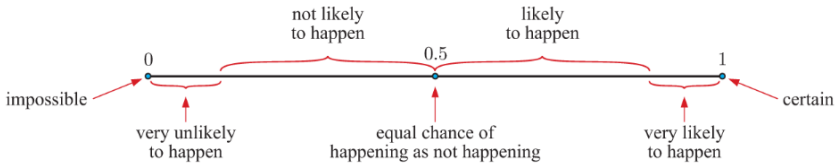
- Dependent events
  - Example:
  - Definition $P(A \cap B) = P(A) \times P(B \mid A)$
  - Experiments and Replacement
  - Questions

2nd half module

- Conditional probability
  - Example:
  - Definition $P(A \mid B) = \frac{n(A \cap B)}{n(B)}$
  - Equally likely $\implies$ conditional probability $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$
  - Questions

1 Notes to Monday 6/1

Probability: In the real world we think of probability as a **chance or likelihood** of some event happening. We assign a number between 0 and 1 to the chance of a event occurring and we call this number the probability.



We can determine probabilities based on: **the results of an experiment** and/or **what we theoretically expect to happen**.

The theory of probability is used in a wide range of fields:

- Biology
- Economics
- Politics
- Sport
- Quality control
- Production planning
- Physics - Quantum mechanics and Statistical mechanics

Experimental Probability: When performing an experiment that involves chance the following information is needed. **Number of trials:** is the total number of times the experiment is performed. **Outcomes:** are the different results possible for one trial of the experiment. **Frequency of an outcome:** is the number of times the outcome is observed. **Relative frequency of an outcome:** is the frequency of the outcome expressed as a fraction or percentage of the total number of trials. $RF = \frac{Frequency}{Numberoftrials}$

Example: Dice probability experiment

- What is the theoretical probability of each face?
- How many trials will you do?
- What outcomes are possible? (Express in set notation)
- Express the frequency of an outcome in your experiment.
- Calculate the relative frequency of an outcome in your experiment.
- (Look at individual experiments vs. the sum of the trials)

In experiments, the relative frequency is the best estimate of the probability of that event occurring, **Relative frequency = Experimental probability**.

Example: Two way table

	Adult	Child	Total
Season ticket holder	1824	779	2603
Not a season ticket holder	3247	1660	4907
Total	5071	2439	7510

$$b_i : P(\text{A child}) = \frac{779 + 1660}{7510} = 0.32 \implies 32\% \tag{1}$$

$$b_{ii} : P(\text{Not a season ticket holder}) = \frac{3247 + 1660}{7510} = 0.65 \implies 65\% \tag{2}$$

$$b_{iii} : P(\text{An adult season ticket holder}) = \frac{1824}{7510} = 0.24 \implies 24\% \tag{3}$$

Questions:

- 10A - 1, 2, 3 (Page 243)
- 10A - 5 (Page 244)
- 10B - 3, 4 (Page 247)

Sample space, events and complementary events

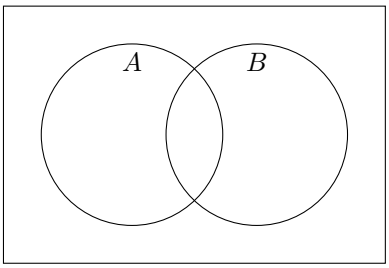
- The **sample space** U is the set of all possible outcomes of an experiment.
- An **event** is a set of outcomes in the sample space that have a particular property.
- The sample space is the **universal set** U.
- The outcomes are the **elements** of the sample space.
- Events are **subsets** of the sample space.
- We use set notation and Venn diagrams to solve probability problems.

Complementary events: Two events are **complementary** if exactly one of the events must occur. If A is an event, then A' is the complementary event of A, or "not A".

Example: Venn diagram Q 10C.3

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\} \tag{4}$$

$$A = \{4, 8, 12, 16\} \tag{5}$$


A
B