BIRKERØD GYMNASIUM,
HF, IB & KOSTSKOLE

IB DIPLOMA PROGRAMME

Mathematics Standard Level SL

Test on Common core material

Test 2 P2

December 2019

50 minutes

Instructions to candidates:

- Write down your name in the space provided.

- Do not open this examination paper until instructed to do so.

- A graphic display calculator is **required** for this test

- Answer all the questions

- Write your answers in the spaces provided.

- You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working.

- Unless otherwise stated in the question, all numerical **answers** must be given **exactly** or rounded to 3 significant figures.

Points:_____/50   *Grade:*

Name:_____Solutions_____

Teacher: (AA, ARC or ANU) _____

1

Question 1. *[total marks: 6]*

The volume of a sphere is $V = \sqrt{\dfrac{S^3}{36\pi}}$, where S is its surface area.

The surface area of a sphere is 500 cm².

(a)   **Calculate** the volume of the sphere. Give your answer correct to **two decimal places**.

[3 marks]

$$V = \sqrt{\frac{500^3}{36\pi}}$$

$$= 1051.3052\ldots \qquad\qquad A1$$

$$= 1051.31 \text{ cm}^3 \quad \text{to} \quad 2 \text{ d.p.} \qquad A2$$

(b)   **Write down** your answer to (a) correct to the nearest integer.   *[1 mark]*

1051

$A1$

(c)   **Write down** your answer to (b) in the form $a \times 10^n$, where $1 \le a < 10$ and $n \in \mathbb{Z}$.

[2 marks]

1.051×10^3

$A1 \qquad A1$

Question 2. *[total marks: 6]*

The grouped frequency table summarises the examination results of a group of students.

Examination result (x)	$0 < x \le 20$	$20 < x \le 40$	$40 < x \le 60$	$60 < x \le 80$	$80 < x \le 100$
Frequency	150	350	1100	1400	200

a)   **Write down** the modal class.

$$60 \le x \le 80$$

[1 mark]

A1

b)   **Write down** the mid-interval value of the modal class.

$$70$$
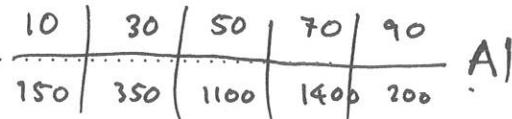
[1 mark]

A1

c)   **Calculate** an estimate of the mean examination result.

[2 marks] (M1)

$$\mu = 57.2$$

10	30	50	70	90
150	350	1100	1400	200

A1

d)   **Calculate** an estimate of the standard deviation, giving your answer correct to **three decimal places**.

[1 mark]

$$\sigma = 18.496$$

A1

Question 3. *[total marks: 6]*

A sequence has general term $u_n = 3n + 1$, for $n \geq 1$.

(a)   **Write down** the first three terms of the sequence. *[1 mark]*

$$4 \; , \; 7 \; , \; 10$$

A1

(b) *[5 marks]*

   (i)   **Expand** as the sum of four terms.

$$\sum_{n=21}^{24} (3n + 1)$$

$$64 + 67 + 70 + 73$$

A2

(A1 – all 4 terms, A1
for sum)

   (ii)   **Calculate.**

$$\sum_{n=21}^{100} (3n + 1)$$

$$14600 \qquad (\text{using} \quad \text{summation})$$

or

$$S_{100} = \frac{100}{2} \left(2 \cdot 4 + 3 (100 - 1) \right) = 15\,250 \qquad \text{(M1) (A1)}$$

$$S_{20} = \frac{20}{2} \left(2 \cdot 4 + 3 (20 - 1) \right) = 650$$

A1

$$S_{100} - S_{20} = 14600$$

4
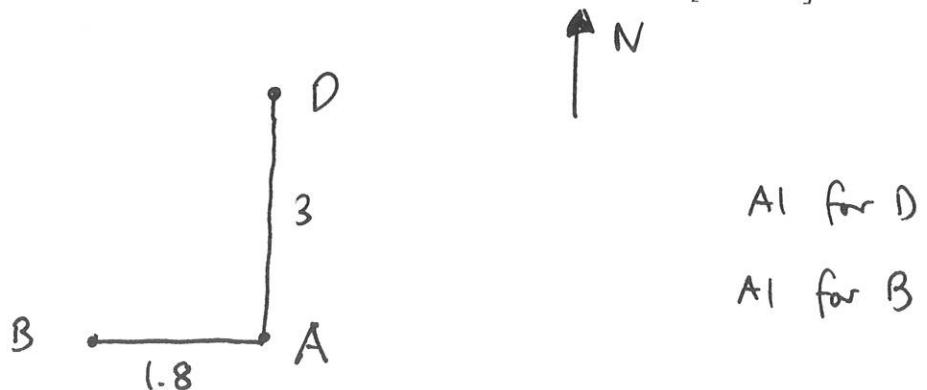
Question 4. *[total marks: 8]*

Andrew is at point A in a park. A deer is 3 km directly north of Andrew, at point D. Brian is 1.8 km due west of Andrew, at point B.

(a)   **Draw** a diagram to represent this information.   *[2 marks]*

D

3

B

1.8

A

↑ N

A1 for D

A1 for B

(b)   **Calculate** the distance between Brian and the deer.   *[2 marks]*

$BD^2 = 1.8^2 + 3^2$ M1

$BD^2 = 12.24$
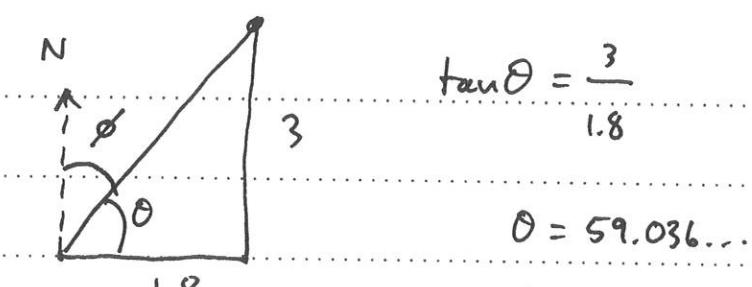
$BD = 3.4985$

$= 3.50$ km   A1

(c)   **Calculate** the bearing of the deer from Brian. Give your answer to 1 decimal place.   *[4 marks]*

N

ø

3

θ

1.8

$\tan\theta = \dfrac{3}{1.8}$ M1

$\theta = 59.036...$ A1

$ø = 90 - 59.036.. = 30.9637..$ M1 A1

Bearing   is $031.0°$

$\left(\text{Alternative, } \tan ø = \dfrac{1.8}{3} \quad M2 \right.$

$\left. ø = 30.96.. \quad A2 \right)$

5

Question 5. [*total marks: 8*]

A sum of $5000 is invested at a compound interest rate of r %, compounded monthly.

(a) **Write** an expression in terms of r and n for the value of the investment after n full years.

[*2 marks*]

MI AI

$$5000 \cdot \left(1 + \frac{r}{100 \times 12}\right)^{12n} = 5000 \cdot \left(1 + \frac{r}{1200}\right)^{12n}$$

After 5 years, the investment has a value of $6100.

(b) **Find** r. [*3 marks*]

$$6100 = 5000 \cdot \left(1 + \frac{r}{1200}\right)^{12n}$$

$$1.22 = \left(1 + \frac{r}{1200}\right)^{60}$$

or use App

N = 60

PV = −5000

FV = 6100

P/Y = 12

(MI) (AI)

$$\sqrt[60]{1.22} = 1 + \frac{r}{1200}$$

$$1.0033196.. = 1 + \frac{r}{1200} \qquad r = 3.98\%$$

AI

Inflation over the 5 years averaged at 2% per year.

(c) **Find** the real value of the investment after 5 years. [*3 marks*]

$$\frac{6100}{1.02^5} = 5524.96$$

MMA2

(MI) (AI) AI

Question 6.   [*total marks: 3*]

From January to September, the mean number of car accidents per month was 630. From October to December, the mean was 810 accidents per month.

What was the mean number of car accidents per month for the whole year?

$$\mu = \frac{630 \times 9 + 810 \times 3}{12} \quad (M1)$$

$$= \frac{8100}{12} \quad A1 \text{ for } 8100$$

$$= 675 \quad A1$$

Question 7. [*total marks: 13*]

An arithmetic sequence is defined as

$$u_n = 135 + 7n, \qquad n = 1, 2, 3, \ldots$$

a)   **Calculate** u_1, the first term in the sequence.

[*1 mark*]

$$135 + 7 = 142 \qquad \text{A1}$$

b)   **Write down** the common difference.

[*1 mark*]

$$7 \qquad \text{A1}$$

S_n is the sum of the first n terms of the arithmetic sequence.

c)   **Find** an expression for S_n. Give your answer in the form $S_n = An^2 + Bn$, where A and B are constants

[*3 marks*]

$$S_n = \frac{n}{2}\left(2 \cdot 142 + 7(n-1)\right) \qquad \text{M1 A1}$$

$$= \frac{n}{2}\left(284 + 7n - 7\right)$$

$$= \frac{n}{2}\left(277 + 7n\right)$$

$$= 138.5n + 3.5n^2 \qquad \text{A1}$$

The sum of the first n terms of the arithmetic sequence is 11 991.

d)   **Find** n.

[*3 marks*]

$$11\,991 = 138.5n + 3.5n^2 \qquad \text{M1}$$

$$0 = 3.5n^2 + 138.5n - 11\,991$$

$$n = 42 \qquad \text{or} \qquad \left(-\frac{571}{7}\right)$$

$$\text{So} \qquad n = 42 \qquad \text{A2}$$

8

The first term, v_1, of a geometric sequence is 20 and its fourth term v_4 is 67.5.

e)   **Show** that the common ratio, r, of the geometric sequence is 1.5.   *[2 marks]*

$$V_4 = 67.5 \implies 67.5 = 20 \times r^3 \qquad M1$$

$$\frac{67.5}{20} = r^3$$

$$3.375 = r^3 \qquad R1$$

$$r = \sqrt[3]{3.375} \qquad = 1.5$$

T_n is the sum of the first n terms of the geometric sequence.

f)   Calculate T_7 , the sum of the first seven terms of the geometric sequence.   *[2 marks]*

$$T_7 = \frac{20\,(1.5^7 - 1)}{1.5 - 1} \qquad (M1)$$

$$= 643.437$$
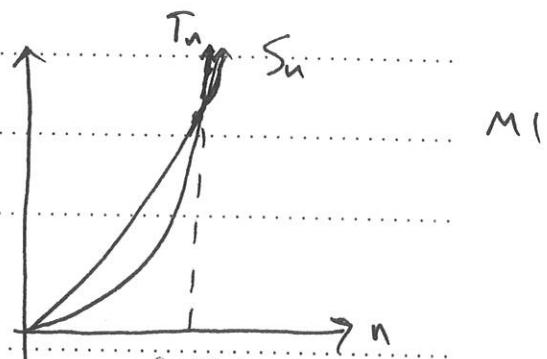
$$= 643 \qquad A1$$

g)   Use your graphic display calculator to **find** the smallest value of n for which $T_n > S_n$.
     [2 marks]

$$T_n = \frac{20\,(1.5^n - 1)}{0.5} \quad \text{or} \quad 40\,(1.5^n - 1)$$

$$S_n = 138.5n + 3.5n^2$$

n	S_n	T_n	
9	1530	1497.7	✗
10	1735	2266.6	✓

9.074..

$$n = 10 \qquad A1$$

M1

9