Measurement

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January 16, 2020

Weeks: 1
Dates: Monday 20/1 and Tuesday 21/1

Monday the 20/1

First Half Module: Probability

- Making predictions using probability
- Birthday paradox
- Questions

Second Half Module:Measurement

- Circles, Arcs and Sectors
- Surface area

Tuesday the 21/1

First Half Module:Measurement

- Volume
- Questions

Second Half Module

- Capacity
- Questions

1 Notes to Monday 20/1

As we have seen previously, the experimental probability is equal to the relative frequency of the event.

$$\begin{aligned} \text{Experimental probability} &= \text{relative frequency of the event} && (1) \\ &= \frac{\text{number of times event occurs}}{\text{number of trials}} && (2) \end{aligned}$$

If we rearrange this equations we get:

$$\text{number of times event occurs} = \text{experimental probability} \times \text{number of trials} \tag{3}$$

So before we were starting with an experiment and using the outcome to say something about the probability. This equation is the opposite! We use a theoretical probability to predict the results.

Definition: If there are n trials of an experiment, and an event has probability p of occurring in each of the trials, then the number of times we expect the event to occur is np.

Example 24 **Self Tutor**

In his basketball career, Michael Jordan made 83.53% of shots from the free throw line. If he had played one more game and had 18 attempts from the free throw line, how many shots would you expect him to have made?

$n = 18$ throws

$p = \text{P(successfully makes free throw)} = 0.8353$

We would expect him to have made $np = 18 \times 0.8353 \approx 15$ shots.

np will not likely be an integer, what do we do? And is this a problem?

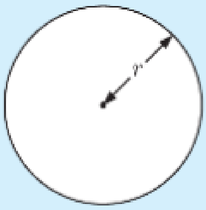
The birthday paradox
Questions:

- 10J - 1,3,5 Page(275-276)

Measurement - Circles, arcs and sectors We will be revising measurement, mostly area and volume of different shapes. The first shape we will be looking at is the circle.

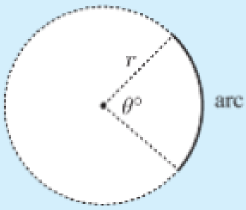
For a **circle** with radius r:

- the **circumference** $C = 2\pi r$
- the **area** $A = \pi r^2$.



An **arc** is a part of a circle which joins any two different points. It can be measured using the angle $\theta°$ subtended by the points at the centre.

Arc length $= \dfrac{\theta}{360} \times 2\pi r$

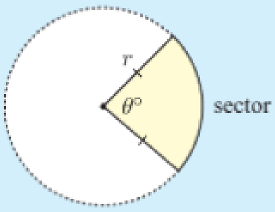


A **sector** is the region between two radii of a circle and the arc between them.

Perimeter $=$ two radii $+$ arc length

$$= 2r + \frac{\theta}{360} \times 2\pi r$$

$$\text{Area} = \frac{\theta}{360} \times \pi r^2$$



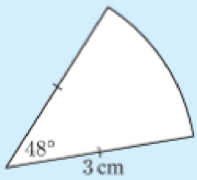
Example 1 **Self Tutor**

For the given figure, find to 3 significant figures:

a the length of the arc

b the perimeter of the sector

c the area of the sector.



a Arc length $= \dfrac{\theta}{360} \times 2\pi r$

$= \frac{48}{360} \times 2\pi \times 3$ cm

≈ 2.51 cm

b Perimeter $= 2r +$ arc length

$\approx 2 \times 3 + 2.51$ cm

≈ 8.51 cm

c Area $= \dfrac{\theta}{360} \times \pi r^2$

$= \frac{48}{360} \times \pi \times 3^2$

$\approx 3.77\ \text{cm}^2$

Surface Area The surface area of a three-dimensional figure with plane faces is the sum of the areas of the faces.

Example 2 **Self Tutor**

The pyramid shown is 10.8 cm high.
Find its surface area.

The net of the pyramid includes one square with side length 13.2 cm, and four isosceles triangles with base 13.2 cm.

Let the height of the triangles be h cm.

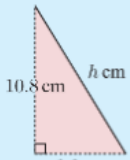
Now $h^2 = 10.8^2 + 6.6^2$ {Pythagoras}

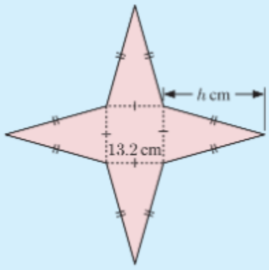
$\therefore\ h = \sqrt{10.8^2 + 6.6^2} \approx 12.66$

$\therefore$ the surface area

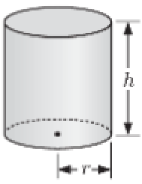
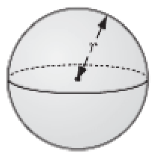
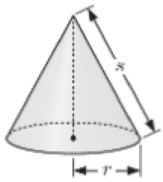
$\approx 13.2^2 + 4 \times (\frac{1}{2} \times 13.2 \times 12.66)\ \text{cm}^2$

$\approx 508\ \text{cm}^2$





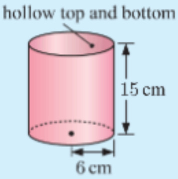
Solids with curved surfaces We have just looked at flat surfaces, but some solids have curved surfaces, these can be calculated.

Cylinder	Sphere	Cone
		
A = curved surface + 2 circular ends $= 2\pi r h + 2\pi r^2$	$A = 4\pi r^2$	A = curved surface + circular base $= \pi r s + \pi r^2$

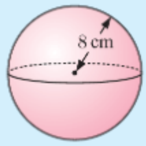
Example 3 — Self Tutor

Find, to 1 decimal place, the outer surface area of:

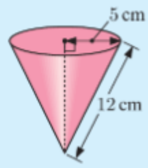
a



b



c



a The cylinder is hollow top and bottom, so we only have the curved surface.

$A = 2\pi rh$
$= 2 \times \pi \times 6 \times 15$
$\approx 565.5 \text{ cm}^2$

b $A = 4\pi r^2$
$= 4 \times \pi \times 8^2$
$\approx 804.2 \text{ cm}^2$

c $A = \pi rs + \pi r^2$
$= \pi \times 5 \times 12 + \pi \times 5^2$
$\approx 267.0 \text{ cm}^2$

Example 4 — Self Tutor

The length of a hollow pipe is three times its radius.

a Write an expression for its outer surface area in terms of its radius r.

b If the outer surface area is 301.6 m^2, find the radius of the pipe.

a Let the radius be r m, so the length is $3r$ m.

Surface area $= 2\pi rh$
$= 2\pi r \times 3r$
$= 6\pi r^2 \text{ m}^2$

b The surface area is 301.6 m^2

$\therefore \; 6\pi r^2 = 301.6$

$\therefore \; r^2 = \dfrac{301.6}{6\pi}$

$\therefore \; r = \sqrt{\dfrac{301.6}{6\pi}} \quad \{\text{as } r > 0\}$

$\therefore \; r \approx 4.00$

The radius of the pipe is 4 m.

Questions:

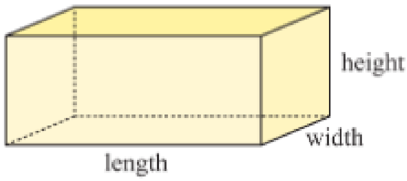
- 6A Odd questions Page(147-148)
- 6B.1 1,2,3,5 Page(149-150)
- 6B.2 1,3,5,7,9 Page(152-153)

2 Notes to Tuesday 21/1

Volume definition: The volume of a solid is the amount of space it occupies.

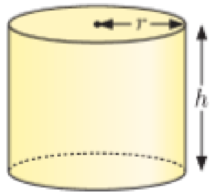
$$\text{Volume} = \text{area of cross-section} \times \text{length} \tag{4}$$

- rectangular prisms



Volume $=$ length $\times$ width $\times$ height

- cylinders

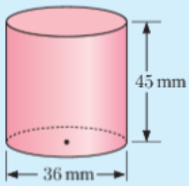


Volume $= \pi r^2 h$

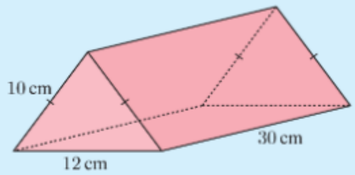
Example 5 **Self Tutor**

Find the volume of:

a



b

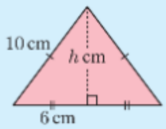


a $V = \pi r^2 h$

$= \pi \times 18^2 \times 45 \text{ mm}^3$

$\approx 45\,800 \text{ mm}^3$

b



Let the prism have height h cm.

$h^2 + 6^2 = 10^2$ {Pythagoras}

$\therefore\ h^2 + 36 = 100$

$\therefore\ h^2 = 64$

$\therefore\ h = 8$ {as $h > 0$}

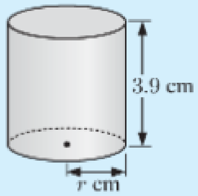
Volume $=$ area of cross-section $\times$ length

$= (\frac{1}{2} \times 12 \times 8) \times 30 \text{ cm}^3$

$= 1440 \text{ cm}^3$

Example 6 | **Self Tutor**

Find, to 3 significant figures, the radius of a cylinder with height 3.9 cm and volume 54.03 cm^3.



$$\begin{aligned} V &= 54.03 \text{ cm}^3 \\ \therefore \quad \pi \times r^2 \times 3.9 &= 54.03 && \{V = \text{area of cross-section} \times \text{height}\} \\ \therefore \quad r^2 &= \frac{54.03}{\pi \times 3.9} && \{\text{dividing both sides by } \pi \times 3.9\} \\ \therefore \quad r &= \sqrt{\frac{54.03}{\pi \times 3.9}} \approx 2.10 && \{\text{as } r > 0\} \end{aligned}$$

The radius is approximately 2.10 cm.

Tapered solids For solids such as pyramids and cones, tapered solids, the following equation is used.

$$\text{Volume} = \frac{1}{3}(\text{area of base} \times \text{height}) \tag{5}$$

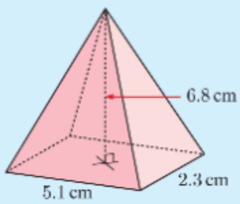
For spheres the volume is given

$$\text{Volume} = \frac{4}{3}\pi r^3 \tag{6}$$

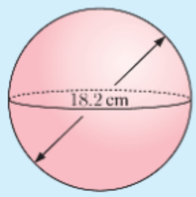
Example 7 | **Self Tutor**

Find the volume of each solid:

a



b



a $V = \frac{1}{3}(\text{area of base} \times \text{height})$
$= \frac{1}{3}(\text{length} \times \text{width} \times \text{height})$
$= \frac{1}{3}(5.1 \times 2.3 \times 6.8) \text{ cm}^3$
$\approx 26.6 \text{ cm}^3$

b $V = \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi \left(\frac{18.2}{2}\right)^3$
$\approx 3160 \text{ cm}^3$

Questions:

- 6C.1 1,2,4,7 Page(155-156)
- 6C.2 1,5,6 Page(160-161)

3 Capacity

Capactiy: The capacity of a container is the quantity of fluid it is capable of holding. The capacity belongs to the container, not to the fluid it's self.

The definition for 1mL of water occupies 1cm^3 of space is a cube with 1cm sides.

Volume	*Capacity*
1 cm^3 $\equiv$	1 mL
1000 cm^3 $\equiv$	1 L
1 m^3 $\equiv$	1 kL
1 m^3 $\equiv$	1000 L

Example 8 — Self Tutor

Find the volume of liquid which will fit in a container with capacity:

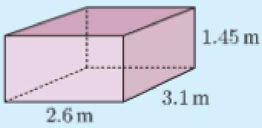
a 9.6 L **b** 3240 L

a 9.6 L
$= (9.6 \times 1000)$ cm^3
$= 9600$ cm^3

b 3240 L
$= (3240 \div 1000)$ m^3
$= 3.24$ m^3

Example 9 — Self Tutor

Find the capacity of a 2.6 m by 3.1 m by 1.45 m tank.



V = length × width × height
$= 2.6 \times 3.1 \times 1.45$ m^3
$= 11.687$ m^3

The tank's capacity is 11.687 kL.

Example 10 **Self Tutor**

17.3 mm of rain falls on a flat rectangular shed roof which has length 10 m and width 6.5 m. All of the water goes into a cylindrical tank with base diameter 4 m. By how many millimetres does the water level in the tank rise?

For the roof: The dimensions of the roof are in m, so we convert 17.3 mm to metres.

$17.3 \text{ mm} = (17.3 \div 1000) \text{ m} = 0.0173 \text{ m}$

The volume of water collected by the roof = area of roof × depth

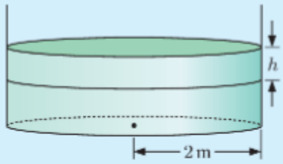
$= 10 \times 6.5 \times 0.0173 \text{ m}^3$

$= 1.1245 \text{ m}^3$

For the tank: The volume added to the tank

$= \text{area of base} \times \text{height}$

$= \pi \times 2^2 \times h \text{ m}^3 = 4\pi \times h \text{ m}^3$



The volume added to the tank must equal the volume which falls on the roof, so

$4\pi \times h = 1.1245$

$\therefore \; h = \frac{1.1245}{4\pi}$ {dividing both sides by 4π}

$\therefore \; h \approx 0.0895 \text{ m}$

$\therefore$ the water level rises by about 89.5 mm.