Trigonometry

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Weeks: 2
Dates: Monday 6/1, Wednesday 8/1, Thursday 9/1, Tuesday 14/1, Thursday 16/1, Friday 17/1

Monday 6/1

1st half module:

- Exam - Anu

2nd half module:

- Math Activity

Wednesday 8/1

1st half module:
Trigonometrical functions

- Sine Ratio

- Cosine Ratio

- Tangent Ratio

2nd half module:
Choosing the right trig function?

- SohCahToa

- How to use the SohCahToa triangles.

- Questions

Thursday 9/1

1st half module:
Recap of the trigonometrical functions

Finding an unknown angle

- Inverse functions

- Questions

Angles of elevation and depression

- Elevation

- Depression

- Questions

2nd half module:
Multi-stage problems

- Triangles split into two right angle triangles.

- Pythagoras' Theorem

- Bearings

- Questions

Tuesday 14/1

1st half module:
  Sine Rule

- Ambiguous case of the sine rule

- Questions

2nd half module:
Cosine Rule

- Questions

Thursday 16/1

1st half module:
  Recap of Right Angles
  Recap of Sine rule and Cosine rule

  2nd half module:
  Area of a Triangle
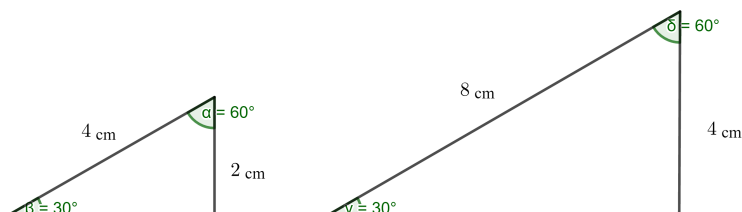
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Friday 17/1

Trigonometry in 3-D:

1 Notes Monday 6/1

2 Notes Wednesday 8/1

The Sine Ratio:

Consider two triangles with similar angles. The ratio will be same regardless of the different lengths. The larger triangle has been enlarged by a scale factor.
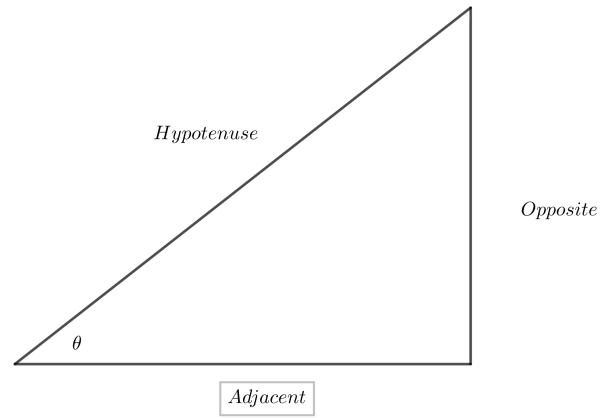
In right angle triangles, the answer obtained by dividing the length opposite the angle by the hypotenuse is called the sine of that angle.

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \tag{1}$$
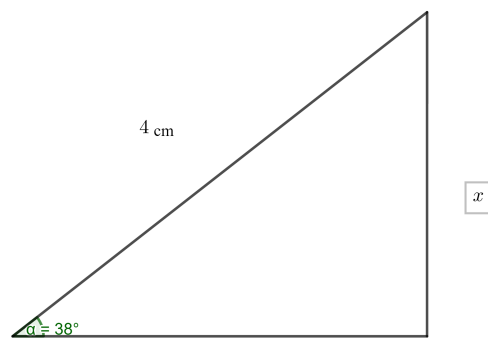
Remember - Calculators set to degrees (DEG)

To calculate a missing length, Isolate the missing length from the sine formula.

Hypotenuse

Opposite

θ

Adjacent

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \qquad (2)$$

$$\implies \text{opposite} = \sin(\theta) \times \text{hypotenuse} \qquad (3)$$

Example:

4 cm

x

$\alpha = 38°$

$$\text{opposite} = \text{hypotenuse} \times \sin(\theta) \tag{4}$$
$$x = 4 \times \sin(38°) \tag{5}$$
$$x = 2.462645901° \approx 2.46° \tag{6}$$

Show that the hypotenuse can also be isolated.

Cosine Ratio:

Instead of trying to find the opposite length, the adjacent length can be found using the cosine ratio.

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} \tag{7}$$
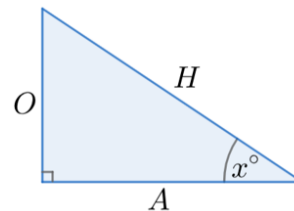$$\implies \text{adjacent} = \text{hypotenuse} \times \cos(\theta) \tag{8}$$

Tangent Ratio:

The last ratio is defined as:

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} \tag{9}$$
$$\implies \text{opposite} = \text{adjacent} \times \tan(\theta) \tag{10}$$

SohCahToa When solving questions the trigonometrical ratio rhyme can be used and triangles can be drawn to help isolate the missing length or angle(but more on that later).

How to use: Cover the quantity you are looking for and the triangle will tell to either multiple or divide the quantities you were given in the question.
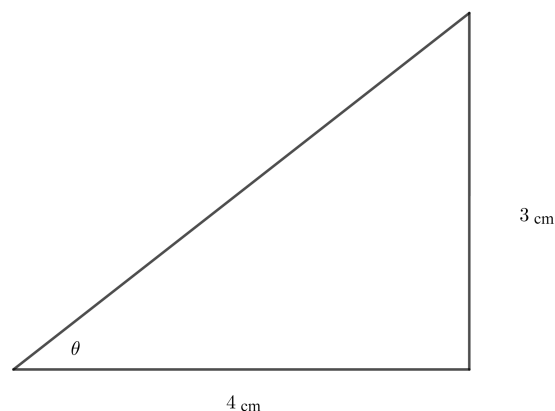
Questions:

- 1 through to 8 (Page 317)

3 Notes Thursday 9/1:

Finding an unknown angle:

- Inverse - "Reverse" function

- Inverse Angle - Calculators -> shift, sin/cos/tan.

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- $\tan(\theta) = \frac{3}{4} \implies \theta = \arctan(\frac{3}{4}) = 36.86989765° \approx 36.9°$

Questions:

- 2,4,6,8,10,12 (Page 319)

Angles of elevation and depression:
Angle of elevation - Above the horizontal line
Angle of depression - Blow the horizontal line.

These problems are a combination of pythagoras' theorem, trigonometrical equations and alternate angles.

Example: A climber is sitting on the summit S of a mountain. He looks down and sees his camp C in the valley below. The direct distance from the summit to the camp is 2200 metres. The summit is at an altitude 420 meters higher than the camp.

a: Calculate the angle of elevation of the summit as seen from the camp. Give your answer to the nearest 0.1°.

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \tag{11}$$

$$\sin(\theta) = \frac{420}{2200} \tag{12}$$

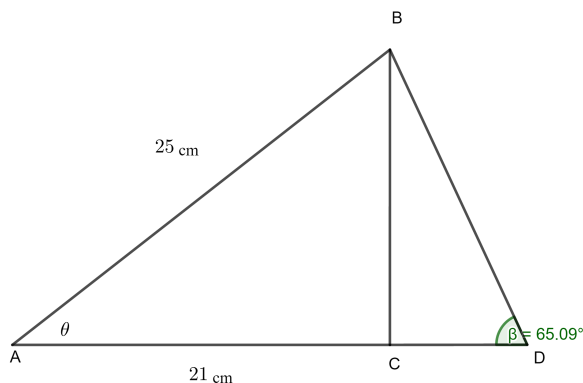$$\theta = \arcsin(\frac{420}{2200}) = 11.005° \approx 11° \tag{13}$$

$$\tag{14}$$

b: Write down the angle of depression of the camp as seen from the summit.
$= 11°$

Questions:

- 1,3,4 (Page. 325-326)

6

Multi-stage problems: Some problems will involve non-right angle triangles, such as:

Now you might be able to see, two right angle triangles!
a: Calculate the height BC, to three significant numbers.
b: Calculate length BD.
c: Calculate θ.

$$c^2 = a^2 + b^2 \tag{15}$$
$$\implies b^2 = c^2 - a^2 \tag{16}$$
$$BC^2 = 25^2 - 21^2 = 184 \tag{17}$$
$$\implies BC = \sqrt{184} = 13.5646 \approx 13.6 \tag{18}$$

Use Soh triangle!

$$\sin(65°) = \frac{BC}{BD} = \frac{13.6}{BD} \tag{19}$$
$$BD = \frac{13.56465997}{\sin(65°)} = 14.96694629 \approx 15.0 \tag{20}$$

$$\cos(\theta) = \frac{21}{25} \tag{21}$$
$$\theta = \arccos(\frac{21}{25}) = 32.85988038 \approx 32.9° \tag{22}$$

Bearing: Absolute bearing refers to the angle between the magnetic North (magnetic bearing) or true North (true bearing) and an object.

Example: A ship leaves its harbour and sails due south for 10km. It then sails due East for 20km then stops. a:How far is the ship from its harbour?

b:The ship wishes to return directly to its habour. On what bearing must it sail?

$$d^2 = 20^2 + 10^2 = 500 \tag{23}$$

$$d = \sqrt{500} = 22.36067978 \approx 22.4km \tag{24}$$

$$\tan(\theta) = \frac{10}{20} = 0.5 \tag{25}$$

$$\theta = \arctan(0.5) = 26.56505118 \approx 26.6° \tag{26}$$

$$\text{Bearing} = 270° + 26.6° = 296.6° \tag{27}$$