Statistics and Probability

$\int_a^b |\Psi(x,t)|^2\,dx =$ Probability of finding the particle between a and b, at time t

$$P_{ab} = \int_a^b \rho(x)\,dx \tag{1}$$
$$1 = \int_{-\infty}^{\infty} \rho(x)\,dx \tag{2}$$
$$\langle x\rangle = \int_{-\infty}^{\infty} x\rho(x)\,dx \tag{3}$$
$$\langle f(x)\rangle = \int_{-\infty}^{\infty} f(x)\rho(x)\,dx \tag{4}$$
$$\sigma^2 = \langle(\delta x)^2\rangle = \langle x^2\rangle - \langle x\rangle^2 \tag{5}$$

Wave Function

$$\text{Normalization: } \int_{-\infty}^{\infty} |\Psi(x,t)|^2\,dx = 1$$

If Ψ is normalized at $t=0$, it stays normalized for all future times

$$\langle x\rangle = \int \Psi^*\,(x)\,\Psi\,dx$$

$$\langle p\rangle = m\frac{d\langle x\rangle}{dt} = -i\hbar\int\left(\Psi^*\frac{\partial\Psi}{\partial x}\right)\,dx = \int\Psi^*\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)\Psi\,dx$$

$$\text{General:}\langle Q(x,p)\rangle = \int \Psi^* Q\left(x, \frac{\hbar}{i}\frac{\partial}{\partial x}\right)\Psi\,dx$$

$$T = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$
$$\text{Kinetic Energy:}\langle T\rangle = -\frac{\hbar^2}{2m}\int\Psi^*\left(\frac{\partial^2\Psi}{\partial x^2}\right)\,dx$$

$$\text{Ehrenfest's TH (V is potential): } \frac{d\langle p\rangle}{dt} = \langle -\frac{\partial V}{\partial x}\rangle$$

$$\text{de Broglie formula: } p = \frac{h}{\lambda} = \frac{2\pi h}{\lambda}$$