# All paths lead to the root

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## All Paths Lead to the Root

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### Abstract

In an attempt to fix the defects of the definition of forward security for Symmetric Searchable Encryption (SSE) schemes, Amjad et al. [2] proposed injection security. This new security property is strictly stronger than most security properties known to date, which makes it particularly challenging to design schemes meeting its requirements. In this work, we show how it is possible to use trees to decorrelate the modification of an index from its effects, hence achieving injection security. In addition to being conceptually simple, our scheme features non-interactive, stateless and mutation-free search operations that allow supporting concurrent readers easily. Finally, the proposed reference implementation is efficient: both Insert and Search operations execute in milliseconds even when operating on an index with up to a million entries and volumes up to a thousand.

### Metadata

**Available format(s)**

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**Category**

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**Publication info**

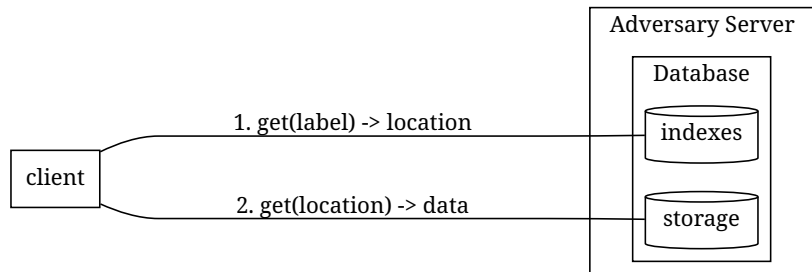Preprint.

**Keywords**

SSE STE EMM

**Contact author(s)**

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**History**

2025-09-18: approved
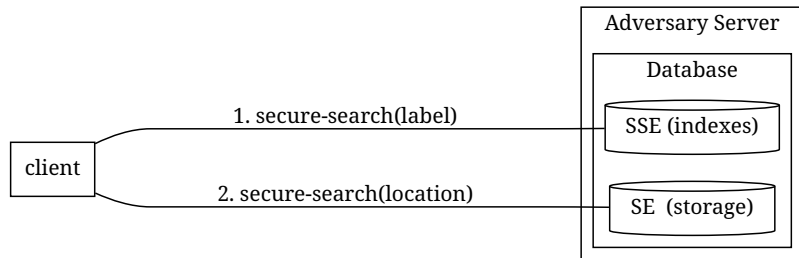
# Searchable Symmetric Encryption (SSE)

# How to retrieve data from an adversary server?



Leaks:

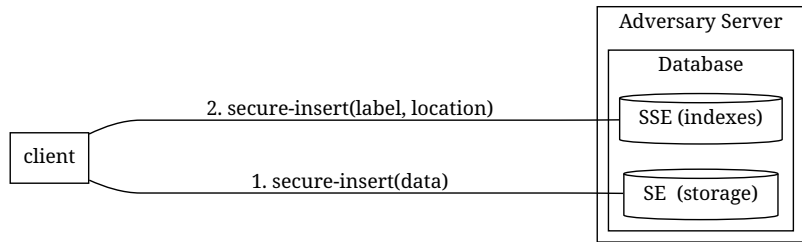1. the label and the location;
2. the location and the data.

# Controlling the leakage



Leaks:

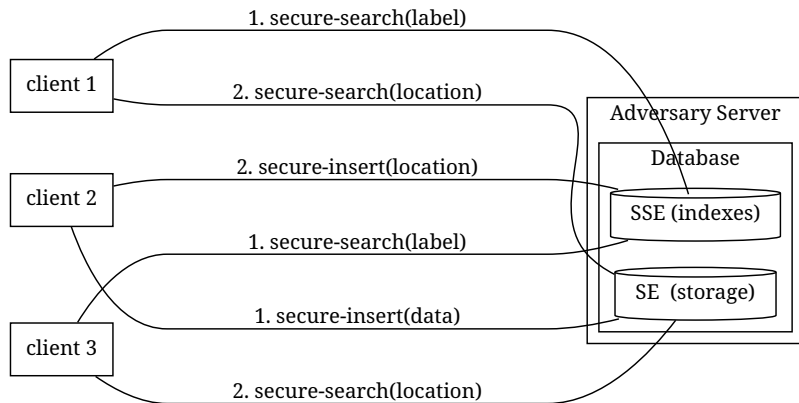1. $\mathcal{L}_{SSE}(label)$;
2. $\mathcal{L}_{SE}(location)$.

# Supporting dynamic states



Leaks:

1. $\mathcal{L}_{SE}(Insert, data)$.
2. $\mathcal{L}_{SSE}(Insert, label, location)$;

# Supporting concurrent queries



client 1

client 2

client 3

1. secure-search(label)

2. secure-search(location)

2. secure-insert(location)

1. secure-search(label)

1. secure-insert(data)

2. secure-search(location)

Adversary Server

Database

SSE (indexes)

SE (storage)

# Structured Encryption

# Index

**Index ADT**:

- ▶ *States*: $\mathbb{L}^{\mathbb{V}^*}$
- ▶ *Operations*:

```
Search :: state -> label -> Set value
Insert :: state -> label -> value -> ()
```

**Index STE**:

- ▶ *States*: $\mathbb{K}^{\mathbb{L}^{\mathbb{V}^*}}$
- ▶ *Operations*:

```
Setup :: () -> (key * IO ())
Search :: key -> label -> IO (Set value)
Insert :: key -> label -> value -> IO ()
```

# Multi-Map (MM)

**MM ADT**:

- ▶ *States*: $\mathbb{L}^{\mathbb{V}^*}$
- ▶ *Operations*:

      Search :: state -> label -> List value
       Insert :: state -> label -> value -> ()

**MM STE**:

- ▶ *States*: $\mathbb{K}^{\mathbb{L}^{\mathbb{V}^*}}$
- ▶ *Operations*:

       Setup :: () -> (key * IO ())
      Search :: key -> label -> IO (List value)
       Insert :: key -> label -> value -> IO ()

# Multi-Map STE to Index STE transformation

▶ Index setup and search are the very MM operations:

```
Index::setup  = MM::setup
Index::insert = MM::insert
```

▶ The result of an Index search is the result of the MM search
without duplicates:

```
Index::search = unique . MM::search
                where
                  unique vs = fold Set::insert
                                    EmptySet vs
```

Why bother implementing a more constrained ADT?

# Semi-dynamic MM to fully-dynamic MM transformation

**Associative ADT**:

*States*: $\mathbb{L}^{\mathbb{T}(\mathbb{V})}$

*Operations*:

Search :: s -> l -> T v

Mutate :: (Mutation M) => s -> l -> M T v -> ()

where:

```
class (Container T) => Mutation T v where
  apply :: v -> (T v) -> (T v)
```

# Semi-dynamic MM to fully-dynamic MM transformation

The fully-dynamic Multi-Map is an associative ADT:

- ▶ Insertion:

  ```
  instance Mutation (MMInsertion List value) where
    (apply) = (Cons)
  ```

- ▶ Deletion:

  ```
  instance Mutation (MMDeletion List value) where
    (apply) = (remove)
                where
                  remove v []    = []
                  remove v [v l] = remove v l
                  remove v [w l] = Cons w (remove v l)
  ```

# Semi-dynamic MM to fully-dynamic MM transformation

*Specialized mutations are T transformations[1]:*
*all we need is to log them!*

**Journaling Multi-Map ADT**:

- *States*: $\mathbb{L}^{(\mathbb{T}(\mathbb{V})^{\mathbb{T}(\mathbb{V})})^*}$
- *Operations*:
    ```
    Search state -> l -> List (Tx T v)
     Insert state -> l -> (Tx T v) -> ()
    ```
  where Tx T v = T v -> T v.

---

[1] More precisely, they form a monoid and can therefore be reduced.

# Semi-dynamic MM to fully-dynamic MM transformation

*Implementing any (fully-dynamic) associative ADT on top
of a (semi-dynamic) multi-map is therefore simple!*

```
search s l      = let transformations = MM::search s l
                  in (reduce transformations) T::empty

mutate s l m v = MM::insert s l (m v)
```
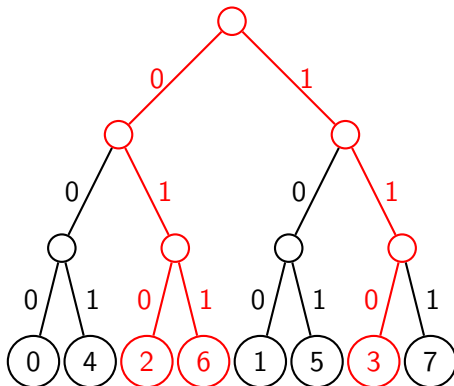
PLOC is actually *simple*!

# Challenge

1. Do not to leak anything during insertion:
   $\mathcal{L}(Insert, label, value) = \bot$

2. Only leak a (meaningless) UID of the label:
   $\mathcal{L}(Search, label) = \mathrm{sp}$

# Search

Simply derive the set of target branches:

- $\mathrm{PRF}(key, cat, 0) = 3 = \mathtt{b011}$
- $\mathrm{PRF}(key, cat, 1) = 2 = \mathtt{b010}$
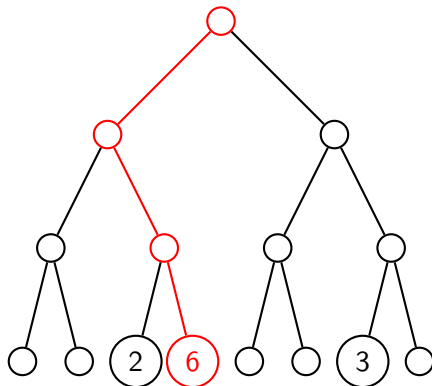- $\mathrm{PRF}(key, cat, 2) = 6 = \mathtt{b110}$

Mind the endianness!

# Insert – Datum

```
type Datum = target * value
```

*A datum must always be stored on its target branch.*

For example with (6 food):
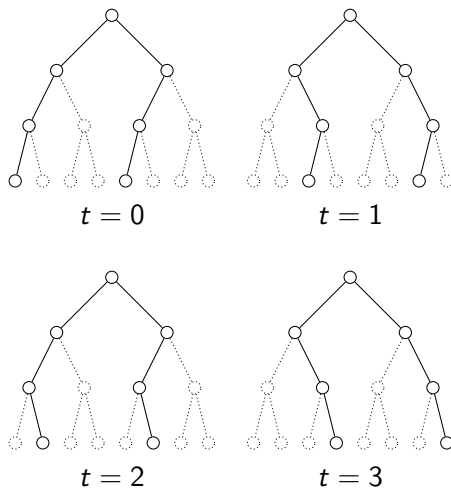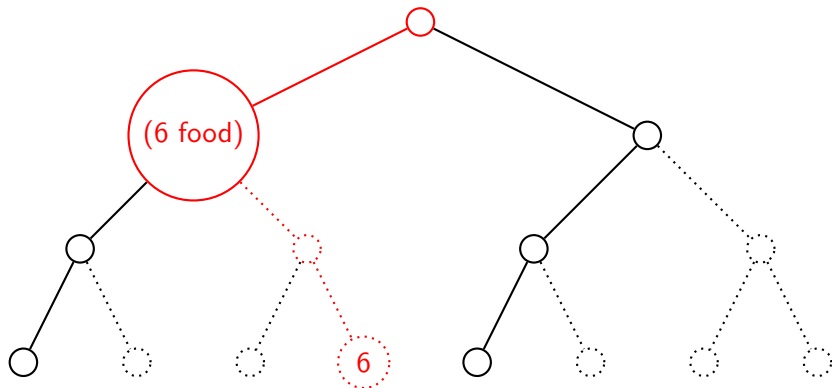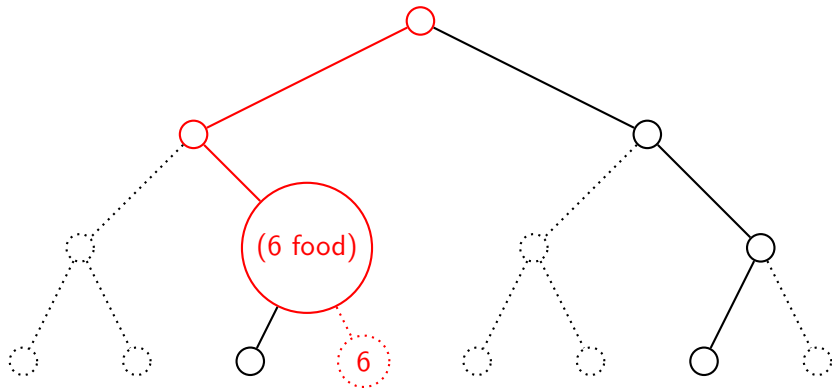
# Insertion – Uniform Scheduling
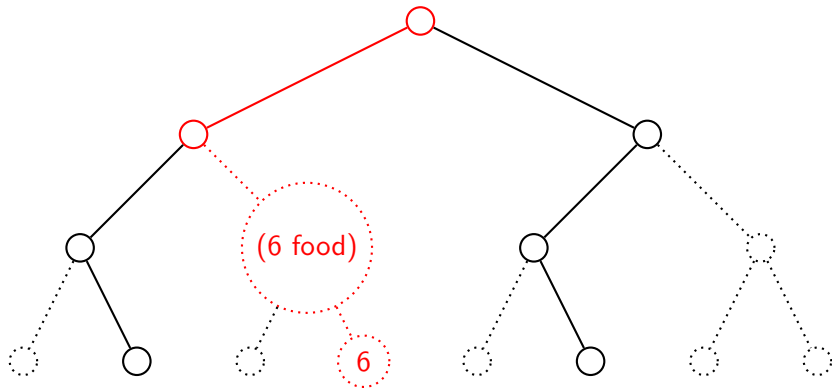


Figure 1: Scheduled subtrees for $N = 8$ and $n = 2$.
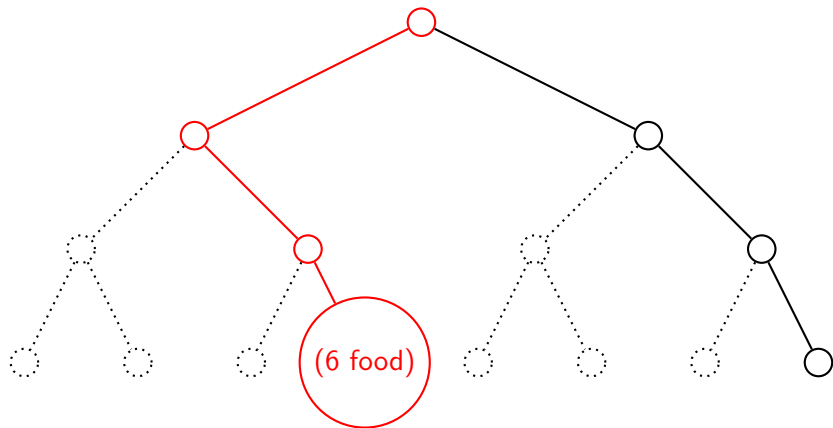
# Insert – Compaction 1

# Insert – Compaction 2

# Insert – Compaction 3



(6 food)

6

*Can the compaction prevent tree overflow?*

# Conclusion – PLOC is *injection-secure*

- $\mathcal{L}(Search, label) = \{target\text{-}branch\}$
- $\mathcal{L}(Insert, label, value) = \bot$

# Conclusion – PLOC is *efficient*

With a *simple* implementation:

| $n \setminus B$ | $2^{10}$ | | $2^{16}$ | | $2^{20}$ | |
|---|---|---|---|---|---|---|
| 16 | 2.1msec | 1.8msec | 26msec | 3.1msec | 0.17sec | 3.8msec |
| 64 | 2.1msec | 5.4msec | 25msec | 11msec | 0.12sec | 13msec |
| 256 | 2.0msec | 18msec | 25msec | 34msec | 0.12sec | 45msec |

Table 1: (*Search Insert*) performances in function of $n$ and $B$ for $V = \sqrt{B}$.

Future works

# Can we improve the performance?

- Search performance is in $O(V)$:
  - can we store more than one datum per target branch? $\Rightarrow O(\frac{V}{m})$

- Search bandwidth is in $O(c \lg B)$:
  - can we reduce the depth by $\lg c$? $\Rightarrow O(\lg B)$

# What about concurrency?

Reliance on an synchronized mutable state due to:

- ▶ MM semantics (order)
  - ▶ implement the index directly?
  - ▶ relax progress property?

- ▶ Uniform scheduling (next scheduled branches)
  - ▶ can compaction work with a random scheduling?
  - ▶ relax progress property?

# What about concurrency?

Reliance on an synchronized mutable state due to:

- MM semantics (order) $O(L) \Rightarrow$ bad
  - ~~implement the index directly?~~
  - relax progress property + relax target selection

- Uniform scheduling (next scheduled branches) $O(1)$
  - ~~can compaction work with a random scheduling?~~
  - relax progress property?

# What about the data-related leakage?

- ▶ Store data directly inside the SSE?
  - ▶ what is the impact on performance?

- ▶ Use an independent scheme with no leakage?
  - ▶ with what performances?

  - ▶ can it be compatible with concurrent queries?

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    **OK** $O(\log N)$
  - ▶ can it be compatible with concurrent queries?
    **NO** requires a lock

Thanks!