Pg. 9

Thursday, June 10, 2021 1:36 PM

25 a) $\varphi\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}x_1-x_2\\-x_2+3x_3\\2x_1\end{pmatrix} = \begin{pmatrix}x_1\\0\\2x_1\end{pmatrix} + \begin{pmatrix}-x_2\\-x_2\\0\end{pmatrix} + \begin{pmatrix}0\\3x_3\\0\end{pmatrix} = \begin{pmatrix}1\\0\\2\end{pmatrix}\cdot x_1 + \begin{pmatrix}-1\\-1\\0\end{pmatrix}\cdot x_2 + \begin{pmatrix}0\\3\\0\end{pmatrix}\cdot x_3$

$$= \underbrace{\begin{pmatrix}1&-1&0\\0&-1&3\\2&0&0\end{pmatrix}}_{M_B(\varphi)} \cdot \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

$$\psi\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}2x_1+2x_3\\2x_1+x_2-x_3\\2x_1+x_2\\2x_1+3x_3\end{pmatrix} = \underbrace{\begin{pmatrix}2&0&2\\2&1&-1\\2&1&0\\2&0&3\end{pmatrix}}_{M_C^B(\psi)} \cdot \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

$$M_C^B(\psi\circ\varphi) = M_C^B(\psi)\cdot M_B^B(\varphi) = M_C^B(\psi)\cdot M_B(\varphi) = \ldots = \begin{pmatrix}6&-2&0\\0&-3&3\\2&-3&3\\8&-2&0\end{pmatrix}$$

b) B Standardbasis $\Rightarrow T_B^{B'} = \begin{pmatrix}1&1&1\\0&1&1\\0&0&1\end{pmatrix}$

$$T_{B'}^B = (T_B^{B'})^{-1} = \ldots = \begin{pmatrix}1&-1&0\\0&1&-1\\0&0&1\end{pmatrix}$$

$$T_C^{C'} = \begin{pmatrix}1&1&1&1\\0&1&1&1\\0&0&1&1\\0&0&0&1\end{pmatrix} \Rightarrow T_{C'}^C = (T_C^{C'})^{-1} = \begin{pmatrix}1&-1&0&0\\0&1&-1&0\\0&0&1&-1\\0&0&0&1\end{pmatrix}$$

c) $M_{B'}(\varphi) = M_{B'}^{B'}(\varphi) = T_{B'}^B \cdot M_B^B(\varphi)\cdot T_B^{B'} \overset{\text{(ausrechnen)}}{=} \begin{pmatrix}1&1&-2\\-2&-3&0\\2&2&2\end{pmatrix}$

$$M_{C'}^{B'}(\psi) = T_{C'}^C \cdot M_C^B(\psi)\cdot T_B^{B'} = \begin{pmatrix}0&-1&2\\0&0&-1\\0&1&-2\\2&2&5\end{pmatrix}$$

26 a) E ist Standardbasis von $\mathbb{R}[x]_{\leq 2}$. T_E^B einfach ablesen.

$B = \{1+2x,\ 3+7x,\ 2+x+x^2\} \Rightarrow T_E^B = \begin{pmatrix}1&3&2\\2&7&1\\0&0&1\end{pmatrix}$

$T_B^E = (T_E^B)^{-1}$. Algorithmus für Inverse:

$$\left(\begin{array}{ccc|ccc}1&3&2&1&0&0\\2&7&1&0&1&0\\0&0&1&0&0&1\end{array}\right) (-2) \rightsquigarrow \left(\begin{array}{ccc|ccc}1&3&2&1&0&0\\0&1&-3&-2&1&0\\0&0&1&0&0&1\end{array}\right) (+3)\ (-2) \rightsquigarrow \left(\begin{array}{ccc|ccc}1&3&0&1&0&-2\\0&1&0&-2&1&3\\0&0&1&0&0&1\end{array}\right) (-3) \rightsquigarrow \left(\begin{array}{ccc|ccc}1&0&0&7&-3&-11\\0&1&0&-2&1&3\\0&0&1&0&0&1\end{array}\right) \Rightarrow T_B^E = \begin{pmatrix}7&-3&-11\\-2&1&3\\0&0&1\end{pmatrix}$$

b) Ansatz: Gesucht sind $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}$ mit:

$$f = a_0 + a_1x + a_2x^2 = \lambda_1(2x+1) + \lambda_2(7x+3) + \lambda_3(x^2+x+2)$$
$$= \ldots =$$
$$= \underbrace{(\lambda_1+3\lambda_2+2\lambda_3)}_{a_0}\cdot 1 + \underbrace{(2\lambda_1+7\lambda_2+\lambda_3)}_{a_1}\cdot x + \underbrace{\lambda_3}_{a_2}\cdot x^2$$

In Form eines LGS:

$$\underbrace{\begin{pmatrix}1&3&2\\2&7&1\\0&0&1\end{pmatrix}}_{T_E^B}\cdot\begin{pmatrix}\lambda_1\\\lambda_2\\\lambda_3\end{pmatrix} = \begin{pmatrix}a_0\\a_1\\a_2\end{pmatrix} \Rightarrow T_E^B\cdot\begin{pmatrix}\lambda_1\\\lambda_2\\\lambda_3\end{pmatrix} = \begin{pmatrix}a_0\\a_1\\a_2\end{pmatrix} \Rightarrow \begin{pmatrix}\lambda_1\\\lambda_2\\\lambda_3\end{pmatrix} = T_B^E\cdot\begin{pmatrix}a_0\\a_1\\a_2\end{pmatrix} = \begin{pmatrix}7&-3&-11\\-2&1&3\\0&0&1\end{pmatrix}\cdot\begin{pmatrix}a_0\\a_1\\a_2\end{pmatrix}$$

$$\Rightarrow \begin{cases}\lambda_1 = 7a_0 - 3a_1 - 11a_2\\ \lambda_2 = -2a_0 + a_1 + 3a_2\\ \lambda_3 = a_2\end{cases}$$