Pg. 10

Thursday, June 17, 2021 1:44 PM

(27) a) $\|u-v\| = \left\|\begin{pmatrix}0\\-2\\-4\end{pmatrix}\right\| = \sqrt{0^2+(-2)^2+(-4)^2} = \sqrt{20} = \sqrt{4\cdot 5} = 2\sqrt{5}$

b) $\cos(\alpha) = \frac{\langle u, w\rangle}{\|u\|\cdot\|w\|} = \frac{1}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{3}$

c)
$$\begin{aligned}
U^\perp &= \{x\in\mathbb{R}^3 \mid \langle x, y\rangle = 0, \forall y = \lambda\cdot u + \mu\cdot v \in \mathbb{R}^3\}\\
&= \{x\in\mathbb{R}^3 \mid \langle x, \lambda\cdot u + \mu\cdot v\rangle = 0, \forall \lambda,\mu\in\mathbb{R}\}\\
&= \{x\in\mathbb{R}^3 \mid \lambda\cdot\langle x, u\rangle + \mu\cdot\langle x, v\rangle = 0, \forall \lambda,\mu\in\mathbb{R}\}\\
&= \{x\in\mathbb{R}^3 \mid \langle x, u\rangle = \langle x, v\rangle = 0\}\\
&= \{x\in\mathbb{R}^3 \mid \underbrace{\left\langle \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}, \begin{pmatrix}1\\0\\-1\end{pmatrix}\right\rangle = \left\langle \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}, \begin{pmatrix}1\\2\\3\end{pmatrix}\right\rangle = 0}\}
\end{aligned}$$

$x_1\cdot 1 + x_2\cdot 0 + x_3\cdot(-1) = 0$

$x_1\cdot 1 + x_2\cdot 2 + x_3\cdot 3 = 0$

$\Leftrightarrow \begin{pmatrix}1&0&-1\\1&2&3\end{pmatrix}\cdot x = 0$ LGS!

$\begin{pmatrix}1&0&-1\\1&2&3\end{pmatrix}\,\big]\cdot(-1) \rightsquigarrow \begin{pmatrix}1&0&-1\\0&2&4\end{pmatrix} \Rightarrow L = \left\{\begin{pmatrix}x_3\\-2x_3\\x_3\end{pmatrix} \mid x_3\in\mathbb{R}\right\}$

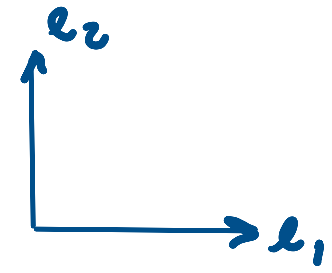
$= \left\langle \begin{pmatrix}1\\-2\\1\end{pmatrix}\right\rangle$

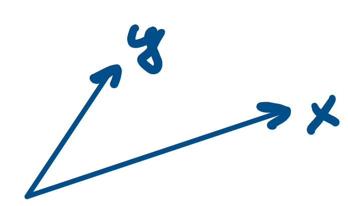
(28) b) $\det(B) = 1\cdot 5\cdot 1 + 2\cdot 7\cdot 0 + 4\cdot 3\cdot(-1) - 0\cdot 5\cdot 4 - (-1)\cdot 7\cdot 1 - 1\cdot 3\cdot 2 = -6.$

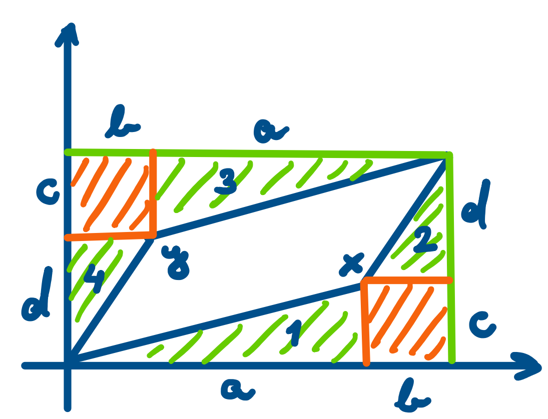
$$\begin{array}{ccc|cc}
1 & 2 & 4 & 1 & 2\\
3 & 5 & 7 & 3 & 5\\
0 & -1 & 1 & 0 & -1
\end{array}$$

d) $\det(D) = \det\begin{pmatrix}4&0&6&3\\0&0&4&0\\2&0&-5&-3\\7&1&-3&-2\end{pmatrix} = (-1)^{4+2}\cdot 1\cdot\det\begin{pmatrix}4&6&3\\0&4&0\\2&-5&-3\end{pmatrix} = (-1)^{2+2}\cdot 4\cdot\det\begin{pmatrix}4&3\\2&-3\end{pmatrix} = -72.$

(29) Voraussetzung: (x,y) haben die gleiche Orientierung wie die Standardbasis (e_1, e_2).







$$\begin{aligned}
F &= (a+b)(c+d) - 2\cdot bc - \Delta_1 - \Delta_2 - \Delta_3 - \Delta_4\\
&= (ac+ad+bc+bd) - 2bc - \tfrac{1}{2}ac - \tfrac{1}{2}bd - \tfrac{1}{2}ac - \tfrac{1}{2}bc\\
&= ad - bc = \det(x\,|\,y)
\end{aligned}$$