Tema 1 - Calcul numeric - an I ID

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Due March 28, 2021 6:59 PM

Instructions

Se se rezolve urmatoarele exercitii. Rezolvarile se vor scrie intr-un fisier (se pot include si poze ale rezolvarilor problemelor pe hartie).

1. Sa se determine o solutie a ecuatiei $f(x) = 0$ unde $f(x) = x^3 - x^2 - x + 1$ folosind metoda lui Newton cu valorea initiala $x_0 = -0.2$ si $epsilon = 0.01$.

2. Sa se determine o solutie a ecuatiei $f(x) = 0$ unde $f(x) = 8x^3 + x^2 + 8x - 3$ folosind metoda secantei cu 0.0 si 0.6 valori initiale si $epsilon = 0.01$.

3. Sa se determine o solutie a ecuatiei $f(x) = 0$ unde $f(x) = x^3 - 4x + 2$ folosind metoda bisectiei cu valorile initiale $a = 0, b = 1, epsilon = 0.01$.

4. Se da functia de la exercitiul 3. Daca $x_1 = 1$, cat este x_2 din metoda lui Newton-Raphson?

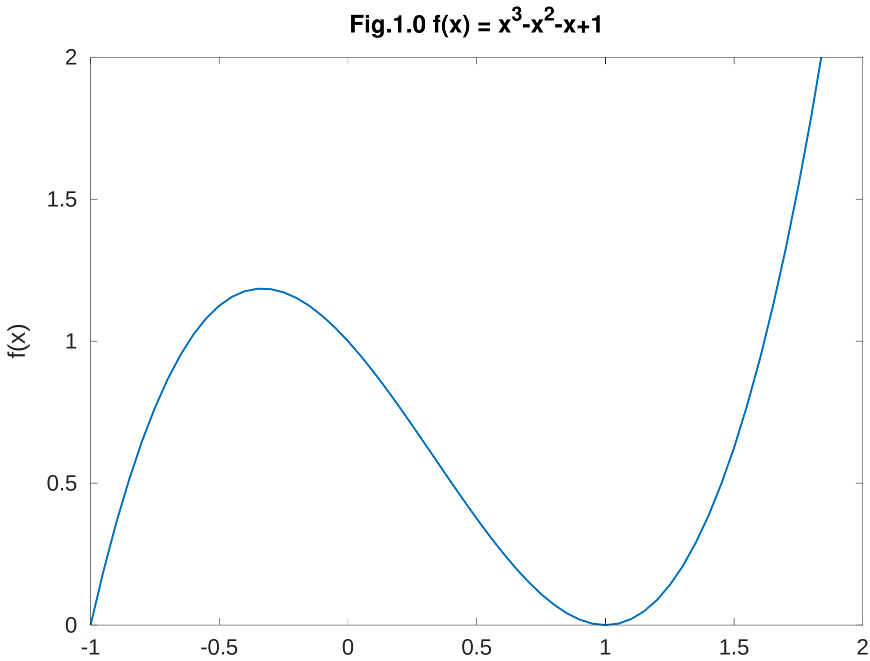
5. Se da functia de la exercitiul 3. Daca $x_0 = 0$ si $x_1 = 1$, cat sunt x_2 si x_3 din metoda secantei?

Rezolvări:

(1) $f(x) = x^3 - x^2 - x + 1, f(x) = 0, x =?$, prin metoda lui Newton-Raphson, având $x_0 = -0.2, \varepsilon = 0.01$.

```
# Cod octave:
f = @(x) x.^3-x.^2-x+1;
df = @(x) 3*x.^2-2*x-1;
epsilon = 0.01;
x_0 = -0.2;
x_1 = x_0 - (f(x_0) / df (x_0));
```

```
while epsilon < abs(f(x_1))
    # Urmatoarea estimare a solutiei
    x_1 = x_0 - (f(x_0) / df (x_0));
    print_graph(f, df, x_0, x_1, iter_count);
    x_0 = x_1;
endwhile
```



Sirul solutiilor aproximate este:
$X = \{2.200000, 1.694737, 1.387033, 1.208029, 1.108694, 1.055712, 1.0557\}$
$x_7 = 1.0557, f(x_7) = 0.00638065, f(x_7) < \varepsilon,$
deci $solutia = x_7 = 1.0557.$

Fig.1.1 x_0 = -0.2, x_1 = 2.2

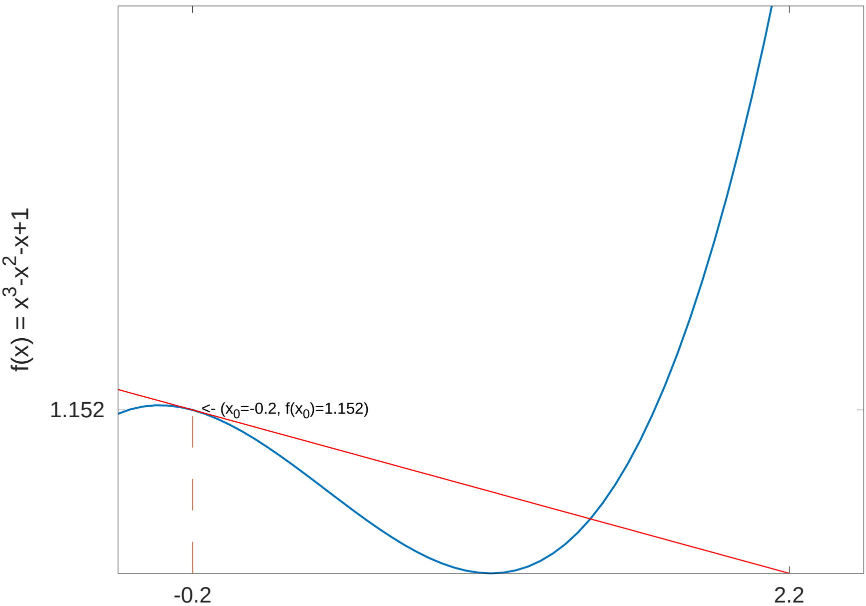


Fig.1.2 x_0 = 2.2, x_1 = 1.6947

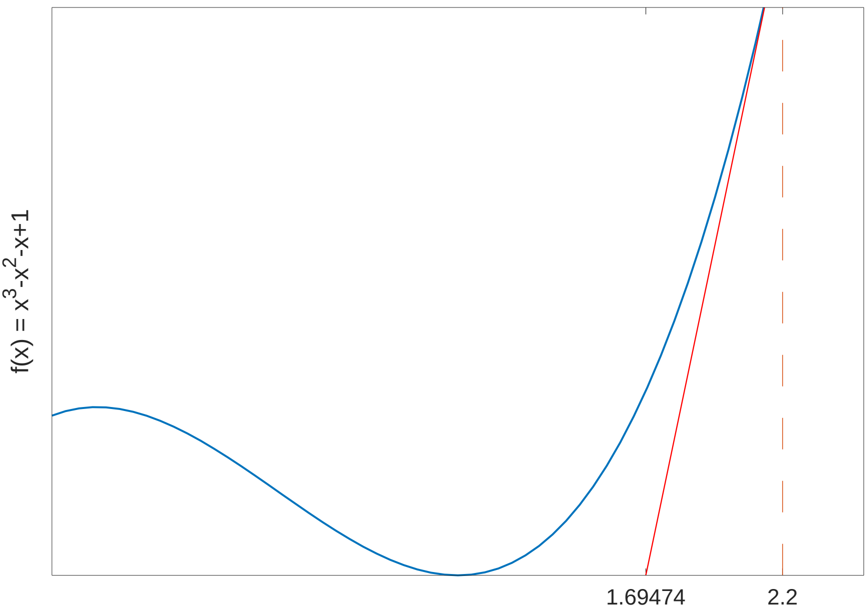


Fig.1.3 x_0 = 1.6947, x_1 = 1.387

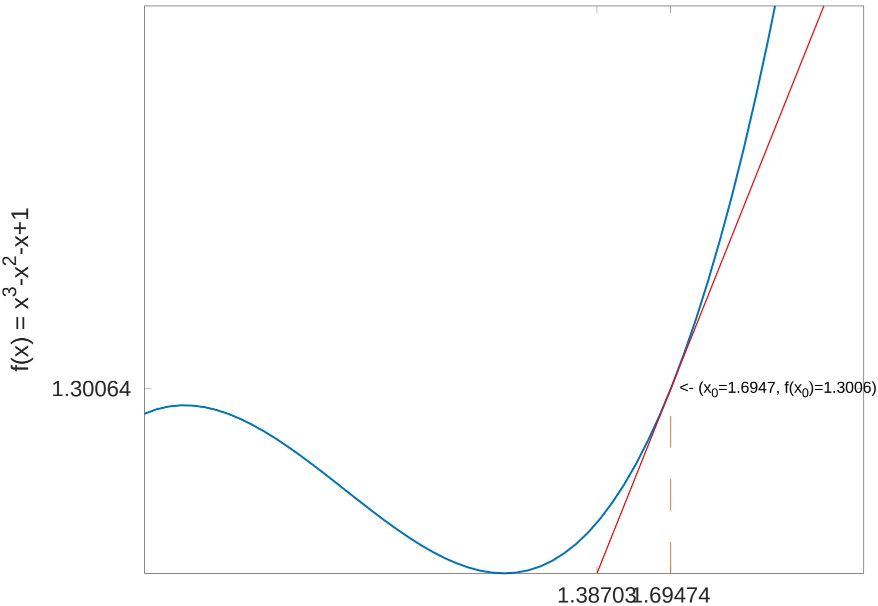


Fig.1.4 x_0 = 1.387, x_1 = 1.208

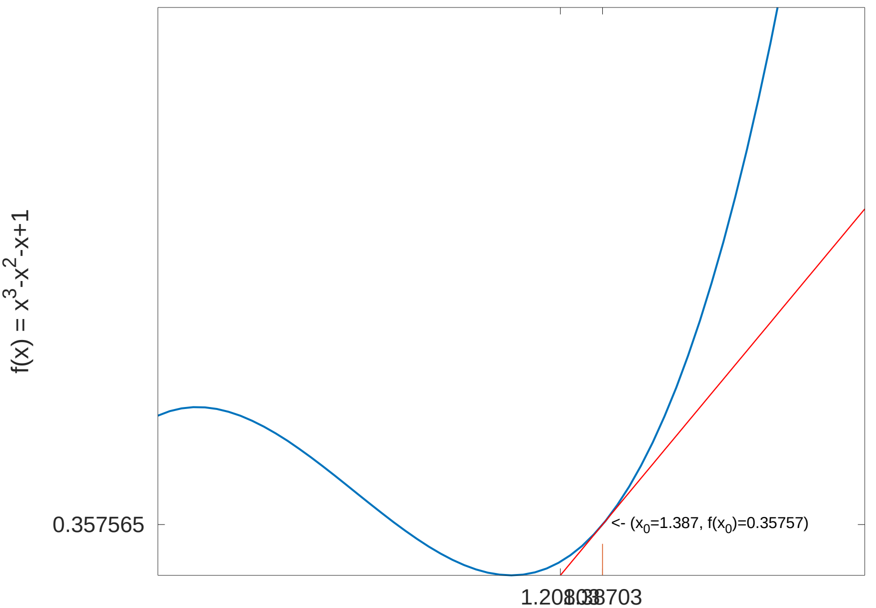


Fig.1.5 x_0 = 1.208, x_1 = 1.1087

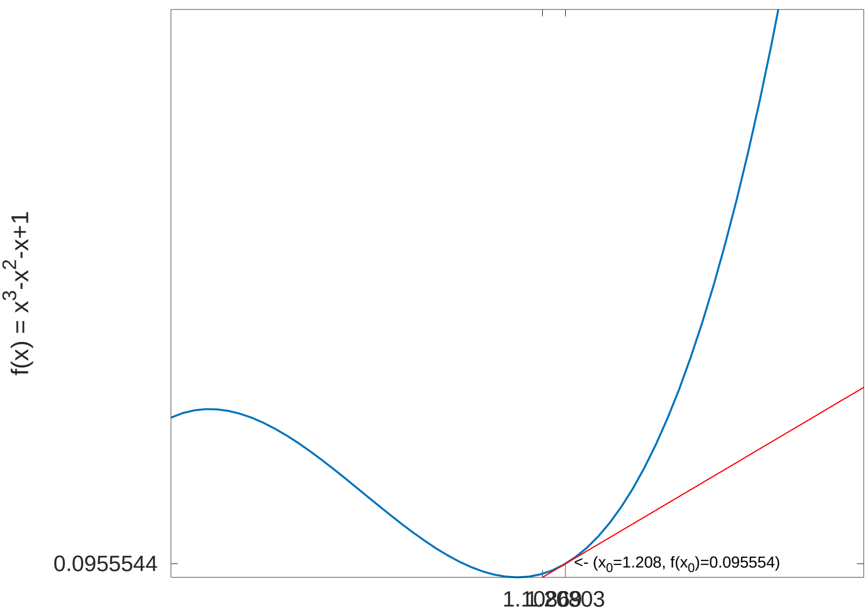


Fig.1.6 x_0 = 1.1087, x_1 = 1.0557

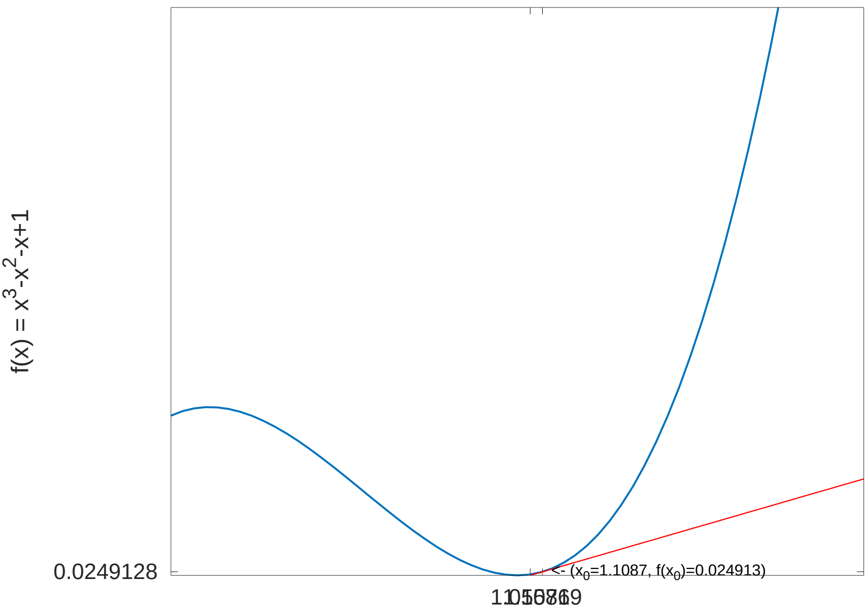
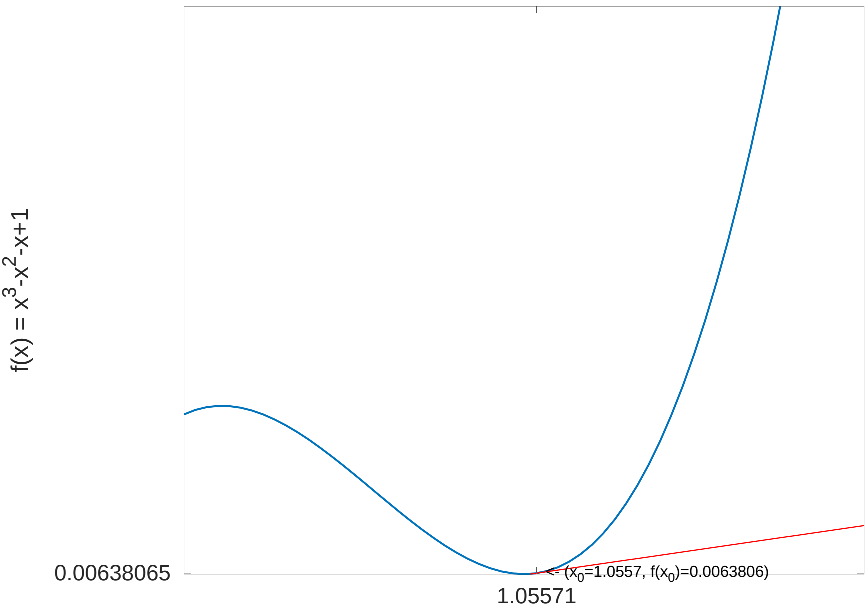


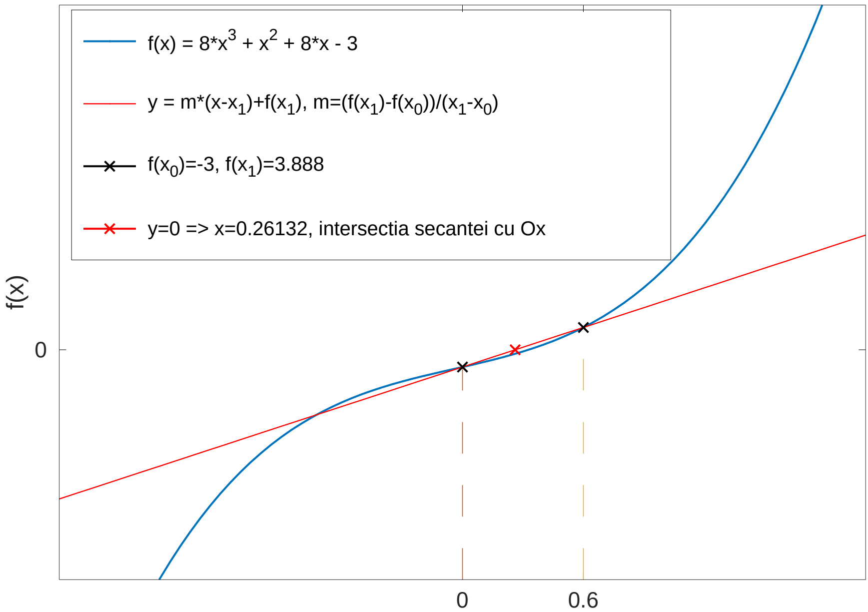
Fig.1.7 $x_0 = 1.0557$, $x_1 = 1.0557$



(2) $f(x) = 8x^3 + x^2 + 8x - 3, f(x) = 0, x =?$, prin metoda secantei, având $0.0, 0.6$ valori initiale si $\varepsilon = 0.01$.

```
# Cod octave:
i = 1;
i_max = 5;
while abs(f(x_0)) > epsilon && i < i_max
    x_1 = x_1 - (f(x_1)*(x_1-x_0))/(f(x_1)-f(x_0));
    print_graph(f, x_0, x_1, i);
    i ++;
endwhile
```

Fig.2.0 x_0 = 0, x_1 = 0.6, f(x_1) = 3.888



i=1	$x_0 = 0.00$	$x_1 = 0.261324$	$f(x_1) = -0.698350$
i=2	$x_0 = 0.00$	$x_2 = 0.340613$	$f(x_2) = 0.157059$
i=3	$x_0 = 0.00$	$x_3 = 0.323668$	$f(x_3) = -0.034631$
i=4	$x_0 = 0.00$	$x_4 = 0.327448$	$f(x_4) = 0.007685$

$f(x_4) = 0.007685 < \varepsilon = 0.01$,
deci $solutia = 0.007685$.

Fig.2.1 x_0 = 0, x_1 = 0.26132, $f(x_1)$ = -0.69835

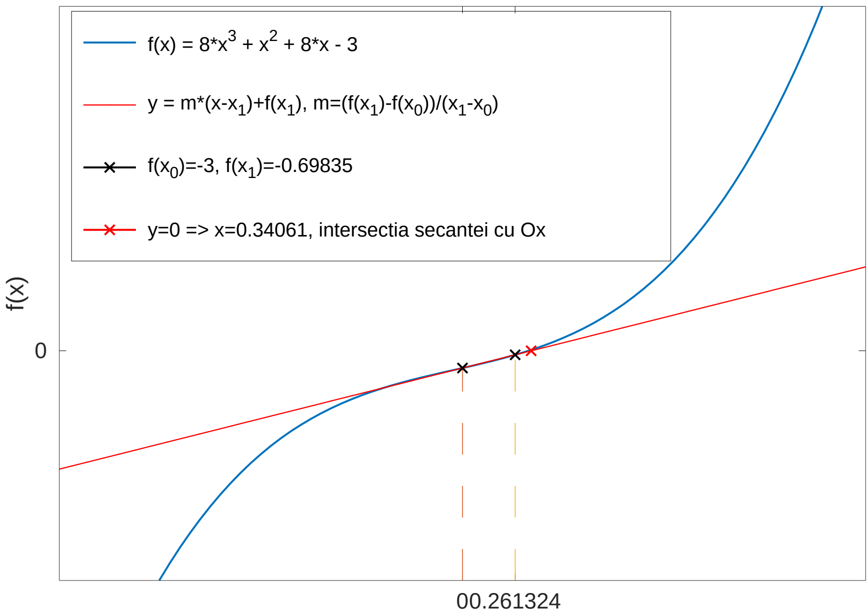


Fig.2.2 x_0 = 0, x_1 = 0.34061, $f(x_1)$ = 0.15706

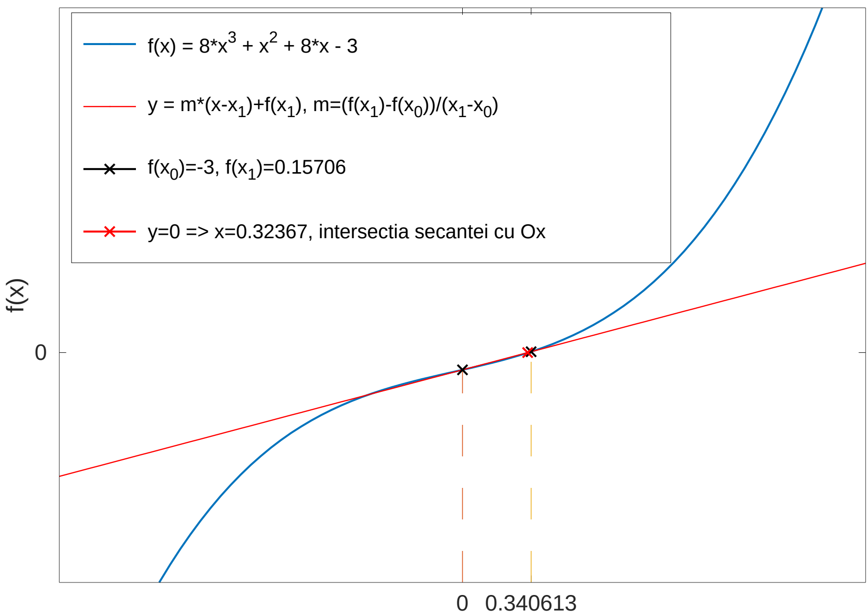


Fig.2.3 x_0 = 0, x_1 = 0.32367, $f(x_1)$ = -0.034631

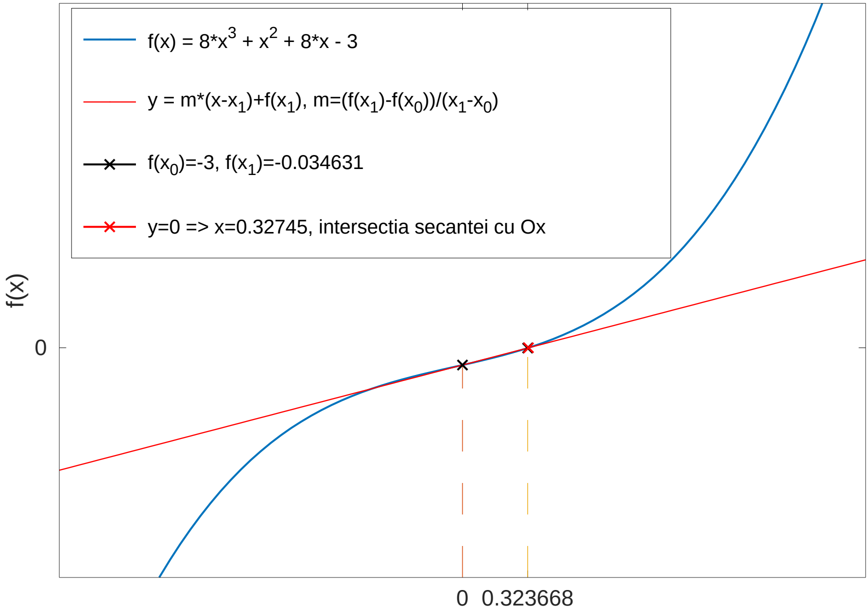
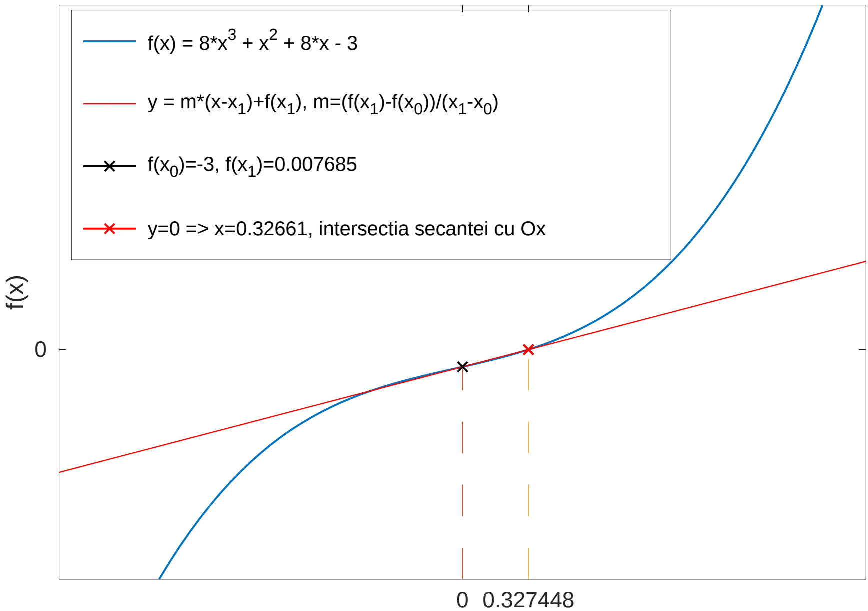


Fig.2.4 x_0 = 0, x_1 = 0.32745, $f(x_1)$ = 0.007685



(3) $f(x) = x^3 - 4x + 2, f(x) = 0, x = ?$, prin metoda bisectiei, având $a = 0, b = 1, \varepsilon = 0.01$.

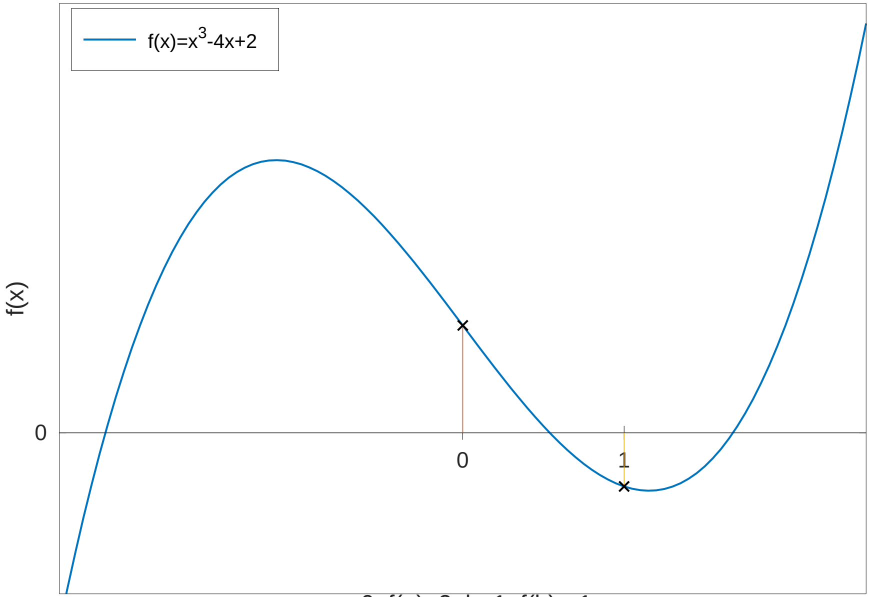
```
# Cod octave
f = @(x) x.^3-4*x+2;
a = 0;
b = 1;
epsilon = 0.01;
x_m = (a+b)/2;
i = 1;
max_i = 10;
while abs(f(x_m)) > epsilon
    if max_i < i
        printf("Solution not found!\n");
        break;
    endif

    printf("i=%d & a=%f & b=%f & x_m=%f & f(x_m)=%f\n",i,a,b,x_m,f(x_m));
    print_graph(f, a, b, i);
    x_m = (a+b)/2;
    if f(x_m)*a < 0
        b = x_m;
    else
        a = x_m;
    endif
    i++;
endwhile
```

$f(a) * f(b) < 0 => Solutia \in [a, b]$

i=1	a=0.000000	b=1.000000	$x_m = 0.500000$	$f(x_m) = 0.125000$
i=2	a=0.500000	b=1.000000	$x_m = 0.500000$	$f(x_m) = 0.125000$
i=3	a=0.500000	b=0.750000	$x_m = 0.750000$	$f(x_m) = -0.578125$
i=4	a=0.500000	b=0.625000	$x_m = 0.625000$	$f(x_m) = -0.255859$
i=5	a=0.500000	b=0.562500	$x_m = 0.562500$	$f(x_m) = -0.072021$
i=6	a=0.531250	b=0.562500	$x_m = 0.531250$	$f(x_m) = 0.024933$
i=7	a=0.531250	b=0.546875	$x_m = 0.546875$	$f(x_m) = -0.023945$
i=8	a=0.539062	b=0.546875	$x_m = 0.539062$	$f(x_m) = 0.000395$

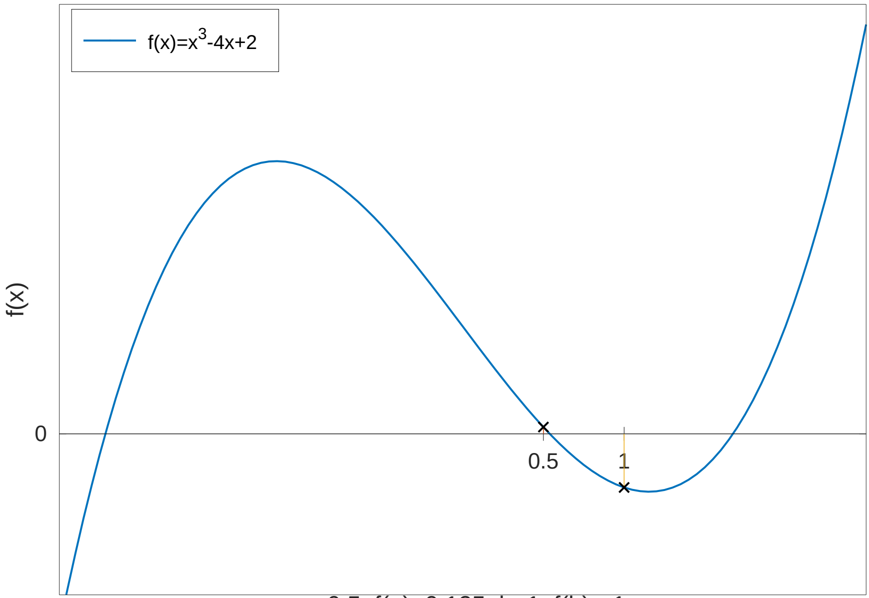
$Solutia = x_m = 0.539062$

Fig.3.1 $f(x)=x^3-4*x+2$



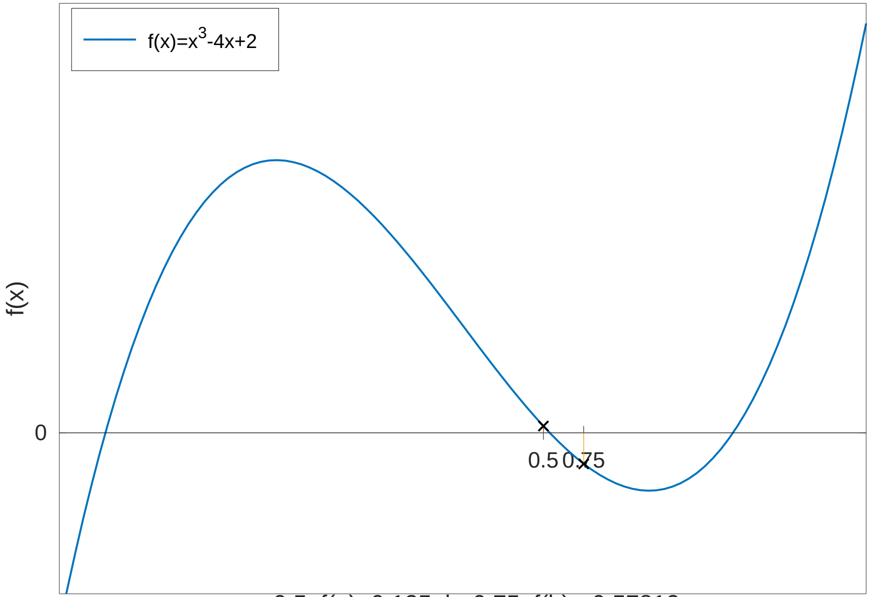
a=0, f(a)=2, b=1, f(b)=-1

Fig.3.2 $f(x)=x^3-4*x+2$



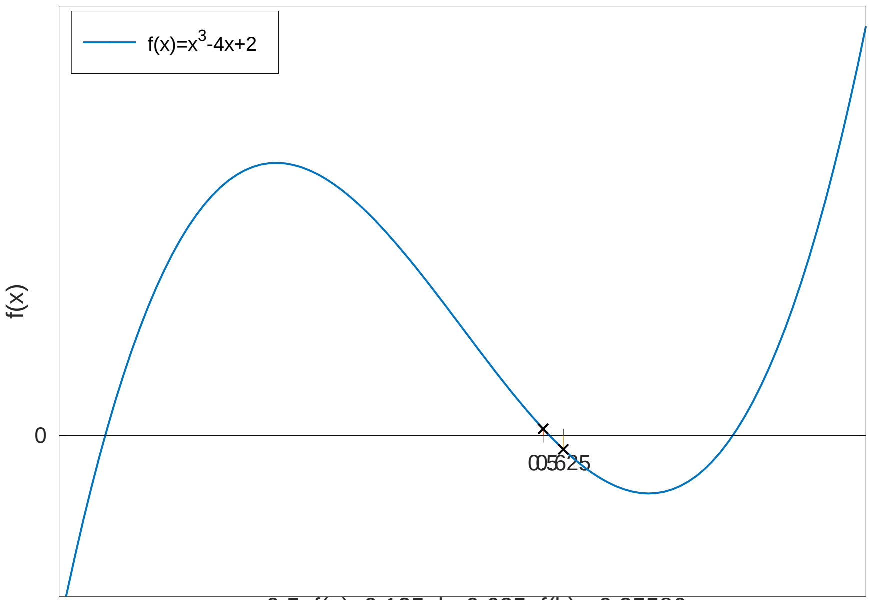
a=0.5, f(a)=0.125, b=1, f(b)=-1

Fig.3.3 $f(x)=x^3-4*x+2$



a=0.5, f(a)=0.125, b=0.75, f(b)=-0.57812

Fig.3.4 $f(x)=x^3-4*x+2$



a=0.5, f(a)=0.125, b=0.625, f(b)=-0.25586

Fig.3.5 $f(x)=x^3-4*x+2$

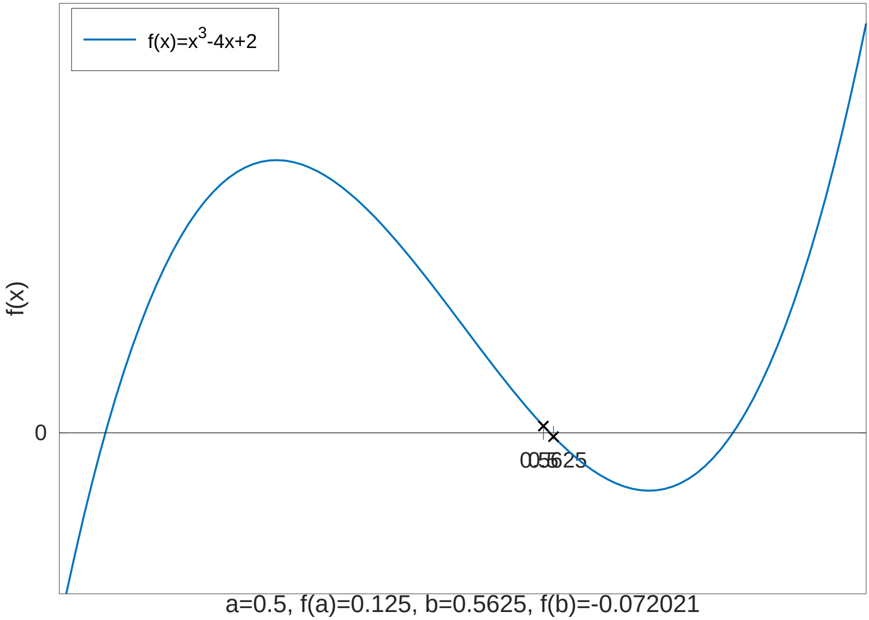


Fig.3.6 $f(x)=x^3-4*x+2$

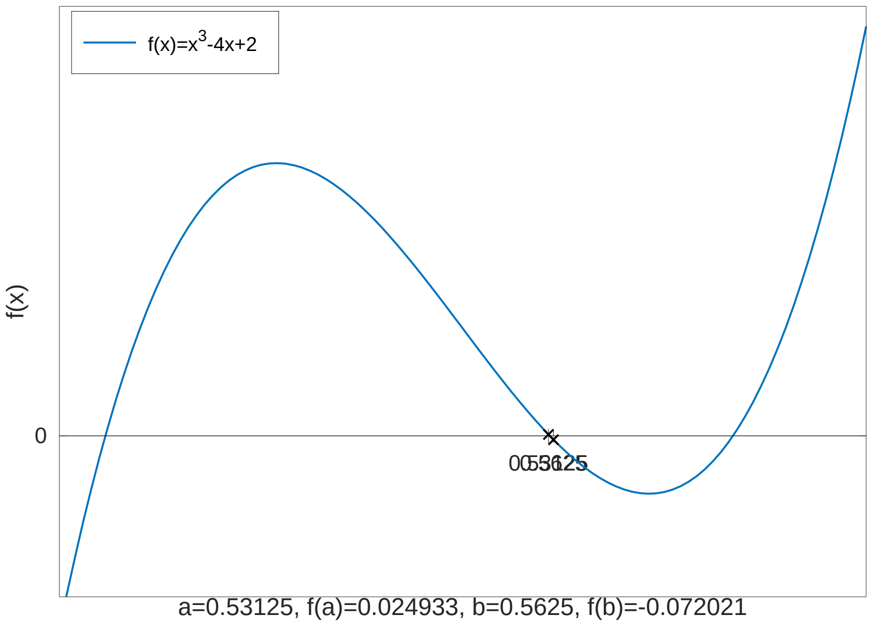


Fig.3.7 $f(x)=x^3-4*x+2$

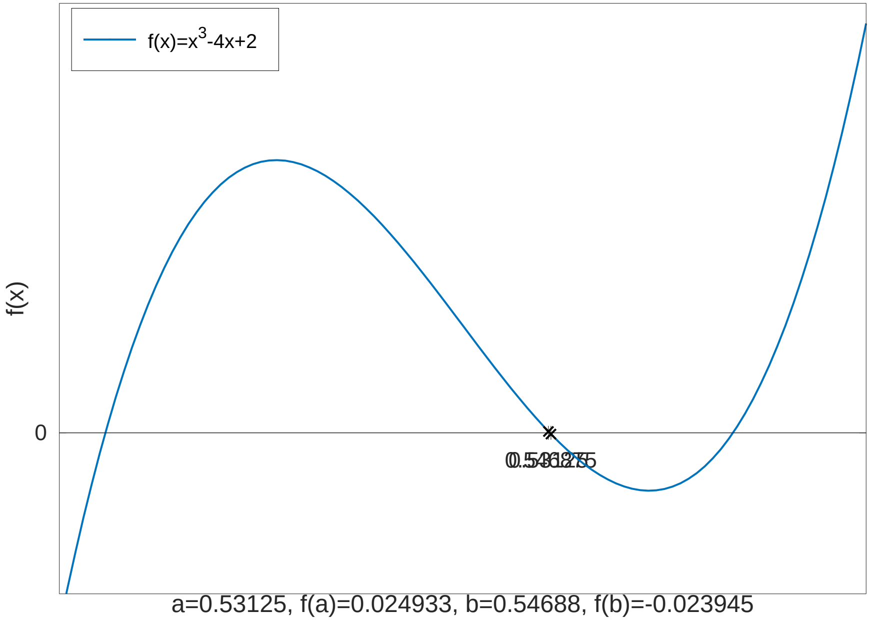
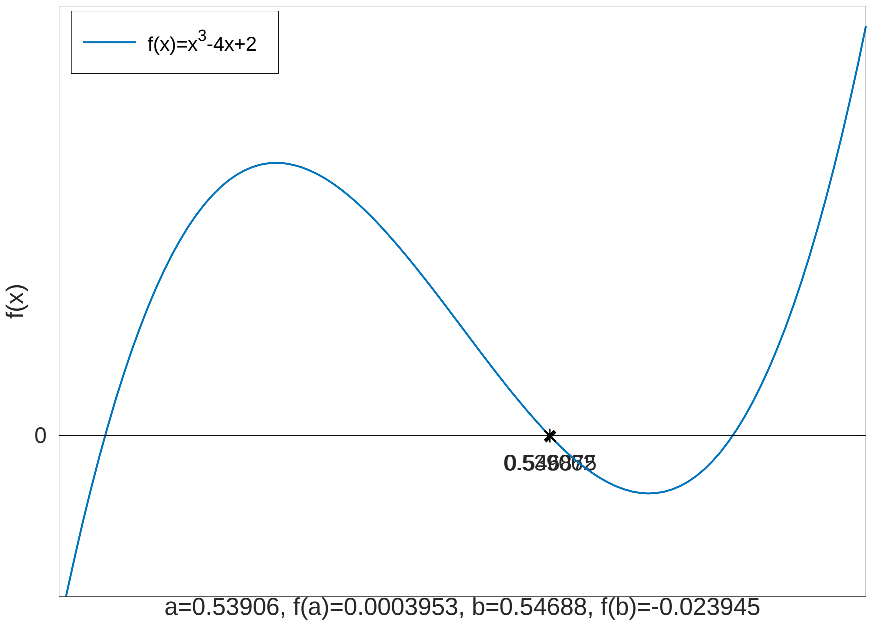


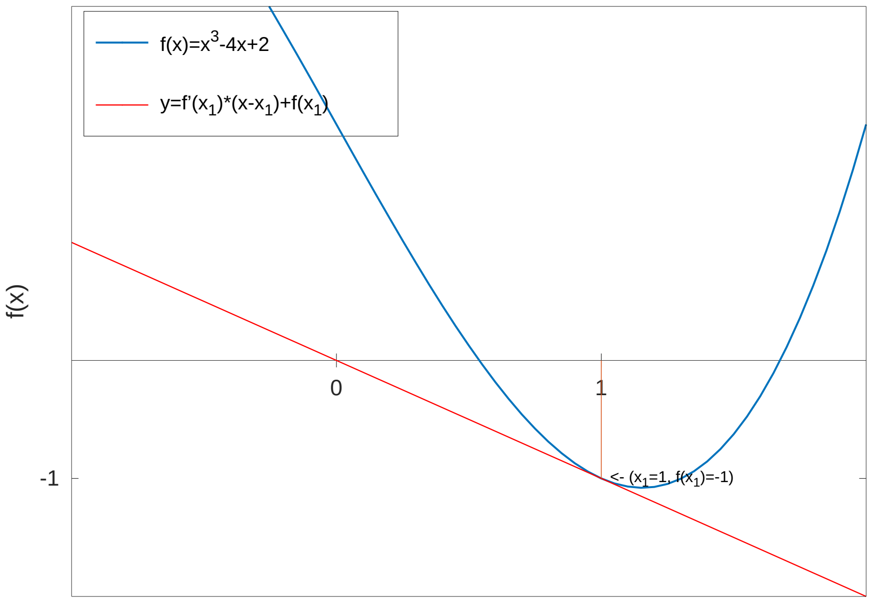
Fig.3.8 $f(x)=x^3-4*x+2$



(4) $f(x) = x^3 - 4x + 2, x_1 = 1, x_2 =?$ prin metoda lui Newton-Raphson.

Punctul $(x_2, 0)$ reprezintă intersecția cu Ox a tangentei prin $(x_1, f(x_1))$. Panta acestei tangente este dată de derivata funcției f pentru $x_1 = 1$. $m = f'(x_1)$. De asemenea aceasta pantă se poate calcula din ecuația tangentei, pentru $m = \frac{f(x_1)-y}{x_1-x_2}, y = 0$.
Din $m = f'(x_1), m = \frac{f(x1)}{x_1-x_2} => f'(x_1) = \frac{f(x_1)}{x_1-x_2}$.
Calculând pentru x_2, avem $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0$.

Fig.4.0 $x_1 = 1, x_2 = 0$




```
# Cod octave
f = @(x) x.^3 - 4*x + 2;
df = @(x) 3*x^2 - 4;
x_1 = 1;
x_2 = x_1 - f(x_1)/df(x_1);
```

(5) $f(x) = x^3 - 4x + 2$ Daca $x_0 = 0$ si $x_1 = 1$, cat sunt x_2 si x_3 din metoda secantei?

Definim relația y în funcție de x ca fiind secanta care trece prin punctele $(x_0, f(x_0)), (x_1, f(x_1))$.

Panta calculată cu aceste puncte este: $m = \frac{f(x_1)-f(x_0)}{x_1-x_0}$.

Panta calculată cu punctul $(x_1, f(x_1))$ și alt punct al secantei este: $m = \frac{f(x_1)-y}{x_1-x}$.

Din $m = \frac{f(x_1)-f(x_0)}{x_1-x_0}, m = \frac{f(x_1)-y}{x_1-x} => \frac{f(x_1)-f(x_0)}{x_1-x_0} = \frac{f(x_1)-y}{x_1-x}$.

Izolându-l pe x obținem:

$x_1 - x = \frac{(f(x_1)-y)(x_1-x_0)}{f(x_1)-f(x_0)} <=> x = x_1 - \frac{(f(x_1)-y)(x_1-x_0)}{f(x_1)-f(x_0)}$.

Definim punctul $(x_2, 0)$ ca fiind intersecția secantei cu Ox, deci $y = 0$.

$y = 0, x = x_2 => x_2 = x_1 - \frac{f(x_1)(x_1-x_0)}{f(x1)-f(x_0)}$.

```
# Cod octave
f = @(x) x^3 - 4*x + 2;
x_0 = 0;
x_1 = 1;
x_2 = x_1 - (f(x_1)*(x_1-x_0))/(f(x_1)-f(x_0));
x_3 = x_2 - (f(x_2)*(x_2-x_1))/(f(x_2-f(x_1)));
printf("$x_2=%.3f, x_3=%.3f$\n", x_2, x_3);
```

Soluțiile calculate sunt: $x_2 = 0.667, x_3 = 4.000$.

Fig.5.0 $x_0 = 0$, $x_1 = 1$, $f(x_1) = -1$