

Tema 1 - Calcul numeric - an I ID

Dinu Eugen Cosmin

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Instructions

Se se rezolve urmatoarele exercitii. Rezolvarile se vor scrie intr-un fisier (se pot include si poze ale rezolvarilor problemelor pe hartie).

1. Sa se determine o solutie a ecuatiei $f(x) = 0$ unde $f(x) = x^3 - x^2 - x + 1$ folosind metoda lui Newton cu valoarea initiala $x_0 = -0.2$ si $\epsilon = 0.01$.

2. Sa se determine o solutie a ecuatiei $f(x) = 0$ unde $f(x) = 8x^3 + x^2 + 8x - 3$ folosind metoda secantei cu 0.0 si 0.6 valori initiale si $\epsilon = 0.01$.

3. Sa se determine o solutie a ecuatiei $f(x) = 0$ unde $f(x) = x^3 - 4x + 2$ folosind metoda bisectiei cu valorile initiale $a = 0, b = 1, \epsilon = 0.01$.

4. Se da functia de la exercitiul 3. Daca $x_1 = 1$, cat este x_2 din metoda lui Newton-Raphson?

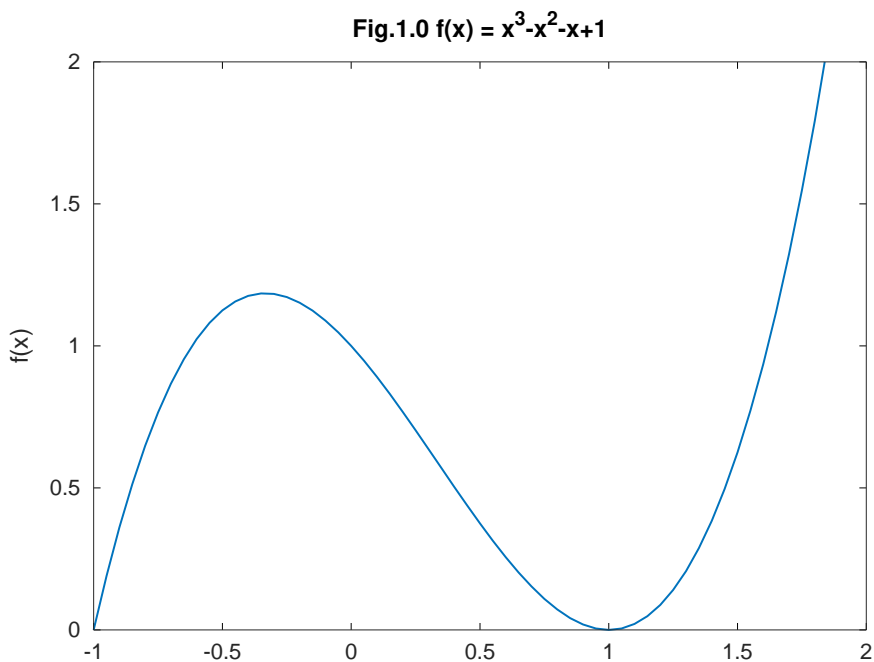
5. Se da functia de la exercitiul 3. Daca $x_0 = 0$ si $x_1 = 1$, cat sunt x_2 si x_3 din metoda secantei?

Rezolvări:

(1) $f(x) = x^3 - x^2 - x + 1, f(x) = 0, x = ?$, prin metoda lui Newton-Raphson, având $x_0 = -0.2, \varepsilon = 0.01$.

```
# Cod octave:
f = @(x) x.^3-x.^2-x+1;
df = @(x) 3*x.^2-2*x-1;
epsilon = 0.01;
x_0 = -0.2;
x_1 = x_0 - (f(x_0) / df (x_0));

while epsilon < abs(f(x_1))
    # Urmatoarea estimare a solutiei
    x_1 = x_0 - (f(x_0) / df (x_0));
    print_graph(f, df, x_0, x_1, iter_count);
    x_0 = x_1;
endwhile
```



Sirul solutiilor approximate este:

$X = \{2.200000, 1.694737, 1.387033, 1.208029, 1.108694, 1.055712, 1.0557\}$

$x_7 = 1.0557, f(x_7) = 0.00638065, f(x_7) < \varepsilon,$

deci *solutia* = $x_7 = 1.0557$.

Fig.1.1 $x_0 = -0.2, x_1 = 2.2$

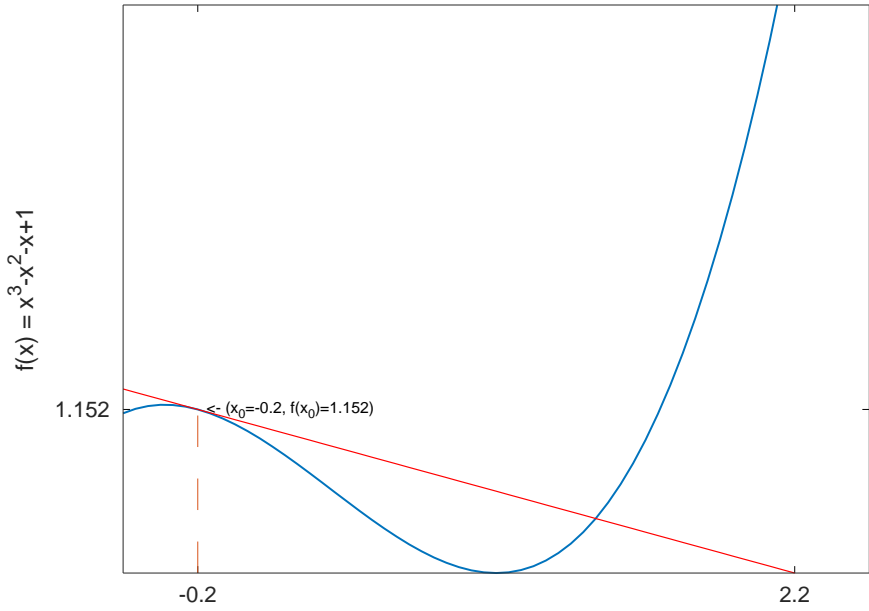


Fig.1.2 $x_0 = 2.2, x_1 = 1.6947$

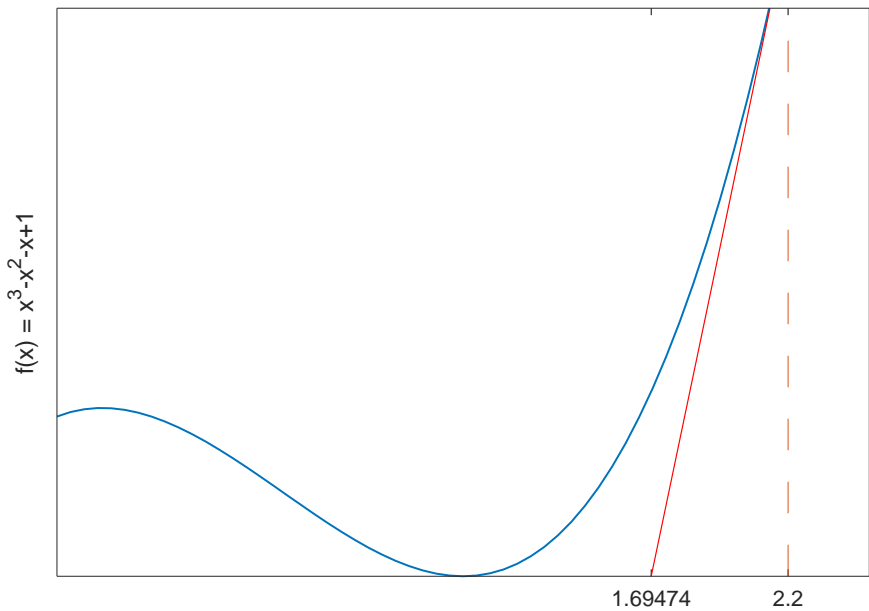


Fig.1.3 $x_0 = 1.6947$, $x_1 = 1.387$

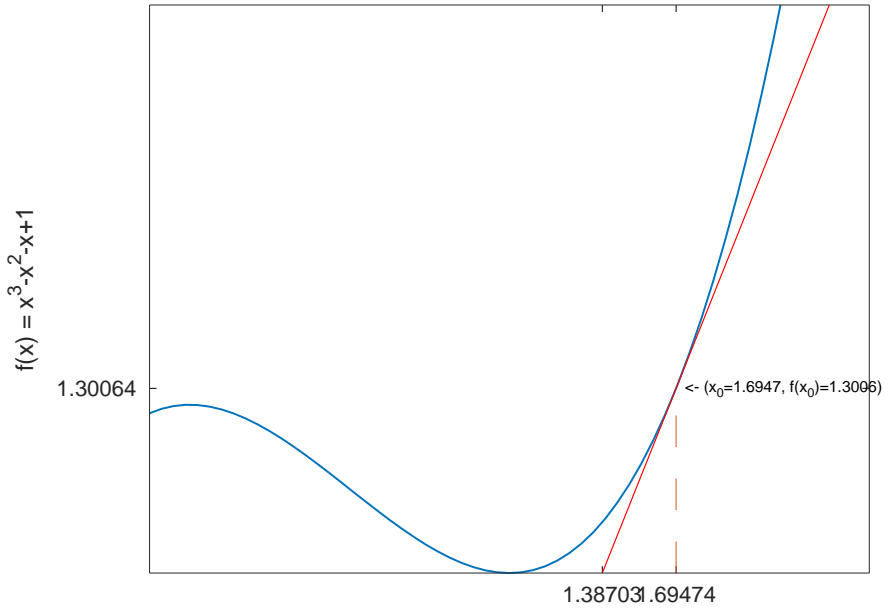


Fig.1.4 $x_0 = 1.387$, $x_1 = 1.208$

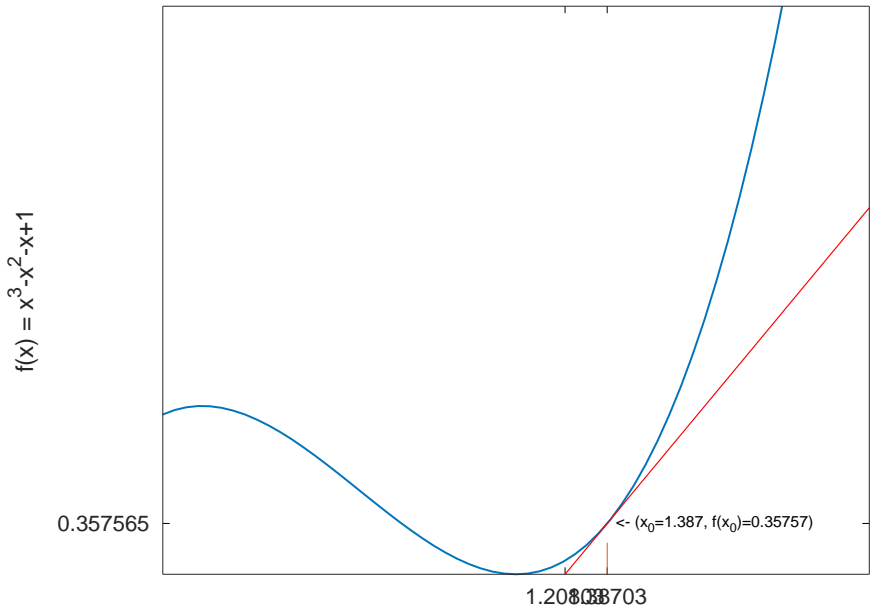


Fig.1.5 $x_0 = 1.208$, $x_1 = 1.1087$

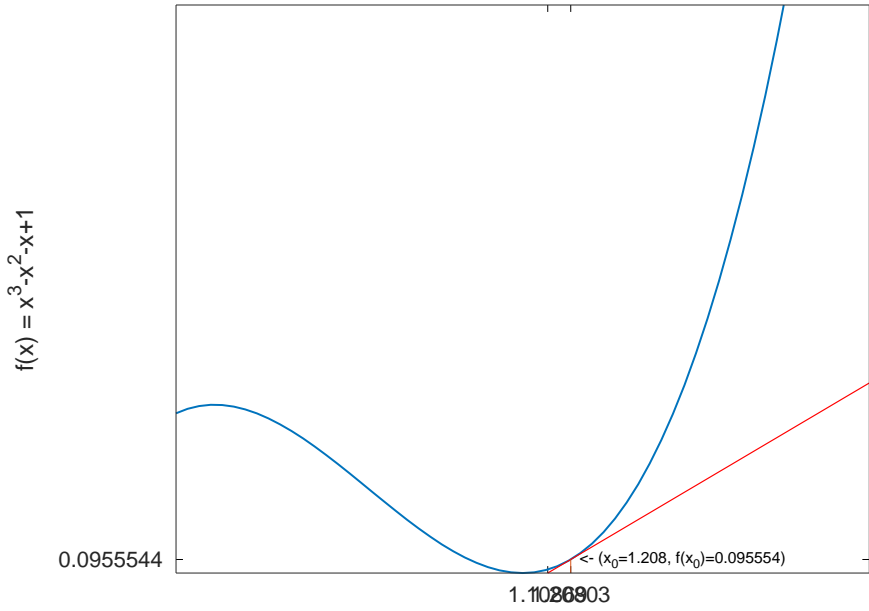


Fig.1.6 $x_0 = 1.1087$, $x_1 = 1.0557$

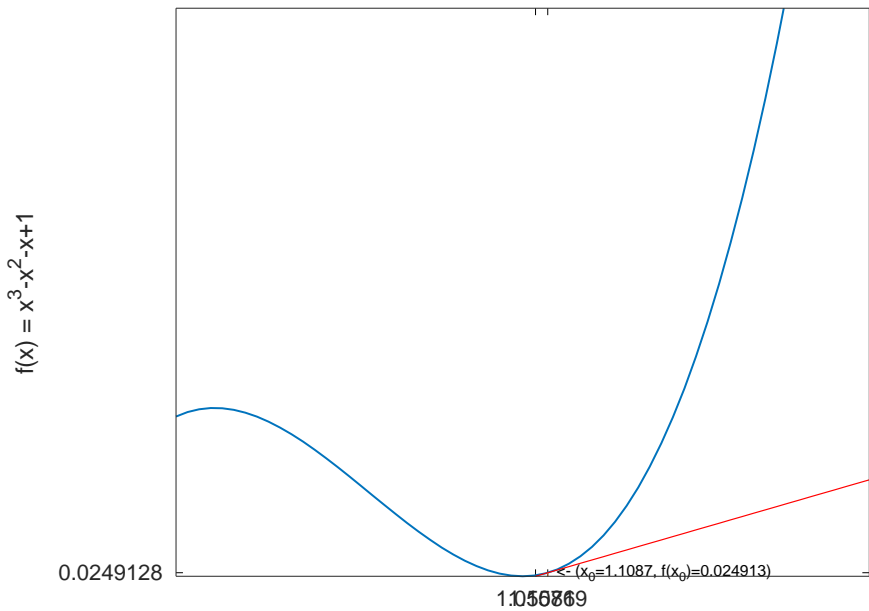
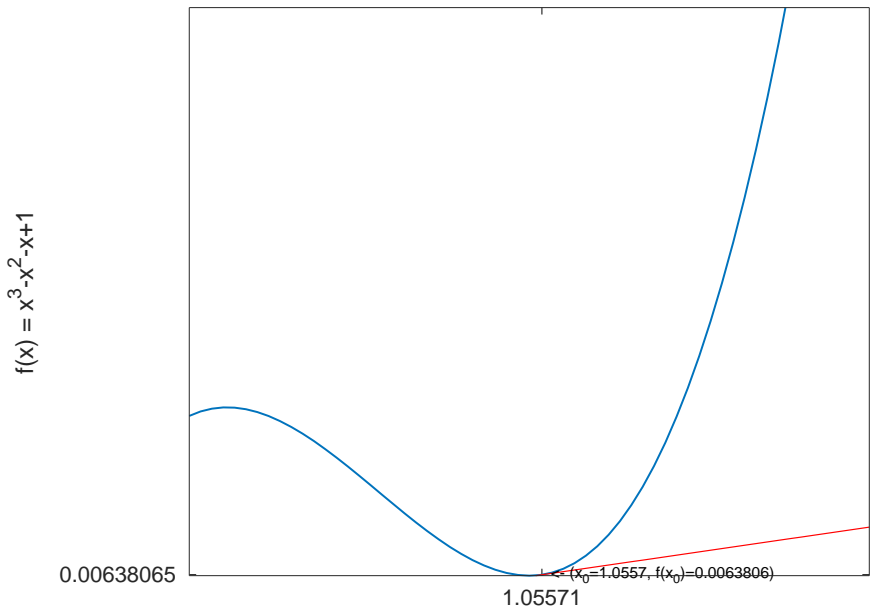


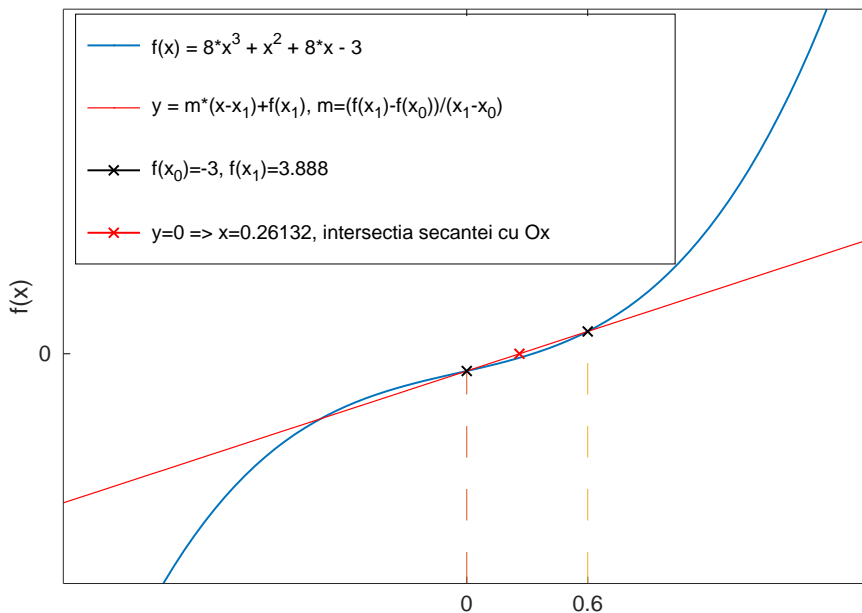
Fig.1.7 $x_0 = 1.0557$, $x_1 = 1.0557$



(2) $f(x) = 8x^3 + x^2 + 8x - 3, f(x) = 0, x = ?$, prin metoda secantei, având 0.0, 0.6 valori initiale si $\varepsilon = 0.01$.

```
# Cod octave:
i = 1;
i_max = 5;
while abs(f(x_0)) > epsilon && i < i_max
    x_1 = x_1 - (f(x_1)*(x_1-x_0))/(f(x_1)-f(x_0));
    print_graph(f, x_0, x_1, i);
    i ++;
endwhile
```

Fig.2.0 $x_0 = 0, x_1 = 0.6, f(x_1) = 3.888$



| | | | |
|-----|--------------|------------------|----------------------|
| i=1 | $x_0 = 0.00$ | $x_1 = 0.261324$ | $f(x_1) = -0.698350$ |
| i=2 | $x_0 = 0.00$ | $x_2 = 0.340613$ | $f(x_2) = 0.157059$ |
| i=3 | $x_0 = 0.00$ | $x_3 = 0.323668$ | $f(x_3) = -0.034631$ |
| i=4 | $x_0 = 0.00$ | $x_4 = 0.327448$ | $f(x_4) = 0.007685$ |

$f(x_4) = 0.007685 < \varepsilon = 0.01$,
deci $solutia = 0.007685$.

Fig.2.1 $x_0 = 0$, $x_1 = 0.26132$, $f(x_1) = -0.69835$

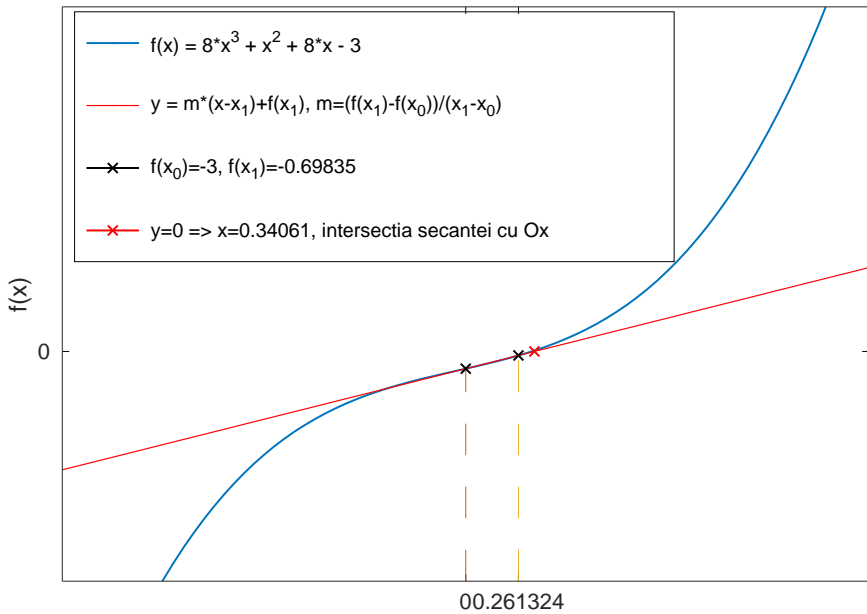


Fig.2.2 $x_0 = 0$, $x_1 = 0.34061$, $f(x_1) = 0.15706$

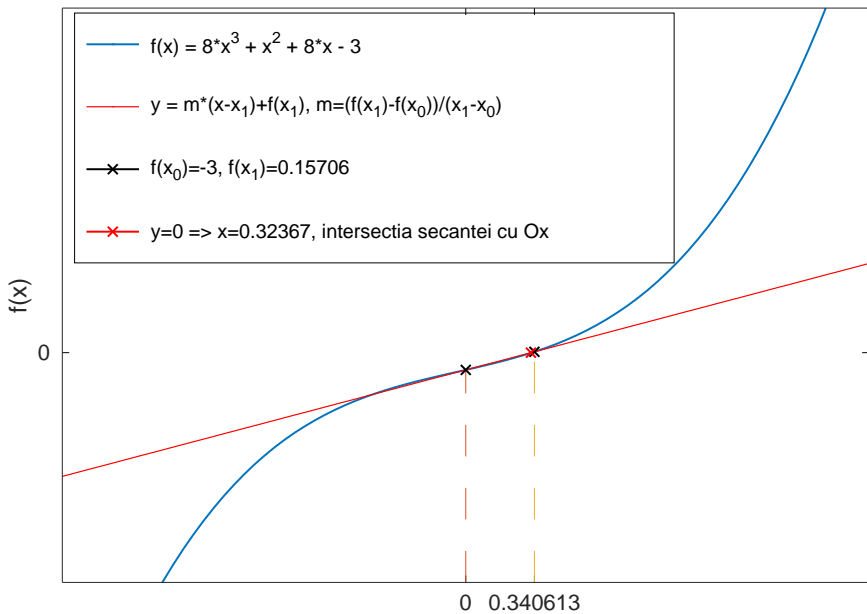


Fig.2.3 $x_0 = 0$, $x_1 = 0.32367$, $f(x_1) = -0.034631$

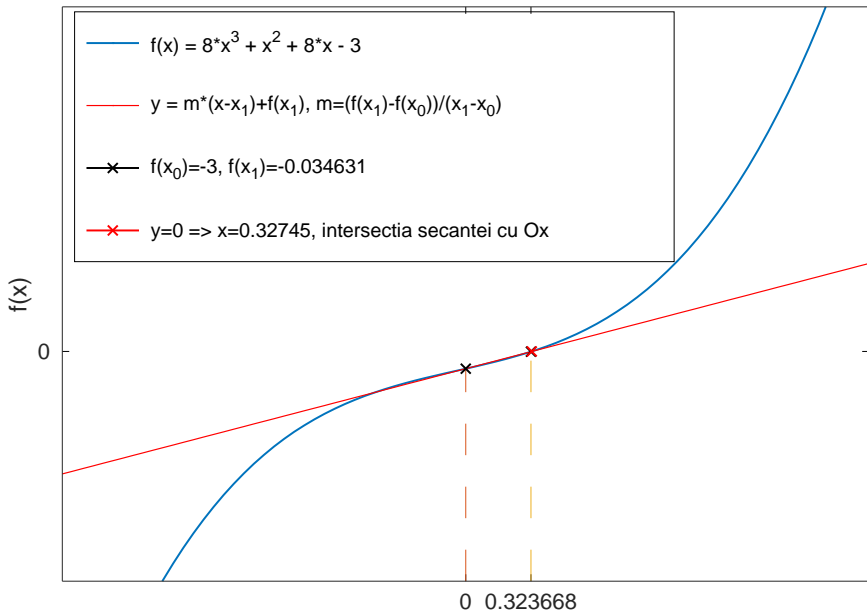
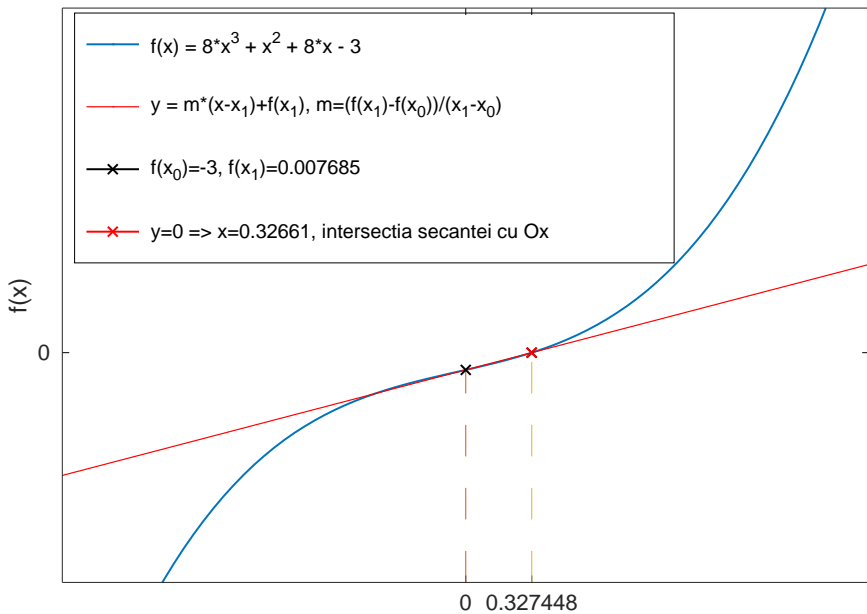


Fig.2.4 $x_0 = 0$, $x_1 = 0.32745$, $f(x_1) = 0.007685$



(3) $f(x) = x^3 - 4x + 2, f(x) = 0, x = ?$, prin metoda bisectiei, având $a = 0, b = 1, \varepsilon = 0.01$.

```
# Cod octave
f = @(x) x.^3-4*x+2;
a = 0;
b = 1;
epsilon = 0.01;
x_m = (a+b)/2;
i = 1;
max_i = 10;
while abs(f(x_m)) > epsilon
    if max_i < i
        printf("Solution not found!\n");
        break;
    endif

    printf("i=%d & a=%f & b=%f & x_m=%f & f(x_m)=%f\n",i,a,b,x_m,f(x_m));
    print_graph(f, a, b, i);
    x_m = (a+b)/2;
    if f(x_m)*a < 0
        b = x_m;
    else
        a = x_m;
    endif
    i++;
endwhile
```

$f(a) * f(b) < 0 \Rightarrow Solutia \in [a, b]$

| | | | | |
|-----|------------|------------|------------------|----------------------|
| i=1 | a=0.000000 | b=1.000000 | $x_m = 0.500000$ | $f(x_m) = 0.125000$ |
| i=2 | a=0.500000 | b=1.000000 | $x_m = 0.500000$ | $f(x_m) = 0.125000$ |
| i=3 | a=0.500000 | b=0.750000 | $x_m = 0.750000$ | $f(x_m) = -0.578125$ |
| i=4 | a=0.500000 | b=0.625000 | $x_m = 0.625000$ | $f(x_m) = -0.255859$ |
| i=5 | a=0.500000 | b=0.562500 | $x_m = 0.562500$ | $f(x_m) = -0.072021$ |
| i=6 | a=0.531250 | b=0.562500 | $x_m = 0.531250$ | $f(x_m) = 0.024933$ |
| i=7 | a=0.531250 | b=0.546875 | $x_m = 0.546875$ | $f(x_m) = -0.023945$ |
| i=8 | a=0.539062 | b=0.546875 | $x_m = 0.539062$ | $f(x_m) = 0.000395$ |

$Solutia = x_m = 0.539062$

Fig.3.1 $f(x)=x^3-4x+2$

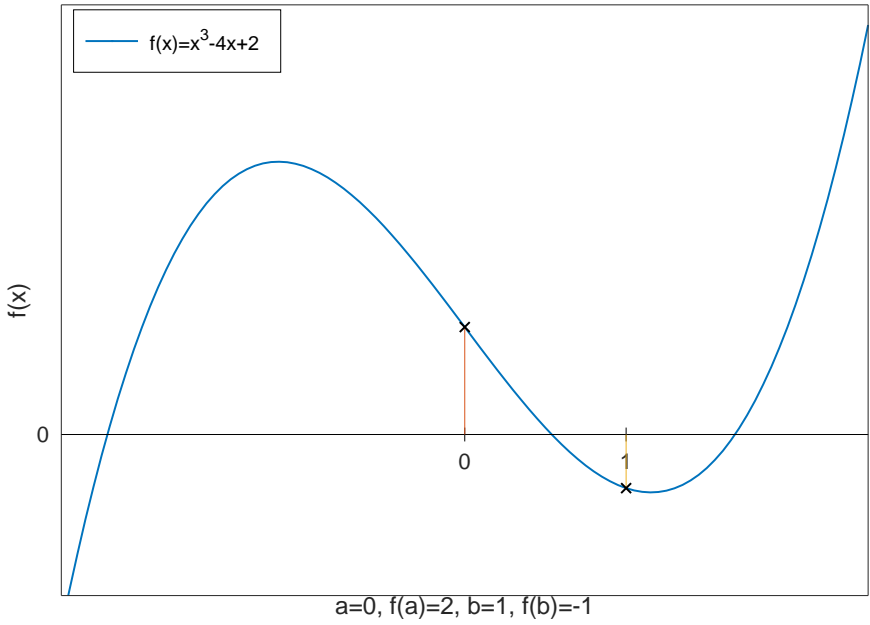


Fig.3.2 $f(x)=x^3-4x+2$

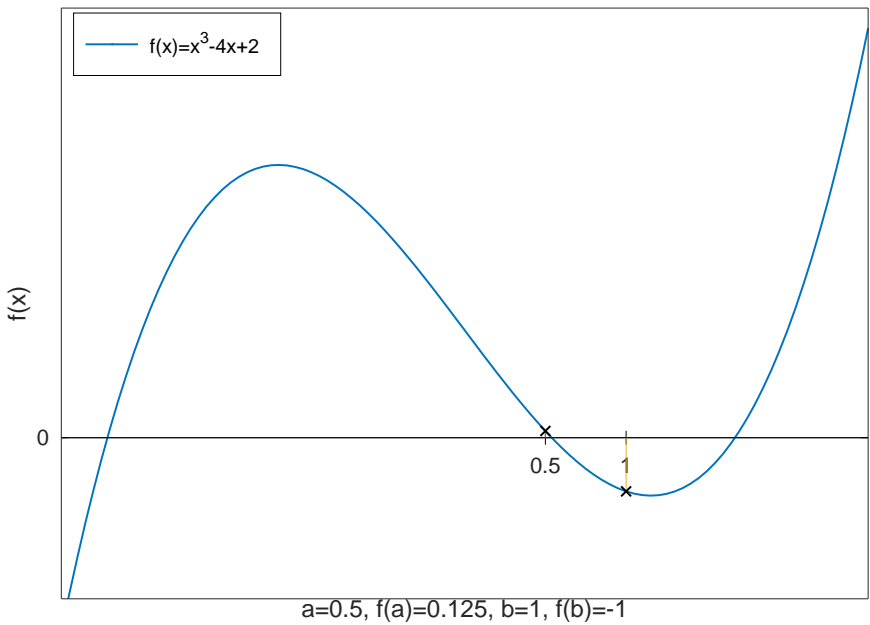


Fig.3.3 $f(x)=x^3-4x+2$

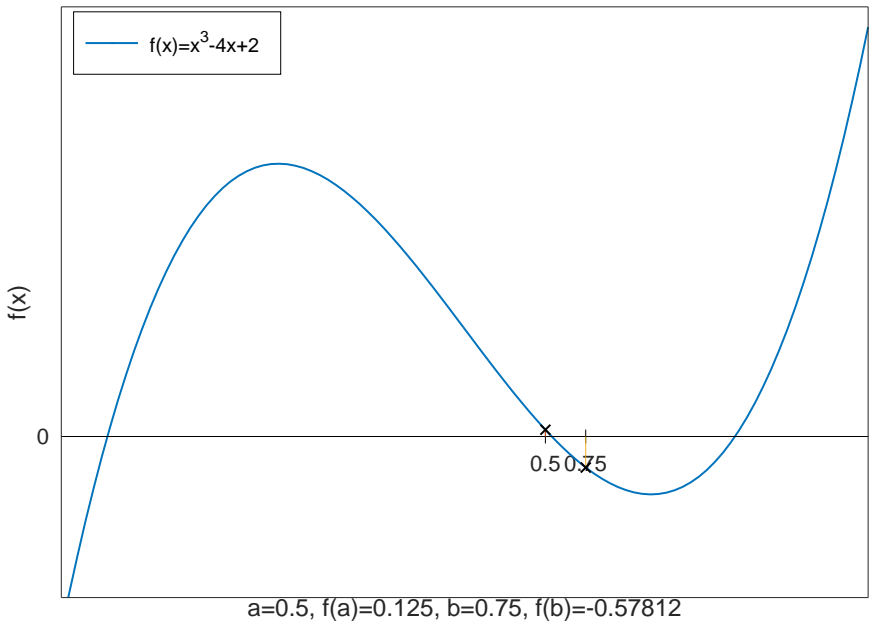


Fig.3.4 $f(x)=x^3-4x+2$

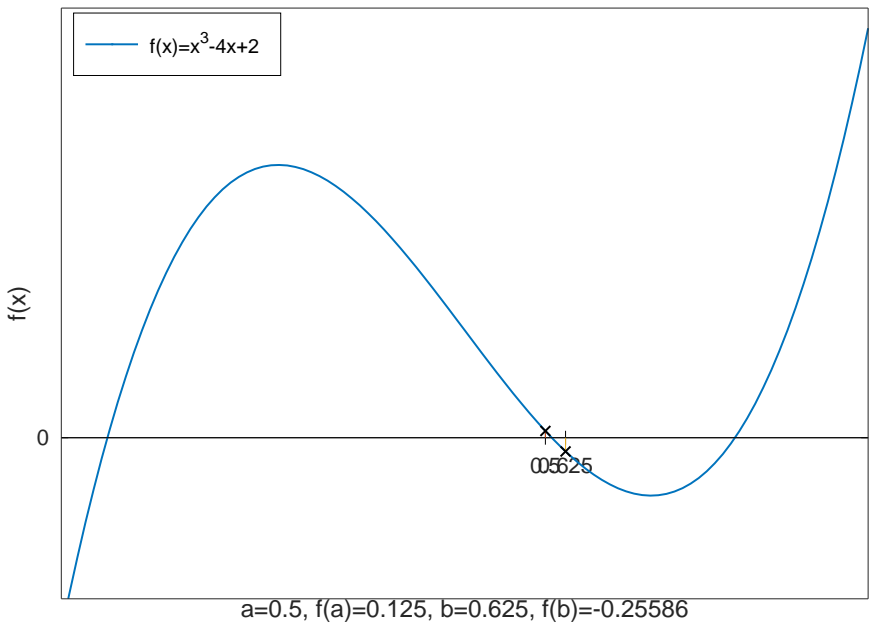


Fig.3.5 $f(x)=x^3-4x+2$

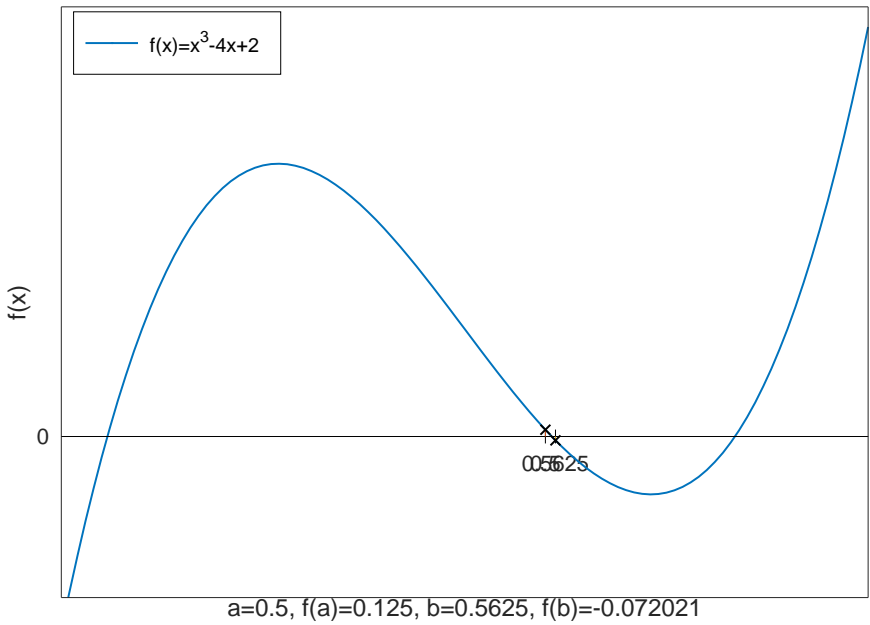


Fig.3.6 $f(x)=x^3-4x+2$

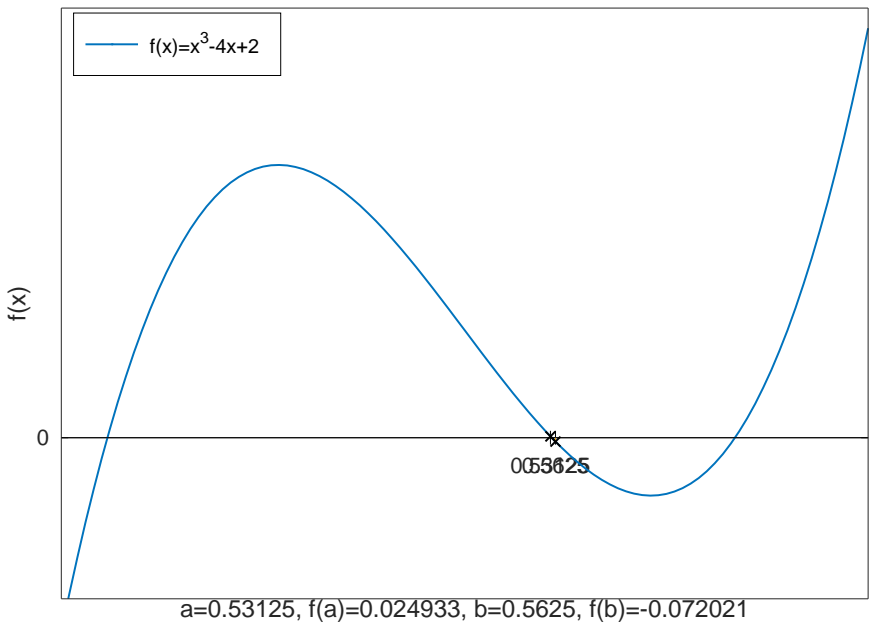


Fig.3.7 $f(x)=x^3-4x+2$

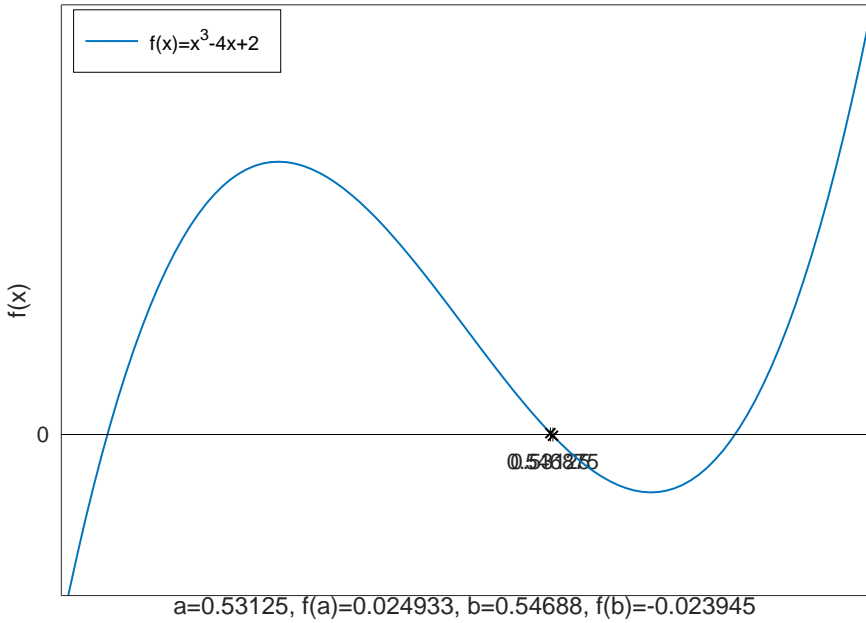
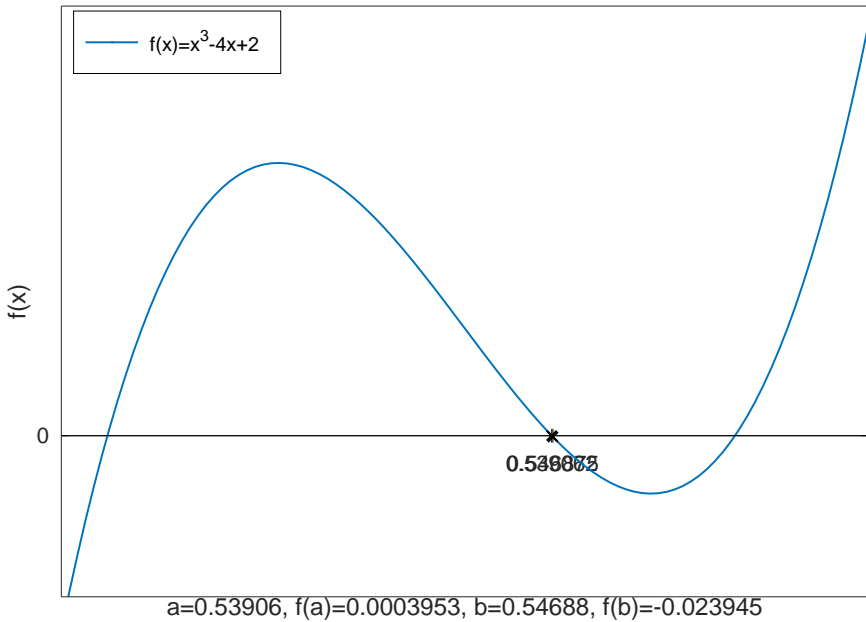


Fig.3.8 $f(x)=x^3-4x+2$



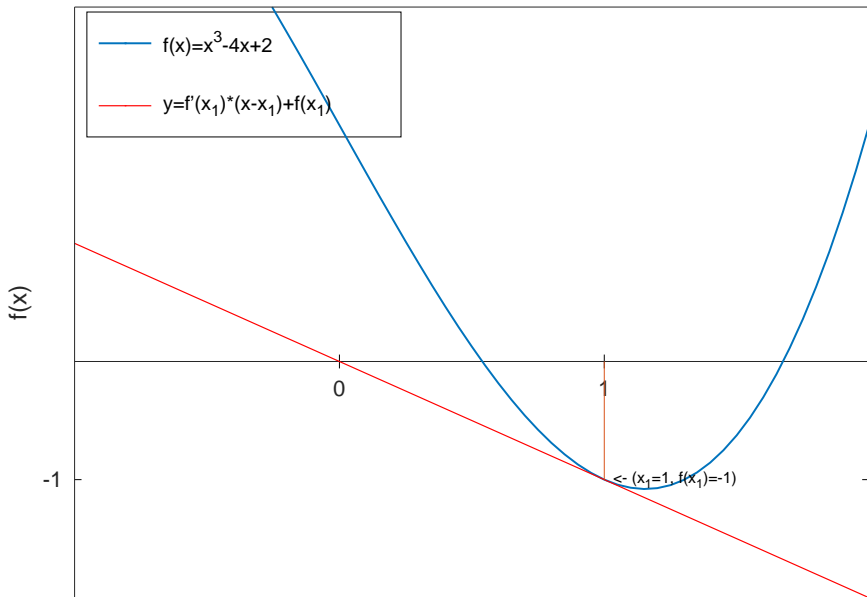
(4) $f(x) = x^3 - 4x + 2, x_1 = 1, x_2 = ?$ prin metoda lui Newton-Raphson.

Punctul $(x_2, 0)$ reprezintă intersecția cu Ox a tangentei prin $(x_1, f(x_1))$. Panta acestei tangente este dată de derivata funcției f pentru $x_1 = 1$. $m = f'(x_1)$. De asemenea aceasta pantă se poate calcula din ecuația tangentei, pentru $m = \frac{f(x_1) - y}{x_1 - x_2}, y = 0$.

Din $m = f'(x_1), m = \frac{f(x_1)}{x_1 - x_2} \Rightarrow f'(x_1) = \frac{f(x_1)}{x_1 - x_2}$.

Calculând pentru x_2 , avem $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0$.

Fig.4.0 $x_1 = 1, x_2 = 0$



```
# Cod octave
f = @(x) x.^3 - 4*x + 2;
df = @(x) 3*x^2 - 4;
x_1 = 1;
x_2 = x_1 - f(x_1)/df(x_1);
```

(5) $f(x) = x^3 - 4x + 2$ Daca $x_0 = 0$ si $x_1 = 1$, cat sunt x_2 si x_3 din metoda secantei?

Definim relația y în funcție de x ca fiind secanta care trece prin punctele $(x_0, f(x_0)), (x_1, f(x_1))$.

Panta calculată cu aceste puncte este: $m = \frac{f(x_1)-f(x_0)}{x_1-x_0}$.

Panta calculată cu punctul $(x_1, f(x_1))$ și alt punct al secantei este: $m = \frac{f(x_1)-y}{x_1-x}$.

Din $m = \frac{f(x_1)-f(x_0)}{x_1-x_0}, m = \frac{f(x_1)-y}{x_1-x} \Rightarrow \frac{f(x_1)-f(x_0)}{x_1-x_0} = \frac{f(x_1)-y}{x_1-x}$.

Izolându-l pe x obținem:

$$x_1 - x = \frac{(f(x_1)-y)(x_1-x_0)}{f(x_1)-f(x_0)} \Leftrightarrow x = x_1 - \frac{(f(x_1)-y)(x_1-x_0)}{f(x_1)-f(x_0)}.$$

Definim punctul $(x_2, 0)$ ca fiind intersecția secantei cu Ox , deci $y = 0$.

$$y = 0, x = x_2 \Rightarrow x_2 = x_1 - \frac{f(x_1)(x_1-x_0)}{f(x_1)-f(x_0)}.$$

```
# Cod octave
f = @(x) x^3 - 4*x + 2;
x_0 = 0;
x_1 = 1;
x_2 = x_1 - (f(x_1)*(x_1-x_0))/(f(x_1)-f(x_0));
x_3 = x_2 - (f(x_2)*(x_2-x_1))/(f(x_2)-f(x_1));
printf("$x_2=%.3f, x_3=%.3f$\n", x_2, x_3);
```

Soluțiile calculate sunt: $x_2 = 0.667, x_3 = 4.000$.

