

CURS 3 ALGEBRĂ S14

(1)

donă demonstrații care ilustrează aplicarea principiilor de inducție formulate în T14 și T15 din cursul 2:

I ~~Arată~~ că suma măsurilor unghiurilor oricărui poligon convex cu n laturi este $(n-2)\pi$.

Leu: Notăm $P(n)$: suma măsurilor unghiurilor oricărui poligon convex cu n laturi e $(n-2)\pi$.

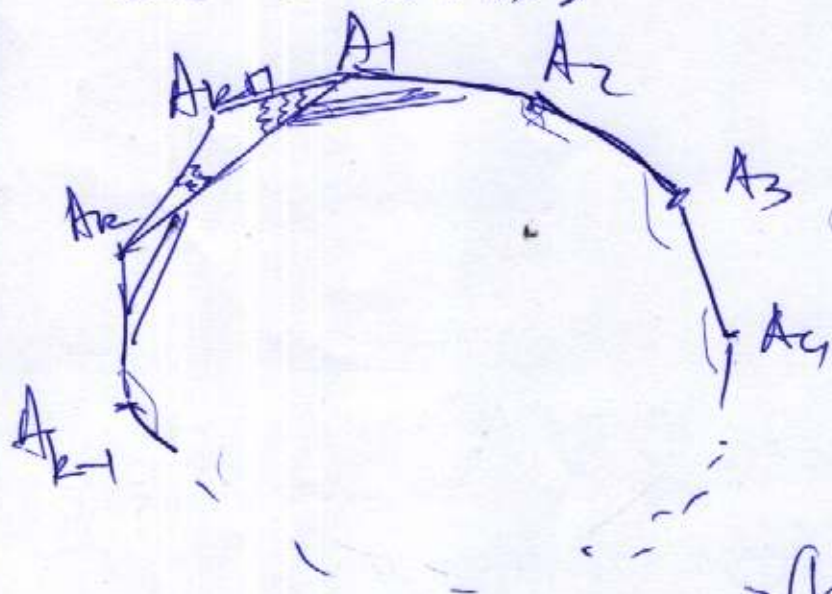
Se știe că suma măsurilor unghiurilor oricărui triunghi e π , deci $P(3)$ e adevărat.

Fie $k \geq 3$. Presupunem că ~~$P(3), P(4), \dots, P(k-1)$~~ $P(k)$ ~~este~~ adevărată \Rightarrow

Orice poligon convex cu k laturi are suma măsurilor $(k-2)\pi$.

Pre A_1, A_2, \dots, A_{k+1} un poligon convex (2)
~~pt care suma unuia~~ sau $k+1$ laturilor.

(vom nota pt fiecare poligon ~~convex~~ convex
 π suma unghiurilor sale
 cu $S(\pi)$).



Atunci:

$$S(A_1 A_2 \dots A_{k+1}) =$$

$$= [S(A_1 A_2 \dots A_k)] +$$

$$S(A_1 A_k A_{k+1}) \geq$$

$$= (k-2) \cdot \pi + \pi = (k+1-2) \cdot \pi$$

$\Rightarrow P(k+1)$ e adevarata,

[cf principiului inductiei matematice] copy
 $P(n)$ e adevarata pt orice $n \geq 3$.

Consideram sirul $(x_n)_n$ definit

prin: $x_1 = 5, x_2 = 35$ si

$$x_n = 6x_{n-1} + 7x_{n-2} \quad \forall n \geq 3.$$

Arata ca $\boxed{x_n \in \mathbb{N}^+}$ $x_n \geq 5 \cdot 7^{n-1}$

Leema: Notăm $P(n): x_n = 5 \cdot 7^{n-1}$ (3)

donc $1^o, x_1 = 5 = 5 \cdot 7^0 = 5 \cdot 7^{1-1} \Rightarrow P(1)$ e adevar.

Pie $k \geq 1$. Presupunem ca $P(1), P(2), \dots, P(k)$ sunt adevarate.

Atunci: $\left\{ \begin{array}{l} \text{Dacă } k \geq 2, \\ x_{k+1} = 6x_k + 7x_{k-1} = \end{array} \right.$

$$6 \cdot 5 \cdot 7^{k-1} + 7 \cdot 5 \cdot 7^{k-2} = 5 \cdot 7^{k-1} (6 + 1) =$$

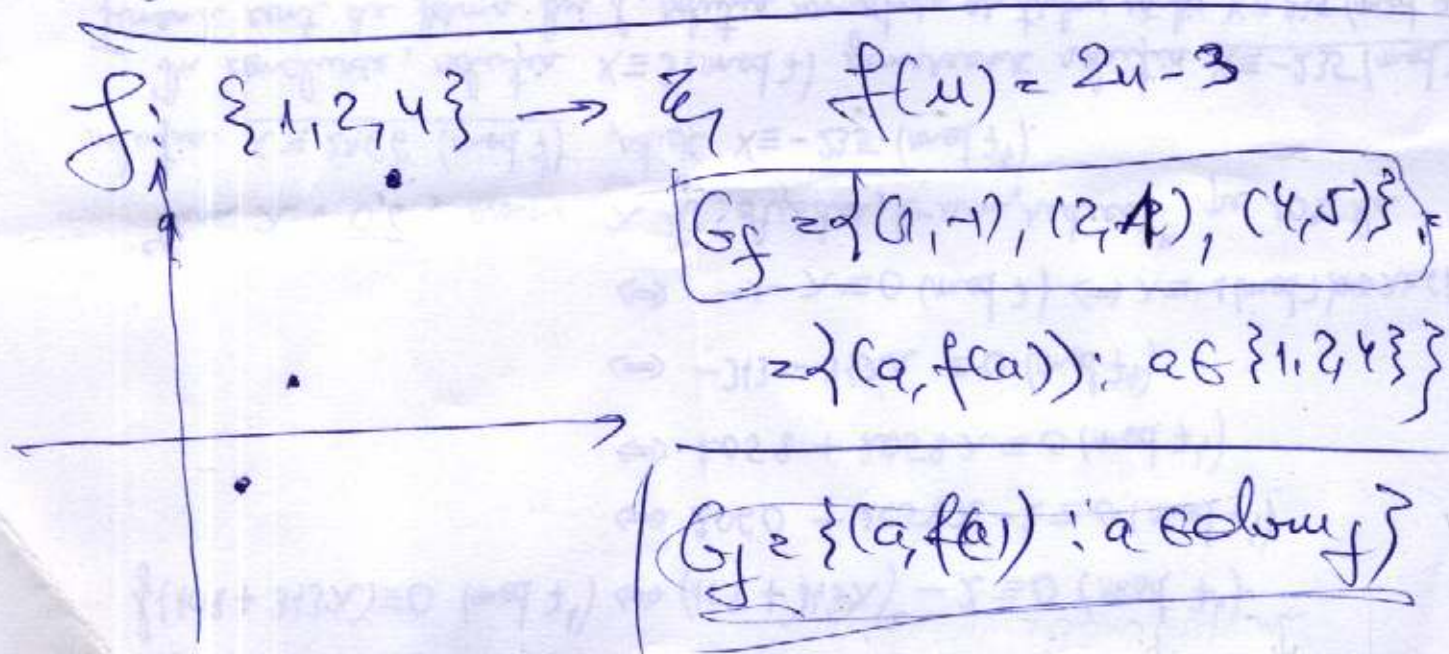
$$5 \cdot 7^{(k+1)-1} \Rightarrow P(k+1) \text{ e adevarat}$$

Dacă $k=1$,

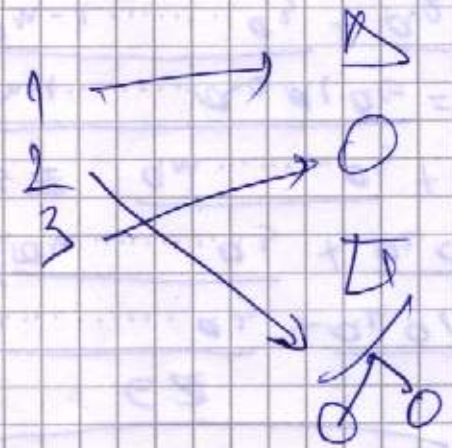
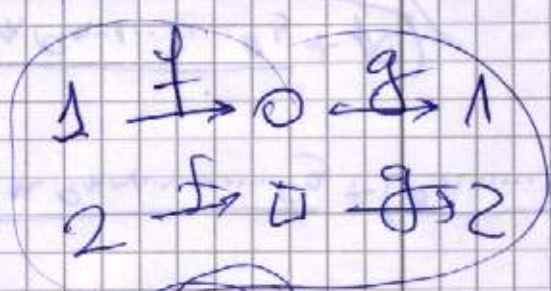
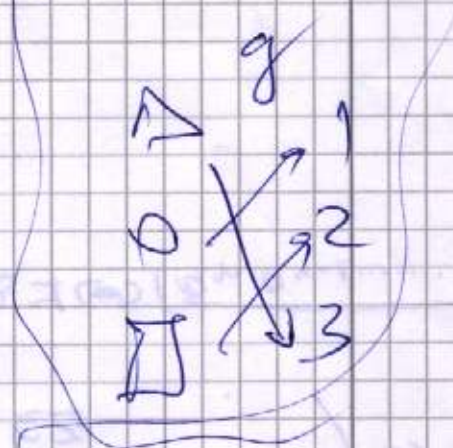
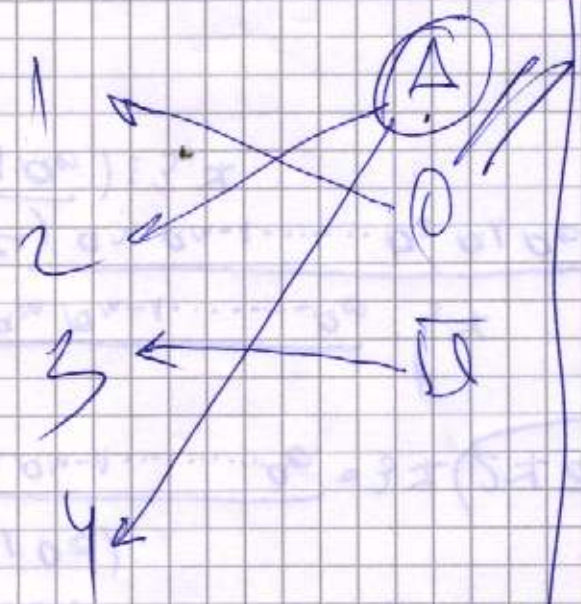
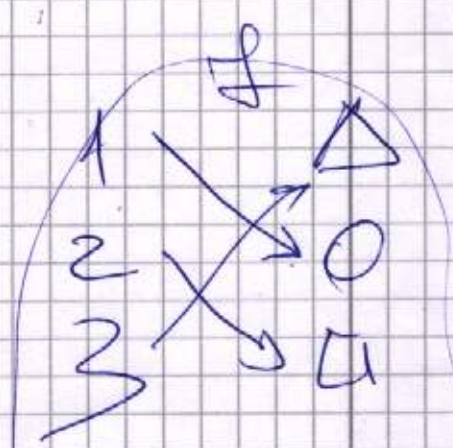
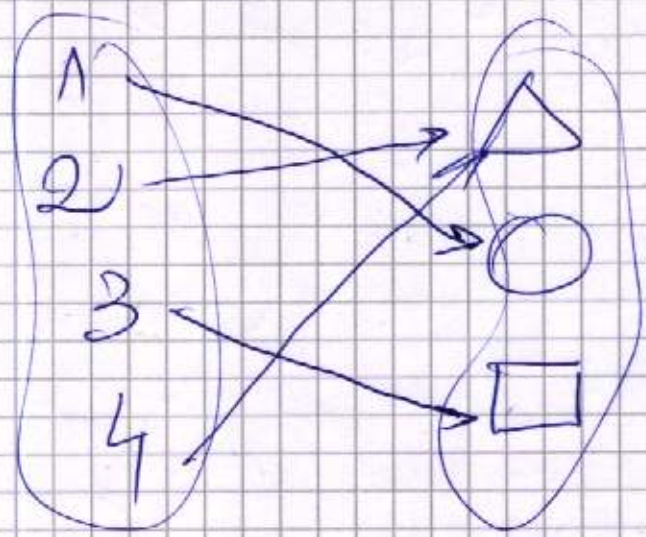
$$x_{k+1} = x_2 = 35 = 5 \cdot 7 = 5 \cdot 7^2 = 5 \cdot 7^{k+1-1} \Rightarrow$$

$P(k+1)$ e adevarat.

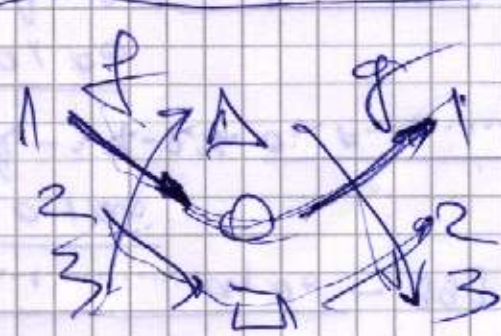
[PASTE]

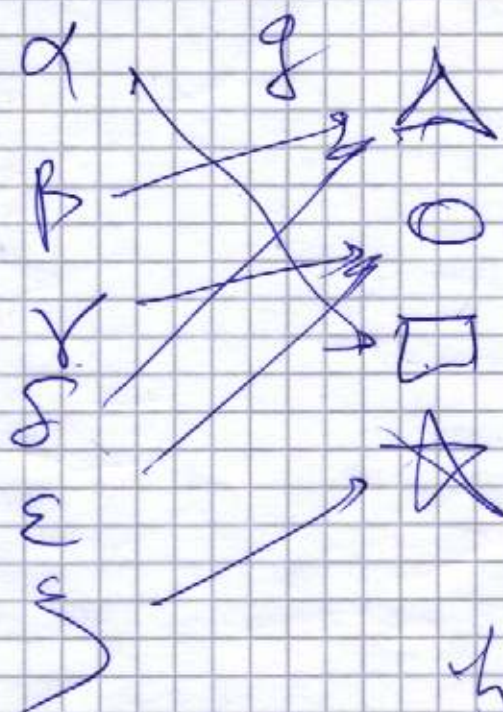
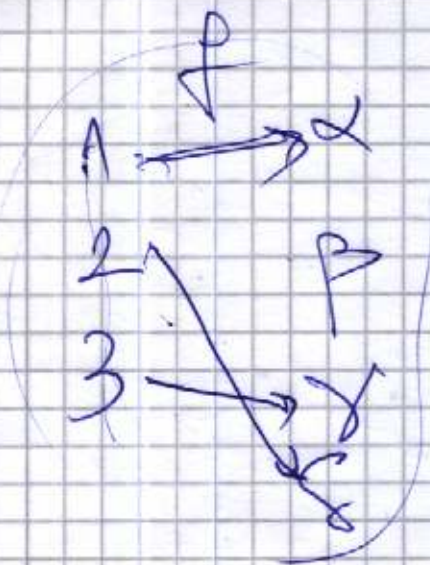


(4)



$g(f(a)) = 1$
 $f(a) = a$
 $\forall a \in \{1, 2, 3\}$





⑤

$$u_1 = g(f(a))$$

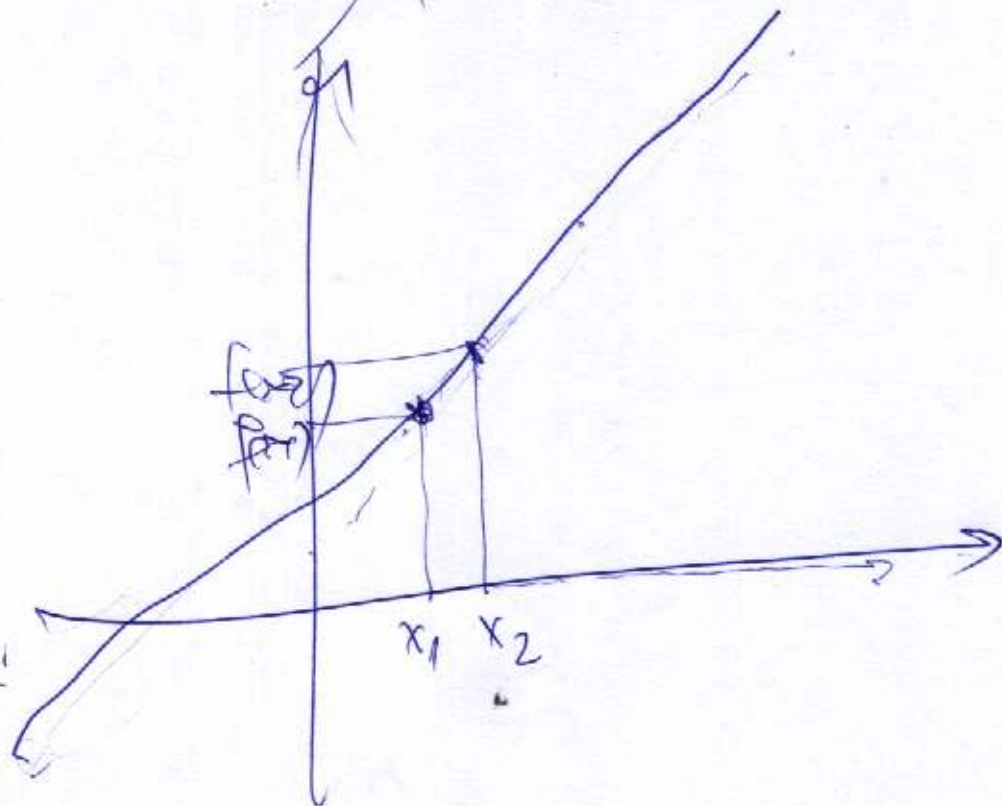
$$u_2 = g(f(b))$$

$$u_3 = g(f(c))$$



$$A \xrightarrow{f} B \subset C \xrightarrow{g} D$$

$$g \circ f: A \rightarrow D \quad (g \circ f)(a) = g(f(a))$$



EXERCISE 1.1

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function. The function f is said to be differentiable at a point $a \in \mathbb{R}$ if there exists a unique real number $f'(a)$ such that

$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = f'(a)$$

The number $f'(a)$ is called the derivative of f at a .

Example: Let $f(x) = x^2$. Then $f'(x) = 2x$.

Proof: We have $f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$. Therefore

$$\frac{f(x+h) - f(x)}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h} = 2x + h$$

As $h \rightarrow 0$, $2x + h \rightarrow 2x$. Hence $f'(x) = 2x$.