Horjea Cosmin-Marian 13.06.2020

143

Examen Geometrie seria 16

1. a)
2. b)
3. b)
4. a)

5. $\Gamma: f(x) = x_1^2 - 8x_1x_2 + 7x_2^2 - 12x_1 - 6x_2 - 9 = 0$

a) $A = \begin{pmatrix} 1 & -4 \\ -4 & 7 \end{pmatrix}$ $\tilde{A} = \begin{pmatrix} 1 & -4 & -6 \\ -4 & 7 & -3 \\ -6 & -3 & -9 \end{pmatrix}$

$\delta = \det A = 7 - 16 = -9 < 0$

$\det \tilde{A} = \Delta = \begin{vmatrix} 1 & -4 & -6 \\ -4 & 7 & -3 \\ -6 & -3 & -9 \end{vmatrix} = -324$

⑥ $(\mathbb{R}^3,+,\cdot)/\mathbb{R}$; $R_0=\{e_1,e_2,e_3\}$ reper canonic $\mathbb{R}^3$

$f\in End(\mathbb{R}^3)$, unde $A=[f]_{R_0,R_0}=\begin{pmatrix}3&-1&1\\2&0&1\\2&-2&3\end{pmatrix}$

$u_1=e_1+e_2$

$u_2=e_2+e_3$

$u_3=e_1+e_2+e_3$

$u=e_1+2e_2+e_3$

a) $f(x)=(3x_1-x_2+x_3,\ 2x_1+x_3,\ 2x_1-2x_2+3x_3)$

$u_1=(1,1,0)$

$f(u_1)=f(1,1,0)=(2,2,0)=2u_1 \Rightarrow u_1$ vector propriu cu $\lambda_1=2$

$u_2=(0,1,1)$

$f(u_2)=f(0,1,1)=(0,1,1)=u_2 \Rightarrow u_2$ vector propriu cu $\lambda_2=1$

$u_3=(1,1,1)$

$f(u_3)=f(1,1,1)=(3,3,3)=3u_3 \Rightarrow u_3$ vector propriu cu $\lambda_3=3$

b) $R=\{u_1,u_2,u_3\}$

$\det\begin{pmatrix}1&1&0\\0&1&1\\1&1&1\end{pmatrix}=1+1+0-0-0-1=1\neq 0 \Rightarrow R$ este SLI.

$\dim R=3=\dim\mathbb{R}^3 \Rightarrow R$ SG în $\mathbb{R}^3$

$\Rightarrow R$ este reper în $\mathbb{R}^3$

$u = e_1 + 2e_2 + e_3 = (1, 2, 1)$

$f(1,2,1) = (2, 3, +1)$

u în reperul R $= \cancel{1 \cdot u_1 + 2 \cdot u_2 + 1 \cdot u_3}$

$\cancel{= (1,1,0) + (0,2,2) + (1,1,1)}$

$\cancel{= (2, 4, 3)}$

$u = a(1,1,0) + b(0,1,1) + c(1,1,1)$

$= a(e_1 + e_2) + b(e_2 + e_3) + c(e_1 + e_2 + e_3)$

$= ae_1 + ae_2 + be_2 + be_3 + ce_1 + ce_2 + ce_3$

$= e_1(a+c) + e_2(a+b+c) + e_3(b+c)$

$\begin{cases} a + c = 1 \\ a + b + c = 2 \\ b + c = 1 \end{cases} \Rightarrow a + 1 = 2 \Rightarrow \boxed{a = 1} \Rightarrow \boxed{c = 0} \Rightarrow b = 1$

$\Rightarrow u = (1, 1, 0)$

$f(u) = 2e_1 + 3e_2 + e_3$

$= a(e_1 + e_2) + b(e_2 + e_3) + c(e_1 + e_2 + e_3)$

$= e_1(a+c) + e_2(a+b+c) + e_3(b+c)$

$\begin{cases} a + c = 2 \\ a + b + c = 3 \\ b + c = 1 \end{cases} \Rightarrow a + 1 = 3 \Rightarrow a = 2$

$a + c = 2 \Rightarrow c = 0$

$b + 0 = 1 \Rightarrow b = 1$

$\Rightarrow$ coord lui $f(u) = (2, 1, 0)$ (în raport cu R)

Scanned with CamScanner

7. $(\mathbb{R}^3, g_0)$ $f: \mathbb{R}^3 \to \mathbb{R}^3$, $f(x) = \frac{1}{3}(x_1+x_2+x_3, x_1+x_2+x_3, x_1+x_2+x_3)$

a) $A = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

$\det A = 0 \Rightarrow f$ nu este bijectivă

$g(x,y) = x_1 y_1 + x_2 y_2 + x_3 y_3 + x_1 y_2 + x_1 y_3 + x_2 y_3$
$+ x_2 y_1 + x_3 y_1 + x_3 y_2$

$g(x,y) = g(y,x) \Rightarrow$ end. simetric.

b) $f_1 = e_1$

$f_2 = e_2 + \frac{\langle f_1, e_2 \rangle}{\langle f_1, f_2 \rangle} \cdot f_1$

$f_3 = e_3 + \frac{\langle f_1, e_3 \rangle}{\langle f_1, e_1 \rangle} \cdot f_1 + \frac{\langle f_2, e_3 \rangle}{\langle f_2, f_2 \rangle} \cdot f_2$

8) $d_1: x_2 = x_3 = 0 \quad d_2 \begin{cases} x_2 - 1 = 0 \\ x_1 = x_3 \end{cases}$

a) $d_1 \begin{cases} x_2 = 0 \\ x_3 = 0 \end{cases}$

$d_1: \frac{x_1}{1} = \frac{x_2}{0} = \frac{x_3}{0} = t \Rightarrow \begin{cases} x_1 = t \\ x_2 = 0 \\ x_3 = 0 \end{cases} \Rightarrow u = (1,0,0)$

$A(0,0,0)$

$d_2: \begin{cases} x_2 - 1 \\ x_1 = x_3 \end{cases} \Rightarrow \begin{cases} x_2 - 1 = 0 \\ x_1 - x_3 = 0 \end{cases}$

$x_3 = s \Rightarrow x_1 = s \Rightarrow x_2 = \cancel{0}\,1$

$d_2: \frac{x_1}{1} = \frac{x_2 - 1}{0} = \frac{x_3}{1} = s \Rightarrow \begin{cases} x_1 = s \\ x_2 = 1 \\ x_3 = s \end{cases}$

$v = (1,0,1)$

$\cancel{A}\,B = (0,1,0)$

$u \neq v \Rightarrow d_1$ şi d_2 necoplanare $\Rightarrow \overrightarrow{AB}(0,1,0)$

$\Delta_C = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = 1 \cdot (-1)^2 \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1 \neq 0$

$P_1(t,0,0) \in d_1 \cap d$

$P_2(s,1,s) \in d_2 \cap d$

$\overrightarrow{P_1P_2} = (s-t, 1, s) \quad u = (1,0,0)$

$v = (1,0,1)$

$\langle \overrightarrow{P_1P_2}, u \rangle = 0 \Rightarrow s - t + 0 + 0 = s - t = 0 \Rightarrow s = t$

$\langle \overrightarrow{P_1P_2}, v \rangle = 0 \Rightarrow s - t + 0 + s = 0 \Rightarrow 2s - t = 0$

$\Rightarrow 2t - t = 0 \Rightarrow t = 0$

$\Rightarrow s = 0$

$s = t = 0 \Rightarrow P_1(0,0,0)$; $P_2(0,1,0)$

$\Rightarrow \overrightarrow{P_1P_2}(0,1,0)$

$d: \frac{x_1}{0} = \frac{x_2 - 1}{0} = \frac{x_3}{0}$

b) ~~dist (P_1P_2)~~

$\left.\begin{array}{l} \text{dist } d_1 d_2 = ? \\ P_1 \in d_1 \\ P_2 \in d_2 \end{array}\right\} \Rightarrow$ $\text{dist}(P_1, P_2) \Rightarrow$

$\|\overrightarrow{P_1P_2}\| = \sqrt{1^2} = 1$

9) $A = [f]_{R_0 R_0} = \frac{1}{7}\begin{pmatrix} -3 & -2 & 6 \\ 6 & -3 & 2 \\ 2 & 6 & 3 \end{pmatrix}$

a) $\det A = \frac{1}{7^3}\begin{vmatrix} -3 & -2 & 6 \\ 6 & -3 & 2 \\ 2 & 6 & 3 \end{vmatrix} = 1$ (2)

$A \cdot A^T = \frac{1}{7^2}\begin{pmatrix} -3 & -2 & 6 \\ 6 & -3 & 2 \\ 2 & 6 & 3 \end{pmatrix} \cancel{\begin{pmatrix} -3 & 6 & 2 \\ 6 & -3 & 6 \\ 2 & 2 & 3 \end{pmatrix}} \begin{pmatrix} -3 & 6 & 2 \\ -2 & -3 & 6 \\ 6 & 2 & 3 \end{pmatrix}$

$= \frac{1}{7^2}\begin{pmatrix} 49 & 0 & 0 \\ 0 & 49 & 0 \\ 0 & 0 & 49 \end{pmatrix} = I_3$ (1)

din 1,2 $\Rightarrow f \in O(\mathbb{R}^3)$ de speta 1

deci f este o rotatie

Scanned with CamScanner