(C1)

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VARIABILE ALEATOARE UNIDIMENSIONALE

MEDIA (v.a. discretă): $\mathbb{E}[X] = \sum_x x f(x)$, unde f este funcția de masă ($f(x) = \mathbb{P}(X=x)$)

Ex: $X \sim \begin{pmatrix} x_1 & x_2 & \dots & x_n \\ p_1 & p_2 & \dots & p_n \end{pmatrix} \Rightarrow \mathbb{E}[X] = x_1p_1 + x_2p_2 + \dots + x_np_n$

a) $\mathbb{E}[aX+bY] = a\mathbb{E}[X] + b\mathbb{E}[Y]$

b) X, Y independente $\Rightarrow \mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y]$, $\mathbb{E}[g(X)h(Y)] = \mathbb{E}[g(X)]\mathbb{E}[h(Y)]$

c) $\mathbb{E}[Y] = \sum_x g(x) f(x)$, unde $g: \mathbb{R} \to \mathbb{R}$ și $Y = g \circ X$

Ex: $X \sim \begin{pmatrix} -2 & -1 & 1 & 3 \\ 1/4 & 1/8 & 1/4 & 3/8 \end{pmatrix}$

$Y = X^2 \sim \begin{pmatrix} 1 & 4 & 9 \\ 3/8 & 1/4 & 3/8 \end{pmatrix}$

$\mathbb{E}[Y] = 1 \cdot \frac{3}{8} + 4 \cdot \frac{1}{4} + 9 \cdot \frac{3}{8} = \frac{23}{8}$, sau $\mathbb{E}[Y] = \sum_x x^2 f(x)$

VARIANȚA: $Var(X) = \mathbb{E}[(X - \mathbb{E}[X])^2]$

a) $Var(aX) = a^2 Var(X)$

b) $Var(X+a) = Var(X)$

c) $X \perp\!\!\!\perp Y \Rightarrow Var(X+Y) = Var(X) + Var(Y)$

d) $\boxed{Var(X) = \mathbb{E}[X^2] - \mathbb{E}^2[X]}$

ABATEREA STANDARD $SD(X) = \sigma = \sqrt{Var(X)}$

MOMENTUL DE ORDIN k: $\mathbb{E}[X^k] = \begin{cases} \sum_x x^k f(x) \\ \int_{-\infty}^{+\infty} x^k f(x)\,dx \end{cases}$

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FORMULE CALCUL PROBABILITĂȚI

$$\begin{cases} \mathbb{P}(A\cup B) = \mathbb{P}(A)+\mathbb{P}(B)-\mathbb{P}(A\cap B) \\ \mathbb{P}\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n}\mathbb{P}(A_i) - \sum_{i<j}\mathbb{P}(A_i\cap A_j) + \sum_{i<j<k}\mathbb{P}(A_i\cap A_j\cap A_k) - \ldots + (-1)^{n-1}\mathbb{P}(A_1\cap A_2\cap\ldots\cap A_n) \end{cases}$$

(Poincare)

$$\mathbb{P}(A\mid B) = \frac{\mathbb{P}(A\cap B)}{\mathbb{P}(B)}$$

$$\mathbb{P}(A_1\cap A_2\cap\ldots\cap A_n) = \mathbb{P}(A_1)\cdot\mathbb{P}(A_2\mid A_1)\cdot\mathbb{P}(A_3\mid A_1\cap A_2)\cdot\ldots\cdot\mathbb{P}(A_n\mid A_1\cap A_2\cap\ldots\cap A_{n-1})$$

$$\mathbb{P}(A\cap B_1)+\mathbb{P}(A\cap B_2)+\mathbb{P}(A\cap B_3) = \mathbb{P}(A\mid B_1)\cdot\mathbb{P}(B_1)+\mathbb{P}(A\mid B_2)\cdot\mathbb{P}(B_2)+\mathbb{P}(A\mid B_3)\cdot\mathbb{P}(B_3),$$

unde $\{B_1, B_2, B_3\}$ partiție a lui Ω (Formula prob. totale)

$$\mathbb{P}(A\mid B) = \frac{\mathbb{P}(B\mid A)\cdot\mathbb{P}(A)}{\mathbb{P}(B)}$$ (Bayes)

$$\mathbb{P}(A) = \mathbb{P}(A\mid B)\cdot\mathbb{P}(B) + \mathbb{P}(A\mid B^c)\cdot\mathbb{P}(B^c)$$

INDEPENDENTA : $X \perp\!\!\!\perp Y \iff \mathbb{P}(X=x_i, Y=y_j) = \mathbb{P}(X=x_i)\cdot\mathbb{P}(Y=y_j),\ \forall i,j$
(v.a. discrete)

DENS. DE REPARTITIE : O functie $f: \mathbb{R} \to \mathbb{R}$ a.î. $\mathbb{P}(X \in A) = \int_A f(x)\,dx$
(v.a. continue)

- $f(x) = \begin{cases} F'(x), & x \in X(\Omega) \\ 0, & x \notin X(\Omega) \end{cases}$, F functia de repartitie

- $F(x) = \int_{-\infty}^{x} f(t)\,dt$

MEDIA : $\mathbb{E}[X] = \int_{-\infty}^{\infty} x f(x)\,dx$, f dens. de repartitie
(v.a. continuă)

$\mathbb{E}[Y] = \int_{-\infty}^{\infty} g(x) f(x)\,dx$, $Y = g \circ X$

- Prop. la fel ca la v.a. discrete

FUNCTIA DE MASĂ CONDIȚIONATĂ LA A : $\mathbb{P}(X=x \mid A) = \dfrac{\mathbb{P}(\{X=x\} \cap A)}{\mathbb{P}(A)} = p_{X|A}$

COVARIANȚA : $Cov(X,Y) = \mathbb{E}[(X-\mathbb{E}[X])(Y-\mathbb{E}[Y])]$
$= \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$

- X, Y necorelate $\iff Cov(X,Y) = 0$
- $X \perp\!\!\!\perp Y \Rightarrow Cov(X,Y) = 0$
- $Cov(aX+b, Y) = a\,Cov(X,Y)$
- $Cov(X+Y, Z) = Cov(X,Z) + Cov(Y,Z)$

COEFICIENTUL DE CORELAȚIE : $\rho(X,Y) = \dfrac{Cov(X,Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}} \in [-1, 1]$

VARIABILA INDICATOR $\mathbb{1}_A(\omega) = \begin{cases} 1, & \omega \in A \\ 0, & \omega \notin A \end{cases}$

C1

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VARIABILE ALEATOARE BIDIMENSIONALE

$X, Y : \Omega \to \mathbb{R}$, $(X, Y) : \Omega \to \mathbb{R}^2$, $(X, Y) \in \{(x_i, y_j) \mid 1 \le i \le m, 1 \le j \le n\}$

DISCRETE

FUNCTIA DE MASĂ : $p_{X,Y}(x, y) = \mathbb{P}(X = x, Y = y)$

- $\sum_x \sum_y p_{X,Y}(x, y) = 1$

FUNCTIA DE REPARTITIE : $F_{X,Y}(x, y) = \mathbb{P}(X \le x, Y \le y)$

$\mathbb{P}((X, Y) \in A \subseteq \mathbb{R}^2) = \sum_{(x,y) \in A} \mathbb{P}(X = x, Y = y)$

REPARTIȚIA MARGINALĂ : $\mathbb{P}(X = x) = \sum_y \mathbb{P}(X = x, Y = y) = p_X(x)$

REPARTIȚIA CONDIȚIONATĂ A LUI X LA Y = y : $p_{X|Y}(x|y) = \mathbb{P}(X = x \mid Y = y) = \frac{p_{X,Y}(x,y)}{p_Y(y)}$

$p_X(x) = \sum_y p_Y(y) \cdot p_{X|Y}(x, y)$ (Formula prob. totale)

$p_{X|Y}(x|y) = \frac{p_X(x)\, p_{Y|X}(y|x)}{\sum_{x'} p_X(x')\, p_{Y|X}(y|x')}$ (Bayes)

MEDIA : $\mathbb{E}[g(X, Y)] = \sum_x \sum_y g(x, y)\, p_{X,Y}(x, y)$, $g : \mathbb{R}^2 \to \mathbb{R}$

- $g(x, y) = ax + by + c \Rightarrow \mathbb{E}[aX + bY + c] = a\mathbb{E}[X] + b\mathbb{E}[Y] + c$

CONTINUE

DENSITATEA COMUNĂ: $f_{(X,Y)}: \mathbb{R}^2 \to [0,\infty)$, $P((X,Y)\in A) = \iint_{(x,y)\in A} f_{(X,Y)}(x,y)\,dx\,dy$, $A \subseteq \mathbb{R}^2$

- $A = [a,b]\times[c,d] \Rightarrow P((X,Y)\in A) = P(a \le X \le b,\ c \le Y \le d) = \int_a^b \int_c^d f_{(X,Y)}(x,y)\,dx\,dy$
- $P((X,Y)\in \mathbb{R}^2) = \iint_{\mathbb{R}^2} f_{(X,Y)}(x,y)\,dx\,dy = 1$

DENSITATEA MARGINALĂ: $f_X(x) = \int_{-\infty}^{\infty} f_{(X,Y)}(x,y)\,dy$

FUNCȚIA DE REPARTIȚIE: $F_{X,Y}(x,y) = P(X \le x,\ Y \le y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f_{X,Y}(u,v)\,du\,dv$

- $f_{(X,Y)}(x,y) = \frac{\partial^2}{\partial x \partial y} F_{X,Y}(x,y)$

MEDIA: $E[g(X,Y)] = \iint_{\mathbb{R}^2} g(x,y)\, f_{X,Y}(x,y)\,dx\,dy$

DENSITATEA CONDIȚIONATĂ A LUI X LA A: $P(X\in B \mid A) = \int_B f_{X|A}(x)\,dx$

INDEPENDENȚA (v.a. continue): $X \perp\!\!\!\perp Y \iff f_{X,Y}(x,y) = f_X(x)\cdot f_Y(y),\ \forall x,y$

- $f_{X,Y}(x,y) = g(x)h(y) \Rightarrow X \perp\!\!\!\perp Y$