Maximization of a Function Using Hill Climbing

Mazilu Cosmin-Alexandru

Abstract

In this report it is examened the maximization of the cubic function $f(x) = x^3 - 60x^2 + 900x + 100$ on the interval $x \in [0, 31]$, using Hill Climbing (HC) algorithms with a 5-bit binary representation of candidate solutions. Both First Improvement (FI) and Best Improvement (BI) HC variants are implemented and analyzed in terms of convergence paths and attraction basins. The results show that both FI and BI reliably converge to the global maximum at $x = 10$, though the two methods vary in terms of convergence speed and exploration behavior. This study provides insights into the impact of HC variant choice on search efficiency in unimodal landscapes.

1 Introduction

This report investigates the maximization of the function $f(x) = x^3 - 60x^2 + 900x + 100$ over the domain $x \in [0, 31]$ using Hill Climbing (HC) algorithms. The test will be on two variants of HC: First Improvement (FI) and Best Improvement (BI), analyzing the attraction basins and performance of each approach.

2 Experimental Setup Description

The function $f(x)$ is a unimodal cubic function within the specified range, with a single maximum at $x = 10$. For the HC algorithm, candidates are encoded as 5-bit binary strings, giving 32 possible values (0 to 31). Each candidate's neighborhood consists of all binary strings at a Hamming distance of 1.

3 Methods

3.1 Hill Climbing Algorithm

The Hill Climbing algorithm is implemented in two variants:

- **First Improvement (FI)**: Selects the first neighboring solution that improves the current solution.
- **Best Improvement (BI)**: Evaluates all neighbors and selects the one with the highest improvement.

3.2 Neighborhood Definition

For a 5-bit candidate solution, the neighborhood consists of all solutions that differ by exactly one bit. This creates a local search around each point, allowing for iterative improvement until convergence.

3.3 Test function

The function is $f : [0, 31] \to \mathbf{R}$, $f(x) = x^3 - 60 \cdot x^2 + 900 \cdot x + 100$. The maximum for this function is obtained for $x = 10$.

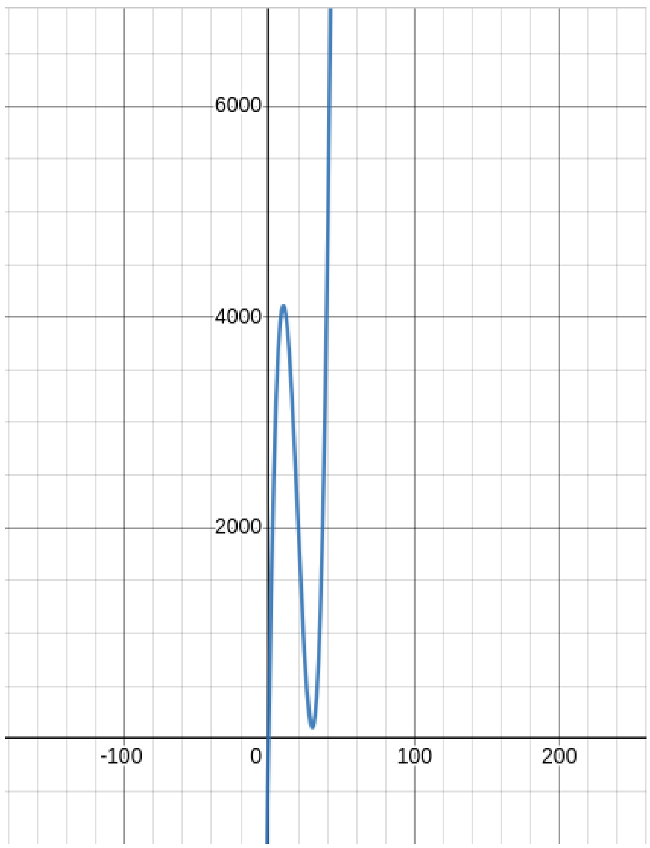

Figure 1:
Illustration of the function [Gra]

x	$f(x)$
0	100
1	941
2	1668
3	2287
4	2804
5	3225
6	3556
7	3803
8	3972
9	4069
10	**4100**
11	4071
12	3988
13	3857
14	3684
15	3475
16	3236
17	2973
18	2692
19	2399
20	2100
21	1801
22	1508
23	1227
24	964
25	725
26	516
27	343
28	212
29	129
30	100
31	131

Table 1: The values of $f(x)$ for $x \in [0, 31]$

4 Results

4.1 First Improvement vs. Best Improvement

x	Best Improvement	First Improvement
00000 (0)	01000 (8)	10000 (16)
00001 (1)	01001 (9)	10001 (17)
00010 (2)	01010 (10)	10010 (18)
00011 (3)	01011 (11)	10011 (19)
00100 (4)	01100 (12)	01100 (12)
00101 (5)	01101 (13)	01101 (13)
00110 (6)	00111 (7)	01110 (14)
00111 (7)	00111 (7)	00111 (7)
01000 (8)	01010 (10)	01100 (12)
01001 (9)	01011 (11)	01011 (11)
01010 (10)	01010 (10)	01010 (10)
01011 (11)	01010 (10)	01010 (10)
01100 (12)	01100 (12)	01100 (12)
01101 (13)	01001 (9)	01001 (9)
01110 (14)	01010 (10)	01010 (10)
01111 (15)	01011 (11)	00111 (7)
10000 (16)	10000 (16)	10000 (16)
10001 (17)	10000 (16)	10000 (16)
10010 (18)	10000 (16)	10000 (16)
10011 (19)	10001 (17)	10001 (17)
10100 (20)	10000 (16)	00100 (4)
10101 (21)	00101 (5)	00101 (5)
10110 (22)	00110 (6)	00110 (6)
10111 (23)	00111 (7)	00111 (7)
11000 (24)	01000 (8)	01000 (8)
11001 (25)	01001 (9)	01001 (9)
11010 (26)	01010 (10)	01010 (10)
11011 (27)	01011 (11)	01011 (11)
11100 (28)	01100 (12)	01100 (12)
11101 (29)	01101 (13)	01101 (13)
11110 (30)	01110 (14)	01110 (14)
11111 (31)	01111 (15)	01111 (15)

Table 2: Comparison between BI and FI for choosing neighborhood.

The difference between this two variants of Hill Climbing is that First Improvement tends to get blocked in local maxima more than Best Improvement, that find the global maxima more times.

4.2 Attraction Basins

The function has 4 local maxima, global maxim being obtained for $x = 10$:

- $x = 10$ with the attraction basin $\{0, 1, 2, 3, 5, 8, 9, 10, 11, 14, 15, 21, 24, 25, 26, 27, 29, 30, 31\}$
- $x = 7$ with the attraction basin $\{6, 7, 22, 23\}$
- $x = 12$ with the attraction basin $\{4, 12, 13, 28\}$
- $x = 16$ with the attraction basin $\{16, 17, 18, 19, 20\}$

5 Conclusion

Hill Climbing is effective for maximizing this unimodal function over the specified domain, with both FI and BI approaches leading to the global optimum at $x = 10$.

References

[Gra] Graph Plotter. `https://www.transum.org/Maths/Activity/Graph/Desmos.asp`.