Trajectory algorithms

Mazilu Cosmin-Alexandru

UAIC
Computer Science

November 3, 2024

Abstract

Finding the optimal solutions, in Computer Science, it's a challenging problem, as it requires a lot of optimizations and much computational resources to find optimal solutions. "In computer science and mathematical optimization, a metaheuristic is a higher-level procedure or heuristic designed to find, generate, tune, or select a heuristic (partial search algorithm) that may provide a sufficiently good solution to an optimization problem. Their use is always of interest when exact or other (approximate) methods are not available or are not expedient, either because the calculation time is too long or because, for example, the solution provided is too imprecise"[Wika]. This report investigates the performance of trajectory optimization algorithms on functions, specifically De Jong, Schwefel's, Rastrigin's, and Michalewicz's functions. The tested algorithms are Hill Climbing (with first improvement, best improvement, and worst improvement variants) and Simulated Annealing, analyzing their efficiency across multiple dimensions. The metaphor behind hill climbing is that you're on a hill (solution space) and want to reach the highest point (optimal solution). You start at a random point on the hill and move in the direction that increases your height (improves your solution) the most. If there's no further way up, you stop because you're likely at a peak, which might be a local or global optimum. "The name of the Simulated Annealing algorithm comes from annealing in metallurgy, a technique involving heating and controlled cooling of a material to alter its physical properties."[Wikb] Results are presented, highlighting the performance differences and implications for optimization in high-dimensional spaces.

0.1 Introduction

The purpose of this report is to explore the effectiveness of different optimization algorithms in finding the minima of several well-known functions, on different dimensions. The chosen algorithms—Hill Climbing and Simulated Annealing—provide distinct approaches in searching local and global optima of a function. A good understanding of these methods, their strengths and weaknesses, and their results is essential for applying optimization techniques in real-world problems.

0.2 Methods

0.2.1 Used algorithms

- **Hill Climbing** is trajectory algorithm used for optimization problem[hc]. It is used to find the best solution in a given space by exploring neighboring solutions of the candidate solution. It starts from a initial candidate solution, selected at random. It generates neighboring solutions by making small modifications to the current solution. Selection of the neighboring solutions differs for every Hill Climbing variant. For Hill Climbing-best improvement (steepest ascent), all the neighboring solutions are generated, and the best of them is selected. Hill climbing-worst improvement generates all the neighboring solutions and chooses the closer but better neighbor solution than candidate solution. Hill Climbing-first improvement (nearest ascent) generates the neighboring solutions one by one and checks and chooses the first neighbor solution that's better than the candidate solution. After selecting the neighbor solution, the candidate solution is replaced by the selected neighbor. This process is repeated until there is no neighbor solution better than the candidate solution, in other words the algorithm reached a local minima. After that it compares the local minima found with the optimal solution found so far. All this steps are repeated until termination condition is met. In this case, the algorithm ends after a maximum number of repetitions or if after a specific number of repetitions the optimal solution doesn't improve.

- **Simulated annealing** (Metropolis algorithm) is a metaheuristic trajectory optimization algorithm, used in finding optimal solutions in large search spaces. The algorithm is constructed by hybridizing one variant of Hill Climbing. In addition, Simulated Annealing has a new mechanic, called temperature [Alg]. The algorithm starts with an initial temperature value which decrease overtime while the algorithm is running. The purpose of the temperature is to make the algorithm accept worse solutions in order to escape from a local optima. The higher the temperature the higher is the chance that a worse solution in accepted. Temperature schedule, which will be presented later, it's a trade-off between the computational time and the quality of solutions. The algorithm ends after a maximum number of repetitions or if after a certain number of iterations the optimal solution doesn't improve.

0.2.2 Representation of solutions

The solutions will be represented as bit strings [Cro24]. The search space will be discretized up to a certain precision 10^{-d}, where d is the precision, the number of decimals of the solutions. For the function $f : [a, b]^n \to R$, the domain $[a, b]$ will be divided into a number of $(b - a) \times 10^d$ equal subintervals, noted as N. For representing all the N values it's necessary a number of ceil($log_2 N$) bits, noted as x. The length of the bit string which represents a candidate solution is the sum of the lengths of all the parameters of the function to optimize. For evaluating a candidate solution (calling the function to optimize) is necessary to decode every parameter represented as bit string to a real number, using the formula:

$$X_{real} = a + \frac{decimal(x_{bits}) \times (b - a)}{(2^x - 1)}$$

A candidate solution is made by random generating a string of bits with the length of x. The neighboring solutions are generated by flipping the bits one by one from the candidate solution.

0.2.3 Tested Functions

- **De Jong 1 Function:** it's a continous, convex and unimodal function, also known as sphere model. The definition of the function is: $f : [-5.12, 5.12]^n \rightarrow R$ where the image of the function is calculated with the following formula:

$$f(\mathbf{x}) = \sum_{i=1}^{n} x_i^2, \quad where \quad x = (x_1, x_2, \ldots, x_n)$$

.

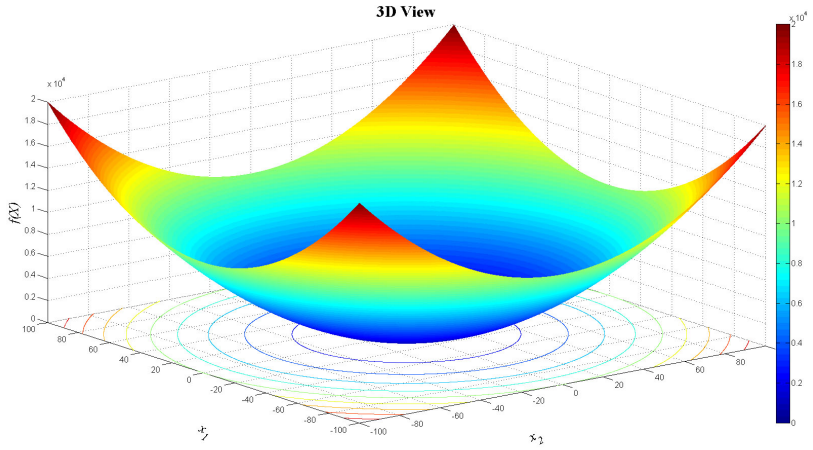

Figure 1: Illustration of the De Jong function. [AR15] [ARa]

- **Schewefel's Function:** is a complex function, having many local minima. The definition of the function is $f : [-500, 500]^n \rightarrow R$ where the image of the function is calculated with the following formula:

$$f(\mathbf{x}) = 418.9829 \times n - \sum_{i=1}^{n} x_i \sin(\sqrt{|x_i|}), \quad where \quad \mathbf{x} = (x_1, x_2, \ldots, x_n)$$

.

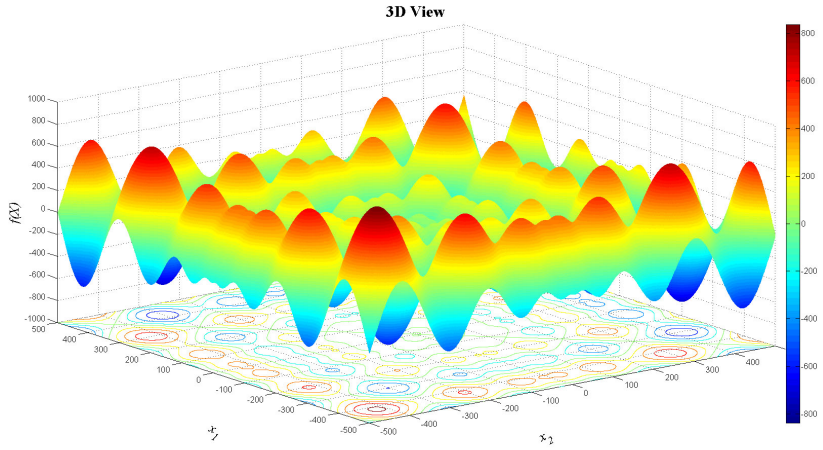

Figure 2
Illustration of the Schwefel's function. [AR15] [ARb]

- **Rastrigin's Function:** is a non-convex, non-linear multimodal function. The definition of the function is: $f : [-5.12, 5.12]^n \to R$ where the image of the function is calculated with the following formula:

$$f(\mathbf{x}) = 10n + \sum_{i=1}^{n} \left[x_i^2 - A\cos(2\pi x_i) \right], \quad where \quad \mathbf{x} = (x_1, x_2, \ldots, x_n)$$

.

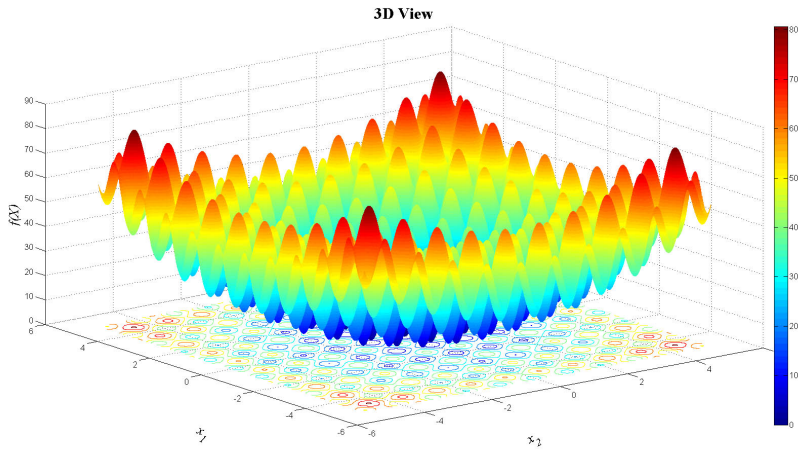

Figure 3
Illustration of the Ratrigin's function. [AR15] [ARc]

- **Michalewicz's Function:** is a multimodal function. The parameter m influences the "steepness" of the valleys or edges. The definition of the function is $f : [0, \pi]^n \to R$ where the image of the function is calculated with the following formula:

$$f(\mathbf{x}) = -\sum_{i=1}^{n} \sin(x_i) \left(\sin\left(\frac{ix_i^2}{\pi} \right) \right)^{2m} \quad where \quad \mathbf{x} = (x_1, x_2, \ldots, x_n) \quad and \quad m \quad is \quad typically \quad 10$$

.

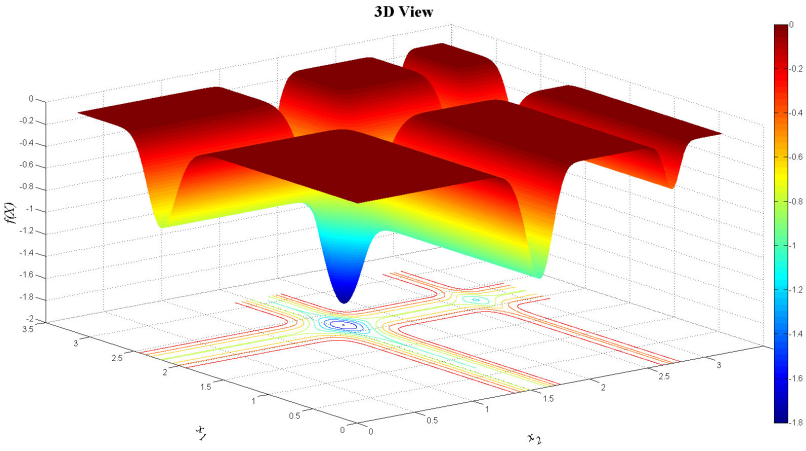

Figure 4
Illustration of the Michalewicz's function. [AR15] [ARd]

0.3 Experiments description

Every pair of (function, algorithm) will be runned 30 times with the fallowing parameters and values:

- Number of dimensions: 5, 10 and 30;
- Precision: 5;
- Max number of repetitions: 1000 (HC), 10000(SA);
- Max repetitions in case of no improvement of the optimal solution found: 1 (for DeJong function), 100 (for Schwefel's), 200 (for Rastrigin's and Michalewicz's);
- Temperature (for Simulated Annealing): the initial temperature is set to 0.99. Temperature is modeled with the followin formula: $T_n = T_{n-1} \times 0.99^t$, where the t is the interation.

0.4 Experimental results

0.4.1 Hill Climbing First Improvement

First Improvement DeJong1						
	n=5		n=10		n=30	
Run	Exec Time	Value	Exec Time	Value	Exec Time	Value
1	0.00411	0.00000	0.01496	0.00000	0.15534	0.00000
2	0.00120	0.00000	0.00887	0.00000	0.17354	0.00000
3	0.00157	0.00000	0.00817	0.00000	0.17006	0.00000
4	0.00128	0.00000	0.00830	0.00000	0.15402	0.00000
5	0.00144	0.00000	0.00729	0.00000	0.15774	0.00000
6	0.00130	0.00000	0.00699	0.00000	0.16282	0.00000
7	0.00126	0.00000	0.00798	0.00000	0.17208	0.00000
8	0.00380	0.00000	0.00980	0.00000	0.17625	0.00000
9	0.00390	0.00000	0.00795	0.00000	0.16212	0.00000
10	0.00332	0.00000	0.00803	0.00000	0.16672	0.00000
11	0.00135	0.00000	0.00742	0.00000	0.16601	0.00000
12	0.00163	0.00000	0.00767	0.00000	0.16051	0.00000
13	0.00301	0.00000	0.00934	0.00000	0.16496	0.00000
14	0.00140	0.00000	0.01677	0.00000	0.17090	0.00000
15	0.00351	0.00000	0.00718	0.00000	0.16617	0.00000
16	0.00325	0.00000	0.00842	0.00000	0.15651	0.00000
17	0.00210	0.00000	0.01379	0.00000	0.17129	0.00000
18	0.00187	0.00000	0.00828	0.00000	0.14933	0.00000
19	0.00120	0.00000	0.01308	0.00000	0.15861	0.00000
20	0.00164	0.00000	0.00896	0.00000	0.16501	0.00000
21	0.00381	0.00000	0.01416	0.00000	0.15504	0.00000
22	0.00332	0.00000	0.00683	0.00000	0.16772	0.00000
23	0.00148	0.00000	0.00712	0.00000	0.15907	0.00000
24	0.00143	0.00000	0.00780	0.00000	0.17053	0.00000
25	0.00187	0.00000	0.01140	0.00000	0.16164	0.00000
26	0.00301	0.00000	0.01000	0.00000	0.14745	0.00000
27	0.00160	0.00000	0.01268	0.00000	0.16139	0.00000
28	0.00153	0.00000	0.01249	0.00000	0.16050	0.00000
29	0.00116	0.00000	0.00828	0.00000	0.17918	0.00000
30	0.00349	0.00000	0.00790	0.00000	0.16814	0.00000

Table 1: Execution Times and Values for DeJong1 Function

First Improvement DeJong1 Function			
	n=5	n=10	n=30
Mean	0.00000	0.00000	0.00000
Standard deviation	0.00000	0.00000	0.00000
Min value	0.00000	0.00000	0.00000
Max value	0.00000	0.00000	0.00000
Mean Exec Time	0.00222	0.00959	0.16368

Table 2: Statistical Metadata for DeJong1 Function

First Improvement Schwefel's Function						
	n=5		n=10		n=30	
Run	Exec Time	Value	Exec Time	Value	Exec Time	Value
1	0.22005	-1963.58457	1.88083	-3620.82082	31.36657	-9972.42699
2	0.25337	-1895.42384	1.37679	-3553.27038	54.46308	-10117.82838
3	0.31470	-2026.44337	2.83632	-3672.20327	63.15746	-10098.03866
4	0.17987	-1895.21718	2.01945	-3671.50693	57.07085	-10074.10627
5	0.23403	-1895.32051	1.21268	-3553.58741	26.88362	-9908.81013
6	0.42240	-1942.13627	1.64915	-3554.96972	27.78871	-9913.44425
7	0.34961	-1845.34767	1.18644	-3528.06021	35.68515	-9900.61932
8	0.23752	-1832.77180	1.37459	-3565.13912	30.38384	-10250.59683
9	0.32962	-1942.03246	1.93691	-3580.02730	50.42015	-10139.37676
10	0.27868	-1832.77303	3.32570	-3621.84907	39.17737	-9897.28977
11	0.18940	-2026.13237	1.20790	-3785.86878	56.70790	-9866.48020
12	0.16568	-1788.87621	1.62400	-3715.19904	31.46630	-9890.92323
13	0.22761	-1942.23438	1.63854	-3571.39790	35.76434	-9841.68690
14	0.22427	-1963.68762	4.02922	-3674.10413	52.61720	-9954.71539
15	0.29545	-1941.54412	1.00224	-3518.21601	54.18685	-10029.35492
16	0.17774	-1895.32085	1.59547	-3599.70145	45.94618	-9913.01538
17	0.43838	-1945.46730	2.16475	-3568.26256	35.18758	-10117.92851
18	0.75831	-1845.24942	1.64398	-3404.65047	34.14007	-9942.71882
19	0.25680	-1949.48353	2.30578	-3608.02521	65.04696	-10187.32236
20	0.27475	-1895.21718	2.03484	-3679.54972	59.28268	-10169.11436
21	0.20702	-1895.32023	1.12520	-3531.87199	46.30176	-10013.26802
22	0.18077	-1929.55637	2.70709	-3571.69131	42.65926	-10536.25260
23	0.19539	-1942.23314	2.31867	-3656.71537	28.38801	-9557.61741
24	0.29330	-1873.76874	1.12968	-3603.39066	44.47309	-10052.89574
25	0.27876	-1907.90121	1.07277	-3514.26767	28.68698	-9837.68939
26	0.28711	-1877.10053	1.52516	-3802.62878	60.93455	-9871.69040
27	0.20410	-1941.43985	3.23874	-3696.18420	45.78391	-9880.74851
28	0.28369	-1992.10418	3.00370	-3706.33460	44.62545	-9959.48925
29	0.19697	-1915.24773	1.56931	-3837.45670	29.93154	-10140.04735
30	0.16877	-1873.17641	2.49165	-3584.35294	27.93248	-9933.83749

Table 3: Execution Times and Values for Schwefel's Function

First Improvement Schwefel's Function			
	n=5	n=10	n=30
Mean	-1913.73706	-3618.37679	-9998.97778
Standard deviation	54.31144	92.17866	169.62926
Min value	-2026.44337	-3837.4567	-10536.2526
Max value	-1788.87621	-3404.65047	-9557.61741
Mean Exec Time	0.27080	1.94091	42.88199

Table 4: Statistical Metadata for Schwefel's Function

First Improvement Rastrigin's Function						
	n=5		n=10		n=30	
Run	Exec Time	Value	Exec Time	Value	Exec Time	Value
1	0.17784	0.00000	0.87176	3.98489	21.04871	30.25265
2	0.17238	0.00000	1.67986	1.98992	60.36824	22.35831
3	0.41703	0.00000	1.98182	4.46156	36.74047	30.26781
4	0.20353	0.00000	1.53782	6.21061	23.83403	28.51371
5	0.38626	0.00000	0.90010	4.94329	28.65278	20.61045
6	0.23586	0.00000	2.01835	2.99498	30.48927	32.32840
7	0.17936	0.00000	2.00231	2.23583	37.13736	33.62691
8	0.35745	0.00000	1.12578	3.97983	40.20069	24.50372
9	0.20970	1.00001	1.18484	1.99497	22.97068	30.84145
10	0.35126	0.00000	1.05922	1.23582	40.42012	32.82067
11	0.27854	0.00000	1.48206	4.97479	24.19294	32.05308
12	0.18082	0.00000	1.45547	4.46662	22.89307	27.12297
13	0.15198	0.00000	1.06156	3.47166	21.34545	27.84632
14	0.40720	0.00000	2.22996	4.46156	37.54857	32.58008
15	0.31945	0.00000	1.52358	2.23078	34.12586	30.16261
16	0.26112	0.00000	0.89669	2.23583	40.29886	32.30266
17	0.17116	0.00000	2.23078	2.23078	23.06366	31.61037
18	0.22235	0.00000	1.74785	3.22574	52.83899	31.52886
19	0.45841	0.00000	0.91255	2.98993	25.66038	26.85137
20	0.17424	0.00000	1.62503	4.70731	25.60464	33.29274
21	0.41724	0.00000	2.24972	4.22070	24.98043	32.09654
22	0.27887	0.00000	1.52852	3.70747	43.06835	26.82612
23	0.27361	0.00000	1.67452	3.22574	25.55663	31.11367
24	0.45757	0.00000	1.28391	5.70244	29.08663	28.79079
25	0.25839	0.00000	1.18523	3.46661	29.76555	27.38479
26	0.29277	0.00000	1.95947	1.18523	29.16694	30.03550
27	0.21434	0.00000	0.97270	5.94330	49.00377	31.32807
28	0.28306	0.00000	1.37141	2.00002	26.65875	24.08211
29	0.16938	0.00000	1.44591	6.23586	18.89162	30.06700
30	0.15041	0.00000	1.82596	3.46661	31.77395	28.65211

Table 5: Execution Times and Values for Rastrin's Function

First Improvement Rastrigin's Function			
	n=5	n=10	n=30
Mean	0.03333	3.60602	29.39506
Standard deviation	0.17951	1.40487	3.22901
Min value	0.00000	1.18523	20.61045
Max value	1.00001	6.23586	33.62691
Mean Exec Time	0.27038	1.50082	31.91291

Table 6: Statistical Metadata for Rastrigin's Function

First Improvement Michalewicz's Function						
	n=5		n=10		n=30	
Run	Exec Time	Value	Exec Time	Value	Exec Time	Value
1	0.23000	-3.69589	1.50268	-8.05845	51.93340	-24.69244
2	0.29765	-3.69303	3.08919	-8.15709	34.14773	-24.60064
3	0.34822	-3.68218	1.15479	-8.19583	39.63000	-24.97615
4	0.21138	-3.69090	1.50652	-8.24985	29.45248	-24.46929
5	0.32963	-3.68219	1.38356	-8.03346	38.57045	-25.12318
6	0.23061	-3.69824	2.99122	-8.13684	26.71644	-24.74443
7	0.19102	-3.69704	1.20359	-7.93496	56.59268	-24.17205
8	0.23558	-3.69445	2.38769	-8.06856	51.44299	-25.10683
9	0.23284	-3.69846	1.18980	-8.13387	31.94784	-24.86940
10	0.24996	-3.69684	2.07093	-8.24863	42.87130	-24.31308
11	0.27854	-3.69819	1.83524	-8.12391	68.61527	-24.54519
12	0.43536	-3.69885	1.76827	-8.18346	45.97276	-24.36908
13	0.28168	-3.69589	2.00788	-8.13519	43.22863	-24.29408
14	0.29075	-3.69879	2.35185	-8.38581	63.08281	-25.02769
15	0.29563	-3.69825	1.52624	-8.32668	76.24133	-24.94329
16	0.27340	-3.69805	1.78963	-8.07905	40.92424	-24.84765
17	0.54984	-3.69529	2.28276	-8.18195	41.36076	-24.32837
18	0.29918	-3.69825	2.87741	-8.27489	28.47432	-23.89163
19	0.21301	-3.69826	3.49240	-8.52691	28.26041	-25.11756
20	0.37213	-3.69742	1.96627	-8.30009	36.20646	-24.22773
21	0.62662	-3.69884	4.07485	-8.19763	66.93441	-25.40706
22	0.21319	-3.69886	1.32656	-8.34381	80.11522	-25.27037
23	0.21326	-3.68160	1.19561	-7.91652	38.66578	-24.58744
24	0.35194	-3.69491	2.06398	-8.23768	37.58608	-25.24476
25	0.33724	-3.69826	1.39747	-8.17119	89.88674	-24.85194
26	0.26643	-3.68219	2.49190	-8.23697	33.81859	-24.22212
27	0.45714	-3.69456	1.79619	-8.13232	29.77417	-23.91997
28	0.32507	-3.69780	1.21621	-8.16325	35.52537	-24.92325
29	0.69865	-3.69684	1.50080	-8.26525	47.52993	-25.70863
30	0.35072	-3.68159	3.39463	-8.25408	64.06368	-24.92407

Table 7: Execution Times and Values for Michalewicz's Function

First Improvement Michalewicz's Function			
	n=5	n=10	n=30
Mean	-3.69439	-8.18847	-24.723979
Standard deviation	0.00586	0.12416	0.43640
Min value	-3.69886	-8.52691	-25.70863
Max value	-3.68159	-7.91652	-23.89163
Mean Exec Time	0.32288	2.02787	46.65240

Table 8: Statistical Metadata for Michalewicz's Function

0.4.2 Hill Climbing Best Improvement

Best Improvement DeJong1						
	n=5		n=10		n=30	
Run	Exec Time	Value	Exec Time	Value	Exec Time	Value
1	0.00613	0.00000	0.01377	0.00000	0.31849	0.00000
2	0.00322	0.00000	0.02253	0.00000	0.31320	0.00000
3	0.00346	0.00000	0.01264	0.00000	0.28734	0.00000
4	0.00204	0.00000	0.01324	0.00000	0.28344	0.00000
5	0.00665	0.00000	0.01587	0.00000	0.30828	0.00000
6	0.00604	0.00000	0.02014	0.00000	0.30003	0.00000
7	0.00228	0.00000	0.02132	0.00000	0.34062	0.00000
8	0.00560	0.00000	0.02399	0.00000	0.29518	0.00000
9	0.00480	0.00000	0.02104	0.00000	0.32584	0.00000
10	0.00432	0.00000	0.02023	0.00000	0.30991	0.00000
11	0.00745	0.00000	0.01461	0.00000	0.30432	0.00000
12	0.00655	0.00000	0.02056	0.00000	0.29067	0.00000
13	0.00223	0.00000	0.02184	0.00000	0.31401	0.00000
14	0.00218	0.00000	0.01520	0.00000	0.30807	0.00000
15	0.00222	0.00000	0.01232	0.00000	0.31904	0.00000
15	0.00210	0.00000	0.01302	0.00000	0.31904	0.00000
16	0.00559	0.00000	0.02031	0.00000	0.30262	0.00000
17	0.00574	0.00000	0.01468	0.00000	0.32546	0.00000
18	0.00259	0.00000	0.01242	0.00000	0.30654	0.00000
19	0.00185	0.00000	0.02090	0.00000	0.30772	0.00000
20	0.00324	0.00000	0.02174	0.00000	0.29813	0.00000
21	0.00204	0.00000	0.01234	0.00000	0.29620	0.00000
22	0.00197	0.00000	0.01797	0.00000	0.27755	0.00000
23	0.00465	0.00000	0.01200	0.00000	0.29171	0.00000
24	0.00533	0.00000	0.01894	0.00000	0.28920	0.00000
25	0.00231	0.00000	0.02077	0.00000	0.29281	0.00000
26	0.00208	0.00000	0.01783	0.00000	0.31103	0.00000
27	0.00605	0.00000	0.01672	0.00000	0.32462	0.00000
28	0.00418	0.00000	0.01840	0.00000	0.29679	0.00000
29	0.00496	0.00000	0.02026	0.00000	0.29683	0.00000
30	0.00443	0.00000	0.01241	0.00000	0.30107	0.00000

Table 9: Execution Times and Values for DeJong1

Best Improvement DeJong1 Function			
	n=5	n=10	n=30
Mean	0.00000	0.00000	0.00000
Standard deviation	0.00000	0.00000	0.00000
Min value	0.00000	0.00000	0.00000
Max value	0.00000	0.00000	0.00000
Mean Exec Time	0.004	0.01741	0.30502

Table 10: Statistical Metadata for DeJong1 Function

Best Improvement Schwefel's Function						
	n=5		n=10		n=30	
Run	Exec Time	Value	Exec Time	Value	Exec Time	Value
1	0.29725	-2094.80951	3.47359	-3918.19215	44.72101	-10624.33781
2	0.45989	-2094.81015	1.82281	-3913.68773	43.69755	-11022.03942
3	0.37088	-2094.70709	2.35574	-3951.22593	92.89173	-11106.90422
4	0.39308	-2094.70585	5.24730	-3975.61563	74.12366	-11225.81427
5	0.32178	-2026.23604	4.79834	-4059.82337	97.09341	-11255.88891
6	0.40186	-2060.57490	2.74606	-3882.75610	88.15183	-10801.86771
7	0.33650	-2060.57523	2.59003	-3939.95081	83.11402	-11255.79971
8	0.40758	-2094.80889	2.76336	-3905.92261	69.10894	-10739.28173
9	0.38923	-2094.81014	2.29593	-3962.93762	50.38746	-11148.62634
10	0.33224	-2094.80890	3.00757	-4002.01445	74.86602	-10939.76248
11	0.26182	-2094.49914	2.43932	-3918.29087	93.36829	-10793.48534
12	0.44460	-2094.70585	2.72451	-3918.39994	83.40258	-10887.47992
13	0.33654	-2026.44304	4.05697	-3884.26576	65.98159	-10995.82660
14	0.28378	-2094.60281	1.97722	-3799.67957	84.21406	-11182.24760
15	0.48643	-2094.70646	2.63477	-3852.64482	41.49002	-10742.44294
16	0.47902	-2067.82348	2.01936	-4162.42753	49.42364	-10820.45406
17	0.54139	-2060.47156	3.28314	-3828.45570	99.84226	-11128.00841
18	0.33879	-1976.36625	5.24596	-4067.15976	68.29416	-10806.34786
19	0.58067	-2067.82408	2.84674	-3883.47371	40.72814	-11328.10411
20	0.54474	-2094.80918	2.10901	-3918.18659	45.77864	-10802.48850
21	0.38310	-2060.57398	2.80862	-4121.04493	41.63585	-11257.81395
22	0.67571	-2094.80889	2.65082	-4036.73229	83.54422	-10761.80927
23	0.47786	-2094.60281	2.97427	-3921.01345	42.06545	-10671.54435
24	0.52268	-2094.91256	2.04201	-4002.29746	57.33925	-10806.67624
25	0.61273	-2067.92713	3.09261	-3914.48745	42.52436	-11182.38891
26	0.41676	-2094.70614	2.03806	-3794.62978	80.32907	-10942.92760
27	0.33976	-2026.44150	2.23669	-3914.38337	77.01776	-10770.04136
28	0.28088	-2094.80951	3.14257	-3928.90880	46.66631	-11015.78179
29	0.60514	-2060.57462	4.05481	-3897.35813	81.10453	-11106.31895
30	0.37751	-2094.70585	1.93304	-3918.29565	44.05271	-10992.51138

Table 11: Execution Times and Values for Schwefel's Function

Best Improvement Schwefel's Function			
	n=5	n=10	n=30
Mean	-2074.91584	-3941.81087	-10966.78591
Standard deviation	29.15409	84.34130	190.20069
Min value	-2094.91318	-4162.42753	-11257.81395
Max value	-1976.36625	-3796.357	-10624.33781
Mean Exec Time	0.42874	2.94312	66.25076

Table 12: Statistical Metadata for Schwefel's Function

Best Improvement Rastrigin's Function						
	n=5		n=10		n=30	
Run	Exec Time	Value	Exec Time	Value	Exec Time	Value
1	0.26013	0.00000	3.47359	3.23585	53.17860	22.33340
2	0.26020	0.00000	1.73037	2.98993	55.98731	24.86145
3	0.26863	0.00000	1.81470	2.23583	47.75973	23.55911
4	0.32301	0.00000	2.76587	0.00000	84.44402	23.00076
5	0.34751	0.99496	4.38606	2.47165	65.74721	20.52912
6	0.25112	0.00000	1.48665	2.98993	63.73100	23.10351
7	0.27680	0.00000	1.84384	1.48665	106.46122	23.14390
8	0.33579	0.00000	2.26821	3.47166	36.57114	18.07229
9	0.53795	0.00000	2.34376	1.98992	50.17492	28.30786
10	0.33224	0.99496	3.31032	2.99498	65.82123	21.13379
11	0.24736	0.00000	1.80215	2.23078	66.45741	24.34439
12	0.26447	0.00000	2.24853	2.99498	40.32637	24.30815
13	0.30430	0.00000	2.81168	2.98993	51.50656	25.83653
14	0.35447	0.00000	1.97722	2.98993	59.27179	17.86141
15	0.57809	0.00000	4.99574	2.23078	68.36305	24.07708
16	0.47902	0.00000	2.23613	3.23079	67.03362	22.57393
17	0.37857	0.00000	2.46606	3.23079	71.51773	22.62056
18	0.41584	0.00000	2.84213	2.47165	78.50611	25.03700
19	0.41771	0.00000	3.03534	4.23081	38.62202	27.02066
20	0.41861	0.00000	4.05393	2.23078	61.16983	21.31319
21	0.38310	0.00000	1.88594	3.46660	69.45578	25.26151
22	0.27501	0.00000	2.09121	4.00003	70.64727	20.36993
23	0.32203	0.00000	2.23503	1.23582	39.59005	24.59063
24	0.40694	0.00000	2.35517	3.22574	33.36846	27.01561
25	0.41062	0.00000	2.61623	0.99496	45.65379	25.98563
26	0.51139	0.00000	1.55289	2.99498	68.39688	17.66213
27	0.26971	0.00000	1.70810	2.23078	77.21157	24.32451
28	0.28663	0.00000	1.96868	2.23078	63.27822	26.22650
29	0.28641	0.00000	2.17610	3.23079	71.87747	20.30341
30	0.32391	0.00000	2.90201	3.46660	44.05271	17.28164

Table 13: Execution Times and Values for Rastrigin's Function

Best Improvement Rastrigin's Function			
	n=5	n=10	n=30
Mean	0.06633	2.51278	23.06865
Standard deviation	0.24819	0.82836	2.88098
Min value	0.00000	1.48665	17.28164
Max value	0.99496	4.99574	28.30786
Mean Exec Time	0.35091	2.76706	60.53943

Table 14: Statistical Metadata for Rastrigin's Function

Best Improvement Michalewicz's Function						
	n=5		n=10		n=30	
Run	Exec Time	Value	Exec Time	Value	Exec Time	Value
1	0.29329	-3.69582	2.92241	-8.32853	73.34371	-25.64903
2	0.39425	-3.69826	3.55060	-8.63569	83.83598	-26.07203
3	0.41998	-3.69786	4.37895	-8.16545	86.67417	-25.58014
4	0.48144	-3.69826	4.95905	-8.56103	101.64337	-25.49970
5	0.58314	-3.69826	5.07899	-8.39643	110.91700	-25.21938
6	0.29473	-3.68219	1.88182	-8.30047	60.65032	-25.39903
7	0.37159	-3.69886	2.40415	-8.47942	66.67528	-25.92071
8	0.39516	-3.69283	2.47376	-8.52305	72.79092	-25.43088
9	0.71674	-3.69886	3.71635	-8.31951	82.02088	-25.30854
10	0.51589	-3.69886	3.74394	-8.54320	107.59938	-26.63946
11	0.32216	-3.69445	2.04463	-8.37624	51.04965	-25.57345
12	0.40040	-3.69886	2.24853	-8.16779	56.74082	-25.38287
13	0.41787	-3.69886	2.81168	-8.25260	70.33554	-25.56874
14	0.52603	-3.69884	3.25013	-8.38378	105.16546	-25.58583
15	0.74717	-3.69885	4.28603	-8.35041	104.09560	-25.91578
16	0.42164	-3.69886	1.86819	-8.32989	53.67721	-25.56322
17	0.43331	-3.69886	2.04343	-8.40369	66.46049	-25.31225
18	0.54530	-3.69826	2.42247	-8.30977	81.71037	-25.33821
19	0.61896	-3.69825	2.46813	-8.23558	82.91384	-25.88654
20	0.32443	-3.69886	3.77682	-8.50252	90.39622	-26.30779
21	0.44489	-3.69818	2.35517	-8.40209	48.89038	-25.51610
22	0.47919	-3.69885	3.17195	-8.55024	56.00378	-25.60897
23	0.55134	-3.69826	2.23503	-8.48725	57.98489	-25.71713
24	0.80689	-3.69879	3.37021	-8.48725	60.75401	-25.52362
25	0.41062	-3.69386	4.36821	-8.46246	69.14401	-25.17724
26	0.42519	-3.69589	2.75393	-8.18843	58.75530	-26.23847
27	0.48559	-3.69879	3.54869	-8.26331	61.34152	-25.93129
28	0.53124	-3.69886	4.66414	-8.25990	62.49364	-25.20858
29	0.28641	-3.69884	5.05820	-8.65284	94.03629	-25.61435
30	0.34634	-3.69884	5.16433	-8.37391	102.40377	-25.80369

Table 15: Execution Times and Values for Michalewicz's Function

Best Improvement Michalewicz's Function			
	n=5	n=10	n=30
Mean	-3.69740	-8.38411	-25.62190
Standard deviation	0.00325	0.12800	0.28674
Min value	-3.69886	-8.65284	-26.30779
Max value	-3.68219	-8.16545	-25.17724
Mean Exec Time	0.48724	3.30066	76.01679

Table 16: Statistical Metadata for Michalewicz's Function

0.4.3 Hill Climbing Worst Improvement

Worst Improvement DeJong1						
	n=5		n=10		n=30	
Run	Exec Time	Value	Exec Time	Value	Exec Time	Value
1	0.13356	1.63842	0.92072	1.63842	23.04920	1.63842
3	0.15576	1.63842	1.01443	1.63842	23.77287	1.63842
4	0.17652	1.63842	1.01723	1.63842	25.85109	1.63842
5	0.18802	1.63842	1.06555	1.63842	26.27783	1.63842
6	0.18853	1.63842	1.07414	1.63842	26.94901	1.63842
7	0.19722	1.63842	1.15004	1.63842	27.20221	1.63842
8	0.20134	1.63842	1.17827	1.63842	33.03097	1.63842
9	0.20374	1.63842	1.24211	1.63842	39.60191	1.63842
10	0.22077	1.63842	1.29467	1.63842	48.67927	1.63842
11	0.25032	0.13149	1.32298	1.63842	20.38076	1.63842
12	0.13723	1.63842	0.86019	1.63842	23.97130	1.63842
13	0.16488	1.63842	0.92093	1.63842	24.17991	1.63842
14	0.16774	1.63842	0.98829	1.63842	24.41074	1.63842
15	0.17254	1.63842	0.99436	1.63842	26.34703	1.63842
16	0.17495	1.63842	1.13427	1.63842	29.37388	1.63842
17	0.18088	1.63842	1.14796	1.63842	34.01603	16.38408
18	0.18188	1.63842	1.20727	1.63842	35.57566	1.63842
19	0.18732	1.63842	1.21093	1.63842	35.86339	1.63842
20	0.19342	1.63842	1.26986	1.63842	39.41592	1.63842
21	0.21768	1.43516	1.30800	1.63842	22.32065	1.63842
22	0.16184	1.63842	1.07392	1.63842	23.59385	1.63842
23	0.17161	1.63842	1.09827	1.63842	24.58912	1.63842
24	0.17508	1.63842	1.15197	1.63842	28.43609	16.38408
25	0.17757	1.63842	1.17155	1.63842	30.91123	1.63842
26	0.18093	1.63842	1.20805	1.63842	31.86240	1.63842
27	0.18293	1.63842	1.24341	1.63842	31.89504	1.63842
28	0.18093	1.63842	1.37751	1.63842	33.39252	1.63842
29	0.18293	1.63842	1.39212	1.63842	35.91008	1.63842
30	0.18439	1.63842	1.50780	1.63842	44.16631	1.63842

Table 17: Execution Times and Values for DeJong1 Function

Worst Improvement DeJong1 Function			
	n=5	n=10	n=30
Mean	1.57734	1.63842	2.69168
Standard deviation	0.28080	0.00000	3.79759
Min value	0.13149	1.63842	1.63842
Max value	1.63842	1.63842	16.38408
Mean Exec Time	0.18250	1.15678	30.17331

Table 18: Statistical Metadata for DeJong1 Function

Worst Improvement Schwefel's Function						
	n=5		n=10		n=30	
Run	Exec Time	Value	Exec Time	Value	Exec Time	Value
1	0.68319	-1300.53279	4.59604	-2637.93644	107.53652	-9047.12721
3	0.92654	-1432.91163	4.61666	-2798.55744	126.24992	-9138.17776
4	0.94528	-1399.08774	4.65842	-2748.94820	128.62090	-8772.95016
5	0.94774	-1253.87452	5.09426	-2758.09061	165.42738	-9332.25426
6	1.00621	-1303.51089	5.12189	-2749.64117	185.37715	-9153.66891
7	1.06205	-1677.21010	5.39825	-3197.92037	185.92121	-9204.09978
8	1.16538	-1432.22408	5.67011	-2893.33828	212.58254	-9044.31722
9	1.27754	-1529.60926	6.26064	-2890.83278	266.19550	-8996.44756
10	1.28223	-1459.73663	6.51767	-2983.50755	296.51308	-9619.54903
11	1.40764	-1543.95600	6.56616	-2842.09464	99.72047	-9105.50074
12	0.69917	-1340.28412	3.84870	-2324.27550	115.38442	-8977.61668
13	0.70240	-1294.95188	6.19440	-2671.35001	121.82721	- 8181.11891
14	0.72940	-1307.89467	6.57458	-2880.08211	123.84535	-7530.48396
15	0.76438	-1307.53225	6.73535	-3021.64725	126.34707	-9034.55174
16	0.89234	-1341.99442	6.75493	-2838.39182	133.90167	-9566.30289
17	0.97434	-1757.07110	7.48608	-3010.16890	140.66444	-9456.20476
18	1.00447	-1579.85630	7.59140	-3076.25888	155.17044	-9292.02273
19	1.04993	-1568.81969	8.18679	-3255.45134	170.67543	-8757.54655
20	1.08931	-1675.96466	8.50850	-2533.80057	199.18074	-9145.10990
21	1.10126	-1522.35448	8.88573	-3010.16890	104.39232	-8714.75900
22	0.42063	-1443.60875	3.08743	-1711.55358	118.45504	-9818.79078
23	0.73006	-1478.06028	3.59981	-2649.63614	137.00498	-9239.20855
24	0.84173	-1480.82202	4.00873	-2752.05328	138.88719	-9515.75480
25	0.87281	-1319.07445	4.03385	-2952.59290	141.29329	-9321.66272
26	0.87996	-1442.61861	4.28041	-2856.02591	151.68149	-8497.00325
27	0.95544	-1671.73552	5.18799	-3163.30324	181.43283	-8415.00431
28	0.96781	-1544.50850	5.21693	-2736.75230	187.08880	-9362.15958
29	1.03279	-1297.02085	5.48949	-2745.31131	237.18359	-9450.32139
30	1.18104	-1355.54801	8.57836	-2916.31685	269.52340	-9794.93079

Table 19: Execution Times and Values for Schwefel's Function

Worst Improvement Schwefel's Function			
	n=5	n=10	n=30
Mean	-1450.42669	-2814.000285	-9085.67744
Standard deviation	135.38488	285.85183	480.58292
Min value	-1757.0711	-3255.45134	-9818.79078
Max value	-1253.87452	-1711.55358	-7530.48396
Mean Exec Time	0.95148	5.81895	163.03739

Table 20: Statistical Metadata for Schwefel's Function

Worst Improvement Rastrigin's Function						
	n=5		n=10		n=30	
Run	Exec Time	Value	Exec Time	Value	Exec Time	Value
1	0.85386	6.96458	5.27676	13.67130	158.75792	60.12555
3	1.02243	6.96458	5.54906	13.20046	177.59632	63.84640
4	1.24455	4.96455	6.09831	17.88728	191.44669	51.51761
5	1.32519	4.96455	7.01665	14.96389	194.92914	60.03115
6	1.55708	7.20039	7.50141	16.88727	228.13226	49.12111
7	1.56254	7.20039	7.76691	17.88728	185.92121	45.96935
8	1.59961	5.96456	8.45660	16.92374	253.37179	47.04444
9	1.63190	6.96458	14.45509	13.96464	255.33536	47.28025
10	1.86316	6.96458	14.73910	13.19968	289.45483	42.19917
11	0.77256	6.96458	16.27648	14.12382	367.50122	53.97332
12	0.97773	4.96455	5.76451	20.88729	135.55218	61.88445
13	1.07986	9.96459	6.27174	15.12384	146.53483	53.04674
14	1.12518	5.96439	7.22152	15.96464	150.35063	46.73320
15	1.34071	5.96456	7.45450	11.20043	180.77622	63.88608
16	1.41011	9.96383	7.52344	24.12160	188.11674	44.33819
17	1.41902	5.96456	8.48314	14.96389	195.42030	68.11800
18	1.44200	5.96456	9.21563	13.96388	216.53219	49.88450
19	1.46412	10.96384	9.49381	17.88728	239.25134	62.96716
20	1.65391	8.96382	12.13859	11.88782	250.09196	56.04372
21	0.74066	6.96458	13.27139	18.19895	332.20481	45.12180
22	0.86343	9.85651	6.28644	13.19968	143.74195	52.10153
23	1.00600	6.96458	6.36481	8.96460	169.41023	59.73131
24	1.27220	5.96456	6.72848	16.08446	180.87987	47.12104
25	1.40909	8.20040	7.51404	21.88654	187.66778	55.43403
26	7.96459	7.96459	7.97983	16.88727	200.05880	51.36009
27	1.49252	6.96458	9.18252	18.12309	204.98770	57.19608
28	1.54148	7.20039	9.27505	14.88803	226.18446	56.89201
29	1.80026	5.96439	9.77089	23.89280	227.79306	42.97007
30	2.00275	5.96456	9.79084	19.19973	272.71050	40.12254

Table 21: Execution Times and Values for Rastrigin's Function

Worst Improvement Rastrigin's Function			
	n=5	n=10	n=30
Mean	7.09673	16.20810	52.96761
Standard deviation	1.53734	3.48686	7.37994
Min value	4.96455	8.9646	40.12254
Max value	10.96384	24.1216	68.118
Mean Exec Time	1.56684	8.71957	212.09352

Table 22: Statistical Metadata for Schwefel's Function

Worst Improvement Michalewicz's Function						
	n=5		n=10		n=30	
Run	Exec Time	Value	Exec Time	Value	Exec Time	Value
1	0.08166	-1.56629	0.14566	-3.29077	0.59555	-5.81494
3	0.09136	-2.78576	0.20762	-3.12039	0.69586	-5.61501
4	0.09252	-1.70314	0.23054	-3.12063	0.72451	-6.72539
5	0.09442	-1.57625	0.24030	-3.72687	0.75621	-6.88242
6	0.12076	-2.27784	0.24034	-3.45423	0.76847	-6.07011
7	0.12129	-2.33897	0.25971	-2.87830	0.81397	-6.15618
8	0.13195	-2.51445	0.27636	-3.54013	0.87251	-6.63409
9	0.15070	-2.16332	0.29492	-3.17693	1.15896	-6.34114
10	0.15874	-1.97379	0.34466	-3.34541	1.72902	-6.09411
11	0.15839	-2.15585	0.35223	-3.47836	1.73471	-6.34837
12	0.06556	-1.77411	0.15703	-2.63715	0.82186	-6.25067
13	0.07846	-1.76959	0.18995	-2.97931	1.00658	-6.77565
14	0.08556	-1.84016	0.20391	-3.38884	1.04553	-6.65090
15	0.08551	-1.82837	0.23987	-2.86321	1.11036	-6.45628
16	0.08683	-1.77462	0.24988	-2.54679	1.21100	-7.36913
17	0.11618	-2.65018	0.25405	-3.19003	1.24299	-6.06689
18	0.11697	-2.38738	0.25712	-2.90013	1.29207	-6.53323
19	0.11988	-2.20507	0.26592	-3.93270	1.31075	-5.66167
20	0.13445	-2.31396	0.26907	-4.21690	1.64831	-6.85341
21	0.16174	-2.88536	0.33805	-4.00599	1.65124	-6.01430
22	0.06071	-2.07520	0.16377	-3.67814	-6.35776	-6.35776
23	0.06182	-2.37825	0.16970	-3.23506	0.83934	-6.24393
24	0.07386	-1.92797	0.17802	-2.83108	0.97144	-6.57229
25	0.07328	-2.15388	0.18195	-3.66274	1.07054	-6.49107
26	0.07605	-2.22846	0.18540	-3.53398	1.19264	-7.12472
27	0.08892	-2.14630	0.20996	-3.14463	1.28765	-8.02400
28	0.11201	-2.17406	0.23123	-3.17753	1.30646	-6.68954
29	0.12973	-2.31359	0.25750	-3.44933	1.36340	-6.62203
30	0.13164	-2.08311	0.28575	-3.48355	1.38518	-6.10639

Table 23: Execution Times and Values for Michalewicz's Function

Worst Improvement Michalewicz's Function			
	n=5	n=10	n=30
Mean	-2.13673	-3.30996	-6.46709
Standard deviation	0.32871	0.39042	0.49593
Min value	-2.88536	-4.2169	-8.024
Max value	10.96384	-2.54679	-5.61501
Mean Exec Time	0.10555	0.23725	1.12882

Table 24: Statistical Metadata for Michalewicz's Function

0.4.4 Simulated Annealing

Simulated Annealing DeJong1 Function						
	n=5		n=10		n=30	
Run	Exec Time	Value	Exec Time	Value	Exec Time	Value
1	0.00121	0.00000	0.00282	0.00000	0.00647	0.00000
2	0.00338	0.00000	0.00256	0.00000	0.01338	0.00000
3	0.00362	0.00000	0.00538	0.00000	0.01554	0.00000
4	0.00368	0.00000	0.00598	0.00000	0.01474	0.00000
5	0.00412	0.00000	0.00629	0.00000	0.01665	0.00000
6	0.00424	0.00000	0.00690	0.00000	0.01559	0.00000
7	0.00549	0.00000	0.00706	0.00000	0.01671	0.00000
8	0.00550	0.00000	0.00729	0.00000	0.01837	0.00000
9	0.00617	0.00000	0.00814	0.00000	0.01935	0.00000
10	0.00596	0.00000	0.00755	0.00000	0.01847	0.00000
11	0.00148	0.00000	0.00332	0.00000	0.00672	0.00000
12	0.00163	0.00000	0.00406	0.00000	0.00956	0.00000
13	0.00445	0.00000	0.00569	0.00000	0.00878	0.00000
14	0.00441	0.00000	0.00597	0.00000	0.01078	0.00000
15	0.00344	0.00000	0.00672	0.00000	0.01257	0.00000
16	0.00292	0.00000	0.00638	0.00000	0.01233	0.00000
17	0.00426	0.00000	0.00674	0.00000	0.01399	0.00000
18	0.00499	0.00000	0.00717	0.00000	0.01340	0.00000
19	0.00283	0.00000	0.00676	0.00000	0.01869	0.00000
20	0.00464	0.00000	0.00665	0.00000	0.01790	0.00000
21	0.00205	0.00000	0.00259	0.00000	0.00802	0.00000
22	0.00132	0.00000	0.00287	0.00000	0.00755	0.00000
23	0.00390	0.00000	0.00286	0.00000	0.00903	0.00000
24	0.00369	0.00000	0.00292	0.00000	0.00936	0.00000
25	0.00291	0.00000	0.00287	0.00000	0.01082	0.00000
26	0.00340	0.00000	0.00599	0.00000	0.01173	0.00000
27	0.00343	0.00000	0.00501	0.00000	0.01063	0.00000
28	0.00524	0.00000	0.00640	0.00000	0.01135	0.00000
29	0.00399	0.00000	0.00754	0.00000	0.01121	0.00000
30	s	0.00000	0.00353	0.00000	0.00969	0.00000

Table 25: Execution Times and Values for DeJong1 Function

Simulated Annealing DeJong1 Function			
	n=5	n=10	n=30
Mean	0.00000	0.00000	0.00000
Standard deviation	0.00000	0.00000	0.00000
Min value	0.00000	0.00000	0.00000
Max value	0.00000	0.00000	0.00000
Mean Exec Time	0.00373	0.00540	0.01264

Table 26: Statistical Metadata for DeJong1 Function

Simulated Annealing Schwefel's Function						
	n=5		n=10		n=30	
Run	Exec Time	Value	Exec Time	Value	Exec Time	Value
1	0.05822	-1907.89581	0.07939	-3184.83851	0.27762	-8726.25454
2	0.05918	-1892.65945	0.11308	-3339.55293	0.28722	-8662.41087
3	0.07604	-1929.55576	0.11467	-3358.47519	0.53573	-8600.37841
4	0.07792	-1787.84231	0.11491	-3124.04749	0.60948	-8730.04800
5	0.07945	-1907.89581	0.14443	-3052.12378	0.62637	-8974.41375
6	0.08605	-1776.49790	0.14873	-3307.01276	0.69992	-8463.37485
7	0.08580	-1805.78639	0.15257	-3228.66343	0.70118	-8986.64213
8	0.08732	-1868.43785	0.15884	-3284.65107	0.78861	-8816.58737
9	0.09975	-1805.99276	0.18088	-3392.28636	0.84461	-8955.53287
10	0.10502	-1929.65880	0.21540	-3328.25201	0.79403	-9180.70195
11	0.05492	-1771.75757	0.11553	-3256.66104	0.42869	-9773.03541
12	0.06528	-1910.02558	0.14904	-3212.17153	0.50297	-8980.38436
13	0.06807	-1771.82042	0.16274	-3529.69716	0.51445	-8751.69227
14	0.07454	-1771.65452	0.16535	-3222.53809	0.51847	-9318.17292
15	0.08381	-1861.08532	0.18448	-3215.37462	0.56556	-9013.85337
16	0.09015	-1788.04901	0.19994	-3225.06679	0.61163	-8831.76818
17	0.09673	-1802.65042	0.20763	-3197.01334	0.65229	-9048.57155
18	0.09778	-1822.07722	0.22698	-3369.80847	0.70159	-8831.97989
19	0.10167	-1707.37893	0.27230	-3322.10737	0.72117	-9025.14289
20	0.12507	-1798.43385	0.09300	-3206.20922	0.73028	-8995.42385
21	0.03552	-1792.17450	0.13344	-3153.20252	0.50707	-8525.55931
22	0.06185	-1753.81349	0.14390	-3345.03554	0.50878	-9131.73610
23	0.06251	-1832.66876	0.14972	-3416.36112	0.51914	-8604.44894
24	0.06422	-1784.23490	0.16696	-3259.50149	0.53505	-8656.75082
25	0.07471	-1788.04964	0.16913	-3122.69057	0.59986	-8708.62838
26	0.07327	-1725.29451	0.18508	-3319.10059	0.62358	-8801.61742
27	0.08676	-1819.18671	0.18651	-3425.56130	0.72771	-8484.30552
28	0.08739	-1741.51108	0.22172	-3286.77189	0.73930	-8907.54711
29	0.09380	-1823.10025	0.24289	-3237.61768	0.79416	-9132.54448
30	0.10146	-1742.15789	0.13765	-3315.15026	1.06239	-9028.47997

Table 27: Execution Times and Values for Schwefel's Function

Simulated Annealing Schwefel's Function			
	n=5	n=10	n=30
Mean	-1813.97824	-3274.584804	-8888.26624
Standard deviation	60.39023	100.41182	268.23234
Min value	-1929.6588	-3529.69716	-9773.03541
Max value	-1707.37893	-3052.12378	-8463.37485
Mean Exec Time	0.08047	0.16456	0.62429

Table 28: Statistical Metadata for Schwefel's Function

Simulated Annealing Rastrigin's Function						
	n=5		n=10		n=30	
Run	Exec Time	Value	Exec Time	Value	Exec Time	Value
1	2.50789	0.00908	7.89083	0.09158	21.42734	28.02784
2	3.54673	0.00978	9.53419	1.19166	29.47141	28.58942
3	4.10104	0.00260	10.30637	1.64249	41.86525	27.26722
4	4.55528	0.01180	10.76559	1.14044	45.45782	31.63052
5	4.78116	0.00599	10.96360	2.21149	45.83838	33.83076
6	5.18758	0.00737	11.14862	2.20149	48.02300	34.39029
7	5.54075	0.01066	11.16513	2.28612	27.74407	28.57579
8	6.37996	0.00729	11.16513	2.28372	50.16146	27.59639
9	7.04077	0.00582	11.75510	1.18447	50.93798	32.55462
10	7.18371	0.00163	14.04355	1.33428	54.84017	29.74062
11	2.91864	0.01466	14.71748	0.33819	55.80583	14.44787
12	3.30090	0.02019	7.30720	0.73850	28.02134	34.66966
13	3.67086	0.00785	8.58474	1.27245	29.35947	20.55578
14	3.97398	0.02300	9.88239	1.22617	33.65861	31.52639
15	4.23755	0.00611	10.58889	1.40354	33.92484	31.45774
16	5.22251	0.00921	11.73387	1.34947	35.97044	28.81303
17	5.61802	0.01111	12.40122	1.18349	43.79178	29.45276
18	5.66794	0.00255	12.79079	1.38156	73.01994	30.99032
19	6.42522	0.00068	14.45029	0.44130	48.04115	27.63970
20	7.29561	0.00346	14.95100	0.52980	40.19483	30.36315
21	3.89434	0.01007	19.22340	0.16748	3 5.51393	30.13447
22	3.84536	0.00159	7.53937	1.12960	33.19477	30.01572
23	4.36169	0.00718	7.75308	1.63318	36.71213	29.89968
24	4.60613	0.00620	7.78616	1.67011	45.26357	26.54602
25	5.18452	0.01021	8.93987	1.48512	47.21455	21.80306
26	5.37133	0.00768	9.01791	2.53979	56.35840	28.46491
27	6.39561	0.00536	9.18319	1.85477	66.65799	28.48948
28	6.52483	0.01033	10.34997	0.57316	21.30880	33.94005
29	6.92136	0.00363	12.16639	1.50928	31.11797	28.88562
30	9.49654	0.00541	13.05568	1.32932	39.88049	22.12454

Table 29: Execution Times and Values for Rastrigin's Function

Simulated Annealing Rastrigin's Function			
	n=5	n=10	n=30
Mean	0.00795	1.31080	28.74744
Standard deviation	0.00495	0.62800	4.26449
Min value	0.00068	0.09158	14.44787
Max value	0.023	2.53979	34.66966
Mean Exec Time	5.19192	11.0387	41.69259

Table 30: Statistical Metadata for Rastrigin's Function

Simulated Annealing Michalewicz's Function						
	n=5		n=10		n=30	
Run	Exec Time	Value	Exec Time	Value	Exec Time	Value
1	2.06483	-3.69884	5.98030	-8.33665	24.28064	-24.32545
2	2.07487	-3.69885	6.44880	-8.44287	24.66028	-24.51342
3	2.96161	-3.62345	6.62434	-8.26185	33.31580	-24.04162
4	2.96616	-3.69885	6.67215	-8.14890	33.32620	-25.59516
5	2.99566	-3.69886	6.68500	-8.38219	33.46975	-24.17475
6	3.03107	-3.69886	6.94435	-8.64906	33.51904	-24.93145
7	3.28996	-3.69886	7.08840	-8.31688	35.85981	-24.01522
8	3.30922	-3.69886	7.30138	-8.28775	36.12381	-24.76561
9	3.33283	-3.69886	7.30365	-8.33061	36.29545	-24.43522
10	3.33391	-3.69886	7.35777	-8.38456	36.51837	-23.87139
11	2.78431	-3.69886	5.67042	-8.48928	27.75591	-24.18864
12	2.88237	-3.69886	5.77028	-8.46556	30.37959	-24.18305
13	2.93020	-3.69886	6.29670	-8.28455	30.69400	-24.27954
14	3.10325	-3.69883	6.55510	-8.35548	30.81770	-24.68546
15	3.24443	-3.69885	6.72967	-8.48232	32.22807	-24.73029
16	3.25357	-3.69885	7.03677	-8.19578	32.27486	-23.85706
17	3.28877	-3.69886	7.11185	-8.32414	32.96394	-24.44323
18	3.31052	-3.69885	7.23037	-8.34350	33.04074	-24.44961
19	3.38526	-3.69886	7.26245	-8.30661	33.09624	-23.93120
20	3.43880	-3.69886	7.38376	-8.38562	33.46990	-25.77466
21	2.20918	-3.69885	4.74939	-8.33468	25.40936	-24.74614
22	2.35782	-3.69886	4.90641	-8.26970	27.27752	-23.69677
23	2.96607	-3.69886	6.59789	-8.54563	29.81066	-24.63561
24	3.04250	-3.69886	6.62486	-8.49269	32.07858	-24.05616
25	3.04322	-3.69886	6.67446	-8.39651	32.22285	-24.28454
26	5.37133	-3.69886	6.70029	-8.55848	32.45612	-24.48575
27	3.07949	-3.69885	7.19543	-8.48123	32.47045	-24.39171
28	3.32461	-3.69846	7.35510	-8.30270	32.81636	-24.72236
29	3.32571	-3.69886	7.36925	-8.42145	33.37194	-24.88006
30	3.34905	-3.69886	7.37486	-8.43958	33.44274	-24.03356

Table 31: Execution Times and Values for Michalewicz's Function

Simulated Annealing Michalewicz's Function			
	n=5	n=10	n=30
Mean	-3.69632	-8.38056	-24.43748
Standard deviation	0.01353	0.10828	0.46230
Min value	-3.69886	-8.64906	-25.77466
Max value	-3.62345	-8.1489	-23.69677
Mean Exec Time	3.10168	6.70004	31.84822

Table 32: Statistical Metadata for Michalewicz's Function

0.4.5 The influence of parameters in the performance of the algorithms

The most influential parameters is the number of dimensions. The higher of the dimensions, the higher is the computational time and the quality of the global minima decreases. Number of iterations increase the running time but it improves the quality of the optimum found. The number of interations of not founding a better solution than the current candidate is a trade-off of optimisation and the risk of loosing a better optim solution, the complex is the function, the higher of this iterations will be.

0.5 Comparisons between algoritms

Overall Simulated Annealing is better than Hill Climbing over every functions and dimensions as its is faster. Hill Climbing excels at finding global minima in convex functions. Simulated Annealing it's good in finding global minima in multimulmodal funcitions, as it can escape from local minima using the temperature.

0.6 Conclusion

In this report, it's explored the performance of various trajectory algorithms, particularly focusing on the comparison between Simulated Annealing and Hill Climbing in the context of optimizing functions such as DeJong, Schewefel's, Rastrigin's Michalewicz's function. Our findings indicate that Simulated Annealing consistently outperforms Hill Climbing across multiple dimensions and functions, demonstrating superior execution times and a greater ability to locate global minima in complex multimodal landscapes. The analysis highlights the importance of dimensionality and iteration count as critical factors influencing the performance of optimization algorithms, where increased dimensions lead to longer computation time.

The adaptability of Simulated Annealing, aided by its temperature parameter, allows it to effectively navigate local minima traps, making it a more robust choice for solving optimization problems characterized by multiple peaks. Conversely, while Hill Climbing may excel in convex functions, its limitations in complex scenarios underscore the necessity for employing metaheuristic approaches like Simulated Annealing when seeking optimal solutions.

In conclusion, the choice of algorithm should be guided by the specific characteristics of the optimization problem at hand, considering factors such as dimensionality, function shape, and desired computational efficiency. Further research could explore hybrid approaches that leverage the strengths of both algorithms to enhance performance across a broader range of optimization challenges.

Bibliography

[Alg] AlgorithAftermoon. Simulated Annealing. https://algorithmafternoon.com/books/simulated_annealing/chapter02/.

[ARa] Ali R. Al-Roomi. Unconstrained Single-Objective Benchmark Functions Repository. https://www.al-roomi.org/benchmarks/unconstrained/n-dimensions/191-sphere-model-spherical-contours-square-sum-harmonic-or-1st-de-jong-s-function.

[ARb] Ali R. Al-Roomi. Unconstrained Single-Objective Benchmark Functions Repository. https://www.al-roomi.org/benchmarks/unconstrained/n-dimensions/176-generalized-schwefel-s-problem-2-26.

[ARc] Ali R. Al-Roomi. Unconstrained Single-Objective Benchmark Functions Repository. https://www.al-roomi.org/benchmarks/unconstrained/n-dimensions/174-generalized-rastrigin-s-function.

[ARd] Ali R. Al-Roomi. Unconstrained Single-Objective Benchmark Functions Repository. https://www.al-roomi.org/benchmarks/unconstrained/n-dimensions/197-michalewicz-s-function.

[AR15] Ali R. Al-Roomi. Unconstrained Single-Objective Benchmark Functions Repository. https://www.al-roomi.org/benchmarks/unconstrained, 2015.

[Cro24] Lect. Dr. Eugen Croitoru. Genethic Algorithms. https://profs.info.uaic.ro/eugen.croitoru/teaching/ga/#Notions02, 2024.

[hc] Hill Climbing. https://www.codecademy.com/resources/docs/ai/search-algorithms/hill-climbing.

[Wika] Wikipedia. Metaheuristic. https://en.wikipedia.org/wiki/Metaheuristic.

[Wikb] Wikipedia. Simulated Annealing. https://en.wikipedia.org/wiki/Simulated_annealing.