A1Q2

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a) I expect that the Manhattan distance heuristic performs better than the misplaced tile heuristic. Intuitively, the former one has a stricter restriction, since the former one requires tiles can move to any position adjacent to it, while the latter one allows tiles move to any positions.
We need to prove that the Manhattan distance heuristic is closer to the true cost than the misplaced tile heuristic. Since we have covered in the lecture that both heuristic are admissible, we only need to show the former one has a larger value than the latter one. That is to say, both heuristics give values fewer than the true cost, so the larger one will be closer.
We can show that, the Manhattan distance heuristic (MDH) always generates a non-smaller value than what the misplaced tile heuristic generates (MTH). Specifically, we will show two things:
   1) For every state, MDH will generate a value that is larger than or equal to what MTH generates, so MDH is no worse than MTH.
   2) There is at least one state that MDH will generate a strict larger value than what MTH gives, so MDH is indeed better than MTH

To show 1), let us consider an arbitrary state. There are only two kinds of tiles in the state: correctly placed tiles or misplaced tiles. It is obvious that both heuristic will give an estimation of zero for correctly placed tiles. For each misplaced tiles, MTH always estimate it as one. If the tile is misplaced, then the target place must be at least one grid away from the current position. In other words, if the target position is adjacent to the current position, then MDT will estimate the cost as one. If the target position is in a different position, then MDT will give an estimation at least two which is larger than what MTH estimates. The fact is true for every misplaced tiles. We add them together to get the final estimation, and since each term generated by MDT is no less than the corresponding term generated by MTH, we can claim that the estimation MDH gives is no worse than what MTH gives.
To show 2), we only need to give an example. Consider the state below:

	1	2
3	4	6
5	7	8

Only tile 5 and 6 are misplaced, so MTH will estimate the cost as two (move tile 5 to position 6, and move tile 6 to position 7)
However, MDH will consider the distance. The Manhattan distance between the position 6 and position 7 is 3. This means that if we want to exchange the position of tile 5 and 6, we must move 6 steps with the relaxation that tiles can be moved to the position next to it. Therefore, MDH will estimate the cost as 6, which is strictly larger than 2.

With 1) and 2), we can claim that the Manhattan distance heuristic is better that the misplaced tile heuristic.

b) We only need to prove that, for every arc $< m, n >$, $h(m) - h(n) \leq cost(m, n)$. In other words, the heuristic estimate of any path cost is always less than the actual cost.
I will claim that both heuristics are monotone, since their definitions do not dependent on special states. I will still give a more rigorous proof in the following.

We can claim that all arcs have the same cost one since in this game, the only admissible move is to move a tile that is adjacent to the empty slot to that empty slot, which is always one move. Therefore, we only need to prove that or every arc $< m, n >$, $h(m) - h(n) \leq 1$

Misplaced tile heuristic is monotone. Suppose we are at an arbitrary state m, and we want to move to another state n by one move (that is, an arc). Since all other tiles are stationary, the estimation from those tiles do not change. The move can only have three results:

1) Move a correctly placed tile to misplaced position: $h(m) - h(n) = -1$
2) Move a misplaced tile to correct position: $h(m) - h(n) = 1$
3) Move a misplaced tile to another misplaced position: $h(m) - h(n) = 0$

Since MTH is related to number of misplaced tiles. We can see that under all situations, the estimation is less than the actual cost 1, so the misplaced tile heuristic is monotone.

Manhattan distance heuristic is monotone. Similarly, suppose we are at an arbitrary state m, and we want to move to another state n by one move (again, also an arc). Since all other tiles are stationary, the estimation from those tiles do not change. We only need to consider the moved tile, or more specifically, its Manhattan distance to the correct position before and after the move. The Manhattan distance before and after will only differ by one.
(Say the tile is at (a, b) originally and the correct position is (c, d), so the Manhattan distance is $|a - c| + |b - d|$. The only possible moves are: $(a + 1, b), (a - 1, b), (a, b + 1), (a, b - 1)$. It is obvious that the Manhattan distance only differs by one)
It could be plus one (moved farther away) or minus one (moved closer), but in either case, $h(m) - h(n) \leq 1$. That means, our claim holds for every arc, so the Manhattan distance heuristic is monotone.