Chapter 4 Review

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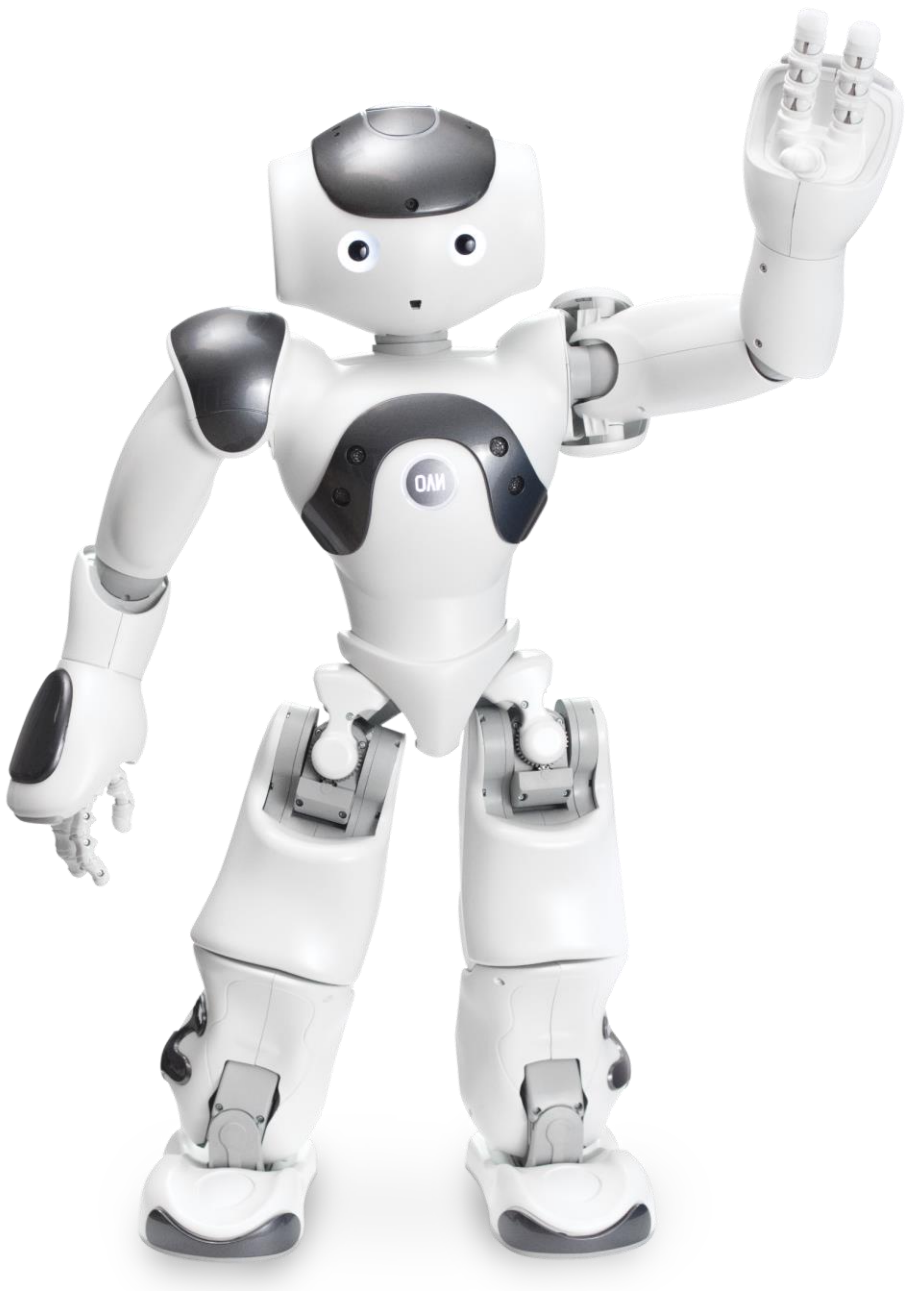

UNIVERSITY OF WATERLOO
FACULTY OF ENGINEERING

CHAPTER 4 REVIEW

What is this chapter all about?

Product of Exponentials (PoE) in Fixed Frame

$$T(\theta) = e^{[\mathcal{S}_1]\theta_1} \cdots e^{[\mathcal{S}_{n-1}]\theta_{n-1}} e^{[\mathcal{S}_n]\theta_n} M$$

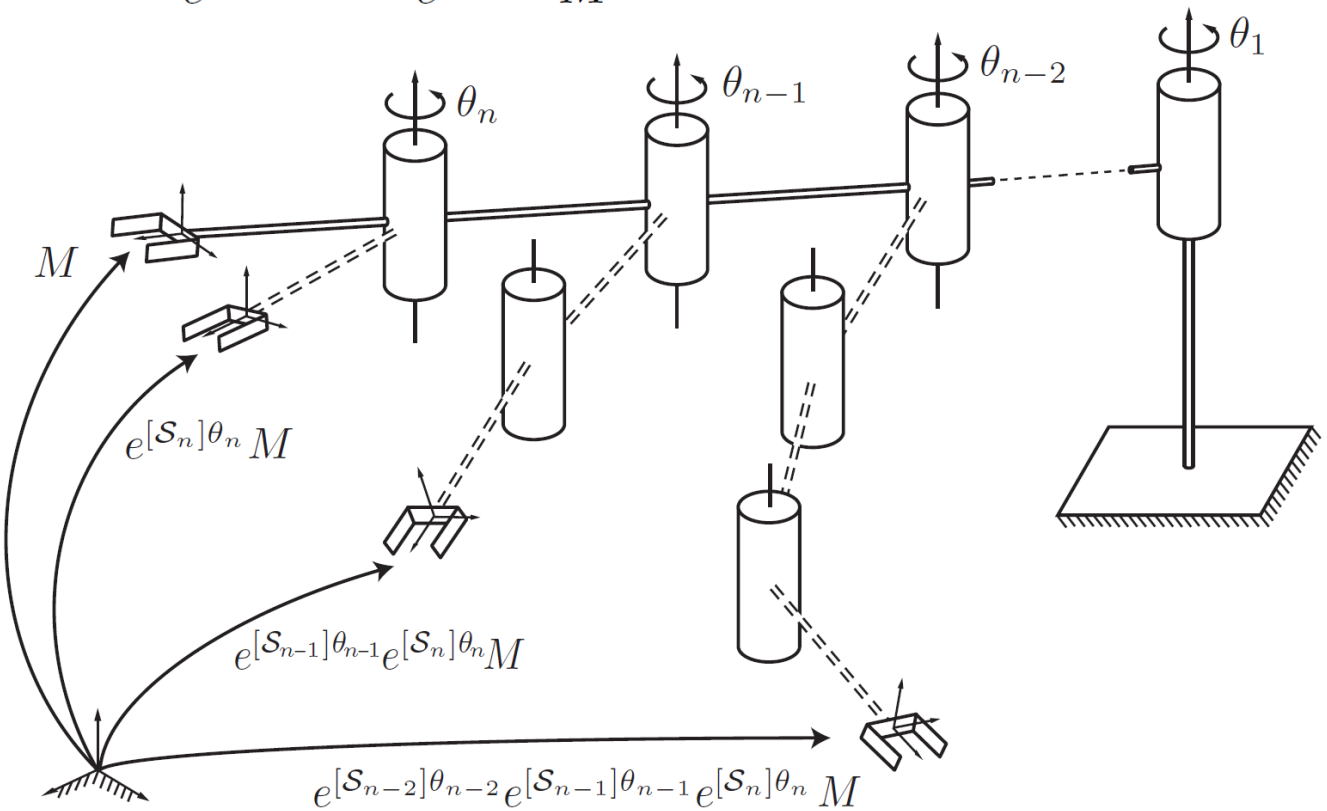

Figure 4.2: Illustration of the PoE formula for an n-link spatial open chain.

PoE in the End Effector (Body) Frame

- Uses Proposition 3.10, summarized as

$$e^{M^{-1}PM} = M^{-1}e^{P}M \Rightarrow Me^{M^{-1}PM} = e^{P}M$$

to convert from the fixed frame formulation to a body frame formulation:

$$\begin{aligned} T(\theta) &= e^{[\mathcal{S}_1]\theta_1} \cdots e^{[\mathcal{S}_n]\theta_n} M \\ &= e^{[\mathcal{S}_1]\theta_1} \cdots M e^{M^{-1}[\mathcal{S}_n]M\theta_n} \\ &= e^{[\mathcal{S}_1]\theta_1} \cdots M e^{M^{-1}[\mathcal{S}_{n-1}]M\theta_{n-1}} e^{M^{-1}[\mathcal{S}_n]M\theta_n} \\ &= M e^{M^{-1}[\mathcal{S}_1]M\theta_1} \cdots e^{M^{-1}[\mathcal{S}_{n-1}]M\theta_{n-1}} e^{M^{-1}[\mathcal{S}_n]M\theta_n} \\ &= M e^{[\mathcal{B}_1]\theta_1} \cdots e^{[\mathcal{B}_{n-1}]\theta_{n-1}} e^{[\mathcal{B}_n]\theta_n}, \end{aligned}$$

Denavit-Hartenberg Parameters

- Forward Kinematics using DH Parameters

1. Locate and label the joint axes as $z_0, \dots, z_{n-1}$ such that q_i acts along/about z_{i-1}.
2. Choose your base frame's x_0 and y_0 axes using the right hand rule. The origin O_0 where these two axes intersect with z_0 may be placed anywhere along the z_0 axis.
3. For every link other than the end effector ($i \in 1:n-1$):
    1. Place the origin O_i of frame i where the common normal of z_{i-1} and z_i intersects z_i. If z_i intersects z_{i-1}, place the origin at the intersection. If z_{i-1} and z_i are parallel, place the origin at joint $i+1$.
    2. Choose x_i extending from O_i along the common normal of z_{i-1} and z_i. If z_i intersects z_{i-1}, choose x_i normal to the plane formed by both z_{i-1} and z_i. Add y_i to complete a right hand frame.
4. Establish the end-effector frame O_n. If the n^{th} joint is revolute, set z_n parallel to z_{n-1}.
5. Create a table of DH parameters using these local frames.
6. For each joint, formulate the transformation matrix $A_i(q_i)$ using the DH parameters.
7. Compute the forward kinematic transformation matrix: ${}^0H_n(\boldsymbol{q}) = A_1(q_1)A_2(q_2)\cdots A_n(q_n)$.

RECOMMENDED EXERCISES

Recommended Exercises

- 4.1 Product of Exponentials (PoE)
  - Exercises: 4.7, 4.8, 4.9
- 4.1 PoE with Angled Joints
  - Exercises: 4.11, 4.12, 4.13
- 4.1 PoE with a Mid-Formula Matrix
  - Exercises: 4.17, 4.20
- 4.5 (Appendix C) DH Parameters
  - Exercises: 4.21

THANK YOU

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