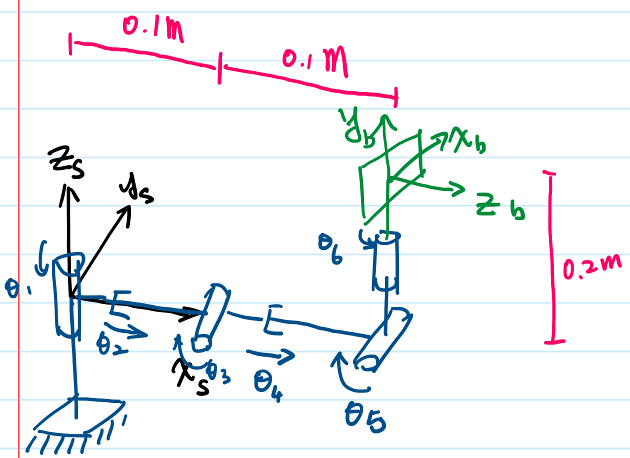
Project5

2022年7月5日 星期二　　下午2:52



joint limits:

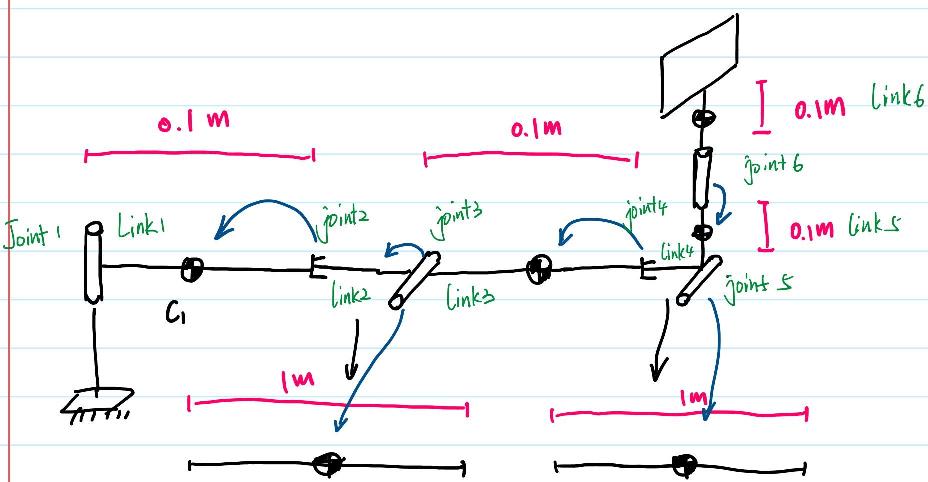
$\theta_1, \theta_3 \in [-\pi/2, \pi/2]$

$\theta_5, \theta_6 \in [-\pi, \pi]$

Since we will have infinite many solns every point, we add constraints:

$\theta_2 = \theta_4$

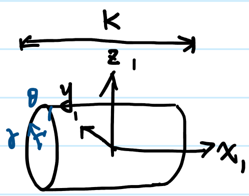
We also want the paddle always perpendicular to $\hat{y}_s$



Since all links in the system is a uniform cylindrical beam:

$R = 0.01m$ $\quad \rho = 10 / \pi R^2 = 10^5/\pi \ kg/m^3$ $\quad \Rightarrow \rho \pi R^2 = 10$

For any beam:



$r \in [0, R]$

$$I_{xx} = \int_B (y^2+z^2)\rho \, dV$$

$$y = r\sin\theta \quad z = r\cos\theta$$

$$= \int_0^{2\pi}\int_0^R\int_{-\frac{k}{2}}^{\frac{k}{2}} (r^2\sin^2\theta + r^2\cos^2\theta)\cdot\rho \; r\,dx\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^R\int_{-\frac{k}{2}}^{\frac{k}{2}} r^3\rho \; dx\,dr\,d\theta = \int_0^{2\pi}\int_0^R r^3\rho k \; dr\,d\theta$$

$$= \int_0^{2\pi} \frac{1}{4}R^4\rho k \, d\theta = \frac{\pi}{2}R^4\rho k$$

$$I_{yy} = \int_B (x^2+z^2)\rho \, dV = \int_0^{2\pi}\int_0^R\int_{-\frac{k}{2}}^{\frac{k}{2}} (x^2+r^2\cos^2\theta)\rho r \; dx\,dr\,d\theta$$

$$I_{yy} = \int_B (x^2+z^2)\rho\, dV = \int_0^{2\pi}\int_0^R\int_{-\frac{k}{2}}^{\frac{k}{2}} (x^2+r^2\cos^2\theta)\rho r\, dx\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^R \frac{1}{12}k^3\rho r + \rho r^3\cos^2\theta k\, dr\, d\theta$$

$$= \int_0^{2\pi} \frac{k^3}{24}\rho R^2 + \frac{R^4}{4}\rho\cos^2\theta k\, d\theta$$

$$= \frac{k^3}{12}\rho\pi R^2 + \frac{R^4}{4}\rho\pi k$$

$$I_{zz} = \int_B (x^2+y^2)\rho dV = \iiint_{-\frac{k}{2}}^{\frac{k}{2}} (x^2+r^2\sin^2\theta)\rho r\, dx\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^R \frac{k^3}{12}\rho r + \rho r^3\sin^2\theta k\, dr\, d\theta$$

$$= \int_0^{2\pi} \frac{k^3}{24}\rho R^2 + \frac{1}{4}\rho R^4 k\sin^2\theta\, d\theta$$

$$= \frac{1}{4}R^4 k\pi\rho + \frac{1}{12}\rho R^2\pi k^3$$

$$I_{xy} = -\int_B xy\rho\, r\, dx\, dr\, d\theta = -\int_0^{2\pi}\int_0^R\int_{-\frac{k}{2}}^{\frac{k}{2}} x r^2\sin\theta\rho\, dx\, dr\, d\theta$$

$$= 0$$

$$I_{xz} = -\int_B xz\rho r\, dx\, dr\, d\theta = 0$$

$$I_{yz} = -\int_B yz\rho r\, dx\, dr\, d\theta = -\int_0^{2\pi}\int_0^R\int_{-\frac{k}{2}}^{\frac{k}{2}} r^3\cos\theta\sin\theta\rho\, dx\, dr\, d\theta$$

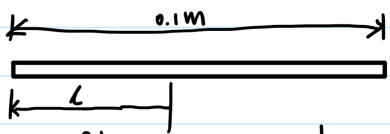
$$= -\int_0^{2\pi}\int_0^R \frac{1}{2}r^3\sin 2\theta\rho k\, dr\, d\theta$$

$$= -\int_0^{2\pi} \frac{1}{8}R^4\sin 2\theta\rho k\, d\theta = 0$$

$\rho\pi R^4 k = 10^{-3}k$

$$I = \begin{bmatrix} \rho\pi R^4\frac{k}{2} & & 0 \\ & \frac{k^3}{12}\rho\pi R^2 + \frac{R^4}{4}\rho\pi k & \\ 0 & & \frac{k^3}{12}\rho\pi R^2 + \frac{R^4}{4}\rho\pi k \end{bmatrix} = \begin{bmatrix} 5\times10^{-4}k & & 0 \\ & \frac{5}{6}k^3 + \frac{5}{2}\times10^{-4}k & \\ 0 & & \frac{5}{6}k^3+\frac{5}{2}\times10^{-4}k \end{bmatrix}$$

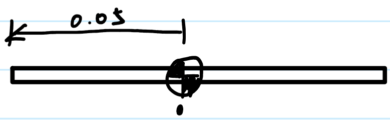
Link 1:

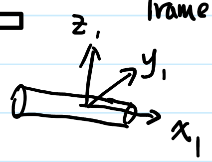


center of mass: $\dfrac{\int_0^{0.1}\rho\pi R^2 l\, dl}{\rho\pi R^2\times 0.1} = \dfrac{\frac{1}{2}(0.1)^2}{\times 0.1} = 0.05\text{m}$

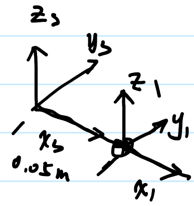
actuator: of joint 2

$k = 0.1$





Frame 1's



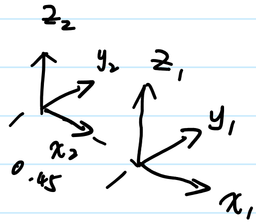
$$I_1 = \begin{bmatrix} 5\times10^{-5} & & 0 \\ & \frac{5}{6}\times10^{-3} + \frac{5}{2}\times10^{-5} & \\ 0 & & \frac{5}{6}\times10^{-3}+\frac{5}{2}\times10^{-5} \end{bmatrix} \approx \begin{bmatrix} 5\times10^{-5} & & 0 \\ & 8.583\times10^{-4} & \\ 0 & & 8.583\times10^{-4} \end{bmatrix}$$

mass $= 10\times 0.1 + 1 = 2$ kg

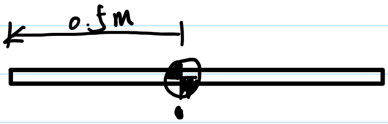
$$M_{0,1} = \begin{bmatrix} I & \begin{matrix}0.05\\0\\0\end{matrix} \\ 0 & 1 \end{bmatrix} \qquad G_1 = \begin{bmatrix} I_1 & 0 \\ 0 & 2I \end{bmatrix}$$

Link 2:

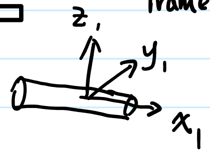
center of mass: $\dfrac{\int_0^1 \rho\pi R^2 l\,dl}{\rho\pi R^2 \times 1} = \dfrac{\frac{1}{2}\,1^2}{1} = 0.5\text{m}$



acturator:
of joint 3
$k = 1$ m



Frame 1,2

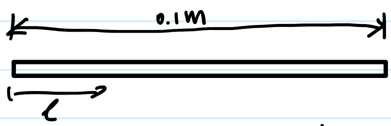


$$I_2 = \begin{bmatrix} 5\times10^{-4} & & 0 \\ & \frac{5}{6}+\frac{5}{2}\times10^{-4} & \\ 0 & & \frac{5}{6}+\frac{5}{2}\times10^{-4} \end{bmatrix} \approx \begin{bmatrix} 5\times10^{-4} & & 0 \\ & 0.833583 & \\ 0 & & 0.833583 \end{bmatrix}$$

mass $= 10\times1 + 1 = 11$ kg

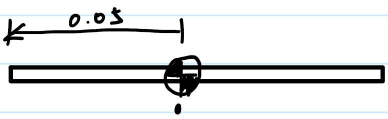
$$M_{1,2} = \begin{bmatrix} I & \begin{matrix}-0.45\\0\\0\end{matrix} \\ 0 & 1 \end{bmatrix} \qquad G_2 = \begin{bmatrix} I_2 & 0 \\ 0 & 11I \end{bmatrix}$$

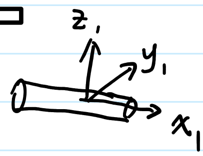
Link 3.



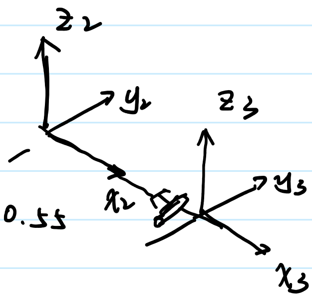
center of mass: $\dfrac{\int_0^{0.1} \rho\pi R^2 l\,dl}{\rho\pi R^2 \times 0.1} = \dfrac{\frac{1}{2}(0.1)^2}{\times 0.1} = 0.05\text{m}$

acturator:
of join4
$k = 0.1$





Frame 3



$$I_3 = \begin{bmatrix} 5\times10^{-5} & & 0 \\ & \frac{5}{6}\times10^{-3}+\frac{5}{2}\times10^{-5} & \\ 0 & & \frac{5}{6}\times10^{-3}+\frac{5}{2}\times10^{-5} \end{bmatrix} \approx \begin{bmatrix} 5\times10^{-5} & & 0 \\ & 8.583\times10^{-4} & \\ 0 & & 8.583\times10^{-4} \end{bmatrix}$$

mass $= 10\times0.1+1 = 2$

$$M_{2,3} = \begin{bmatrix} I & \begin{matrix}0.55\\0\\0\end{matrix} \\ 0 & 1 \end{bmatrix} \qquad G_3 = \begin{bmatrix} I_3 & 0 \\ 0 & 2I \end{bmatrix}$$

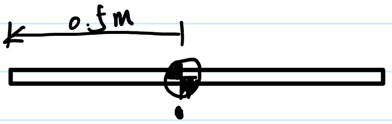
Link 4

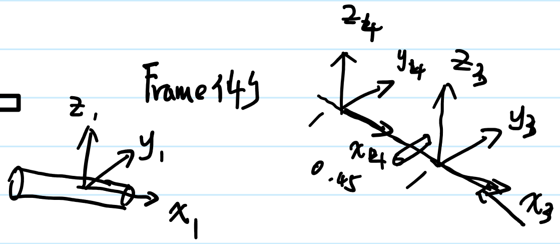
center of mass: $\int^1 \rho\pi R^2 l\,dl \quad \frac{1}{2}\,1^2 \quad = 0.5\text{m}$

center of mass: $\dfrac{\int_0^1 \rho\pi R^2 l\,dl}{\rho\pi R^2 \times 1} = \dfrac{\frac{1}{2}\,1^2}{1} = 0.5\,m$

actuator: of joint 5

$k = 1\ m$





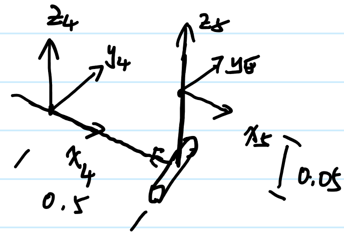
$$I_4 = \begin{bmatrix} 5\times10^{-4} & & 0 \\ & \frac{5}{6}+\frac{5}{2}\times10^{-4} & \\ 0 & & \frac{5}{6}+\frac{5}{2}\times10^{-4} \end{bmatrix} \approx \begin{bmatrix} 5\times10^{-4} & & 0 \\ & 0.833583 & \\ 0 & & 0.833583 \end{bmatrix}$$

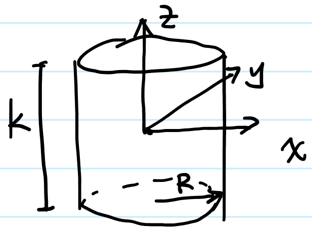
mass $= 10\times1 + 1 = 11$ kg

$$M_{3,4} = \begin{bmatrix} I & \begin{matrix}-0.45\\0\\0\end{matrix} \\ 0 & 1 \end{bmatrix} \qquad G_4 = \begin{bmatrix} I_4 & 0 \\ 0 & 11I \end{bmatrix}$$

Link 5:

center of mass: at length $= 0.05\,m$.





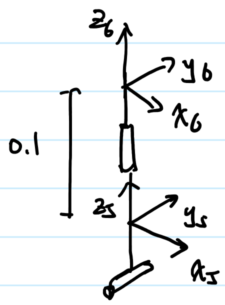
$$I_5 = \begin{bmatrix} \frac{5}{6}\times10^{-3}+\frac{5}{2}\times10^{-5} & & 0 \\ & \frac{5}{6}\times10^{-3}+\frac{5}{2}\times10^{-5} & \\ 0 & & 5\times10^{-5} \end{bmatrix} \approx \begin{bmatrix} 8.583\times10^{-4} & & 0 \\ & 8.583\times10^{-4} & \\ 0 & & 5\times10^{-5} \end{bmatrix}$$

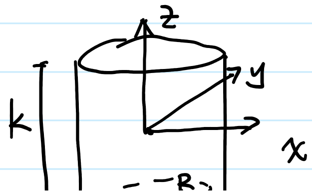
mass $= 10\times0.1+1 = 2$

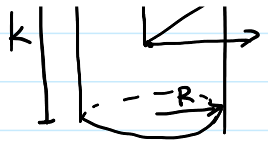
$$M_{4,5} = \begin{bmatrix} I & \begin{matrix}0.5\\0\\0.05\end{matrix} \\ 0 & 1 \end{bmatrix} \qquad G_5 = \begin{bmatrix} I_5 & \\ & 2I \end{bmatrix}$$

Link 6:

center of mass: at length $= 0.05\,m$.





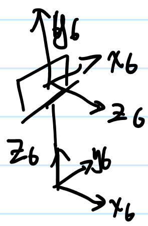


$$I_6 = \begin{bmatrix} \frac{5}{6}\times10^{-3}+\frac{5}{2}\times10^{-5} & & 0 \\ & \frac{5}{6}\times10^{-3}+\frac{5}{2}\times10^{-5} & \\ 0 & & 5\times10^{-5} \end{bmatrix} \approx \begin{bmatrix} 8.583\times10^{-4} & & 0 \\ & 8.583\times10^{-4} & \\ 0 & & 5\times10^{-5} \end{bmatrix}$$

$$\text{mass} = 10\times0.1 = 1$$

$$M_{5,6} = \begin{bmatrix} I & \begin{matrix} 0 \\ 0 \\ 0.1 \end{matrix} \\ 0 & 1 \end{bmatrix} \qquad G_5 = \begin{bmatrix} I_6 & \\ & I \end{bmatrix}$$

$M_{6,7}$



$$M_{6,7} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.05 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$