SCI238 Assignment 3

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Question 1

a

To determine the distance with trigonometric parallax, we can calculate the distance in parsecs:

$$\begin{aligned} d &= \frac{1}{0.1\underline{0} \text{ arcsec}} \\ &= 1\underline{0} \text{ parsec} \\ &= 2.\underline{0}62650 \times 10^6 \text{ A.U.} \end{aligned}$$

So these stars are about $2.\underline{1} \times 10^6$ A.U. away from the solar system.

b

The two stars have a constant distance apart from each other, thus we can conclude that their orbits are a circle around their common center of mass. Therefore, their semi-major axis can be calculated using the distance from them to the solar system:

$$\begin{aligned} a &= \frac{1}{2} \times 5.\underline{0} \text{ arcsec} \times 2.\underline{0}62650 \times 10^6 \\ &= \frac{1}{2} \times \frac{5.\underline{0}}{206265} \times 2.\underline{0}62650 \times 10^6 \\ &= 2\underline{5} \text{ A.U.} \end{aligned}$$

c

The stars move 90 degrees for 60 years, so we can calculate the period as follows:

$$\begin{aligned} P &= \frac{6\underline{0}}{\frac{9\underline{0}\,\text{deg}}{360\,\text{deg}}} \\ &= 2\underline{4}0 \text{ years} \end{aligned}$$

Since the stars have identical brightness and colours, they have identical mass, and we can then apply Kepler's third law:

$$\begin{aligned} & (m_1 + m_2)P^2 = (a_1 + a_2)^3 \\ \iff & (2m)P^2 = (2a)^3 \\ \iff & (2m) = \frac{(2a)^3}{P^2} \\ \iff & m = 4 \times \frac{2\underline{5}^3}{2\underline{4}0^2} \\ \iff & m = 4 \times \frac{\underline{1}5625}{\underline{5}7600} \\ \iff & m = 4 \times 0.\underline{2}712673611 \\ \iff & m = \underline{1}.085069444 \, M_\odot \end{aligned}$$

Therefore, the mass of each star is about 1 solar mass (we lost one significant figure due to square/cubic operation).

Question 2

a

The distance can be calculated from the brightness formula

$$b = \frac{L}{4\pi d^2}$$
$$\iff d = \sqrt[2]{\frac{L}{4\pi b}}$$

For lower luminosity G2V stars, the luminosity value is 3.0×10^{26} W, and we use the brightness 3.58×10^{-12} W/m^2. Insert those values, we get

$$\begin{aligned} d &= \sqrt[2]{\frac{3.\underline{0} \times 10^{26}}{4\pi \times 3.5\underline{8} \times 10^{-12}}} \\ &= \sqrt[2]{6.\underline{6}68503202 \times 10^{36}} \\ &= 2.5\underline{8}2344517 \times 10^{18}\text{m} \end{aligned}$$

So the distance to this star will be around $2.5\underline{8} \times 10^{18}$m if it is one of the lower luminosity G2V stars.

b

We use the same formula, but the luminosity value is 4.8×10^{26} W for higher luminosity G2V stars.

$$\begin{aligned} d &= \sqrt[2]{\frac{4.\underline{8} \times 10^{26}}{4\pi \times 3.5\underline{8} \times 10^{-12}}} \\ &= \sqrt[2]{1.\underline{0}66960512 \times 10^{37}} \\ &= 3.2\underline{6}643615 \times 10^{18}\text{m} \end{aligned}$$

So the distance to this star will be around $3.2\underline{7} \times 10^{18}$m if it is one of the higher luminosity G2V stars.

c

The "best estimate" can be calculated by the average of the higher and lower values, which is

$$\begin{aligned} d_{avg} &= \frac{2.5\underline{8}2344517 \times 10^{18} + 3.2\underline{6}643615 \times 10^{18}}{2} \\ &= 2.9\underline{2}4390334 \times 10^{18}\text{m} \end{aligned}$$

And the uncertainty is:

$$\begin{aligned} &((2.9\underline{2}4390334 \times 10^{18} - 2.5\underline{8}2344517 \times 10^{18}) + (3.2\underline{6}643615 \times 10^{18} - 2.9\underline{2}4390334 \times 10^{18}))/2 \\ =&3.\underline{4}20458165 \times 10^{17}\text{m} \end{aligned}$$

We calculate the relative uncertainty:

$$\frac{3.\underline{4}20458165 \times 10^{17}}{2.9\underline{2}4390334 \times 10^{18}} = 1\underline{1}.69631196\%$$

So the uncertainty in distance is about $3.\underline{4} \times 10^{17}$m, or about $1\underline{2}\%$ relatively.

d

The uncertainty for brightness is

$$
\begin{aligned}
&((3.5\underline{8} \times 10^{-12} - (3.5\underline{8} - 0.0\underline{2}) \times 10^{-12}) - ((3.5\underline{8} + 0.0\underline{2}) \times 10^{-12} - 3.5\underline{8} \times 10^{-12}))/2 \\
=&\underline{2} \times 10^{-14} \mathrm{W/m^2}
\end{aligned}
$$

We calculate the relative uncertainty for luminosity:

$$\frac{\underline{2} \times 10^{-14}}{3.5\underline{8} \times 10^{-12}} = 0.\underline{5}5865922\%$$

The difference between them is about:

$$\frac{1\underline{1}.69631196\%}{0.\underline{5}5865922\%} = \underline{2}0.93639833$$

The distance uncertainty and the uncertainty in brightness measurement are not the same. In fact, the difference is pretty large. The distance uncertainty is about $\underline{2} \times 10^1$ times larger than the uncertainty in brightness measurement.