A1

2023年1月28日 星期六 下午3:45

1.

a) Opposition happens, so the object must be superior

synodic period $S = 1.4$ years

equation: $S = 1 + S/P \Rightarrow 1.4 = 1 + \frac{1.4}{P}$

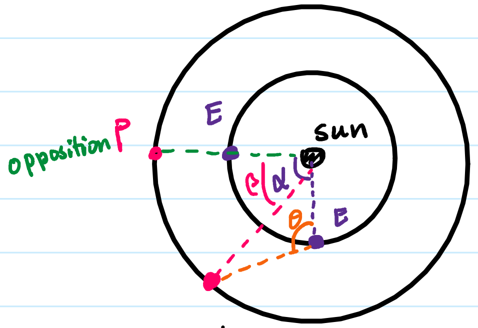
$P = 1.4/0.40 = 3.5$ years

b) By Kepler's Law: $P^2 = a^3$

$a^3 = 3.5^2 \Rightarrow a = \sqrt[3]{3.5^2} = 2.305218146\ldots$ A.U.

The semi-major axis is about 2.3 A.U.

c)



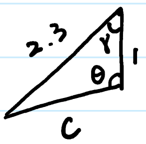
3 months means $\frac{3}{12} = 0.25$ year

$\alpha = \frac{0.25}{1} \times 360 = 90°$

$\beta = \frac{0.25}{3.5} \times 360 = 25.71428571\ldots°$

$\alpha - \beta = 64.28571429\ldots°$

solve the triangle by the sun, the object and the Earth:



$\gamma = \alpha - \beta$

by cosine law: $c^2 = 2.305218146^2 + 1^2 - 2 \times 2.305218146 \times 1 \times \cos(64.28571429°)$

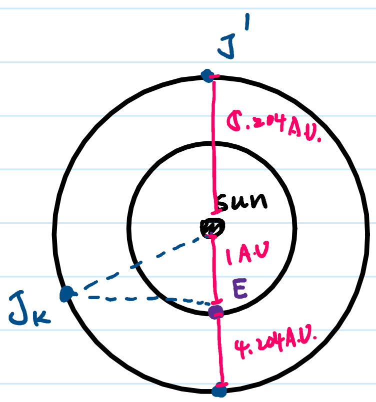
$= 4.313637364\ldots$ A.U.

$c = \sqrt{4.313637364} = 2.076929793\ldots$ A.U.

$\cos\theta = \frac{4.313637364 + 1 - 2.305218146^2}{2 \times 1 \times 2.076929793} = -9.46918497 \times 10^{-5}$

$\theta = \cos^{-1}(-9.46918497 \times 10^{-5}) = 90.00542544°$ So the elongation is 9.0×10^{1} degree

2.



The orbit radius:

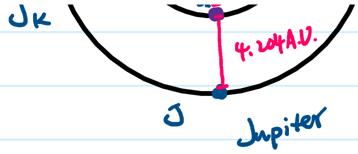
$r_J = 5.204$ A.U.

$r_e = 1$ A.U.

For any arbitrary position of Jupiter J_K, We have:

$SJ_K + SE \geq EJ_K$

$SE + EJ_K \geq SJ_K$



$SJ_k + SE \geq EJ_k$

$SE + EJ_k \geq SJ_k$

by the triangle. SEJ_k

Therefor : $\begin{cases} EJ_k \geq SJ_k - SE = 5.204 - 1 = 4.204\ A.U. \\ EJ_k \leq SJ_k + SE = 5.204 + 1 = 6.204\ A.U. \end{cases}$

The angular size of Jupiter is approximately: $\dfrac{\text{diameter of J}}{EJ_k}$

Therefore, the maximum angular size of Jupiter happens at J

$$\frac{2 \times 7.1492 \times 10^4\ km}{4.204\ A.U.} \times \frac{1\ A.U.}{149597871\ km} \approx 2.2735228 43 \times 10^{-4}\ rad \times \frac{180\ deg}{\pi\ rad}$$

$$\approx 0.0130263264\ deg \times \frac{3600\ arc\ sec}{1\ deg}$$

$$\approx 46.89477487\ arcsec$$

The minimum angular size is at J'

$$\frac{2 \times 7.1492 \times 10^4\ km}{6.204\ A.U.} \times \frac{1\ A.U.}{149597871\ km} \approx 1.54060123 \times 10^{-4}\ rad \times \frac{180\ deg}{\pi\ rad}$$

$$\approx 0.00882 69948\ deg \times \frac{3600\ arcsec}{1\ deg}$$

$$\approx 31.77718128\ arcsec$$

Therefore, the maximum angular size is 46.89'' and the minimum angular size is 31.78''

3. Doppler shift formula: $\dfrac{V_{radial}}{c} = \dfrac{\Delta\lambda}{\lambda_{rest}}$

$$V_{radial} = \frac{656.1 - 656.3}{656.3} \times 2.99792458 \times 10^8 = -91258.3599\ m/s$$

$$= -91.3583599\ km/s$$

The star is moving towards the observer at 91.36 km/s