# SCI238 Assignment 4

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# Question 1

## a

To determine the mass at these five points, we use the equation $M_R = v_{circ}^2 R/G$. We also conver the unit to SI by:

$$1kpc = 1 \times 10^3 pc = 3.086 \times 10^{19}\text{m} \quad 1\text{km/s} = 1 \times 10^3\text{m/s}$$

Then the unit is given by:

$$\frac{(m/s)^2 \cdot m}{N \cdot m^2/kg^2} = \frac{(m/s)^2 \cdot m}{kg \cdot m/s^2 \cdot m^2/kg^2} = kg$$

For the first point:

$$\begin{aligned} M_R &= \frac{v_r^2 R}{G} \\ &= \frac{(20\underline{0} \times 10^3)^2 \times 5.0\underline{0} \times 3.086 \times 10^{19}}{6.67 \times 10^{-11}} \\ &= 9.2\underline{5}3373313 \times 10^{40} kg \end{aligned}$$

For the second point:

$$\begin{aligned} M_R &= \frac{v_r^2 R}{G} \\ &= \frac{(25\underline{3} \times 10^3)^2 \times 10.\underline{0} \times 3.086 \times 10^{19}}{6.67 \times 10^{-11}} \\ &= 2.9\underline{6}1495862 \times 10^{41} kg \end{aligned}$$

For the third point:

$$\begin{aligned} M_R &= \frac{v_r^2 R}{G} \\ &= \frac{(26\underline{1} \times 10^3)^2 \times 15.\underline{0} \times 3.086 \times 10^{19}}{6.67 \times 10^{-11}} \\ &= 4.7\underline{2}7617826 \times 10^{41} kg \end{aligned}$$

For the fourth point:

$$\begin{aligned} M_R &= \frac{v_r^2 R}{G} \\ &= \frac{(25\underline{9} \times 10^3)^2 \times 20.\underline{0} \times 3.086 \times 10^{19}}{6.67 \times 10^{-11}} \\ &= 6.2\underline{0}7255352 \times 10^{41} kg \end{aligned}$$

For the fifth point:

$$\begin{aligned} M_R &= \frac{v_r^2 R}{G} \\ &= \frac{(26\underline{8} \times 10^3)^2 \times 25.\underline{0} \times 3.086 \times 10^{19}}{6.67 \times 10^{-11}} \\ &= 8.3\underline{0}7678561 \times 10^{41} kg \end{aligned}$$

So at these five points, the estiamted masses are about: $9.2\underline{5} \times 10^{40} kg$, $2.9\underline{6} \times 10^{41} kg$, $4.7\underline{3} \times 10^{41} kg$, $6.2\underline{1} \times 10^{41} kg$, $8.3\underline{1} \times 10^{41} kg$, respectively.

### b

The two stars have a constant distance apart from each other, thus we can conclude that their orbits are a circle around their common center of mass. Therefore, their semi-major axis can be calculated using the distance from them to the solar system:

$$\begin{aligned} a &= \frac{1}{2} \times 5.\underline{0} \text{ arcsec} \times 2.\underline{0}62650 \times 10^6 \\ &= \frac{1}{2} \times \frac{5.\underline{0}}{206265} \times 2.\underline{0}62650 \times 10^6 \\ &= 2\underline{5} \text{ A.U.} \end{aligned}$$

### c

The stars move 90 degrees for 60 years, so we can calculate the period as follows:

$$\begin{aligned} P &= \frac{6\underline{0}}{\frac{9\underline{0}\,\text{deg}}{360\,\text{deg}}} \\ &= 2\underline{4}0 \text{ years} \end{aligned}$$

Since the stars have identical brightness and colours, they have identical mass, and we can then apply Kepler's third law:

$$\begin{aligned} & (m_1 + m_2)P^2 = (a_1 + a_2)^3 \\ \iff & (2m)P^2 = (2a)^3 \\ \iff & (2m) = \frac{(2a)^3}{P^2} \\ \iff & m = 4 \times \frac{2\underline{5}^3}{2\underline{4}0^2} \\ \iff & m = 4 \times \frac{\underline{1}5625}{\underline{5}7600} \\ \iff & m = 4 \times 0.\underline{2}712673611 \\ \iff & m = \underline{1}.085069444\ M_\odot \end{aligned}$$

Therefore, the mass of each star is about 1 solar mass (we lost one significant figure due to square/cubic operation).

## Question 2

### a

The distance can be calculated from the brightness formula

$$\begin{aligned} & b = \frac{L}{4\pi d^2} \\ \iff & d = \sqrt[2]{\frac{L}{4\pi b}} \end{aligned}$$

For lower luminosity G2V stars, the luminosity value is $3.0 \times 10^{26}$ W, and we use the brightness $3.58 \times 10^{-12}$ W/m$^2$. Insert those values, we get

$$\begin{aligned} d &= \sqrt[2]{\frac{3.\underline{0} \times 10^{26}}{4\pi \times 3.5\underline{8} \times 10^{-12}}} \\ &= \sqrt[2]{6.\underline{6}68503202 \times 10^{36}} \\ &= 2.5\underline{8}2344517 \times 10^{18}\text{m} \end{aligned}$$

So the distance to this star will be around $2.5\underline{8} \times 10^{18}$m if it is one of the lower luminosity G2V stars.

## b

We use the same formula, but the luminosity value is $4.8 \times 10^{26}$ W for higher luminosity G2V stars.

$$\begin{aligned} d &= \sqrt[2]{\frac{4.\underline{8} \times 10^{26}}{4\pi \times 3.5\underline{8} \times 10^{-12}}} \\ &= \sqrt[2]{1.\underline{0}66960512 \times 10^{37}} \\ &= 3.2\underline{6}643615 \times 10^{18}\text{m} \end{aligned}$$

So the distance to this star will be around $3.2\underline{7} \times 10^{18}$m if it is one of the higher luminosity G2V stars.

## c

The "best estimate" can be calculated by the average of the higher and lower values, which is

$$\begin{aligned} d_{avg} &= \frac{2.5\underline{8}2344517 \times 10^{18} + 3.2\underline{6}643615 \times 10^{18}}{2} \\ &= 2.9\underline{2}4390334 \times 10^{18}\text{m} \end{aligned}$$

And the uncertainty is:

$$\begin{aligned} &((2.9\underline{2}4390334 \times 10^{18} - 2.5\underline{8}2344517 \times 10^{18}) + (3.2\underline{6}643615 \times 10^{18} - 2.9\underline{2}4390334 \times 10^{18}))/2 \\ =&3.\underline{4}20458165 \times 10^{17}\text{m} \end{aligned}$$

We calculate the relative uncertainty:

$$\frac{3.\underline{4}20458165 \times 10^{17}}{2.9\underline{2}4390334 \times 10^{18}} = 1\underline{1}.69631196\%$$

So the uncertainty in distance is about $3.\underline{4} \times 10^{17}$m, or about $1\underline{2}\%$ relatively.

## d

The uncertainty for brightness is

$$\begin{aligned} &((3.5\underline{8} \times 10^{-12} - (3.5\underline{8} - 0.0\underline{2}) \times 10^{-12}) - ((3.5\underline{8} + 0.0\underline{2}) \times 10^{-12} - 3.5\underline{8} \times 10^{-12}))/2 \\ =&\underline{2} \times 10^{-14}\text{W/m}^2 \end{aligned}$$

We calculate the relative uncertainty for luminosity:

$$\frac{\underline{2} \times 10^{-14}}{3.5\underline{8} \times 10^{-12}} = 0.\underline{5}5865922\%$$

The difference between them is about:

$$\frac{1\underline{1}.69631196\%}{0.\underline{5}5865922\%} = \underline{2}0.93639833$$

The distance uncertainty and the uncertainty in brightness measurement are not the same. In fact, the difference is pretty large. The distance uncertainty is about $\underline{2} \times 10^1$ times larger than the uncertainty in brightness measurement.