A2

February 15, 2023

```python
import numpy as np
import matplotlib.pyplot as plt
```

0.1 Question 1

Let's drive the equation for the helium line:

$$
\begin{aligned}
T &= 75 \times m \frac{M_p/M_E}{R_p/R_E} \\
&= 75 \times 4 \frac{M}{R} \\
&= 300 \frac{M}{R}
\end{aligned}
$$

This is an euqation for a straight line w.r.t $\frac{M}{R}$, we can choose two points where $\frac{M}{R}$ equals 10^{-2} and 10^2, plug them into the function, we get two points: $(0.01, 3), (100, 30000)$.

In the code below, we generate many points between 10^{-2} and 10^2 and use the above equation to calculate the temperature limit, and draw them.

```python
Mercury = "Mercury"
Venus = "Venus"
Earth = "Earth"
Mars = "Mars"
Jupiter = "Jupiter"
Saturn = "Saturn"
Uranus = "Uranus"
Neptune = "Neptune"

RADIUS = 0
MASS = 1
TEMP = 2
```

```python
planets_data = {
    Mercury: [2440, 3.30e23, 167],
    Venus: [6052, 4.87e24, 464],
    Earth: [6378, 5.97e24, 15],
    Mars: [3394, 6.42e23, -65],
```

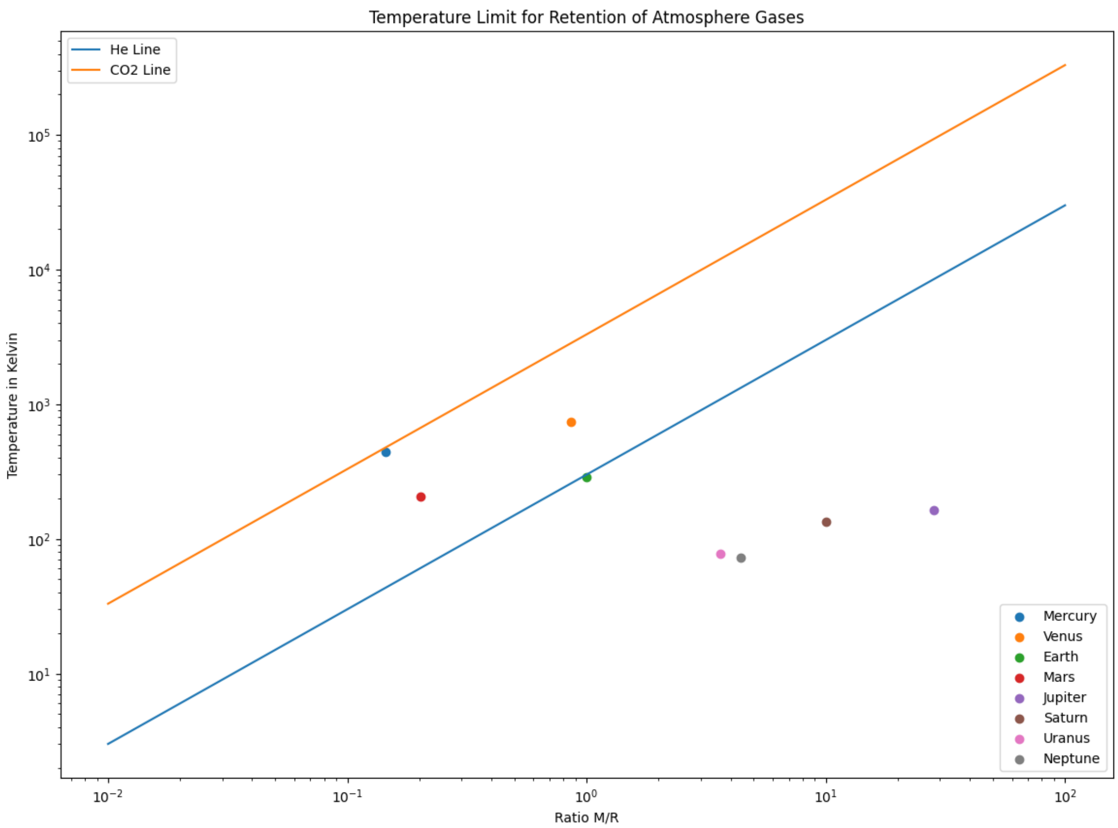
```
    Jupiter: [71492, 1.90e27, -110],
    Saturn: [60268, 5.68e26, -140],
    Uranus: [25559, 8.68e25, -195],
    Neptune: [24766, 1.02e26, -200]
}

for _, data in planets_data.items():
    data[TEMP] += 273
```

[ ]:
```
x_input = np.logspace(-2, 2, 500)
plt.figure(figsize=(14,10))
HeLine, = plt.loglog(x_input, 75 * 4 * x_input, label="He Line")
CO2Line, = plt.loglog(x_input, 75 * 44 * x_input, label="CO2 Line")
first_legend = plt.legend(handles=[HeLine, CO2Line], loc="upper left")
ax = plt.gca().add_artist(first_legend)
points = []
for planet, data in planets_data.items():
    x = (data[MASS] / planets_data[Earth][MASS]) / (data[RADIUS] /
 ↪planets_data[Earth][RADIUS])
    y = data[TEMP]
    pt = plt.scatter(x, y, label=f"{planet}")
    points.append(pt)

plt.legend(handles=points, loc="lower right")

plt.xlabel("Ratio M/R")
plt.ylabel("Temperature in Kelvin")
plt.title("Temperature Limit for Retention of Atmosphere Gases")
plt.show()
```



b) From the graph above, we can see that all planets are below the temperature limit of the CO_2 line, which means they are all able to retain CO_2 gas if we only consider the average temperature. Mercury, Venus and Mars are above the helium line, so they can not retain helium gas in their atmosphere. The rest of the planets can hold helium, so they can pretty much hold almost every element in their atmosphere since heliums is lighter than most of the gases. However, the temperatures we used here are just average temperature of the planet. If we further consider the temperature distribution along the planet, it is possible that these planets may still retain little of those gases. We can see from lecture 8 that almost all those planets have helium in the atmosphere, but Venus and Earth has very little. As for the carbon dioxide, it is possible that most of them in the giant palnets are forzen to solid because the temperature their is so low (they are beyong the frost line)

If we consider the Earth point and the Mercury point, we can see that those two points are very close to the gas lines, slightly below them. By the theory, it should mean that the Earth and the Mercury can reatin helium and CO_2 respectively. However, again, we used the average temperature here. And since they are so close to the line, it is very likely that the actual temperate becomes higher than then limit, causing them lose the gases. For example, our Earth is pretty lack of helium even though the graph tells us that we are able to retain it.

0.2 Question 2

1. The ratio of brightness directly equal to the ratio of areas. The ratio of brightness is 0.015. The radius of the star should be equal to that of the Sun, which is about $6.957 * 10^8\ m$. According to the formula $\frac{\pi r^2}{\pi R^2} =$ ratio, the radius of this planet can be calculated as:

$$
\begin{aligned}
r &= \sqrt{(0.01\underline{5} \times 10^{-2}) \times (6.95\underline{7} \times 10^8)^2} \\
&= \sqrt{(0.01\underline{5} \times 10^{-2}) \times (4.8\underline{3}99849 \times 10^{17})} \\
&= \sqrt{7.\underline{2}5997735 \times 10^{13}} \\
&= 8.5\underline{2}055007 \times 10^6
\end{aligned}
$$

   Therefore, the radius of this planet is around $8.5\underline{2} \times 10^6$ meters.

2. We observe this brightness decrement every 12 days, so we can conclude that the period of the planet is about 12 days. Since the spectrum of the star is identical to that of the Sun, we can say that the star is identical to the Sun thus we can apply the Kepler's rule.

   Period of planet in years:

$$
\frac{1\underline{2}}{365} = 0.03\underline{2}8767123\ years
$$

   Applying Keplars law, we get the axis is:

$$
\begin{aligned}
a &= \sqrt[3]{(0.03\underline{2}8767123)^2} \\
&= \sqrt[3]{(0.03\underline{2}8767123)^2} \\
&= \sqrt[3]{0.00\underline{1}0808782} \\
&= 0.1\underline{0}26263585
\end{aligned}
$$

> Therefore, the semi-major axis of the planet's orbit is about $0.1\underline{0}$ A.U.