SCI238 Assignment 4

Cosmo Zhao (20761282)

March 29, 2023

Question 1

a

To determine the mass at these five points, we use the equation $M_R = v_{circ}^2 R/G$. We also conver the unit to SI by:

$$1kpc = 1 \times 10^3 pc = 3.086 \times 10^{19}\text{m} \quad 1\text{km/s} = 1 \times 10^3\text{m/s}$$

Then the unit is given by:

$$\frac{(m/s)^2 \cdot m}{N \cdot m^2/kg^2} = \frac{(m/s)^2 \cdot m}{kg \cdot m/s^2 \cdot m^2/kg^2} = kg$$

For the first point:

$$\begin{aligned} M_R &= \frac{v_r^2 R}{G} \\ &= \frac{(20\underline{0} \times 10^3)^2 \times 5.0\underline{0} \times 3.086 \times 10^{19}}{6.67 \times 10^{-11}} \\ &= 9.2\underline{5}3373313 \times 10^{40} kg \end{aligned}$$

For the second point:

$$\begin{aligned} M_R &= \frac{v_r^2 R}{G} \\ &= \frac{(25\underline{3} \times 10^3)^2 \times 10.\underline{0} \times 3.086 \times 10^{19}}{6.67 \times 10^{-11}} \\ &= 2.9\underline{6}1495862 \times 10^{41} kg \end{aligned}$$

For the third point:

$$\begin{aligned} M_R &= \frac{v_r^2 R}{G} \\ &= \frac{(26\underline{1} \times 10^3)^2 \times 15.\underline{0} \times 3.086 \times 10^{19}}{6.67 \times 10^{-11}} \\ &= 4.7\underline{2}7617826 \times 10^{41} kg \end{aligned}$$

For the fourth point:

$$\begin{aligned} M_R &= \frac{v_r^2 R}{G} \\ &= \frac{(25\underline{9} \times 10^3)^2 \times 20.\underline{0} \times 3.086 \times 10^{19}}{6.67 \times 10^{-11}} \\ &= 6.2\underline{0}7255352 \times 10^{41} kg \end{aligned}$$

For the fifth point:

$$\begin{aligned} M_R &= \frac{v_r^2 R}{G} \\ &= \frac{(26\underline{8} \times 10^3)^2 \times 25.\underline{0} \times 3.086 \times 10^{19}}{6.67 \times 10^{-11}} \\ &= 8.3\underline{0}7678561 \times 10^{41} kg \end{aligned}$$

So at these five points, the estiamted masses are about: $9.2\underline{5} \times 10^{40} kg$, $2.9\underline{6} \times 10^{41} kg$, $4.7\underline{3} \times 10^{41} kg$, $6.2\underline{1} \times 10^{41} kg$, $8.3\underline{1} \times 10^{41} kg$, respectively.

b

The loglog plot is as follows, with a linear regression line.

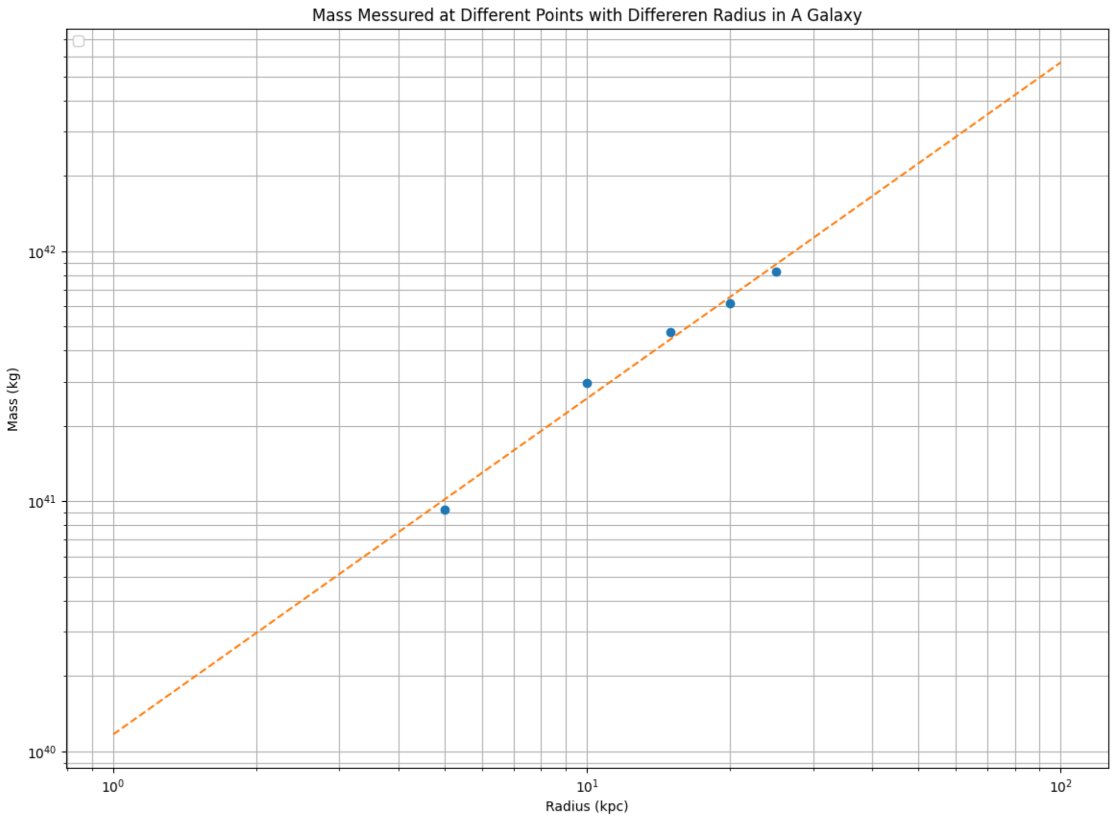

Figure 1: Mass measured at points in this galaxy with different radius and volocities

c

The picture above is drawn by matplotlib in Python. We can see that the plotted graph roughly forms a straight line. Then we used linear regression method with least square difference to plot the prediction line. The parameter given by linear regression on this line is: slope 1.34, y-intersection: 92.3.

Let's consider the equation given in the background:

$$M_R = 4\pi\rho_0 R^{3-\alpha}$$
$$\iff \log M_R = (3-\alpha)\log R + \log 4\pi\rho_0$$

In our loglog plot, y is actually $\log M_R$, x is actually $\log R$. Therefore, with the linear regression line, we can conclude that:

$$3-\alpha = 1.3\underline{4}$$
$$\iff \alpha = 1.6\underline{6}$$

Our estimation of the value of α is $1.6\underline{6}$.

Question 2

a

Let's calculate the Hubble constant from each galaxy:

A	$\frac{31\underline{0}km/s}{14.\underline{0}Mpc} = 22.\underline{1}4285714\ km/(s \cdot Mpc)$
B	$\frac{13\underline{5}0km/s}{62.\underline{0}Mpc} = 21.\underline{7}7419355\ km/(s \cdot Mpc)$
C	$\frac{32\underline{5}0km/s}{14\underline{3}Mpc} = 22.\underline{7}2727273\ km/(s \cdot Mpc)$
D	$\frac{40\underline{7}0km/s}{18\underline{9}Mpc} = 21.\underline{5}3439153\ km/(s \cdot Mpc)$
E	$\frac{55\underline{7}0km/s}{26\underline{9}Mpc} = 20.\underline{7}063197\ km/(s \cdot Mpc)$

Then, let's take the average of those values to estimate:

$$H_0 = \frac{22.\underline{1}4285714 + 21.\underline{7}7419355 + 22.\underline{7}2727273 + 21.\underline{5}3439153 + 20.\underline{7}063197}{5} = 21.\underline{7}7700693$$

Therefore, the Hubble Costant in this Universe should be about $21.\underline{8}\ km/(s \cdot Mpc)$.

b

Since we assume constant expansion (constant velocity), the age of this universe should be the inverse of Hubble Constant

$$\begin{aligned}
&\frac{1}{21.\underline{7}7700693\ km/(s \cdot Mpc)} \frac{1\ km}{1000\ m} \frac{3.086 \times 10^{22}\ m}{1\ Mpc} \\
=&1.4\underline{1}7090976 \times 10^{18} s \\
=&1.4\underline{1}7090976 \times 10^{18} s \frac{1\ yr}{31556952\ sec} \frac{1\ Gyr}{10^9 yr} \\
=&44.\underline{9}0582538\ Gyr
\end{aligned}$$

Therefore, the age of the universe is about $44.\underline{9}\ Gyr$