PaintArcs 2.0: Simulating FITS files of galaxy-galaxy strong lensing

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March 18, 2022

Outline

PaintArcs 2.0

- PaintArcs 2.0 is a python code capable of producing simulated strong lensing images, similar to those taken by ground-based telescopes.

- It uses a Sérsic profile to simulate the surface brightness of a source-galaxy and applies the lens equation of a user chosen lens model to generate the lensed images.

- There are 8 physical lens models for now. We intend to add more models in the future.

- The PaintArcs 2.0 code consists of 13 python modules: *add_noise.py*, *config_parser.py*, *convert_arcsec_pix.py*, *convolve.py*, *get_sersic_b_n.py*, *header_funcs.py*, *imcp.py*, *lensing_models.py*, *paint_arcs_pipeline.py*, *paint_arcs_v2.py*, *regexp.py*, *segmentation_ids.py* and *wcs_conversion.py*.

PaintArcs 2.0

- The most important files are $lensing\_models.py$, $paint\_arcs\_v2.py$ and $paint\_arcs\_pipeline.py$. The first one contains the lens models and the elliptical radial profile representing the source, the middle one produces the pixelation of our model into a FITS file and the last one is just the pipeline of our code.

- Most Strong Lensing simulation codes use ray tracing techniques even when dealing with analytical models for the lens en source. In PaintArcs 2.0 we simply map the source to the lens plane using the lens equation in an analytic way, which makes it a very fast code.

- **Point Lens**: Exterior solution to any spherically symmetric mass distribution (e.g. stars and black holes). Its deflection angle is given by the famous result:

$$\boxed{\hat{\alpha} = \frac{4GM}{c^2 r}}. \tag{1}$$

- **Application**: It us widely used for microlensing by single lenses.

- **Singular Isothermal Sphere (SIS)**: The mass components of this lens behave like particles of an isothermal ideal gas in hydrostatic equilibrium. It is possible to show that the radial density profile of this model is given by:

$$\boxed{\rho(r) = \frac{\sigma_v^2}{2\pi G r^2}}, \tag{2}$$

where σ_v is the one-dimensional particle velocity dispersion, G the Newton's constant of gravitation.

- **Application**: This density profile produces flat rotation curves. Furthermore it provides a good description of the total (Dark Matter plus baryonic) density profile of elliptical and lenticular galaxies.

- **Singular Isothermal Ellipsoid (SIE)**: It has the same density profile as the SIS, however, instead of spherical symmetry, it has the shape of an ellipsoid[1]. Its density profile is a generalization of the SIS model:

$$\boxed{\rho(\xi_1, \xi_2, z) = \frac{\sigma_v^2}{2\pi G} \frac{\sqrt{f}}{(\xi_1^2 + f^2\xi_2^2 + z^2)}}. \tag{3}$$

- **Application**: It models a gravitational lens in the galactic scale. Its use is the same as for the SIS, but with the addition of the mass distribution ellipticity, which makes the model more realistic.

[1] $f = 1 - \varepsilon_\Sigma$, where ε_Σ is the ellipticity of the profile projection.

- **Singular Isothermal Elliptical Potential (SIEP)**: It has the same profile of the SIS, however, instead of having elliptical symmetry in the profile of matter itself like the SIE, it has en elliptical potential. For this, we just do the coordinate transformation:

$$\boxed{\begin{aligned} & x \rightarrow \bar{x} = x\sqrt{1 + \varepsilon \cos 2\phi} \\ & \Phi(x) \rightarrow \Phi_\varepsilon = \Phi(\bar{x}) \end{aligned}} \tag{4}$$

- **Application**: For small ellipticities, this model is reduced to the SIE model through the relation $\varepsilon_\Sigma = 3\varepsilon_\Phi$. For its simplicity, the SIEP can be used instead of the SIE thanks to the relationship between these two models.

- For the models with axial symmetry, that is, the **Point Lens** and the **SIS**, we have that the **Lens Equation**[2] of the two models is given by (5) and (6), respectively.

$$\begin{cases} \hat{\vec{\alpha}}(\vec{\theta}) = \frac{4GM}{c^2 D_{OL}} \frac{\vec{\theta}}{\theta^2} \\ \\ \theta_E = \sqrt{\frac{4GM}{c^2} \frac{D_{LS}}{D_{OS} D_{OL}}} \end{cases} \quad \Rightarrow \quad \boxed{\vec{\beta} = \left(1 - \frac{\theta_E^2}{\theta^2}\right) \vec{\theta}} \tag{5}$$

$$\begin{cases} \hat{\vec{\alpha}}(\vec{\theta}) = \frac{4\pi\sigma_v^2}{c^2} \frac{\vec{\theta}}{\theta} \\ \\ \theta_E = \frac{4\pi\sigma_v^2}{c^2} \frac{D_{LS}}{D_{OS}} \end{cases} \quad \Rightarrow \quad \boxed{\vec{\beta} = \left(1 - \frac{\theta_E}{\theta}\right) \vec{\theta}} \tag{6}$$

[2] D_{OL} is the angular diameter distance between the observer and the lens.

- For the models with elliptical symmetry, that is, the **SIE** and the **SIEP**, we have that the **Lens Equation** in Cartesian form of the two models is given by (7) and (8) , respectively.

$$\begin{cases} \beta_1 = \theta_1 - \theta_E \frac{\sqrt{f}}{\sqrt{1-f^2}} \operatorname{arcsenh}\left(\frac{\sqrt{1-f^2}}{f} \frac{\theta_1}{\sqrt{\theta_1^2+\theta_2^2}} \right) \\ \\ \beta_2 = \theta_2 - \theta_E \frac{\sqrt{f}}{\sqrt{1-f^2}} \operatorname{arcsen}\left(\sqrt{1-f^2} \frac{\theta_2}{\sqrt{\theta_1^2+\theta_2^2}} \right) \end{cases} \tag{7}$$

$$\begin{cases} \beta_1 = \left(1 - \frac{\theta_E \sqrt{1+\varepsilon}}{\sqrt{\theta_1^2 + \frac{1-\varepsilon}{1+\varepsilon}\theta_2^2}} \right) \theta_1 \\ \\ \beta_2 = \left(1 - \frac{\theta_E \sqrt{1-\varepsilon}}{\sqrt{\theta_2^2 + \frac{1+\varepsilon}{1-\varepsilon}\theta_1^2}} \right) \theta_2 \end{cases} \tag{8}$$

- **Models with external shear and external convergence**: In all four models mentioned, it is possible to add sources of angular perturbations to the potential in a linear way, we just have to use the so-called external convergence and external shear in the lens equation:

$$\begin{cases} \beta_1 = (1 - \kappa - \gamma)\theta_1 - \frac{D_{LS}}{D_{OS}}\hat{\alpha}_1(\theta_1, \theta_2) \\ \\ \beta_2 = (1 - \kappa + \gamma)\theta_2 - \frac{D_{LS}}{D_{OS}}\hat{\alpha}_2(\theta_1, \theta_2) \end{cases} \tag{9}$$

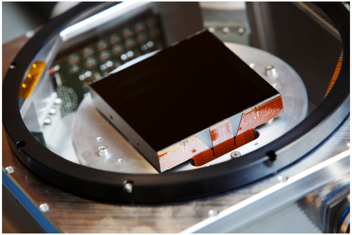
- **Application**: It is used to model tidal effects caused by objects close to our main lens, for example, a galaxy perturbing the gravitational potential of another galaxy acting as a lens.

- The PaintArcs 2.0 uses 8 models, but the focus of this code is on the SIS.

Astronomical Images

- Digital images are nothing more than 2D matrices whose components are called pixels. In Astronomy, telescope images are captured by **CCDs** (Charge Coupled Devices), which convert photons from celestial objects into electric charge (signal).

- The file format used to save these images is the **FITS** (Flexible Image Transport System) one, which not only store data about the signal strength at each pixel but also preserve data such as position in the sky, exposure time, filter, etc. in the so-called **header**.



Sources

- We want to simulate astronomical images of the phenomenon of gravitational lensing on the galactic scale, for this we need a surface brightness profile that mimics the light of a galaxy as it is captured by CCDs. For this, we use the Sérsic profile:

$$\boxed{I(R) = I_0 \exp\left\{-b_n \left(\frac{R}{R_e}\right)^{1/n}\right\}} \tag{10}$$

- n: Sérsic index, defines the shape of the profile;
- R_e: Effective Sérsic radius, contains half of the galaxy's light;
- I_0: $I_0 = I_e e^{b_n}$, is the intensity at the center of the galaxy;
- b_n: Constant that only depends on the n parameter;
- R: Radial profile of the lensed galaxy.

Elliptical sources

- In this work, both the lens and the source are galaxies, these in general have elliptical shapes when seen by telescopes, so we can use this geometry to model our source. The elliptical profile is given by:

$$\boxed{\begin{aligned} R^2 = (1-\varepsilon_S)[(\beta_1 - S_1)\cos\phi_e + (\beta_2 - S_2)\sin\phi_e]^2 \\ +(1+\varepsilon_S)[(\beta_2 - S_2)\cos\phi_e - (\beta_1 - S_1)\sin\phi_e]^2 \end{aligned}} \qquad \begin{aligned} (11) \\ (12) \end{aligned}$$

- R_0: Effective radius of the source;

- ε_S: Source's ellipticity;

- ϕ_e: Angle of the major axis with respect to β_1, ellipse's orientation;

- (S_1, S_2): Cartesian position of the center of the ellipse.

- Our simulation method consists of 3 basic elements: the elliptical source, the lens equation and the surface brightness. The lens equation distorts the elliptical profile creating the geometric shapes of the lensed images and the Sérsic profile mimics the brightness of galaxies.

$$R_0(\beta_1, \beta_2) \;\rightarrow\; R_0(\theta_1, \theta_2) \;\Rightarrow\; I(R_0) = I(\theta_1, \theta_2) \tag{13}$$

- We can apply this to any of the 8 models mentioned, we just have to make the change $(\beta_1, \beta_2) \rightarrow (\theta_1, \theta_2)$ in (11) through the lens equation. In the case of the SIS, the radial profile is given by:

$$\boxed{\begin{aligned} R_0^2 = (1-\varepsilon_s)&\left[(\theta_1\cos\phi_e + \theta_2\sin\phi_e)\left(1 - \frac{\theta_E}{\sqrt{\theta_1^2+\theta_2^2}}\right) - S_1\cos\phi_e - S_2\sin\phi_e\right]^2 \\ + (1+\varepsilon_s)&\left[(\theta_2\cos\phi_e - \theta_1\sin\phi_e)\left(1 - \frac{\theta_E}{\sqrt{\theta_1^2+\theta_2^2}}\right) + S_1\sin\phi_e - S_2\cos\phi_e\right]^2 \end{aligned}} \tag{14}$$

- At a first level, the method explained here is suitable for any model, as it is not necessary to solve the lens equation to find a solution $\theta = \theta(\beta_1, \beta_2)$.

- Our surface brightness profile is a continuous function, however, a digital image is discrete. The process of digitalizing an image consists of approximating the signal at each pixel and estimating the error of this transition from continuous to discrete.

- In total, we have 15 input parameters in this code:

$$[R_0, S_1, S_2, \phi_e, n, R_e, \varepsilon_S, I_0, \varepsilon_\Sigma, \kappa, \gamma, \theta_E, x_{1,0}, x_{2,0}, model] \qquad (15)$$

I_0 can be determined from the apparent magnitude m of the image or chosen by the user as in our pedagogical examples below.

Setting I_0

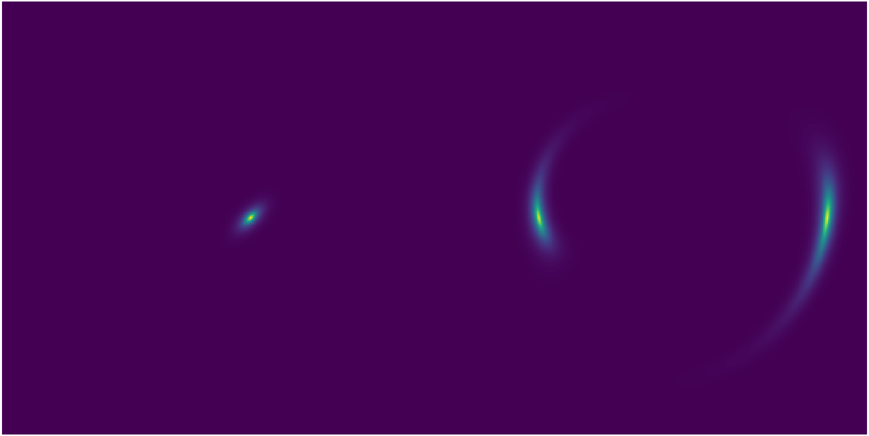
- To define I_0 from m, the function I must be normalized such that the total brightness of the image is equal to the total sign $S_{total} = 10^{-0.4(m-m_{zpt})}$, where m_{zpt} is the zero-point magnitude. So:

$$I_0 = \frac{10^{-0,4(m-m_{zpt})}}{\iint_A \exp\left\{-b_n\left(\frac{R_0(\theta_1,\theta_2)}{R_e}\right)^{1/n}\right\} dA} \tag{16}$$

- The integral in (15) is over the area of the lensed object, in which case it is necessary to know the analytical solution of the lens equation to determine the limits of integration of the various possible angular regimes. However, we have already done this for SIS in our past work.
- The PaintArcs 2.0 has two modes, a realistic one, which generates the images formed by the SIS (input: m and m_{zpt}), and a "pedagogical" version, which generates images of the other 7 models (input: I_0).

Example

- Image of the gravitational arcs of the SIS model with elliptical source produced by PaintArcs 2.0:



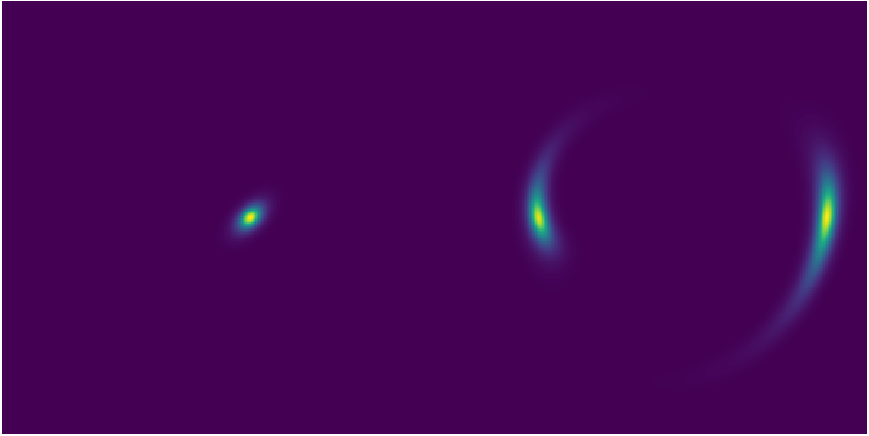
Adding the effect of the PSF

- **Seeing**: Cumulative effects caused by atmospheric refraction, movement of telescopes and incorrect focus. Causes luminous points to be mapped onto a 2D light distribution, with approximately elliptical shapes through the so-called PSF (Point Spread Function).

- To simulate these effects, we just convolute the total signal with the PSF, in our case a normalized two-dimensional Gaussian, to generate the observed signal:

$$\begin{cases} S_{obs} = S_{total} * PSF \\ \\ PSF = \exp\left\{-\frac{\theta^2}{2\sigma^2}\right\} \Big/ \iint_A \exp\left\{-\frac{\theta^2}{2\sigma^2}\right\} dA \end{cases} \tag{17}$$

- Addition of the seeing effect in the image of the gravitational arcs of the SIS model with elliptical source:



Adding noise

- **Poisson noise**: Astronomical sources emit few photons, so multiple exposures of equal integration times will yield different values of N_e (photon count), whose variation occurs according to a Poisson distribution:

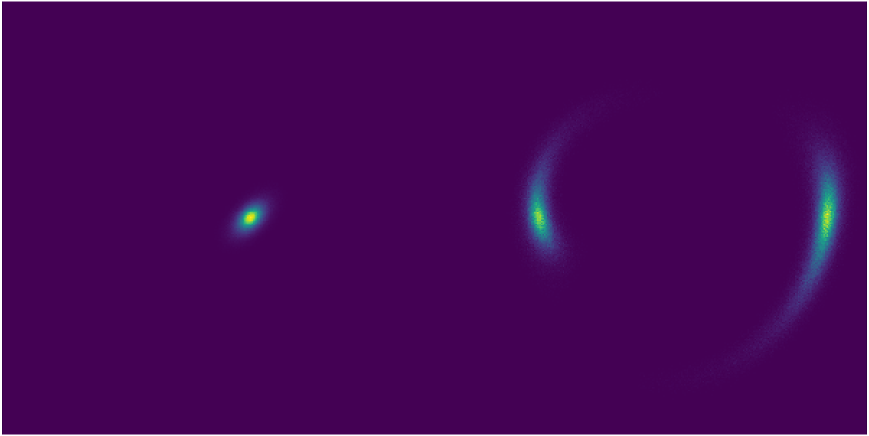
$$P(x, \mu) = \mu^x \frac{e^{-\mu}}{x!}. \tag{18}$$

As in this distribution the variance is equal to the expected value, and assuming N_e is now the expected value at total integration time, we have:

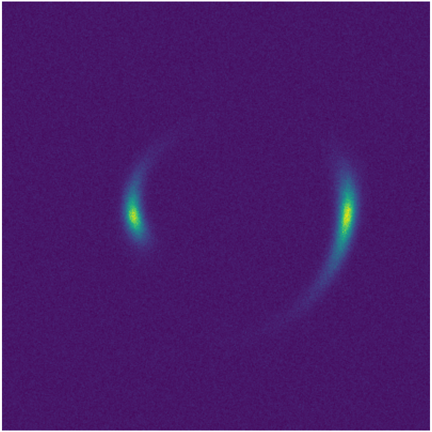
$$\sigma = \sqrt{N_e}. \tag{19}$$

- The noise in the photon count of an image is nothing more than this deviation. There are several reasons why this fluctuation occurs: the discrete nature of light, problems with the CCD etc.

- Addition of CCD noise and seeing effect in the image of SIS gravitational arcs with elliptical source:



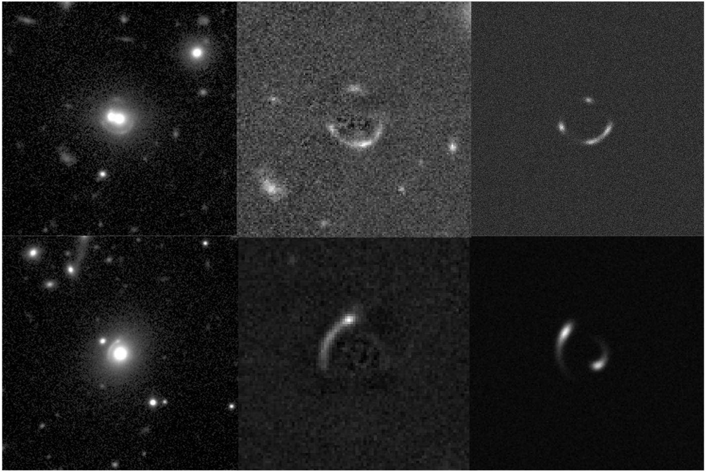
- Image and header of the SIS gravitational arcs with elliptical source, plus the noise in the arcs, a background noise and the effect of seeing:



Arquivo Editar Fonte

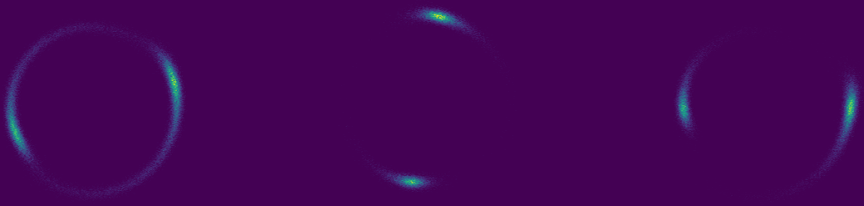
```
SIMPLE  =                    T / conforms to FITS standard
BITPIX  =                   64 / array data type
NAXIS   =                    2 / number of array dimensions
NAXIS1  =                  401
NAXIS2  =                  401
MODEL   = 'SIS     '           / Singular Isothermal Sphere
ERES    =                  0.8 / Effective Radius of the Source (arcsec)
ELLS    =                  0.6 / Ellipticity of the Source
AOES    =                   45 / Angular Orientation of the Source (degrees)
XPCEC   =                 4.93 / X Position of the Center of the Source (arcsec)
YPCEC   =                  4.0 / Y Position of the Center of the Source (arcsec)
XPCI    =                    4 / X Position of the Center of the Image (arcsec)
YPCI    =                    4 / Y Position of the Center of the Image (arcsec)
NINDEX  =                    1 / Arcs Sersic Profile index
RE      =                  0.2 / Arcs Sersic Effective Radius (arcsec)
IO      =    556.9651346179913 / Surface Brightness in the Center of the Source
SMAG    =                 18.0 / Arcs Magnitude
MAG_ZPT =                31.83 / Zero Point Magnitude
ESM     =                  0.0 / Ellipticity of the SIE or SIEP Model
EC      =                  0.0 / External Convergence
ES      =                  0.0 / External Shear
ER      =                    4 / Einstein Radius
ASIG    =    340408.1897010009 / Total Signal of the Arcs
ASIGDIST=    340404.5781198823 / Signal of the Arcs Distributed by PaintArcs
FLUXERR = 1.06095600160942E-05 / Error on the Distributed Arcs Flux
APSF    =                 20.0 / Arcs PSF (arcsec)
BKG     =                   25 / Background Mean Value
END
```

- llustrative comparison between simulations (SIE and SIS) and real images of gravitational arcs candidates obtained by the CBPF group in the CS82 survey.

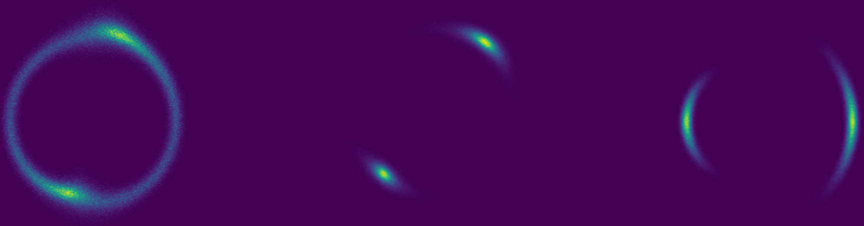


More examples

Lente Pontual

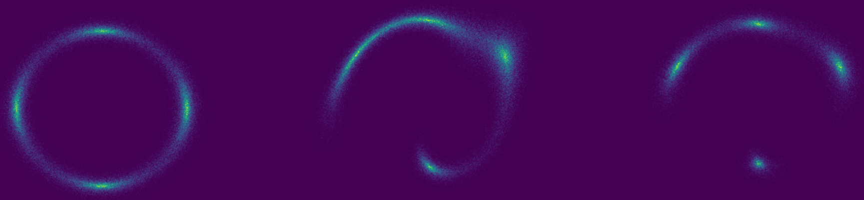


SIS

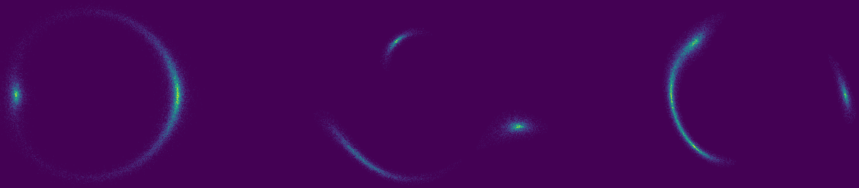


More examples

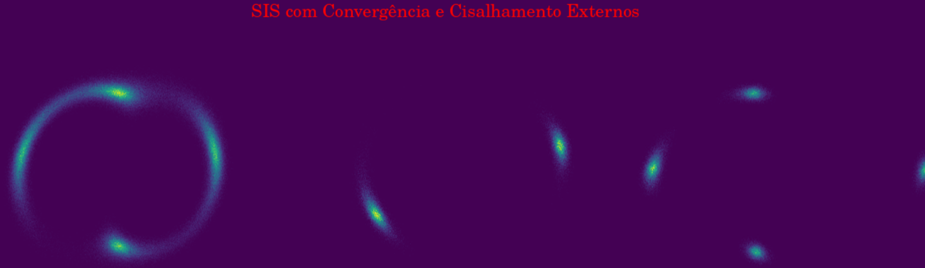
SIE



SIEP



More examples



More examples

References

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Figures

1 Image of the 1919 solar eclipse in Sobral. Taken from $https://www.eso.org/public/brazil/images/potw1926a/?fbclid = IwAR16fShpjWK5vkx1At8Ef_RrCGY8bwJJMVzyrZ246$ $2xy9oWcNmWSt_2yf_w$ - Page 2.

2 Image of a CCD of the ESPRESSO instrument on the ESO VLT. Taken from $https://www.eso.org/public/blog/50 - years - of - ccds/$. - Page 19.