Part II
Microlensing

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PPGCosmo

OAC Observatorio Astronómico de Córdoba

UNC Universidad Nacional de Córdoba

IATE

Lens Equation and Point Mass

Point Mass Lens

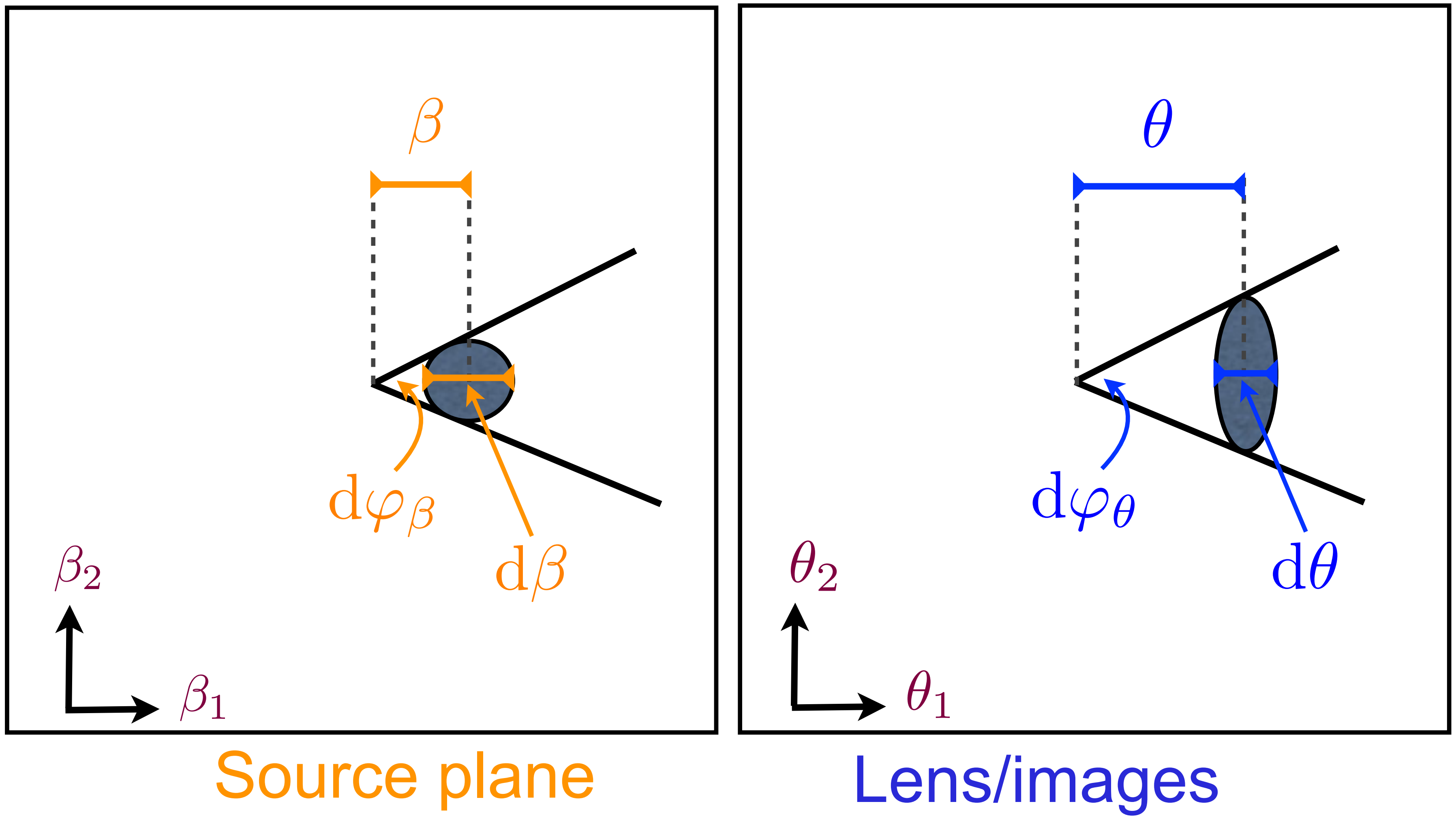
Point mass $$\hat{\vec{\alpha}} = \frac{4GM}{c^2 \xi}$$

Reduced deflection angle $$\vec{\alpha} = \frac{D_{LS}}{D_{OS} D_{OL}} \frac{4GM}{c^2 \theta}$$

Lens equation $$\beta = \theta - \frac{\theta_E^2}{\theta}$$

Einstein angle $$\theta_E = \sqrt{\frac{D_{LS}}{D_{OS} D_{OL}} \frac{4GM}{c^2}}$$

Magnification in the axially symmetric case



Radial length	$\mathrm{d}\beta$	$\mathrm{d}\theta$	Radial magnification	$\dfrac{\mathrm{d}\theta}{\mathrm{d}\beta}$
Tangential length	$\beta\,\mathrm{d}\varphi_\beta$	$\theta\,\mathrm{d}\varphi_\theta$	Tangential magnification	$\underbrace{\dfrac{\mathrm{d}\varphi_\theta}{\mathrm{d}\varphi_\beta}}_{=1}\dfrac{\theta}{\beta}$

Images and magnification

Lens equation $$\beta = \theta - \frac{\theta_E^2}{\theta}$$

Solutions $$\theta_{1,2} = \frac{1}{2}\left(\beta \pm \sqrt{\beta^2 + 4\theta^2}\right)$$

Magnification $$\mu = \frac{\theta}{\beta}\frac{d\theta}{d\beta}$$

Images and magnification

Lens equation $$\beta = \theta - \frac{\theta_E^2}{\theta}$$

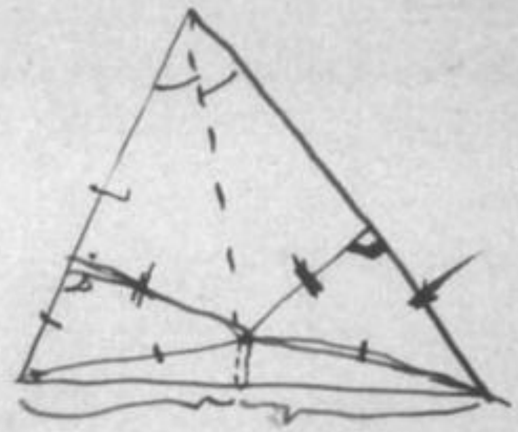
Solutions $$\theta_{1,2} = \frac{1}{2}\left(\beta \pm \sqrt{\beta^2 + 4\theta^2}\right)$$

Magnification $$\mu_{1,2} = \left(1 - \left[\frac{\theta_E}{\theta_{1,2}}\right]^4\right)^{-1} = \frac{1}{2} \pm \frac{u^2 + 2}{2u\sqrt{u^2 + 4}}$$

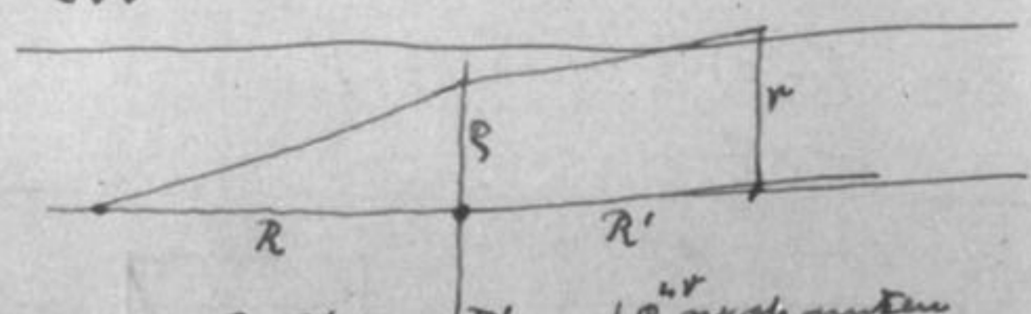
Distance in units of the Einstein angle $$u = \beta/\theta_E$$

Total magnification $$\mu = |\mu_1| + |\mu_2| = \frac{u^2 + 2}{u\sqrt{u^2 + 4}}$$

Alle Dreiecke sind gleichschenklig.



Leo Kestenberg
Berlin-Halensee,
Joachim Friedrichstr. 33.



$$r = \varrho \frac{R+R'}{R} - \frac{R'\alpha}{\varrho}$$

ϱ nach unten negativ. Dann gilt auch für stark abgebeugte Strahl.

$$r_0 = \varrho_0 - \frac{1}{\varrho_0} \quad \ldots (1)$$

$$\boxed{\varrho_0^2 = \varrho^2 \frac{R+R'}{RR'\alpha}}$$

Ersetzt.
$$r = \cdot - \frac{R\alpha}{\varrho} = \cdot - \frac{R\alpha}{\varrho_0}\sqrt{\frac{R+R'}{RR'\alpha}}$$
$$= \cdot - \frac{1}{\varrho_0}\sqrt{\frac{R}{R'}(R+R')\alpha}$$

$$\left.\begin{array}{l} \boxed{r_0 = r\sqrt{\dfrac{R'}{R(R+R')\alpha}}} \\ \boxed{\varrho_0 = \varrho\sqrt{\dfrac{R+R'}{RR'\alpha}}} \end{array}\right\} (2)$$

1) gibt zwei Wurzeln für ϱ_0

Von hier an Index 0 weggelassen.

$$2 + r^2 = \varrho^2 + \frac{1}{\varrho^2}$$
$$f = \varphi + \frac{\pi^2}{\varphi}$$
$$df = \left(1 - \frac{\pi^2}{\varphi^2}\right)d\varphi = \left(1 - \frac{1}{\varrho^4}\right)d\varphi$$
$$\mathcal{H}\,df = \pm H\,d\varphi$$
$$\mathcal{H} = \pm \frac{H}{1 - \frac{1}{\varrho^4}}$$
$$\mathcal{H}_{tot} = H\left\{\frac{1}{1 - \frac{1}{\varrho_1^4}} + \frac{1}{\frac{1}{\varrho_2^4} - 1}\right\} \quad \ldots (3)$$

Klammer gibt relative Helligkeit. (im ∞ = 1)

$$r = \frac{1}{x} - x$$

$$\{\ \} = \frac{1}{1 - x_1^4} + \frac{1}{x_2^4 - 1}$$

DISCUSSION

LENS-LIKE ACTION OF A STAR BY THE DEVIATION OF LIGHT IN THE GRAVITATIONAL FIELD

SOME time ago, R. W. Mandl paid me a visit and asked me to publish the results of a little calculation, which I had made at his request. This note complies with his wish.

The light coming from a star A traverses the gravitational field of another star B, whose radius is R_o. Let there be an observer at a distance D from B and at a distance x, small compared with D, from the extended central line $\overline{AB}$. According to the general theory of relativity, let α_o be the deviation of the light ray passing the star B at a distance R_o from its center.

For the sake of simplicity, let us assume that $\overline{AB}$ is large, compared with the distance D of the observer from the deviating star B. We also neglect the eclipse (geometrical obscuration) by the star B, which indeed is negligible in all practically important cases. To permit this, D has to be very large compared to the radius R_o of the deviating star.

It follows from the law of deviation that an observer situated exactly on the extension of the central line $\overline{AB}$ will perceive, instead of a point-like star A, a luminius circle of the angular radius β around the center of B, where

$$\beta = \sqrt{\alpha_0 \frac{R_0}{D}}.$$

It should be noted that this angular diameter β does not decrease like $1/D$, but like $1/\sqrt{D}$, as the distance D increases.

Of course, there is no hope of observing this phenomenon directly. First, we shall scarcely ever approach closely enough to such a central line. Second, the angle β will defy the resolving power of our instruments. For, α_o being of the order of magnitude of one second of arc, the angle R_o/D, under which the deviating star B is seen, is much smaller. Therefore, the light coming from the luminous circle can not be distinguished by an observer as geometrically different from that coming from the star B, but simply will manifest itself as increased apparent brightness of B.

The same will happen, if the observer is situated at a small distance x from the extended central line $\overline{AB}$. But then the observer will see A as two point-like light-sources, which are deviated from the true geometrical position of A by the angle β, approximately.

The apparent brightness of A will be increased by the lens-like action of the gravitational field of B in the ratio q. This q will be considerably larger than unity only if x is so small that the observed positions of A and B coincide, within the resolving power of our instruments. Simple geometric considerations lead to the expression

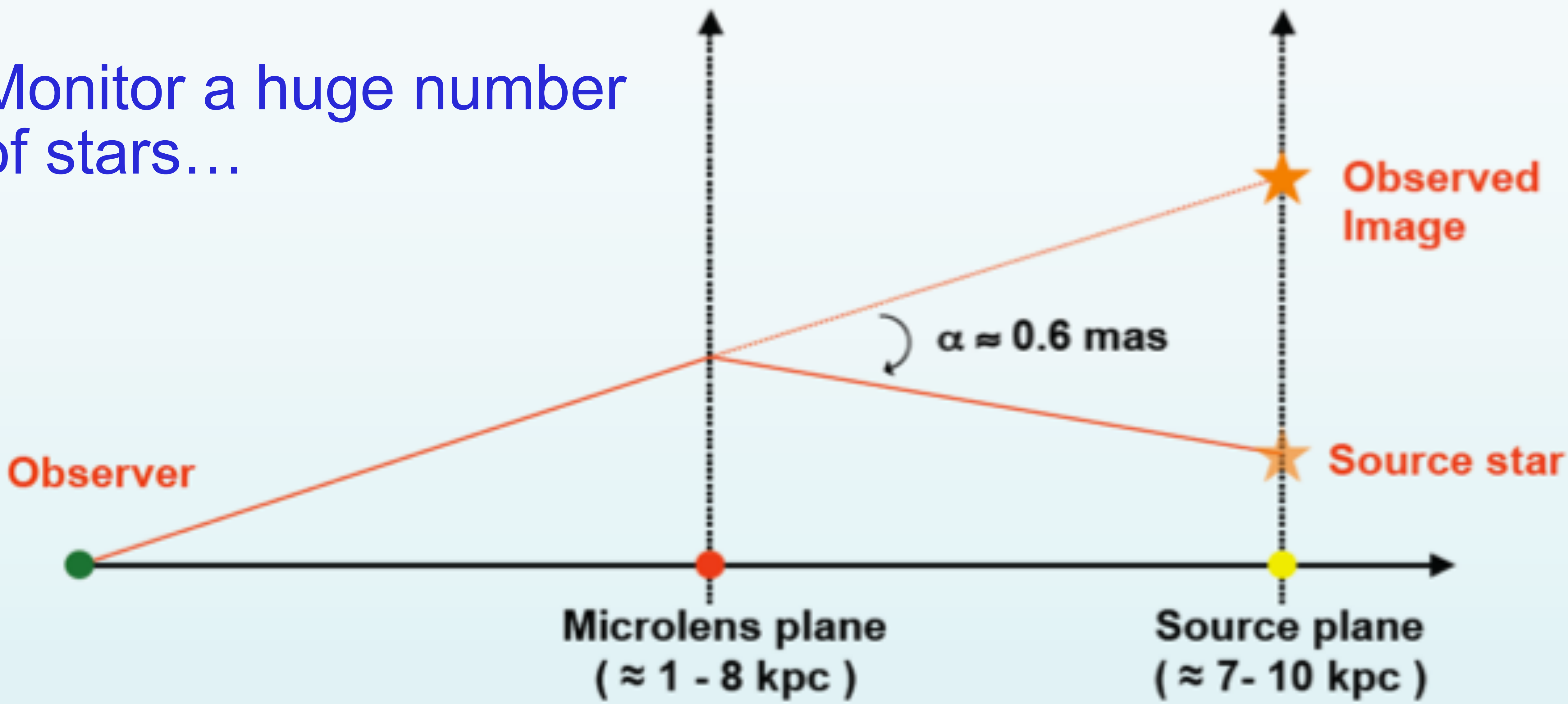
$$q = \frac{l}{x} \cdot \frac{1 + \frac{x^2}{2l^2}}{\sqrt{1 + \frac{x^2}{4l^2}}},$$

where

$$l = \sqrt{\alpha_0 D R_0}.$$

Galactic microlensing

Monitor a huge number of stars…



Microlensing light curve

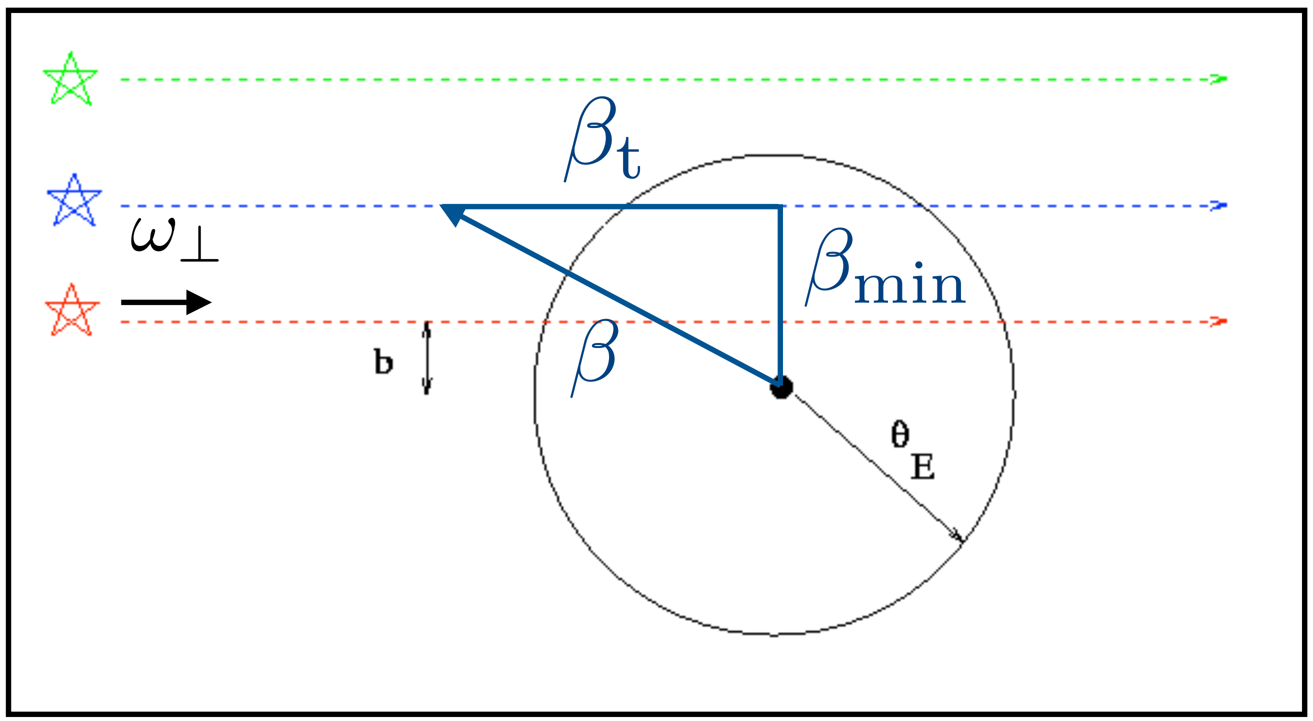
Point lens point source magnification:

$$\mu = \frac{u^2 + 2}{u\sqrt{u^2 + 4}} \qquad u = \beta/\theta_E$$

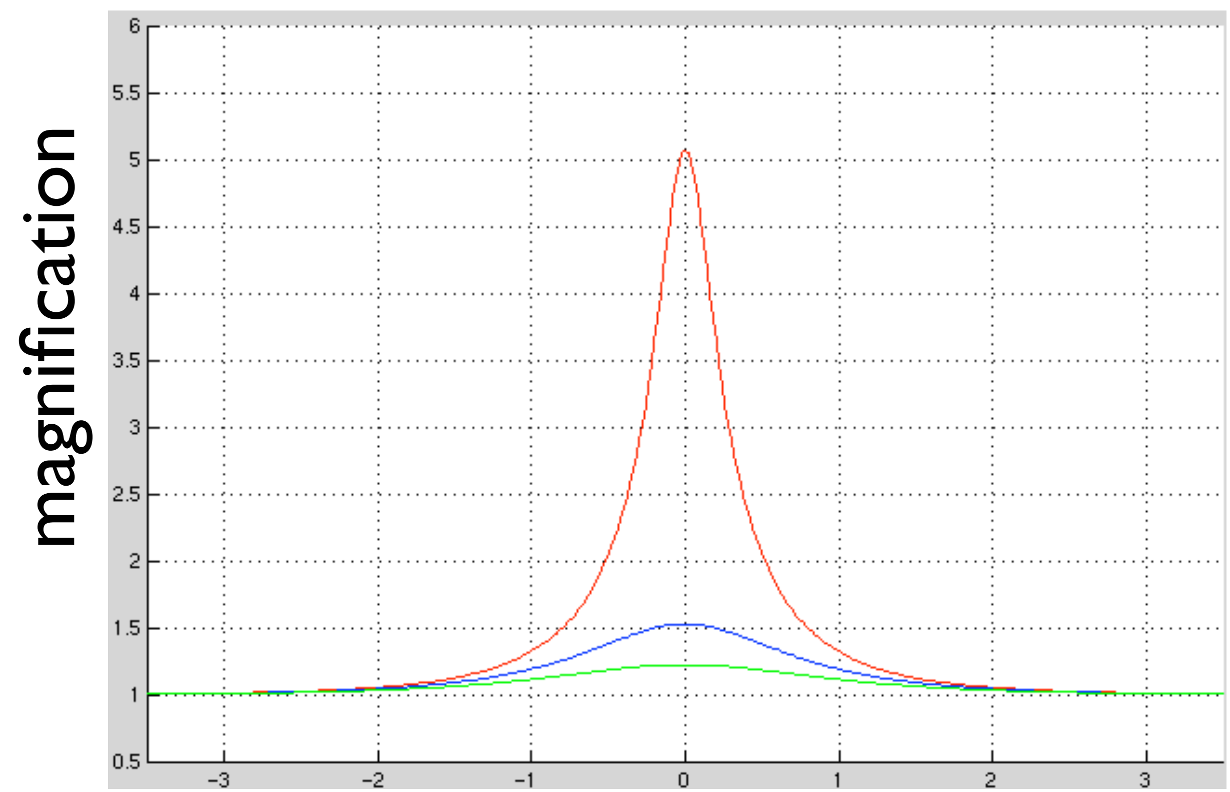
Einstein radius (angle):

$$\theta_E = \sqrt{\frac{D_{LS}}{D_{OS} D_{OL}} \frac{4GM}{c^2}}$$

$\omega_\perp$: angular velocity between lens and source



Source plane



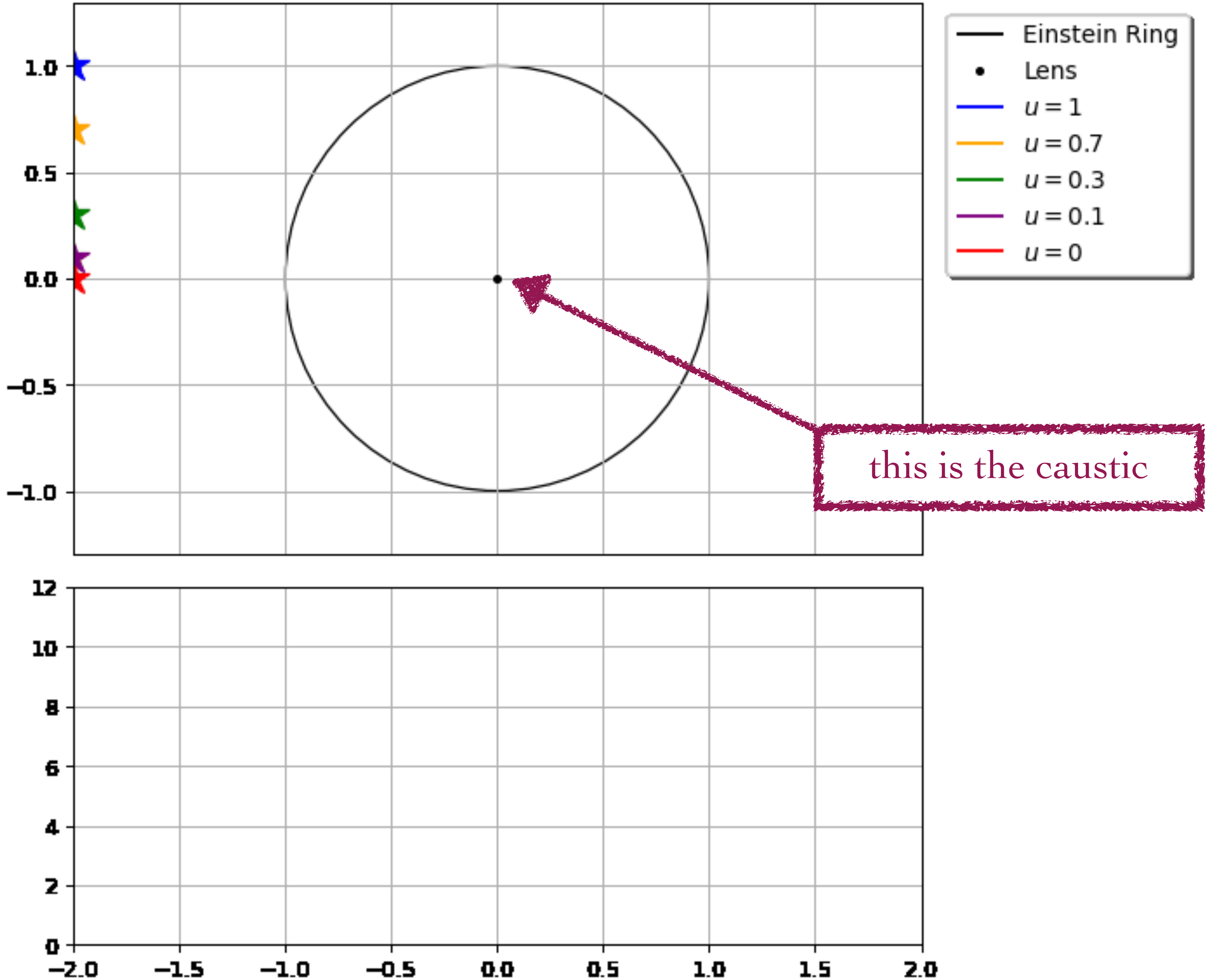
Time in Einstein time units

$$\beta_t = \omega_\perp t = \frac{v_\perp}{D_{OL}} t$$

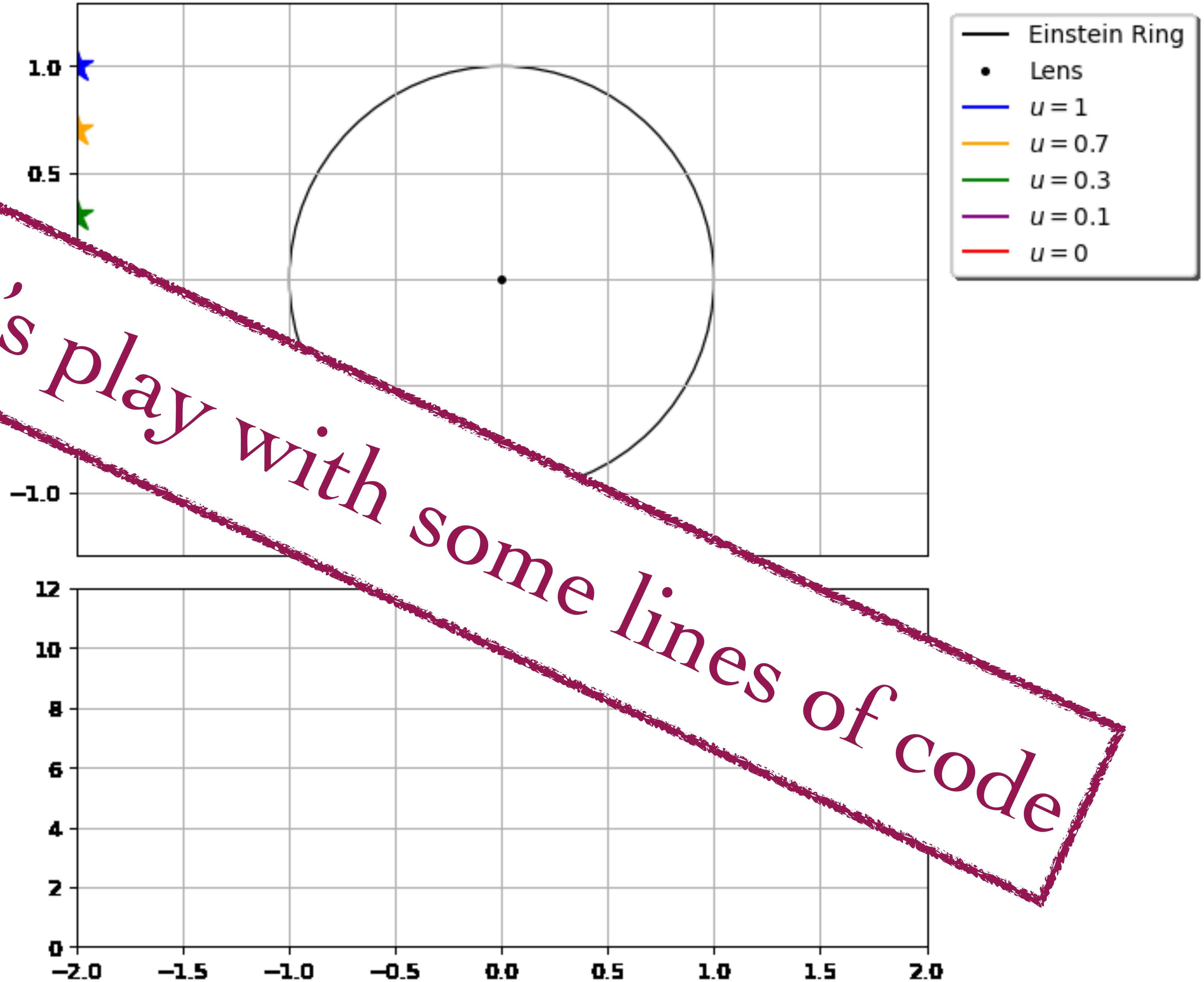
$$u(t) = \sqrt{u_{\mathrm{min}}^2 + \left(\frac{v_\perp t}{\theta_E D_{OL}}\right)^2} = \sqrt{u_{\mathrm{min}}^2 + \left(\frac{t}{t_E}\right)^2}$$

$$\boxed{t_E = \frac{\theta_E D_{OL}}{v_\perp}}$$

Microlensing of point source by a point mass



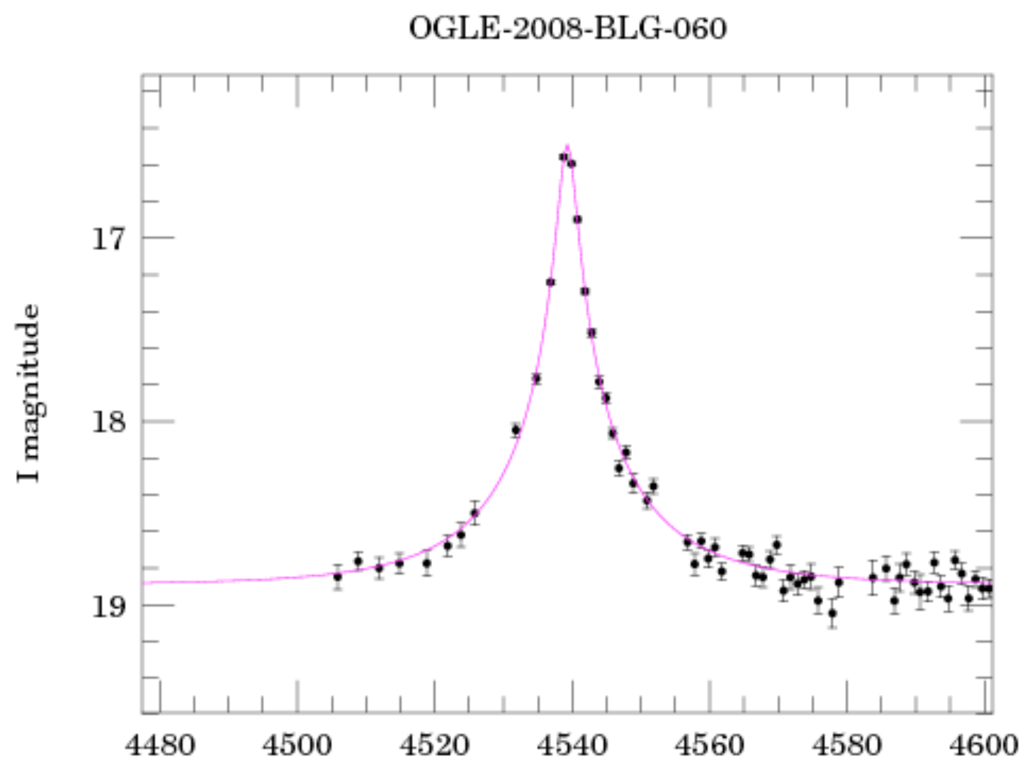
Microlensing of point source by a point mass

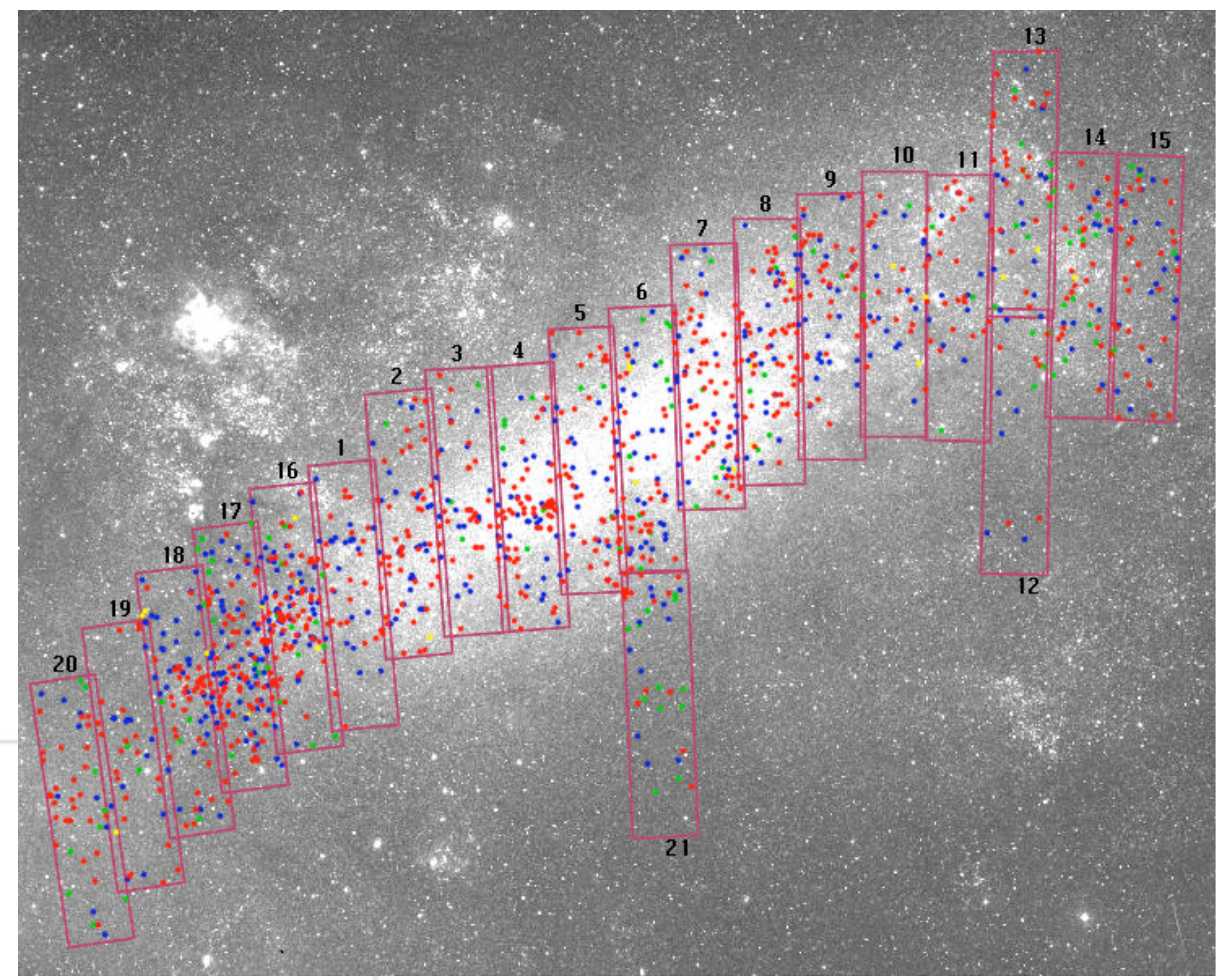


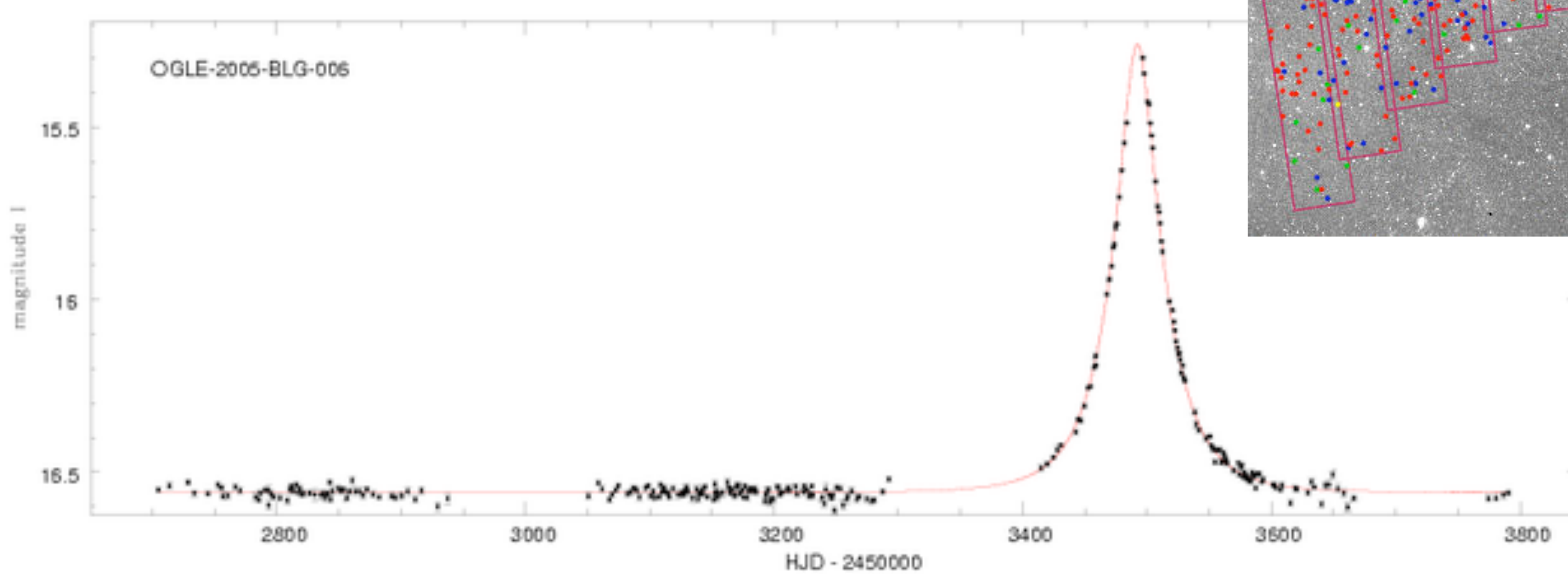
Let's play with some lines of code

Einstein was wrong
Microlensing observations

Projects MACHO, OGLE, EROS... (+ MOA, Kepler, HSC…)







$$t_{\mathrm{E}} \equiv D_{\mathrm{d}}\, \theta_{\mathrm{E}} / v_{\perp}$$

Microlensing by a binary lens

- Lens equation

$$\vec{\beta} = \vec{\theta} - \mu_A(\vec{\theta} - \vec{\theta}_A)\frac{\theta_E^2}{|\vec{\theta} - \vec{\theta}_A|^2} - \mu_B(\vec{\theta} - \vec{\theta}_B)\frac{\theta_E^2}{|\vec{\theta} - \vec{\theta}_B|^2}$$

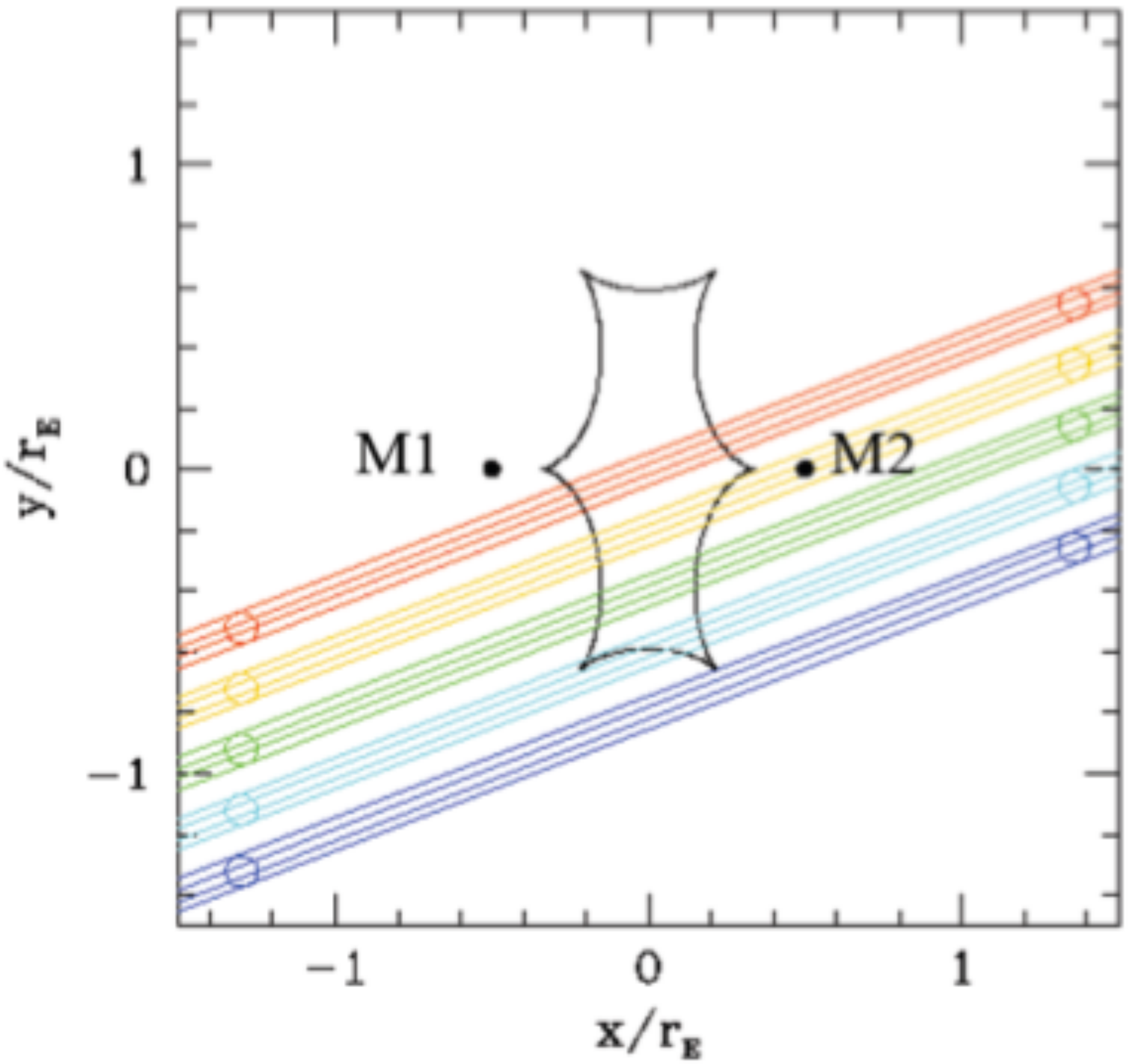
$$\mu_i = \frac{M_i}{M_1 + M_2}$$

θ_E : Einstein radius (angle) from the total mass $M_1 + M_2$

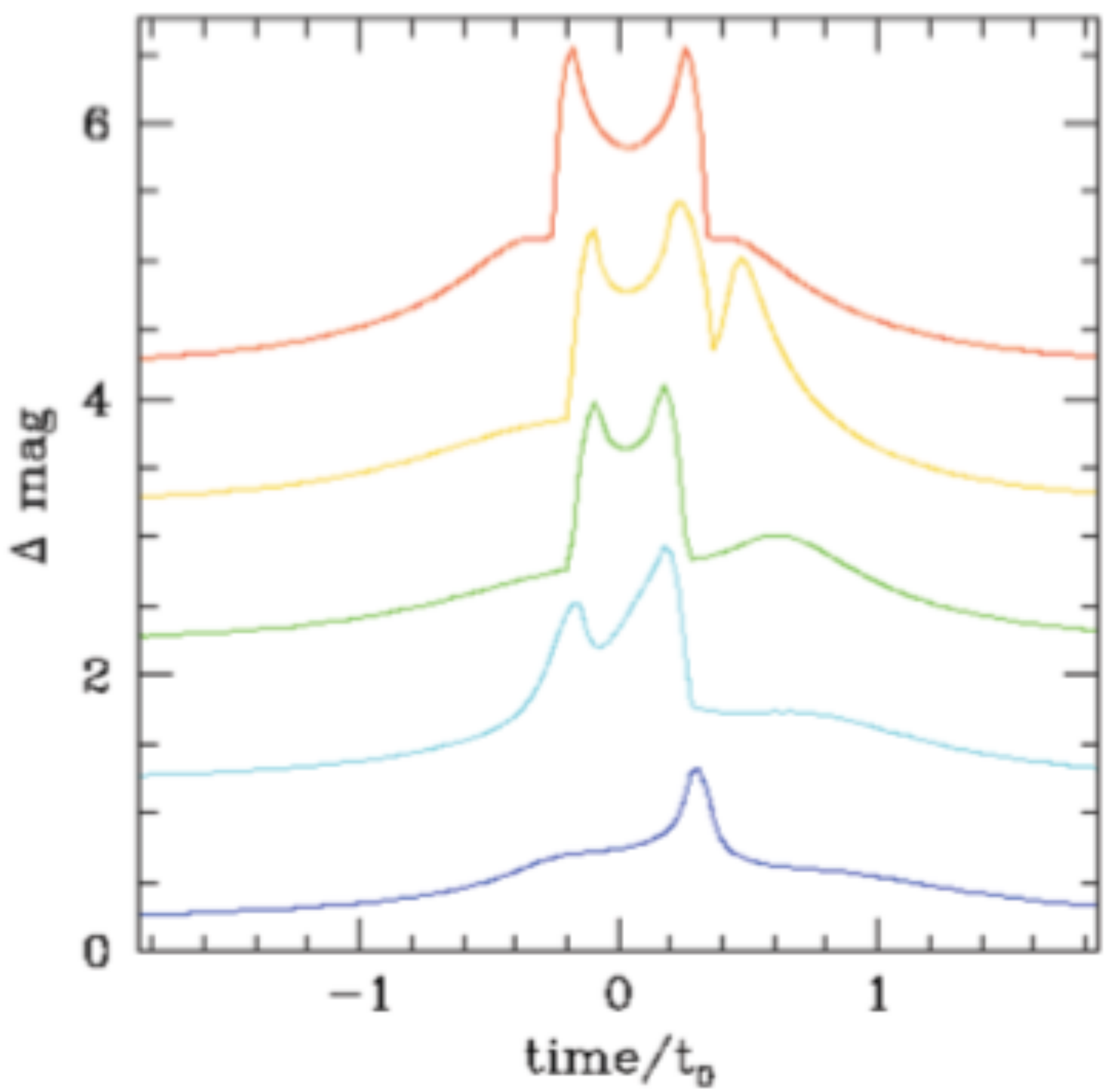
- Important to include finite source size effects

Microlensing by a binary lens

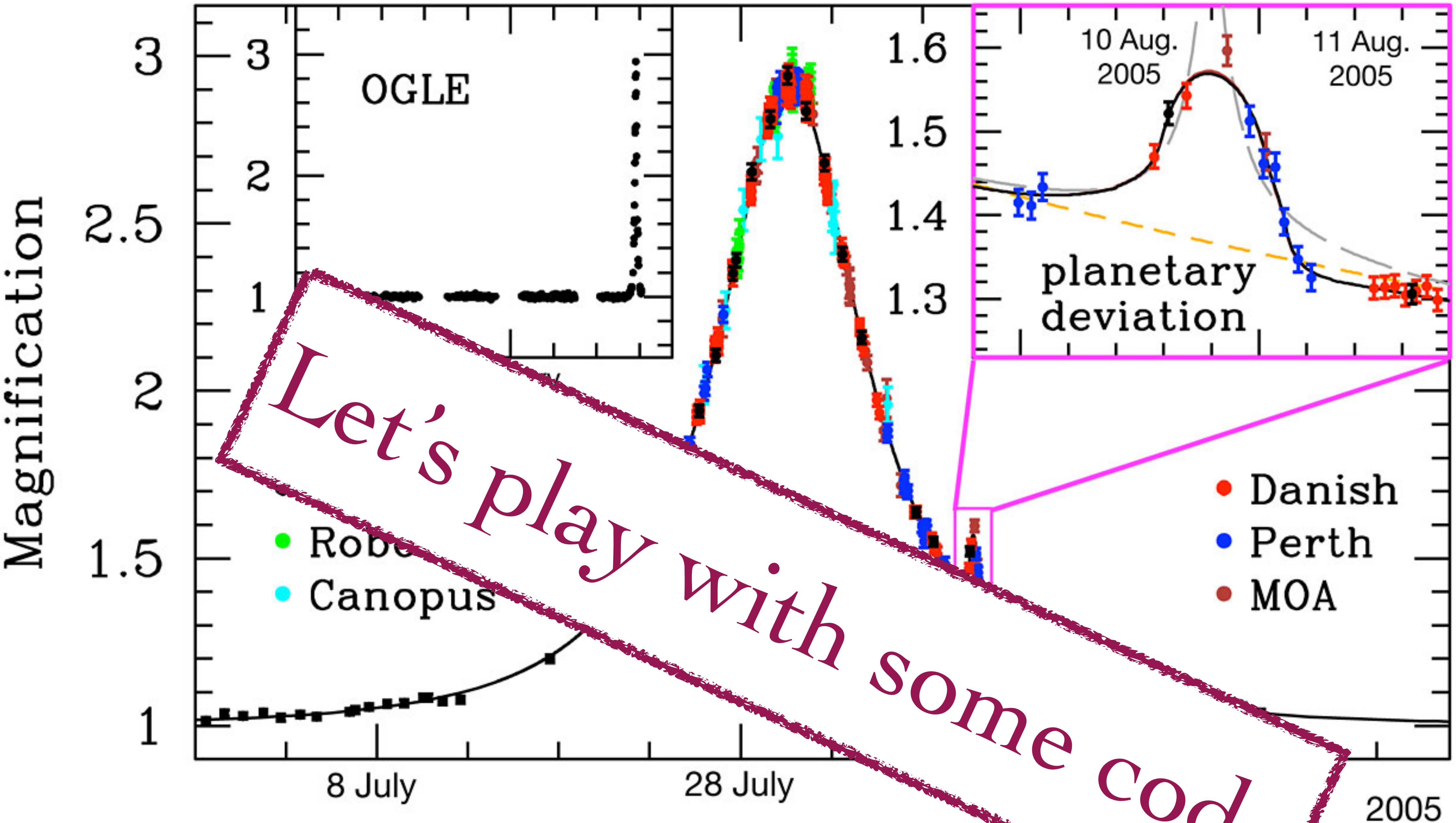
- Caustic structure



- Light curves



Exoplanets



https://www.eso.org/public/news/eso0603/

Let's play with some code

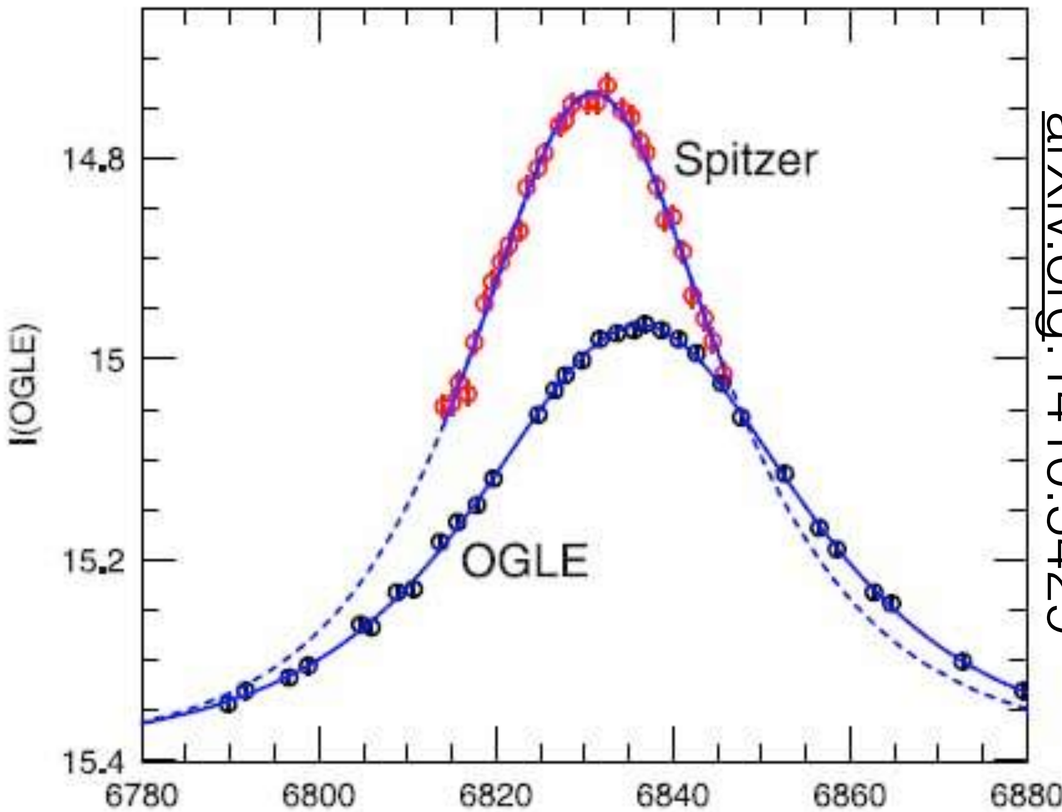
http://exoplanet.eu/catalog/ in 18/04/2022

- Over 170 planetary systems detected
- "Earth like" region in parameter space
- Combination with other methods

$$t_{\rm E} = D_{\rm d}\,\theta_{\rm E}/v_\perp$$

break degeneracy:
proper motions, parallax

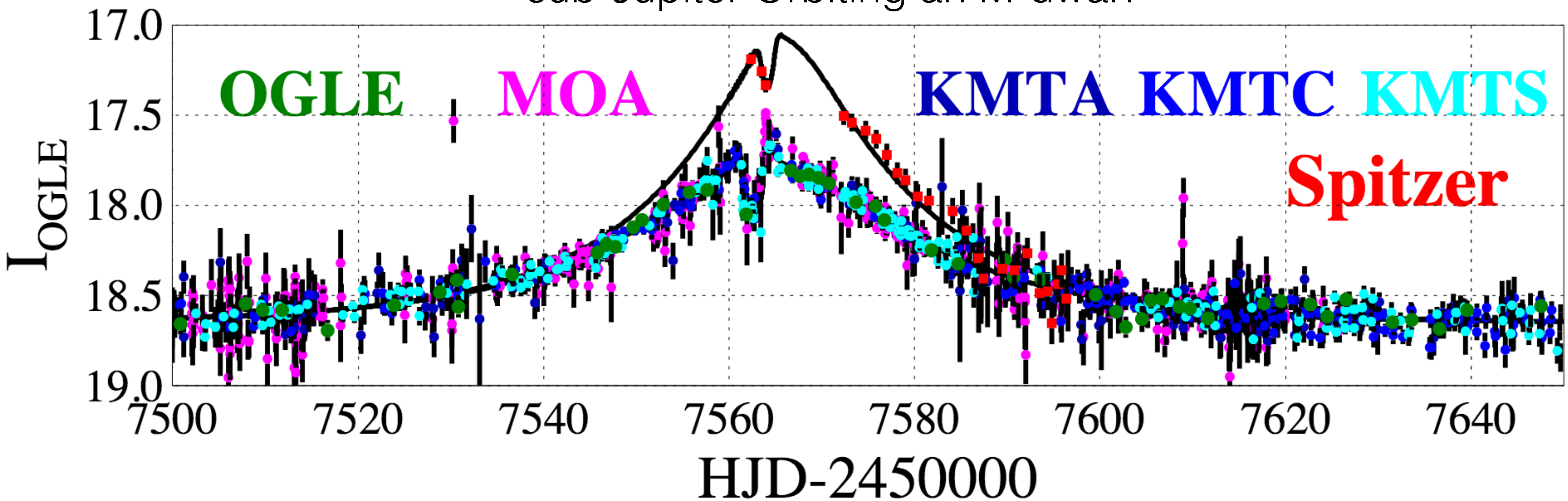
Microlensing parallax



arXiv.org:1410.5429

- Time and magnification effect
- No astrometry
- Parallax of a dark object
- Breaks degeneracies
- arXiv:1904.07789: dark lenses with parallax (WD, NS, BH; PBH?)

sub-Jupiter Orbiting an M-dwarf



arXiv:1801.05806

Limits on the abundance of compact objects as Dark Matter

- Model for the Dark Matter distribution in our galaxy (data and simulations): $\rho_{DM}(r,\theta,\phi)$
- Assumption: DM in the form of condensed objects (DCO/MACHO) of single mass M: $n(r,\theta,\phi) = \dfrac{\rho_{DM}(r,\theta,\phi)}{M}$
- Can predict the rate of microlensing events as a function of the lens mass and observational conditions
- Assume a fraction *f* of DM in the form of DCO
- Observed number of events sets limits on *f*
- Mass scale accessible depends on the time scale of the data and the distances involved

Limits on the abundance of compact objects as Dark Matter

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- Observed number of events [illegible] f
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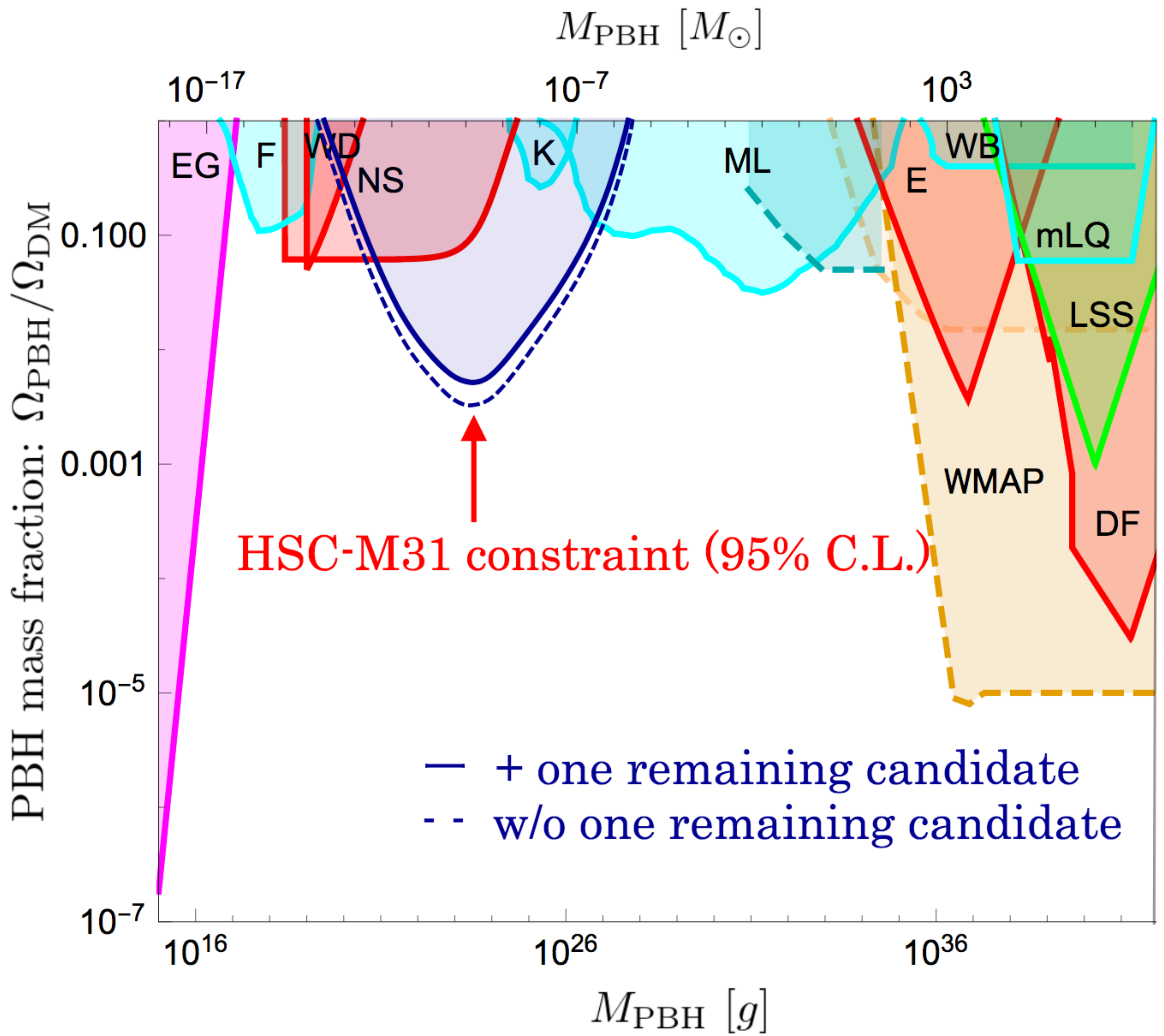
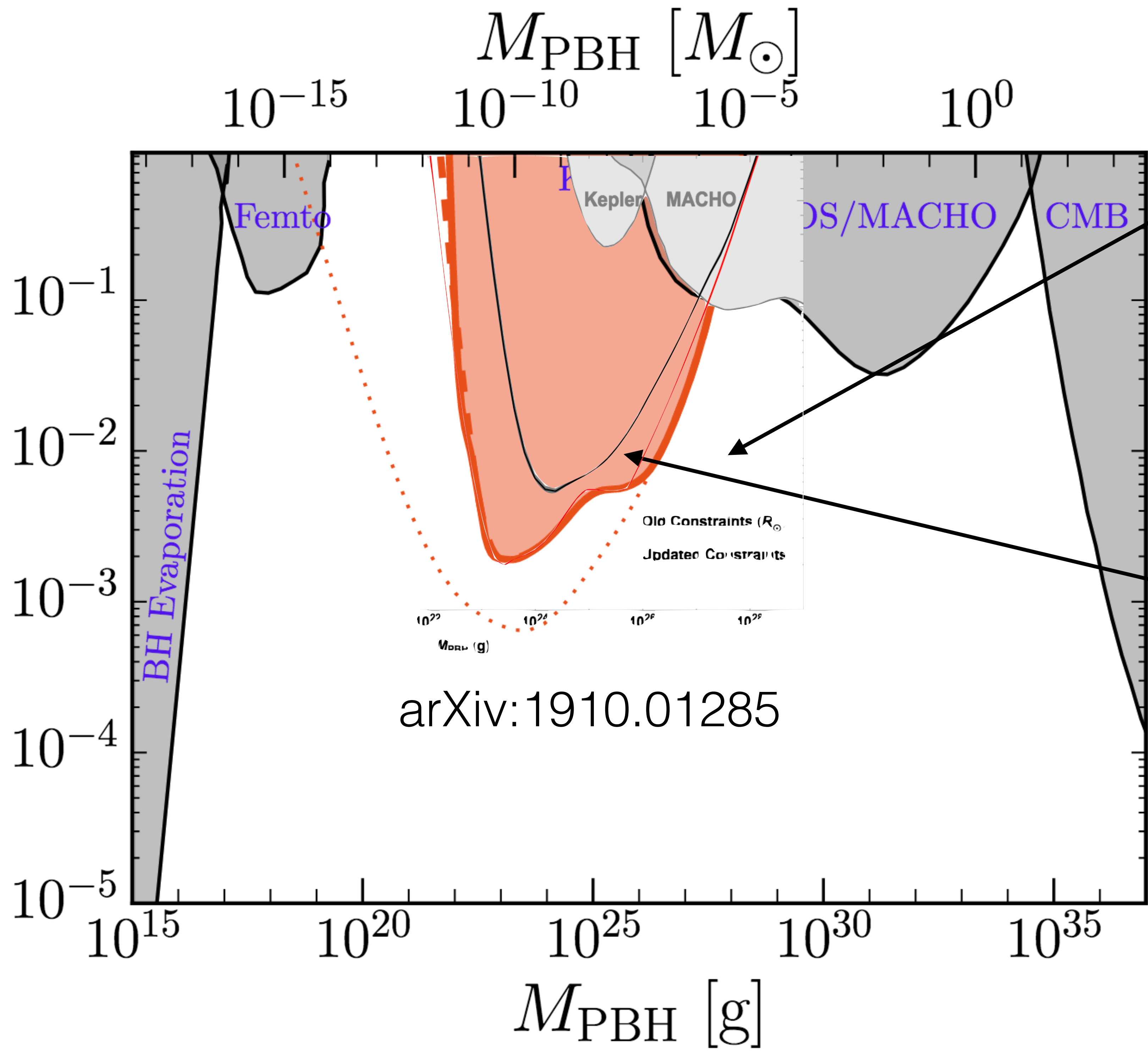
Let's see this in practice with some code

Limits on DM in the form of DCO in ~ 2017



arXiv:1701.02151v1

Limits on DM in the form of DCO in ~ 2019

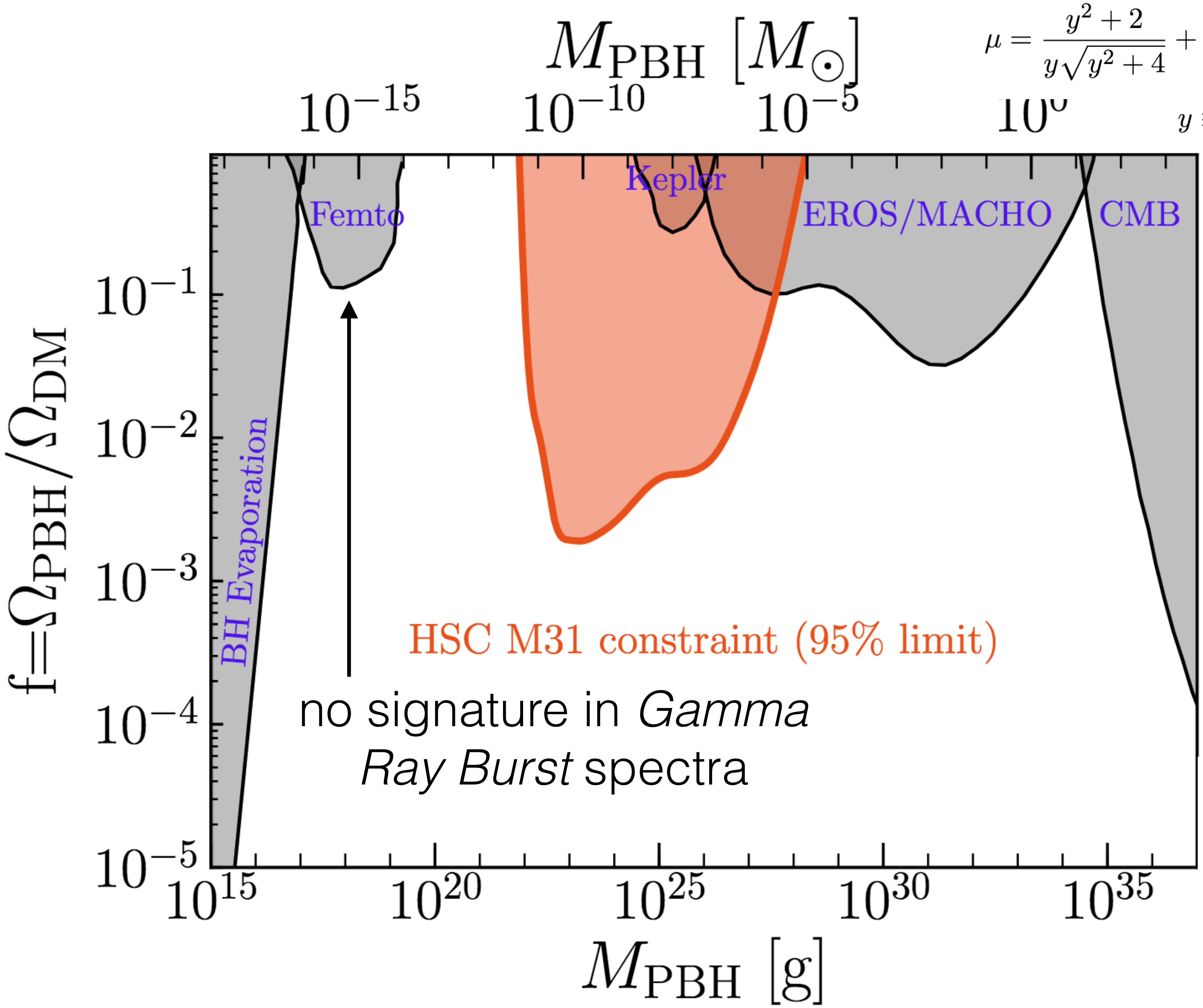


Considering finite source effects (+ wave optics)

Considering a realistic distribution of star sizes in M31 the limits are even less restrictive

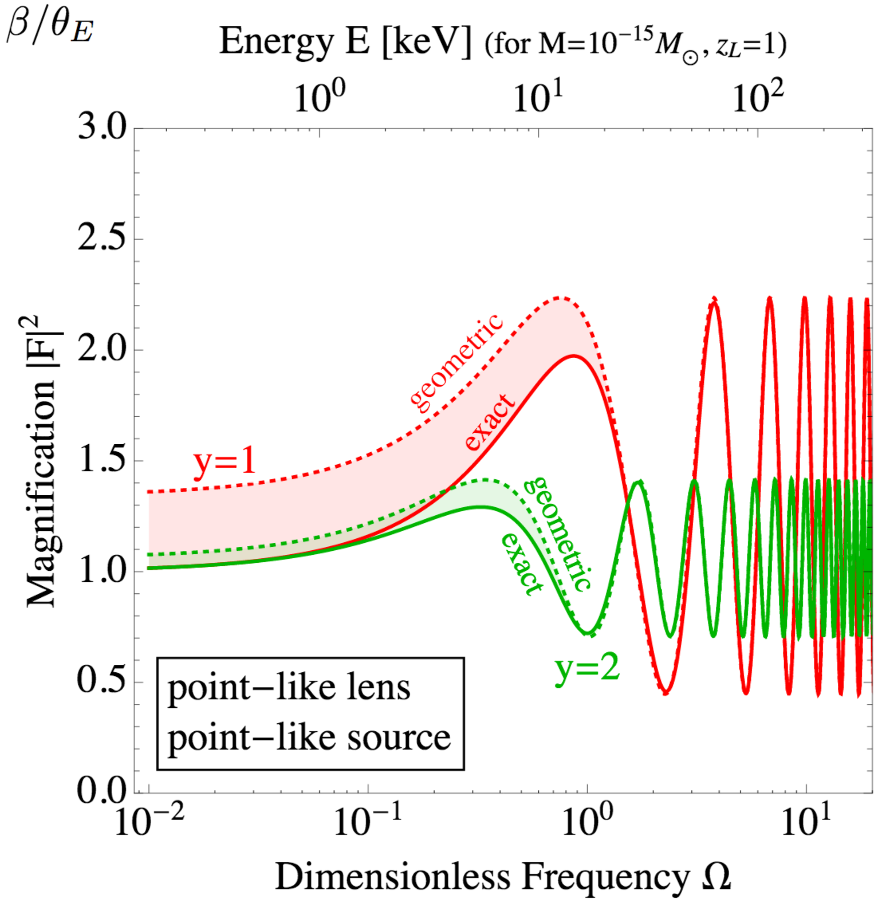
arXiv:1701.02151v3

Femtolensing: wave optics



$$\mu = \frac{y^2+2}{y\sqrt{y^2+4}} + \frac{2}{y\sqrt{y^2+4}}\sin\left(\Omega\left[\frac{y\sqrt{y^2+4}}{2} + \log\left|\frac{y+\sqrt{y^2+4}}{y-\sqrt{y^2+4}}\right|\right]\right)$$

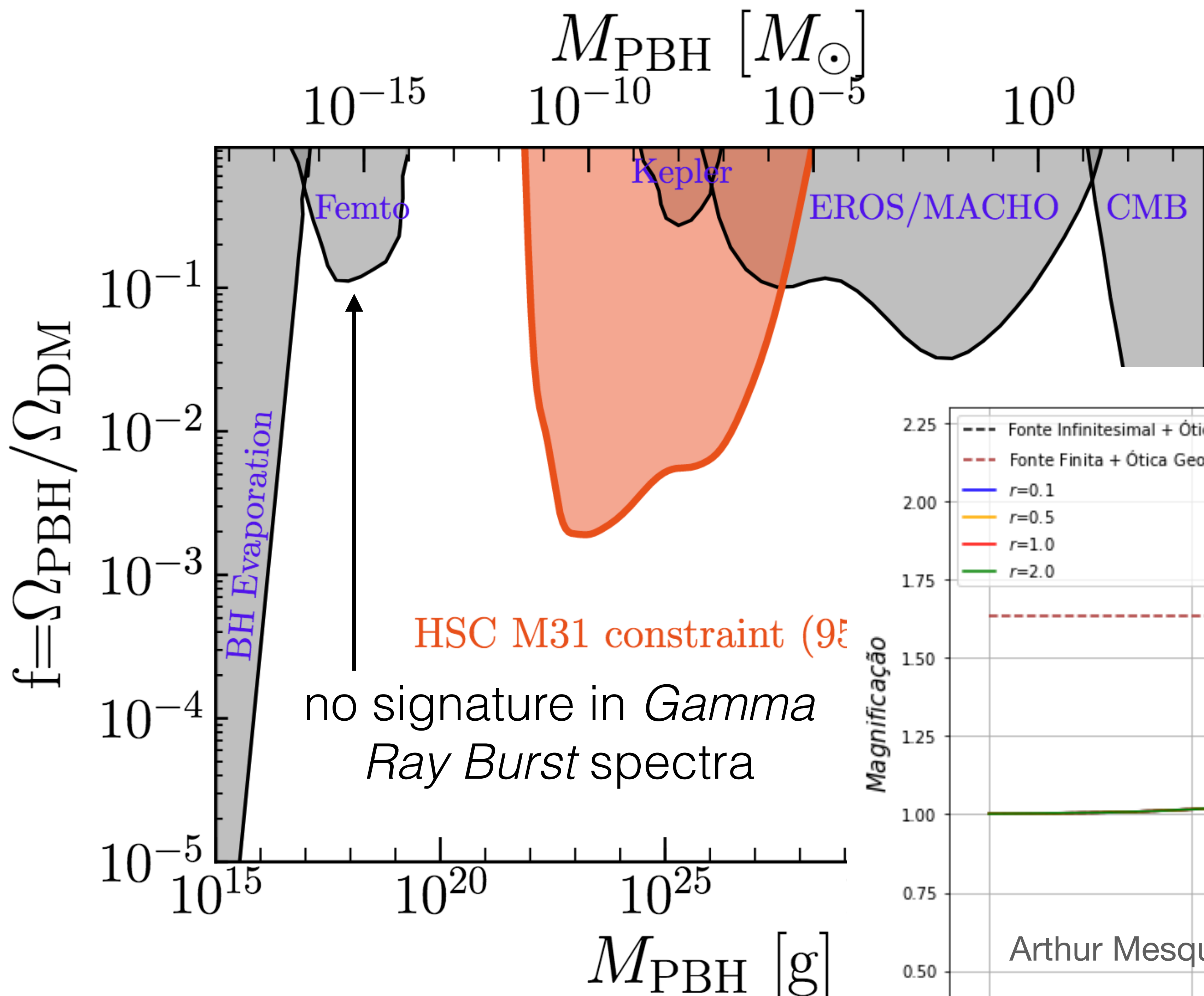
$y \equiv \beta/\theta_E$



$$\Omega \equiv \frac{1}{c}\frac{D_S D_L}{D_{LS}}\theta_0^2(1+z_L)\,\omega \equiv \Delta t_0\,\omega$$

arXiv:1701.02151v3

Femtolensing: wave optics

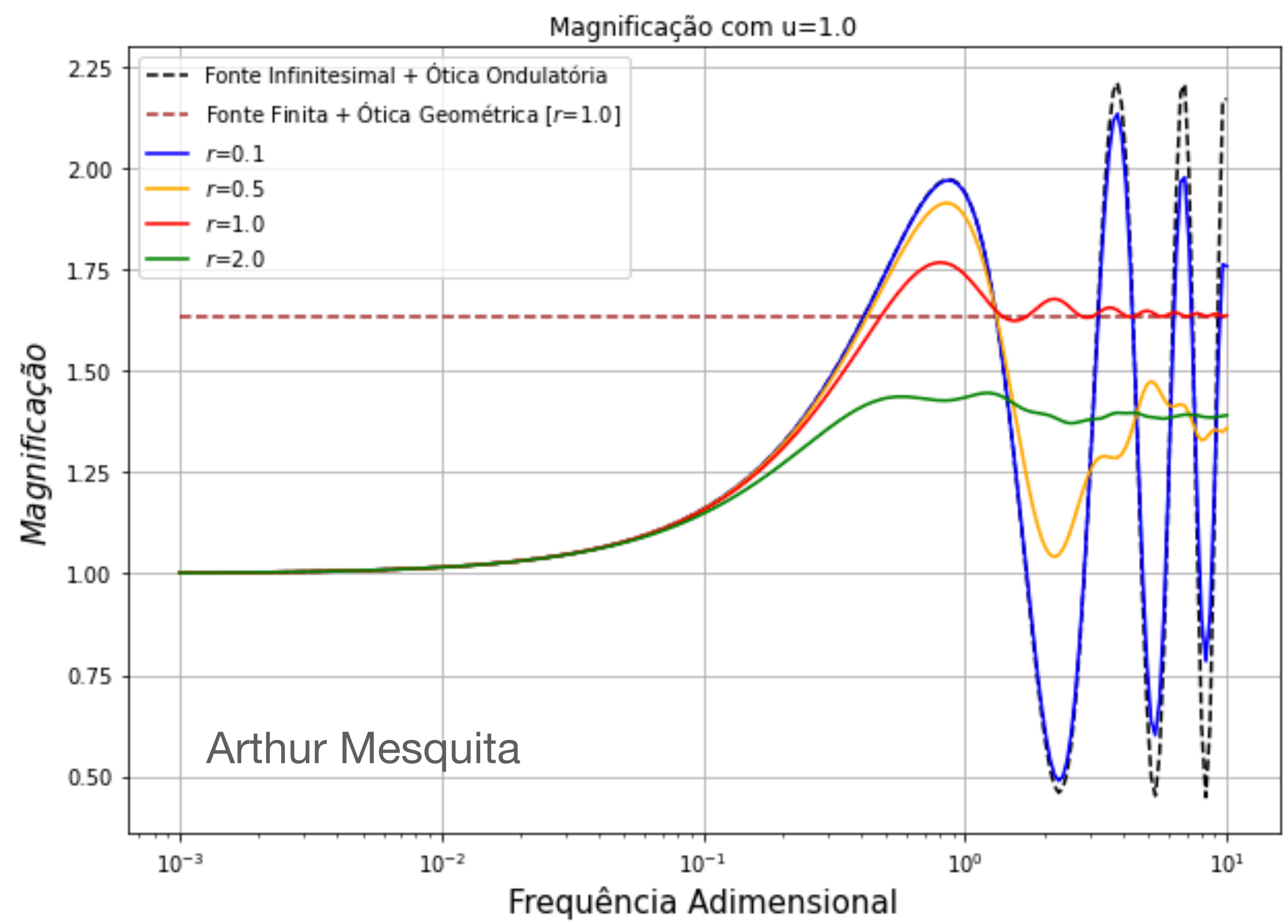


arXiv:1701.02151v3

Finite size effects destroy the signal!

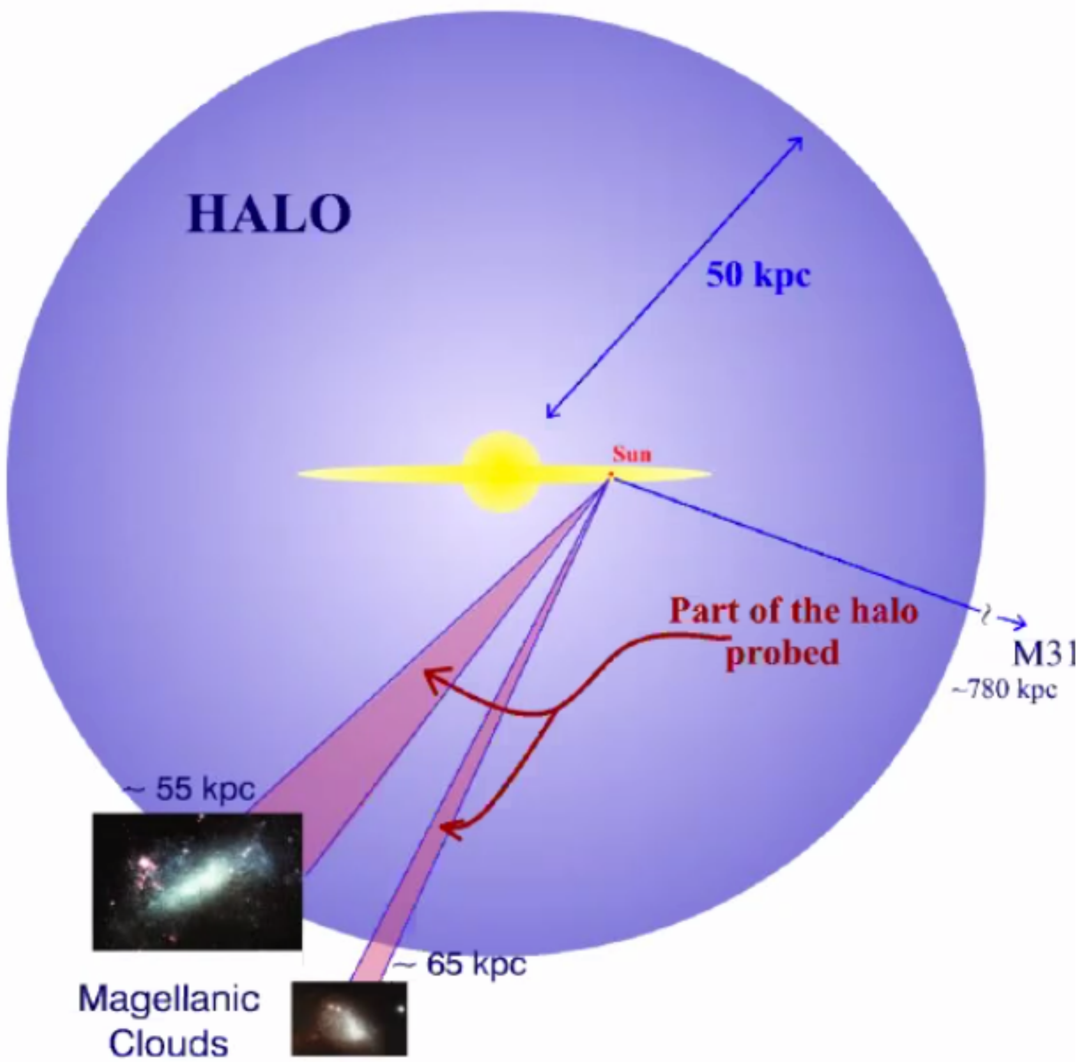
arXiv:1807.11495

Projections for femtolensing of GRB: arXiv:1807.11495



Future: microlensing with Vera Rubin LSST

Adapted from Tristan Blaineau



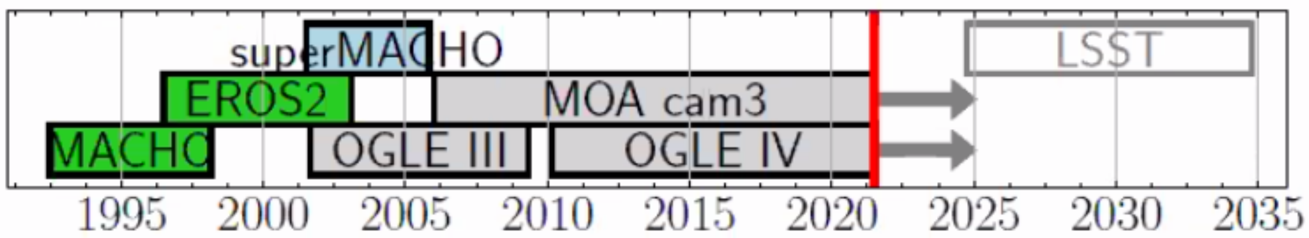
Up to now:

- Search restricted to regions of higher stellar density: Magellanic clouds, bulge, M31
- Use of small telescopes (~ 1m)

With LSST (+Roman, Euclid, etc.):

- Search in the whole celestial sphere (specially in the disk, but on streams, GC…)
- Modern telescope with large aperture (6,67 m), high image quality and large FoV in an excellent site!
- Large range of cadences: large mass range!
- Stronger constraints on DCO
- Planet detection
- Constraints on BH populations, planet populations, etc.

+ Strong lensing by galaxies and clusters, lensing of supernovae, quasar microlensing, weak lensing from galaxies to the Large Scale Structure of the Universe, search for optical counterparts of gravitational waves

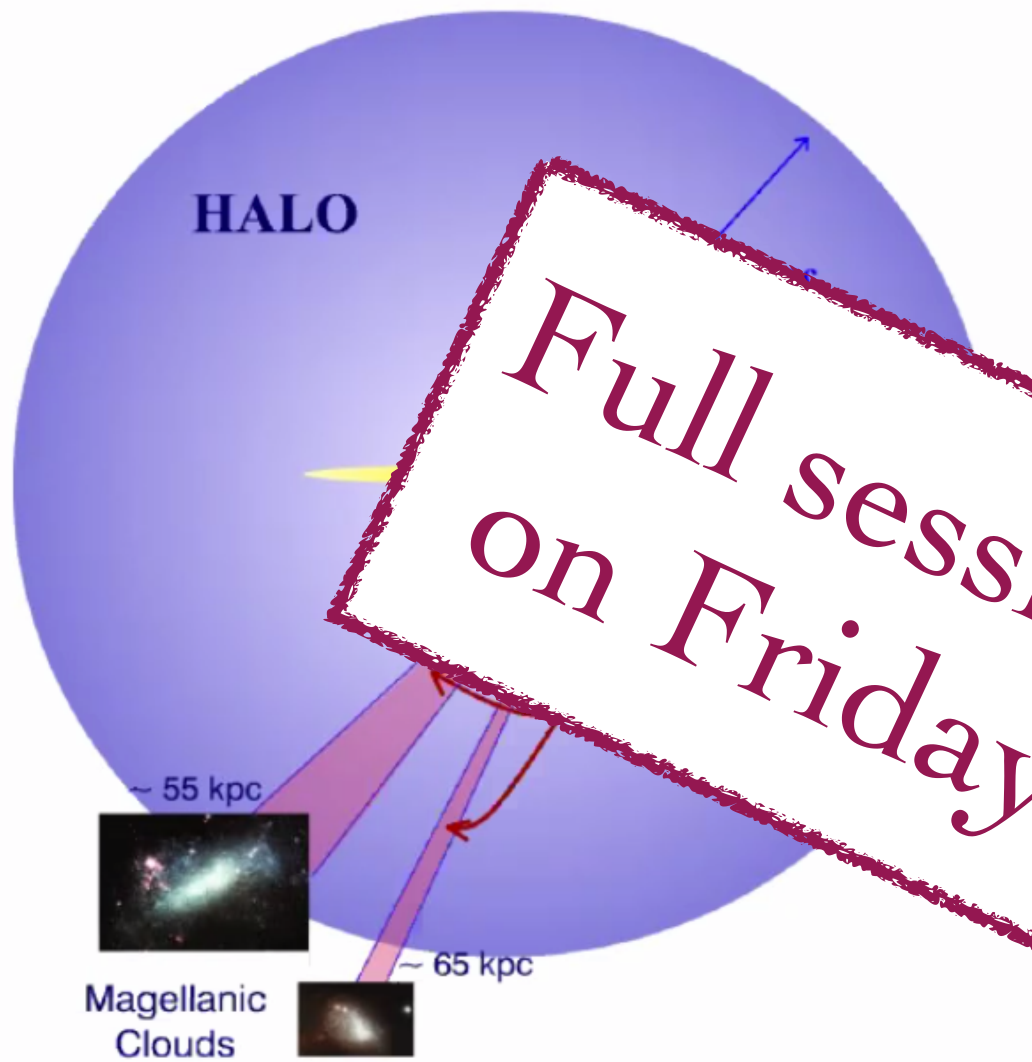


Observation time span of the surveys toward the LMC.

Future: microlensing with Vera Rubin LSST

VERA C. RUBIN OBSERVATORY

Adapted from Tristan Blaineau



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- …CO
- …ations, planet

Full session on LSST on Friday afternoon

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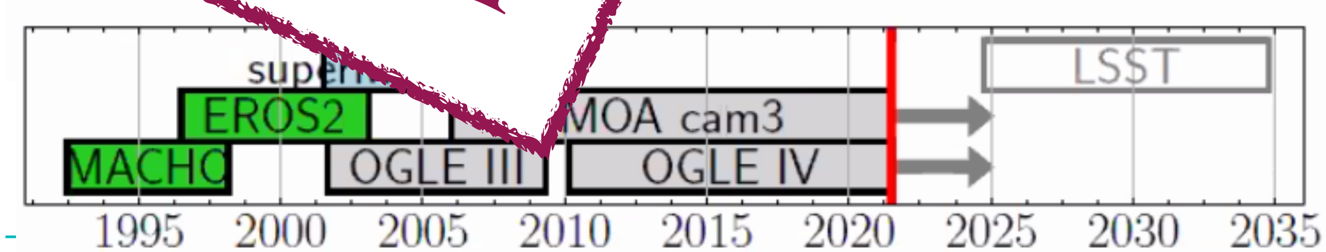

Observation time span of the surveys toward the LMC.

Challenges for Microlensing

- Detection and identification
  - How to identify microlensing light curves among many transients? (thousands per day for LSST)
  - How to classify (in real-time) interesting events to follow-up?
  - Account for blending (at LSST depths!)
  - Pixel lensing (extra-galactic sources)
- What can we learn from the microlensing event distribution?
  - Dark Matter, BH population, planet abundance, etc.
  - Derive predictions for models (including extended lenses)
- Theory, data/statistics, computing (ML), etc.
- New era for microlensing with Rubin, Roman and more