Part III
Strong Lensing

Martín Makler
ICAS/IFICI/CONICET&UNSAM & CBPF

+ Renan Alves, João França, Elizabeth Gonzalez, Arthur Mesquita, Giulya Souza, Eduardo Valadão, Anibal Varela

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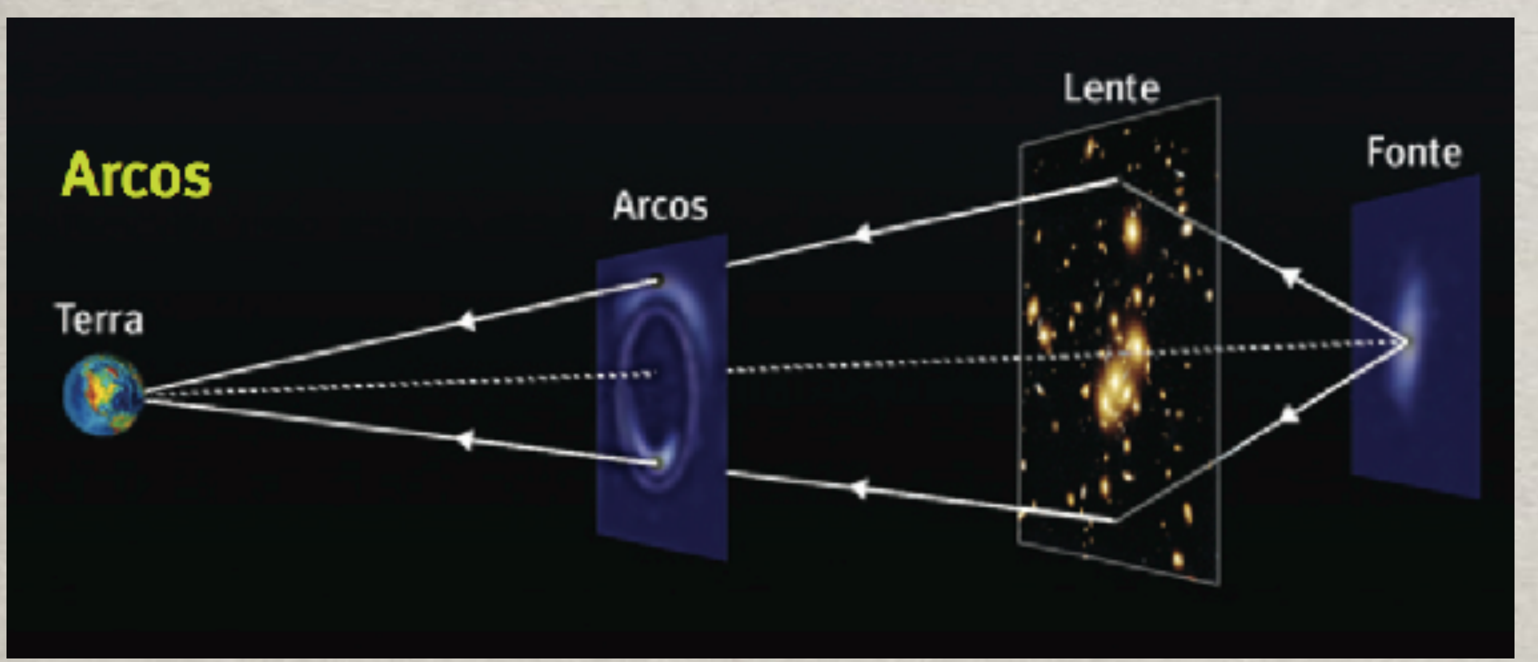
Strong Lensing

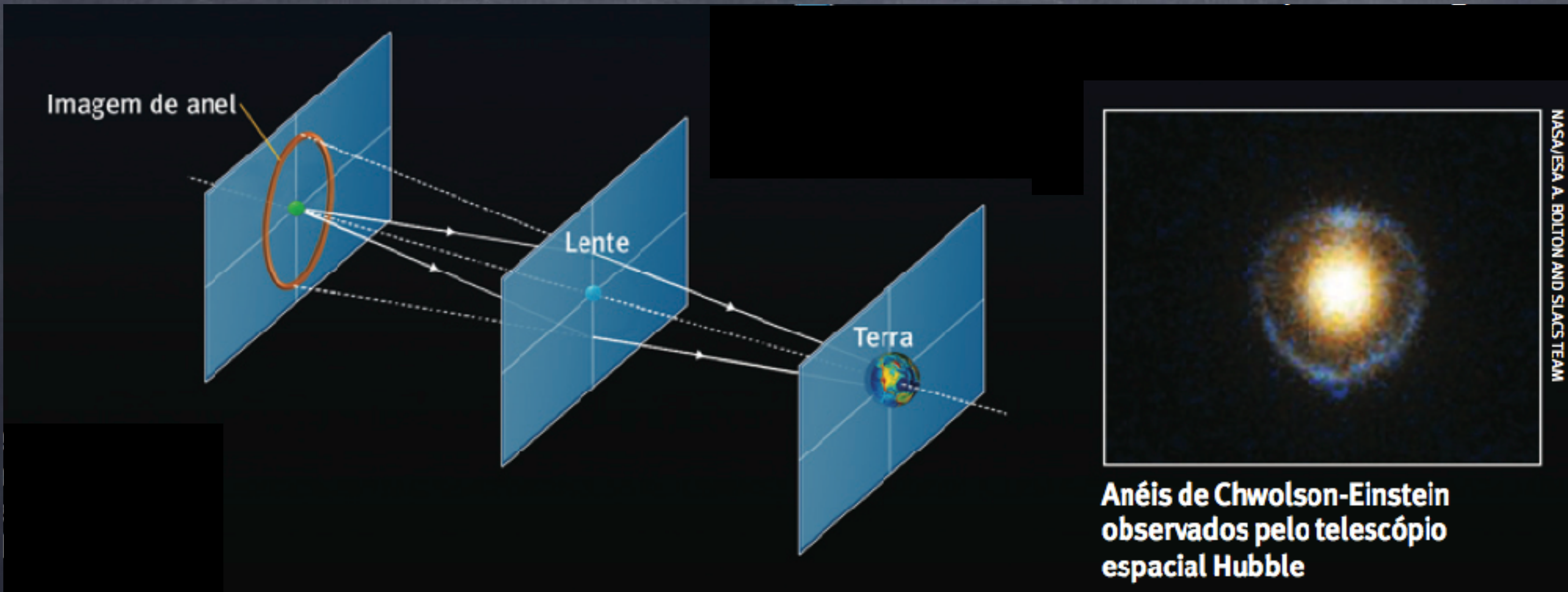
- Multiple images, strong distortions, large magnifications, time delays
- Null geodesics
  - surface brightness conservation
  - achromatic

  } → Gravitational telescopes
  - Unique probe of inner structure of galaxy clusters ➔ DM, *b*
  - Provide complementary cosmological probes and tests of gravity



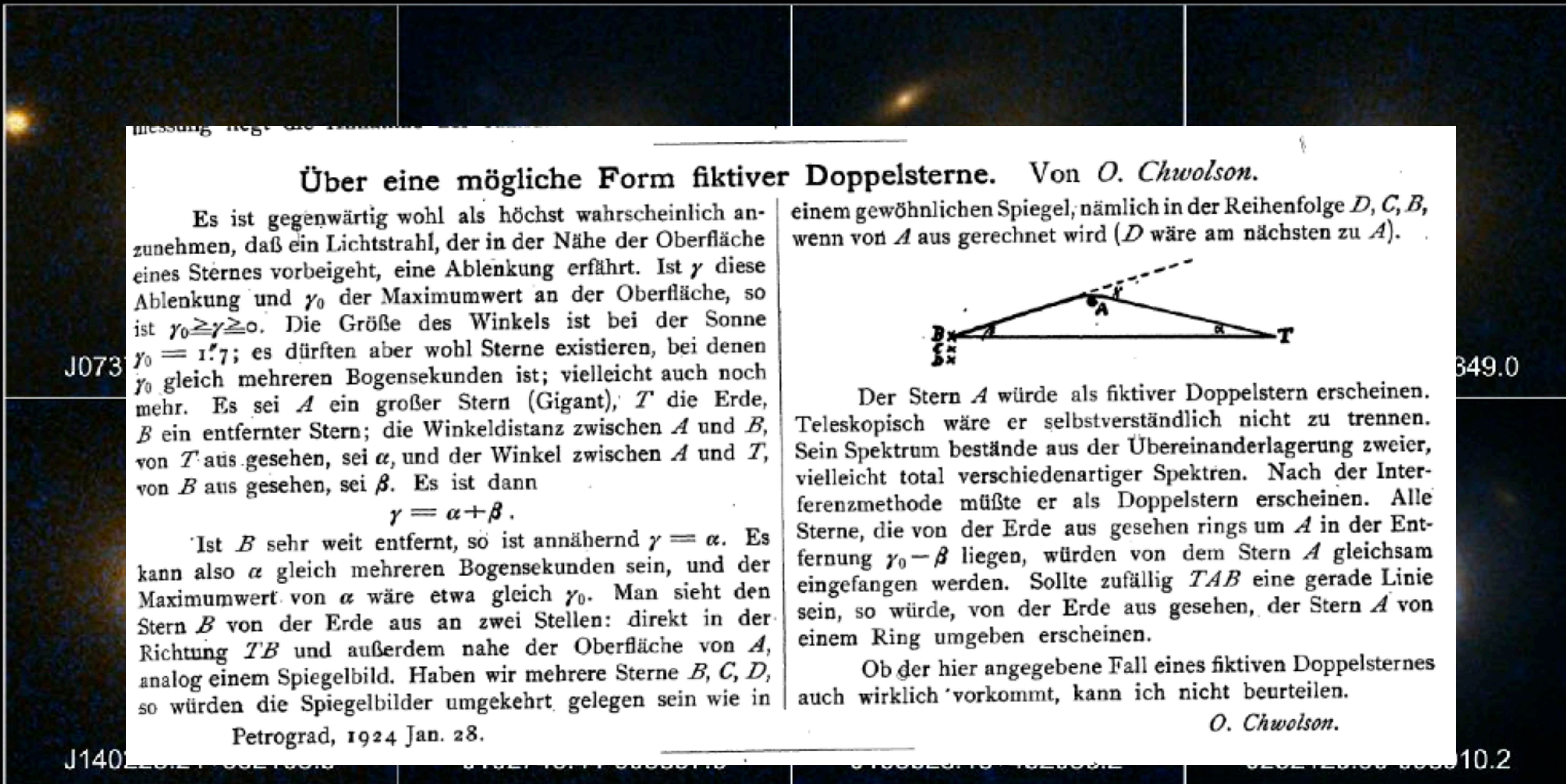
strong lensing, **weak gravity**

“Chwolson-Einstein rings”



Anéis de Chwolson-Einstein observados pelo telescópio espacial Hubble

Chwolson-Einstein rings



Über eine mögliche Form fiktiver Doppelsterne. Von *O. Chwolson.*

Es ist gegenwärtig wohl als höchst wahrscheinlich anzunehmen, daß ein Lichtstrahl, der in der Nähe der Oberfläche eines Sternes vorbeigeht, eine Ablenkung erfährt. Ist γ diese Ablenkung und γ_0 der Maximumwert an der Oberfläche, so ist $\gamma_0 \geqq \gamma \geqq 0$. Die Größe des Winkels ist bei der Sonne $\gamma_0 = 1''7$; es dürften aber wohl Sterne existieren, bei denen γ_0 gleich mehreren Bogensekunden ist; vielleicht auch noch mehr. Es sei A ein großer Stern (Gigant), T die Erde, B ein entfernter Stern; die Winkeldistanz zwischen A und B, von T aus gesehen, sei α, und der Winkel zwischen A und T, von B aus gesehen, sei β. Es ist dann

$$\gamma = \alpha + \beta .$$

Ist B sehr weit entfernt, so ist annähernd $\gamma = \alpha$. Es kann also α gleich mehreren Bogensekunden sein, und der Maximumwert von α wäre etwa gleich γ_0. Man sieht den Stern B von der Erde aus an zwei Stellen: direkt in der Richtung TB und außerdem nahe der Oberfläche von A, analog einem Spiegelbild. Haben wir mehrere Sterne B, C, D, so würden die Spiegelbilder umgekehrt gelegen sein wie in einem gewöhnlichen Spiegel, nämlich in der Reihenfolge D, C, B, wenn von A aus gerechnet wird (D wäre am nächsten zu A).

Der Stern A würde als fiktiver Doppelstern erscheinen. Teleskopisch wäre er selbstverständlich nicht zu trennen. Sein Spektrum bestände aus der Übereinanderlagerung zweier, vielleicht total verschiedenartiger Spektren. Nach der Interferenzmethode müßte er als Doppelstern erscheinen. Alle Sterne, die von der Erde aus gesehen rings um A in der Entfernung $\gamma_0 - \beta$ liegen, würden von dem Stern A gleichsam eingefangen werden. Sollte zufällig TAB eine gerade Linie sein, so würde, von der Erde aus gesehen, der Stern A von einem Ring umgeben erscheinen.

Ob der hier angegebene Fall eines fiktiven Doppelsternes auch wirklich vorkommt, kann ich nicht beurteilen.

O. Chwolson.

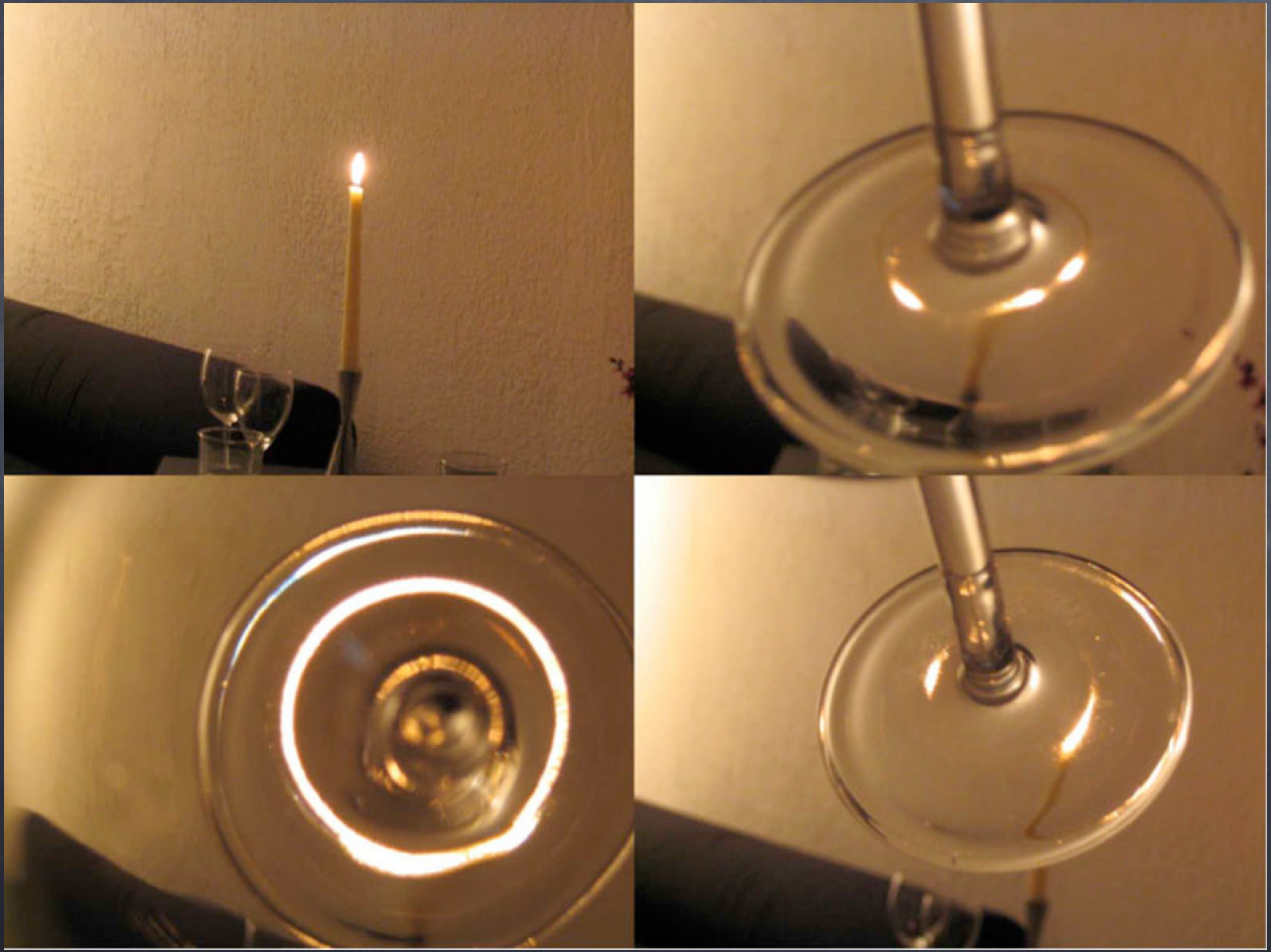
Petrograd, 1924 Jan. 28.

Einstein Ring Gravitational Lenses

Hubble Space Telescope • Advanced Camera for Surveys

NASA, ESA, A. Bolton (Harvard-Smithsonian CfA), and the SLACS Team

STScI-PRC05-32



Galaxy Clusters

Strong Lensing
Macrolensing

A Relativistic Eclipse

What Might be Seen from a Planet Conveniently Placed Near the Companion of Sirius . . . Perfect Tests of General Relativity that are Unavailable

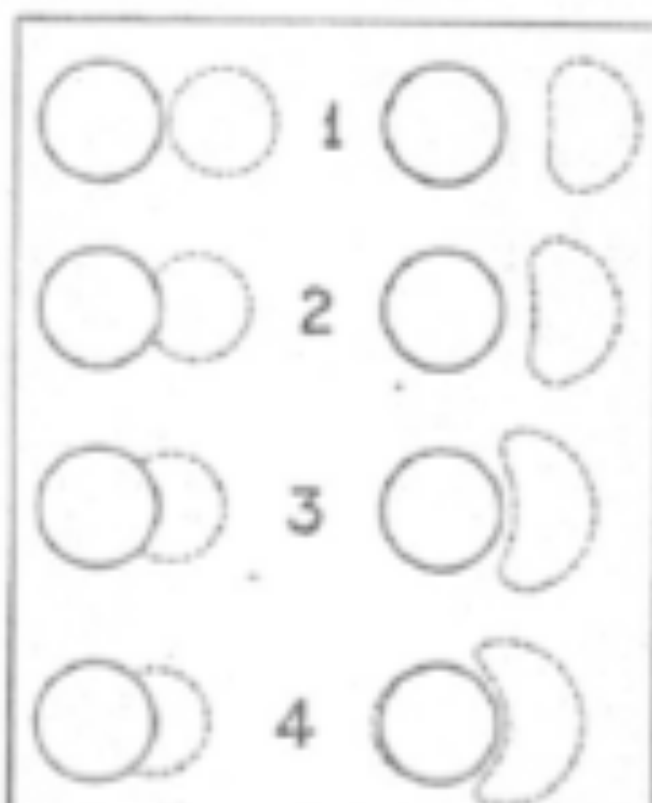
By HENRY NORRIS RUSSELL, Ph. D.

Chairman of the Department of Astronomy and Director of the Observatory at Princeton University. Research Associate of the Mount Wilson Observatory of the Carnegie Institution of Washington. President of the American Astronomical Society.

IT is a familiar aphorism that the theory of relativity, despite its enormous importance, both in physics and philosophy, may be forgotten in ordinary practical life. There is a good reason. In almost every known case its results agree so closely with those of the older "classical" theories that very accurate observations are required to distinguish between them. Thus, for example, the famous Michelson-Morley experiment requires long series of the

No one predicted gravitational arcs?

stein himself.[1] From a point exactly in line ... be resolved by even the greatest tele-



for it. The focusing effect, however, ... For a star ... mathematical point, with no angular diameter at all, the increase in brightness would be infinite. But any real star must have a finite angular size, however small, and for such a star the increase in brightness, though it might be large, would have a limit.

If a bright star passed in front of a fainter one, its own light would drown out the effect; but if one of the faint red stars, which are really more abundant than any other sort, should get directly

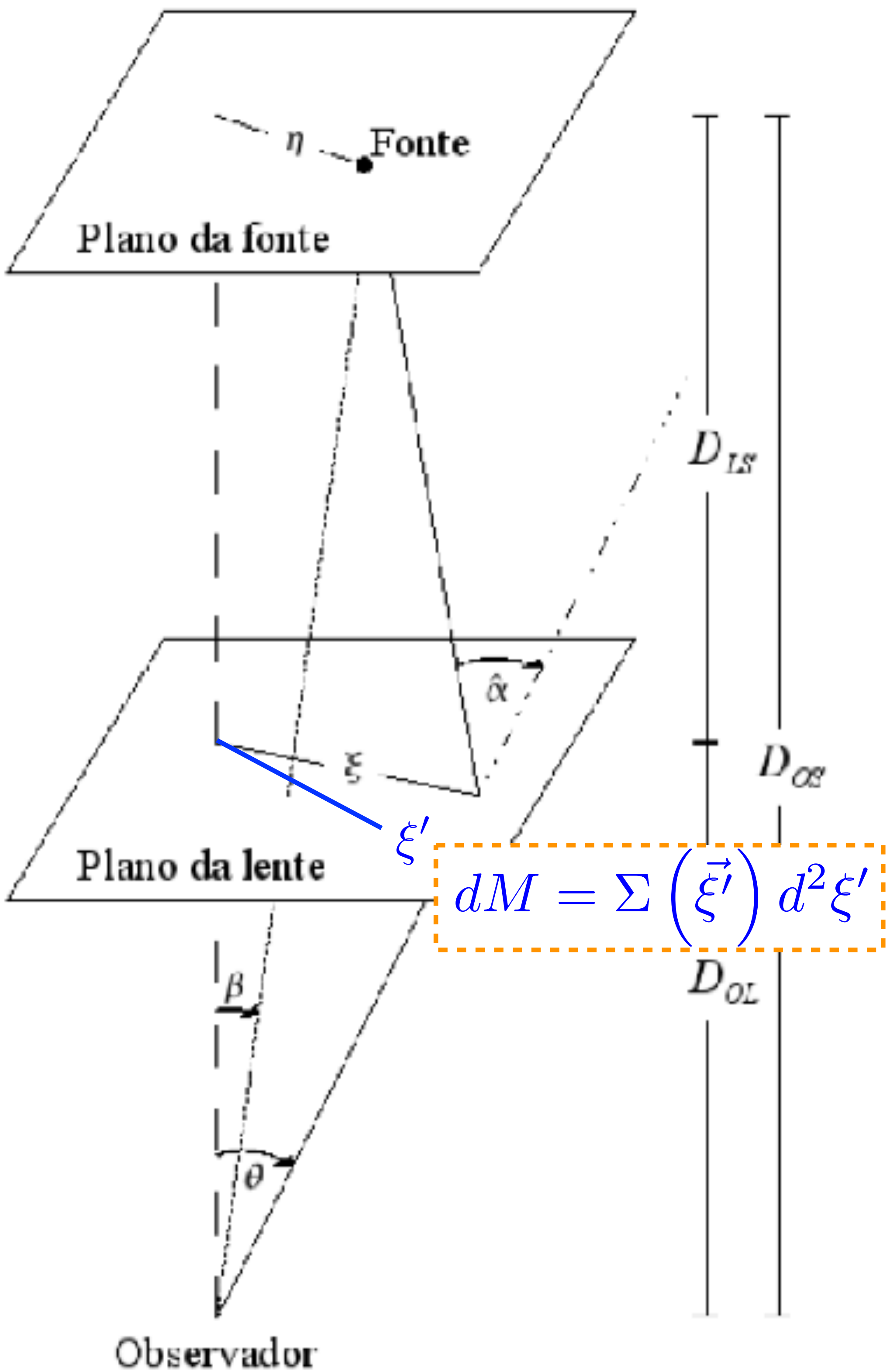
My hearty thanks are due to Professor Einstein, who permitted me to see the manuscript of his note before its publication.—*Princeton University Observatory, December 2, 1936.*

is, a million miles or so—from the line through the center of the two stars. The

A Bit of theory/modelling:

The realm of extended Lenses and extended sources

Extended Lenses



Point mass $$\hat{\vec{\alpha}} = \frac{4GM}{c^2\xi}$$

Surface mass density

$$\Sigma\left(\vec{\xi}\right) = \int_0^\infty dz\rho\left(\vec{\xi}, z\right)$$

Contribution of the area element

$$d\hat{\vec{\alpha}} = \frac{4G}{c^2}\Sigma\left(\vec{\xi'}\right) d^2\xi' \frac{\vec{\xi} - \vec{\xi'}}{|\vec{\xi} - \vec{\xi'}|^2}$$

Deflection angle

$$\hat{\vec{\alpha}} = \frac{4G}{c^2}\int d^2\xi'\Sigma\left(\vec{\xi'}\right) \frac{\vec{\xi} - \vec{\xi'}}{|\vec{\xi} - \vec{\xi'}|^2}$$

Extended Lenses

Projected potential $$\psi\left(\vec{\xi}\right) = \int dz \varphi(\vec{\xi}, z)$$

Deflection angle $$\hat{\vec{\alpha}} = \frac{2}{c^2}\vec{\nabla}_\xi \psi\left(\vec{\xi}\right)$$

Reduced deflection angle

$$\vec{\alpha} = \hat{\vec{\alpha}}\frac{D_{LS}}{D_{OS}} = \frac{2}{c^2}\frac{D_{LS}}{D_{OS}D_{OL}}\vec{\nabla}_\theta \psi\left(\vec{\theta}\right)$$

Lensing potential $$\Psi(\vec{x}) \equiv \frac{2}{c^2}\frac{D_{LS}}{D_{OS}D_{OL}}\psi$$

Lens equation $$\vec{\beta} = \vec{\theta} - \vec{\alpha}\left(\vec{\theta}\right) = \boxed{\vec{\theta} - \vec{\nabla}_\theta \Psi\left(\vec{\theta}\right)}$$

Lens equation with axial symmetry

Deflection angle $$\vec{\hat{\alpha}} = \frac{4GM(\xi)}{c^2 \xi} \hat{\xi}$$

Lens equation $$\vec{\beta} = \vec{\theta} - \vec{\hat{\alpha}} \frac{D_{LS}}{D_{OS}} = \vec{\theta} - \frac{4GM(\theta)}{c^2 \theta^2 D_{OL}} \vec{\theta} \frac{D_{LS}}{D_{OS}}$$

$$\vec{\beta} = \left(1 - \frac{4GM(\theta) D_{LS}}{c^2 D_{OL} D_{OS}} \frac{1}{\theta^2} \right) \vec{\theta}$$

Einstein angle $\left(\vec{\beta} = 0 \right)$

$$\theta_E = \sqrt{\frac{D_{LS}}{D_{OS} D_{OL}} \frac{4GM(\theta_E)}{c^2}}$$

$\Rightarrow$ Mass estimate at $\theta < \theta_E$

Singular Isothermal Sphere

Singular Isothermal Sphere

$$\rho(r) = \frac{\sigma_v^2}{2\pi G r^2}$$

Surface density (projected)

$$\Sigma(\xi) = \frac{\sigma_v^2}{2\pi G} 2 \int_0^\infty \frac{dz}{\xi^2 + z^2} = \frac{\sigma_v^2}{2G\xi}$$

Mass within a radius ξ

$$M(\xi) = \int_0^\xi \Sigma(\xi') \, 2\pi\xi' \, \mathrm{d}\xi' = \sigma_v^2 \frac{\pi}{G} \xi$$

Deflection angle

$$\Rightarrow \quad \hat{\vec{\alpha}} = \frac{4GM(\xi)}{c^2 \xi} \hat{\xi} = 4\pi \left(\frac{\sigma_v}{c}\right)^2 \hat{\xi}$$

constant!

Lens Equation

$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}} \qquad \hat{\vec{\alpha}} = 4\pi \left(\frac{\sigma_v}{c} \right)^2 \hat{\xi}$$

$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}} = \underbrace{\boxed{4\pi \left(\frac{\sigma_v}{c} \right)^2 \frac{D_{LS}}{D_{OS}}}}_{\theta_E} \hat{\xi} = \theta_E \, \hat{\xi}$$

Lens Equation

$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}} \qquad \hat{\vec{\alpha}} = 4\pi \left(\frac{\sigma_v}{c}\right)^2 \hat{\xi}$$

$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}} = 4\pi \left(\frac{\sigma_v}{c}\right)^2 \frac{D_{LS}}{D_{OS}} \hat{\xi} = \theta_E \, \hat{\xi}$$

$$\vec{\beta} = \vec{\theta} - \theta_E \, \hat{\theta} = \left(1 - \frac{\theta_E}{\theta}\right) \vec{\theta}$$

$$\Rightarrow \quad \beta = \left| 1 - \frac{\theta_E}{\theta} \right| \theta$$

Solutions: 1) if $1 - \dfrac{\theta_E}{\theta} > 0$

then $\beta = \left(1 - \dfrac{\theta_E}{\theta}\right) \theta = \theta - \theta_E \Rightarrow \boxed{\theta = \beta + \theta_E}$

Lens Equation

$$\beta = \left|1 - \frac{\theta_E}{\theta}\right| \theta$$

Solutions: I) if $1 - \frac{\theta_E}{\theta} > 0$

then $\beta = \left(1 - \frac{\theta_E}{\theta}\right)\theta = \theta - \theta_E \Rightarrow \boxed{\theta = \beta + \theta_E}$

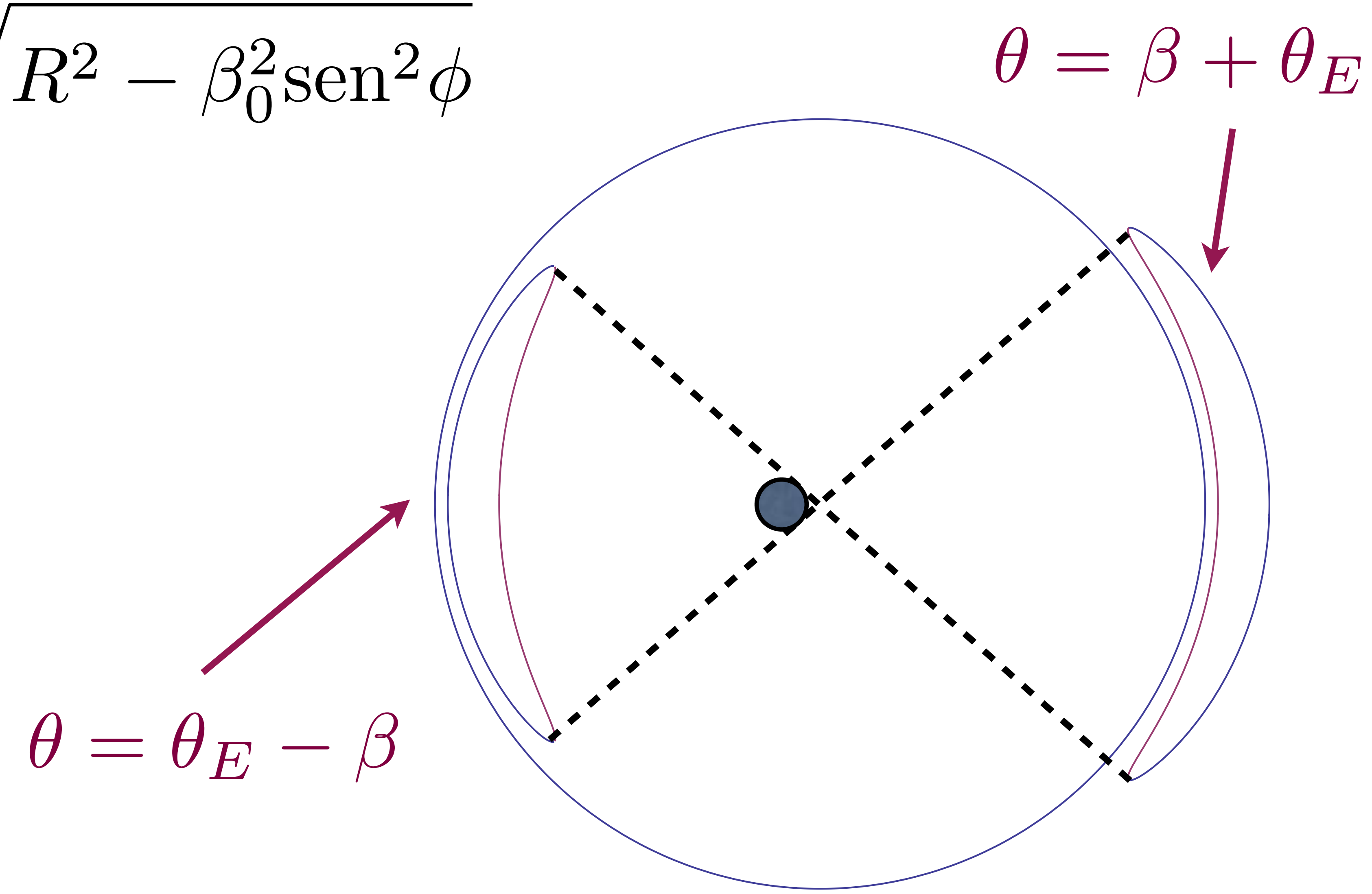
II) if $1 - \frac{\theta_E}{\theta} < 0$

then $\beta = -\left(1 - \frac{\theta_E}{\theta}\right)\theta = \theta_E - \theta \Rightarrow \boxed{\theta = \theta_E - \beta}$

Circular source

Circle in the source plane: $\left|\vec{\beta}-\vec{\beta}_0\right| = R$

$$\beta = \beta_0 \cos\phi \pm \sqrt{R^2 - \beta_0^2 \mathrm{sen}^2\phi}$$



in general $\beta = \beta_0 \cos(\phi - \phi_0) \pm \sqrt{R^2 - \beta_0^2 \mathrm{sen}^2(\phi - \phi_0)}$

Other lens models

Implementes in *gravlens* and several codes (+ Sérsic, Einasto, etc.)

Model	N_r	Density $\rho(r)$	Surface Density $\kappa(r)$
Point mass	0	$\delta(\mathbf{x})$	$\delta(\mathbf{x})$
Power law or α-models	2	$\left(s^2+r^2\right)^{(\alpha-3)/2}$	$\left(s^2+r^2\right)^{(\alpha-2)/2}$
Isothermal ($\alpha=1$)	1	$\left(s^2+r^2\right)^{-1}$	$\left(s^2+r^2\right)^{-1/2}$
$\alpha=-1$	1	$\left(s^2+r^2\right)^{-2}$	$\left(s^2+r^2\right)^{-3/2}$
Pseudo-Jaffe	2	$\left(s^2+r^2\right)^{-1}\left(a^2+r^2\right)^{-1}$	$\left(s^2+r^2\right)^{-1/2}-\left(a^2+r^2\right)^{-1/2}$
King (approximate)	1	$\cdots$	$2.12\left(0.75r_s^2+r^2\right)^{-1/2}$ $-1.75\left(2.99r_s^2+r^2\right)^{-1/2}$
de Vaucouleurs	1	$\cdots$	$\exp\left[-7.67(r/R_e)^{1/4}\right]$
Hernquist	1	$r^{-1}\left(r_s+r\right)^{-3}$	see eq. (47)
NFW	1	$r^{-1}\left(r_s+r\right)^{-2}$	see eq. (53)
Cuspy NFW	2	$r^{-\gamma}\left(r_s+r\right)^{\gamma-3}$	see eq. (57)
Cusp	3	$r^{-\gamma}\left(r_s^2+r^2\right)^{(\gamma-n)/2}$	see eq. (64)
Nuker	4	$\cdots$	see eq. (71)
Exponential disk	1	$\cdots$	$\exp[-r/R_d]$
Kuzmin disk	1	$\cdots$	$\left(r_s^2+r^2\right)^{-3/2}$

Elliptical models

Replace $\theta \to \sqrt{q_1\theta_1^2 + q_2\theta_2^2}$

- in the potential (pseudo-elliptic models)
- in the projected density (elliptical models)

Example: pseudo-elliptic isothermal model with a core

$$\Psi(\theta_1, \theta_2) = \frac{\Psi_0}{\theta_c}\sqrt{(1-\epsilon)\theta_1^2 + (1+\epsilon)\theta_2^2 + \theta_c^2}$$

Model for the source

Surface brightness distribution of the source

Example: Sérsic profile

$$\boxed{I(R) = I_0 \exp\left\{-b_n \left(\frac{R}{R_e}\right)^{1/n}\right\}}$$

Elliptical brightness distribution

$$\begin{aligned} R^2 = &(1-\varepsilon_S)[(\beta_1 - S_1)\cos\phi_e + (\beta_2 - S_2)\sin\phi_e]^2 \\ &+(1+\varepsilon_S)[(\beta_2 - S_2)\cos\phi_e - (\beta_1 - S_1)\sin\phi_e]^2 \end{aligned}$$

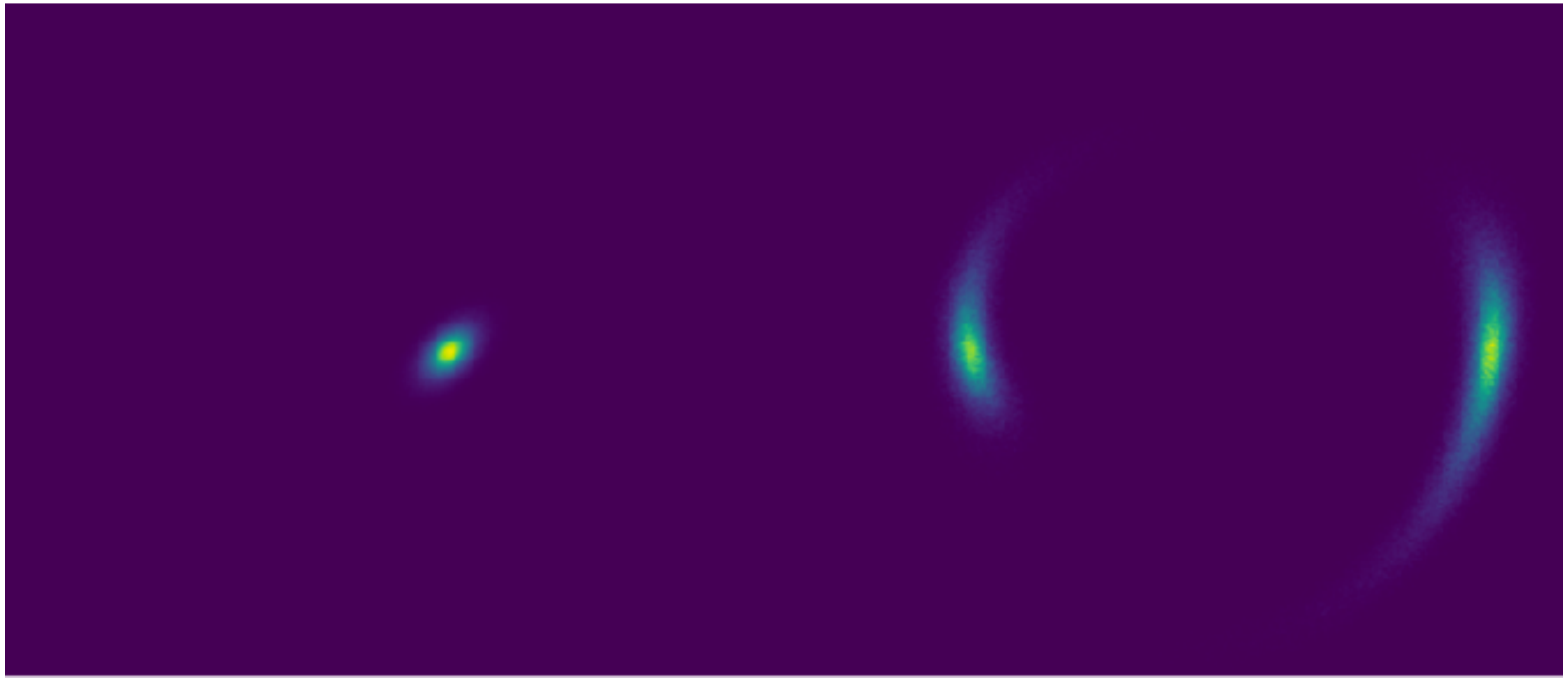
Simulating Strong Lensing Images

Map the brightness distribution of the source to the lens plane

$$I\left(\vec{\theta}\right) = I\left(\vec{\beta}\left(\vec{\theta}\right)\right)$$

Use the lens equation (no need to solve it!)

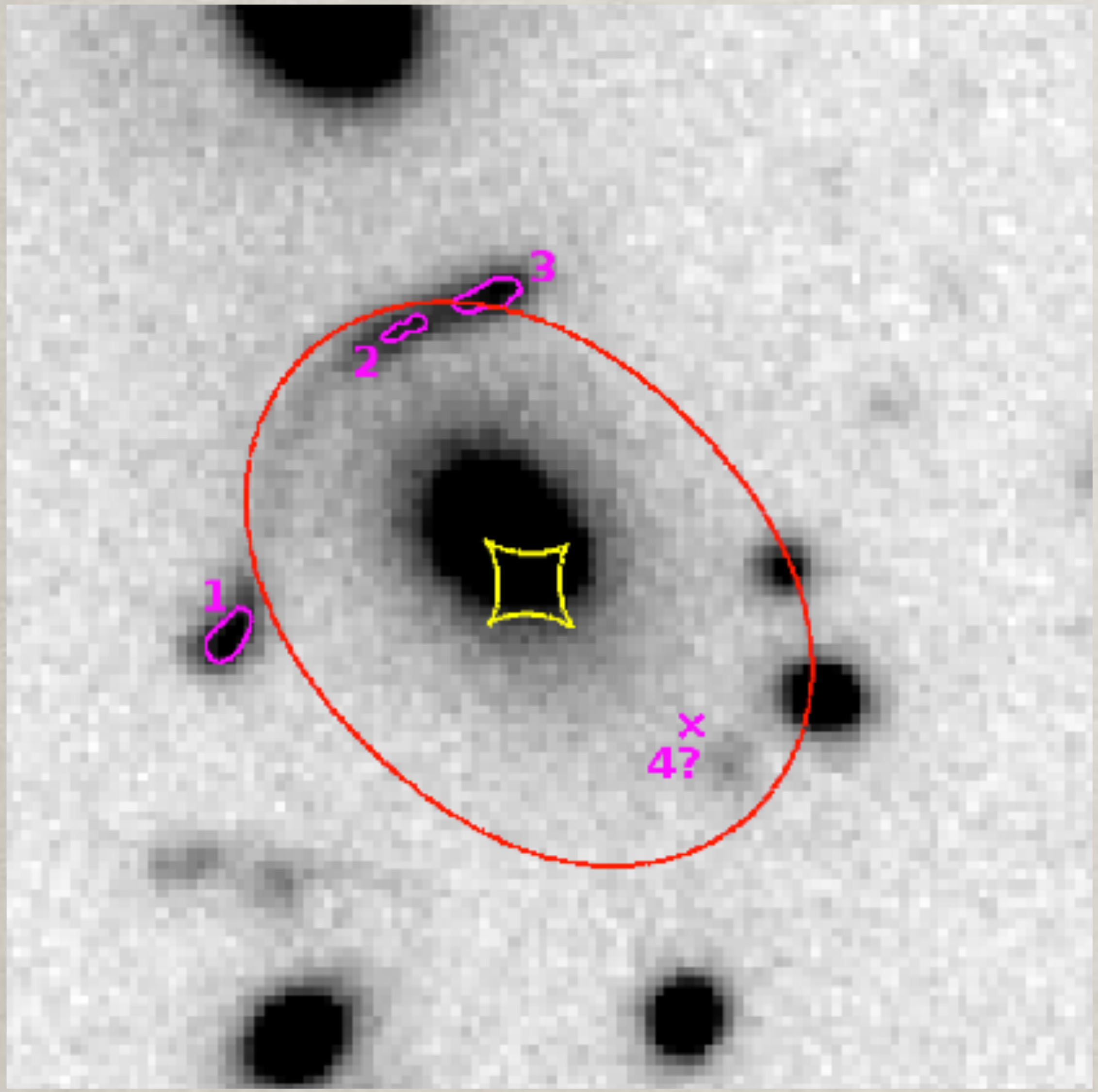
Add PSF and noise



AddArcs v2.0

Let's see this in practice with some code

Inverse Modeling: Mapping the Mass



Use systems of multiple images to determine the lensing potential

$$\chi^2_{\mathrm{lente}} := \sum_i \left(\frac{\vec{\theta}_i^{\mathrm{obs}} - \vec{\theta}^{\mathrm{mod}} \left(\vec{\beta}, \vec{\Pi} \right)}{\sigma_i^{\mathrm{obs}}} \right)^2$$

Multiple image positions

Error on image positions

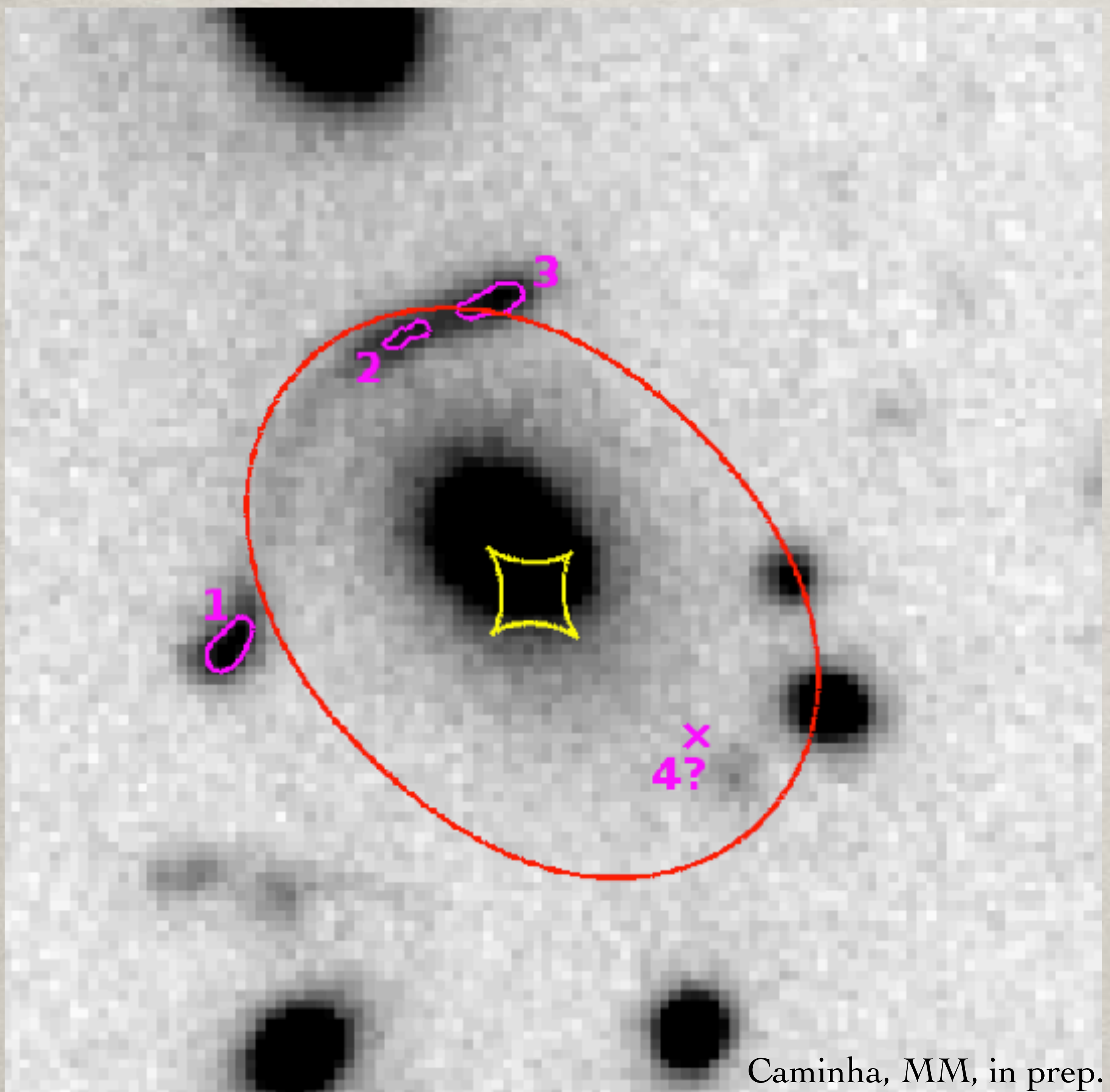
Methods: parametric (often "mass traces light"), free form

The more multiple images, the more constrains

Cluster x Galaxy scales

- Combination with independent mass constraints (e.g., x-ray, Sunyaev Zel'dovich, velocity dispersions) yields limits on cosmology or gravity

Inverse Modeling for system SOGRAS0041-0043



$$2.35^{+0.03}_{-0.14} \times 10^{14} M_\odot$$

Modelling with lenstool (Jullo, Kneib)

Fit 1: 3 images

Fit 2: 4 images

	Fit 1	Fit 2
$\sigma_v[km/s]$	622^{-11}_{-13}	642^{+3}_{-3}
$\theta_{or}[°]$	$135.2^{+0.7}_{-0.8}$	$135.2^{+1.5}_{-1.3}$
$x_{\rm lente}['']$	$0.50^{+0.2}_{-0.2}$	$0.50^{+0.05}_{-0.06}$
$y_{\rm lente}['']$	$-0.76^{+0.17}_{-0.15}$	$-0.86^{+0.06}_{-0.03}$
ε	—	$0.13^{+0.02}_{-0.03}$

~ 0.9'' displacement between central galaxy and center of mass distribution (*Zitrin et al. 2012*)

Error estimate from simulations (Caminha et al. in prep.):

~ 8% bias in mass

~ 5% statistical errors

Modeling the full light distribution of the images

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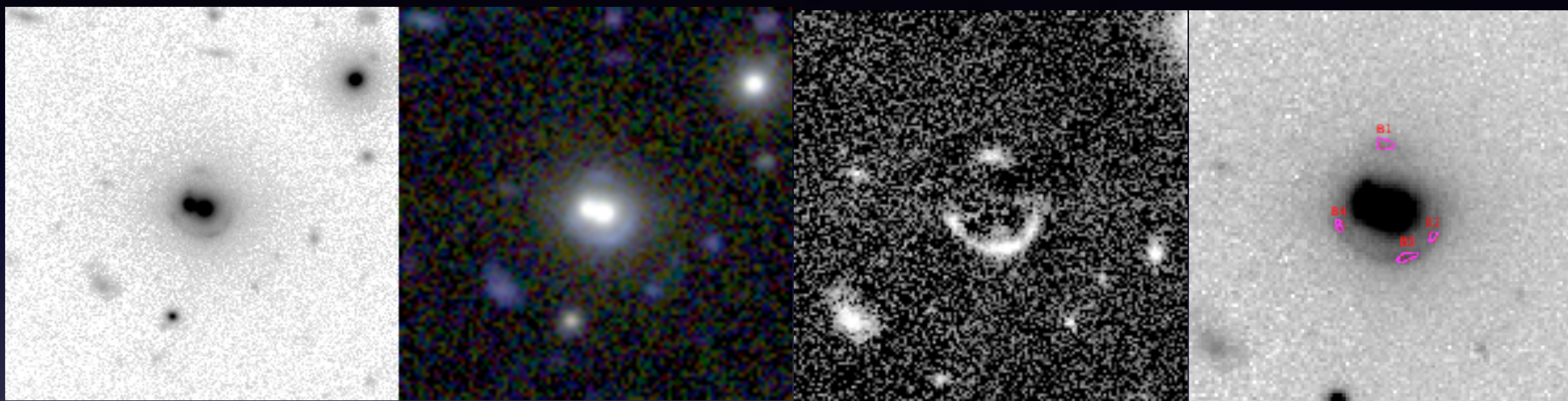
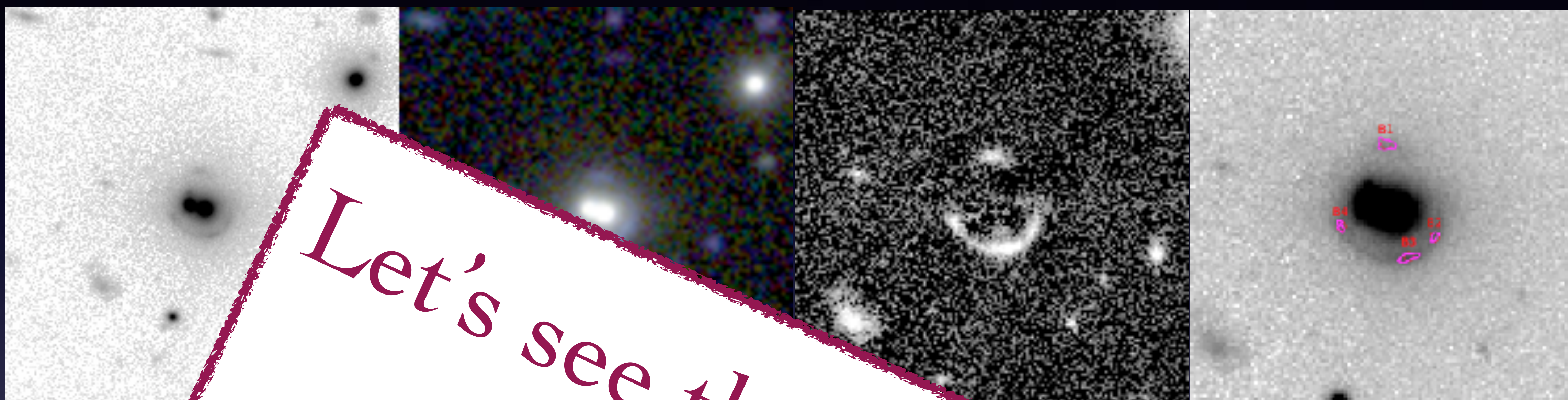


Anna Niemiec

- Instead of peak/multiple images, use the full information of the images
- Allows one to reconstruct source properties (often parametrically)
- Remove contamination from lens galaxy (with galfit) and mask other objects

Modeling the full light distribution of the images

CS82SL01:36:39+00:08:18



Anna Niemiec

Let’s see this in practice with some code

- Instead o … he full information of t …
- Allows one to reconstru … es (often parametrically)
- Remove contamination from lens galaxy (with galfit) and mask other objects

Example Applications:

- Finding substructure: Dark Matter

- Einstein rings: testing modified gravity

- Galaxy clusters: probing the background cosmology

Strong lensing and substructures

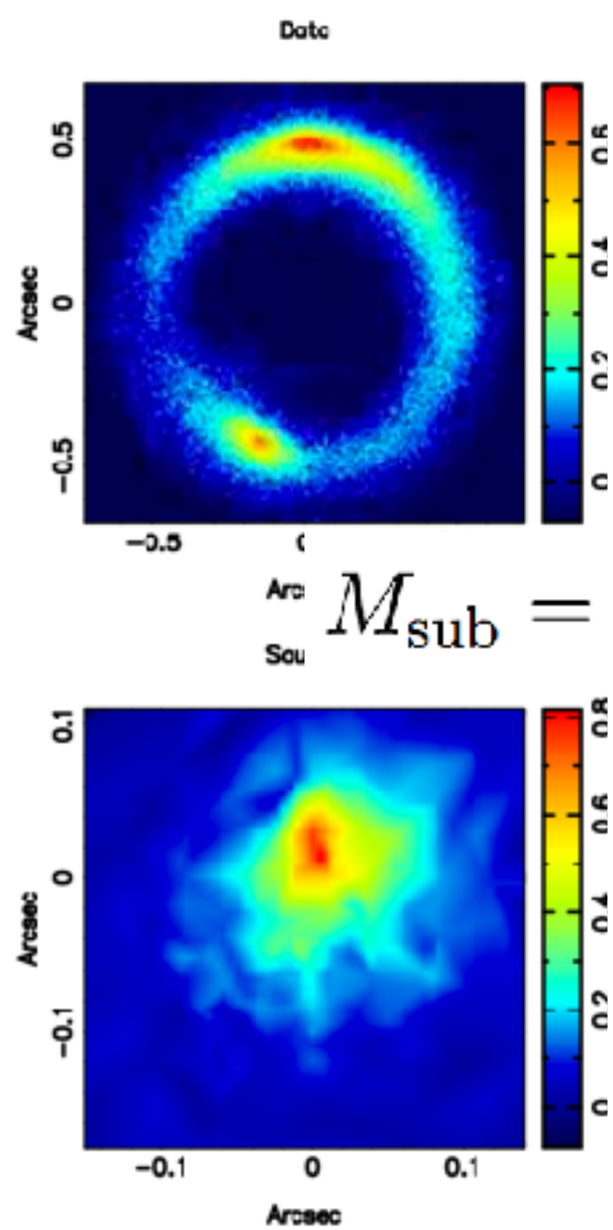
High sensitivity to small perturbations due to the caustic structure

MENU nature

Gravitational detection of a low-mass dark satellite galaxy at cosmological distance

S. Vegetti, D. J. Lagattuta, J. P. McKean, I

Nature **481**, 341–343 (19 January 2012)



- “Aside from direct or indirect detection of the dark matter particles themselves, Einstein ring systems currently offer the best astrophysical test of the nature of the dark matter” (Li et al. 2016)
- measurements of approximately 100 strong lens systems with a detection limit of $M_{low} = 10^7\ h^{-1}\ M_\odot$ would clearly distinguish CDM from WDM in the case where this consists of 7 keV sterile neutrinos

One gravitational potential or two? Forecasts and tests

By Edmund Bertschinger*

Phil. Trans. R. Soc. A (2011) **369**, 4947–4961
doi:10.1098/rsta.2011.0369

conformal newtonian metric (choices and assumptions):

$$ds^2 = a^2(\tau)[-(1+2\Phi)\,d\tau^2 + (1-2\Psi)\gamma_{ij}\,dx^i\,dx^j]$$

for General Relativity (for *standard* matter components) in general

$$\Phi = \Psi$$

slip parameter

$$\gamma = \frac{\Phi}{\Psi}$$

$\gamma = 1$ compatible with GR

$\gamma \neq 1$ GR ruled out

One gravitational potential or two? Forecasts and tests

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$$\mathrm{d}s^2 = a^2(\tau)[-(1+2\Phi)\,\mathrm{d}\tau^2 + (1-2\Psi)\gamma_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j]$$

geodesics

$$\frac{1}{a}\frac{\mathrm{d}(a\boldsymbol{v})}{\mathrm{d}\tau} = -\boldsymbol{\nabla}\Phi, \quad v^2 \ll 1 \text{ (CDM)} \longrightarrow \text{Jean's equation}$$

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}\tau} = -\boldsymbol{\nabla}_\perp(\Phi+\Psi), \quad v^2 = 1 \text{ (photons)} \longrightarrow \text{lensing}$$

kinematics: Φ

deflection angle: $\Phi + \Psi$

on ~ 100 kph
galaxy scales

from galaxy
velocity dispersion

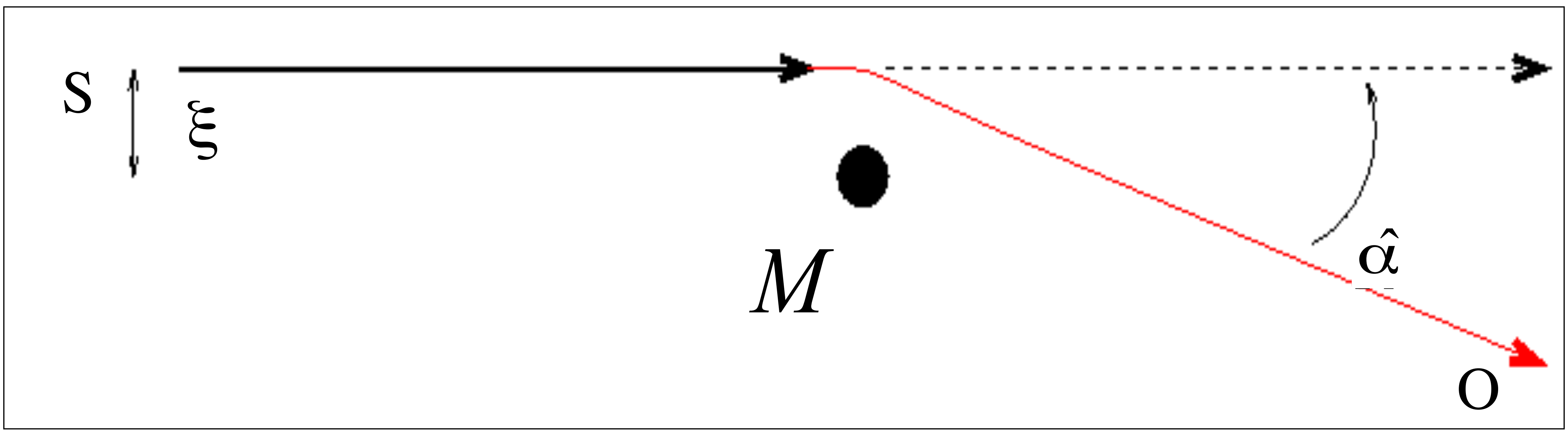
from strong lensing

Bending of light by gravity

Null geodesic,
Fermat principle

$$ds^2 = \left(1 + \frac{2\phi}{c^2}\right) c^2 dt^2 - \left(1 - \frac{2\phi}{c^2}\right) d\sigma^2$$

$$\frac{d\sigma}{dt} := c' = \sqrt{\frac{1 + 2\phi/c^2}{1 - 2\phi/c^2}} \simeq c\left(1 + \frac{2\phi}{c^2}\right)$$

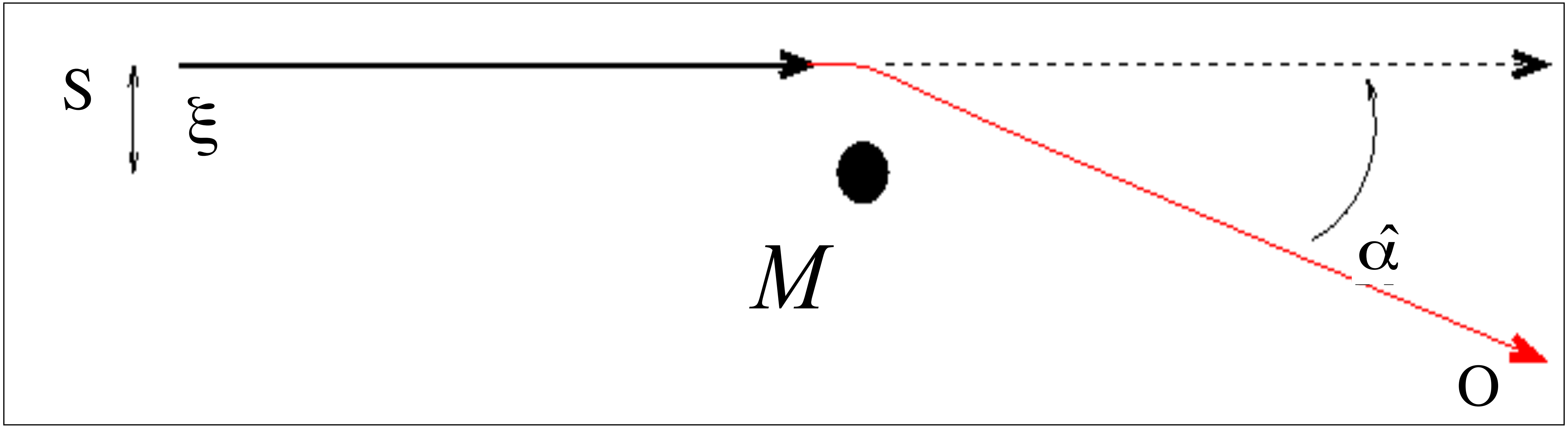


Bending of light by (modified) gravity

Null geodesic,
Fermat principle

$$ds^2 = \left(1 + \frac{2\psi}{c^2}\right) c^2 dt^2 - \left(1 - \frac{2\phi}{c^2}\right) d\sigma^2$$

peculiar gravitational potentials
(in GR $\psi = \phi$)



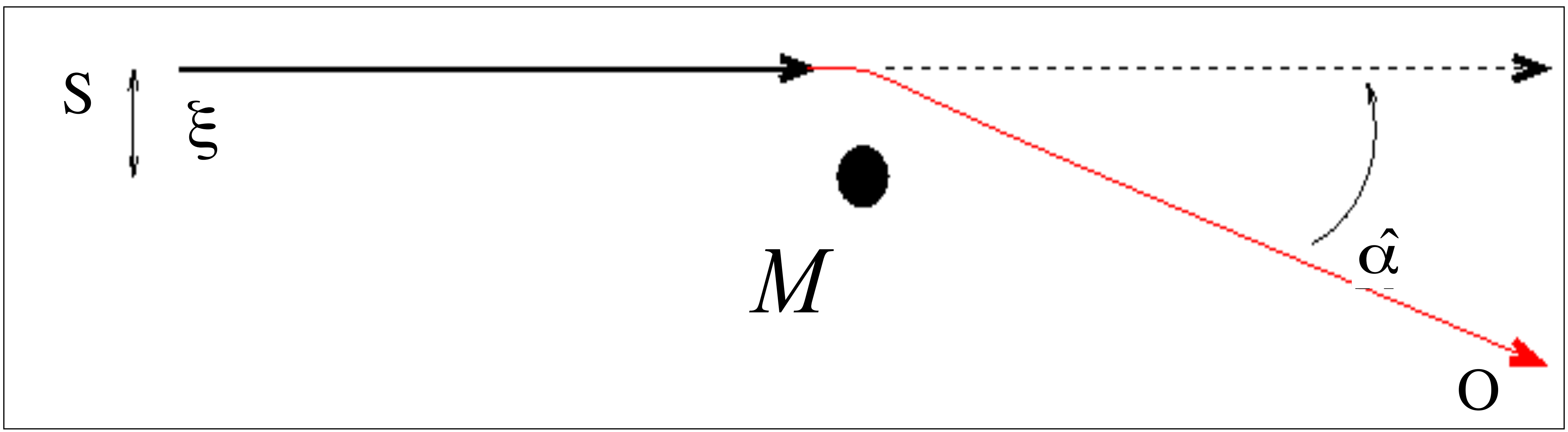
Bending of light by (modified) gravity

Null geodesic,
Fermat principle

$$ds^2 = \left(1 + \frac{2\psi}{c^2}\right) c^2 dt^2 - \left(1 - \frac{2\phi}{c^2}\right) d\sigma^2$$

$$\frac{d\sigma}{dt} = c' = c\sqrt{\frac{1 + \frac{2\psi}{c^2}}{1 - \frac{2\phi}{c^2}}} \simeq c\left(1 + \frac{\psi + \phi}{c^2}\right)$$

$$\frac{\phi}{\psi} = \gamma$$



Dynamical mass obtained from

$$\nabla^2 \psi = 4\pi G \rho$$

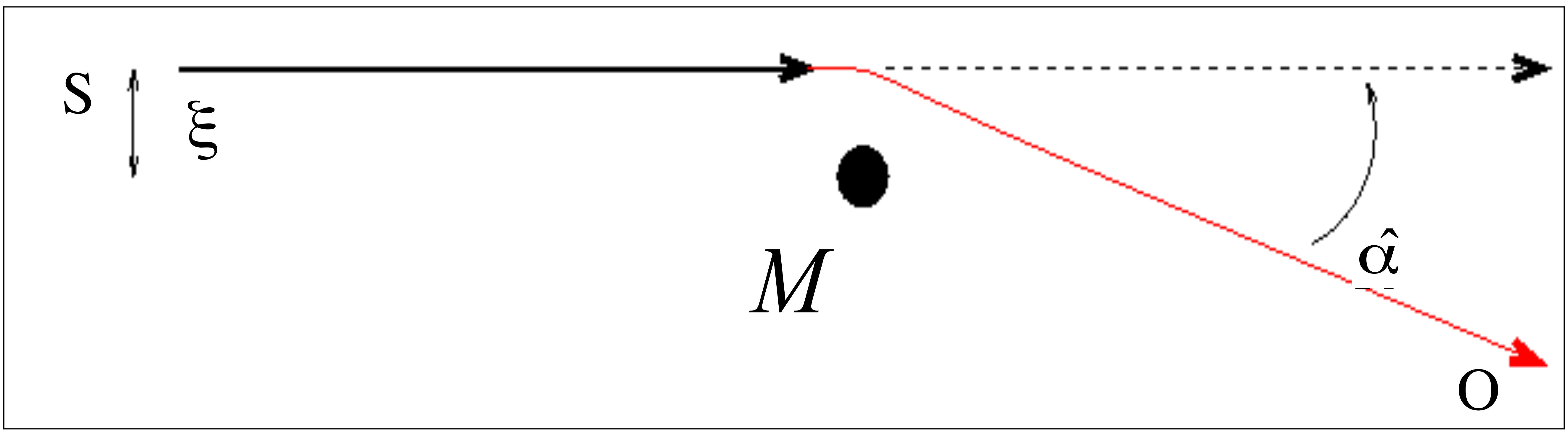
Use combination of lensing + dynamics to test gravity

Bending of light by (modified) gravity

Null geodesic,
Fermat principle

$$ds^2 = \left(1 + \frac{2\psi}{c^2}\right) c^2 dt^2 - \left(1 - \frac{2\phi}{c^2}\right) d\sigma^2$$

$$\frac{d\sigma}{dt} = c' = c\sqrt{\frac{1 + \frac{2\psi}{c^2}}{1 - \frac{2\phi}{c^2}}} \simeq c\left(1 + \frac{\psi + \phi}{c^2}\right) = c\left(1 + \frac{2\phi}{c^2}\left[\frac{1+\gamma}{2}\right]\right)$$



Dynamical mass obtained from

$$\nabla^2 \psi = 4\pi G \rho$$

Use combination of lensing + dynamics to test gravity

Basic picture

Singular Isothermal Sphere profile: $\rho(r) = \frac{\sigma_v^2}{2\pi G r^2}$

slip parameter
$$\gamma = \frac{\Phi}{\Psi}$$

Observed stellar velocity dispersion $\sigma_{\rm obs}$ probes Φ

Lensing yields $\sigma^2_{\rm lens}$ from $\Phi + \Psi$:

$$\sigma^2_{\rm lens} = \left(\frac{1+\gamma}{2}\right)\sigma^2_{\rm obs}$$

Real world:

- Non isothermal elliptical models
- Anisotropic velocity dispersion (assumptions)
- Seeing and aperture corrections

on ~ 100 kph
galaxy scales

kinematics: Φ

from galaxy
velocity dispersion

deflection angle: $\Phi + \Psi$

from strong lensing

Galaxy kinematics

The Jeans equation described the motion of a collection of stars. In spherical symmetry,

$$\frac{d}{dr}\left(\nu(r)\sigma_r^2(r)\right) + \frac{2\beta(r)}{r}\nu(r)\sigma_r^2(r) = \nu(r)\frac{d\Phi}{dr}, \tag{8}$$

where

$$\frac{d\Phi}{dr} = \frac{GM(r)}{r^2}, \quad \beta(r) := 1 - \frac{\sigma_t^2}{\sigma_r^2}, \tag{9}$$

where $M(r)$ is the total mass inside a sphere of radius r.
For $\beta =$ constant,

$$\sigma_r^2(r) = \frac{G}{r^{2\beta}\nu(r)}\int_r^\infty (r')^{2\beta-2}\nu(r')M(r')dr'. \tag{10}$$

The total mass contained within a sphere with radius r is,

$$M(r) = 4\pi\int_0^r (r')^2\rho(r')dr' = 4\pi\frac{r^{(3-\alpha)}}{3-\alpha}\frac{\rho_0}{r_0^{-\alpha}} \tag{11}$$

The actual velocity dispersion measured by observations is given by,

$$\bar{\sigma}_*^2 := \frac{\int_0^\infty dRRw(R)\int_{-\infty}^\infty dz\nu(r)\left(1-\beta\frac{R^2}{r^2}\right)\sigma_r^2(r)}{\int_0^\infty dRRw(R)\int_{-\infty}^\infty dz\nu(r)}, \tag{19}$$

where $w(R)$ is the convolution of the atmospheric seeing σ_{atm} and the fiber aperture θ_{ap}.

$$\begin{aligned}\bar{\sigma}_*^2 &= \left[\frac{2}{1+\gamma}\frac{c^2}{4}\frac{D_S}{D_{LS}}\theta_E\right]\frac{2}{\sqrt{\pi}}\frac{(2\tilde{\sigma}_{\mathsf{atm}}^2/\theta_E^2)^{1-\alpha/2}}{\xi-2\beta}\\ &\times\left[\frac{\lambda(\xi)-\beta\lambda(\xi+2)}{\lambda(\alpha)\lambda(\delta)}\right]\frac{\Gamma(\frac{3-\xi}{2})}{\Gamma(\frac{3-\delta}{2})},\end{aligned} \tag{20}$$

where

$$\tilde{\sigma}_{\mathsf{atm}} \approx \sigma_{\mathsf{atm}}\sqrt{1+\chi^2/4+\chi^4/40}, \quad \chi = \theta_{\mathsf{ap}}/\sigma_{\mathsf{atm}}. \tag{21}$$

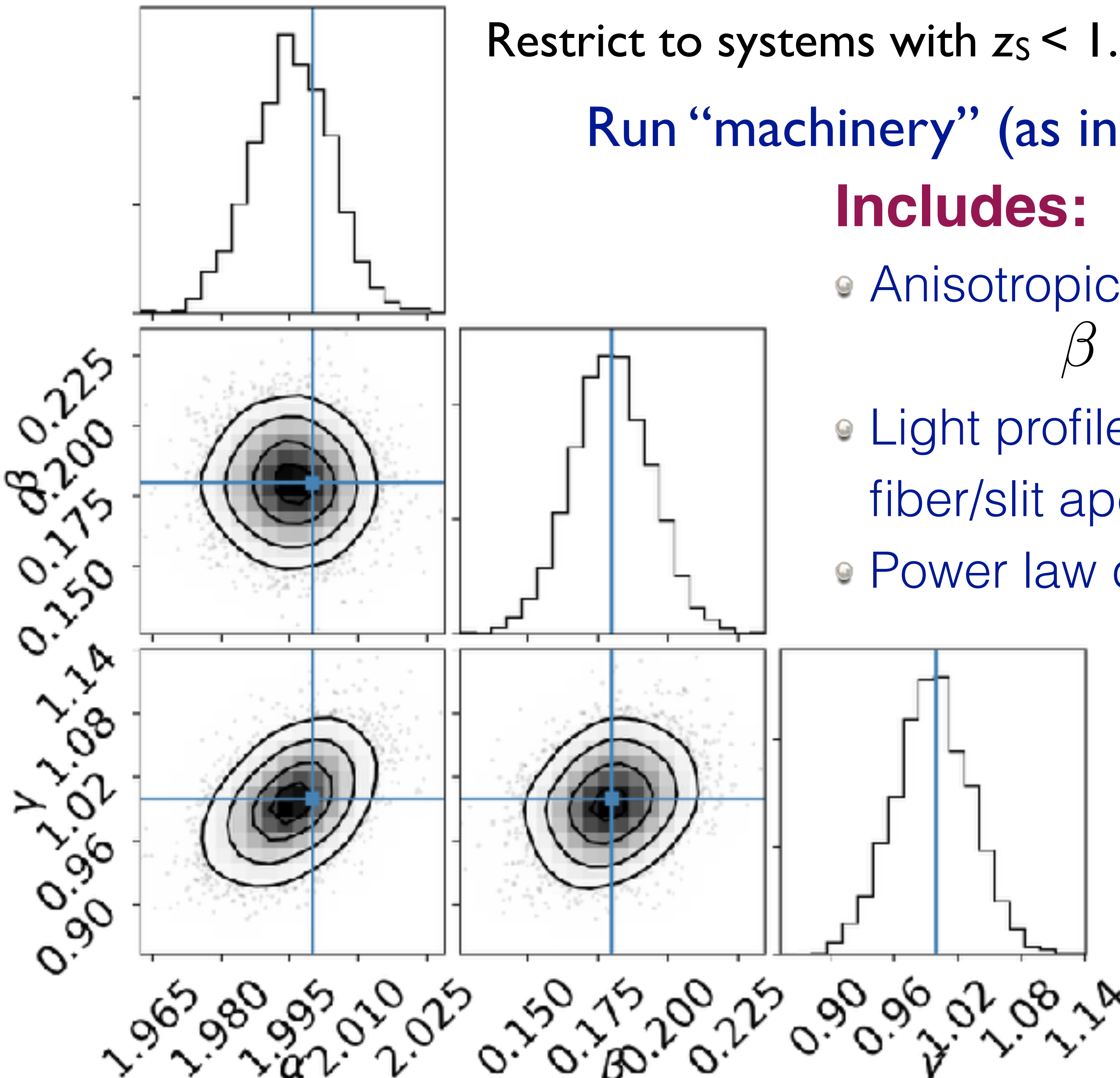
Archival Data

From (beta, public) “masterlens” catalog of 8577 SL systems

Galaxy-Galaxy systems, with known z_L, z_S, σ_v and lens modelling

Restrict to systems with $z_S < 1.5$: total of 110 systems

Crisnejo et al.



Run “machinery” (as in Schwab+2010, Cao+2017)

Includes:

- Anisotropic velocity dispersion
$$\beta = 1 - \sigma_t^2/\sigma_r^2$$
- Light profiles, seeing and fiber/slit aperture effects $\nu(r) = \nu_0 \left(\frac{r}{r_0}\right)^{-\delta}$
- Power law density profile $\rho(r) \propto r^{-\alpha}$
- Priors on
$$\beta = 0.180^{+0.014}_{-0.014}$$
$$\alpha = 1.995^{+0.009}_{-0.009}$$

Slip parameter

$$\gamma = 1.001^{+0.022}_{-0.023}$$

Archival Data

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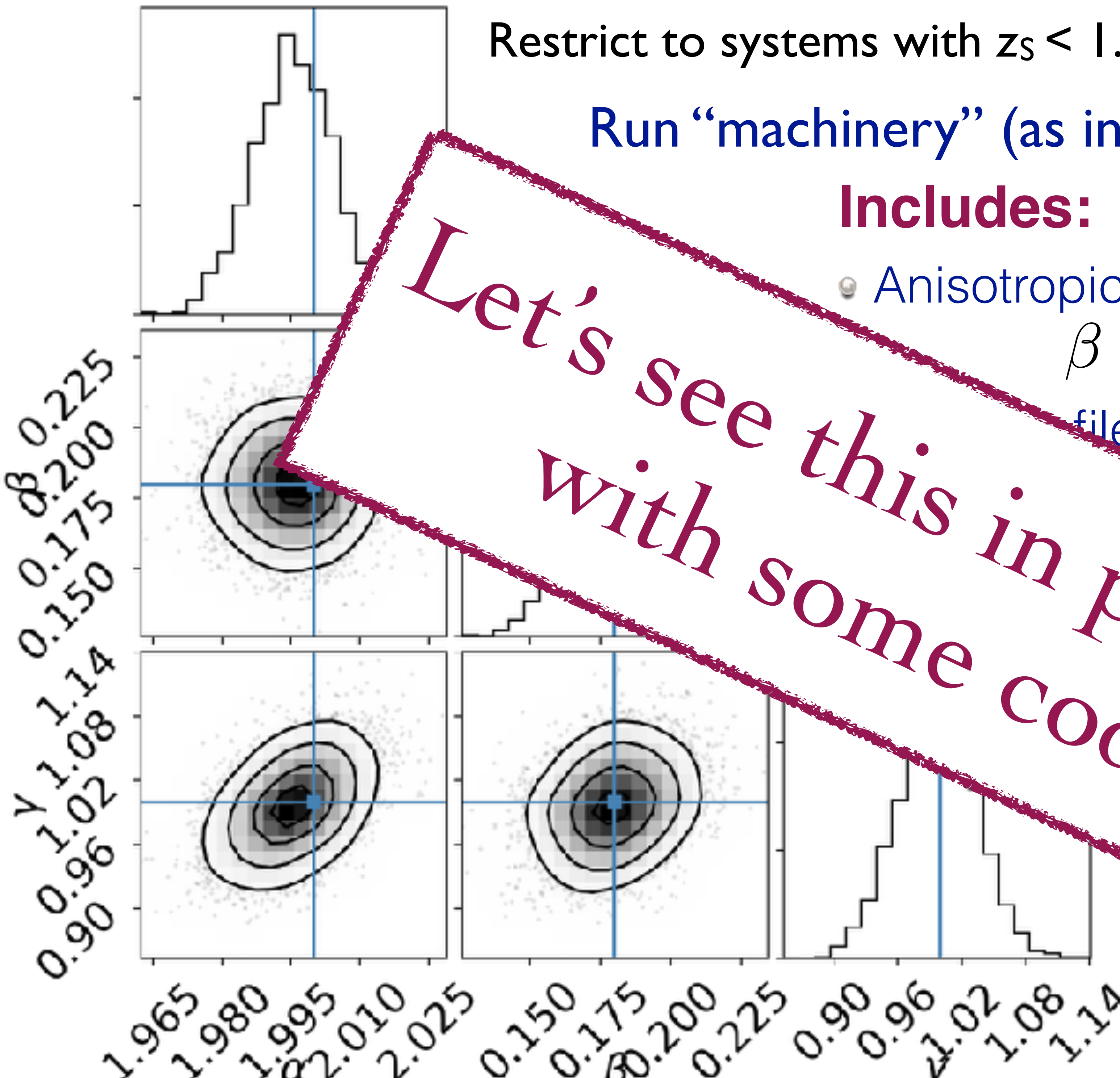


Run “machinery” (as in Schwab+2010, Cao+2017)

Includes:

- Anisotropic velocity dispersion

$$\beta = 1 - \sigma_t^2/\sigma_r^2$$

- files, seeing and
  e effects

$$\nu(r) = \nu_0 \left(\frac{r}{r_0}\right)^{-\delta}$$

file $\rho(r) \propto r^{-\alpha}$

$$= 0.180^{+0.014}_{-0.014}$$

$$\alpha = 1.995^{+0.009}_{-0.009}$$

parameter

$$\gamma = 1.001^{+0.022}_{-0.023}$$

Let’s see this in practice
with some code

Part IV
Weak Lensing

Martín Makler
ICAS/IFICI/CONICET&UNSAM & CBPF

+ Renan Alves, João França, Elizabeth Gonzalez, Giulya Souza, Eduardo Valadão, Anibal Varela

FoF 2022

ICAS

ICIFI

CBPF

OAC Observatorio Astronómico de Córdoba

UNC Universidad Nacional de Córdoba

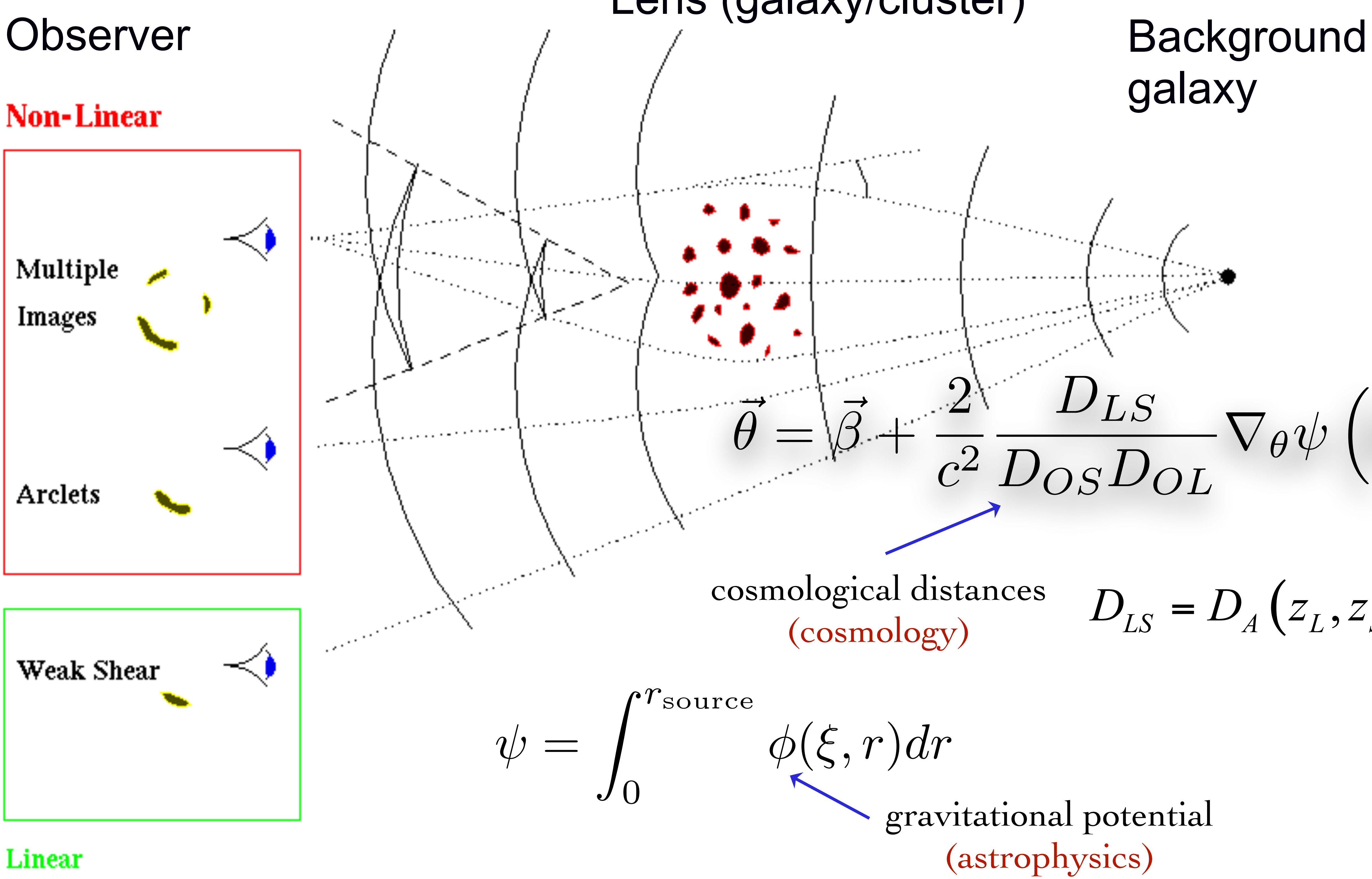
IATE

UNSAM

CONICET

PPGCosmo

Weak and Strong Lensing Effects



$$\vec{\theta} = \vec{\beta} + \frac{2}{c^2}\frac{D_{LS}}{D_{OS}D_{OL}}\nabla_\theta \psi\left(\vec{\theta}\right)$$

cosmological distances
(cosmology)

$$D_{LS} = D_A\left(z_L, z_S\right)\ldots$$

$$\psi = \int_0^{r_{\mathrm{source}}} \phi(\xi, r)dr$$

gravitational potential
(astrophysics)

Lens Mapping

- mapping image → source

$$\frac{\partial \beta_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial^2 \Psi(\vec{\theta})}{\partial \theta_i \partial \theta_j}$$

- single plane

$$\Psi = \frac{2}{c^2} \frac{D_{LS}}{D_{OS} D_{OL}} \int \phi(\xi, z) dz$$

gravitacional potential

cosmological distances

$$D_{LS} = D_A(z_L, z_S)...$$

- autovalores:

$$\mu_1 = \frac{1}{1-\kappa+\gamma}, \mu_2 = \frac{1}{1-\kappa-\gamma}$$

- local magnifications and axis ratio:

$$\mu = \mu_1 \, \mu_2 \qquad r = \left| \frac{\mu_1}{\mu_2} \right|$$

- critical surface density

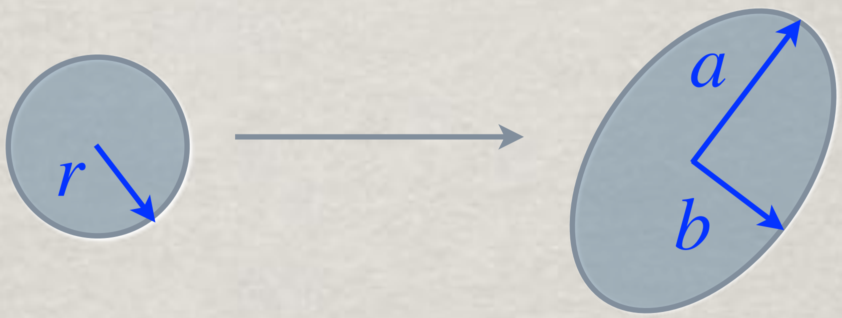
$$\Sigma_{\text{crit}} = \frac{c^2}{4\pi G} \frac{D_{OS}}{D_{OL} D_{LS}}$$

- convergence

$$\kappa = \frac{\Sigma}{\Sigma_{\text{crit}}}$$

Linear Mapping

- Circular sources



$$a = \left(\frac{1}{1-\kappa-\gamma}\right) r$$

$$b = \left(\frac{1}{1-\kappa+\gamma}\right) r$$

- Magnifications

$$\mu = \frac{A_{\mathrm{imagem}}}{A_{\mathrm{fonte}}} = \left[(1-\kappa)^2 - \gamma^2\right]^{-1}$$

- Ellipticity

$$\boxed{\epsilon := \frac{a-b}{a+b} = \frac{\gamma}{1-\kappa} =: g}$$

Real sources

- Center of the object (image)

$$\bar{\theta}_i = \frac{\int \mathrm{d}^2\theta q_I \left[I\left(\theta\right)\right] \theta_i}{\int \mathrm{d}^2\theta q_I \left[I\left(\theta\right)\right]}$$

- Second order momenta

$$Q_{ij} = \frac{\int d^2\theta q_I[I(\theta)](\theta_i - \bar{\theta}_i)(\theta_j - \bar{\theta}_j)}{\int d^2\theta q_I[I(\theta)]}$$

- Area

$$\Omega = \left(Q_{11}Q_{22} - Q_{12}^2\right)^{1/2}$$

- Ellipticity

$$\epsilon := \frac{Q_{11} - Q_{22} + 2\,i\,Q_{12}}{Q_{11} + Q_{22} + 2\left(Q_{11}Q_{22} - Q_{12}^2\right)^{1/2}}$$

Real sources

- Second order momenta

$$Q_{ij} = \frac{\int d^2\theta q_I[I(\theta)](\theta_i - \bar{\theta}_i)(\theta_j - \bar{\theta}_j)}{\int d^2\theta q_I[I(\theta)]}$$

- Area

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$$\epsilon := \frac{Q_{11} - Q_{22} + 2\,i\,Q_{12}}{Q_{11} + Q_{22} + 2\left(Q_{11}Q_{22} - Q_{12}^2\right)^{1/2}}$$

- Linear distortion

$$\epsilon_S = \frac{\epsilon_I - g}{1 - g^*\epsilon_I} \qquad g_i := \frac{\gamma_i(\theta)}{1 - \kappa(\theta)} \qquad |g| \leq 1$$

Weak Lensing

- Slight shear in background galaxies (change in axis + size)
- Weak lensing regime

$$\epsilon = \epsilon_I = \epsilon_S + g$$

- "weak lensing fundamental theorem":

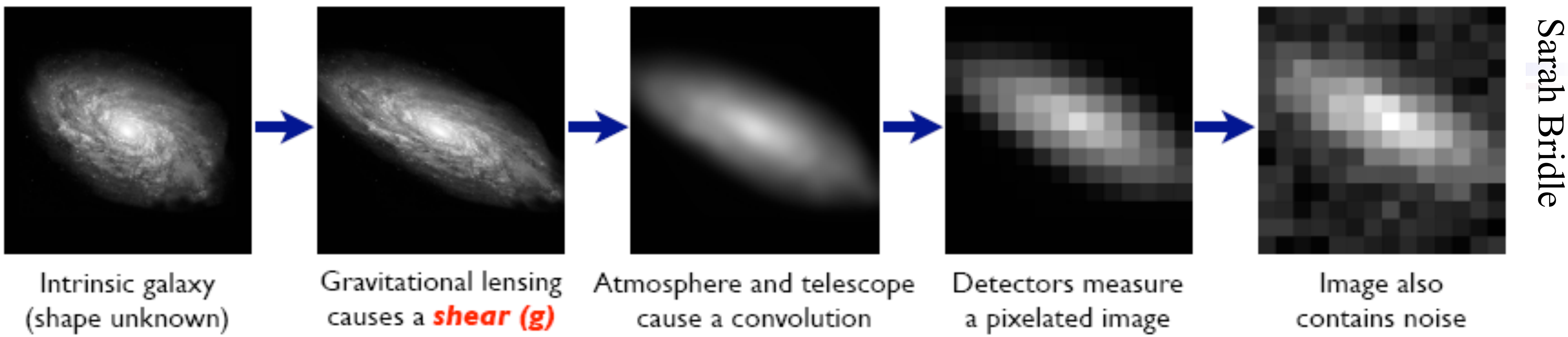
$$\boxed{\langle \epsilon \rangle = g}$$

- but

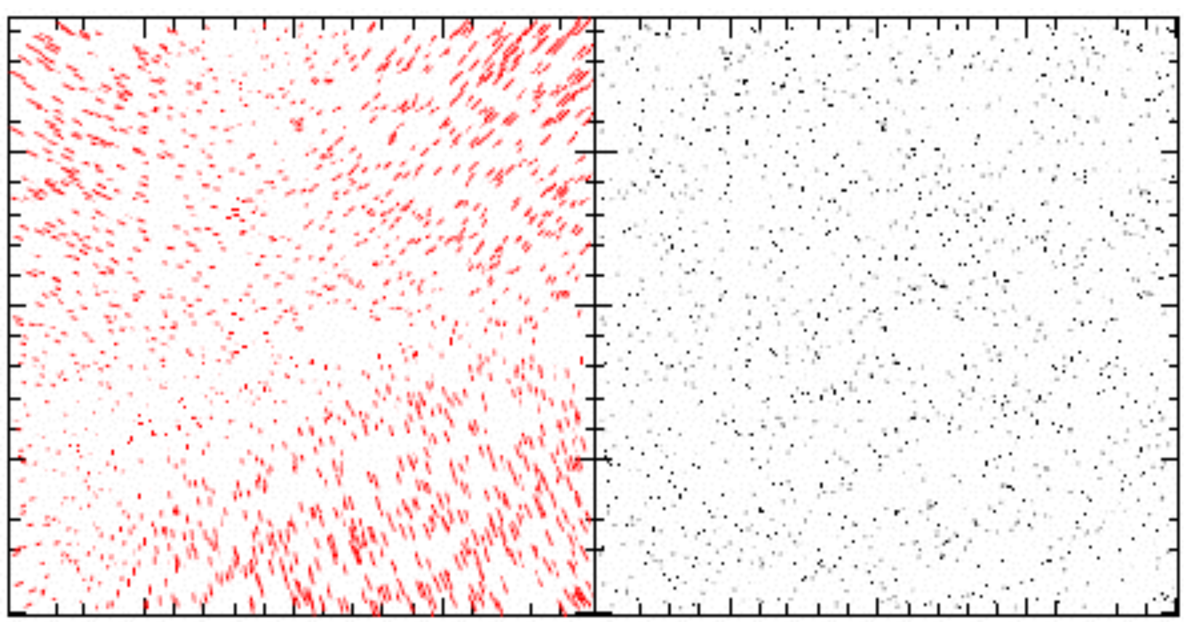
$$g \ll \epsilon$$

- observational, instrumental, computational and theoretical challenges!

Technical challenges

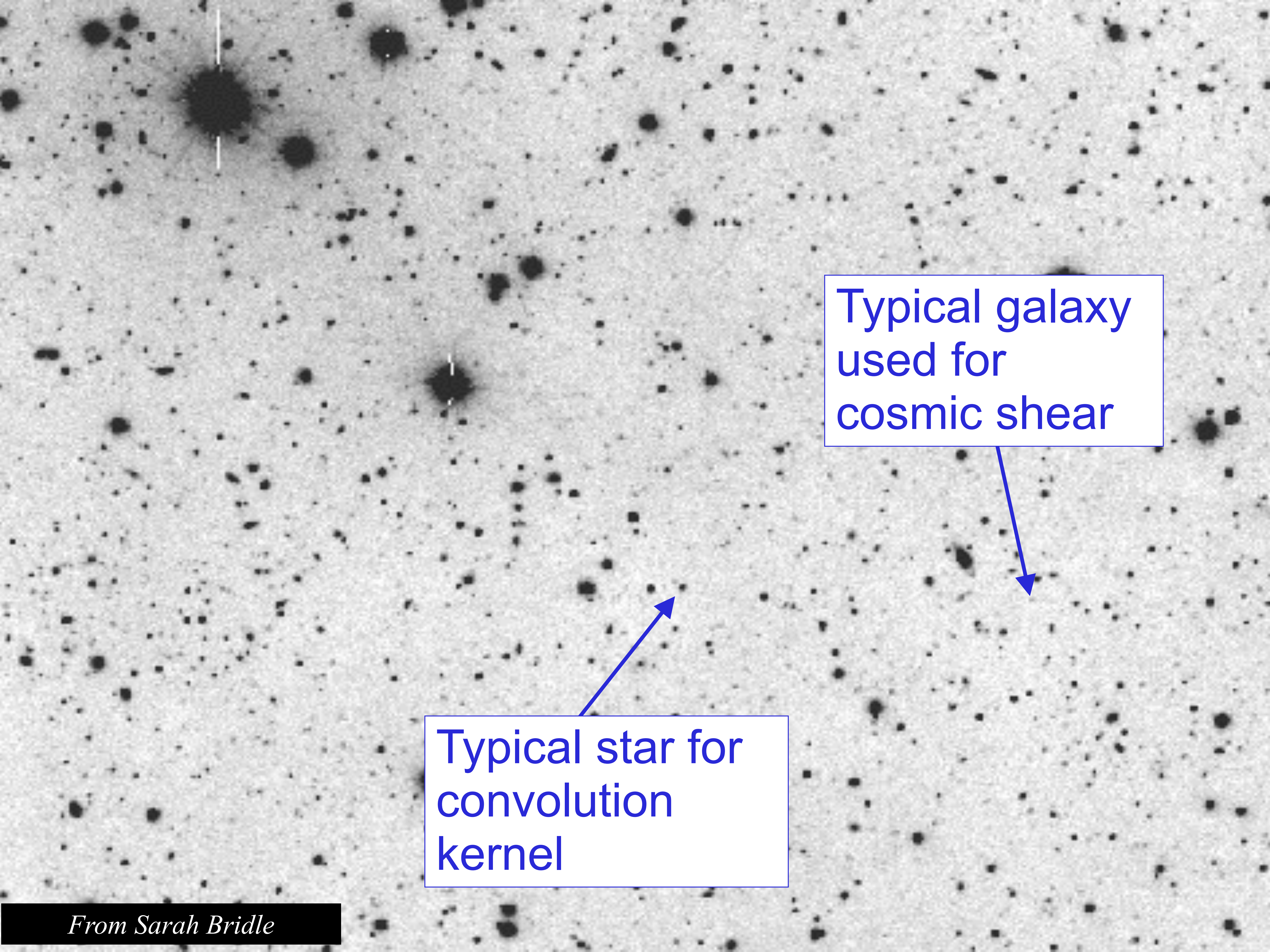


- Model the PSF distortion (many sources)
  - measure a large number of stars in the field



CFHTLS Deep 1
Gavazzi, Soucail 2006

- Difficulties: saturated stars, charge transfer efficiency, halos, tracking...


Typical galaxy used for cosmic shear
Typical star for convolution kernel
From Sarah Bridle

Measuring weak lensing

- Signal $\langle \varepsilon_I \rangle = \gamma$ $\qquad (\varepsilon_I = \varepsilon_S + \gamma)$
- Noise $\sigma_\varepsilon = \langle |\varepsilon_S{}^2| \rangle^{1/2} \sim 0.3 \gg \gamma$
- Win over the noise by averaging over a large number of galaxies

Regime	γ	$\gamma/\sigma_\varepsilon$	N_{gal} for S/N~ 1
weak lensing by clusters	0.03	0.1	10^2
galaxy-galaxy lensing	0.003	0.01	10^4
cosmic shear	0.001	0.003	10^5

Much more galaxies for precision measurements needed.

(de Kilbinger)

2 Regimes and Methods

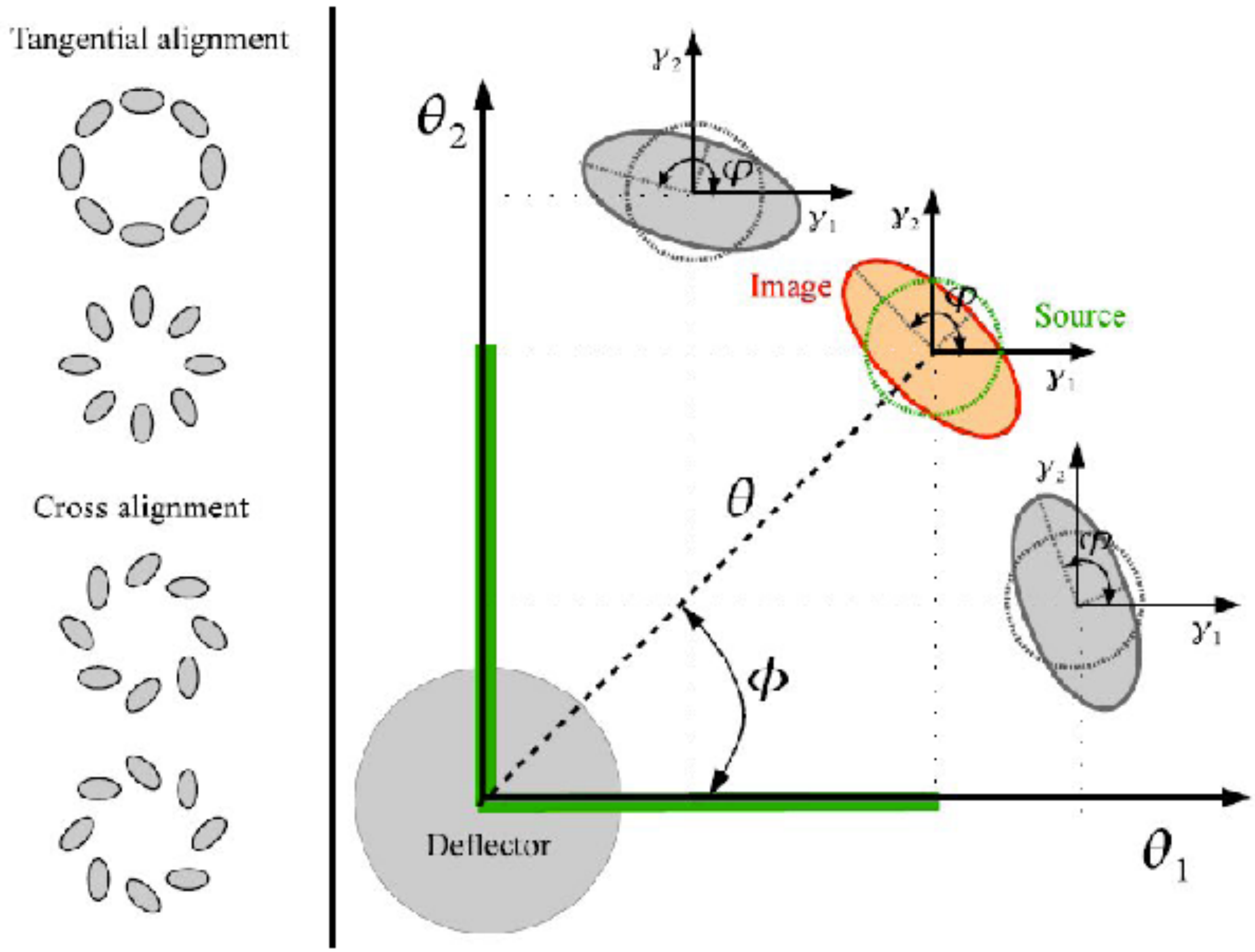
Lensing by galaxies and clusters

- Larger signal
- Center of reference
- Model/profile fitting
- Individual objects or *Stacking* of the signal

Large-scale structure

- Convergence maps (also in clusters)
- Correlations:
  - power spectrum, correlation function
  - among different probes, z-bins, CMB, etc.

Components of the Shear



Figueiró 2011

Mean shear in circles

$$< \gamma_t(\theta) >= \bar{\kappa}(\theta) - < k(\theta) >$$

Mean shear and radial profiles

It is possible to show that

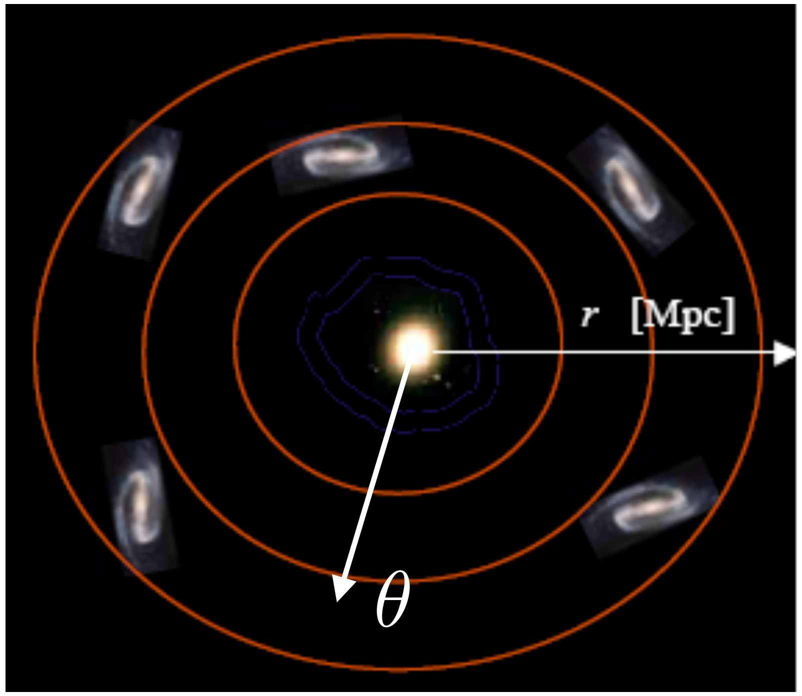
$$< \gamma_t(\theta) >= \bar{\kappa}(\theta) - < k(\theta) >$$

mean along a circle of the tangential component of the shear

mean within a disk of radius θ

mean on the circle

In practice: mean un anulii (radial bins)



Stacking of the signal

Need (and possibility) to increase the S/*N*

Combine data on many galaxies or clusters: mean signal

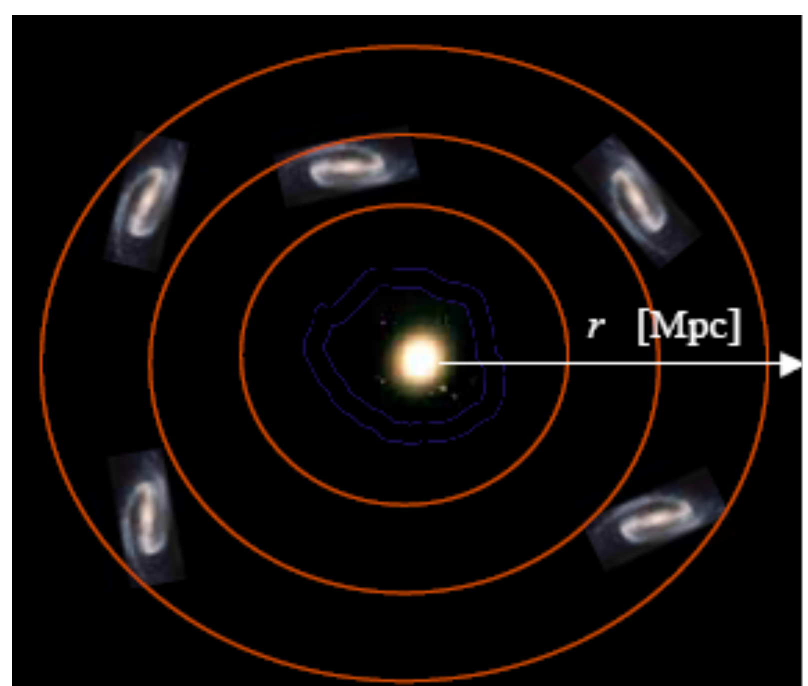
Physical signal and models: $\Sigma(r)$ $\longrightarrow$ multiply by $\Sigma_{\rm crit}$

$$\Sigma_{\rm crit} \times < \gamma_t(\theta) >= \bar{\kappa}(\theta) - < k(\theta) > \times \Sigma_{\rm crit}$$

$$\boxed{\Sigma_{\rm crit} \times < \gamma_t(r) >= \bar{\Sigma}(r) - < \Sigma(r) > := \Delta\Sigma(r)}$$

redshifts

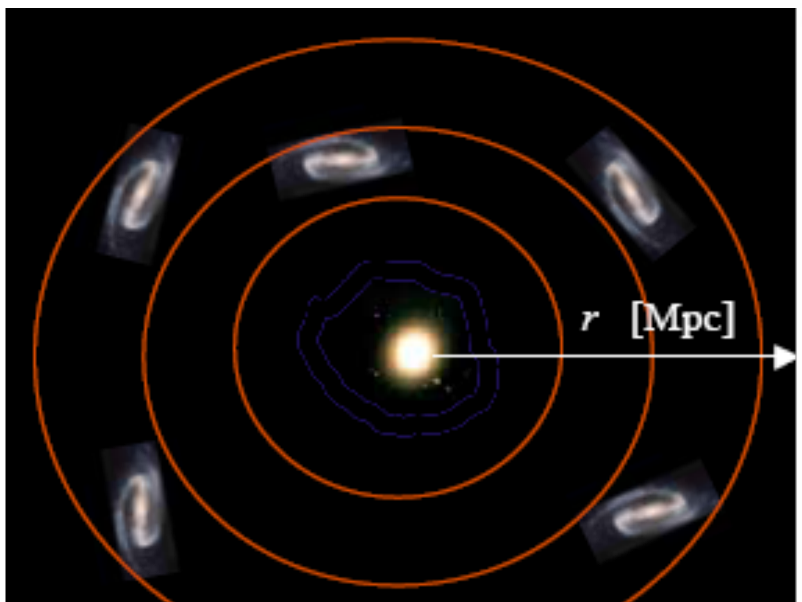
shapes



model

Mass reconstruction in clusters (radial profile)

- Measure the tangential shear to get $\Delta\Sigma$



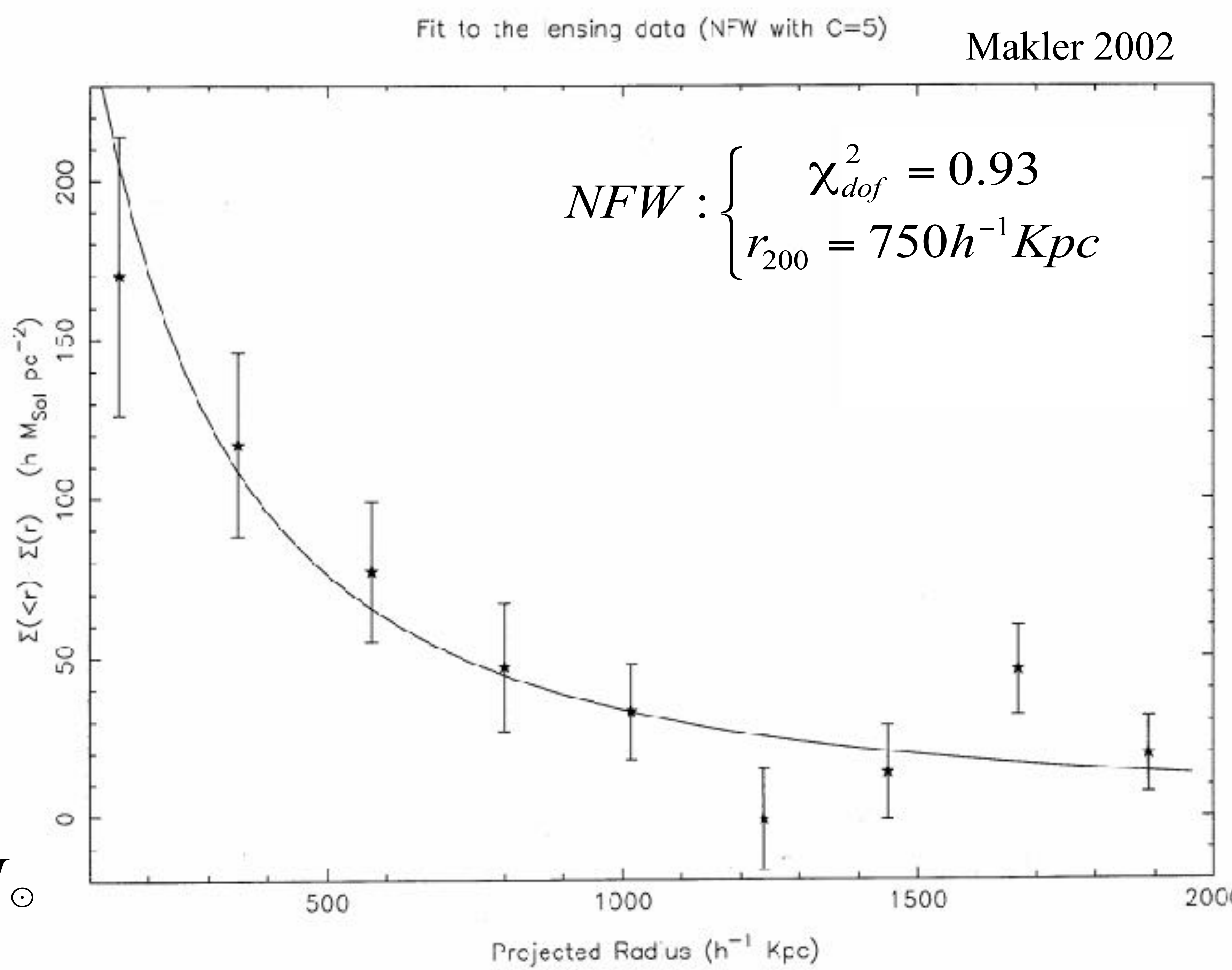
$$\Delta\Sigma(r) \equiv \bar{\Sigma}(<r) - \bar{\Sigma}(r) = \Sigma_{crit} \times \gamma_t(r)$$

$$\Sigma_{crit} = \frac{c^2}{4\pi G}\frac{D_{OS}}{D_{OL}D_{LS}}$$

- Masses:

$$\bar{M}_{NFW} = (1.0 \pm 0.2)10^{14} h^{-1} M_{\odot}$$

$$\frac{\Delta M_{200}}{M_{200}} \simeq 2.5\%$$



42 clusters (RASS/SDSS), Sheldon, et al., ApJ 554, 88 (2001)

Modeling the mass profile

Interpretation with the halo model: halos x galaxies correlation

- ☑ 1-halo term: matter density in the halo

Exemple: NFW $\rho(r) = \dfrac{\rho_0}{(r/r_s)\left(1 + r/r_s\right)^2}$ compute $\Sigma(r)$

- ☑ 2-halo term: correlation with other halos (large scale structure) $\rho(r) = b\bar{\rho}_m \xi(r)$ where

$$\xi(r) = \frac{1}{2\pi^2}\int dk\, k^2 P(k)\frac{\sin(kr)}{kr} \qquad b(\nu) = 1 - A\frac{\nu^a}{\nu^a + \delta_c^a} + B\nu^b + C\nu^c$$

$$\nu = \delta_c/\sigma(M) = 1.686/\sigma(M) \qquad \sigma(R) = \frac{1}{2\pi^2}\int dk\, k^2 P(k,z)\hat{W}^2(k,R)$$

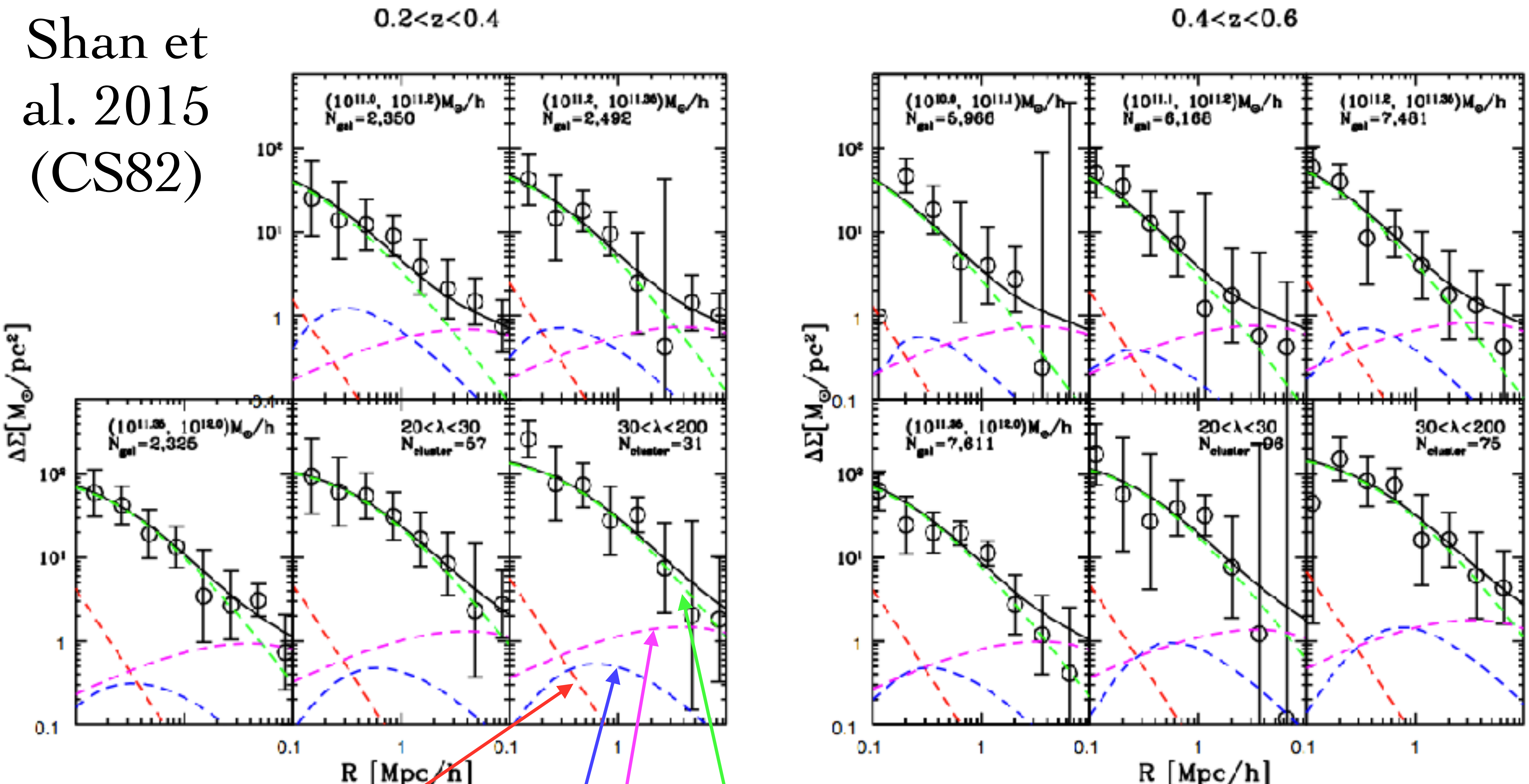
$$R = (3M/4\pi\bar{\rho}_m)^{1/3}$$

(see mass function)

- ☑ Term for the *offset* of the profile with respect to other center
- ☑ Central potential (central galaxy, e.g. SIS)

Example: Cluster mass calibration

Shan et al. 2015 (CS82)



1-halo

2-halo

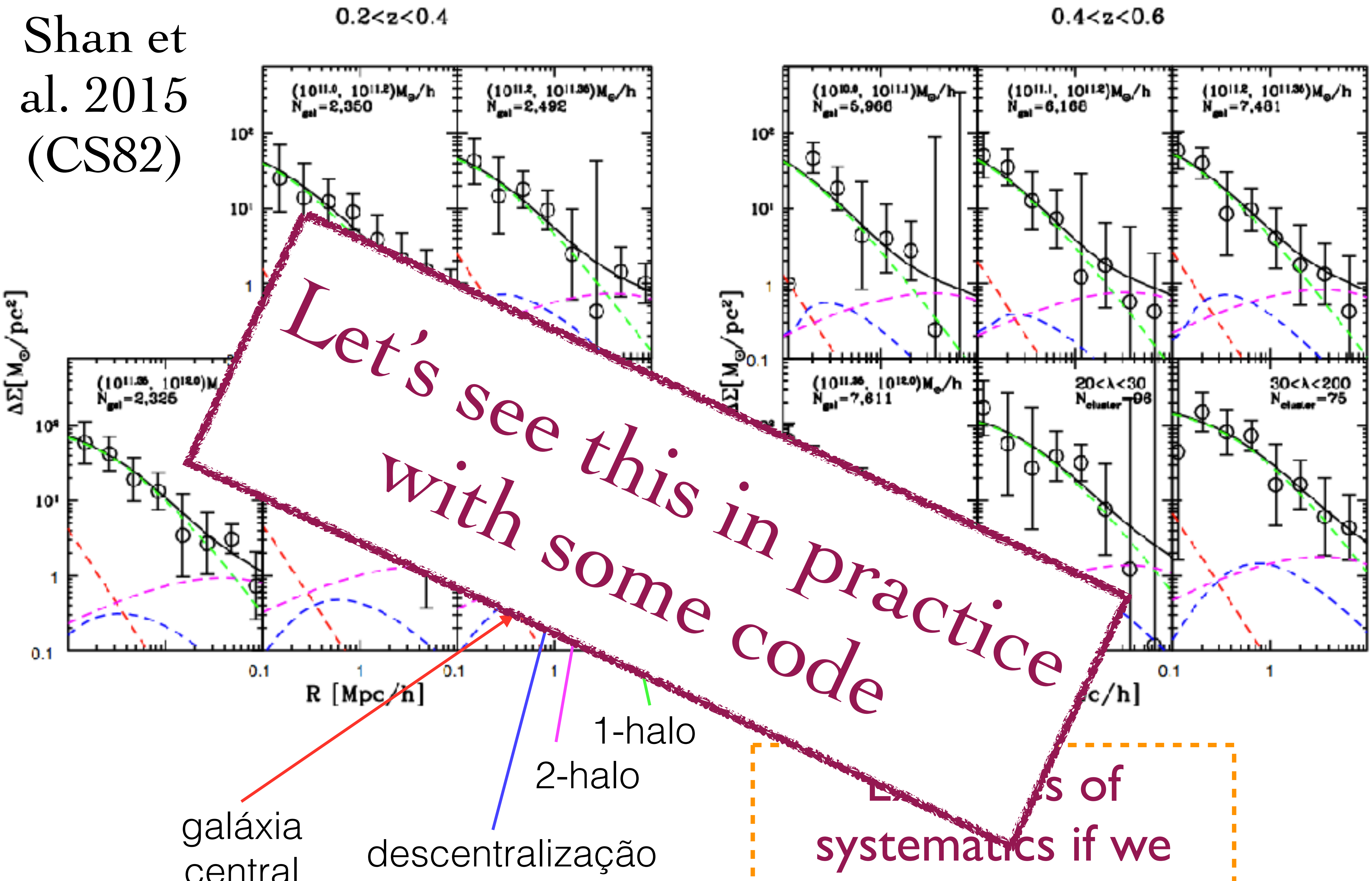
galáxia central

descentralização

Examples of systematics if we ignore these effects

Mass calibration

Shan et al. 2015 (CS82)



Let's see this in practice with some code

Ex… s of systematics if we ignore these effects

Gravitational Lensing: a tool for astrophysics and cosmology

- From planets to the Large Scale Structure of the Universe
- Essentially 21st century science, with many discoveries in the last 10 years
- These were by no means review talks on the main topic of lensing
  - Illustrations for pedagogical purposes
  - A few worked examples and use cases that can be followed at a basic level, reproduced and run in the example code provided in the repository
- Many many subjects not addressed, such as
  - Time delays, $H0$, QSO (micro-)lensing
  - Implications for DM properties
  - Astrophysical consequences of profiles, etc.
  - Gravitational telescope
  - Arcfinding, automated modeling, ML applications

Gravitational Lensing: a tool for astrophysics and cosmology

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- Interdisciplinary field involving from fundamental physics to data reduction, including image processing, statistics, simulations, theory and semi-analytic modeling
- Lots of room for interesting work to do
- New dynamics of large projects

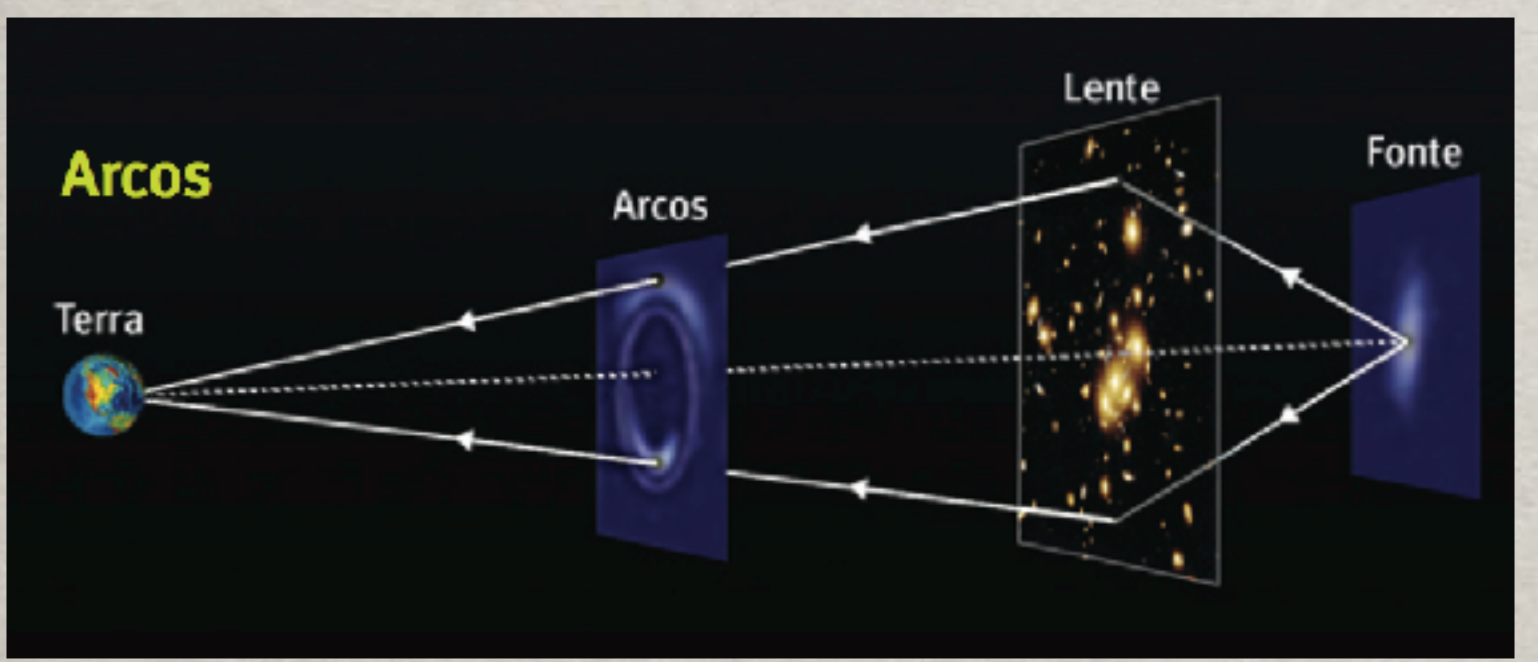

Thank You

Strong Lensing

- Multiple images, strong distortions, large magnifications, time delays
- Null geodesics
  - surface brightness conservation
  - achromatic

  } → Gravitational telescopes

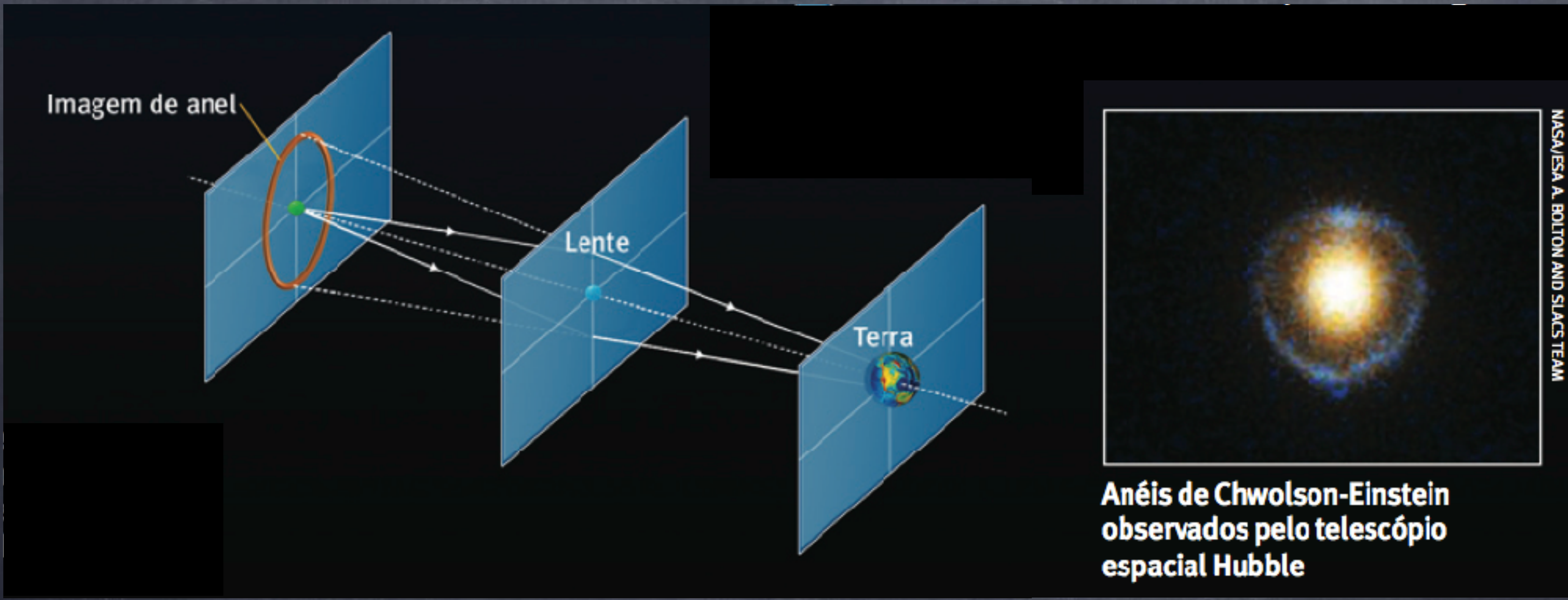
  - Unique probe of inner structure of galaxy clusters ➔ DM, *b*
  - Provide complementary cosmological probes and tests of gravity



strong lensing, **weak gravity**

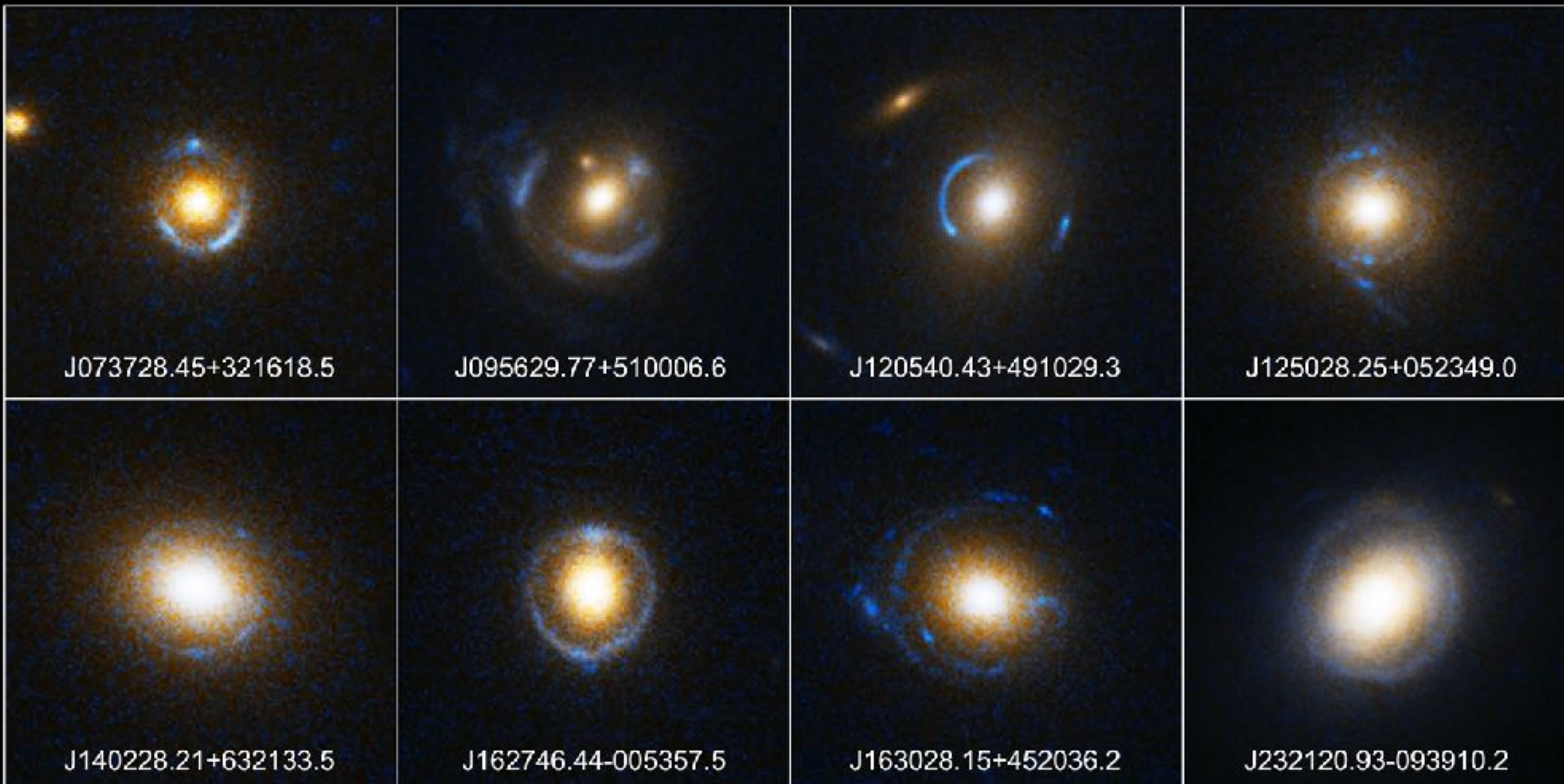
Gravitational arcs

"Chwolson-Einstein rings"



Anéis de Chwolson-Einstein observados pelo telescópio espacial Hubble

“Anillos de Chwolson-Einstein”



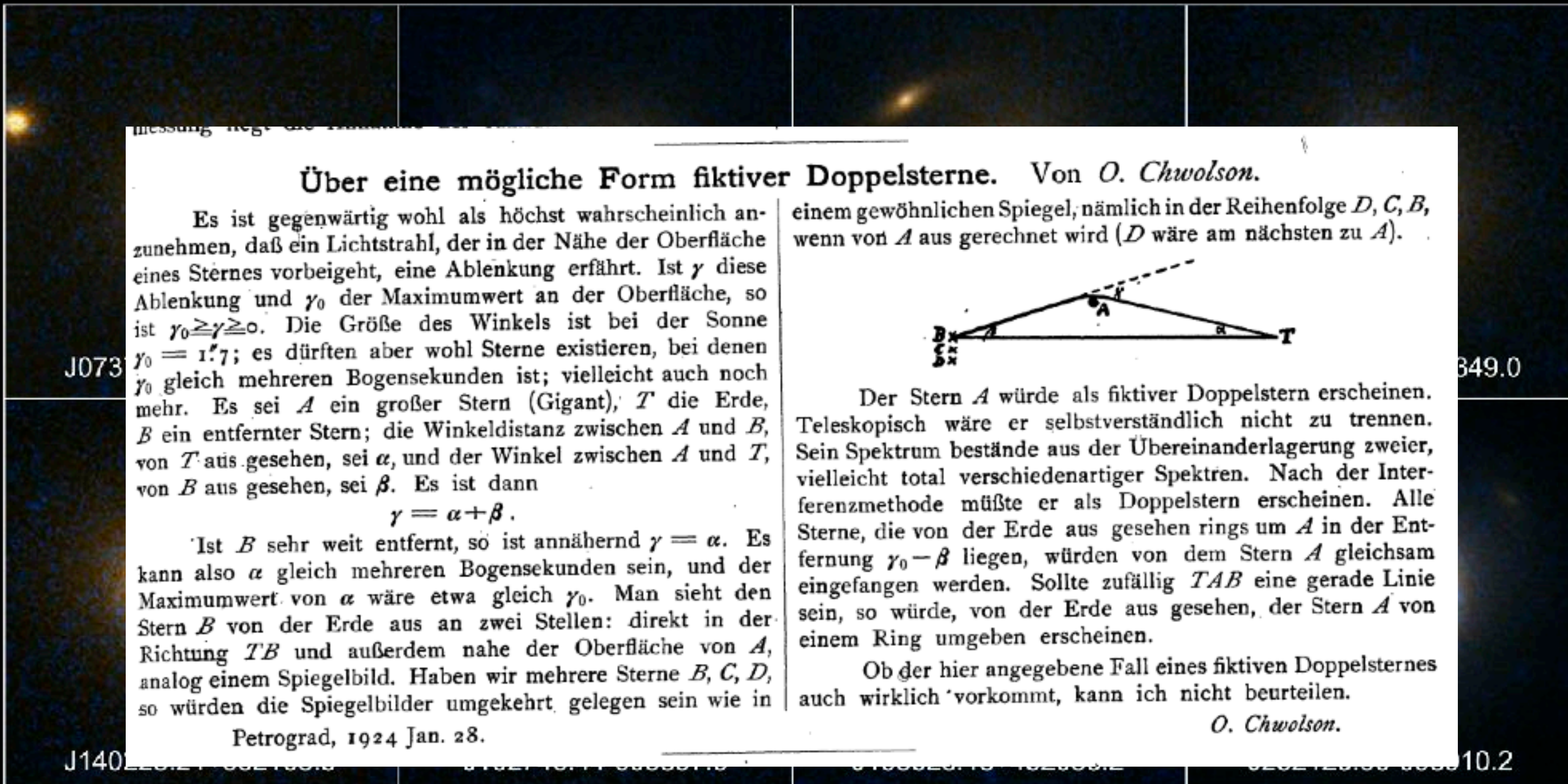
Einstein Ring Gravitational Lenses

Hubble Space Telescope • Advanced Camera for Surveys

NASA, ESA, A. Bolton (Harvard-Smithsonian CfA), and the SLACS Team

STScI-PRC05-32

Chwolson-Einstein rings



Über eine mögliche Form fiktiver Doppelsterne. Von *O. Chwolson.*

Es ist gegenwärtig wohl als höchst wahrscheinlich anzunehmen, daß ein Lichtstrahl, der in der Nähe der Oberfläche eines Sternes vorbeigeht, eine Ablenkung erfährt. Ist γ diese Ablenkung und γ_0 der Maximumwert an der Oberfläche, so ist $\gamma_0 \geq \gamma \geq 0$. Die Größe des Winkels ist bei der Sonne $\gamma_0 = 1''7$; es dürften aber wohl Sterne existieren, bei denen γ_0 gleich mehreren Bogensekunden ist; vielleicht auch noch mehr. Es sei A ein großer Stern (Gigant), T die Erde, B ein entfernter Stern; die Winkeldistanz zwischen A und B, von T aus gesehen, sei α, und der Winkel zwischen A und T, von B aus gesehen, sei β. Es ist dann

$$\gamma = \alpha + \beta .$$

Ist B sehr weit entfernt, so ist annähernd $\gamma = \alpha$. Es kann also α gleich mehreren Bogensekunden sein, und der Maximumwert von α wäre etwa gleich γ_0. Man sieht den Stern B von der Erde aus an zwei Stellen: direkt in der Richtung TB und außerdem nahe der Oberfläche von A, analog einem Spiegelbild. Haben wir mehrere Sterne B, C, D, so würden die Spiegelbilder umgekehrt gelegen sein wie in einem gewöhnlichen Spiegel, nämlich in der Reihenfolge D, C, B, wenn von A aus gerechnet wird (D wäre am nächsten zu A).

Der Stern A würde als fiktiver Doppelstern erscheinen. Teleskopisch wäre er selbstverständlich nicht zu trennen. Sein Spektrum bestände aus der Übereinanderlagerung zweier, vielleicht total verschiedenartiger Spektren. Nach der Interferenzmethode müßte er als Doppelstern erscheinen. Alle Sterne, die von der Erde aus gesehen rings um A in der Entfernung $\gamma_0 - \beta$ liegen, würden von dem Stern A gleichsam eingefangen werden. Sollte zufällig TAB eine gerade Linie sein, so würde, von der Erde aus gesehen, der Stern A von einem Ring umgeben erscheinen.

Ob der hier angegebene Fall eines fiktiven Doppelsternes auch wirklich vorkommt, kann ich nicht beurteilen.

O. Chwolson.

Petrograd, 1924 Jan. 28.

Einstein Ring Gravitational Lenses

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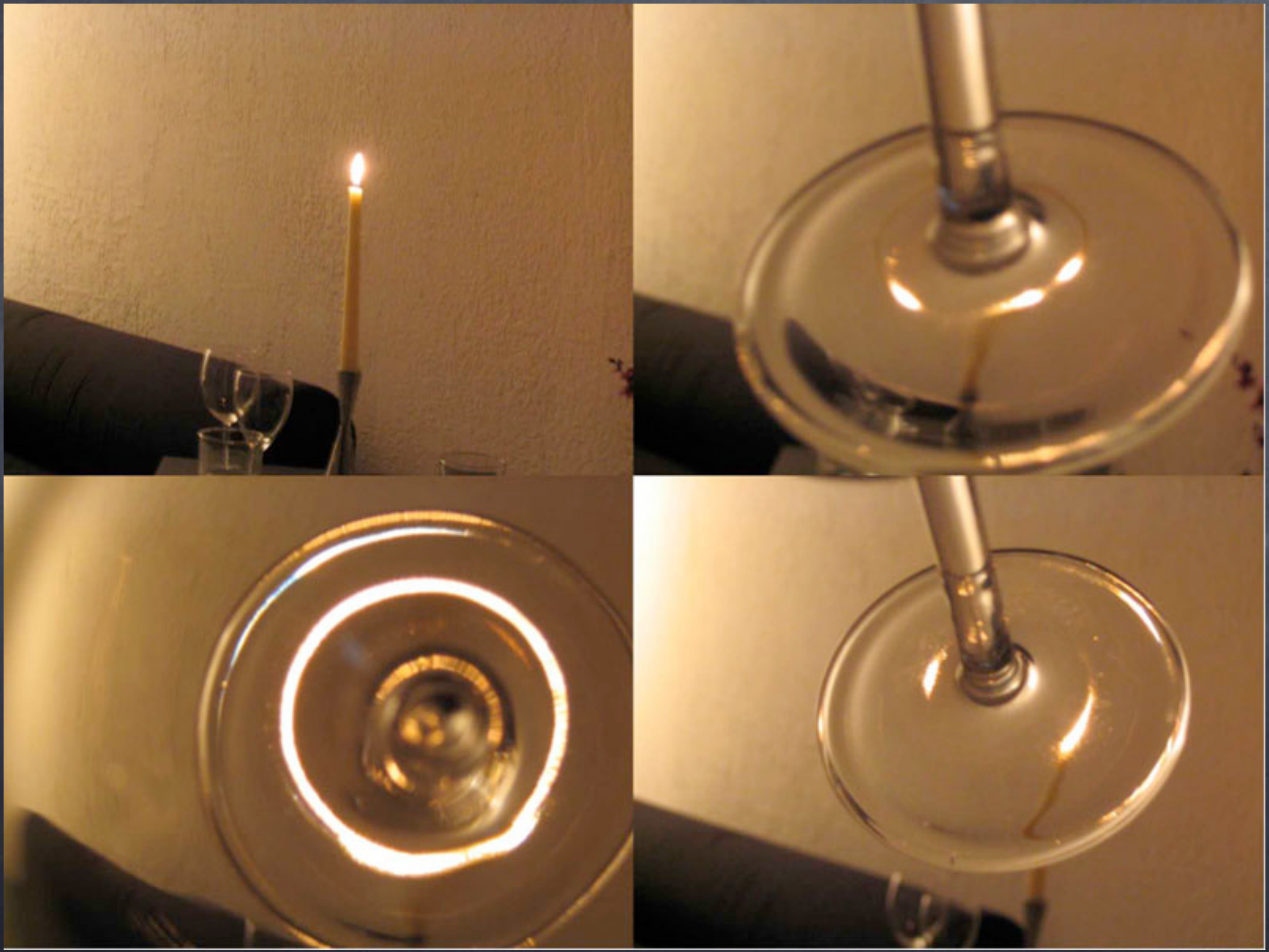



Galaxy Clusters
Strong Lensing
Macrolensing

A Relativistic Eclipse

What Might be Seen from a Planet Conveniently Placed Near the Companion of Sirius . . . Perfect Tests of General Relativity that are Unavailable

By HENRY NORRIS RUSSELL, Ph. D.

Chairman of the Department of Astronomy and Director of the Observatory at Princeton University. Research Associate of the Mount Wilson Observatory of the Carnegie Institution of Washington. President of the American Astronomical Society.

IT is a familiar aphorism that the theory of relativity, despite its enormous importance, both in physics and philosophy, may be forgotten in ordinary practical life. There is a good reason. In almost every known case its results agree so closely with those of the older "classical" theories that very accurate observations are required to distinguish between them. Thus, for example, the famous Michelson-Morley experiment requires long series of the

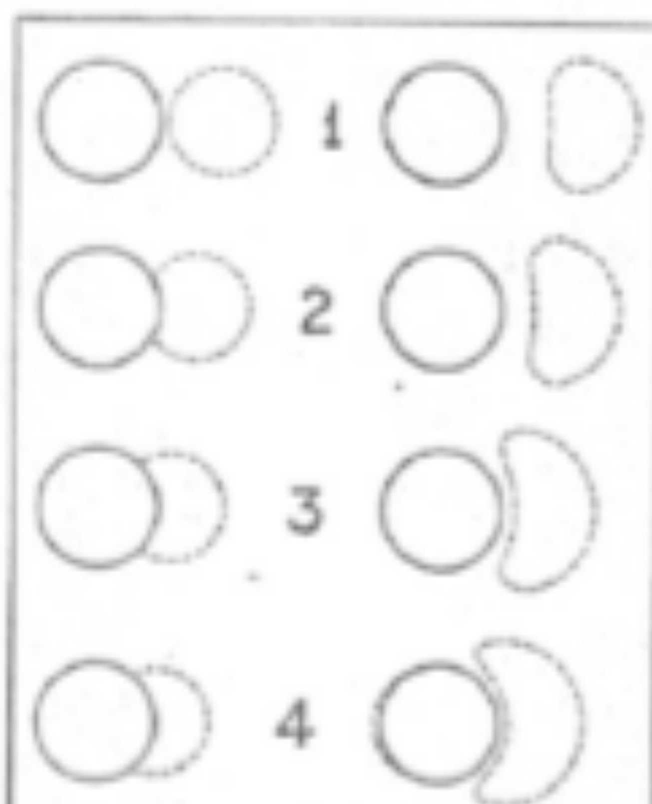
cludes gravitation in its scope) are of this sort. The advance of the perihelion of Mercury ...

No one predicted gravitational arcs?

stein himself.[1] From a point exactly in line the distant star would appear to be ... be resolved by even the greatest tele-

for it. The focusing effect, however, would concentrate the light of the dis- ... For a star ... mathematical point, with no angular diameter at all, the increase in brightness would be infinite. But any real star must have a finite angular size, however small, and for such a star the increase in brightness, though it might be large, would have a limit.

If a bright star passed in front of a fainter one, its own light would drown out the effect; but if one of the faint red stars, which are really more abundant than any other sort, should get directly



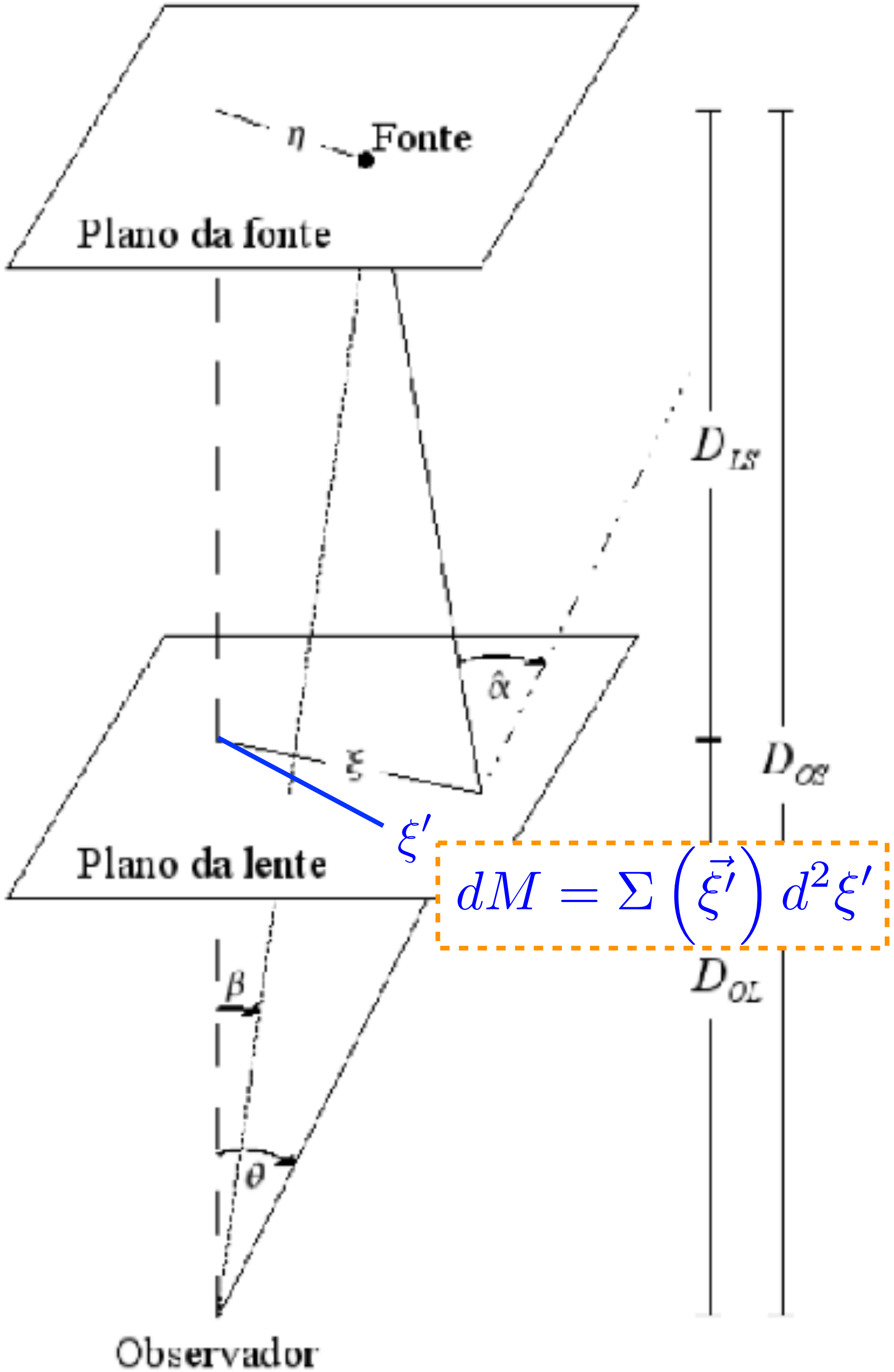
My hearty thanks are due to Professor Einstein, who permitted me to see the manuscript of his note before its publication.—*Princeton University Observatory, December 2, 1936.*

is, a million miles or so—from the line through the center of the two stars. The

A Bit of theory/modelling:

The realm of extended Lenses and extended sources

Extended Lenses



Point mass $\hat{\vec{\alpha}} = \dfrac{4GM}{c^2\xi}$

Surface mass density

$$\Sigma\left(\vec{\xi}\right) = \int_0^\infty dz\rho\left(\vec{\xi}, z\right)$$

Contribution of the area element

$$d\hat{\vec{\alpha}} = \frac{4G}{c^2}\Sigma\left(\vec{\xi'}\right) d^2\xi' \frac{\vec{\xi} - \vec{\xi'}}{|\vec{\xi} - \vec{\xi'}|^2}$$

Deflection angle

$$\hat{\vec{\alpha}} = \frac{4G}{c^2}\int d^2\xi'\Sigma\left(\vec{\xi'}\right) \frac{\vec{\xi} - \vec{\xi'}}{|\vec{\xi} - \vec{\xi'}|^2}$$

Extended Lenses

Projected potential $\psi\left(\vec{\xi}\right)=\int dz\varphi(\vec{\xi},z)$

Deflection angle $\hat{\vec{\alpha}}=\frac{2}{c^2}\vec{\nabla}_\xi\psi\left(\vec{\xi}\right)$

Reduced deflection angle

$$\vec{\alpha}=\hat{\vec{\alpha}}\frac{D_{LS}}{D_{OS}}=\frac{2}{c^2}\frac{D_{LS}}{D_{OS}D_{OL}}\vec{\nabla}_\theta\psi\left(\vec{\theta}\right)$$

Lensing potential $\Psi(\vec{x})\equiv\frac{2}{c^2}\frac{D_{LS}}{D_{OS}D_{OL}}\psi$

Lens equation $\vec{\beta}=\vec{\theta}-\vec{\alpha}\left(\vec{\theta}\right)=\boxed{\vec{\theta}-\vec{\nabla}_\theta\Psi\left(\vec{\theta}\right)}$

Lens equation with axial symmetry

Deflection angle $$\hat{\vec{\alpha}} = \frac{4GM(\xi)}{c^2\xi}\hat{\xi}$$

Lens equation $$\vec{\beta} = \vec{\theta} - \hat{\vec{\alpha}}\frac{D_{LS}}{D_{OS}} = \vec{\theta} - \frac{4GM(\theta)}{c^2\theta^2 D_{OL}}\vec{\theta}\frac{D_{LS}}{D_{OS}}$$

$$\vec{\beta} = \left(1 - \frac{4GM(\theta)D_{LS}}{c^2 D_{OL} D_{OS}}\frac{1}{\theta^2}\right)\vec{\theta}$$

Einstein angle $\left(\vec{\beta} = 0\right)$

$$\theta_E = \sqrt{\frac{D_{LS}}{D_{OS}D_{OL}}\frac{4GM\left(\theta_E\right)}{c^2}}$$

⇨ Mass estimate at $\theta < \theta_E$

Singular Isothermal Sphere

Singular Isothermal Sphere

$$\rho(r) = \frac{\sigma_v^2}{2\pi G r^2}$$

Surface density (projected)

$$\Sigma(\xi) = \frac{\sigma_v^2}{2\pi G} 2 \int_0^\infty \frac{dz}{\xi^2 + z^2} = \frac{\sigma_v^2}{2G\xi}$$

Mass within a radius ξ

$$M(\xi) = \int_0^\xi \Sigma(\xi') \, 2\pi\xi' \, \mathrm{d}\xi' = \sigma_v^2 \frac{\pi}{G} \xi$$

Deflection angle

$$\Rightarrow \quad \hat{\vec{\alpha}} = \frac{4GM(\xi)}{c^2 \xi} \hat{\xi} = 4\pi \left(\frac{\sigma_v}{c}\right)^2 \hat{\xi}$$

constant!

Lens Equation

$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}} \qquad\qquad \hat{\vec{\alpha}} = 4\pi \left(\frac{\sigma_v}{c}\right)^2 \hat{\xi}$$

$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}} = \boxed{4\pi \left(\frac{\sigma_v}{c}\right)^2 \frac{D_{LS}}{D_{OS}}} \hat{\xi} = \theta_E \, \hat{\xi}$$

$$\vec{\beta} = \vec{\theta} - \theta_E \, \hat{\theta} = \left(1 - \frac{\theta_E}{\theta}\right) \vec{\theta}$$

$$\Rightarrow \quad \beta = \left| 1 - \frac{\theta_E}{\theta} \right| \theta$$

Solutions: 1) if $1 - \dfrac{\theta_E}{\theta} > 0$

then $\beta = \left(1 - \dfrac{\theta_E}{\theta}\right) \theta = \theta - \theta_E \Rightarrow \boxed{\theta = \beta + \theta_E}$

Lens Equation

$$\beta = \left|1 - \frac{\theta_E}{\theta}\right| \theta$$

Solutions: I) if $1 - \frac{\theta_E}{\theta} > 0$

then $\beta = \left(1 - \frac{\theta_E}{\theta}\right)\theta = \theta - \theta_E \Rightarrow \boxed{\theta = \beta + \theta_E}$

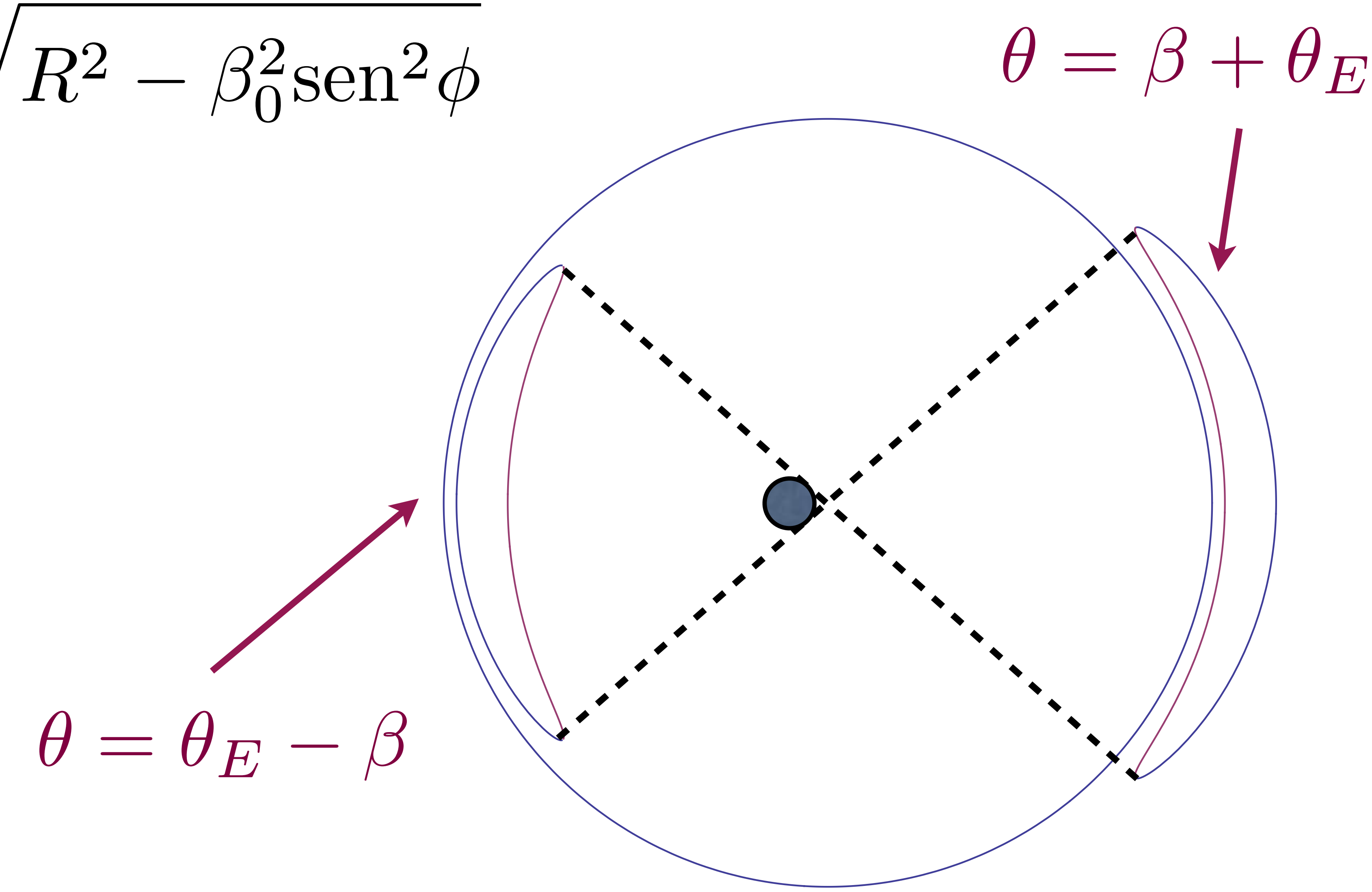
II) if $1 - \frac{\theta_E}{\theta} < 0$

then $\beta = -\left(1 - \frac{\theta_E}{\theta}\right)\theta = \theta_E - \theta \Rightarrow \boxed{\theta = \theta_E - \beta}$

Circular source

Circle in the source plane: $\left|\vec{\beta} - \vec{\beta}_0\right| = R$

$$\beta = \beta_0 \cos\phi \pm \sqrt{R^2 - \beta_0^2 \mathrm{sen}^2\phi}$$



in general $\beta = \beta_0 \cos(\phi - \phi_0) \pm \sqrt{R^2 - \beta_0^2 \mathrm{sen}^2(\phi - \phi_0)}$

Other lens models

Implementes in *gravlens* and several codes (+ Sérsic, Einasto, etc.)

Model	N_r	Density $\rho(r)$	Surface Density $\kappa(r)$
Point mass	0	$\delta(\mathbf{x})$	$\delta(\mathbf{x})$
Power law or α-models	2	$\left(s^2+r^2\right)^{(\alpha-3)/2}$	$\left(s^2+r^2\right)^{(\alpha-2)/2}$
Isothermal ($\alpha=1$)	1	$\left(s^2+r^2\right)^{-1}$	$\left(s^2+r^2\right)^{-1/2}$
$\alpha=-1$	1	$\left(s^2+r^2\right)^{-2}$	$\left(s^2+r^2\right)^{-3/2}$
Pseudo-Jaffe	2	$\left(s^2+r^2\right)^{-1}\left(a^2+r^2\right)^{-1}$	$\left(s^2+r^2\right)^{-1/2}-\left(a^2+r^2\right)^{-1/2}$
King (approximate)	1	$\cdots$	$2.12\left(0.75r_s^2+r^2\right)^{-1/2}$ $-1.75\left(2.99r_s^2+r^2\right)^{-1/2}$
de Vaucouleurs	1	$\cdots$	$\exp\left[-7.67(r/R_e)^{1/4}\right]$
Hernquist	1	$r^{-1}\left(r_s+r\right)^{-3}$	see eq. (47)
NFW	1	$r^{-1}\left(r_s+r\right)^{-2}$	see eq. (53)
Cuspy NFW	2	$r^{-\gamma}\left(r_s+r\right)^{\gamma-3}$	see eq. (57)
Cusp	3	$r^{-\gamma}\left(r_s^2+r^2\right)^{(\gamma-n)/2}$	see eq. (64)
Nuker	4	$\cdots$	see eq. (71)
Exponential disk	1	$\cdots$	$\exp[-r/R_d]$
Kuzmin disk	1	$\cdots$	$\left(r_s^2+r^2\right)^{-3/2}$

Elliptical models

Replace $\theta \to \sqrt{q_1\theta_1^2 + q_2\theta_2^2}$

- in the potential (pseudo-elliptic models)
- in the projected density (elliptical models)

Example: pseudo-elliptic isothermal model with a core

$$\Psi(\theta_1, \theta_2) = \frac{\Psi_0}{\theta_c}\sqrt{(1-\epsilon)\theta_1^2 + (1+\epsilon)\theta_2^2 + \theta_c^2}$$

Model for the source

Surface brightness distribution of the source

Example: Sérsic profile

$$\boxed{I(R) = I_0 \exp\left\{-b_n \left(\frac{R}{R_e}\right)^{1/n}\right\}}$$

Elliptical brightness distribution

$$\begin{aligned} R^2 = {} & (1-\varepsilon_S)[(\beta_1 - S_1)\cos\phi_e + (\beta_2 - S_2)\sin\phi_e]^2 \\ & +(1+\varepsilon_S)[(\beta_2 - S_2)\cos\phi_e - (\beta_1 - S_1)\sin\phi_e]^2 \end{aligned}$$

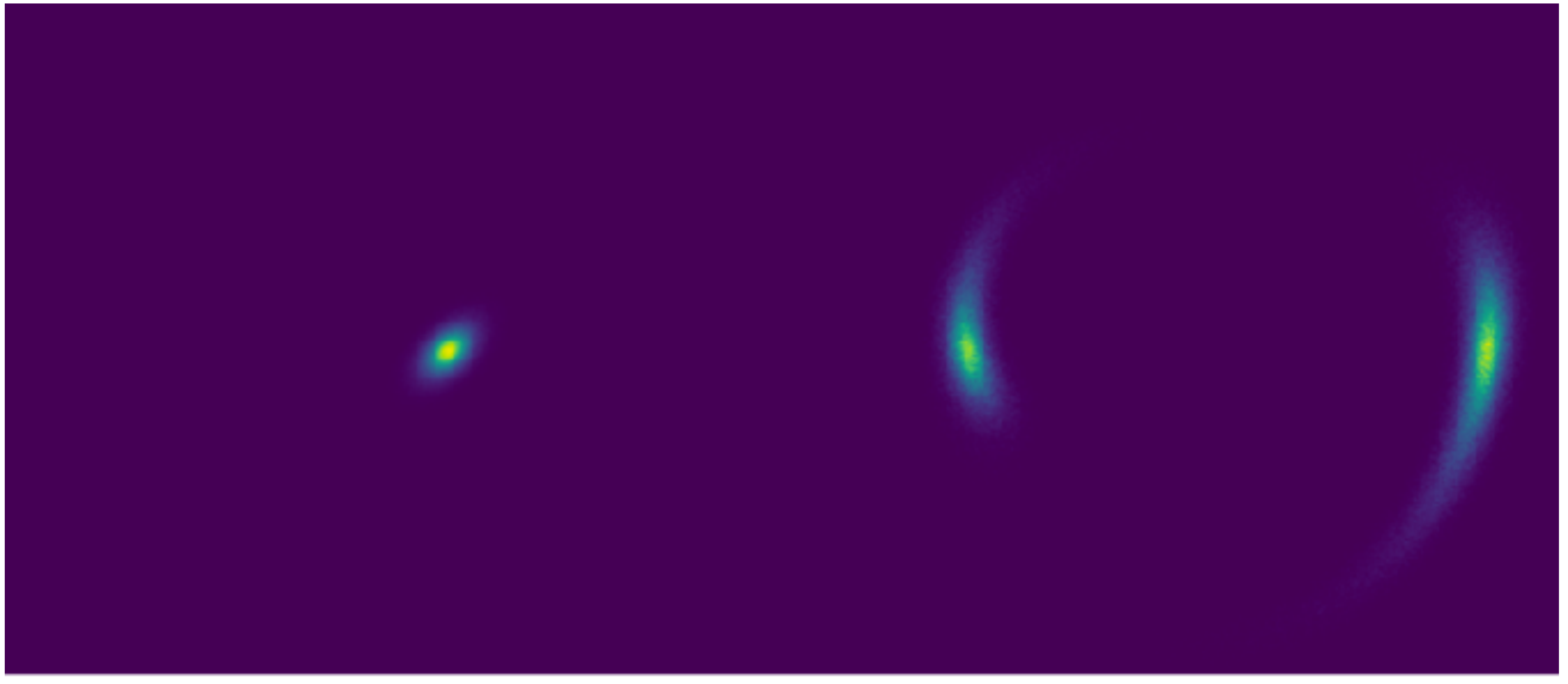
Simulating Strong Lensing Images

Map the brightness distribution of the source to the lens plane

$$I\left(\vec{\theta}\right) = I\left(\vec{\beta}\left(\vec{\theta}\right)\right)$$

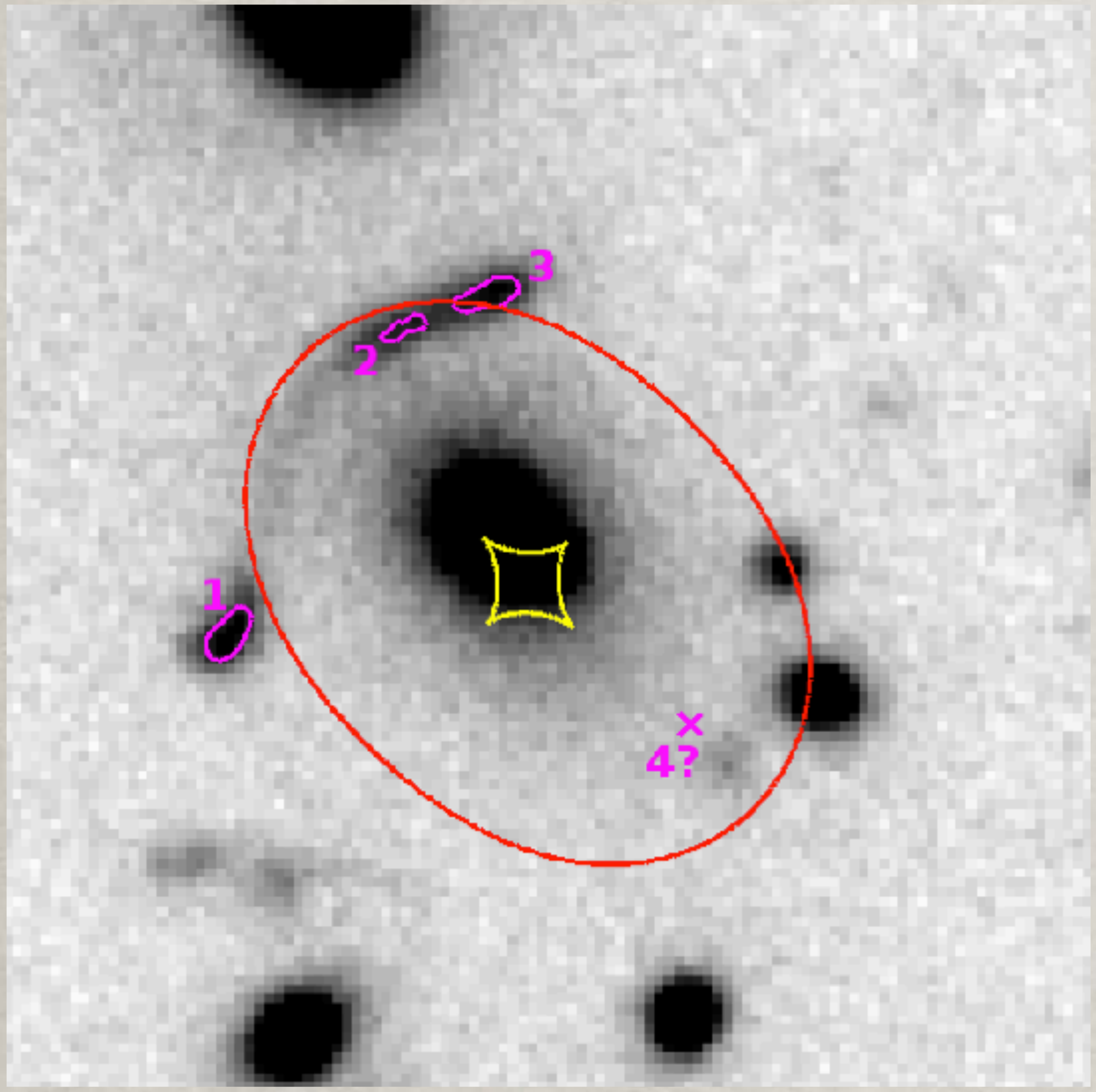
Use the lens equation (no need to solve it!)

Add PSF and noise




AddArcs v2.0
SIE
Let's see this in practice
with some code

Inverse Modeling: Mapping the Mass



Use systems of multiple images to determine the lensing potential

$$\chi^2_{\text{lente}} := \sum_i \left(\frac{\vec{\theta}_i^{\text{obs}} - \vec{\theta}^{\text{mod}}\left(\vec{\beta}, \vec{\Pi}\right)}{\sigma_i^{\text{obs}}} \right)^2$$

Multiple image positions

Error on image positions

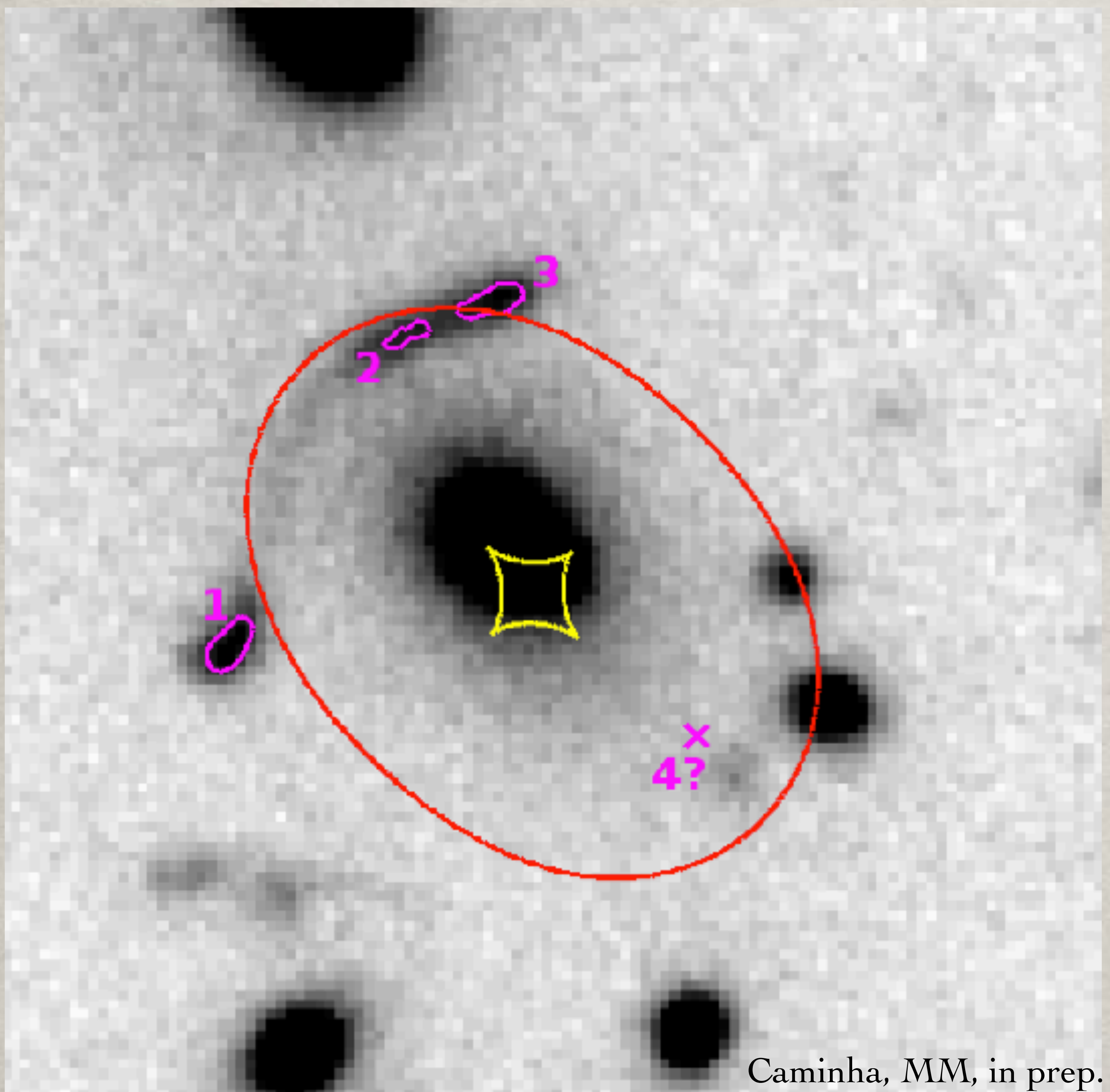
Methods: parametric (often "mass traces light"), free form

The more multiple images, the more constrains

Cluster x Galaxy scales

- Combination with independent mass constraints (e.g., x-ray, Sunyaev Zel'dovich, velocity dispersions) yields limits on cosmology or gravity

Inverse Modeling for system SOGRAS0041-0043



Caminha, MM, in prep.

$$2.35^{+0.03}_{-0.14} \times 10^{14} M_{\odot}$$

Modelling with lenstool (Jullo, Kneib)

Fit 1: 3 images

Fit 2: 4 images

	Fit 1	Fit 2
$\sigma_v[km/s]$	622^{-11}_{-13}	642^{+3}_{-3}
$\theta_{or}[°]$	$135.2^{+0.7}_{-0.8}$	$135.2^{+1.5}_{-1.3}$
$x_{\rm lente}['']$	$0.50^{+0.2}_{-0.2}$	$0.50^{+0.05}_{-0.06}$
$y_{\rm lente}['']$	$-0.76^{+0.17}_{-0.15}$	$-0.86^{+0.06}_{-0.03}$
ε	—	$0.13^{+0.02}_{-0.03}$

~ 0.9'' displacement between central galaxy and center of mass distribution (*Zitrin et al. 2012*)

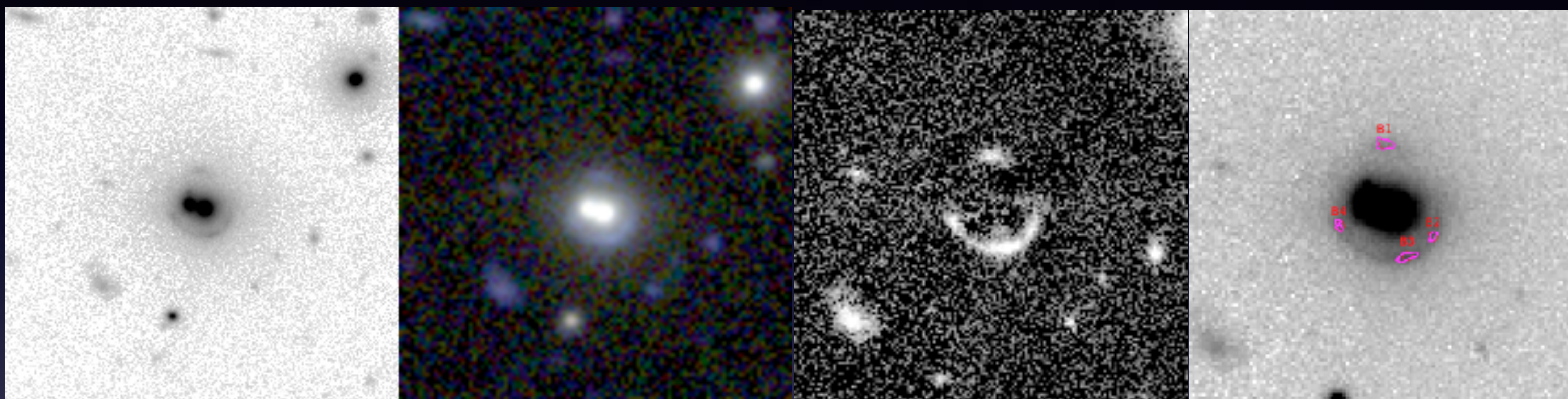
Error estimate from simulations (Caminha et al. in prep.):

~ 8% bias in mass

~ 5% statistical errors

Modeling the full light distribution of the images

CS82SL01:36:39+00:08:18

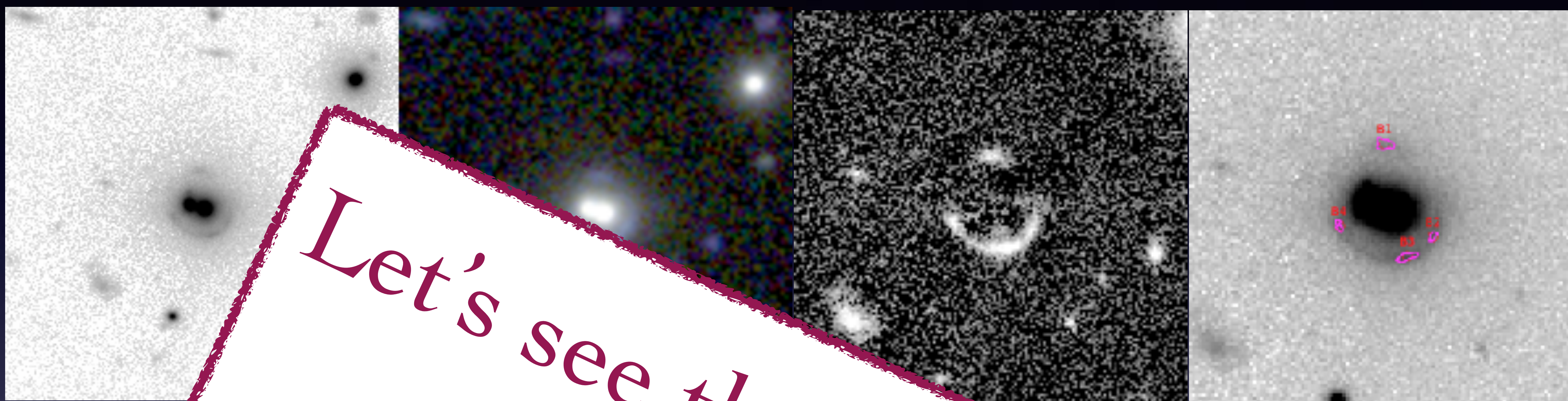


Anna Niemiec

- Instead of peak/multiple images, use the full information of the images
- Allows one to reconstruct source properties (often parametrically)
- Remove contamination from lens galaxy (with galfit) and mask other objects

Modeling the full light distribution of the images

CS82SL01:36:39+00:08:18



Anna Niemiec

Let's see this in practice with some code

- Instead o… he full information o…
- Allows one to reconstru… es (often parametrically)
- Remove contamination from lens galaxy (with galfit) and mask other objects

Example Applications:

- Finding substructure: Dark Matter
- Einstein rings: testing modified gravity
- Galaxy clusters: probing the background cosmology

Strong lensing and substructures

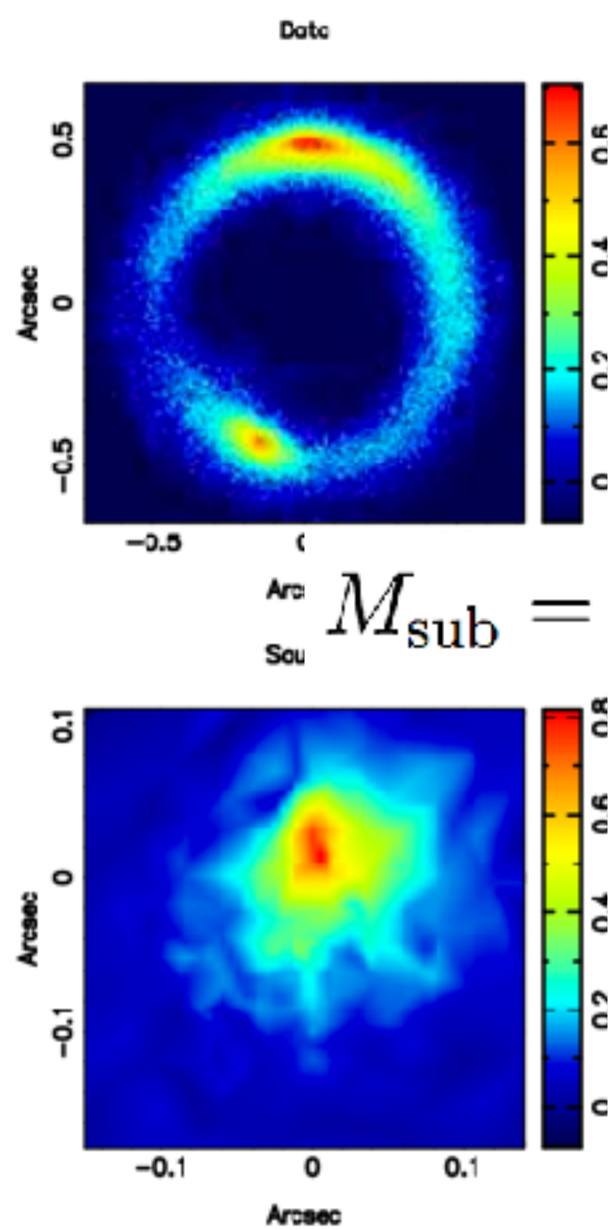
High sensitivity to small perturbations due to the caustic structure

MENU nature

Gravitational detection of a low-mass dark satellite galaxy at cosmological distance

S. Vegetti, D. J. Lagattuta, J. P. McKean,

Nature **481**, 341–343 (19 January 2012)



- “Aside from direct or indirect detection of the dark matter particles themselves, Einstein ring systems currently offer the best astrophysical test of the nature of the dark matter” (Li et al. 2016)
- measurements of approximately 100 strong lens systems with a detection limit of $M_{low} = 10^7\ h^{-1}\ M_\odot$ would clearly distinguish CDM from WDM in the case where this consists of 7 keV sterile neutrinos

One gravitational potential or two? Forecasts and tests

By Edmund Bertschinger*

Phil. Trans. R. Soc. A (2011) **369**, 4947–4961
doi:10.1098/rsta.2011.0369

conformal newtonian metric (choices and assumptions):

$$ds^2 = a^2(\tau)[-(1+2\Phi)\,d\tau^2 + (1-2\Psi)\gamma_{ij}\,dx^i\,dx^j]$$

for General Relativity (for *standard* matter components) in general

$$\Phi = \Psi$$

slip parameter

$$\boxed{\gamma = \frac{\Phi}{\Psi}}$$

$\gamma = 1$ compatible with GR

$\gamma \neq 1$ GR ruled out

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$$\mathrm{d}s^2 = a^2(\tau)[-(1+2\Phi)\,\mathrm{d}\tau^2 + (1-2\Psi)\gamma_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j]$$

geodesics

$$\frac{1}{a}\frac{\mathrm{d}(a\boldsymbol{v})}{\mathrm{d}\tau} = -\boldsymbol{\nabla}\Phi, \quad v^2 \ll 1 \text{ (CDM)} \longrightarrow \text{Jean's equation}$$

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}\tau} = -\boldsymbol{\nabla}_\perp(\Phi+\Psi), \quad v^2 = 1 \text{ (photons)} \longrightarrow \text{lensing}$$

kinematics: Φ

deflection angle: $\Phi + \Psi$

on ~ 100 kph galaxy scales

from galaxy velocity dispersion

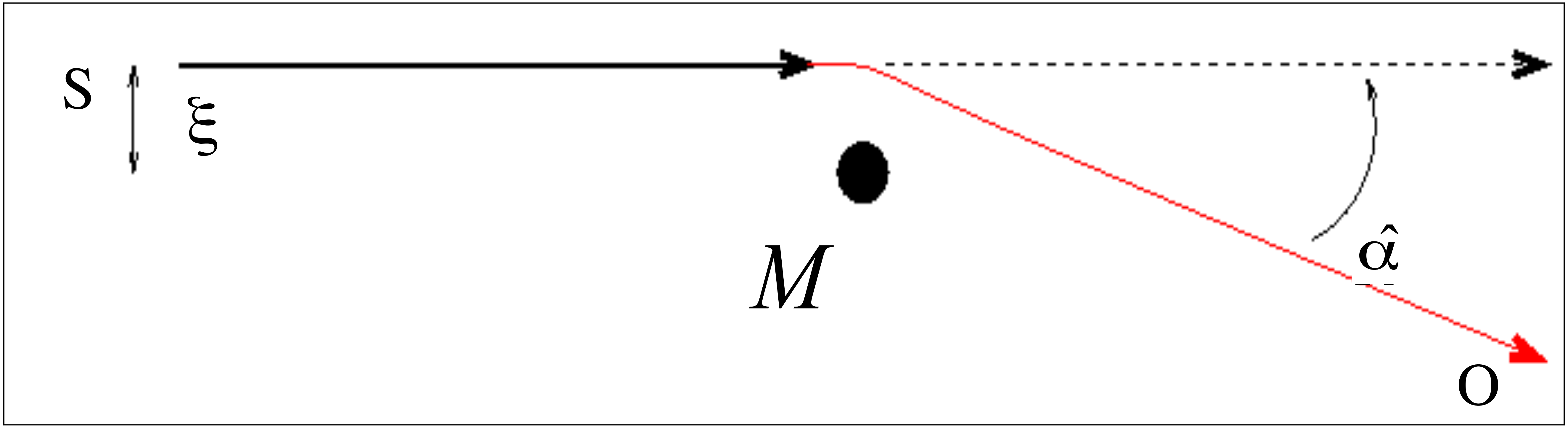
from strong lensing

Bending of light by gravity

Null geodesic,
Fermat principle

$$ds^2 = \left(1 + \frac{2\phi}{c^2}\right) c^2 dt^2 - \left(1 - \frac{2\phi}{c^2}\right) d\sigma^2$$

$$\frac{d\sigma}{dt} := c' = \sqrt{\frac{1 + 2\phi/c^2}{1 - 2\phi/c^2}} \simeq c\left(1 + \frac{2\phi}{c^2}\right)$$

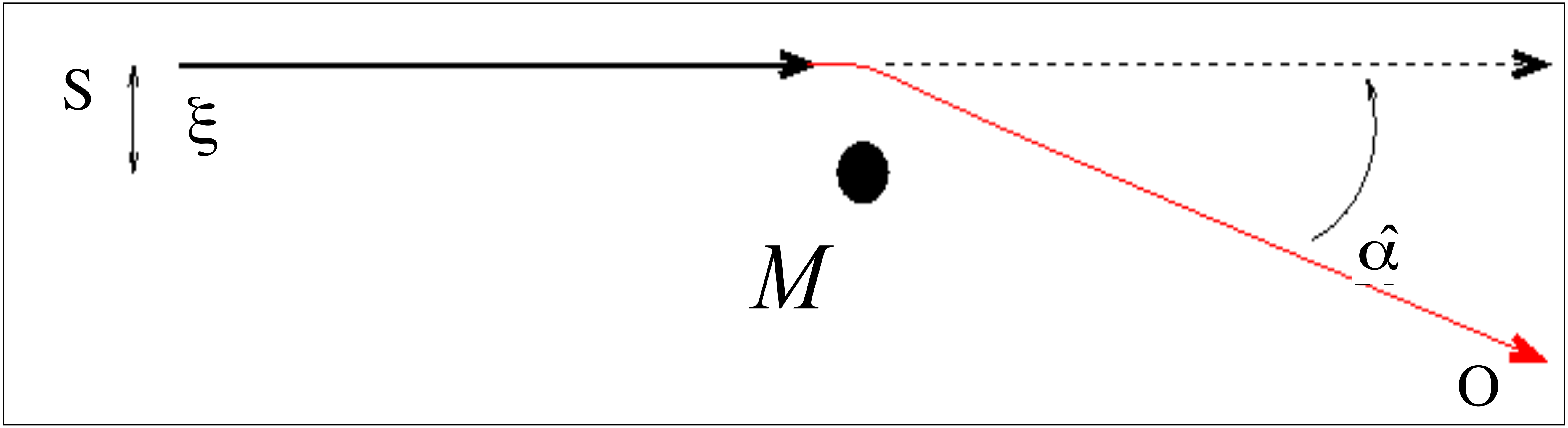


Bending of light by (modified) gravity

Null geodesic,
Fermat principle

$$ds^2 = \left(1 + \frac{2\psi}{c^2}\right) c^2 dt^2 - \left(1 - \frac{2\phi}{c^2}\right) d\sigma^2$$

peculiar gravitational potentials
(in GR $\psi = \phi$)

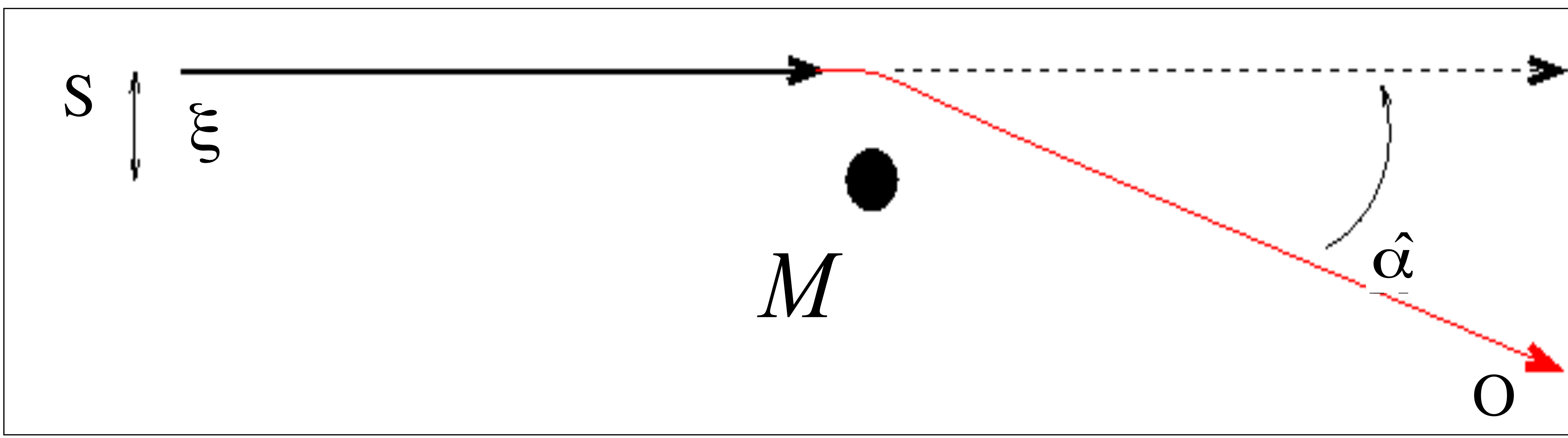


Bending of light by (modified) gravity

Null geodesic,
Fermat principle

$$ds^2 = \left(1+\frac{2\psi}{c^2}\right)c^2dt^2 - \left(1-\frac{2\phi}{c^2}\right)d\sigma^2$$

$$\frac{d\sigma}{dt} = c' = c\sqrt{\frac{1+\frac{2\psi}{c^2}}{1-\frac{2\phi}{c^2}}} \simeq c\left(1+\frac{\psi+\phi}{c^2}\right) = c\left(1+\frac{2\phi}{c^2}\left[\frac{1+\gamma}{2}\right]\right)$$



Dynamical mass obtained from

$$\nabla^2\psi = 4\pi G\rho$$

Use combination of lensing + dynamics to test gravity

Basic picture

Singular Isothermal Sphere profile: $\rho(r) = \frac{\sigma_v^2}{2\pi G r^2}$

slip parameter

$$\gamma = \frac{\Phi}{\Psi}$$

Observed stellar velocity dispersion $\sigma_{\rm obs}$ probes Φ

Lensing yields $\sigma^2_{\rm lens}$ from $\Phi + \Psi$:

$$\sigma^2_{\rm lens} = \left(\frac{1+\gamma}{2}\right)\sigma^2_{\rm obs}$$

Real world:

- Non isothermal elliptical models
- Anisotropic velocity dispersion (assumptions)
- Seeing and aperture corrections

on ~ 100 kph
galaxy scales

kinematics: Φ

from galaxy
velocity dispersion

deflection angle: $\Phi + \Psi$

from strong lensing

Galaxy kinematics

The Jeans equation described the motion of a collection of stars. In spherical symmetry,

$$\frac{d}{dr}\left(\nu(r)\sigma_r^2(r)\right) + \frac{2\beta(r)}{r}\nu(r)\sigma_r^2(r) = \nu(r)\frac{d\Phi}{dr}, \tag{8}$$

where

$$\frac{d\Phi}{dr} = \frac{GM(r)}{r^2}, \quad \beta(r) := 1 - \frac{\sigma_t^2}{\sigma_r^2}, \tag{9}$$

where $M(r)$ is the total mass inside a sphere of radius r.

For $\beta =$ constant,

$$\sigma_r^2(r) = \frac{G}{r^{2\beta}\nu(r)} \int_r^\infty (r')^{2\beta-2}\nu(r')M(r')dr'. \tag{10}$$

The total mass contained within a sphere with radius r is,

$$M(r) = 4\pi \int_0^r (r')^2\rho(r')dr' = 4\pi \frac{r^{(3-\alpha)}}{3-\alpha}\frac{\rho_0}{r_0^{-\alpha}} \tag{11}$$

The actual velocity dispersion measured by observations is given by,

$$\bar{\sigma}_*^2 := \frac{\int_0^\infty dR R w(R) \int_{-\infty}^\infty dz \nu(r) \left(1 - \beta \frac{R^2}{r^2}\right) \sigma_r^2(r)}{\int_0^\infty dR R w(R) \int_{-\infty}^\infty dz \nu(r)}, \tag{19}$$

where $w(R)$ is the convolution of the atmospheric seeing σ_{atm} and the fiber aperture θ_{ap}.

$$\begin{aligned} \bar{\sigma}_*^2 = & \left[\frac{2}{1+\gamma} \frac{c^2}{4} \frac{D_S}{D_{LS}} \theta_E\right] \frac{2}{\sqrt{\pi}} \frac{(2\tilde{\sigma}_{\mathsf{atm}}^2/\theta_E^2)^{1-\alpha/2}}{\xi - 2\beta} \\ & \times \left[\frac{\lambda(\xi) - \beta\lambda(\xi+2)}{\lambda(\alpha)\lambda(\delta)}\right] \frac{\Gamma(\frac{3-\xi}{2})}{\Gamma(\frac{3-\delta}{2})}, \end{aligned} \tag{20}$$

where

$$\tilde{\sigma}_{\mathsf{atm}} \approx \sigma_{\mathsf{atm}} \sqrt{1 + \chi^2/4 + \chi^4/40}, \quad \chi = \theta_{\mathsf{ap}}/\sigma_{\mathsf{atm}}. \tag{21}$$

Archival Data

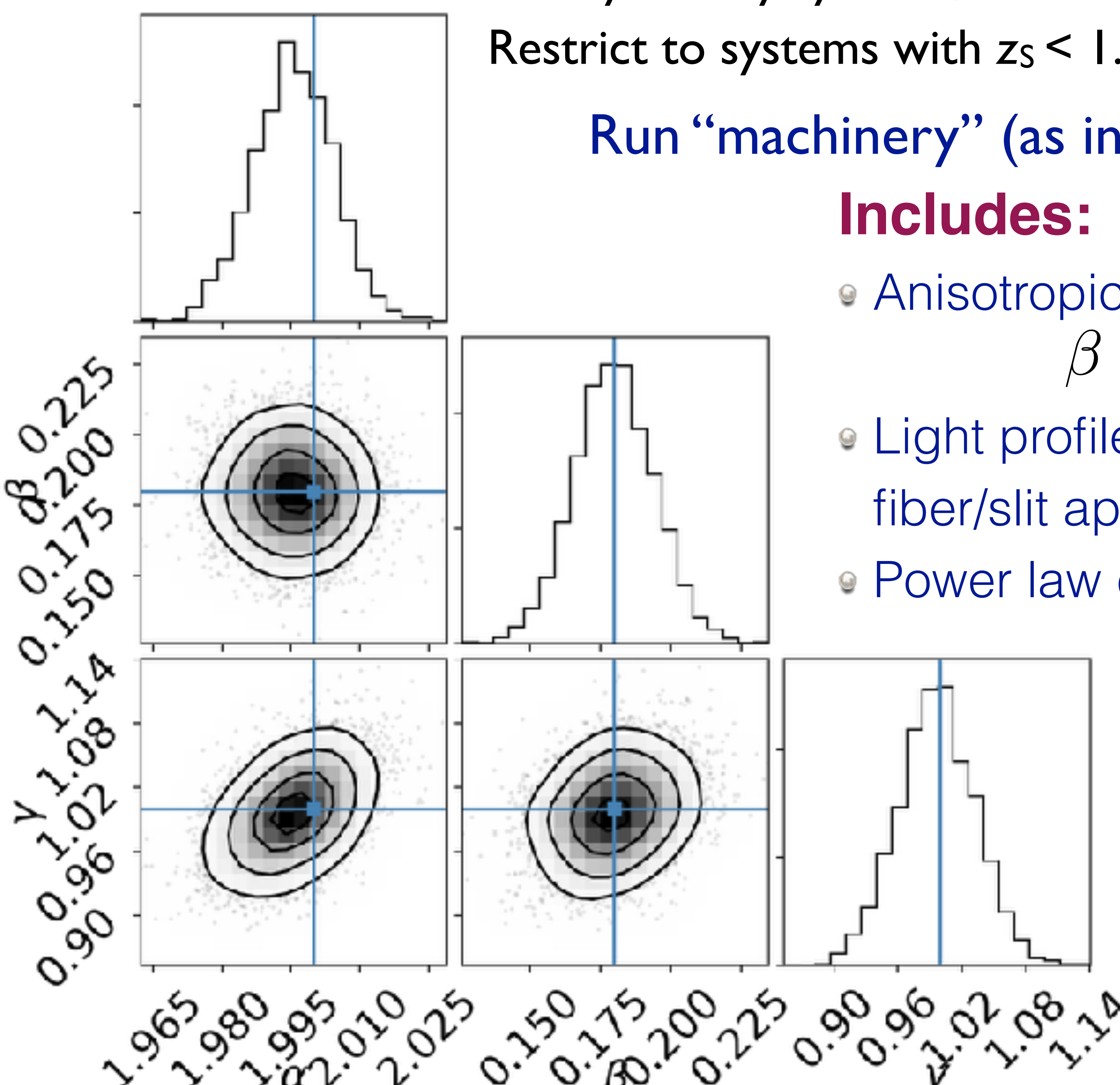
From (beta, public) “masterlens” catalog of 8577 SL systems

Galaxy-Galaxy systems, with known z_L, z_S , σ_v and lens modelling

Restrict to systems with z_S < 1.5 : total of 110 systems

Run “machinery” (as in Schwab+2010, Cao+2017)

Crisnejo et al.



Includes:

- Anisotropic velocity dispersion

$$\beta = 1 - \sigma_t^2/\sigma_r^2$$

- Light profiles, seeing and fiber/slit aperture effects $\nu(r) = \nu_0 \left(\frac{r}{r_0}\right)^{-\delta}$
- Power law density profile $\rho(r) \propto r^{-\alpha}$
- Priors on

$$\beta = 0.180^{+0.014}_{-0.014}$$

$$\alpha = 1.995^{+0.009}_{-0.009}$$

Slip parameter

$$\gamma = 1.001^{+0.022}_{-0.023}$$

Archival Data

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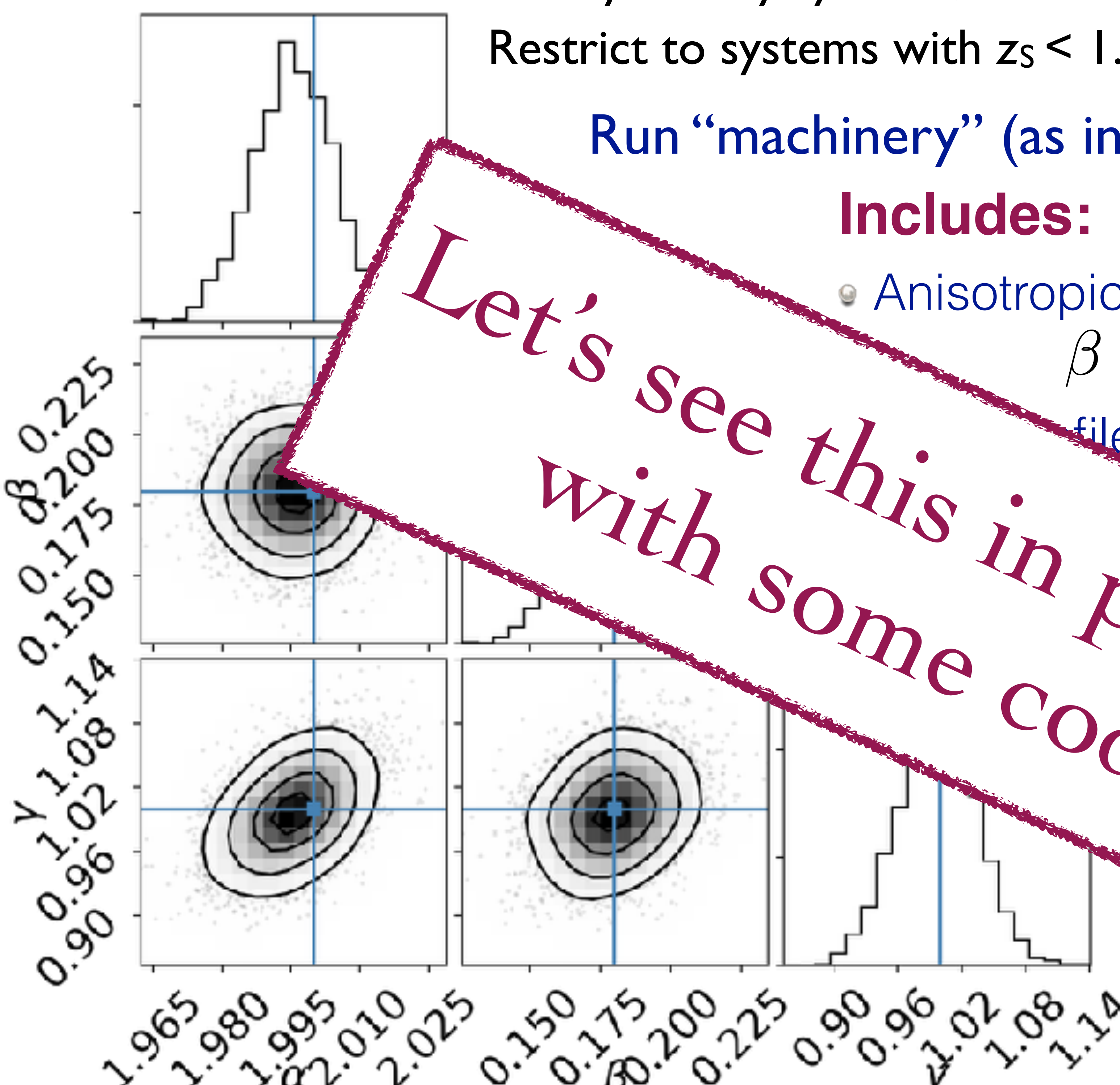


Run "machinery" (as in Schwab+2010, Cao+2017)

Includes:

- Anisotropic velocity dispersion

$$\beta = 1 - \sigma_t^2 / \sigma_r^2$$

files, seeing and $\nu(r) = \nu_0 \left(\frac{r}{r_0}\right)^{-\delta}$

e effects

file $\rho(r) \propto r^{-\alpha}$

$= 0.180^{+0.014}_{-0.014}$

$\alpha = 1.995^{+0.009}_{-0.009}$

parameter

$\gamma = 1.001^{+0.022}_{-0.023}$

Let's see this in practice with some code

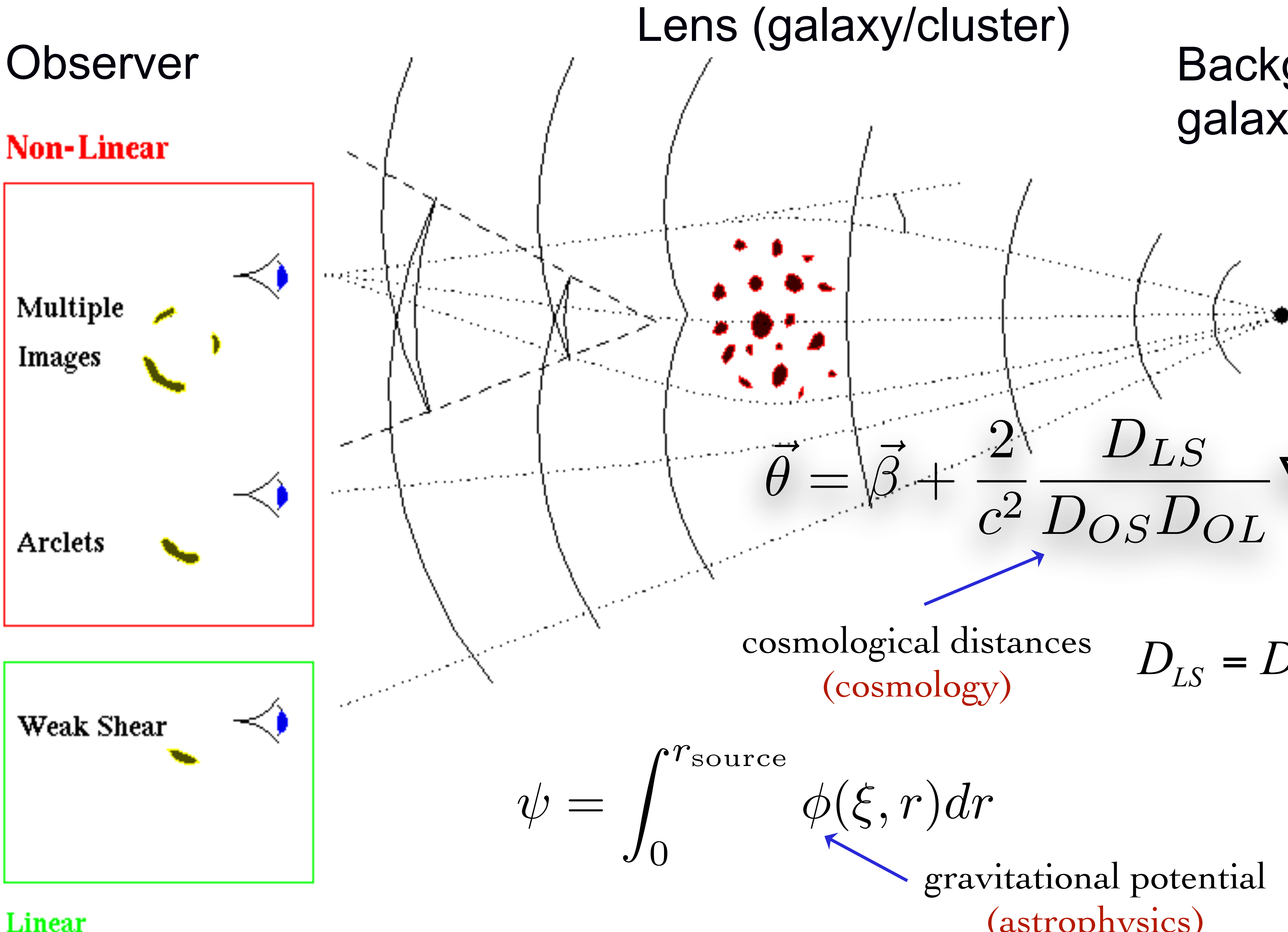
Part IV
Weak Lensing

Martín Makler

ICAS/IFICI/CONICET&UNSAM & CBPF

+ Renan Alves, João França, Elizabeth Gonzalez,
Giulya Souza, Eduardo Valadão, Anibal Varela

Weak and Strong Lensing Effects



$$\vec{\theta} = \vec{\beta} + \frac{2}{c^2} \frac{D_{LS}}{D_{OS} D_{OL}} \nabla_\theta \psi \left(\vec{\theta}\right)$$

cosmological distances
(cosmology)

$$D_{LS} = D_A \left(z_L, z_S\right)\ldots$$

$$\psi = \int_0^{r_{\mathrm{source}}} \phi(\xi, r) dr$$

gravitational potential
(astrophysics)

Lens Mapping

- mapping image → source

$$\frac{\partial \beta_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial^2 \Psi(\vec{\theta})}{\partial \theta_i \partial \theta_j}$$

- single plane

$$\Psi = \frac{2}{c^2} \frac{D_{LS}}{D_{OS} D_{OL}} \int \phi(\xi, z)\, dz$$

gravitacional potential

cosmological distances

$$D_{LS} = D_A(z_L, z_S) \ldots$$

- autovalores:

$$\mu_1 = \frac{1}{1 - \kappa + \gamma}, \mu_2 = \frac{1}{1 - \kappa - \gamma}$$

- local magnifications and axis ratio:

$$\mu = \mu_1 \, \mu_2 \qquad r = \left| \frac{\mu_1}{\mu_2} \right|$$

- critical surface density

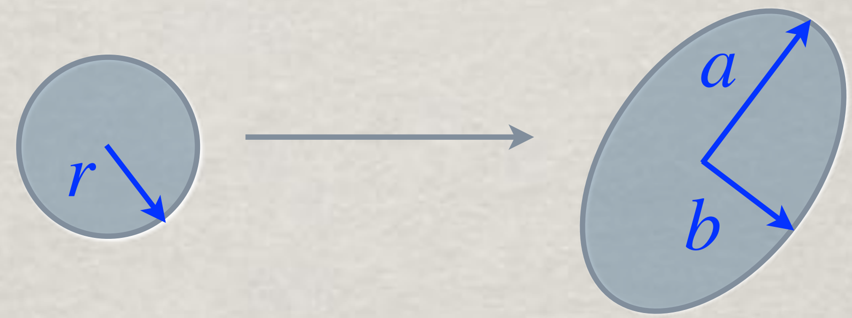
$$\Sigma_{\text{crit}} = \frac{c^2}{4\pi G} \frac{D_{OS}}{D_{OL} D_{LS}}$$

- convergence

$$\kappa = \frac{\Sigma}{\Sigma_{\text{crit}}}$$

Linear Mapping

- Circular sources



$$a = \left(\frac{1}{1-\kappa-\gamma}\right) r$$

$$b = \left(\frac{1}{1-\kappa+\gamma}\right) r$$

- Magnifications

$$\mu = \frac{A_{\text{imagem}}}{A_{\text{fonte}}} = \left[(1-\kappa)^2 - \gamma^2\right]^{-1}$$

- Ellipticity

$$\boxed{\epsilon := \frac{a-b}{a+b} = \frac{\gamma}{1-\kappa} =: g}$$

Real sources

- Center of the object (image)

$$\bar{\theta}_i = \frac{\int \mathrm{d}^2\theta q_I \left[I\left(\theta\right)\right] \theta_i}{\int \mathrm{d}^2\theta q_I \left[I\left(\theta\right)\right]}$$

- Second order momenta

$$Q_{ij} = \frac{\int d^2\theta q_I[I(\theta)](\theta_i - \bar{\theta}_i)(\theta_j - \bar{\theta}_j)}{\int d^2\theta q_I[I(\theta)]}$$

- Area

$$\Omega = \left(Q_{11}Q_{22} - Q_{12}^2\right)^{1/2}$$

- Ellipticity

$$\epsilon := \frac{Q_{11} - Q_{22} + 2\, i\, Q_{12}}{Q_{11} + Q_{22} + 2\left(Q_{11}Q_{22} - Q_{12}^2\right)^{1/2}}$$

Real sources

- Second order momenta

$$Q_{ij} = \frac{\int d^2\theta q_I[I(\theta)](\theta_i - \bar{\theta}_i)(\theta_j - \bar{\theta}_j)}{\int d^2\theta q_I[I(\theta)]}$$

- Area

$$\Omega = \left(Q_{11}Q_{22} - Q_{12}^2\right)^{1/2}$$

- Ellipticity

$$\epsilon := \frac{Q_{11} - Q_{22} + 2\,i\,Q_{12}}{Q_{11} + Q_{22} + 2\left(Q_{11}Q_{22} - Q_{12}^2\right)^{1/2}}$$

- Linear distortion

$$\epsilon_S = \frac{\epsilon_I - g}{1 - g^*\epsilon_I} \qquad g_i := \frac{\gamma_i(\theta)}{1 - \kappa(\theta)} \qquad |g| \leq 1$$

Weak Lensing

- Slight shear in background galaxies (change in axis + size)
- Weak lensing regime

$$\epsilon = \epsilon_I = \epsilon_S + g$$

- "weak lensing fundamental theorem":

$$\boxed{\langle \epsilon \rangle = g}$$

- but

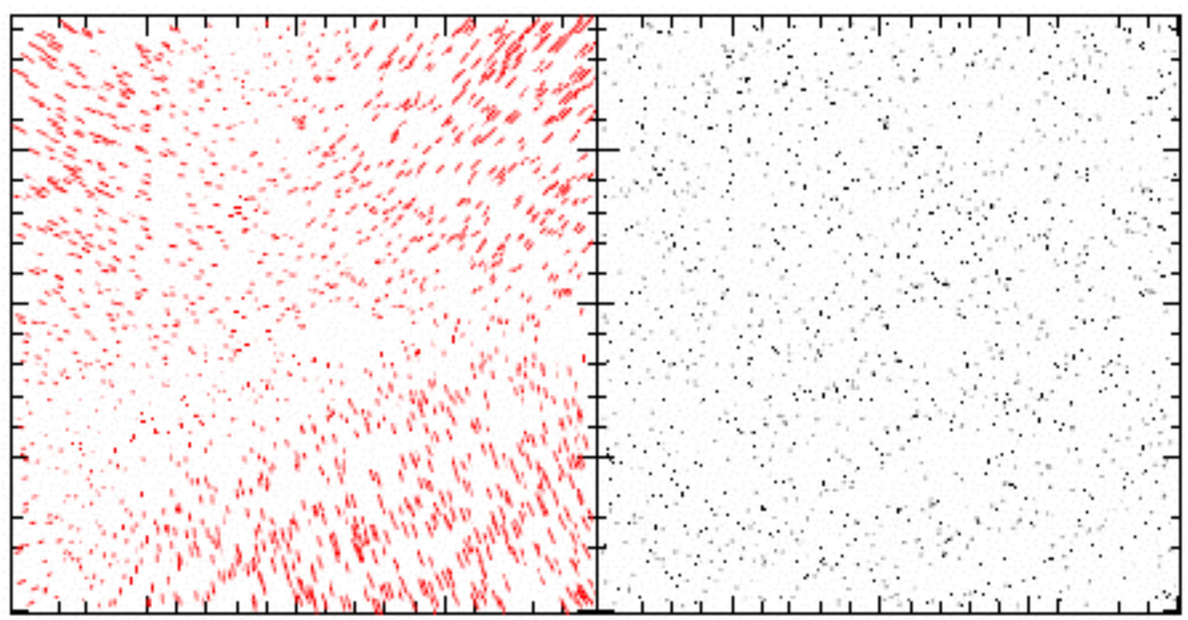
$$g \ll \epsilon$$

- observational, instrumental, computational and theoretical challenges!

Technical challenges

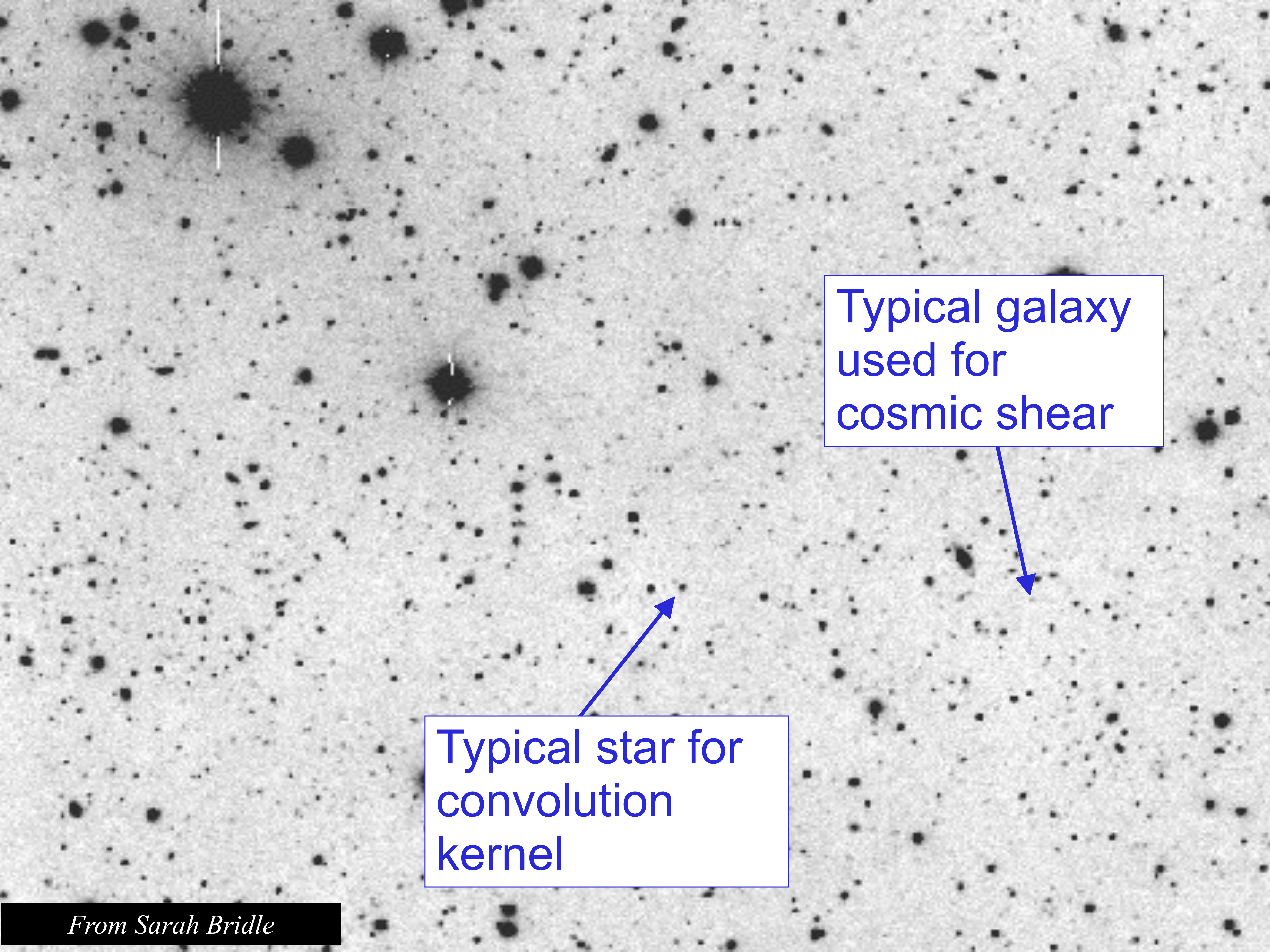
Sarah Bridle

- Model the PSF distortion (many sources)
  - measure a large number of stars in the field



CFHTLS Deep 1
Gavazzi, Soucail 2006

- Difficulties: saturated stars, charge transfer efficiency, halos, tracking...


Typical galaxy used for cosmic shear
Typical star for convolution kernel
From Sarah Bridle

Measuring weak lensing

- Signal $\langle \varepsilon_I \rangle = \gamma \qquad (\varepsilon_I = \varepsilon_S + \gamma)$
- Noise $\sigma_\varepsilon = \left\langle \left| \varepsilon_S{}^2 \right| \right\rangle^{1/2} \sim 0.3 \gg \gamma$
- Win over the noise by averaging over a large number of galaxies

Regime	γ	$\gamma/\sigma_\varepsilon$	$N_{\rm gal}$ for S/N~ 1
weak lensing by clusters	0.03	0.1	10^2
galaxy-galaxy lensing	0.003	0.01	10^4
cosmic shear	0.001	0.003	10^5

Much more galaxies for precision measurements needed.

(de Kilbinger)

2 Regimes and Methods

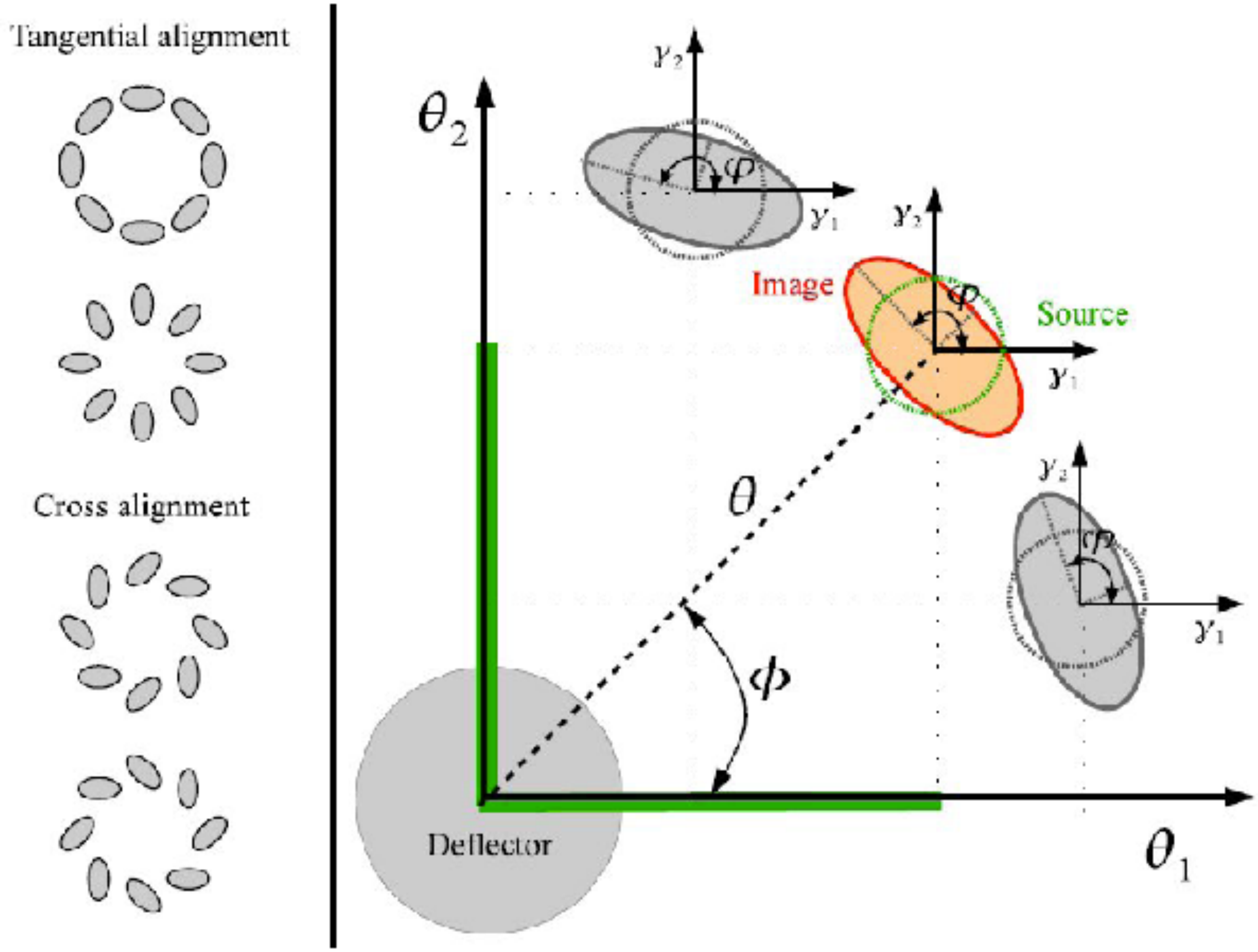
Lensing by galaxies and clusters

- Larger signal
- Center of reference
- Model/profile fitting
- Individual objects or *Stacking* of the signal

Large-scale structure

- Convergence maps (also in clusters)
- Correlations:
    - power spectrum, correlation function
    - among different probes, z-bins, CMB, etc.

Components of the Shear



Figueiró 2011

Mean shear in circles

$$< \gamma_t(\theta) >= \bar{\kappa}(\theta) - < k(\theta) >$$

Mean shear and radial profiles

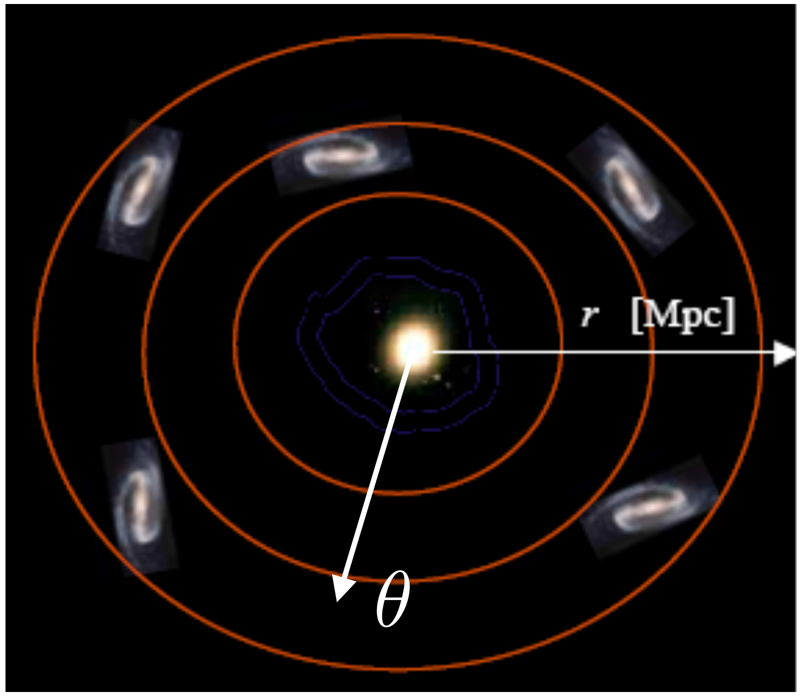
It is possible to show that

$$< \gamma_t(\theta) > = \bar{\kappa}(\theta) - < k(\theta) >$$

mean along a circle of the tangential component of the shear

mean within a disk of radius θ

mean on the circle



In practice: mean un anulii (radial bins)

Stacking of the signal

Need (and possibility) to increase the S/*N*

Combine data on many galaxies or clusters: mean signal

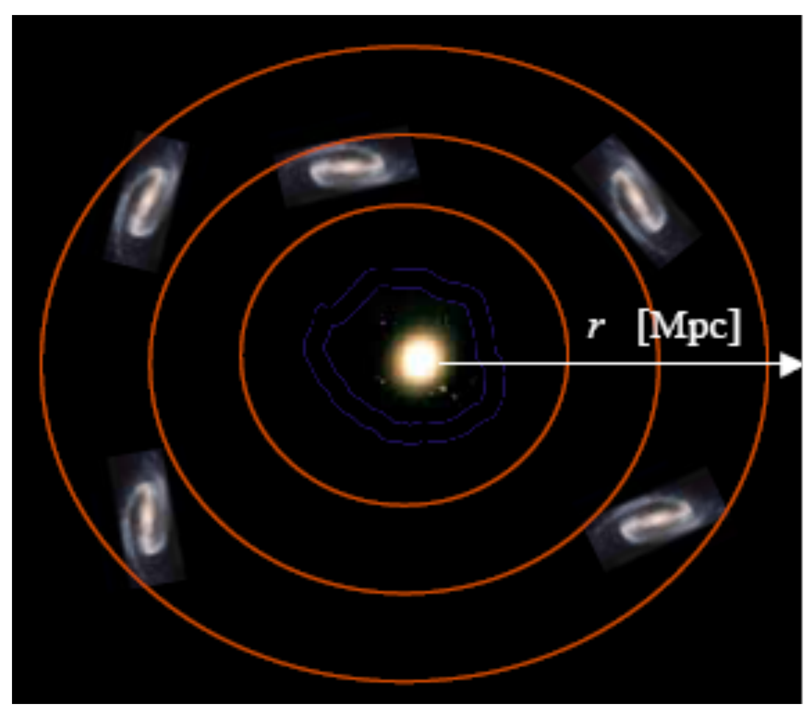
Physical signal and models: $\Sigma(r) \longrightarrow$ multiply by $\Sigma_{\rm crit}$

$$\Sigma_{\rm crit} \times < \gamma_t(\theta) > = \bar{\kappa}(\theta) - < k(\theta) > \times \Sigma_{\rm crit}$$

$$\boxed{\Sigma_{\rm crit} \times < \gamma_t(r) > = \bar{\Sigma}(r) - < \Sigma(r) > := \Delta\Sigma(r)}$$

redshifts

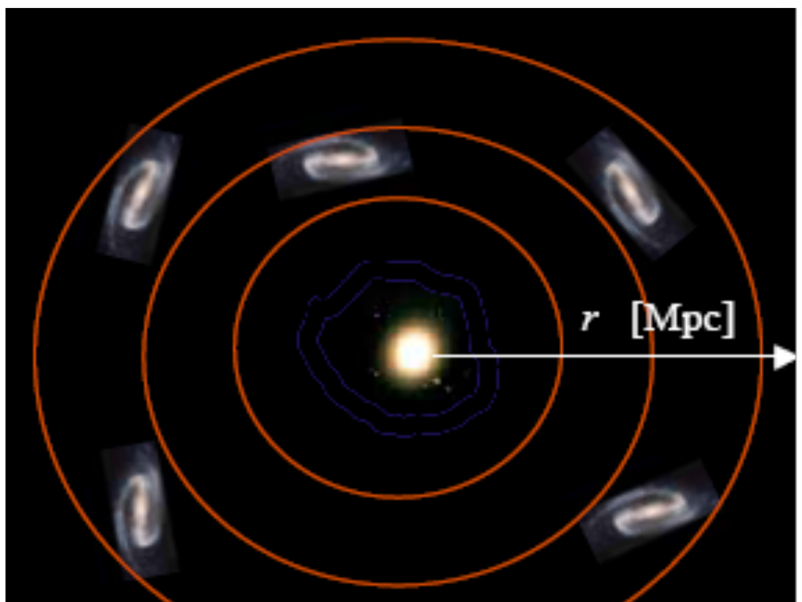
shapes



model

Mass reconstruction in clusters (radial profile)

- Measure the tangential shear to get $\Delta\Sigma$



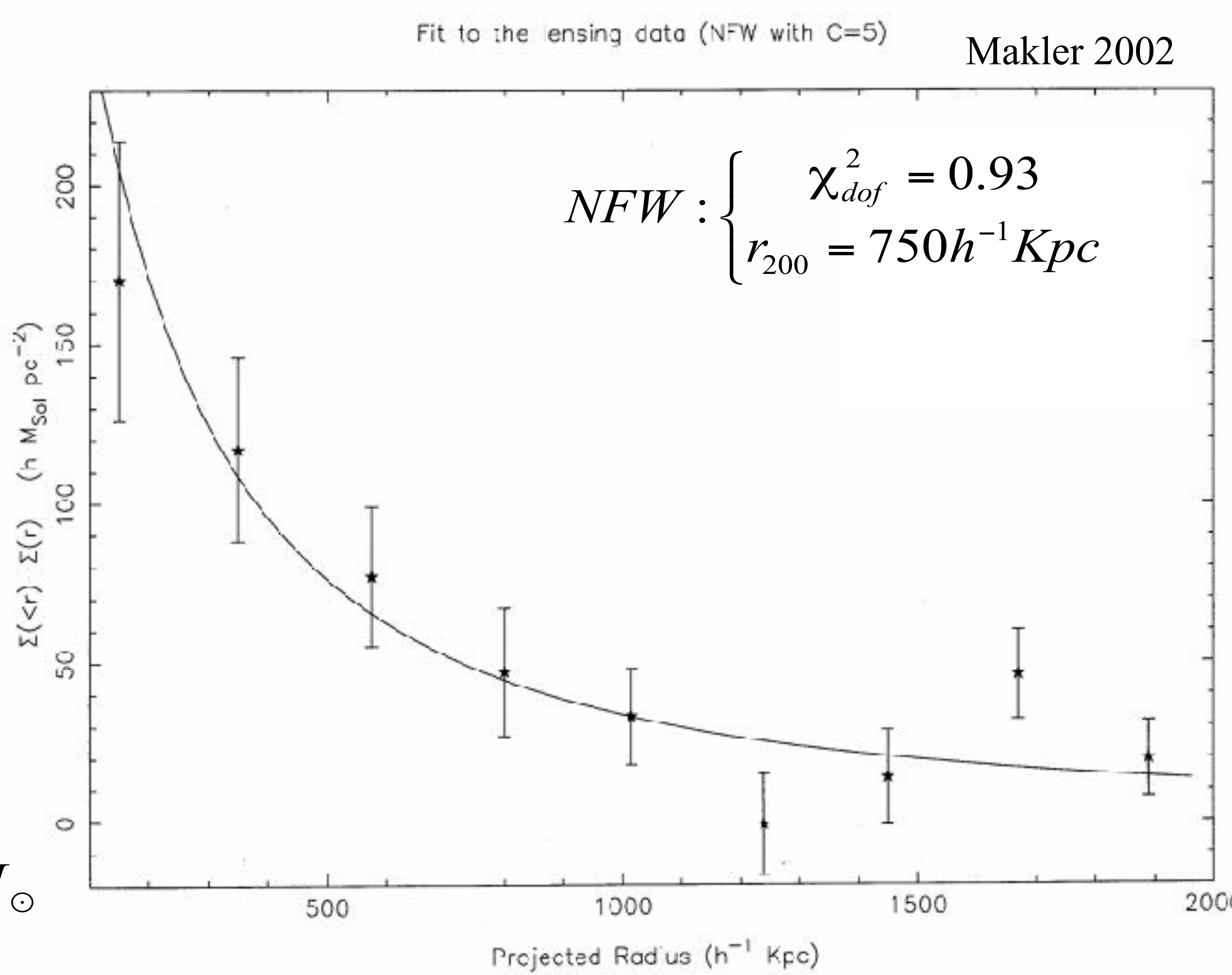
$$\Delta\Sigma(r) \equiv \bar{\Sigma}(<r) - \bar{\Sigma}(r) = \Sigma_{crit} \times \gamma_t(r)$$

$$\Sigma_{crit} = \frac{c^2}{4\pi G}\frac{D_{OS}}{D_{OL}D_{LS}}$$

- Massas:

$$\bar{M}_{NFW} = (1.0 \pm 0.2)10^{14}h^{-1}M_\odot$$

$$\frac{\Delta M_{200}}{M_{200}} \simeq 2.5\%$$



42 aglomerados (RASS/SDSS), Sheldon, et al., ApJ 554, 88 (2001)

Modeling the mass profile

Interpretation with the halo model: halos x galaxies correlation

- ☑ 1-halo term: matter density in the halo

Exemple: NFW $$\rho(r) = \frac{\rho_0}{(r/r_s)\,(1+r/r_s)^2}$$ compute $\Sigma(r)$

- ☑ 2-halo term: correlation with other halos (large scale structure) $\rho(r) = b\bar{\rho}_m \xi(r)$ where

$$\xi(r) = \frac{1}{2\pi^2}\int dk\, k^2 P(k) \frac{\sin(kr)}{kr} \qquad b(\nu) = 1 - A\frac{\nu^a}{\nu^a + \delta_c^a} + B\nu^b + C\nu^c$$

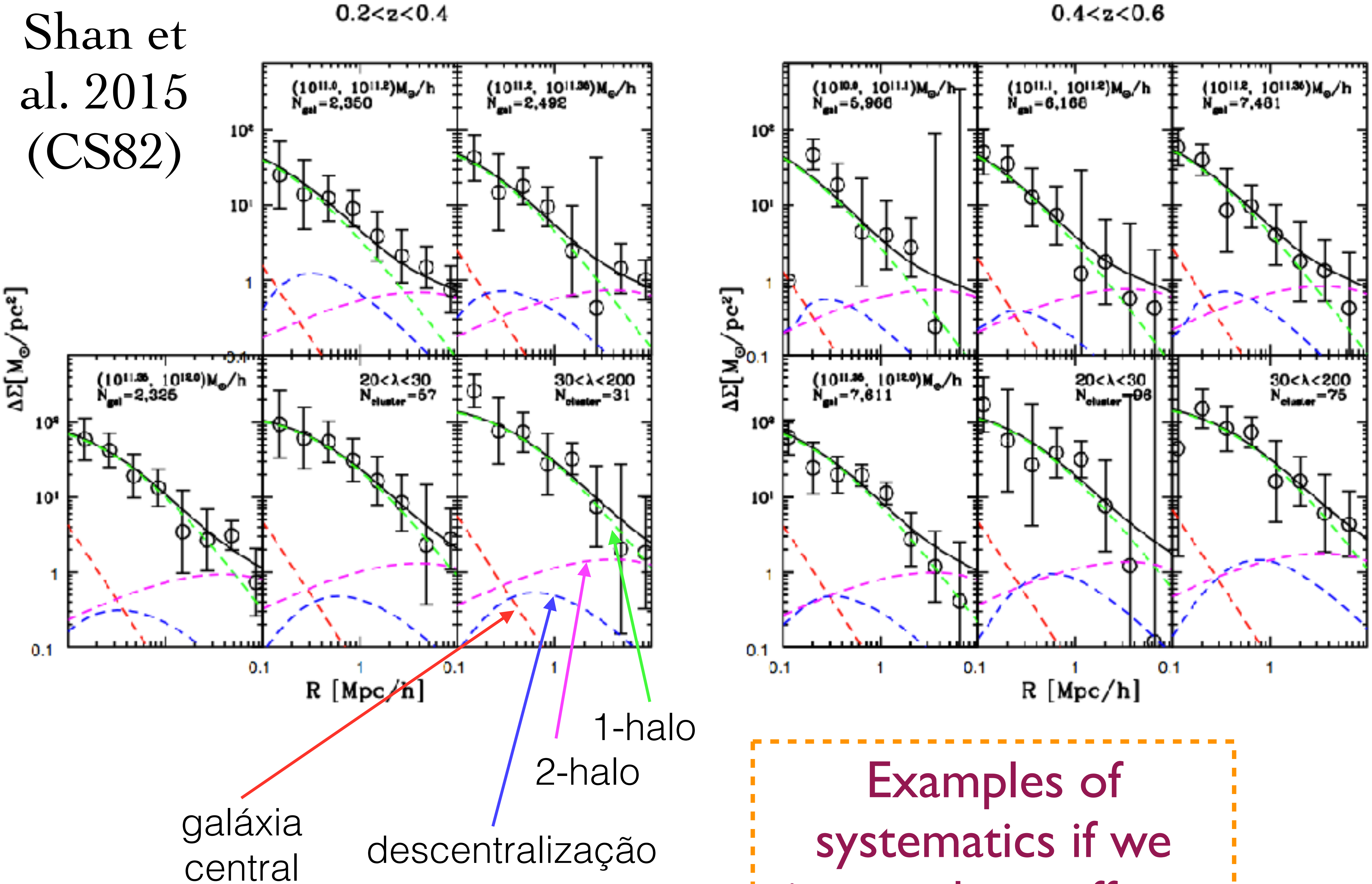
$$\nu = \delta_c/\sigma(M) = 1.686/\sigma(M) \qquad \sigma(R) = \frac{1}{2\pi^2}\int dk\, k^2 P(k,z)\hat{W}^2(k,R)$$

$$R = (3M/4\pi\bar{\rho}_m)^{1/3}$$ (see mass function)

- ☑ Term for the *offset* of the profile with respect to other center
- ☑ Central potential (central galaxy, e.g. SIS)

Example: Cluster mass calibration

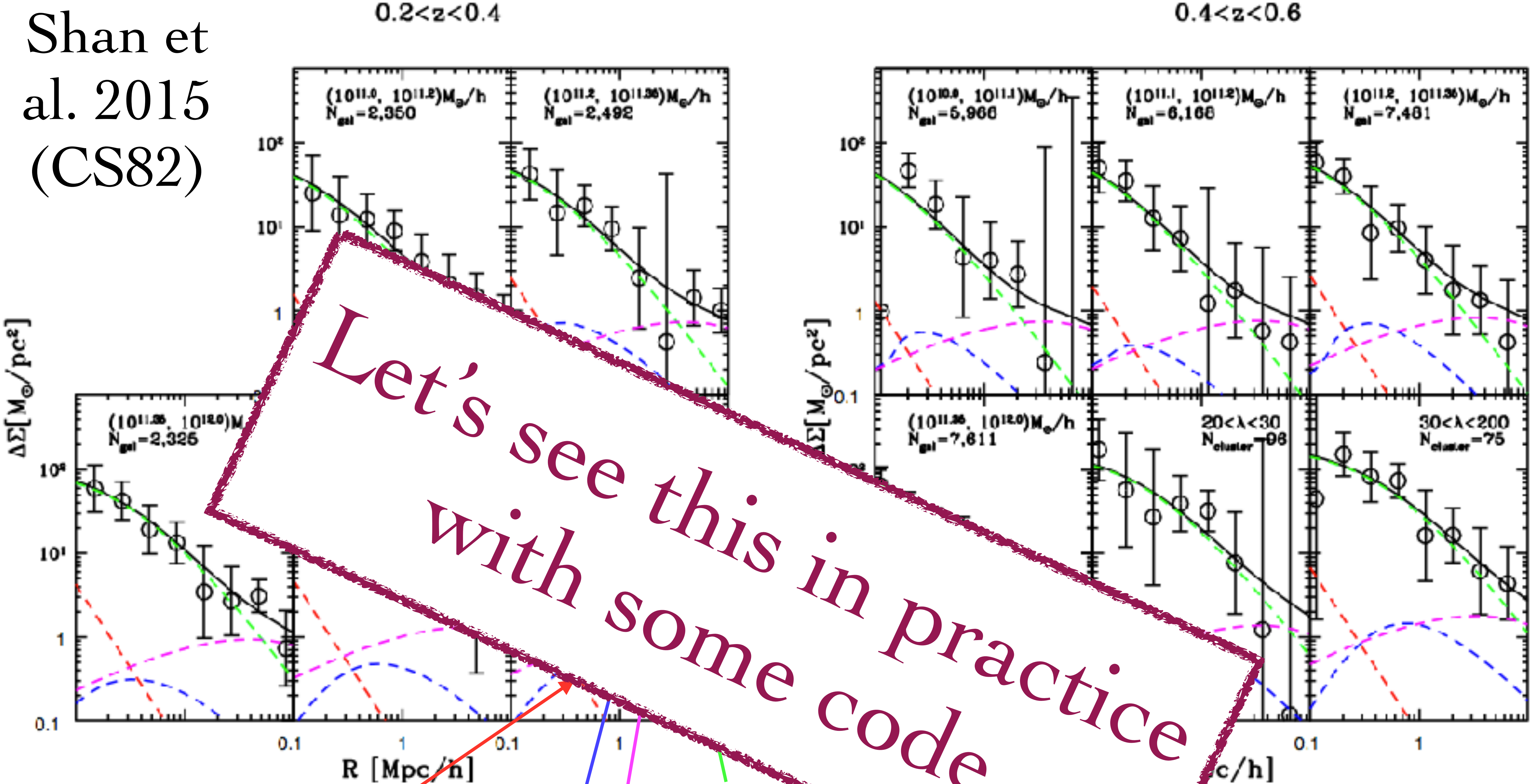
Shan et al. 2015 (CS82)



Examples of systematics if we ignore these effects

Mass calibration

Shan et al. 2015 (CS82)



1-halo

2-halo

galáxia central

descentralização

Examples of systematics if we ignore these effects

Gravitational Lensing: a tool for astrophysics and cosmology

- From planets to the Large Scale Structure of the Universe
- Essentially 21st century science, with many discoveries in the last 10 years
- These were by no means review talks on the main topic of lensing
  - Illustrations for pedagogical purposes
  - A few worked examples and use cases that can be followed at a basic level, reproduced and run in the example code provided in the repository
- Many many subjects not addressed, such as
  - Time delays, $H0$, QSO (micro-)lensing
  - Implications for DM properties
  - Astrophysical consequences of profiles, etc.
  - Gravitational telescope
  - Arcfinding, automated modeling, ML applications

Gravitational Lensing: a tool for astrophysics and cosmology

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- Interdisciplinary field involving from fundamental physics to data reduction, including image processing, statistics, simulations, theory and semi-analytic modeling
- Lots of room for interesting work to do
- New dynamics of large projects


Thank You