

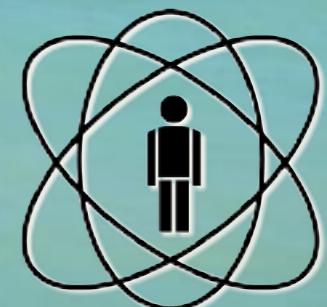
PART III

STRONG LENSING

MARTÍN MAKLER

ICAS/IFIFI/CONICET & UNSAM & CBPF

+ Renan Alves, João França, Elizabeth Gonzalez, Arthur Mesquita, Giulya Souza, Eduardo Valadão, Anibal Varella



Observatorio
Astronómico
de Córdoba



UNC

Universidad
Nacional
de Córdoba



UNSAM

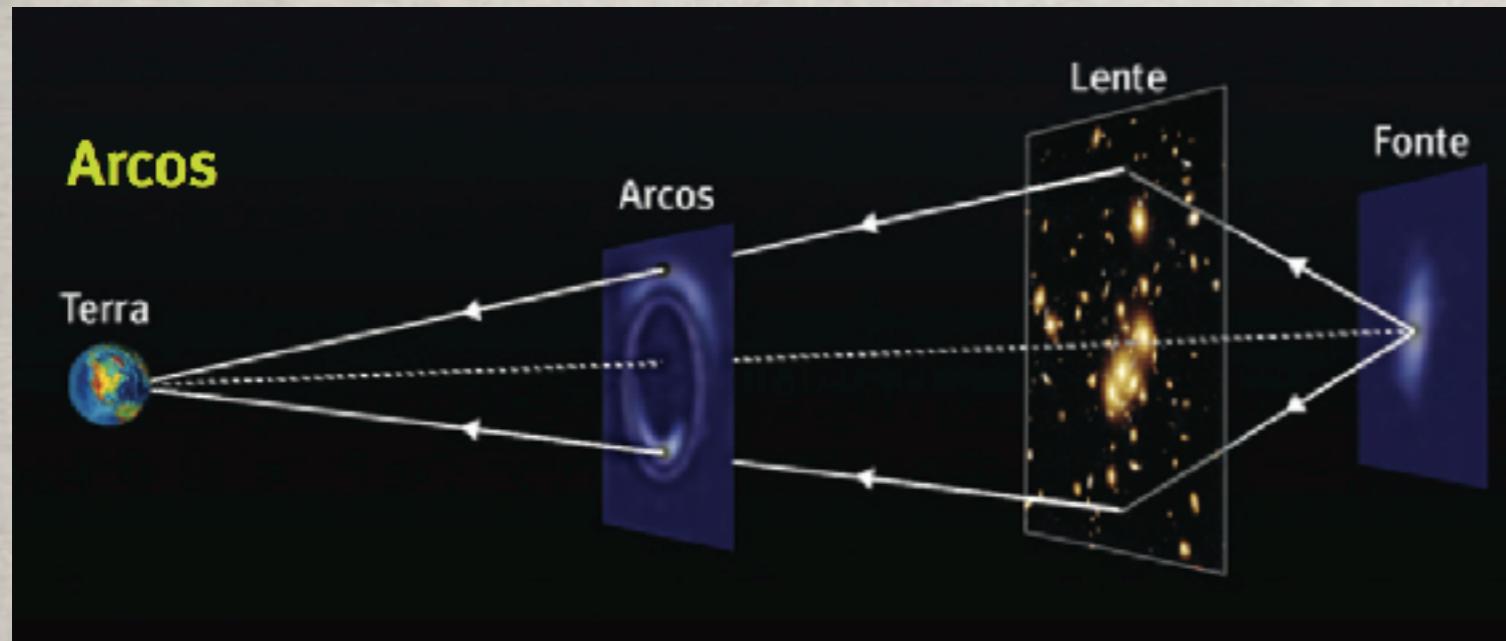


I A T E



STRONG LENSING

- Multiple images, strong distortions, large magnifications, time delays
 - Null geodesics
 - surface brightness conservation
 - achromatic
 - Unique probe of inner structure of galaxy clusters → DM, b
 - Provide complementary cosmological probes and tests of gravity
- } → **Gravitational telescopes**

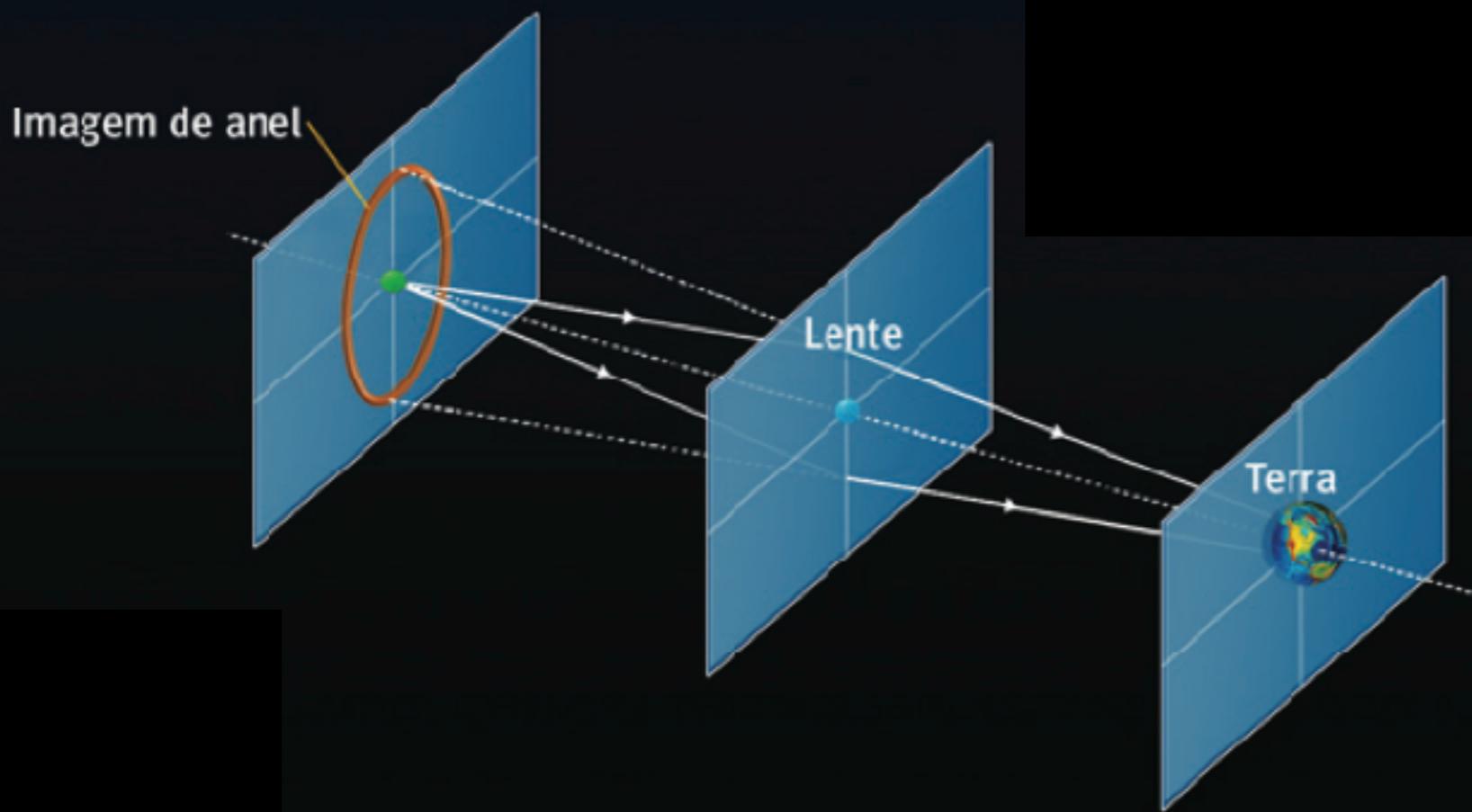


strong lensing, **weak gravity**



Gravitational arcs

“Chwolson-Einstein rings”

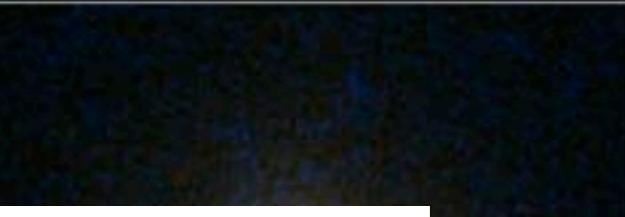
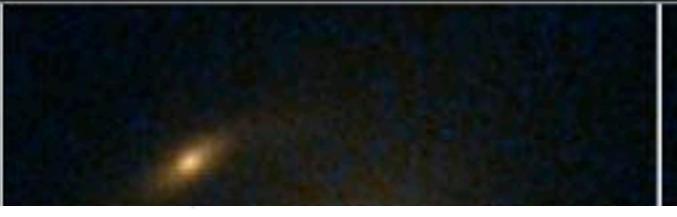
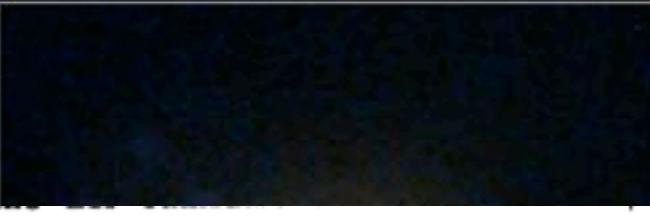


NASA/JESA A. BOLTON AND SLACS TEAM

Anéis de Chwolson-Einstein
observados pelo telescópio
espacial Hubble

Chwolson-Einstein rings

J073



messung liegt die Position

Über eine mögliche Form fiktiver Doppelsterne. Von O. Chwolson.

Es ist gegenwärtig wohl als höchst wahrscheinlich anzunehmen, daß ein Lichtstrahl, der in der Nähe der Oberfläche eines Sternes vorbeigeht, eine Ablenkung erfährt. Ist γ diese Ablenkung und γ_0 der Maximumwert an der Oberfläche, so ist $\gamma_0 \geq \gamma \geq 0$. Die Größe des Winkels ist bei der Sonne $\gamma_0 = 1.^{\circ}7$; es dürften aber wohl Sterne existieren, bei denen γ_0 gleich mehreren Bogensekunden ist; vielleicht auch noch mehr. Es sei A ein großer Stern (Gigant); T die Erde, B ein entfernter Stern; die Winkeldistanz zwischen A und B , von T aus gesehen, sei α , und der Winkel zwischen A und T , von B aus gesehen, sei β . Es ist dann

$$\gamma = \alpha + \beta.$$

Ist B sehr weit entfernt, so ist annähernd $\gamma = \alpha$. Es kann also α gleich mehreren Bogensekunden sein, und der Maximumwert von α wäre etwa gleich γ_0 . Man sieht den Stern B von der Erde aus an zwei Stellen: direkt in der Richtung TB und außerdem nahe der Oberfläche von A , analog einem Spiegelbild. Haben wir mehrere Sterne B, C, D , so würden die Spiegelbilder umgekehrt gelegen sein wie in

Petrograd, 1924 Jan. 28.

einem gewöhnlichen Spiegel; nämlich in der Reihenfolge D, C, B , wenn von A aus gerechnet wird (D wäre am nächsten zu A).



Der Stern A würde als fiktiver Doppelstern erscheinen. Teleskopisch wäre er selbstverständlich nicht zu trennen. Sein Spektrum bestände aus der Übereinanderlagerung zweier, vielleicht total verschiedenartiger Spektren. Nach der Interferenzmethode müßte er als Doppelstern erscheinen. Alle Sterne, die von der Erde aus gesehen rings um A in der Entfernung $\gamma_0 - \beta$ liegen, würden von dem Stern A gleichsam eingefangen werden. Sollte zufällig TAB eine gerade Linie sein, so würde, von der Erde aus gesehen, der Stern A von einem Ring umgeben erscheinen.

Ob der hier angegebene Fall eines fiktiven Doppelsternes auch wirklich vorkommt, kann ich nicht beurteilen.

O. Chwolson.

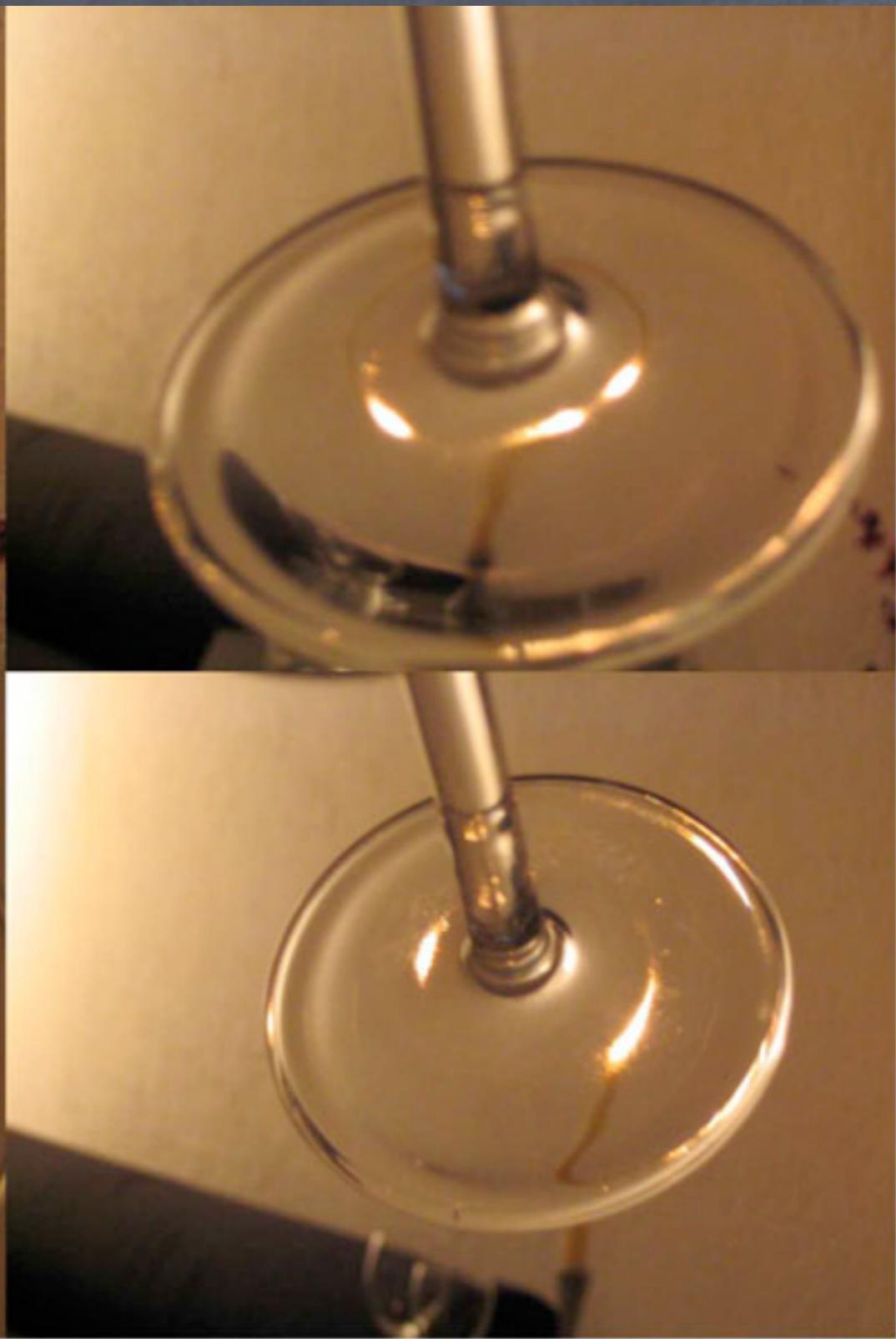
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Einstein Ring Gravitational Lenses
Hubble Space Telescope • Advanced Camera for Surveys



GALAXY CLUSTERS



STRONG LENSING
MACROLENSING

A RELATIVISTIC ECLIPSE

IT is a familiar aphorism that the theory of relativity, despite its enormous importance, both in physics and philosophy, may be forgotten in ordinary practical life. There is a good reason. In almost every known case its results agree so closely with those of the older "classical" theories that very accurate observations are required to distinguish between them. Thus, for example, the famous Michelson-Morley experiment requires four series of the

What Might be Seen from a Planet Conveniently Placed Near the Companion of Sirius . . . Perfect Tests of General Relativity that are Unavailable

By HENRY NORRIS RUSSELL, Ph. D.

Chairman of the Department of Astronomy and Director of the Observatory at Princeton University; Research Associate of the Mount Wilson Observatory of the Carnegie Institution of Washington; President of the American Astronomical Society.

FEBRUARY • 1937

SCIENTIFIC AMERICAN

77

cludes gravitation in its scope) are of this sort. The advance of the perihelion of Mercury provides an increase, but definitely finite, in brightness when the star passes directly between the Sun and Earth.

Einstein himself,¹ From a point exactly in

line the distance one would expect to be resolved by even the greatest tele-

for it. The focusing effect, however,

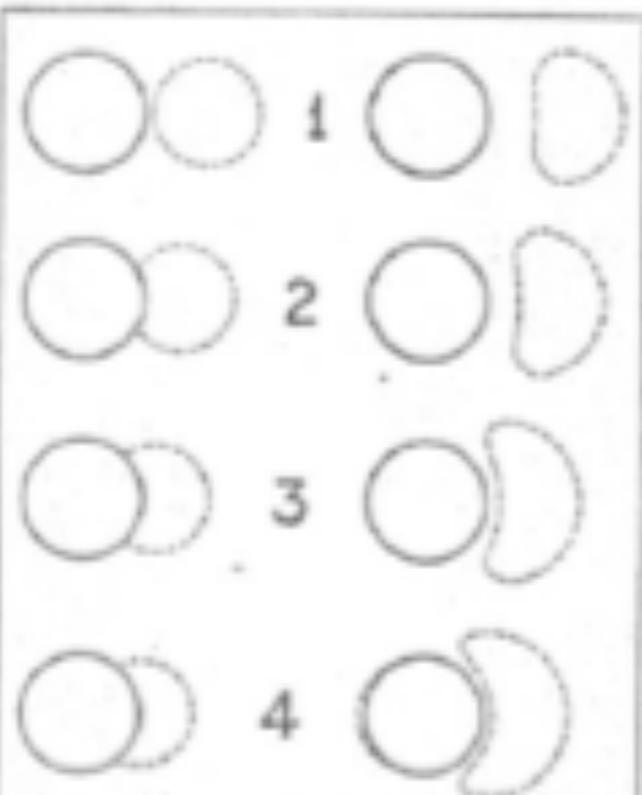
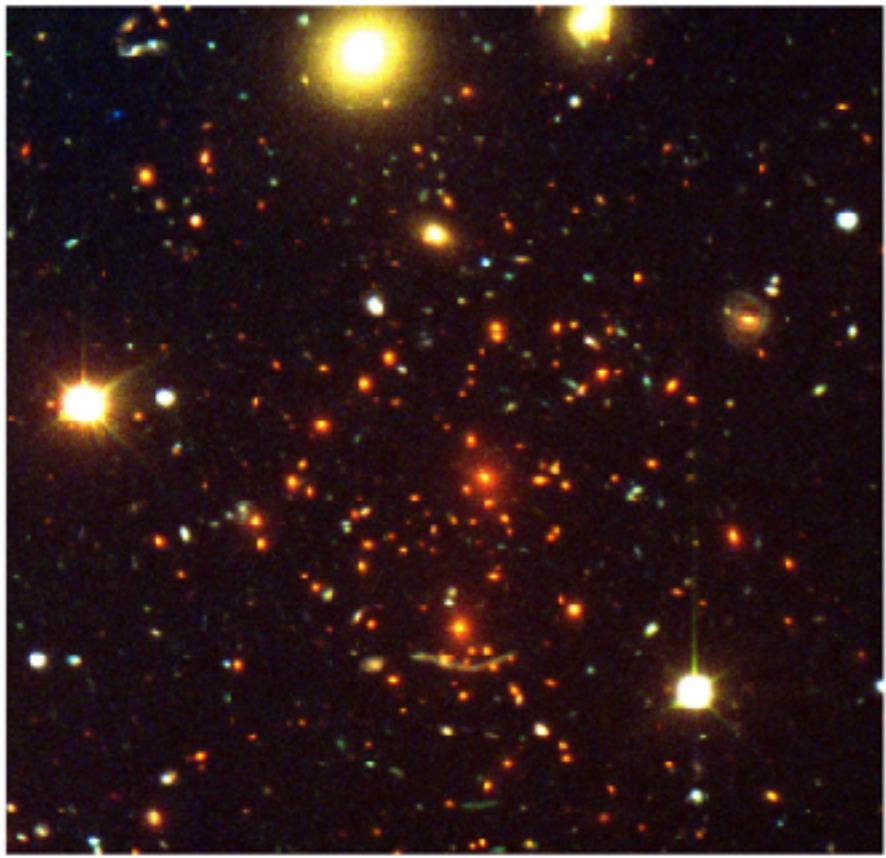
is not so great as that of the light of the stars, For a star

of mathematical

precision, with no angular diameter at all, the increase in brightness would be infinite. But any real star must have a finite angular size, however small, and for such a star the increase in brightness, though it might be large, would have a limit.

If a bright star passed in front of a fainter one, its own light would drown out the effect; but if one of the faint red stars, which are really more abundant than any other sort, should get directly

My hearty thanks are due to Professor Einstein, who permitted me to see the manuscript of his note before its publication.—*Princeton University Observatory, December 2, 1936.*



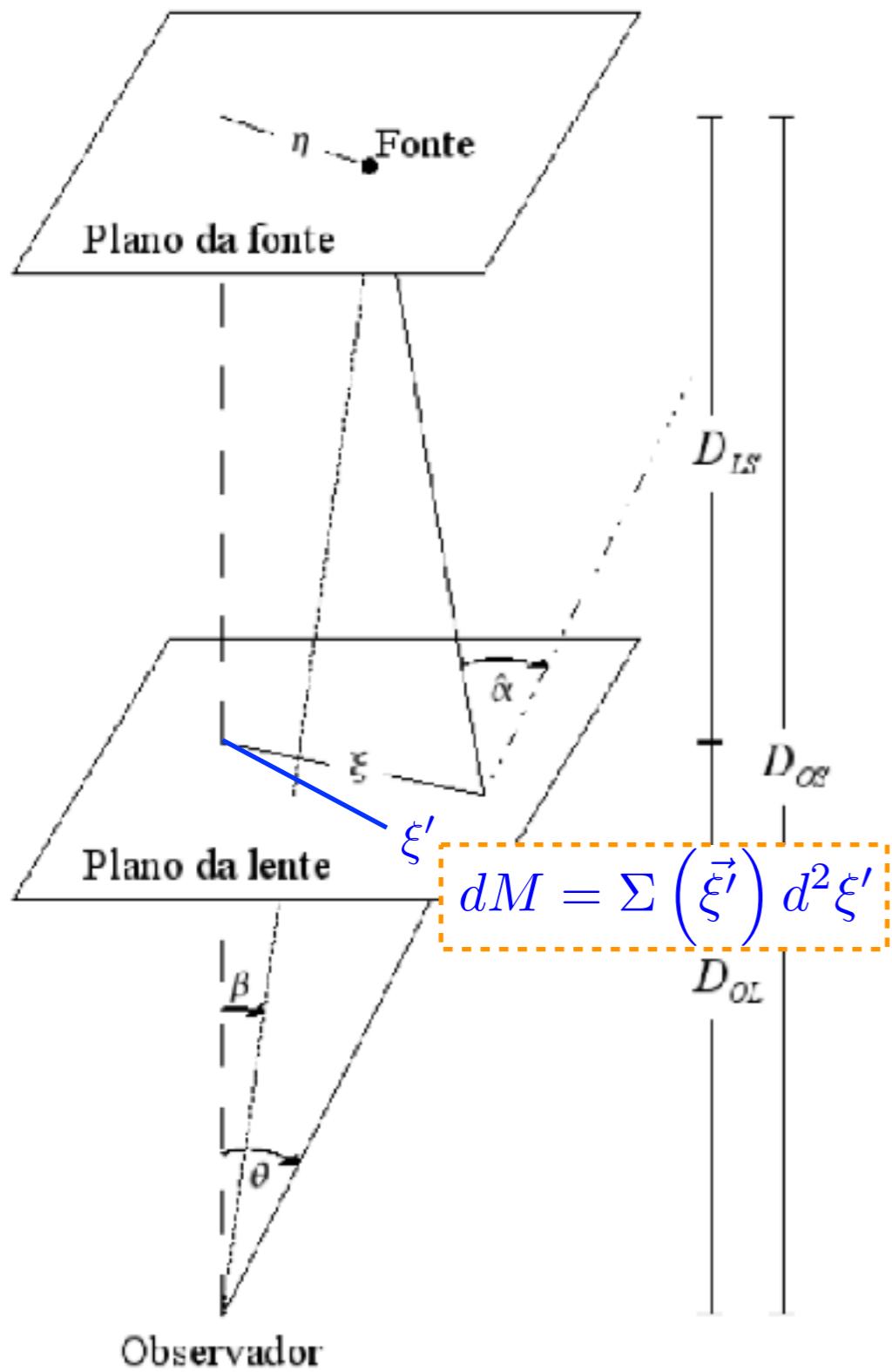
is, a million miles or so—from the Sun through the center of the two stars. The

The background of the slide is a photograph of a beach. The foreground shows light brown sand with some small debris. The middle ground is filled with the ocean, showing white-capped waves breaking. The sky above is a clear, pale blue.

A BIT OF THEORY/MODELLING:

**THE REALM OF EXTENDED
LENSES AND EXTENDED
SOURCES**

Extended Lenses



Point mass

$$\hat{\vec{\alpha}} = \frac{4GM}{c^2\xi}$$

Surface mass density

$$\Sigma(\vec{\xi}) = \int_0^\infty dz \rho(\vec{\xi}, z)$$

Contribution of the area element

$$d\hat{\vec{\alpha}} = \frac{4G}{c^2} \Sigma(\vec{\xi}') d^2\xi' \frac{\vec{\xi} - \vec{\xi}'}{|\vec{\xi} - \vec{\xi}'|^2}$$

Deflection angle

$$\hat{\vec{\alpha}} = \frac{4G}{c^2} \int d^2\xi' \Sigma(\vec{\xi}') \frac{\vec{\xi} - \vec{\xi}'}{|\vec{\xi} - \vec{\xi}'|^2}$$

Extended Lenses

Projected potential $\psi(\vec{\xi}) = \int dz \varphi(\vec{\xi}, z)$

Deflection angle $\hat{\vec{\alpha}} = \frac{2}{c^2} \vec{\nabla}_{\xi} \psi(\vec{\xi})$

Reduced deflection angle

$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}} = \frac{2}{c^2} \frac{D_{LS}}{D_{OS} D_{OL}} \vec{\nabla}_{\theta} \psi(\vec{\theta})$$

Lensing potential

$$\Psi(\vec{x}) \equiv \frac{2}{c^2} \frac{D_{LS}}{D_{OS} D_{OL}} \psi$$

Lens equation

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta}) = \boxed{\vec{\theta} - \vec{\nabla}_{\theta} \Psi(\vec{\theta})}$$

Lens equation with axial symmetry

Deflection angle

$$\hat{\vec{\alpha}} = \frac{4GM(\xi)}{c^2\xi} \hat{\xi}$$

Lens equation

$$\vec{\beta} = \vec{\theta} - \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}} = \vec{\theta} - \frac{4GM(\theta)}{c^2\theta^2 D_{OL}} \vec{\theta} \frac{D_{LS}}{D_{OS}}$$

$$\vec{\beta} = \left(1 - \frac{4GM(\theta)D_{LS}}{c^2 D_{OL} D_{OS}} \frac{1}{\theta^2} \right) \vec{\theta}$$

Einstein angle

$$(\vec{\beta} = 0)$$

$$\theta_E = \sqrt{\frac{D_{LS}}{D_{OS} D_{OL}} \frac{4GM(\theta_E)}{c^2}}$$

Mass estimate at $\theta < \theta_E$

A wide-angle photograph of a tropical beach. The foreground is a light tan sandy beach with some scattered small rocks and debris. The middle ground is filled with the vibrant turquoise water of the ocean, with gentle waves breaking near the shore. The background is a clear, pale blue sky meeting the horizon.

SINGULAR ISOTHERMAL SPHERE

Singular Isothermal Sphere

$$\rho(r) = \frac{\sigma_v^2}{2\pi G r^2}$$

Surface density (projected)

$$\Sigma(\xi) = \frac{\sigma_v^2}{2\pi G} 2 \int_0^\infty \frac{dz}{\xi^2 + z^2} = \frac{\sigma_v^2}{2G\xi}$$

Mass within a radius ξ

$$M(\xi) = \int_0^\xi \Sigma(\xi') 2\pi \xi' d\xi' = \sigma_v^2 \frac{\pi}{G} \xi^2$$

Deflection angle

$$\rightarrow \hat{\vec{\alpha}} = \frac{4GM(\xi)}{c^2 \xi} \hat{\xi} = 4\pi \left(\frac{\sigma_v}{c} \right)^2 \xi \quad \text{constant!}$$

Lens Equation

$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}}$$

$$\hat{\vec{\alpha}} = 4\pi \left(\frac{\sigma_v}{c} \right)^2 \hat{\xi}$$

$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}} = \boxed{4\pi \left(\frac{\sigma_v}{c} \right)^2 \frac{D_{LS}}{D_{OS}} \hat{\xi}} = \theta_E \hat{\xi}$$

θ_E

Lens Equation

$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}}$$

$$\hat{\vec{\alpha}} = 4\pi \left(\frac{\sigma_v}{c} \right)^2 \hat{\xi}$$

$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}} = 4\pi \left(\frac{\sigma_v}{c} \right)^2 \frac{D_{LS}}{D_{OS}} \hat{\xi} = \theta_E \hat{\xi}$$

$$\vec{\beta} = \vec{\theta} - \theta_E \hat{\theta} = \left(1 - \frac{\theta_E}{\theta} \right) \vec{\theta}$$

→ $\beta = \left| 1 - \frac{\theta_E}{\theta} \right| \theta$

Solutions: I) if $1 - \frac{\theta_E}{\theta} > 0$

then $\beta = \left(1 - \frac{\theta_E}{\theta} \right) \theta = \theta - \theta_E \Rightarrow \boxed{\theta = \beta + \theta_E}$

Lens Equation

$$\beta = \left| 1 - \frac{\theta_E}{\theta} \right| \theta$$

Solutions: I) if $1 - \frac{\theta_E}{\theta} > 0$

then $\beta = \left(1 - \frac{\theta_E}{\theta} \right) \theta = \theta - \theta_E \Rightarrow \boxed{\theta = \beta + \theta_E}$

II) if $1 - \frac{\theta_E}{\theta} < 0$

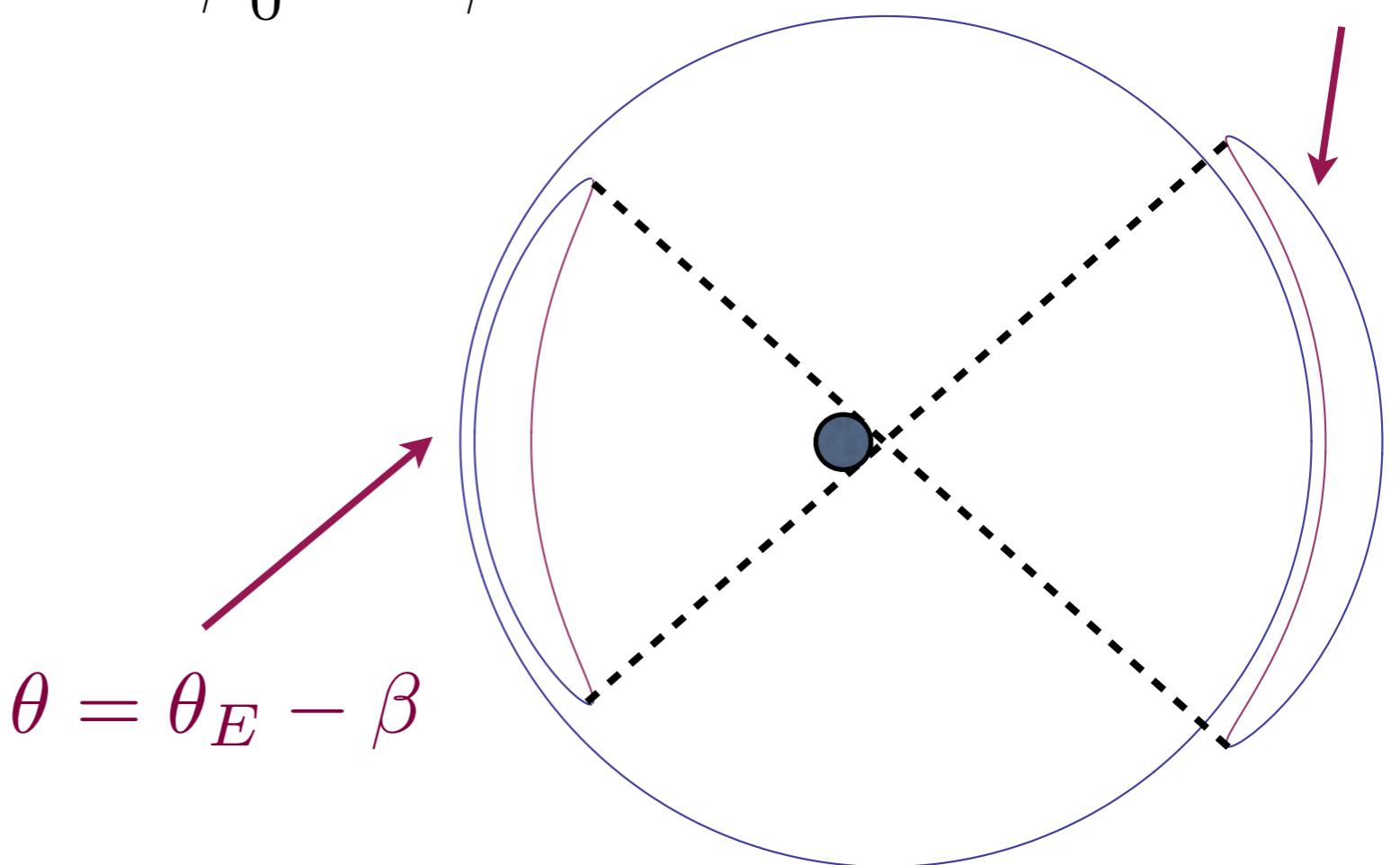
then $\beta = - \left(1 - \frac{\theta_E}{\theta} \right) \theta = \theta_E - \theta \Rightarrow \boxed{\theta = \theta_E - \beta}$

Circular source

Circle in the source plane: $|\vec{\beta} - \vec{\beta}_0| = R$

$$\beta = \beta_0 \cos \phi \pm \sqrt{R^2 - \beta_0^2 \sin^2 \phi}$$

$$\theta = \beta + \theta_E$$



in general $\beta = \beta_0 \cos(\phi - \phi_0) \pm \sqrt{R^2 - \beta_0^2 \sin^2(\phi - \phi_0)}$

Other lens models

Implemented in *gravlens* and several public codes (+ Sérsic, Einasto, etc.)

Model	N_r	Density $\rho(r)$	Surface Density $\kappa(r)$
Point mass	0	$\delta(\mathbf{x})$	$\delta(\mathbf{x})$
Power law or α -models	2	$(s^2 + r^2)^{(\alpha-3)/2}$	$(s^2 + r^2)^{(\alpha-2)/2}$
Isothermal ($\alpha = 1$)	1	$(s^2 + r^2)^{-1}$	$(s^2 + r^2)^{-1/2}$
$\alpha = -1$	1	$(s^2 + r^2)^{-2}$	$(s^2 + r^2)^{-3/2}$
Pseudo-Jaffe	2	$(s^2 + r^2)^{-1} (a^2 + r^2)^{-1}$	$(s^2 + r^2)^{-1/2} - (a^2 + r^2)^{-1/2}$
King (approximate)	1	...	$2.12 (0.75r_s^2 + r^2)^{-1/2}$ $-1.75 (2.99r_s^2 + r^2)^{-1/2}$
de Vaucouleurs	1	...	$\exp [-7.67(r/R_e)^{1/4}]$
Hernquist	1	$r^{-1} (r_s + r)^{-3}$	see eq. (47)
NFW	1	$r^{-1} (r_s + r)^{-2}$	see eq. (53)
Cuspy NFW	2	$r^{-\gamma} (r_s + r)^{\gamma-3}$	see eq. (57)
Cusp	3	$r^{-\gamma} (r_s^2 + r^2)^{(\gamma-n)/2}$	see eq. (64)
Nuker	4	...	see eq. (71)
Exponential disk	1	...	$\exp[-r/R_d]$
Kuzmin disk	1	...	$(r_s^2 + r^2)^{-3/2}$

Elliptical models

Replace radial coordinate: $\theta \rightarrow \sqrt{q_1 \theta_1^2 + q_2 \theta_2^2}$

- in the potential (pseudo-elliptic models)
- in the projected density (elliptical models)

Example: pseudo-elliptic isothermal model
with a core

$$\Psi(\theta_1, \theta_2) = \frac{\Psi_0}{\theta_c} \sqrt{(1 - \epsilon)\theta_1^2 + (1 + \epsilon)\theta_2^2 + \theta_c^2}$$

Model for the source

Surface brightness distribution of the source

Example: Sérsic profile

$$I(R) = I_0 \exp \left\{ -b_n \left(\frac{R}{R_e} \right)^{1/n} \right\}$$

Elliptical brightness distribution
source centered at (S_1, S_2)

$$\begin{aligned} R^2 &= (1 - \varepsilon_S)[(\beta_1 - S_1) \cos \phi_e + (\beta_2 - S_2) \sin \phi_e]^2 \\ &\quad + (1 + \varepsilon_S)[(\beta_2 - S_2) \cos \phi_e - (\beta_1 - S_1) \sin \phi_e]^2 \end{aligned}$$

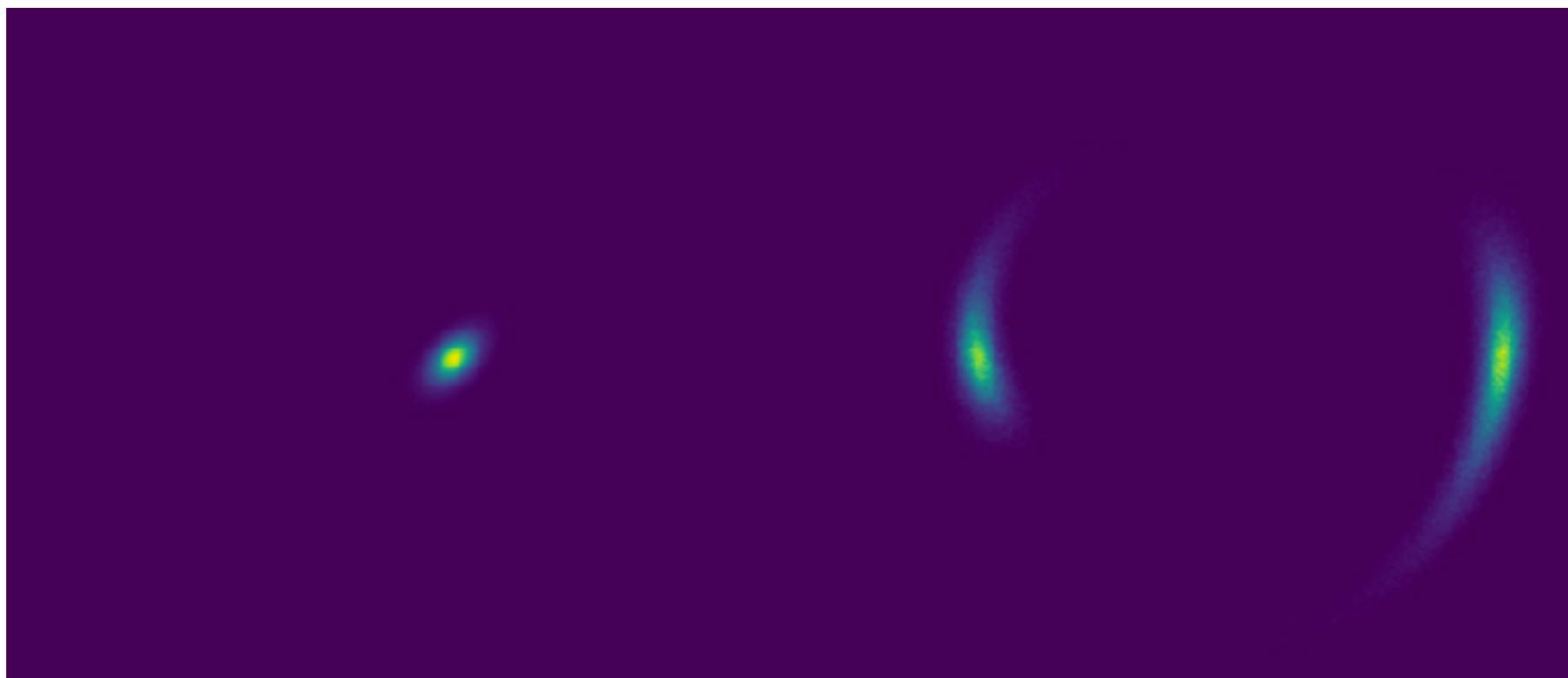
Simulating Strong Lensing Images

Map the brightness distribution of the source to the lens plane

$$I(\vec{\theta}) = I(\vec{\beta}(\vec{\theta}))$$

Use the lens equation (no need to solve it!)

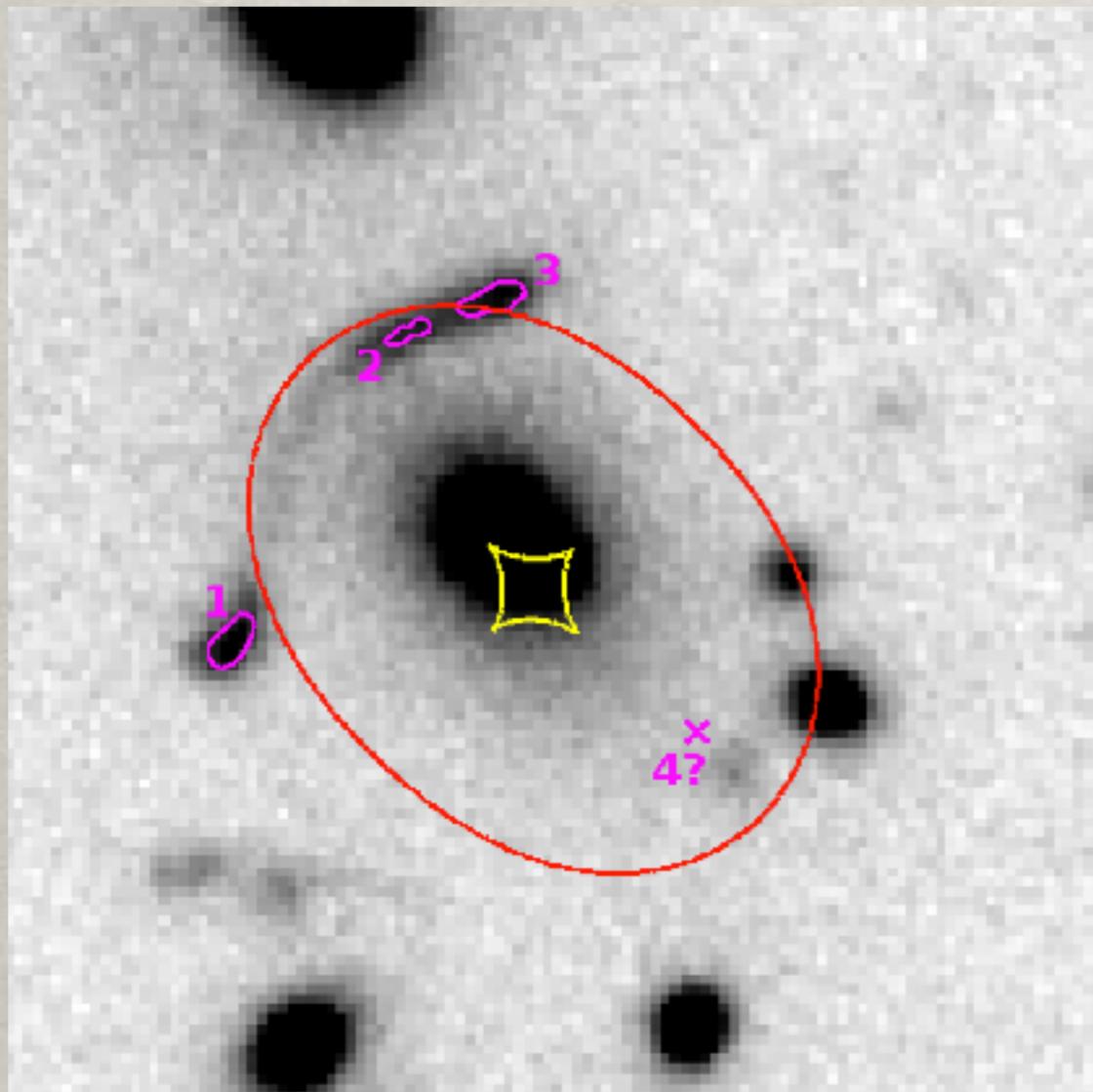
Add PSF and noise



AddArcs v2.0

*Let's see this in practice
with some code*

INVERSE MODELING: MAPPING THE MASS



Use systems of multiple images to determine the lensing potential

$$\chi^2_{\text{lente}} := \sum_i \left(\frac{\vec{\theta}_i^{\text{obs}} - \vec{\theta}^{\text{mod}}(\vec{\beta}, \vec{\Pi})}{\sigma_i^{\text{obs}}} \right)^2$$

Multiple image positions

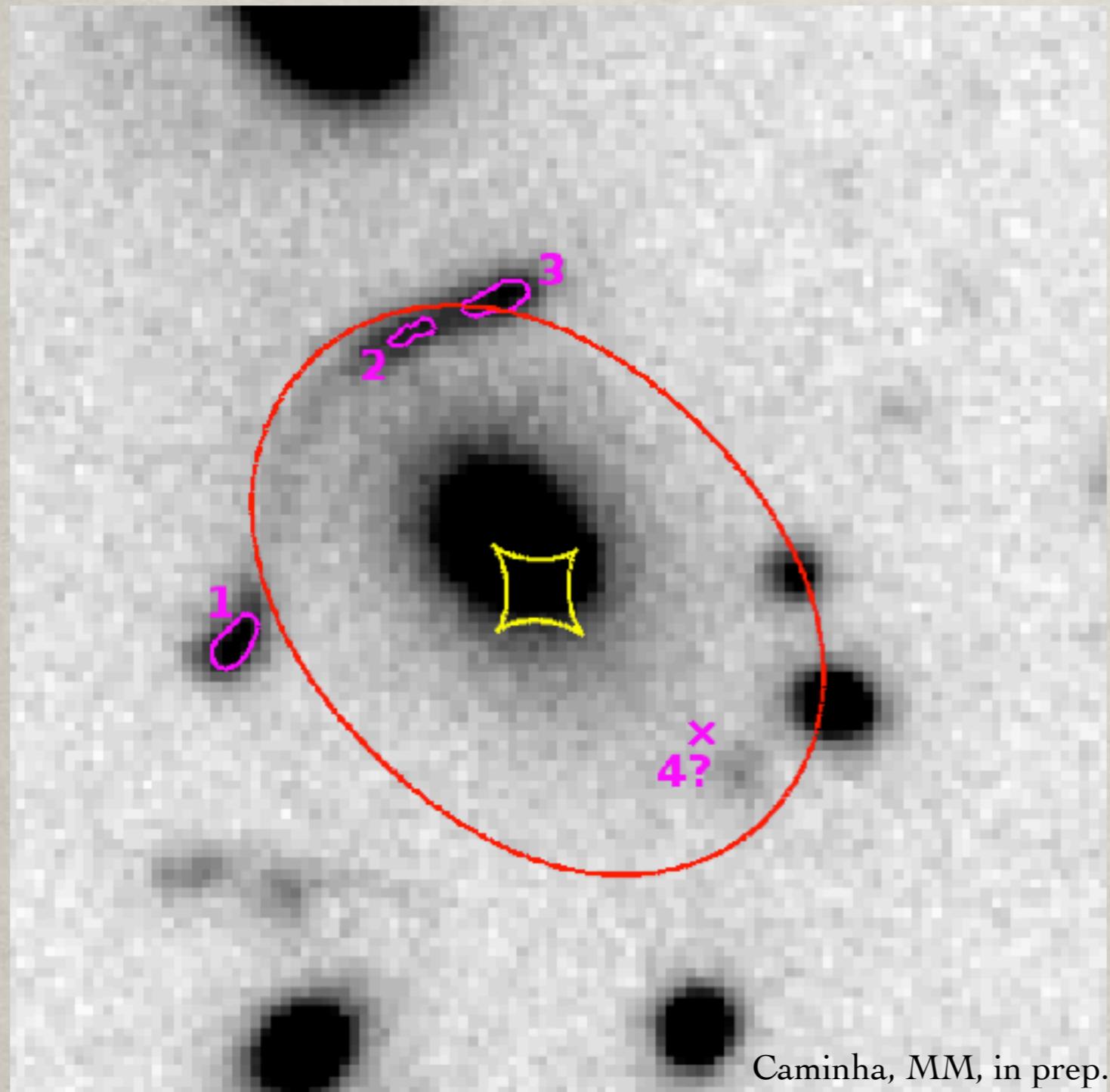
Error on image positions

Methods: parametric (often “mass traces light”), free form

The more multiple images, the more constraints
Cluster x Galaxy scales

- Combination with independent mass constraints (e.g., x-ray, Sunyaev Zel'dovich, velocity dispersions) yields limits on cosmology or gravity

INVERSE MODELING FOR SYSTEM SOGRASO04 1-OO43



$$2.35_{-0.14}^{+0.03} \times 10^{14} M_{\odot}$$

Modelling with lenstool
(Jullo, Kneib)

Fit 1: 3 images

Fit 2: 4 images

	Fit 1	Fit 2
$\sigma_v [km/s]$	622_{-13}^{+11}	642_{-3}^{+3}
$\theta_{\text{or}} [\circ]$	$135.2_{-0.8}^{+0.7}$	$135.2_{-1.3}^{+1.5}$
$x_{\text{lente}} ["]$	$0.50_{-0.2}^{+0.2}$	$0.50_{-0.06}^{+0.05}$
$y_{\text{lente}} ["]$	$-0.76_{-0.15}^{+0.17}$	$-0.86_{-0.03}^{+0.06}$
ε	—	$0.13_{-0.03}^{+0.02}$

~ 0.9" displacement between central galaxy and center of mass distribution
(Zitrin et al. 2012)

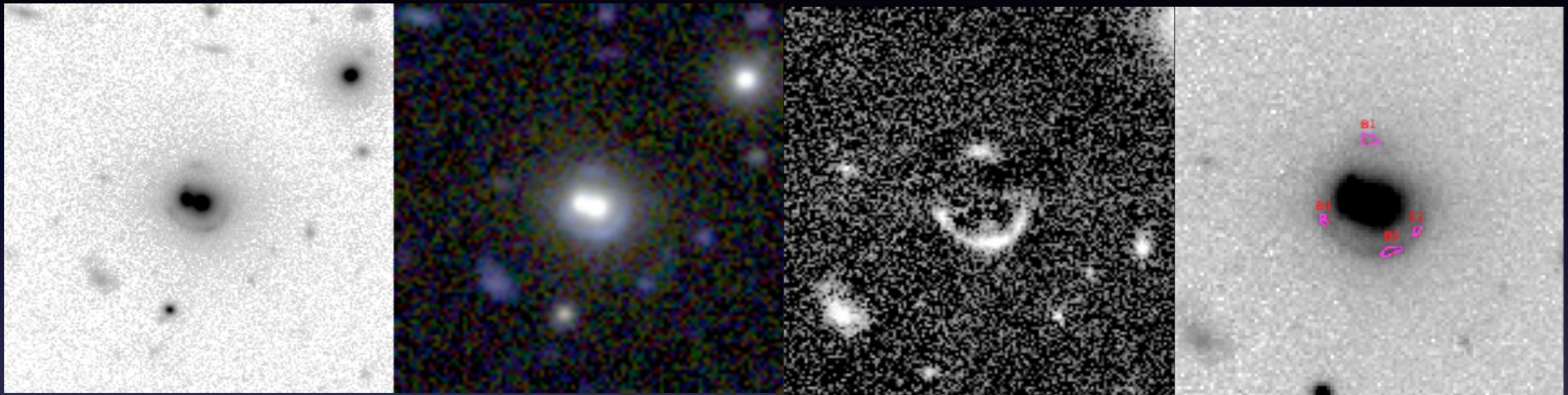
Error estimate from simulations
(Caminha et al. in prep.):

~ 8% bias in mass

~ 5% statistical errors

Modeling the full light distribution of the images

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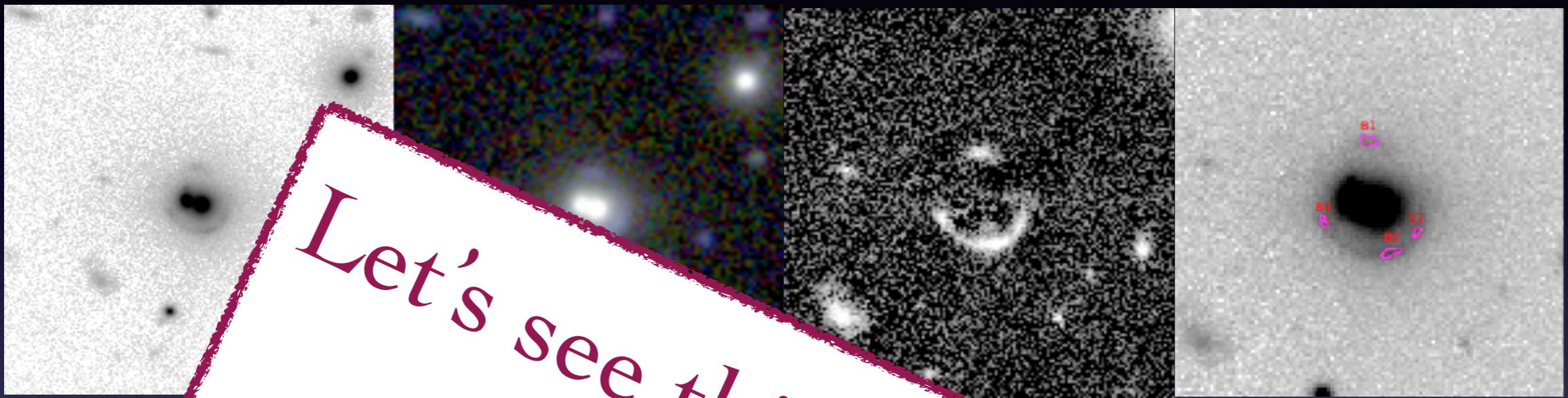


Anna Niemiec

- Instead of peak/multiple images, use the full information of the images
- Allows one to reconstruct source properties (often parametrically)
- Remove contamination from lens galaxy (with `galfit`) and mask other objects

Modeling the full light distribution of the images

CS82SL01:36:39+00:08:18



- Instead of ~~...with some code~~ the full information on ~~...with some code~~ allows one to reconstruct ~~...with some code~~ (often parametrically)
- Remove contamination from lens galaxy (with `galfit`) and mask other objects

Let's see this in practice

EXAMPLE APPLICATIONS:

- Finding substructure: Dark Matter
- Einstein rings: testing modified gravity
- Galaxy clusters: probing the background cosmology

Strong lensing and substructures

High sensitivity to small perturbations due to the caustic structure

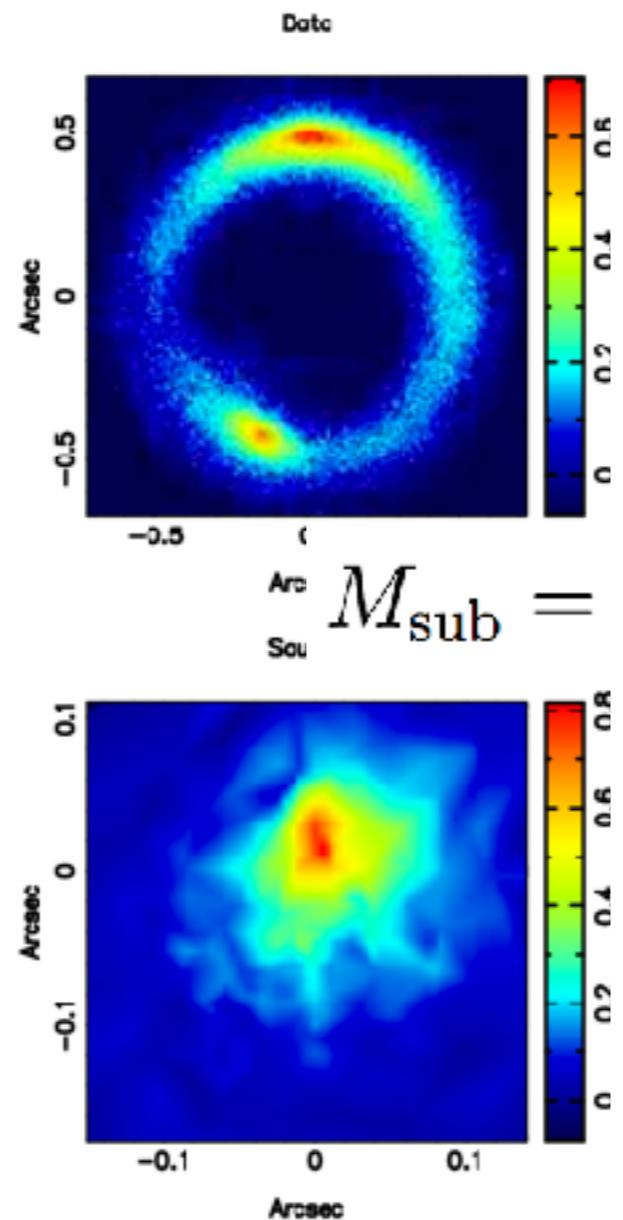
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nature
International Journal of Science

Gravitational detection of a low-mass dark satellite galaxy at cosmological distance

S. Vegetti , D. J. Lagattuta, J. P. McKean, I.

Nature 481, 341–343 (19 January 2012) |



- “Aside from direct or indirect detection of the dark matter particles themselves, Einstein ring systems currently offer the best astrophysical test of the nature of the dark matter” (Li et al. 2016)
- measurements of approximately 100 strong lens systems with a detection limit of $M_{\text{low}} = 10^7 h^{-1} M_\odot$ would clearly distinguish CDM from WDM in the case where this consists of 7 keV sterile neutrinos

One gravitational potential or two? Forecasts and tests

Phil. Trans. R. Soc. A (2011) **369**, 4947–4961

doi:10.1098/rsta.2011.0369

BY EDMUND BERTSCHINGER*

conformal newtonian metric (choices and assumptions):

$$ds^2 = a^2(\tau)[-(1 + 2\Phi)d\tau^2 + (1 - 2\Psi)\gamma_{ij}dx^i dx^j]$$

for General Relativity (for standard matter components) in general

$$\Phi = \Psi$$

slip parameter

$$\gamma = \frac{\Phi}{\Psi}$$

$\gamma = 1$ compatible with GR

$\gamma \neq 1$ GR ruled out

One gravitational potential or two? Forecasts and tests

Phil. Trans. R. Soc. A (2011) **369**, 4947–4961
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BY EDMUND BERTSCHINGER*

conformal newtonian metric (choices and assumptions):

$$ds^2 = a^2(\tau) [-(1 + 2\Phi) d\tau^2 + (1 - 2\Psi)\gamma_{ij} dx^i dx^j]$$

geodesics

$$\frac{1}{a} \frac{d(av)}{d\tau} = -\nabla\Phi, \quad v^2 \ll 1 \text{ (CDM)} \longrightarrow \text{Jean's equation}$$

$$\frac{d\mathbf{v}}{d\tau} = -\nabla_{\perp}(\Phi + \Psi), \quad v^2 = 1 \text{ (photons)} \longrightarrow \text{lensing}$$

kinematics: Φ

on ~ 100 kph
galaxy scales

from galaxy
velocity dispersion

deflection angle: $\Phi + \Psi$

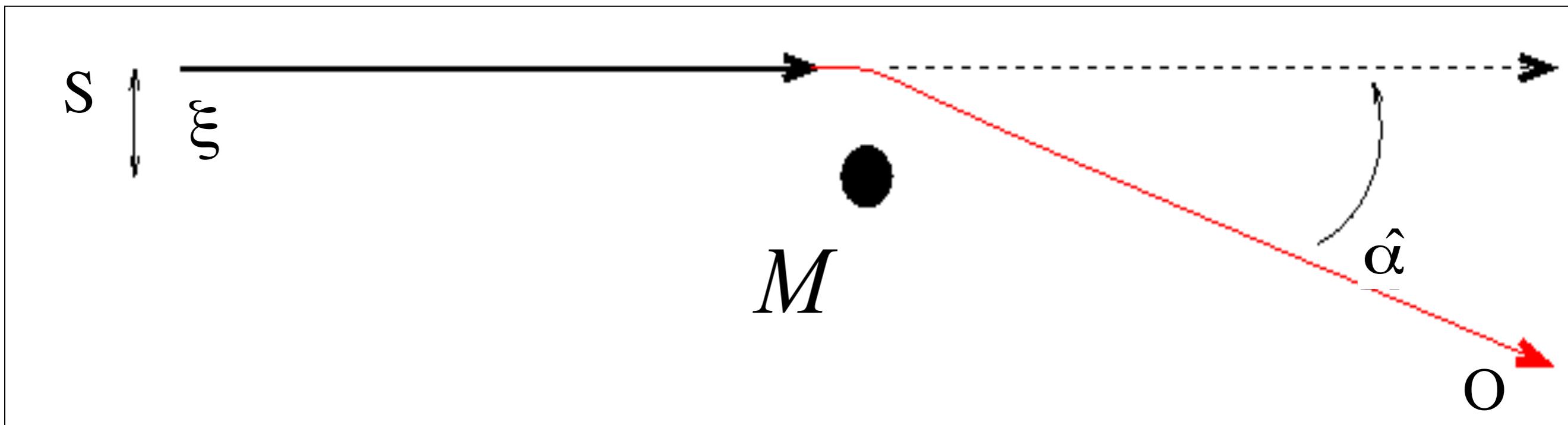
from strong lensing

BENDING OF LIGHT BY GRAVITY

Null geodesic,
Fermat principle

$$ds^2 = \left(1 + \frac{2\phi}{c^2}\right) c^2 dt^2 - \left(1 - \frac{2\phi}{c^2}\right) d\sigma^2$$

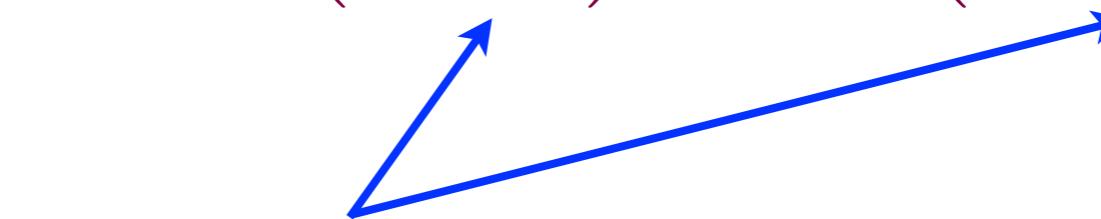
$$\frac{d\sigma}{dt} := c' = \sqrt{\frac{1 + 2\phi/c^2}{1 - 2\phi/c^2}} \simeq c \left(1 + \frac{2\phi}{c^2}\right)$$



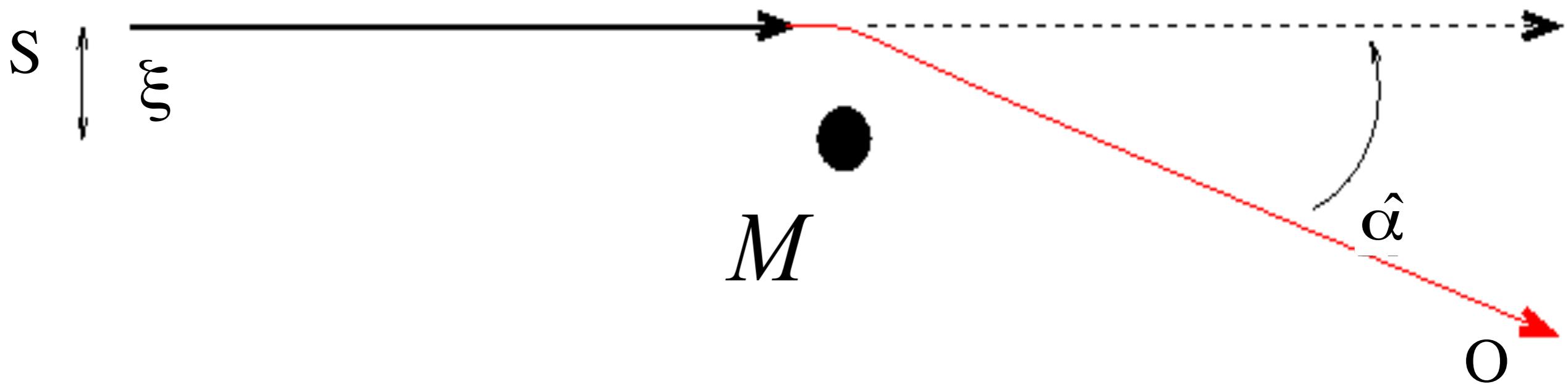
BENDING OF LIGHT BY (MODIFIED) GRAVITY

Null geodesic,
Fermat principle

$$ds^2 = \left(1 + \frac{2\psi}{c^2}\right) c^2 dt^2 - \left(1 - \frac{2\phi}{c^2}\right) d\sigma^2$$



peculiar gravitational potentials
(in GR $\psi = \phi$)



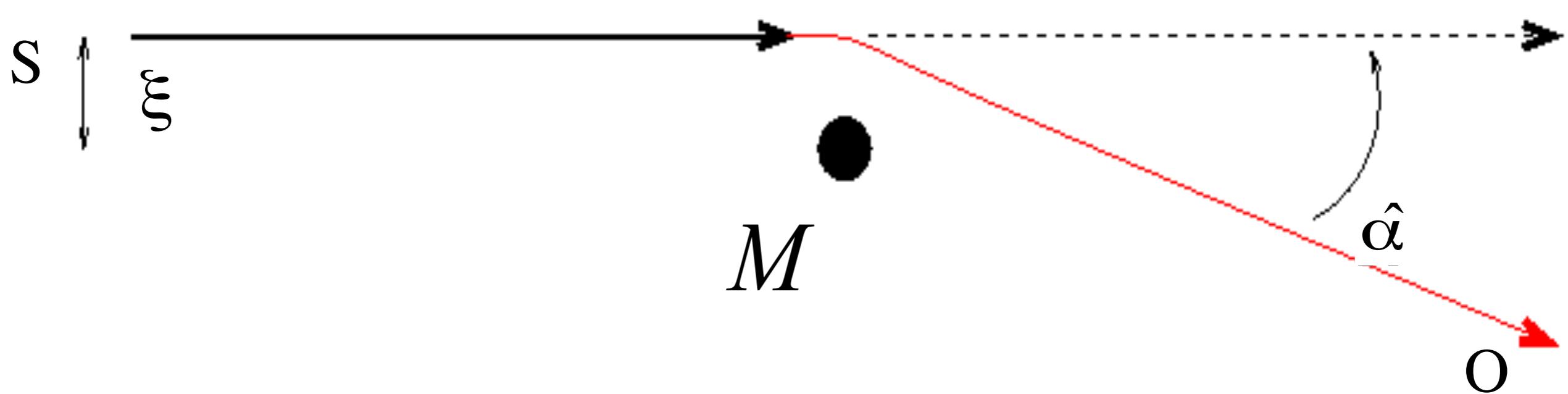
BENDING OF LIGHT BY (MODIFIED) GRAVITY

Null geodesic,
Fermat principle

$$ds^2 = \left(1 + \frac{2\psi}{c^2}\right) c^2 dt^2 - \left(1 - \frac{2\phi}{c^2}\right) d\sigma^2$$

$$\frac{d\sigma}{dt} = c' = c \sqrt{\frac{1 + \frac{2\psi}{c^2}}{1 - \frac{2\phi}{c^2}}} \simeq c \left(1 + \frac{\psi + \phi}{c^2}\right)$$

$$\frac{\phi}{\psi} = \gamma$$



Dynamical mass obtained from

$$\nabla^2 \psi = 4\pi G \rho$$

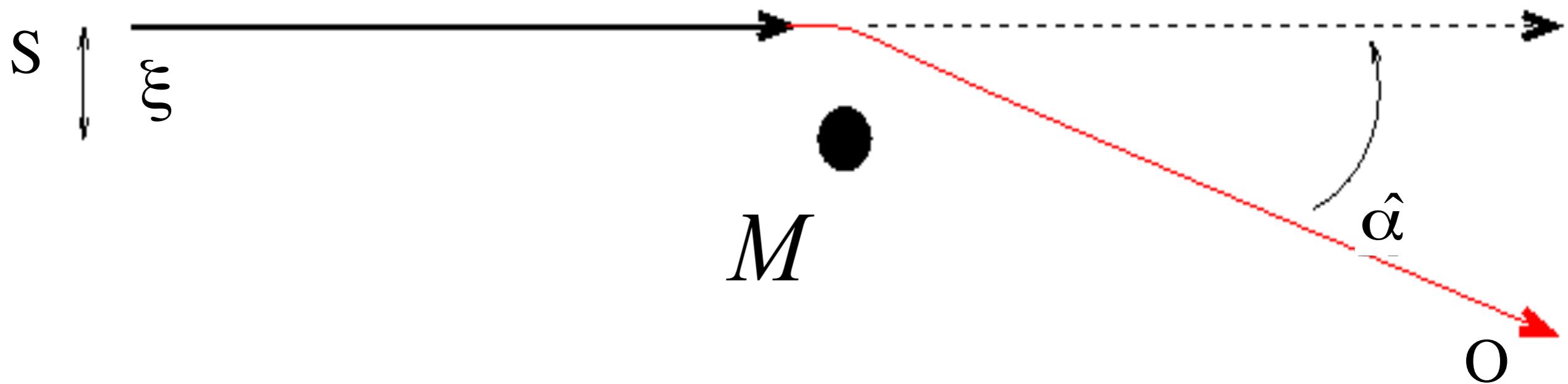
Use combination of lensing +
dynamics to test gravity

BENDING OF LIGHT BY (MODIFIED) GRAVITY

Null geodesic,
Fermat principle

$$ds^2 = \left(1 + \frac{2\psi}{c^2}\right) c^2 dt^2 - \left(1 - \frac{2\phi}{c^2}\right) d\sigma^2$$

$$\frac{d\sigma}{dt} = c' = c \sqrt{\frac{1 + \frac{2\psi}{c^2}}{1 - \frac{2\phi}{c^2}}} \simeq c \left(1 + \frac{\psi + \phi}{c^2}\right) = c \left(1 + \frac{2\phi}{c^2} \left[\frac{1 + \gamma}{2}\right]\right)$$



Dynamical mass obtained from

$$\nabla^2 \psi = 4\pi G \rho$$

Use combination of lensing +
dynamics to test gravity

Basic picture

Singular Isothermal Sphere profile: $\rho(r) = \frac{\sigma_v^2}{2\pi G r^2}$

Observed stellar velocity dispersion σ_{obs} probes Φ

slip parameter
$$\gamma = \frac{\Phi}{\Psi}$$

Lensing yields σ_{lens}^2 from $\Phi + \Psi$:

$$\sigma_{\text{lens}}^2 = \left(\frac{1 + \gamma}{2} \right) \sigma_{\text{obs}}^2$$

Real world:

- Non isothermal elliptical models
- Anisotropic velocity dispersion (assumptions)
- Seeing and aperture corrections

on ~ 100 kph
galaxy scales

kinematics: Φ
from galaxy
velocity dispersion

deflection angle: $\Phi + \Psi$
from strong lensing

Galaxy kinematics

The Jeans equation described the motion of a collection of stars. In spherical symmetry,

$$\frac{d}{dr} (\nu(r) \sigma_r^2(r)) + \frac{2\beta(r)}{r} \nu(r) \sigma_r^2(r) = \nu(r) \frac{d\Phi}{dr}, \quad (8)$$

where

$$\frac{d\Phi}{dr} = \frac{GM(r)}{r^2}, \quad \beta(r) := 1 - \frac{\sigma_t^2}{\sigma_r^2}, \quad (9)$$

where $M(r)$ is the total mass inside a sphere of radius r .

For $\beta = \text{constant}$,

$$\sigma_r^2(r) = \frac{G}{r^{2\beta} \nu(r)} \int_r^\infty (r')^{2\beta-2} \nu(r') M(r') dr'. \quad (10)$$

The total mass contained within a sphere with radius r is,

$$M(r) = 4\pi \int_0^r (r')^2 \rho(r') dr' = 4\pi \frac{r^{(3-\alpha)}}{3-\alpha} \frac{\rho_0}{r_0^{-\alpha}} \quad (11)$$

The actual velocity dispersion measured by observations is given by,

$$\bar{\sigma}_*^2 := \frac{\int_0^\infty dR R w(R) \int_{-\infty}^\infty dz \nu(r) \left(1 - \beta \frac{R^2}{r^2}\right) \sigma_r^2(r)}{\int_0^\infty dR R w(R) \int_{-\infty}^\infty dz \nu(r)}, \quad (19)$$

where $w(R)$ is the convolution of the atmospheric seeing σ_{atm} and the fiber aperture θ_{ap} .

$$\begin{aligned} \bar{\sigma}_*^2 = & \left[\frac{2}{1+\gamma} \frac{c^2}{4} \frac{D_S}{D_{LS}} \theta_E \right] \frac{2}{\sqrt{\pi}} \frac{(2\tilde{\sigma}_{\text{atm}}^2/\theta_E^2)^{1-\alpha/2}}{\xi - 2\beta} \\ & \times \left[\frac{\lambda(\xi) - \beta\lambda(\xi+2)}{\lambda(\alpha)\lambda(\delta)} \right] \frac{\Gamma(\frac{3-\xi}{2})}{\Gamma(\frac{3-\delta}{2})}, \end{aligned} \quad (20)$$

where

$$\tilde{\sigma}_{\text{atm}} \approx \sigma_{\text{atm}} \sqrt{1 + \chi^2/4 + \chi^4/40}, \quad \chi = \theta_{\text{ap}}/\sigma_{\text{atm}}. \quad (21)$$

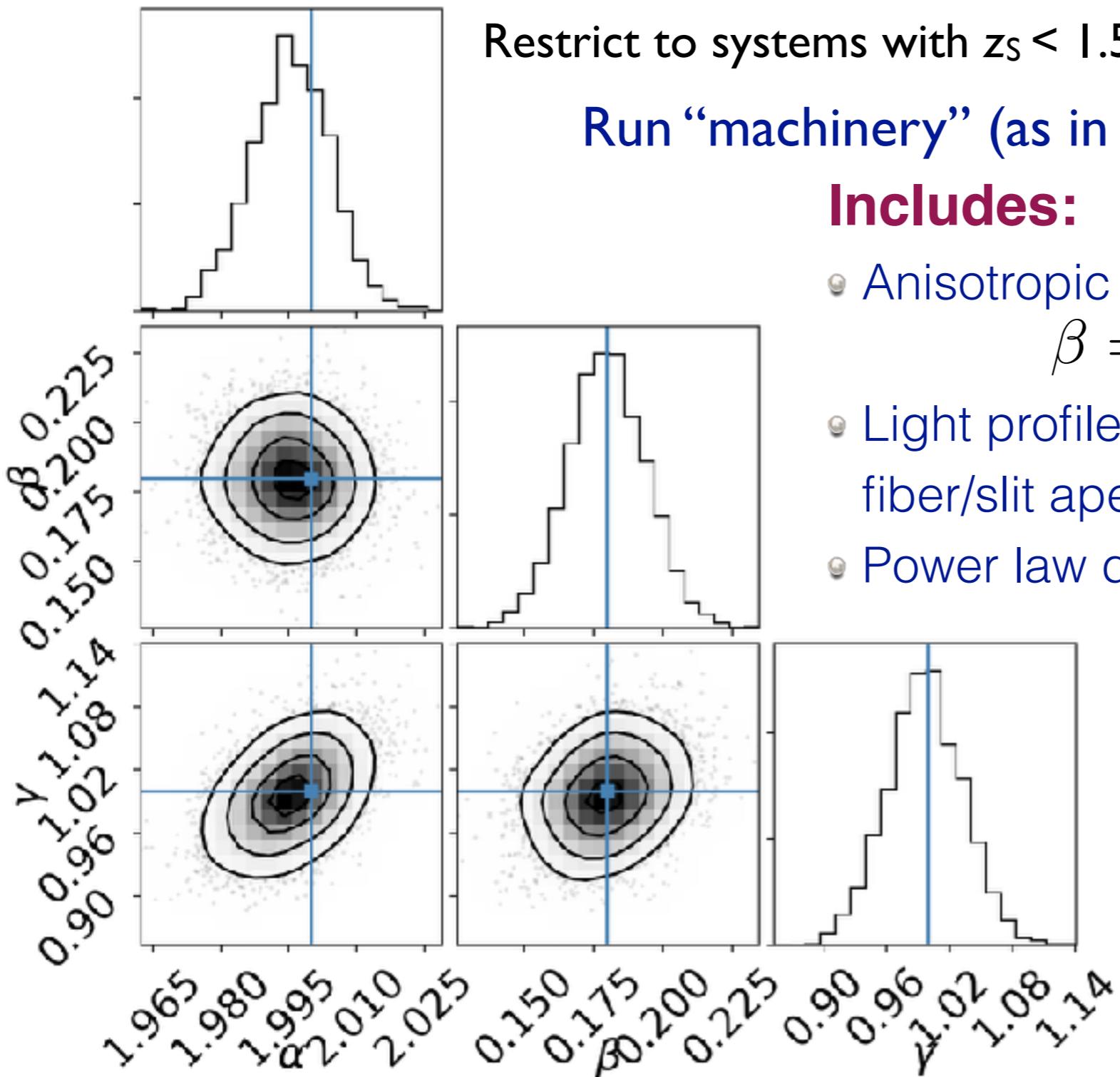
Archival Data

From (beta, public) “masterlens” catalog of 8577 SL systems
Galaxy-Galaxy systems, with known z_L, z_s, σ_v and lens modelling
Restrict to systems with $z_s < 1.5$: total of 110 systems

Run “machinery” (as in Schwab+2010, Cao+2017)

Includes:

- Anisotropic velocity dispersion $\beta = 1 - \sigma_t^2 / \sigma_r^2$
 - Light profiles, seeing and fiber/slit aperture effects
 - Power law density profile $\rho(r) \propto r^{-\alpha}$
- Priors on
- $$\beta = 0.180^{+0.014}_{-0.014}$$
- $$\alpha = 1.995^{+0.009}_{-0.009}$$

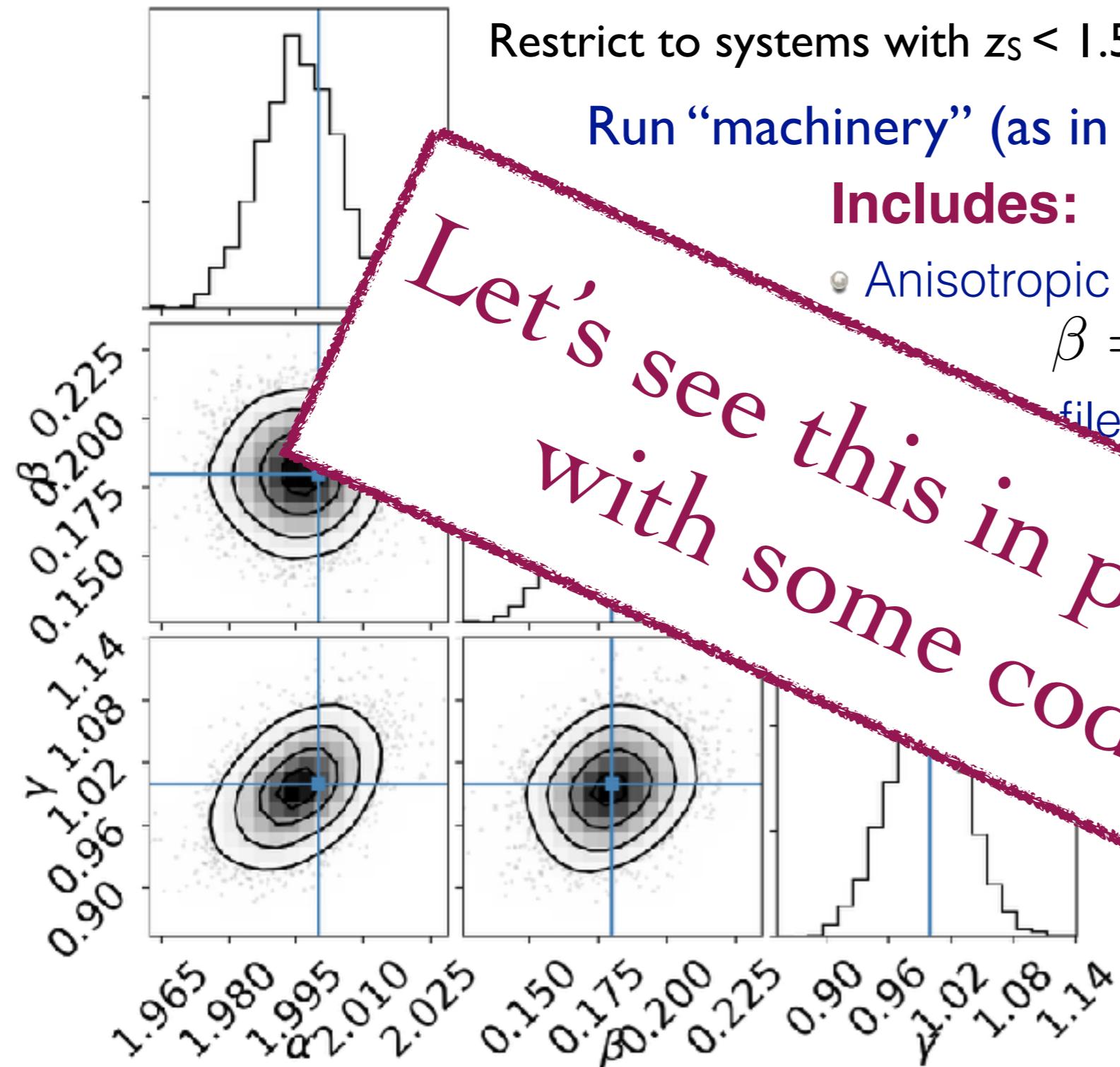


Slip parameter

$$\gamma = 1.001^{+0.022}_{-0.023}$$

Archival Data

From (beta, public) “masterlens” catalog of 8577 SL systems
Galaxy-Galaxy systems, with known z_L, z_s, σ_v and lens modelling
Restrict to systems with $z_s < 1.5$: total of 110 systems



Run “machinery” (as in Schwab+2010, Cao+2017)

Includes:

- Anisotropic velocity dispersion

$$\beta = 1 - \sigma_t^2 / \sigma_r^2$$

files, seeing and

shear effects

$$\nu(r) = \nu_0 \left(\frac{r}{r_0} \right)^{-\delta}$$

$$\rho(r) \propto r^{-\alpha}$$

$$= 0.180^{+0.014}_{-0.014}$$

$$\alpha = 1.995^{+0.009}_{-0.009}$$

parameter

$$\gamma = 1.001^{+0.022}_{-0.023}$$

PART IV

WEAK LENSING

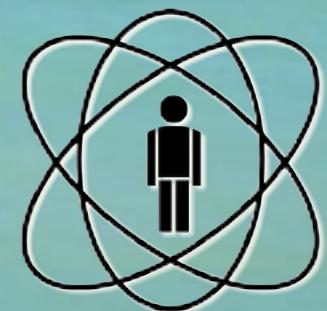
MARTÍN MAKLER

ICAS/IFI/CONICET & UNSAM & CBPF

+ RENAN ALVES, JOÃO FRANÇA, ELIZABETH GONZALEZ,
GIULYA SOUZA, EDUARDO VALADÃO, ANIBAL VARELA



ICIFI



Observatorio
Astronómico
de Córdoba



UNC

Universidad
Nacional
de Córdoba

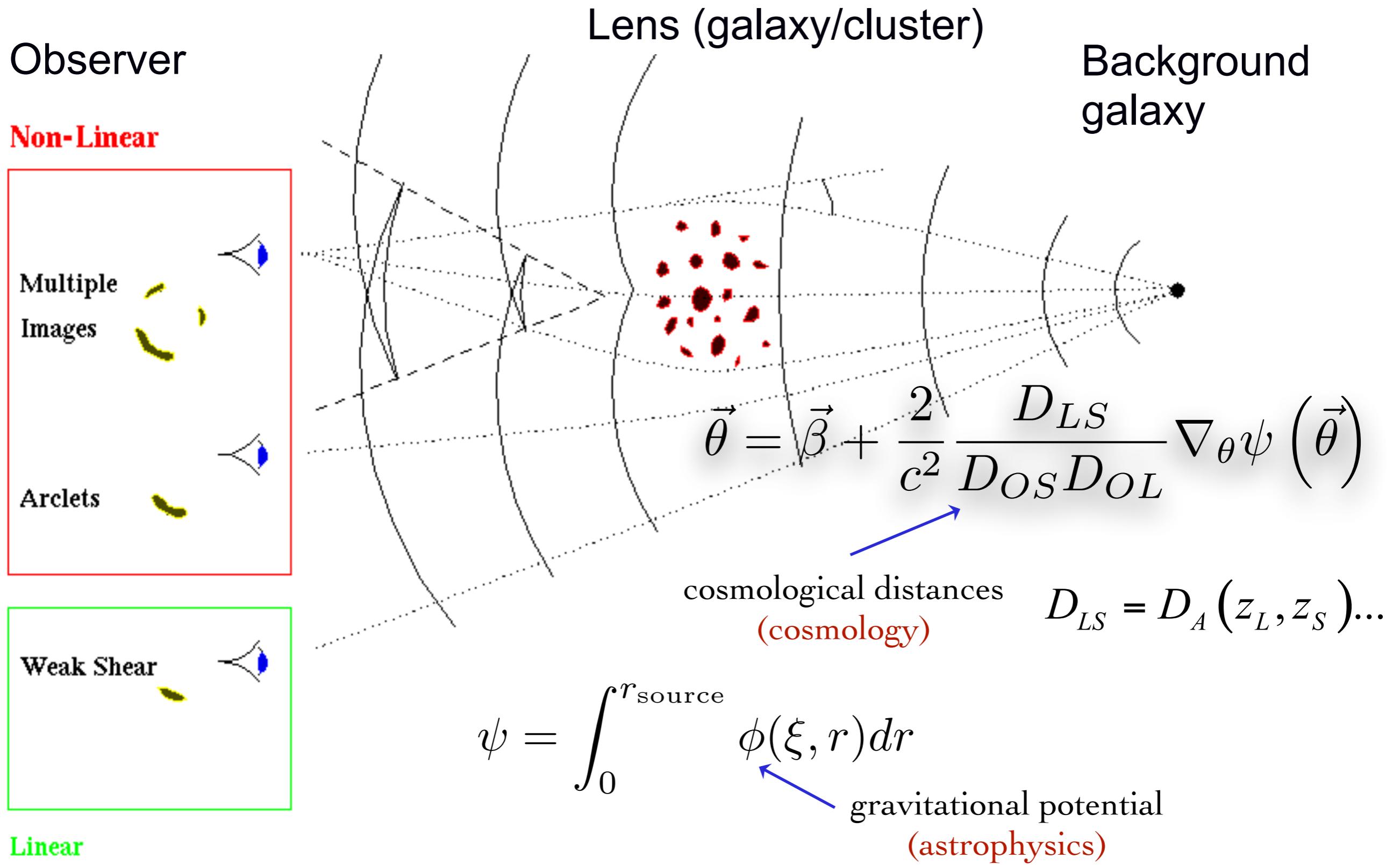


UNSAM

CONICET



Weak and Strong Lensing Effects



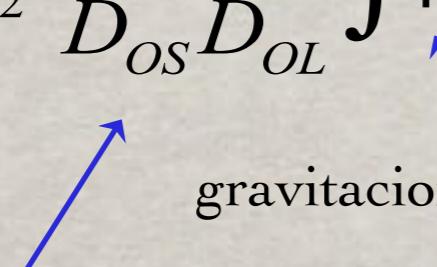
LENS MAPPING

- ▶ mapping image → source

$$\frac{\partial \beta_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial^2 \Psi(\vec{\theta})}{\partial \theta_i \partial \theta_j}$$

- ▶ single plane

$$\Psi = \frac{2}{c^2} \frac{D_{LS}}{D_{OS} D_{OL}} \int \phi(\xi, z) dz$$



 cosmological distances gravitacional potential

$$D_{LS} = D_A(z_L, z_S) \dots$$

- ▶ autovalores:

$$\mu_1 = \frac{1}{1-\kappa+\gamma}, \mu_2 = \frac{1}{1-\kappa-\gamma}$$

- ▶ local magnifications and axis ratio:

$$\mu = \mu_1 \mu_2 \quad r = \left| \frac{\mu_1}{\mu_2} \right|$$

- ▶ critical surface density

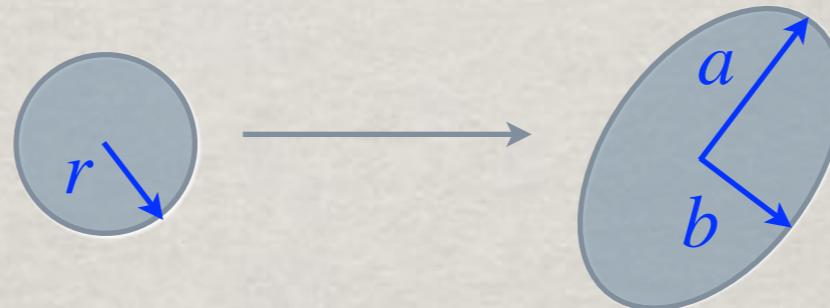
$$\Sigma_{\text{crit}} = \frac{c^2}{4\pi G} \frac{D_{OS}}{D_{OL} D_{LS}}$$

- ▶ convergence

$$\kappa = \frac{\Sigma}{\Sigma_{\text{crit}}}$$

LINEAR MAPPING

- Circular sources



$$a = \left(\frac{1}{1 - \kappa - \gamma} \right) r$$
$$b = \left(\frac{1}{1 - \kappa + \gamma} \right) r$$

- Magnifications

$$\mu = \frac{A_{\text{imagem}}}{A_{\text{fonte}}} = [(1 - \kappa)^2 - \gamma^2]^{-1}$$

- Ellipticity

$$\epsilon := \frac{a - b}{a + b} = \frac{\gamma}{1 - \kappa} =: g$$

REAL SOURCES

- Center of the object (image)

$$\bar{\theta}_i = \frac{\int d^2\theta q_I [I(\theta)] \theta_i}{\int d^2\theta q_I [I(\theta)]}$$

- Second order momenta

$$Q_{ij} = \frac{\int d^2\theta q_I [I(\theta)] (\theta_i - \bar{\theta}_i)(\theta_j - \bar{\theta}_j)}{\int d^2\theta q_I [I(\theta)]}$$

- Area

$$\Omega = (Q_{11}Q_{22} - Q_{12}^2)^{1/2}$$

- Ellipticity

$$\epsilon := \frac{Q_{11} - Q_{22} + 2i Q_{12}}{Q_{11} + Q_{22} + 2(Q_{11}Q_{22} - Q_{12}^2)^{1/2}}$$

REAL SOURCES

• Second order momenta

$$Q_{ij} = \frac{\int d^2\theta q_I[I(\theta)](\theta_i - \bar{\theta}_i)(\theta_j - \bar{\theta}_j)}{\int d^2\theta q_I[I(\theta)]}$$

• Area

$$\Omega = (Q_{11}Q_{22} - Q_{12}^2)^{1/2}$$

• Ellipticity

$$\epsilon := \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22} + 2(Q_{11}Q_{22} - Q_{12}^2)^{1/2}}$$

• Linear distortion

$$\epsilon_S = \frac{\epsilon_I - g}{1 - g^* \epsilon_I} \quad g_i := \frac{\gamma_i(\theta)}{1 - \kappa(\theta)} \quad |g| \leq 1$$

WEAK LENSING

- Slight shear in background galaxies (change in axis + size)
- Weak lensing regime

$$\epsilon = \epsilon_I = \epsilon_S + g$$

- “weak lensing fundamental theorem”:

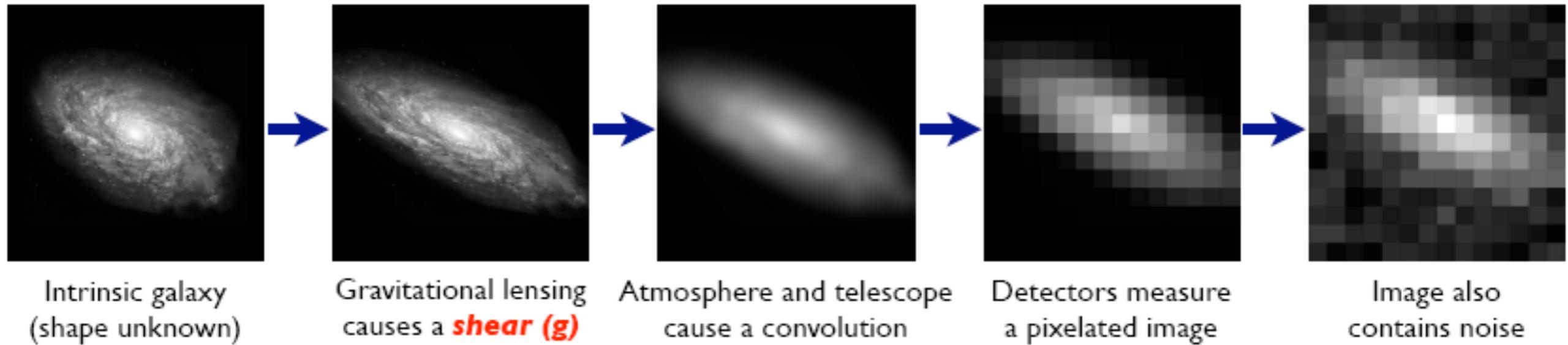
$$\langle \epsilon \rangle = g$$

- but

$$g \ll \epsilon$$

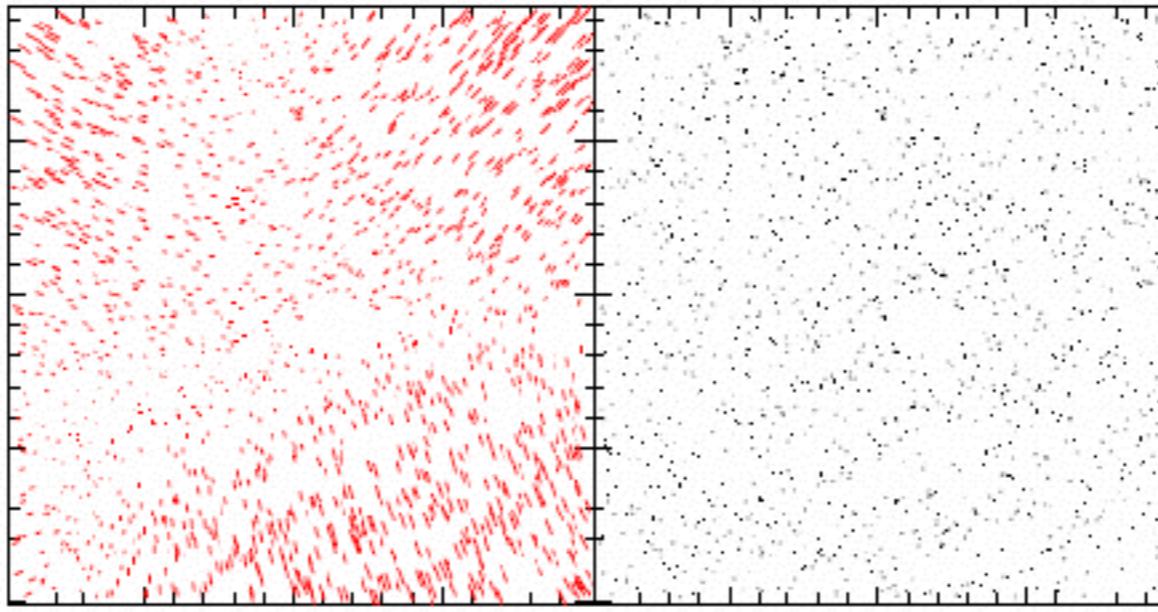
- observational, instrumental, computational and theoretical challenges!

Technical challenges



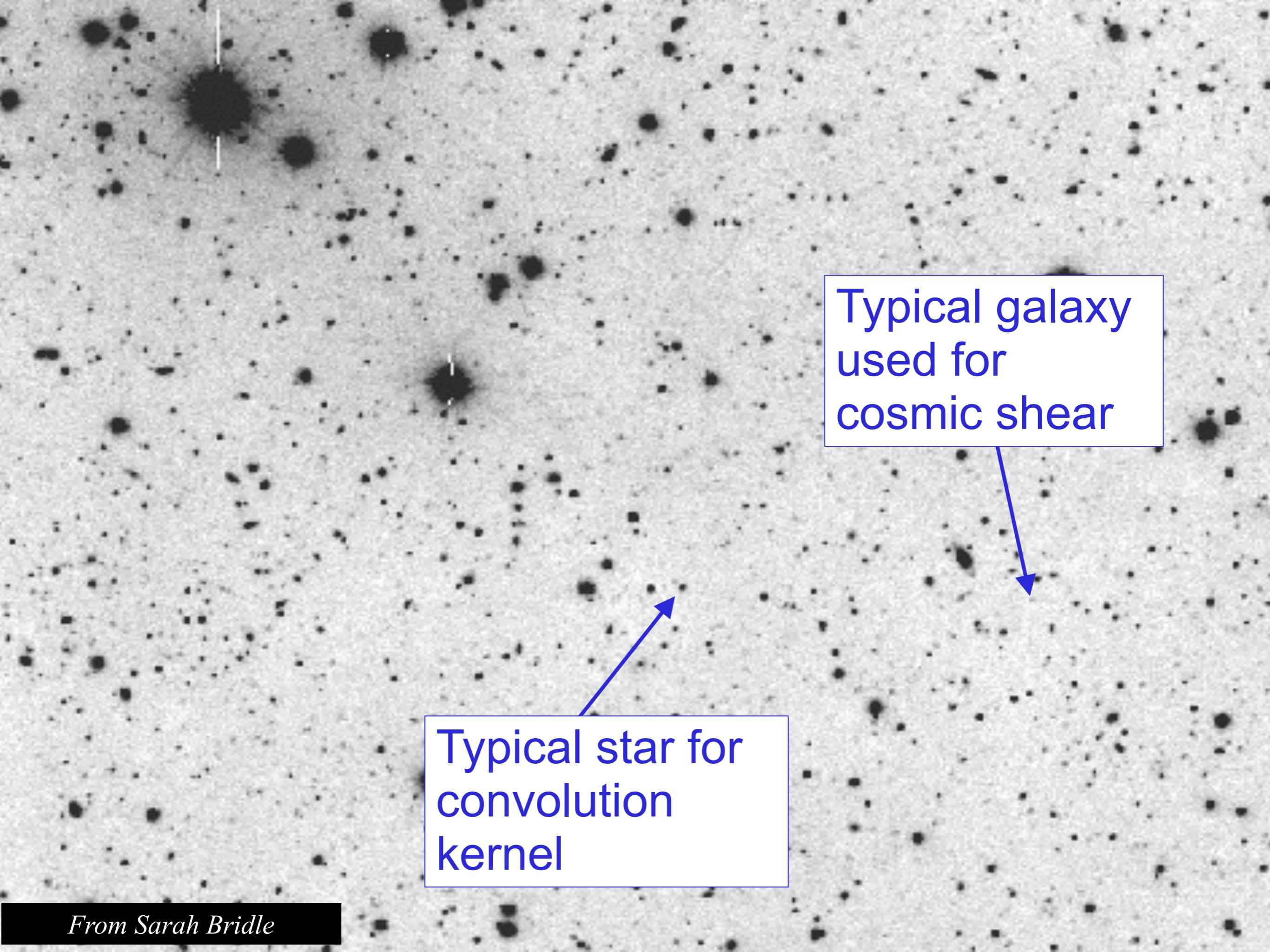
Sarah Bridle

- Model the PSF distortion (many sources)
 - measure a large number of stars in the field



CFHTLS Deep 1
Gavazzi, Soucail 2006

- Difficulties: saturated stars, charge transfer efficiency, halos, tracking...



Typical galaxy
used for
cosmic shear

Typical star for
convolution
kernel

Measuring weak lensing

- Signal $\langle \varepsilon_I \rangle = \gamma$ ($\varepsilon_I = \varepsilon_S + \gamma$)
- Noise $\sigma_\varepsilon = \langle |\varepsilon_S|^2 \rangle^{1/2} \sim 0.3 \gg \gamma$
- Win over the noise by averaging over a large number of galaxies

Regime	γ	$\gamma/\sigma_\varepsilon$	N_{gal} for S/N ~ 1
weak lensing by clusters	0.03	0.1	10^2
galaxy-galaxy lensing	0.003	0.01	10^4
cosmic shear	0.001	0.003	10^5

Much more galaxies for precision measurements needed.

(de Kilbinger)

2 Regimes and Methods

Lensing by galaxies and clusters

- Larger signal
- Center of reference
- Model/profile fitting
- Individual objects or *Stacking* of the signal

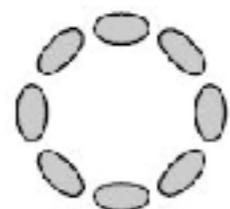
Large-scale structure

- Convergence maps (also in clusters)
- Correlations:
 - power spectrum, correlation function
 - among different probes, z-bins, CMB, etc.

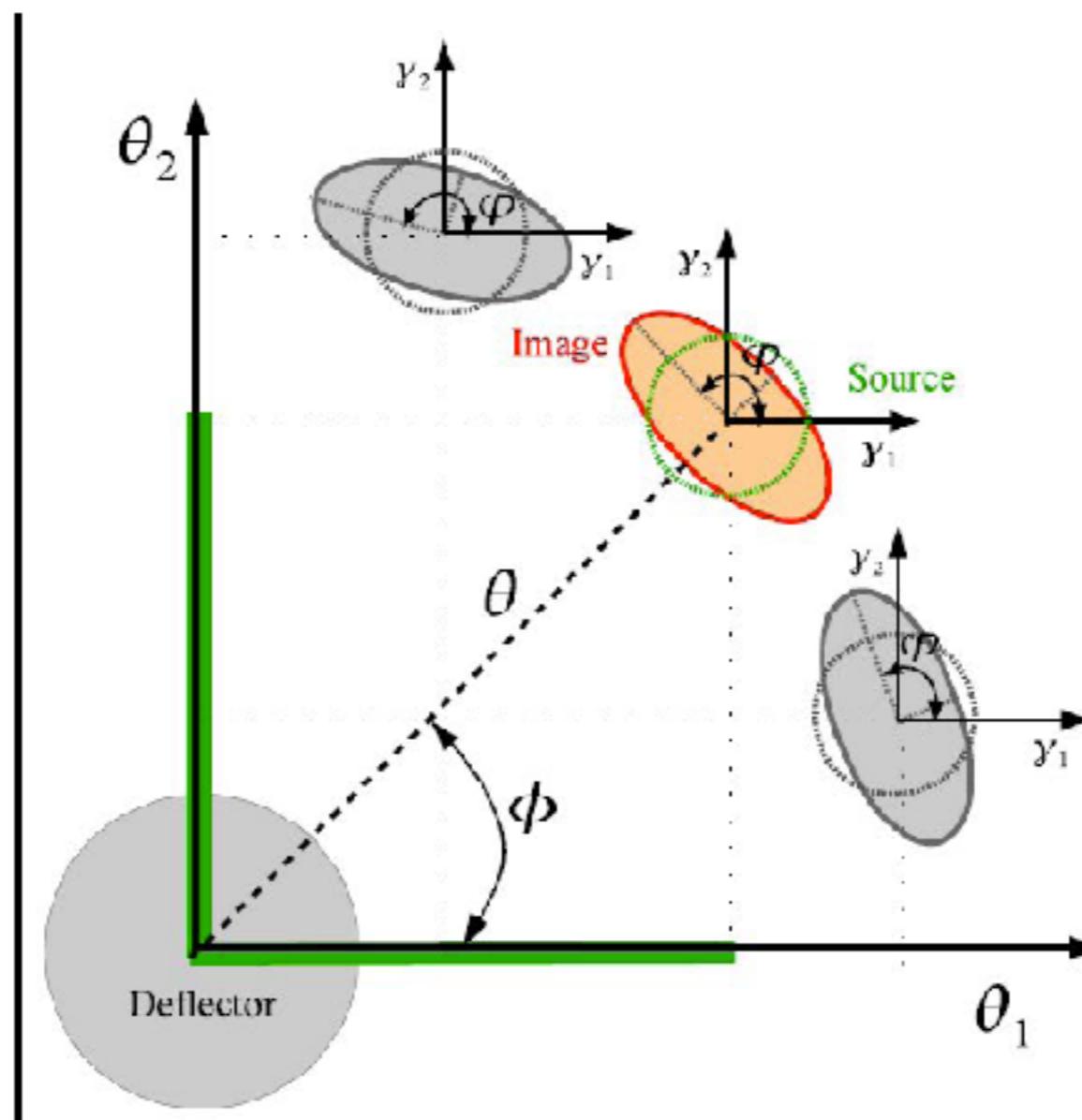
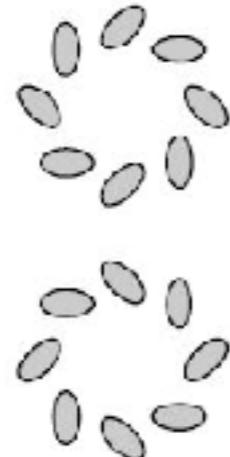
Components of the Shear

Figueiró 2011

Tangential alignment



Cross alignment



Mean shear in circles

$$\langle \gamma_t(\theta) \rangle = \bar{\kappa}(\theta) - \langle k(\theta) \rangle$$

Mean shear and radial profiles

It is possible to show that

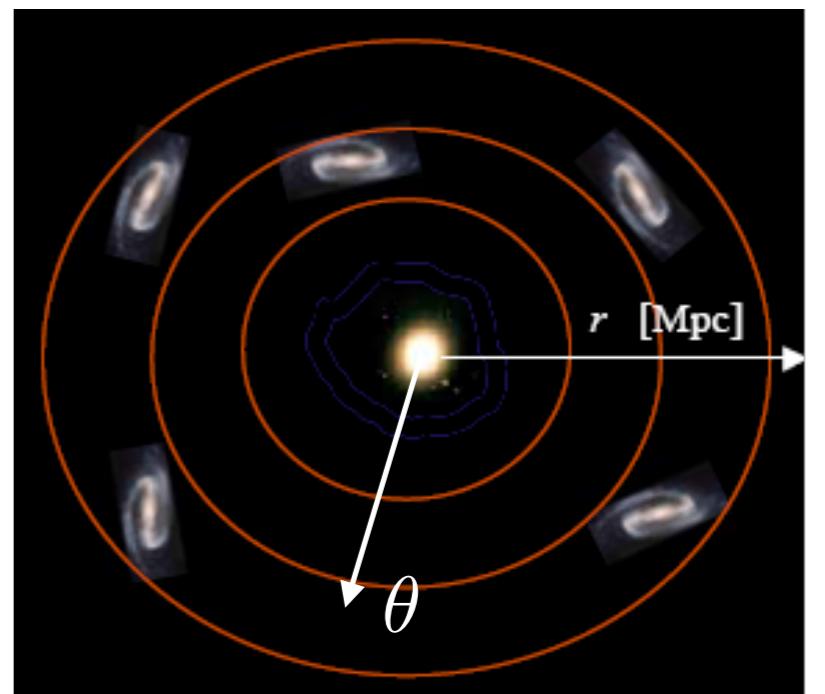
$$\langle \gamma_t(\theta) \rangle = \bar{\kappa}(\theta) - \langle k(\theta) \rangle$$

mean along a circle of the tangential component of the shear

mean within a disk of radius θ

mean on the circle

In practice: mean un anulii
(radial bins)



Stacking of the signal

Need (and possibility) to increase the S/N

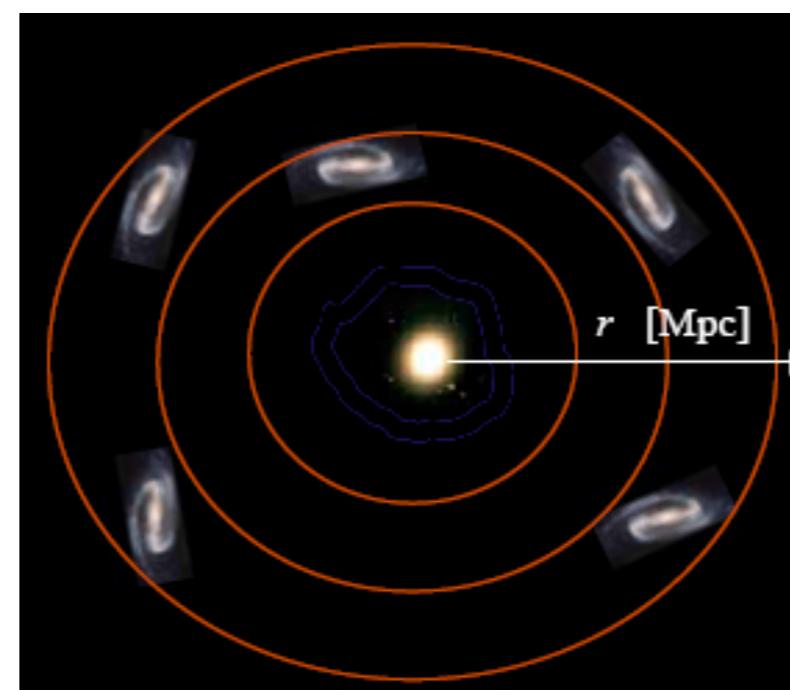
Combine data on many galaxies or clusters: mean signal

Physical signal and models: $\Sigma(r) \rightarrow$ multiply by Σ_{crit}

$$\Sigma_{\text{crit}} \times \langle \gamma_t(\theta) \rangle = \bar{\kappa}(\theta) - \langle k(\theta) \rangle \times \Sigma_{\text{crit}}$$

$$\Sigma_{\text{crit}} \times \langle \gamma_t(r) \rangle = \bar{\Sigma}(r) - \langle \Sigma(r) \rangle := \Delta\Sigma(r)$$

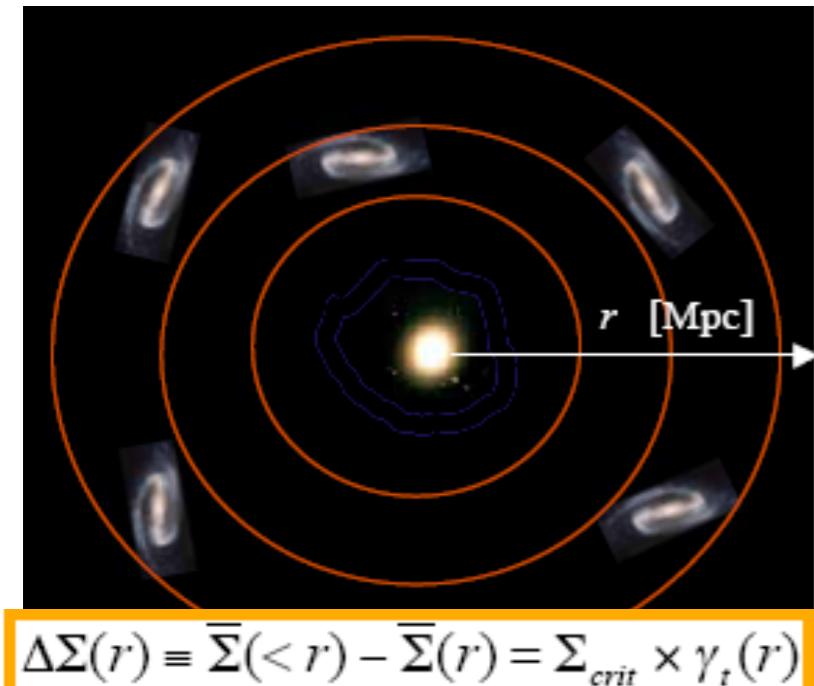
↓
redshifts ↓
shapes



↓
model

Mass reconstruction in clusters (radial profile)

- Measure the tangential shear to get $\Delta\Sigma$

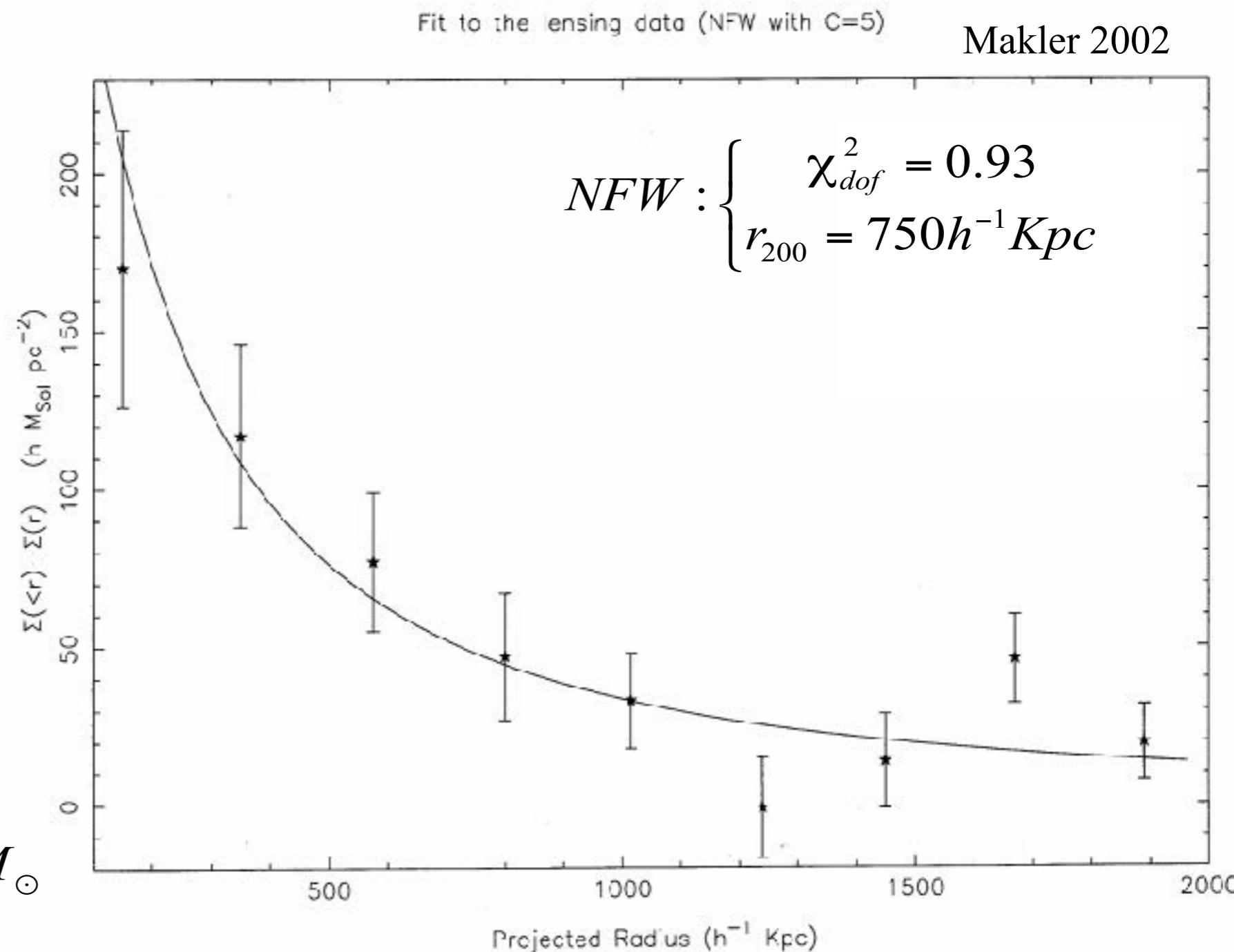


$$\Sigma_{crit} = \frac{c^2}{4\pi G} \frac{D_{OS}}{D_{OL} D_{LS}}$$

- Masses:

$$\bar{M}_{NFW} = (1.0 \pm 0.2) 10^{14} h^{-1} M_\odot$$

$$\frac{\Delta M_{200}}{M_{200}} \simeq 2.5\%$$



42 clusters (RASS/SDSS), Sheldon, et al., ApJ 554, 88 (2001)

Modeling the mass profile

Interpretation with the halo model: halos x galaxies correlation

- 1-halo term: matter density in the halo

Exemple: NFW $\rho(r) = \frac{\rho_0}{(r/r_s)(1+r/r_s)^2}$ compute $\Sigma(r)$

- 2-halo term: correlation with other halos
(large scale structure) $\rho(r) = b\bar{\rho}_m\xi(r)$ where

$$\xi(r) = \frac{1}{2\pi^2} \int dk k^2 P(k) \frac{\sin(kr)}{kr} \quad b(\nu) = 1 - A \frac{\nu^a}{\nu^a + \delta_c^a} + B\nu^b + C\nu^c$$

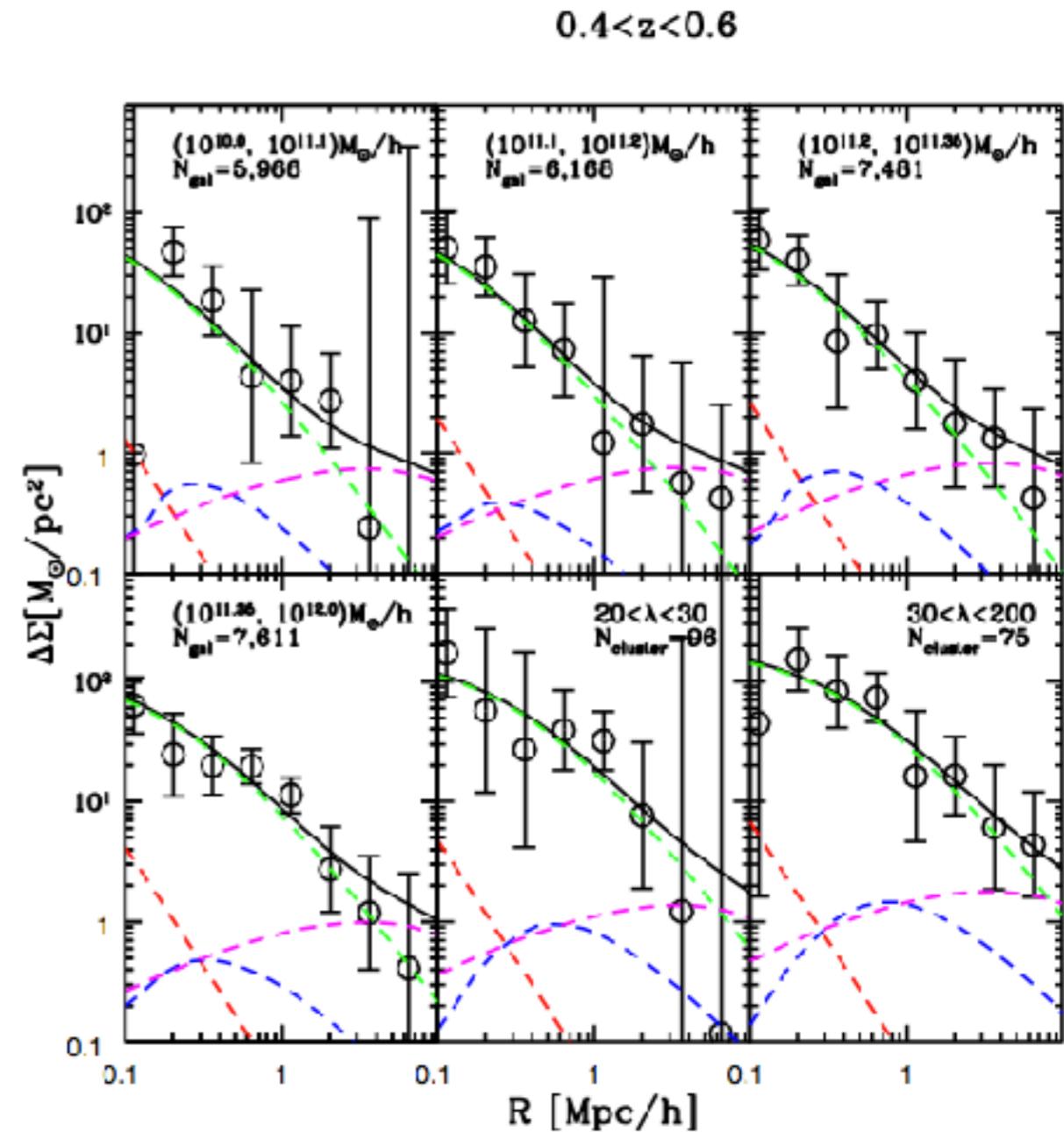
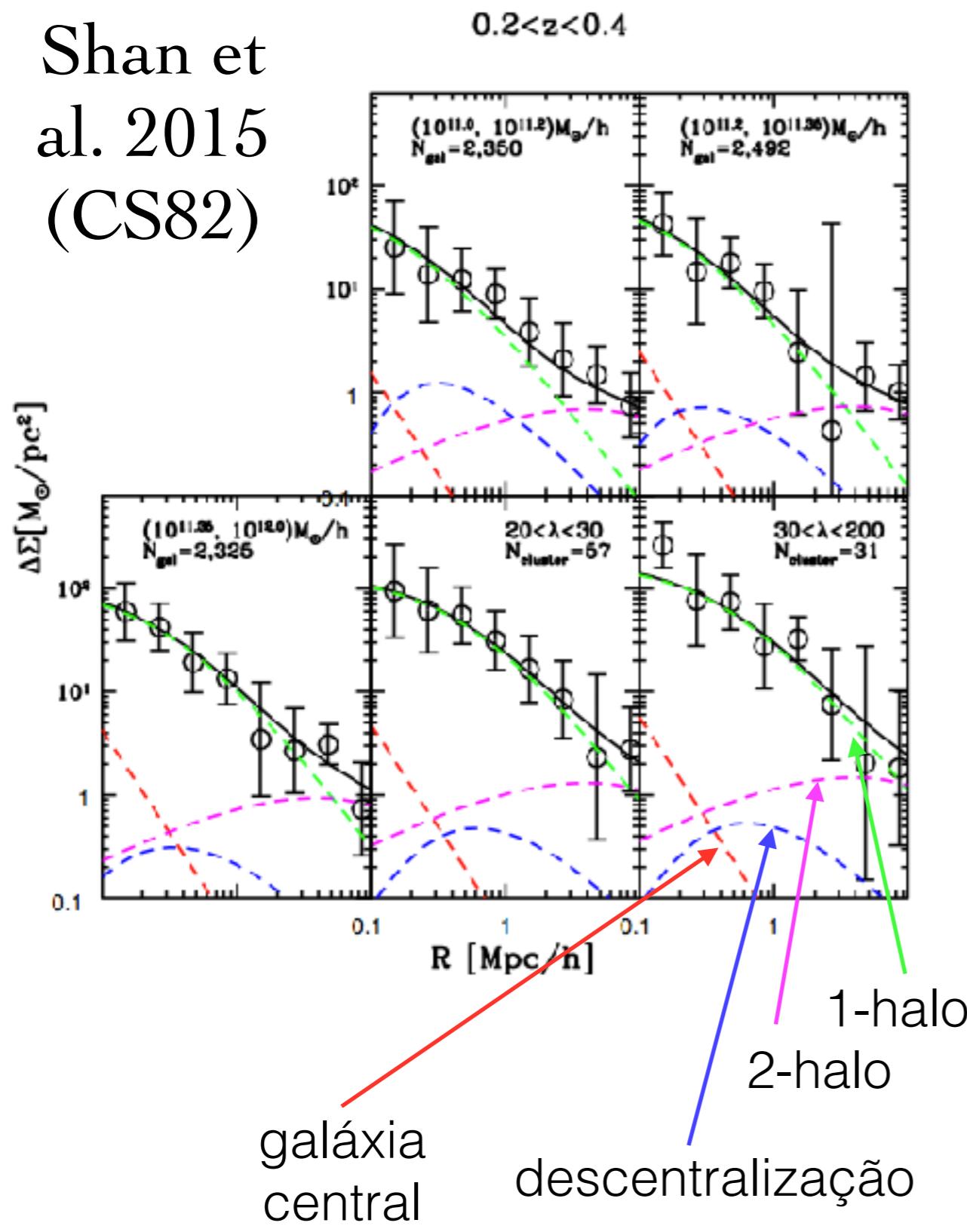
$$\nu = \delta_c/\sigma(M) = 1.686/\sigma(M) \quad \sigma(R) = \frac{1}{2\pi^2} \int dk k^2 P(k, z) \hat{W}^2(k, R)$$

$$R = (3M/4\pi\bar{\rho}_m)^{1/3} \quad (\text{see mass function})$$

- Term for the offset of the profile with respect to other center
- Central potential (central galaxy, e.g. SIS)

Example: Cluster mass calibration

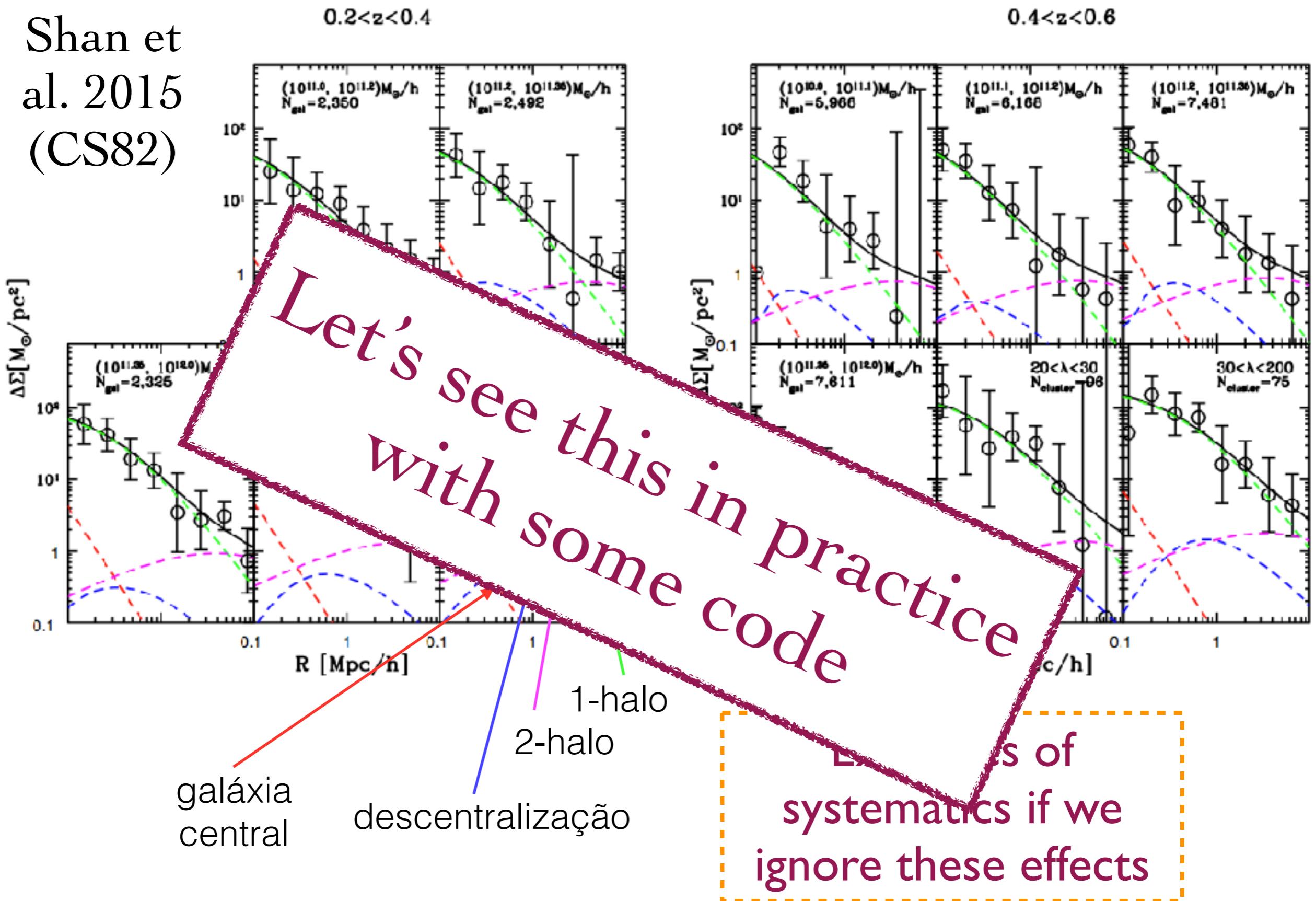
Shan et
al. 2015
(CS82)



Examples of
systematics if we
ignore these effects

Mass calibration

Shan et
al. 2015
(CS82)



Gravitational Lensing: a tool for astrophysics and cosmology

- From planets to the Large Scale Structure of the Universe
- Essentially 21st century science, with many discoveries in the past 10 years
- These were by no means review talks on the main topics of lensing
 - Illustrations for pedagogical purposes
 - A few worked examples and use cases that can be followed at a basic level, reproduced and run in the example code provided in the repository
- Many many subjects not addressed, such as
 - Time delays, H_0 , QSO (micro-)lensing
 - Implications for DM properties
 - Astrophysical consequences of profiles, etc.
 - Gravitational telescope
 - Arcfinding, automated modeling, ML applications

Gravitational Lensing: a tool for astrophysics and cosmology

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 - A few worked examples and use cases that can be followed at a basic level, reproduced and run in the example code provided in the repository
- **Interdisciplinary** field involving from fundamental physics to data reduction, including image processing, statistics, simulations, theory and semi-analytic modeling
- Lots of room for interesting work to do
- New dynamics of large projects

Thank You

