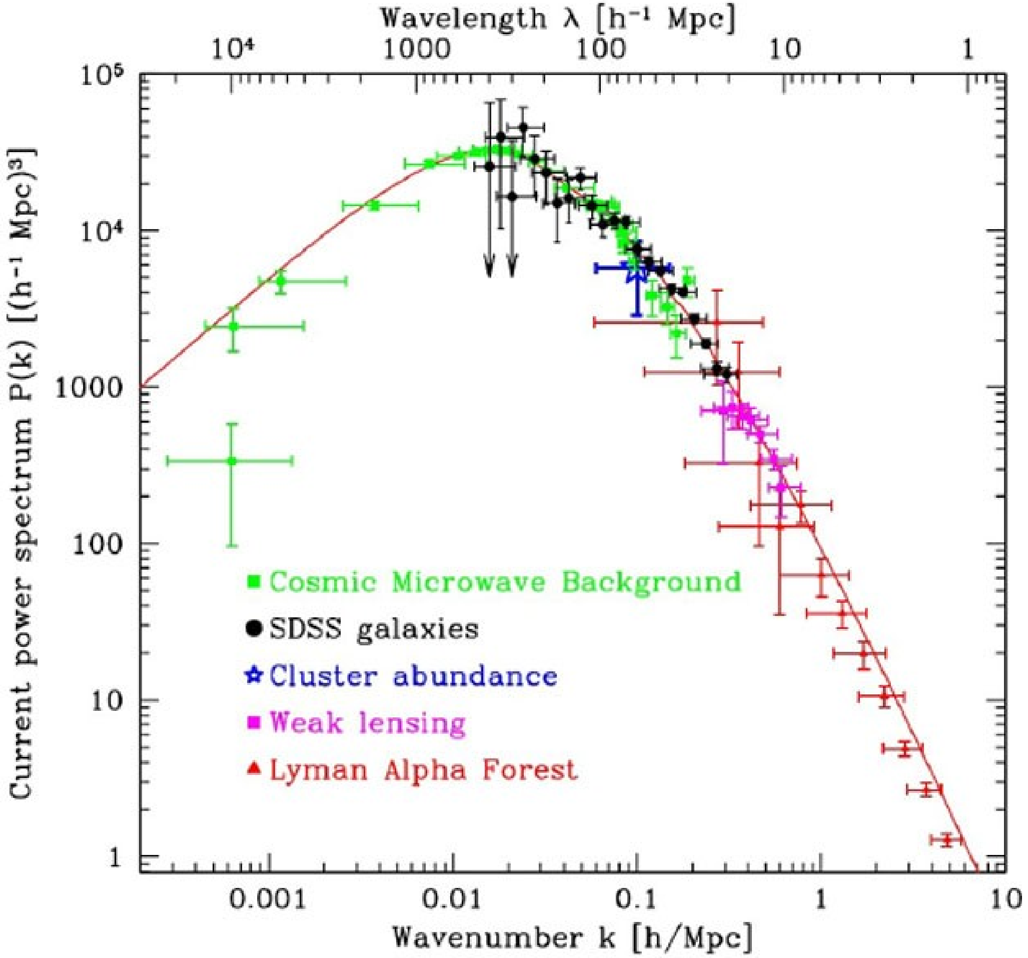
Data analysis in Cosmology



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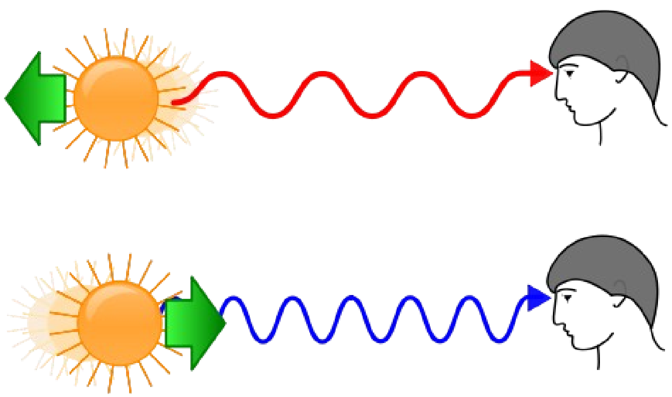
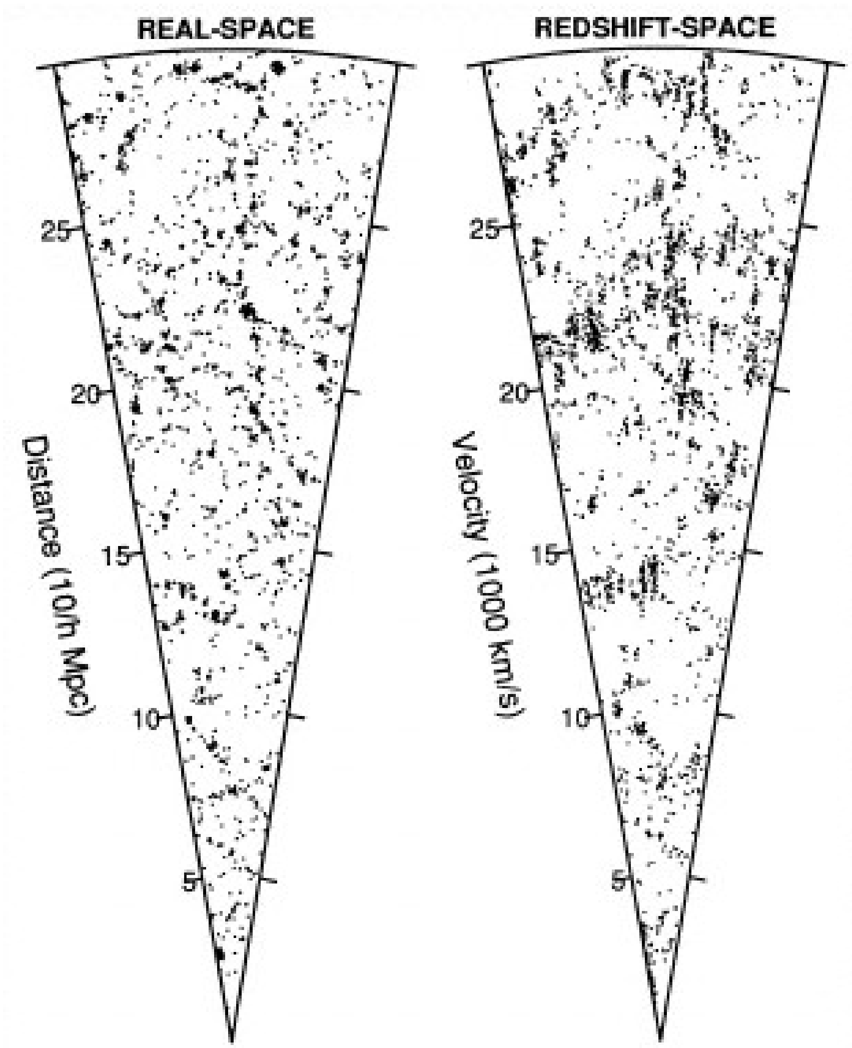
Redshift space

- Observations



$$z = \frac{\lambda_{obs} - \lambda_{emit}}{\lambda_{emit}}$$

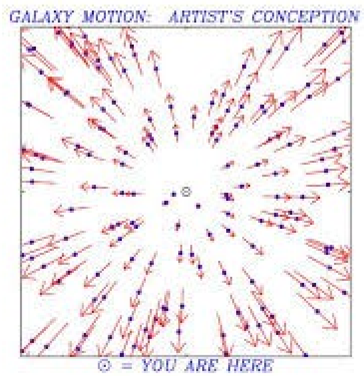


Redshift space

- Two contributions $1 + z_{obs} = (1 + z_{cos})\left(1 + \frac{v_{||}}{c}\right)$

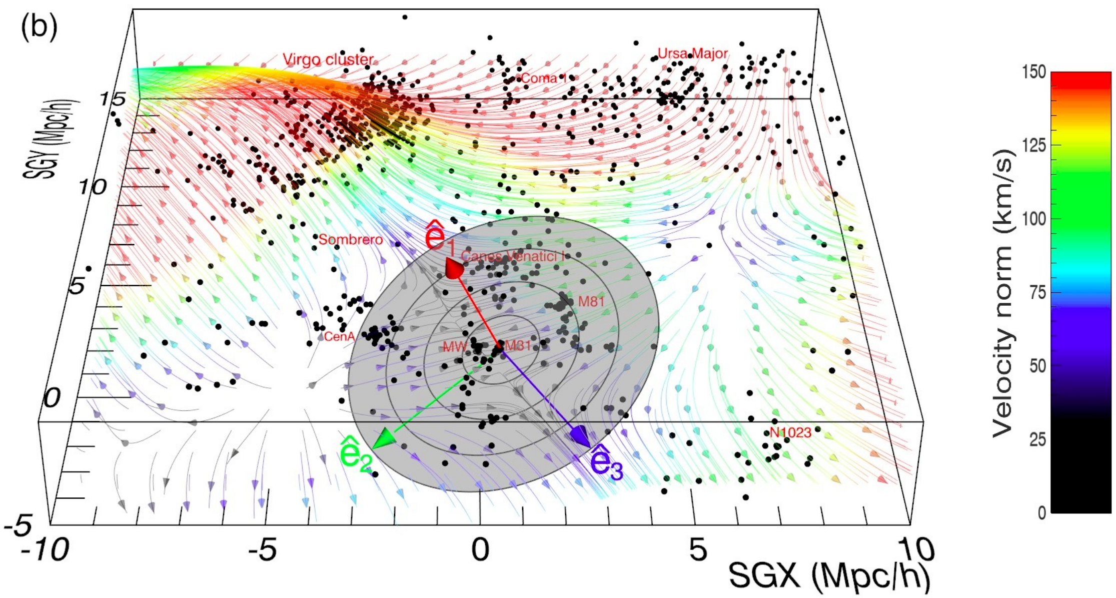
~ 0.5

$\sim 10^{-3}$



$$v_{||} = \mathbf{v} \cdot \hat{\mathbf{r}}$$

$$1 + z_{cos} = \frac{\lambda_{obs}}{\lambda_{emit}} = \frac{a_{now}}{a_{emit}}$$



Redshift space

- Two contributions $$1 + z_{obs} = (1 + z_{cos}) \left(1 + \frac{v_{||}}{c}\right)$$

In comoving coordinates (*z* = *H r*)

$$\mathbf{s} = \mathbf{r} + \frac{(1 + z_{cos}) v_{||}}{H(z_{cos})} \hat{\mathbf{r}}$$

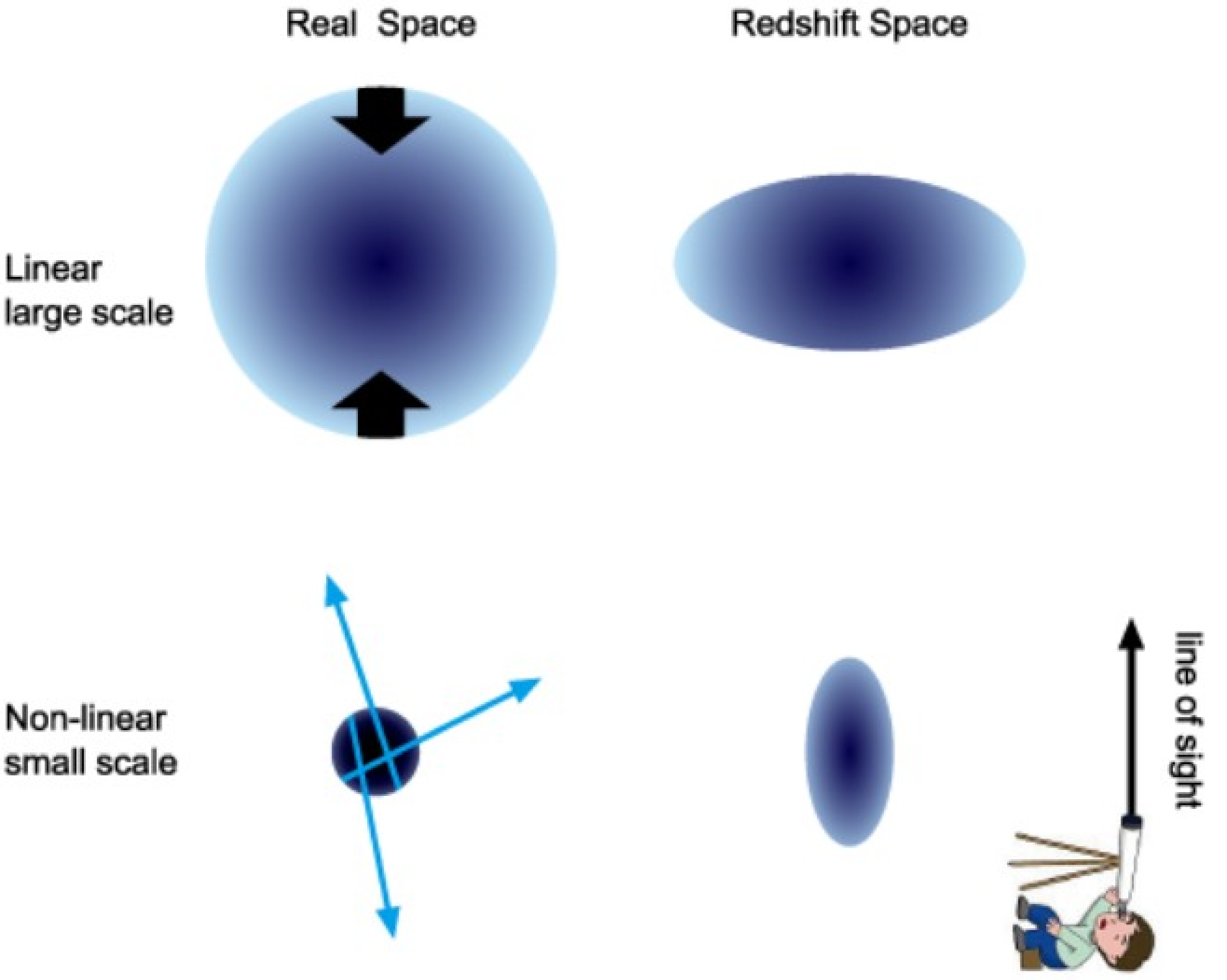
$$r(z) = H_0^{-1} \int_0^z \frac{dz'}{\sqrt{\Omega_m (1+z)^3 + (1 - \Omega_m)}} .$$

$$r(0.5) = 1.32 Gpc/h$$

$$\left. \frac{(1 + z_{\rm cos}) v_{||}(\boldsymbol{r})}{H(z_{\rm cos})} \right|_{z_{\rm cos}=0.5} \simeq 1.18 \frac{v_{||}}{100\,{\rm km/s}} \,[{\rm Mpc}/h],$$

Eventhough small produces distorsions in clustering

Redshift space distorsions (RSD)



Kimika Saito

RSD

$$\mathbf{z} = H_0 \mathbf{r} \left(1 + \frac{v_{||}}{H_0 r}\right)$$

Mass preservation

$$\rho_z(z) d^3 z = \rho(r) d^3 r$$

$$(1 + \delta(z)) d^3 z = (1 + \delta(r)) d^3 r$$

$$H_0 = 1$$

$$\frac{d^3 r}{d^3 z} = \left(\frac{r}{z}\right)^2 \frac{dr}{dz} = \frac{1}{(1 + v_r/r)^2} \left(1 + \frac{\partial v_r}{\partial r}\right)^{-1}$$

$$1 + \delta^{(z)}(\mathbf{z}) = \frac{r^2}{(r + v_r^2)} \left(1 + \frac{\partial v_r}{\partial r}\right)^{-1} [1 + \delta(\mathbf{r})] .$$

To first order

$$\delta^{(z)}(\mathbf{z}) \approx \delta(\mathbf{r}) - \cancel{2\frac{v_r}{r}} - \frac{\partial v_r}{\partial r} .$$

RSD

Plane parallel approximation

$$\hat{\mathbf{r}} \simeq const$$

In Fourier space

$$\mathbf{v_k} = v_\mathbf{k} \hat{\mathbf{k}} \,.$$

$$\frac{\partial v_r}{\partial r} \to i(\hat{\mathbf{r}} \cdot \mathbf{k})(\hat{\mathbf{r}} \cdot \hat{\mathbf{k}}) v_\mathbf{k}$$

$$\delta_\mathbf{k}^{(z)} \approx \delta_\mathbf{k} - i(\hat{\mathbf{r}} \cdot \hat{\mathbf{k}})^2 k v_\mathbf{k} \,.$$

$$\mu \equiv \cos(\theta)$$

RSD

$$\beta = \frac{f}{b} = \frac{\text{Growth}}{\text{Bias}}$$

Power spectrum

From perturbation theory

$$-ikv_{\mathbf{k}} = aH\beta\delta_{\mathbf{k}} \approx \beta\delta_{\mathbf{k}},$$

$$\delta^{(z)}_{\mathbf{k}} \approx \delta_{\mathbf{k}} - i(\hat{\mathbf{r}}\cdot\hat{\mathbf{k}})^2 k v_{\mathbf{k}}. \quad \Rightarrow \quad \delta^{(z)}_{\mathbf{k}} \approx [1 + \beta(\cos\vartheta_{\mathbf{k}})^2]\delta_{\mathbf{k}}.$$

$$\boxed{P^{(z)}(\mathbf{k}) \equiv V\langle|\delta^{(z)}_{\mathbf{k}}|^2\rangle = (1 + 2\beta\cos^2\vartheta_{\mathbf{k}} + \beta^2\cos^4\vartheta_{\mathbf{k}})P(k)}$$

To appreciate differences expand in Legendre basis

$$P^{(z)}(k, \cos\vartheta_{\mathbf{k}}) = \sum_{\ell} P^{(z)}_{\ell}(k) P_{\ell}(\cos\vartheta_{\mathbf{k}}),$$

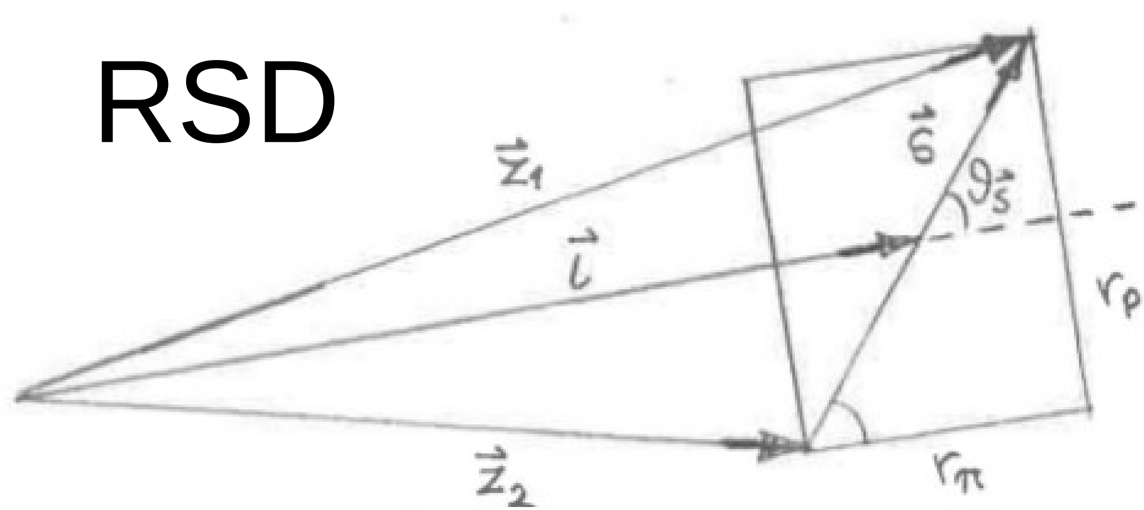
$$P^{(z)}_{\ell}(k) \equiv \frac{2\ell+1}{2}\int_{-1}^{1} d\cos\vartheta_{\mathbf{k}} P_{\ell}(\cos\vartheta_{\mathbf{k}}) P^{(z)}(k, \cos\vartheta_{\mathbf{k}})$$

$$\Rightarrow$$

$$P^{(z)}_0(k) = (1 + \tfrac{2}{3}\beta + \tfrac{1}{5}\beta^2)P(k)$$
$$P^{(z)}_2(k) = (\tfrac{4}{3}\beta + \tfrac{4}{7}\beta^2)P(k)$$
$$P^{(z)}_4(k) = \tfrac{8}{35}\beta^2 P(k).$$

Others are zero

RSD



Correlation function

$$
\begin{aligned}
\mathbf{l} &\equiv \tfrac{1}{2}(\mathbf{z}_1 + \mathbf{z}_2) && \text{line-of-sight vector} \\
\mathbf{s} &\equiv \mathbf{z}_1 - \mathbf{z}_2 && \text{separation vector in redshift space} \\
r_\pi &\equiv \hat{\mathbf{l}} \cdot \mathbf{s} && \text{parallel component of separation} \\
r_p &\equiv \sqrt{s^2 - r_\pi^2} && \text{perpendicular component of separation} \\
\mu &\equiv \frac{r_\pi}{s} = \cos\vartheta_{\mathbf{s}} && \text{direction cosine}
\end{aligned}
$$

In the plane parallel approx. only depends on $\xi(z_1, z_2) \sim \xi(s, \mu)$

$$\delta_{\mathbf{k}}^{(z)} \approx [1 + \beta(\cos\vartheta_{\mathbf{k}})^2]\delta_{\mathbf{k}} = \left[1 + \beta\left(\frac{k_z}{k}\right)^2\right]\delta_{\mathbf{k}}\,, \quad \xrightarrow{\text{FT}} \quad \delta^{(z)}(\mathbf{s}) \approx \left[1 + \beta\left(\frac{\partial}{\partial z}\right)^2 \left(\nabla^2\right)^{-1}\right]\delta(\mathbf{x})$$

Use Green functions and with some algebra get the 2PCF

RSD

Correlation function in Legendre basis

$$\xi^{(z)}(s,\mu) \;=\; \sum_\ell \xi_\ell^{(z)}(s) P_\ell(\mu)\,.$$

$$J_\ell(x) \;\equiv\; \int_0^x \xi(y) y^{\ell-1} dy\,,$$

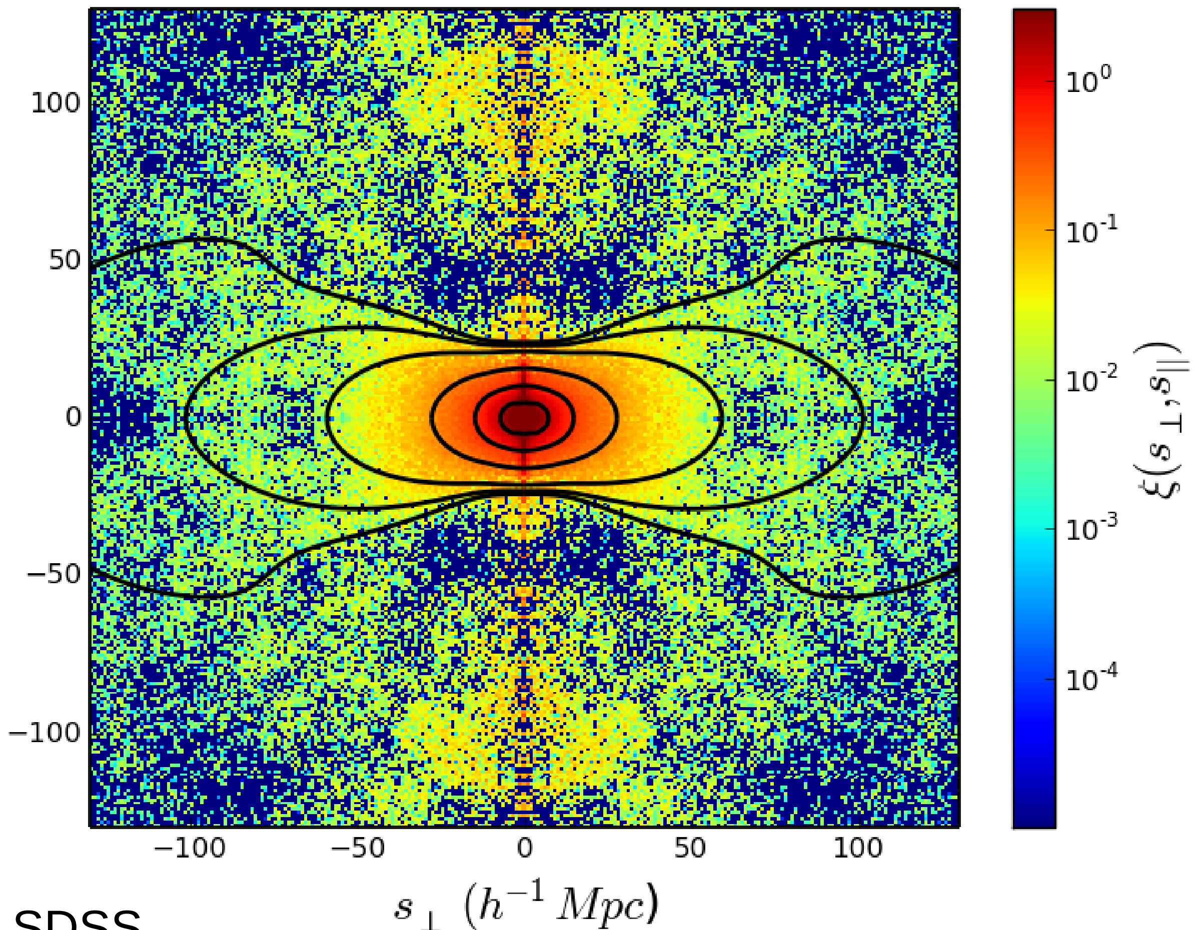
First multipoles are

$$\begin{aligned}
\xi_0^{(z)}(s) &= \left(1+\tfrac{2}{3}\beta+\tfrac{1}{5}\beta^2\right)\xi(s) \\
\xi_2^{(z)}(s) &= \left(\tfrac{4}{3}\beta+\tfrac{4}{7}\beta^2\right)\xi(s) + \left(-4\beta-\tfrac{12}{7}\beta^2\right)\frac{J_3(s)}{s^3} \;=\; \left(\tfrac{4}{3}\beta+\tfrac{4}{7}\beta^2\right)\left(\xi-\bar{\xi}\right) \\
\xi_4^{(z)}(s) &= \tfrac{8}{35}\beta^2\xi(s) + \tfrac{12}{7}\beta^2\frac{J_3(s)}{s^3} - 4\beta^2\frac{J_5(s)}{s^5}\,,
\end{aligned}$$

$$\bar{\xi}(R) \;\equiv\; \frac{3}{R^3}\int_0^R \xi(r) r^2 dr \;\equiv\; \frac{3}{R^3} J_3(R)\,,$$

RSD

Correlation function in Legendre basis $$\xi^{(z)}(s,\mu) = \sum_{\ell} \xi_{\ell}^{(z)}(s) P_{\ell}(\mu)\,.$$



SDSS