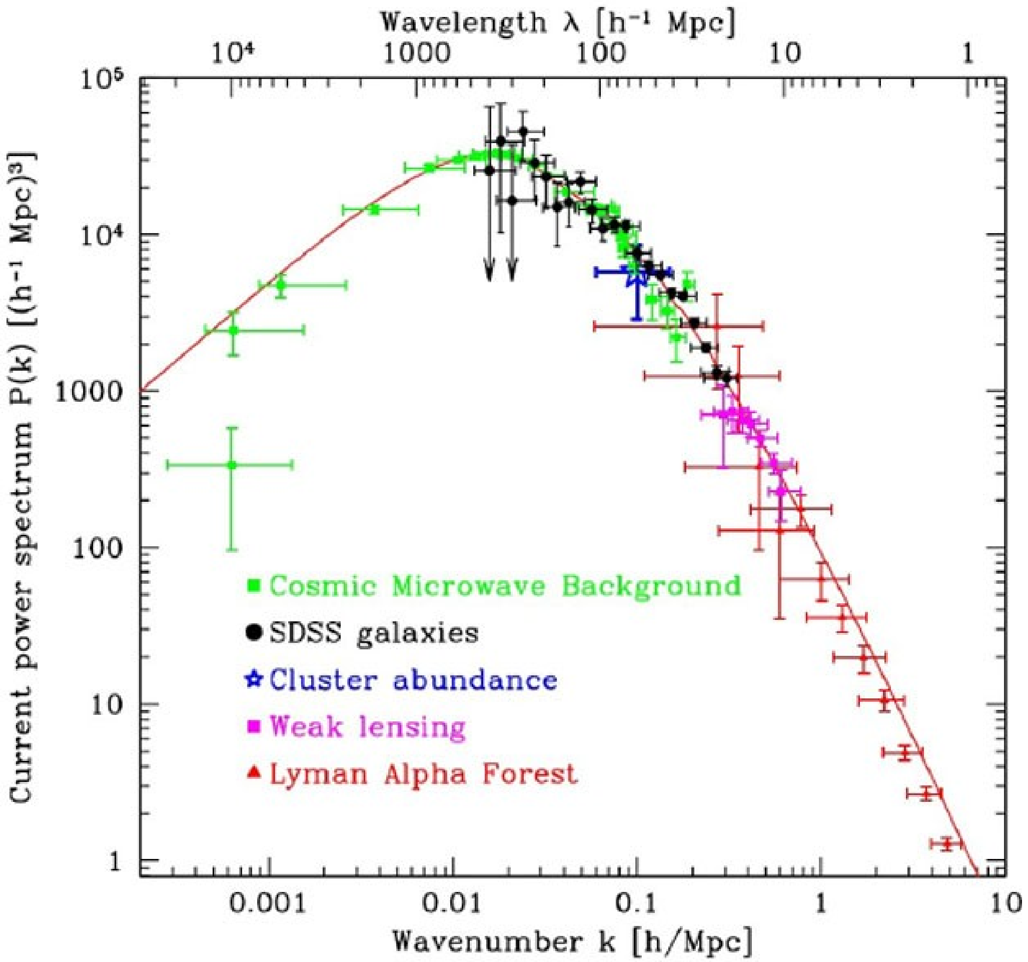
Two point correlation function

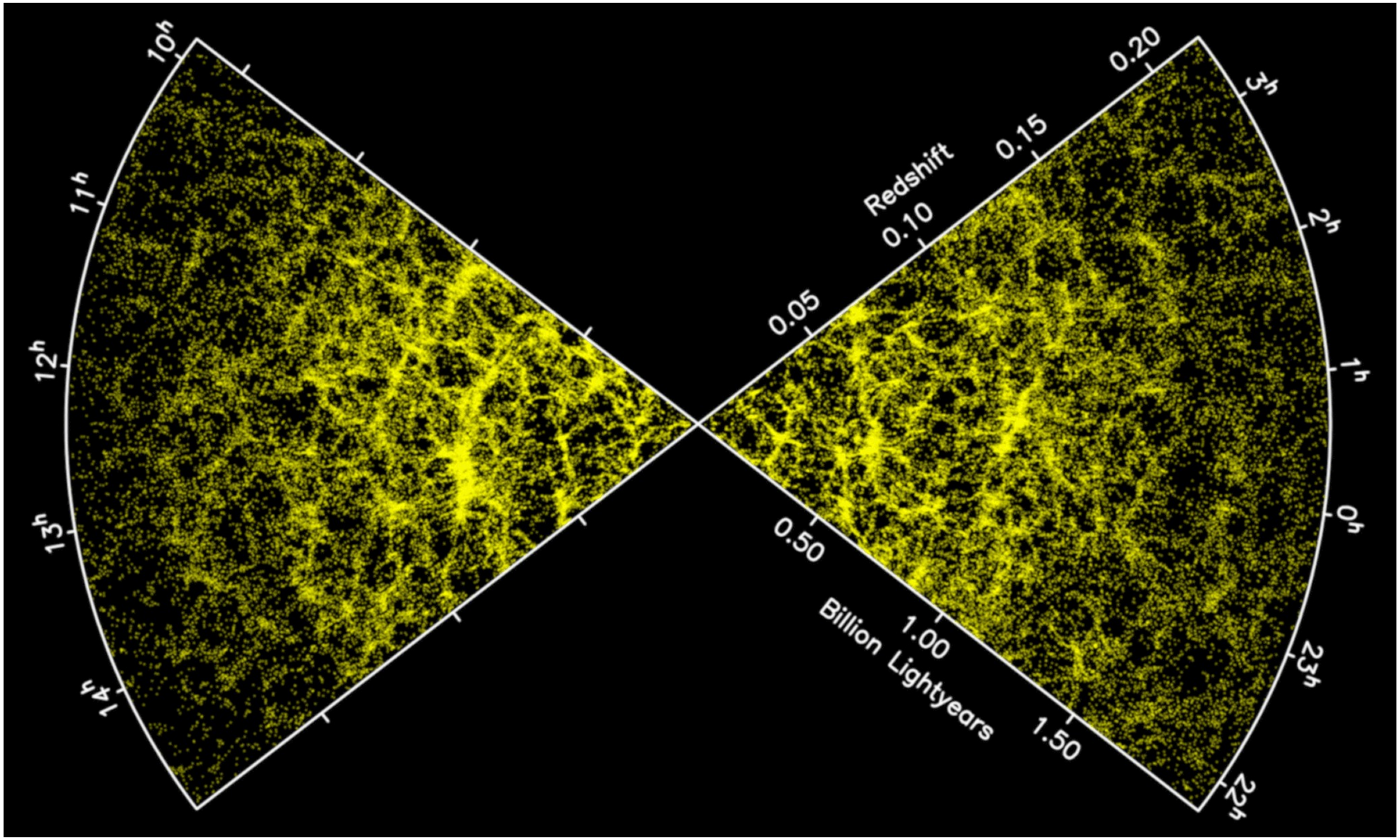


Gustavo Niz

U. de Guanajuato

2018

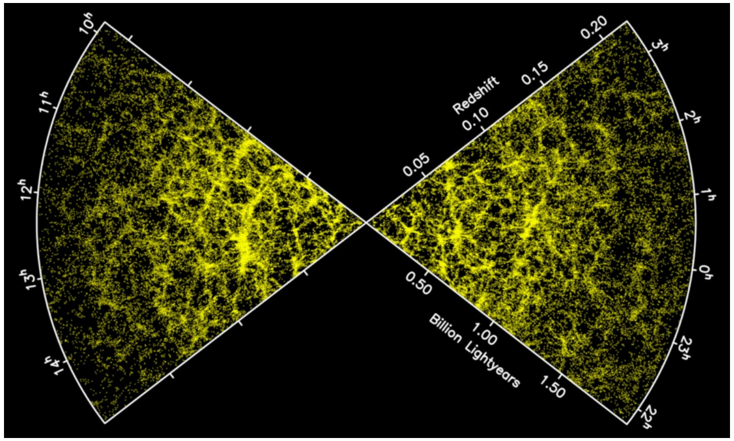
LSS

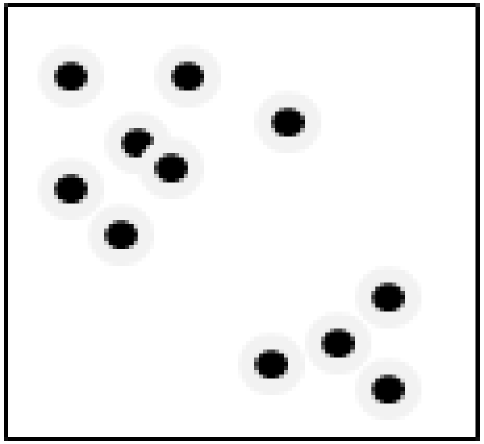


$$\rho = \bar{\rho}(1 + \delta)$$

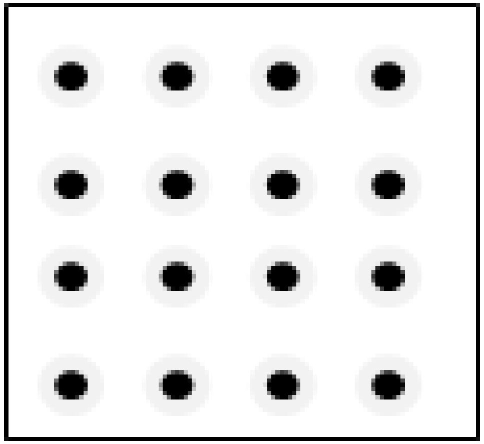
$$\delta \ll 1$$

Info in the LSS

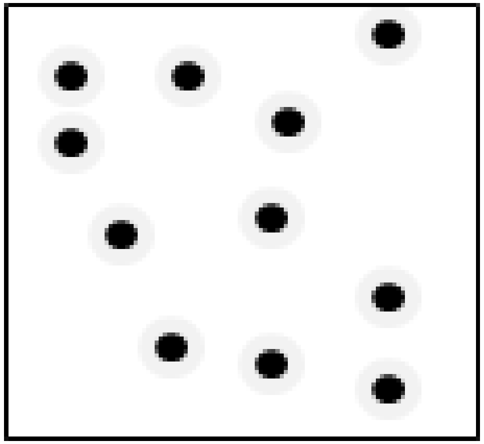




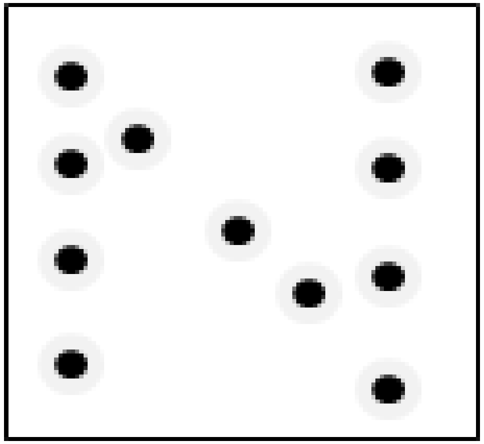
Clusters



Ordered



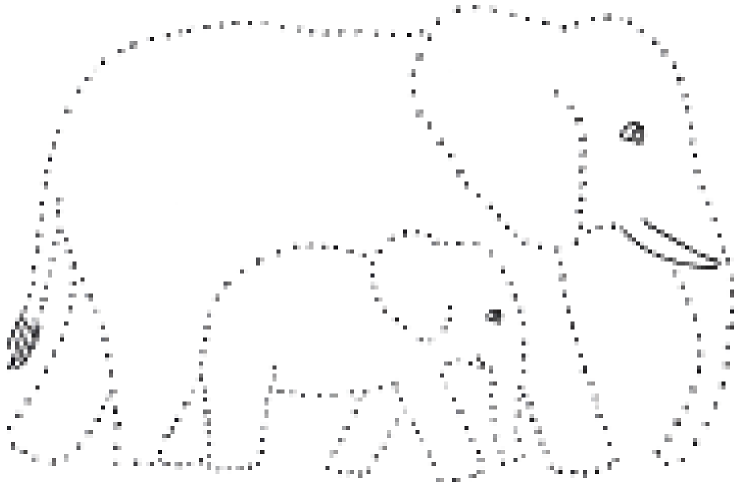
Random

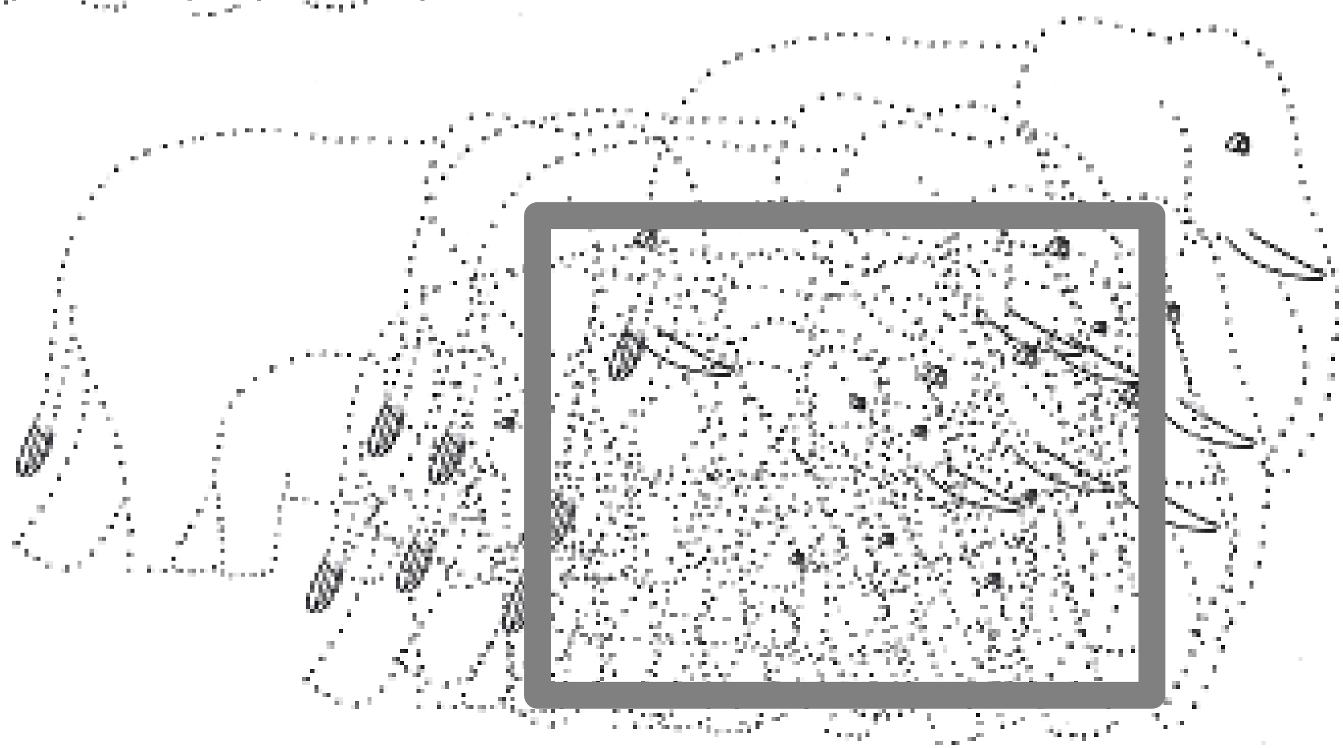


Patterns

Does the distribution of galaxies in the LSS contains info like this?

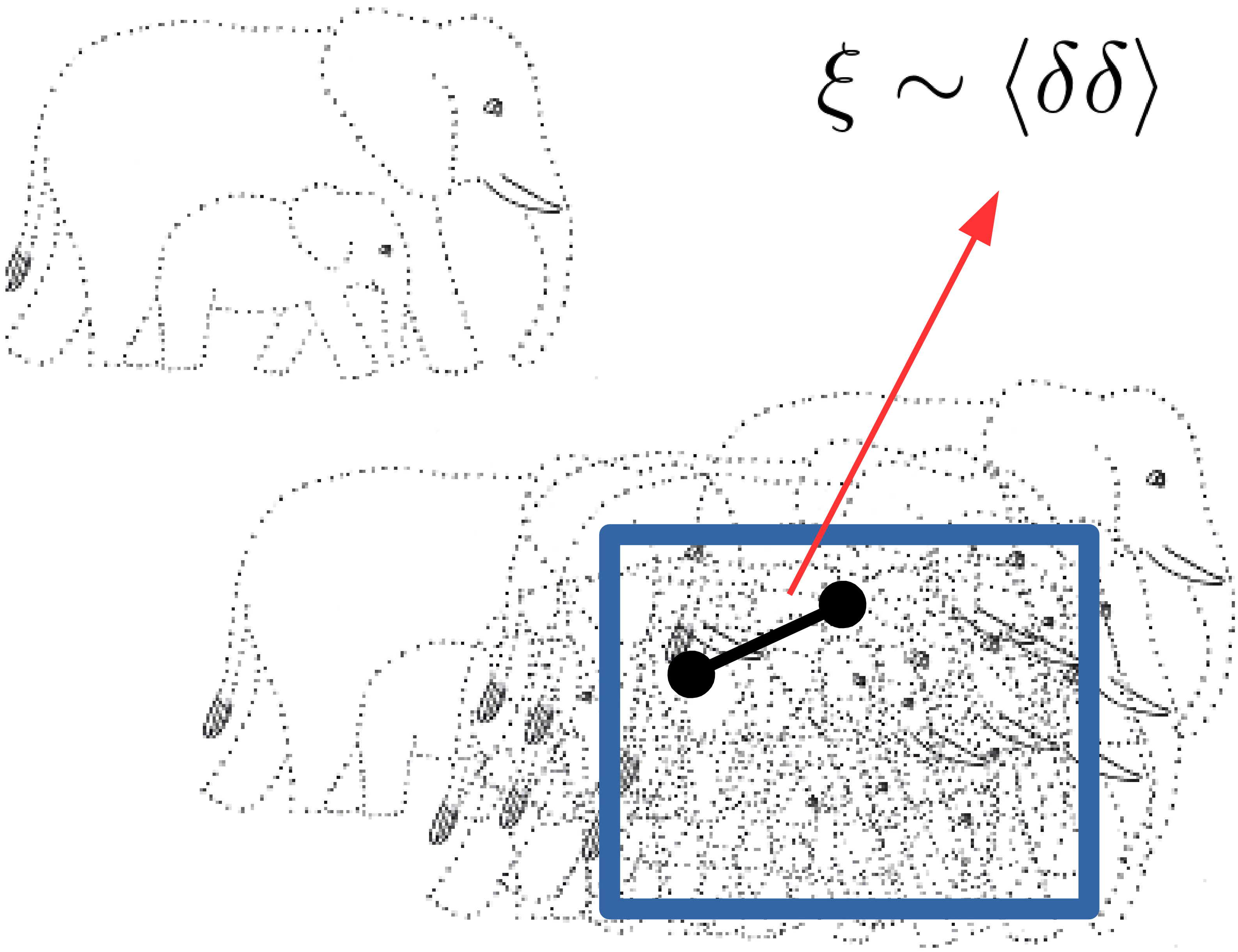
Understading distributions



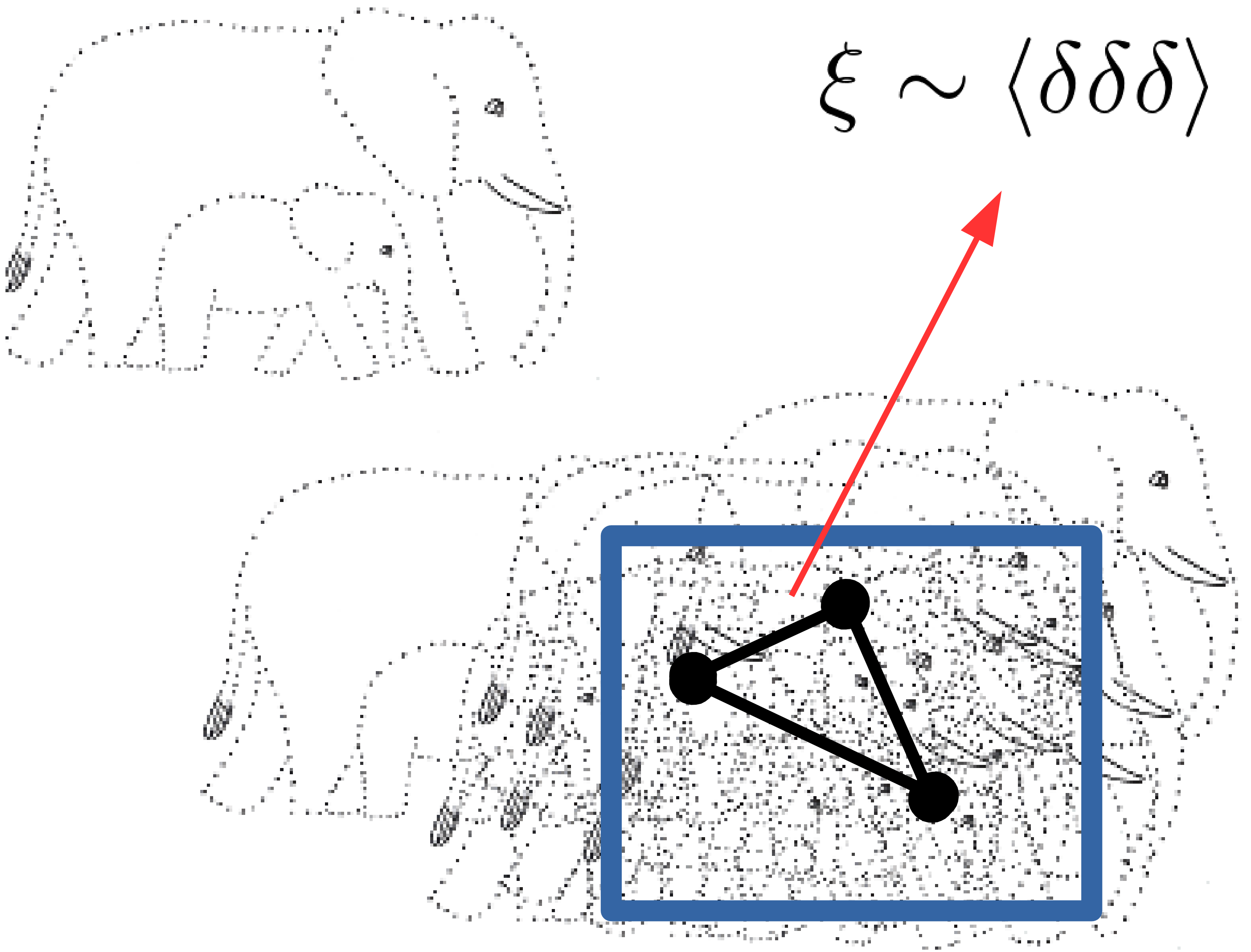


Understading distributions

$$\xi \sim \langle \delta\delta \rangle$$



Understading distributions

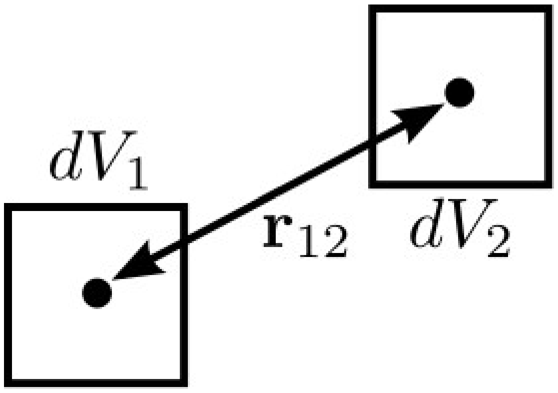


$$\xi \sim \langle \delta\delta\delta \rangle$$

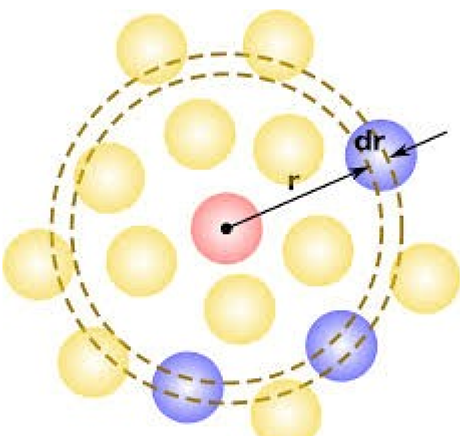
Formally

Usual to consider N-point correlation functions

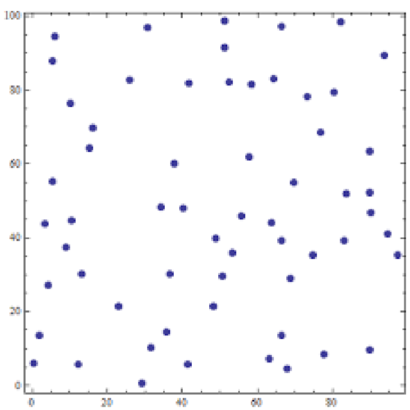
- N=2 (2pcf)



$$dP = \bar{n}^2(1+\xi^{(2)}(\mathbf{r_{12}}))dV_1 dV_2$$

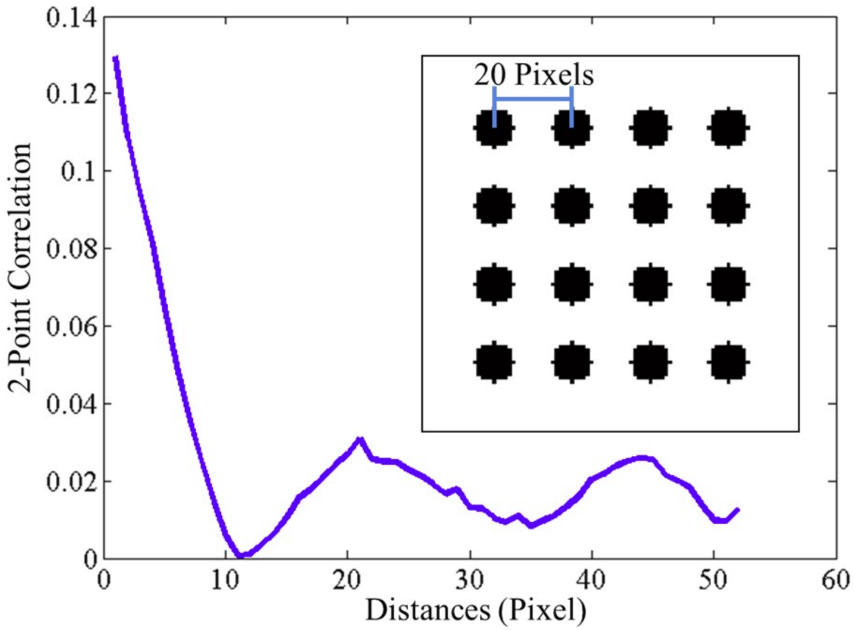


Excess correlation
over the random pairs

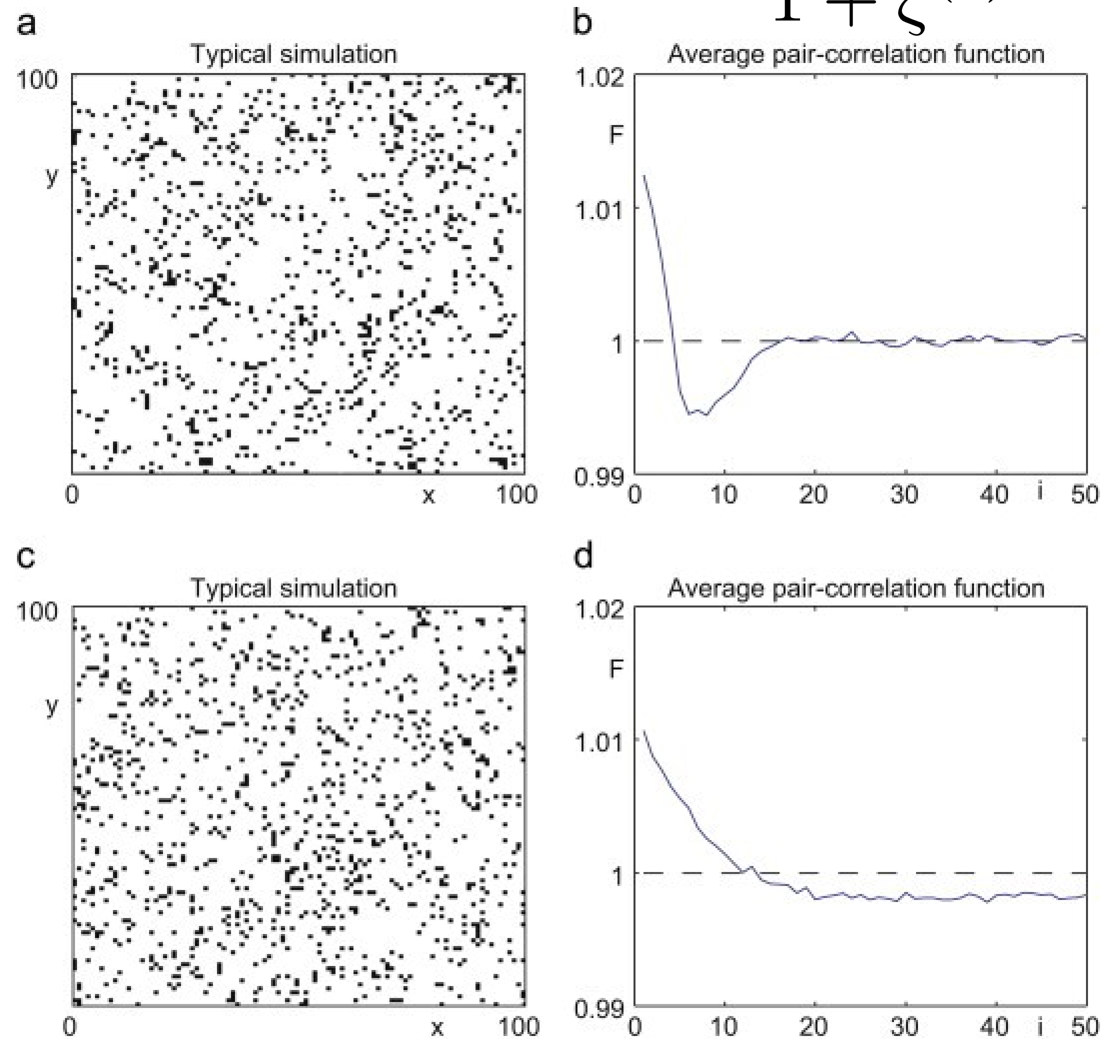


2PCF

- examples

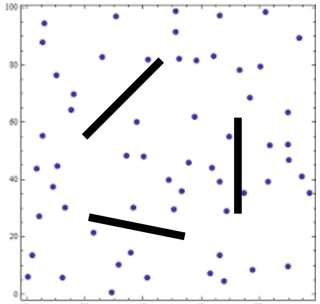


$$1 + \xi^{(2)}$$

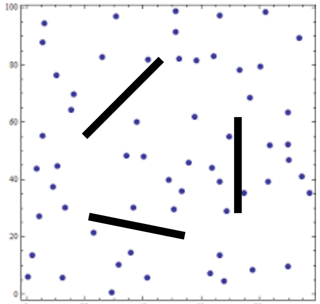


2PCF

Practial way to do it over sample data
(Estimator of Peebles-Hauser)



D



R

$$\xi^{(2)}(r) = \frac{DD(r)}{RR(r)} - 1 = \frac{DD(r) - RR(r)}{RR(r)}$$

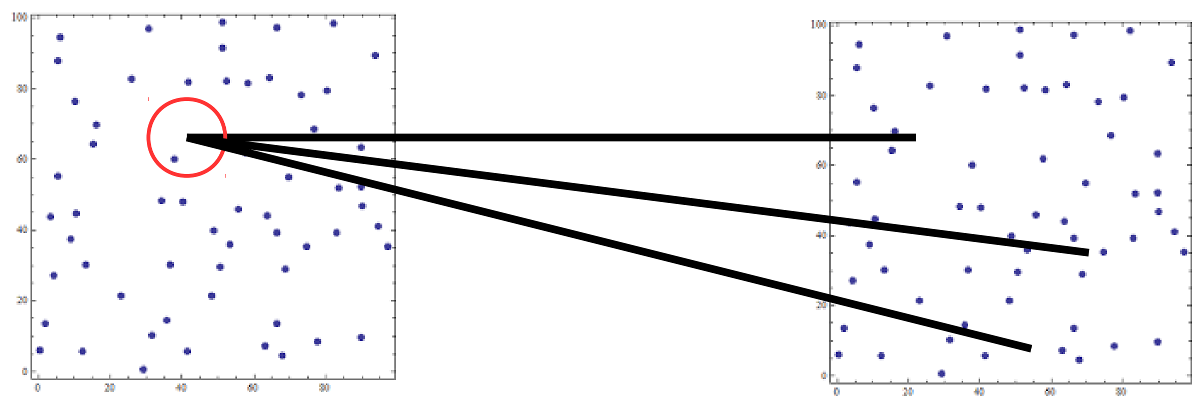
If no. of particles in R and D are not equal:

$$\xi^{(2)} = \frac{N_D(N_D - 1)}{N_R(N_R - 1)} \frac{DD}{RR} - 1 \equiv \frac{1}{N_{est}^2} \frac{DD}{RR} - 1$$

2PCF

Algorithm & computing time

D or R



For i = 1 to n

For j = 1 to n

distance(r)

histo(bin of r)+=1

scales as

$$\mathcal{O}(n^2)$$

Can do better (eg. kd tree, FFT)

$$\mathcal{O}(n\log(n))$$

2PCF

Biased estimator!

$$\delta = \frac{n - \bar{n}}{\bar{n}}$$

Sample is volume finite

$$\langle\ ,\ \rangle = \int dV$$

- *W(r)*: indicator function

$$no.\ points = \langle W(r) n(r) \rangle$$

2PCF

Uncertainity in mean density and 1-pt function

$$\bar{\delta} = \frac{\langle W(r)\delta(r)\rangle}{\langle W(r)\rangle} \qquad \Psi = \frac{\langle\langle W(r)W(r)\delta(r)\rangle\rangle}{\langle\langle W(r)W(r)\rangle\rangle}$$

2-pt function is

$$\xi^2(r) = \frac{\langle\langle W(r)W(r)\delta(r)\delta(r)\rangle\rangle}{\langle\langle W(r)W(r)\rangle\rangle}$$

$$\xi_{PH}^{(2)} = \frac{\xi_{true}^{(2)}(x_1, x_2) + \Psi(x_1) + \Psi(x_2) - 2\bar{\delta} - \bar{\delta}^2}{(1+\bar{\delta})^2}$$

2PCF

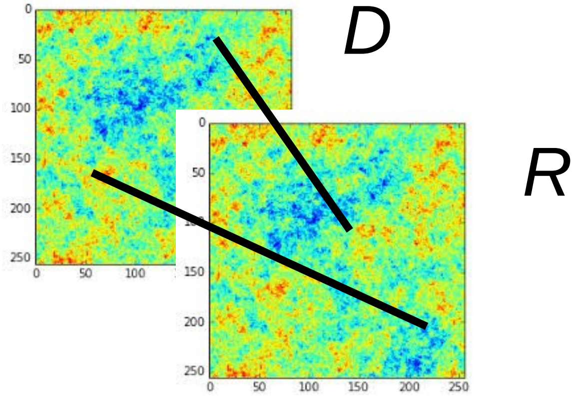
Better estimators

Hamilton

$$\xi_H^{(2)}(r) = \frac{DD(r)RR(r)}{DR(r)^2}$$

Landy-Szalay

$$\xi_{LZ}^{(2)}(r) = \frac{DD(r) + RR(r) - 2DR(r)}{RR(r)}$$



Biased estimator properties

Davis-Peebles

$$\xi^{(2)}_{DP}(r) = \frac{1}{N_{est}}\frac{DD(r)}{DR(r)} - 1 \quad \Rightarrow \quad \xi^{(2)}_{DP} = \frac{\xi^{(2)}_{true} + \Psi(1-\bar{\delta}) - \bar{\delta}}{(1+\Psi)(1+\bar{\delta})}$$

Hamilton

$$\xi^{(2)}_{H}(r) = \frac{DD(r)RR(r)}{DR(r)^2}$$

Corrections are second order in $\bar{\delta}$ or higher in the numerator for both!

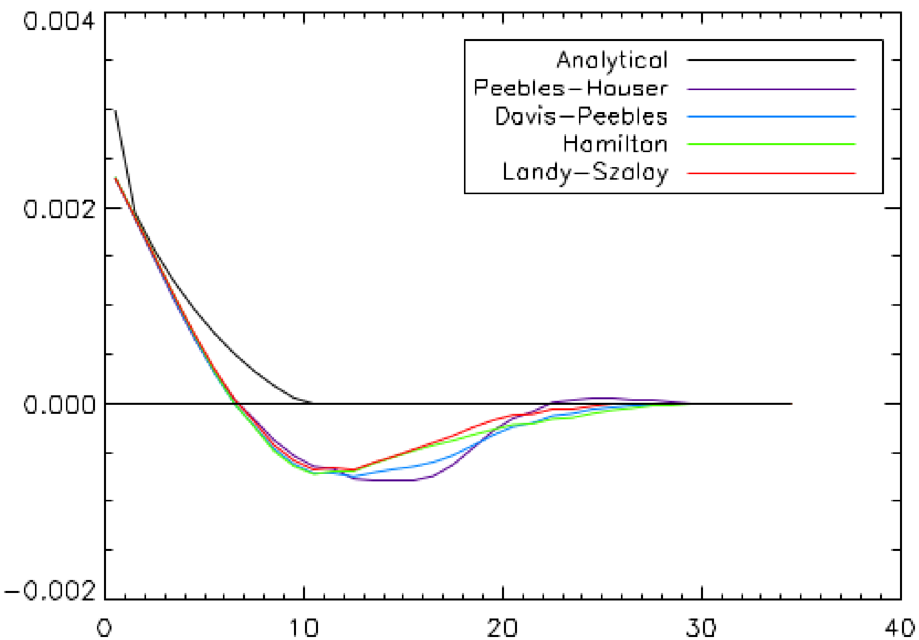
Landy-Szalay

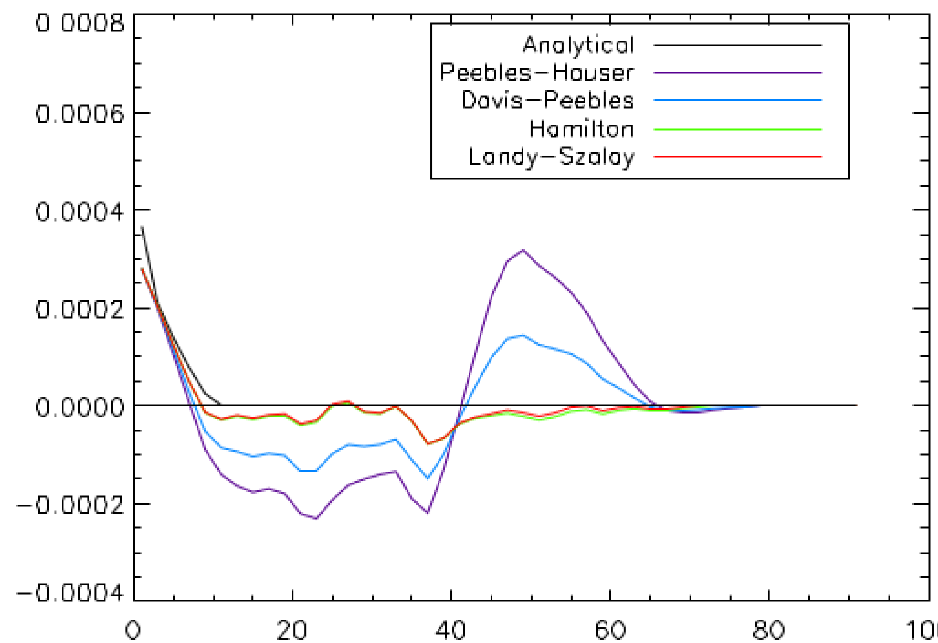
$$\xi^{(2)}_{LZ}(r) = 1 + \frac{1}{N^2_{est}}\frac{DD(r)}{RR(r)} - 2\frac{1}{N_{est}}\frac{DR(r)}{RR(r)}$$

Integral constraint

Over a finite box, we don't use the real average density, which leads to a correction in the estimator, at first order, given by

$$\xi_X^{(2)} = \xi_{true}^{(2)} - K \qquad K = \frac{1}{V^2}\int_V\int_V \xi_X^{(2)}(x_1, x_2)d^3x_1 d^3x_2$$





Further reading: https://arxiv.org/abs/1311.3280 https://arxiv.org/abs/1009.1232

Next lecture: power spectrum

Fourier transform

$$\xi(\mathbf{x}) = \frac{1}{(2\pi)^d} \int_V P(\mathrm{k}) e^{-i\mathbf{k}\cdot\mathbf{x}} d^d\mathbf{k}$$

Dimensionless

$$\Delta^2(\mathbf{k}) \sim k^d P(\mathbf{k})$$

Dimensionfull

	Correlation function	Power spectrum
Stat. Isotropy	$\xi(\mathbf{r}) = \xi(r)$	$P(\mathbf{k}) = P(k)$
	Intuitive	**Theory**
Modes (linear)	dependent	independent

Systematics drive them apart these Fourier cousins