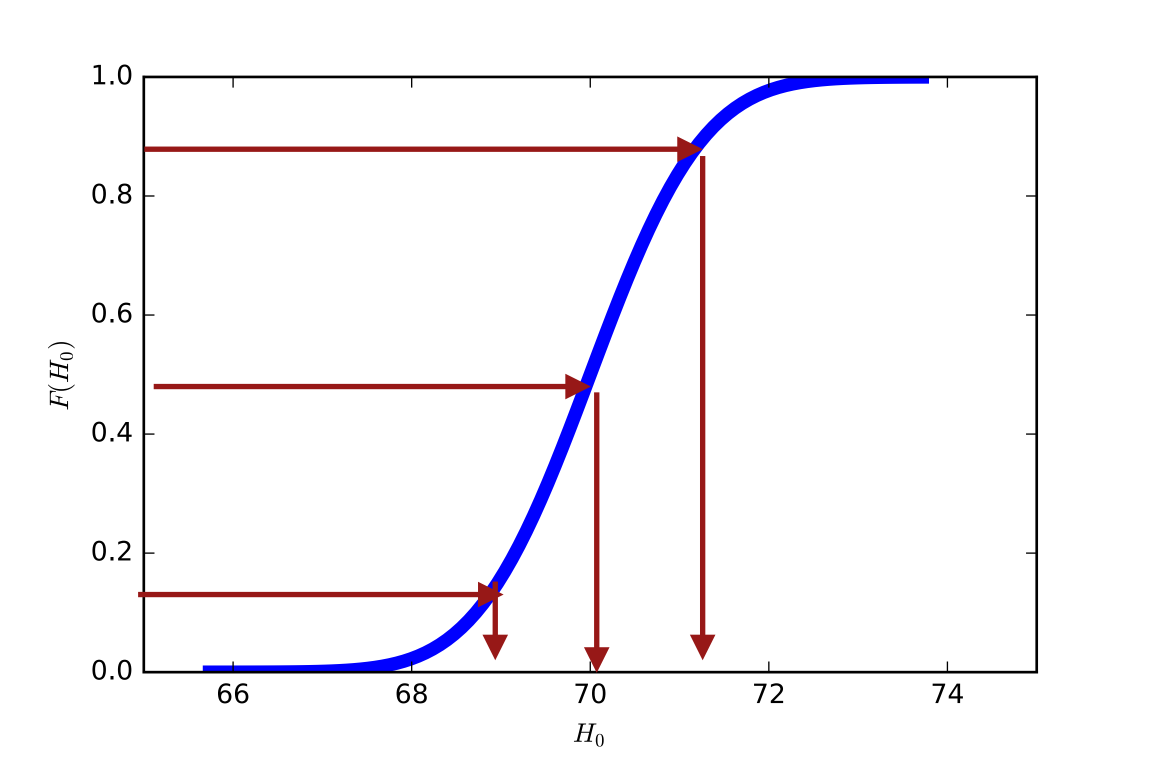
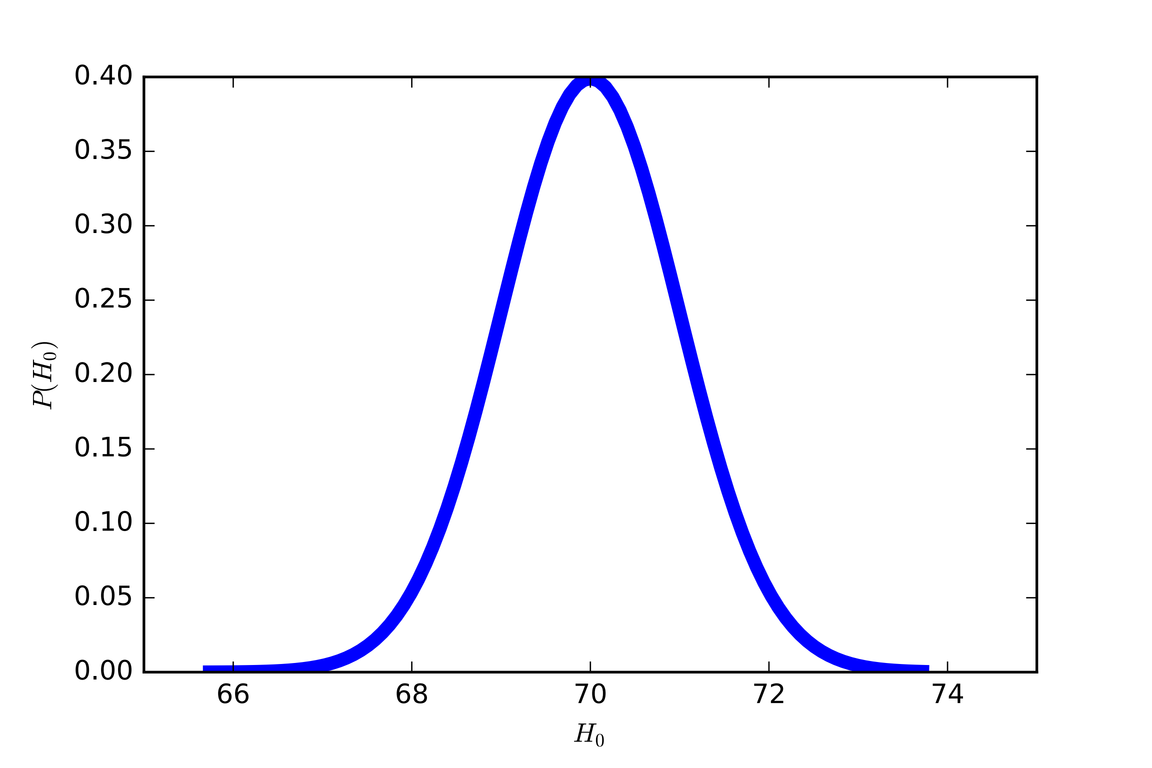
How to sample a PDF

- Depending on the programing language you are using it can be more or less difficult. But simple method is by using the CDF.



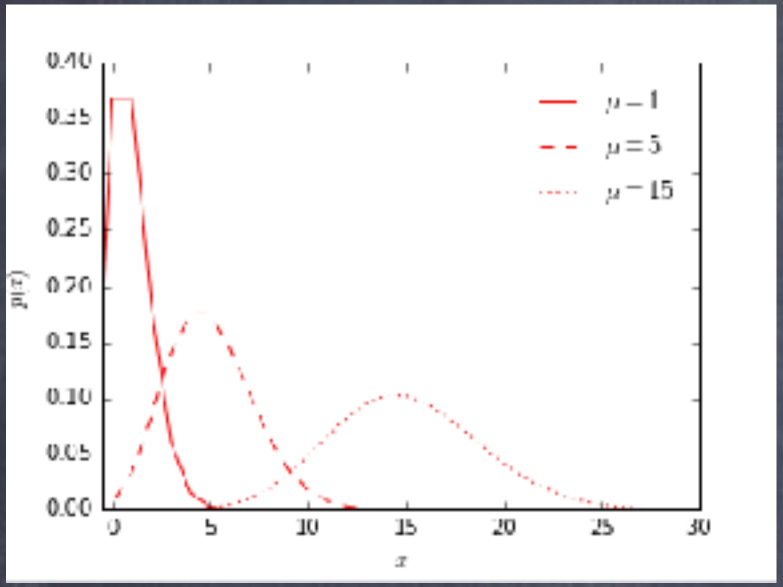


$$F(x) = \int_{-\infty}^{x} f(x')dx'$$

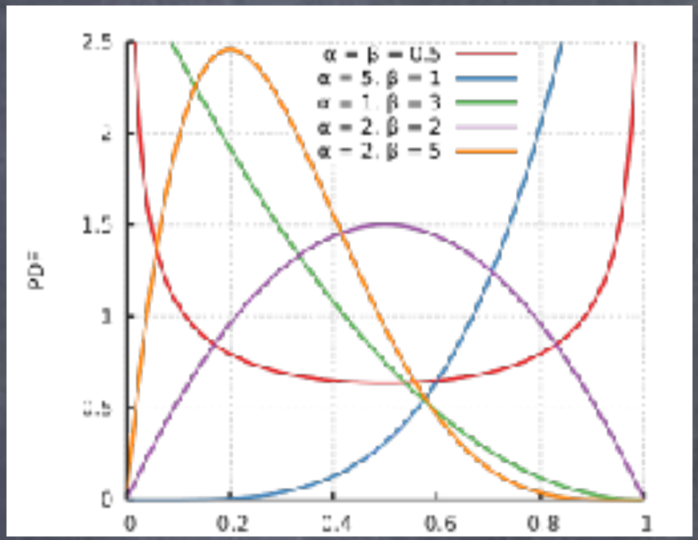
Exercise: Find the cumulative distribution function for the Gaussian Distribution, and reproduce the plots. Choose a random number between 0 and 1, and use the CDF to assign the corresponding value of H0. Generate as many as you want, and make the histogram of H0 to verify you did it right. Use a mean of 70 and a sigma=2.

Other Probability distribution

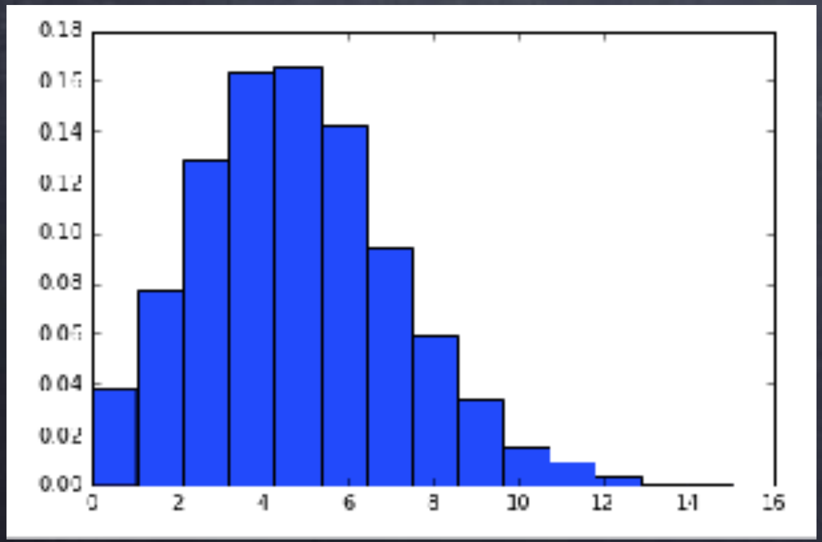
- Binomial, Poisson and χ^2 distribution can be approximated, for large numbers, by a Gaussian distribution. Other distributions



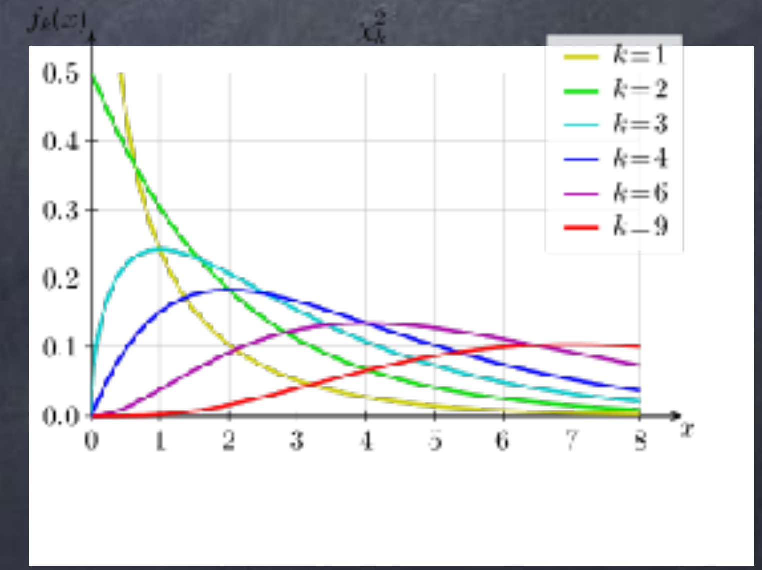
- Poisson



- Beta distribution



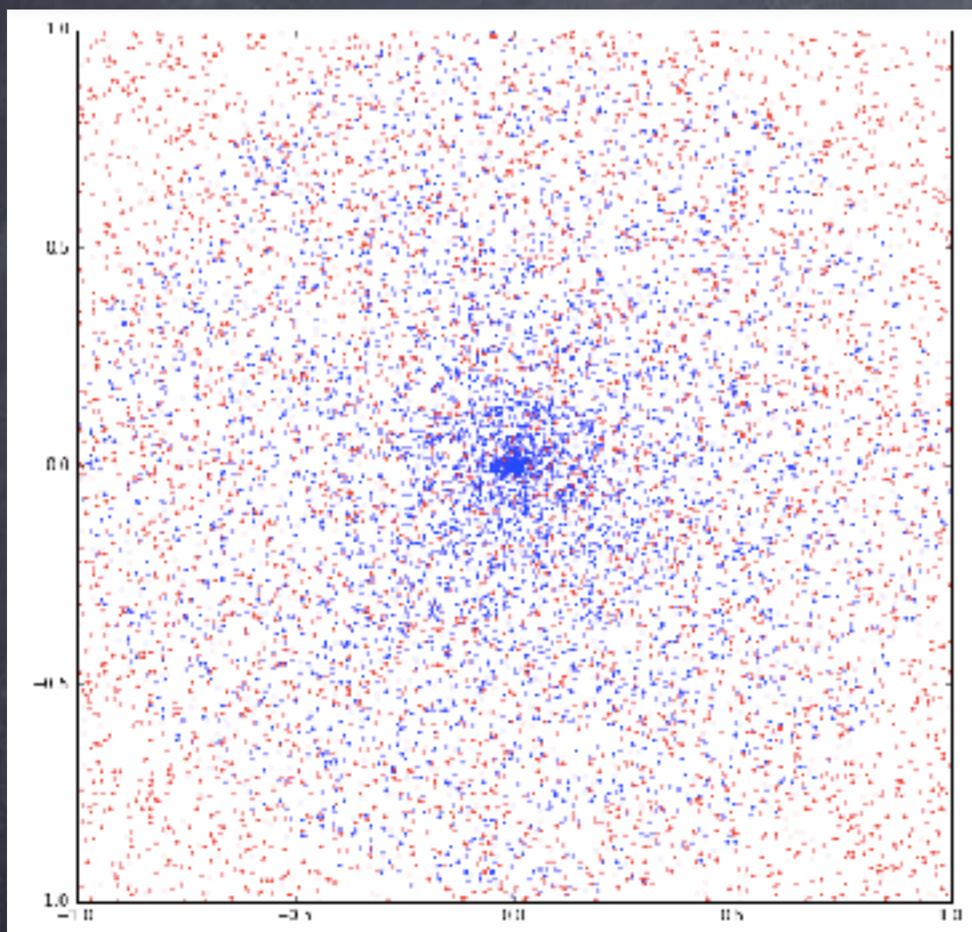
- Binomial



- χ^2 distribution

Draw samples from a specific distribution. Transformation of variables.

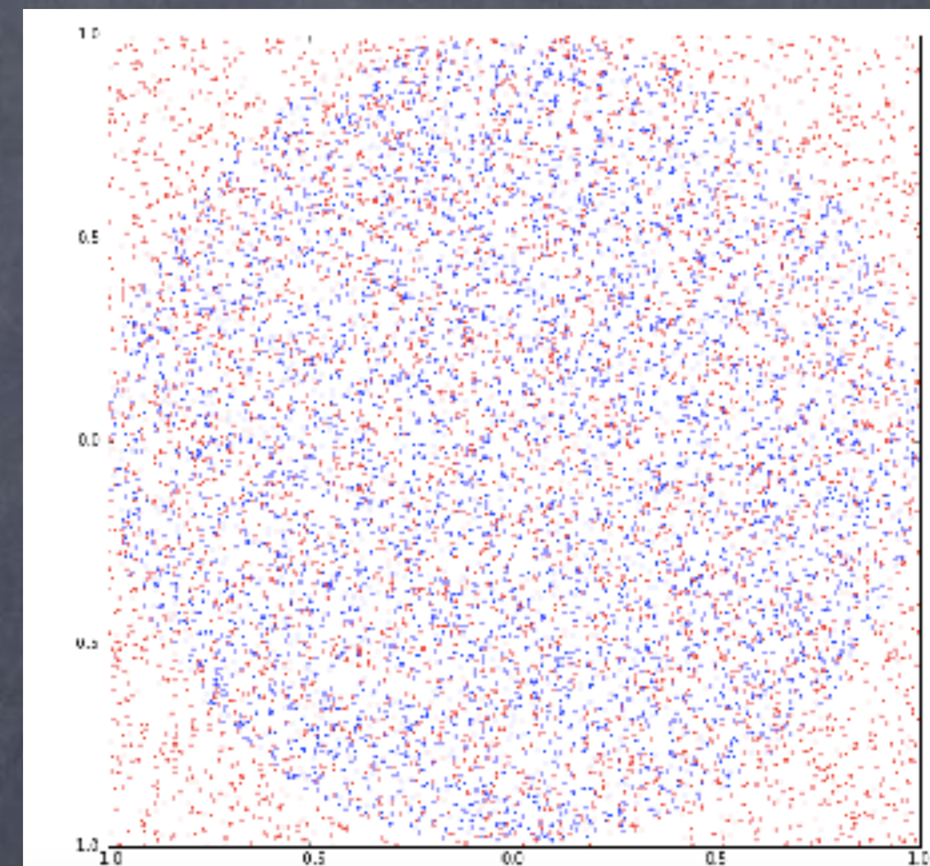
(Ej. 2D)



non-Uniform for r<1

1) generate r and theta, from U.

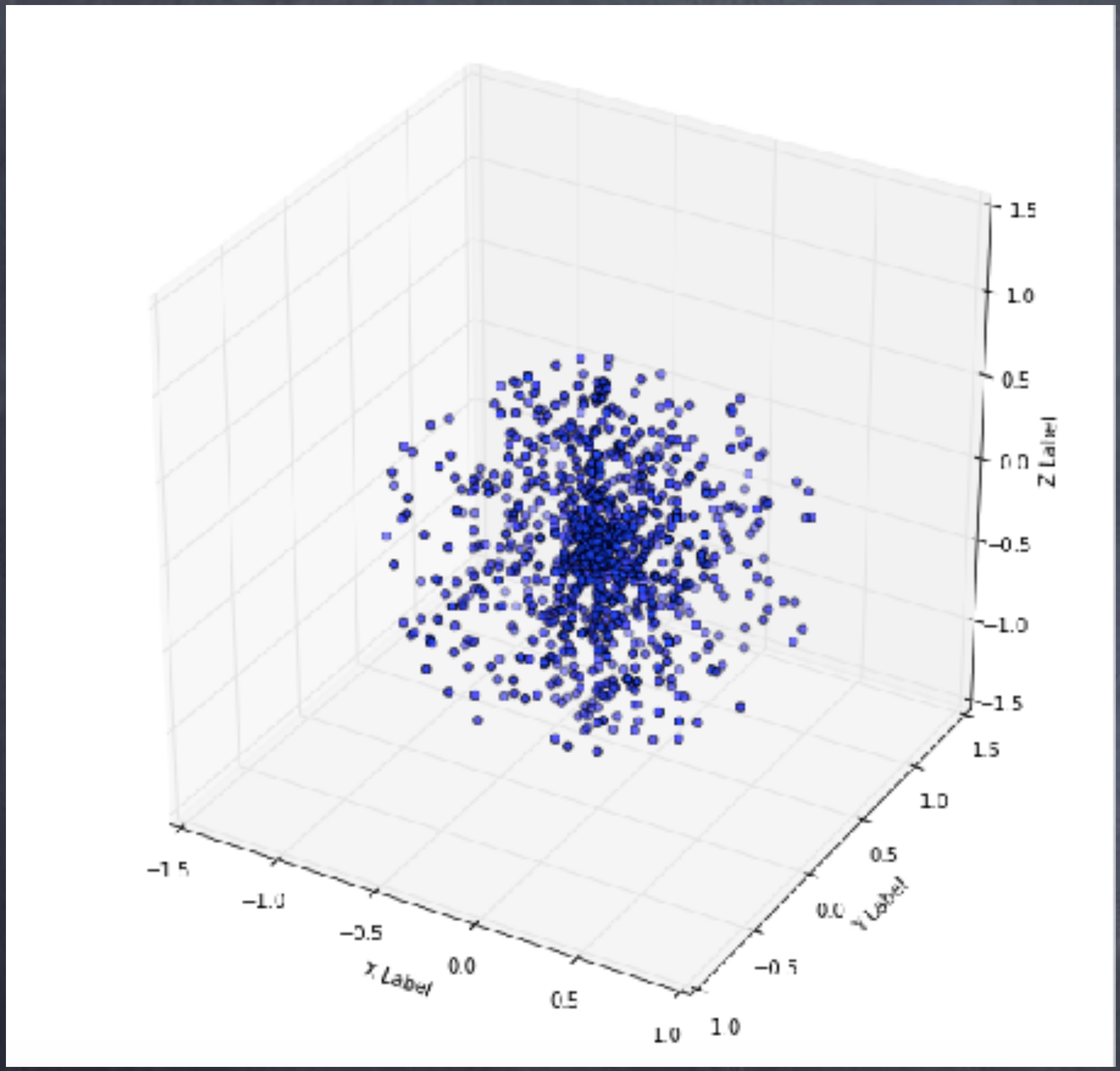
!=

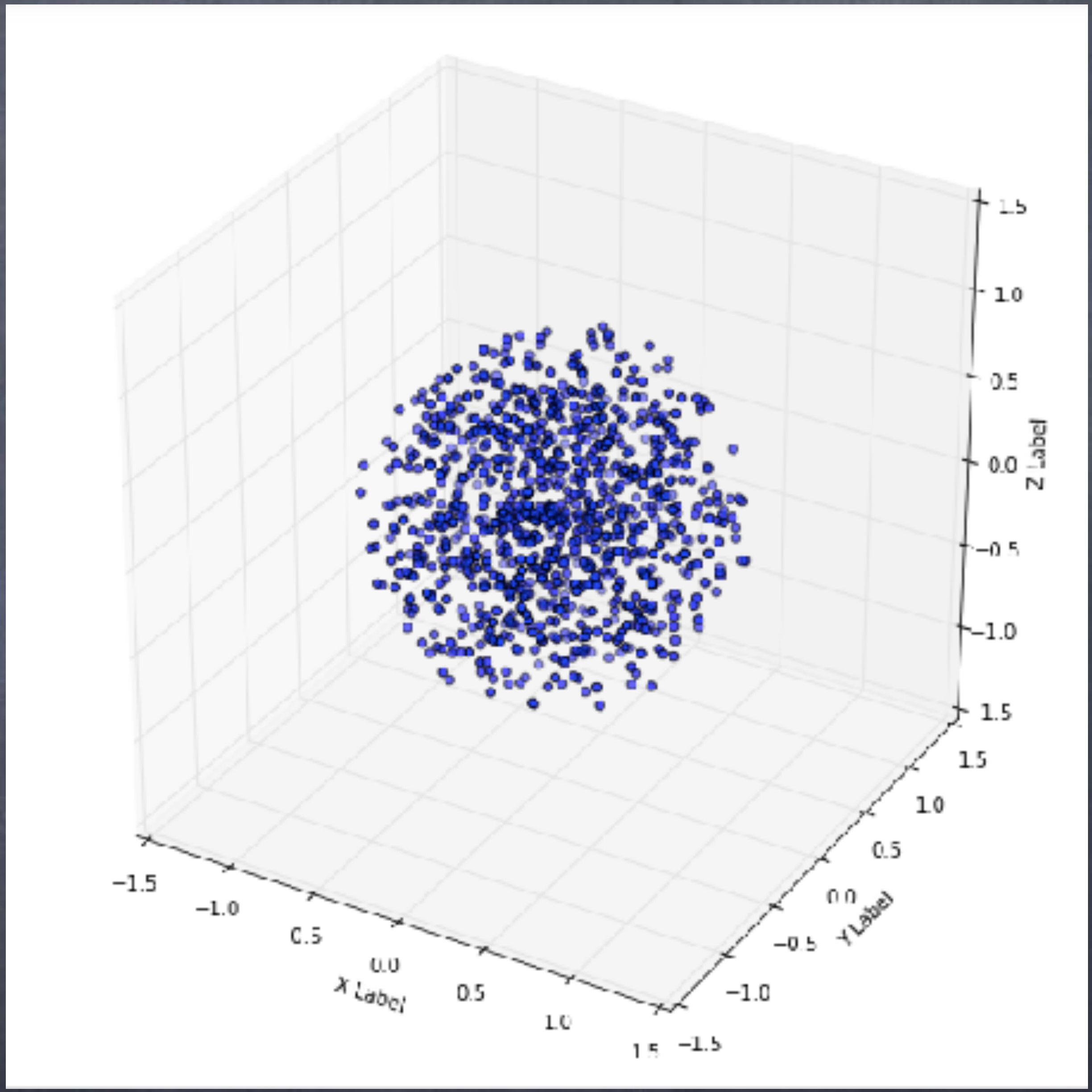


Uniform for r<1

1) generate x,y from U so that r=sqry(x^2+y^2)<=1
2) generate sqrt[r] and theta, from U.

Draw samples from a specific distribution. Transformation of variables.





1) generate r and theta, phi from U.

1) generate x,y from U
r=sqry(x^2+y^2+z^2)<=1
2) generate sqrt[r], theta and z=cos(phi) from U.

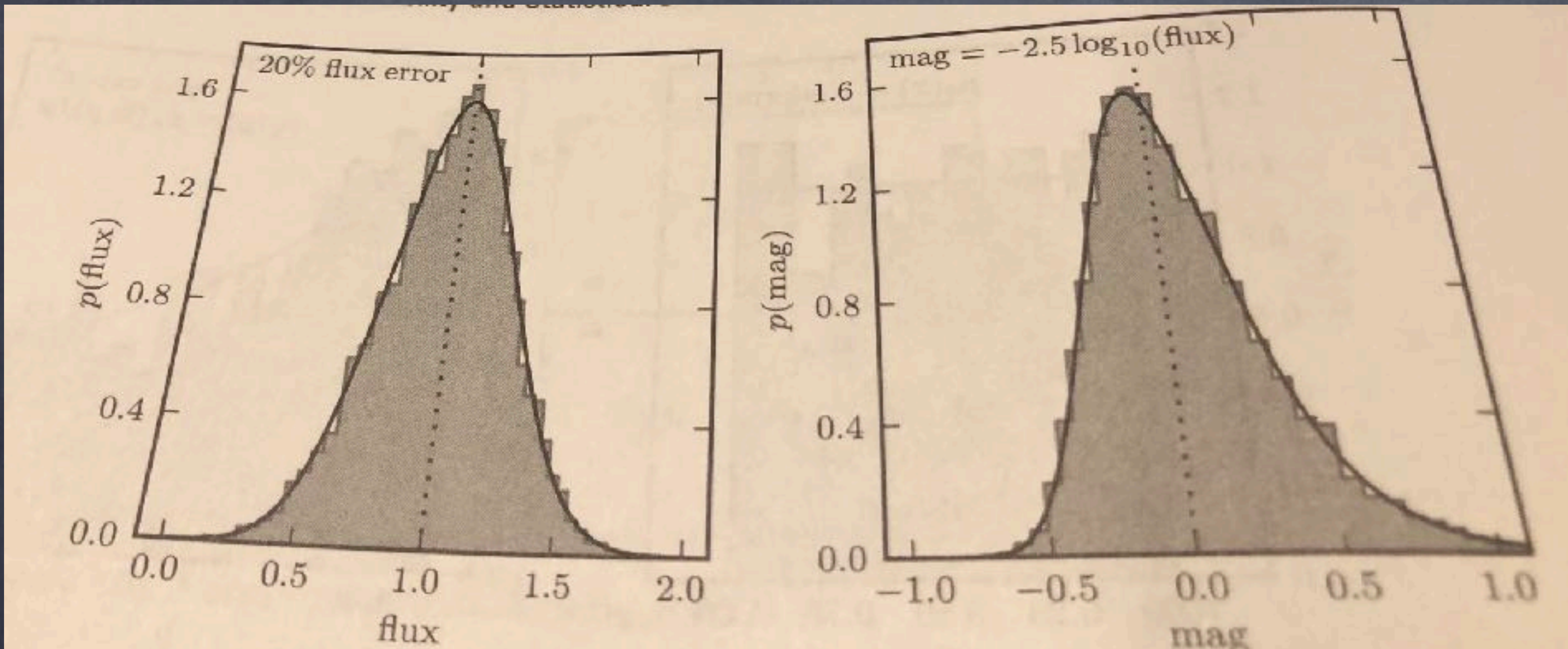
Transformation of variables.

- Any function of a random variable is a random variable itself.

- Sometimes we measure a variable x, but the interesting final result is y(x). In we know the PDF p(x), what is the PDF p(y)?, where y=Phi(x).

$$p(y) = p[\Phi^{-1}(y)] \left| \frac{d\Phi^{-1}(y)}{dy} \right|$$

Eg. Flux Vs Mag



Exercise:

1.- If $y = \Phi(x) = \exp(x)$ and p(x)=1 for $0 \leq x \leq 1$ (a uniform distribution). What is the resultant distribution for y.

2.- Si el flujo, en la imagen, sigue una distribución Gaussiana, ¿cómo es la distribución en magnitud?. Usa un método de muestreo para reproducir las gráficas

Exercise

- Reproduce the plots of the Uniform distribution of points inside a circle, and a sphere.

Likelihood

- The probability , under the assumption of a model/theory, to observe the data as was actually obtained.

$$\mathcal{L} \longrightarrow P(\mathrm{Data}, \mathrm{Model})$$

- For data that can be thought as samples of a sequence of normal random variables that have a mean and a variance, the likelihood is Gaussian

$$\mathcal{L} \propto \prod_{i}^{n} \frac{1}{2\pi\sigma_i^2} \exp\left(-\frac{(x_i - \mu)^2}{2\sigma_i^2}\right)$$

mu will be the expected mean given our model

- A minimization of the Chi-square correspond to the maximization of the likelihood.

Gaussian Likelihood

$$-\ln(\mathcal{L}(\vec{x},\vec{y}|\vec{\theta})) \propto \frac{1}{2}\sum_i \left(\frac{(y_i - \lambda(x_i,\vec{\theta}))^2}{\sigma_i^2}\right)$$

In terms of data points and parameters.
Lambda is our model for y_i

How do we maximize the likelihood if there 2,3, or more parameters...?

How do we maximize the likelihood if we have a complex model?

What if the likelihood is not Gaussian...?

MonteCarlo Markov Chain

Draw random samples and accept them or reject them according to the likelihood.

Metropolis algorithm

If the likelihood of a new sample is higher than the previous one we accept the sample and save it. new—>old

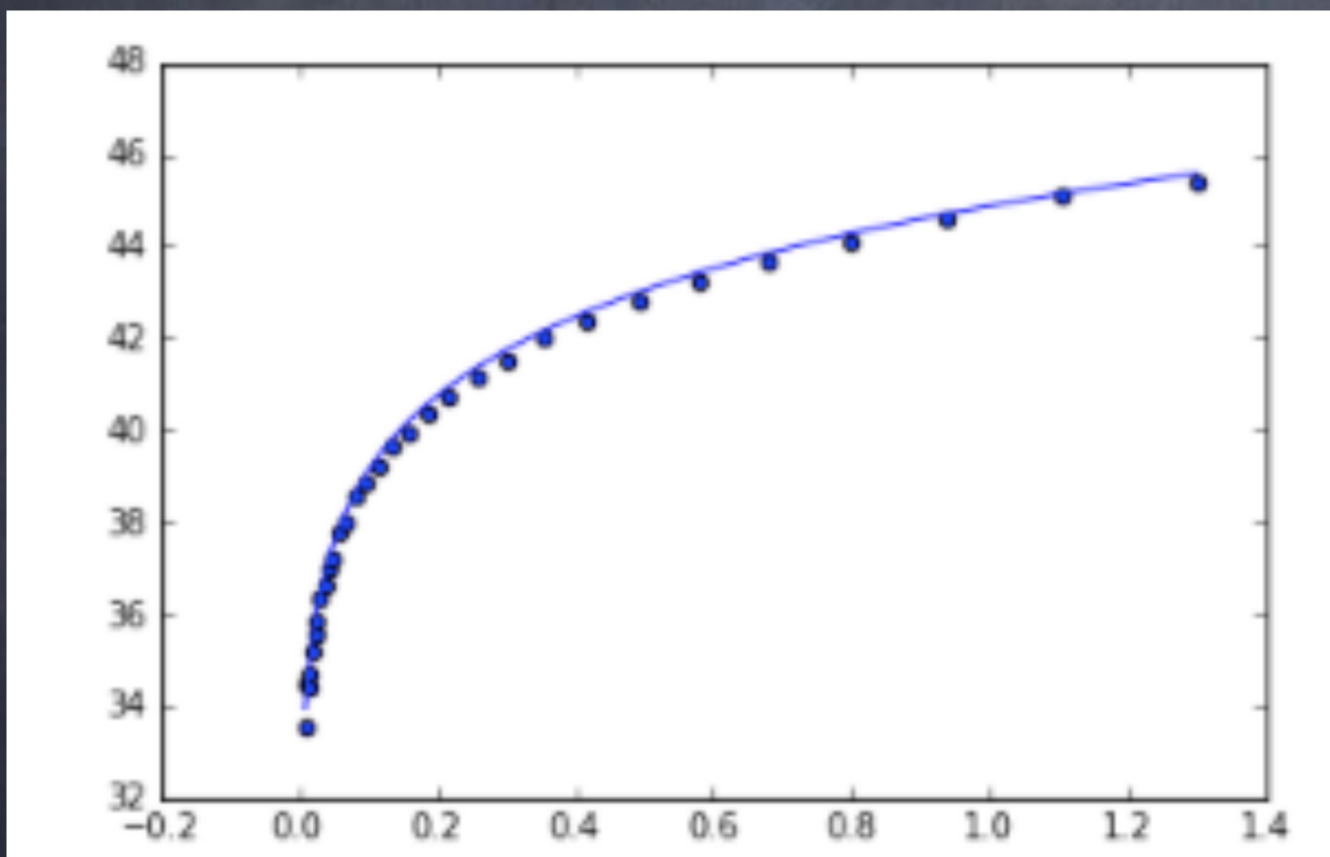
If the likelihood of a new sample is lower than the previous one, we then draw a random number between 0-1, if the sample likelihood is larger than such number then we accept it, not otherwise.

Draw a new sample and start again….

After many steps, look at the resultant distribution (the chains) of parameters, i.e., the likelihood..

Look at the burning period and the convergence….

Ej. Find cosmological parameters with SuperNova Data (you'll work the simplest example)



$$\mu = 25 - 5Log_{10}(H_0/100) + 5Log_{10}(D_L/Mpc)$$

$$D_L = \frac{(1+z)c}{H_0\sqrt{|1-\Omega|}} S_k(r), \text{ donde,}$$

$$r(z) = \sqrt{|1-\Omega|} \int_0^z \frac{dz'}{\sqrt{\Omega_m(1+z')^3 + \Omega_\Lambda + (1-\Omega)(1+z')^2}} \text{Mpc}$$

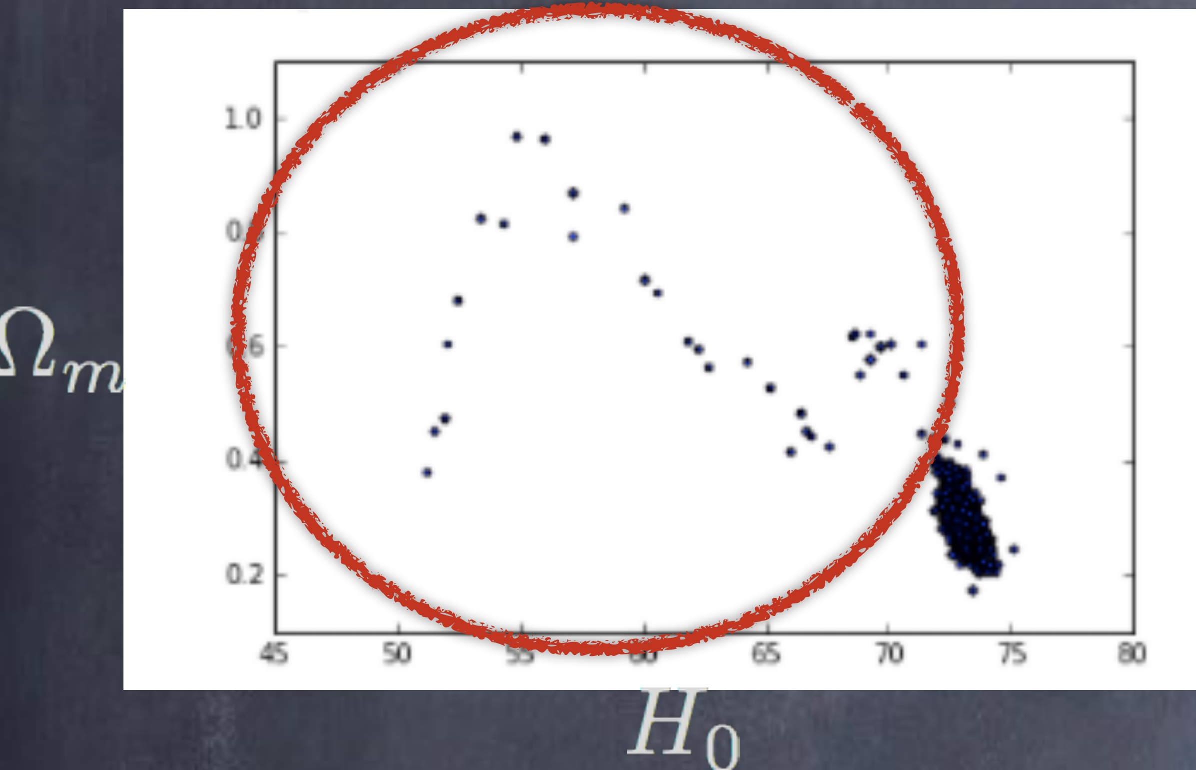
Approximate Solution

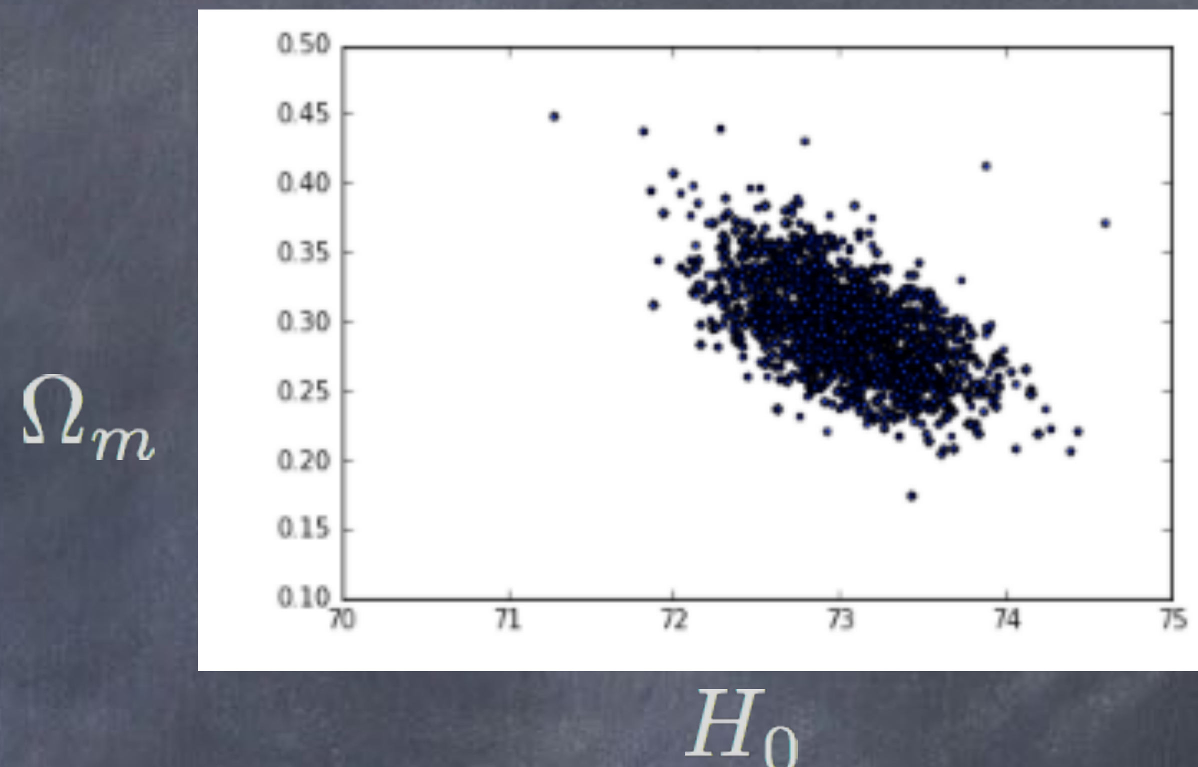
$$D_L = \frac{c}{H_0}(1+z)[\eta(1,\Omega_m) - \eta(1/(1+z),\Omega_m)]$$

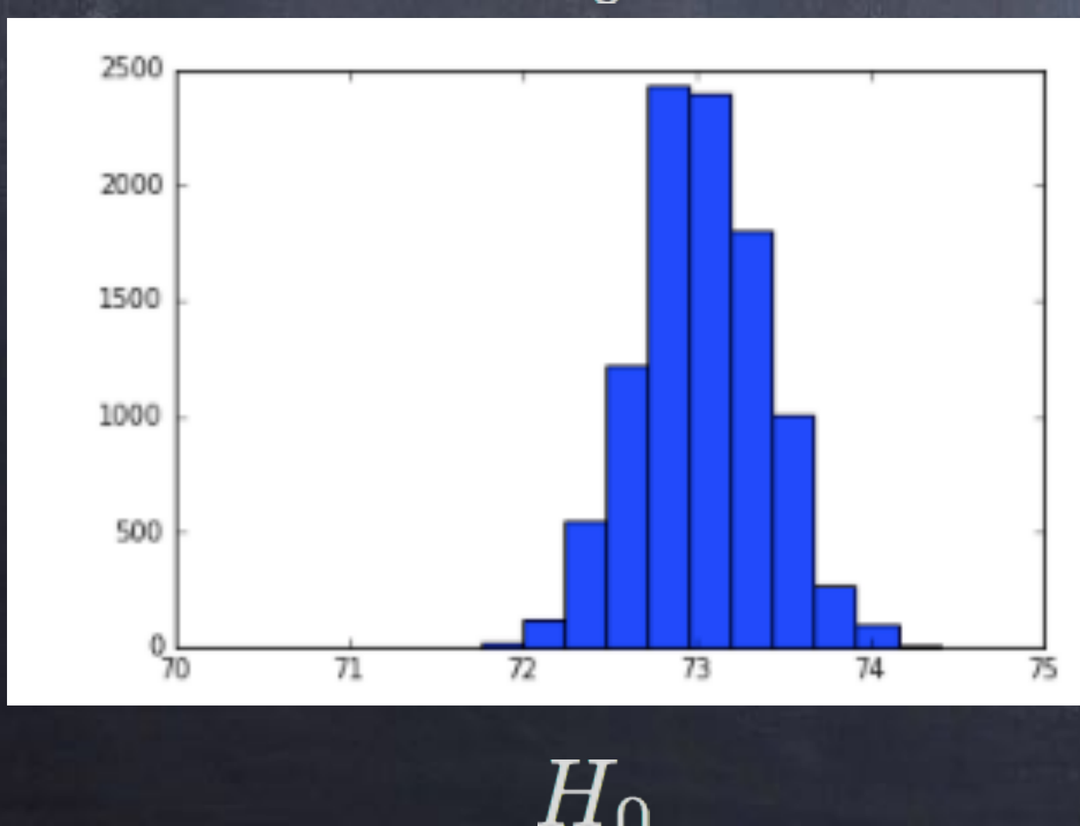
$$\eta(a,\Omega_m) = 2\sqrt{s^3+1}\,[a^{-4} - 0.1540 s\, a^{-3} + 0.4304 s^2 a^{-2} + 0.19097 s^3 a^{-1} + 0.066941 s^4]^{-1/8}$$

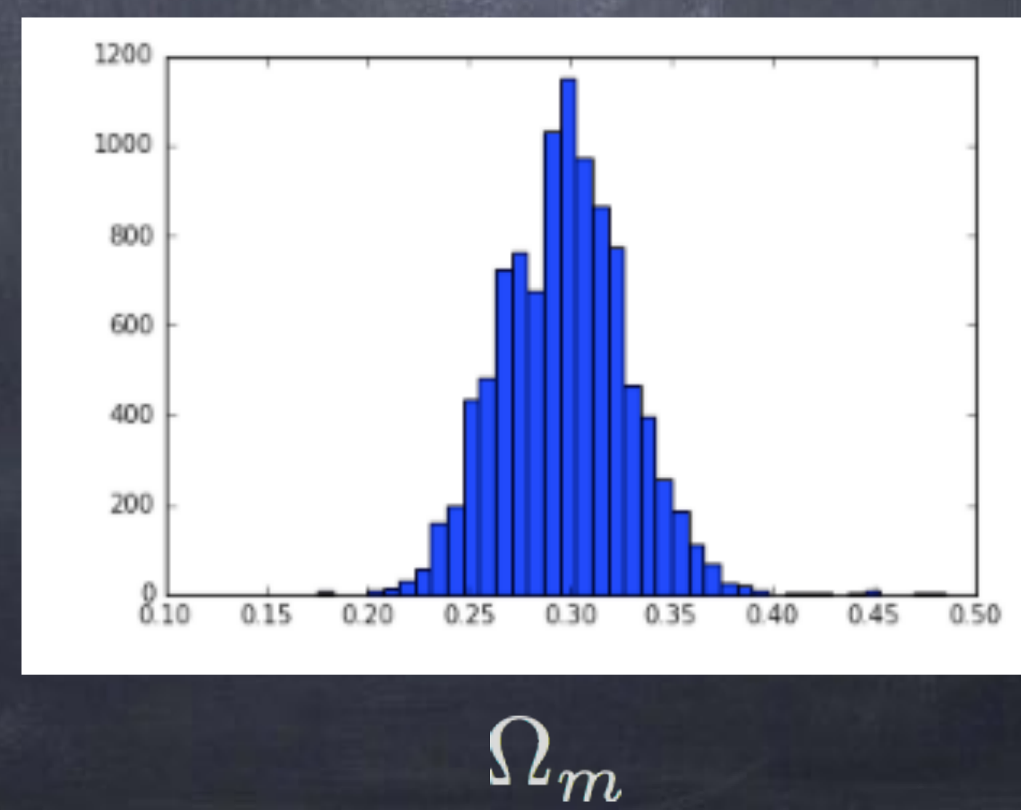
$$s^3 = (1-\Omega_m)/\Omega_m$$

Walker


Ω_m
1.0
0.2
45 50 55 60 65 70 75 80
H_0


Ω_m
0.50 0.45 0.40 0.35 0.30 0.25 0.20 0.15 0.10
70 71 72 73 74 75
H_0


2500 2000 1500 1000 500 0
70 71 72 73 74 75
H_0


1200 1000 800 600 400 200 0
0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50
Ω_m