IQM CHALLENGE

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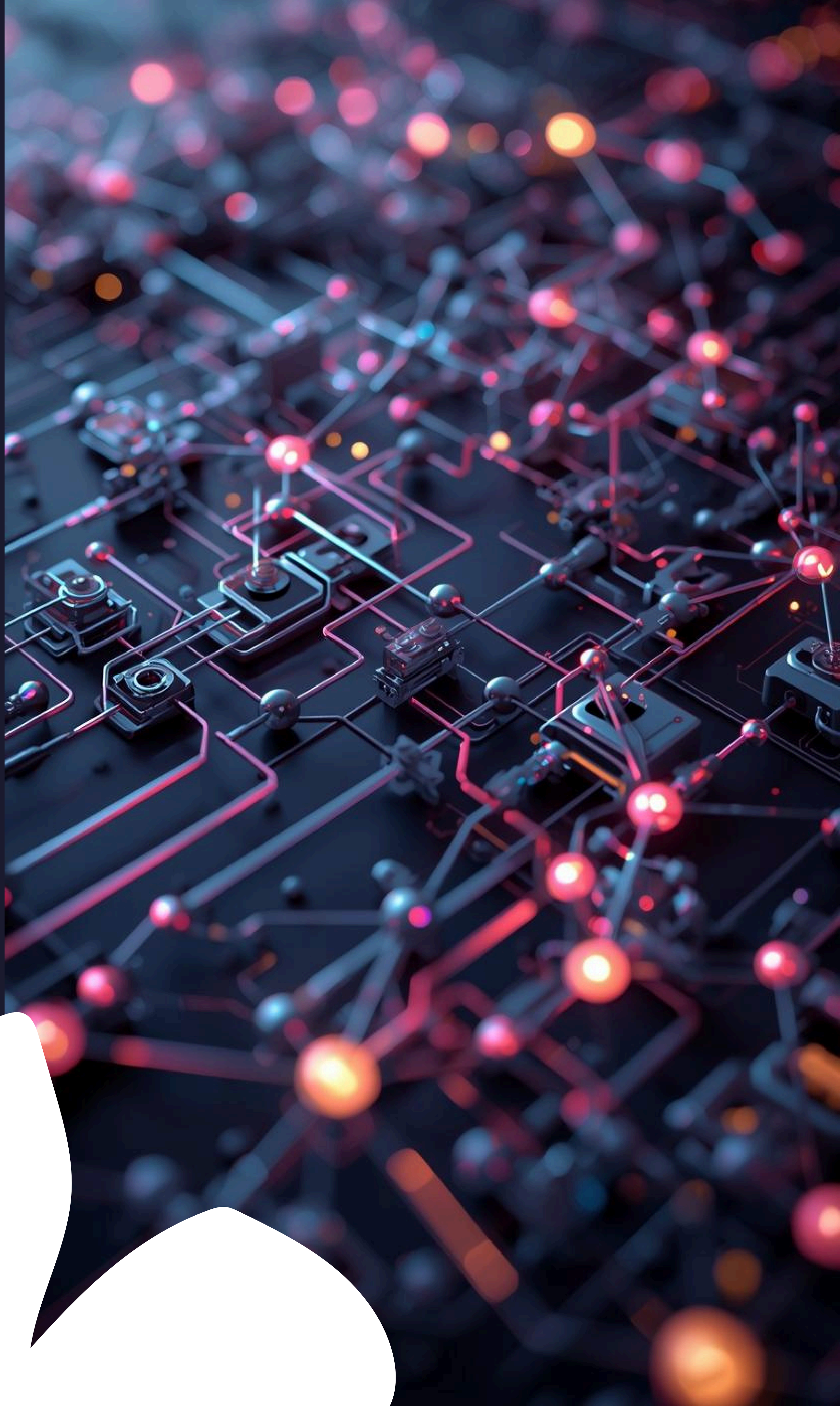


Introduction to Quantum Entanglement

SETTING THE FOUNDATION

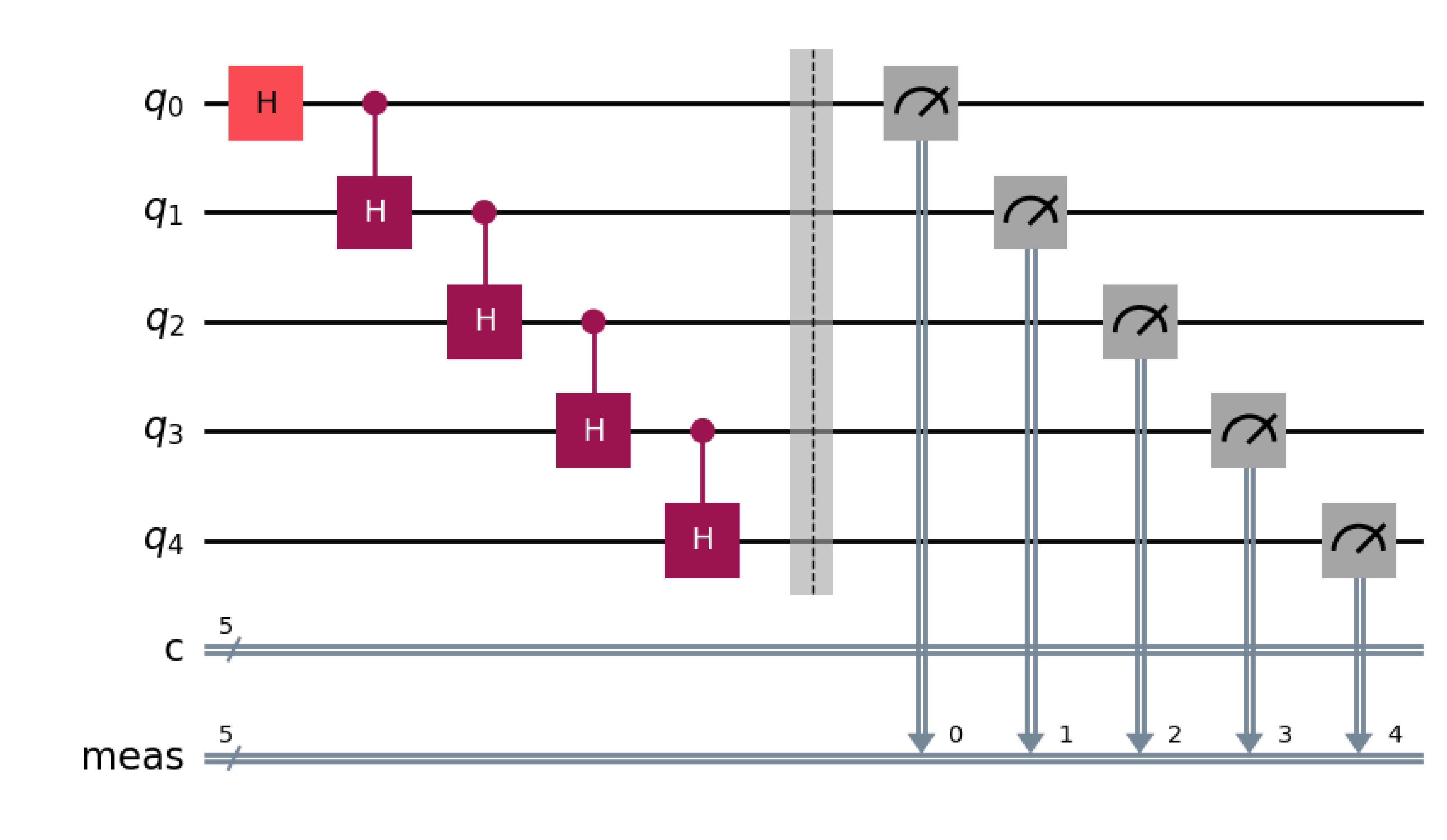
Simply put, quantum entanglement is when measuring one qubit allows you to obtain information about another qubit.

We have seen this is the case in the CSHS inequality. In this project, we attempted a variety of approaches to prove that quantum entanglement is actually present, and we present 4 main ways:

- Statistical significance
- Expanding CSHS to four qubits
- Calculating Multipartite Entanglement Measures
- Using an Entanglement Witness

Constructing a circuit for statistical significance

Quantum State Construction: Consider n qubits such that for all $0 \leq i \leq n-2$, there exists a Hadamard control Gate between qubit i and qubit $i+1$. For the first qubit (qubit 0), there is a normal Hadamard gate applied to it as well. Suppose $n = 5$.



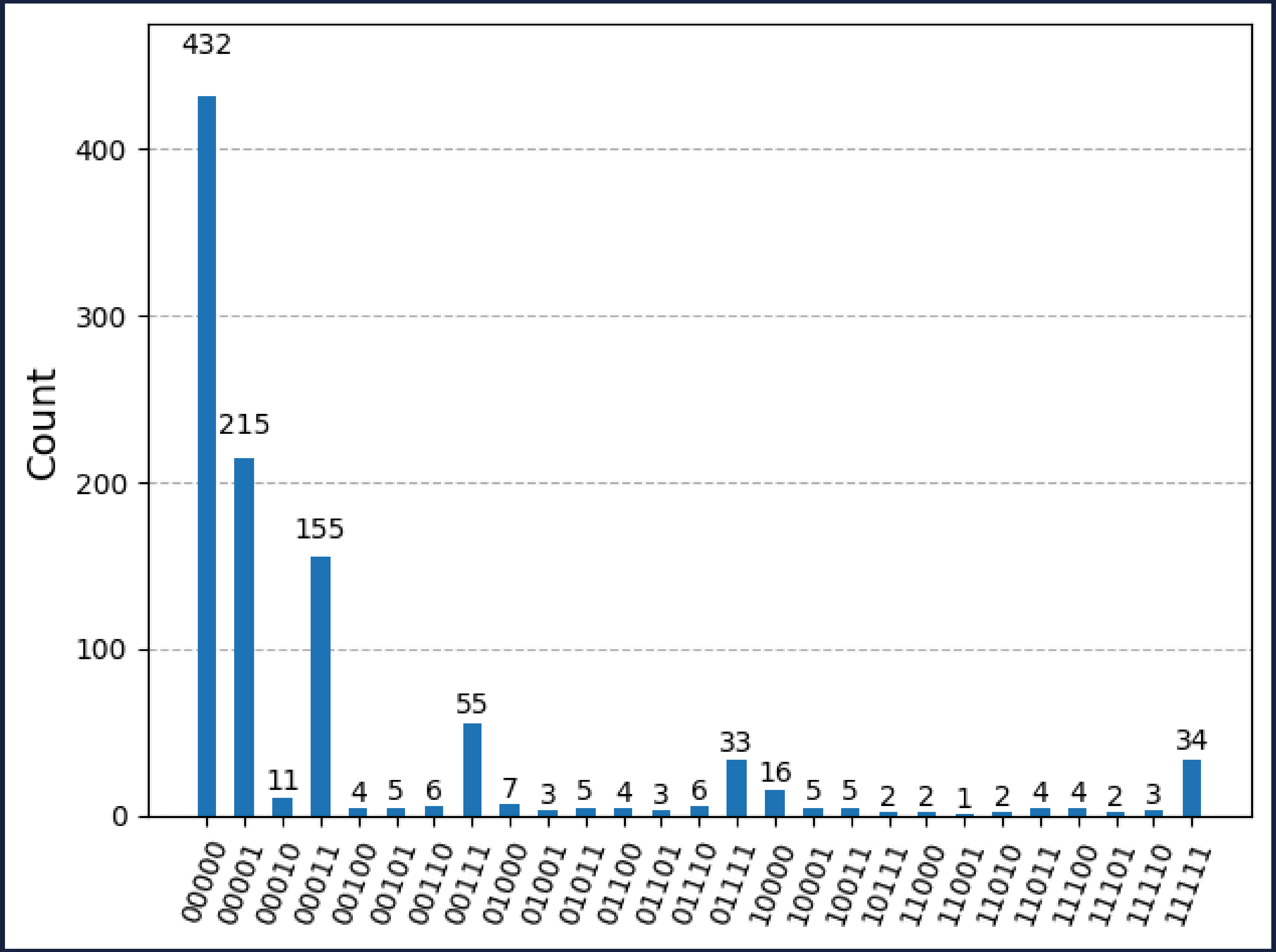
Analyzing Circuit

After the first Hadamard gate, q_0 has equal probability of either $|0\rangle$ or $|1\rangle$ states, so the controlled Hadamard gate on q_1 is equally like to be activated or not, leaving a $\frac{1}{2}$ chance of $|1\rangle$ state if q_0 is $|1\rangle$. Thus, $|q_0q_1\rangle = \frac{(\sqrt{2}|00\rangle + 0\cdot|01\rangle + 1\cdot|10\rangle + 1\cdot|11\rangle}{2}$. Note that any qubit q_n can only take the $|1\rangle$ state if q_{n-1} is also $|1\rangle$. By induction, this means the qubits $q_0, q_1, \ldots q_{n-1}$ must all have taken the $|1\rangle$ state.

It follows that for an n qubit system, the probability of measuring the i-th qubit to be $|1\rangle$ is $\frac{1}{2^{i+1}}$.

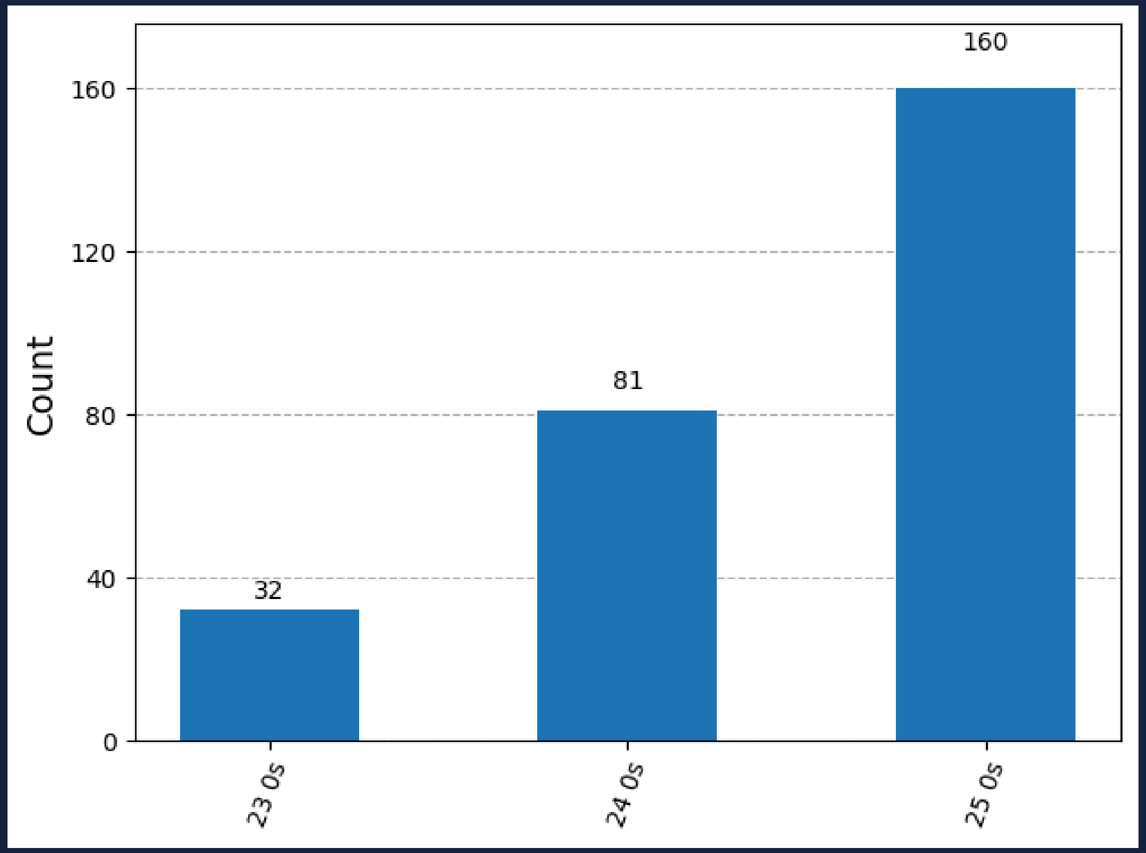
Based on our analysis, the possible states for n = 5 are 00000, 00001, 00011, 00111, 01111, and 11111, where our format is |q4q3q2q1q0> (we do this since Qiskit prints the output in reverse order).

Results



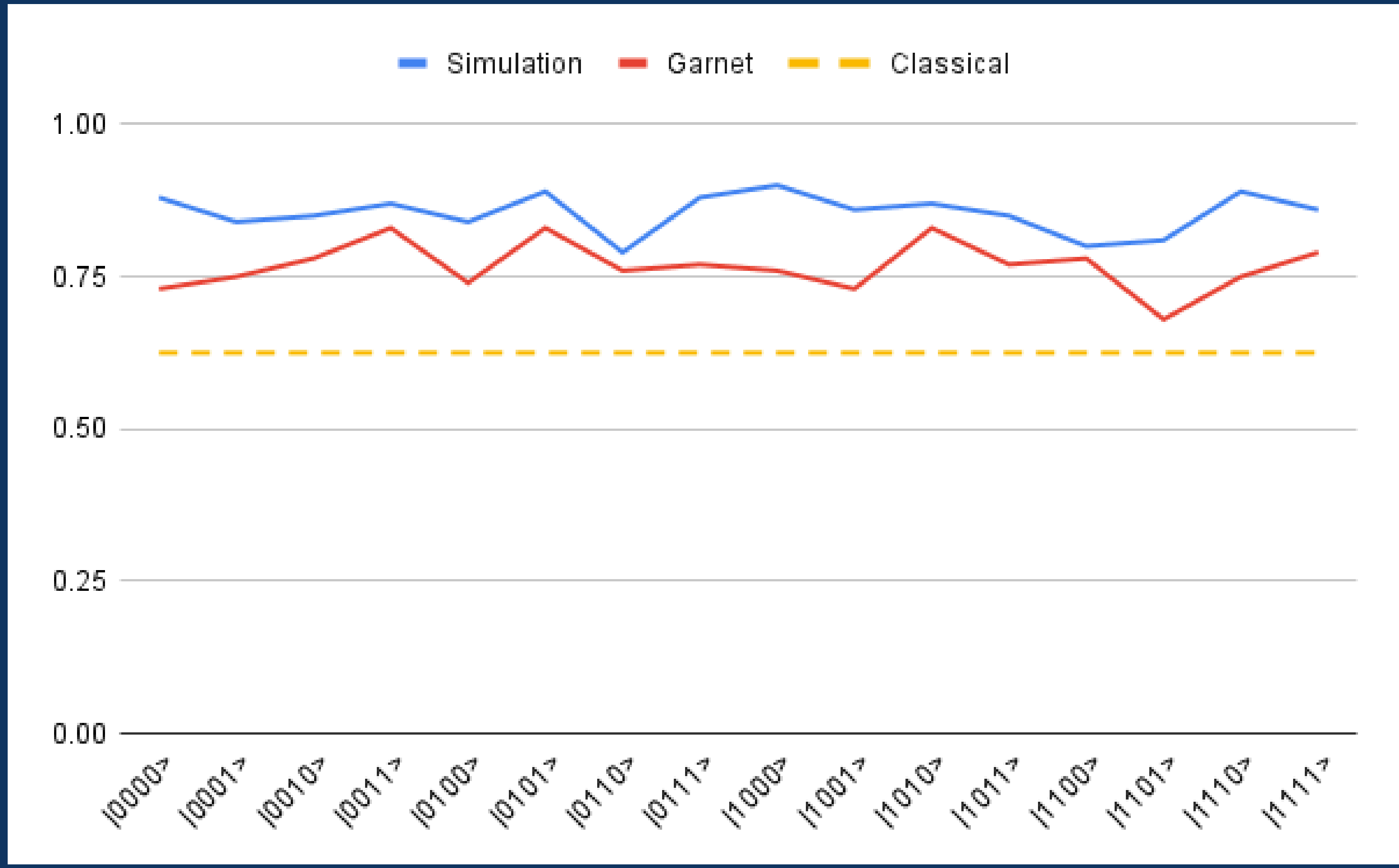
Notes:

- Noise needs to be measured in some of the other states being measured
- This multipartite entanglement is essentially a more advanced version of GHZ with more peaks to check for verification statistically, other than just two states.
- The peaks are scalable, but the probability ratios aren't.



Implementing CSHS 4 Qubit Game

WE NEXT EXPLORED THE CSHS GAME INTO 4 QUBITS TO FURTHER SUPPORT THE RESULTS AS DEMONSTRATED. WE FOLLOWED A PAPER (FOUR-QUBIT CHSH GAME BY JUSSEAU ET AL)

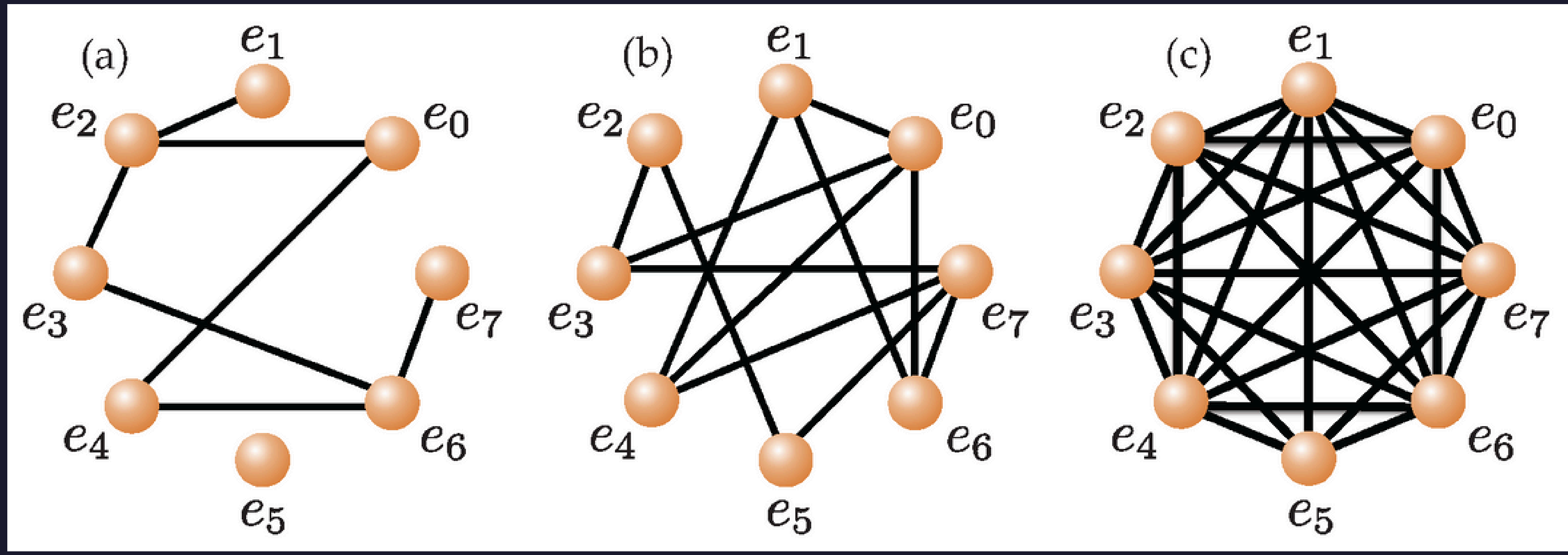


- Running the CHSH circuit on a real quantum processor (e.g., IQM Resonance) shows performance slightly below ideal (~85%) due to noise, decoherence, and hardware imperfections.
- The CHSH game extends naturally to more than two players by replacing $|\Phi^+\rangle$ with an n-qubit GHZ state: $|GHZ_n\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes n} + |1\rangle^{\otimes n})$
- The number of possible boolean functions $f(x_1, \ldots, x_n)$ grows exponentially, making it increasingly difficult to find functions that exhibit a quantum-classical performance gap.

Calculating Multipartite Entanglement Measures

$$I(A:B|E) = S(AE) + S(BE) - S(ABE) - S(E)$$

$$E_{sq}^{c}(\rho_{A_1,A_2,\cdots,A_m}) = \inf I\left(A_1 : A_2 : \cdots : A_m|E\right)$$



VARIOUS APPROACHES - SCALE DIFFERENTLY, UNIQUE QUALITIES

Following some of the work of "Multipartite entanglement measures: a review" (Ma, Li, Shang), we determined approximate upper and lower bounds for multipartite entanglement (ME) measures using conditional mutual information (CMI).
Tried using partial and complete tomography and classical shadows to estimate (reduced) density matrices and calculate entropy.

Also referenced

- "Benchmarking Multipartite Entanglement Generation with Graph States" (Zander, Becker)
- "Entanglement in a 20-Qubit Superconducting Quantum Computer" (Mooney, Hill, Hollenberg)
- "Predicting Many Properties of a Quantum System from Very Few Measurements" (Huang, Kueng, Preskill)

Using Entanglement Witnesses

A witness can be used to prove entanglement exists without fully measuring the quantum state. For any separable state, the expectation of W will be non-negative. Therefore, if after measurement a state yields a negative value for the expectation of W the state must be entangled.

In terms of experimental design, part of the experiment can be to utilize a portfolio of different entanglement witnesses for different states. For example, we can implement an entanglement witness for GHZ states, W states, Dicke states, and so on.

For the W state and cluster state, the attempts at specialized entanglement witnesses ran into some issues. A more generalized approach of computing entropy and using it to show the existence of entropy could sometimes be used instead.

$$\mathrm{Tr}(W\sigma) \geq 0 \quad \mathrm{Tr}(W\rho) < 0$$

W is a Hermitian operator acting on $\mathcal{H}_A \otimes \mathcal{H}_B$

$$W = \frac{1}{2}\mathbf{1} - |\mathrm{GHZ}_n\rangle\langle\mathrm{GHZ}_n|$$

Common Genuine Multipartite Entanglement (GME) Witness for GHZ state

$$\mathrm{Witness} = \frac{1}{2} - \frac{1}{2}\left(p_{\mathrm{GHZ}} + x_{\mathrm{corr}}\right)$$

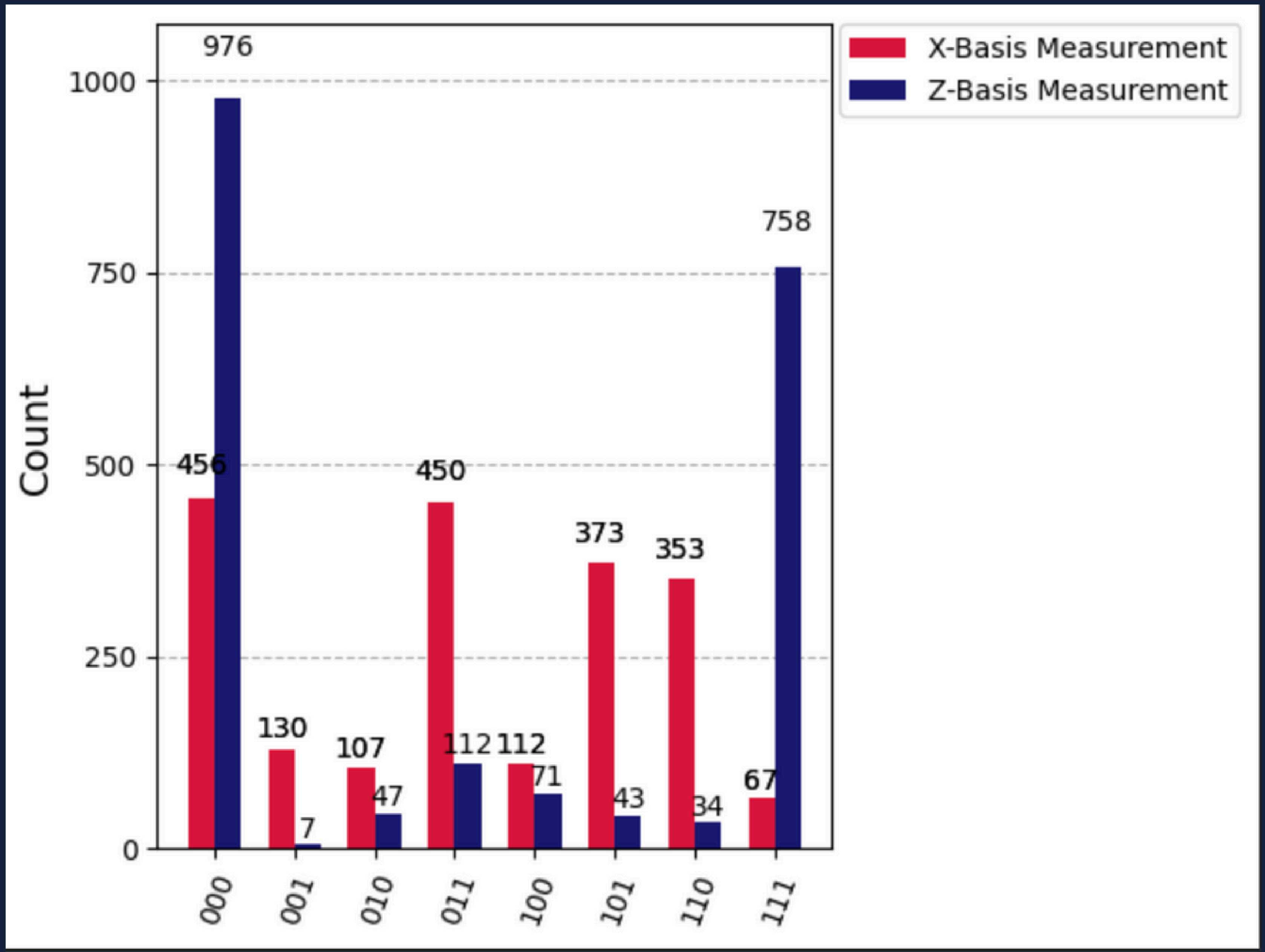
Combine probability that the state is in the GHZ state and measure of correlation in the X basis

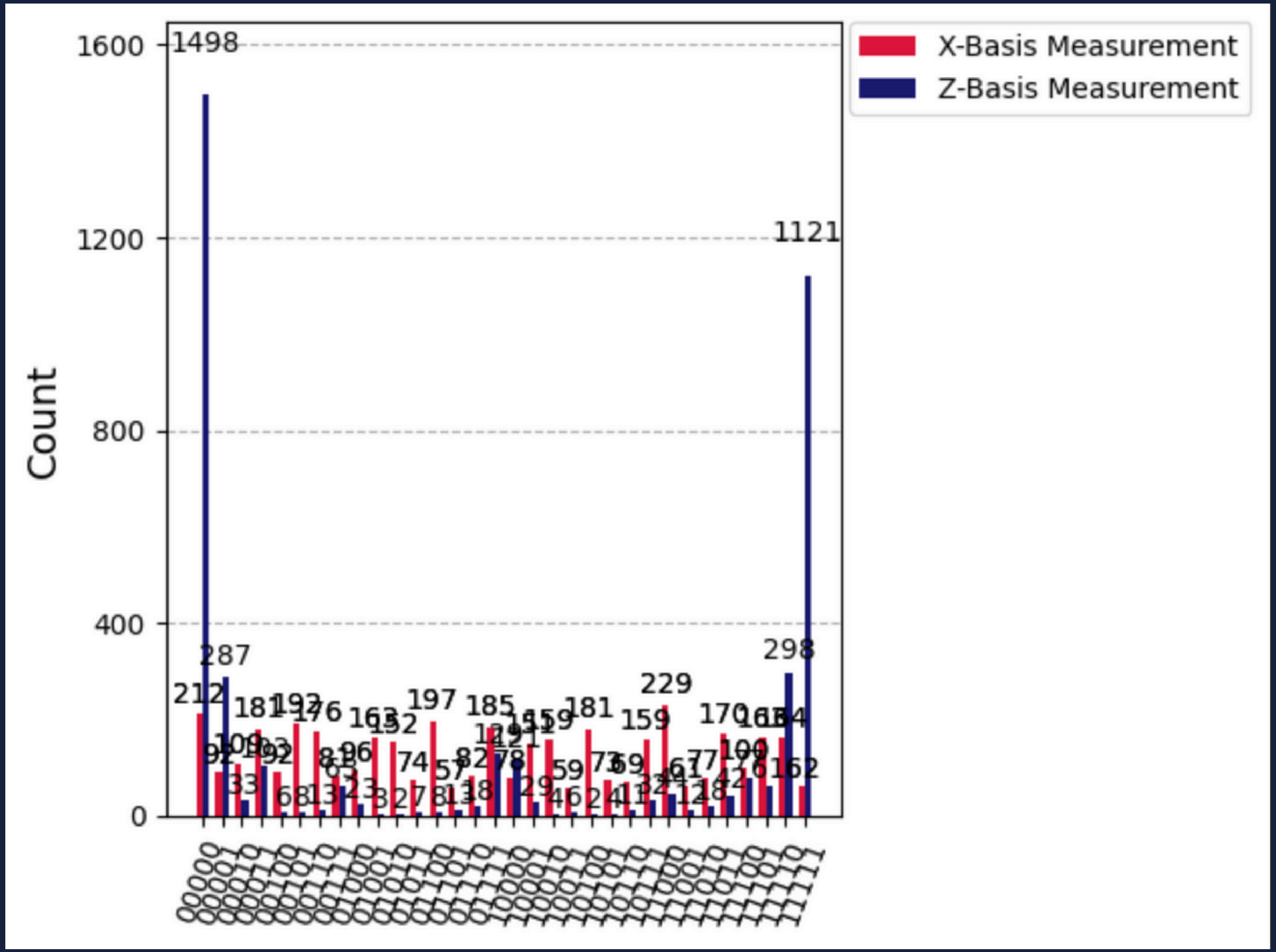
Results (GHZ)

Managing Noise:

- Reducing circuit depth by using tree-style entanglement: fewer gates leads to less error
- Use partial measurements when calculating X correlation
- Or, weighted sampling to estimate for $\langle X^{\otimes n} \rangle$

X CORRELATION: desirable states are 000, 011, 101, 110





Thank you! Questions?