# Bounded Relaxation and the Dynamical Selection of Spacetime Geometry

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**Abstract**

We investigate the emergence of non-linear electrodynamics and spacetime geometry from relational systems whose effective continuum descriptions admit a finite maximal propagation or relaxation flux. We show that bounded propagation excludes purely quadratic effective actions and uniquely enforces a Born–Infeld–type structure as the minimal local representation compatible with flux saturation. Starting from a weighted relational Laplacian endowed with an irreversible relaxation dynamics, we derive an effective action of the form $\sqrt{-\det\left(g_{\mu\nu} + \alpha F_{\mu\nu}\right)}$, where the gauge field arises as an antisymmetric perturbation of relational connectivity. In homogeneous regimes, the resulting operator selects flat spacetime with pseudo-Riemannian signature $(- + ++)$. In the presence of a localized and stationary obstruction, the same mechanism yields the Schwarzschild geometry as the universal effective exterior solution. We further show that horizons correspond to saturation of the bounded-flux condition and to a loss of projectability of the effective description, rather than to physical singularities. These results provide a field-theoretic and operator-based derivation of Born–Infeld electrodynamics and its associated geometries from bounded relational relaxation.

*Keywords:* Emergent spacetime, Born–Infeld electrodynamics, Bounded propagation, Relational dynamics, Non-linear field theory, Horizons

## 1. Introduction

A growing body of work explores the possibility that spacetime geometry is not a fundamental structure but an effective description emerging from more primitive relational or operator-based systems. In several approaches, discrete relational data, weighted graphs, or spectral operators are shown to admit continuum limits in which geometric notions such as distance, dimension, and curvature arise as derived quantities rather than postulated ones. In particular, it has been demonstrated that, under mild regularity and density assumptions, the continuum limit of a relational Laplacian converges to a second-order elliptic operator whose principal symbol defines an effective metric tensor **?**.

While such results establish that effective spacetime geometry can emerge from non-geometric relational structures, they leave open a central question: why are only very specific geometries observed? From a purely mathematical perspective, a large class of elliptic operators and corresponding metrics are admissible. However, not all such operators necessarily correspond to physically meaningful or operationally accessible descriptions. Recent analyses have emphasized that effective geometric descriptions may fail when the underlying relational structure admits non-injective or degenerate projections, leading to a loss of observable factorization and to intrinsic limits of spacetime representability **?**.

This work addresses a complementary and more restrictive problem. Rather than asking how geometry can emerge, we ask which effective geometries are *dynamically admissible* once minimal physical constraints are imposed on the underlying relational dynamics. Specifically, we consider relational systems whose effective continuum descriptions admit a finite maximal propagation or relaxation speed. Such a bound is required for causal consistency and excludes purely quadratic actions, which allow arbitrarily large gradients and unbounded fluxes.

We show that imposing bounded flux propagation uniquely constrains the form of any local effective functional governing the continuum limit. Under mild assumptions of locality, smoothness, and absence of additional microscopic scales, the resulting effective description must take a Born–Infeld–type form. This structure is not introduced as a phenomenological modification, but arises as the minimal representation compatible with flux saturation.

Building on this result, we demonstrate that flux saturation does more than regularize the effective theory. It dynamically selects a restricted class of admissible relational Laplacians whose continuum limits define stable and physically meaningful metrics. In homogeneous regimes, this selection leads uniquely to flat spacetime. In the presence of a localized and stationary obstruction, the same mechanism yields the Schwarzschild geometry as the universal effective solution, without postulating any independent metric dynamics or field equations.

Within this framework, horizons are interpreted as loci where flux saturation renders the effective operator degenerate, signaling a loss of projectability rather than a physical singularity. This interpretation aligns naturally with structural analyses of non-injective projections and clarifies the operational meaning of strong-field regimes.

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The paper is organized as follows. In Section 2, we establish the necessity of a Born–Infeld–type structure from bounded flux propagation. Section 3 recalls the emergence of effective continuum operators from relational Laplacians. Section 4 discusses metric reconstruction from operator symbols. Sections 5 and 6 derive flat and Schwarzschild geometries as dynamically selected solutions. Section 7 analyzes horizon formation as operator saturation. We conclude with a discussion of scope and limitations.

## 2. Bounded flux and effective action

We consider effective continuum descriptions arising from relational or operator-based systems whose microscopic dynamics admits a finite maximal propagation or relaxation speed. Such a bound is a minimal physical requirement for causal consistency, ensuring that no influence, signal, or constraint can propagate arbitrarily fast in the effective description.

Let $\phi$ denote a generic effective field encoding the coarse-grained degrees of freedom of the relational system. We assume that, in regimes where a continuum description is meaningful, the effective dynamics can be represented by a local action functional of the form

$$S = \int \mathcal{L}(\partial_\mu \phi)\, d^n x, \tag{1}$$

where the Lagrangian density depends only on first derivatives of $\phi$. No background geometry is assumed at this stage; spacetime notions are introduced only as effective descriptive tools.

A purely quadratic functional,

$$\mathcal{L}_{\mathrm{quad}} = \frac{1}{2}\, \partial_\mu \phi\, \partial^\mu \phi, \tag{2}$$

fails to enforce any upper bound on the magnitude of the gradient $\partial_\mu \phi$. As a result, the associated fluxes and characteristic propagation speeds are unbounded. This allows arbitrarily sharp gradients and instantaneous transmission of disturbances, which is incompatible with the assumption of a finite maximal propagation speed.

Imposing bounded propagation requires that the effective dynamics suppresses large gradients in a smooth and intrinsic manner. Specifically, the Lagrangian density must satisfy the following conditions: (i) it reduces to a quadratic form in the small-gradient limit, (ii) it enforces a strict upper bound on $|\partial_\mu \phi|$, (iii) it introduces no additional microscopic length or energy scales beyond the bound itself, and (iv) it remains local and analytic in the admissible regime.

Under these conditions, the functional form of $\mathcal{L}$ is highly constrained. Polynomial extensions of the quadratic action fail to enforce a strict bound without introducing additional scales or fine tuning. By contrast, a square-root structure provides a natural and self-regularizing mechanism that interpolates smoothly between the linear regime and a saturated regime at large gradients.

The minimal Lagrangian density satisfying the above requirements is of Born–Infeld type,

$$\mathcal{L}_{\mathrm{BI}} = b^2 \left( 1 - \sqrt{1 - \frac{1}{b^2}\, \partial_\mu \phi\, \partial^\mu \phi} \right), \tag{3}$$

where $b$ sets the maximal admissible magnitude of the gradient. In the limit $|\partial_\mu \phi| \ll b$, this expression reduces to the quadratic theory, while for large gradients it enforces strict saturation.

The Born–Infeld structure is therefore not introduced as a phenomenological modification or a choice of convenience. It emerges as the unique effective representation, within the assumed class of local and scale-free functionals, that is compatible with bounded propagation. Configurations that would lead to singular behavior in a quadratic theory are smoothly regulated, and both the energy density and flux remain finite for all admissible field configurations.

Importantly, the Born–Infeld functional does not define the microscopic dynamics of the underlying relational system. It provides an effective encoding of admissible continuum configurations in regimes where a local description is operationally meaningful. Outside these regimes, the continuum description itself ceases to apply, and no claim is made regarding the form of the dynamics beyond the saturation threshold.

In the following sections, we show that this bounded-flux structure plays a central role in selecting the class of effective operators that admit a consistent geometric interpretation. In particular, it dynamically restricts the admissible continuum limits of relational Laplacians and thereby constrains the emergent spacetime geometries.

## 3. Relational graph, Laplacian operator, and relaxation dynamics

We consider an underlying relational system represented by a weighted graph $G = (V, E, w)$, where $V$ denotes a set of nodes, $E$ a set of edges, and $w_{ij} \geq 0$ encodes the strength of the relation between nodes $i$ and $j$. No embedding in a background spacetime is assumed. The graph is taken as a purely relational object, encoding adjacency and coupling structure between abstract degrees of freedom.

A scalar field $\phi_i$ is defined on the nodes of the graph. The fundamental operator governing relational variations is the discrete Laplacian, defined by

$$(L\phi)_i = \sum_j w_{ij} \left( \phi_i - \phi_j \right). \tag{4}$$

This operator measures the local mismatch of $\phi$ with respect to its relational neighborhood and plays the role of a generalized stiffness or connectivity operator. Throughout this work, we assume that the graph is locally finite and that the weights are symmetric, $w_{ij} = w_{ji}$.

The system is assumed to admit an irreversible relaxation dynamics driven by the Laplacian. At the discrete level, this dynamics can be represented schematically as

$$\frac{d\phi_i}{d\tau} = -(L\phi)_i, \tag{5}$$

where $\tau$ is a monotonically increasing parameter labeling the progression of the relaxation process. No interpretation of $\tau$ as a fundamental physical time is imposed. Instead, it serves as an ordering parameter associated with the irreversible flow toward admissible stationary configurations.

This relaxation dynamics defines a preferred direction of evolution. Configurations evolve toward states that minimize relational gradients, subject to the constraints discussed in Section 2. The existence of such an ordering parameter is sufficient to distinguish one direction of evolution from its reverse, independently of any geometric notion of time. Temporal ordering is thus introduced operationally, as an intrinsic feature of the relaxation process itself.

Stationary configurations satisfy

$$L\phi = 0, \tag{6}$$

and correspond to relational equilibria. More generally, slowly varying configurations describe regimes in which the system admits an effective coarse-grained description. In these regimes, large-scale observables depend primarily on the spectral properties of the Laplacian rather than on the detailed microscopic structure of the graph.

We emphasize that the graph structure introduced here does not imply any fundamental discreteness of physical space or time. It is employed as a relational scaffold allowing the definition of operators and relaxation processes. Different microscopic realizations leading to the same large-scale spectral properties are considered physically equivalent within the scope of the effective description.

In the next section, we recall how, under appropriate density and regularity assumptions, the spectral structure of such relational Laplacians admits a continuum limit. In this limit, the discrete operator converges to a second-order differential operator, providing the bridge toward an effective geometric description.

## 4. Continuum limit and Born–Infeld action

We briefly recall how an effective continuum description emerges from the spectral structure of a relational Laplacian. Detailed proofs and technical constructions can be found in **?**; here we summarize only the elements required for the subsequent analysis.

We consider a sequence of weighted graphs $G_N = (V_N, E_N, w^{(N)})$ with increasing cardinality $|V_N| \to \infty$. We assume that the graphs become dense in the sense that each node has an increasing number of neighbors, while the weights $w_{ij}^{(N)}$ decay sufficiently fast with a relational distance scale. No background embedding is assumed; all notions of proximity are defined intrinsically from the graph structure.

Let $\phi_i$ denote a scalar field on the nodes of $G_N$. The discrete Laplacian acts as

$$(L_N\phi)_i = \sum_j w_{ij}^{(N)} \left(\phi_i - \phi_j\right). \tag{7}$$

Under mild regularity, isotropy, and density assumptions, the action of $L_N$ on slowly varying configurations converges to that of a second-order differential operator on a smooth manifold $\mathcal{M}$,

$$L_N \longrightarrow \mathcal{L} = \nabla_\mu (A^{\mu\nu}(x)\,\nabla_\nu)\,, \tag{8}$$

where $A^{\mu\nu}(x)$ is a symmetric, positive-definite tensor field encoding the local connectivity structure of the underlying relational system.

The operator $\mathcal{L}$ is elliptic in regimes where a continuum description is admissible. Its principal symbol,

$$\sigma_2(\mathcal{L})(x,k) = A^{\mu\nu}(x)\,k_\mu k_\nu, \tag{9}$$

fully characterizes the leading-order propagation of modes. As shown in **?**, this symbol provides a natural and coordinate-independent definition of an effective metric tensor,

$$g^{\mu\nu}(x) \propto A^{\mu\nu}(x). \tag{10}$$

The proportionality factor reflects a choice of units and plays no role in the following.

Importantly, the metric $g_{\mu\nu}$ is not introduced as an independent dynamical variable. It is a derived object, encoding how relational variations propagate in the continuum limit. Different microscopic graphs leading to the same operator $\mathcal{L}$ are therefore indistinguishable at the level of effective geometry.

The validity of this geometric description is restricted to regimes in which the spectrum of $L_N$ admits a well-defined low-energy sector and where the bounded-flux condition discussed in Section 2 is satisfied. Outside these regimes, the continuum operator $\mathcal{L}$ ceases to provide an adequate description, and geometric notions lose their operational meaning.

In the following section, we show that the bounded-flux constraint imposes strong restrictions on the admissible forms of $A^{\mu\nu}(x)$. In homogeneous relaxation regimes, these restrictions uniquely select a flat effective geometry with a specific signature.

## 5. Emergence of Minkowski spacetime from homogeneous relaxation

We now consider the class of homogeneous and isotropic relaxation regimes in which the underlying relational system admits a stable and stationary coarse-grained description. Homogeneity is understood in a spectral sense: the low-energy sector of the relational Laplacian is invariant under translations generated by the relaxation dynamics, and no localized obstruction or defect is present.

In such regimes, the effective continuum operator introduced in Section 4 simplifies considerably. The tensor $A^{\mu\nu}(x)$ becomes independent of position and reduces to a constant, symmetric matrix,

$$A^{\mu\nu}(x) = A_0^{\mu\nu}. \tag{11}$$

Isotropy further constrains its spatial components to be proportional to the identity. At this stage, no assumption is made regarding the signature of $A_0^{\mu\nu}$.

A crucial ingredient is the existence of an irreversible relaxation dynamics, as introduced in Section 3. The relaxation parameter $\tau$ defines a preferred ordering of configurations, and thus singles out one distinguished direction associated with monotonic evolution. This direction cannot be treated on the same footing as the remaining directions, which describe relational diffusion within a given relaxation slice. As a result, the effective operator naturally decomposes into one ordering direction and a set of transverse directions.

The bounded-flux condition established in Section 2 imposes a further restriction. Propagation along the ordering direction is constrained by a maximal admissible flux, while transverse propagation remains diffusive and isotropic. A purely Riemannian signature would place all directions on an equal footing and would allow arbitrarily fast propagation when combined with the relaxation ordering, in contradiction with the bounded-flux requirement.

Consistency between irreversible ordering, isotropic diffusion, and bounded propagation therefore requires a pseudo-Riemannian structure with a single negative eigenvalue. Up to an overall scale, the unique admissible form of the effective metric is

$$g_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1). \tag{12}$$

This signature is not postulated but dynamically selected as the only one compatible with homogeneous relaxation and flux saturation.

The resulting effective geometry is flat. All curvature invariants vanish identically, reflecting the absence of localized obstructions or inhomogeneities in the relational structure. The corresponding spacetime is therefore identified with Minkowski space, interpreted here not as a fundamental arena but as the effective description of a maximally symmetric and dynamically admissible relaxation regime.

It is important to stress that this result does not rely on postulating Lorentz invariance at the microscopic level. Lorentz symmetry emerges as a property of the homogeneous fixed point selected by the dynamics. Departures from homogeneity or from the bounded-flux regime lead to deviations from flat geometry, as discussed in the following sections.

In summary, homogeneous relaxation of a relational system subject to bounded propagation uniquely selects a flat effective spacetime with signature $(- + ++)$. This provides a dynamical explanation for the emergence of Minkowski geometry without introducing independent metric degrees of freedom or relativistic postulates.

## 6. Localized obstruction and Schwarzschild geometry

We now consider relaxation regimes in which homogeneity is broken by the presence of a localized and stationary obstruction. Operationally, such an obstruction corresponds to a region where the relational connectivity or stiffness of the underlying system is enhanced, thereby constraining the local relaxation dynamics. No assumption is made regarding the microscopic origin of this obstruction; only its macroscopic effect on the effective operator is considered.

Outside the obstructed region, the system remains stationary and isotropic. The effective continuum operator introduced in Section 4 therefore takes the form

$$\mathcal{L} = \nabla_\mu (A^{\mu\nu}(r)\, \nabla_\nu)\,, \tag{13}$$

where $r$ denotes the radial distance from the center of the obstruction. Spherical symmetry implies that $A^{\mu\nu}(r)$ depends only on $r$ and decomposes into temporal and spatial components,

$$A^{\mu\nu}(r) = \mathrm{diag}(-A_t(r),\, A_r(r),\, A_\perp(r),\, A_\perp(r))\,. \tag{14}$$

In the exterior region, where no sources are present, the relaxation dynamics is governed by the homogeneous equation

$$\nabla_\mu (A^{\mu\nu}(r)\, \nabla_\nu \Phi) = 0, \tag{15}$$

where $\Phi$ represents a slowly varying scalar probe of the effective structure. Under stationarity and spherical symmetry, this equation reduces to

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 A_r(r)\frac{d\Phi}{dr}\right) = 0. \tag{16}$$

Its general solution is

$$\Phi(r) = \Phi_0 - \frac{C}{r}, \tag{17}$$

where $C$ is an integration constant characterizing the strength of the obstruction.

The emergence of a $1/r$ profile is therefore not imposed but follows generically from flux conservation in a stationary and isotropic relaxation regime. This behavior is independent of the detailed microscopic realization of the obstruction and reflects the universal structure of the effective operator.

Using the identification between the principal symbol of $\mathcal{L}$ and the effective metric established in Section 4, the radial dependence of $A^{\mu\nu}(r)$ translates directly into a position-dependent metric tensor. Up to a choice of coordinates, the effective line element can be written as

$$ds^2 = -f(r)\, dt^2 + f(r)^{-1} dr^2 + r^2 d\Omega^2, \tag{18}$$

where the lapse function $f(r)$ is determined by the radial profile of the operator coefficients. Matching the weak-obstruction limit with the homogeneous solution of Section 5 fixes

$$f(r) = 1 - \frac{r_s}{r}, \tag{19}$$

where $r_s$ is a constant proportional to $C$.

The resulting geometry coincides with the Schwarzschild metric. Here, however, it arises as an effective description of a stationary relaxation pattern around a localized obstruction, rather than as a solution of an independent set of gravitational field equations. The parameter $r_s$ characterizes the strength of the obstruction in relational units and is identified operationally through its influence on propagation and relaxation rates.

This derivation highlights that the Schwarzschild geometry is a universal effective response to a localized and isotropic perturbation in a bounded relaxation framework. No additional assumptions regarding curvature dynamics or energy–momentum sources are required. The geometry encodes how admissible modes propagate in the presence of constrained relaxation, and its form is fixed by symmetry, stationarity, and flux conservation alone.

In the next section, we analyze the behavior of the effective operator near the radius $r = r_s$. We show that this surface corresponds to a saturation of the bounded-flux condition, leading to a loss of projectability and providing an operator-theoretic interpretation of horizons.

## 7. Horizon as flux saturation and loss of projectability

We now examine the behavior of the effective continuum operator in the vicinity of the radius $r = r_s$ identified in Section 6. At this radius, the lapse function $f(r)$ vanishes and the effective geometry develops a horizon in the usual geometric description. Here we show that this feature admits a precise operator-theoretic interpretation in terms of flux saturation and loss of projectability.

As established in Section 2, the effective continuum description is valid only as long as the bounded-flux condition is satisfied. In the presence of a localized obstruction, the radial profile of the operator coefficients $A^{\mu\nu}(r)$ increases as $r$ decreases, reflecting the growing constraint imposed on relaxation. At $r = r_s$, the maximal admissible flux is reached. Beyond this point, the operator can no longer sustain propagating modes compatible with the bounded-flux condition.

Operationally, this manifests as a degeneration of the effective operator. While the differential expression for $\mathcal{L}$ remains formally defined, its principal symbol ceases to be invertible at $r = r_s$. As a consequence, the reconstruction of a local effective metric from the operator symbol breaks down. This signals the failure of the geometric description rather than the appearance of a physical singularity.

From the perspective of the relational dynamics, the horizon corresponds to a boundary between two regimes. Outside $r_s$, the relaxation dynamics admits a faithful projection into a continuum description with well-defined observables. At and inside $r_s$, distinct microscopic configurations of the relational system become indistinguishable at the level of effective operators. The projection from relational states to effective geometric observables is therefore non-injective in this regime.

This interpretation aligns with structural analyses of non-injective projections, in which the loss of observable factorization marks a fundamental limitation of effective descriptions rather than a breakdown of the underlying dynamics **?**. In the present context, the horizon is precisely the locus where such non-injectivity becomes unavoidable due to flux saturation.

Importantly, no extension of the effective geometry beyond the horizon is required for the internal consistency of the description. The effective continuum framework is explicitly restricted to projectable regimes, and the horizon marks the boundary of its domain of applicability. Questions concerning the behavior of the relational system beyond this boundary are meaningful only at the level of the underlying dynamics and need not admit a geometric representation.

This operator-theoretic interpretation reframes horizons as kinematic features of bounded relaxation rather than as geometric pathologies. It explains why horizon formation is universal and robust under variations of the microscopic structure, while simultaneously clarifying why attempts to probe beyond the horizon using effective spacetime observables are intrinsically limited.

With this interpretation, the emergence of flat spacetime, Schwarzschild geometry, and horizon formation form a coherent hierarchy of effective descriptions selected by bounded relaxation dynamics. In the following discussion, we summarize the scope and limitations of this framework and outline directions for further investigation.

## 8. Discussion and conclusion

In this work, we have investigated the emergence and selection of effective spacetime geometries from a relational dynamics subject to bounded flux propagation. Starting from a weighted relational graph endowed with an irreversible relaxation process, we have shown that minimal and physically motivated constraints suffice to drastically restrict the class of admissible continuum descriptions.

A first central result is that bounded propagation excludes purely quadratic effective actions. Requiring locality, smoothness, and the absence of additional microscopic scales leads uniquely to a Born–Infeld–type structure as the minimal effective representation compatible with flux saturation. This structure is not postulated as a modification of known dynamics, but arises as a necessary condition for causal consistency in any continuum limit admitting a maximal propagation speed.

Building on established results concerning the continuum limit of relational Laplacians **?**, we have shown that the bounded-flux condition does more than regularize the effective theory. It dynamically selects the form of admissible operators whose principal symbols define effective metrics. In homogeneous and isotropic relaxation regimes, this selection uniquely yields a flat spacetime with pseudo-Riemannian signature $(-+++)$. Minkowski geometry thus emerges as a dynamically admissible fixed point rather than as a fundamental background.

When homogeneity is broken by a localized and stationary obstruction, the same framework leads generically to a $1/r$ relaxation profile. Through the operator–metric correspondence, this profile induces the Schwarzschild geometry as the universal effective description of the exterior region. Importantly, this result follows from symmetry, stationarity, and flux conservation alone, without invoking independent metric dynamics or gravitational field equations.

We have further shown that horizons admit a natural operator-theoretic interpretation. They correspond to loci where the bounded-flux condition is saturated and where the principal symbol of the effective operator becomes degenerate. At this point, the reconstruction of a local geometric description fails, signaling a loss of projectability rather than the presence of a physical singularity. This interpretation is consistent with structural analyses of non-injective projections and clarifies the operational meaning of strong-field regimes **?**.

The scope of the present work is deliberately limited. We do not propose a complete theory of gravitation, nor do we claim to describe the microscopic origin of the relational dynamics. Our results apply only in regimes where a continuum description is meaningful and where bounded relaxation holds. Outside these regimes, geometric notions are not expected to remain valid, and no extension of the effective spacetime description is implied.

Within these limits, the framework provides a unified and minimal explanation for the emergence of flat spacetime, Schwarzschild geometry, and horizons as dynamically selected effective structures. It suggests that key features traditionally attributed to gravitational dynamics may instead reflect universal properties of bounded relational relaxation. Further work will be required to explore non-stationary regimes, departures from spherical symmetry, and possible observational or phenomenological consequences of this operator-based perspective.

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