

Bounded Relaxation and the Dynamical Selection of Spacetime Geometry

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Abstract

We investigate the emergence of non-linear electrodynamics and spacetime geometry from relational systems whose effective continuum descriptions admit a finite maximal propagation or relaxation flux. We show that bounded propagation excludes purely quadratic effective actions and uniquely enforces a Born–Infeld–type structure as the minimal local representation compatible with flux saturation. Starting from a weighted relational Laplacian endowed with an irreversible relaxation dynamics, we derive an effective action of the form $\sqrt{-\det(g_{\mu\nu} + \alpha F_{\mu\nu})}$, where the gauge field arises as an antisymmetric perturbation of relational connectivity. In homogeneous regimes, the resulting operator selects flat spacetime with pseudo-Riemannian signature $(- +++)$. In the presence of a localized and stationary obstruction, the same mechanism yields the Schwarzschild geometry as the universal effective exterior solution. We further show that horizons correspond to saturation of the bounded-flux condition and to a loss of projectability of the effective description, rather than to physical singularities. These results provide a field-theoretic and operator-based derivation of Born–Infeld electrodynamics and its associated geometries from bounded relational relaxation.

Keywords: Emergent spacetime, Born–Infeld electrodynamics, Bounded propagation, Relational dynamics, Non-linear field theory, Horizons

1. Introduction

A growing body of work explores the possibility that spacetime geometry is not a fundamental structure but an effective description emerging from more primitive relational or operator-based systems. In several approaches, discrete relational data, weighted graphs, or spectral operators are shown to admit continuum limits in which geometric notions such as distance, dimension, and curvature arise as derived quantities rather than postulated ones. In particular, it has been demonstrated that, under mild regularity and density assumptions, the continuum limit of a relational Laplacian converges to a second-order elliptic operator whose principal symbol defines an effective metric tensor Beau (2026b).

While such results establish that effective spacetime geometry can emerge from non-geometric relational structures, they leave open a central question: why are only very specific geometries observed? From a purely mathematical perspective, a large class of elliptic operators and corresponding metrics are admissible. However, not all such operators necessarily correspond to physically meaningful or operationally accessible descriptions. Recent analyses have emphasized that effective geometric descriptions may fail when the underlying relational structure admits non-injective or degenerate projections, leading to a loss of observable factorization and to intrinsic limits of spacetime representability Beau (2026a).

This work addresses a complementary and more restrictive problem. Rather than asking how geometry can emerge, we

ask which effective geometries are *dynamically admissible* once minimal physical constraints are imposed on the underlying relational dynamics. Specifically, we consider relational systems whose effective continuum descriptions admit a finite maximal propagation or relaxation speed. Such a bound is required for causal consistency and excludes purely quadratic actions, which allow arbitrarily large gradients and unbounded fluxes.

We show that imposing bounded flux propagation uniquely constrains the form of any local effective functional governing the continuum limit. Under mild assumptions of locality, smoothness, and absence of additional microscopic scales, the resulting effective description must take a Born–Infeld–type form. This structure is not introduced as a phenomenological modification, but arises as the minimal representation compatible with flux saturation.

Building on this result, we demonstrate that flux saturation does more than regularize the effective theory. It dynamically selects a restricted class of admissible relational Laplacians whose continuum limits define stable and physically meaningful metrics. In homogeneous regimes, this selection leads uniquely to flat spacetime. In the presence of a localized and stationary obstruction, the same mechanism yields the Schwarzschild geometry as the universal effective solution, without postulating any independent metric dynamics or field equations.

Within this framework, horizons are interpreted as loci where flux saturation renders the effective operator degenerate, signaling a loss of projectability rather than a physical singularity. This interpretation aligns naturally with structural analyses of non-injective projections and clarifies the operational meaning of strong-field regimes.

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The paper is organized as follows. In Section 2, we establish the necessity of a Born–Infeld–type structure from bounded flux propagation. Section 3 recalls the emergence of effective continuum operators from relational Laplacians. Section 4 discusses metric reconstruction from operator symbols. Sections 5 and 6 derive flat and Schwarzschild geometries as dynamically selected solutions. Section 7 analyzes horizon formation as operator saturation. We conclude with a discussion of scope and limitations.

2. Bounded relaxation and effective action

We consider effective continuum descriptions arising from relational systems whose microscopic dynamics admits a finite maximal propagation or relaxation flux. Such a bound is a minimal physical requirement for causal consistency: no influence, constraint, or excitation can propagate arbitrarily fast in the effective theory.

In a continuum regime, the dynamics of slowly varying configurations can be encoded in a local action functional

$$S = \int d^4x \mathcal{L}, \quad (1)$$

where \mathcal{L} depends on the effective fields and their first derivatives. From the perspective of field theory, the central question is therefore the following: which local Lagrangian densities are compatible with bounded propagation?

A purely quadratic action, such as the Maxwell form

$$\mathcal{L}_{\text{quad}} \propto F_{\mu\nu}F^{\mu\nu}, \quad (2)$$

admits arbitrarily large field gradients. As a result, the associated fluxes and characteristic propagation scales are unbounded. This behavior is incompatible with the existence of a finite maximal relaxation or transport capacity at the microscopic level.

Imposing bounded propagation requires the effective Lagrangian density to satisfy three minimal conditions: (i) it reduces to a quadratic form in the weak-field limit, (ii) it enforces a strict upper bound on admissible field invariants, and (iii) it introduces no additional microscopic scales beyond the saturation scale itself. Polynomial extensions of quadratic actions fail to meet these requirements without fine tuning or the introduction of auxiliary parameters.

Under these assumptions, the effective Lagrangian density is uniquely constrained to take a Born–Infeld–type form. Anticipating the operator derivation developed in the following sections, the resulting effective action can be written as

$$S_{\text{eff}} = \beta^2 \int d^4x \left(\sqrt{-\det(g_{\mu\nu} + \frac{1}{\beta}F_{\mu\nu})} - \sqrt{-\det(g_{\mu\nu})} \right), \quad (3)$$

where $g_{\mu\nu}$ is the effective metric and $F_{\mu\nu}$ an antisymmetric field strength. The parameter β sets the maximal admissible field magnitude and fixes the saturation scale of the theory.

In the weak-field regime $|F_{\mu\nu}| \ll \beta$, the expansion of Eq. (3) yields

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + O(\beta^{-2}), \quad (4)$$

showing that standard Maxwell electrodynamics is recovered universally as the low-flux approximation of bounded relaxation. Non-linear corrections become relevant only when the microscopic saturation scale is probed.

Crucially, the Born–Infeld structure is not introduced here as a phenomenological modification of electrodynamics. It arises as the minimal local representation compatible with bounded propagation. This observation can be formulated as a no-go statement: any local effective action admitting a finite maximal flux cannot be purely quadratic in field gradients. Born–Infeld electrodynamics therefore represents the unique natural completion of Maxwell theory under bounded relaxation.

In the present framework, the parameter β is not free. It encodes the maximal transport or relaxation capacity of the underlying relational system and is fixed by its microscopic connectivity density and stiffness. In this sense, β plays the role of an ultraviolet field scale, analogous to a Planck-scale bound, although no specific identification is assumed at this stage.

The remainder of the paper establishes how the effective action (3) emerges dynamically from a bounded relational Laplacian and how this structure selects physically admissible space-time geometries.

3. Relational graph, Laplacian operator, and relaxation dynamics

We consider an underlying relational system represented by a weighted graph $G = (V, E, w)$, where V denotes a set of nodes, E a set of edges, and $w_{ij} \geq 0$ encodes the strength of the relation between nodes i and j . No embedding in a background space-time is assumed. The graph is taken as a purely relational object, encoding adjacency and coupling structure between abstract degrees of freedom.

A scalar field ϕ_i is defined on the nodes of the graph. The fundamental operator governing relational variations is the discrete Laplacian, defined by

$$(L\phi)_i = \sum_j w_{ij} (\phi_i - \phi_j). \quad (5)$$

This operator measures the local mismatch of ϕ with respect to its relational neighborhood and plays the role of a generalized stiffness or connectivity operator. Throughout this work, we assume that the graph is locally finite and that the weights are symmetric, $w_{ij} = w_{ji}$.

The system is assumed to admit an irreversible relaxation dynamics driven by the Laplacian. At the discrete level, this dynamics can be represented schematically as

$$\frac{d\phi_i}{d\tau} = -(L\phi)_i, \quad (6)$$

where τ is a monotonically increasing parameter labeling the progression of the relaxation process. No interpretation of τ as a fundamental physical time is imposed. Instead, it serves as an ordering parameter associated with the irreversible flow toward admissible stationary configurations.

This relaxation dynamics defines a preferred direction of evolution. Configurations evolve toward states that minimize relational gradients, subject to the constraints discussed in Section 2. The existence of such an ordering parameter is sufficient to distinguish one direction of evolution from its reverse, independently of any geometric notion of time. Temporal ordering is thus introduced operationally, as an intrinsic feature of the relaxation process itself.

Stationary configurations satisfy

$$L\phi = 0, \quad (7)$$

and correspond to relational equilibria. More generally, slowly varying configurations describe regimes in which the system admits an effective coarse-grained description. In these regimes, large-scale observables depend primarily on the spectral properties of the Laplacian rather than on the detailed microscopic structure of the graph.

We emphasize that the graph structure introduced here does not imply any fundamental discreteness of physical space or time. It is employed as a relational scaffold allowing the definition of operators and relaxation processes. Different microscopic realizations leading to the same large-scale spectral properties are considered physically equivalent within the scope of the effective description.

In the next section, we recall how, under appropriate density and regularity assumptions, the spectral structure of such relational Laplacians admits a continuum limit. In this limit, the discrete operator converges to a second-order differential operator, providing the bridge toward an effective geometric description.

4. Continuum limit and Born–Infeld action

We briefly recall how an effective continuum description and its associated action emerge from the spectral structure of a bounded relational Laplacian. Detailed proofs of the continuum limit and operator convergence can be found in Beau (2026b); here we summarize only the elements required to establish the effective field-theoretic description.

We consider a sequence of weighted graphs $G_N = (V_N, E_N, w^{(N)})$ with increasing cardinality $|V_N| \rightarrow \infty$. The graphs are assumed to become dense in the sense that each node has an increasing number of neighbors, while the weights $w_{ij}^{(N)}$ decay sufficiently fast with an intrinsic relational distance. No background embedding is assumed; all notions of proximity are defined intrinsically from the graph structure.

Let ϕ_i denote a scalar degree of freedom on the nodes of G_N . The discrete Laplacian acts as

$$(L_N\phi)_i = \sum_j w_{ij}^{(N)} (\phi_i - \phi_j). \quad (8)$$

Under mild regularity, isotropy, and density assumptions, the action of L_N on slowly varying configurations converges to that of a second-order differential operator on a smooth manifold \mathcal{M} ,

$$L_N \longrightarrow \mathcal{L} = \nabla_\mu (A^{\mu\nu}(x) \nabla_\nu), \quad (9)$$

where $A^{\mu\nu}(x)$ is a symmetric tensor encoding the local connectivity and stiffness of the underlying relational system.

In admissible continuum regimes, the operator \mathcal{L} is elliptic. Its principal symbol,

$$\sigma_2(\mathcal{L})(x, k) = A^{\mu\nu}(x) k_\mu k_\nu, \quad (10)$$

fully characterizes the leading-order propagation of modes. As shown in Beau (2026b), this symbol provides a natural, coordinate-independent definition of an effective metric tensor,

$$g^{\mu\nu}(x) \propto A^{\mu\nu}(x), \quad (11)$$

where the proportionality factor reflects a choice of units.

Crucially, the effective metric $g_{\mu\nu}$ is not introduced as an independent dynamical variable. It is a derived object, encoding how relational variations propagate in the continuum limit. Different microscopic graphs leading to the same operator \mathcal{L} are therefore indistinguishable at the level of effective geometry.

When antisymmetric perturbations of the relational connectivity are included, the continuum operator acquires an additional two-form contribution that enters naturally as a field strength $F_{\mu\nu}$. As discussed in Section 2, bounded relaxation constrains the admissible gradients of these perturbations. At the level of the effective continuum description, this constraint is most naturally implemented at the level of the action rather than the equations of motion.

The resulting effective action takes a Born–Infeld form,

$$S_{\text{eff}} = \beta^2 \int d^4x \left(\sqrt{-\det(g_{\mu\nu} + \frac{1}{\beta} F_{\mu\nu})} - \sqrt{-\det(g_{\mu\nu})} \right), \quad (12)$$

where β sets the maximal admissible relational flux. In the weak-field regime, this action reduces to the standard Maxwell form, while at large field strengths it enforces saturation of the effective dynamics.

The validity of this effective action is restricted to regimes in which the spectrum of L_N admits a well-defined low-energy sector and the bounded-flux condition holds. Outside these regimes, the continuum operator \mathcal{L} ceases to provide an adequate description, and geometric or field-theoretic notions lose their operational meaning.

In the following section, we show that the bounded-relaxation constraint further restricts the admissible forms of $A^{\mu\nu}(x)$. In homogeneous regimes, these restrictions uniquely select a flat effective geometry with a specific signature.

5. Emergence of Minkowski Spacetime: Spectral Stability and Signature Selection

We now demonstrate that the class of homogeneous and isotropic relaxation regimes uniquely selects a flat pseudo-Riemannian metric. In this framework, homogeneity is defined spectrally: the low-energy sector of the relational Laplacian \mathcal{L} is invariant under the translation group, implying that the effective tensor $A^{\mu\nu}(x)$ reduces to a constant matrix $A_0^{\mu\nu}$.

The selection of the signature follows from the requirement of *causal consistency* between the irreversible relaxation dynamics

and the bounded-flux condition. Let τ be the relaxation parameter defining the monotonic evolution of the relational system. The effective propagator associated with the bounded-flux operator must satisfy a well-posed initial value problem relative to τ .

Consider the effective dispersion relation derived from the saturated regime of Eq. (4). In a local coordinate basis where the ordering direction is ∂_τ , the bounded propagation requirement translates into a constraint on the characteristic cone of the operator. If we denote k_0 as the frequency conjugate to τ and \mathbf{k} the spatial momenta, the stability of the relaxation process requires that the operator remains hyperbolic. A purely Riemannian signature (+ +++) would lead to an elliptic operator, where perturbations propagate instantaneously across the entire relational structure, violating the bounded-flux postulate established in Section 2.

Conversely, a signature with multiple time-like directions would lead to ultra-hyperbolic equations, which are generically unstable and fail to support a well-defined causal structure. Consequently, the only spectral configuration that allows for a stable, monotonic relaxation under a finite maximal flux is the one where the ordering direction τ possesses a sign opposite to the diffusive spatial directions. By normalizing the maximal relaxation speed to $c = 1$, the effective metric is dynamically fixed to:

$$g_{\mu\nu} = \eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1). \quad (13)$$

This result implies that Lorentz invariance is an *emergent symmetry* of the homogeneous fixed point of the relaxation dynamics. The Minkowski metric is not a background stage but the unique effective representation of a maximally symmetric relational state at equilibrium.

Crucially, the emergence of the (- +++) signature provides a physical justification for the use of Wick rotations in standard field theory: here, the ‘‘Euclidean’’ sector describes the relational connectivity at a fixed relaxation slice, while the Lorentzian signature describes the dynamical unfolding of these relations. In this light, the null-interval $ds^2 = 0$ represents the saturation of the relational flux, where the information transfer reaches the bound imposed by the microscopic connectivity of χ .

In the absence of localized obstructions, all curvature invariants $R_{\mu\nu\rho\sigma}$ vanish, and the geometry is identically flat. We shall see in the next section how localized defects break this symmetry and generate the Schwarzschild effective geometry through the same saturation mechanism.

6. Localized obstruction and Schwarzschild geometry

We now consider relaxation regimes in which homogeneity is broken by the presence of a localized and stationary obstruction. Operationally, such an obstruction corresponds to a region where the relational connectivity or stiffness of the underlying system is enhanced, thereby constraining the local relaxation dynamics. No assumption is made regarding the microscopic origin of this obstruction; only its macroscopic effect on the effective operator is considered.

Outside the obstructed region, the system remains stationary and isotropic. The effective continuum operator introduced in Section 4 therefore takes the form

$$\mathcal{L} = \nabla_\mu (A^{\mu\nu}(r) \nabla_\nu), \quad (14)$$

where r denotes the radial distance from the center of the obstruction. Spherical symmetry implies that $A^{\mu\nu}(r)$ depends only on r and decomposes into temporal and spatial components,

$$A^{\mu\nu}(r) = \text{diag}(-A_t(r), A_r(r), A_\perp(r), A_\perp(r)). \quad (15)$$

In the exterior region, where no sources are present, the relaxation dynamics is governed by the homogeneous equation

$$\nabla_\mu (A^{\mu\nu}(r) \nabla_\nu \Phi) = 0, \quad (16)$$

where Φ represents a slowly varying scalar probe of the effective structure. Under stationarity and spherical symmetry, this equation reduces to

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 A_r(r) \frac{d\Phi}{dr} \right) = 0. \quad (17)$$

Its general solution is

$$\Phi(r) = \Phi_0 - \frac{C}{r}, \quad (18)$$

where C is an integration constant characterizing the strength of the obstruction.

The emergence of a $1/r$ profile is therefore not imposed but follows generically from flux conservation in a stationary and isotropic relaxation regime. This behavior is independent of the detailed microscopic realization of the obstruction and reflects the universal structure of the effective operator.

Using the identification between the principal symbol of \mathcal{L} and the effective metric established in Section 4, the radial dependence of $A^{\mu\nu}(r)$ translates directly into a position-dependent metric tensor. Up to a choice of coordinates, the effective line element can be written as

$$ds^2 = -f(r) dt^2 + f(r)^{-1} dr^2 + r^2 d\Omega^2, \quad (19)$$

where the lapse function $f(r)$ is determined by the radial profile of the operator coefficients. Matching the weak-obstruction limit with the homogeneous solution of Section 5 fixes

$$f(r) = 1 - \frac{r_s}{r}, \quad (20)$$

where r_s is a constant proportional to C .

The resulting geometry coincides with the Schwarzschild metric. Here, however, it arises as an effective description of a stationary relaxation pattern around a localized obstruction, rather than as a solution of an independent set of gravitational field equations. The parameter r_s characterizes the strength of the obstruction in relational units and is identified operationally through its influence on propagation and relaxation rates.

This derivation highlights that the Schwarzschild geometry is a universal effective response to a localized and isotropic perturbation in a bounded relaxation framework. No additional assumptions regarding curvature dynamics or energy-momentum

sources are required. The geometry encodes how admissible modes propagate in the presence of constrained relaxation, and its form is fixed by symmetry, stationarity, and flux conservation alone.

In the next section, we analyze the behavior of the effective operator near the radius $r = r_s$. We show that this surface corresponds to a saturation of the bounded-flux condition, leading to a loss of projectability and providing an operator-theoretic interpretation of horizons.

7. Horizon as flux saturation and loss of projectability

We now examine the behavior of the effective continuum description in the vicinity of the radius $r = r_s$ identified in Section 6. At this radius, the lapse function $f(r)$ vanishes and the effective geometry develops a horizon in the standard geometric representation. We show that, in the present framework, this phenomenon admits a precise and purely operator-theoretic interpretation in terms of flux saturation.

As established in Section 2, the effective action and the associated continuum operator remain valid only as long as the bounded-flux condition is satisfied. In the presence of a localized obstruction, the radial dependence of the operator coefficients $A^{\mu\nu}(r)$ reflects the increasing constraint imposed on relational relaxation. As r decreases, admissible gradients approach the maximal value set by the saturation scale β . At $r = r_s$, this bound is reached.

Beyond this point, the effective operator can no longer sustain propagating modes compatible with bounded relaxation. While the differential expression for \mathcal{L} remains formally defined, its principal symbol becomes degenerate at $r = r_s$. As a result, the reconstruction of a local effective metric from the operator symbol ceases to be well-defined. This breakdown signals the loss of validity of the geometric description rather than the presence of a physical singularity.

From the perspective of the effective action, the horizon corresponds to the locus where the Born–Infeld saturation becomes operative. At this point, the non-linear structure of the action enforces a maximal admissible field strength, preventing divergences in energy density and flux. As in standard Born–Infeld electrodynamics, the saturation mechanism ensures the finiteness of physically relevant quantities and excludes point-like singular sources.

From the viewpoint of the underlying relational dynamics, the horizon separates two qualitatively distinct regimes. Outside r_s , the relaxation dynamics admits a faithful projection into a continuum description with well-defined geometric and field-theoretic observables. At and inside r_s , distinct microscopic configurations of the relational system become indistinguishable at the level of effective operators. The projection from relational states to effective spacetime observables is therefore non-injective in this regime.

This interpretation is consistent with structural analyses of non-injective effective descriptions, in which the loss of observable factorization reflects a limitation of the effective description rather than a breakdown of the underlying dynamics Beau

(2026a). In the present context, the horizon is precisely the locus where such non-injectivity becomes unavoidable as a consequence of bounded flux propagation.

Importantly, no extension of the effective spacetime geometry beyond the horizon is required for internal consistency. The continuum description is explicitly restricted to projectable regimes, and the horizon marks the boundary of its domain of applicability. Questions concerning the behavior of the relational system beyond this boundary are meaningful only at the level of the underlying dynamics and need not admit a geometric or field-theoretic representation.

This operator-based interpretation reframes horizons as kinematic consequences of bounded relaxation rather than as geometric pathologies. It explains both the universality of horizon formation and the robustness of Schwarzschild geometry under variations of the microscopic relational structure, while clarifying why attempts to probe beyond the horizon using effective spacetime observables are intrinsically limited.

8. Discussion and conclusion

In this work, we have investigated how effective spacetime geometries and non-linear field dynamics can emerge from a relational system subject to bounded flux propagation. Starting from a weighted relational graph endowed with an irreversible relaxation process, we have shown that minimal and physically motivated constraints suffice to severely restrict the class of admissible continuum descriptions.

A first central result is that bounded propagation excludes purely quadratic effective actions. Requiring locality, smoothness, and the absence of additional microscopic scales uniquely selects a Born–Infeld–type structure as the minimal effective action compatible with flux saturation. In this sense, Born–Infeld electrodynamics is not introduced as a phenomenological modification of Maxwell theory, but arises as the unique natural completion enforced by bounded relaxation. Standard Maxwell dynamics is recovered universally as the weak-flux limit of this structure.

Building on established results concerning the continuum limit of relational Laplacians Beau (2026b), we have shown that the bounded-flux constraint does more than regularize the effective theory. It dynamically selects the class of admissible differential operators whose principal symbols define effective spacetime metrics. In homogeneous and isotropic relaxation regimes, this selection uniquely yields flat spacetime with pseudo-Riemannian signature $(- + + +)$. Minkowski geometry thus appears as a dynamically admissible fixed point rather than as a fundamental background assumption.

When homogeneity is broken by a localized and stationary obstruction, the same framework generically produces a $1/r$ relaxation profile. Through the operator–metric correspondence, this profile induces the Schwarzschild geometry as the universal effective description of the exterior region. Remarkably, this result follows from symmetry, stationarity, and bounded flux conservation alone, without invoking independent metric dynamics or gravitational field equations.

We have further shown that horizons admit a precise operator-theoretic interpretation. They correspond to loci where the bounded-flux condition is saturated and where the principal symbol of the effective operator becomes degenerate. At this point, the reconstruction of a local geometric description fails, signaling a loss of projectability rather than the presence of a physical singularity. This interpretation is consistent with structural analyses of non-injective effective descriptions and clarifies the operational meaning of strong-field regimes Beau (2026a).

The scope of the present work is deliberately limited. We do not propose a complete theory of gravitation, nor do we address the microscopic origin of the relational dynamics itself. Our results apply only in regimes where a continuum description is meaningful and where bounded relaxation holds. Outside these regimes, geometric and field-theoretic notions are not expected to remain valid, and no extension of the effective spacetime description is implied.

Within these limits, the framework provides a unified and minimal explanation for the emergence of flat spacetime, Schwarzschild geometry, and horizon formation as dynamically selected effective structures. It suggests that key features traditionally attributed to gravitational dynamics may instead reflect universal properties of bounded relational relaxation. Further work will be required to explore non-stationary regimes, departures from spherical symmetry, and potential phenomenological consequences of this operator-based perspective.

References

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