Charge, Mass, and Inertia as Saturated Responses in a Pre-Geometric Relational Description

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Abstract

We investigate the structural origin of mass, electric charge, and inertial response within a pre-geometric relational framework in which physical observables arise through a generally non-injective projection onto effective spacetime descriptions. In this setting, geometric and dynamical quantities are not fundamental but emerge as regime-dependent responses of an underlying relational description subject to intrinsic saturation bounds.

We show that mass and charge can be interpreted as distinct symmetry realizations of a single bounded-response mechanism. Mass corresponds to an isotropic inhibition of relational relaxation, while electric charge arises as an oriented saturation of the same underlying flux. Inertia is shown to arise as the linear-response limit of a bounded projective update process, rather than as an intrinsic resistance to motion.

In addition to the conceptual analysis, we provide an explicit effective realization of the saturation mechanism in Lagrangian form. We show that a finite projective capacity naturally induces Born–Infeld–type nonlinearities, from which mass-like spectral gaps, bounded inertial response, and $\boldsymbol{U(1)}$ charge structures arise as symmetry consequences of saturation.

The proposed framework does not modify Standard Model dynamics and does not introduce additional fundamental fields. Standard Model phenomenology is recovered as the unsaturated, linear-response regime of the effective description, while the existence of finite saturation scales implies necessary high-gradient limits.

The results suggest that mass, charge, and inertia represent complementary limits of a single saturated relational mechanism, providing a unified and falsifiable structural basis for inertial, gravitational, and electromagnetic response.

Keywords: Pre-geometric frameworks; emergent spacetime; mass and charge; inertia; bounded response; non-injective projection; relational description; saturation mechanisms; Born–Infeld-type dynamics; foundations of physics

1 Introduction

The physical notions of mass, electric charge, and inertia occupy a central role in modern physics. They enter as fundamental parameters in both classical and quantum theories, yet their conceptual origin remains largely unexplained.

In the Standard Model, mass and charge are introduced as intrinsic properties of elementary fields, with numerical values fixed by empirical input. While the Higgs mechanism provides a dynamical account of mass generation for gauge bosons and fermions, it does not address why mass exists as a property, nor why inertial response accompanies it [1]. Electric charge, similarly, is treated as a primitive coupling constant, constrained by gauge symmetry but not derived from a deeper structural principle [2].

From a foundational perspective, inertia presents an additional conceptual challenge. The resistance of a system to acceleration is usually taken as a defining feature of matter, yet its relation to gravitational mass and its possible emergence from more primitive structures have been the subject of long-standing debate [3, 4].

A parallel line of inquiry has emerged in approaches to quantum gravity and pre-geometric physics. In these frameworks, spacetime geometry, and sometimes even locality and causality, are not assumed as fundamental, but are reconstructed as effective descriptions from more primitive relational or algebraic structures [5, 6]. Within such approaches, familiar physical quantities may acquire an emergent or regime-dependent status.

Recent work has emphasized that effective physical descriptions often arise through projections from an underlying configuration space to observable spacetime variables. When such projections are non-injective, multiple underlying configurations correspond to the same effective observable state, leading to intrinsic information loss at the descriptive level [7]. This structural feature has been shown to account for the breakdown of classical probabilistic factorization in quantum correlations, without invoking nonlocal dynamics [8].

In parallel, bounded-response mechanisms have been extensively studied in effective field theories. Born–Infeld electrodynamics provides a paradigmatic example in which divergences are regulated by intrinsic saturation of the field response [9]. Related saturation phenomena appear in gravitational and cosmological contexts, where effective responses cease to scale linearly beyond certain thresholds.

Motivated by these developments, we examine whether mass, electric charge, and inertial response can be understood as manifestations of a single structural mechanism, and whether such an interpretation imposes non-trivial constraints on effective physical descriptions. Specifically, we consider a pre-geometric relational framework in which physical observables arise through a projection subject to intrinsic saturation bounds.

Within this setting, mass is interpreted as an isotropic inhibition of relational relaxation, while electric charge corresponds to an oriented or asymmetric saturation of the same underlying relational flux. Inertia then emerges as the linear-response limit of a bounded projective update process, reflecting the finite resolvability of changes in relational configurations rather than a primitive dynamical property.

The purpose of this work is not to modify the Standard Model or to introduce new fundamental fields. Instead, it is to identify structural constraints on how familiar physical quantities can consistently arise as effective descriptors within a projection-limited framework. In addition to the conceptual analysis, we provide an explicit effective realization of the proposed saturation mechanism in Lagrangian form, showing that bounded relational projection naturally induces Born–Infeld–type nonlinearities and associated symmetry structures.

The paper is organized as follows. Section 2 introduces the relational and projective framework underlying the analysis. Section 3 develops the classification of saturated effective responses and shows how mass, electric charge, and inertia arise as distinct symmetry realizations within this framework. Section 5 provides a formal effective derivation of the saturation mechanism and its physical consequences. Section 4 examines the consistency of this interpretation with Standard Model phenomenology and identifies necessary high-gradient limits. We conclude in Section 6 with a discussion of conceptual implications, limitations, and possible directions for further investigation.

2 Relational Framework and Projective Description

In this section, we summarize the minimal relational and projective framework required for the present analysis. The detailed construction of this framework is developed in companion works, including the relational and spectral reconstruction of effective geometry [10], the formal role of non-injective projection in effective descriptions [7], and the emergence of bounded-response regimes relevant for mass, charge, and inertia [11]. Here we restrict attention to the structural assumptions that constrain how mass, charge, and inertia can consistently arise as effective descriptors.

2.1 Underlying relational description

We assume that effective spacetime observables arise from an underlying relational description, in which configurations encode admissible correlations between abstract degrees of freedom. No spacetime manifold, metric, or field content is assumed at this level.

Explicit realizations of such relational descriptions, including spectral reconstructions of effective geometry from correlation structure alone, have been developed in companion work [10]. The present paper does not rely on the details of those constructions, but adopts their structural conclusion: spacetime geometry is an effective description, not a fundamental input.

The underlying relational description should not be interpreted as a hidden-variable completion of existing theories. It defines a space of admissible configurations whose effective description is intrinsically coarse-grained.

Configurations at this level are not endowed with local coordinates, temporal ordering, or intrinsic geometric meaning. These notions arise only when relational configurations admit a stable and approximately injective representation.

2.2 Projection to effective observables

Observable physical quantities are defined through a projection

$$\Pi : \Omega \to \mathcal{O}, \tag{1}$$

where Ω denotes the space of underlying relational configurations and $\mathcal{O}$ the space of effective observables accessible within spacetime descriptions.

A key structural feature of this projection is that it is generally non-injective. Distinct underlying configurations may correspond to the same effective observable state. This identification is not an approximation, but an intrinsic property of the descriptive mapping.

The physical consequences of non-injective projection have been analyzed in detail in the context of quantum correlations [7]. There it was shown that non-injectivity provides a sufficient mechanism for the failure of classical probabilistic factorization, without invoking nonlocal dynamics or hidden-variable assumptions.

In the present work, the same projective structure is shown to govern inertial and interaction-related quantities. Mass, charge, and inertia are therefore treated as effective descriptors defined on equivalence classes of underlying relational configurations.

2.3 Saturation and bounded response

A central structural requirement of the present framework is that the projection Π admits an intrinsic saturation. Beyond a certain threshold, additional variations in the underlying relational configuration cannot be resolved within the effective description.

This bounded-response behavior has been studied in detail in companion work [11], where it was shown that saturation naturally leads to effective Born–Infeld-type dynamics and to the dynamical selection of stable geometric regimes.

Here, saturation is not introduced as a modification of fundamental dynamics. It is interpreted as a limitation of the effective description itself, reflecting a finite projective resolution. As a result, effective responses cannot be extrapolated linearly under arbitrarily large relational gradients.

Operationally, this implies that departures from linear behavior are not contingent features, but necessary consequences of finite projective capacity.

2.4 Effective descriptors and regime dependence

Physical quantities such as mass, electric charge, and inertia are treated as effective descriptors defined within the projected space $\mathcal{O}$. They characterize how the effective description responds to variations of the underlying relational configuration.

These quantities are inherently regime dependent. In domains where the projection is approximately injective and unsaturated, classical descriptions are recovered and effective responses behave linearly. In saturated regimes, departures from linearity arise as a direct consequence of projective limitations.

Importantly, the present framework does not modify the operational content of the Standard Model in experimentally tested regimes. It provides a structural reinterpretation of existing quantities while constraining the conditions under which their linear extrapolation remains valid.

Within this perspective, distinctions between mass, charge, and inertial response correspond to different symmetry classes of saturated effective behavior. This classification is developed in the following sections.

3 Symmetry Classes of Saturated Effective Responses

In this section, we identify the symmetry classes of effective physical behavior that necessarily arise once saturation of the projective description is taken into account. The goal is not to introduce new physical entities, but to show that mass, electric charge, and inertial response correspond to the only stable symmetry realizations of a bounded effective response, without invoking additional fundamental degrees of freedom.

3.1 Linear and saturated response regimes

When the projection from underlying relational configurations to effective observables is approximately injective, variations at the underlying level are faithfully reflected in the effective description. In this regime, effective responses scale linearly with the magnitude of relational gradients.

Such linear behavior underlies the standard formulation of classical field theories. Small perturbations lead to proportionate responses, and superposition principles apply.

However, when relational gradients exceed the resolving capacity of the effective description, the projection necessarily enters a saturated regime. Beyond this threshold, additional variations at the underlying level no longer produce distinct effective outcomes.

This transition from linear to saturated response is structural rather than dynamical. It reflects a limitation of descriptive resolution, not a modification of underlying laws. Bounded-response regimes of this type are known to give rise to Born–Infeld-like effective dynamics in a variety of contexts [9, 11].

3.2 Isotropic saturation and effective mass

We first consider saturated responses that preserve isotropy in the effective description. In this case, saturation suppresses relational variations uniformly in all directions.

An isotropic inhibition of effective response leads necessarily to behavior characteristic of inertial and gravitational mass. The effective description resists changes in motion independently of direction, yielding a scalar parameter that quantifies the degree of saturation.

From this perspective, mass is not introduced as an intrinsic substance. It appears as a measure of how strongly the effective description inhibits relational reconfiguration once saturation is reached.

This interpretation is consistent with the equivalence between inertial and gravitational mass. Both correspond to isotropic limitations of effective response, differing only in the context in which the response is probed. The universality of free fall follows as a direct consequence of the symmetry of the saturation mechanism.

3.3 Oriented saturation and effective charge

We now consider saturated responses that break isotropy while preserving locality and stability of the effective description. In this class, saturation is directionally biased with respect to relational gradients.

Such oriented saturation necessarily leads to behavior characteristic of electric charge. The effective response distinguishes between opposing directions, giving rise to attractive and repulsive interactions depending on orientation.

Charge thus appears as a signed quantity associated with asymmetric saturation of relational flux. The existence of both positive and negative charges reflects the presence of two stable orientations of the saturated response.

Importantly, this interpretation does not require introducing charge as a primitive coupling. It arises as a symmetry-breaking mode of the same bounded-response mechanism that gives rise to mass.

The long-range character of electromagnetic interactions follows from the fact that oriented saturation does not induce isotropic suppression, allowing extended field-like behavior within the effective description.

3.4 Inertia as finite-resolution response

Inertial behavior emerges when changes in motion probe the finite update capacity of the effective description. Acceleration corresponds to a demand for rapid reconfiguration of relational correlations within the projected space.

When such reconfiguration remains well below the projective capacity, the effective response is linear. In this regime, resistance to acceleration scales proportionally with the applied stress, reproducing the standard inertial relation as a linear-response approximation.

As the required rate of reconfiguration approaches the resolving limit of the projection, the effective response saturates. Beyond this point, additional relational variation cannot be resolved instantaneously, and the projected dynamics exhibits a bounded update behavior.

In this view, inertia is not a fundamental property attached to matter. It is the linear-response limit of a bounded projective update process, reflecting the finite resolvability of variations in relational configurations within the effective description.

This interpretation preserves all empirical content of relativistic kinematics while reinterpreting inertial resistance as a consequence of finite descriptive capacity rather than a primitive dynamical postulate.

3.5 Unified classification

The analysis above yields an exhaustive classification of effective physical responses permitted by a bounded projective description.

Isotropic saturated responses correspond to mass. Oriented saturated responses correspond to charge. Finite-resolution resistance to reconfiguration corresponds to inertia.

All three arise from the same structural mechanism: saturation of the effective description under non-injective projection. Their distinction follows from symmetry properties of saturation rather than from distinct underlying substances or fields.

This classification preserves the operational definitions of mass and charge used in the Standard Model. It reinterprets their origin while constraining the space of admissible effective responses.

In the following section, we examine the consistency of this framework with established phenomenology and show that bounded-response symmetry classes imply necessary limits on linear extrapolations.

4 Consistency with Standard Model Phenomenology

In this section, we examine the compatibility of the proposed structural interpretation with established phenomenology of the Standard Model. The purpose is not to derive Standard Model parameters, but to ensure that the framework does not conflict with known experimental facts or operational definitions. We further identify necessary physical consequences that must follow if the saturated-response interpretation is meaningful.

4.1 Status of mass in the Standard Model

In the Standard Model, particle masses arise through the Higgs mechanism, which endows fermions and gauge bosons with effective mass terms via spontaneous symmetry breaking. This mechanism successfully accounts for the dynamical generation of mass and its role in particle interactions.

The present framework does not challenge this description. Instead, it addresses a logically distinct question: why mass appears as a universal scalar measure of inertial and gravitational response.

Within the proposed interpretation, the Higgs mechanism determines how mass values are assigned within the effective description, while the structural origin of mass as an isotropic saturated response explains why such a parameter has the physical meaning it does. The two perspectives operate at different conceptual levels and are therefore complementary rather than competing.

4.2 Electric charge and gauge structure

Electric charge in the Standard Model is associated with local gauge invariance and conserved currents. Its quantization and coupling structure are fixed by the underlying gauge symmetry and anomaly cancellation requirements.

The framework developed here does not modify gauge symmetry or charge conservation. It reinterprets electric charge as an effective manifestation of oriented saturation in the projective response, without altering its operational role.

From this perspective, gauge invariance constrains how oriented saturation can appear consistently within the effective description. The existence of discrete charge values reflects the stability of specific oriented saturation modes, rather than an arbitrary assignment of coupling constants.

Importantly, this interpretation does not predict deviations from known electromagnetic phenomena in the weak-field regime. All standard results of quantum electrodynamics are recovered in the unsaturated and approximately injective limit.

4.3 Inertia, relativistic dynamics, and equivalence

Relativistic dynamics treats inertia as a fundamental response encoded in the energy–momentum relation. The equivalence between inertial and gravitational mass is experimentally well established and constitutes a cornerstone of relativistic physics.

Within the present framework, this equivalence arises naturally from the isotropy of saturated response. Both inertial resistance to acceleration and gravitational response correspond to the same structural limitation of effective reconfiguration. Inertia itself is shown to emerge as the linear-response limit of a bounded projective update process, while gravitational response probes the same saturation mechanism through spacetime geometry.

No modification of relativistic kinematics is implied. Lorentz invariance remains an effective symmetry of the projected description, valid in regimes where the projection is approximately injective and unsaturated.

4.4 Absence of observable deviations at accessible scales

A crucial consistency requirement is the absence of observable deviations from Standard Model predictions in experimentally tested regimes. The framework satisfies this requirement by construction.

Saturation effects become relevant only when relational gradients approach the resolving capacity of the effective description. In present-day particle physics experiments, electromagnetic and inertial responses remain well within the linear regime, and no saturation is probed.

As a result, the framework predicts no departures from established cross sections, decay rates, or precision tests of quantum electrodynamics and electroweak theory. All Standard Model phenomenology is recovered as the linear-response limit of the effective description.

4.5 High-energy consistency and saturation limits

The absence of deviations at accessible scales does not imply that effective responses can be extrapolated linearly to arbitrarily high gradients. If mass, charge, and inertia arise as saturated responses constrained by a finite projective bound b, then strictly unbounded growth of effective couplings would render the framework internally inconsistent.

The consistency of the saturated-response interpretation therefore requires the existence of regimes in which perturbative extrapolations must fail. Such deviations are not additional assumptions or phenomenological predictions, but necessary consequences of introducing a finite projective capacity.

An empirical indication that such bounded behavior already exists is provided by the well-established Schwinger limit, which sets an upper threshold on sustainable electromagnetic field invariants before vacuum instability occurs. In the present framework, this threshold is interpreted as the operational manifestation of a finite projective bandwidth of the vacuum.

We emphasize that the present work does not address the microscopic dynamics of vacuum instability or pair production. The detailed realization of saturation effects in strong-field quantum electrodynamics will be developed in a companion paper. Here, the Schwinger limit serves solely as an existence proof that unbounded extrapolation of effective responses is physically untenable.

This interpretation is explicitly falsifiable. If effective couplings can be shown to grow without bound under arbitrarily strong field gradients, with no indication of saturation or inflection, then the bounded-response framework developed here must be rejected.

4.6 Relation to other bounded-response frameworks

Bounded-response mechanisms have previously been considered in both electromagnetic and gravitational contexts. Born–Infeld electrodynamics provides a well-known example in which saturation regulates divergent field strengths without spoiling low-energy phenomenology [9].

The present framework generalizes this idea conceptually. Rather than introducing bounded response as a modification of specific field equations, saturation is interpreted as a generic limitation of effective descriptions arising from non-injective projection.

This shift in perspective allows mass, charge, and inertia to be treated on equal footing, as different symmetry realizations of the same structural mechanism.

4.7 Summary

The proposed interpretation is fully consistent with Standard Model phenomenology. It does not alter gauge structure, particle content, or dynamical equations in experimentally tested regimes.

At the same time, it identifies necessary high-gradient limits in which linear extrapolations of effective responses must break down. Standard Model physics is recovered as the unsaturated regime of a bounded projective description.

In the following section, we discuss conceptual implications, limitations, and possible directions for further investigation.

5 Formal Derivation of Saturated Responses

This section adds the minimal mathematical structure missing from the current manuscript. It provides an explicit effective action for the projected relaxation degree of freedom and derives mass-like and charge-like symmetry classes as consequences of a finite projective capacity. The goal is not to postulate new fundamental fields, but to demonstrate that a bounded relational update flux necessarily induces a Born–Infeld-type effective completion in the observable description.

5.1 Relational flux and projective capacity

We represent the local relational update content by a rank-two tensor density

$$J_{\mu\nu} \equiv \partial_\mu \phi \, \partial_\nu \phi, \tag{2}$$

where ϕ is an effective scalar encoding the relaxation mode that remains after projection. The essential assumption is that the projection Π has a finite capacity. Operationally, this means that the observable description cannot resolve arbitrarily large values of relational flux invariants constructed from $J_{\mu\nu}$. We encode this limitation by introducing a saturation scale b with dimensions of a flux density.

The minimal Lorentz-invariant completion that enforces boundedness while recovering the standard quadratic kinetic term at low flux is the Dirac–Born–Infeld-type Lagrangian

$$\mathcal{L}_{\mathrm{BI}}(\phi) \equiv b^2 \left(1 - \sqrt{1 - \frac{\partial_\mu\phi\, \partial^\mu\phi}{b^2}}\right). \tag{3}$$

Equivalently, the associated action can be written as

$$S_\Pi[\phi] = \int d^4x\, \mathcal{L}_{\mathrm{BI}}(\phi) = -b^2 \int d^4x \left(\sqrt{-\det\left(\eta_{\mu\nu} - b^{-2}\partial_\mu\phi\, \partial_\nu\phi\right)} - 1\right), \tag{4}$$

where the determinant form makes explicit that saturation is controlled by an effective stretching of the metric volume element as the relational flux approaches b.

For weak gradients, $\partial\phi \ll b$, Eq. (3) expands as

$$\mathcal{L}_{\mathrm{BI}}(\phi) = \frac{1}{2}\partial_\mu\phi\, \partial^\mu\phi + \frac{1}{8b^2}\left(\partial_\mu\phi\, \partial^\mu\phi\right)^2 + \mathcal{O}\big(b^{-4}\big)\,, \tag{5}$$

so that the unsaturated regime reproduces the standard linear-response kinetic term, with nonlinear corrections suppressed by the saturation scale b.

5.2 Isotropic saturation and the emergence of a mass scale

We now demonstrate how an effective mass scale arises when saturation is isotropic. Isotropy here means that saturation is reached through a scalar invariant of the flux,

$$X \equiv \partial_\mu\phi\, \partial^\mu\phi, \tag{6}$$

rather than through direction-dependent tensor components.

Equation (3) alone does not generate a mass term. A mass scale appears when isotropic saturation locks the system near a finite-flux background. We model this locking by requiring that the projected description maintains a preferred saturation level $X_\star$ through a weak restoring term representing the elastic cost of reconfiguring an already saturated relational neighborhood. The minimal effective completion reads

$$\mathcal{L}_{\mathrm{iso}} \equiv \mathcal{L}_{\mathrm{BI}}(\phi) - \frac{\kappa}{2}\left(X - X_\star\right)^2, \tag{7}$$

where $\kappa > 0$ encodes the stiffness of the saturation constraint in the projected description. This term does not introduce an independent interaction channel. It encodes the elastic response of the projection once saturation has been reached.

Let $\phi = \phi_0 + \delta\phi$, with $X[\phi_0] = X_\star$. Linearizing the Euler–Lagrange equation for $\delta\phi$ yields a Klein–Gordon form

$$\big(\Box + m_{\mathrm{eff}}^2\big)\, \delta\phi = 0, \tag{8}$$

with an effective mass gap

$$m_{\text{eff}}^2 = 2\kappa\, X_\star \left(\frac{\partial X}{\partial \phi}\right)^2\Bigg|_{\phi_0} \times \mathcal{Z}^{-1}(X_\star, b), \tag{9}$$

where $\mathcal{Z}(X_\star, b)$ denotes the DBI renormalization of the kinetic term evaluated on the saturated background. As $X_\star \to b^2$, $\mathcal{Z}$ diverges, reflecting the progressive loss of sensitivity of the projected dynamics to further increases of relational flux. The resulting low-energy spectrum exhibits a finite gap with dispersion relation

$$E^2 = p^2c^2 + m_{\text{eff}}^2 c^4. \tag{10}$$

Equation (10) provides the sought mathematical statement. An isotropically saturated projected description admits a mass-like descriptor because saturation induces an elastic strain with a finite energetic cost for local reconfiguration. Mass is therefore not a primitive substance, but the effective label of a persistent isotropic projective strain.

5.3 Inertia as the linear limit of saturated update rates

We now demonstrate that the second law of motion, $F = ma$, emerges as the linear-response limit of a bounded projective update process. In the Cosmochrony framework, a change in the effective state of an observable $x \in \mathcal{O}$ requires a reconfiguration of the underlying relational substrate χ. This reconfiguration is limited by a finite projective bandwidth B_Π, which provides a temporal manifestation of the saturation bound b.

Let $\mathcal{O}(t)$ denote the effective description of a physical system. The rate at which this description can be updated in response to a relational stress $\mathcal{F}$ is therefore bounded. At the effective level, this constraint can be modeled by a saturated response equation of the form

$$\frac{d^2\mathcal{O}}{dt^2} = a_\star\, \mathcal{S}\left(\frac{\mathcal{F}}{m_{\text{eff}} a_\star}\right), \tag{11}$$

where $m_{\text{eff}} \sim b/c^2$ is the effective mass scale derived in Section 5.2, $a_\star$ is the characteristic acceleration associated with saturation, and $\mathcal{S}$ is a smooth odd saturation function satisfying $\mathcal{S}(x) \to x$ for $|x| \ll 1$ and $|\mathcal{S}(x)| \to 1$ for $|x| \gg 1$. A convenient representative choice is $\mathcal{S}(x) = \tanh(x)$, though the specific functional form is not essential for the argument.

In the low-stress regime, $\mathcal{F} \ll m_{\text{eff}} a_\star$, Eq. (**??**) reduces to

$$\frac{d^2\mathcal{O}}{dt^2} \approx a_\star \left(\frac{\mathcal{F}}{m_{\text{eff}} a_\star}\right) = \frac{\mathcal{F}}{m_{\text{eff}}}, \tag{12}$$

which recovers the Newtonian relation $F = m_{\text{eff}} a$ as the first-order expansion of the saturated response. Inertia therefore appears not as a fundamental resistance to motion, but as the linear scaling factor of a bounded projective update process.

Beyond the linear regime, when the required acceleration approaches $a_\star$, the response saturates. This reflects the impossibility of resolving arbitrarily rapid changes of the relational configuration within a finite projective bandwidth.

This dynamical formulation admits a direct interpretation in terms of finite observational resolvability. Let B_Π denote the maximal rate at which the projected description can update observables. For an effective observable $\mathcal{O}$, the minimal constraint reads

$$\left\| \frac{d\mathcal{O}}{dt} \right\| \leq B_\Pi. \tag{13}$$

Equivalently, changes of the projection satisfy the scaling

$$\Delta\Pi \sim \frac{1}{b}\frac{d\mathcal{O}}{dt}, \tag{14}$$

where b acts as a flux normalization converting an attempted rapid change of the effective state into a finite projective strain.

In this interpretation, inertia is the temporal lag of the projection Π. It measures the finite time required for the effective description to track a change in the underlying relational structure. The existence of a characteristic acceleration scale $a_\star$ thus follows directly from the bounded-response hypothesis. In low-density or weakly constrained environments, this scale can dominate the effective dynamics, producing deviations from linear inertia while preserving the Newtonian limit at high accelerations.

5.4 Oriented saturation and the emergence of a local $U(1)$ symmetry

We next consider the oriented class, in which saturation preserves a scalar bound while allowing an internal phase degeneracy. We introduce a complex relaxation mode

$$\psi \equiv \rho\, e^{i\theta}, \tag{15}$$

and impose that saturation fixes the modulus ρ near a preferred value $\rho_\star$, while leaving the phase θ as a locally underdetermined coordinate of the projection. The projected description is therefore insensitive to local internal rotations, expressed by the invariance

$$\theta(x) \to \theta(x) + \alpha(x). \tag{16}$$

Local invariance (12) necessitates the introduction of a compensating connection. We define the covariant derivative

$$D_\mu \psi \equiv (\partial_\mu - iqA_\mu)\,\psi, \tag{17}$$

so that $D_\mu\psi$ transforms covariantly under $\psi \to e^{i\alpha(x)}\psi$. The effective Lagrangian for the oriented saturated sector takes the minimal form

$$\mathcal{L}_{\rm ori} = b^2\left(1 - \sqrt{1 - \frac{D_\mu\psi\, D^\mu\psi^*}{b^2}}\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{18}$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$.

The group implied by Eq. (12) is $U(1)$, as it is the unique continuous one-parameter compact symmetry preserving a scalar saturation bound. In this construction, electric charge is the Noether generator associated with phase rotations in the oriented saturated flux sector. Charge is therefore not postulated as an intrinsic property, but labels a symmetry class of bounded projective response.

This dynamical formulation admits a direct interpretation in terms of finite projective bandwidth.

Finally, we connect inertial response to a finite update bandwidth of the projection. Let B_Π denote the maximal rate at which the projected description can update observables. For an observable $\mathcal{O}$, the minimal constraint reads

$$\left\| \frac{d\mathcal{O}}{dt} \right\| \leq B_\Pi. \tag{19}$$

Equivalently, changes of the projection satisfy the scaling

$$\Delta\Pi \sim \frac{1}{b} \frac{d\mathcal{O}}{dt}, \tag{20}$$

where b acts as a flux normalization converting an attempted rapid change of the effective state into a finite projective strain.

Inertial behavior arises when external forcing demands an update rate approaching the bound. In this regime, the projected dynamics cannot follow instantaneously. The resulting lag manifests as an effective resistance to acceleration, consistent with the interpretation of inertia as a response limited by finite projective capacity rather than as a primitive force.

5.5 Immediate falsifiability

The formalism above yields a direct falsifiability criterion. The effective action (4) ceases to define a real projected description when the invariant exceeds the saturation bound,

$$\partial_\mu \phi \, \partial^\mu \phi > b^2, \tag{21}$$

so the framework is ruled out if an experimentally established regime requires a strictly linear, unsaturated continuation beyond the scale b without any sign of instability, saturation, or nonlinear completion. Anchoring this bound to known strong-field thresholds provides a direct operational test of the bounded-response hypothesis.

6 Discussion, Limitations, and Outlook

In this final section, we summarize the conceptual implications of the proposed framework, clarify its limitations, and outline possible directions for future investigation. While the analysis remains primarily structural, it now includes an explicit effective realization of the saturation mechanism. This allows physical constraints to be formulated at the level of an effective Lagrangian, rather than at the level of interpretation alone.

6.1 Conceptual implications

A central implication of the present analysis is that mass, electric charge, and inertia need not be treated as fundamentally distinct physical primitives. Instead, they may be understood as different symmetry realizations of a single structural mechanism, namely the saturation of effective response under non-injective projection.

From this perspective, the apparent diversity of physical properties reflects differences in how the effective description responds to relational variation, rather than differences in underlying substance. Scalar, oriented, and finite-bandwidth response modes correspond respectively to mass, charge, and inertial behavior, with inertia arising as the linear-response limit of a bounded projective update process.

The explicit effective construction introduced in Section 5 shows that these distinctions can be realized within a single bounded-response framework, without modifying the operational definitions of mass and charge. In particular, mass appears as a spectral gap induced by isotropic saturation, while electric charge emerges as the generator of a local phase degeneracy associated with oriented saturation. Inertia, in turn, is shown to correspond to the first-order expansion of a saturated update dynamics.

This interpretation provides a unified conceptual basis for the equivalence between inertial and gravitational mass and for the signed nature of electric charge, while preserving their empirical roles within established physical theories.

6.2 Relation to existing foundational approaches

The framework developed here is compatible with a wide range of foundational approaches in which spacetime and physical observables are regarded as effective constructs. It does not rely on a specific microscopic ontology and can be embedded in different relational, background-independent, or pre-geometric settings.

Unlike approaches that postulate new degrees of freedom or modified fundamental dynamics, the present work operates at the level of effective description. Its contribution is to isolate a minimal structural mechanism that constrains how familiar physical notions can consistently arise within effective theories.

In this sense, the framework complements rather than replaces existing formulations. It provides an interpretative and constraining layer that may coexist with standard quantum field theory and relativistic dynamics, while clarifying the domain of validity of linear-response extrapolations.

6.3 Limitations of the present work

The present analysis is subject to several important limitations.

First, no microscopic dynamics of the underlying relational description is specified. As a result, the framework does not predict numerical values for particle masses, charges, or coupling constants. These remain empirical inputs determined by effective theories operating in the unsaturated regime.

Second, the analysis does not provide quantitative predictions for high-energy or strong-field deviations from Standard Model behavior. However, the effective formalism developed in Section 5 identifies a well-defined functional departure from linear inertia and a characteristic acceleration scale associated with saturation, beyond which strictly linear extrapolations necessarily fail.

Third, the framework does not address questions related to particle generations, flavor structure, or symmetry-breaking patterns within the Standard Model. Its scope is restricted to the structural origin and effective interpretation of mass, charge, and inertia.

Finally, although the framework implies the existence of saturation regimes at extreme relational gradients, such regimes are only indirectly accessible in current experimental settings. Direct tests are therefore most naturally associated with future strong-field or high-gradient probes.

6.4 Possible extensions and outlook

Despite these limitations, the framework suggests several concrete directions for further investigation.

One natural extension is the construction of explicit relational or spectral models in which the projective bound and its associated saturation scale can be derived rather than postulated. Such developments could bridge the gap between structural necessity and quantitative prediction.

Another important direction concerns the detailed study of strong-field and high-gradient regimes, where saturation effects are expected to become operational. Clarifying the relation between the present bounded-response framework and known nonlinear phenomena in quantum field theory may help identify experimentally relevant signatures.

More broadly, the analysis invites reconsideration of the status of physical parameters traditionally regarded as fundamental. If mass and charge are effective descriptors tied to finite descriptive capacity, their role in physical theories may be understood as constrained rather than primitive.

6.5 Concluding remarks

We have proposed a unified structural interpretation of mass, electric charge, and inertia as effective manifestations of saturated response under non-injective projection. The framework is fully consistent with Standard Model phenomenology and does not modify its dynamical content in experimentally tested regimes.

By providing an explicit effective realization of the saturation mechanism, the present work goes beyond purely interpretative unification and introduces well-defined physical constraints. In particular, the derivation of inertia as the linear limit of a bounded update process implies necessary high-gradient limits, rendering the framework falsifiable in principle.

While deliberately modest in its quantitative claims, the analysis establishes a coherent theoretical basis for further exploration of the structural origin of physical properties.

Appendices

Acknowledgements. The author acknowledges the use of large language models as a supportive tool for refining language, structure, and internal consistency during the development of this manuscript. All conceptual contributions, theoretical choices, and interpretations remain the sole responsibility of the author.

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