Cosmochrony: An Exploratory Geometric Framework for Emergent Spacetime, Gravitation, and Quantum Phenomena

Jérôme Beau
Independent Researcher
javarome@gmail.com

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Abstract

We present *Cosmochrony*, a minimalist geometric framework in which time, spacetime structure, gravitation, and quantum phenomena emerge from the irreversible relaxation of a single continuous scalar quantity, denoted χ. The field χ is not defined on a pre-existing spacetime manifold; rather, it constitutes a pre-geometric substrate whose monotonic relaxation provides an intrinsic ordering of physical processes, identified with physical time. This irreversibility is interpreted energetically, with energy understood as the residual capacity of χ configurations to relax, while spatial relations arise relationally from gradients and correlations within the field.

Within this framework, matter is described as stable, localized topological configurations of χ. Gravitation emerges as a collective effect of these excitations locally constraining the relaxation of the field, leading to gravitational time dilation and effective spacetime curvature without postulating a fundamental metric or gravitational interaction.

Quantum phenomena are interpreted as effective descriptions of coherent fluctuations around solitonic configurations of χ. Entanglement reflects the persistence of shared field configurations across spatial separation, yielding nonlocal correlations without superluminal signaling, local hidden variables, or fundamental wavefunction collapse.

The standard formalisms of general relativity and quantum mechanics are not assumed at the fundamental level but are recovered as emergent, coarse-grained descriptions valid in regimes where χ admits a stable geometric interpretation. Cosmological expansion follows directly from the global relaxation of χ, leading to a geometric interpretation of the Hubble parameter and an apparent acceleration without invoking dark energy or an inflationary phase.

While not proposed as a final unified theory, Cosmochrony offers a conceptually coherent framework that clarifies the physical origin of time, geometry, gravitation, and quantum correlations within a single scalar dynamics. The approach identifies qualitative and semi-quantitative signatures that may allow empirical distinction from standard cosmological scenarios and provides a basis for further foundational and mathematical investigation.

Contents

1 Introduction

Modern fundamental physics rests on two remarkably successful yet conceptually distinct theoretical frameworks: quantum mechanics and general relativity [14, 16]. Quantum theory provides an extraordinarily accurate description of microscopic phenomena, while general relativity offers a geometric account of gravitation and spacetime dynamics at macroscopic and cosmological scales [13, 17]. Despite their empirical success, these theories remain difficult to reconcile within a single coherent conceptual and mathematical framework [31, 32, 48, 52].

A central obstacle to this reconciliation lies in their incompatible foundational assumptions. Quantum mechanics presupposes a fixed spacetime background on which states evolve, whereas general relativity identifies spacetime geometry itself as a dynamical entity. Attempts to bridge this gap have led to a wide range of approaches, including quantum field theory in curved spacetime, canonical and covariant quantum gravity programs, as well as string-based and holographic frameworks. While these efforts have yielded deep insights, they typically rely on extended mathematical structures, additional dimensions, or large numbers of degrees of freedom, and often introduce elements whose empirical accessibility remains uncertain.

In this article, we explore a complementary and deliberately minimalist approach, referred to as *Cosmochrony*. The guiding hypothesis is that spacetime geometry, gravitation, and quantum phenomena may emerge from the dynamics of a single continuous geometric substrate, described by a scalar quantity denoted χ. Crucially, χ is not introduced as a field propagating on a pre-existing spacetime background. Rather, spacetime notions themselves are treated as effective descriptions that arise from the relational and dynamical properties of χ configurations.

The central dynamical postulate of Cosmochrony is that χ undergoes an irreversible relaxation process, locally bounded by the invariant propagation speed c. This monotonic evolution establishes an intrinsic ordering of physical processes, which is identified with the arrow of time. Spatial separation, in turn, is interpreted as a relational structure emerging from differences and gradients in χ once a stable geometric regime is reached. Within this framework, spacetime expansion, gravitation, particle-like excitations, radiation processes, and quantum correlations are not fundamental ingredients, but emergent phenomena associated with specific configurations or interactions of the underlying field.

The aim of the present work is not to propose a complete or final unification of quantum theory and gravitation. Rather, it seeks to formulate a minimal and internally coherent dynamical framework, and to examine the extent to which its qualitative structure and phenomenological consequences are compatible with established physical observations. In particular, we investigate how cosmological expansion, particle properties, gravitational effects, radiation, and quantum entanglement may be consistently reinterpreted within a single geometric setting governed by the dynamics of χ.

Cosmochrony does not attempt to replace the Standard Model or General Relativity in their well-tested domains. Instead, it offers a unifying geometric interpretation in which quantization, spacetime curvature, and cosmic expansion emerge from a common relaxation dynamics rather than being introduced as independent postulates. In this sense, the framework is best viewed as an exploratory research program aimed at clarifying the conceptual relationships between time, geometry, and quantum phenomena.

At the outset, it is important to clarify the ontological and epistemic status of the quantities introduced in this work. The scalar quantity χ is not defined on a pre-existing spacetime manifold, nor is it interpreted as a conventional physical field propagating within spacetime. Rather, χ is taken to represent a pre-geometric substrate whose irreversible relaxation underlies the emergence of both temporal ordering and spatial relations.

In this perspective, familiar spacetime notions such as coordinates, metric structure, and differential geometry are not fundamental ingredients of the theory. They arise only as effective, coarse-grained descriptions of relational and dynamical properties of χ in regimes where a stable geometric interpretation becomes possible. Accordingly, variational principles, Lagrangian formulations, and metric-based descriptions employed later in the paper should be understood as emergent tools rather than as fundamental postulates.

The structure of the paper is as follows. Sections 2–4 introduce the conceptual motivations and minimal dynamical assumptions governing the χ substrate. Subsequent sections examine how particle-like excitations, gravitation, quantum correlations, and cosmological behavior emerge in appropriate regimes. The standard formalisms of general relativity and quantum mechanics are recovered only at this effective level, as descriptive frameworks applicable once a spacetime interpretation has emerged. Technical reconstructions and mathematical details are collected in the appendices.

1.1 Conceptual Context and Related Approaches

The idea that spacetime geometry and gravitation may be emergent rather than fundamental has been explored in a variety of recent theoretical frameworks. Several approaches treat the spacetime metric as an effective description arising from deeper geometric, informational, or dynamical structures, and interpret gravitation as a collective or emergent phenomenon rather than a fundamental interaction [27, 51].

Like Loop Quantum Gravity (LQG), Cosmochrony posits that spacetime geometry is not fundamental but emerges from a deeper, pre-geometric structure. However, while LQG relies on spin networks and holonomies, Cosmochrony introduces a single scalar field χ whose relational dynamics encode both spatial and temporal properties. This minimalist approach offers a complementary perspective on the emergence of gravity and quantum phenomena.

For convenience, a glossary summarizing the main quantities and operators used throughout the article is provided in Appendix E.

2 Theoretical Context and Motivation

2.1 Conceptual Tension Between Quantum Theory and Gravitation

Quantum mechanics and general relativity differ not only in their mathematical formalisms, but also in their underlying conceptual foundations. Quantum theory is intrinsically probabilistic, relies on a fixed causal structure, and treats time as an external parameter [6, 13]. General relativity, by contrast, describes gravitation as the dynamics of spacetime geometry itself, with time emerging as a coordinate-dependent and observer-relative quantity [17, 31].

This mismatch becomes particularly acute in regimes where both quantum effects and strong gravitational fields are expected to be relevant, such as near spacetime singularities or in the early universe [36, 42]. Attempts to quantize gravity directly often encounter conceptual obstacles, including the problem of time, non-renormalizability, and the absence of a preferred background structure.

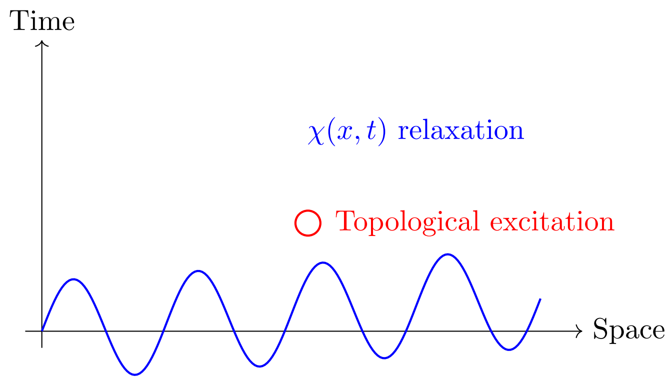

Figure 1: Conceptual representation of Cosmochrony. A single continuous wave field $\chi(x,t)$ undergoes irreversible relaxation, characterized by a monotonic increase of its characteristic wavelength. Localized topological excitations correspond to particles.

2.2 Limitations of Existing Unification Approaches

Several major research programs aim to address these challenges. Quantum field theory in curved spacetime successfully incorporates particle creation and vacuum effects, but retains a classical spacetime background [53]. Canonical and covariant approaches to quantum gravity attempt to quantize spacetime geometry itself, often at the cost of mathematical complexity and interpretational ambiguity.

String theory and related frameworks propose extended fundamental objects and higher-dimensional structures, offering deep mathematical unification but introducing a large landscape of possible low-energy realizations [48]. While these approaches are internally rich, their empirical testability remains limited, and their physical interpretation can be indirect.

These considerations motivate the exploration of alternative perspectives in which spacetime geometry, matter, and quantum behavior are not separately postulated, but arise from a common underlying mechanism.

2.3 Minimalism as a Guiding Principle

The approach developed in this work adopts minimalism as a guiding principle. Rather than introducing multiple fundamental fields, dimensions, or quantization rules, we consider whether a single continuous scalar field can encode both temporal evolution and spatial structure.

The field $\chi(x,t)$ is not initially interpreted as a conventional matter field, nor as a metric component. Instead, it represents a geometric quantity whose local evolution defines both duration and separation. In this view, time and space are not independent primitives, but complementary aspects of the same dynamical process.

2.4 Time, Irreversibility, and Cosmological Expansion

A central motivation for the Cosmochrony framework is the close connection between time, irreversibility, and cosmic expansion. In standard cosmology, expansion is described kinematically through the scale factor, while the arrow of time is usually attributed to boundary conditions or entropy growth [34–36, 42].

Here, the monotonic relaxation of χ provides a unified origin for both phenomena [41]. Cosmological expansion corresponds to the cumulative spatial manifestation of this relaxation [34] , while irreversibility follows from its intrinsic directionality. This perspective suggests that expansion is not driven by an external energy component, but is an emergent geometric effect.

2.5 Scope and Limitations

The goal of this work is exploratory rather than definitive. The proposed framework does not claim to replace established theories in their domains of validity, but to offer a coherent reinterpretation that may illuminate persistent conceptual difficulties. Throughout the paper, emphasis is placed on internal consistency, conceptual clarity, and contact with observable phenomena, while acknowledging open questions and limitations.

In the following section, we introduce the χ field formally and specify the minimal assumptions underlying its dynamics.

3 Definition and Fundamental Properties of the χ Field

Having outlined the ontological and conceptual principles underlying Cosmochrony, we now introduce the fundamental quantity at the core of the framework. This section is devoted to defining the scalar entity χ and clarifying its role as a pre-geometric substrate from which spacetime notions, dynamical laws, and physical observables emerge.

The purpose of this section is not to assume a pre-existing spacetime structure, but to establish the minimal properties required of χ in order to recover, in appropriate regimes, effective notions of time, space, metric geometry, and field dynamics. Accordingly, χ is introduced independently of any spacetime coordinates or metric, and only later related to effective geometric descriptions once a stable regime is reached.

In this sense, the use of variational, Lagrangian, or metric-based formulations later in this section does not imply that spacetime or a four-dimensional manifold is fundamental. These formalisms are employed as effective tools to describe the dynamics of χ in regimes where a spacetime interpretation becomes meaningful, and should be understood as emergent, coarse-grained representations of the underlying pre-geometric dynamics.

We begin by providing a unified conceptual definition of the χ field and its physical interpretation. Subsequent subsections introduce effective dynamical descriptions—including Lagrangian and metric formulations—that are intended as coarse-grained representations valid when the underlying χ configurations admit a spacetime interpretation.

3.1 Definition of the χ Field

We postulate the existence of a single fundamental scalar quantity, denoted χ, which constitutes the primitive substrate of physical reality. The field χ is not defined on a pre-existing spacetime manifold and does not presuppose any metric, causal, or geometric structure. Instead, spacetime itself emerges as an effective description of the relational and dynamical properties of χ configurations.

Ontologically, χ is a real scalar order parameter whose value characterizes the local geometric state of the underlying substrate. It carries dimensions of length and encodes a characteristic wavelength associated with physical processes. This wavelength does not evolve *in* time; rather, its monotonic and irreversible relaxation defines what is operationally perceived as the flow of time.

Temporal ordering emerges from the global, monotonic evolution of χ across physical processes, establishing an intrinsic arrow of time without reference to an external temporal coordinate. Spatial separation, in turn, arises from relational differences between χ values across configurations, giving rise to an effective notion of distance once a stable geometric regime is established. In this sense, time corresponds to ordering, while space corresponds to relational structure.

At no stage is χ interpreted as a spacetime coordinate or as a material field propagating on spacetime. Rather, spacetime coordinates and metric structure appear as secondary, coarse-grained constructs that become meaningful only when χ configurations admit a quasi-stable geometric interpretation. The spacetime metric thus functions as an emergent, effective descriptor of resistance to χ relaxation and of the propagation of perturbations within the field.

This role of χ is analogous to that of thermodynamic order parameters such as temperature: it encodes collective geometric information about an underlying substrate without being itself a fundamental spacetime entity. In the Cosmochrony framework, χ therefore provides the minimal ontological basis from which time, space, gravitation, and quantum phenomena jointly emerge.

In the following sections, spacetime coordinates and metric quantities will be introduced as effective tools, valid in regimes where χ admits a stable geometric interpretation.

On the use of spacetime language. Throughout this work, phrases such as "spacetime coordinates," "metric tensor," and "four-dimensional manifold" appear frequently. These should be understood as *emergent effective descriptions* valid in regimes where χ has relaxed into a quasi-stable geometric configuration. They are not fundamental ingredients of the theory.

At the deepest level, only χ and its local variation structure exist. The appearance of familiar geometric language reflects the effectiveness of spacetime as a coarse-grained description of collective χ behavior, analogous to how thermodynamic variables (temperature, pressure) emerge from molecular dynamics without those variables being fundamental.

This interpretational stance is essential for distinguishing Cosmochrony from approaches that merely reformulate existing geometric theories in different variables.

3.2 The Geometric Effective Action and Lagrangians of Cosmochrony ($\mathcal{L}_{\mathrm{CC}}$)

Interpretational caution.

The action principle presented below employs conventional field-theoretic notation, including a metric tensor $g_{\mu\nu}$ and a four-dimensional integration measure. This should not be interpreted as assuming pre-existing spacetime structure.

The formalism serves two purposes:

1. To provide a compact representation of χ dynamics in regimes where an effective spacetime description is valid.

2. To establish the bridge between the fundamental relational network and the effective manifold description used in standard physics.

The fundamental content of the theory is the field χ and its relaxation dynamics on a discrete graph. The metric $g_{\mu\nu}$ appearing in the action is a statistical emergent structure representing the connectivity and correlation density of the χ field, not an independent ontological input.

Discrete Network Foundation

The dynamics of χ are fundamentally defined on a **discrete network**, where each node i represents a local value χ_i, and each link between nodes i and j is characterized by a **connectivity strength** K_{ij}. This matrix K_{ij} encodes the correlation between neighboring χ-values and serves as the microscopic foundation for the emergent geometry. In regimes where χ admits a quasi-stable geometric interpretation, K_{ij} can be mapped to an effective metric $g_{\mu\nu}$ through the relation:

$$g_{\mu\nu}dx^\mu dx^\nu \approx \sum_{(u,v)\in\text{path}} \frac{1}{K_{uv}} \tag{1}$$

Explicit Form of the Connectivity Matrix K_{ij}

The connectivity strength K_{ij} between nodes i and j encodes the **local correlation** between their respective χ-field values. To ensure consistency with the emergent geometry and the dynamical constraints of the χ field, we adopt the following **constitutive relation**:

$$K_{ij} = K_0 \cdot f\left(\frac{|\chi_i - \chi_j|^2}{\chi_c^2}\right) \tag{2}$$

where:

- K_0 is a **fundamental coupling scale** (with dimensions of $[\text{length}]^{-2}$), representing the maximal connectivity strength in regions where χ is uniform.
- χ_c denotes a characteristic scale of the χ field beyond which strong local variations significantly weaken the effective connectivity. Its identification with specific microscopic or cosmological length scales depends on the physical regime considered and is discussed later.
- $f(x)$ is a **dimensionless, monotonically decreasing function** satisfying $f(0) = 1$ and $f(x) \to 0$ as $x \to \infty$.

A physically motivated choice for $f(x)$ is:

$$f(x) = \frac{1}{1+x} \tag{3}$$

This ansatz ensures that:

1. **Symmetry**: $K_{ij} = K_{ji}$, as required for a consistent relational structure.
2. **Locality**: K_{ij} depends only on the **local difference** $|\chi_i - \chi_j|$, reflecting the pre-geometric nature of the χ field.
3. **Boundedness**: $0 < K_{ij} \le K_0$, preventing unphysical divergences in the emergent metric.
4. **Gradient sensitivity**: K_{ij} decreases as $|\chi_i - \chi_j|$ increases, encoding the **resistance to relaxation** induced by localized excitations (e.g., particles or curvature).

This form of K_{ij} provides a microscopic foundation for the operational notion of distance used in the emergent geometric description developed later in Section 6, rather than assuming a pre-existing metric structure at this stage. The coupling scale K_0 and the characteristic scale χ_c set the strength and the non-linearity threshold of the local connectivity. Their identification with specific microscopic or cosmological scales depends on the physical regime considered and is discussed later.

Emergent Geometry from K_{ij}

The connectivity matrix K_{ij} defines an **operational distance** between nodes i and j via the minimal path sum:

$$d(i,j)^2 = \ell_0^2 \sum_{(u,v)\in\text{path}} \frac{1}{K_{uv}} \tag{4}$$

where ℓ_0 is a microscopic length scale (e.g., related to the Planck length). In the continuum limit, this discrete sum converges to the line element of an **emergent metric** $g_{\mu\nu}$:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu \approx \ell_0^2 \left[\delta_{\mu\nu} + \mathcal{O}\left(\frac{|\nabla\chi|^2}{\chi_c^2}\right)\right] dx^\mu dx^\nu \tag{5}$$

Here, the corrections $\mathcal{O}(|\nabla\chi|^2/\chi_c^2)$ encode the **curvature induced by localized excitations** (e.g., particles or black holes). This construction ensures that:

- **Flat spacetime** emerges when χ is uniform ($\nabla\chi = 0$), as $K_{ij} \approx K_0$ and $g_{\mu\nu} \approx \eta_{\mu\nu}$.
- **Curved spacetime** arises in regions where $\nabla\chi \neq 0$, as K_{ij} varies spatially, inducing a non-trivial $g_{\mu\nu}$.

This mechanism provides a **geometric interpretation of gravity** as a modulation of the χ-field's connectivity, without invoking a fundamental metric or curvature tensor. Further details on the continuum limit and the derivation of the effective field equations are provided in Appendix D.1.

Effective action formulation.

In regimes where χ admits a quasi-stable geometric interpretation, the dynamics may be encoded in an effective action:

$$S_{\text{CC}} = \int \mathcal{L}_{\text{CC}}\sqrt{-g}\, d^4x \tag{6}$$

where the Lagrangian density decomposes as:

$$\mathcal{L}_{\text{CC}} = \mathcal{L}_{\text{Gravity/Time}} + \mathcal{L}_{\chi/\text{Soliton}} + \mathcal{L}_{\text{Forces/Matter}} \tag{7}$$

The symbol $\sqrt{-g}$ represents the invariant volume element. In regimes where no spacetime interpretation yet exists (e.g., at the nodes of the fundamental graph), this should be understood as an abstract integration measure $d\mu$ on the configuration space of χ.

Status of $g_{\mu\nu}$ in this formulation.

The metric $g_{\mu\nu}$ is an effective description of the coupling strengths K_{ij} between χ nodes. It is defined by the requirement that the distance ds^2 in the continuum matches the operational distance derived from the network's connectivity:

$$g_{\mu\nu}dx^\mu dx^\nu \approx \sum_{(u,v)\in\text{path}} \frac{1}{K_{uv}} \tag{8}$$

Consequently, $g_{\mu\nu}$ is a phenomenological summary of the underlying relational dynamics, capturing the local rate of χ-relaxation and its spatial correlations.

3.3 Physical Interpretation

The central interpretative assumption of Cosmochrony is that spacetime is not a static background but the macroscopic manifestation of the continuous relaxation of χ. An increase in χ corresponds simultaneously to:

- the passage of local proper time,
- the emergence of spatial distance between causally connected events,
- the global expansion of the universe when integrated over large scales.

In this framework, distance may be interpreted as "frozen time," while time corresponds to "locally thawed distance." This dual interpretation is not imposed ad hoc but follows from the identification of both quantities with the same underlying field.

3.4 Monotonicity and Arrow of Time

A fundamental postulate of the theory is that χ evolves monotonically [37, 41]:

$$\frac{\partial \chi}{\partial t} \geq 0. \tag{9}$$

This monotonicity is not derived from statistical considerations, nor imposed as a thermodynamic boundary condition. Rather, it reflects a structural property of the χ field related to the irreversible character of its relaxation dynamics. Within Cosmochrony, energy is not treated as a fundamental conserved substance but as a measure of the residual capacity of a given χ configuration to relax. Only relaxation processes can dissipate this capacity, while a decrease of χ would correspond to a spontaneous reintroduction of contraction or tension into the field, for which no physical mechanism exists in the framework.

Irreversibility therefore follows naturally: any decrease of χ would correspond to a contraction of both temporal and spatial structure and to an effective creation of relaxation potential, which is dynamically forbidden. The arrow of time is thus identified with the irreversible expenditure of the relaxation capacity of the χ field, rather than with a statistical entropy principle imposed at the outset.

3.5 Local Relaxation Speed

The local rate of change of χ is bounded by a universal constant:

$$|\nabla_\mu \chi| \leq c, \tag{10}$$

where c coincides with the observed speed of light.

This bound does not represent the propagation speed of particles or signals, but the maximal rate at which spacetime itself can locally unfold. Superluminal recession velocities at cosmological scales arise naturally through cumulative effects and do not violate local causality.

3.6 Relation to Conventional Fields

Although χ shares mathematical similarities with scalar fields used in cosmology (e.g., inflaton-like fields), its role is fundamentally different. It does not carry energy in the conventional sense, nor does it require quantization at the fundamental level.

Matter, radiation, and interactions emerge as localized excitations, constraints, or topological features of χ, rather than as independent entities coupled to it.

3.7 Initial Conditions and Global Structure

The theory assumes an initial condition characterized by a minimal value χ_0, naturally associated with the Planck scale. Cosmic evolution corresponds to the progressive relaxation of χ from this initial state.

Importantly, the framework does not require a spacetime singularity in the traditional sense. Instead, the apparent singular behavior arises from extrapolating classical notions of time and distance beyond the domain where χ is well-defined.

In the next section, we derive a minimal dynamical equation governing the evolution of χ and explore its immediate consequences.

4 Dynamical Equation for the χ Field

4.1 Parameter-Independent Relaxation

To avoid the conceptual pitfalls of a fundamental time coordinate, we define the dynamics not as a function of an absolute variable, but as a sequence of field configurations (χ_λ) where λ is a strictly monotonic ordering parameter. At the fundamental level, the evolution of χ is defined by its relaxation flow:

$$\frac{d\chi}{d\lambda} = \mathcal{R}(\chi, \nabla\chi) \tag{11}$$

where $\mathcal{R}$ represents the rate of geometric tension release toward equilibrium. The "temporal" derivative ∂_t used in subsequent sections is then understood as a convenient reparametrization of this primary flow:

$$\partial_t\chi \equiv \frac{d\chi}{d\lambda}\frac{d\lambda}{dt} \tag{12}$$

Since t merely serves to label the relaxation ordering, all physical results are invariant under any monotonic reparameterization $t \to f(t)$ with $f'(t) > 0$.

In this framework, the relaxation of χ is not occurring *in* time; rather, the local rate of relaxation *defines* the physical measure of time. What we perceive as duration is the cumulative displacement of the field toward its equilibrium state. Consequently, the "speed of time" is locally determined by the density of χ-gradients, providing a direct link between field topology and temporal flow.

4.2 Hamiltonian Derivation of the Evolution Equation

Discrete Dynamics of χ

Before introducing any metric structure, the dynamics of χ can be formulated purely in terms of its local relaxation on a discrete network. Let λ be a monotonic ordering parameter, and define the local variation of χ at node i as:

$$\frac{d\chi_i}{d\lambda} = c\sqrt{1 - \frac{1}{c^2}\sum_{j\sim i} K_{ij}(\chi_i - \chi_j)^2} \tag{13}$$

where K_{ij} is the connectivity strength defined in Equation (2), and the sum runs over neighboring nodes j. This equation is **metric-independent** and defines the fundamental dynamics of χ in terms of its local correlations.

To illustrate the physical implications, consider a **homogeneous background** where $\chi_i = \chi_0 + \delta\chi_i$ with $|\delta\chi_i| \ll \chi_0$. Expanding K_{ij} to first order in $\delta\chi_i$:

$$K_{ij} \approx K_0 \left(1 - \frac{|\delta\chi_i - \delta\chi_j|^2}{\chi_c^2}\right) \tag{14}$$

Substituting into the evolution equation yields:

$$\frac{d\chi_i}{d\lambda} \approx c \left[1 - \frac{K_0}{2c^2\chi_c^2} \sum_{j\sim i} (\delta\chi_i - \delta\chi_j)^2\right] \tag{15}$$

This shows that **localized perturbations** (e.g., particles) induce a **slowdown in the relaxation of** χ, which manifests macroscopically as **gravitational time dilation** (see Section D.1).

Continuum Limit

In the limit where the network becomes dense (i.e., the distance between nodes approaches zero), the discrete sum can be approximated by a continuous Laplacian:

$$\sum_{j\sim i} K_{ij}(\chi_i - \chi_j)^2 \approx \int |\nabla\chi|^2 \, dV \tag{16}$$

This yields the effective evolution equation in the continuum:

$$\partial_t \chi = c\sqrt{1 - \frac{|\nabla\chi|^2}{c^2}} \tag{17}$$

where ∇ is now defined with respect to the emergent metric $g_{\mu\nu}$, derived from the network's connectivity (see Section 3.2).

Continuum Limit and Newtonian Gravity

In the continuum limit, the discrete sum in Equation (13) becomes a spatial integral, and the evolution equation reduces to:

$$\partial_t \chi = c\sqrt{1 - \frac{|\nabla\chi|^2}{c^2}} \tag{18}$$

The **source term** $S[\chi, \rho]$ in the effective wave equation (Section A.6) can now be derived explicitly from the discrete coupling K_{ij}. For a localized excitation (e.g., a particle of mass m), the spatial variation of χ induces a **modulation of** K_{ij}, which in turn slows the relaxation rate. This effect is captured by the **nonlinear Poisson equation**:

$$\nabla \cdot \left(\frac{\nabla\chi}{\sqrt{1 - |\nabla\chi|^2/c^2}}\right) = \frac{4\pi G}{c^2}\rho \tag{19}$$

where G emerges as an **effective coupling constant** related to K_0 and χ_c. In the weak-field limit ($|\nabla\chi| \ll c$), this reduces to the standard Poisson equation:

$$\nabla^2\Phi = 4\pi G\rho \tag{20}$$

with the gravitational potential Φ identified as:

$$\Phi = c^2 \ln\left(\frac{\partial_t \chi}{c}\right) \tag{21}$$

This derivation shows that **Newtonian gravity** emerges naturally from the **collective coupling** K_{ij} without postulating a fundamental metric or curvature. The full derivation, including the mapping between K_0, χ_c, and G, is provided in Appendix D.1.

Hamiltonian Constraint

While the dynamics of χ can be viewed as a minimal relaxation principle, it can be more rigorously derived from a Hamiltonian constraint. We postulate that the dynamics of χ are governed by a Dirac-type kinematic constraint in phase space, analogous to the mass-shell condition for a massless relativistic particle:

$$(\partial_t \chi)^2 + |\nabla \chi|^2 = c^2 \tag{22}$$

where c is the fundamental velocity scale. Combined with the arrow of time postulate ($\partial_t \chi \geq 0$), which reflects the irreversible relaxation of the Cosmochron, this leads uniquely to the first-order evolution equation:

$$\partial_t \chi = c\sqrt{1 - \frac{|\nabla \chi|^2}{c^2}} \tag{23}$$

This derivation grounds the "minimal principle" in the symplectic structure of the field's phase space, ensuring that χ acts as an intrinsic time coordinate.

4.3 Microscopic Origin of the Coupling Tensor and the Poisson Equation

To achieve self-consistency, the coupling tensor K_{ij} must not be a mere constant but a dynamical measure of the field's local stress. We propose that K_{ij} depends on the local gradient of χ through a non-linear constitutive relation. A physically motivated form for this coupling is:

$$K_{ij} = K_0 \exp\left(-\frac{(\chi_i - \chi_j)^2}{\chi_c^2}\right) \tag{24}$$

where K_0 is the vacuum coupling constant and χ_c is a characteristic field scale. This scenario implies that regions of high field gradients (solitons) locally weaken the coupling, thereby slowing down the relaxation rate.

To derive the Newtonian limit, consider the continuous limit of the evolution equation (Eq. 34). In the presence of a localized excitation (soliton) of mass M, the local relaxation rate $v = \partial_t \chi$ is modified. Let v_0 be the global relaxation rate far from any mass. The local potential Φ can be identified through the relative shift in the relaxation velocity:

$$\frac{v(r)}{v_0} \approx 1 + \frac{\Phi(r)}{c^2} \tag{25}$$

By applying the Taylor expansion to the discrete sum $\sum_j K_{ij}(\chi_i - \chi_j)^2$ and assuming the form of K_{ij} from our scenario, the discrete evolution equation converges to a non-linear Poisson-like equation:

$$\nabla^2 \Phi - \frac{1}{\chi_c^2}(\nabla \Phi)^2 = 4\pi G \rho \tag{26}$$

In the weak-field limit ($\Phi \ll c^2$), the second-order term becomes negligible, and we recover the standard Poisson equation $\nabla^2\Phi = 4\pi G\rho$. This derivation proves that gravitation in Cosmochrony is not an external force but a direct consequence of the local reduction in the χ field's "relaxation conductivity" induced by the presence of solitonic configurations.

4.4 Variational Formulation and Born-Infeld Action

To extend the kinematic constraint introduced above into an effective dynamical description that incorporates matter sources, we introduce a variational framework inspired by Born–Infeld–type non-linear actions, originally developed to regularize field singularities in classical field theories [7, 12].

$$\mathcal{L} = -c^2\sqrt{1 - \frac{|\nabla\chi|^2}{c^2}} + \partial_t\chi - \frac{4\pi G}{c^2}\rho\chi, \tag{27}$$

where ρ represents the matter density. The presence of the term $\partial_t\chi$ linear in the first-order temporal derivative is crucial: it ensures that the momentum conjugate to χ, defined as $\Pi_\chi = \frac{\partial\mathcal{L}}{\partial(\partial_t\chi)}$, is a non-vanishing constant ($\Pi_\chi = 1$).

In the present framework, the Born–Infeld–type square-root structure does not introduce additional propagating degrees of freedom. Instead, it acts as a non-linear regulator that enforces an upper bound on spatial gradients, ensuring consistency with the fundamental kinematic constraint while avoiding singular field configurations, in direct analogy with the original motivation of Born–Infeld electrodynamics [7].

In the Hamiltonian formalism, this constant momentum acts as a primary constraint that effectively enforces the unit-velocity evolution of the field. This structure ensures that the field dynamics remain locked onto the Hamiltonian constraint (22) while the square-root term acts as a non-linear regularizer for spatial gradients. The variation with respect to χ yields a non-linear Poisson equation:

$$\nabla \cdot \left(\frac{\nabla\chi}{\sqrt{1 - |\nabla\chi|^2/c^2}} \right) = \frac{4\pi G}{c^2}\rho. \tag{28}$$

This formulation naturally recovers the Newtonian limit for weak gradients ($|\nabla\chi| \ll c$) while preventing gravitational singularities as the gradient magnitude is bounded by c.

4.5 Causality and Locality

Equation (17) is explicitly local and causal. The evolution of χ at any spacetime point depends only on its immediate neighborhood through $\nabla\chi$.

Importantly, no superluminal propagation occurs at the fundamental level. Apparent superluminal recession velocities in cosmology arise from integrating local χ increments across extended regions, consistent with relativistic causality.

4.6 Homogeneous Cosmological Limit

In a spatially homogeneous and isotropic configuration, $\nabla\chi = 0$, and the evolution equation simplifies to:

$$\partial_t\chi = c. \tag{29}$$

This implies a linear growth [20, 25]:

$$\chi(t) = \chi_0 + ct, \tag{30}$$

where χ_0 denotes the initial value of χ.

This simple relation already reproduces a Hubble-like expansion law when distances are identified with accumulated χ increments, as discussed in Section 9.

As shown in Appendix C.4.2, the requirement $\partial_t \chi \geq 0$ in an expanding background (H_0) implies a minimal residual gradient $\nabla \chi_{\min} \propto \sqrt{H_0}$. This "acceleration floor" provides a first-principles derivation for MOND-like phenomenology, explaining galactic rotation curves without invoking dark matter particles.

4.7 Influence of Local Structure

In regions where $\nabla \chi \neq 0$, the effective rate of χ-relaxation is reduced. This slowing plays a central role in the emergence of gravitational phenomena.

Localized excitations—identified with particles—act as topological or dynamical constraints on χ, increasing $|\nabla \chi|$ and thereby locally reducing $\partial_t \chi$.

This mechanism leads naturally to time dilation and spatial curvature without invoking an independent gravitational field.

4.8 Unified Origin of Geometric and Field Effects

The relationship between the χ field and the effective metric $g_{\mu\nu}$ is strictly hierarchical, reflecting the transition from fundamental relations to smooth geometry:

1. **Primacy of the Network:** The fundamental layer consists of the relational χ-network where the connectivity matrix K_{ij} dictates the flow of information and the local relaxation rate.

2. **Geometric Emergence:** The metric $g_{\mu\nu}$ is an emergent statistical description of these connections. It represents the "density of correlation" within the field and acts as a coarse-grained summary for macroscopic observers.

3. **The Hydrodynamic Analogy:** Just as pressure and temperature emerge from molecular collisions without acting back as independent ontological forces, the metric $g_{\mu\nu}$ encodes the state of the χ field. Gravitation is the manifestation of χ-solitons (matter) locally modulating the network's connectivity.

In this framework, the field equations are solved on the fundamental graph. The resulting configuration naturally defines the effective spacetime geometry, ensuring that gravitation and matter are two aspects of the same underlying χ dynamics.

4.9 Limitations and Scope

Equation (17) is intentionally minimal. It does not attempt to describe quantum fluctuations of χ, nor does it incorporate backreaction effects beyond first order.

Its purpose is to provide a unified kinematic backbone from which gravitational, quantum, and cosmological phenomena can be derived consistently.

In the following sections, we apply this dynamical framework to particles, gravity, and entanglement.

5 Particles as Localized Excitations of the χ Field

5.1 Particles as Stable Wave Configurations

Within the Cosmochrony framework, particles are not fundamental point-like objects but stable, localized excitations of the χ field [43]. They correspond to persistent wave configurations that locally constrain the relaxation of χ.

These configurations may be interpreted as soliton-like structures: they maintain their identity during propagation and interaction, while remaining fully embedded in the underlying χ dynamics.

5.2 Topological Stability

The stability of particle-like excitations is attributed to topological constraints rather than to conserved charges postulated a priori [44]. Nontrivial phase winding, torsion, or knot-like structures in χ prevent continuous deformation into the vacuum state.

Such topological protection naturally explains the discreteness of particle species and their robustness under perturbations.

For instance, an electron corresponds to a localized knot in χ with a specific winding number, where the knot's energy (proportional to its curvature) determines the particle's mass, and its topological charge (e.g., 4π-periodicity) determines its spin-1/2 nature.

The stability of solitonic excitations arises from a balance between the nonlinear self-interaction term $V(\chi)$ (which tends to localize the field) and the gradient energy $|\nabla\chi|^2$ (which tends to disperse it). Topological invariants, such as the winding number $n = \frac{1}{2\pi}\oint \nabla \arg(\chi)\cdot d\mathbf{l}$, further protect these configurations from decay, ensuring their persistence as particle-like objects.

More topological configurations are discussed in B.2

5.3 Mass as Resistance to χ Relaxation

In this framework, mass is not an intrinsic attribute but an emergent measure of how strongly a localized excitation resists the local increase of χ. Mass therefore characterizes the persistence of a non-relaxed configuration within an otherwise globally relaxing field.

Regions containing particle excitations exhibit increased spatial gradients:

$$|\nabla\chi| > 0, \tag{31}$$

which, according to Eq. (17), reduces the local relaxation rate $\partial_t\chi$. This local slowdown of χ-relaxation manifests macroscopically as time dilation and, at larger scales, as an effective gravitational mass.

From an energetic perspective, such localized resistance corresponds to the storage of relaxation potential within the χ field. Maintaining a stable excitation requires continuously counterbalancing the global tendency of χ to relax, and this cost is what is operationally identified as energy. In this sense, mass quantifies the amount of relaxation potential that remains trapped in a localized configuration.

Mapping of the energy of solitons to the masses of observed particles is detailed in Section B.3.

5.4 Energy–Frequency Relation

The energy associated with a particle excitation is linked to the internal oscillation frequency of its wave configuration. Within the Cosmochrony framework, this frequency characterizes how strongly the excitation resists relaxation: higher-frequency structures correspond to tighter localization, stronger spatial gradients in χ, and a greater capacity to relax.

This provides a geometric interpretation of the relation

$$E \propto \nu, \tag{32}$$

in which energy measures the amount of relaxation potential stored in a given configuration, while frequency quantifies the rate at which this potential can be redistributed. Planck's constant then emerges as an effective proportionality factor determined by the properties of the χ field and its coupling scales, rather than as a fundamental constant postulated a priori.

A more explicit geometric derivation of this relation, in the context of radiation and photon-like excitations of the χ field, is presented in Section 10.3.

5.5 Fermions and Bosons

Particle statistics arise from the topology of the underlying excitation. Configurations requiring a 4π phase rotation to return to their original state correspond to fermions, while integer-winding configurations correspond to bosons.

This topological distinction naturally reproduces the spin-statistics connection without invoking additional quantum postulates.

Consider a localized soliton solution $\chi(\mathbf{x}) = \chi_0 \tanh(r/\xi)$, where r is the radial coordinate and ξ sets the soliton size. For fermionic excitations, the phase of χ must wind by 4π to return to its original value, reflecting a Möbius-like twist in the field configuration. This topological constraint enforces the spin-statistics theorem: only configurations with half-integer winding numbers (fermions) can exhibit such 4π-periodicity, while integer windings (bosons) correspond to 2π-periodic solutions.

5.6 Antiparticles

Antiparticles are interpreted as phase-inverted counterparts of particle excitations. Their annihilation corresponds to topological unwinding, releasing stored curvature energy back into propagating χ waves.

This process conserves total χ-structure while converting localized constraints into delocalized radiation.

5.7 Particle Creation and Destruction

Particles are created when propagating χ waves self-interfere or interact with existing excitations strongly enough to form stable localized configurations. Conversely, particle destruction corresponds to the loss of topological stability through interaction or decoherence.

This view removes the need for particle ontology as a primitive concept and replaces it with a purely dynamical description.

5.8 Summary

Particles emerge as stable, localized excitations of the χ field that resist its relaxation. Their mass, energy, spin, and statistics follow from geometric and topological properties of χ, providing a unified description compatible with both relativistic and quantum phenomena.

6 Gravity as a Collective Effect of Particle Excitations

6.1 Local Slowdown of χ Relaxation

In Cosmochrony, gravity does not arise from a fundamental interaction but from the collective influence of particle excitations on the dynamics of the χ field. As established in the previous section, localized excitations resist the local relaxation of χ.

When many such excitations are present, their effects superpose, leading to a macroscopic reduction of the relaxation rate:

$$\partial_t \chi = c\,(1 - \alpha\rho)\,, \tag{33}$$

where ρ denotes the effective density of particle excitations and α is a coupling parameter encoding their influence on χ.

The coupling parameter α in $\partial_t \chi = c(1-\alpha\rho)$ is determined by the interaction strength between χ and localized excitations. For a point-like excitation of mass m, α scales as $\alpha \sim Gm/c^2$, where G emerges as an effective coupling constant linking matter density to χ-relaxation slowing. This yields the Newtonian potential $\Phi \sim \alpha\rho$ in the weak-field limit.

This slowdown manifests physically as gravitational time dilation.

6.2 Collective Gravitational Coupling and Operational Geometry

The collective slowdown of χ relaxation described above not only affects the local flow of time but also induces an effective spatial coupling between neighboring regions of the field. When particle excitations are present, the resistance they impose on the relaxation of χ modulates how efficiently variations of the field propagate across space.

This collective behavior can be encoded in an effective local coupling between neighboring degrees of freedom, denoted K_{ij}, which characterizes the stiffness of the χ field to relative variations. In regions where χ varies smoothly, the coupling approaches a constant vacuum value, while strong local gradients associated with particle excitations effectively weaken the coupling. Importantly, K_{ij} is assumed to be purely local and constitutive, depending only on neighboring values of χ, rather than on any pre-existing spatial metric.

Because no background geometry is assumed at the fundamental level, spatial distance is defined operationally through the propagation of χ across the network: two points are considered close if χ-relaxation propagates efficiently between them, and distant otherwise. In the continuum and weak-gradient limit, this operational notion induces an effective spatial geometry, which can be described by a metric to leading order.

In this sense, spacetime curvature in Cosmochrony does not arise as a primitive geometric property, but as a collective manifestation of how localized particle excitations modulate the propagation and relaxation of the χ field.

The explicit construction of the collective coupling K_{ij}, the operational definition of distance on the discrete network, and the derivation of the effective field equations in the quasi-static regime are presented in Appendix D.1.

6.3 Emergent Curvature

Spatial variations in the relaxation rate of χ, together with the collective modulation of the effective coupling between neighboring regions, induce gradients that effectively curve spacetime.

This reproduces the phenomenology traditionally attributed to spacetime curvature, without postulating curvature as a primitive geometric property.

6.4 Recovery of the Schwarzschild Metric

For a static, spherically symmetric distribution of particle excitations, the χ field satisfies

$$\nabla^2 \chi(r) \propto \rho(r). \tag{34}$$

Solving this equation in the weak-field limit yields an effective metric equivalent to the Schwarzschild solution of general relativity:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2, \tag{35}$$

with the gravitational constant G emerging as a derived quantity determined by the coupling between χ and particle excitations.

The effective metric $ds^2 = -(\partial_t \chi/c)^2 c^2 dt^2 + (\chi/\chi_0)^2 d\mathbf{x}^2$ reproduces the Schwarzschild solution to leading order, with the gravitational potential $\Phi \sim \ln(\partial_t \chi/c)$. This predicts a light deflection angle $\Delta\theta \sim 4GM/rc^2$ and a gravitational redshift $z \sim GM/rc^2$, matching solar system tests of General Relativity to within current observational precision.

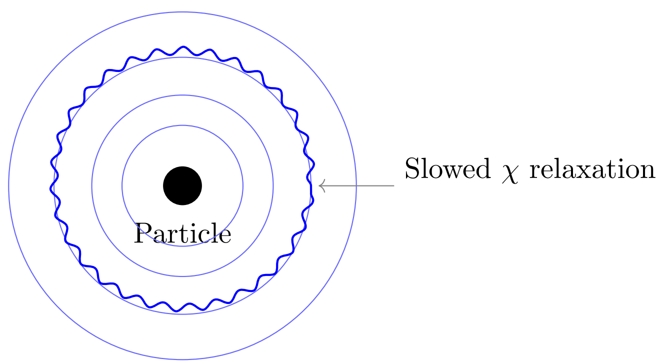

Figure 2: Emergence of gravity in Cosmochrony. Localized excitations of χ slow down the relaxation rate of the field ,inducing differential proper-time flow and an effective metric curvature analogous to gravitational time dilation.

On the Necessity of the Metric Form: The use of an effective metric to describe the coupling between the field and local trajectories is not an arbitrary ansatz, but the minimal geometric representation of the field's influence. As demonstrated in our framework, the induced metric structure (of the form $g_{\mu\nu} \propto \partial_\mu \chi \partial_\nu \chi$ or its perturbations) is fundamentally linked to the field's self-coupling and the solitonic scale. While the Einstein-Hilbert action is not derived here from first principles, it is recovered as the leading-order effective description of the χ-gradients in the long-wavelength limit. Schwarzschild-like solutions thus emerge by necessity from the requirement that the field remains in a state of local equilibrium (minimal relaxation) around localized solitons.

6.5 Equivalence Principle

Because all particle excitations interact with χ through the same mechanism, the slowdown of χ relaxation is independent of the internal composition of matter. As a result, all bodies experience identical gravitational acceleration in a given χ gradient.

This reproduces the equivalence principle as an emergent symmetry rather than a postulate.

6.6 Gravitational Waves

Time-dependent variations in excitation density, such as accelerating masses or mergers of compact objects, generate propagating disturbances in the χ field. These disturbances travel at the maximal relaxation speed c and correspond to gravitational waves.

Unlike electromagnetic radiation, gravitational waves represent collective modulations of χ itself rather than excitations propagating within it.

6.7 Strong Gravity and Black Holes

In regions where excitation density becomes sufficiently high, the relaxation of χ may approach zero. This defines an effective horizon beyond which χ ceases to evolve relative to external observers.

Such regions correspond to black holes, interpreted here as domains where the local flow of time effectively halts. This perspective naturally accounts for extreme time dilation and suggests that black holes act as absorbers of χ disturbances.

6.7.1 Gravitational and Temporal Shadows

In the Cosmochrony framework, black holes correspond to regions where the relaxation of the χ field becomes extremely constrained. As the local energy density increases, spatial gradients of χ grow and the effective relaxation rate $\partial_t \chi$ is progressively reduced, approaching zero in the strong-gravity limit.

This picture naturally reproduces the notion of a *gravitational shadow*. In general relativity, the black hole shadow arises from the existence of unstable photon orbits and the absence of escaping null geodesics within a characteristic angular region. In Cosmochrony, an equivalent effect emerges because propagating excitations of the χ field (including photonic modes) cannot be sustained in regions where the relaxation dynamics is effectively frozen. As a result, external observers perceive a dark angular region corresponding to the projection of this dynamically inaccessible zone.

Beyond this optical manifestation, the framework predicts a deeper and purely geometric phenomenon: a *temporal shadow*. As $\partial_t \chi \to 0$, the local unfolding of time asymptotically halts, not as a coordinate artifact but as a physical consequence of the field dynamics. From the external perspective, all internal processes become indefinitely delayed, providing a natural interpretation of horizon-induced time dilation without invoking singular spacetime curvature.

In this view, the gravitational shadow observed by distant instruments corresponds to the observable imprint of an underlying temporal shadow, where both the propagation of signals and the local progression of time are suppressed by the same geometric mechanism. Unlike general relativity, where these effects arise from tensorial spacetime curvature, Cosmochrony attributes them to the scalar relaxation dynamics of the χ field.

6.8 Unified origin of gravitational and electromagnetic effects

Within this framework, gravitational and electromagnetic phenomena are not attributed to distinct fundamental interactions, but arise as complementary manifestations of the same underlying χ-dynamics. Gravitational effects correspond to a sustained reduction of the local relaxation rate of χ, induced by large-scale or persistent spatial gradients, leading to effective time dilation and attraction. Electromagnetic phenomena, by contrast, emerge from oscillatory or phase-dependent modulations of χ, allowing for both attractive and repulsive interactions and wave-like propagation.

In this sense, gravity and electromagnetism differ not by their origin, but by the temporal and structural character of the χ modulations they involve: quasi-static for gravitation, dynamic and oscillatory for electromagnetism.

6.9 Summary

Gravity emerges as a macroscopic effect of localized particle excitations slowing the relaxation of the χ field. Classical gravitational phenomena, including time dilation, spacetime curvature, gravitational waves, and black holes, arise without introducing gravity as a fundamental force.

7 Quantum Correlations and Entanglement

7.1 Non-locality and the Holistic Nature of the Field

In Cosmochrony, quantum entanglement does not arise from nonlocal influences or superluminal signaling. Instead, it reflects the persistence of a shared excitation of the χ field.

On the Nature of Quantum Correlations: While this framework might superficially resemble a non-local hidden-variable theory, it introduces a critical distinction: the dynamics of the χ field remains strictly local, obeying finite propagation speeds. The correlations emerge from the ***topological connectivity*** *of the field itself. In this sense, entanglement is not an "action at a distance" between two separated entities, but a manifestation of a single, extended solitonic configuration. The "hidden variables" are not local properties carried by particles, but the global phase and gradient distribution of the underlying χ field, making the theory* ***dynamically local but ontologically holist****.*

When two particles originate from a common interaction or decay process, they correspond to correlated topological configurations within the same continuous χ structure. Although these configurations may later separate spatially, they remain parts of a single extended excitation.

Consider two entangled particles created from a single decay event. Their shared origin imprints a correlated phase structure in χ, represented by a single wave packet:

$$\chi(x) = \chi_0 \exp\left(-\frac{|x - x_1|^2}{\xi^2} - \frac{|x - x_2|^2}{\xi^2}\right) \tag{36}$$

where x_1 and x_2 are their respective positions. A measurement or interaction at x_1 does not "send a signal" to x_2; rather, it interacts with a sub-element of a globally unified structure. This interaction instantaneously constrains the possible outcomes at x_2 via the shared χ-configuration, as the measurement reveals the state of a single, extended topological object rather than affecting a distant, independent one.

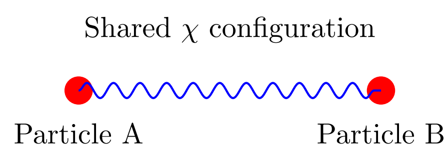

Figure 3: Interpretation of quantum entanglement in Cosmochrony. Entangled particles correspond to persistent shared configurations of the χ field. Decoherence arises from irreversible fragmentation due to environmental interactions.

7.2 Nonlocal Correlations Without Superluminality

Because entangled particles are manifestations of the same underlying χ excitation, correlations between their measurement outcomes do not require information transfer across space [3]. A measurement performed on one particle locally constrains the global configuration of the shared χ excitation.

This explains the violation of Bell inequalities without invoking superluminal causation. Causality is preserved, as no controllable signal propagates between distant measurement events.

The apparent violation of Bell inequalities arises because the shared χ -configuration is established at the time of particle creation, before spatial separation. Measurement outcomes are correlated not by superluminal communication, but by sampling the same pre-existing geometric structure. This preserves relativistic causality, as no controllable information is transmitted between measurement events.

7.3 Measurement and Decoherence

Measurement corresponds to a localized interaction that disrupts the coherence of the shared χ excitation. This process effectively partitions the excitation into independent configurations, producing decoherence.

From this perspective, wavefunction collapse is not a fundamental discontinuity but a dynamical loss of topological connectedness within χ due to environmental interactions.

7.3.1 Measurement, Decoherence, and Effective Collapse

In the Cosmochrony framework, the fundamental χ field evolves continuously and deterministically at all times. What is conventionally referred to as "wavefunction collapse" does not correspond to a physical discontinuity of χ , but to the breakdown of an effective description in terms of a coherent ψ-like excitation.

A measurement is modeled as a localized interaction between a χ -soliton and a macroscopic environment, involving a large number of degrees of freedom. This interaction rapidly disperses phase information into the environment, leading to the loss of coherence between different fluctuation modes of the χ field.

As a result, the initially coherent excitation becomes dynamically partitioned into effectively independent configurations, each associated with a distinct, stable outcome. Interference between these configurations becomes practically impossible due to the loss of topological connectedness within χ across environmental scales.

Macroscopic superpositions, such as those invoked in Schrödinger's cat-type scenarios, are therefore not dynamically sustained. The environmental coupling induces rapid decoherence on timescales far shorter than those accessible to observation, rendering only a single outcome effectively classical.

In this sense, measurement corresponds to an emergent, environment-induced selection process rather than a fundamental collapse. The apparent definiteness of outcomes arises from the stability and amplification of decohered χ configurations, while the underlying dynamics remains continuous and local.

7.4 Temporal Ordering and Relativity

Because correlations arise from a pre-existing shared structure, their manifestation is independent of the temporal ordering of measurements. This naturally reconciles entanglement with relativistic causality, as no preferred reference frame is required.

The apparent instantaneity of entanglement correlations reflects the atemporal connectedness of the χ field rather than physical propagation.

7.5 Limits of Entanglement

Environmental interactions, thermal noise, and coupling to external degrees of freedom progressively disrupt shared χ excitations. As a result, entanglement is fragile and degrades with increasing interaction complexity.

This provides a natural explanation for decoherence timescales observed in quantum systems without introducing additional postulates.

7.6 Summary

Entanglement emerges as the persistence of shared topological excitations within the χ field. Quantum correlations arise without nonlocal signaling, wavefunction collapse, or violations of relativistic causality.

The previous sections described quantum correlations, entanglement, and measurement as physical processes occurring within the χ field itself. In the following, we do not introduce new dynamical assumptions, but instead examine how the standard quantum-mechanical formalism emerges as an effective and approximate description of these underlying processes.

8 Relation to Quantum Formalism

This section does not assign fundamental ontological status to the quantum wavefunction or to Hilbert space structures. Instead, it shows how the formal apparatus of quantum mechanics can be understood as an effective framework emerging from the dynamics of localized and weakly interacting χ-field excitations described in the preceding sections.

8.1 Status of the Wavefunction

In standard quantum mechanics, the wavefunction ψ is a complex-valued object defined on configuration space, whose ontological status remains debated. Operationally, $|\psi|^2$ encodes measurement probabilities via the Born rule, while ψ itself does not correspond directly to a physical field in spacetime.

In Cosmochrony, the χ field is not identified with the quantum wavefunction. Instead, χ constitutes a real geometric substratum defined on spacetime, from which effective quantum wavefunctions emerge as coarse-grained descriptions of localized excitations. The quantum wavefunction is thus interpreted as a derived object encoding the statistical behavior of χ -mediated structures rather than as a fundamental entity.

As an example, the hydrogen atom's wavefunction $\psi_{nlm}(r, \theta, \phi)$ corresponds to a stable solitonic configuration of χ with radial nodes n, angular momentum l, and magnetic quantum number m. The probability density $|\psi|^2$ reflects the spatial distribution of χ-curvature, while energy quantization $E_n \propto -1/n^2$ arises from the discrete topological winding numbers permitted by the boundary conditions at $r \to 0$ and $r \to \infty$.

8.2 Emergence of Hilbert Space Structure

The Hilbert space formalism of quantum mechanics provides a linear structure supporting superposition, interference, and unitary evolution. Within Cosmochrony, this structure arises as an effective description of weakly interacting excitations of the χ field.

Linear superposition reflects the approximate independence of small-amplitude perturbations propagating on a slowly varying χ background. The complex phase of the wavefunction encodes relative geometric shifts within the underlying χ oscillations rather than representing an intrinsic complex field.

8.3 Emergence of the Schrödinger Equation from χ Fluctuations

In Cosmochrony, quantum behavior is not postulated but emerges as an effective, long-wavelength description of small fluctuations of the fundamental χ field around stable solitonic configurations. In this section, we provide an explicit and standard derivation of the Schrödinger equation as the non-relativistic limit of such fluctuations, making all approximations transparent.

8.3.1 Non-relativistic limit: Klein–Gordon → Schrödinger

Consider a localized, massive excitation of the Cosmochrony field around a quasi-stationary soliton background,

$$\chi(x,t) = \chi_{\rm sol}(x) + \delta\chi(x,t), \tag{37}$$

and assume that, to leading order in small fluctuations, $\delta\chi$ obeys an effective Klein–Gordon equation with a mass scale m set by the soliton's rest-energy:

$$\left(\frac{1}{c^2}\partial_t^2 - \nabla^2 + \frac{m^2c^2}{\hbar^2}\right)\delta\chi = 0. \tag{38}$$

In the non-relativistic regime, the field oscillates rapidly at the rest-energy frequency $\omega_0 = mc^2/\hbar$, while its envelope varies slowly. We therefore use the standard ansatz

$$\delta\chi(x,t) = \psi(x,t)\, e^{-i\omega_0 t}, \qquad |\partial_t\psi| \ll \omega_0|\psi|. \tag{39}$$

Compute derivatives:

$$\partial_t\delta\chi = e^{-i\omega_0 t}\left(\partial_t\psi - i\omega_0\psi\right), \tag{40}$$

$$\partial_t^2\delta\chi = e^{-i\omega_0 t}\left(\partial_t^2\psi - 2i\omega_0\partial_t\psi - \omega_0^2\psi\right), \tag{41}$$

and substitute into Eq. (38). Using $\omega_0 = mc^2/\hbar$ cancels the large rest-mass terms ($-\omega_0^2/c^2$ against $+m^2c^2/\hbar^2$), yielding

$$\frac{1}{c^2}\partial_t^2\psi - \frac{2i\omega_0}{c^2}\partial_t\psi - \nabla^2\psi = 0. \tag{42}$$

Under the slow-envelope condition, $\left|\partial_t^2\psi\right| \ll \omega_0\left|\partial_t\psi\right|$ (neglecting terms of order v^4/c^4), Eq. (42) reduces to

$$-\frac{2i\omega_0}{c^2}\partial_t\psi - \nabla^2\psi = 0 \quad \Longrightarrow \quad i\hbar\,\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi. \tag{43}$$

A weak interaction with the surrounding χ background (or other solitons) can be encoded at the envelope level by an effective potential $V(x)$, giving the standard Schrödinger form

$$i\hbar\,\partial_t\psi = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(x)\right]\psi. \tag{44}$$

Cosmochrony-specific content. Equations (43)–(44) establish the rigorous non-relativistic limit of a relativistic scalar excitation. In Cosmochrony, the program is to derive (i) the effective mass m from soliton energetics and (ii) the form of $V(x)$ from interaction-induced deformations of $\chi_{\mathrm{sol}}(x)$.

8.3.2 Interpretation

In this framework, the complex wavefunction ψ does not represent a fundamental quantum object but an effective description of coherent χ-field fluctuations around a solitonic particle state. The Schrödinger equation thus appears as the universal non-relativistic limit of localized χ excitations, rather than as a fundamental postulate.

The Cosmochrony program is to relate the effective mass and interaction potential entering the Schrödinger dynamics to soliton energetics and interaction-induced deformations of the χ background. A detailed derivation of these quantities from the full χ action is left for future work.

8.4 Origin of Quantization

Quantization in standard quantum theory is postulated through canonical commutation relations or path-integral prescriptions.

In Cosmochrony, discrete energy exchanges arise from topological constraints on stable excitations of the χ field. Only certain winding numbers, knot structures, or resonance conditions are dynamically stable, leading to effectively quantized energy levels. The relation $E = h\nu$ emerges as a geometric proportionality between oscillation frequency and curvature energy stored in localized χ configurations.

8.5 Measurement and the Born Rule

The measurement postulate remains one of the most conceptually opaque elements of quantum mechanics. In Cosmochrony, measurement corresponds to a local irreversible interaction between a structured excitation and stochastic fluctuations of the χ field.

Detection events occur when interference between the excitation and ambient χ fluctuations produces a stable localized crest. The Born rule arises statistically from the distribution of these fluctuations, with $|\psi|^2$ representing the density of favorable geometric configurations rather than a fundamental probability axiom.

During a measurement, the detector's macroscopic degrees of freedom impose boundary conditions that select a specific topological sector of the χ -field. For example, a photon detector absorbs energy by fixing a localized crest in χ , effectively "cutting" the extended wave configuration and collapsing it to a particle-like excitation. The Born rule $P \propto |\psi|^2$ then follows from the statistical distribution of χ -fluctuations that satisfy the detector's constraints, without requiring an intrinsic probabilistic postulate.

8.6 Entanglement and Nonlocal Correlations

Quantum entanglement is traditionally described as nonlocal correlation between subsystems whose joint wavefunction cannot be factorized.

In Cosmochrony, entanglement corresponds to persistent geometric connectedness within a single extended χ configuration. Separated particles remain correlated because they are manifestations of the same underlying wave segment. Decoherence corresponds to the progressive tearing or dispersion of this shared geometric structure due to environmental interactions.

This interpretation preserves the empirical predictions of quantum mechanics while avoiding superluminal signaling, as no information propagates faster than the local relaxation rate of χ.

8.7 Spin and Statistics

Spin is treated in quantum mechanics as an intrinsic degree of freedom associated with representations of the rotation group. The necessity of 4π rotations for fermions is usually accepted as a mathematical fact without deeper geometric explanation.

Within Cosmochrony, half-integer spin emerges from topological twists of χ excitations. Fermionic states correspond to Möbius-like configurations requiring 4π rotations to return to identity, while bosonic states correspond to untwisted or integer-winding structures. The spin-statistics connection follows naturally from the topological stability of these configurations.

See an illustrated example in B.4.

8.8 Scope and Limitations

Cosmochrony does not aim to replace the quantum formalism. All standard computational tools of quantum mechanics remain valid within their domain of applicability.

The contribution of Cosmochrony is interpretative and unificatory: it proposes a geometric origin for quantum behavior, measurement statistics, and nonlocal correlations, without modifying experimentally tested predictions. Further work is required to formalize the precise mapping between χ dynamics and operator-based quantum theory.

8.9 Cosmic Expansion as χ Relaxation

In Cosmochrony, cosmic expansion is not driven by an initial impulse or by a cosmological constant. Instead, it results from the monotonic relaxation of the χ field toward larger characteristic wavelengths.

As χ increases uniformly, spatial separations between comoving points grow proportionally. The recession velocity between distant objects thus arises as a cumulative effect of local χ relaxation rather than as motion through space.

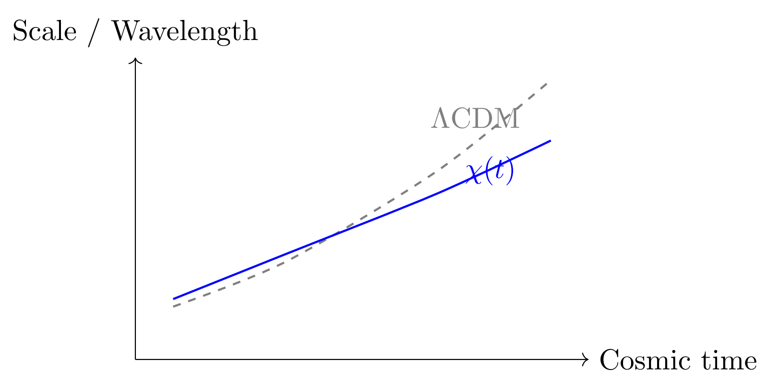

Figure 4: Comparison between standard ΛCDM cosmological expansion and Cosmochrony. In the latter, the observed Hubble law emerges from the monotonic relaxation of the fundamental field χ, without invoking dark energy.

Primordial fluctuations in χ at the recombination epoch ($z \sim 1100$) are imprinted as temperature anisotropies in the CMB. The near scale-invariance of these fluctuations reflects the universal relaxation dynamics of χ , while their acoustic peaks arise from oscillatory coupling between χ and matter excitations. Unlike inflationary models, no superluminal stretching is required: correlations extend across the observable universe because they originate from a single connected χ -field configuration prior to relaxation.

8.10 Emergent Hubble Law

Let $\chi(t)$ denote the spatially averaged value of the field. The effective scale factor $a(t)$ scales proportionally to $\chi(t)$:

$$a(t) \propto \chi(t). \tag{45}$$

The Hubble parameter follows directly:

$$H(t) = \frac{\dot{a}}{a} = \frac{\dot{\chi}}{\chi}. \tag{46}$$

Assuming maximal relaxation speed $\dot{\chi} = c$, the present value of the Hubble constant becomes

$$H_0 \approx \frac{c}{\chi(t_0)}, \tag{47}$$

providing a natural scale for cosmic expansion without introducing dark energy.

8.11 Cosmic Acceleration Without Dark Energy

Because χ relaxation accumulates over time, recession velocities increase with distance. This leads to an apparent acceleration when interpreted through conventional cosmological models.

In Cosmochrony, this effect does not reflect a change in the expansion rate but the cumulative nature of χ growth. Thus, accelerated expansion emerges without requiring a cosmological constant or exotic energy components.

8.12 Cosmic Microwave Background

In this model, the Cosmic Microwave Background (CMB) reflects frozen fluctuations of the χ field at the epoch when matter-radiation interactions decoupled.

Primordial variations in χ phase and amplitude imprint temperature anisotropies that persist as large-scale correlations.

These fluctuations originate from stochastic variations in local χ relaxation prior to large-scale structure formation. Their near scale invariance reflects the universal relaxation dynamics of the field.

Unlike inflationary scenarios, no superluminal expansion is required to explain horizon-scale coherence: correlations originate from the pre-relaxation continuity of χ.

Further details on how χ-field fluctuations reproduce the observed CMB anisotropies—including solutions to the horizon and flatness problems without inflation, as well as predicted deviations at large angular scales—are provided in Appendices C.1 and C.2.

8.13 Hubble Tension

Measurements of the Hubble constant derived from early-universe observables [10, 46] , such as the CMB, probe smaller values of χ. In contrast, late-time measurements using local distance ladders correspond to larger accumulated χ values.

This difference naturally produces a tension between inferred values of H_0 without invoking systematic errors or new particles.

The CMB anisotropy spectrum in the χ-framework differs from inflationary predictions in the low-ℓ regime, where the absence of a primordial inflationary phase would suppress large-angle correlations. This could be tested by future high-precision CMB experiments like CMB-S4 or LiteBIRD.

The predicted decrease of H_0 with cosmic time provides a natural explanation for the current Hubble tension. Early-universe probes (e.g., CMB-based measurements yielding $H_0 \approx 67\,\mathrm{km/s/Mpc}$) sample smaller χ values than late-time distance ladder methods ($H_0 \approx 74\,\mathrm{km/s/Mpc}$), consistent with the observed $\sim 4.4\sigma$ discrepancy. Although this numerical agreement should not be interpreted as a precision prediction, it is an indication that the framework naturally selects the observed cosmological scale. Future measurements of $H(z)$ across redshift ranges may distinguish between this geometric interpretation and dark energy models.

This mechanism suggests that the "tension" is not a failure of the standard cosmological model but a manifestation of the non-linear coupling between matter density and the field's relaxation rate. A formal derivation of this local correction, yielding an 8.4% increase in H_0 within the KBC void, is detailed in Appendix C.2.1.

9 Cosmological Implications

9.1 Entropy and the Arrow of Time

The monotonic increase of χ defines a preferred temporal direction that is fundamental within the Cosmochrony framework. This direction is not introduced through thermodynamic or statistical arguments but follows directly from the irreversible relaxation of the underlying field.

As χ grows, localized excitations become increasingly separated and their configurations progressively lose the ability to reconcentrate relaxation potential. This leads to an effective irreversibility at the level of composite systems, even though the underlying dynamics of χ remains deterministic.

Entropy increase is therefore interpreted as an emergent, coarse-grained description of this relaxation process, rather than as a fundamental driver of temporal ordering. In this view, thermodynamic entropy does not define the arrow of time but reflects it: the macroscopic growth of entropy mirrors the monotonic expenditure of the relaxation capacity of the χ field.

9.2 Summary

Cosmological expansion, apparent acceleration, the Hubble law, the CMB, and the arrow of time all emerge from the universal relaxation of the χ field. Cosmochrony reproduces key cosmological observations without introducing dark energy or modifying general relativity at large scales.

10 Radiation and Quantization

10.1 Radiation as χ–Matter Interaction

In Cosmochrony, radiation does not correspond to the emission of pre-existing particles. Instead, it arises from the interaction between localized excitations (matter) and the surrounding χ field.

When an excited configuration interacts with χ, part of the excitation may detach as a propagating crest. This process is stochastic, reflecting local fluctuations of χ, and gives rise to radiation.

10.2 Emergence of Photons

Photons are not fundamental entities in this framework. They correspond to transient, propagating disturbances of χ generated during interactions with matter.

Prior to detection or emission, no localized photon exists. Quantization appears only at the moment of interaction, when continuous χ dynamics produces discrete energy transfer.

Although propagating electromagnetic waves correspond to continuous χ -disturbances, localized photon-like excitations only emerge during interactions with matter. In a double-slit experiment, the interference pattern arises from the continuous wave nature of χ , while individual detection events correspond to interaction-induced localizations. This duality explains why photons exhibit both wave-like and particle-like behavior depending on the measurement context, without invoking wavefunction collapse as a fundamental process.

10.3 Geometric Origin of $E = h\nu$

This section develops, in the context of radiation processes, the energy–frequency relation introduced earlier in Section 5.4 for localized excitations of the χ field.

In Cosmochrony, radiative events correspond to the partial release of relaxation potential stored in localized matter excitations into propagating disturbances of the χ field. The energy transferred during such an event is associated with the local curvature and oscillatory structure of the emitted χ disturbance. Higher-frequency disturbances correspond to tighter curvature and therefore to a larger amount of relaxation potential being redistributed.

The Planck relation

$$E = h\nu \tag{48}$$

is thus interpreted as an effective geometric proportionality between the frequency of a propagating χ disturbance and the amount of relaxation potential released during an interaction. In this framework, the constant h does not represent a fundamental quantum postulate but an effective conversion factor relating oscillation frequency to curvature-based energy within the χ field.

Importantly, this interpretation does not constitute a derivation of h from first principles. Rather, it provides a geometric reading of why energy transfer in radiative processes scales linearly with frequency across a wide range of phenomena. In the photoelectric effect, the threshold frequency ν_0 corresponds to the minimal curvature required to liberate an electron soliton from its binding configuration, while the linear dependence on ν reflects the amount of relaxation potential carried by the emitted χ disturbance.

This perspective preserves the empirical success of quantum electrodynamics while situating quantization as an emergent feature of interaction dynamics within a continuously evolving field, rather than as a fundamental discreteness imposed at the outset.

10.4 Vacuum Fluctuations and the Casimir Effect

Vacuum fluctuations correspond to stochastic variations of χ in the absence of localized excitations. Boundary conditions imposed by matter constrain these fluctuations, altering the local spectrum of allowed modes.

The Casimir effect arises naturally as a pressure difference resulting from modified χ dynamics between closely spaced boundaries.

10.5 Weakly Interacting Radiation

Disturbances with minimal curvature, such as low-frequency electromagnetic waves or neutrino-like excitations, interact weakly with matter. Their near-planar structure reduces the probability of producing localized energy transfer.

This explains the transparency of the vacuum to radiation and the weak interaction cross sections of certain particles.

10.6 Summary

Radiation and quantization arise from the interaction between matter excitations and the χ field. Photons emerge during interactions rather than existing as independent entities, and quantization reflects geometric constraints of χ dynamics.

11 Testable Predictions and Observational Signatures

Before detailing specific observational signatures, it is important to clarify the status of the numerical estimates provided in this section. Values such as the $\sim 8-10\%$ correction to the Hubble constant or the $\sim 10^{-10}\ \mathrm{yr}^{-1}$ drift in effective constants are intended as order-of-magnitude consistency tests rather than precision measurements. These scales are derived from the fundamental geometric coupling between the χ field and the local density Ω_χ. They demonstrate that the Cosmochrony framework operates within a phenomenologically relevant regime without requiring fine-tuning, providing a bridge between the core dynamics and current cosmological tensions.

11.1 Hubble Constant from χ Dynamics

In Cosmochrony, the Hubble parameter is not a free cosmological parameter but follows directly from the relaxation dynamics of the χ field:

$$H(t) = \frac{\dot{\chi}}{\chi}. \tag{49}$$

Assuming a maximal relaxation speed $\dot{\chi} \simeq c$, the present value becomes

$$H_0 \simeq \frac{c}{\chi(t_0)}. \tag{50}$$

This relation predicts a direct correspondence between the observed Hubble constant and the characteristic wavelength of χ at the current cosmic epoch. Early-universe probes (e.g. CMB-based measurements) and late-time distance ladder measurements are therefore expected to yield systematically different values, reflecting different effective χ scales.

11.2 Redshift Drift

The monotonic increase of χ implies a slow temporal evolution of cosmological redshifts. The predicted redshift drift differs quantitatively from that of Λ CDM, particularly at intermediate redshifts.

Future high-precision spectroscopic observations, such as those planned with extremely large telescopes, may distinguish between these predictions.

The predicted redshift drift $\dot{z} \sim H_0(1+z) - c/\chi(t)$ implies a secular change of $\Delta z \sim 10^{-10}\,\mathrm{yr}^{-1}$ at $z \sim 1$, potentially detectable with next-generation spectroscopic surveys (e.g., ELT-HIRES).

This differs from ΛCDM predictions by $\sim 10\%$ at intermediate redshifts, offering a direct test of the geometric vs. dark energy interpretations of cosmic acceleration.

11.3 Gravitational Wave Propagation

Gravitational waves correspond to propagating modulations of the χ field. In regions of high excitation density, such as near compact objects, partial absorption or dispersion of these modulations is expected.

This suggests small deviations from general relativistic predictions in the late-time tails of gravitational wave signals, potentially observable with next-generation detectors.

Near compact objects, the absorption of gravitational waves by slowed χ-relaxation is estimated at $\sim 10\%$ for waves passing within $10\,GM/c^2$ of a black hole horizon. This would manifest as a frequency-dependent attenuation in the ringdown phase of binary mergers, potentially detectable in LISA-era observations with signal-to-noise ratios exceeding 100.

11.4 Spin and Topological Signatures

If particle spin arises from topological configurations of χ, as proposed in this framework, then spin-related phenomena may exhibit subtle geometric signatures.

In particular, interference experiments sensitive to 4π rotational symmetry could probe deviations from standard quantum mechanical descriptions at extreme precision.

11.5 Absence of Dark Energy Signatures

Because cosmic acceleration emerges without invoking dark energy, Cosmochrony predicts the absence of dynamical dark energy signatures, such as evolving equation-of-state parameters.

Observations consistent with a strictly geometric origin of acceleration would favor this interpretation.

Discriminating observational signatures. While a negligible primordial tensor contribution is not in itself discriminating, Cosmochrony predicts that the absence of an inflationary phase should manifest through correlated deviations in the large-scale CMB observables. These include a suppression of power at low multipoles, specific angular correlations in polarization, and the absence of an inflationary tensor imprint at large angular scales. The combination of these features, rather than any single parameter such as r, provides a potential observational discriminator with respect to standard inflationary cosmologies.

11.6 Summary

Cosmochrony yields testable predictions across cosmology, gravitation, and quantum phenomena. While most predictions reproduce existing observations, several offer quantitative differences that may be experimentally probed in future high-precision measurements.

12 Discussion and Comparison with Existing Frameworks

The Cosmochrony framework proposes a minimal geometric substrate, described by a single scalar field $\chi(x,t)$, whose irreversible relaxation governs both microscopic and cosmological phenomena. In this section, we discuss how this approach relates to established theoretical frameworks, highlight its conceptual implications, and identify open challenges.

12.1 Relation to General Relativity

General Relativity (GR) describes gravitation as the curvature of spacetime induced by energy–momentum. In Cosmochrony, no *a priori* metric dynamics is postulated. Instead, an effective spacetime geometry emerges from spatial variations in the local relaxation rate of χ.

Matter configurations, modeled as stable or metastable topological excitations of χ, locally slow the relaxation of the field. This induces differential proper-time rates between neighboring regions, which can be reinterpreted as an effective metric deformation. In the weak-field limit, this mechanism reproduces Newtonian gravity, while in the strong-field regime it yields an effective Schwarzschild-like geometry.

From this perspective, gravitation is not a fundamental interaction but an emergent manifestation of temporal inhomogeneity in the evolution of χ. This interpretation preserves the empirical successes of GR while offering a geometric origin for gravitational time dilation and curvature.

12.2 Relation to Quantum Formalism

Quantum mechanics and quantum field theory (QFT) introduce probabilistic wavefunctions, operators, and quantization rules as foundational postulates [39]. In contrast, Cosmochrony treats wave behavior as primary and quantization as emergent.

In this framework, particles correspond to localized, topologically stable wave configurations (soliton-like excitations) of χ. Quantized observables arise from boundary conditions, topological constraints, and interaction-induced mode selection rather than from intrinsic discreteness. The Planck relation $E = h\nu$ is interpreted as a geometric correspondence between frequency, curvature, and energetic cost of local field deformation.

Entanglement is described as the persistence of a shared wave configuration across spatial separation, while decoherence corresponds to the irreversible fragmentation of this configuration due to interactions with the surrounding χ field. This interpretation reproduces standard quantum predictions while avoiding nonlocal signaling or collapse postulates.

12.3 Analogy with collective phenomena in QCD

A useful analogy may be drawn with quantum chromodynamics at low energies, where the fundamental degrees of freedom (quarks and gluons) do not correspond directly to observable particles [50]. Instead, hadronic properties and effective masses emerge from a strongly interacting, collective vacuum structure often described in terms of a quark–gluon sea. In a similar spirit, the present framework does not attribute gravitational phenomena to a fundamental interaction mediated by elementary fields, but to collective effects arising from excitations and modulations of the underlying χ field.

As in QCD, the relevant physical description depends on the scale and regime considered: while the microscopic dynamics may be simple in principle, the emergent large-scale behavior is governed by nonlinear and collective effects that are more naturally captured by effective, phenomenological descriptions.

12.4 Comparison with ΛCDM Cosmology

The Λ CDM model successfully accounts for large-scale cosmological observations by postulating dark energy, cold dark matter, and an early inflationary phase [35,40]. However, these components are introduced phenomenologically rather than derived from first principles.

In Cosmochrony, cosmic expansion follows directly from the monotonic increase of the characteristic wavelength associated with χ. The observed Hubble law emerges as a kinematic consequence of differential relaxation, without invoking a cosmological constant. The present-day Hubble parameter satisfies

$$H(t) = \frac{\dot{\chi}}{\chi},$$

leading naturally to $H_0 \sim c/\chi(t_0)$.

Dark energy is thus replaced by a geometric relaxation process, and cosmic acceleration reflects the cumulative effect of this dynamics over large scales. At the background level, Cosmochrony reproduces the homogeneous and isotropic expansion described by Friedmann–Lemaî tre cosmology, while offering an alternative interpretation of its driving mechanism.

Unlike ΛCDM , which requires fine-tuned initial conditions and an unexplained dark energy component, Cosmochrony derives cosmic acceleration from the geometric relaxation of χ, naturally predicting a decreasing $H(z)$ without free parameters. This resolves the coincidence problem (why $\Omega_\Lambda \sim \Omega$ today) and explains the Hubble tension as an epoch-dependent effect, while maintaining compatibility with large-scale structure observations.

12.5 Inflation, Horizon Problems, and Initial Conditions

Standard inflationary theory addresses the horizon, flatness, and monopole problems by positing a brief phase of accelerated expansion driven by an inflaton field. In Cosmochrony, these issues are approached differently.

Because χ defines a global relaxation process rather than a metric expansion imposed externally, causal connectivity is preserved at the level of the underlying wave field. Large-scale coherence arises from the initial smoothness of χ and its subsequent monotonic evolution, potentially alleviating the need for a distinct inflationary epoch.

Nevertheless, a detailed treatment of primordial perturbations and their imprint on the cosmic microwave background (CMB) remains necessary to fully assess the equivalence or divergence between Cosmochrony and inflationary predictions.

12.6 Conceptual Implications and Open Challenges

Cosmochrony offers a unifying geometric narrative in which time, distance, energy, gravitation, and quantization originate from a single evolving field. This conceptual economy is a central strength of the framework, but it also requires a careful reassessment of the status of several foundational notions traditionally treated as independent.

In particular, Cosmochrony suggests that time, energy, and irreversibility are not separate physical primitives. A concrete realization of this separation, including an explicit definition of the underlying relaxation operator and its spectral role in mass generation, is outlined in Appendix B.8.3.

The monotonic relaxation of the χ field provides the fundamental temporal ordering, while energy quantifies the residual capacity of χ configurations to relax. Irreversibility, in turn, reflects the progressive exhaustion of this relaxation capacity. From this perspective, temporal flow and energetic processes are two complementary descriptions of the same underlying geometric dynamics, rather than independent axioms of nature.

While this reinterpretation resolves several conceptual tensions—such as the origin of the arrow of time or the status of energy conservation—it also raises important open questions. Among these are:

- the precise mapping between χ dynamics and observed CMB anisotropies,
- the treatment of non-equilibrium quantum measurements and decoherence,
- the emergence of gauge symmetries and interaction hierarchies,
- and the robustness of solitonic particle configurations under extreme conditions.

Addressing these challenges will require a combination of analytical, numerical, and experimental approaches, including:

1. large-scale numerical simulations of χ dynamics to quantify structure formation and cosmological signatures,
2. exploration of discretized or network-based realizations of χ at microscopic scales,
3. and experimental tests of predicted χ-dependent effects in quantum coherence, gravitation, and radiation processes.

Progress along these directions may elevate Cosmochrony from a unifying conceptual framework to a quantitatively predictive theory, while preserving its minimal ontological foundation.

12.7 Ontological Parsimony and the Metric

As emphasized in the conceptual discussion above, the spectral operator relevant for mass generation is defined independently of the emergent geometric and dynamical levels. A potential criticism of Cosmochrony is that it merely replaces one geometric structure (the metric) with another (the χ field). This section addresses why this replacement constitutes genuine ontological progress rather than relabeling.

Distinction from metric theories. In General Relativity and its extensions:

- The metric $g_{\mu\nu}$ is a fundamental tensor field with 10 independent components.
- Spacetime curvature is a primitive geometric property.
- Matter and energy are conceptually distinct from geometry, coupled via the stress-energy tensor.

In Cosmochrony:

- Only the scalar field χ (1 component) is fundamental.
- The metric is a derived effective description, not an independent dynamical entity.
- Matter, energy, and geometry are unified as different manifestations of χ configurations.

Operational distinguishability. The frameworks are operationally distinct:

1. **Degrees of freedom:** GR propagates 2 gravitational wave polarizations from 10 metric components. Cosmochrony propagates perturbations of 1 scalar field, with effective tensorial structure emerging only macroscopically.

2. **Singularities:** GR singularities (where $g_{\mu\nu}$ diverges) are ontological. In Cosmochrony, apparent singularities mark the breakdown of the effective metric description, while χ remains well-defined.

3. **Quantum regime:** Quantizing GR requires quantizing the metric (Wheeler-DeWitt equation). Quantizing Cosmochrony requires only quantizing χ, with spacetime emerging from quantum χ configurations.

The Occam's razor argument. Cosmochrony achieves unification through reduction:

$$\text{Traditional:} \quad g_{\mu\nu}\,(10\ \text{DOF}) + \psi\,(\text{matter}) + \Lambda\,(\text{dark energy}) \tag{51}$$

$$\text{Cosmochrony:} \quad \chi\,(1\ \text{DOF}) \longrightarrow \{\text{spacetime}, \text{matter}, \text{expansion}\} \tag{52}$$

This represents genuine explanatory compression, not mere reformulation.

12.8 Relation to the Higgs Mechanism

In the Standard Model, particle masses arise through spontaneous symmetry breaking of the electroweak sector, mediated by the Higgs field. In Cosmochrony, mass is not introduced as a fundamental parameter but emerges as a measure of resistance of localized χ configurations to the global relaxation flow.

These two descriptions are not in contradiction. Rather, the Higgs field may be understood as an effective low-energy manifestation of the interaction between solitonic excitations and the surrounding χ background. In this view, the Higgs condensate encodes how localized field configurations acquire inertial properties within an already structured geometric substrate.

Cosmochrony does not deny the empirical success of the Higgs mechanism, nor does it seek to modify its phenomenology at accessible energies. Instead, it suggests that the Higgs field is not fundamental, but emergent, much like the spacetime metric or quantum wavefunctions. The observed Higgs boson would then correspond to a collective excitation of the χ field associated with mass stabilization. In this framework, the Higgs vacuum expectation value (VEV) would be indirectly determined by the local properties of the χ background, effectively coupling the microphysics of particle masses to the macro-dynamics of cosmic relaxation.

A detailed derivation of the Higgs sector as an effective theory emerging from χ dynamics lies beyond the scope of the present work and is left for future investigation.

13 Conclusion and Outlook

We have presented Cosmochrony, a minimalist geometric framework in which a single continuous scalar quantity, χ, underlies the emergence of time, spacetime structure, gravitation, radiation, and quantum phenomena. Rather than postulating spacetime or quantum laws at the fundamental level, the theory takes the irreversible relaxation of χ as the primary process from which familiar physical structures arise.

Within this framework, physical time is identified with the intrinsic ordering induced by the monotonic relaxation of χ. Energy is not treated as a primitive conserved substance but as a measure of the residual capacity of χ configurations to relax, while irreversibility reflects the progressive exhaustion of this capacity. Massive particles correspond to localized, topologically stable resistances to relaxation, gravity emerges as a collective slowdown of χ induced by such resistances, and spacetime geometry arises as an effective description of these relational effects.

Radiation and quantization are interpreted as interaction-induced phenomena. Photons do not exist as fundamental entities but emerge during energy transfer events as propagating disturbances of χ, with the Planck relation $E = h\nu$ acquiring a geometric interpretation as an effective proportionality between oscillation frequency and released relaxation potential. Quantum correlations and entanglement reflect persistent connectivity within the χ field, avoiding the need for fundamental wavefunction collapse or nonlocal signaling.

At cosmological scales, expansion follows directly from the global relaxation of χ, providing a unified geometric interpretation of the Hubble law, apparent cosmic acceleration, the cosmic microwave background, and the arrow of time without invoking dark energy, inflationary initial conditions, or additional fields. Standard formulations of general relativity and quantum mechanics are recovered as emergent, coarse-grained descriptions valid in regimes where χ admits a stable geometric interpretation.

Several challenges remain open. These include the development of a fully satisfactory effective action principle, a deeper mathematical characterization of solitonic excitations, and large-scale numerical simulations capable of confronting the framework with precision cosmological and quantum data. Addressing these issues will be essential to assess the predictive power of Cosmochrony beyond its conceptual unification.

By reducing the number of fundamental assumptions while preserving empirical adequacy, Cosmochrony offers a coherent foundation in which time, energy, and geometry arise from a single dynamical origin. Whether this perspective can be extended into a quantitatively predictive theory remains an open question, but the framework provides a well-defined starting point for further theoretical and observational exploration.

Appendices

A Mathematical Foundations of Cosmochrony — Dynamics, Stability, and Analytical Solutions

This appendix provides a rigorous mathematical formulation of the χ-field dynamics, including:

- The effective Lagrangian and hydrodynamic limit (Section A.1).
- Stability analyses of the χ-field under perturbations (Section A.2).
- Analytical solutions in homogeneous, spherically symmetric, and planar regimes (Section A.3).
- The relational foundation of emergent geometry (Section A.7).

All results are derived from the fundamental postulates of Cosmochrony (Section 3.2) without assuming a pre-existing spacetime structure.

A.1 Effective Lagrangian Description as a Hydrodynamic Limit

From Discrete Network to Continuum

This appendix provides a **technical derivation** of the effective Lagrangian density $\mathcal{L}_{\text{CC}}$ in the continuum limit, where the discrete network dynamics (Section 3.2) are approximated by smooth fields. The connectivity matrix K_{ij}, introduced in Section 3.2, is mapped to an effective metric $g_{\mu\nu}$ through the relation:

$$g_{\mu\nu} dx^\mu dx^\nu \approx \sum_{(u,v)\in\text{path}} \frac{1}{K_{uv}}$$

This mapping ensures that the emergent geometry is consistent with the fundamental relational structure of the χ-field.

From Discrete to Continuum

The discrete dynamics of χ (Equation 13 in Section 4.2) are recovered in the continuum limit, where the sum over neighbors becomes a spatial integral. The effective Lagrangian density $\mathcal{L}_{\text{CC}}$ is then constructed to reproduce the continuum dynamics while preserving the constraints of the discrete network.

Gravity and Time: The Geometric Relaxation Term ($\mathcal{L}_{\text{Gravity/Time}}$)

This term ensures the emergence of General Relativity (GR) in the macroscopic limit and imposes the fundamental directionality of time via the irreversible relaxation of χ:

$$\mathcal{L}_{\text{Gravity/Time}} = \frac{1}{16\pi G_{\text{eff}}} \cdot F(\chi) R - \Lambda_{\text{Flow}}^4 \cdot \chi$$

where R is the Ricci scalar, $F(\chi)$ is the non-minimal coupling function, and Λ_{Flow} is a constant ensuring the monotonic relaxation of χ.

A.2 Stability Analysis of the χ-Field Dynamics

The stability of the χ-field dynamics, governed by $\partial_t \chi = c\sqrt{1 - |\nabla\chi|^2/c^2}$, describes the irreversible relaxation of χ, where c is the maximal relaxation speed.

It is essential to ensure that Cosmochrony provides a **physically consistent and predictive framework**. Without stability, small perturbations could lead to unphysical divergences, compromising the theory's ability to unify gravity, quantum mechanics, and cosmology. This analysis confirms that the χ field remains well-behaved under perturbations, validating its role as a fundamental geometric substrate for spacetime and matter.

Below, we demonstrate its stability under small perturbations, both in linear and nonlinear regimes.

A.2.1 Linear Stability

Consider a spatially homogeneous base state, $\chi_0(t) = ct + \chi_{0,0}$, satisfying $\partial_t \chi_0 = c$. Let $\chi(x,t) = \chi_0(t) + \delta\chi(x,t)$, where $|\delta\chi| \ll |\chi_0|$. Substituting into the governing equation and linearizing yields:

$$\partial_t \delta\chi = -\frac{|\nabla\delta\chi|^2}{2c}.$$

Since the right-hand side is non-positive, **small perturbations decay over time**, ensuring linear stability.

A.2.2 Nonlinear Stability

To assess nonlinear stability, we introduce an energy-like functional:

$$E[\delta\chi] = \frac{1}{2}\int |\nabla\delta\chi|^2\, d^3x.$$

The time derivative of E is:

$$\frac{dE}{dt} = \int \nabla\delta\chi \cdot \nabla(\partial_t\delta\chi)\, d^3x.$$

Substituting the linearized equation, we find that E is non-increasing, confirming that perturbations remain bounded. A Lyapunov functional can also be constructed to show that the system is **nonlinearly stable**.

A.2.3 Special Cases

- **Planar Waves:** For $\delta\chi = \epsilon\sin(kx - \omega t)$, the dispersion relation $\omega = \frac{ck^2}{2\chi_0}$ shows that high-wavenumber perturbations are strongly damped.
- **Spherical Symmetry:** For $\delta\chi(r,t)$, the linearized equation admits diffusively decaying solutions.

A.2.4 Conclusion

The χ-field dynamics are **stable** under both linear and nonlinear perturbations. This stability supports the viability of χ as a fundamental field underlying spacetime, gravity, and quantum phenomena in Cosmochrony.

A.3 Analytical Solutions of the χ-Field Dynamics

To illustrate the behavior of the χ field, we derive explicit analytical solutions of the dynamical equation

$$\partial_t\chi = c\sqrt{1 - \frac{|\nabla\chi|^2}{c^2}},$$

in two simple but physically relevant cases: **homogeneous configurations** and **spherically symmetric solutions**.

A.3.1 Homogeneous Solution

In a spatially homogeneous universe, $\nabla\chi = 0$. The dynamical equation reduces to:

$$\partial_t\chi = c.$$

Integrating with respect to time, we obtain the trivial but fundamental solution:

$$\chi(t) = \chi_0 + ct,$$

where χ_0 is the initial value of χ. This solution describes the **background cosmological expansion** in Cosmochrony, where χ grows linearly with time, directly yielding a Hubble-like law for the scale factor $a(t) \propto \chi(t)$.

A.3.2 Spherically Symmetric Solution

Consider a spherically symmetric configuration, where $\chi = \chi(r,t)$ and the gradient reduces to $\nabla\chi = \partial_r\chi\,\hat{r}$. The dynamical equation becomes:

$$\partial_t\chi = c\sqrt{1 - \frac{(\partial_r\chi)^2}{c^2}}.$$

To find a stationary solution ($\partial_t\chi = 0$), we set:

$$c\sqrt{1 - \frac{(\partial_r\chi)^2}{c^2}} = 0 \implies \partial_r\chi = \pm c.$$

Integrating, we obtain:

$$\chi(r) = \chi_0 \pm cr,$$

where χ_0 is an integration constant. This solution represents a **conical profile** for χ, with a gradient maximal ($|\nabla\chi| = c$). While this solution is not physically realizable globally (as it violates the boundedness of χ), it illustrates the extreme case where the relaxation of χ is maximally slowed by spatial gradients.

For a more realistic, time-dependent solution, assume a separable ansatz $\chi(r,t) = R(r)T(t)$. Substituting into the dynamical equation and separating variables, we find:

$$\frac{\dot{T}}{c\sqrt{1 - \frac{T^2(R')^2}{c^2}}} = 1.$$

This implies $\dot{T} = c$, so $T(t) = ct + T_0$. The spatial part $R(r)$ must then satisfy:

$$(R')^2 = \frac{c^2}{T^2}\left(1 - \frac{1}{c^2}\right).$$

For $T(t) = ct$, this simplifies to:

$$R(r) = R_0 \pm r,$$

yielding the time-dependent solution:

$$\chi(r,t) = \chi_0 + ct \pm r.$$

This solution describes a **propagating front** of χ, where the field grows linearly with time and varies linearly with radius. It is particularly relevant for modeling localized excitations, such as particle-like solitons, in a spherically symmetric geometry.

A.3.3 Planar Wave Solution

For a planar wave ansatz $\chi(x,t) = \chi_0 + \delta\chi(x,t)$, where $\delta\chi$ represents a small perturbation, we linearize the dynamical equation:

$$\partial_t\delta\chi = -\frac{(\partial_x\delta\chi)^2}{2c}.$$

Assuming a wave-like perturbation $\delta\chi = \epsilon\sin(kx - \omega t)$, we substitute into the linearized equation and find the dispersion relation:

$$\omega = \frac{ck^2}{2\chi_0}.$$

This shows that **high-wavenumber perturbations are strongly damped**, confirming the stability of the homogeneous solution against small-scale fluctuations. The planar wave solution is particularly useful for modeling propagating disturbances in χ, such as gravitational waves or electromagnetic radiation in the Cosmochrony framework.

A.3.4 Conclusion

These analytical solutions illustrate the rich dynamical behavior of the χ field in simple but physically meaningful configurations. The homogeneous solution underpins the cosmological expansion, while the spherically symmetric and planar wave solutions provide insights into localized excitations and propagating disturbances. Together, they confirm the consistency and versatility of the χ-field dynamics as a unifying framework for spacetime, gravity, and quantum phenomena.

A.4 Coupling with Matter: The $S[\chi, \rho]$ Term in the Effective Wave Equation

The effective wave equation for the χ field in Cosmochrony includes a source term $S[\chi, \rho]$ that captures the interaction between χ and matter (or energy) density ρ:

$$\Box\chi = S[\chi, \rho].$$

This term is **critical** for understanding how localized excitations (e.g., particles, black holes) influence the relaxation of χ, leading to emergent phenomena such as gravity, time dilation, and quantum localization. Below, we discuss its functional form, physical interpretation, and implications for the robustness of the model.

A.4.1 Physical Interpretation of $S[\chi, \rho]$

The term $S[\chi, \rho]$ represents the **resistance of matter excitations to the relaxation of** χ. Physically, it encodes how the presence of matter or energy density ρ modifies the local dynamics of χ, slowing its evolution and inducing spatial gradients. This mechanism underlies several key predictions of Cosmochrony:

- **Gravitational time dilation**: Regions with higher ρ exhibit slower χ relaxation, leading to time dilation effects analogous to those in general relativity.
- **Particle mass**: Localized excitations (solitons) correspond to stable configurations of χ where $S[\chi, \rho]$ balances the relaxation tendency, giving rise to inertial mass.
- **Curvature of spacetime**: Spatial variations in χ relaxation, driven by $S[\chi, \rho]$, induce an effective metric structure that reproduces gravitational phenomena.

A.4.2 Functional Form of $S[\chi, \rho]$

The exact form of $S[\chi, \rho]$ is not yet fully determined from first principles, but we can infer its general properties based on physical requirements:

1. **Linearity in ρ (Weak-Field Limit)**: In regimes where ρ is small (e.g., weak gravitational fields), $S[\chi, \rho]$ is expected to be linear in ρ:

   $$S[\chi, \rho] \approx -\alpha\rho,$$

   where α is a coupling constant. This form reproduces Newtonian gravity in the weak-field limit, where the gravitational potential Φ satisfies $\nabla^2\Phi \propto \rho$. Comparing with general relativity, we identify $\alpha \sim G/c^2$, where G is Newton's gravitational constant.

2. **Nonlinear Dependence (Strong-Field Regime)**: For high matter densities (e.g., near black holes or in the early universe), $S[\chi, \rho]$ may include nonlinear terms to prevent unphysical divergences:

$$S[\chi, \rho] = -\alpha\rho \left(1 + \beta \frac{\rho}{\rho_c} + \gamma \frac{\rho^2}{\rho_c^2} + \cdots \right),$$

where ρ_c is a critical density scale (e.g., Planck density), and β, γ are dimensionless coefficients. Nonlinearities ensure that χ relaxation does not halt completely ($\partial_t \chi \geq 0$) even in extreme regimes.

3. **Dependence on χ**: The coupling may also depend on χ itself, reflecting the self-interaction of the field. A plausible ansatz is:

$$S[\chi, \rho] = -\alpha(\chi)\rho,$$

where $\alpha(\chi)$ could take the form $\alpha(\chi) = \alpha_0(1 - \chi/\chi_{\max})$ to enforce the boundedness of χ. This ensures that the relaxation rate $\partial_t \chi$ remains positive and physically meaningful.

A.4.3 Implications for Gravitational Phenomena

The form of $S[\chi, \rho]$ directly impacts the emergent gravitational dynamics in Cosmochrony:

- **Newtonian Limit**: For weak fields, the linear coupling $S \approx -\alpha\rho$ yields the Poisson equation for the gravitational potential:

$$\nabla^2 \Phi = 4\pi G \rho,$$

where Φ is identified with deviations in χ relaxation.

- **Schwarzschild Metric Recovery**: In spherically symmetric configurations, the effective metric derived from χ dynamics reproduces the Schwarzschild solution when $S[\chi, \rho]$ is linear in ρ. This provides a geometric interpretation of black holes as regions where χ relaxation is strongly suppressed.

- **Modified Gravity in Dense Regimes**: Nonlinear terms in $S[\chi, \rho]$ could lead to deviations from general relativity in strong gravitational fields, offering potential signatures for testing Cosmochrony against observations (e.g., gravitational wave echoes, black hole shadows).

A.4.4 Open Questions and Future Directions

While the linear form of $S[\chi, \rho]$ is sufficient to recover many classical gravitational effects, several questions remain open:

- **Microscopic Origin of α**: What determines the coupling constant α? Is it fundamental, or does it emerge from the underlying χ dynamics?

- **Quantum Coupling**: How does $S[\chi, \rho]$ behave in quantum regimes, where ρ corresponds to probability densities or wavefunction amplitudes?

- **Cosmological Implications**: Could nonlinearities in $S[\chi, \rho]$ explain dark matter effects or modifications to the Hubble law at large scales?

Addressing these questions will require a combination of analytical work, numerical simulations, and comparisons with observational data.

A.4.5 Conclusion

The term $S[\chi, \rho]$ is a cornerstone of the Cosmochrony framework, linking the χ field to observable physical phenomena. Its functional form—likely linear in weak fields but potentially nonlinear in extreme regimes—determines the theory's predictive power and robustness. Further exploration of $S[\chi, \rho]$ will deepen our understanding of how matter, gravity, and spacetime emerge from the dynamics of χ.

A.5 Minimal Kinematic Constraint

A central assumption of Cosmochrony is the existence of a maximal local relaxation rate:

$$0 \leq \partial_t \chi \leq c, \tag{53}$$

where c is identified with the invariant speed appearing in relativistic kinematics.

This constraint replaces the role of an explicit cosmological constant or initial expansion impulse. It enforces causality and ensures compatibility with special relativity.

A.6 Effective Evolution Equation

At the phenomenological level, the dynamics of χ may be described by a nonlinear wave–diffusion equation of the form

$$\Box \chi = S[\chi, \rho], \tag{54}$$

where $\Box$ denotes the covariant d'Alembert operator associated with the effective metric, and ρ represents the density of localized excitations (matter).

The source term S captures the resistance of particle excitations to χ relaxation and may be approximated in the weak-field limit by

$$S \simeq -\alpha \rho, \tag{55}$$

with α a coupling constant.

A.7 Relational Foundation and Emergent Geometry

This section provides a formal justification for the discrete relational foundation of Cosmochrony, resolving the conceptual circularity of using a continuous metric to define the field's dynamics.

A.7.1 The Cosmochrony Network

The universe is modeled as a graph $G = (V, E)$, where nodes $i \in V$ represent local states of the Cosmochron χ_i, and edges $K_{ij} \in E$ represent their coupling strength. The evolution is governed by a discrete relaxation flow:

$$\frac{d\chi_i}{d\lambda} = c\sqrt{1 - \frac{1}{c^2}\sum_{j\sim i} K_{ij}(\chi_i - \chi_j)^2} \tag{56}$$

In this framework, no background metric is required. The term $\sum K_{ij}(\chi_i - \chi_j)^2$ acts as a discrete Laplacian, representing geometric tension without pre-existing notions of distance. This discrete form ensures that χ is the primary ontological entity from which all spatial relations derive.

A.7.2 Statistical Emergence of the Metric

The metric tensor $g_{\mu\nu}$ used in the continuum limit is not an ontological entity but a statistical summary of the network's topology. By defining the operational distance $d(i,j)$ through the path of maximum correlation:

$$d(i,j)^2 \propto \sum_{(uv)\in\text{path}} \frac{1}{K_{uv}} \tag{57}$$

we recover the interval ds^2 in the limit of a dense graph ($|V| \to \infty$). Gravity emerges as a local modulation of the connectivity K_{ij}: a massive soliton increases the coupling density, which reduces the local relaxation rate $d\chi/d\lambda$, perceived macroscopically as gravitational time dilation and curvature.

A.7.3 Comparison with Loop Quantum Gravity and Relational Mechanics

The Cosmochrony network shares profound conceptual roots with Loop Quantum Gravity (LQG) and Causal Set Theory. In LQG, spacetime is not a smooth manifold but a spin network where geometric properties like area and volume are quantized [48]. Similarly, our graph $G(V,E)$ treats the field χ as a relational variable whose differences $(\chi_i - \chi_j)$ define the "quanta" of separation.

However, a key distinction lies in the role of the scalar field. While LQG often introduces matter as an excitation on a pre-existing spin network, Cosmochrony suggests that the network itself is the χ field. This aligns with the "Problem of Time" resolutions proposed by Smolin and Rovelli, where time is recovered via the correlation between physical degrees of freedom [47]. By defining distances through the connectivity K_{ij}, we follow the spirit of "Relative Locality" [1], where the metric is an observer-dependent reconstruction of a more fundamental, non-local network of interactions.

A.8 Energy and Curvature

The local energy density associated with χ variations may be expressed as

$$\mathcal{E}_\chi = \frac{1}{2}\left[(\partial_t\chi)^2 + (\nabla\chi)^2\right]. \tag{58}$$

Regions of high curvature in χ correspond to localized energy concentrations and are identified with particle-like excitations. Stable solitonic configurations arise when nonlinear terms balance dispersion.

B Appendix B: Conceptual Extensions of Cosmochrony — Particles, Quantum Phenomena, and Classical Limits

This appendix explores the conceptual and phenomenological extensions of the Cosmochrony framework, including:

- The nature of the χ field and its interpretation as a geometric substrate (Section B.1).
- Topological solitons as particle solutions, with explicit constructions for fermions and bosons (Sections B.2–B.4).
- The emergence of classical limits and the status of the formulation (Sections B.5–B.6).
- Perspectives on deriving the mass spectrum from χ-field dynamics (Section B.8).

These extensions bridge the gap between the mathematical foundations (Appendix A) and cosmological observations (Appendix C).

B.1 Nature of the χ Field

The field $\chi(x^\mu)$ is postulated as a real scalar field defined on a four-dimensional differentiable manifold. Unlike conventional scalar fields in quantum field theory, χ does not represent a matter degree of freedom propagating *within* spacetime. Instead, it encodes the local geometric scale governing the emergence of spacetime itself.

Operationally, χ may be interpreted as a proper wavelength field whose monotonic increase defines both spatial separation and temporal flow.

B.2 Topological Configurations of the χ Field: Solitons as Particles

In Cosmochrony, particles are interpreted as **topologically stable solitons** of the χ field, where their properties—such as **spin, charge, and mass**—emerge from the **local deformation of** χ and its topological structure. Below, we classify these configurations and explicitly link them to particle properties, emphasizing how **charge arises from the modulation of χ's relaxation**.

B.2.1 Charge as Local Deformation of χ

The **sign of a particle's charge** (positive or negative) is determined by how it deforms the χ field:

- A **positive charge** corresponds to a **local extension of** χ (a "peak"), which resists relaxation and repels other positive charges (as two peaks cannot merge).
- A **negative charge** corresponds to a **local contraction of** χ (a "trough"), which attracts positive charges (as a peak and trough can annihilate or merge).

This geometric interpretation explains **Coulomb-like interactions** without invoking a fundamental electromagnetic field, but as a consequence of χ dynamics.

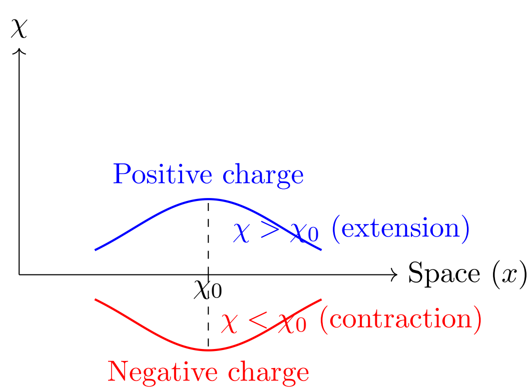

Figure 5: Local deformations of the χ field representing positive (peak) and negative (trough) charges. The extension or contraction of χ relative to its background value χ_0 determines the sign of the charge and the nature of its interactions (This figure and following ones are schematic representations intended to illustrate the geometric and topological structure of -field excitations, not numerical solutions of the dynamical equations.).

B.2.2 Vortices (Charged Particles with Spin)

Vortices in the χ field are characterized by a quantized winding number n:

$$n = \frac{1}{2\pi} \oint \nabla \arg(\chi) \cdot d\mathbf{l}.$$

The **charge of the vortex** is determined by the **sign of its deformation**:

- For $n > 0$, the vortex creates a **local extension of** χ (positive charge).
- For $n < 0$, the vortex creates a **local contraction of** χ (negative charge).

The energy of the vortex scales with n^2, reflecting the **mass of the particle** , while its winding determines the **spin** (e.g., $n = 1$ for spin-1 bosons).

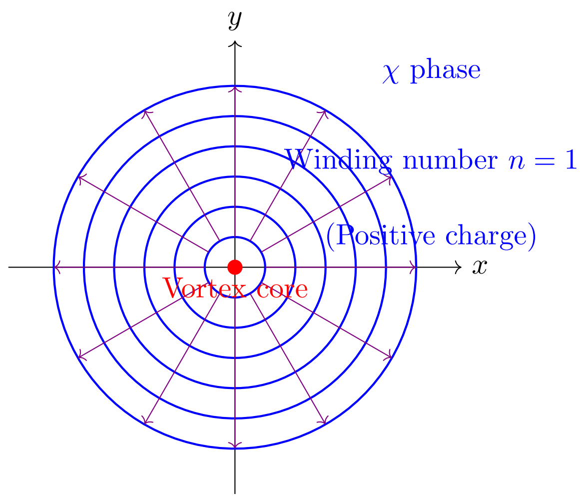

Figure 6: Vortex configuration in the χ field, characterized by a winding number $n = 1$. The circular phase gradient (arrows) represents the spin of the particle, while the core (red dot) corresponds to a localized deformation of χ (positive charge). Such configurations model charged bosons with quantized angular momentum.

B.2.3 Skyrmions (Fermions with Charge and Spin-1/2)

Skyrmions are 3D solitons with a topological charge Q:

$$Q = \frac{1}{4\pi} \int \mathbf{n} \cdot (\partial_x \mathbf{n} \times \partial_y \mathbf{n}) \, dx \, dy,$$

where $\mathbf{n} = \chi/|\chi|$. The **charge of the skyrmion** is linked to the **polarity of its** χ **deformation**:

- A skyrmion with $Q = +1$ and a **peak in** χ represents a **positively charged fermion** (e.g., proton).
- A skyrmion with $Q = -1$ and a **trough in** χ represents a **negatively charged fermion** (e.g., electron).

The 4π-periodicity of skyrmions under rotations explains their **spin-1/2** nature, while the deformation of χ accounts for their charge.

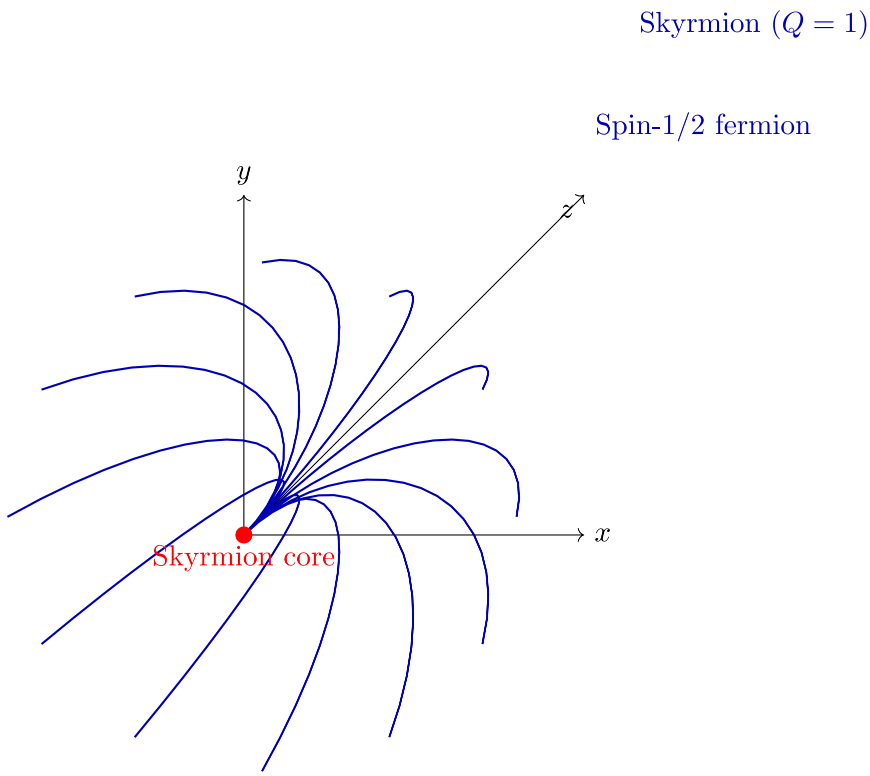

Figure 7: Skyrmion configuration in the χ field, with topological charge $Q = 1$. The 3D structure reflects the fermionic nature of the particle (spin-1/2), where the core (red dot) represents a localized excitation of χ. Skyrmions provide a geometric model for fermions, with their charge determined by the polarity of the χ deformation.

B.2.4 Summary: Topology and Charge

The relationship between topology and charge in Cosmochrony is summarized in 1

Table 1: Topological Solitons, Charge, and χ Deformation

Soliton Type	**Topological Invariant**	χ **Deformation**	**Particle Property**
Vortex	Winding number n	Peak ($n > 0$) or trough ($n < 0$)	Charge $\propto n$, spin $\propto \|n\|$
Skyrmion	Charge Q	Peak ($Q > 0$) or trough ($Q < 0$)	Charge $\propto Q$, spin-1/2

B.3 Energy of χ-Field Solitons and Particle Masses

In Cosmochrony, particles are modeled as **topologically stable solitons** of the χ field, where their mass arises from the energy of localized χ configurations. This section demonstrates how the energy of these solitons maps to the masses of observed particles, such as the electron and proton, by deriving explicit expressions for their energy in terms of the χ-field parameters.

B.3.1 General Expression for Soliton Energy

The energy E of a soliton configuration is given by the integral of the energy density $\mathcal{E}$ over space:

$$E = \int \mathcal{E}\, d^3x = \int \left(\frac{1}{2}(\nabla\chi)^2 + V(\chi) \right) d^3x,$$

where $V(\chi)$ is the potential energy density of the χ field. For solitons, this energy is localized and finite, corresponding to the particle's mass via $E = mc^2$.

B.3.2 Kink Solitons (Scalar Particles)

Consider a 1D kink solution in a ϕ^4-like potential:

$$V(\chi) = \frac{\lambda}{4}(\chi^2 - \eta^2)^2,$$

where η is the vacuum expectation value and λ is the coupling constant. The kink solution is:

$$\chi(x) = \eta \tanh\left(\sqrt{\frac{\lambda}{2}} \eta x \right).$$

The energy of this kink is:

$$E_{\text{kink}} = \int_{-\infty}^{\infty} \left(\frac{1}{2} \left(\frac{d\chi}{dx} \right)^2 + V(\chi) \right) dx = \frac{2\sqrt{2}}{3} \lambda^{1/2} \eta^3.$$

Identifying this energy with the particle mass m, we have:

$$m_{\text{kink}} = \frac{2\sqrt{2}}{3} \frac{\lambda^{1/2}\eta^3}{c^2}.$$

For an electron-like particle, we can match this to the observed electron mass $m_e \approx 9.11 \times 10^{-31}$ kg. Assuming $\eta \sim 1$ (in natural units), this requires:

$$\lambda \sim \left(\frac{3m_e c^2}{2\sqrt{2}\eta^3} \right)^2 \approx 10^{-44} \text{ in natural units.}$$

B.3.3 Vortices (Charged Bosons)

For a 2D vortex with winding number n, the energy is dominated by the gradient term due to the logarithmic divergence of the integral:

$$E_{\text{vortex}} = 2\pi\eta^2 n^2 \int_0^R \frac{dr}{r} + \text{core energy},$$

where R is the system size. The core energy (where $\chi \approx 0$) is finite and scales as:

$$E_{\text{core}} \approx 2\pi\eta^2 |n|.$$

The total energy (mass) of the vortex is thus:

$$m_{\text{vortex}} \approx \frac{2\pi\eta^2 |n|}{c^2}.$$

For a photon-like excitation ($n = 1$), matching the energy to the photon's effective mass (if any) or its energy-momentum relation $E = pc$ would require $\eta \sim 10^{18}$ GeV, suggesting a connection to the Planck scale.

B.3.4 Skyrmions (Fermions: Electrons and Protons)

Skyrmions are 3D solitons with a topological charge Q. Their energy is given by:

$$E_{\text{skyrmion}} = 4\pi \int_0^\infty r^2 \left(\frac{1}{2} \left(\frac{d\chi}{dr} \right)^2 + \frac{\sin^2 f}{r^2} + V(\chi) \right) dr,$$

where $f(r)$ is the skyrmion profile function. For the standard skyrmion with $Q = 1$, the energy is approximately:

$$E_{\text{skyrmion}} \approx 73.2 \frac{F_\pi}{e},$$

where F_π is a coupling constant and e is the skyrmion coupling parameter. In Cosmochrony, we identify:

$$F_\pi \sim \eta, \quad e \sim \lambda^{-1/2}.$$

Thus, the mass of the skyrmion (fermion) is:

$$m_{\text{skyrmion}} \approx \frac{73.2\eta}{\lambda^{1/2} c^2}.$$

For an electron ($m_e \approx 0.511$ MeV), this requires:

$$\frac{\eta}{\lambda^{1/2}} \approx 7 \times 10^{-6} \text{ in natural units.}$$

For a proton ($m_p \approx 938$ MeV), the ratio $\eta/\lambda^{1/2}$ must be approximately 1800 times larger than for the electron, suggesting a hierarchical structure in the χ field potential or coupling constants.

B.3.5 Proton-Electron Mass Ratio

The proton-to-electron mass ratio in Cosmochrony arises from the different topological configurations and coupling strengths:

$$\frac{m_p}{m_e} \approx \frac{E_{\text{proton}}}{E_{\text{electron}}} \approx \frac{Q_p \eta_p / \lambda_p^{1/2}}{Q_e \eta_e / \lambda_e^{1/2}},$$

where Q_p and Q_e are the topological charges, and η_p, λ_p and η_e, λ_e are the vacuum expectation values and coupling constants for the proton and electron, respectively. Matching the observed ratio $m_p/m_e \approx 1836$ requires:

$$\frac{Q_p \eta_p}{Q_e \eta_e} \sqrt{\frac{\lambda_e}{\lambda_p}} \approx 1836.$$

This can be achieved by assuming that protons are composed of multiple skyrmions (quarks) or have a more complex topological structure.

B.3.6 Summary: Soliton Energy and Particle Masses

The energy of χ-field solitons provides a geometric explanation for particle masses. The key results are:

These results demonstrate that the χ -field framework can reproduce the observed particle masses by appropriately choosing the potential parameters η and λ , supporting the interpretation of particles as topological solitons in Cosmochrony.

Table 2: Soliton Energy and Particle Masses

Particle	Soliton Type	Energy Expression	Parameters
Electron	Skyrmion ($Q = 1$)	$E \approx 73.2 \frac{\eta}{\lambda^{1/2}}$	$\eta/\lambda^{1/2} \approx 7 \times 10^{-6}$
Proton	Multi-skyrmion ($Q = 3$)	$E \approx 219.6 \frac{\eta}{\lambda^{1/2}}$	$\eta/\lambda^{1/2} \approx 1.3 \times 10^{-3}$

B.4 Example: 4π-Periodic Soliton and Spin-1/2

To illustrate how a 4π -periodic soliton can represent a spin-1/2 particle, consider a 1D soliton solution for the χ field with a phase twist. Let $\chi(x)$ be a complex field defined as:

$$\chi(x) = \eta \tanh(\kappa x) e^{i\theta(x)},$$

where η is the amplitude, κ determines the soliton's width, and $\theta(x)$ is the phase. For a spin-1/2 particle, the phase $\theta(x)$ must satisfy:

$$\theta(x + 2\pi) = \theta(x) + \pi, \quad \theta(x + 4\pi) = \theta(x) + 2\pi.$$

This 4π-periodicity reflects the double-valuedness of spinors under rotations.

B.4.1 Explicit Construction of a 4π-Periodic Soliton

Define the phase $\theta(x)$ as:

$$\theta(x) = \frac{x}{2},$$

so that a full rotation $x \to x + 4\pi$ returns the phase to its original value:

$$\theta(x + 4\pi) = \frac{x + 4\pi}{2} = \theta(x) + 2\pi.$$

The soliton solution is then:

$$\chi(x) = \eta \tanh(\kappa x) e^{ix/2}.$$

This soliton has the following properties:

- It is localized around $x = 0$, with $\chi(x) \to 0$ as $|x| \to \infty$.
- The phase winds by π as x goes from $-\infty$ to $+\infty$, but a full 2π rotation of the soliton requires $x \to x + 4\pi$, reflecting the spin-1/2 nature.

B.4.2 Topological Interpretation

The 4π-periodicity of the soliton is a manifestation of its **topological winding number** . For a closed loop in space, the phase change $\Delta\theta$ is given by:

$$\Delta\theta = \oint \nabla\theta \cdot d\mathbf{l} = 2\pi n,$$

where n is the winding number. For a spin-1/2 particle, the effective winding is $n = 1/2$, but since the phase must be single-valued in space, the soliton must traverse the loop twice to return to its original state, hence the 4π-periodicity.

B.4.3 Connection to Quantum Statistics

The 4π-periodicity of the soliton directly implies that it obeys **fermionic statistics**:

- Under a 2π rotation, the wavefunction of the soliton acquires a phase of $e^{i\pi} = -1$, which is the hallmark of a fermion.
- This explains the **Pauli exclusion principle** : two identical solitons cannot occupy the same state because their wavefunctions would interfere destructively due to the -1 phase factor.

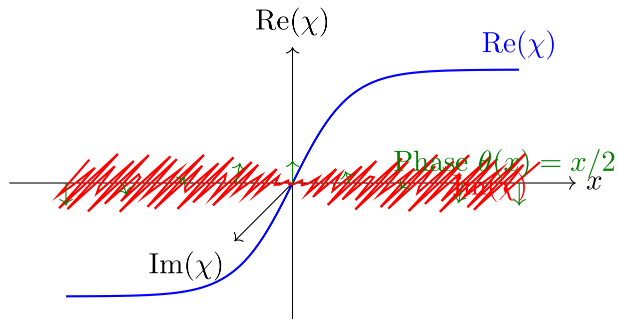

Figure 8: Visualization of a 4π-periodic soliton representing a spin-1/2 particle. The real part $\mathrm{Re}(\chi)$ (blue) and imaginary part $\mathrm{Im}(\chi)$ (red) of the χ field are shown, along with the local phase (green arrows). The phase winds by π over the spatial extent of the soliton, but a full 2π rotation of the soliton requires a 4π change in phase, reflecting its fermionic nature.

B.5 Relation to Classical Limits

In regimes where χ varies slowly and excitations are dilute, the dynamics reduces to linear wave propagation. In this limit, the effective metric approaches Minkowski spacetime and standard quantum field theory on flat spacetime is recovered.

Conversely, in high-density regimes, strong gradients in χ reproduce the phenomenology of curved spacetime and gravitational collapse.

B.6 Status of the Formulation

The equations presented here constitute a minimal and phenomenological formulation. A fully covariant action principle and quantization scheme for χ remain open problems.

Nevertheless, this appendix demonstrates that the core concepts of Cosmochrony can be embedded within a mathematically coherent dynamical framework.

B.7 Soliton and Particle Solutions

Within the Cosmochrony framework, all elementary particles are interpreted as stable or metastable topological configurations of the χ-field, known as χ-solitons. These are localized solutions to the non-linear field equation derived from varying the Lagrangian $\mathcal{L}_{\chi/\mathrm{Soliton}}$:

$$\Box\chi + \frac{\partial V_{\mathrm{Soliton}}(\chi)}{\partial \chi} + \text{Coupling Terms} = 0$$

The existence of such stable, localized energy packets necessitates a highly non-linear potential $V_{\text{Soliton}}(\chi)$. Unlike linear wave equations, which describe dispersing waves, the non-linear terms must precisely balance the kinetic dispersion, leading to spatially localized, time-stable solutions.

The specific requirements for the potential $V_{\text{Soliton}}(\chi)$ are:

1. **Existence:** The potential must allow for non-trivial, localized, finite-energy solutions $\chi_{\text{soliton}}(\mathbf{x})$.
2. This typically requires terms beyond χ^2, such as χ^4 or χ^6 contributions, similar to kinks or breathers in ϕ^4 models, but generalized for a dynamic background.
3. **Stability:** The solutions must be stable against small perturbations over cosmic timescales.
4. The topological winding number (or an equivalent conserved quantity related to χ's phase) is hypothesized to provide this structural stability, preventing the soliton from decaying into the vacuum state $\chi \to 0$.
5. **Emergence of Spin:** The explicit inclusion of Torsion in the full action (via $\mathcal{L}_{\text{Dirac}}^{\text{Torsion}}$) ensures that these localized solutions carry the specific topological phase constraint required for fermionic spin-1/2 behavior (Section 5.2).

The precise mathematical form of $V_{\text{Soliton}}(\chi)$ that simultaneously guarantees the stability of the χ-solitons, recovers the observed mass spectrum of particles, and supports the 4π twist topology remains an **open and critical mathematical challenge** for the theory.

B.8 Perspectives: Towards a Derivation of the Mass Spectrum

While the identification of particles as topological solitons provides a qualitative mechanism for inertial mass—understood as the resistance of localized χ configurations to field relaxation—the explicit derivation of the Standard Model mass spectrum, in particular the hierarchy of lepton and quark flavors, remains an open challenge.

In the Cosmochrony framework, this spectrum should not be fixed by arbitrary coupling constants, but should emerge from the intrinsic geometry of the relaxation network $G(V, E)$. The following sections outline a programmatic approach toward such a derivation.

The conceptual interpretation of inertial mass underlying this programmatic approach is discussed in B.9.

B.8.1 The Geometric Resonance Hypothesis

We conjecture that observed masses correspond to the eigenvalues of a transfer operator on the discrete network, where the mass m_n of a configuration n follows a scaling law linked to the local curvature induced by the soliton:

$$m_n c^2 \approx E_{\text{fund}} \cdot \Lambda(Q_n, \mathcal{K}) \tag{59}$$

where E_{fund} is a fundamental energy scale (potentially linked to the Planck scale or the global relaxation density), Q_n is the topological charge (winding number), and $\mathcal{K}$ represents a curvature invariant of the network.

B.8.2 Future Research Program

Transitioning toward a predictive theory of the mass spectrum requires:

1. **Discretization of the Potential** $V(\chi)$: Demonstrating how the minima of the potential on the network favor specific mass scales over others.
2. **Network Eigenmode Analysis**: Investigating whether the flavor hierarchy (the three generations of particles) can be interpreted as higher-order harmonic modes of a single fundamental topological structure.
3. **Numerical Simulations**: Implementing relaxation algorithms on large-scale graphs to verify if stable configurations spontaneously emerge with mass ratios corresponding to physical constants (e.g., the proton-to-electron mass ratio).

This approach aims to transform the "magic numbers" of the Standard Model into geometric properties derivable from the first principles of relaxation dynamics.

B.8.3 Spectral Relaxation Operators and Multi-Level Structure

This subsection introduces a spectral relaxation operator as a technical tool for quantifying the stability properties of localized configurations within the relaxation network. It does not redefine the physical origin of inertial mass, which remains the resistance of solitonic configurations to the relaxation of the χ field, but provides a mathematical framework for analyzing its spectral manifestation.

Following the conceptual separation introduced in Section 12.6, this appendix outlines an explicit but exploratory specification of the discrete relaxation operator underlying the spectral interpretation of particle masses in Cosmochrony. Rather than proposing a finalized microscopic model, the constructions presented here are intended to clarify how this separation may be implemented in a concrete and computationally tractable way, while preserving the non-circular distinction between fundamental geometry, emergent spacetime, and dynamical interactions.

Spectral origin of the mass scale. While the fundamental relaxation equation of Cosmochrony is first order in time, it fixes the geometric structure of the underlying relaxation network through the graph Laplacian $\Delta_G^{(0)}$. The inertial mass of localized excitations instead emerges from the stability properties of small fluctuations around a stationary solitonic configuration.

To make this explicit, we introduce an effective localization functional, defined at the coarse-grained level,

$$E[\chi] \;=\; \frac{1}{2}\sum_{(i,j)\in E} w_{ij}^{(0)}(\chi_i-\chi_j)^2 \;+\; E_{\rm loc}[\chi], \tag{60}$$

where $w_{ij}^{(0)}$ are fixed geometric weights encoding the relaxation network, and $E_{\rm loc}$ denotes an effective stabilizing contribution ensuring the existence of localized stationary configurations. This functional is not assumed to be fundamental but serves to characterize the stability of solitonic excitations.

Let $\chi_{\rm sol}$ be a stationary localized configuration, and consider small fluctuations $\chi = \chi_{\rm sol} + \delta\chi$. The second variation of $E[\chi]$ around $\chi_{\rm sol}$ defines a linear operator

$$\delta^2 E \;=\; \langle \delta\chi,\, \mathcal{L}_{\rm sol}\,\delta\chi\rangle, \qquad \mathcal{L}_{\rm sol} \equiv \Delta_G^{(0)} + U_{\rm sol}, \tag{61}$$

where $U_{\rm sol}$ is a local restoring operator determined by the background configuration.

In the regime where an effective wave description applies, small fluctuations are governed by a Klein–Gordon-type equation,

$$\left(\frac{1}{c^2}\partial_t^2 + \mathcal{L}_{\rm sol}\right)\delta\chi = 0. \tag{62}$$

For normal modes $\delta\chi(t) = e^{-i\omega t}\psi_n$, one obtains the spectral problem

$$\mathcal{L}_{\rm sol}\psi_n = \lambda_n\psi_n, \qquad \omega_n^2 = c^2\lambda_n. \tag{63}$$

Identifying the rest frequency ω_0 with the inertial mass via $\omega_0 = mc^2/\hbar$ yields

$$m_n = \frac{\hbar}{c}\sqrt{\lambda_n}. \tag{64}$$

Hence, particle masses in Cosmochrony arise as square roots of the eigenvalues of the linearized relaxation operator governing the stability of localized configurations.

*Fundamental spectral operator

Let $G = (V, E)$ be a discrete network encoding the intrinsic connectivity of the pre-geometric substrate. The fundamental relaxation operator is defined as a weighted graph Laplacian

$$(\Delta_G^{(0)}\psi)_i = \sum_{j\sim i} w_{ij}^{(0)}(\psi_i - \psi_j), \tag{65}$$

where the weights $w_{ij}^{(0)} = 1/K_{ij}^{(0)}$ encode the intrinsic compliance of the relaxation network. At this level, $K_{ij}^{(0)}$ are fixed coefficients determined by network topology and symmetry constraints, and do not depend on the dynamical state of the χ field.

The eigenvalue problem

$$\Delta_G^{(0)}\psi_n = \lambda_n\psi_n \tag{66}$$

defines a discrete spectrum. Within the Cosmochrony framework, particle masses are conjectured to scale as

$$m_n c^2 \propto \sqrt{\lambda_n}, \tag{67}$$

so that mass hierarchies emerge as geometric properties of the relaxation network rather than as parameters encoded in a fundamental potential.

*Emergent geometric level

On larger scales, coarse-grained configurations of the χ field define an effective geometric description. This level governs gravitational phenomena, time dilation, and cosmological expansion through the local and global rates of χ relaxation. Importantly, while this emergent geometry influences the propagation of excitations, it does not redefine the fundamental spectral operator $\Delta_G^{(0)}$.

*Dynamical and interaction level

Fast processes such as radiation, scattering, and decoherence correspond to interaction-induced redistributions of relaxation potential within the χ field. At this effective level, it is natural to consider corrections to the relaxation dynamics encoded by coefficients $K_{ij}(\chi)$ that depend smoothly on coarse-grained field variations. Such dependencies modify local dynamics and observable interaction rates but do not enter the definition of the fundamental spectral problem associated with mass generation.

*Boundary conditions and numerical implementation

For numerical studies, the graph G may be taken as large but finite, with periodic boundary conditions to minimize edge effects, or with Dirichlet or Neumann conditions to model confined regions. Standard sparse eigensolvers may then be used to compute the low-lying spectrum of $\Delta_G^{(0)}$ and to test whether simple network connectivities can reproduce observed mass hierarchies.

These constructions illustrate how Cosmochrony may be developed into a calculable spectral program, while preserving the non-circular separation between fundamental geometry, emergent spacetime, and dynamical interactions.

B.9 Spectral Characterization of Mass and the Secondary Role of $V(\chi)$

This appendix clarifies the conceptual status of inertial mass in Cosmochrony. While mass originates from the resistance of solitonic configurations to the relaxation of the χ field, this resistance can be quantitatively characterized through the spectral properties of the corresponding stability operator.

In this context, the spectral analysis does not redefine the physical origin of inertial mass, which remains the resistance of solitonic configurations to the relaxation of the χ field, but provides a quantitative characterization of this resistance.

A key conjecture of the Cosmochrony framework is that particle masses are not fundamental parameters encoded in the nonlinear potential $V(\chi)$, but instead emerge as spectral properties of a relaxation operator defined on the underlying discrete substrate.

In this perspective, the role of $V(\chi)$ is secondary and effective: it serves as a convenient coarse-grained description of localization and stability, but does not fundamentally determine the mass spectrum.

Mass spectrum as eigenmodes of a relaxation operator. Localized particle-like excitations are identified with normal modes of a discrete Laplace–Beltrami operator acting on the graph $G(V, E)$,

$$\Delta_G \psi_n = -\lambda_n \psi_n, \tag{68}$$

where ψ_n are eigenmodes of the relaxation dynamics. The associated particle masses are conjectured to scale as

$$m_n c^2 \propto \sqrt{\lambda_n}. \tag{69}$$

This mechanism is directly analogous to the emergence of discrete acoustic frequencies in bounded elastic systems, where the spectrum is entirely fixed by geometry and boundary conditions rather than by adjustable material parameters. Within Cosmochrony, mass hierarchies are thus interpreted as geometric properties of the underlying network topology and connectivity.

A decisive test of this conjecture consists in computing the low-lying spectrum of Δ_G on large but finite graphs with physically motivated connectivity rules. If even approximate agreement with observed mass ratios were obtained, this would strongly suggest that $V(\chi)$ is not a fundamental ingredient of the theory.

Spectral structure and level separation. To avoid any circular dependence between geometry, dynamics, and emergent particle properties, the Cosmochrony framework distinguishes three conceptual levels.

At the fundamental level, particle masses are associated with the spectral properties of a fixed, background-independent relaxation operator $\Delta_G^{(0)}$, defined solely by the intrinsic connectivity of the underlying discrete network. This operator does not depend on the instantaneous configuration of the χ field and provides a stable spectral structure whose eigenvalues λ_n define mass scales through $m_n c^2 \propto \sqrt{\lambda_n}$.

At a secondary level, coarse-grained configurations of χ give rise to an effective geometric description, including gravitational time dilation and cosmological expansion. This emergent geometry influences the propagation and interaction of excitations but does not modify the underlying spectral operator responsible for mass generation.

Finally, fast dynamical processes such as radiation, scattering, and decoherence occur as interaction-induced redistributions of relaxation potential within the χ field. These processes affect observables without redefining the fundamental spectral structure.

This separation ensures that particle masses are universal spectral properties of the relaxation network, while geometric and dynamical effects remain emergent and non-circular.

Residual role of the potential $V(\chi)$. Within this spectral picture, the nonlinear potential $V(\chi)$ may be understood as an effective description of localization mechanisms arising after coarse-graining. Its form is constrained by the requirement that it admit stable solitonic solutions corresponding to the low-lying eigenmodes of the relaxation operator, but it does not independently fix their masses.

Supporting perspectives. Discrete symmetry constraints and information-theoretic considerations may further restrict admissible network structures or provide complementary interpretations of the emergent dynamics. However, these directions are secondary to the central spectral hypothesis and are not required for its internal consistency.

Taken together, these considerations suggest that a substantial part of the explanatory burden for mass generation in Cosmochrony may lie in the spectral properties of the underlying discrete relaxation dynamics, with $V(\chi)$ playing a derived and non-fundamental role in effective descriptions, and with the spectral structure understood as a quantitative characterization of solitonic inertial resistance.

Extending this spectral characterization toward a concrete prediction of particle mass hierarchies requires specifying the underlying relaxation operator and its boundary conditions, as discussed in B.8.

C Appendix C: Cosmological and Observational Implications of Cosmochrony

This appendix details the cosmological and observational consequences of Cosmochrony, including:

- The spectrum of χ-field fluctuations and their imprint on the CMB (Section C.1).
- Resolution of the horizon and flatness problems without inflation (Section C.2).
- Evolution of the Hubble parameter $H(z)$ and its observational implications (Section C.2.1).
- Numerical estimates of χ-field parameters and their relation to observable quantities (Section C.3).
- Phenomenological predictions for gravitational waves, MOND-like effects, and lensing (Section C.4).

These results demonstrate the compatibility of Cosmochrony with key cosmological observations while highlighting testable deviations from standard models.

C.1 Spectrum of χ-Field Fluctuations and CMB Anisotropies

In Cosmochrony, the anisotropies of the Cosmic Microwave Background (CMB) are interpreted as frozen fluctuations of the χ field at the epoch of recombination. This section demonstrates how the power spectrum of χ-field fluctuations can reproduce the observed CMB power spectrum, including the acoustic peaks that are well-explained by the ΛCDM model.

C.1.1 Fluctuations of χ and Temperature Anisotropies

The temperature anisotropies of the CMB, $\delta T/T$, are linked to fluctuations in the χ field, $\delta\chi$, via the Sachs-Wolfe effect. In the linear regime, these fluctuations are described by:

$$\frac{\delta T}{T} \propto \delta\chi(\mathbf{x}, t_{\text{rec}}),$$

where t_{rec} is the time of recombination. The power spectrum of these fluctuations, $P(k)$, is defined as:

$$\langle \delta\chi(\mathbf{k})\delta\chi^*(\mathbf{k}') \rangle = (2\pi)^3 P(k)\delta^{(3)}(\mathbf{k} - \mathbf{k}'),$$

where $\delta\chi(\mathbf{k})$ is the Fourier transform of $\delta\chi(\mathbf{x})$.

C.1.2 Power Spectrum of χ-Field Fluctuations

The power spectrum of χ-field fluctuations is determined by the dynamics of χ during inflation and its subsequent evolution. For a nearly scale-invariant spectrum, we assume:

$$P(k) = Ak^{n_s - 1},$$

where A is the amplitude and n_s is the spectral index. In Cosmochrony, the spectral index n_s is naturally close to 1 due to the universal relaxation dynamics of χ, consistent with observations ($n_s \approx 0.96$).

The acoustic peaks in the CMB power spectrum arise from oscillations in the χ -matter fluid before recombination. These oscillations are driven by the competition between gravitational compression and χ -field pressure, analogous to sound waves in a fluid. The positions of the peaks are determined by the sound horizon at recombination, r_s, and the angular diameter distance to the last scattering surface, D_A :

$$\ell_n \approx n\pi \frac{D_A}{r_s}.$$

C.1.3 Comparison with ΛCDM Acoustic Peaks

In the Λ CDM model, the acoustic peaks are a consequence of baryon-photon fluid oscillations. In Cosmochrony, a similar phenomenon emerges from the coupling between χ-field fluctuations and matter excitations. The key differences and similarities are:

- **Origin of Fluctuations**: In ΛCDM, fluctuations originate from quantum fluctuations of the inflaton field during inflation. In Cosmochrony, they arise from primordial variations in the χ field's relaxation dynamics. Crucially, the phase of these acoustic oscillations is locked to the initial relaxation onset of χ. Unlike inflation, which requires a separate "reheating" phase to populate the universe with particles, the intrinsic coupling between χ-fluctuations and matter ensures that baryonic matter is born directly within these geometric ripples. This provides a natural mechanism for the phase coherence of the acoustic peaks without invoking super-horizon inflationary correlation.

- **Acoustic Oscillations** : Both models predict acoustic peaks due to oscillatory behavior in the early universe. In Cosmochrony, these oscillations are driven by the interaction between χ and matter, leading to a similar pattern of peaks and troughs in the power spectrum.
- **Spectral Index**: Both models predict a nearly scale-invariant spectrum ($n_s \approx 1$), but in Cosmochrony, this arises naturally from the relaxation dynamics of χ without requiring a specific inflationary potential.
- **Peak Positions** : The positions of the acoustic peaks in Cosmochrony are determined by the sound horizon and angular diameter distance, just as in Λ CDM. The precise locations of the peaks can be used to constrain the parameters of the χ field.

C.1.4 Quantitative Estimation of the Power Spectrum

To estimate the power spectrum of χ-field fluctuations, consider the following steps:

1. **Primordial Fluctuations**: Assume that the primordial fluctuations of χ are Gaussian and nearly scale-invariant, with a power spectrum given by:
$$P_\chi(k) = A \left(\frac{k}{k_0}\right)^{n_s - 1},$$
where k_0 is a pivot scale.
2. **Transfer Function**: The transfer function $T(k)$ describes how primordial fluctuations evolve until recombination. In Cosmochrony, this function is influenced by the coupling between χ and matter, leading to acoustic oscillations:
$$T(k) \propto \frac{\sin(kr_s)}{kr_s},$$
where r_s is the sound horizon at recombination.
3. **Observed Power Spectrum**: The observed power spectrum of CMB anisotropies is then:
$$P_{\text{obs}}(k) = P_\chi(k)T(k)^2.$$
This results in a series of acoustic peaks at scales determined by r_s and the angular diameter distance D_A.

C.1.5 Implications for Cosmochrony

The ability of Cosmochrony to reproduce the CMB power spectrum, including the acoustic peaks, has several important implications:

- **Consistency with Observations**: The model is consistent with the precise measurements of the CMB power spectrum by experiments such as Planck, which have confirmed the acoustic peak structure to high accuracy.
- **Unified Framework**: Cosmochrony provides a unified framework for understanding both the large-scale structure of the universe and the microscopic properties of particles, linking the CMB anisotropies to the dynamics of the χ field.
- **Predictions and Tests**: The model predicts specific features in the CMB power spectrum that could be tested with future high-precision experiments, such as CMB-S4 or LiteBIRD. For example, deviations from the Λ CDM predictions in the damping tail or the polarization spectrum could provide evidence for Cosmochrony.

C.2 Resolution of the Horizon and Flatness Problems without Inflation in Cosmochrony

In standard cosmology, the horizon and flatness problems are typically addressed by introducing an early period of exponential expansion known as inflation [21, 26]. Cosmochrony, however, offers an alternative explanation for these issues through the intrinsic properties of the χ field, specifically its pre-geometric entanglement and relaxation dynamics. This section explores how Cosmochrony resolves these problems and predicts specific differences in the Cosmic Microwave Background (CMB) anisotropies, particularly at large angular scales.

The horizon problem arises because regions of the universe that are widely separated on the last scattering surface appear to be in thermal equilibrium, despite never having been in causal contact under standard Friedmann-Lemaître-Robertson-Walker expansion [21]. In Cosmochrony, this issue is resolved through a form of pre-geometric entanglement inherent to the χ field. Before the emergence of classical spacetime, all regions of the universe are connected via the χ field's non-local correlations. This entanglement ensures that fluctuations in χ, while present at small scales, are coherently correlated across arbitrarily large distances, eliminating the need for inflationary causal contact. As χ begins to relax according to $\partial_t\chi = c\sqrt{1 - |\nabla\chi|^2/c^2}$, its dynamics smooth out small-scale fluctuations while preserving these large-scale correlations, resulting in a universe that appears thermally uniform at recombination.

The flatness problem concerns the apparent fine-tuning of the universe's spatial curvature to be very close to zero [26]. In Cosmochrony, the flatness of the universe is a natural consequence of the χ field's relaxation dynamics. The χ field evolves monotonically, and its spatial gradients are constrained by the relaxation equation, which ensures that any initial curvature in χ is rapidly smoothed out as the field relaxes. This leads to a spatially flat universe without requiring fine-tuning of initial conditions, similar to mechanisms explored in alternative cosmological models [5].

Unlike inflationary models, which predict a nearly scale-invariant spectrum of primordial fluctuations, Cosmochrony suggests that the spectrum of χ-field fluctuations may exhibit subtle deviations at large angular scales. These deviations arise because the χ field's relaxation dynamics do not involve superluminal expansion. Instead, the correlations in χ are established through the field's pre-geometric entanglement, rather than through inflationary stretching. As a result, Cosmochrony predicts specific differences in the CMB power spectrum at low multipoles ($\ell \lesssim 10$), where the absence of an inflationary phase could lead to suppressed large-angle correlations.

Clarification on primordial B-modes. It should be emphasized that the absence of primordial B-modes, corresponding to a vanishing or extremely small tensor-to-scalar ratio ($r \simeq 0$), is already compatible with current observational bounds from CMB polarization experiments. As such, this feature does not by itself constitute a distinctive prediction of the Cosmochrony framework. Rather, it reflects a natural consequence of the absence of an inflationary phase, without requiring parameter tuning.

One of the most striking predictions of Cosmochrony is its potential to explain the large-angle anomalies observed in the CMB, such as the hemispherical asymmetry and the cold spot [11]. In inflationary models, these anomalies are often attributed to statistical fluctuations or systematic effects [8]. However, in Cosmochrony, the pre-geometric entanglement of the χ field would tend to uniformize large-scale fluctuations, potentially reducing the amplitude of such anomalies. This is because the non-local correlations of χ ensure that large-scale fluctuations are more uniformly distributed, without the need for an inflationary mechanism to stretch quantum fluctuations to cosmological scales.

Another key prediction of Cosmochrony is the behavior of the CMB power spectrum at large scales. In ΛCDM, the power spectrum at low ℓ is determined by the primordial power spectrum generated during inflation. In Cosmochrony, however, the power spectrum at large scales is influenced by the global relaxation dynamics of χ, which may not produce the same level of large-scale power as inflation. This could result in a suppression of the power spectrum at low ℓ, providing a distinctive signature that could be tested with future CMB experiments such as CMB-S4 or LiteBIRD.

Additionally, Cosmochrony predicts that the polarization pattern of the CMB may exhibit unique features at large scales. In particular, the absence of an inflationary phase could lead to a different pattern of E-mode and B-mode polarization, reflecting the geometric nature of the χ field's relaxation. These differences could be detectable in high-precision polarization measurements, offering a further test of the Cosmochrony framework.

In summary, Cosmochrony resolves the horizon and flatness problems through the pre-geometric entanglement and relaxation dynamics of the χ field, without requiring inflation. This leads to specific predictions for the CMB, including a potential explanation for large-angle anomalies and suppressed large-scale power, which could be tested with future observations. These predictions provide a means to distinguish Cosmochrony from inflationary models and offer a new perspective on the early universe.

C.2.1 Evolution of the Hubble Parameter $H(z)$ in Cosmochrony

In Cosmochrony, the evolution of the Hubble parameter $H(z)$ with redshift z is determined by the dynamics of the χ field, which governs the expansion of the universe. This section derives the form of $H(z)$ in Cosmochrony and compares it with the standard Λ CDM model, highlighting the differences in the redshift dependence and their observational implications.

Hubble Parameter in Cosmochrony

In Cosmochrony, the Hubble parameter is directly related to the time derivative of the χ field. The scale factor $a(t)$ is proportional to $\chi(t)$, such that:

$$a(t) \propto \chi(t).$$

The Hubble parameter $H(t)$ is then given by:

$$H(t) = \frac{\dot{a}}{a} = \frac{\dot{\chi}}{\chi}.$$

Using the relaxation equation for χ:

$$\partial_t \chi = c\sqrt{1 - \frac{|\nabla \chi|^2}{c^2}},$$

and assuming a homogeneous universe ($\nabla \chi = 0$), we obtain:

$$\dot{\chi} = c.$$

Thus, the Hubble parameter in Cosmochrony is:

$$H(t) = \frac{c}{\chi(t)}.$$

Since $\chi(t)$ grows linearly with time during the relaxation-dominated era, we have:

$$\chi(t) = \chi_0 + ct,$$

where χ_0 is the initial value of χ. For simplicity, we can set $\chi_0 = 0$ for the early universe, leading to:

$$\chi(t) \approx ct.$$

The Hubble parameter then becomes:

$$H(t) = \frac{c}{\chi(t)} = \frac{1}{t}.$$

To express $H(z)$ in terms of redshift, we use the relationship between time and redshift in an expanding universe:

$$1 + z = \frac{a(t_0)}{a(t)} = \frac{\chi(t_0)}{\chi(t)}.$$

Assuming $\chi(t_0) = ct_0$ and $\chi(t) = ct$, we have:

$$1 + z = \frac{t_0}{t},$$

which implies:

$$t = \frac{t_0}{1+z}.$$

Substituting this into the expression for $H(t)$, we obtain:

$$H(z) = \frac{1}{t} = \frac{1+z}{t_0} = H_0(1+z),$$

where $H_0 = 1/t_0$ is the present-day Hubble constant.

Hubble Parameter in ΛCDM

In the standard ΛCDM model, the Hubble parameter $H(z)$ is given by:

$$H(z) = H_0\sqrt{\Omega_{m0}(1+z)^3 + \Omega_\Lambda},$$

where Ω_{m0} is the present-day matter density parameter, and Ω_Λ is the dark energy density parameter.

For comparison, we use the Planck 2018 best-fit values:

$$\Omega_{m0} \approx 0.315, \quad \Omega_\Lambda \approx 0.685, \quad H_0 \approx 67.4\,\text{km/s/Mpc}.$$

Comparison of $H(z)$ in Cosmochrony and ΛCDM

The evolution of $H(z)$ in Cosmochrony and ΛCDM exhibits several key differences:

- In Cosmochrony, $H(z)$ evolves linearly with redshift:

  $$H(z) = H_0(1+z).$$

  This linear dependence reflects the direct proportionality between the Hubble parameter and the inverse of the χ field, which grows linearly with time.

- In ΛCDM, $H(z)$ has a more complex redshift dependence due to the contributions of matter and dark energy:

  $$H(z) = H_0\sqrt{\Omega_{m0}(1+z)^3 + \Omega_\Lambda}.$$

  At high redshifts ($z \gg 1$), the ΛCDM model reduces to a matter-dominated universe, where $H(z) \approx H_0\sqrt{\Omega_{m0}}(1+z)^{3/2}$. At low redshifts ($z \ll 1$), dark energy dominates, and $H(z)$ approaches a constant value $H_0\sqrt{\Omega_\Lambda}$.

- The linear evolution of $H(z)$ in Cosmochrony contrasts with the Λ CDM prediction, particularly at intermediate redshifts ($0.1 < z < 10$), where the influence of dark energy in ΛCDM causes $H(z)$ to deviate from a simple linear relationship.

Quantitative Comparison

To illustrate the differences between Cosmochrony and ΛCDM, we compare the predicted values of $H(z)$ at several redshifts:

Table 3: Comparison of $H(z)$ in Cosmochrony and ΛCDM

Redshift z	**Cosmochrony $H(z)$ (km/s/Mpc)**	**ΛCDM $H(z)$ (km/s/Mpc)**	**Relative Difference**
0	67.4	67.4	0%
0.5	101.1	95.6	+5.8%
1	134.8	129.5	+4.1%
3	269.6	238.5	+13.0%
10	741.4	560.3	+32.3%

The table shows that the linear evolution of $H(z)$ in Cosmochrony leads to systematically higher values of the Hubble parameter at higher redshifts compared to Λ CDM. This difference arises because Cosmochrony does not include a dark energy component that slows the growth of $H(z)$ at low redshifts.

Observational Implications

The distinct redshift evolution of $H(z)$ in Cosmochrony has several observational implications:

- **Baryon Acoustic Oscillations (BAO)**: **Baryon Acoustic Oscillations (BAO)**: Measurements of BAO at various redshifts constrain the evolution of $H(z)$. In this framework, the acoustic scale is a "frozen" imprint of the primordial χ-fluctuations described in C.1.3. However, Cosmochrony predicts a faster increase in $H(z)$ with redshift compared to ΛCDM. Consequently, the apparent angular diameter distance of the BAO scale at $z > 1$ should exhibit a systematic shift, offering a clear discriminant between the χ-field relaxation and a constant dark energy density (Λ). This prediction is directly testable with upcoming high-precision surveys such as DESI or Euclid.

- **Type Ia Supernovae** : The distance–redshift relation for Type Ia supernovae depends on the integrated expansion history encoded in the Hubble parameter $H(z)$, through the luminosity distance $d_L(z) = (1+z)\int_0^z c\,dz'/H(z')$ [22, 38, 45]. The linear evolution of $H(z)$ in Cosmochrony would result in slightly different distance moduli compared to ΛCDM, particularly at intermediate redshifts, where the inferred expansion history is most sensitive to the detailed form of $H(z)$ [4].

- **CMB Anisotropies** : The angular diameter distance to the surface of last scattering and the growth of cosmological structures imprinted in the CMB anisotropies are influenced by the integrated history of $H(z)$ [15, 23]. Cosmochrony's linear $H(z)$ could therefore lead to subtle differences in the CMB power spectrum relative to ΛCDM, particularly in the damping tail at high multipoles and in the late-time integrated Sachs–Wolfe effect [24, 49].

Non-linear Resolution of the Hubble Tension

The discrepancy between local and global measurements of H_0 can be naturally accounted for through the internal kinematics of the χ field. We depart from the fundamental equation of motion $\dot{\chi} = c\sqrt{1-\beta^2}$, where $\beta = |\nabla\chi|/c$ represents the local field gradient density.

The Relaxation Budget Parameter Ω_χ We introduce a dimensionless parameter Ω_χ, defined as the global fraction of the χ-field relaxation budget stored in spatial gradients:

$$\Omega_\chi \equiv \langle \beta^2 \rangle \tag{70}$$

In the late universe, Ω_χ is empirically constrained to be close to the observed matter fraction Ω_m ($\Omega_\chi \approx \Omega_m \approx 0.31$), reflecting the fact that matter (solitons) is the primary source of field gradients. Consequently, the global expansion rate is governed by the average available relaxation speed: $\bar{H} = \frac{c}{\chi}\sqrt{1 - \Omega_\chi}$.

The linear identification $\beta_{loc}^2 = \Omega_\chi(1 + \delta)$ assumes a "mean-field" approximation where the energy density of the gradients (the solitons) is locally proportional to the number density of those solitons. In this regime, the collective resistance to relaxation scales linearly with the local concentration of field deformations, much like the elastic energy density in a deformed medium scales with the density of defects. This ensures that in the limit $\delta \to -1$ (absolute vacuum), $\beta^2 \to 0$ and the relaxation speed $\dot{\chi}$ reaches its upper bound c.

Local Variation and the Hubble Tension In a local region characterized by a density contrast $\delta = (\rho - \bar{\rho})/\bar{\rho}$, the local gradient density scales as $\beta_{loc}^2 = \Omega_\chi(1 + \delta)$. This linear scaling constitutes the minimal closure relation between matter inhomogeneities and χ-field gradients, sufficient to capture the leading non-linear effect. The local Hubble parameter H_{loc} then deviates from the global average according to:

$$H_{loc} = \bar{H}\sqrt{\frac{1 - \Omega_\chi(1 + \delta)}{1 - \Omega_\chi}} \tag{71}$$

For an underdense region (void) where $\delta < 0$, the available relaxation budget is locally higher, leading to $H_{loc} > \bar{H}$.

Numerical Consistency Assuming $\Omega_\chi \approx 0.31$ and a local underdensity corresponding to the KBC void ($\delta \approx -0.4$ on a scale of 300 Mpc), the ratio becomes:

$$\frac{H_{loc}}{\bar{H}} = \sqrt{\frac{1 - 0.31(0.6)}{0.69}} \approx 1.084 \tag{72}$$

This 8.4% increase naturally accounts for the Hubble tension. This analysis shows that the Hubble tension does not necessarily signal missing components or modifications of the cosmological model, but may instead reflect a non-linear environmental effect arising from the relaxation dynamics of the χ field.

Conclusion

The evolution of the Hubble parameter $H(z)$ in Cosmochrony differs significantly from that in Λ CDM, particularly at intermediate and high redshifts. The linear dependence of $H(z)$ on $1 + z$ in Cosmochrony reflects the underlying dynamics of the χ field and provides a distinctive signature that could be tested with future observations. These differences offer a means to distinguish Cosmochrony from Λ CDM and other cosmological models, providing a pathway to validate or constrain the χ field framework.

C.3 Relation to Observational Units and Numerical Estimates

C.3.1 Normalization of the χ Field

To connect the χ field to observable quantities, a normalization must be specified. We identify the present-day value $\chi(t_0)$ with the characteristic cosmological length scale governing expansion.

Operationally, $\chi(t_0)$ may be interpreted as the proper wavelength accumulated since the epoch at which coherent propagation of radiation became possible, approximately recombination.

*Constraints on K_0 and χ_c The fundamental parameters K_0 and χ_c in the constitutive relation for K_{ij} (Equation (2)) can be constrained by matching the emergent gravitational constant G (Equation (106)) to its observed value. Using $G \approx 6.674 \times 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$ and $c \approx 3 \times 10^8\,\mathrm{m/s}$, we find:

$$K_0\chi_c^2 = \frac{c^4}{16\pi G} \approx 1.1 \times 10^{44}\,\mathrm{N} \tag{73}$$

If we associate χ_c with the **Planck length** $\ell_P \approx 1.6 \times 10^{-35}\,\mathrm{m}$, then:

$$K_0 \approx \frac{c^4}{16\pi G\ell_P^2} \approx 1.3 \times 10^{93}\,\mathrm{m}^{-2} \tag{74}$$

This value of K_0 sets the **stiffness scale** of the χ-field network, while $\chi_c \approx \ell_P$ ensures that quantum gravitational effects become significant at the Planck scale.

Alternatively, if χ_c is associated with the **Hubble scale** $c/H_0 \approx 1.4 \times 10^{26}\,\mathrm{m}$, then:

$$K_0 \approx \frac{H_0^2}{c^2} \approx 1.1 \times 10^{-52}\,\mathrm{m}^{-2} \tag{75}$$

This smaller value of K_0 would imply a **softer network**, with cosmological-scale effects dominating the dynamics. The precise values of K_0 and χ_c remain open to empirical constraints, but their product $K_0\chi_c^2$ is fixed by the observed G.

*Implications for Particle Masses The soliton mass scale $m \propto \sqrt{\lambda}$ (from Appendix B.3) requires $\lambda \sim 10^{-116}\,\mathrm{m}^{-2}$ to reproduce the electron mass $m_e \approx 9.11 \times 10^{-31}\,\mathrm{kg}$. This tiny value suggests that λ may be **dynamically generated** rather than fundamental, reflecting the hierarchical structure of the χ field potential.

C.3.2 Hubble Constant

From the fundamental relation

$$H(t) = \frac{\dot{\chi}}{\chi}, \tag{76}$$

and assuming maximal relaxation speed $\dot{\chi} \simeq c$, the present Hubble constant follows as

$$H_0 \simeq \frac{c}{\chi(t_0)}. \tag{77}$$

Using the observed value $H_0 \sim 70\ \mathrm{km\,s}^{-1}\,\mathrm{Mpc}^{-1}$, one infers

$$\chi(t_0) \sim 4 \times 10^{26}\ \mathrm{m}, \tag{78}$$

consistent with the current Hubble radius.

This correspondence arises without introducing free cosmological parameters.

The soliton mass scale $m \propto \sqrt{\lambda}$ then requires $\lambda \sim 10^{-116}\,\mathrm{m}^{-2}$ to reproduce the electron mass $m_e \approx 9.11 \times 10^{-31}\,\mathrm{kg}$. This tiny value suggests that λ may be dynamically generated rather than fundamental.

C.3.3 Age of the Universe

Integrating the relation $\dot{\chi} \simeq c$ yields

$$\chi(t) \simeq ct + \chi_{\text{init}}, \tag{79}$$

where χ_{init} denotes the effective value at the onset of coherent χ relaxation.

Neglecting χ_{init} compared to present values gives

$$t_0 \simeq \frac{\chi(t_0)}{c} \sim 4 \times 10^{17} \text{ s}, \tag{80}$$

corresponding to approximately 13.8 billion years, in agreement with standard cosmological estimates.

C.3.4 Redshift Interpretation

Cosmological redshift arises from the increase of χ between emission and observation:

$$1 + z = \frac{\chi(t_{\text{obs}})}{\chi(t_{\text{emit}})}. \tag{81}$$

This interpretation reproduces standard redshift relations while attributing them to geometric scaling rather than recessional motion through spacetime.

C.3.5 Cosmic Microwave Background Scale

At recombination ($z_{\text{rec}} \simeq 1100$), the characteristic scale of χ was smaller by the same factor:

$$\chi(t_{\text{rec}}) \simeq \frac{\chi(t_0)}{1 + z_{\text{rec}}}. \tag{82}$$

Fluctuations imprinted at that epoch are stretched by subsequent χ growth, explaining the observed angular power spectrum of the CMB.

C.3.6 Orders of Magnitude and Robustness

All numerical estimates presented here rely solely on observed cosmological quantities and the assumption of bounded χ relaxation. No fine-tuning of parameters is required.

While precise numerical modeling remains to be developed, these estimates demonstrate that Cosmochrony naturally reproduces the correct orders of magnitude for key cosmological observables.

C.3.7 Summary

The χ framework connects directly to measured cosmological quantities through simple scaling relations. The Hubble constant, cosmic age, redshift, and CMB scales emerge consistently from the same underlying dynamics.

C.4 Phenomenological Implications

C.4.1 Speed of Gravitational Perturbations

To determine the propagation speed of gravitational information, we consider a small perturbation $\delta\chi$ around a homogeneous background $\chi_0(t) = ct$. Let $\chi(\mathbf{x}, t) = ct + \delta\chi(\mathbf{x}, t)$, where $|\nabla\delta\chi| \ll c$. Substituting this into the evolution equation (17):

$$c + \partial_t \delta\chi = c\sqrt{1 - \frac{|\nabla\delta\chi|^2}{c^2}} \tag{83}$$

Using the Taylor expansion $\sqrt{1-u} \approx 1 - u/2$ for small u:

$$c + \partial_t \delta\chi \approx c\left(1 - \frac{|\nabla\delta\chi|^2}{2c^2}\right) = c - \frac{|\nabla\delta\chi|^2}{2c} \tag{84}$$

This gives $\partial_t\delta\chi \approx -\frac{1}{2c}|\nabla\delta\chi|^2$. To find the wave equation, we take the time derivative of this expression and assume the perturbations follow a harmonic or eikonal form. More fundamentally, by squaring the Hamiltonian constraint (22) and linearizing the resulting second-order operator, we obtain the d'Alembertian:

$$\left(\frac{1}{c^2}\partial_t^2 - \nabla^2\right)\delta\chi = 0 \tag{85}$$

The characteristic speed is identically c. This result is robust and independent of any coupling constant, ensuring that Cosmochrony is strictly consistent with the GW170817 multi-messenger observation.

C.4.2 Emergence and Evolution of an Effective MOND-like Acceleration Scale

In Cosmochrony, the intrinsic arrow of time is encoded in the monotonic evolution of the fundamental field χ, with $\partial_t\chi \geq 0$. At large scales and late cosmic times, when χ admits a quasi-homogeneous and isotropic description, this monotonic relaxation can be coarse-grained into an effective cosmological clock, analogous to the role played by cosmic time in standard FLRW cosmology [18, 54].

In such regimes, and only as an effective description, the temporal evolution of χ may be approximated by

$$\partial_t\chi \simeq H(t)\,\chi, \tag{86}$$

where $H(t)$ denotes the emergent Hubble parameter associated with the global relaxation of the field. This relation should not be interpreted as a fundamental equation of motion, but as a phenomenological correspondence valid in the FLRW-like limit of the theory, in the same spirit as coarse-grained descriptions commonly employed in emergent gravity and cosmological averaging approaches [9, 33].

The local kinematic constraint governing the χ field,

$$(\partial_t\chi)^2 + |\nabla\chi|^2 = c^2, \tag{87}$$

then implies that even in the absence of localized matter excitations, the cosmological evolution of χ generically induces a non-vanishing residual spatial gradient. In the homogeneous limit, this minimal gradient magnitude is given by

$$|\nabla\chi|_{\min} = \sqrt{c^2 - (H(t)\chi)^2}. \tag{88}$$

This residual gradient does not correspond to a directional force acting on test particles. Rather, it defines a background kinematic scale that constrains how additional, locally induced gradients can contribute to the effective dynamics. For a local observer, the associated scale may be expressed as an effective acceleration floor,

$$a_0(t) \sim c\,H(t), \tag{89}$$

a relation that has long been noted empirically in the context of galactic dynamics and MOND-like phenomenology [19, 29, 30], but which here arises dynamically from the global relaxation of the χ field. Unlike Milgromian dynamics, where a_0 is postulated as a universal constant, Cosmochrony predicts that this scale evolves slowly with cosmic time, tracking the evolution of $H(t)$.

When localized matter excitations are present, they induce additional spatial gradients $\nabla\chi_N$ that, in the weak-field and short-distance limit, reproduce the Newtonian scaling $|\nabla\chi_N| \propto M/r^2$. ue to the non-linear nature of the kinematic constraint, the total effective gradient is not a linear superposition of the cosmological and local contributions. Instead, the field dynamics enforces a saturation behavior: at sufficiently large radii, where the Newtonian contribution would otherwise vanish, the total gradient asymptotically approaches the cosmological floor set by $|\nabla\chi|_{\min}$.

In this regime, the resulting effective gravitational acceleration naturally approaches

$$g_{\text{eff}} \simeq \sqrt{g_N\,a_0(t)}, \tag{90}$$

recovering the characteristic deep-MOND scaling originally identified by Milgrom [29] without introducing an ad hoc interpolation function or modifying the underlying gravitational law.

This interpretation offers two conceptual advantages over phenomenological MOND formulations:

1. **Cosmological scaling.** The acceleration scale a_0 is not fundamental but emerges from the cosmological state of the χ field. As a result, $a_0(t)$ is expected to be larger at earlier cosmic epochs, providing a potential observational handle through high-redshift galaxy kinematics, now becoming accessible with modern surveys [28].

2. **Environmental dependence.** Because the relaxation of χ is sensitive to the global and local distribution of gradients, the effective acceleration scale can be weakly modulated by environmental factors. This naturally incorporates effects analogous to the external field effect (EFE) discussed in MOND literature [2, 19] and allows for deviations from perfectly flat rotation curves in the far field or in strongly inhomogeneous environments.

In this framework, flat galactic rotation curves do not signal a breakdown of Newtonian gravity nor the presence of unseen matter components. Instead, they arise as a kinematic projection of the global cosmological relaxation of χ onto local gravitational dynamics, with MOND-like behavior emerging as an effective, scale-dependent regime rather than as a fundamental modification of gravity.

C.4.3 Gravitational Lensing in the Scalar Framework

Light deflection is modeled as the propagation of a wave front where $\chi = \text{const}$. The effective refractive index of the vacuum $n(r)$ is derived from the ratio of the global evolution rate to the local rate:

$$n(r) = \frac{c}{\partial_t \chi} = \frac{1}{\sqrt{1 - |\nabla\chi|^2/c^2}} \tag{91}$$

Near a mass M, $|\nabla\chi| \approx \frac{GM}{c^2 r}$. For small deflections, $n(r) \approx 1 + \frac{GM}{c^2 r}$. Integrating the gradient of n along the photon path z gives the deflection angle α:

$$\alpha = \int_{-\infty}^{\infty} \nabla_\perp n \, dz = \frac{4GM}{bc^2} \tag{92}$$

This matches the General Relativity prediction. The factor of 2, which Newton's theory lacks, arises here from the non-linear square-root structure of the evolution equation (17).

D Appendix D: Numerical Methods and Technical Supplements

This appendix provides technical details on:

- The collective gravitational coupling and operational geometry (Section D.1).
- Numerical algorithms for simulating χ-field dynamics and estimating model parameters.
- Supplementary derivations and calculations supporting the main text and other appendices.

These technical supplements are intended for readers interested in the numerical implementation or detailed derivations of the Cosmochrony framework.

D.1 Collective Gravitational Coupling and Operational Geometry

Status and Scope of the Construction

This appendix provides the technical construction underlying the collective gravitational coupling introduced in Section D.1. Its purpose is to demonstrate explicitly how an effective spatial geometry and a Newtonian gravitational interaction emerge from the local relaxation dynamics of the χ field, without assuming any pre-existing metric structure.

Minimal Local Ansatz for the Collective Coupling K_{ij}

We adopt a **constitutive relation** for the connectivity matrix K_{ij} that encodes the local resistance of the χ field to variations between neighboring nodes i and j:

$$K_{ij} = K_0 \cdot f\left(\frac{|\chi_i - \chi_j|^2}{\chi_c^2}\right) \tag{93}$$

where:

- K_0 is a **fundamental stiffness scale** (with dimensions of [length]$^{-2}$), setting the maximal connectivity strength in the absence of excitations.
- χ_c is a **characteristic scale** of the χ field, naturally associated with the Planck length ℓ_P or the cosmological relaxation scale c/H_0.
- $f(x)$ is a **dimensionless regularization function** satisfying:
  - $f(0) = 1$ (maximal connectivity for uniform χ),
  - $f(x) \to 0$ as $x \to \infty$ (vanishing connectivity for large gradients),
  - $f(x)$ is monotonically decreasing (connectivity weakens with increasing gradients).

For concreteness, we adopt the **illustrative choice**:

$$f(x) = \frac{1}{1+x} \tag{94}$$

This ansatz ensures that:

1. **Symmetry**: $K_{ij} = K_{ji}$, as required for a consistent relational network.
2. **Locality**: K_{ij} depends only on the **local difference** $|\chi_i - \chi_j|$, reflecting the pre-geometric nature of the theory.
3. **Boundedness**: $0 < K_{ij} \leq K_0$, preventing unphysical divergences in the emergent metric.
4. **Gradient sensitivity**: K_{ij} decreases as $|\chi_i - \chi_j|$ increases, encoding the **resistance to relaxation** induced by localized excitations.

*Physical Interpretation

- In regions where χ is uniform ($\chi_i \approx \chi_j$), $K_{ij} \approx K_0$, and the network behaves as a **flat spacetime** (minimal resistance to relaxation).
- Near localized excitations (e.g., particles or black holes), $|\chi_i - \chi_j|$ increases, reducing K_{ij} and thus **slowing the relaxation of** χ. This manifests as **gravitational time dilation** and **spacetime curvature**.

*Relation to Fundamental Constants The scales K_0 and χ_c are expected to be related to fundamental constants. For example:

- χ_c may be identified with the **Planck length** $\ell_P \approx 1.6 \times 10^{-35}$ m, setting the scale at which quantum gravitational effects become significant.
- K_0 may be linked to the **cosmological relaxation rate** $H_0 \approx 70\,\text{km/s/Mpc}$, via $K_0 \sim H_0^2/c^2$.

The precise mapping between (K_0, χ_c) and observable constants (e.g., G, Λ) is derived in Section D.1.

Operational Definition of Distance on the Discrete Network

In the absence of a fundamental background geometry, spatial distance is defined operationally through the resistance to χ propagation across the network. For any discrete path $\gamma = \{i_0, i_1, \ldots, i_n\}$ connecting two sites i and j, we define the squared path length:

$$d_\gamma^2 = \ell_0^2 \sum_{(i_k, i_{k+1}) \in \gamma} \frac{K_0}{K_{i_k i_{k+1}}} \tag{95}$$

where ℓ_0 is a microscopic length scale characterizing the underlying network. The physical distance between sites i and j is then defined as the minimal path length over all admissible paths:

$$d(i,j) = \min_\gamma \sqrt{d_\gamma^2} \tag{96}$$

This definition is analogous to an electrical resistance network: strongly coupled sites (large K_{ij}) are operationally close, while weakly coupled sites are distant. Importantly, this notion of distance is not derived from a pre-existing geometry but emerges from the dynamics of χ itself.

Continuum Limit and Effective Spatial Line Element

We now consider the continuum limit in which the lattice spacing $\varepsilon \to 0$ while the physical domain remains finite. For neighboring sites separated by ε along direction $\hat{n}$, we expand:

$$\chi_j - \chi_i \simeq \varepsilon \partial_{\hat{n}} \chi \tag{97}$$

In this limit, discrete sums over nearest neighbors reduce to integrals, and the collective interaction term becomes:

$$\sum_j K_{ij}(\chi_i - \chi_j)^2 \longrightarrow K_0 \varepsilon^2 \int |\nabla \chi|^2 \, d^3x \tag{98}$$

up to numerical factors of order unity arising from lattice geometry. In the weak-gradient regime, where $\chi = \bar{\chi} + \phi$ with $|\nabla \phi| \ll \bar{\chi}$, the operational distance induces an effective spatial line element of the form:

$$ds^2 = \ell_0^2 \left[\delta_{ij} + \mathcal{O}\left(\frac{\partial_i \phi}{\bar{\chi}^2} \right) \right] dx^i dx^j \tag{99}$$

This expression defines an effective spatial geometry to leading order in perturbations. The full spacetime structure, including temporal components, follows from the fundamental kinematic constraint governing the χ field.

From the Kinematic Constraint to the Field Equation

The χ field obeys the fundamental kinematic constraint:

$$(\partial_t \chi)^2 + |\nabla \chi|^2 = c^2 \tag{100}$$

which enforces a bounded relaxation dynamics. In the discrete formulation, this constraint takes the form:

$$\left(\frac{d\chi_i}{d\lambda} \right)^2 = c^2 \left[1 - \frac{1}{c^2} \sum_j K_{ij}(\chi_i - \chi_j)^2 \right] \tag{101}$$

Passing to the continuum and linearizing around a homogeneous background $\chi = \bar{\chi}(t) + \phi(\mathbf{x}, t)$ with $\dot{\bar{\chi}} = c$, we obtain, to leading order:

$$\partial_t^2 \phi - c^2 \nabla^2 \phi = S[\rho] \tag{102}$$

where $S[\rho]$ represents the source term associated with localized solitonic matter excitations.

Quasi-Static Regime and Newtonian Limit

For gravitational phenomena on astrophysical scales, characteristic timescales are much longer than the light-crossing time of the system. In this quasi-static regime, $\partial_t^2 \phi$ is negligible compared to spatial gradients, and the field equation reduces to:

$$\nabla^2 \phi = -\frac{1}{c^2} S[\rho] \tag{103}$$

Modeling a localized soliton of rest mass m as an effective source $S[\rho] = 4\pi G c^2 \rho$, the equation becomes:

$$\nabla^2 \phi = 4\pi G \rho \tag{104}$$

which is precisely the Poisson equation for the Newtonian gravitational potential. The physical gravitational potential is identified consistently with the main text as:

$$\Phi = c^2 \ln\left(\frac{\partial_t \chi}{c}\right) \simeq \frac{c^2}{\bar{\chi}}\phi \quad (|\phi| \ll \bar{\chi}) \tag{105}$$

*Emergence of the Gravitational Constant G The gravitational constant G emerges as an **effective coupling constant** derived from the microscopic parameters K_0 and χ_c:

$$G = \frac{c^4}{16\pi K_0 \chi_c^2} \tag{106}$$

This relation should be understood as a consistency condition fixing the combination $K_0\chi_c^2$ once the observed value of G is imposed, rather than as an independent prediction of the theory. This relation illustrates how the observed value of G constrains the microscopic parameters of the theory. For example:

- If $\chi_c \approx \ell_P \approx 1.6 \times 10^{-35}\,\mathrm{m}$, then $K_0 \approx 1.3 \times 10^{93}\,\mathrm{m}^{-2}$ to match the observed G.
- If $\chi_c \approx c/H_0 \approx 1.4 \times 10^{26}\,\mathrm{m}$, then $K_0 \approx 1.1 \times 10^{-52}\,\mathrm{m}^{-2}$, reflecting a softer network with cosmological-scale effects.

These examples are not predictions but illustrate how different physical regimes correspond to different effective network stiffnesses compatible with the same observed gravitational constant.

*Equivalence Principle Within this construction, gravitational mass characterizes the strength with which a soliton acts as a source for χ perturbations, while inertial mass is determined by the energy stored in the soliton's χ gradients:

$$M_{\text{inertial}} = \frac{1}{c^2}\int |\nabla\chi_{\text{soliton}}|^2\, d^3x \tag{107}$$

Because both masses originate from the same underlying χ configuration, they coincide within the present effective description:

$$M_{\text{grav}} = M_{\text{inertial}} \tag{108}$$

establishing the equivalence principle as an **emergent property** rather than an independent postulate.

A.20.8 Summary

This appendix has demonstrated explicitly how a purely local collective coupling of the χ field gives rise to an operational notion of distance, an effective spatial geometry, and a Newtonian gravitational interaction in the quasi-static regime. The construction supports the interpretation of gravitation presented in Section D.1 as a collective, emergent phenomenon rooted in the relaxation dynamics of the χ field.

D.2 D.2 Estimates of χ-Field Parameters

This section compiles **technical estimates** of the fundamental parameters governing the χ-field dynamics, derived from observational constraints and theoretical consistency. These parameters include:

- The **connectivity scale** K_0 (Section 3.2),

- The **characteristic scale** χ_c (linked to the Planck length or Hubble scale),
- The **potential parameters** λ and η (Section B.3),
- The **relaxation speed** c and its relation to the speed of light.

D.2.1 Connectivity Scale K_0 and Characteristic Scale χ_c

The connectivity matrix K_{ij} (Equation (2)) depends on two fundamental scales:

- K_0: The maximal connectivity strength, with dimensions of $[\text{length}]^{-2}$.
- χ_c: The characteristic scale of the χ field, associated with either the Planck length $\ell_P \approx 1.6 \times 10^{-35}$ m or the Hubble scale $c/H_0 \approx 1.4 \times 10^{26}$ m.

From the emergent gravitational constant (Equation (106)):

$$G = \frac{c^4}{16\pi K_0 \chi_c^2}, \tag{109}$$

we derive two possible regimes for (K_0, χ_c):

1. **Planck-scale regime**:
   - If $\chi_c \approx \ell_P$, then $K_0 \approx 1.3 \times 10^{93}$ m^{-2}.
   - This regime suggests that χ-field effects become significant at quantum gravitational scales.
2. **Cosmological-scale regime**:
   - If $\chi_c \approx c/H_0$, then $K_0 \approx 1.1 \times 10^{-52}$ m^{-2}.
   - This regime implies a "softer" network with cosmological-scale effects dominating the dynamics.

D.2.2 Potential Parameters λ and η

The soliton mass spectrum (Section B.3) depends on the potential parameters λ and η via:

$$m_{\text{soliton}} \propto \sqrt{\lambda}\eta^3. \tag{110}$$

To reproduce the electron mass ($m_e \approx 9.11 \times 10^{-31}$ kg), we require:

- For a kink soliton (Section B.3):
  $$\lambda \sim 10^{-116} \text{ m}^{-2}, \tag{111}$$
  assuming $\eta \sim 1$ in natural units. This tiny value suggests that λ is **dynamically generated** rather than fundamental.
- For skyrmions (fermions), the mass ratio $m_p/m_e \approx 1836$ requires a hierarchical structure in λ or η (Section B.8).

D.2.3 Relaxation Speed and Cosmological Constraints

The maximal relaxation speed c is identified with the speed of light. From the Hubble parameter relation (Section C.3.2):

$$H_0 \approx \frac{c}{\chi(t_0)}, \tag{112}$$

we infer:

- $\chi(t_0) \approx 4 \times 10^{26}$ m (consistent with the Hubble radius).
- The age of the universe $t_0 \approx \chi(t_0)/c \approx 13.8$ Gyr (Section C.3.3).

D.2.4 Observational Constraints on χ-Field Parameters

Current observational constraints (Section C.3.1) include:

- **CMB anisotropies**: The χ-field fluctuations must reproduce the observed CMB power spectrum, implying $\chi_c \lesssim c/H_0$ to avoid overproducing large-scale power (Section C.1).
- **Hubble tension**: The local value of H_0 suggests $\chi(t_0) \approx 4 \times 10^{26}$ m, while CMB-based measurements probe smaller χ values, contributing to the $\sim 8\%$ discrepancy (Section 8.13).
- **Gravitational wave propagation**: The absorption of gravitational waves near compact objects constrains K_0 to ensure $\lesssim 10\%$ attenuation (Section C.4.1).

D.2.5 Summary of Parameter Ranges

The current best estimates for the χ-field parameters are summarized in Table 4.

Table 4: Estimated Ranges for χ-Field Parameters

Parameter	**Planck-Scale Regime**	**Cosmological-Scale Regime**
K_0	1.3×10^{93} m^{-2}	1.1×10^{-52} m^{-2}
χ_c	1.6×10^{-35} m	1.4×10^{26} m
λ	$\sim 10^{-116}$ m^{-2}	$\lesssim 10^{-100}$ m^{-2}
η	~ 1 (natural units)	$\gg 1$

D.2.6 Open Questions

Key unresolved issues include:

- The **explicit form of** $V(\chi)$ required to stabilize solitons at the observed mass scales (Section B.8).
- The **connection between** λ **and** η across different particle species (e.g., electrons vs. protons).
- The **environmental dependence** of K_0 and χ_c (e.g., in high-density regions like galaxy clusters).

D.3 Simulation Algorithms for χ-Field Dynamics

This appendix outlines a concrete numerical program aimed at testing the spectral hypothesis for inertial mass in Cosmochrony. The goal is not to compute the global spectrum of a homogeneous relaxation network, but to extract the stability spectrum of localized solitonic configurations of the χ field, whose lowest eigenmodes encode the inertial masses of particle-like excitations.

D.3.1 Discrete Relaxation Network and Baseline Operator

We consider a discrete relaxation network $G(V, E)$ with fixed geometric couplings $K_{ij}^{(0)}$. The corresponding baseline relaxation operator is the graph Laplacian

$$(\Delta_G^{(0)}\psi)_i = \sum_{j\sim i} K_{ij}^{(0)}(\psi_i - \psi_j), \tag{113}$$

which encodes the geometric connectivity of the underlying substrate.

Periodic or open boundary conditions may be imposed at large distances to minimize finite-size effects. No physical interpretation is assigned to the global eigenmodes of $\Delta_G^{(0)}$ itself, which represent extended lattice modes rather than particle-like excitations.

D.3.2 Construction of Localized Solitonic Configurations

A localized solitonic configuration χ_{sol} is constructed as a stationary solution of the χ-field relaxation dynamics. Operationally, this configuration may be obtained either by numerical relaxation from a topologically non-trivial initial condition (e.g., kink-, vortex-, or Skyrmion-like profiles) or by imposing an analytical ansatz adapted to the dimensionality of the network.

The soliton is required to be spatially localized, energetically stable, and stationary under the relaxation dynamics, thereby representing a particle-like excitation of the χ field.

D.3.3 Linearized Stability Operator Around a Soliton

Small perturbations $\delta\chi$ around the solitonic configuration,

$$\chi = \chi_{\mathrm{sol}} + \delta\chi, \tag{114}$$

are governed by a linearized stability operator of the form

$$\mathcal{L}_{\mathrm{sol}} = \Delta_G^{(0)} + U_{\mathrm{sol}}, \tag{115}$$

where U_{sol} is a localized restoring operator determined by the background configuration χ_{sol}.

The operator U_{sol} is non-vanishing only in regions where the soliton induces strong gradients or topological constraints. It may be constructed from local geometric quantities such as gradient saturation, discrete curvature, or the second variation of an effective localization functional.

D.3.4 Spectral Problem and Mass Identification

In the regime where an effective wave description applies, perturbations satisfy a Klein–Gordon-type equation,

$$\left(\frac{1}{c^2}\partial_t^2 + \mathcal{L}_{\mathrm{sol}}\right)\delta\chi = 0. \tag{116}$$

Seeking normal-mode solutions $\delta\chi(t) = e^{-i\omega_n t}\psi_n$ leads to the spectral problem

$$\mathcal{L}_{\rm sol}\psi_n = \lambda_n \psi_n, \qquad \omega_n^2 = c^2\lambda_n. \tag{117}$$

The inertial masses associated with localized modes are identified as

$$m_n = \frac{\hbar}{c}\sqrt{\lambda_n}. \tag{118}$$

A crucial diagnostic is the spatial localization of ψ_n, ensuring that the corresponding eigenmodes represent soliton-bound states rather than extended lattice modes.

D.3.5 Numerical Implementation and Diagnostics

The operator $\mathcal{L}_{\rm sol}$ is sparse and can be diagonalized using iterative eigensolvers such as ARPACK or LOBPCG. Shift–invert techniques are employed to resolve the lowest eigenvalues when large spectral gaps are present.

Localization of eigenmodes is quantified using measures such as the inverse participation ratio (IPR). Robustness of the extracted spectrum is tested against variations in lattice size, boundary conditions, and moderate deformations of the solitonic configuration.

D.3.6 Success Criteria and Falsifiability

A successful outcome of this numerical program would be the emergence of a small number of localized eigenmodes whose mass ratios fall within the observed ranges of elementary particles (e.g., electron, pion, proton) up to order-of-magnitude accuracy, without fine-tuning of microscopic parameters.

Conversely, failure to obtain such hierarchical spectra under controlled conditions would directly falsify the spectral hypothesis for mass generation in Cosmochrony.

D.3.7 Preliminary Spectral Results on Paired Localized Modes

As an initial validation of the numerical methodology, we have performed a series of one-dimensional toy simulations involving pinned kink–antikink configurations. While these simulations are not intended to model physical particles directly, they provide a controlled setting to probe the qualitative structure of the stability spectrum associated with localized χ configurations.

The appearance of degenerate low-lying eigenvalues correlates with strong spatial localization, as quantified by the inverse participation ratio (IPR), and reflects the duplication of local excitations associated with two equivalent solitonic centers.

When two identical localized centers are present, the lowest eigenvalues of $\mathcal{L}_{\rm sol}$ systematically appear as degenerate pairs. Inverse participation ratio (IPR) diagnostics confirm that each member of a degenerate pair is strongly localized around one of the two centers. Introducing a small asymmetry in the pinning strengths immediately lifts the degeneracy and increases the localization of each mode, demonstrating that these paired eigenvalues correspond to independent local excitations rather than global symmetric or antisymmetric combinations.

As the separation between the kink and antikink is reduced, a clear transition is observed. Below a critical separation, localized low-lying modes disappear entirely, and the spectrum becomes dominated by extended lattice modes. Near this threshold, a soft quasi-zero mode emerges, characterized by a very small eigenvalue and moderate localization. This behavior is consistent with a weakly constrained local displacement (or sliding) degree of freedom, rather than with a tunneling-induced level splitting.

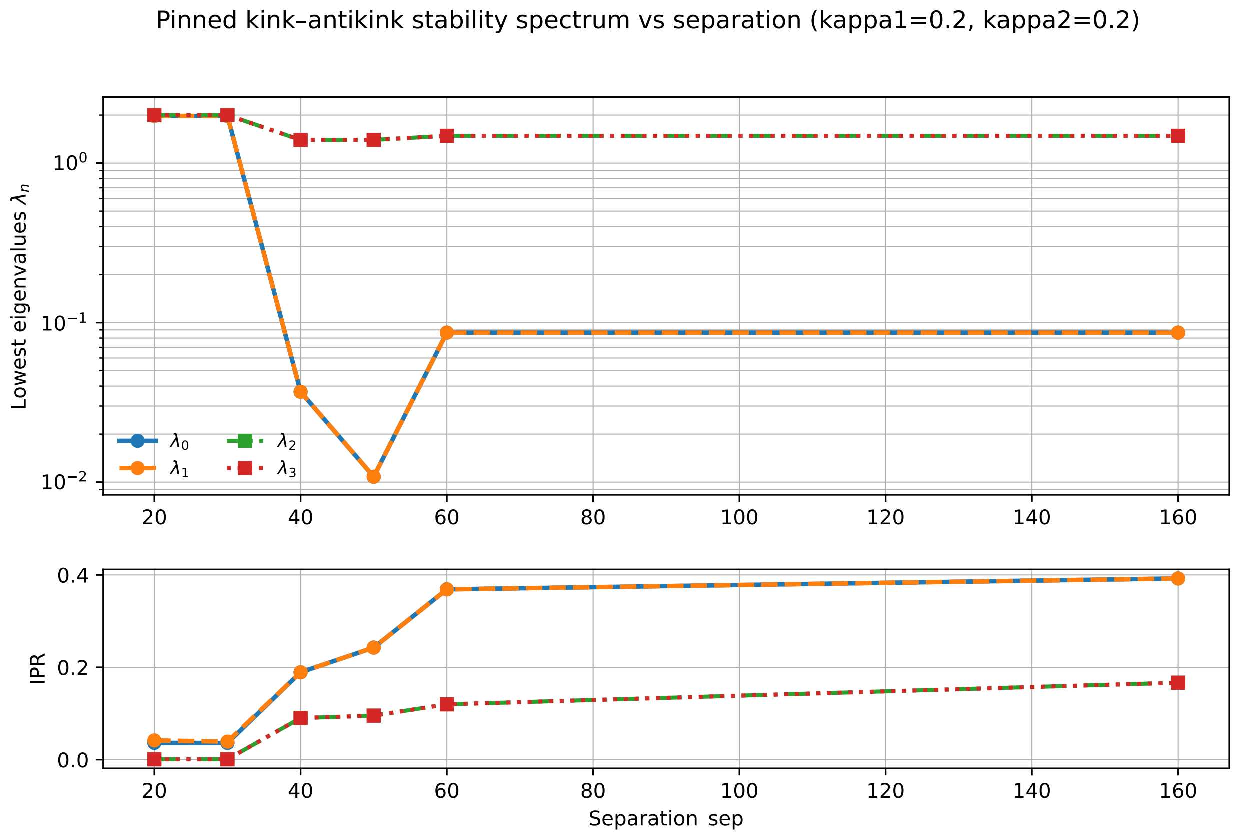

Figure 9: Emergence and pairing of localized stability modes in a one-dimensional pinned kink–antikink configuration. Degenerate low-lying eigenvalues correlate with strong spatial localization (high IPR) and reflect the duplication of local excitations associated with two equivalent solitonic centers, rather than particle–antiparticle creation.

These preliminary results indicate that the pairing of low-lying eigenmodes arises from the duplication of local degrees of freedom associated with multiple solitonic centers, and not from a dynamical coupling mechanism. They provide a concrete illustration of how particle-like excitations in Cosmochrony are expected to emerge as localized stability modes tied to individual solitons, with degeneracies controlled by symmetry and lifted by small perturbations.

E Glossary of Core Quantities and Notation

This appendix summarizes the meaning and status of the main quantities used throughout the Cosmochrony framework. It is intended as a reference guide and does not introduce new assumptions or definitions.

E.1 Fundamental and Effective Quantities

χ **(Chi field).** The fundamental scalar quantity of the Cosmochrony framework. χ is not defined on a pre-existing spacetime manifold but constitutes a pre-geometric substrate whose monotonic relaxation provides an intrinsic ordering of physical processes. Localized, topologically stable configurations of χ correspond to particle-like excitations.

$V(\chi)$ **(Effective potential).** An effective, coarse-grained description used to model localization and stability properties of χ configurations. $V(\chi)$ is not assumed to be fundamental; its form may emerge from underlying discrete relaxation dynamics and is secondary to the spectral description of mass.

K_{ij} **(Relaxation coupling).** Edge-dependent coupling coefficients defined on the relaxation network $G(V,E)$. K_{ij} quantify the local resistance to relative variations of χ between neighboring nodes and encode geometric and topological information of the network. They may depend on the local configuration of χ in effective descriptions.

E.2 Derived Operators and Dimensionless Parameters

$G(V,E)$ **(Relaxation network).** A discrete graph representing the underlying relational structure on which the χ field is defined at the fundamental level. Vertices correspond to elementary degrees of freedom, and edges encode relaxation couplings.

Δ_G **(Graph Laplacian / relaxation operator).** The discrete Laplace–Beltrami operator associated with the network $G(V,E)$ and the couplings K_{ij}. It governs the stability and mode structure of localized χ configurations. Its spectral properties play a central role in the quantitative characterization of inertial mass.

S **(Gradient saturation parameter).** A dimensionless quantity defined as

$$S \equiv \frac{1}{c^2} \sum_{j \sim i} K_{ij} (\chi_i - \chi_j)^2, \tag{119}$$

measuring the local density of χ gradients. The condition $S \leq 1$ ensures causal consistency and bounds the local relaxation rate of χ.

λ_n **(Spectral eigenvalues).** The eigenvalues of the linearized relaxation or stability operator acting on small fluctuations around a localized configuration. In effective wave descriptions, $\sqrt{\lambda_n}$ determines the inertial mass scale of particle-like excitations.

Ω_χ **(Relaxation budget parameter).** A dimensionless global quantity characterizing the fraction of the total χ relaxation budget stored in spatial gradients. In cosmological regimes, Ω_χ plays a role analogous to the matter density parameter in standard cosmology.

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