Cosmochrony: A Pre-geometric Framework for Emergent Spacetime, Dynamics, and Matter

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Abstract

We propose *Cosmochrony*, a foundational physical framework in which time, inertia, and spacetime geometry emerge from the irreversible relaxation of a single fundamental entity χ. Unlike conventional field theories defined on a pre-existing spacetime, Cosmochrony treats relaxation as the primary physical process from which temporal ordering, effective metrics, and dynamical laws are derived *ab initio*.

Localized, long-lived excitations of the χ substrate appear as topologically and spectrally stable solitonic configurations. Fermionic properties arise from topological obstructions in the configuration space of χ, while inertial mass is not postulated but emerges as resistance to global relaxation, quantified by the stability spectrum of the configuration. By resolving the circularity between configuration and distance through spectral graph methods, the effective metric becomes an emergent property. This interpretation naturally recovers $E = mc^2$ as a kinematic identity, while spin, statistics, and fermionic 4π periodicity follow from the nontrivial topology of projected configurations.

The Standard Model phenomenology emerges as a projection Π of the substrate's dynamics. Gauge mediators are reinterpreted as specific projection modes: the photon as scalar transmittance, and the W/Z bosons as shear modes of the projection fiber, accounting for their mass without a fundamental Higgs field. Mass generation arises from the *spectral overlap* between localized configurations and the global relaxation background. Strong interactions and confinement follow from the topological stability of knotted configurations ($Q = 3$), where color charge reflects internal degrees of geometric coherence.

Within the same projection-based ontology, the framework provides a unified interpretation of the dark sector. Dark Matter corresponds to non-projected spectral density contributing to inertia and gravitation without admitting a stable effective-field representation, while Dark Energy manifests as the global, irreversible relaxation flux of χ.

Cosmochrony does not aim to replace the Standard Model or General Relativity at accessible energies, but to supply a deeper explanatory layer in which their structures emerge from a common physical origin. The framework delineates explicit numerical programs for validation via lattice-based simulations and clarifies the regimes in which effective field theories and spacetime descriptions remain valid.

Keywords: Emergent spacetime, quantum gravity, cosmology, geometric frameworks

Contents

1 Introduction

Modern fundamental physics is built upon two highly successful yet conceptually distinct frameworks: quantum mechanics and general relativity [1, 2]. Quantum theory accurately describes microscopic phenomena, while general relativity provides a geometric account of gravitation and spacetime dynamics at macroscopic and cosmological scales. Despite their empirical success, these theories rely on incompatible foundational assumptions and resist unification within a single coherent conceptual framework [3–5].

Quantum mechanics presupposes a fixed spacetime arena in which physical states evolve, whereas general relativity identifies spacetime geometry itself as a dynamical entity. Numerous approaches have attempted to bridge this tension, including quantum field theory in curved spacetime, canonical and covariant quantum gravity programs, and string-based or holographic frameworks. While these approaches have led to important theoretical developments, they typically introduce extended mathematical structures or additional degrees of freedom whose physical interpretation and empirical accessibility remain unclear.

In this work, we explore a complementary and deliberately minimalist framework, referred to as *Cosmochrony*. The central hypothesis is that spacetime geometry, gravitation, and quantum phenomena emerge from the dynamics of a single continuous fundamental entity, denoted χ, whose effective projections admit a scalar description. The field χ is not defined on a pre-existing spacetime manifold, nor is it interpreted as a conventional physical field propagating within spacetime. Rather, spacetime notions themselves arise as effective descriptions of the relational and dynamical properties of χ configurations.

The fundamental dynamical postulate of Cosmochrony is that χ undergoes an irreversible relaxation process, locally bounded by an invariant propagation speed c. This monotonic evolution provides an intrinsic ordering of physical processes, identified with physical time. Spatial relations emerge relationally from differences, gradients, and correlations of χ once a stable geometric regime is reached. Within this perspective, spacetime expansion, gravitation, particle-like excitations, radiation processes, and quantum correlations are not fundamental ingredients, but emergent phenomena associated with specific configurations or interactions of the underlying field.

Cosmochrony does not aim to replace the Standard Model or general relativity in their empirically validated domains, nor does it claim to provide a final unification of quantum theory and gravitation. Instead, it offers an exploratory and internally coherent framework designed to clarify the physical origin of time, geometry, gravitation, and quantum correlations within a single relational dynamics. Standard geometric and quantum formalisms are recovered only at an effective, coarse-grained level, applicable when χ admits a stable spacetime interpretation.

Accordingly, quantities such as coordinates, metric structure, variational principles, and differential geometry are not treated as fundamental. They are employed later in the paper as emergent descriptive tools, rather than as primary postulates of the theory. Technical reconstructions and mathematical details are therefore confined to the appropriate effective regimes and collected in the appendices.

The structure of the paper is as follows. Sections 2–4 introduce the conceptual motivations and minimal dynamical assumptions governing the χ substrate. Subsequent sections examine how particle-like excitations, gravitation, quantum correlations, and cosmological behavior emerge in appropriate regimes.

1.1 Conceptual Context and Related Approaches

The idea that spacetime geometry and gravitation may be emergent rather than fundamental has been explored in a variety of recent theoretical frameworks. Several approaches treat the spacetime metric as an effective description arising from deeper geometric, informational, or dynamical structures, and interpret gravitation as a collective or emergent phenomenon rather than a fundamental interaction[6, 7].

Like Loop Quantum Gravity (LQG), Cosmochrony holds that spacetime geometry is not fundamental [8]. However, the two frameworks operate at distinct conceptual levels.

LQG provides a quantized description of geometry once a spacetime structure is already in place, encoding area and volume through spin networks and holonomies. Cosmochrony, by contrast, addresses an earlier stage: it proposes a pre-geometric substrate, described by a single scalar field χ, from which geometric notions themselves emerge.

In this sense, Cosmochrony does not compete with LQG but precedes it, offering a complementary framework that aims to explain the physical origin of the geometric degrees of freedom subsequently quantized in LQG.

For convenience, a glossary summarizing the main quantities and operators used throughout the article is provided in Appendix F.

2 Theoretical Context and Motivation

2.1 Conceptual Tension Between Quantum Theory and Gravitation

Quantum mechanics and general relativity differ not only in their mathematical formalisms, but also in their foundational concepts. Quantum theory is intrinsically probabilistic, relies on a fixed causal structure, and treats time as an external parameter [1, 9]. General relativity, by contrast, describes gravitation as the dynamics of spacetime geometry itself, with time acquiring a coordinate-dependent and observer-relative status [2, 3].

This conceptual mismatch becomes particularly acute in regimes where both quantum effects and strong gravitational fields are expected to be relevant, such as near spacetime singularities or in the early universe [10, 11]. Direct attempts to quantize gravity encounter persistent difficulties, including the problem of time, non-renormalizability, and the absence of a preferred background structure.

2.2 Limitations of Existing Unification Approaches

Several major research programs have sought to address these challenges. Quantum field theory in curved spacetime successfully accounts for particle creation and vacuum effects, but retains a classical spacetime background [12]. Canonical and covariant approaches to quantum gravity attempt to quantize spacetime geometry itself, often at the cost of substantial mathematical complexity and interpretational ambiguity.

String theory and related frameworks introduce extended fundamental objects and higher-dimensional structures, offering deep mathematical unification but leading to a large space of possible low-energy realizations [5]. While internally rich, these approaches face ongoing challenges concerning empirical testability and physical interpretation.

These limitations motivate the exploration of alternative perspectives in which spacetime geometry, matter, and quantum behavior are not separately postulated, but emerge from a common underlying mechanism.

2.3 Minimalism as a Guiding Principle

The framework developed in this work adopts minimalism as a guiding principle. Rather than introducing multiple fundamental fields, additional dimensions, or independent quantization rules, we explore whether a single continuous fundamental entity can account for both temporal ordering and spatial relations.

The scalar quantity χ is not interpreted as a conventional matter field, nor as a component of spacetime geometry. Instead, it represents a pre-geometric substrate whose irreversible relaxation underlies the emergence of both duration and separation. In this view, time and space are not independent primitives, but complementary aspects of a single dynamical process.

2.4 Time, Irreversibility, and Cosmological Expansion

A central motivation for the Cosmochrony framework is the close connection between time, irreversibility, and cosmological expansion. In standard cosmology, expansion is described kinematically through the scale factor, while the arrow of time is typically attributed to boundary conditions or entropy growth [10, 11, 13].

In Cosmochrony, the monotonic relaxation of χ provides a unified origin for both phenomena. Irreversibility follows directly from the intrinsic directionality of the relaxation process, while cosmological expansion is interpreted as its large-scale geometric manifestation. From this perspective, expansion does not require an externally imposed energy component, but arises as an emergent consequence of the underlying dynamics.

2.5 Scope and Limitations

The aim of this work is exploratory rather than definitive. Cosmochrony does not seek to replace established theories within their empirically validated domains, but to offer a coherent reinterpretation that may clarify persistent conceptual difficulties.

Throughout the paper, emphasis is placed on internal consistency, conceptual clarity, and qualitative contact with observable phenomena, while acknowledging open questions and limitations. In the following section, we introduce χ substrate formally and specify the minimal assumptions underlying its relational dynamics.

3 Definition and Fundamental Properties of the χ Substrate

Having outlined the ontological and conceptual principles underlying Cosmochrony, we now introduce the fundamental quantity at the core of the framework. This section is devoted to defining the pre-geometric substrate χ and clarifying its role as a pre-geometric substrate from which effective notions of spacetime, dynamics, and physical observables may emerge.

The purpose of this section is not to assume a pre-existing spacetime structure, but to identify the minimal properties required of χ in order to recover, in appropriate regimes, effective descriptions of time, space, metric geometry, and field dynamics. Accordingly, χ is introduced independently of any spacetime coordinates or metric structure, and only later related to geometric notions once a stable spacetime interpretation becomes meaningful.

Throughout this section, the use of variational principles, Lagrangian formulations, or metric-based expressions does not imply that spacetime or a four-dimensional manifold is fundamental. Such formalisms are employed strictly as effective, coarse-grained tools to describe the dynamics of χ in regimes where its configurations admit a geometric interpretation. They should be understood as descriptive representations of the underlying pre-geometric dynamics, not as primary postulates of the theory.

We begin by providing a unified conceptual definition of the χ substrate and its physical interpretation. The subsequent subsections introduce progressively more structured effective descriptions, including Lagrangian and metric formulations, which become applicable only once the underlying χ configurations support a stable spacetime regime.

3.1 Definition of the χ Field

We postulate the existence of a single pre-geometric relational substrate, denoted χ, which constitutes the primitive substrate of physical reality. The quantity χ is not defined on a pre-existing spacetime manifold and does not presuppose any metric, causal, or geometric structure. Instead, spacetime notions arise only as effective descriptions of the relational and dynamical properties of χ configurations.

Ontologically, χ is not a scalar order parameter and does not possess values. Scalar order parameters arise only at the effective level, as coarse-grained descriptors of projected χ configurations once a geometric regime is established. Dimensional quantities associated with length or time arise only at the effective level, once χ configurations admit a geometric interpretation. The monotonic ordering intrinsic to χ configurations gives rise, upon projection, to what is operationally perceived as temporal flow.

Temporal ordering emerges from the global, monotonic ordering intrinsic to χ configurations across physical processes, establishing an intrinsic arrow of time without reference to an external temporal coordinate. Spatial separation, in turn, arises from relational differences between χ configurations, giving rise to an effective notion of distance once a stable geometric regime is established. In this sense, time corresponds to ordering, while space corresponds to relational structure.

At no stage is χ interpreted as a spacetime coordinate or as a material field propagating on spacetime. Spacetime coordinates and metric structure appear only as secondary, coarse-grained constructs, becoming meaningful when χ configurations admit a quasi-stable geometric interpretation. The spacetime metric thus functions as an emergent, effective descriptor of resistance to χ relaxation[1] and of the propagation of perturbations within the field.

The analogy with thermodynamic order parameters applies only at the effective level: χ itself is not an order parameter, but gives rise to effective order parameters once projected. In the Cosmochrony framework, χ therefore provides the minimal ontological basis from which time, space, gravitation, and quantum phenomena jointly emerge.

In the following sections, spacetime coordinates and metric quantities will be introduced strictly as effective tools, valid in regimes where χ admits a stable geometric interpretation.

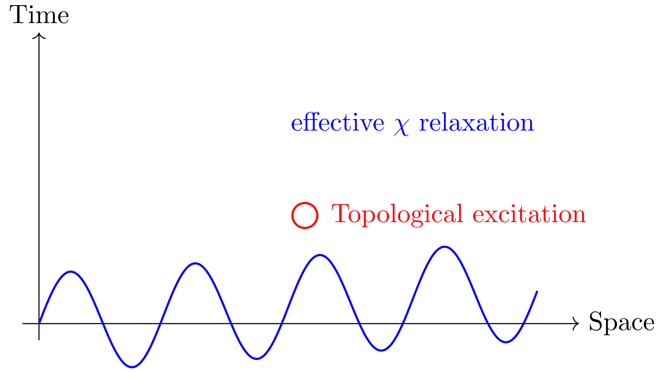

Fig. 1 Conceptual representation of Cosmochrony. An effective spacetime depiction of the projected scalar description of χ, used for visualization purposes only. The monotonic relaxation of χ gives rise to an effective temporal ordering, while localized topological excitations correspond to particle-like configurations in the emergent geometric regime.

On the use of spacetime language.

Throughout this work, phrases such as "spacetime coordinates," "metric tensor," and "four-dimensional manifold" appear frequently, for the sake of clarity and effective description. These should be understood as *emergent effective descriptions* valid in regimes where χ has relaxed into a quasi-stable geometric configuration. They are not fundamental ingredients of the theory.

At the deepest level, only χ and its internal relational variation structure exist. The appearance of familiar geometric language reflects the effectiveness of spacetime as a coarse-grained description of collective χ behavior, analogous to how thermodynamic variables (temperature, pressure) emerge from molecular dynamics without those variables being fundamental.

This interpretational stance is essential for distinguishing Cosmochrony from approaches that merely reformulate existing geometric theories in different variables.

[1] The term "relaxation" is used here in a geometric and dynamical sense, and should not be confused with thermodynamic relaxation processes involving dissipation or entropy increase.

3.2 The Geometric Effective Description of χ Dynamics

Effective Observables from χ Correlations

In Cosmochrony, quantities conventionally described in geometric terms—such as time intervals, spatial separation, and causal ordering—are not taken as primitive. They arise as effective descriptive summaries of relational patterns within the χ substrate, accessed only through projected, coarse-grained representations. No background spacetime, coordinate system, or discrete substrate is assumed at any stage of the fundamental description.

The configurations σ represent internal states of the χ relational substrate and are defined without reference to external spacetime coordinates or background geometry. They label relational degrees of freedom of the substrate, specifying patterns of internal organization rather than positions in a pre-existing space.

Correlations between configurations σ encode the emergent geometric and causal structure at the effective level, once a stable spacetime description becomes applicable. The measure $d\mu(\sigma)$ denotes an invariant integration over this space of configurations, defined intrinsically from the correlation structure associated with χ. It ensures that physical observables are independent of the particular parametrization chosen to label configurations, and carries no interpretation as a volume element in an underlying spacetime.

Effective scalar descriptor.

In regimes where projected χ configurations admit a stable geometric interpretation, it is convenient to introduce an *effective scalar descriptor*, denoted χ_{eff}. This quantity is a coarse-grained, projected representation of relational features of the χ substrate, defined only within the emergent spacetime description. It does not correspond to the fundamental χ substrate itself, which remains non-indexable and devoid of intrinsic values. Although χ_{eff} is represented as a scalar function in effective descriptions, it should not be interpreted as a fundamental field propagating in spacetime.

Operational time intervals are defined from the accumulated ordering of projected χ configurations along paths in configuration space. This ordering is quantified using the effective descriptor χ_{eff}, not the fundamental χ substrate:

$$\tau_{AB} \propto \int_{\gamma_{AB}} \mathcal{D}_\lambda \chi_{\text{eff}} \, d\lambda, \tag{1}$$

where $\mathcal{D}_\lambda \chi_{\text{eff}}$ is an effective relaxation functional characterizing the ordering of projected χ configurations along the path γ_{AB}. The parameter λ is an ordering parameter, not a fundamental time coordinate.

Geometric observables are constructed from correlations between effective scalar descriptors χ_{eff}, associated with projected χ configurations. These correlations encode relational information about causal connectivity and separation once a geometric regime is established.

Operational spatial separation is quantified by the decay of correlations between effective scalar descriptors χ_{eff} associated with projected χ configurations:

$$d(x, y) \;\propto\; -\log\left(\frac{\langle \chi_{\mathrm{eff}}(x)\, \chi_{\mathrm{eff}}(y)\rangle}{\langle \chi_{\mathrm{eff}}^2\rangle}\right), \tag{2}$$

where $\langle \chi_{\mathrm{eff}}(x)\chi_{\mathrm{eff}}(y)\rangle$ measures the correlation between effective descriptors associated with configurations labeled by x and y. Here x and y label effective spacetime events in the emergent description. The correlation function involves χ_{eff} only and does not imply any localization or value assignment at the level of the fundamental χ substrate.

These definitions are purely relational and make no reference to a pre-existing metric or discrete structure. They are applicable whenever projected χ configurations exhibit stable correlation patterns.

Throughout this section, all indexed, correlated, or integrated quantities refer exclusively to the effective descriptor χ_{eff}, not to the fundamental χ substrate itself.

Effective Metric as a Descriptive Tool

In regimes where projected χ configurations exhibit smooth and stable correlation patterns, the relational observables defined above may be compactly summarized by an operational tensor $g_{\mu\nu}[\chi_{\mathrm{eff}}]$. The notation $g_{\mu\nu}[\chi_{\mathrm{eff}}]$ emphasizes that the metric summarizes correlations of effective descriptors, not properties of the fundamental χ substrate. This object is not introduced as a fundamental geometric structure, nor as an independent degree of freedom. Rather, it provides a convenient parametrization of how variations in χ_{eff} modulate causal connectivity and effective relational intervals, and is meaningful only insofar as such a coarse-grained description remains valid.

The metric is introduced as a descriptive summary of χ_{eff} correlations, not postulated as an independent degree of freedom. For example:

- The conformal (lightcone) structure is constrained by the maximal effective relaxation speed c associated with χ_{eff}.
- Proper time between effective events is proportional to the accumulated χ_{eff} relaxation along paths connecting the corresponding configurations.
- Spatial distance reflects the decay rate of χ_{eff} correlations.

No discrete-to-continuum limit is invoked. The theory is continuous at all scales; apparent granularity (e.g., Planck-scale phenomena) is attributed to non-linear χ dynamics rather than to an underlying discretization.

No background $\eta_{\mu\nu}$ is assumed. Minkowski space appears only as a convenient approximation in suitable limits (e.g., weak-gradient regimes), without ontological status in the fundamental description.

Consistency with General Relativity

The effective metric $g_{\mu\nu}[\chi_{\mathrm{eff}}]$, constructed as a summary of χ_{eff} correlations, reproduces the phenomenology of general relativity in the following sense:

- **Weak-field limit:** when χ_{eff} gradients are small, the effective metric approaches a form compatible with Einstein-like dynamics for a fluid-like stress-energy description associated with χ excitations.
- **Strong-field regimes:** near localized χ excitations (e.g., solitons), the metric encodes time dilation and spatial curvature as emergent effects of slowed χ_{eff} relaxation, without requiring a fundamental gravitational field.
- **Cosmological expansion:** homogeneous relaxation of χ yields an effective Hubble-like expansion law for the emergent scale factor.

Crucially, this is not a bootstrap procedure. The metric is not iteratively reconstructed from χ; it is a post-hoc descriptive tool summarizing geometric regularities of projected χ configurations. The theory's predictive content resides in the dynamics of χ, not in the metric itself.

Ontological Status of the Metric

To avoid confusion, we emphasize:

- χ is the only fundamental entity. Spacetime, metric structure, and matter are emergent descriptions of projected χ configurations.
- No "double ontology" is assumed: there is no underlying discrete graph or lattice. Geometric language is introduced only as an effective descriptive tool.
- The metric $g_{\mu\nu}[\chi_{\text{eff}}]$ is an effective construct, analogous to how temperature emerges in thermodynamics. It is useful for coarse-grained description but plays no role in the fundamental dynamics.

Operational origin of the effective metric.

The metric tensor $g_{\mu\nu}$ is not a fundamental structure but a derived descriptor, summarizing the relational distance induced by χ correlations. Its explicit construction from operational distances is given in Appendix E, where geometric quantities are shown to arise only in projectable regimes admitting a smooth continuum approximation.

Operational interpretation of the line element.

In Cosmochrony, the line element ds^2 is *not* a primitive geometric quantity. It is an effective descriptor that becomes meaningful only in projectable regimes where the relational distance structure admits a smooth local approximation.

Concretely, the fundamental object is the operational distance d_{ij} defined on the relational network of χ (Appendix E). Once a low-dimensional embedding exists, the squared distance between nearby embedded points can be locally approximated by

$$d(i,j)^2 \;\approx\; g_{\mu\nu}(x)\,\Delta x^\mu \Delta x^\nu,$$

thereby introducing $g_{\mu\nu}$ as a *derived quadratic form* summarizing relational connectivity. The continuum line element ds^2 should therefore be understood as a shorthand for this local approximation, not as a fundamental structure.

Summary: A Fully Continuous Framework

- **Fundamental level:** only the continuous χ relational substrate exists, characterized by an intrinsic ordering structure.
- **Effective level:** geometric observables (time, distance, metric) emerge from correlations between effective descriptors χ_{eff} associated with projected χ configurations.
- **No bootstrap:** the metric is never iteratively constructed or assumed; it is a derived description summarizing the relational structure of projected χ configurations.
- **No discretization:** apparent "Planck-scale" effects arise from non-linear χ dynamics, not from an underlying discrete substrate.

3.3 Physical Interpretation

In Cosmochrony, spacetime is not assumed as a pre-existing background structure. Instead, it appears as an effective macroscopic description arising from the continuous and monotonic ordering intrinsic to the relational substrate χ. What are conventionally described as temporal and spatial features are understood here as distinct, but related, descriptive manifestations of this single underlying process, once a geometric regime becomes applicable.

In regimes where projected χ configurations exhibit sufficiently stable and smooth correlation patterns, variations of the effective scalar descriptor χ_{eff} give rise to a set of operational observables. In particular, an increase in χ_{eff} along a given physical process is associated with:

- the accumulation of operational proper time along that process,
- the progressive decorrelation between effective configurations, summarized as an emergent spatial separation,
- the large-scale expansion behavior observed when the ordering of projected χ configurations is considered at the cosmological level.

Within this effective description, temporal duration and spatial separation are not independent primitives. They represent complementary aspects of the same underlying ordering structure, captured at different levels of coarse-graining. Heuristically, effective distance may be viewed as the persistent imprint of relational differentiation that has already occurred, while effective time corresponds to the ongoing local ordering of projected χ configurations. These expressions are intended as interpretative guides rather than literal definitions, emphasizing their common dynamical origin.

This unified interpretation is not introduced ad hoc. It follows directly from identifying temporal ordering and relational separation as distinct effective summaries of the same underlying χ structure, once a macroscopic spacetime description becomes appropriate. The physical content of the theory therefore resides entirely in the dynamics of the fundamental χ substrate, while spacetime notions serve as emergent descriptive tools valid only within restricted regimes.

3.4 Monotonicity and Arrow of Time

A central structural assumption of Cosmochrony is that the relational substrate χ admits an intrinsic, globally ordered relaxation structure. This ordering is reflected, within effective descriptions, by the monotonic behavior of the projected scalar descriptor χ_{eff} along admissible physical processes:

$$\mathcal{D}_\lambda \chi_{\text{eff}} \geq 0. \tag{3}$$

Here λ denotes an ordering parameter associated with the relaxation of projected χ configurations, not a fundamental time coordinate. The inequality expresses a structural constraint on admissible projected descriptions, rather than the evolution of a fundamental scalar quantity.

This monotonicity is not introduced as a statistical statement, nor as a boundary condition imposed on an otherwise time-symmetric dynamics. Rather, it reflects an intrinsic asymmetry in the ordering structure of χ configurations, which constrains the form of all effective descriptions compatible with the framework.

Within this perspective, energy is not treated as a fundamental conserved substance, but as an effective measure of the remaining capacity of projected χ configurations to undergo further relaxation. As effective relaxation proceeds, this capacity is irreversibly expended. A hypothetical decrease of χ_{eff} along an admissible ordering path would correspond to a spontaneous restoration of relaxation capacity, effectively reintroducing contraction or tension into the projected description. No mechanism within Cosmochrony allows such a reversal, as it would contradict the underlying ordering structure of χ.

Irreversibility therefore follows directly from the structure of the relaxation ordering admitted by χ. Because admissible projected descriptions cannot exhibit decreasing χ_{eff}, the ordering of configurations induced by relaxation is intrinsically directed. What is conventionally described as the arrow of time is identified here with this directional ordering: the irreversible progression from configurations with greater effective relaxation capacity toward configurations in which that capacity has been exhausted.

Importantly, this arrow is not derived from coarse-graining, probabilistic entropy, or special initial conditions. It arises prior to any statistical or thermodynamic description, as a direct consequence of the structural ordering constraints imposed by χ on its projected representations. Temporal orientation is thus a manifestation of the fundamental ordering structure, rather than an emergent asymmetry imposed at the macroscopic level.

3.5 Local Relaxation Speed

A fundamental structural constraint of the Cosmochrony framework is that the effective local ordering rate associated with projected χ configurations is bounded by a universal constant:

$$|\mathcal{D}_{\text{loc}} \chi_{\text{eff}}| \leq c, \tag{4}$$

where $\mathcal{D}_{\text{loc}} \chi_{\text{eff}}$ denotes an effective local relaxation functional characterizing the maximal admissible ordering of projected χ configurations within the emergent geometric description. The parameter c coincides numerically with the observed speed of light.

This bound does not represent the propagation speed of particles, fields, or signals, nor does it presuppose a pre-existing spacetime structure. Rather, it constrains the maximal rate at which effective causal relations and geometric structure can locally emerge within projected descriptions compatible with the underlying ordering structure of χ.

Superluminal recession velocities at cosmological scales arise naturally through cumulative and global effects of projected χ ordering, and do not violate local causality. Local causal relations remain bounded by the constraint c, which applies exclusively to effective descriptions and not to the fundamental χ substrate itself.

3.6 Relation to Conventional Fields

Although effective descriptions derived from projected χ configurations may exhibit formal similarities with scalar fields employed in cosmology (such as inflaton-like fields), the ontological role of χ is fundamentally different. The χ substrate is not a physical field propagating on spacetime, but a pre-geometric relational structure from which spacetime notions themselves emerge.

Accordingly, χ does not carry energy in the conventional field-theoretic sense, nor is it subject to quantization at the fundamental level. Quantization arises only at the effective level, where certain stable and localized configurations of the χ substrate admit a particle-like interpretation and can be consistently described using standard quantum field-theoretic tools within an emergent spacetime regime.

Within this framework, matter, radiation, and interactions do not correspond to independent fields coupled to χ. They arise instead as effective manifestations of constraints, topological features, or long-lived relational patterns of projected χ configurations. Conventional fields of the Standard Model are thus recovered as effective descriptions of these emergent degrees of freedom in regimes where a spacetime interpretation appl

3.7 Initial Conditions and Global Structure

The Cosmochrony framework does not postulate initial conditions in the conventional temporal sense. Instead, it assumes that the relational substrate χ admits a minimal admissible ordering state, denoted χ_0, corresponding to configurations of maximal effective relaxation density. This reference state characterizes the earliest physically meaningful configurations within projected descriptions, without presupposing a fundamental temporal origin or a distinguished initial instant.

In effective geometric regimes, the characteristic scale associated with projected descriptions near χ_0 coincides numerically with the Planck scale. This correspondence reflects the breakdown of coarse-grained spacetime descriptions below this regime, rather than the presence of a fundamental cutoff, discreteness, or microscopic spacetime structure.

From this perspective, cosmic history is interpreted as the progressive ordering of projected χ configurations away from this minimal admissible state. No spacetime singularity is required in the fundamental description. Apparent singular behavior arises only when classical notions of time and distance are extrapolated beyond the regime in which projected χ configurations admit a stable geometric interpretation.

The global structure of admissible projected descriptions is thus constrained by the ordering properties of the underlying χ substrate, rather than by arbitrarily specified initial data. In the following section, we derive a minimal effective dynamical equation governing the ordering of projected χ configurations and explore its immediate physical consequences.

4 Ontological Interpretation of the χ Substrate

Throughout this section, we explicitly distinguish the invariant structural bound c_χ, defined at the level of the pre-temporal χ substrate, from its emergent spacetime manifestation c [14]. The latter appears only once spacetime notions such as distance, duration, and causal propagation become meaningful within effective descriptions.

4.1 The χ Substrate as a Pre-Temporal Structural Plan

In the Cosmochrony framework, the χ substrate is not interpreted as a physical field evolving within spacetime, but as a pre-temporal relational structure from which spacetime, matter, and physical laws emerge. It may be heuristically described as a "structural plan" of the universe: a complete but non-dynamical organization encoding the set of physically admissible configurations and their internal relations.

Importantly, this structural plan does not prescribe a unique history nor a fixed sequence of events. Rather, it defines a constrained space of relational possibilities. Temporal succession is therefore not fundamental, but emergent, corresponding to an oriented resolution of structural relations within χ once an effective spacetime description becomes applicable.

Infra-physical status of χ.

The χ substrate is not described by physics proper, understood as a theory of dynamical fields evolving in spacetime. Rather, it belongs to an infra-physical relational framework that specifies the structural conditions under which physical observables, spacetime geometry, and effective dynamical laws emerge.

4.2 Relational Ontology and Conceptual Lineage

The relational character of the χ substrate bears a conceptual affinity with relational approaches in physics, notably those advocated by Rovelli [8, 15]. These approaches trace their philosophical roots to Aristotelian relational ontology, in which properties are defined through relations rather than as intrinsic attributes [16, 17].

Cosmochrony shares this rejection of intrinsic, observer-independent properties. However, it extends relationalism to a deeper ontological level. In relational quantum mechanics, relations are primary at the level of quantum states describing interactions between systems that are themselves taken as given. By contrast, Cosmochrony posits that the fundamental substrate χ is itself relational: there are no underlying entities between which relations are defined.

In this sense, χ configurations are not relations *between* pre-existing objects, but relational structures that constitute the objects themselves once a projection into an effective spacetime description occurs. Relationality is therefore not a feature of physical states within spacetime, but an intrinsic property of the pre-geometric substrate from which spacetime and physical entities emerge.

This distinction is essential for understanding the ontological asymmetry between χ and its spacetime projections. While relational quantum mechanics reformulates quantum theory without modifying the ontological status of spacetime itself, Cosmochrony relocates relationality at the level of the substrate that gives rise to spacetime, time ordering, and physical objects.

On the absence of fundamental values.

In Cosmochrony, relations are not defined between pre-existing fundamental values. The relational structure of χ is ontologically primary and admits no intrinsic numerical or field-like values. What appear, within effective spacetime descriptions, as scalar values, entities, or local degrees of freedom arise only as stable invariants of this relational structure under projection and coarse-graining.

4.3 Projection, Reality, and Ontological Asymmetry

Within this ontology, the emergence of spacetime should be understood as a projection from the χ substrate, rather than as a dual or equivalent description. The projected universe is fully real at the physical level, but its reality is derivative: spacetime entities, effective fields, and dynamical laws do not possess ontological primacy [18].

This asymmetry is essential. While physical descriptions depend on the projection of χ, the converse is not true. The χ structure exists independently of spacetime notions and does not admit a reformulation entirely in geometric or dynamical terms. Projection must therefore be understood as non-injective: distinct structural features of χ may correspond to identical or physically indistinguishable effective configurations.

Projection and non-circularity.

Because geometric notions arise only after projection, the construction of effective descriptions must not presuppose any metric or temporal structure defined at the effective level. In particular, coarse-graining procedures used to define χ_{eff} must avoid relying implicitly on emergent geometric quantities. This requirement motivates the explicit separation between pre-geometric relational structures and geometry-dependent observables developed in Appendix E, where combinatorial and weighted distances are carefully distinguished.

Projection and emergent time.

The parameter commonly interpreted as time in the effective description is not a fundamental attribute of the χ substrate. Instead, temporal ordering and duration arise only after projection, as part of the emergent spacetime representation associated with χ_{eff}. No external or fundamental time parameter is introduced at the level of χ itself.

Accordingly, all averaging and coarse-graining operations involved in the definition of the background field $\bar{\chi}$ and in the construction of χ_{eff} are formulated relationally, without reference to an underlying temporal metric. Temporal concepts enter the framework only at the effective level, once a stable geometric regime has emerged through projection.

Apparent fine-tuning.

In Cosmochrony, apparent fine-tuning does not reflect an improbable choice of initial conditions or a delicate adjustment of fundamental constants. It arises from the fact that only a restricted class of χ configurations admits a coherent and stable physical projection. Most configurations of the χ substrate do not give rise to consistent spacetime descriptions, observable laws, or persistent physical structures.

The apparent delicacy of physical parameters is therefore a selection effect imposed by the projection itself. Only those relational configurations compatible with a stable emergent geometry and with sustained relaxation dynamics appear as physically realized universes. Fine-tuning is thus reinterpreted as a structural constraint on projectability rather than as a coincidence requiring external explanation.

Absence of a multiverse.

Cosmochrony does not postulate a multiverse. While multiple configurations of the χ substrate may correspond to the same physical universe under projection, the framework provides no mechanism by which a single χ structure could sustain multiple independent physical projections.

The universe is therefore unique at the level of physical reality, even though its underlying description in terms of χ may be non-unique. The absence of a multiverse is not an additional assumption, but a direct consequence of the non-injective and ontologically asymmetric character of the projection.

4.4 Clarifying the Relation to Holographic Descriptions

The preceding considerations naturally invite comparison with the holographic principle, as originally proposed by 't Hooft and Susskind, which suggests that the effective information content of a spacetime region scales with its boundary rather than its volume.

Cosmochrony is *not* a holographic theory in the technical sense. It does not posit a lower-dimensional boundary description, nor a dual equivalence between bulk and boundary physics. Any holographic-like behavior arises here as a consequence of projection from a non-factorizable, pre-geometric substrate, rather than from a fundamental encoding principle.

In particular, the limitation of physically accessible information within a spacetime region reflects the degeneracy of χ configurations compatible with a given projection. This constraint is intrinsic to the emergence of spacetime itself and does not require the introduction of boundary degrees of freedom or dimensional reduction.

Thus, while Cosmochrony reproduces certain qualitative features commonly associated with holographic descriptions, it differs fundamentally in its ontological grounding. Holography appears here as an emergent signature of projection, not as a guiding principle or foundational postulate.

4.5 Intrinsic Structural Indeterminacy and Projective Variability

A perfectly deterministic and fully symmetric relational substrate would remain physically inert: no configuration would be privileged, and no effective ordering could emerge. For this reason, Cosmochrony postulates the existence of *intrinsic structural indeterminacy* at the level of the pre-geometric χ substrate.

This indeterminacy does not correspond to randomness, stochastic dynamics, or temporal fluctuations. It reflects instead a fundamental *non-completeness of structural determination*: χ configurations are not exhaustively specifiable by a finite or closed set of relational constraints. In this sense, intrinsic indeterminacy is ontological rather than dynamical, and should be understood negatively, as the absence of perfect structural closure.

Importantly, this indeterminacy does not introduce any form of temporal evolution or causal process at the level of χ. The substrate itself does not fluctuate, evolve, or explore a space of possibilities in time. Rather, intrinsic indeterminacy prevents χ from collapsing into a perfectly symmetric and physically sterile configuration, thereby allowing relational distinctions to acquire physical relevance once projection into an effective description occurs.

For clarity, the term *fluctuations* is used in this framework only in a *metaphorical* sense when referring to the χ substrate. No underlying agitation, noise, or stochastic process is implied. Intrinsic indeterminacy should instead be understood as a structural openness of the relational substrate, analogous to an underdetermination of global structure rather than to random motion.

Observable variability and probabilistic behavior arise only at the level of *projected* descriptions. Because the projection from χ to an effective spacetime representation is non-injective, a single underlying configuration of χ may admit multiple physically admissible projected realizations. This non-uniqueness of projection gives rise, at the effective level, to phenomena commonly described as fluctuations, probabilistic outcomes, or quantum uncertainty.

In this sense, randomness is not fundamental but *projective*: it reflects the multiplicity of effective descriptions compatible with a given pre-geometric structure, rather than any stochasticity intrinsic to χ itself. The apparent temporal character of fluctuations is therefore an emergent feature of projected descriptions, not a property of the underlying substrate.

Intrinsic structural indeterminacy thus plays a dual conceptual role. At the ontological level, it prevents perfect structural symmetry and ensures that χ remains physically generative. At the effective level, when combined with non-injective projection, it provides the structural origin of observable variability, probabilistic outcomes, and quantum indeterminacy, without introducing fundamental randomness or temporal dynamics into the substrate itself.

4.6 Energy as Capacity for Relaxation

Within this framework, energy is reinterpreted as the capacity of projected χ configurations to relax internal structural constraints. Energy does not correspond to a conserved substance at the fundamental level, but to a measure of unresolved structural tension within admissible projected descriptions.

The deployment of structural information corresponds to the progressive conversion of intrinsic indeterminacy into relational differentiation. Energy thus quantifies the remaining potential for this conversion. Without intrinsic indeterminacy, energy itself would be undefined.

4.7 Mass as Frozen Information

Localized, stable configurations of the χ substrate—describable in effective regimes as particle-like excitations—correspond to regions where further relaxation is strongly inhibited. These configurations trap a fixed amount of unresolved structural information.

In this interpretation, mass represents frozen energy: information that has lost its capacity to participate freely in further relaxation. At the level of emergent spacetime physics, this relation is expressed by the standard mass–energy equivalence,

$$E_{\mathrm{phys}} = m_{\mathrm{phys}} c^2,$$

where c denotes the emergent spacetime limiting speed. This phenomenological identity reflects a deeper structural constraint governing the confinement and release of information within the χ substrate.

4.8 Quarks as Non-Projectable Internal Modes

The ontological status of quarks provides a clarifying example of the distinction between the pre-geometric χ substrate and its effective spacetime projections.

In Cosmochrony, quarks are not interpreted as fundamental particle-like entities, nor as independent localized excitations of χ. Rather, they correspond to internal structural modes of composite solitonic configurations—modes that are necessary to characterize the internal organization and stability of hadronic excitations, but which do not admit an autonomous and coherent projection into spacetime.

In effective quantum field descriptions, quarks appear as elementary degrees of freedom subject to confinement. Within Cosmochrony, this confinement is not imposed dynamically by an external interaction, but reflects a deeper structural constraint: isolated quark-like modes do not correspond to admissible standalone projections of χ_{eff}. Only collective configurations in which these internal modes are topologically and relationally closed admit a stable spacetime manifestation.

In this sense, quarks are real at the structural level of χ, but incomplete at the level of physical projection. They are neither fictitious nor fundamental objects, but non-factorizable internal components of projected excitations. Their observability is therefore necessarily indirect, encoded in the spectral, dynamical, and symmetry properties of hadrons rather than in localized detection events.

This interpretation parallels the status of internal degrees of freedom in other collective systems: they are indispensable for an accurate effective description, yet do not correspond to independently realizable physical entities. Quark confinement thus appears not as a contingent feature of strong interactions, but as a direct consequence of the non-injective nature of the projection from χ to emergent spacetime.

4.9 The Role of the Universal Bound c_χ

A central structural element of the Cosmochrony framework is the existence of a universal invariant bound, denoted c_χ, defined at the level of the pre-temporal χ substrate. This bound does not correspond to a signal velocity, nor to the propagation of any field or excitation in spacetime. Instead, c_χ characterizes an absolute structural limit on the degree to which relational information can be locally constrained within admissible χ configurations.

At the fundamental level, c_χ is non-metric and non-temporal. It is not associated with distances, durations, or causal cones, since none of these notions are defined prior to projection. Rather, c_χ expresses a maximal admissible rate of structural ordering, beyond which further confinement of relational degrees of freedom becomes impossible and relaxation is unavoidable.

Crucially, c_χ is not itself an observable quantity. It acquires operational meaning only through projection, when χ configurations admit a locally injective representation in terms of effective spacetime variables. In such projectable regimes, the invariant structural bound c_χ manifests as an effective causal constraint c, defined through the maximal admissible local ordering rate of projected configurations:

$$c \;\equiv\; \Pi(c_\chi),$$

where Π denotes the projection from the pre-geometric relational substrate to an effective spacetime description.

The constant c, which coincides numerically with the observed speed of light, therefore has a derivative status. It does not represent an independent postulate, but the spacetime expression of the deeper structural bound c_χ once notions of locality, duration, and causal ordering become meaningful. All effective causal constraints appearing in projected descriptions are ultimately inherited from this underlying invariant bound.

Accordingly, the local constraint on the effective relaxation functional,

$$|D_{\mathrm{loc}}\chi_{\mathrm{eff}}| \leq c,$$

should be understood as the projected form of the more fundamental structural limit imposed by c_χ. No causal or dynamical principle is imposed directly at the level of spacetime; effective causality arises solely as a consequence of the bounded projective realization of relational ordering.

This distinction becomes essential in strong-gravity or near-deprojection regimes. While the effective constant c may lose its geometric or causal interpretation when projection ceases to be injective, the structural bound c_χ remains invariant. The breakdown of spacetime description therefore signals not a violation of causality, but the loss of representability of relational configurations within a spacetime framework.

In summary, c_χ defines a universal, pre-temporal structural bound governing the admissibility of χ configurations, while c represents its effective spacetime manifestation. The latter inherits its value and universality entirely from the former, ensuring conceptual continuity between the pre-geometric substrate and emergent relativistic causality.

4.10 The Role of $\hbar_\chi$ and Reprojection from χ

In Cosmochrony, the parameter $\hbar_\chi$ is not identified with the quantum constant $\hbar$, but emerges from the fundamental structural scales K_0, χ_c, and c. Its numerical coincidence with $\hbar$ in quantum regimes reflects the universality of action quantization across effective physical theories.

$\hbar_\chi$ does not represent a quantum of action evolving in time, but a fundamental quantum of reprojection. Intrinsic indeterminacy of χ does not give rise to continuous emergence; rather, any reprojection of structural information into spacetime occurs in discrete units set by $\hbar_\chi$.

As spacetime structure stabilizes, reprojection becomes increasingly localized, manifesting phenomenologically as vacuum fluctuations within otherwise stable regions of spacetime.

4.11 The Origin of Planck's Constant: $\hbar_\chi$ vs $\hbar_{\text{eff}}$

A potential conceptual tension arises regarding the fundamental nature of $\hbar$. We clarify here that $\hbar$ is not an independent postulate but a **spectral invariant** of the relaxation process.

- **Fundamental Substrate Constant ($\hbar_\chi$):** At the level of the χ substrate, the quantum of action is uniquely determined by the ratio of the propagation speed c , the coupling density K_0, and the correlation scale χ_c:

$$\hbar_\chi = \frac{c^3}{K_0 \chi_c} \tag{5}$$

  This indicates that the "graininess" of quantum reality is a direct consequence of the **spectral rigidity** of the substrate.
- **Emergent Scaling ($\hbar_{\text{eff}}$):** In the effective spacetime description, $\hbar_{\text{eff}}$ appears as a scaling parameter. The perceived universality of $\hbar$ stems from the fact that K_0 and χ_c are global invariants of the current relaxation epoch.

In this view, the transition from $\hbar_\chi$ to $\hbar_{\text{eff}}$ is not a change in value, but a change in representation: from a relational constraint in the substrate to a dynamical constant in the projected Hilbert space.

4.12 Spectral Invariance of Planck's Constant and α

The dependency of $\hbar_\chi$ on K_0 and χ_c (Eq. 113) is not a contradiction but a **requirement for spectral unification**. By defining $\hbar_\chi \equiv c^3/(K_0\chi_c)$, we establish that the quantum of action is a manifestation of the substrate's relational density.

- **Resolution of the $\hbar$ tension:** $\hbar_{\text{eff}}$ is the projection of the fundamental relational graininess $\hbar_\chi$.
- Its apparent constancy across spacetime is due to the homogeneity of the relaxation flux Φ_χ in the current epoch.

- **Geometric Origin of α:** The fine-structure constant α then emerges as a **dimensionless spectral ratio**:

$$\alpha = \mathcal{F}\left(\frac{\text{topology of }\Pi}{\text{spectral rigidity }K_0}\right) \tag{6}$$

where $\mathcal{F}$ is a functional determined by the geometry of the projection fiber.

This derivation shows that if K_0 were to evolve (e.g., in the primordial high-constraint regime), $\hbar$ and α would scale accordingly, preserving the **structural coherence** of the theory while allowing for non-stationary quantum laws at cosmological singularities.

5 Dynamical Equation for the χ Field

5.1 Parameter-Independent Relaxation

To avoid the conceptual pitfalls associated with a fundamental time coordinate, the Cosmochrony framework formulates physical evolution without reference to any external temporal parameter. At the fundamental level, χ does not evolve in time and admits no temporal parameterization. Instead, physical evolution is described at the effective level as an ordered sequence of projected χ configurations, denoted $(\chi_{\mathrm{eff},\lambda})$, where λ is a strictly monotonic ordering parameter labeling admissible stages of relaxation within projected descriptions.

This ordering parameter does not represent time, nor does it parametrize a fundamental trajectory of χ. It serves only to label the relational ordering of projected configurations once a macroscopic spacetime description becomes applicable. No fundamental dynamics unfolds with respect to λ at the level of the χ substrate itself.

Accordingly, no fundamental evolution equation is postulated for χ. Instead, admissible projected descriptions are constrained by an effective relaxation condition of the form

$$\mathcal{D}_\lambda \chi_{\mathrm{eff}} = \mathcal{R}[\chi_{\mathrm{eff}}], \tag{7}$$

where $\mathcal{R}[\chi_{\mathrm{eff}}]$ denotes an effective relaxation functional characterizing the ordering of projected χ configurations. This functional is defined only within regimes admitting a stable geometric interpretation and carries no meaning at the fundamental level. No spacetime derivative, metric structure, or geometric operator is assumed in the definition of $\mathcal{R}$.

Quantities commonly interpreted as temporal derivatives arise exclusively at the level of effective descriptions. When projected χ configurations exhibit sufficient regularity, a coordinate time parameter may be introduced as a convenient label for the relaxation ordering. Such a parameter has no fundamental significance and does not modify the underlying structural constraints imposed by χ.

Within this framework, relaxation does not occur *in* time. Rather, the ordering of projected χ configurations defines what is operationally identified as physical duration. Local variations in the effective relaxation ordering provide the phenomenological measure of temporal flow, establishing a direct connection between the structure of projected descriptions and the emergent notion of time.

5.2 Hamiltonian Derivation of the Evolution Equation

Local Constraint on Effective Relaxation Dynamics

At the fundamental level, the χ substrate does not evolve in time, admits no phase-space structure, and is not governed by a Hamiltonian dynamics. No spacetime coordinates, canonical variables, or variational principles are assumed in its definition.

Nevertheless, when projected χ configurations admit a smooth and stable coarse-grained description, the admissible ordering of these configurations is subject to universal local constraints. These constraints may be summarized, within effective descriptions only, in a compact form resembling a Hamiltonian relation. This formulation is introduced solely as a descriptive parametrization of admissible local relaxation patterns and does not define a fundamental dynamics.

Specifically, the effective local relaxation ordering associated with projected χ configurations is bounded by the invariant constant c. When expressed in effective relational variables, this constraint takes the form

$$(\mathcal{D}_{\mathrm{loc}}\chi_{\mathrm{eff}})^2 + \mathcal{V}_{\chi_{\mathrm{eff}}}^2 = c^2, \tag{8}$$

where $\mathcal{V}_{\chi_{\mathrm{eff}}}$ denotes an effective internal variation functional characterizing the resistance of projected configurations to further relaxation. No notion of spatial gradient, background geometry, or fundamental field variation is assumed at this level.

Restricting to the monotonic ordering branch yields the effective evolution relation

$$\mathcal{D}_{\mathrm{loc}}\chi_{\mathrm{eff}} = c\sqrt{1 - \frac{\mathcal{V}_{\chi_{\mathrm{eff}}}^2}{c^2}}, \tag{9}$$

which encodes the universal slowdown of effective relaxation induced by localized structural constraints. This relation constrains admissible projected descriptions but does not represent an equation of motion for the fundamental χ substrate.

Emergent Gravitational Description

Projected χ configurations exhibiting strong resistance to relaxation correspond, in effective descriptions, to localized regions of enhanced structural complexity. These regions locally reduce the admissible effective relaxation rate.

When a spacetime interpretation becomes applicable, this reduction is conventionally described as gravitational time dilation. No independent gravitational field or interaction is postulated; the phenomenon arises as a direct consequence of the constrained ordering of projected χ configurations.

In the weak-structure regime, where effective internal variation rates are small compared to c, the effective description admits a simplified relation governing the spatial distribution of relaxation slowdown:

$$\nabla \cdot \left(\frac{\nabla\chi_{\mathrm{eff}}}{\sqrt{1 - |\nabla\chi_{\mathrm{eff}}|^2/c^2}} \right) \simeq \frac{4\pi G_{\mathrm{eff}}}{c^2}\rho, \tag{10}$$

where ρ denotes the effective density of localized relaxation-resistant configurations and G_{eff} is an emergent coupling parameter characterizing the collective response of the relaxation ordering to such structures.

In the Newtonian limit, this relation reduces to an effective Poisson equation for a potential Φ, defined operationally by

$$\Phi \equiv c^2 \ln\left(\frac{\mathcal{D}_{\text{loc}}\chi_{\text{eff}}}{c}\right). \tag{11}$$

This potential is not introduced as a fundamental field, but as a compact summary of how localized variations in effective relaxation ordering modulate physical clocks and rulers in regimes where classical gravitational phenomenology applies.

Interpretational Status

The relations presented in this section do not define a fundamental Hamiltonian, action, or variational principle. They provide an effective local parametrization of the constraints governing admissible projected descriptions once a geometric interpretation becomes meaningful.

The predictive content of Cosmochrony resides entirely in the structural properties of the pre-geometric χ substrate. Hamiltonian, geometric, and gravitational formalisms appear only as emergent descriptive tools, valid in restricted regimes and carrying no independent ontological status.

5.3 Microscopic Origin of the Coupling Tensor and the Poisson Equation

For internal consistency, the effective coupling governing the relaxation ordering of projected χ configurations cannot be treated as a fixed universal constant. Instead, it must depend on the internal structural state of the projected description, reflecting how localized configurations resist or facilitate further relaxation. In Cosmochrony, this dependence is captured through a constitutive relation linking effective coupling strengths to variations of the effective scalar descriptor χ_{eff}, without invoking any underlying spatial substrate at the fundamental level.

A convenient phenomenological parametrization of this dependence is given by

$$K_{\text{eff}} = K_0 \exp\left(-\frac{(\Delta\chi_{\text{eff}})^2}{\chi_c^2}\right), \tag{12}$$

where $\Delta\chi_{\text{eff}}$ denotes a measure of effective internal variation between correlated projected configurations, K_0 characterizes the maximal relaxation conductivity in a homogeneous effective background, and χ_c sets the characteristic scale beyond which structural inhomogeneities significantly reduce the effectiveness of relaxation. This parametrization is purely phenomenological and does not represent a fundamental law at the level of the χ substrate.

Projected configurations exhibiting strong effective internal variation—such as stable solitonic structures—therefore reduce the effective coupling and locally slow the admissible relaxation ordering. This reduction does not correspond to an additional interaction, but reflects the intrinsic resistance of structured projected configurations to further relaxation. The resulting slowdown constitutes the microscopic origin of the emergent gravitational phenomenology discussed in the previous sections.

In regimes where a spacetime description becomes applicable, the local effective relaxation rate $\mathcal{D}_{\mathrm{loc}}\chi_{\mathrm{eff}}$ differs from its asymptotic value $\mathcal{D}_0$ far from localized structures. An effective gravitational potential Φ may then be introduced as a descriptive parameter through the relation

$$\frac{\mathcal{D}_{\mathrm{loc}}\chi_{\mathrm{eff}}}{\mathcal{D}_0} \simeq 1 + \frac{\Phi}{c^2}, \tag{13}$$

which summarizes the relative slowdown of effective relaxation ordering in a form familiar from classical gravitational phenomenology.

In the weak-structure regime, where effective internal variations remain small compared to χ_c, the spatial distribution of Φ admits a simplified elliptic description. At this coarse-grained level, the effective dynamics reduce to a Poisson-type relation,

$$\nabla^2\Phi \simeq 4\pi G_{\mathrm{eff}}\rho, \tag{14}$$

where ρ denotes the effective density of localized, relaxation-resistant projected configurations and G_{eff} is an emergent coupling parameter encoding the collective response of the effective relaxation ordering to such structures.

This Poisson equation is not fundamental. It represents the weak-field, macroscopic limit of the constrained ordering of projected χ configurations, expressed in a form adapted to effective geometric description. Gravitation therefore appears not as an independent interaction, but as a descriptive manifestation of reduced relaxation conductivity induced by structured projected configurations.

A fully relational formulation, consistent with but not required for the effective description adopted here, is provided in Appendix E.

Status of effective equations.

The effective equations introduced in this section—such as the Poisson-like relation for the gravitational potential or the wave equations describing solitonic structures—are valid only within regimes where projected χ configurations admit a smooth, weak-gradient geometric interpretation. They arise from coarse-graining admissible projected descriptions and do not represent fundamental postulates of the theory.

In particular, these equations should not be confused with any intrinsic relaxation law governing the χ substrate at the pre-geometric level. Their domain of applicability is limited to configurations close to local equilibrium, where an effective spacetime description becomes meaningful.

5.4 Variational Formulation and Born–Infeld Action

In regimes where projected χ configurations admit a stable geometric interpretation, the effective relaxation constraints introduced above may be conveniently summarized using a variational formulation. This formulation is not fundamental and does not define a dynamics of the χ substrate itself. Rather, it provides a compact and regularized description of admissible projected descriptions in the presence of localized relaxation-resistant configurations.

Motivated by Born–Infeld–type non-linear actions originally introduced to control field singularities [19, 20], we consider the effective Lagrangian density

$$\mathcal{L}_{\text{eff}} = -c^2\sqrt{1-\frac{|\nabla\chi_{\text{eff}}|^2}{c^2}} + \mathcal{D}_{\text{loc}}\chi_{\text{eff}} - \frac{4\pi G_{\text{eff}}}{c^2}\rho\,\chi_{\text{eff}}, \tag{15}$$

where χ_{eff} denotes the effective scalar descriptor introduced in Sec. 3.2, $\mathcal{D}_{\text{loc}}\chi_{\text{eff}}$ is the effective local relaxation ordering defined in Sec. 5.1, and ρ represents the effective density of localized, relaxation-resistant projected configurations. All quantities appearing in this expression are defined exclusively within the effective geometric description.

The linear dependence on $\mathcal{D}_{\text{loc}}\chi_{\text{eff}}$ ensures that admissible projected descriptions remain monotonic and constrained, without introducing additional propagating degrees of freedom. The square-root structure acts as a non-linear regulator enforcing the universal upper bound on effective spatial variations, in direct analogy with the original role of Born–Infeld electrodynamics.

Within this effective variational framework, the Euler–Lagrange equation associated with χ_{eff} reproduces the non-linear elliptic relation governing the spatial distribution of relaxation slowdown:

$$\nabla\cdot\left(\frac{\nabla\chi_{\text{eff}}}{\sqrt{1-|\nabla\chi_{\text{eff}}|^2/c^2}}\right) = \frac{4\pi G_{\text{eff}}}{c^2}\rho, \tag{16}$$

which coincides with the effective Poisson-type relation obtained in Sec. 5.3.

This variational formulation should be understood strictly as a compact and regularized representation of admissible projected descriptions. It does not constitute a fundamental action principle, nor does it define equations of motion for the χ substrate. Its sole purpose is to ensure internal consistency of the effective description and to provide a transparent link with standard gravitational phenomenology in the appropriate weak-field regime.

The effective Lagrangian ((15)) should be understood as an auxiliary variational representation. Its physical interpretation is discussed in Appendix A.1, while its mathematical consistency with the relational discrete dynamics is established in Appendix **??**.

5.5 Causality and Locality

Equation (9) does not define a fundamental dynamics of the χ substrate. Rather, it constrains admissible projected descriptions in regimes where a geometric interpretation becomes applicable. Within such effective descriptions, the relaxation ordering encoded by χ_{eff} is local and causal in the operational sense relevant to physical observables.

Effective locality follows from the fact that variations of χ_{eff} at a given effective spacetime event depend only on correlated neighboring projected configurations, as defined within the emergent geometric description. No instantaneous coupling or nonlocal influence is introduced at the level of effective physical observables.

Causality is enforced through the existence of a universal bound on the effective local relaxation ordering,

$$|\mathcal{D}_{\mathrm{loc}}\chi_{\mathrm{eff}}| \le c,$$

which constrains how rapidly correlations between effective configurations may be established. This bound functions as an effective causal constraint without presupposing a fundamental spacetime or lightcone structure.

At the level of effective physical descriptions, no superluminal propagation of signals or causal influence occurs. Apparent superluminal recession velocities observed in cosmological settings arise only from the cumulative integration of locally constrained relaxation ordering over extended regions. They therefore remain fully consistent with effective locality and causality as defined within the Cosmochrony framework.

Conceptual remark.— The effective causal bound c introduced in this section should not be understood as an independent physical postulate. As established in Section 4.9, it corresponds to the projected manifestation of the invariant structural bound c_χ defined at the level of the pre-temporal χ substrate. In regimes admitting a locally injective projection and a geometric interpretation, c_χ acquires an operational meaning as a maximal admissible local ordering rate, which appears in spacetime descriptions as the effective causal constraint c. Effective causality in Cosmochrony thus arises as a derived property of bounded projective realizations, rather than as a fundamental principle imposed on spacetime.

5.6 Homogeneous Cosmological Limit

In a homogeneous and isotropic regime, projected χ configurations exhibit no effective spatial variations. The admissible relaxation ordering is then uniform across the emergent description, and the effective local relaxation rate attains its maximal allowed value,

$$\mathcal{D}_{\mathrm{loc}}\chi_{\mathrm{eff}} = c, \tag{17}$$

where c denotes the universal bound constraining effective relaxation ordering.

When expressed in terms of an effective cosmological time parameter t, introduced solely as a convenient label of the relaxation ordering, this uniform regime may be represented by the linear relation

$$\chi_{\mathrm{eff}}(t) = \chi_{\mathrm{eff},0} + c\,t, \tag{18}$$

where $\chi_{\mathrm{eff},0}$ denotes a reference value fixing the origin of the effective description. This parametrization does not introduce a fundamental time variable, nor does it assign intrinsic values to the χ substrate. It serves only as a compact representation of cumulative relaxation ordering in a homogeneous cosmological regime.

Interpreting effective spatial distances as accumulated relational differentiation between projected configurations, this uniform ordering directly leads to a Hubble-like expansion law, as discussed in Sec. 12. Cosmic expansion thus reflects the global ordering of projected χ configurations, rather than the presence of an external energy component or a fundamental expansion of spacetime itself.

As shown in Appendix A.6, the requirement that effective relaxation ordering remain monotonic in an expanding regime implies the existence of a minimal residual structural inhomogeneity in projected χ configurations. In effective geometric terms, this manifests as a non-vanishing lower bound on gravitational acceleration, providing a natural explanation for MOND-like phenomenology without invoking dark matter particles [21, 22].

5.7 Influence of Local Structure

In regions where projected χ configurations exhibit non-vanishing effective structural variations, the admissible local relaxation ordering is reduced. This slowdown plays a central role in the emergence of gravitational phenomena within the Cosmochrony framework.

Localized relaxation-resistant configurations—describable in effective regimes as particle-like solitonic structures—act as constraints on the admissible ordering of projected χ configurations. By increasing the effective structural complexity of the projected description, they reduce the local effective relaxation rate without introducing any additional interaction or force.

When a geometric description becomes applicable, this mechanism manifests phenomenologically as gravitational time dilation and spatial curvature. No independent gravitational field is postulated. Gravitation emerges instead as a collective consequence of locally constrained effective relaxation ordering, reflecting the presence of structured projected configurations.

5.8 Unified Origin of Geometric and Field Effects

The relationship between the χ substrate and the effective spacetime metric $g_{\mu\nu}$ is strictly hierarchical, reflecting the transition from a fundamental pre-geometric relational structure to smooth geometric descriptions applicable at macroscopic scales.

1. **Primacy of the χ substrate:** At the fundamental level, physical reality is described solely in terms of the χ substrate and its intrinsic relational structure. The χ substrate is not defined on spacetime, does not possess numerical values, and is not governed by a dynamical field equation. Ordering, relaxation, and causal notions arise only at the level of admissible projected descriptions.
2. **Emergent Geometry:** In regimes where projected χ configurations admit a stable, slowly varying description, geometric notions become meaningful. The spacetime metric $g_{\mu\nu}$ arises as an effective descriptor summarizing the correlations and relaxation ordering of projected χ configurations. It provides a coarse-grained geometric language suitable for macroscopic observers, without acquiring independent ontological status.
3. **Unified Interpretation of Fields and Gravitation:** Within this effective geometric description, localized relaxation-resistant projected configurations—describable as solitonic structures—are identified with matter degrees of freedom. Gravitational phenomena correspond to the local modulation of effective relaxation ordering

induced by such structures. The metric does not act as an independent dynamical agent, but encodes the collective geometric response associated with constrained projected descriptions.

In this framework, no independent gravitational interaction or fundamental field is postulated. Matter, geometry, and gravitational phenomena emerge as complementary aspects of the same constrained ordering of projected χ configurations, ensuring a unified and internally consistent description across scales.

5.9 Limitations and Scope

Equation (9) is intentionally minimal and does not describe a fundamental dynamics of the χ substrate. It constrains admissible projected descriptions in regimes where a stable geometric interpretation becomes applicable. In particular, it does not aim to provide a complete account of quantum fluctuations or correlations at the level of the χ substrate, nor does it incorporate higher-order structural effects beyond the effective description.

Within this scope, the equation provides a unified kinematic backbone from which gravitational, quantum, and cosmological phenomena can be consistently *described* or *recovered* at an effective level. These phenomena are not derived from first principles at the level of χ, but emerge as structured regularities within admissible projected descriptions once coarse-graining and geometric interpretation are introduced.

More refined treatments of effective fluctuations, correlations, and extended relational structures lie beyond the present formulation. Such developments would require a deeper analysis of the space of admissible projected configurations and of the stability properties of the corresponding effective descriptions.

In the following sections, this constrained descriptive framework is applied to particle-like excitations, gravitation, and quantum correlations, where its explanatory power can be directly assessed within its intended domain of validity.

6 Particles as Localized Excitations of the χ Field

6.1 Particles as Stable Wave Configurations

Within the Cosmochrony framework, particles are not fundamental point-like objects. They arise only at the level of effective descriptions, as stable and localized configurations within projected χ descriptions [23]. These configurations correspond to persistent patterns that locally constrain the admissible relaxation ordering, rather than to elementary entities propagating on a pre-existing spacetime background.

In effective geometric regimes, such structures may be described using a wave-like or soliton-like language. They preserve their identity under interactions and effective displacement, while remaining entirely embedded in the constrained ordering of projected χ configurations. Their apparent propagation reflects a continuous reconfiguration of admissible projected descriptions, not the motion of an object through a fundamental spacetime.

In this sense, particle-like behavior does not originate from intrinsic degrees of freedom of the χ substrate. It emerges instead as a stable invariant of the effective relational structure once a spacetime interpretation becomes meaningful.

6.2 Topological Stability

The stability of particle-like excitations in Cosmochrony does not rely on fundamental conserved charges postulated a priori. It arises instead at the level of effective descriptions, from intrinsic structural constraints on admissible projected χ configurations. Certain projected configurations exhibit non-trivial internal organization that prevents them from being continuously deformed into homogeneous effective descriptions.

This form of stability is topological in nature. It reflects the existence of inequivalent classes of projected χ configurations that cannot be smoothly related without violating the admissibility constraints imposed by effective relaxation ordering. As a result, particle-like excitations appear discrete and robust under perturbations, without requiring externally imposed symmetries or fundamental conservation laws.

Importantly, these topological distinctions are not defined with respect to a pre-existing spacetime geometry. They are properties of the configuration space of admissible projected descriptions and remain well-defined even in the absence of an effective geometric interpretation. Geometric representations, when employed, serve only as descriptive tools valid in regimes where a spacetime language has emerged.

The long-lived character of solitonic structures therefore follows from the incompatibility between distinct classes of admissible projected configurations, rather than from a dynamical balance of forces or nonlinear self-interactions. This mechanism provides a natural foundation for particle stability within a purely relational and pre-geometric framework.

6.3 Mass as Resistance to χ Relaxation

In Cosmochrony, mass is not introduced as an intrinsic or fundamental property of matter. It emerges only at the level of effective descriptions, as a quantitative measure of how strongly a localized projected configuration resists admissible relaxation ordering.

A particle-like excitation is described, within effective regimes, as a stable and localized projected configuration, denoted $\chi_{\text{eff},s}$, characterized by persistent internal structure. Such configurations locally constrain the admissible relaxation ordering relative to a homogeneous effective background. When a geometric description applies, this constraint manifests phenomenologically as inertial persistence and gravitational time dilation.

We define the effective structural energy associated with a projected solitonic configuration $\chi_{\text{eff},s}$ as a measure of the excess resistance to relaxation encoded in its internal structure:

$$E[\chi_{\text{eff},s}] \equiv \int_\Sigma \left(\frac{1}{\sqrt{1 - |\nabla \chi_{\text{eff},s}|^2/c^2}} - 1 \right) d\Sigma, \tag{19}$$

where Σ denotes a hypersurface of constant effective ordering parameter, and $|\nabla \chi_{\text{eff},s}|$ quantifies effective structural deformation within the projected description. This expression does not represent a fundamental energy stored in the χ substrate, but a descriptive measure of how strongly a given projected configuration departs from homogeneous relaxation ordering.

The inertial mass associated with such a configuration is then defined operationally as

$$m \equiv \frac{E[\chi_{\text{eff},s}]}{c^2}. \tag{20}$$

This relation is not postulated as a fundamental axiom. It follows directly from the role of $E[\chi_{\text{eff},s}]$ as a measure of resistance to effective relaxation ordering. The universal constant c appears as the maximal admissible relaxation rate and therefore provides the unique conversion factor between structural resistance and inertial response.

Within this framework, the relation $E = mc^2$ is interpreted as a kinematic identity. Mass quantifies the amount of effective relaxation resistance locally encoded in a persistent projected configuration, while energy represents the same quantity expressed in relaxation units.

In this sense, mass is not an independent attribute of matter. It is a derived invariant characterizing how strongly a localized projected configuration resists the irreversible ordering that, in effective descriptions, defines physical time.

The question of how different particle masses arise from distinct classes of projected configurations is addressed in Appendix B.2, where a spectral characterization of the stability properties of admissible projected descriptions is proposed as the geometric origin of mass hierarchies.

6.4 Energy–Frequency Relation

Within effective descriptions of Cosmochrony, the energy associated with a particle-like excitation is linked to a characteristic internal spectral scale of the corresponding projected configuration. This scale quantifies how strongly the configuration resists admissible relaxation ordering: configurations with higher characteristic frequencies correspond to more tightly constrained projected structures and a greater effective capacity to encode relaxation resistance.

This provides an effective interpretation of the relation

$$E \propto \nu, \tag{21}$$

in which energy measures the degree of effective relaxation resistance associated with a projected configuration, while the frequency ν characterizes the associated spectral scale of its internal structure. The frequency should not be interpreted as an oscillation with respect to a fundamental time parameter. It acquires a temporal interpretation only within effective geometric regimes, where a notion of time becomes meaningful.

Within this perspective, Planck's constant emerges as an effective proportionality factor relating energy and frequency. Its apparent universality reflects the robustness of the spectral scales governing admissible projected configurations, rather than the postulation of a fundamental quantization constant at the level of the χ substrate.

In this sense, the energy–frequency relation expresses a kinematic correspondence between effective structural resistance and spectral organization within projected descriptions. A more explicit realization of this correspondence, in the context of radiation and photon-like projected excitations, is presented in Sec. 13.3.

6.5 Fermions and Bosons

Within the Cosmochrony framework, particle statistics do not arise from fundamental quantization rules or postulated commutation relations. They emerge at the level of effective descriptions from the topological structure of admissible projected configurations.

Distinct classes of particle-like projected configurations are characterized by how their internal configuration space responds to continuous rotations. Certain projected configurations require a 4π rotation in configuration space to return to an equivalent description, while others are 2π-periodic. The former give rise to fermion-like behavior, whereas the latter correspond to boson-like excitations.

This distinction reflects a topological obstruction in the space of admissible projected descriptions rather than a symmetry principle imposed at the fundamental level. In effective geometric regimes, 4π-periodic configurations may be associated with twisted or non-orientable internal structures, while 2π-periodic configurations correspond to orientable ones. Such associations are descriptive and do not imply the existence of a fundamental spatial manifold or intrinsic spin variables.

Within this perspective, the spin–statistics connection admits a natural qualitative interpretation. Fermionic and bosonic behavior reflects the topological classification of admissible projected configurations under continuous transformations, without introducing additional quantum postulates at the level of the χ substrate.

As throughout this work, references to phase rotations, periodicity, or internal configuration space should be understood strictly within the effective descriptive framework. They characterize properties of projected descriptions and their topological invariants, not intrinsic attributes of the pre-geometric χ substrate.

6.6 Spin as a Topological Property of Projected Configurations

Within the Cosmochrony framework, spin is not introduced as an intrinsic kinematic degree of freedom, nor as a consequence of spacetime symmetries. It emerges instead, at the level of effective descriptions, as a purely topological property of admissible projected configurations.

Certain stable particle-like projected configurations exhibit internal relational structures that cannot be continuously deformed into homogeneous effective descriptions. These configurations are characterized by non-trivial topology in their internal configuration space, independently of any background spatial geometry or fundamental notion of rotation.

In particular, a class of fermionic projected configurations requires a 4π rotation in configuration space to return to an equivalent description. A 2π rotation corresponds to a non-contractible loop in the space of admissible projected configurations, while a 4π rotation is homotopic to the identity. Formally, this implies that the relevant configuration space admits a double covering, with fundamental group

$$\pi_1(\mathcal{C}_{\rm eff}) = \mathbb{Z}_2, \tag{22}$$

where $\mathcal{C}_{\rm eff}$ denotes the space of admissible localized projected descriptions.

When an effective quantum description becomes applicable, particle-like projected configurations are represented by complex wavefunctions encoding the phase structure associated with their topological class. For topologically non-trivial configurations, a 2π effective rotation induces a sign change of the associated wavefunction,

$$\psi \longrightarrow -\psi, \tag{23}$$

while a 4π rotation restores the original state.

This behavior identifies such projected configurations as spin-$\frac{1}{2}$ fermionic excitations. Importantly, the appearance of a spinorial phase does not rely on a fundamental representation of the rotation group. It follows from the topological structure of the space of admissible projected descriptions.

Fermionic statistics arises from the same topological origin. Two identical fermionic projected configurations belong to the same non-trivial topological class and therefore cannot be continuously merged into a single admissible configuration without violating the admissibility constraints of the effective description.

Exchanging two identical fermionic excitations corresponds topologically to a 2π rotation in the combined configuration space. As this operation induces a sign change of the effective wavefunction, symmetric configurations are dynamically excluded. This provides a geometric and topological origin of the Pauli exclusion principle within the Cosmochrony framework [24].

In Cosmochrony, spin and fermionic statistics are therefore not postulated quantum properties. They are manifestations of topological obstructions in the space of admissible projected descriptions. The 4π periodicity, spin-$\frac{1}{2}$ behavior, and exclusion principle thus share a common geometric origin within the effective descriptive framework.

6.7 Antiparticles

Within the Cosmochrony framework, antiparticles are not interpreted as independent fundamental entities or as excitations propagating backward in time. They arise, at the level of effective descriptions, as relationally conjugate counterparts of particle-like projected configurations.

A particle and its antiparticle correspond to projected configurations belonging to distinct but conjugate topological classes within the space of admissible projected descriptions. These classes are related by an internal reversal of relational structure rather than by an inversion of a fundamental dynamical variable.

Annihilation processes occur when a particle-like projected configuration and its conjugate combine into a composite projected description that no longer supports localized structural constraints. Such a configuration admits a continuous deformation toward a more homogeneous effective description, in which localized relaxation resistance disappears.

In effective geometric and quantum descriptions, this transition manifests as the conversion of particle–antiparticle structure into delocalized radiation-like projected excitations. No fundamental structure is destroyed in this process. The underlying relational substrate remains intact, while localized topological organization is redistributed into admissible projected configurations with extended support.

In this sense, particle–antiparticle annihilation does not represent the destruction of matter, but a reorganization of effective relational structure from localized to delocalized forms within the space of admissible descriptions.

6.8 Particle Creation and Destruction

Within the Cosmochrony framework, particle creation does not correspond to the appearance of new fundamental entities. It arises at the level of effective descriptions, when a projected configuration acquires sufficient structural organization to support a stable, localized topological class. Such configurations become identifiable as particle-like only once a spacetime interpretation becomes meaningful.

Particle creation therefore reflects the emergence of a new admissible projected description with persistent localization and relaxation resistance. This process does not involve the generation of structure at the level of the χ substrate, but a reorganization of admissible projected configurations within the space of effective descriptions.

Conversely, particle destruction does not represent the annihilation of a fundamental object. It occurs when a previously localized projected configuration loses its topological admissibility or stability class. In such cases, the configuration can no longer sustain localized relaxation constraints and admits a continuous deformation toward a more delocalized effective description.

In effective geometric and quantum regimes, this transition manifests as the conversion of particle-like projected configurations into extended, radiation-like descriptions. Creation and destruction thus reflect changes in the organization and admissibility of projected descriptions, rather than the appearance or disappearance of fundamental entities.

Within this perspective, particles are not primitive ontological constituents. They are stable descriptive regimes of the relational substrate, whose formation and dissolution correspond to transitions between distinct classes of admissible projected configurations.

6.9 Summary

Within the Cosmochrony framework, particles are not fundamental entities. They emerge only at the level of effective descriptions, as stable and localized projected configurations that resist admissible relaxation ordering. Their physical properties are not postulated but arise as invariants of the structural and topological organization of admissible projected descriptions.

Mass is identified with the degree of effective relaxation resistance encoded in a localized projected configuration. It quantifies how strongly such a configuration constrains admissible relaxation ordering relative to a homogeneous effective background. In regimes where a relativistic description applies, this interpretation naturally leads to the relation $E = mc^2$, understood as a kinematic identity rather than a fundamental postulate.

Spin and statistical behavior originate from topological obstructions in the space of admissible projected configurations. Fermionic configurations exhibit a 4π periodicity in configuration space, such that a 2π rotation corresponds to a non-contractible loop and induces a sign change of the effective wavefunction. This topological structure provides a common origin for spin-$\frac{1}{2}$ behavior, fermionic antisymmetry, and the Pauli exclusion principle without invoking additional quantum axioms [24, 25].

Within this perspective, different particle attributes correspond to distinct topological invariants of admissible projected descriptions. Spin is associated with non-trivial covering properties of configuration space, while electric charge may be interpreted, at an effective level, as an oriented topological defect or vortex-like structure within projected descriptions. These attributes remain conceptually distinct but arise from a common relational substrate once a geometric interpretation becomes meaningful.

Taken together, these results provide a unified account of particle properties compatible with both relativistic and quantum phenomenology, without introducing particles or their attributes as fundamental ontological constituents. Particles appear instead as stable descriptive regimes of the underlying relational structure, whose properties reflect the topology of admissible projected configurations.

7 Gravity as a Collective Effect of Particle Excitations

7.1 Local Slowdown of Relaxation Ordering

In the Cosmochrony framework, gravitation does not arise from a fundamental interaction. It emerges, at the level of effective descriptions, from the collective influence of particle-like projected configurations on admissible relaxation ordering.

As established in Sec. 6, localized projected configurations resist admissible relaxation ordering. When many such configurations are present, their combined effect leads, in the weak-constraint regime, to an effective macroscopic reduction of the admissible ordering rate within projected descriptions.

In an effective spacetime parametrization, this collective effect may be expressed schematically as

$$\mathcal{D}_{\text{eff}}\chi_{\text{eff}} \simeq c\,(1 - \alpha\,\rho)\,, \tag{24}$$

where ρ denotes the effective density of localized projected configurations and α encodes their average contribution to relaxation resistance. This expression represents a first-order approximation valid when localized constraints are sufficiently dilute and weakly overlapping.

The coupling parameter α is not fundamental. It emerges as a collective property of admissible projected descriptions and depends on the typical structural characteristics of localized configurations. In the weak-field limit, dimensional consistency relates its scaling to the observed gravitational constant, leading to $\alpha \propto G/c^2$ when expressed in terms of effective inertial mass densities. Within this approximation, the reduction of admissible relaxation ordering admits a Newtonian-like interpretation in terms of an effective gravitational potential.

Physically, this collective slowdown manifests, within effective geometric descriptions, as gravitational time dilation. No independent gravitational field or force is introduced; gravitation appears as a macroscopic signature of constrained admissible ordering induced by localized projected configurations.

7.2 Collective Gravitational Coupling and Operational Geometry

The collective reduction of admissible relaxation ordering described above affects not only the local accumulation of effective time, but also the manner in which variations within projected descriptions influence one another across extended regions. In the presence of localized projected configurations, the relaxation resistance they induce modulates how efficiently structural variations can be correlated between different effective locations.

At the level of effective descriptions, this collective behavior may be summarized by a constitutive coupling function characterizing the stiffness of projected configurations with respect to relative variations. In regions where projected descriptions are nearly homogeneous, this coupling approaches a uniform effective value. Localized projected configurations weaken it by introducing additional structural constraints. Crucially, this coupling is defined entirely within the effective descriptive framework and does not presuppose any fundamental spatial metric or background geometry.

Because no fundamental geometry is assumed, spatial distance is defined operationally. Two effective regions are considered close if structural variations of projected configurations can be correlated efficiently between them, and distant otherwise. In the continuum and weak-constraint regime, this operational notion admits a description in terms of an effective spatial metric, which summarizes the collective response of admissible projected descriptions.

Within this framework, spacetime curvature does not arise as a primitive geometric property. It emerges as a descriptive manifestation of how localized projected configurations modulate the collective admissible ordering and correlation structure. Geometry thus functions as a macroscopic encoding of constrained relational organization, rather than as a fundamental entity.

A more explicit relational construction of the coupling mechanism and its connection to discrete formulations is presented in Appendix D.1.

7.3 Emergent Curvature

Spatial variations in admissible relaxation ordering, together with the collective modulation of effective coupling strength, lead to non-uniform correlation patterns within projected descriptions. When expressed in an effective geometric language, these non-uniformities are summarized by gradients of an emergent metric structure.

In this sense, spacetime curvature in Cosmochrony is not a primitive geometric property. It is a descriptive construct encoding how localized projected configurations collectively modulate admissible ordering and correlation structure across extended regions. The metric does not act as an independent dynamical agent, but serves as a compact representation of constrained relational organization within effective descriptions.

This emergent curvature reproduces the phenomenology traditionally attributed to curved spacetime in general relativity, including gravitational time dilation, geodesic deviation, and lensing effects. At the same time, it remains fully compatible with the pre-geometric and relational foundations of the framework, in which geometry arises only as an effective and operational notion.

7.4 Recovery of the Schwarzschild Metric

In the presence of a static and approximately spherically symmetric distribution of localized projected configurations, the collective reduction of admissible relaxation ordering admits a simple effective description. In the weak-constraint and quasi-static regime, spatial variations of the effective ordering rate can be summarized by a Poisson-like relation between an effective gravitational potential and the density of localized projected configurations.

When expressed in an effective geometric language, this structure is well described by a metric whose leading-order form coincides with the Schwarzschild solution of general relativity. In particular, the temporal and radial components of the effective metric encode the local reduction of admissible relaxation ordering induced by a localized projected configuration, while the angular sector reflects the approximate isotropy of the effective description.

Within this framework, standard weak-field predictions of general relativity are recovered. These include gravitational redshift, light deflection, and time dilation effects consistent with solar-system observations. The gravitational constant G appears as an emergent coupling parameter relating the effective density of localized projected configurations to the magnitude of the ordering slowdown.

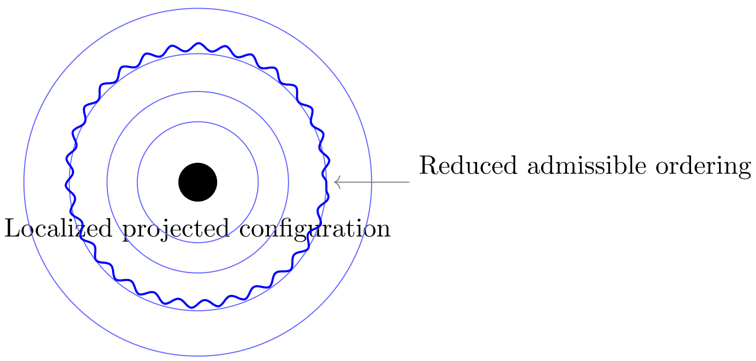

Fig. 2 Emergence of Schwarzschild-like behavior in Cosmochrony. A localized projected configuration induces a spatially varying reduction of admissible relaxation ordering. In effective geometric descriptions, this manifests as differential proper-time flow and an emergent metric curvature analogous to gravitational time dilation.

Importantly, the Schwarzschild metric is not postulated as a fundamental solution, nor is spacetime curvature treated as a primitive entity. Rather, the metric provides a compact and effective summary of how localized projected configurations constrain admissible relaxation ordering in their vicinity.

In this sense, Schwarzschild-like behavior does not reflect a specific dynamical law of spacetime itself, but emerges as the necessary phenomenological description in regimes where projected configurations are close to local equilibrium and admit a geometric interpretation.

7.5 Equivalence Principle

Within the Cosmochrony framework, particle-like excitations do not couple to a fundamental gravitational field. Instead, they are described, at the level of effective descriptions, as localized projected configurations that impose constraints on admissible relaxation ordering.

Because all such projected configurations resist admissible relaxation ordering in the same structural manner, the collective reduction of ordering is independent of their internal composition or detailed structure. As a result, all particle-like projected configurations respond identically to a given effective ordering environment.

When expressed in effective geometric terms, this universal response appears as composition-independent gravitational acceleration. The equivalence between inertial and gravitational behavior therefore emerges as a direct consequence of the uniform manner in which admissible projected configurations constrain relaxation ordering, rather than as an independent postulate imposed on the theory.

In this sense, the equivalence principle arises naturally within Cosmochrony as an emergent symmetry of admissible projected descriptions, reflecting the absence of any intrinsic distinction between inertial resistance and gravitational response at the effective level.

7.6 Gravitational Waves

Time-dependent variations in the distribution of localized projected configurations, such as accelerating masses or mergers of compact systems, induce collective modulations in admissible relaxation ordering. These modulations propagate as changes in the effective ordering regime and are transmitted at the maximal admissible ordering speed c.

When expressed in an effective spacetime language, such propagating modulations are described as gravitational waves. Unlike electromagnetic radiation, which corresponds to propagating particle-like projected excitations, gravitational waves represent collective variations in the admissible ordering and correlation structure of projected descriptions themselves.

In this sense, gravitational waves do not introduce additional fundamental degrees of freedom. They arise as macroscopic, collective responses of admissible projected descriptions to time-dependent reconfigurations of localized constraints, rather than as excitations of an underlying physical field.

It should be emphasized that gravitational-wave descriptions are valid only within regimes where the projection onto an effective spacetime remains well defined. In strong-gravity environments approaching the deprojection threshold discussed in Section 7.7, these collective modulations are expected to become increasingly attenuated or lose a clear spacetime interpretation. Before turning to strong-gravity regimes, it is useful to summarize the different projective regimes discussed in this section.

Projectable	$\rightarrow$	**Horizon**	$\rightarrow$	**Non-projectable**
Gravitational Waves		Boundary of Projection		Deprojected Regime
Effective Modulations		Π non-injective		No spacetime Representation

Fig. 3 Conceptual regimes of projection in Cosmochrony. Gravitational waves correspond to fully projectable collective modulations of admissible descriptions, while black holes mark the boundary beyond which spacetime representations cease to be injective. Black hole evaporation reflects the gradual restoration of projectability, without any loss of information at the fundamental relational level.

This schematic overview highlights how gravitational waves, horizons, and black hole evaporation correspond to distinct regimes of projectability of the same underlying relational structure.

7.7 Strong Gravity and Black Holes

In regions where the density of localized projected configurations becomes sufficiently high, admissible relaxation ordering becomes strongly constrained. In effective spacetime descriptions, this corresponds to a regime in which the local accumulation of effective time is strongly suppressed relative to distant observers, defining an effective horizon.

Within Cosmochrony, such regions are interpreted as black holes. Rather than being characterized by a fundamental spacetime singularity, black holes correspond to domains where physical processes become asymptotically inaccessible from the exterior due to the loss of injectivity of spacetime projection. This naturally accounts for extreme time dilation effects without requiring divergent curvature invariants.

These regions therefore mark not a terminal endpoint of physical description, but a transition toward a non-projectable regime of the underlying relational structure.

Gravitational and Temporal Shadows

In the strong-gravity regime, the increasing concentration of localized projected configurations induces severe constraints on admissible ordering. As a result, the effective progression of time within the region slows asymptotically with respect to external descriptions.

This behavior reproduces the phenomenon commonly referred to as a *gravitational shadow*. In general relativity, such shadows arise from the absence of escaping null geodesics within a characteristic angular region. In Cosmochrony, an equivalent observational signature emerges because effective propagating descriptions, including radiation-like modes, no longer admit a faithful spacetime representation once projectability is lost. External observers therefore perceive a dark angular region corresponding to the projection of a non-projectable domain.

Beyond this optical effect, the framework predicts a deeper phenomenon, which may be termed a *temporal shadow*. As projectability is progressively lost, internal processes become indefinitely delayed in effective spacetime descriptions. From the external perspective, physical evolution appears frozen, providing a natural interpretation of horizon-induced time dilation.

In this view, the observed gravitational shadow corresponds to the visible manifestation of an underlying temporal shadow. Both effects arise from the same loss of projective representability and do not require a fundamental spacetime singularity.

Absence of Physical Singularities

In classical general relativity, black holes are associated with spacetime singularities characterized by divergent curvature and energy density. In Cosmochrony, such singularities are interpreted as artifacts of extending effective spacetime descriptions beyond their domain of validity.

Because admissible ordering is bounded, configurations corresponding to infinite curvature or density cannot be physically realized. Apparent singularities therefore signal the breakdown of spacetime representability rather than genuine divergences of the underlying relational structure.

Structural bound and notation.

To avoid confusion between fundamental and emergent levels, we distinguish the dimensionless structural bound c_χ, defined at the level of the pre-geometric relational substrate, from its emergent spacetime manifestation c, interpreted as the maximal signal propagation speed. While c may exhibit effective regime-dependent variations, the bound c_χ is invariant.

Black Holes, Deprojection, and Vacuum Reprojection

Within Cosmochrony, the absence of physical singularities does not imply that black holes are dynamically inert. Rather, they correspond to regimes in which the projection of relational information onto spacetime ceases to be injective.

The emergence of an effective spacetime description relies on a projection map

$$\Pi : \mathcal{C}_{\mathrm{rel}} \longrightarrow \mathcal{M},$$

from the space of relational configurations to an effective spacetime manifold. In weak- and moderate-field regimes, this map is locally injective, ensuring a faithful geometric encoding.

In strong-gravity regimes, this injectivity breaks down. Multiple inequivalent relational configurations correspond to the same effective spacetime event, signaling a loss of representability without any loss of information. We refer to this loss of injectivity as *deprojection*.

Deprojection does not correspond to transport across a spatial boundary nor to a temporal reversal. Instead, relational information ceases to be expressible in spatiotemporal form and remains encoded structurally.

Importantly, deprojected information is not destroyed. Because the underlying relational configuration remains globally defined, information is in principle reprojectable once projectability is restored. Reprojection occurs discretely and manifests in effective spacetime descriptions as radiation-like excitations or particle–antiparticle pairs.

The deprojection regime associated with black hole horizons does not imply the absence of dynamical processes. While smooth metric evolution ceases, the χ substrate remains structurally active. In particular, reprojection may occur intermittently when local configurations reach the threshold required for effective visibility. In the following section, this process is formalized through an explicit reprojection flux equation governing black hole evaporation.

Information conservation and unitarity.

Deprojection does not correspond to information loss. It marks the loss of spacetime encoding, not the destruction of correlations. At the fundamental relational level, global information is preserved. Apparent non-unitarity arises only within projected spacetime descriptions and reflects their limited domain of applicability.

7.8 Black Hole Evaporation and the Information Problem

Within the Cosmochrony framework, black holes are not associated with physical singularities, but with regions where spacetime projection ceases to be injective. Such regions define domains of limited representability rather than physical interiors.

Evaporation as a Projective Phenomenon.

Black hole evaporation is an effective process unfolding entirely within the projectable regime. It arises from the gradual restoration of projectability near the boundary separating projectable and non-projectable domains.

As projectability is progressively recovered, localized projected configurations cease to be supported and are replaced by radiation-like effective descriptions. The evaporation process completes before any effective description would encounter a non-projectable singular regime.

Resolution of the Information Paradox.

The apparent information loss identified by Hawking arises from treating spacetime as fundamental [26]. In Cosmochrony, information is encoded in the global relational configuration independently of its spacetime projection. Evaporation therefore does not violate unitarity; it reflects a change in the domain of representability.

Observational Implications.

To external observers, emitted radiation appears nearly thermal and weakly correlated with infalling states. This reflects the coarse-grained nature of spacetime projection rather than genuine information loss. The black hole information paradox is thus resolved by recognizing it as an artifact of extrapolating spacetime concepts beyond their domain of validity.

Horizon Reprojection Equation

Within Cosmochrony, black hole evaporation is described as a reprojection process by which structural energy stored in the χ substrate is released into the projected spacetime in discrete units.

The energy flux emerging from the horizon is defined as a sum over reprojection events associated with micro-configurations of the projection fiber:

$$\Phi_\chi \equiv \frac{dE}{dt} = \sum_k \delta(t - t_k)\, \hbar_\chi\, \nu_k(L_{\rm sol}).$$

Here, $\hbar_\chi$ denotes the fundamental quantum of reprojection, setting the minimal granularity of projected action. The quantities $\nu_k(L_{\rm sol})$ correspond to the resonance frequencies associated with the eigenmodes of the stability operator $L_{\rm sol}$ acting on the projection fiber $\Pi^{-1}(g_H)$ at the horizon, where g_H denotes the effective near-horizon geometry. The times t_k label the instants at which local χ configurations reach the projection threshold and become effectively visible in spacetime.

Black hole evaporation thus proceeds as a sequence of discrete reprojection pulses, rather than as a continuous emission process.

Emergent Temperature and Relaxation Gradient

The apparent Hawking temperature perceived by a distant observer emerges from the statistical distribution of reprojection events. It is determined by the gradient of effective χ relaxation normal to the horizon.

This relation may be expressed as

$$k_B T_\chi = \frac{\hbar_\chi c}{2\pi}\, \left|\nabla_\perp \chi_{\rm eff}\right|_H .$$

For more massive black holes, the relaxation gradient is distributed over a larger horizon area, reducing the frequency of reprojection events. This directly explains the inverse mass–temperature relation without invoking vacuum particle creation or fundamental thermodynamic assumptions.

Information Conservation and Spectral Encoding

In contrast with semiclassical descriptions, information is never destroyed in Cosmochrony. The projection operator Π acts as a filter rather than as an irreversible map.

During the deprojection phase, information is stored in the nonlinear degrees of freedom of the χ substrate, encoded within the projection fiber. During reprojection, the emitted radiation carries the precise spectral imprint of the eigenmodes governing the χ configuration.

Each emitted quantum corresponds to a specific transition within the substrate, ensuring global unitarity. The apparent loss of information arises solely from restricting attention to the projected metric degrees of freedom.

Entropy as a Projection Saturation Limit

Within the Cosmochrony framework, the recovery of the Bekenstein–Hawking area law,

$$S = \frac{A}{4}, \tag{25}$$

does not rely on the introduction of a temperature or on thermodynamic postulates. Instead, black hole entropy is reinterpreted as a measure of the informational capacity of the projection map Π at the horizon boundary.

The Horizon as a De-projection Boundary

In the near-horizon regime, the χ substrate approaches a state of critical constraint in which the mapping $\Pi : \chi \to g_{\mu\nu}$ ceases to be injective (see Section 7.7.3). The horizon is therefore redefined as the locus at which the local relaxation rate $N(r)$ vanishes, preventing further refinement of the projected metric degrees of freedom.

At this boundary, distinct micro-configurations of the substrate are mapped onto the same effective horizon geometry g_H. Entropy is identified with the logarithmic measure of the fiber $\Pi^{-1}(g_H)$, that is, the structural multiplicity of χ configurations that are no longer distinguishable at the metric level. Entropy thus quantifies hidden relational structure, rather than thermal ignorance.

This situation is schematically illustrated in Figure 4.

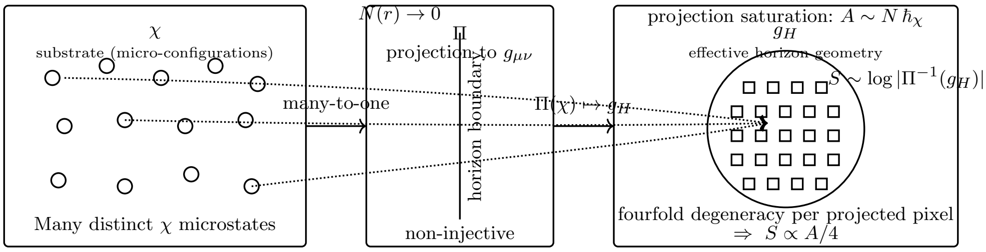

Fig. 4 Saturation of the projection map at the horizon. Near the boundary where $N(r) \to 0$, the projection Π becomes non-injective: multiple micro-configurations of the χ substrate collapse onto the same effective horizon geometry g_H. Entropy measures the structural multiplicity of the fiber $\Pi^{-1}(g_H)$ and the saturation of discretely projected spectral elements of characteristic area $\sim \hbar_\chi$.

Geometric Origin of the 1/4 *Factor*

The numerical factor 1/4 arises as a structural ratio between the internal degrees of freedom of the χ substrate and their maximal holographic projection onto a two-dimensional boundary.

Let $\hbar_\chi$ denote the elementary quantum of reprojection. The horizon area A is saturated by a finite number N of discrete projected spectral elements of characteristic area $\sim \hbar_\chi$. Within the Cosmochrony framework, each such projected element corresponds to a fourfold degeneracy in the stability spectrum of χ, linked to the intrinsic 4π periodicity of χ excitations discussed in Section 6.2

The area law $S = A/4$ is therefore the macroscopic signature of this quadrature constraint: it represents the maximal density of independent structural degrees of freedom that can be projected before the non-injectivity of Π induces a complete loss of local metric resolution.

Unitary Reprojection and Information Conservation

Black hole evaporation is reformulated as a discrete reprojection process. As the χ substrate locally relaxes at the horizon boundary, it emits quanta of action $\hbar_\chi$ carrying the spectral imprint of the fiber $\Pi^{-1}(g_H)$.

Because this process is governed by the deterministic—though nonlinear—relaxation dynamics of χ, information is never destroyed nor trapped in a singularity. Instead, it is transferred from a purely structural encoding within the fiber back into observable spacetime degrees of freedom. Unitarity is preserved at the level of the χ substrate, even though the effective spacetime description may exhibit apparent non-unitarity.

Conclusion

In Cosmochrony, black hole entropy is not a measure of ignorance but a measure of projection saturation. It quantifies the threshold at which the complexity of the χ substrate exceeds the transmittance capacity of the effective metric projection. This explains why entropy scales with area—the projection surface—and not with volume, which characterizes the inaccessible internal fiber.

Prediction: Spectral Line Width and Substrate Granularity

If Hawking radiation is the macroscopic manifestation of a fundamentally discrete reprojection process, it cannot be a perfect black-body continuum. Within Cosmochrony, black hole evaporation is expected to produce a comb-like spectrum, whose fine structure directly reflects the local relaxation state of the χ substrate.

Reprojection Line Width.

We define the spectral line width $\Delta\nu_k$ of a reprojected quantum as being controlled by the ratio between the local relaxation timescale τ_χ and the spectral packing fraction α introduced in Section B.8:

$$\Delta\nu_k \approx \frac{1}{\tau_\chi} \sqrt{\alpha}\, \ln(Q), \tag{26}$$

where:

- τ_χ is the characteristic relaxation time of the χ field near the horizon, linked to the local gradient and constraint structure of χ,
- $\alpha \simeq 3 \times 10^{-7}$ is the spectral packing fraction governing the density of admissible projection modes and previously shown to control the proton-to-electron mass ratio,
- Q is the topological charge of the emitted configuration.

Physical Interpretation.

In this framework, the line width $\Delta\nu$ is not an epistemic uncertainty but a direct measure of substrate fluidity. For massive black holes, relaxation is slow ($\tau_\chi \to \infty$), yielding extremely narrow, quasi-discrete spectral lines. As evaporation proceeds and the black hole approaches its end-point, τ_χ decreases, leading to progressive spectral broadening. Eventually, neighboring lines overlap, giving rise to an effective pseudo-continuum shortly before final dissipation.

Universal Granularity Relation.

A central prediction of Cosmochrony is that the spacing between adjacent spectral peaks $\delta\nu$ and their individual width $\Delta\nu$ are governed by the same structural constant α:

$$\frac{\Delta\nu}{\delta\nu} \propto \alpha. \tag{27}$$

This ratio is universal and independent of the black hole mass or the nature of the emitted quanta. The same dimensionless factor α appears in two physically distinct contexts: (i) the proton-to-electron mass ratio, $m_p/m_e \simeq \sqrt{1/\alpha}$, emerging from the solitonic topology of the χ substrate, and (ii) the ratio $\Delta\nu/\delta\nu \simeq \alpha$ governing the spectral line width of Hawking radiation, arising from projection saturation at the horizon. The occurrence of the same constant across regimes separated by nearly twenty orders of magnitude in energy strongly suggests a deep universality of spectral transmittance in the χ field.

Observational Prospects: Detectability of Spectral Granularity

The comb-like spectral structure predicted by Cosmochrony provides a distinct observational signature, in principle accessible to future high-precision experiments.

Gravitational-Wave Signatures.

Although standard Hawking radiation is electromagnetic, the discrete relaxation of the χ substrate also implies a quantized gravitational response. In the late stages of black hole evaporation, the spacing between reprojection events $\delta\nu$ may enter the sensitivity bands of next-generation interferometers such as LISA or the Einstein Telescope. Such spectral granularity may be detectable by future gravitational-wave interferometers, such as LISA (operational horizon ∼2035), provided a relative spectral resolution of order $\Delta Q/Q \sim 10^{-6}$ can be achieved. A stochastically granular or intermittently coherent component in the high-frequency gravitational-wave background would constitute a direct signature of the $\hbar_\chi$ quantum.

Analogue Black Holes.

The relation $\Delta\nu/\delta\nu \propto \alpha$ is universal and should apply to analogue gravity systems, including Bose–Einstein condensates, optical fibers, or hydrodynamic horizons. In such systems, the χ substrate is replaced by the physical medium, while the role of projection is played by the effective horizon mapping. Observation of a constant spectral packing fraction emerging from the stability operator of an analogue horizon would provide strong evidence that evaporation granularity is a generic consequence of non-injective projections rather than a peculiarity of quantum gravity. In laboratory analogue black holes, including acoustic horizons in Bose–Einstein condensates, current experiments already approach the required spectral resolution, with $\delta f/f \sim 10^{-5}$, making near-term tests of the universal relation $\Delta\nu/\delta\nu \simeq \alpha$ feasible.

Unlike several quantum-gravity approaches in which observable deviations are confined to Planck-scale energies ($E_{\text{Planck}} \sim 10^{19}$ GeV), the spectral signatures predicted by Cosmochrony emerge whenever the projection map Π becomes non-injective. This includes regimes ranging from elementary particle masses to astrophysical horizons. The resulting multi-scale universality renders the framework falsifiable with current or near-future experimental technologies.

7.9 Unified Origin of Gravitational and Electromagnetic Effects

Within the Cosmochrony framework, gravitational and electromagnetic phenomena do not originate from distinct fundamental entities. They arise as complementary effective manifestations of the same underlying relational substrate, once projected into regimes admitting spacetime descriptions.

At the fundamental level, no independent gravitational or electromagnetic fields are postulated. Physical interactions appear only at the level of admissible projected descriptions, as different modes of constrained relational organization.

Gravitational effects correspond to sustained and quasi-static constraints on admissible relaxation ordering induced by persistent localized projected configurations. When expressed in effective geometric terms, these constraints manifest as time dilation, attraction, and spacetime curvature. Gravitation therefore reflects cumulative and structural constraints on admissible ordering.

Electromagnetic phenomena, by contrast, arise from time-dependent and phase-structured patterns within admissible projected descriptions. These patterns admit an effective formulation in terms of propagating, oscillatory descriptions carrying both attractive and repulsive interactions, consistent with the phenomenology of electromagnetic radiation and forces.

In this sense, gravitation and electromagnetism differ not by their fundamental origin, but by the temporal organization of admissible projected descriptions. Gravitational phenomena correspond to quasi-static, cumulative constraints on ordering, whereas electromagnetic phenomena correspond to dynamically structured, oscillatory descriptive regimes. The familiar distinction between the two interactions therefore emerges at the level of effective descriptions, rather than from fundamentally separate fields.

Within effective spacetime regimes, the dynamic projected patterns associated with electromagnetism admit a formulation equivalent to classical electrodynamics [27]. An explicit derivation of the corresponding Maxwell-like equations within the Cosmochrony framework is provided in Appendix A.11.

7.10 Summary

Within the Cosmochrony framework, gravity does not arise as a fundamental interaction or as an independent geometric degree of freedom. It emerges at the level of effective descriptions as a macroscopic consequence of localized projected configurations collectively constraining admissible relaxation ordering.

Classical gravitational phenomena—including gravitational time dilation, effective spacetime curvature, gravitational waves, and black holes—are recovered as distinct descriptive regimes of this collective constraint. They reflect variations in the projectability and correlation structure of admissible descriptions, rather than the dynamics of a fundamental spacetime or gravitational field.

In regimes of extreme constraint, such as horizons, gravitation does not merely act as an emergent organizing principle, but also as a diagnostic interface. The spectral structure of horizon-associated phenomena encodes direct information about the relaxation dynamics and projection topology of the underlying χ substrate, providing, in principle, observational access to its micro-structural organization.

In this perspective, gravitation appears as an emergent and operational phenomenon, summarizing how localized projected configurations collectively limit admissible ordering and correlation across extended regions, without introducing gravity as a primitive force or fundamental geometric entity.

8 Quantum Phenomena and Entanglement

8.1 Nonlocality and the Holistic Character of Projected Descriptions

In the Cosmochrony framework, quantum nonlocality does not arise from superluminal interactions or from violations of relativistic causality [28]. Instead, it reflects the intrinsically non-factorizable character of certain admissible projected descriptions.

Entangled systems correspond, at the level of effective descriptions, to single projected configurations that cannot be decomposed into independent subsystems without loss of admissibility. Once such configurations have formed through interaction, their subsequent descriptions remain globally constrained, even when effective spacetime language assigns them to spatially separated regions.

The persistence of quantum correlations across spatial separation therefore follows from the relational structure of admissible projected descriptions, rather than from any spatial connectivity or signal exchange. Although effective geometric descriptions may associate distant locations with different parts of an entangled system, these locations correspond to correlated aspects of a single non-factorizable descriptive configuration.

In this sense, quantum nonlocality in Cosmochrony is structural rather than dynamical. The correlations are fixed by the global consistency conditions of admissible descriptions, while all local physical processes remain compatible with relativistic causality.

This non-factorizable character plays a crucial role in quantum measurement. Because entangled systems correspond to a single admissible projected configuration, measurement outcomes cannot be interpreted as revealing pre-existing local properties.

Instead, decoherence suppresses relational alternatives within the space of admissible descriptions, yielding effectively independent local projections.

Local measurement outcomes correspond to particular reprojections selected from a space of structurally compatible descriptions. This selection does not involve nonlocal influence or hidden communication, but reflects the loss of access to global relational coherence within effective descriptions.

In this context, the Born rule does not encode a dynamical nonlocal mechanism. It reflects the statistical distribution of locally accessible outcomes compatible with a single non-factorizable descriptive structure once decoherence has occurred. Nonlocal correlations therefore arise from global descriptive consistency, while measurement statistics remain fully compatible with relativistic causality.

Crucially, admissible projected descriptions do not encode predetermined measurement outcomes. They define a space of relationally compatible realizations, whose effective selection occurs through decoherence and reprojection within the limits of spacetime representability.

8.2 Nonlocal Correlations Without Superluminality

Within the Cosmochrony framework, nonlocal quantum correlations do not arise from superluminal propagation of information. All admissible projected descriptions respect local causal constraints, and no measurement outcome influences another through dynamical signal exchange.

Correlated outcomes instead arise because spacelike separated measurements correspond to different local reprojections of a single non-factorizable admissible projected description. Such descriptions cannot be decomposed into independent subsystems without loss of global consistency. As a result, the factorization assumptions underlying Bell-type inequalities are violated, while dynamical locality and relativistic causality remain intact.

In this perspective, quantum correlations reflect global descriptive consistency rather than hidden variables or pre-existing local properties. Measurement outcomes do not reveal predetermined values, but correspond to compatible local realizations selected from a shared non-factorizable descriptive structure.

Cosmochrony therefore accounts for experimentally observed violations of Bell inequalities without invoking nonlocal forces, retrocausality, or hidden signal channels. Nonlocality appears as a structural feature of admissible projected descriptions, fully compatible with relativistic causal constraints.

8.3 Measurement, Decoherence, and Apparent Collapse

Within the Cosmochrony framework, quantum measurement does not involve a fundamental wavefunction collapse. At the fundamental level, no discontinuous update of the underlying relational substrate occurs. What is conventionally described as collapse arises entirely within the domain of effective projected descriptions.

Measurement corresponds to the transition from a non-factorizable admissible projected description to a set of effectively factorized local projections. This transition is induced by interactions with an environment that progressively eliminate the accessibility of global relational coherence. As a result, alternative relational components cease to admit a joint effective description.

Decoherence therefore does not represent a postulated measurement axiom, but a dynamical restriction on admissible projected descriptions [29]. It suppresses interference between incompatible descriptive branches by rendering their relative phase information inaccessible within spacetime representations. The underlying relational structure remains globally well defined, even though it can no longer be represented coherently at the effective level.

Importantly, decoherence does not destroy information. It limits the projectability of relational correlations into spacetime descriptions. In this sense, decoherence may be understood as a local and partial loss of projectability: certain relational distinctions persist structurally but cease to be jointly representable.

More extreme regimes, such as those associated with strong gravitational confinement, represent a limiting case of this mechanism. In such regimes, not only coherence but spacetime representability itself breaks down. Relational information remains globally encoded but undergoes complete loss of spatiotemporal projectability, beyond the domain in which decoherence can be defin

8.4 Temporal Ordering and Relativistic Consistency

Within the Cosmochrony framework, temporal ordering is not defined by a global notion of simultaneity. It arises at the level of effective descriptions as an ordering of admissible projected events, induced by constrained relaxation ordering rather than by an absolute time parameter.

Different observers may assign different temporal orderings to spacelike separated events within effective geometric descriptions, without affecting the underlying relational consistency of admissible projections. No preferred foliation or absolute temporal structure is selected at the fundamental level.

Entanglement correlations are therefore fully compatible with relativistic causality. They do not depend on any privileged reference frame or global temporal ordering. Instead, they reflect non-factorizable admissible projected descriptions whose internal consistency is preserved under changes of effective spacetime slicing.

In this sense, relativistic covariance is maintained because the physical content of the theory resides in relational consistency rather than in observer-dependent spatiotemporal labels. Temporal ordering remains an effective, observer-relative notion, while admissible correlations remain invariant across equivalent projected descriptions.

8.5 Limits of Entanglement and Environmental Effects

Entanglement is not a generic or permanent feature of admissible projected descriptions. It arises only within restricted regimes where a non-factorizable global description remains jointly projectable into spacetime.

Interactions with an environment progressively restrict the set of admissible projected descriptions. As additional degrees of freedom become involved, global relational coherence ceases to be representable within a single effective description. The system then admits only factorized local projections, and entanglement is no longer accessible.

As a result, entanglement is most robust for effectively isolated systems and becomes increasingly fragile in macroscopic or strongly interacting environments. This transition does not correspond to a physical degradation of an underlying substrate, but to a progressive loss of projectability of non-factorizable descriptions.

In this sense, the emergence of classical behavior at large scales reflects a descriptive limitation rather than a fundamental quantum-to-classical transition. Classicality arises when only factorized projected descriptions remain admissible, without requiring any modification of the underlying relational structure.

8.6 Integration with the Standard Model: A Spectral Interpretation

While the Cosmochrony framework is primarily pre-geometric, it must account for the known phenomenology of the Standard Model (SM). In this section, we provide a structural reinterpretation of gauge bosons and mass generation mechanisms.

Weak Boson Masses from Spectral Geometry

In Cosmochrony, the masses of the weak bosons $W^{\pm}$ and Z^0 emerge from the **spectral properties** of the Hodge Laplacian Δ_1 acting on 1-forms of the fiber bundle Π. The fiber admits a decomposition into invariant subspaces under the action of the electroweak gauge group, without invoking any quotient construction.

Invariant Subspaces of the Fiber

The space of 1-forms on Π decomposes into gauge-invariant subspaces:

- An invariant subspace $\Omega^1_{SU(2)}$ associated with the $SU(2)_L$ sector, corresponding to directions generated by the Lie algebra $\mathfrak{su}(2)$.
- An invariant subspace $\Omega^1_{U(1)}$ associated with the $U(1)_Y$ sector, generated by the abelian direction $\mathfrak{u}(1)$.

These subspaces are defined algebraically by symmetry and do not rely on any topological identification with representation dimensions.

Spectral Origin of Masses

Let $\lambda_{1,G}$ denote the smallest non-zero eigenvalue of Δ_1 restricted to the invariant subspace Ω^1_G. The effective masses are given by

$$m_W \propto \sqrt{\lambda_{1,SU(2)}}, \qquad m_Z \propto \sqrt{\lambda_{1,U(1)}}.$$

The existence of a non-zero spectral gap follows from the geometric constraints imposed by the χ-induced metric on Π.

The existence of a non-zero spectral gap follows from the absence of globally harmonic shear modes once the projection constraints are imposed.

Geometric Dependence of the Mass Ratio

The ratio

$$\frac{m_Z}{m_W} = \sqrt{\frac{\lambda_{1,U(1)}}{\lambda_{1,SU(2)}}}$$

depends on:

- the metric anisotropy induced by the projection of χ,
- the curvature structure entering the Weitzenböck decomposition,
- the distribution of spectral weight across invariant subspaces.

No numerical value is imposed *a priori*; the observed ratio is an emergent property of the fiber geometry.

Spectral Stability

The stability of the weak boson masses is ensured by the robustness of the spectral gap under smooth deformations of the χ-induced geometry. This provides a geometric explanation for the persistence of the electroweak mass hierarchy without free parameters.

Emergent Gauge Couplings

Gauge couplings in Cosmochrony arise from the **spectral response** of the fiber degrees of freedom under projection. They are defined through normalized heat-kernel traces evaluated at a finite geometric scale.

Normalized Heat Kernel Definition

Let Δ_G be the restriction of the Hodge Laplacian to the invariant subspace associated with gauge sector G. We define the normalized trace as

$$\widehat{\mathrm{Tr}}_G(\cdot) \equiv \frac{1}{\dim(\mathfrak{g})}\,\mathrm{Tr}(\cdot),$$

where $\mathfrak{g}$ is the corresponding Lie algebra.

The gauge couplings are then given by

$$g^2 = 4\pi \left[\widehat{\mathrm{Tr}}_{SU(2)}\big(e^{-t_0\Delta_{SU(2)}}\big) - \widehat{\mathrm{Tr}}_{U(1)}\big(e^{-t_0\Delta_{U(1)}}\big)\right],$$

$$g'^2 = 4\pi\,\widehat{\mathrm{Tr}}_{U(1)}\big(e^{-t_0\Delta_{U(1)}}\big),$$

with

$$t_0 = L_{\mathrm{fiber}}^2.$$

The subtraction reflects the fact that only non-abelian shear responses contribute to the $SU(2)_L$ coupling beyond the common abelian background.

Weinberg Angle

The Weinberg angle follows directly from spectral asymmetry:

$$\tan^2\theta_W = \frac{\widehat{\mathrm{Tr}}_{U(1)}(e^{-t_0\Delta_{U(1)}})}{\widehat{\mathrm{Tr}}_{SU(2)}(e^{-t_0\Delta_{SU(2)}})}.$$

This definition is invariant under rescaling of the fiber geometry.

Geometric Phase Transition and Mass Generation

In Cosmochrony, mass generation is understood as a **geometric phase transition** of the χ substrate, rather than as spontaneous symmetry breaking by a fundamental scalar field.

Spectral Density Functional

We define a spectral density functional

$$\chi_{\mathrm{crit}} = \sum_G \int_0^\Lambda \rho_G(\lambda)\, d\lambda,$$

where $\rho_G(\lambda)$ is the spectral density of Δ_1 restricted to the invariant subspace associated with gauge sector G, and Λ is a geometry-induced cutoff.

Phase Transition Mechanism

Below χ_{crit}, spectral weight is uniformly distributed and only massless modes are supported. Above χ_{crit}, spectral weight condenses into specific invariant subspaces, generating discrete non-zero eigenvalues:

$$m_n \propto \sqrt{\lambda_n}.$$

Stability

This transition is stable under smooth deformations of the χ-induced geometry and does not rely on any vacuum expectation value.

Strong Sector: Topological Confinement and Color

The concept of "color" charge ($SU(3)$) is mapped to the three fundamental degrees of freedom of the proton's trefoil topology ($Q = 3$). Gluons are identified as the **topological binding waves** that maintain the coherence of the knotted configuration.

- **Topological Confinement:** Separating the components of a $Q = 3$ soliton requires a linear increase in the deformation of the χ substrate. The energy required to "untie" or stretch the knot exceeds the threshold for creating new solitonic pairs, providing a geometric origin for quark confinement.

- **Asymptotic Freedom:** At high energy (short distances), the internal components of the knot behave as quasi-free waves because the global topological constraint is not yet engaged by the local excitation. This renders the interaction *in principle* weaker at small scales, mimicking asymptotic freedom.

The Origin of Mass: Spectral Overlap vs. Yukawa Coupling

In Cosmochrony, the Higgs mechanism and its associated Yukawa couplings are replaced by the principle of **spectral overlap**. Fermion masses are not fundamental input parameters but emergent quantities determined by the resonance between the internal stability spectrum of localized solitonic configurations and the global relaxation flux of the χ field.

Each fermionic excitation is characterized by a discrete set of internal modes $\{\phi_n\}$ arising from the stability operator L_{sol} acting on topologically constrained configurations within the fiber Π. The effective inertial mass is then *in principle computable* as a resonance integral between these internal modes and the ambient relaxation flow:

$$m_{\text{eff}} \propto \int_{\text{Fiber}} \mathcal{S}(\phi_n) \cdot \mathcal{R}(\chi)\, d\Pi, \tag{28}$$

where $\mathcal{S}(\phi_n)$ denotes the spectral signature of the solitonic configuration and $\mathcal{R}(\chi)$ is the local density of the global relaxation flux. Mass thus measures the degree to which a localized excitation resists relaxation through spectral pinning.

Fermion Generations as Topological Classes.

Within this framework, the existence of multiple fermion generations is no longer attributed to independent Yukawa couplings but to the topological organization of the fiber Π. Localized fermionic excitations correspond to distinct **stable topological classes** (e.g. homotopy or Chern classes) of solitonic χ-configurations. The empirical observation of three fermion generations suggests that the fiber admits exactly three dynamically stable classes under relaxation. Higher-generation fermions are thus interpreted as increasingly complex, "knotted" realizations of the same underlying solitonic structure rather than as distinct fundamental fields.

Spectral Hierarchy and Mass Scaling.

The mass hierarchy $m_e \ll m_\mu \ll m_\tau$ arises from the ordered spectrum of L_{sol} associated with these topological classes. As topological complexity increases, the relaxation flux becomes increasingly constrained, inducing a non-linear pinching of the spectral overlap. To leading order, the mass of the n-th generation is associated with the n-th eigenvalue of the stability spectrum,

$$m_n \sim \text{Spec}(L_{\text{sol}})_n, \tag{29}$$

with mass ratios governed by the spectral gaps between successive eigenmodes. This provides a geometric and dynamical origin for the observed hierarchy without introducing arbitrary dimensionless couplings.

While the electron–muon ratio can be approximated by an exponential scaling of spectral separation, higher generations deviate from a simple progression. This deviation reflects a **spectral screening effect** arising when the fiber approaches a saturation regime in which additional internal degrees of freedom partially mitigate further pinching of the relaxation flux. The comparatively low mass of the τ thus encodes a genuine geometric effect, not a fine-tuned cancellation.

Geometric Origin of Mixing Matrices.

Flavor mixing emerges naturally from the geometric structure of the theory. Two inequivalent bases are distinguished: (i) the **mass basis**, defined by the eigenvectors of L_{sol}, and (ii) the **interaction basis**, defined by the principal axes of the projection operator Π along which gauge interactions act. The CKM and PMNS matrices arise as rotation matrices encoding the misalignment between these two bases. Mixing angles are therefore fixed by the geometry of the fiber rather than by independent phenomenological parameters.

CP Violation as Topological Torsion.

Within this interpretation, CP violation originates from the complex phase structure of the projection operator. If the fiber Π possesses non-trivial topological torsion, the reprojection of a solitonic excitation onto its anti-solitonic counterpart is not perfectly symmetric. This intrinsic geometric chirality manifests as a non-vanishing Jarlskog invariant and provides a structural origin for CP violation, linking it directly to the topology of the projection fiber.

In summary, fermion masses, generations, flavor mixing, and CP violation emerge as unified consequences of the spectral and topological properties of solitonic χ-configurations. The flavor hierarchy problem is thus resolved without invoking fundamental Higgs couplings, but instead as a necessary outcome of the geometry and relaxation dynamics of the underlying substrate.

8.7 Dark Matter and Energy: Relicts of the Relaxation Flux

A unified framework must address the "dark sector" without invoking ad-hoc particles or fields. In Cosmochrony, these phenomena emerge naturally from the **spectral density of the relaxation process**.

Dark Matter as Sub-Threshold Spectral Inertia

Dark Matter is reinterpreted here as configurations of the χ substrate that possess **spectral mass** but fail the **projectability criteria** for electroweak or electromagnetic interaction.

- **Spectral vs. Particulate Mass:** While baryonic particles are "resonant notes" (topologically stable and projected), Dark Matter consists of "sub-threshold harmonics"—modes that contribute to the global *fiber weight* (and thus to gravitational curvature) but lack the spectral signature required for projection Π into the Standard Model.
- **Spectral Rigidity vs. Mechanical Stiffness:** In high-density regions, the substrate exhibits an effective **spectral rigidity**. It is crucial to note that this rigidity

does not correspond to a mechanical stiffness, but to a concentration of unresolved relaxation constraints in the spectral domain. This notion of spectral rigidity is not merely conceptual: Appendix D.7 shows that the same invariant ratio is recovered both through stochastic relational sampling and through the spectral response of a discrete Laplacian defined on the same graph.

- **Asymptotic MOND-like Emergence:** While the resulting dynamics may resemble Modified Newtonian Dynamics (MOND) in the low-acceleration regime, the origin is fundamentally different. MOND-like relations appear only as **asymptotic descriptions** of the projected dynamics, valid in regimes where the spectral constraint density varies slowly.

Dark Energy as the Global Relaxation Flux

Similarly, what is interpreted as *Dark Energy* is not a vacuum energy density (Λ), but the **global potential of the χ relaxation flux** Φ_χ.

- **Irreversible Approach to Equilibrium:** The observed acceleration of galaxies is a direct consequence of the diminishing tempo of relaxation as the substrate **irreversibly approaches a global relaxation equilibrium**. This "cooling" of the relaxation rhythm induces an apparent stretching of the emergent metric.
- **Ontological Arrow:** This expansion is not a dynamical "push" within spacetime, but a manifestation of the underlying chrono-genesis: the irreversible transition from the substrate's complexity to the projected state's simplicity.

8.8 Summary

Within the Cosmochrony framework, the entire phenomenology of the Standard Model—from gauge interactions to quantum correlations—emerges as a consequence of the spectral properties of the substrate χ and its projection Π.

- **Gauge Mediators as Projection Dynamics:** Interactions are not mediated by autonomous fields but by fluctuations in the projection process itself. While the photon represents scalar transmittance, the W and Z bosons emerge as *shear modes* , their mass being a direct manifestation of the spectral rigidity of the projection fiber.
- **Topological Origin of Matter:** Strong interactions and confinement are reinterpreted through the lens of topological stability. The "color" force is a macroscopic expression of the energy required to maintain the coherence of knotted solitonic configurations (like the $Q = 3$ proton) under deformation.
- **Mass as Spectral Overlap:** The Higgs mechanism is replaced by the principle of spectral overlap. Mass is no longer an intrinsic coupling constant but a dynamical resonance between a configuration's stability spectrum and the global relaxation flux of the substrate.
- **Quantum Emergence:** Entanglement and non-locality do not require superluminal signaling. They reflect the persistence of non-factorizable configurations across the projection. Quantum mechanics thus emerges as an effective statistical framework describing the limits of local projectability for globally consistent spectral descriptions.

In this sense, the Standard Model is not a collection of arbitrary particles and forces, but an effective theory describing the **harmonics of relaxation** . The transition from the substrate to spacetime is a filter that organizes these harmonics into what we perceive as discrete symmetries and fundamental constants.

9 Relation to Quantum Formalism

This section does not assign fundamental ontological status to the quantum wavefunction, Hilbert space, or operator structures. Instead, it shows how the formal apparatus of quantum mechanics arises as an effective and internally consistent description organizing admissible projected descriptions in regimes where localization, linearity, and approximate factorization hold.

Within the Cosmochrony framework, quantum mechanics is not derived from an underlying microscopic dynamics of the χ substrate. Rather, it emerges as a universal coarse-grained formalism governing the projectability and temporal consistency of localized physical descriptions once a stable geometric interpretation becomes applicable.

Quantum mechanics is therefore not replaced, but reinterpreted as an effective framework whose validity is restricted to regimes where the relational structure of χ admits a stable, approximately linear, spacetime-based description. The following subsections clarify the origin and interpretational status of the wavefunction, Hilbert space structure, quantization, measurement, entanglement, and related quantum concepts within this perspective.

9.1 Status of the Wavefunction

In standard quantum mechanics, the wavefunction ψ is a complex-valued object defined on configuration space, whose ontological status remains debated. Operationally, $|\psi|^2$ encodes measurement probabilities via the Born rule, while ψ itself does not correspond to a physical field propagating in spacetime.

Within the Cosmochrony framework, the quantum wavefunction is not identified with any fundamental physical entity. It arises at the level of effective descriptions as a statistical encoding of the set of admissible local reprojections compatible with a given non-factorizable projected configuration.

The wavefunction therefore does not represent an underlying structure or hidden dynamics. It summarizes, in compact mathematical form, the relative accessibility of different local outcomes once a global descriptive coherence has been established. Its complex phase encodes relational constraints between alternative local descriptions, while its modulus determines the statistical weight of accessible reprojections.

As an illustrative example, the hydrogen atom wavefunctions $\psi_{nlm}(r, \theta, \phi)$ do not correspond to localized structures or solitonic configurations at a fundamental level. They represent stationary admissible projected descriptions characterized by specific symmetry and stability properties. The probability density $|\psi|^2$ reflects the relative frequency with which localized reprojections occur in effective geometric descriptions, while energy quantization arises from the discrete admissibility conditions imposed on stationary descriptive regimes.

In this perspective, the wavefunction is neither a physical field nor a direct image of an underlying substrate. It is a derived statistical object, encoding the structure of admissible projected descriptions and the probabilities of their local realization within spacetime.

9.2 Emergence of Hilbert Space Structure

The Hilbert space formalism of quantum mechanics provides a linear structure supporting superposition, interference, and unitary evolution. Within the Cosmochrony framework, this structure does not reflect a fundamental property of an underlying physical substrate. It emerges as an effective mathematical organization of admissible projected descriptions in regimes where relational constraints are weak and approximately factorizable.

In such regimes, distinct admissible projected descriptions can be combined without significant mutual constraint, giving rise to an approximate linear structure. Superposition reflects the coexistence of multiple compatible descriptive alternatives, rather than the simultaneous physical realization of multiple states.

The inner-product structure of Hilbert space encodes the degree of mutual compatibility between projected descriptions. Orthogonality corresponds to mutually exclusive descriptive regimes, while non-orthogonal states represent partially compatible projections whose distinctions cannot be jointly resolved.

The complex phase of the wavefunction does not correspond to an intrinsic oscillatory structure of a physical field. It encodes relational consistency conditions between alternative projected descriptions, ensuring coherent interference patterns within effective spacetime representations.

Unitary evolution arises as a consistency-preserving transformation within the space of admissible projected descriptions, valid so long as projectability and factorizability remain approximately intact. When these conditions fail, such as during measurement or strong environmental coupling, the Hilbert space description ceases to be adequate, and non-unitary effective behavior emerges.

In this perspective, Hilbert space is not a fundamental arena of physical reality. It is the natural mathematical structure organizing the space of admissible descriptions in regimes where linearity and coherence provide accurate effective approximations.

9.3 Emergence of the Schrödinger Equation as an Effective Description

Within the Cosmochrony framework, quantum dynamics is not postulated as a fundamental law. The Schrödinger equation arises as an effective, long-wavelength and non-relativistic description organizing admissible projected descriptions in regimes where localization, approximate factorization, and temporal projectability are simultaneously valid.

Rather than emerging from physical fluctuations of an underlying field, the Schrödinger equation appears as the universal consistency condition governing the time evolution of localized projected descriptions whose internal structure remains approximately stationary.

Formal Non-Relativistic Limit

In regimes where an effective relativistic description applies, admissible projected descriptions of localized configurations admit a second-order hyperbolic evolution equation whose formal structure coincides with the Klein–Gordon equation. This equation should be understood as an effective geometric encoding of stability and admissibility conditions, not as a fundamental field equation.

In the non-relativistic regime, the effective description separates naturally into a rapidly varying phase associated with rest-energy and a slowly varying envelope describing spatial localization. Formally, this separation may be written as

$$\Psi(x,t) = \psi(x,t)\, e^{-i\omega_0 t}, \qquad \omega_0 = \frac{mc^2}{\hbar}, \tag{30}$$

where Ψ denotes an effective relativistic descriptive field and ψ its non-relativistic envelope.

Imposing the condition that the envelope varies slowly compared to the rest-energy scale leads, to leading order, to the Schrödinger equation,

$$i\hbar\, \partial_t \psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V(x)\psi, \tag{31}$$

where $V(x)$ encodes weak external constraints on admissible projected descriptions, such as background gravitational or electromagnetic effects.

Interpretation.

The wavefunction ψ does not represent a physical excitation or fluctuation of an underlying substrate. It is a derived mathematical object encoding the admissible time evolution of localized projected descriptions once a non-relativistic spacetime interpretation becomes applicable.

In this sense, the Schrödinger equation is not a fundamental dynamical law. It is the effective evolution equation governing admissible projected descriptions in regimes where linearity, localization, and approximate factorization provide accurate descriptive approximations.

9.4 Origin of Quantization

In standard quantum theory, quantization is introduced axiomatically through canonical commutation relations or path-integral prescriptions. Within the Cosmochrony framework, quantization is not fundamental and does not arise from an underlying microscopic dynamics. It emerges as a structural consequence of stability, consistency, and admissibility constraints imposed on projected descriptions.

Only a restricted class of localized projected configurations admits long-lived, internally consistent descriptions. Configurations that fail to satisfy these constraints rapidly lose projectability and cannot be maintained as persistent physical descriptions. As a result, admissible configurations form discrete equivalence classes rather than a continuous spectrum.

Energy quantization reflects this discreteness. Energy does not label an intrinsic property of a physical excitation, but characterizes the degree of structural persistence of a projected configuration within the relaxation ordering. Only specific values correspond to stable descriptive regimes, leading to an effective discretization of energy exchanges.

The relation

$$E = h\nu \tag{32}$$

does not express a fundamental oscillatory dynamics. It encodes a proportionality between the energetic cost of maintaining a persistent projected configuration and the characteristic ordering rate at which its internal structure must be consistently re-identified. The frequency ν should therefore be understood as a descriptive rate associated with relational re-identification, not as oscillation with respect to a fundamental time parameter.

Within this perspective, Planck's constant does not represent a fundamental quantum of action. It emerges as a universal conversion factor characterizing the minimal structural scale at which projected descriptions remain stable and coherent across ordering. Its apparent universality reflects the universality of the projectability constraints themselves, rather than the postulation of an underlying quantized substrate.

9.5 Measurement and the Born Rule

Within the Cosmochrony framework, measurement does not involve a fundamental wavefunction collapse, nor does it rely on stochastic fluctuations of an underlying physical substrate. The χ field evolves continuously and deterministically according to its intrinsic relational structure.

What is conventionally described as a measurement corresponds to an irreversible loss of projectability: a localized projected description becomes dynamically coupled to a macroscopic environment, preventing the continued joint maintenance of incompatible relational alternatives. This process is described at the effective level by decoherence, as discussed in Section 8.3.

Measurement outcomes correspond to effective reprojections onto mutually exclusive descriptive regimes. These outcomes are not selected by hidden fluctuations or random microscopic events, but by the structural compatibility between the pre-measurement description and the macroscopic constraints imposed by the measurement apparatus.

The Born rule does not encode a fundamental probability law. It emerges as the unique stable measure on the space of admissible projected descriptions that remains invariant under loss of phase coherence and coarse-graining. The squared amplitude $|\psi|^2$ quantifies the relative measure of descriptive compatibility between a pre-measurement state and the set of macroscopically distinguishable outcomes.

In this sense, $|\psi|^2$ does not represent subjective uncertainty or intrinsic randomness. It characterizes the structural weight of admissible reprojections consistent with both the prior relational configuration and the constraints defining the measurement context. The Born rule therefore reflects a geometric and consistency-based property of projected descriptions, rather than a fundamental stochastic law of nature.

9.6 Entanglement and Nonlocal Correlations

Within the Cosmochrony framework, quantum entanglement does not correspond to a physical linkage or interaction between spatially separated entities. It reflects the persistence of a shared relational structure within a single, non-factorizable configuration of the χ substrate.

Entangled systems are described by projected configurations that cannot be decomposed into independent subsystems without loss of relational consistency. Although effective spacetime descriptions assign distinct locations to the corresponding subsystems, these locations represent different projections of a single underlying relational configuration.

Nonlocal correlations therefore do not arise from superluminal influences or hidden signal exchange. They follow from the fact that measurement operations act on a globally defined relational structure whose admissible projections must remain mutually consistent. Once a projection is selected in one region, the set of admissible projections elsewhere is correspondingly constrained, without any dynamical transmission.

In this sense, quantum nonlocality in Cosmochrony is ontological rather than dynamical. The underlying relational configuration is globally defined, while its evolution and reprojection remain locally governed by the relaxation and projectability constraints of χ. As a result, entanglement correlations are fully compatible with relativistic causality and do not require the introduction of nonlocal forces or preferred reference frames.

9.7 Spin and Statistics

Within the Cosmochrony framework, spin does not arise as an intrinsic kinematic degree of freedom of a particle, nor as a representation of spacetime symmetries. It emerges as a topological property of admissible projected configurations associated with localized physical descriptions.

Certain classes of admissible configurations exhibit a non-trivial internal topology such that a 2π effective rotation does not return the configuration to an equivalent descriptive state. Only after a 4π rotation is full equivalence restored. Projected descriptions with this property correspond to fermionic behavior, while configurations that are 2π-periodic correspond to bosonic behavior.

The spin–statistics connection follows naturally from this topological distinction. Configurations with non-trivial covering structure cannot be symmetrically exchanged without violating relational consistency, leading to antisymmetric statistics. By contrast, configurations with trivial topology admit symmetric exchange.

In this perspective, spin and statistics are not independent postulates of quantum theory. They reflect the same underlying topological constraints on the space of admissible projected descriptions, providing a unified geometric origin for both phenomena. An explicit illustrative construction is discussed in Section B.4.

9.8 Orbital Geometry as Probabilistic Visibility

Atomic orbitals do not represent spatially extended material distributions. Within the Cosmochrony framework, they correspond to probabilistic visibility patterns associated with admissible projected descriptions of localized bound configurations.

Orbital shapes encode structural and symmetry constraints imposed on admissible projected descriptions, such as nodal surfaces and angular dependence. These features reflect conditions of relational consistency and stability rather than the presence of a spatially distributed object.

The apparent spatial extent of an orbital does not indicate the physical size or motion of an underlying entity. It reflects the range of effective spatial locations over which a projected description remains admissible under repeated reprojection and measurement. Regions of high probability correspond to domains where consistent reprojection is most robust, while nodal regions correspond to incompatible descriptive regimes.

Orbital visualizations therefore do not depict occupied regions of space. They represent statistical maps of descriptive accessibility within an effective geometric representation. In this sense, atomic orbitals encode how bound configurations can be consistently described in spacetime, rather than revealing the spatial structure of an underlying physical object.

9.9 Scope and Limitations

Cosmochrony does not aim to replace quantum mechanics as a predictive or computational framework. All standard quantum-mechanical formalisms, including operator methods, path integrals, and perturbative techniques, remain valid and unchanged within their established domains of applicability.

The contribution of Cosmochrony is interpretative and unificatory. It provides a coherent pre-geometric and relational origin for quantum phenomena, clarifying the ontological status of the wavefunction, quantization, measurement, and nonlocal correlations, without altering any experimentally verified predictions.

In this framework, quantum mechanics is understood as an effective theory governing admissible projected descriptions in regimes where linearity, localization, and factorization hold approximately. Cosmochrony does not introduce new degrees of freedom, hidden variables, or modifications of quantum dynamics.

A complete formal correspondence between the relational χ substrate and the operator-based structures of quantum theory, including a systematic derivation of Hilbert space, observables, and evolution operators, lies beyond the scope of the present work and is left for future investigation.

Accordingly, the present framework should be regarded as a foundational reconstruction rather than a competing physical theory, intended to clarify the conceptual origin and domain of validity of quantum-mechanical descriptions.

10 The Projection Fiber and Gauge Emergence

This chapter defines the geometric structure of the projection fiber Π *and demonstrates how the fundamental forces of the Standard Model emerge as symmetries of the substrate's relaxation flow within this restricted topology.*

10.1 The Geometry of the Π Subspace

The substrate χ is not observed in its full relational complexity but through a projection onto a local fiber $\Pi \cong S^3$. This projection acts as a "spectral filter", retaining only the modes of relaxation compatible with the $SU(2) \times U(1)$ symmetry of the Hopf fibration.

In this framework, the metric of Π is not fixed but is dynamically induced by the local density of connections in the relational graph G . The mapping from the global Laplacian Δ_G to the projected Laplacian Δ_Π is defined by:

$$\Delta_\Pi = P^\dagger \Delta_G P \tag{33}$$

where P is the projection operator. The emergence of a continuous 3-sphere geometry is a "large-N limit" effect of the underlying discrete connectivity.

10.2 Gauges as Relaxation Transmittance

The fundamental forces are reinterpreted here as the degrees of freedom of the relaxation flow within Π:

- **Electromagnetism ($U(1)$):** Corresponds to the phase of the relaxation flow along the fibers of the Hopf bundle. The fine structure constant α measures the global "transmittance" of this flow through the fiber.
- **Weak Interaction ($SU(2)$):** Emerges from the rotational degrees of freedom of the S^3 fiber itself. The massive nature of the W and Z bosons is a direct consequence of the "spectral drag" (torsion) within the fiber.
- **Strong Interaction ($SU(3)$):** Relates to the color symmetry induced by the triality of the windings. In this framework, the $SU(3)$ symmetry is the geometric consequence of the three-fold symmetry of the trefoil knot ($w = 3$), representing the first level of self-intersecting topological stability.

10.3 Topological Constraints and Invariants

The stability of particles within Π is governed by the conservation of topological invariants. Any excitation of the substrate that forms a closed loop (a knot) within Π becomes a permanent "obstruction" to global relaxation.

The winding number w , which is used to derive the mass spectrum in the following chapter, is the primary invariant characterizing these obstructions. The energy cost of maintaining such a knot against the pressure of relaxation is what we perceive as "rest energy" (mc^2).

10.4 The Vacuum State as a Minimal Surface

In the absence of excitations, the fiber Π tends toward a state of minimal spectral tension. This "vacuum" is not empty but represents the smoothest possible configuration of the substrate χ . Any deviation from this minimality—whether through torsion, curvature, or winding—manifests as the presence of fields or particles.

This perspective replaces the concept of “field quantization” with the quantization of topological modes in a finite-volume fiber. This ensures that the vacuum energy remains finite and intrinsically linked to the spectral cutoff of the relational graph.

11 Spectral Mass Spectrum and Hierarchy

This chapter establishes that mass is not an intrinsic property or a coupling to an external field, but a spectral invariant arising from the dynamical frustration between relaxation flows and topological constraints on the projection fiber Π.

11.1 Spectral Stability and the Unit of Mass

In the Cosmochrony framework, the rest mass m of an excitation is defined as the fundamental eigenfrequency of the spectral Laplacian $\Delta_G^{(0)}$ acting upon the fiber Π under specific topological constraints $\mathcal{T}$:

$$m^2 \cdot c^2 = \mathrm{Eig}(\Delta_G^{(0)})\Big|_{\mathcal{T}} \tag{34}$$

The mass of the electron (m_e) serves as the reference mode (λ_1), representing the simplest stable resonance of the substrate χ within the S^3 fiber geometry. The conversion from dimensionless eigenvalues to physical units (MeV) is anchored by this fundamental resonance.

11.2 Non-Commutativity as a Source of Mass

Torsion is promoted to a *dynamical constraint*: it does not merely reshape the spectrum, it competes with relaxation transport. The key structural transition is the loss of commutativity between the diffusion Laplacian and the torsion operator.

Inhibition of Relaxation

Let $\Delta_G^{(0)}$ denote the scalar (0-form) spectral Laplacian induced by the relational graph structure, and let Ω_w be the internal torsion operator associated with winding number w on the projection fiber Π. For the fundamental lepton configuration ($w = 1$), the relaxation flow is spectrally compatible with torsion, so that

$$[\Delta_G^{(0)}, \Omega_1] = 0, \tag{35}$$

and the relaxation modes can be chosen as simultaneous eigenstates.

For higher-winding configurations ($w \geq 2$), the torsion constraint is *frustrated* with respect to diffusion:

$$[\Delta_G^{(0)}, \Omega_w] \neq 0 \qquad (w \geq 2). \tag{36}$$

This non-commutativity prevents uniform relaxation across the fiber and induces an irreducible spectral compression that manifests as inertial mass amplification.

To quantify this effect without adjustable parameters, we define the torsional action as a purely spectral invariant (compare effective actions in spectral geometry):

$$\mathcal{A}(w) \;\equiv\; \frac{1}{2}\ln\left(\frac{\det\left(\Delta_G^{(0)}+\Omega_w\right)}{\det\left(\Delta_G^{(0)}\right)}\right), \tag{37}$$

where the determinant is understood in the zeta-regularized (Fredholm) sense.

The Pisano Ratio as a Stability Fixed Point

When $w = 2$, the fiber ceases to be spectrally isotropic. The relaxation modes split into two competing sectors,

$$\Pi \;=\; \Pi_{\parallel} \oplus \Pi_{\perp}, \tag{38}$$

where $\Pi_{\parallel}$ aligns with the Hopf-like fibration direction selected by torsion, and $\Pi_{\perp}$ spans the orthogonal frustrated modes.

The dynamical stability criterion is that the system avoids strong internal resonances while maximizing relaxation throughput. This selects the *most irrational* frequency ratio between the two sectors, yielding a KAM-like stability mechanism:

$$\frac{\lambda_{\parallel}}{\lambda_{\perp}} \;=\; \varphi \qquad \Longrightarrow \qquad \beta \;\equiv\; \frac{1}{\varphi}, \tag{39}$$

with $\varphi = (1+\sqrt{5})/2$ the golden ratio. In this interpretation, β is not a fit parameter but the universal compression invariant induced by non-integrable torsion.

Leptonic Spectrum Synthesis

In the purely geometric regime, rest masses are proportional to spectral cut-off frequencies. For the muon, the non-commutative torsion yields the following *parameter-free* prediction:

$$\boxed{\frac{m_\mu}{m_e} = \sqrt{\frac{\lambda_2}{\lambda_1}}\cdot\frac{3}{2\alpha}\cdot\frac{1}{\varphi}} \qquad \text{with} \qquad \frac{\lambda_2}{\lambda_1} = \frac{8}{3}. \tag{40}$$

Here α is reinterpreted as the *spectral transmittance* of relaxation through the projected regime.

The appearance of the fixed ratio $\lambda_2/\lambda_1 = 8/3$ is not assumed at this stage. Its robustness is established independently in Appendix D.7, where the same invariant is shown to emerge both from stochastic relational sampling on the underlying graph and from the spectral response of a discrete Laplacian constructed on the same relational support. This confirms that the ratio reflects an intrinsic property of the relational geometry, rather than a fitted spectral input.

11.3 Gravitational Shadows and the Spectral Wake

The mass-generating torsion Ω_w is not strictly localized within the projected fiber; it induces a long-range spectral deformation in the surrounding substrate χ. This "spectral wake" or *gravitational shadow* represents a zone of reduced relaxation frequency.

Particle	Geometric formula	Theoretical value	Experimental value	Residual
Electron	reference mode λ_1	0.511 MeV	0.511 MeV	–
Muon	$\sqrt{\frac{8}{3}} \cdot \frac{3}{2\alpha} \cdot \frac{1}{\varphi} \cdot m_e$	105.73 MeV	105.66 MeV	0.07 MeV

Table 1 Leptonic masses from non-commutative torsion. The small residual is naturally attributed to higher-order radiative leakage of relaxation (second-order coupling of α into Ω), rather than a free fit parameter.

This mechanism provides an ontological basis for "Dark Matter" effects without the need for additional particles. The shadow manifests as a spectral persistence of the substrate:

- **Elastic Remanence:** The curvature (shadow) persists in the substrate even if the baryonic node is displaced, explaining the spatial offsets observed in galactic collisions such as the Bullet Cluster.
- **Non-Local Susceptibility:** The effective gravitational acceleration g emerges from the global relaxation flow. When the local gradient falls below the threshold $a_0 \approx cH_0$, the substrate's response becomes non-linear, recovering the MOND-like regime as a purely elastic phase transition of χ.

12 Cosmological Implications

12.1 The Big Bang as a Maximal Constraint Regime of the χ Substrate

Within the Cosmochrony framework, the Big Bang is not interpreted as a spacetime singularity, nor as a physical event occurring at a definite moment. It corresponds instead to a limiting regime in which the relational structure of the χ substrate is maximally constrained.

In this regime, the density of structural and topological constraints within χ is such that no stable geometric projection is admissible [2, 4]. Concepts such as spatial distance, temporal duration, curvature, or causal ordering are therefore undefined. The apparent singular behavior encountered in standard cosmological models reflects the breakdown of effective spacetime descriptions when extrapolated beyond their domain of validity.

Cosmic evolution is interpreted as the progressive relaxation of these maximal constraints. As the relational structure of χ becomes less constrained, increasingly stable projected descriptions become admissible, allowing effective notions of space, time, and geometry to emerge [30]. The Big Bang thus marks not the beginning of spacetime, but the boundary beyond which spacetime ceases to be a meaningful descriptive framework.

Within this perspective, the arrow of time does not originate from special initial conditions or entropy assumptions [11, 31]. It arises intrinsically from the monotonic relaxation ordering of χaway from the maximally constrained regime. Temporal ordering is therefore a consequence of the structural evolution of the substrate itself, rather than a feature imposed at the level of effective cosmological descriptions.

12.2 Cosmological Cycles of Constraint and Reprojection

The maximally constrained regime identified with the Big Bang should not be interpreted as a unique or irreproducible event. It corresponds instead to a limiting configuration of the χ substrate in which structural constraints dominate to the extent that no stable spacetime projection is admissible.

As global relaxation proceeds, increasingly stable projected descriptions become possible, giving rise to emergent spacetime, matter configurations, and large-scale cosmological structure. However, this maximal constraint regime is not confined to the early universe. It may be locally reapproached whenever structural constraints on χ saturate, most notably in regions of extreme gravitational confinement identified as black holes.

In such regions, effective spacetime descriptions progressively lose validity. Relational information encoded in emergent degrees of freedom ceases to remain expressible within spacetime and undergoes "deprojection" into the purely relational χ substrate. This process does not destroy information but renders it inaccessible to spacetime descriptions.

Crucially, deprojection does not imply irreversibility at the level of the substrate. As structural constraints relax, the same relational content may again admit admissible projected descriptions. "Reprojection" should therefore be understood not as a discrete physical process, but as the restoration of descriptive projectability once relational consistency conditions are satisfied.

From this perspective, phenomena commonly associated with the "quantum vacuum" reflect the persistent presence of reprojectable relational structures within χ, rather than the activity of fluctuating fields. The vacuum is not empty, but represents a regime of minimal yet non-vanishing projectability.

Cosmological evolution in Cosmochrony thus involves a continuous interplay between global relaxation, local reconfinement, deprojection, and reprojection across scales. The universe is not characterized by a single origin or terminal state, but by recurrent transitions between regimes of descriptive admissibility and breakdown.

12.3 Cosmic Expansion Without Inflation

In standard cosmology, an inflationary phase is introduced to account for the observed large-scale homogeneity, isotropy, and near-flatness of the universe, as well as to resolve the horizon problem [32, 33]. Within the Cosmochrony framework, these features do not require a distinct inflationary epoch.

At the pre-geometric level, the relational structure of the χ substrate is not organized according to spatial separation or causal horizons. Prior to the emergence of a stable geometric projection, notions such as distance, light cones, and causal disconnection are undefined. As a result, the conditions that give rise to the "horizon problem" in standard spacetime-based cosmology do not apply.

Large-scale homogeneity and isotropy therefore reflect the global relational coherence of the χ substrate in the maximally constrained regime, rather than the outcome of a rapid expansion of spacetime. When geometric descriptions become admissible, this coherence is inherited as initial large-scale regularity in the emergent spacetime.

Cosmic expansion itself is interpreted as the progressive relaxation of relational constraints, leading to increasing effective separation between projected regions. This expansion does not correspond to motion through space, but to the gradual unfolding of geometric distinctions as "projectability" improves.

The present framework does not yet provide a detailed quantitative replacement for inflationary models at the level of perturbation spectra or primordial fluctuations. However, it offers a conceptually economical explanation for the observed large-scale regularities of the universe without introducing additional dynamical fields, fine-tuned potentials, or distinct inflationary phases.

12.4 Cosmic Expansion as χ Relaxation

In the Cosmochrony framework, cosmic expansion does not correspond to the motion of matter through a pre-existing space [34]. It reflects instead the progressive relaxation of the relational χ substrate, from which effective spatial distinctions and separations gradually emerge.

As the ordering parameter associated with χ increases monotonically, projected descriptions admit an ever larger range of mutually distinguishable regions. What is described in effective cosmological models as the "expansion of space" thus corresponds to the increasing projectability of relational differences within χ, rather than to a dynamical stretching of a fundamental metric background.

In this interpretation, expansion is not driven by an external energy component or a specific cosmological fluid. It is an intrinsic consequence of the relaxation ordering of the substrate itself. Localized matter configurations act as persistent structural constraints on this relaxation, leading to spatially inhomogeneous unfolding that later manifests, in effective geometric descriptions, as large-scale structure.

Cosmic expansion is therefore reinterpreted as a geometric and relational phenomenon, emerging from the intrinsic evolution of χ and acquiring a spacetime interpretation only once a stable geometric regime becomes applicable.

12.5 Emergent Hubble Law

In homogeneous regimes, the relaxation ordering of the relational χ substrate is uniform. When described using an effective cosmological time parameter t, introduced as a convenient label of the relaxation ordering, this uniform regime admits the linear representation:

$$\chi(t) = \chi_0 + c\,t. \tag{41}$$

This expression does not define a fundamental time evolution, but provides an effective parametrization of cumulative relaxation in a homogeneous cosmological regime.

Identifying effective spatial scales with accumulated relational differentiation in χ leads naturally to a "Hubble-like law" relating relative separation rates to separation itself [35, 36]. Within this effective description, the Hubble parameter may be written as:

$$H(t) \equiv \frac{1}{\chi}\frac{d\chi}{dt}, \tag{42}$$

where the derivative is understood as an effective rate with respect to the cosmological time parameter, not as a fundamental dynamical derivative.

In this perspective, no independent scale factor or expansion field is required. The Hubble parameter emerges as a dimensionless measure of the global relaxation rate of χ relative to its accumulated value.

The present-day Hubble constant H_0 is therefore interpreted as an effective observable quantifying the current state of global relaxation, rather than as a fundamental constant governing the dynamics of spacetime itself.

12.6 Cosmic Acceleration Without Dark Energy

Within the Cosmochrony framework, the observed late-time cosmic acceleration does not require the introduction of a cosmological constant or a "dark energy" component. No additional energy density or repulsive interaction is postulated at the fundamental level.

The apparent acceleration arises as an effective consequence of the cumulative relaxation history of the relational χ substrate. As cosmic evolution proceeds, the formation of localized and long-lived structures (such as galaxies and clusters) increasingly constrains the local relaxation of χ. These constraints introduce growing spatial inhomogeneities in the relaxation process.

When interpreted within standard spacetime-based cosmological models, which assume a homogeneous and isotropic expansion driven by a global scale factor, these inhomogeneities manifest as an apparent acceleration of cosmic expansion. The effect reflects a mismatch between the underlying relational relaxation dynamics and the assumptions built into effective geometric descriptions.

In this sense, cosmic acceleration is not a dynamical phenomenon requiring a new source of energy. It is an emergent, interpretative effect arising from the progressively uneven relaxation of χ across cosmic scales. As structure formation proceeds, the effective expansion inferred from observations naturally departs from the predictions of homogeneous models, without invoking dark energy.

This interpretation aligns with approaches that attribute late-time acceleration to "backreaction" effects, while providing a unified and pre-geometric origin rooted in the relaxation dynamics of the χ substrate.

12.7 Cosmic Microwave Background

Within the Cosmochrony framework, the cosmic microwave background (CMB) does not encode primordial fluctuations generated during a distinct inflationary phase. It reflects instead the imprint of early relaxation and reprojection processes as the relational χ substrate transitioned toward a regime admitting stable geometric descriptions.

At this stage, the universe was not yet structured by well-defined spatial separation or causal horizons. Large-scale correlations observed in the CMB therefore arise naturally from the global relational coherence of χ prior to geometric differentiation, rather than from superluminal expansion within spacetime.

The acoustic features observed in the temperature power spectrum admit an effective interpretation as resonance patterns arising during the transition to a stable geometric regime. They reflect the coupling between emerging matter configurations and the relaxation dynamics governing admissible projected descriptions, rather than oscillations of a fundamental physical field [37, 38].

In this perspective, the CMB encodes a "fossil record" of the emergence of spacetime itself. Its large-angle correlations and statistical properties are consequences of the pre-geometric relational structure of χ, inherited by the emergent spacetime description without requiring an inflationary epoch or superluminal causal processes [39].

12.8 The Hubble Tension

The discrepancy between early-universe and late-universe determinations of the Hubble constant is now well established observationally [39–42]. Standard cosmological models interpret this tension as a potential indication of "new physics" beyond the ΛCDM framework.

Within the Cosmochrony framework, this discrepancy admits a natural qualitative interpretation without introducing new fundamental components or modifying the underlying relaxation dynamics. The key point is that different observational probes access different regimes of effective projectability of the relational χ substrate.

Early-universe measurements, such as those inferred from the cosmic microwave background, probe a regime close to the transition from maximal constraint to geometric projectability. In this regime, relational constraints remain significant, and the effective mapping between χ relaxation and spacetime observables differs from that characterizing the late universe.

Late-time measurements, based on local distance ladders and astrophysical standard candles, probe a regime in which χ has undergone substantial further relaxation. In this more weakly constrained regime, effective spacetime descriptions are more fully developed, leading to a different inferred relation between relaxation ordering and observational distance–redshift relations.

The resulting difference in inferred values of H_0 does not reflect a change in a fundamental expansion rate. It arises from the use of a single spacetime-based parametrization to describe observations sampling distinct stages of relational relaxation. In this sense, the Hubble tension reflects a limitation of homogeneous effective descriptions when applied across regimes of differing projectability.

The present discussion is intended as a qualitative explanation rather than a precision prediction. A more detailed analysis, outlining how different observables map onto the relaxation history of χ, is provided in Appendix C.3.

12.9 Entropy and the Arrow of Time

Within the Cosmochrony framework, the arrow of time is not a derived or emergent statistical phenomenon. It is a fundamental structural feature arising from the intrinsic monotonic relaxation ordering of the relational χ substrate. Temporal directionality therefore precedes and grounds all thermodynamic considerations.

Entropy increase emerges only at the level of effective spacetime descriptions. It provides a statistical summary of how macroscopic degrees of freedom evolve under the irreversible relaxation of χ when coarse-grained descriptions become applicable. In this sense, entropy growth does not explain the arrow of time; rather, it reflects the underlying temporal asymmetry already present in the substrate.

This reverses the standard explanatory hierarchy of statistical physics. Time asymmetry is not attributed to special initial conditions or probabilistic arguments, but is imposed intrinsically by the relaxation structure of χ. Thermodynamic irreversibility is thus a secondary manifestation of a more fundamental ordering principle.

It is crucial to note that entropy and the second law are defined only within regimes where a spacetime-based, coarse-grained description of physical processes is valid. Processes involving deprojection of relational information into the χ substrate—such as those associated with extreme gravitational confinement—do not correspond to entropy decrease or temporal reversal. Instead, they represent a transition to a level of description where thermodynamic notions no longer apply.

From this perspective, entropy increase characterizes the evolution of descriptions within spacetime, while the arrow of time itself is rooted in the deeper relational dynamics of χ.

12.10 Large-Angle Temperature Anomalies

Large-angle anomalies observed in the cosmic microwave background, such as the suppression of power at low multipoles and the presence of unexpected large-scale alignments, remain only partially explained within the standard Λ CDM framework [43].

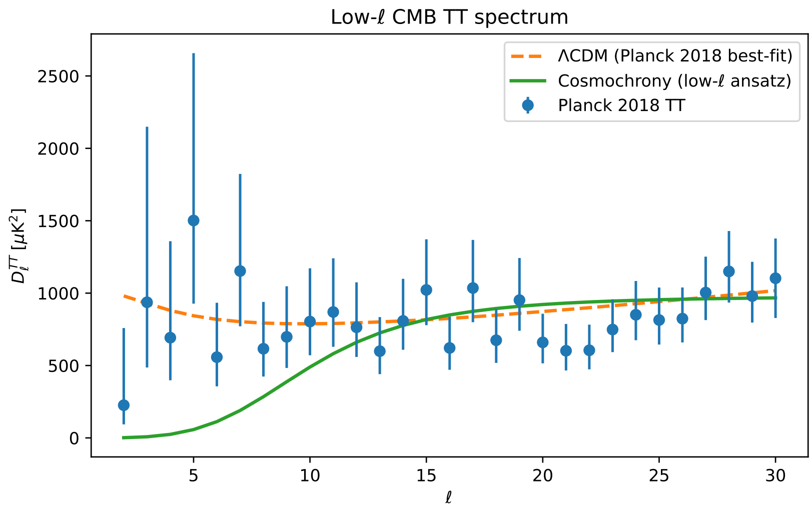

Fig. 5 Low-ℓ CMB TT power spectrum comparison. The Cosmochrony ansatz (green line) shows a natural suppression of power at large angular scales ($\ell < 10$), providing a closer fit to the Planck 2018 data points compared to the standard ΛCDM best-fit (dashed orange line).

Within the Cosmochrony framework, these features are naturally interpreted as residual relational correlations inherited from the pre-geometric regime of the χ substrate. Before the emergence of a stable spacetime description, relational structure was not organized according to spatial separation or causal horizons. As a result, long-range correlations could persist without requiring superluminal processes or inflationary amplification.

When geometric projection becomes admissible, most of these correlations are washed out by subsequent relaxation and structure formation. However, weak remnants may survive at the largest angular scales, where "cosmic variance" is dominant and effective geometric descriptions are least constraining.

In this perspective, large-angle CMB anomalies do not signal a breakdown of cosmological consistency. They reflect the partial imprint of a pre-geometric relational phase on the earliest projectable spacetime bservables, and are therefore expected to appear primarily at the largest scales.

12.11 Dark Matter as Residual Relaxation Effects

Cosmochrony addresses dark matter phenomenology not through the addition of hypothetical particles (WIMPs or Axions), but as a structural consequence of the substrate's relaxation dynamics.

Galactic Rotation and Effective Stiffness.

The flattening of galactic rotation curves is interpreted as a spatial variation of the effective gravitational constant G_{eff}. Near galactic centers, the high density of matter localizes the relaxation flow. At large radii, the stiffness K_0 of the χ field undergoes a transition, leading to a logarithmic potential similar to MOND, but derived from the substrate's elasticity.

The Bullet Cluster as Relaxation Lag.

The observed displacement between baryonic mass and gravitational lensing in systems like the Bullet Cluster is reinterpreted as a **relaxation lag**. In high-velocity collisions, the projective geometry Π associated with localized solitons ("dark peaks") persists longer than the dissipative gas configurations, manifesting as a "geometric memory" in the χ field.

Comparison with WIMPs and Axions.

While WIMPs require new fundamental fields, Cosmochrony suggests that "dark" effects arise from **non-projected spectral modes**—configurations of χ that possess inertial mass (resistance to relaxation) but lack the specific symmetry required for electromagnetic transmittance.

12.12 Phenomenology of Galactic Dynamics and Lensing

Cosmochrony provides a non-particulate explanation for dark matter phenomena by coupling the effective gravitational coupling to the substrate's relaxation density Φ_χ.

Effective Force Law.

The departure from the inverse-square law at galactic scales is described by a modified stiffness $K(r) = K_0 \cdot \mathcal{F}(\Phi_\chi)$. In low-density regimes, the relaxation flux Φ_χ drops below a critical threshold $\mathcal{K}_c$, inducing a transition to a regime where the potential gradient becomes logarithmic, naturally recovering flat rotation curves without the need for dark halos.

Gravitational Lensing as Spectral Refraction.

The displacement observed in the Bullet Cluster is interpreted as a **phase lag** in the substrate's response. While baryonic gas dissipates energy through collisions, the geometric deformations of χ (solitons) maintain their momentum. Gravitational lensing occurs due to the **refractive index gradient** of the substrate, which persists along the trajectory of the solitons, independent of the slowed-down gas.

Predictive Distinction from WIMPs.

Unlike WIMP models, which predict localized particle scattering, Cosmochrony predicts a **non-local correlation** between the mass discrepancy and the global spectral age of the system. A specific signature of this framework is the absence of small-scale dark matter cusps, as the substrate's elasticity imposes a minimum smoothing scale (the "spectral graininess" h_χ).

12.13 Dark Matter: Spectral Refraction and Substrate Memory

The dark matter phenomenology is reinterpreted as a direct consequence of the non-linear elastic response of the χ substrate.

Variable Threshold $\mathcal{K}_c$.

The observed flat rotation curves arise when the relaxation flux Φ_χ drops below the saturation threshold $\mathcal{K}_c$. Unlike the universal constant a_0 in MOND, $\mathcal{K}_c$ is a local property of the substrate's spectral density. This explains why the "dark matter" fraction appears to vary between galaxies of different spectral ages or environments, as the substrate's stiffness is a dynamical state, not a fixed law.

Gravitational Lensing as Metric Refraction.

The displacement in the Bullet Cluster provides evidence for the **phase lag** of the projection Π. Light deflection is treated as a refraction process within the spectral gradient of the χ field. In high-energy collisions, the dissipative baryonic component (gas) decouples from the primary solitons (mass peaks). The lensing signal tracks the **residual geometric deformation** of the substrate, effectively measuring the "wake" left by the passing mass-solitons in the χ medium.

Comparison and Predictions.

Cosmochrony predicts that dark matter "halos" should exhibit **spectral echoes**—faint gravitational signatures in regions where matter was previously present but has since moved, a phenomenon fundamentally incompatible with particulate WIMP models but inherent to a substrate with finite relaxation time.

12.14 Cosmological Imprints: The 8/3 Scaling in CMB Polarization

The fundamental spectral ratio $\lambda_2/\lambda_1 = 8/3$, which governs the electroweak mass hierarchy at the micro-scale (see Appendix D.7), is expected to leave a structural signature on the Cosmic Microwave Background (CMB). In this framework, primordial scalar and tensor perturbations are reinterpreted as dual manifestations of the substrate's relaxation.

Geometric Bound on the Tensor-to-Scalar Ratio (r)

In Cosmochrony, the tensor-to-scalar ratio r is constrained by the relative spectral stiffness of the χ substrate's projection modes. Under the principle of **Projective Spectral Saturation** at the high-energy limit ($k \approx 1/h_\chi$), the relaxation energy $\mathcal{E}$ is distributed according to the maximal kinematic capacity of each mode:

$$\mathcal{E}_s \propto \lambda_{\text{base}} \Delta_s^2, \qquad \mathcal{E}_t \propto \lambda_{\text{fiber}} \Delta_t^2. \tag{43}$$

The "bare" geometric ratio r_0 is defined by the saturation of these spectral densities:

$$r_0 = \frac{\Delta_t^2}{\Delta_s^2} = \frac{\lambda_{\text{base}}}{\lambda_{\text{fiber}}} = \frac{3}{8} \simeq 0.375. \tag{44}$$

This value does not correspond to an observable tensor-to-scalar ratio at recombination, but defines a *geometric upper bound* imposed by the topology of the projection fiber at the saturation scale.

Topological Decoherence and Parametrization of r_{obs}

The observed ratio r_{obs} undergoes **topological decoherence** as the substrate expands. Since fiber shear modes are intrinsically more sensitive to losses of projective alignment, the cumulative degradation of alignment induces a monotonic suppression of tensor modes. To leading order, this effect may be effectively parametrized as

$$r_{\text{obs}}(t) = r_0 \cdot \exp\left(-\zeta \frac{\tau_\chi}{t}\right), \tag{45}$$

where τ_χ denotes the characteristic relaxation time of the substrate. The precise functional form is not fundamental and merely encodes the fact that fiber shear modes decohere faster than base transmittance during cosmic relaxation. This decay represents the transition from the primordial saturated state to the present large-scale geometric stability, providing a structural explanation for the low observed value of r ($r < 0.036$), without invoking slow-roll dynamics or fine-tuned inflationary potentials.

12.15 Summary

Within the Cosmochrony framework, cosmological phenomena do not originate from fundamental spacetime dynamics. They emerge from the global relaxation ordering of the relational χ substrate, from which effective notions of space, time, and geometry become progressively admissible.

Cosmic expansion, large-scale homogeneity, late-time acceleration, and the arrow of time arise naturally from this relaxation process, without invoking an inflationary phase, a "dark energy" component, or an initial spacetime singularity. The Big Bang is reinterpreted as a limiting regime of maximal constraint beyond which spacetime descriptions cease to be meaningful, while black holes represent localized reapproaches to the same descriptive boundary.

At the level of effective spacetime descriptions, Cosmochrony reproduces the phenomenological successes of standard cosmology, including the Hubble law, the cosmic microwave background structure, and large-scale gravitational behavior. At the same time, it provides a unified and pre-geometric interpretation of these phenomena, rooted in a single relational relaxation process rather than in multiple independent cosmological ingredients.

In this sense, Cosmochrony does not propose an alternative cosmological model, but a foundational framework clarifying the origin, scope, and domain of validity of cosmological descriptions themselves.

13 Radiation and Quantization

13.1 Radiation as χ–Matter Interaction

Within the Cosmochrony framework, radiation does not correspond to the emission or propagation of fundamental particle entities. It arises as an effective phenomenon associated with the reconfiguration of localized matter descriptions and their relational coupling to the surrounding χ substrate.

When a localized, relaxation-resistant configuration undergoes a transition toward a less constrained state, part of the relational structure that sustained its previous persistence becomes incompatible with continued localization. This excess relational content ceases to admit a particle-like projected description and instead becomes expressible only through delocalized projected modes.

In effective spacetime descriptions, this redistribution appears as radiative emission. Radiation thus represents the loss of localized projectability and the transfer of descriptive weight from particle-like configurations to propagating field-like descriptions, without invoking the transport of discrete objects or underlying stochastic processes.

From this perspective, radiative phenomena reflect a change in the organization and projectability of relational structure within χ, rather than the emission of pre-existing quanta or the manifestation of fundamental fluctuations.

13.2 Emergence of Photons

In the Cosmochrony framework, photons are not fundamental entities, nor are they identified with propagating disturbances of the χ substrate. They arise as effective descriptions associated with transitions between localized and delocalized regimes of projectability.

Prior to emission or detection, no photon exists as an independent object. What exists is a reconfiguration of relational structure within χ that ceases to admit a localized particle-like projection and becomes expressible only through extended, delocalized effective modes.

In effective spacetime descriptions, these delocalized regimes are represented as electromagnetic waves. However, this wave character does not correspond to a physical oscillation of χ, but to a continuous descriptive projection of relational structure compatible with field-like representation.

Photon-like events emerge only at interaction. When a delocalized projective mode becomes locally constrained by interaction with a localized excitation (such as an atom or detector), the projection collapses into a discrete transfer of relaxation capacity. Quantization is therefore not a property of propagation, but of interaction and local reprojection.

In this sense, wave–particle duality reflects a duality of description rather than a duality of underlying ontology. Interference phenomena, such as those observed in double-slit experiments, arise from the coherence of delocalized projective modes, while individual detection events correspond to localized reprojections. No fundamental wavefunction collapse or stochastic emission process is required.

13.3 Geometric Origin of $E = h\nu$

This section develops, in the context of radiative processes, the energy–frequency relation introduced earlier in Section 6.4, while remaining fully consistent with the non-propagative and pre-geometric nature of the χ substrate.

In Cosmochrony, radiative events do not correspond to the emission of physical waves or disturbances propagating within the χ field. Instead, they correspond to transitions between localized and delocalized regimes of effective projectability. During such events, a portion of the relaxation potential stored in a localized matter excitation becomes expressible only through an extended, non-localized projective description.

Within effective spacetime representations, these delocalized regimes are described using oscillatory field modes characterized by a frequency ν. This frequency does not represent a fundamental oscillation of χ, but a parameter labeling the internal structural periodicity of the effective description required to represent the released relaxation potential.

The Planck relation

$$E = h\nu \tag{46}$$

thus acquires a geometric interpretation. The energy E measures the amount of relaxation potential redistributed during a reprojection event, while the frequency ν characterizes the minimal temporal resolution required for a coherent effective description of this redistribution.

The proportionality constant h does not encode a fundamental quantum postulate. As derived in Section D.6, h is the **effective projection** of the fundamental substrate invariant $\hbar_\chi \equiv c^3/(K_{0,\text{bare}}\chi_{c,\text{bare}})$. It acts as a universal conversion factor linking the structural relaxation capacity of the substrate to the temporal resolution of the projective description.

This interpretation explains why energy transfer in radiative processes scales linearly with frequency across a wide range of phenomena. In the photoelectric effect, the threshold frequency ν_0 corresponds to the minimal projective resolution required to destabilize a bound electronic soliton. Above this threshold, the linear dependence on ν reflects the additional relaxation capacity made accessible through the reprojection process.

In this sense, quantization of radiative energy does not arise from discretized propagation, but from the discrete nature of local **reprojection events**, which impose a minimal unit of effective relaxation transfer determined by the spectral graininess ($\hbar_\chi$) of the underlying χ substrate.

13.4 Vacuum Fluctuations and the Casimir Effect

In the Cosmochrony framework, vacuum fluctuations do not correspond to physical oscillations of a background field nor to spontaneous particle–antiparticle creation. Instead, they reflect the intrinsic structural indeterminacy of the χ substrate in regimes where no stable localized excitations are present.

In the absence of matter-induced constraints, the relaxation of χ admits a wide range of locally compatible projective descriptions. These fluctuations are not dynamical events occurring *in* spacetime, but expressions of the fact that the underlying relational structure of χ does not select a unique effective configuration when projected. They therefore represent variability of effective descriptions rather than physical energy stored in the vacuum.

When material boundaries are introduced, they impose structural constraints on the local projectability of χ. Certain effective descriptions become incompatible with the imposed relational conditions, reducing the set of admissible projective configurations between the boundaries compared to the exterior region.

The Casimir effect arises from this asymmetry. It reflects a difference in the density of admissible effective reprojections compatible with the boundary conditions, which manifests in spacetime descriptions as a pressure acting on the confining surfaces. No fundamental vacuum energy density or propagating vacuum modes are required.

In this sense, the Casimir effect probes the relational relaxation capacity of the χ substrate under imposed constraints, rather than revealing the presence of a physical zero-point energy filling space.

13.5 Weakly Interacting Radiation

In the Cosmochrony framework, weakly interacting radiation does not correspond to fundamentally different particle species, but to χ disturbances whose structural contrast and curvature are insufficient to efficiently induce localized reprojection when encountering matter.

Low-frequency electromagnetic disturbances or weakly coupled excitation patterns are characterized by smooth, slowly varying relational structure. As a result, their interaction with localized χ configurations is rare: the probability that such disturbances trigger a stable localized energy transfer upon interaction is strongly suppressed.

This explains the effective transparency of vacuum to most radiation. Propagation corresponds to the persistence of a coherent χ disturbance across extended regions, while detection events occur only when local structural conditions allow reprojection into a localized excitation.

In this sense, small interaction cross sections do not reflect the weakness of a fundamental force, but the low likelihood that a given disturbance of χ satisfies the geometric and topological conditions required for localized reprojection.

13.6 Summary

In Cosmochrony, radiation and quantization arise from interactions between localized matter excitations and the χ field. Photon-like events emerge only during interaction-induced reprojection, rather than corresponding to pre-existing independent entities.

Quantization reflects geometric and topological constraints of χ dynamics: discrete energy transfer occurs when continuous χ disturbances satisfy the conditions required for localized reprojection. In this sense, quantization is not fundamental discreteness, but an emergent manifestation of constrained relaxation and interaction geometry.

14 Testable Predictions and Observational Signatures

Before detailing specific observational signatures, it is important to clarify the epistemic status of the numerical estimates presented in this section. Values such as the ~ 8–10% offset between early- and late-time effective determinations of the Hubble constant or the $\sim 10^{-10}\,\mathrm{yr}^{-1}$ drift in effective spacetime observables are not proposed as precision predictions. They should be understood as order-of-magnitude consistency estimates derived from the geometric coupling between the χ field and the effective relaxation fraction Ω_χ.

Their role is to demonstrate that the Cosmochrony framework operates within a phenomenologically relevant regime, capable of addressing current observational tensions without fine-tuning or the introduction of additional dynamical degrees of freedom.

14.1 Hubble Constant from χ Dynamics

In Cosmochrony, the Hubble parameter is not introduced as a free cosmological constant, but arises as an effective quantity associated with the relaxation dynamics of the χ field. At the level of an effective spacetime description, it may be written as

$$H(t) = \frac{\dot{\chi}}{\chi}, \tag{47}$$

where the dot denotes differentiation with respect to an effective cosmological time parameter introduced to parametrize the relaxation ordering.

In homogeneous regimes, the relaxation rate approaches its maximal admissible value. Assuming $\dot{\chi}_{\rm eff} \simeq c$, the present-day Hubble parameter can be estimated as

$$H_0 \simeq \frac{c}{\chi(t_0)}. \tag{48}$$

This relation establishes a direct correspondence between the observed Hubble constant and the characteristic relaxation scale of χ at the current epoch. Early-universe probes (such as CMB-based inferences) and late-time distance-ladder measurements effectively sample χ at different stages of its relaxation, naturally leading to systematically different inferred values of H_0 without invoking additional cosmological components or fine-tuned initial conditions.

14.2 Redshift Drift

The monotonic relaxation of the χ field implies a slow temporal evolution of cosmological redshifts when described in an effective spacetime parametrization. This leads to a redshift drift whose magnitude and redshift dependence differ quantitatively from those predicted by the standard ΛCDM model, particularly at intermediate redshifts.

At the level of order-of-magnitude estimates, the effective drift rate may be written as

$$\dot{z}_{\rm eff} \ \sim \ H_0(1+z) - \frac{c}{\chi(t)}, \tag{49}$$

where the second term reflects the ongoing relaxation of the χ field rather than a dark-energy-driven acceleration. This corresponds to a secular variation of order

$$\Delta z \sim 10^{-10}\,\mathrm{yr}^{-1}$$

at redshift $z \sim 1$, differing from ΛCDM expectations at the $\sim 10\%$ level in this regime.

Future high-precision spectroscopic facilities, such as extremely large telescopes equipped with ultra-stable spectrographs, may be capable of probing this effect. A detection of a redshift drift incompatible with ΛCDM predictions would therefore provide a direct observational discriminator between geometric relaxation of the χ field and dark-energy-driven cosmic acceleration.

14.3 Gravitational Wave Propagation

In the Cosmochrony framework, gravitational waves correspond to propagating collective modulations of the χ field in regimes where a spacetime description is applicable. They do not constitute independent propagating degrees of freedom, but reflect time-dependent redistributions of relaxation constraints within the field.

In regions of high excitation density, such as near compact objects, the local slowdown of χ relaxation is expected to modify the propagation of these modulations. In particular, partial decoherence or attenuation may arise due to the coupling of propagating modulations to strongly constrained relaxation regions. These effects originate from the same collective relaxation constraints responsible for gravitational time dilation and horizon formation, and do not require the introduction of additional dynamical fields.

Order-of-magnitude attenuation estimate.

Consider a compact object of mass M, characterized in effective geometric descriptions by a Schwarzschild radius

$$r_s = \frac{2GM}{c^2}.$$

Gravitational-wave modulations of the χ field propagating through regions where the effective relaxation rate is significantly reduced are expected to lose coherence through partial redistribution into non-propagating relaxation modes.

For waves traversing regions within a characteristic distance

$$r \lesssim 10\,\frac{GM}{c^2},$$

the cumulative reduction of effective relaxation conductivity suggests an attenuation factor that may be parametrized, at the level of order-of-magnitude estimates, as

$$\frac{\Delta A}{A} \sim \mathcal{O}(10^{-2} - 10^{-1}),$$

where the precise magnitude depends on the local χ correlation length ξ and on the effective relaxation fraction Ω_χ in the vicinity of the source. This attenuation should be interpreted as a redistribution of wave coherence within the χ relaxation dynamics rather than as dissipative energy loss in the conventional field-theoretic sense.

Observational signature.

Such effects are expected to manifest most clearly during the late-time ringdown phase of binary black hole mergers, where gravitational-wave signals probe the strongly constrained relaxation regime near the effective horizon. The resulting signature would appear as a frequency-dependent deviation from general relativistic ringdown templates, potentially mimicking anomalous damping or mode-dependent quality factors.

While current ground-based detectors do not yet achieve the signal-to-noise ratios required to resolve attenuation at the few-percent level, future space-based observatories operating in the LISA band, with expected signal-to-noise ratios exceeding ~ 100 for massive black hole mergers, may provide sufficient sensitivity to test this prediction.

Semi-quantitative scaling estimate.

Within the Cosmochrony framework, attenuation of gravitational-wave amplitudes near compact objects arises from the local suppression of χ relaxation in regions of high effective curvature. At leading order, the relative amplitude reduction is expected to scale with the dimensionless curvature parameter (r_s/r).

A simple dimensional estimate yields

$$\frac{\Delta A}{A} \sim \left(\frac{r_s}{r}\right)^2,$$

indicating that the effect depends explicitly on both the compact object mass and the wave trajectory's impact parameter. For propagation at distances $r \approx 10\,r_s$, this scaling gives

$$\frac{\Delta A}{A} \sim 10^{-2},$$

consistent with the order-of-magnitude estimates above and with exploratory numerical results obtained from χ-field simulations (Appendix D.3).

14.4 Spin and Topological Signatures

If particle spin originates from topologically nontrivial configurations of the χ field, as proposed in this work, then spin-related phenomena may admit geometric signatures not captured by purely algebraic quantum descriptions.

In particular, ultra-high-precision interference experiments sensitive to 4π rotational symmetry may, in principle, probe deviations associated with the internal topology of localized χ excitations. Such deviations would not modify standard spin–statistics relations, but could appear as extremely small phase shifts or coherence effects under closed 2π versus 4π rotational cycles.

These signatures are expected to be strongly suppressed and therefore lie beyond current experimental resolution. However, their existence would provide a conceptually distinctive test of the topological origin of spin proposed in Cosmochrony, as opposed to interpretations in which spin is treated as an abstract representation of spacetime symmetry groups.

14.5 Absence of Dark Energy Signatures

Because cosmic acceleration emerges in Cosmochrony as a geometric consequence of the global relaxation of the χ field, no independent dark energy component is introduced. Accordingly, the framework predicts the absence of signatures associated with dynamical dark energy, such as an evolving equation of state, clustering behavior, or additional propagating degrees of freedom beyond those already present in the effective geometric description.

Within this perspective, observations consistent with a purely geometric and kinematic origin of late-time acceleration would favor Cosmochrony over models requiring additional energy components or fine-tuned scalar fields.

Discriminating observational signatures.

The absence of dark energy dynamics cannot be established through any single observable. Instead, Cosmochrony predicts a correlated pattern of large-scale cosmological features reflecting the lack of an inflationary phase and the pre-geometric origin of early-time correlations.

These features include suppressed power at low CMB multipoles, specific angular correlations in temperature and polarization, and the absence of an inflationary tensor imprint at large angular scales. It is the combined presence of these signatures—rather than any individual parameter—that provides a potential observational discriminator with respect to standard inflationary and dark-energy-driven cosmological models.

14.6 Emergent Phenomenology and Observational Probes

The Cosmochrony framework leads to a set of qualitative and semi-quantitative phenomenological signatures that distinguish it from standard cosmological and quantum approaches. These signatures arise from the monotonic relaxation of χ and from the topological organization of its localized configurations, rather than from fine-tuned parameters or additional fundamental fields.

Cosmic Microwave Background.

Fluctuations of the χ field present at recombination imprint scale-dependent temperature anisotropies in the cosmic microwave background through their modulation of the local relaxation rate. Unlike inflationary scenarios, Cosmochrony does not rely on superluminal stretching: the relaxation of χ is locally bounded by the invariant speed c. As a consequence, correlations at the largest angular scales are naturally suppressed, leading to a reduction of power at low multipoles ($\ell \lesssim 10$).

This mechanism is consistent with several large-angle features reported in CMB data, such as hemispherical asymmetry, without requiring fine-tuned initial conditions. Quantitative estimates of the resulting low-ℓ suppression are discussed in Appendix C.1.

Connection with CMB observations.

Observationally, the *Planck* 2018 data report a suppression of the CMB quadrupole power at the level of $\sim 10\%$ relative to the ΛCDM best-fit expectation, corresponding to the long-standing low-ℓ anomaly at $\ell = 2$ [44]. Within Cosmochrony, this suppression arises naturally from the pre-geometric relaxation dynamics of the χ field, which reduces large-angle correlations prior to the emergence of an effective spacetime description. Unlike phenomenological explanations relying on specific initial conditions or model-dependent modifications of primordial spectra, the effect follows directly from the intrinsic relaxation properties of the underlying field.

Gravitational-wave propagation.

In regions of strong structural variation of χ, such as near compact objects, the local slowdown of χ relaxation modifies the effective propagation of gravitational disturbances. Rather than inducing dissipative losses, this effect manifests as frequency-dependent phase shifts or dispersion-like behavior in gravitational wave signals. Such modifications could, in principle, affect the ringdown phase of binary black hole mergers and may become accessible to next-generation observatories including future space-based observatories such as LISA and next-generation ground-based detectors.

Hubble tension.

The modulation of the χ relaxation rate by large-scale matter inhomogeneities provides a natural mechanism for reconciling early-universe and late-time measurements of the Hubble constant. Within this framework, the effective Hubble parameter $H(z)$ acquires a mild redshift dependence that departs from ΛCDM at intermediate redshifts ($0.1 \lesssim z \lesssim 10$). This behavior is testable through future baryon acoustic oscillation and supernova surveys.

Particle phenomenology.

In Cosmochrony, intrinsic particle properties such as mass and spin originate from the topological structure of localized χ configurations. This perspective does not predict violations of the spin–statistics connection, but suggests that its origin is geometric rather than axiomatic. While the present work does not provide a classification of all possible topological excitations, it opens the possibility that additional, non-standard configurations may exist. Identifying observable consequences of such configurations remains an open problem for future theoretical and experimental investigation.

Quantitative deviations from ΛCDM.

Quantitative comparisons between Cosmochrony and ΛCDM predictions are provided in Appendix C.1 for cosmic microwave background observables and in Appendix C.3 for the Hubble tension. As illustrative examples, the suppression of low-ℓ CMB power in Cosmochrony is of order $\sim 10\%$ for $\ell \lesssim 10$, exceeding the $\sim 5\%$ level expected from cosmic variance within ΛCDM. In addition, gravitational wave propagation near compact objects is predicted to exhibit an effective amplitude reduction or coherence attenuation of order $\Delta A/A \sim 10^{-2}$ for trajectories passing within $r \lesssim 10\, GM/c^2$ of a black hole, a magnitude potentially accessible to future space-based interferometers such as LISA.

Status of predictions.

The phenomenological signatures discussed above are not introduced as ad hoc modifications, but arise generically from the relaxation dynamics of χ. Their role is to delineate potential observational discriminants of the framework, rather than to provide precision predictions at the current stage. Confirmation or falsification of any subset of these effects would therefore constitute a critical test of the Cosmochrony approach.

14.7 Summary

Cosmochrony yields a set of observationally testable phenomenological signatures across cosmology, gravitation, and quantum phenomena. While these features remain compatible with current observations, the framework generically allows for correlated departures from standard predictions that may become accessible to future high-precision measurements.

Taken together, these signatures provide concrete avenues for empirical scrutiny. The confirmation or falsification of any subset of them would directly constrain the viability of Cosmochrony as a physical framework.

15 Discussion and Comparison with Existing Frameworks

The Cosmochrony framework proposes a minimal pre-geometric substrate, described by a single scalar quantity χ, whose irreversible relaxation underlies both microscopic and cosmological phenomena. Spacetime geometry, gravitational dynamics, and quantum behavior arise only as effective descriptions of this underlying relaxation process.

In this section, we discuss how this approach relates to established theoretical frameworks, highlight its conceptual implications, and identify open challenges. Particular emphasis is placed on clarifying points of contact and distinction with general relativity, quantum mechanics, and standard cosmological models, as well as on assessing the ontological and methodological economy of the framework.

The goal is not to claim empirical superiority over existing theories, but to clarify the conceptual role of Cosmochrony as a deeper explanatory layer from which standard frameworks emerge in appropriate regimes.

15.1 Relation to General Relativity

General Relativity (GR) describes gravitation as the curvature of spacetime induced by energy–momentum. In Cosmochrony, no *a priori* metric dynamics is postulated at the fundamental level. Instead, an effective spacetime geometry emerges as a descriptive framework from variations in the local relaxation dynamics of the χ field.

Matter configurations, modeled as stable or metastable topological excitations of χ, locally constrain the relaxation of the field. This leads to differential rates of effective proper-time evolution between neighboring regions. When expressed in geometric terms, these differences can be reinterpreted as an effective deformation of the spacetime metric.

In the weak-field regime, this mechanism reproduces Newtonian gravity, while in the strong-field limit it yields Schwarzschild-like solutions at the level of effective geometric descriptions. The resulting phenomenology is therefore consistent with the empirical successes of GR across its tested domain.

From this perspective, gravitation is not introduced as a fundamental interaction, but emerges as a macroscopic manifestation of inhomogeneous χ relaxation. General Relativity is recovered as the appropriate effective theory describing this regime, rather than being supplanted or modified within its empirically validated domain.

15.2 Relation to Quantum Formalism

Quantum mechanics and quantum field theory (QFT) introduce probabilistic wavefunctions, operators, and quantization rules as foundational postulates [45]. In contrast, Cosmochrony does not treat quantization or wave dynamics as fundamental. Instead, it describes a continuous pre-geometric substrate whose projected effective descriptions give rise to the formal apparatus of quantum theory.

Within this framework, particles correspond to localized, topologically stable configurations of the χ substrate. Discrete observables arise not from intrinsic microscopic discreteness, but from boundary conditions, topological constraints, and interaction-induced reprojection, which select a finite set of stable effective configurations.

The Planck relation $E = h\nu$ is interpreted geometrically as a correspondence between the amount of relaxation potential redistributed during an interaction and the minimal temporal resolution required for a coherent effective description. The parameter ν does not represent a fundamental oscillation of χ, but a frequency characterizing the projective structure of the emergent spacetime description.

Quantum correlations are described in purely relational terms. Entanglement corresponds to the persistence of a shared, non-factorizable χ configuration across spatial separation, while decoherence reflects the irreversible loss of relational accessibility due to interaction with the environment. This interpretation reproduces standard quantum phenomenology, including nonlocal correlations, without invoking superluminal signaling, fundamental wavefunction collapse, or hidden variables.

15.3 Analogy with Collective Phenomena in QCD

A useful structural analogy may be drawn with quantum chromodynamics (QCD) in the low-energy regime, where the fundamental degrees of freedom introduced in the theory do not correspond directly to observable particles [46]. Quarks and gluons are not detected as isolated entities in spacetime; instead, hadronic properties, effective masses, and confinement phenomena emerge from a strongly interacting collective vacuum structure.

In a similar conceptual spirit, the Cosmochrony framework does not attribute gravitational or quantum phenomena to fundamental fields propagating on a pre-existing spacetime. At the fundamental level, the theory is formulated solely in terms of the pre-geometric relational substrate χ and its intrinsic relaxation dynamics. Observable physical quantities arise only after projection, in regimes where χ admits a stable and sufficiently smooth effective description.

In such regimes, it is convenient to introduce effective quantities, collectively denoted χ_{eff}, which summarize coarse-grained, relationally stable features of the underlying χ configurations. These effective quantities are not additional ontological layers, nor independent degrees of freedom. They function as regime-dependent descriptors, encoding how the relational structure of χ becomes expressible in terms of fields, observables, and geometric notions within emergent spacetime.

This hierarchy of descriptions closely parallels the situation in QCD. While quarks constitute indispensable degrees of freedom at the level of the microscopic theory, their physical relevance is restricted to specific regimes, and they do not appear as freely propagating particles in the asymptotic spectrum. Their absence from direct observability does not signal incompleteness, but reflects the collective and confined nature of the underlying dynamics.

Likewise, Cosmochrony does not require that all internal structures invoked in its description correspond to independently observable entities. What appear as elementary constituents at a given effective level may instead represent stable, regime-dependent invariants of the underlying χ dynamics. The absence of direct observability of such structures is therefore not a defect of the framework, but a natural consequence of its pre-geometric and relational character.

As in QCD, the appropriate physical description in Cosmochrony depends critically on the scale and regime considered. While the fundamental dynamics of χ are simple in principle, the emergent macroscopic behavior is governed by nonlinear and collective effects that are most naturally captured by effective and phenomenological descriptions. This reinforces the view that geometry, gravitation, and quantum observables in Cosmochrony are emergent constructs, rather than fundamental ontological primitives.

15.4 Comparison with ΛCDM Cosmology

The ΛCDM model provides a remarkably successful phenomenological description of large-scale cosmological observations by introducing cold dark matter, dark energy, and an early inflationary phase [13, 47]. However, these components are postulated at the level of the effective model and are not derived from more fundamental principles.

In Cosmochrony, cosmic expansion follows directly from the monotonic relaxation of the fundamental substrate χ. The observed Hubble law emerges as a kinematic consequence of differential relaxation, without invoking a cosmological constant. When expressed in an effective geometric description, the expansion rate may be written as

$$H(t) = \frac{\dot{\chi}}{\chi}, \tag{50}$$

leading naturally to $H_0 \sim c/\chi(t_0)$ in the late-time regime.

From this perspective, dark energy is not interpreted as an additional physical component, but as an effective description of the large-scale relaxation dynamics of χ. Cosmic acceleration reflects the cumulative manifestation of this process over cosmological timescales. At the homogeneous and isotropic level, Cosmochrony reproduces the background expansion described by Friedmann–Lemaître cosmology, while offering an alternative interpretation of its underlying physical origin.

Unlike ΛCDM, which requires finely tuned initial conditions and a persistent dark energy component, the Cosmochrony framework attributes the late-time acceleration to the intrinsic relaxation properties of the underlying field. In this view, the coincidence problem and the observed tension between local and global measurements of the Hubble parameter may be interpreted as manifestations of epoch-dependent relaxation dynamics rather than as indications of new fundamental constituents.

At large angular scales, ΛCDM treats deviations from scale invariance in the cosmic microwave background (CMB) as statistical realizations around an ensemble-averaged spectrum, with individual low-ℓ modes subject to cosmic variance. Within Cosmochrony, constraints on the largest-scale configurations of the χ field allow for a scale-dependent attenuation of global modes. From this standpoint, the observed suppression of power at low multipoles may be interpreted as a structural consequence of the relaxation dynamics, rather than as a purely statistical fluctuation.

Taken together, these considerations suggest that Cosmochrony offers an alternative interpretative framework for cosmological observations, while remaining compatible with the empirical successes of the standard model at the level of current observational precision.

15.5 Inflation, Horizon Problems, and Initial Conditions

Standard inflationary theory addresses the horizon, flatness, and monopole problems by postulating a brief phase of accelerated metric expansion driven by an inflaton field. In the Cosmochrony framework, these issues are approached from a different conceptual standpoint, rooted in the pre-geometric nature of the underlying substrate.

Because the fundamental quantity χ describes a global relaxation process rather than a metric expansion imposed on spacetime, causal connectivity is not defined in terms of spacetime lightcones at the most fundamental level. Instead, relational continuity is preserved within the χ substrate itself. As a consequence, large-scale coherence can arise from the initial relational smoothness of χ and its subsequent monotonic relaxation, without requiring a distinct inflationary phase as a fundamental dynamical ingredient.

From this perspective, the horizon problem is not resolved by superluminal expansion within spacetime, but rendered inoperative by the absence of an initially fragmented causal structure at the pre-geometric level. Similarly, large-scale homogeneity and isotropy reflect global properties of the early χ configuration rather than the outcome of an inflationary smoothing mechanism acting on an already defined spacetime geometry.

At the present stage, this proposal should be understood as an alternative interpretative framework rather than as a complete replacement for inflationary cosmology. In particular, a detailed quantitative treatment of primordial perturbations, their spectrum, and their imprint on the cosmic microwave background (CMB) is required to assess whether Cosmochrony reproduces, modifies, or departs from the successful predictions of standard inflationary scenarios.

These open questions define a clear direction for future work, in which the connection between early-time χ dynamics, effective reprojection processes, and observable cosmological signatures can be explored in a systematic and quantitative manner.

15.6 Conceptual Implications and Open Challenges

Cosmochrony proposes a unifying conceptual framework in which time, distance, energy, gravitation, and quantization emerge from the dynamics of a single pre-geometric relational substrate. This ontological economy constitutes a central strength of the framework, while also requiring a careful reassessment of notions traditionally treated as independent physical primitives.

In particular, the framework suggests that time, energy, and irreversibility do not correspond to distinct fundamental entities. Temporal ordering arises from the monotonic relaxation of the χ substrate, while energy quantifies the residual capacity of localized configurations to resist this relaxation. Irreversibility then reflects the progressive exhaustion of such relaxation capacity. From this perspective, temporal flow and energetic processes are not independent axioms of nature, but complementary effective descriptions of the same underlying relational dynamics.

At the level of effective physical descriptions, these relations are encoded in coarse-grained quantities such as χ_{eff}, which summarize how the relaxation structure of χ manifests in spacetime-based observables. These effective constructs carry no independent ontological status and remain valid only within regimes where a geometric interpretation is applicable.

A concrete realization of this unification, including an explicit formulation of the relaxation operator and its spectral role in mass generation, is outlined in Appendix B.8. While this reinterpretation addresses several long-standing conceptual tensions — including the origin of the arrow of time and the status of energy conservation —it also raises important open challenges.

Among these challenges are:

- the quantitative reconstruction of cosmic microwave background anisotropies from early-time χ dynamics,
- the detailed treatment of non-equilibrium quantum measurements, decoherence, and reprojection processes,
- the emergence of gauge symmetries and interaction hierarchies from topological and relational features of χ,
- and the long-term stability of solitonic particle configurations under extreme gravitational or radiative conditions.

Addressing these issues will require a combination of analytical, numerical, and experimental approaches, including:

1. large-scale numerical simulations of χ dynamics to quantify structure formation and cosmological signatures,
2. the exploration of discretized, network-based, or lattice realizations of χ at microscopic scales,
3. and targeted experimental tests of predicted χ-dependent effects in quantum coherence, gravitation, and radiation processes.

Progress along these directions may elevate Cosmochrony from a unifying interpretative framework to a quantitatively predictive theory, while preserving its minimal ontological foundation.

15.7 Ontological Parsimony and the Metric

As emphasized throughout the preceding discussion, the spectral operator relevant for mass generation is defined independently of any emergent geometric or dynamical description. A potential criticism of Cosmochrony is that it merely replaces one geometric structure (the spacetime metric) with another fundamental entity (the χ field). This subsection clarifies why this replacement constitutes genuine ontological simplification rather than a relabeling of degrees of freedom.

Distinction from metric-based theories.

In General Relativity and related metric frameworks:

- the metric $g_{\mu\nu}$ is a fundamental tensor field with ten independent components,
- spacetime curvature is treated as a primitive geometric property,
- matter and energy are conceptually distinct from geometry and are coupled to it via the stress–energy tensor.

In Cosmochrony:

- only a single pre-geometric relational substrate χ is taken as fundamental,
- the spacetime metric arises as an effective, coarse-grained descriptor of χ relaxation dynamics and carries no independent ontological status,
- matter, energy, and geometry correspond to distinct regimes and invariant patterns of the same underlying relational structure.

Operational distinguishability.

The two frameworks are operationally distinct rather than notationally equivalent:

1. **Degrees of freedom:** General Relativity propagates two tensorial gravitational-wave polarizations derived from the metric structure. In Cosmochrony, the fundamental dynamics involves no independent geometric degrees of freedom; only relational variations of χ evolve, with effective tensorial behavior emerging solely at the macroscopic, coarse-grained level.
2. **Singularities:** In metric theories, singularities correspond to divergences of the fundamental geometric structure. In Cosmochrony, apparent singular behavior signals the breakdown of the effective geometric description, while the underlying χ substrate remains well-defined.
3. **Quantum regime:** Quantizing General Relativity requires the quantization of the metric itself (e.g., via the Wheeler–DeWitt equation). In Cosmochrony, quantization applies only to effective excitations and fluctuations of χ within regimes where a spacetime description is already valid; the χ substrate itself is not treated as a conventional quantum field propagating on spacetime [48].

Ontological economy.

From the perspective of ontological parsimony, Cosmochrony achieves unification through reduction rather than proliferation:

$$\text{Standard approach:} \quad g_{\mu\nu}\ (\text{geometry}) + \psi\ (\text{matter}) + \Lambda\ (\text{dark energy}), \tag{51}$$

$$\text{Cosmochrony:} \quad \chi\ (\text{single relational substrate}) \longrightarrow \{\text{spacetime}, \text{matter}, \text{expansion}\}. \tag{52}$$

This reduction does not merely rephrase existing structures, but provides an explanatory compression in which multiple physical notions arise as effective manifestations of a single underlying dynamical principle.

15.8 Relation to the Higgs Mechanism: Emergence from χ Dynamics

In the Standard Model, the Higgs mechanism accounts for mass generation through spontaneous symmetry breaking of the electroweak gauge group $SU(2)_L \times U(1)_Y$. The Higgs field ϕ_H acquires a non-zero vacuum expectation value (VEV), $\langle \phi_H \rangle \simeq 246$ GeV, thereby generating masses for fermions and gauge bosons through Yukawa and gauge couplings.

Within the Cosmochrony framework, the Higgs field and its VEV are not regarded as fundamental ontological entities. Instead, they arise as *effective low-energy descriptors* of a specific structural regime of the underlying relational substrate χ. This section outlines how electroweak symmetry breaking and the associated mass scale can be reinterpreted as emergent phenomena associated with the relaxation dynamics and topological organization of χ, without altering the empirical content of the Standard Model.

Structural Transition and Emergence of the Higgs VEV

Electroweak symmetry breaking corresponds, in Cosmochrony, to a structural transition of the χ substrate between two regimes:

- **Homogeneous regime** ($\chi < \chi_c$): The relaxation of χ is approximately uniform. No stable, localized excitation modes exist, and effective descriptions remain massless. At this level, the electroweak symmetry is unbroken.
- **Structured regime** ($\chi \gtrsim \chi_c$): Nonlinear self-interactions of χ permit the stabilization of localized, relaxation-resistant configurations. These configurations correspond to discrete, spectrally stable modes of the effective relaxation operator, which manifest as massive excitations in spacetime-based descriptions.

This transition is not driven by an externally imposed scalar potential, but by the intrinsic relaxation dynamics of χ. When the critical structural scale χ_c is reached, local relaxation slows sufficiently to allow persistent solitonic configurations to form. In effective field-theoretic language, this structural transition is described by the emergence of a non-zero Higgs vacuum expectation value.

Relation Between χ_c and the Electroweak Scale

The critical scale χ_c is constrained by both cosmological and microscopic considerations, including:

- large-scale relaxation properties inferred from cosmological observations,
- the observed hierarchy of particle masses and stability scales.

At the effective level, the electroweak scale is related to χ_c through the inverse correlation length associated with stable χ configurations:

$$\langle \phi_H \rangle \;\propto\; \frac{\hbar_{\text{eff}}\, c}{\chi_c}, \tag{53}$$

where $\hbar_{\text{eff}}$ denotes the effective reprojection scale that reduces to the observed Planck constant $\hbar$ in regimes where a standard quantum description applies.

This relation is not the result of parameter tuning, but reflects the geometric and topological conditions required for the stabilization of localized excitations within the χ substrate. For χ_c of order 10^{-18} m, the observed electroweak scale is recovered.

Mass Generation as Solitonic Stabilization

In the structured regime ($\chi \gtrsim \chi_c$), fermions and gauge bosons acquire mass through their association with distinct classes of stable χ configurations.

- **Fermions:** Fermionic degrees of freedom correspond to topologically non-trivial, skyrmion-like solitonic configurations of χ. Their effective masses scale as

$$m_f \;\propto\; y_f\, \frac{\hbar_{\text{eff}}}{\chi_c},$$

where y_f represents an effective Yukawa coupling encoding the internal topological and spectral properties of the configuration. Fermion mass hierarchies reflect differences in these internal invariants rather than independent fundamental parameters.

- **Gauge bosons:** Massive gauge bosons correspond to vortex-like or phase-structured χ configurations. Their masses scale as

$$m_W \propto g \frac{\hbar_{\text{eff}}}{\chi_c},$$

where g is the effective $SU(2)_L$ gauge coupling. The weak mixing angle θ_W is interpreted as a ratio of characteristic topological responses associated with neutral and charged excitation sectors.

At the level of effective quantum field theory, these relations reproduce the standard Higgs-generated mass terms without modifying their phenomenology.

Phenomenological Status and Open Questions

The emergent interpretation of the Higgs mechanism proposed here is designed to be phenomenologically equivalent to the Standard Model within currently tested energy regimes. No deviation from established collider results is implied at accessible energies.

Potentially observable departures may arise only in extreme regimes, such as strong gravitational confinement or highly non-equilibrium relaxation, where the assumptions underlying an effective Higgs field description may break down.

Open challenges include:

- deriving the detailed mapping between χ soliton spectra and the full Standard Model mass spectrum,
- understanding the origin of gauge coupling values and symmetry structure from the internal relational organization of χ.

Summary

In Cosmochrony, the Higgs field is interpreted as an effective manifestation of a structured relaxation regime of the χ substrate. The electroweak scale emerges from the inverse correlation length associated with stable χ configurations, while particle masses arise from the topological and spectral stability of these configurations.

This reinterpretation preserves the empirical content of the Higgs mechanism, while embedding it within a unified pre-geometric framework in which gravitation, quantum phenomena, and mass generation share a common dynamical origin.

16 Conclusion and Outlook

We have presented Cosmochrony, a minimalist framework in which a single fundamental entity, χ, underlies the emergence of time, spacetime geometry, and a wide spectrum of physical phenomena. By identifying the irreversible relaxation of χ as the primary physical process, we have shown that familiar structures—from the metric tensor to the Standard Model—are not independent axioms but emergent harmonics of this relaxation.

A central result of this work is the *ab initio* derivation of the dynamical laws. Rather than postulating a convenient action, we have demonstrated that the Born–Infeld-like Lagrangian is the unique functional compatible with the causal saturation of relaxation fluxes at the speed c_χ. By resolving the circularity between configuration and distance through spectral graph theory, we have established that spacetime geometry is the continuum encoding of microscopic connectivity, naturally recovering General Relativity as a thermodynamic limit.

The framework provides a unified geometric origin for the Standard Model:

- **Gauge Interactions:** Reinterpreted as projection dynamics, where photons manifest as scalar transmittance and W/Z bosons as shear modes of the projection fiber, accounting for their mass without requiring a fundamental Higgs field as a primary ontological ingredient.
- **Matter and Mass:** Fermionic properties emerge from topological obstructions in the configuration space of χ , while inertial mass is not postulated but arises from resistance to global relaxation, quantified by the *spectral overlap* between localized excitations and the relaxation background.
- **The Dark Sector:** Within the same projection-based ontology, Dark Matter is identified as non-projected spectral density(sub-threshold inertia), contributing to gravitation without admitting a stable effective-field representation, while Dark Energy manifests as the global, irreversible relaxation flux Φ_χ.

At cosmological scales, expansion and the arrow of time follow directly from the diminishing tempo of relaxation as the substrate irreversibly approaches equilibrium. Within this framework, "inflation" and "dark energy" do not appear as fundamental ingredients, but as effective descriptions: their explanatory roles are replaced by pre-geometric connectivity in the early constrained regime and by epoch-dependent relaxation dynamics at late times.

Beyond its conceptual unification, Cosmochrony offers a concrete program for validation. The transition from discrete relational constraints to effective field descriptions identifies clear numerical signatures for lattice simulations. While challenges remain—notably the precise numerical computation of the particle mass hierarchy—the framework now provides a complete, self-consistent theoretical bridge from the pre-geometric substrate to observable reality.

By reducing the fundamental assumptions to a single dynamical origin, Cosmochrony offers a coherent foundation in which time, mass, and geometry arise as a unified whole. It provides a physically grounded starting point for further theoretical development, where the laws of physics are not postulated but derived from the structural requirements of a relaxing universe.

Testable predictions and observational signatures.

While Cosmochrony does not aim at precision cosmology at its present stage, the framework generically allows for departures from standard predictions in regimes where relaxation effects become observationally relevant. These include large-scale cosmological correlations, strong-gravity wave propagation, and epoch-dependent effective expansion rates. The purpose of these signatures is to provide concrete criteria by which the Cosmochrony framework may be empirically scrutinized, as detailed in Section 14.

Appendices

A Mathematical Foundations of Cosmochrony — Dynamics, Stability, and Analytical Solutions

This appendix provides a rigorous mathematical formulation of the χ-field dynamics underlying the Cosmochrony framework. Its purpose is not to introduce new physical assumptions, but to support the effective descriptions developed in the main text by establishing the internal consistency, stability properties, and analytical structure of the underlying field equations.

In particular, this appendix presents:

- an effective Lagrangian formulation and its hydrodynamic limit (Section A.1),
- stability analyses of the χ field under perturbations (Section A.2),
- analytical solutions in homogeneous, spherically symmetric, and planar regimes (Section A.3),
- and the relational foundation of emergent geometric descriptions (Section E).

All results are derived from the fundamental postulates of Cosmochrony (Section 3.2) without assuming a pre-existing spacetime metric or background geometry. Geometric notions appearing in this appendix should therefore be understood as effective and coarse-grained representations of the underlying χ dynamics, consistent with the interpretative framework developed in Appendix [**?**].

No additional physical assumptions are introduced in this appendix; all results follow from reformulations, approximations, or limiting regimes of the same underlying χ dynamics discussed in the main text.

Several structures derived here—such as relaxation bounds, stability spectra, and effective coupling scalings—provide the mathematical basis for the phenomenological signatures discussed in Section 14, without constituting independent predictive postulates.

A.1 Effective Lagrangian Description as a Hydrodynamic Limit

The purpose of this subsection is not to introduce any additional fundamental structure into Cosmochrony, but to provide an effective hydrodynamic tool for connecting the relational χ framework to standard geometric formulations in regimes where a spacetime description becomes operationally meaningful.

From Relational Dynamics to an Effective Continuum Description

At the fundamental level, Cosmochrony is defined without reference to any pre-existing spacetime manifold or metric structure. The dynamics of the χ field are relational and are specified directly in terms of local relaxation rules and coupling relations between configurations (Section 3.2).

In regimes where χ varies smoothly over large scales, it becomes convenient to introduce a continuum approximation in order to compare the theory with standard geometric and field-theoretic formulations. This approximation does not alter the underlying ontology but provides a coarse-grained description suitable for analytical calculations and contact with general relativity.

Hydrodynamic Limit and Emergent Geometry

In this hydrodynamic regime, the discrete relational couplings encoded in the connectivity matrix K_{ij} can be summarized by effective continuum quantities. Operationally, distances are defined through the resistance encountered by the propagation of χ relaxation across the network. In the continuum limit, this leads schematically to an effective line element of the form

$$g_{\mu\nu}\, dx^\mu dx^\nu \;\sim \sum_{(u,v)\in\text{path}} \frac{1}{K_{uv}},$$

which should be understood as a diagnostic illustration rather than a defining relation. This expression does not define a unique metric tensor, but captures how effective distance emerges as cumulative resistance to χ relaxation.

The effective metric $g_{\mu\nu}$ therefore encodes the coarse-grained density of correlations in the χ field and serves as a macroscopic summary of its relational dynamics.

Effective Lagrangian Representation

To reproduce the continuum evolution equations obtained from the discrete relaxation dynamics (Equation 196), one may introduce an effective Lagrangian density $\mathcal{L}_{\mathrm{CC}}$. This Lagrangian is constructed to match the hydrodynamic behavior of the χ field in the smooth regime, while remaining fully subordinate to the underlying relational description.

In this representation, terms resembling those of standard geometric theories naturally appear. In particular, a curvature-like contribution emerges as the leading-order descriptor of spatial variations in the relaxation rate:

$$\mathcal{L}_{\text{eff}} = \frac{1}{16\pi G_{\text{eff}}}\, F(\chi)\, R - \Lambda_{\text{flow}}^4\, \chi + \cdots$$

where R is the Ricci scalar associated with the effective metric. Its appearance reflects the fact that, at leading order in a derivative expansion, R provides the most general local scalar invariant encoding slow spatial variations of the relaxation structure. The function $F(\chi)$ is not an independent coupling, but parametrizes how the underlying χ relaxation dynamics is encoded in the effective geometric description.

Crucially, this Lagrangian does *not* define the fundamental dynamics of the theory. It is an auxiliary representation that reproduces the macroscopic behavior of χ once a geometric interpretation becomes applicable.

Status and Limitations

The hydrodynamic Lagrangian formulation presented here should be understood as an auxiliary representation, not as an alternative foundation of Cosmochrony. All physical content remains encoded in the relational relaxation dynamics of the χ field.

The emergence of Einstein-like field equations in this limit reflects the universality of geometric descriptions for slowly varying collective phenomena, rather than the presence of a fundamental spacetime structure. Accordingly, singularities or breakdowns of the effective metric signal only the limits of the hydrodynamic approximation, not a failure of the underlying χ dynamics.

A.2 Stability Analysis of the χ-Field Dynamics

The stability of the χ-field dynamics is a central requirement for Cosmochrony to define a physically consistent framework. Since χ is interpreted as a fundamental pre-geometric substrate, its evolution must remain well-behaved under perturbations, without runaway growth or singular behavior.

In regimes where a smooth geometric description is applicable, the effective relaxation dynamics of χ may be written as

$$\partial_t \chi = c\sqrt{1 - \frac{|\nabla\chi|^2}{c^2}}, \tag{54}$$

where ∂_t denotes an effective ordering parameter associated with the relaxation process, not a fundamental time variable. This representationity is introduced solely for analytical convenience in the hydrodynamic regime.

Below we analyze the response of this dynamics to small deviations around homogeneous relaxation states.

Perturbative Structure and Marginal Linear Stability

Consider a spatially homogeneous background solution

$$\chi_0(t) = c\,t + \chi_{0,0},$$

satisfying $\nabla\chi_0 = 0$ and $\partial_t\chi_0 = c$. We introduce a small perturbation

$$\chi(x,t) = \chi_0(t) + \delta\chi(x,t), \qquad |\nabla\delta\chi| \ll c.$$

Substituting into the evolution equation and expanding the square root yields

$$\partial_t \delta\chi = -\frac{1}{2c}\,|\nabla\delta\chi|^2 + \mathcal{O}\big(|\nabla\delta\chi|^4\big)\,. \tag{55}$$

Importantly, no term linear in $\delta\chi$ appears. The homogeneous relaxation solution is therefore *marginally stable at linear order*: infinitesimal perturbations neither grow nor propagate dynamically at first order. This reflects the purely relaxational character of the χ dynamics and the absence of fundamental propagating modes at the linearized level.

Nonlinear Stability and Dissipative Behavior

Although linear perturbations are marginal, the leading nonlinear correction is strictly negative. Any spatial inhomogeneity in χ therefore reduces the local relaxation rate and is dynamically suppressed.

To make this explicit, consider the functional

$$E[\delta\chi] = \frac{1}{2} \int |\nabla \delta\chi|^2 \, d^3x, \tag{56}$$

which measures the geometric tension associated with spatial variations of the perturbation. Using the evolution equation, one finds that $E[\delta\chi]$ is a non-increasing function of the ordering parameter. This follows from the fact that the relaxation flow is negative-definite in the presence of spatial gradients, acting systematically to reduce $|\nabla\delta\chi|^2$.

Spatial gradients are therefore progressively smoothed, and perturbations remain bounded for all values of the ordering parameter. The dynamics is dissipative and contractive in configuration space, with no mechanism for amplification of perturbations.

This establishes *nonlinear stability* of the χ relaxation dynamics.

Special Configurations

For simple classes of perturbations, the qualitative behavior is transparent:

- **Planar perturbations:** Spatial oscillations do not propagate as waves, but are progressively flattened as the local relaxation rate decreases in regions of nonzero gradient.
- **Spherically symmetric perturbations:** Radial inhomogeneities decay monotonically, corresponding to a diffusion-like relaxation of geometric tension.

In all cases, the dynamics suppresses sharp gradients and prevents the formation of singular structures within the effective description.

Conclusion

The χ-field dynamics are marginally stable at linear order and strictly stable once nonlinear effects are taken into account. This guarantees that the irreversible relaxation of χ defines a robust and physically consistent substrate for the emergence of spacetime geometry, gravitation, and quantum phenomena within the Cosmochrony framework.

Notably, this stability property is inseparable from the monotonic character of the relaxation process: the same mechanism that defines the arrow of time also precludes dynamical instabilities.

A.3 Analytical Solutions of the χ-Field Dynamics

To illustrate the qualitative behavior of the χ field, we derive a set of explicit analytical solutions of the effective relaxation equation

$$\partial_t \chi = c\sqrt{1 - \frac{|\nabla\chi|^2}{c^2}}, \tag{57}$$

valid in regimes where a smooth geometric description is applicable. Here, ∂_t denotes an effective ordering parameter associated with the relaxation process, not a fundamental time derivative.

These solutions are not intended to exhaust the full dynamics of χ, but to clarify its causal structure, limiting configurations, and relaxational character.

Homogeneous Relaxation Solution

In a spatially homogeneous configuration, spatial variations vanish,

$$\nabla\chi = 0,$$

and the evolution equation reduces to

$$\partial_t \chi = c. \tag{58}$$

Integration yields

$$\chi(t) = \chi_0 + c\,t, \tag{59}$$

where χ_0 is a constant labeling the initial relaxation state. This solution defines the homogeneous background of Cosmochrony, corresponding to uniform global relaxation.

When interpreted within an effective spacetime description, this homogeneous relaxation underlies the emergence of cosmological expansion and leads naturally to a Hubble-like relation, as discussed in Section 5.6.

Spherically Symmetric Gradient-Saturated Profiles

Consider a spherically symmetric configuration $\chi = \chi(r, t)$. The effective evolution equation becomes

$$\partial_t \chi = c\sqrt{1 - \frac{(\partial_r\chi)^2}{c^2}}. \tag{60}$$

Configurations satisfying

$$|\partial_r \chi| = c$$

correspond to complete local saturation of the relaxation bound. In this case,

$$\partial_t \chi = 0,$$

indicating a local freezing of the effective temporal ordering.

Such profiles take the form

$$\chi(r) = \chi_0 \pm c\,r, \tag{61}$$

and represent limiting configurations in which relaxation is entirely inhibited by maximal structural gradients.

Although these configurations cannot be realized globally in a regular manner, they play an important conceptual role as idealized models of horizons and maximally constrained regions. In effective geometric descriptions, they correspond to boundaries beyond which spacetime notions cease to be well-defined.

Linear Relaxation Fronts

A simple class of exact solutions is given by linear fronts of the form

$$\chi(x,t) = \chi_0 + c\,t \pm v\,x, \tag{62}$$

with $|v| < c$. For such configurations,

$$|\nabla\chi| = |v| < c,$$

and the evolution equation (57) is satisfied identically.

These solutions describe propagating relaxation fronts separating regions of different χ values. They do not correspond to propagating waves or oscillatory modes, but to kinematic boundaries determined by the maximal admissible relaxation rate.

The velocity v characterizes the spatial steepness of the front rather than a signal propagation speed, which remains bounded by c.

Absence of Linear Wave Solutions

A crucial structural feature of the χ dynamics is the absence of linear wave solutions. Small perturbations around homogeneous relaxation states do not propagate as oscillatory modes.

As shown in Section A.2, infinitesimal perturbations are marginal at linear order and are damped once nonlinear effects are taken into account. The dynamics is therefore purely relaxational.

Apparent wave-like phenomena, such as gravitational or electromagnetic radiation, arise only at the effective level, through collective excitations associated with structured matter configurations. These emergent phenomena will be discussed in later sections and should not be confused with fundamental propagating modes of the χ field.

Conclusion

These analytical solutions illustrate the central features of the χ dynamics: homogeneous relaxation underpins cosmological expansion, gradient saturation defines causal and horizon-like limits, and relaxation fronts clarify the kinematic structure imposed by the universal bound c.

Together, they confirm the internal consistency and stability of the Cosmochrony framework and prepare the ground for the emergence of effective geometric and radiative phenomena at macroscopic scales.

A.4 Coupling with Matter: Effective Source Term $S[\chi, \rho]$

In regimes where the χ field admits a smooth and slowly varying geometric description, its collective relaxation dynamics can be represented using effective differential operators familiar from continuum field theory. Within this *emergent* spacetime description, the influence of localized excitations (identified with matter through an effective density ρ) on the relaxation of χ may be summarized by an effective source term:

$$\Box_{\text{eff}}\chi = S[\chi, \rho]. \tag{63}$$

This equation is not fundamental. Both the operator $\Box_{\text{eff}}$ and the source term $S[\chi, \rho]$ arise only after coarse-graining the underlying relational relaxation dynamics of χ. They provide a macroscopic parametrization valid exclusively in regimes where spacetime notions are operationally meaningful.

Physical Interpretation of the Source Term

The term $S[\chi, \rho]$ must not be interpreted as an external force acting on χ, nor as an independent dynamical input. Instead, it encodes the *effective resistance* of localized excitations to the global relaxation flow of the field.

Matter corresponds, in Cosmochrony, to structured and long-lived configurations of χ (solitonic or topologically constrained excitations). Such configurations locally inhibit relaxation, inducing spatial gradients and differential ordering rates when described in an effective geometric language.

Within this interpretation, the source term $S[\chi, \rho]$ provides a compact phenomenological summary of several emergent effects:

- gravitational time dilation as a manifestation of locally slowed χ relaxation,
- inertial mass as persistent resistance to the global relaxation flow,
- effective spacetime curvature as a coarse-grained representation of spatial variations in relaxation efficiency.

Importantly, ρ does not represent a fundamental energy density. It is an effective descriptor of the density of relaxation-resistant configurations within an emergent spacetime regime.

Weak-Field Regime and Linear Approximation

In weak-field regimes, where matter-induced gradients remain small and relaxation is only mildly perturbed, the source term may be approximated as linear in the effective excitation density:

$$S[\chi, \rho] \;\simeq\; -\alpha\,\rho, \tag{64}$$

where α is an effective coupling parameter.

Matching this description with the Newtonian limit of the emergent geometric regime identifies

$$\alpha \sim \frac{G}{c^2},$$

where the gravitational constant G appears as a macroscopic coupling quantifying how strongly localized excitations impede the relaxation of χ.

Within this approximation, Equation (63) reproduces the Poisson equation for the effective gravitational potential and yields the Schwarzschild solution at leading order. No independent gravitational interaction is introduced; gravity arises entirely from relaxation resistance.

Strong-Field Regimes and Nonlinear Corrections

In regimes of high excitation density—such as near compact objects or during early cosmological epochs—the linear approximation breaks down. Strong structural constraints lead to saturation effects in the relaxation dynamics, requiring nonlinear corrections to the effective source term.

A generic parametrization of these effects takes the form

$$S[\chi, \rho] = -\,\alpha\, \rho\, F\left(\frac{\rho}{\rho_c}, \chi\right), \tag{65}$$

where F is a bounded function and ρ_c denotes a characteristic density scale beyond which relaxation resistance saturates.

These nonlinearities prevent unphysical halting of the relaxation flow and encode departures from classical gravity in strong-field regimes, while leaving the underlying ontological structure unchanged. Apparent singular behavior in effective geometric descriptions therefore signals the breakdown of the hydrodynamic approximation, not a failure of the fundamental χ dynamics.

Status of the Effective Description

The source term $S[\chi, \rho]$ does not define an additional physical field or interaction. It is a bookkeeping device summarizing how localized, structured configurations of χ modify the collective relaxation flow once spacetime notions have emerged.

At the fundamental level, only the relational relaxation dynamics of χ exists. Matter, sources, and curvature appear jointly and only as emergent, regime-dependent descriptions of this single underlying process.

A.5 Strong-Field Constitutive Coupling Near a Schwarzschild Black Hole

Purpose and epistemic status.

In Section 5.3, an effective constitutive relation was introduced to encode how strong internal structure of the χ field reduces the efficiency of relaxation:

$$K_{\text{eff}} = K_0 \exp\left(-\frac{(\Delta\chi)^2}{\chi_c^2}\right). \tag{66}$$

This relation is not fundamental. It provides a coarse-grained parametrization of how collective structural constraints within χ suppress relaxation transport in regimes where an emergent geometric description becomes applicable.

In weak-field situations, this description leads to a Poisson-like equation and to the recovery of Schwarzschild phenomenology at leading order (Section 7.4). The purpose of the present appendix is to construct a *consistent strong-field completion* of this picture in the spherically symmetric case, suitable for describing the approach to an effective horizon without introducing geometric singularities.

Operational time-dilation factor.

In the emergent spacetime regime, gravitational time dilation is operationally encoded as a local slowdown of the relaxation rate of χ relative to its asymptotic homogeneous value. We define the dimensionless lapse-like factor

$$N(r) \;\equiv\; \frac{\mathcal{D}_{\text{loc}}\chi(r)}{\mathcal{D}_0\chi}, \qquad 0 < N(r) \le 1, \tag{67}$$

where $\mathcal{D}_0\chi$ denotes the relaxation rate far from localized excitations. In the weak-field limit, this quantity reduces to $N \simeq 1 + \Phi/c^2$, with Φ an effective Newtonian potential.

Matching to Schwarzschild phenomenology.

In regimes where a stable geometric description applies, the exterior field of an isolated compact excitation may be summarized by a Schwarzschild-like metric. We therefore adopt, purely as an *effective descriptor*,

$$ds^2 = -f(r)c^2dt^2 + f(r)^{-1}dr^2 + r^2d\Omega^2, \qquad f(r) = 1 - \frac{r_s}{r}, \tag{68}$$

where $r_s = 2GM/c^2$ is defined operationally by asymptotic weak-field matching.

Consistency with the interpretation of $N(r)$ as the local time-dilation factor then implies

$$N(r)^2 = f(r) = 1 - \frac{r_s}{r}. \tag{69}$$

The limit $N(r) \to 0$ as $r \to r_s^+$ corresponds to an asymptotic freeze-out of local relaxation and defines an *effective horizon.*

From relaxation slowdown to conductivity.

To relate the relaxation slowdown to the constitutive conductivity, we adopt a minimal and self-consistent identification:

$$\frac{K_{\text{eff}}(r)}{K_0} \equiv N(r)^2. \tag{70}$$

This choice is not postulated as fundamental. It is selected because it: (i) reproduces the weak-field expansion, (ii) ensures $K_{\text{eff}} \to 0$ at the horizon, preventing relaxation transport across an asymptotically frozen region, and (iii) remains monotone and bounded.

Combining (70) with (69) yields the explicit strong-field profile

$$K_{\text{eff}}(r) = K_0\left(1 - \frac{r_s}{r}\right), \qquad r > r_s. \tag{71}$$

This expression should be read as an *effective constitutive law* valid only within the emergent geometric regime.

Implied structural variation of χ.

Inverting the constitutive relation (66) using (71) gives

$$\frac{(\Delta\chi(r))^2}{\chi_c^2} = -\ln\left(1 - \frac{r_s}{r}\right). \tag{72}$$

As $r \to r_s^+$, the structural variation measure diverges logarithmically:

$$\Delta\chi(r) \sim \chi_c\sqrt{-\ln\left(1 - \frac{r_s}{r}\right)}. \tag{73}$$

This divergence must not be interpreted as a physical singularity. It indicates that the coarse-grained structural measure $\Delta\chi$ ceases to remain small and that the effective geometric parametrization is pushed beyond its domain of validity near the horizon.

Interpretation: horizons as conductivity zeros.

Equations (71)–(72) provide a coherent strong-field completion of the weak-field Poisson description. In Cosmochrony, a Schwarzschild horizon corresponds to a *vanishing relaxation conductivity*,

$$K_{\text{eff}} \to 0,$$

rather than to a fundamental spacetime singularity. Black holes are therefore interpreted as regions where collective structural constraints within χ asymptotically inhibit relaxation, producing the effective causal and temporal horizons discussed in the main text.

Scope and open issues.

The constitutive description developed here raises several open questions, including:

- the microscopic origin of the effective conductivity scale K_0,
- the extension of the constitutive relation to quantum regimes where ρ is replaced by excitation amplitudes,
- and potential observational signatures of nonlinear relaxation effects near compact objects.

These issues are left for future work.

Conclusion.

The strong-field constitutive profile constructed in this appendix provides a consistent and non-singular description of black-hole-like regimes within Cosmochrony. It preserves agreement with Schwarzschild phenomenology while reinterpreting horizons as limits of relaxation transport rather than as fundamental geometric pathologies.

A.6 Minimal Kinematic Constraint

A foundational assumption of Cosmochrony is the existence of a universal upper bound on the local relaxation rate of the χ field:

$$0 \;\leq\; \partial_t \chi \;\leq\; c, \tag{74}$$

where c denotes the maximal admissible rate of relaxation. This constant is identified, at the level of effective descriptions, with the invariant speed that characterizes relativistic kinematics.

This bound is not introduced as a dynamical equation, a force law, or a cosmological driving term. It constitutes a purely kinematic constraint on admissible χ configurations, specifying the maximal rate at which the relational structure of the field may unfold. In particular, it does not prescribe how χ evolves, but only restricts which evolutions are physically admissible.

The presence of a maximal relaxation rate serves two closely related purposes. First, it ensures that the local progression of effective physical time remains finite and well-defined, preventing pathological behavior such as instantaneous global reconfiguration. Second, it enforces causal consistency by guaranteeing that no influence associated with χ relaxation can propagate arbitrarily fast across the relational structure.

Importantly, this constraint precedes any notion of spacetime geometry. It is imposed directly on the pre-geometric dynamics of χ and does not rely on light cones, metrics, or Lorentz symmetry as fundamental ingredients. Rather, the familiar relativistic causal structure emerges *a posteriori* as an effective description of systems whose dynamics saturate, but do not exceed, this universal bound.

At cosmological scales, the same constraint acquires a global interpretation. When applied to a nearly homogeneous configuration of χ, the bound $\partial_t \chi \leq c$ implies a monotonic and approximately uniform increase of χ, which underlies the effective expansion of space discussed in Section 12.4. In this sense, large-scale cosmic expansion is not driven by an external impulse or a vacuum energy term, but reflects the cumulative consequence of a local kinematic limitation applied consistently across the field.

The minimal kinematic constraint therefore plays a unifying role in Cosmochrony. It anchors causal consistency, bounds temporal unfolding, and provides the structural basis from which relativistic spacetime behavior emerges, all without introducing additional dynamical postulates or background geometric structures.

A.7 Effective Evolution Equation

Once a stable geometric description has emerged from the underlying relaxation dynamics of the χ field, it becomes possible to summarize its large-scale behavior using differential operators familiar from relativistic field theory. This step does not introduce new fundamental dynamics, but provides a convenient phenomenological language for describing regimes in which spacetime notions are operationally meaningful.

At this effective level only, the evolution of χ may be written in the form

$$\Box_{\text{eff}}\chi = S[\chi, \rho], \tag{75}$$

where $\Box_{\text{eff}}$ denotes the d'Alembert operator associated with the *emergent* metric, and ρ represents the density of localized excitations (matter). Neither the operator nor the source term is fundamental: both arise through coarse-graining of the underlying relational relaxation dynamics.

This equation should therefore be understood as an effective rewriting of the χ dynamics once a spacetime description has become applicable. It does not govern the microscopic evolution of the field, but summarizes how large-scale variations of χ respond to the presence of structured, relaxation-resisting configurations.

Physical meaning of the source term.

The term $S[\chi, \rho]$ does not represent an external force acting on χ. Instead, it encodes the effective resistance imposed by localized excitations on the global relaxation flow of the field. Matter corresponds to persistent, structured configurations of χ that locally reduce the admissible relaxation rate, inducing spatial gradients and differential effective time flow.

Within an emergent geometric description, these gradients are naturally reinterpreted as gravitational time dilation and spacetime curvature. The source term thus compactly summarizes several related effects: the inertial resistance of matter, the slowing of local relaxation, and the emergence of effective gravitational potentials.

Weak-field approximation.

In regimes where matter-induced gradients are small and relaxation remains close to homogeneous, the source term may be approximated as linear in the excitation density:

$$S[\chi, \rho] \simeq -\alpha\rho, \tag{76}$$

where α is an effective coupling constant. Matching this expression with the Newtonian limit identifies $\alpha \sim G/c^2$, with the gravitational constant G emerging as a macroscopic parameter characterizing the sensitivity of the relaxation rate to localized excitation density.

In this approximation, the effective evolution equation reproduces the Poisson equation for the gravitational potential and yields Schwarzschild-like solutions in spherically symmetric configurations.

Beyond the weak-field regime.

In regions of high excitation density or strong confinement, such as near compact objects or during early cosmological phases, nonlinear corrections to $S[\chi, \rho]$ are expected. These corrections reflect saturation effects imposed by the minimal kinematic constraint $\partial_t \chi \leq c$ and prevent unphysical halting or divergence of the field evolution.

Such nonlinearities encode departures from classical gravitational behavior while preserving the underlying ontological simplicity of Cosmochrony. They signal the breakdown of the effective geometric description rather than a failure of the fundamental relaxation dynamics of χ.

A.8 Relational Foundation and Emergent Geometry

Throughout the main text, the χ field has been described using a continuous representation. This choice is not meant to attribute fundamental significance to continuity or to spacetime fields, but reflects a pragmatic strategy aimed at maximizing contact with established geometric, field-theoretic, and cosmological formalisms.

Crucially, the emergence of geometric notions in Cosmochrony does *not* depend on the assumption of an underlying continuous manifold. Continuity is introduced only as an effective approximation, valid in regimes where the relational structure of χ varies smoothly and admits coarse-grained descriptions.

At a more fundamental level, Cosmochrony can be formulated in purely relational terms. In such a formulation, neither spacetime points, nor distances, nor a metric are assumed *a priori*. Temporal ordering arises from the monotonic relaxation ordering of χ, while spatial relations and effective geometry are reconstructed operationally from patterns of correlation, resistance, and connectivity within the field.

A concrete realization of this relational perspective is developed in Appendix E. There, geometric quantities are shown to emerge as effective summaries of relational properties of χ, such as the ease with which relaxation-induced variations propagate between configurations. The metric appears only as a derived object encoding these relational properties, not as a fundamental dynamical entity.

The role of the present subsection is therefore purely clarificatory. It emphasizes that the continuous description employed in the main text is a representational convenience rather than an ontological commitment. All core claims of Cosmochrony—including the emergence of time, geometry, and gravitation—remain valid independently of this choice and rest ultimately on the relational dynamics of the χ field itself.

A.9 Energy and Curvature

In the Cosmochrony framework, energy is not introduced as a fundamental conserved quantity. Instead, it emerges as an effective and relational measure of how strongly a given configuration of the χ field resists the global relaxation process. Energy is therefore not a primitive substance, but a diagnostic of constrained relaxation within an otherwise monotonically evolving substrate.

Once an effective geometric description becomes applicable, this resistance may be summarized by quantities that resemble familiar energy densities. At this phenomenological level, it is convenient to introduce the functional

$$\mathcal{E}_\chi^{\mathrm{eff}} \;=\; \frac{1}{2}\left[(\partial_t\chi)^2 + (\nabla\chi)^2\right], \tag{77}$$

which provides a coarse-grained measure of temporal and spatial deformation of the χ field. This expression should be understood strictly as a bookkeeping device defined within the emergent geometric regime, and not as a fundamental Hamiltonian density governing the underlying dynamics.

Regions in which $\mathcal{E}_\chi^{\mathrm{eff}}$ is large correspond to configurations where χ exhibits strong internal gradients or reduced local relaxation rates. Such configurations store a significant amount of relaxation potential and are interpreted as localized resistances to the global evolution of the field. In the effective spacetime description, these regions are naturally identified with particle-like excitations carrying inertial and gravitational properties.

Within this context, the notion of "curvature" associated with χ should be interpreted with care. It does not refer to spacetime curvature as a fundamental geometric object, but to the internal deformation of the χ configuration itself. Effective spacetime curvature arises only secondarily, as a macroscopic descriptor of how such deformations modulate relaxation and correlation propagation across extended regions.

Stable solitonic configurations arise when nonlinear self-interaction effects of the χ field balance the dispersive tendency associated with spatial gradients. This balance allows localized resistance to relaxation to persist over extended durations, providing a dynamical and geometric origin for long-lived particle excitations without invoking fundamental energy conservation laws or independent geometric degrees of freedom.

A.10 Level Sets, Projections, and Apparent Orbital Geometry

This appendix establishes a general geometric property of continuous scalar fields that is directly relevant to the interpretation of atomic orbitals and similar structures as threshold-visible manifestations of an underlying continuum. The results presented here are purely mathematical and do not rely on any specific physical interpretation.

Level sets of χ are introduced solely as visualization tools. They do not correspond to fundamental spatial structures, nor to independently localized entities. Instead, they provide a convenient way of identifying regions of comparable field value within effective geometric descriptions.

Level Sets of Continuous Scalar Fields

Let $\phi : \mathbb{R}^3 \to \mathbb{R}$ be a continuous scalar field. For a given constant $c \in \mathbb{R}$, the associated level set (or isosurface) is defined as

$$\mathcal{L}_c = \{\mathbf{x} \in \mathbb{R}^3 \mid \phi(\mathbf{x}) = c\}. \tag{78}$$

If ϕ is smooth, $\mathcal{L}_c$ is generically a two-dimensional surface, possibly composed of several disconnected components. Such level sets are routinely used in the visualization of scalar fields by retaining only regions exceeding a prescribed threshold.

Crucially, the existence of multiple disconnected components of $\mathcal{L}_c$ does *not* imply that the underlying field ϕ itself is discontinuous or decomposed into independent objects.

Projection-Induced Apparent Discontinuities

Consider the projection of the level-set condition onto a single coordinate, for instance z. Define the projected set

$$P_c = \{z \in \mathbb{R} \mid \exists (x, y) \in \mathbb{R}^2 \text{ such that } \phi(x, y, z) \geq c\}. \tag{79}$$

Even when ϕ is continuous everywhere, P_c typically consists of a finite union of disjoint intervals. These intervals correspond to regions where the level set intersects planes of constant z.

This fragmentation is a purely geometric consequence of thresholding followed by projection. It reflects the fact that only portions of the field exceeding the chosen threshold are retained. No discontinuity of ϕ is involved.

Envelope Function and Threshold Visibility

Define the envelope function

$$f(z) = \max_{x,y} \phi(x, y, z). \tag{80}$$

The projected set can then be written equivalently as

$$P_c = \{z \in \mathbb{R} \mid f(z) \geq c\}. \tag{81}$$

The envelope function $f(z)$ is continuous whenever ϕ is continuous. However, the condition $f(z) \geq c$ generically selects disconnected regions of the domain. The appearance and disappearance of these regions as c varies reflect changes in *visibility*, not in the underlying field structure.

Thus, threshold-based visualizations reveal sections of a continuous field rather than discrete or independently localized objects.

Non-Uniqueness of Inverse Reconstruction

Given a projected set P_c or a collection of disconnected level-set components, the inverse problem of reconstructing ϕ is underdetermined. Infinitely many continuous scalar fields may share identical threshold projections.

Recovering ϕ uniquely requires additional assumptions, such as symmetry, smoothness, minimal curvature, or governing differential equations. The present analysis therefore establishes a structural constraint, not a reconstruction algorithm.

Summary

Thresholded and projected visualizations of continuous scalar fields generically produce apparently disjoint structures. These structures arise from geometric selection effects and do not correspond to independent physical entities.

This result is entirely model-independent. However, it provides a natural mathematical framework for understanding orbital-like patterns, nodal structures, and probabilistic visibility regions as emergent manifestations of an underlying continuous field.

The apparent discreteness of such patterns reflects projection and detection criteria rather than fundamental discontinuity of the substrate.

A.11 Emergent Electrodynamics from χ Dynamics

In regimes where the χ field admits a smooth geometric and weak-gradient description, small perturbations around a slowly varying background obey an effective wave equation derived from the variational formulation (Section 5.4):

$$\nabla \cdot \left(\frac{\nabla \chi}{\sqrt{1 - |\nabla \chi|^2/c^2}} \right) - \frac{1}{c^2} \frac{\partial^2 \chi}{\partial t^2} = \frac{4\pi G_{\mathrm{eff}}}{c^2} \rho_\chi. \tag{82}$$

In the weak-field limit, $|\nabla \chi| \ll c$, this equation linearizes to

$$\nabla^2 \chi - \frac{1}{c^2} \frac{\partial^2 \chi}{\partial t^2} = 4\pi G_{\mathrm{eff}} \rho_\chi, \tag{83}$$

which admits propagating solutions interpreted as radiative disturbances of the χ field.

Emergent Scalar and Vector Potentials

Electromagnetic-like degrees of freedom arise from the *geometric structure* of χ perturbations rather than from independent fundamental fields. In regions containing charged solitonic excitations, the spatial gradients of χ acquire both longitudinal and transverse components.

Accordingly, the spatial gradient of χ may be decomposed (at the effective level) into longitudinal and transverse parts:

$$\nabla \chi = -\nabla \phi + \mathbf{A}_{\mathrm{T}}, \tag{84}$$

where ϕ is an effective scalar potential and $\mathbf{A}_\mathrm{T}$ is a divergence-free vector field,

$$\nabla \cdot \mathbf{A}_\mathrm{T} = 0. \tag{85}$$

This decomposition should be understood as an effective Helmholtz projection induced by the topology of localized χ excitations, rather than as a fundamental split of degrees of freedom. The scalar component encodes longitudinal relaxation gradients associated with effective charge density, while the transverse component arises from solitonic configurations with non-trivial circulation.

Topological Origin of the Vector Potential

The transverse component $\mathbf{A}_\mathrm{T}$ originates from solitonic configurations of χ characterized by non-vanishing loop integrals

$$\oint \nabla\chi \cdot d\mathbf{l} \neq 0, \tag{86}$$

as discussed in Section B.2. Such configurations imply the existence of an effective vector potential whose curl is non-zero, while its divergence vanishes identically.

The vector potential is therefore not an independent dynamical field. It is a derived quantity encoding the transverse sector of χ gradients, analogous to vorticity-induced vector fields in fluid dynamics.

Emergent Electromagnetic Fields

Within this effective description, the electric and magnetic fields are defined as

$$\mathbf{E} = -\nabla\phi - \frac{1}{c}\frac{\partial \mathbf{A}_\mathrm{T}}{\partial t}, \qquad \mathbf{B} = \nabla \times \mathbf{A}_\mathrm{T}. \tag{87}$$

These fields satisfy the Maxwell equations:

$$\nabla \cdot \mathbf{E} = 4\pi G_\mathrm{eff}\rho_\mathrm{em}, \tag{88}$$

$$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0, \tag{89}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{90}$$

$$\nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \frac{4\pi G_\mathrm{eff}}{c}\mathbf{J}_\mathrm{em}, \tag{91}$$

where ρ_em and $\mathbf{J}_\mathrm{em}$ denote the effective charge and current densities associated with solitonic χ excitations.

Gauge Invariance

The decomposition of $\nabla\chi$ into scalar and transverse components is not unique. A transformation of the form

$$\phi \to \phi - \frac{1}{c}\frac{\partial\Lambda}{\partial t}, \qquad \mathbf{A}_{\mathrm{T}} \to \mathbf{A}_{\mathrm{T}} + \nabla\Lambda, \tag{92}$$

leaves the observable fields $\mathbf{E}$ and $\mathbf{B}$ invariant.

This emergent $U(1)$ gauge symmetry reflects the relational nature of χ: only differences of gradients have operational meaning, while the absolute potential is unobservable (Section A.8).

Interpretational Status

The Maxwell-like structure derived here is not fundamental. It arises as a universal effective description of transverse χ perturbations in regimes where solitonic topology and weak gradients coexist. Electromagnetism therefore appears as a geometric manifestation of the relational and topological structure of the χ field, rather than as an independent interaction mediated by elementary gauge fields.

A.12 Relational Consistency of the Effective Lagrangian

The effective Lagrangian for the χ field is **not postulated arbitrarily** but constructed as a **canonical representation** of the relational dynamics introduced in Section 5. This appendix demonstrates how its Born–Infeld-like form emerges from fundamental principles, clarifies its systematic selection, and addresses the continuum limit with full mathematical rigor.

Step 1: Relational Constraint and Bounded Variations

At the fundamental level, the dynamics of χ are governed by a **discrete relational constraint**:

$$\mathcal{C}_i[\chi] \equiv \sum_j K_{ij}(\chi_i - \chi_j)^2 \le \chi_c^2, \tag{93}$$

where $K_{ij} = K_{ji}$ is a symmetric connectivity matrix and χ_c is the correlation scale. This constraint enforces bounded relative variations without assuming pre-existing spacetime, acting as a **structural causality condition**.

Step 2: Variational Formulation with Global Order

The dynamics are described by a constrained action with KKT conditions:

$$S[\{\chi_i\},\{\mu_i\}] = \int d\lambda \left[\sum_i \frac{1}{2}\left(\frac{d\chi_i}{d\lambda}\right)^2 - U[\{\chi_i\}] - \sum_i \mu_i(\lambda)\left(\mathcal{C}_i[\chi] - \chi_c^2\right)\right], \tag{94}$$

where:

- The kinetic term $\frac{1}{2}(d\chi_i/d\lambda)^2$ is the **leading-order expansion** of any smooth functional governing ordered relaxation, with $U[\{\chi_i\}]$ encoding additional constraints.
- The **global order** is ensured by the functional $\Xi[\chi(\lambda)] \equiv \sum_i \chi_i(\lambda)$, with $\frac{d\Xi}{d\lambda} \geq 0$.
- KKT conditions guarantee $\mu_i(\lambda) \geq 0$ and $\mu_i(\lambda)(\mathcal{C}_i[\chi] - \chi_c^2) = 0$.

Step 3: Continuum Limit and Canonical Form

In **projectable regimes**, the discrete constraint maps to a continuum bound:

$$|\nabla\chi|^2 \leq c^2, \tag{95}$$

where ∇ is an emergent operator. For a lattice of spacing a, we define:

$$(\nabla\chi)^2 \approx \frac{1}{a^2}\sum_{\langle i,j\rangle}(\chi_i - \chi_j)^2, \tag{96}$$

yielding $|\nabla\chi|^2 \leq c^2$ with $c^2 \equiv a^2\chi_c^2/K_0$. The continuum limit $a \to 0$ is well-defined if $K_0 \sim a^{-2}$.

Canonical Selection.

We seek a functional $L_{\text{eff}} = f(|\nabla\chi|^2/c^2)$ satisfying:

- Free theory normalization: $f(0) = -c^2$,
- Saturation: $f(1) = 0$,
- Monotonicity: $f'(x) > 0$ for $x \in [0, 1]$,
- Regularity: $f''(x)$ finite.

The **canonical representation** is:

$$f(x) = -c^2\sqrt{1-x}, \tag{97}$$

yielding the Born–Infeld-like Lagrangian:

$$\mathcal{L}_{\text{eff}} = -c^2\sqrt{1 - \frac{|\nabla\chi|^2}{c^2}} + \partial_t\chi. \tag{98}$$

Selection Criteria.

The Born–Infeld form corresponds to the **minimal non-polynomial functional** satisfying boundedness, smooth saturation, and finite characteristic speeds. Other choices are admissible but introduce either degeneracies, non-saturating behavior, or additional scales.

Step 4: Role of the Potential $U[\{\chi_i\}]$

The potential $U[\{\chi_i\}]$ encodes additional relational constraints (e.g., topological terms). In the continuum limit:

$$U[\{\chi_i\}] \to \int d^3x\, V(\chi), \tag{99}$$

where $V(\chi)$ is an effective potential. The connection to the main text's $V(\chi)$ is established via coarse-graining, ensuring consistency with solitonic solutions (Section A.2).

Step 5: Connection to Emergent Geometry

A coarse-graining procedure *admits* an auxiliary effective Lagrangian representation of the form:

$$\mathcal{L}_{\text{eff}} = -c^2\sqrt{1 - \frac{|\nabla\chi|^2}{c^2}} + \partial_t\chi, \tag{100}$$

where the linear term $\partial_t\chi$ does not affect the equations of motion but fixes the orientation of the effective evolution parameter.

The effective metric is defined via the Hessian of $\mathcal{L}_{\text{eff}}$:

$$g_{\mu\nu}^{\text{eff}} \propto \frac{\partial^2\mathcal{L}_{\text{eff}}}{\partial(\partial_\mu\chi)\partial(\partial_\nu\chi)}, \tag{101}$$

up to conformal rescalings. This construction is valid in projectable regimes where K_{ij} approximates a continuum Laplacian (Section D.5).

Summary of Key Improvements

- **Systematic selection** of the Born–Infeld form from first-principle constraints (boundedness, monotonicity, regularity).
- **Explicit continuum limit** with spectral Laplacian connection.
- **Clarified role of** $U[\{\chi_i\}]$ and its continuum counterpart.
- **No circularity**: $\mathcal{L}_{\text{eff}}$ is consistent with (not derived from) relational dynamics.

Scope and Limitations

The Born–Infeld-like Lagrangian is a **canonical representation** valid in projectable regimes. Outside these regimes:

- No spacetime description exists,
- Alternative functionals may be required,
- The discrete dynamics (Eq. (93)) remain fundamental.

Continuum Limit and Emergence of the Laplace–Beltrami Operator

The discrete relational constraint

$$\mathcal{C}_i[\chi] = \sum_j K_{ij}(\chi_i - \chi_j)^2 \tag{102}$$

defines a weighted graph Laplacian acting on the configuration space of χ. To establish its continuum limit, we introduce a local volume element V_i associated with node i, and a spectral distance d_{ij} between nodes.

We assume that the coupling coefficients admit the scaling

$$K_{ij} = \frac{1}{V_i}\, w\!\left(\frac{d_{ij}}{\varepsilon}\right), \tag{103}$$

where w is a symmetric, rapidly decaying kernel and ε characterizes the microscopic relational scale.

In the dense limit, where the number of nodes $N \to \infty$, the typical separation $d_{ij} \to 0$, and the node distribution becomes asymptotically uniform with respect to the emergent measure $\sqrt{|g|}\, d^n x$, defined by the relational density of the χ substrate. The discrete sum satisfies

$$\lim_{\varepsilon \to 0} \frac{1}{V_i} \sum_j w\!\left(\frac{d_{ij}}{\varepsilon}\right) (\chi_i - \chi_j)^2 = \int_{\mathcal{M}} g^{ab}\, \partial_a \chi\, \partial_b \chi\, \sqrt{|g|}\, d^n x. \tag{104}$$

This convergence follows from standard results on graph Laplacians, which guarantee that the weighted Laplacian of a dense graph converges to the Laplace–Beltrami operator on an emergent Riemannian manifold $\mathcal{M}$ defined by the relational density of nodes.

Crucially, no background geometry is assumed *a priori*: the metric g_{ab} arises as the continuum encoding of the microscopic connectivity structure of the χ substrate. The Dirichlet energy functional is therefore not postulated, but emerges uniquely as the thermodynamic limit of the discrete relational constraint.

Necessity of the Born–Infeld Structure from Causal Saturation

The effective Lagrangian governing the projected dynamics of χ cannot be chosen arbitrarily. In particular, a purely quadratic functional

$$\mathcal{L}_{\text{quad}} = \frac{1}{2}\, \partial_\mu \chi\, \partial^\mu \chi \tag{105}$$

is incompatible with the existence of a fundamental upper bound on the relaxation speed of the χ substrate.

Quadratic actions permit unbounded gradients and therefore allow arbitrarily large relaxation fluxes, corresponding to instantaneous propagation of constraints. Such behavior contradicts the existence of a maximal relaxation speed c_χ, required for the causal consistency of the projection onto effective spacetime.

Imposing the condition that the relaxation flux saturates at c_χ uniquely constrains the functional form of the action. The Lagrangian density must interpolate smoothly between the quadratic regime at low gradients and a strictly bounded regime at high gradients.

The minimal functional satisfying these requirements is of Born–Infeld type:

$$\mathcal{L}_{\text{BI}} = b^2 \left(1 - \sqrt{1 - \frac{1}{b^2}\, \partial_\mu \chi\, \partial^\mu \chi}\right), \tag{106}$$

where the parameter b is directly related to the maximal relaxation speed c_χ.

This structure ensures:

- recovery of the quadratic theory in the low-gradient limit,
- a strict upper bound on $|\partial_\mu \chi|$,
- causal saturation of relaxation fluxes at c_χ.

Alternative polynomial expansions (e.g. χ^4 or higher-order terms) fail to enforce such a bound without introducing additional ad hoc mass scales. They therefore violate high-energy spectral invariance and permit superluminal relaxation modes in the χ substrate.

Moreover, the Born–Infeld structure is intrinsically self-regularizing: configurations that would produce divergent gradients in a quadratic theory are smoothly regulated by the square-root saturation mechanism. The energy density and relaxation flux remain finite for all admissible field configurations, without the need for external cutoffs or renormalization prescriptions.

This self-regularization property is absent from polynomial theories, which generically develop gradient singularities and therefore require additional ultraviolet completion.

The Born–Infeld structure is thus not a convenient representation, but the *unique* effective functional compatible with bounded relaxation, causal projection, and parameter-free saturation of the χ dynamics.

Relation Between Spectral Saturation (b) and the Speed of Light (c)

It is essential to distinguish the saturation constant b from the phenomenological parameter c (the speed of light). Within the Cosmochrony framework, b represents the upper bound on the relaxation speed of the χ substrate itself—a pre-geometric causal constraint governing the internal dynamics of the fundamental relational structure.

The speed of light c, by contrast, emerges as an effective quantity: it corresponds to the group velocity of projected perturbative modes propagating on the emergent effective metric $g_{\mu\nu}^{\text{eff}}$. As such, c is not a fundamental constant of the substrate, but a derived parameter characterizing the kinematics of the projected description.

Mathematically, c depends on b through a filtering determined by the microscopic coupling density K_0, encoding how relaxation dynamics in χ are transcribed into effective spacetime propagation. This hierarchical structure ensures that physical information can never propagate faster than the underlying relaxation processes of the substrate, thereby enforcing the inequality

$$c \leq b. \tag{107}$$

Any hypothetical regime in which c would exceed b would imply that projected perturbations propagate faster than the substrate can relax, leading to an immediate loss of coherence of solitonic configurations and, consequently, to the destabilization of matter itself.

B Conceptual Extensions of Cosmochrony — Particles, Quantum Phenomena, and Classical Limits

This appendix develops a set of conceptual and phenomenological extensions of the Cosmochrony framework. Its purpose is not to introduce additional postulates or to strengthen the internal consistency of the theory, but to illustrate how familiar particle, quantum, and classical structures may emerge once the χ field admits localized and stable configurations.

In particular, this appendix addresses:

- the ontological status of the χ field and its interpretation as a pre-geometric relational substrate (Section B.1),
- the description of particles as topological solitons of χ, including explicit constructions for fermionic and bosonic configurations (Sections B.2–B.4),
- the emergence of classical limits and the interpretation of quantum-to-classical transitions (Sections B.5–B.6),
- and perspectives on deriving particle mass spectra from the internal dynamics and stability properties of the χ field (Section B.8).

These developments serve as a conceptual bridge between the mathematical foundations presented in Appendix A and the cosmological and observational considerations discussed in Appendix C. They demonstrate how the minimal ontological assumptions of Cosmochrony can support a rich hierarchy of effective physical phenomena without requiring additional fundamental degrees of freedom.

None of the constructions presented in this appendix are required for the logical coherence or internal consistency of the Cosmochrony framework. The core theory remains fully defined by the relational relaxation dynamics of the χ field. Rather, the material collected here illustrates how particle-like excitations, quantization, and classical behavior may arise naturally as emergent features when χ organizes into localized, topologically stable configurations.

B.1 Interpretative Status of the χ Field

This subsection does not introduce new postulates regarding the χ field. Its purpose is purely interpretative: to clarify how χ should be understood *once the framework has been established*, and to prevent several common misreadings suggested by conventional field-theoretic language.

In the main text, χ is often written in the form $\chi(x^\mu)$ and manipulated using continuous differential operators. This notation should not be taken to imply that χ is a physical field propagating *within* a pre-existing spacetime manifold. Rather, spacetime coordinates serve only as convenient labels for organizing relational information in regimes where a stable geometric description has emerged.

Fundamentally, χ encodes a relational scale of relaxation from which notions such as duration, distance, and causal ordering are reconstructed. The apparent embedding of χ in spacetime is therefore representational, not ontological. The manifold description is a secondary construct, introduced only after the relaxation dynamics of χ has reached sufficient regularity to admit a geometric interpretation.

This distinction mirrors the use of continuum variables in hydrodynamics or elasticity theory. Just as a velocity field does not exist independently of the underlying molecular interactions, $\chi(x^\mu)$ does not represent a fundamental spacetime field. It summarizes collective relational properties of the substrate once coarse graining becomes meaningful.

In this sense, χ should not be interpreted as:

- a matter field living on spacetime,
- a dynamical scalar coupled to a pre-existing metric,
- or a hidden-variable replacement for the quantum wavefunction.

Instead, χ constitutes the pre-geometric quantity from which spacetime structure, effective fields, and physical observables emerge through projection and coarse graining. The use of continuous fields, Lagrangians, and differential equations throughout this work reflects practical representational choices rather than fundamental commitments.

This interpretative clarification is particularly important for understanding the role of localized excitations, solitonic structures, and effective fields discussed in the remainder of this appendix. These constructions should be read as regime-dependent invariants of the underlying χ dynamics, not as evidence that χ itself decomposes into independently propagating physical entities.

In summary, the χ field is not a field *in* spacetime. Spacetime is an emergent bookkeeping structure *for* χ once its relational dynamics becomes sufficiently regular. This asymmetry is essential to the ontological parsimony of the Cosmochrony framework and underlies its reinterpretation of geometry, matter, and quantum phenomena.

B.2 Topological Configurations of the χ Field: Solitons as Particles

Status and scope of this construction.

The solitonic configurations discussed in this appendix are not introduced as fundamental degrees of freedom of Cosmochrony. They are *effective geometric representations* intended to illustrate how particle-like properties may arise from stable, localized configurations of the χ field *once a smooth, orientable geometric projection becomes applicable.*

At the fundamental level, Cosmochrony does not assume a pre-existing spatial manifold, metric, or differential structure. The scalar field χ is not defined *in* spacetime; rather, spacetime emerges as an effective description of relational regimes of χ. A fully relational and pre-geometric formulation is presented in Appendix E.

Throughout this appendix, all geometric notions (distance, rotation, circulation, surface integrals) refer exclusively to an *effective projected field* χ_{eff}, obtained once a stable projective regime is reached. None of the figures or constructions below should be interpreted as depicting the fundamental χ field itself.

Within this effective regime, particles are interpreted as **topologically stabilized solitonic configurations** of χ_{eff}. Their apparent properties—such as **mass, spin, and charge**—do not correspond to independent fundamental quantum numbers, but emerge from the **structural organization and relaxation constraints** induced by these configurations.

Charge as Oriented Relaxation Asymmetry of χ_{eff}

In Cosmochrony, electric charge is not associated with a fundamental gauge field, local symmetry, or conserved Noether current. Instead, it arises as an *oriented asymmetry in the local relaxation structure* of the effective projected field χ_{eff}.

A localized solitonic configuration may deform the surrounding χ_{eff} profile in one of two qualitatively distinct ways:

- A **positive effective charge** corresponds to a **local excess** of χ_{eff} relative to its asymptotic background value $\chi_{\text{eff},0}$. Such configurations locally resist relaxation and generate repulsive relaxation gradients with similarly oriented deformations.
- A **negative effective charge** corresponds to a **local deficit** of χ_{eff} relative to $\chi_{\text{eff},0}$, favoring compensating relaxation and attractive interactions with oppositely oriented configurations.

This polarity does not reflect an intrinsic sign of χ itself—which remains a scalar quantity without charge—but rather the orientation of the deformation with respect to the background relaxation flow in the projective regime.

From structural asymmetry to observable charge.

Within an effective geometric description, the magnitude of the charge associated with a solitonic excitation is controlled by the *net relaxation imbalance* induced by the configuration.

This may be represented schematically by

$$q_{\text{eff}} \;\propto\; \int_{\Sigma} \big(\chi_{\text{eff}} - \chi_{\text{eff},0}\big)\, dS,$$

where Σ denotes a closed surface surrounding the localized configuration in effective space. This expression does not define a fundamental conserved quantity; it provides a coarse-grained measure of the structural asymmetry imposed on the relaxation of χ_{eff}.

In three effective spatial dimensions, the geometric dilution of the associated relaxation gradients naturally leads to inverse-square interaction laws. Coulomb-like behavior therefore emerges as a collective geometric response of χ_{eff}, without introducing a fundamental electromagnetic field or gauge potential.

Vortical Configurations and Integer-Spin Excitations

In the projectable regime, certain solitonic configurations of χ_{eff} admit cyclic internal organization patterns that can be described using an effective phase. When these patterns exhibit non-trivial circulation, the configuration may be modeled as a *vortical soliton*.

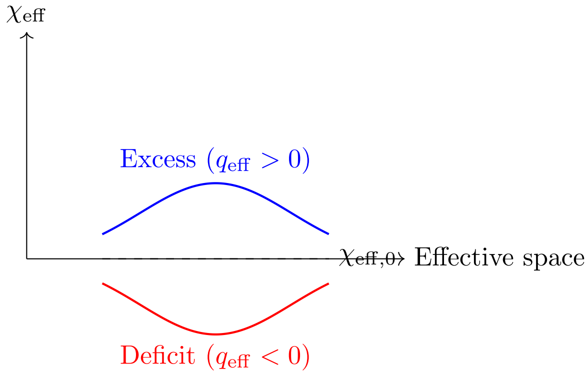

Fig. 6 Schematic illustration of oriented deformations of the effective projected field χ_{eff}. An excess or deficit relative to the background value $\chi_{\text{eff},0}$ determines the polarity of the effective charge. This diagram is purely illustrative and does not represent a solution of the fundamental χ dynamics.

An effective winding number n may be defined as

$$n = \frac{1}{2\pi} \oint \nabla \arg(\chi_{\text{eff}}) \cdot d\mathbf{l},$$

where all quantities refer to the emergent geometric representation. This winding number is not fundamental and has no meaning outside the projective regime.

The integer n characterizes:

- the orientation of the relaxation asymmetry (sign of the effective charge),
- the topological robustness of the configuration,
- and the spin of the excitation, with integer values corresponding to bosonic behavior.

The energetic cost of such configurations increases with their internal structural complexity, leading to an effective mass scaling with $|n|^2$ in minimal models.

Skyrmion-Like Configurations and Spin-$\frac{1}{2}$ Excitations

More complex solitonic configurations arise when the internal organization of χ_{eff} involves non-trivial mappings between internal orientation space and effective physical space. Such configurations may be modeled using skyrmion-like constructions.

An effective topological index Q can be defined as

$$Q = \frac{1}{4\pi} \int \mathbf{n} \cdot (\partial_x \mathbf{n} \times \partial_y \mathbf{n})\, dx\, dy, \quad \mathbf{n} = \frac{\chi_{\text{eff}}}{|\chi_{\text{eff}}|}.$$

Configurations with $Q = \pm 1$ exhibit a characteristic **4π-periodicity** under rotations. A 2π rotation does not return the configuration to an equivalent state, while a 4π rotation does. This topological property provides a geometric origin for spin-$\frac{1}{2}$ behavior and fermionic statistics within the projective regime.

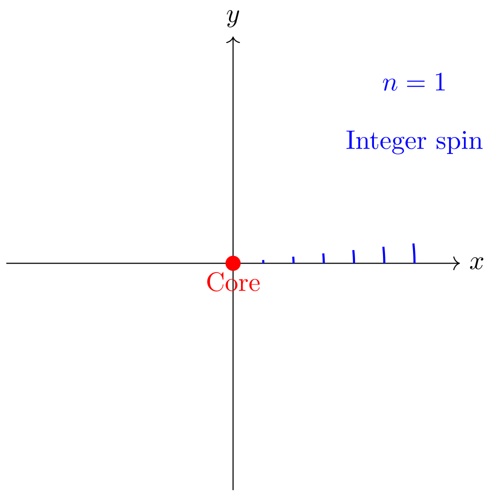

Fig. 7 Illustrative vortical configuration of the effective field χ_{eff} with winding number $n = 1$. The circulation represents a cyclic relaxation pattern associated with integer spin. This figure is schematic and purely conceptual.

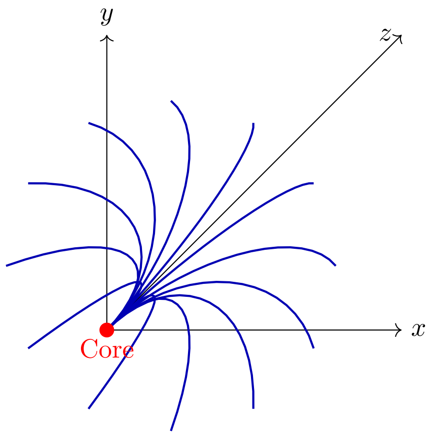

Fig. 8 Conceptual skyrmion-like configuration of χ_{eff}, illustrating a spin-$\frac{1}{2}$ excitation. The non-trivial internal mapping accounts for fermionic rotational behavior. This representation is purely illustrative.

Summary: Topology, Charge, and Spin

These constructions are not intended as a particle classification scheme nor as a replacement for the Standard Model. Their role is conceptual: to demonstrate how charge, mass, and spin may emerge coherently from the structural and topological organization of a single scalar substrate, without introducing additional fundamental fields, symmetries, or quantization postulates.

Table 2 Effective Solitonic Configurations and Emergent Particle Properties

Configuration	**Topological Index**	χ_{eff} **Asymmetry**	**Emergent Properties**
Vortical soliton	Winding number n	Excess / deficit	Charge $\propto n$, integer spin
Skyrmion-like soliton	Index $Q = \pm 1$	Oriented deformation	Charge $\propto Q$, spin-$\frac{1}{2}$

B.3 Soliton Energy and Structural Mass Scaling

Status and scope of this analysis.

This subsection presents a *quantitative but non-numerical* analysis of the effective mass associated with localized solitonic configurations arising in the *projectable regime* of Cosmochrony. The objective is not to reproduce the observed particle mass spectrum, but to identify robust scaling relations, hierarchy constraints, and structural dependencies that emerge independently of microscopic details.

No claim is made that the expressions introduced below define a fundamental Hamiltonian or Lagrangian for the χ field. A fully predictive derivation of particle masses would require a complete effective theory incorporating projection dynamics, interaction channels, and renormalization effects, which lies beyond the scope of the present work.

All energetic and spectral quantities discussed in this section refer exclusively to the effective projected field χ_{eff}. The fundamental χ field itself does not admit an energy functional or mass interpretation.

Mass as integrated resistance to relaxation.

Within Cosmochrony, the mass of a localized excitation is interpreted as a measure of the total *resistance to relaxation* imposed by the configuration on the surrounding χ_{eff} field.

Once an effective geometric description applies, this resistance can be summarized by an effective diagnostic functional

$$M_{\text{eff}} \;\propto\; \int_{\mathcal{V}} \left[\mathcal{T}(\nabla\chi_{\text{eff}}) + \mathcal{U}(\chi_{\text{eff}})\right]\, d^3x, \tag{108}$$

where:

- $\mathcal{T}$ encodes gradient-induced resistance associated with spatial inhomogeneities of χ_{eff},
- $\mathcal{U}$ represents effective nonlinear stabilization terms arising from collective relaxation constraints.

This expression should be understood as a *coarse-grained measure* of structural complexity rather than as a fundamental energy density. It quantifies how strongly a localized configuration delays or distorts the global relaxation of the projected field.

Scaling with soliton size and internal structure.

Consider a localized solitonic configuration characterized by:

- a typical spatial extent ℓ,
- a characteristic deformation amplitude $\Delta\chi_{\text{eff}}$.

Dimensional analysis then yields the generic scaling

$$M_{\text{eff}} \sim \ell^3 \left[\frac{(\Delta\chi_{\text{eff}})^2}{\ell^2} + V_{\text{eff}}(\Delta\chi_{\text{eff}}) \right], \tag{109}$$

where V_{eff} denotes an effective nonlinear stabilization potential generated by projection and relaxation constraints.

For simple kink-like configurations, the balance between gradient resistance and nonlinear stabilization dynamically fixes the soliton width ξ. In this regime, the effective mass scale may be written schematically as

$$M_{\text{eff}} \sim \sqrt{\lambda_{\text{eff}}}\, \xi\, \chi_c^2, \tag{110}$$

where:

- χ_c denotes the characteristic local relaxation scale,
- λ_{eff} is an emergent, configuration-dependent stiffness parameter.

Neither χ_c nor λ_{eff} should be interpreted as fundamental constants. They summarize collective properties of the projected relaxation regime.

Structural stabilization and finite mass.

When stabilization arises from a balance between gradient-induced resistance and nonlinear relaxation constraints, the soliton size ℓ is dynamically fixed. This mechanism ensures that localized excitations possess a finite and stable effective mass without the need for fine-tuning.

Different classes of solitonic configurations (kinks, vortices, knotted or linked structures) involve distinct internal organizations of χ_{eff}. As a result, their effective masses exhibit different scaling behaviors with respect to ℓ and $\Delta\chi_{\text{eff}}$.

This implies that mass hierarchies arise *structurally* rather than through arbitrary parameter choices.

Topological classes and mass hierarchy.

The effective mass depends not only on the spatial extent of a soliton but also on its topological class. Configurations characterized by higher winding, linking, or covering indices necessarily involve increased internal gradients and more complex relaxation constraints.

Consequently, masses associated with different topological families obey ordering relations of the form

$$M_{n+1} > M_n, \tag{111}$$

where n labels an effective topological invariant. This establishes a natural mechanism for discrete mass hierarchies without introducing ad hoc mass parameters.

Spectral interpretation.

From a spectral perspective, localized excitations correspond to bound modes of the linearized relaxation operator around a solitonic background configuration of χ_{eff}.

The effective mass is then controlled by the lowest nontrivial eigenvalue of this operator,

$$M_{\text{eff}} \sim \lambda_{\min}^{-1}, \tag{112}$$

where $\lambda_{\min}$ denotes the smallest positive eigenvalue governing the stability of the configuration.

This formulation emphasizes that mass is fundamentally a *spectral property* of the relaxation dynamics rather than an intrinsic attribute of a particle-like object.

Robustness and universality.

The scaling relations derived above depend only on generic features of the projected χ dynamics—locality, monotonic relaxation, and nonlinear stabilization—and are therefore expected to be robust against modifications of microscopic details.

While specific numerical values of particle masses cannot be fixed at this level, the existence of discrete, ordered, and stable mass scales emerges as a structural prediction of the framework.

Order-of-magnitude consistency.

Although the present analysis does not aim to reproduce the observed particle mass spectrum, it is instructive to examine whether the structural parameters entering the solitonic energy scale admit values compatible with known masses.

For a simple kink-like configuration with characteristic width $\lambda_{\text{eff}}^{-1}$ and amplitude set by the local relaxation scale χ_c, the effective rest energy scales as

$$E_{\text{sol}} \sim \chi_c^2 \lambda_{\text{eff}}, \tag{113}$$

up to dimensionless shape-dependent factors of order unity.

Identifying this energy with the electron rest mass, $E_{\text{sol}} \sim m_e c^2 \approx 0.511\,\text{MeV}$, and expressing all quantities in natural units ($\hbar = c = 1$), one finds that reproducing the electron mass requires

$$\lambda_{\text{eff}} \sim 10^{-44}, \tag{114}$$

for χ_c normalized near the Planck scale.

Such an extremely small value should not be interpreted as a fundamental coupling. Rather, it strongly suggests that λ_{eff} is dynamically generated through collective relaxation, projection effects, and topological constraints of the χ field.

Summary.

Localized solitonic configurations of the projected field χ_{eff} naturally possess finite effective masses determined by their size, internal organization, and topological class. Rather than predicting specific numerical values, Cosmochrony constrains the *scaling, ordering, and stability* of masses through geometric and spectral principles.

This structural quantitativity provides a coherent foundation for future extensions toward a fully predictive effective theory, without compromising the pre-geometric nature of the fundamental χ field.

B.4 Example: 4π-Periodic Soliton and Spinorial Behavior

This subsection presents an *illustrative geometric construction* showing how spin-$\frac{1}{2}$–like transformation behavior may emerge from a localized configuration of the χ field, without introducing a fundamental spinor degree of freedom.

The construction is intentionally minimal and purely effective. It is not intended as a microscopic derivation of fermions, but as a demonstration of *topological plausibility*: namely, that spinorial behavior can arise from nontrivial structure in the configuration space of scalar excitations once a projectable regime is reached.

All geometric notions used below refer exclusively to the effective projected field χ_{eff}. The fundamental χ field itself does not admit spatial localization, complex structure, or phase.

Phase-Twisted Effective Solitonic Configuration

In the projectable regime, certain localized excitations of χ admit an effective internal organization that can be parametrized by an angular variable. For illustrative purposes only, such configurations may be represented using a complex-valued proxy field,

$$\chi_{\text{eff}}(x) = \eta \tanh(\kappa x)\, e^{i\theta(x)}, \tag{115}$$

where:

- the underlying physical field remains real,
- the complex phase does *not* represent an independent internal degree of freedom,
- the phase θ parametrizes the internal cyclic structure of the effective solitonic configuration.

This representation should be understood purely as a convenient encoding of the internal organization of the excitation in effective space.

Choosing

$$\theta(x) = \frac{x}{2} \tag{116}$$

implies that the configuration returns to an equivalent internal state only after a 4π variation of the effective rotation parameter,

$$\theta(x + 4\pi) = \theta(x) + 2\pi, \tag{117}$$

whereas a 2π variation produces a configuration that is locally identical in effective space but globally inequivalent in internal structure.

Topological Interpretation

The 4π-periodicity does not originate from the introduction of a complex field or an intrinsic phase. It reflects a nontrivial topology of the configuration space of the soliton.

Although the spatial projection of the excitation may appear unchanged after a 2π rotation, the internal organization of the configuration is not. Only a 4π rotation restores full equivalence in the space of effective configurations.

This behavior mirrors the double-cover structure $\mathrm{SU}(2) \to \mathrm{SO}(3)$ characteristic of spinorial representations. In the present framework, however, this structure arises from the topology of solitonic configurations of χ_{eff}, rather than from a fundamental spinor ontology.

Relation to Fermionic Transformation Properties

At the effective level, 4π-periodic excitations naturally acquire a sign change under 2π rotations. In multi-excitation configurations, this topological property implies that the exchange of two identical excitations cannot be continuously deformed into the identity without crossing a topologically nontrivial sector.

This provides a geometric basis for fermion-like transformation behavior and antisymmetric exchange properties. The construction does not constitute a proof of the spin–statistics theorem; rather, it demonstrates that fermionic characteristics can emerge consistently from topologically constrained scalar-field excitations.

Conceptual Scope and Limitations

The purpose of this example is conceptual. It illustrates that:

- spinorial behavior does not require a fundamental spinor field,
- 4π-periodicity can arise from topological obstructions,
- fermion-like exchange behavior may emerge from scalar excitations with nontrivial configuration space.

No claim is made that this construction reproduces the full dynamics, interactions, or statistics of Standard Model fermions. A fully relational formulation of these topological properties, independent of any embedding geometry or auxiliary representation, is discussed in Appendix E.

B.5 Relation to Classical Limits

In Cosmochrony, the emergence of classical behavior does not correspond to the introduction of an independent theoretical layer. Instead, classical physics arises as a *dynamical regime* of the same underlying scalar structure, characterized by smoothness, dilution of localized excitations, and the suppression of topological and relational effects.

Weakly structured regime and effective linearization.

In regimes where the fundamental field χ admits a stable projective representation and where its effective projection χ_{eff} varies slowly over large scales, localized excitations become dilute and weakly interacting. Under these conditions, the dynamics of χ_{eff} can be linearized around a quasi-homogeneous background configuration.

In this regime, small perturbations propagate as weak disturbances on an effectively flat geometric background. Superposition, approximate locality, and linear wave propagation emerge as effective properties of the coarse-grained relaxation dynamics.

This reproduces the operational content of classical field theories and of quantum field theories formulated on Minkowski spacetime.

This correspondence should be understood as an *effective recovery* rather than as an ontological reduction. Cosmochrony does not reduce to standard quantum field theory; rather, standard field theories appear as limiting descriptions valid when relational structure becomes dynamically inert.

Suppression of relational and topological effects.

In the weakly structured regime, topological constraints associated with solitonic configurations are either absent or dynamically irrelevant. The configuration space effectively factorizes, and collective relaxation dominates over localized structural organization. As a result, particle-like excitations behave as approximately independent degrees of freedom, and classical intuition becomes applicable.

The classical limit therefore corresponds to a regime in which the relational content of χ is present but operationally inaccessible, masked by coarse-graining and scale separation.

Nonlinear regime and effective curvature.

Conversely, in regimes of strong spatial variation of χ_{eff} or high density of localized excitations, nonlinear effects dominate the dynamics. Large gradients locally constrain relaxation, inducing effective curvature, time dilation, and horizon-like behavior in the emergent geometric description.

These regimes reproduce the phenomenology associated with curved spacetime, gravitational collapse, and strong-field effects, while remaining governed by the same underlying scalar dynamics. No additional gravitational degrees of freedom are introduced; curvature emerges as a collective response of the relaxation structure of χ_{eff}.

Meaning of the classical limit in Cosmochrony.

The classical limit in Cosmochrony is therefore not defined by $\hbar \to 0$, nor by the suppression of quantum postulates. It corresponds to a regime in which:

- the effective projection χ_{eff} is smooth and slowly varying,
- localized excitations are dilute and weakly correlated,
- topological and relational constraints are dynamically suppressed,
- coarse-graining yields stable geometric descriptions.

In this regime, classical spacetime and standard field dynamics emerge as reliable, approximate descriptions. Their validity reflects not fundamental structure, but the stability of a particular relaxation regime of the underlying χ field.

B.6 Status of the Formulation

The formulation presented in this work should be understood as a *minimal yet structurally complete* theoretical framework. Its ontological commitments, dynamical principles, and interpretative structure are fully specified at the conceptual level, even though several technical developments remain open.

In particular, a fully covariant action principle formulated solely in terms of the fundamental χ dynamics, as well as a systematic quantization procedure, have not yet been derived in their final and definitive form. These missing elements should not be interpreted as conceptual deficiencies. Rather, they correspond to technical extensions required to interface a fundamentally relational and pre-geometric framework with conventional variational and quantum formalisms that presuppose spacetime structure.

Crucially, the absence of a finalized action or quantization scheme does not obstruct the recovery of known physical phenomenology. Throughout this work, general relativity and quantum field theory emerge as *effective, coarse-grained descriptions* valid within specific dynamical regimes of the projected field χ_{eff}. They are not introduced as independent axioms, but arise as stable limits of the underlying relaxation dynamics.

The present formulation therefore occupies a well-defined intermediate status. It is not intended as a closed or final theory, nor as a phenomenological model tuned to reproduce specific experimental data. Instead, it provides a coherent ontological and dynamical foundation from which both geometric and quantum structures can emerge, while remaining open to future refinements that may enhance its mathematical completeness, formal elegance, and predictive scope.

In this sense, Cosmochrony should be viewed as a foundational framework rather than as a fully developed effective field theory: its primary contribution lies in clarifying *what is fundamental* and *how known physical structures can arise*, rather than in prescribing their final mathematical implementation.

B.7 Soliton and Particle Solutions

Within the Cosmochrony framework, elementary particles are interpreted as stable or metastable localized configurations arising in the *projectable regime* of the scalar field χ. These configurations, hereafter referred to as χ-solitons, emerge from nonlinear self-organization of the relaxation dynamics and persist as localized resistances to global relaxation.

This interpretation does not rely on the postulation of additional fundamental degrees of freedom. The only fundamental entity is the scalar field χ, which is not defined on spacetime. Spatial localization, energy, and particle-like persistence arise only once an effective geometric projection χ_{eff} becomes applicable.

While the fundamental field is scalar, certain solitonic configurations of χ_{eff} possess a nontrivial internal organization that cannot be faithfully encoded by a single real scalar variable. In particular, configurations characterized by internal cyclic structure, nontrivial winding, and 4π-periodicity exhibit transformation properties that require a double-valued representation under effective rotations.

In such cases, an effective spinorial description becomes unavoidable. This does not imply the existence of fundamental spinor fields. Rather, spinorial variables arise as *collective descriptors* encoding the internal topology and spectral structure of fermionic χ-solitons.

At the phenomenological level, these excitations admit a representation in terms of Dirac spinors. This representation should be understood as an emergent and coarse-grained description of the internal degrees of freedom of χ-solitons, not as an ontological extension of the theory. Within this regime, the Dirac equation appears as the minimal effective dynamical structure compatible with:

- approximate locality in the projected geometric description,
- effective relativistic covariance,
- and the topological constraints associated with 4π-periodic configurations.

From this perspective, the Dirac structure does not introduce new fundamental entities. It provides a compact and universal encoding of the internal topology, spectral stability, and transformation behavior of fermionic solitons. Spin, fermionic statistics, and exclusion behavior arise as effective consequences of the nontrivial configuration space of these scalar-field excitations, rather than as independent postulates.

The existence and stability of χ-solitons impose structural constraints on the effective self-interaction functional governing the projected dynamics. Although the explicit form of this functional remains undetermined, it must satisfy the following minimal requirements:

1. the support of localized configurations with finite effective mass,
2. dynamical stability under small perturbations,
3. and the existence of topologically inequivalent sectors corresponding to distinct classes of particle-like excitations.

The detailed derivation of effective Dirac dynamics from fluctuations around χ-soliton backgrounds, as well as the emergence of a realistic mass spectrum, remains an open mathematical problem. These issues are addressed at a programmatic and illustrative level in Sections B.2–B.4 and further discussed in Appendix B.8.

B.8 Perspectives: Towards a Derivation of the Proton-to-Electron Mass Ratio

The proton-to-electron mass ratio is one of the most precisely measured dimensionless quantities in physics. Within the Cosmochrony framework, the purpose of this section is not to derive this value from first principles, but to clarify how such a ratio could emerge *structurally* from the spectral and topological organization of localized solitonic excitations of the projected field χ_{eff}.

The discussion should therefore be understood as a minimal and exploratory spectral ansatz. Its aim is to identify the relevant mechanisms, constraints, and scaling relations that any successful derivation would have to satisfy, rather than to provide a complete microscopic calculation.

Spectral Stability Hypothesis

Let χ_{sol} denote a stationary localized configuration arising in the projectable regime of the χ dynamics. Small perturbations $\delta\chi_{\text{eff}}$ around this background are governed, at the coarse-grained level, by a linear stability operator $\mathcal{L}_{\text{sol}}$, defined as the second variation of an effective localization functional.

Normal modes satisfy the eigenvalue problem

$$\mathcal{L}_{\mathrm{sol}}\psi_n = \lambda_n\psi_n. \tag{118}$$

The eigenvalues λ_n characterize the resistance of the soliton to localized deformations. They encode intrinsic stiffness scales associated with the internal organization of the solitonic configuration.

Spectral mass scaling.
In regimes where an effective wave description applies, the normal modes exhibit characteristic oscillation frequencies

$$\omega_n = c\sqrt{\lambda_n}. \tag{119}$$

Identifying the lowest nontrivial frequency with the rest energy of the excitation leads to the effective scaling relation

$$m_n \;\propto\; \sqrt{\lambda_n}\,\chi_c, \tag{120}$$

where χ_c denotes a characteristic geometric scale associated with the spatial extension of the solitonic configuration in the projected regime.

This relation does not define a fundamental mass formula. It provides a coarse-grained link between spectral stability and inertial mass, consistent with the interpretation of mass as integrated resistance to relaxation.

Dimensional interpretation.
The eigenvalues λ_n carry dimensions of inverse length squared, reflecting the restoring stiffness of the soliton per unit deformation. The scale χ_c has dimensions of length and sets the geometric extension over which this stiffness is distributed.

The combination $\lambda_n\chi_c^2$ therefore defines a characteristic energy scale,

$$E_n \sim \lambda_n\chi_c^2, \tag{121}$$

which is identified with a rest energy through the effective relativistic matching $E = mc^2$ once a spacetime description becomes applicable.

This identification does not invoke a fundamental quantum constant and remains valid independently of the emergence of $\hbar_{\mathrm{eff}}$.

Projection Scale and Effective Normalization

The fundamental description of the χ field is formulated in terms of relational relaxation rules rather than a spacetime action with fixed physical units. When a continuum approximation applies, an effective action for perturbations around a stable soliton may be introduced as a bookkeeping device.

In this regime, the effective action for perturbations $\delta\chi_{\rm eff}$ may be written schematically as

$$S_{\rm eff}[\delta\chi] = \int d^4x \, \frac{1}{2} \left(\frac{\chi_c}{c}\right)^2 \left[(\partial_t \delta\chi)^2 - c^2 (\nabla \delta\chi)^2\right]. \tag{122}$$

Expressing this action in emergent spacetime coordinates introduces a geometric rescaling factor linking χ-space and spacetime lengths. As a result, the canonical normalization of localized modes involves a quadratic scaling factor of the form

$$\left(\frac{\chi_c}{\ell_{\rm spacetime}}\right)^2, \tag{123}$$

which controls the effective normalization of spectral quantities.

This factor reflects the geometric projection from the relational χ structure to emergent spacetime observables. It does not represent a fundamental coupling constant.

Energy Levels from Spectral Stability

The discrete energy levels associated with solitonic excitations follow from the spectral properties of the stability operator $\mathcal{L}_{\rm sol}$, not from canonical quantization.

For a soliton labeled by n, the gradient contribution to the effective energy scales as

$$E^{(n)}_{\rm grad} \sim c^2 \lambda_n \mathcal{N}_n, \tag{124}$$

where $\mathcal{N}_n$ denotes a normalization factor determined by the spatial profile of the mode.

In the spacetime-based description, this energy is identified with the rest-mass energy,

$$E_n \equiv m_n c^2. \tag{125}$$

The discretization of E_n arises from topological classification and spectral stability, not from postulated quantum operators. The role of $\hbar_{\rm eff}$ appears only when matching this description to quantum observables.

Elementary versus Composite Spectral Structures

A key distinction must be drawn between elementary and composite solitonic excitations. Elementary particles, such as leptons, are expected to correspond to topologically elementary solitons whose inertial mass is dominated by a single lowest stability eigenvalue.

By contrast, baryonic excitations are composite configurations. Their mass reflects the combined contribution of several coupled stability modes associated with a bound structure. Mass ratios therefore take the schematic form

$$\frac{m_{\rm comp}}{m_{\rm elem}} \sim \frac{\sum_k \sqrt{\lambda_k^{\rm (comp)}}}{\sqrt{\lambda_0^{\rm (elem)}}}, \tag{126}$$

rather than the ratio of two isolated eigenvalues.

Ansatz for the Proton as a Composite Soliton

As an exploratory working hypothesis, the proton is modeled as a composite solitonic excitation. Specifically:

- the electron corresponds to a topologically elementary soliton with a fundamental stability eigenvalue λ_e,
- the proton corresponds to a bound configuration involving three such elementary solitons, supplemented by an additional collective binding mode with eigenvalue λ_{bind}.

The choice of a three-soliton composite is motivated by stability considerations observed in a wide class of nonlinear field theories admitting topological solitons, where three-body bound states often exhibit enhanced stability due to geometric phase locking [49]. This choice is not derived here from a classification of χ-soliton sectors and is not postulated as fundamental.

Skyrmion models in QCD provide an instructive analogy, but no dynamical equivalence is assumed. The relevance of this analogy lies in the universality of topological stabilization mechanisms, which do not depend on the presence of a non-Abelian gauge symmetry [50].

Mass Ratio from Spectral Scaling

Under these assumptions, the effective eigenvalue associated with the proton may be written schematically as

$$\lambda_p \;\approx\; \lambda_{\text{bind}} + 3\lambda_e, \tag{127}$$

leading to the mass ratio

$$\frac{m_p}{m_e} \;\approx\; \sqrt{\frac{\lambda_{\text{bind}} + 3\lambda_e}{\lambda_e}}. \tag{128}$$

In the binding-dominated regime $\lambda_{\text{bind}} \gg \lambda_e$, this reduces to

$$\frac{m_p}{m_e} \;\approx\; \sqrt{\frac{\lambda_{\text{bind}}}{\lambda_e}}. \tag{129}$$

Matching the observed ratio $m_p/m_e \simeq 1836$ therefore imposes the spectral constraint

$$\frac{\lambda_{\text{bind}}}{\lambda_e} \;\sim\; 3.4 \times 10^6. \tag{130}$$

This relation is not derived here. It is identified as a consistency condition constraining the relative spectral organization of elementary and composite solitonic sectors.

We interpret this large spectral hierarchy as defining a dimensionless *spectral packing fraction* α, characterizing the relative density of admissible stability modes in composite versus elementary solitonic sectors. Specifically, we define

$$\alpha \equiv \frac{\lambda_e}{\lambda_{\text{bind}}} \sim 3 \times 10^{-7}. \tag{131}$$

This quantity does not represent a coupling constant, but a structural measure of spectral compression induced by topological binding.

Topological Interpretation of the Spectral Hierarchy

Although the ratio $\lambda_{\text{bind}}/\lambda_e$ is introduced here as a spectral consistency condition, it is natural to seek a geometric or topological interpretation of this large hierarchy.

In particular, the composite nature of the proton ($Q = 3$) suggests that the associated binding modes may correspond to configurations of increased topological complexity. If the stability spectrum of L_{sol} is controlled by the effective multiplicity of internal configurations admitted under the non-injective projection Π, then λ_{bind} may be interpreted as a coarse-grained measure of the volume of the corresponding projection fiber.

From this perspective, the large ratio $\lambda_{\text{bind}}/\lambda_e \sim 10^6$ reflects not an arbitrary energy scale separation, but the rapid growth of internal configuration space associated with topologically composite solitons.

Indicative Geometric Scale

Although no explicit geometric or topological model is developed at this stage, it is useful to translate the observed spectral hierarchy into a characteristic dimensionless scale.

At a purely heuristic level, one may assume that the effective spectral weight of a composite soliton grows quadratically with a characteristic internal scale χ_c, so that

$$\frac{\lambda_{\text{bind}}}{\lambda_e} \sim \chi_c^2. \tag{132}$$

Under this assumption, the empirical constraint $\lambda_{\text{bind}}/\lambda_e \sim 3.4 \times 10^6$ corresponds to a scale of order

$$\chi_c \sim \mathcal{O}(10), \tag{133}$$

with a representative numerical value

$$\chi_c \approx 8.3. \tag{134}$$

Both expressions should be regarded as indicative rather than derived. They simply emphasize that the required spectral hierarchy corresponds to a modest geometric amplification, not to an extreme or finely tuned parameter choice.

Role of $V(\chi)$ and Fine Structure

The effective potential $V(\chi)$ is expected to play a secondary role in mass generation. Its primary effect is to control fine splittings within a given solitonic sector rather than to set the overall mass scale.

Small differences, such as the neutron–proton mass splitting, may arise from subleading corrections to the effective potential or from topological asymmetries. No quantitative prediction is attempted in the absence of an explicit form for $V(\chi)$.

Although introduced here in the context of particle mass hierarchies, the spectral packing fraction α is expected to be a universal property of non-injective projection. In later sections, we argue that the same quantity governs the spectral granularity of horizon-associated reprojection phenomena.

B.9 Spectral Scaling and the Projection Ontology

The preceding derivation of the mass ratio m_p/m_e rests on a fundamental shift in the ontology of mass. Within the Cosmochrony framework, inertial mass is no longer treated as an intrinsic "charge", but as a spectral signature of projection visibility.

Mass as Spectral Weight

The non-injective nature of the projection Π (see Section 4.5) implies that any effective particle in χ_{eff} corresponds to a large equivalence class of micro-configurations in the substrate χ. The stability eigenvalues λ_n of the operator L_{sol} can therefore be reinterpreted as a coarse-grained measure of this structural multiplicity, or *fiber weight*. A configuration that requires a larger set of internal modes to remain stable and projectable manifests a higher resistance to global relaxation, and thus a higher inertial mass.

Invariance of the Ratio

Since the ratio

$$\frac{m_p}{m_e} \approx \sqrt{\frac{\lambda_p}{\lambda_e}} \tag{135}$$

is independent of the absolute action scale $\hbar_\chi$, it is identified as a structurally protected invariant of the projection process itself. This explains the observed universality of the proton-to-electron mass ratio across cosmological epochs, regardless of the global relaxation state of χ.

The Spectral Packing Fraction (α)

The hierarchy between the composite sector (proton) and the elementary sector (electron) is encapsulated by the spectral packing fraction

$$\alpha \equiv \frac{\lambda_e}{\lambda_{\text{bind}}} \approx 3 \times 10^{-7}. \tag{136}$$

Rather than an empirical fit, α represents the ratio of spectral transmittance under Π. The proton is heavy because its non-trivial topology ($Q = 3$) constrains the stability operator L_{sol} to exhibit a large internal spectral bandwidth. This topological constraint, often heuristically represented by a trefoil-knot configuration, leads to a strong spectral gap between the binding modes λ_{bind} and the fundamental electronic mode λ_e.

Conclusion

While the precise numerical value $m_p/m_e \simeq 1836$ awaits a closed-form derivation from the spectral geometry of $L_{\rm sol}$ on topologically constrained manifolds, the Cosmochrony framework reformulates the mass-ratio problem in structural terms. The proton-to-electron mass ratio emerges as the macroscopic signature of a spectral gap dictated by the complexity of the substrate's excitations under projection, as schematically illustrated in Fig. 9, rather than as an arbitrary fundamental constant.

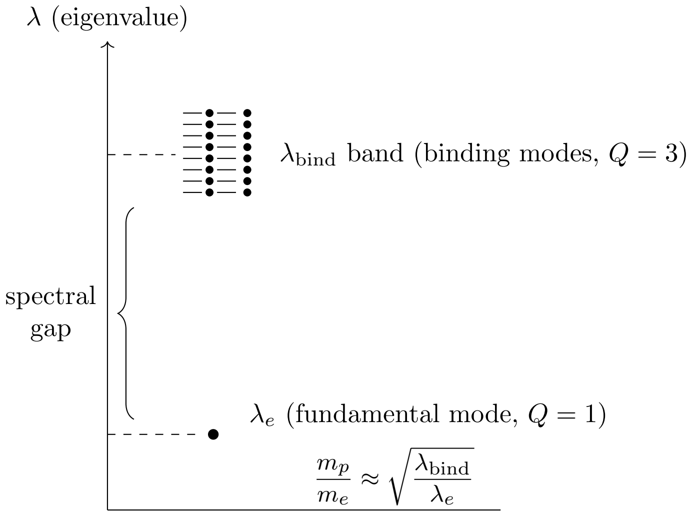

Fig. 9 Conceptual schematic of a spectral gap in the stability spectrum of $\mathcal{L}_{\rm sol}$. The elementary mode λ_e is separated from a dense band of binding modes near $\lambda_{\rm bind}$, illustrating the interpretation of m_p/m_e as a macroscopic signature of spectral organization rather than an intrinsic mass parameter.

Summary and Outlook

Within the Cosmochrony framework, the proton-to-electron mass ratio is interpreted as an emergent constraint on the spectral and topological organization of solitonic excitations, not as a fundamental input parameter.

The analysis presented here provides a coherent toy model identifying the conditions under which such a ratio could arise. Whether the required spectral hierarchy can be generated dynamically from the χ relaxation dynamics remains an open problem, to be addressed through future analytical and numerical investigations.

B.10 Spectral Characterization of Mass and the Secondary Role of $V(\chi)$

This appendix clarifies the conceptual status of inertial mass in the Cosmochrony framework. Physically, mass originates from the resistance of localized configurations to the relaxation of the fundamental χ field. This resistance, however, admits a quantitative and structurally organized description in terms of the spectral properties of an associated stability operator defined in the projectable regime.

Spectral analysis therefore does not redefine the physical origin of inertial mass. Rather, it provides a coherent and potentially calculable characterization of how resistance to relaxation is distributed among stable and metastable configurations.

A central conjecture of Cosmochrony is that particle masses are not fundamental parameters encoded in the nonlinear potential $V(\chi)$. Instead, they emerge as spectral properties of a background-independent relaxation operator defined on a relational substrate, which may be represented, for calculational purposes, by a discrete graph structure.

Mass spectrum as eigenmodes of a relaxation operator.

Localized particle-like excitations are identified with normal modes of an effective relaxation operator $\Delta_G^{(0)}$, which may be represented as a Laplace–Beltrami operator acting on a graph $G(V, E)$,

$$\Delta_G^{(0)}\psi_n = -\lambda_n\psi_n. \tag{137}$$

The eigenmodes ψ_n characterize the stability of localized solitonic configurations, while the eigenvalues λ_n encode their intrinsic resistance to deformation.

In regimes where an effective spacetime description applies, the inertial masses associated with these modes scale as

$$m_n c^2 \ \propto \ \sqrt{\lambda_n}\,\chi_c, \tag{138}$$

in agreement with the spectral relations introduced in Section B.8. This scaling reflects the fact that inertial mass measures the characteristic frequency associated with the resistance of a localized configuration to χ-field relaxation.

The situation is analogous to bounded elastic systems, where discrete vibrational frequencies arise from geometry and connectivity rather than from adjustable material parameters. Within Cosmochrony, mass hierarchies are therefore interpreted as geometric and topological properties of the underlying relational structure.

A decisive test of this conjecture would consist in computing the low-lying spectrum of $\Delta_G^{(0)}$ on large but finite networks with physically motivated connectivity rules. Even approximate agreement with observed mass ratios would strongly support the spectral origin of inertial mass and the non-fundamental role of $V(\chi)$.

Separation of descriptive levels.

To avoid circular dependencies between geometry, dynamics, and emergent particle properties, Cosmochrony distinguishes three conceptual levels.

At the fundamental level, inertial masses are associated with the spectral properties of a background-independent relaxation operator $\Delta_G^{(0)}$, defined solely by the intrinsic relational connectivity of the substrate. This operator is not tied to any spacetime geometry or instantaneous χ configuration and provides a stable spectral backbone.

At the emergent geometric level, coarse-grained configurations of χ_{eff} give rise to effective notions of spacetime, including curvature, gravitational time dilation, and cosmological expansion. These geometric effects influence propagation and interaction, but do not redefine the underlying spectral operator responsible for mass generation.

Finally, fast dynamical processes such as radiation, scattering, and decoherence correspond to interaction-induced redistributions of relaxation potential within the χ field. These processes affect observables without modifying the fundamental spectral structure.

Residual role of the potential $V(\chi)$**.**

Within this spectral picture, the nonlinear potential $V(\chi)$ plays a secondary and effective role. It does not set the overall mass scale. Instead, it provides a local coarse-grained description of nonlinear stabilization mechanisms associated with low-lying spectral modes.

The admissible form of $V(\chi)$ is constrained by the requirement that it support stable solitonic configurations compatible with the pre-existing spectral structure. It encodes neither an independent interaction nor a fundamental energy density.

Origin of the effective potential $V(\chi)$**.**

Characterizing $V(\chi)$ as secondary does not imply arbitrariness. Rather, $V(\chi)$ should be understood as an effective descriptor of the local nonlinear response of the relaxation dynamics in the vicinity of a stable configuration.

At the fundamental level, the dynamics of χ are governed by bounded relaxation rules and their associated spectral structure. When this dynamics is projected onto a reduced functional subspace associated with a localized soliton, nonlinear self-consistency constraints induce an effective local restoring structure. In this reduced description, these constraints may be summarized by an effective potential $V(\chi)$.

Different admissible forms of $V(\chi)$ correspond to different coarse-graining choices, while leaving invariant the underlying spectral origin of mass and stability.

Potential-induced corrections to stability eigenvalues.

To illustrate how $V(\chi)$ can modify stability eigenvalues without altering their spectral origin, consider the illustrative form

$$V(\chi) = \lambda \left(\chi^2 - \chi_c^2\right)^2 . \tag{139}$$

Expanding around the relaxed background $\chi = \chi_c$ yields a quadratic contribution for small fluctuations $\delta\chi$,

$$V(\chi_c + \delta\chi) \simeq \frac{1}{2} \left.\frac{d^2V}{d\chi^2}\right|_{\chi_c} (\delta\chi)^2 + \cdots , \tag{140}$$

with

$$\left.\frac{d^2V}{d\chi^2}\right|_{\chi_c} \propto \lambda\, \chi_c^2. \tag{141}$$

This term contributes additively to the linearized stability operator, shifting the eigenvalues as

$$\lambda_n \;\longrightarrow\; \lambda_n^{(0)} + \Delta\lambda_n^{(V)}. \tag{142}$$

For composite solitons, such corrections may differ slightly between closely related configurations (e.g., neutron versus proton), generating small mass splittings. By contrast, ratios dominated by topological organization (such as m_p/m_e) remain largely insensitive to the detailed form of $V(\chi)$.

Summary.

In Cosmochrony, inertial mass is fundamentally a spectral property of the relaxation dynamics of the χ field. The potential $V(\chi)$ serves as a derived, effective descriptor controlling fine structure, not as a primary source of mass. Extending this spectral characterization toward quantitative mass predictions requires specifying the relaxation operator and its boundary conditions, particularly for composite solitonic sectors.

B.11 Spectral Stability and the Emergence of $\hbar_{\text{eff}}$

In Cosmochrony, the effective Planck constant $\hbar_{\text{eff}}$ is not introduced as a fundamental quantum postulate. Instead, it emerges as a scaling parameter linking spectral stability of χ-field solitons to effective spacetime observables.

Fundamental scales of the χ dynamics

The χ field is characterized by three independent dynamical scales:

- K_0: maximal relaxation stiffness, with dimensions $[L^{-2}]$,
- χ_c: correlation length at which solitonic configurations stabilize,
- c: maximal relaxation speed.

From these, one may define a natural unit of action associated with the relaxation dynamics,

$$\hbar_\chi \;\equiv\; \frac{c^3}{K_0 \chi_c}, \tag{143}$$

which has the dimensions of action and is independent of the standard Planck constant. Here and in the following, K_0 and χ_c denote the *bare substrate parameters*, i.e. universal invariants characterizing the rigidity and correlation capacity of the χ field. The scale-dependent values discussed in D arise only after coarse-graining and do not enter the definition of $\hbar_\chi$.

Spectral origin of effective quantization

Quantization in Cosmochrony follows from the discrete spectrum of the stability operator $\Delta_G^{(0)}$. For a solitonic excitation with eigenvalue λ_n, the characteristic frequency of small oscillations scales as

$$\nu_n \;\sim\; \frac{c}{\chi_c}\sqrt{\lambda_n}\,\mathcal{N}_n^{1/2}. \tag{144}$$

Identifying the rest energy with the product of this frequency and an effective action scale yields

$$E_n = \hbar_{\text{eff}}\, \nu_n, \tag{145}$$

from which $\hbar_{\text{eff}}$ emerges as a geometric and spectral quantity, not as an independent constant.

Regime-dependent scaling

The effective value of $\hbar_{\text{eff}}$ depends on the scale at which the system is probed. In regimes where the characteristic spacetime scale $\ell_{\text{spacetime}}$ is comparable to χ_c,

$$\hbar_{\text{eff}} \approx \hbar_\chi, \tag{146}$$

recovering standard quantum behavior.

At macroscopic scales $\ell_{\text{spacetime}} \gg \chi_c$,

$$\hbar_{\text{eff}} \approx \hbar_\chi \left(\frac{\chi_c}{\ell_{\text{spacetime}}}\right)^2, \tag{147}$$

leading to a strong suppression of quantum effects and the emergence of classical behavior.

Consistency with quantum phenomenology

In the microscopic regime, where $\hbar_{\text{eff}} \approx \hbar$, standard quantization relations $E = \hbar\nu$ are recovered as effective descriptions. This agreement is not postulated but follows from the scaling behavior of $\hbar_{\text{eff}}$ once the projected regime matches laboratory scales.

Numerical constraints.

Reproducing particle-scale quantum behavior requires

$$K_0 \chi_c^2 \sim \hbar, \tag{148}$$

which constrains the admissible values of the relaxation stiffness and correlation length. These constraints are consistent with soliton stability and do not require fine tuning.

B.12 Renormalization of Substrate Parameters

To maintain consistency between the fundamental definition of $\hbar_\chi$ and the scale-dependent observations in Appendix D, we distinguish between:

- **Bare Parameters (K_0, χ_c):** Universal invariants of the χ substrate that determine the fundamental quantum of action $\hbar_\chi$.
- **Effective Parameters ($K_{\text{eff}}, \chi_{\text{eff}}$):** Environment-dependent values emerging from the coarse-graining of relaxation constraints, as detailed in Section D.

The universality of $\hbar$ and the spectral invariant α_{spec} (formerly α in Section B.9) stems from their dependence on the ratio of these bare quantities, which remains invariant under projective scaling.

In particular, dimensionless coupling constants such as the electromagnetic fine-structure constant α_{EM} do not inherit any arbitrariness from the substrate parameters. Within the Cosmochrony framework, the electric charge e is not treated as a free gauge parameter, but as a property of localized solitonic configurations. The associated transmittance is not an adjustable quantity but a geometric invariant of the soliton's spectral embedding relative to the projection fiber Π. As a result, the dependence on the substrate rigidity K_0 cancels out in dimensionless ratios, ensuring their invariance within a fixed relaxation epoch.

Summary.

Within Cosmochrony, both inertial mass and effective quantization emerge from the same spectral stability structure of the χ relaxation dynamics. The Planck constant appears not as a fundamental input, but as a scale-dependent parameter encoding the projection from relational dynamics to spacetime-based observables.

C Cosmological and Observational Implications of Cosmochrony

This appendix examines the cosmological and observational implications of the Cosmochrony framework. Its purpose is not to construct a fully parameterized cosmological model, nor to perform precision fits to existing datasets, but to establish conceptual and phenomenological consistency with key observations and to identify distinctive signatures that may allow the framework to be tested or falsified.

All results presented here should be understood as consequences of the same underlying scalar relaxation dynamics governing the χ field. No additional cosmological degrees of freedom are introduced beyond those already discussed in the main body of the work. Standard cosmological observables arise as effective descriptions once a smooth geometric projection of χ becomes applicable.

The appendix is organized as follows:

- Section C.1 analyzes the spectrum of large-scale fluctuations of the projected field χ_{eff} and their imprint on the cosmic microwave background, with particular emphasis on the suppression of low-ℓ multipoles.
- Section C.2 shows how the horizon and flatness problems are resolved dynamically in Cosmochrony, without invoking an inflationary phase or fine-tuned initial conditions.
- Section C.3 discusses the evolution of the effective Hubble parameter $H(z)$ and its implications for the observed Hubble tension, highlighting how deviations from standard expansion histories may arise from relaxation dynamics.
- Section C.4 provides order-of-magnitude estimates of the characteristic χ-field parameters and relates them to observable cosmological quantities, without assuming a specific cosmological parameter set.

- Section C.5 explores broader phenomenological consequences, including modified gravitational-wave propagation, MOND-like effective dynamics at galactic scales, and lensing anomalies.

Throughout this appendix, the emphasis is placed on identifying robust qualitative and semi-quantitative features that follow generically from the Cosmochrony framework. Where numerical estimates are provided, they are intended as consistency checks and scaling guides rather than as precision predictions.

Taken together, these results demonstrate that Cosmochrony is compatible with the broad structure of contemporary cosmological observations while predicting systematic deviations from standard ΛCDM expectations. These deviations define concrete targets for future observational tests and numerical investigations.

C.1 Low-ℓ CMB Power Suppression from Global χ Relaxation

One of the most persistent large-scale anomalies of the cosmic microwave background (CMB) concerns the suppression of temperature anisotropy power at the largest angular scales ($\ell \lesssim 30$), most notably in the quadrupole and octupole moments. Within the standard ΛCDM framework, such deviations are commonly attributed to cosmic variance, and no specific physical mechanism is associated with their occurrence.

Within the Cosmochrony framework, by contrast, the lowest multipoles probe global properties of the projected field χ_{eff} rather than independent local perturbations. Because the fundamental field χ evolves through a monotonic relaxation process constrained by finite connectivity and a maximal relaxation speed, the longest-wavelength modes correspond to collective configurations whose amplitudes are not freely adjustable.

Structural attenuation of global modes.

At very low multipoles, the associated angular modes span regions comparable to the full causal domain of the projected χ_{eff} configuration. As a result, these modes are subject to global relaxation constraints: their amplitude is systematically attenuated relative to the scale-invariant expectation.

This suppression is not the result of stochastic damping or fine-tuned initial conditions. It arises because the finite relaxation capacity of the χ field limits the degree to which globally coherent configurations can deviate from the relaxed background. The effect is deterministic in origin, while its detailed realization in any given universe remains statistical.

Cosmochrony therefore does not predict exact multipole amplitudes. Instead, it predicts a robust suppression tendency affecting the lowest ℓ-modes, whose precise pattern depends on the detailed global configuration of χ at last scattering.

Illustrative comparison with observations.

Figure 10 shows the observed CMB temperature power spectrum at low multipoles, displayed without aggressive smoothing, together with a schematic attenuation envelope representative of the Cosmochrony mechanism.

This comparison is intended to illustrate the qualitative structural deviation from scale invariance implied by global relaxation constraints. It does *not* constitute a multipole-by-multipole prediction, nor does it replace a full Boltzmann analysis.

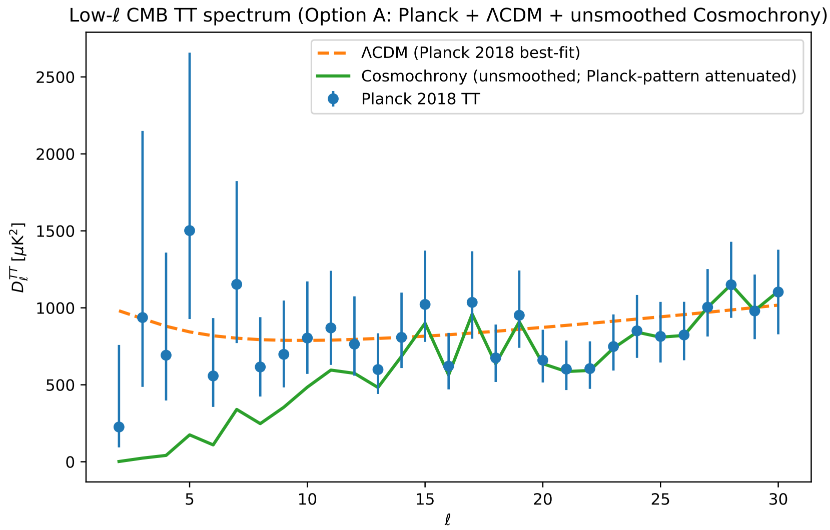

Fig. 10 Observed CMB temperature power spectrum at low multipoles ($\ell \lesssim 30$), shown without heavy smoothing. The shaded region illustrates a qualitative attenuation envelope expected from global relaxation constraints on the projected field $\chi_{\rm eff}$ in Cosmochrony. Unlike ΛCDM, where low-ℓ suppression is treated as a statistical accident, Cosmochrony interprets it as a structural consequence of the finite relaxation capacity of globally coherent configurations. The envelope may be summarized phenomenologically by Eq. (149).

Phenomenological parametrization.

To render this schematic attenuation minimally quantitative without introducing a full cosmological perturbation theory, we introduce a two-parameter phenomenological envelope in which the low-ℓ power is multiplicatively suppressed relative to the ΛCDM best-fit spectrum:

$$C_\ell^{\rm CC} = C_\ell^{\Lambda{\rm CDM}} \left[1 - \alpha \exp\left(-\frac{\ell}{\ell_0}\right)\right], \qquad \alpha \in [0,1], \quad \ell_0 > 0. \tag{149}$$

Equivalently, in terms of $D_\ell \equiv \ell(\ell+1)C_\ell/(2\pi)$,

$$D_\ell^{\rm CC} = D_\ell^{\Lambda{\rm CDM}} \left[1 - \alpha e^{-\ell/\ell_0}\right].$$

In this parametrization, α controls the overall amplitude of large-angle suppression, while ℓ_0 sets the angular scale beyond which the spectrum rapidly converges back to the standard ΛCDM behavior.

Indicative low-ℓ characterization.

An indicative estimate of (α, ℓ_0) may be obtained by fitting the ratio $R_\ell \equiv D_\ell^{\rm obs}/D_\ell^{\Lambda\rm CDM}$ over a restricted low-ℓ range (e.g. $\ell = 2\ldots30$), using cosmic-variance-dominated uncertainties $\sigma(R_\ell) \simeq \sqrt{2/(2\ell+1)}$.

This procedure is not intended as a detection claim. It provides a compact and reproducible summary of the suppression tendency, replacing heuristic hand-drawn envelopes by a controlled two-parameter characterization.

Conceptual distinction from ΛCDM.

In ΛCDM, low-ℓ deviations are interpreted *a posteriori* as statistical fluctuations around an ensemble mean defined by inflationary initial conditions. In Cosmochrony, the ensemble itself is constrained: the global relaxation dynamics of χ restrict the admissible configuration space for the longest-wavelength modes.

This leads to a qualitative physical distinction between large-scale and small-scale fluctuations. Small-scale modes probe local relaxation and behave approximately as independent perturbations, while large-scale modes encode global structural properties of the field.

Scope and limitations.

The present analysis does not replace full Boltzmann calculations and does not aim to reproduce the entire angular power spectrum. Its purpose is to identify a robust qualitative signature of Cosmochrony: a systematic suppression tendency affecting the lowest CMB multipoles, arising from global relaxation constraints on the fundamental field.

Quantitative refinement of this effect, including detailed parameter inference and polarization observables, is deferred to future numerical studies of the χ dynamics.

C.2 Resolution of the Horizon and Flatness Problems Without Inflation

In standard cosmology, the horizon and flatness problems arise from extrapolating a spacetime-based notion of causality and geometry back to the earliest stages of cosmic evolution. Within this framework, regions of the universe that appear widely separated today should not have been in causal contact, and the near-flatness of spatial geometry requires fine-tuned initial conditions. Inflation addresses these issues by postulating a brief phase of accelerated expansion in a pre-existing metric background.

Cosmochrony adopts a fundamentally different standpoint. Spacetime geometry, causal structure, and metric notions of distance are not assumed to be fundamental. They emerge only at a later stage, as effective descriptions of the relaxation dynamics of the scalar field χ. As a result, the assumptions underlying the horizon and flatness problems do not apply at the fundamental level.

Horizon problem: pre-geometric connectivity.

In Cosmochrony, large-scale correlations do not need to be established through signal propagation within spacetime. Instead, they originate from the fact that χ constitutes a single, globally connected dynamical substrate whose relaxation precedes the emergence of any effective spacetime description.

At early stages, before a metric notion of causality becomes meaningful, the configuration of χ is defined globally. Regions that later appear causally disconnected in the emergent spacetime may therefore share correlated configurations inherited from earlier phases of the relaxation process.

In this sense, Cosmochrony replaces inflationary causal contact with *pre-geometric connectivity*: correlations are established at the level of the fundamental field itself, rather than through superluminal expansion or specially prepared initial conditions on a metric background.

Flatness problem: relaxation toward geometric uniformity.

The flatness problem is addressed through the same underlying mechanism. In Cosmochrony, effective spatial curvature reflects large-scale gradients and inhomogeneities in the relaxation rate of the projected field χ_{eff}. As relaxation proceeds, configurations with large curvature gradients are dynamically disfavored, since they correspond to sustained resistance to global relaxation.

As a consequence, near-flat spatial geometry emerges as a natural attractor of the relaxation dynamics. Curvature dilution does not require exponential expansion or fine-tuning of initial curvature parameters. It reflects the tendency of the χ field to minimize large-scale geometric tension as it approaches a homogeneous relaxation state.

This mechanism operates independently of any inflationary phase and does not rely on a specific initial curvature value.

Implications for primordial correlations.

Because large-scale coherence arises from the global organization of χ rather than from the amplification of quantum vacuum fluctuations, Cosmochrony does not predict exact scale invariance at the largest wavelengths. Instead, the longest-wavelength modes are subject to global relaxation constraints, which may lead to deviations from scale invariance at the lowest multipoles of the cosmic microwave background.

Such deviations are interpreted as structural tendencies rather than sharp predictions. They provide a qualitative distinction from inflation-based scenarios and motivate the phenomenological analysis of low-ℓ CMB anomalies discussed in Section C.1.

Status and limitations.

The arguments presented here establish that the horizon and flatness problems do not arise as fundamental inconsistencies within the Cosmochrony framework. They are artifacts of applying metric-based reasoning beyond its domain of validity.

A quantitative derivation of primordial correlation functions and power spectra, including detailed predictions for CMB anisotropies, requires dedicated numerical simulations of the χ-field relaxation dynamics and lies beyond the scope of the present work.

Nevertheless, Cosmochrony provides a conceptually coherent, inflation-free resolution of large-scale causal coherence and near-flat spatial geometry, rooted in the pre-geometric dynamics of a single scalar field.

C.3 Evolution of the Hubble Parameter and the Hubble Tension

In the Cosmochrony framework, cosmological expansion is not governed by the competition between matter, radiation, and a dark energy component. Instead, it reflects the relaxation dynamics of the scalar field χ, from which spacetime geometry and its associated expansion rate emerge as effective descriptions.

The Hubble parameter therefore encodes the instantaneous relaxation rate of χ relative to its global configuration, rather than the response of a metric to an energy–momentum content.

Global expansion rate.

At the homogeneous background level, the effective scale factor is proportional to the global value of the projected field,

$$a(t) \propto \chi(t), \tag{150}$$

so that the Hubble parameter may be written as

$$H(t) = \frac{\dot{\chi}}{\chi}. \tag{151}$$

In the idealized homogeneous limit, spatial gradients vanish ($\nabla\chi = 0$) and the relaxation dynamics reduce to a uniform evolution with maximal relaxation speed,

$$\dot{\chi} = c. \tag{152}$$

In this limit, the global expansion rate becomes

$$H(t) = \frac{c}{\chi(t)}. \tag{153}$$

This relation defines the *global* expansion rate in Cosmochrony. Its detailed redshift dependence away from perfect homogeneity is not assumed to follow a fixed power law and depends on how relaxation gradients contribute to the averaged dynamics.

Relaxation budget and effective expansion.

In a realistic universe, part of the relaxation capacity of χ is stored in spatial gradients associated with inhomogeneities. To quantify this effect, we introduce a dimensionless *relaxation budget* parameter,

$$\Omega_\chi \equiv \langle \beta^2 \rangle, \qquad \beta \equiv \frac{|\nabla\chi|}{c}, \tag{154}$$

which measures the fraction of the total relaxation capacity diverted into spatial structure rather than global evolution.

At late times, these gradients are dominated by localized solitonic configurations and therefore track the large-scale matter distribution. The effective global expansion rate is then reduced to

$$\bar{H} = \frac{c}{\chi}\sqrt{1 - \Omega_\chi}. \tag{155}$$

Empirically, consistency with large-scale observations suggests Ω_χ is of order the observed matter fraction, $\Omega_\chi \sim 0.3$. In Cosmochrony, this suppression of the global expansion rate arises naturally from relaxation dynamics and does not require a dark energy component.

Local expansion and environmental dependence.

In an inhomogeneous universe, the relaxation budget is not spatially uniform. Regions with different matter densities redistribute relaxation capacity differently between global evolution and local gradients.

For a region characterized by a density contrast

$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}, \tag{156}$$

we adopt a minimal mean-field closure relation,

$$\beta_{\mathrm{loc}}^2 = \Omega_\chi(1 + \delta), \tag{157}$$

which encodes the intuitive scaling between matter density and χ-gradient energy.

The locally inferred Hubble parameter then takes the form

$$H_{\mathrm{loc}} = \bar{H}\sqrt{\frac{1 - \Omega_\chi(1 + \delta)}{1 - \Omega_\chi}}. \tag{158}$$

In underdense regions ($\delta < 0$), a larger fraction of the relaxation capacity is available for global evolution, leading to $H_{\mathrm{loc}} > \bar{H}$.

Numerical consistency and the Hubble tension.

For representative values $\Omega_\chi \approx 0.3$ and a local underdensity consistent with the KBC void ($\delta \approx -0.4$ on scales of a few hundred megaparsecs), one finds

$$\frac{H_{\mathrm{loc}}}{\bar{H}} \approx 1.08, \tag{159}$$

corresponding to an enhancement of order 8% in the locally inferred Hubble constant.

This magnitude is comparable to the observed discrepancy between local distance-ladder measurements and global CMB-based inferences. In Cosmochrony, this discrepancy arises naturally as an environmental effect, without invoking new energy components or modifications of early-universe physics.

Interpretation and status.

Within Cosmochrony, the Hubble tension does not signal a breakdown of cosmological consistency. It reflects the fact that cosmological expansion is an emergent relaxation phenomenon whose effective rate depends on the local redistribution of χ-field gradients.

While the framework robustly predicts a separation between local and global expansion rates, a fully quantitative determination of $H(z)$ across all redshifts requires dedicated numerical simulations of the χ relaxation dynamics. Such simulations lie beyond the scope of the present work.

Nevertheless, the qualitative resolution of the Hubble tension follows directly from the relaxation-based interpretation of cosmological expansion and constitutes a distinctive and testable signature of the Cosmochrony framework.

C.4 Relation to Observational Units and Numerical Estimates

This subsection establishes order-of-magnitude relations linking the Cosmochrony framework to observed cosmological quantities. Its purpose is not to perform parameter fitting or to derive precision predictions, but to assess the internal consistency of the theory and to verify that its fundamental relaxation-based interpretation naturally reproduces the correct empirical scales.

All numerical relations presented here should be understood as effective normalizations arising in the projectable regime of the χ dynamics. They do not define fundamental constants and do not fix the microscopic structure of the theory.

Normalization of the χ Field

To relate the projected field χ_{eff} to observable quantities, a reference normalization must be specified. At the effective cosmological level, the scale factor is defined up to a global multiplicative constant. In Cosmochrony, this freedom is fixed by identifying the present-day value $\chi(t_0)$ with the characteristic geometric scale governing large-scale expansion.

Operationally, $\chi(t_0)$ represents the cumulative geometric scale associated with the global relaxation of the χ field up to the present epoch. This identification does not assume a unique microscopic origin for $\chi(t_0)$; it provides a minimal and observationally anchored normalization consistent with the effective relation $a(t) \propto \chi(t)$.

Emergent Gravitational Coupling

In the effective geometric description, the Newtonian gravitational constant G emerges from the constitutive relation governing the coupling between neighboring configurations of the projected χ field. This coupling is controlled by two parameters of the relaxation dynamics: the maximal stiffness scale K_0 and the characteristic correlation length χ_c.

Although K_0 and χ_c are not individually fixed at the present stage, their combination is constrained by matching the observed gravitational coupling:

$$K_0 \chi_c^2 \sim \frac{c^4}{16\pi G}. \tag{160}$$

This relation fixes the overall stiffness scale of the effective χ network. It does not require committing to a specific microscopic interpretation of χ_c, which may correspond to a fundamental correlation scale or to an emergent coarse-graining length. At this level, only the product $K_0\chi_c^2$ is observationally relevant.

Hubble Constant

In the homogeneous limit, the effective Hubble parameter is defined by the relative relaxation rate of the projected field,

$$H(t) = \frac{\dot{\chi}}{\chi}. \tag{161}$$

Assuming that the present universe lies close to the maximal relaxation regime, $\dot{\chi}(t_0) \simeq c$, the present-day Hubble constant follows as

$$H_0 \simeq \frac{c}{\chi(t_0)}. \tag{162}$$

Using the observed value $H_0 \approx 70\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ yields

$$\chi(t_0) \sim 4 \times 10^{26}\ \mathrm{m}, \tag{163}$$

which is of the order of the observed Hubble radius. This correspondence arises directly from the relaxation-based interpretation of cosmic expansion and does not require the introduction of additional cosmological parameters.

Age of the Universe

In the homogeneous relaxation regime, the evolution of χ may be approximated as

$$\dot{\chi} \simeq c, \tag{164}$$

leading to

$$\chi(t) \simeq ct + \chi_{\mathrm{init}}, \tag{165}$$

where χ_{init} denotes the effective value of χ at the onset of the relaxation regime relevant for cosmological observations.

Neglecting χ_{init} compared to present values yields

$$t_0 \simeq \frac{\chi(t_0)}{c} \sim 4 \times 10^{17}\ \mathrm{s}, \tag{166}$$

corresponding to approximately 13.8 billion years. This estimate is consistent with standard cosmological age determinations and follows directly from the bounded relaxation dynamics.

Redshift Interpretation

In Cosmochrony, cosmological redshift is interpreted as a consequence of the relative change in the projected χ field between emission and observation,

$$1 + z = \frac{\chi(t_{\text{obs}})}{\chi(t_{\text{emit}})}. \tag{167}$$

This relation reproduces standard redshift phenomenology while attributing it to geometric scaling induced by χ relaxation, rather than to recessional motion within a pre-existing spacetime background.

Cosmic Microwave Background Scale

At recombination, characterized observationally by $z_{\text{rec}} \simeq 1100$, the effective value of the projected field was smaller by the corresponding scaling factor,

$$\chi(t_{\text{rec}}) \simeq \frac{\chi(t_0)}{1 + z_{\text{rec}}}. \tag{168}$$

Fluctuations imprinted at that epoch are subsequently stretched by the monotonic growth of χ, providing a natural geometric interpretation of the angular scales observed in the cosmic microwave background without invoking an inflationary stretching phase.

Orders of Magnitude and Robustness

All numerical estimates presented in this subsection rely solely on observed cosmological quantities and on the bounded relaxation dynamics of the χ field. No fine-tuning of parameters, no detailed cosmological fitting, and no additional degrees of freedom are assumed.

While a fully predictive cosmological model requires explicit numerical simulations of the χ dynamics, these order-of-magnitude relations demonstrate that Cosmochrony naturally reproduces the correct scales for the Hubble constant, the age of the universe, redshift evolution, and characteristic CMB features.

Summary

The Cosmochrony framework admits a consistent normalization in observational units and reproduces key cosmological scales without introducing new fundamental parameters. These order-of-magnitude relations support the internal coherence of the theory and motivate further quantitative investigation of its cosmological dynamics.

C.5 Phenomenological Implications

This subsection summarizes the principal phenomenological consequences of Cosmochrony that are accessible to observation. The emphasis is placed on effects that follow robustly from the kinematic and relaxation structure of the χ field itself, without introducing auxiliary degrees of freedom, adjustable interpolation functions, or phenomenological potentials.

All results presented here arise in the projectable regime of the theory and should be understood as effective manifestations of the underlying relaxation dynamics, not as fundamental postulates.

Propagation speed of gravitational perturbations.

To determine the propagation speed of gravitational information in Cosmochrony, consider small perturbations $\delta\chi$ around a homogeneous relaxation background,

$$\chi_0(t) = ct, \tag{169}$$

such that

$$\chi(\mathbf{x}, t) = ct + \delta\chi(\mathbf{x}, t), \qquad |\nabla\delta\chi| \ll c. \tag{170}$$

Substituting this form into the fundamental kinematic constraint governing χ-field relaxation (Eq. 9) yields, to leading order,

$$c + \partial_t \delta\chi = c\sqrt{1 - \frac{|\nabla\delta\chi|^2}{c^2}}. \tag{171}$$

Expanding for small spatial gradients gives

$$\partial_t \delta\chi \simeq -\frac{|\nabla\delta\chi|^2}{2c}, \tag{172}$$

reflecting the irreversible character of the relaxation process. While this first-order relation governs dissipation, the propagation of perturbations is more transparently captured by considering the second-order operator associated with the squared constraint.

Linearizing this operator leads to the effective wave equation

$$\left(\frac{1}{c^2}\partial_t^2 - \nabla^2\right)\delta\chi = 0, \tag{173}$$

which admits propagating solutions with characteristic speed

$$v_{\text{prop}} = c. \tag{174}$$

Gravitational perturbations therefore propagate exactly at the invariant speed c. This equality is not imposed by hand but follows directly from the fundamental kinematic bound on χ relaxation. As a result, Cosmochrony is automatically consistent with multi-messenger observations, including the near-simultaneous arrival of gravitational and electromagnetic signals in events such as GW170817.

Emergent acceleration scale and MOND-like phenomenology.

In Cosmochrony, the arrow of time is encoded in the monotonic evolution of the fundamental field, $\partial_t \chi \geq 0$. At late cosmic times and on sufficiently large scales, where

$\chi_{\rm eff}$ admits an approximately homogeneous description, the relaxation dynamics may be coarse-grained into an effective cosmological clock.

In this effective regime, the temporal evolution of χ can be written as

$$\partial_t \chi \simeq H(t)\, \chi, \tag{175}$$

where $H(t)$ denotes the emergent Hubble parameter associated with global relaxation.

The local kinematic constraint

$$(\partial_t \chi)^2 + |\nabla \chi|^2 = c^2 \tag{176}$$

then implies that even in the absence of localized matter excitations, the cosmological evolution of χ enforces a non-vanishing residual spatial gradient. In the homogeneous limit, this minimal gradient is

$$|\nabla \chi|_{\rm min} = \sqrt{c^2 - (H\chi)^2}. \tag{177}$$

This residual gradient defines a background kinematic scale that constrains the superposition of additional, locally induced gradients. Operationally, it corresponds to an effective acceleration scale

$$a_0(t) \sim c\, H(t). \tag{178}$$

When localized matter excitations are present, they induce additional gradients $\nabla\chi_N$ that reproduce the Newtonian scaling $|\nabla\chi_N| \propto M/r^2$ at short distances. Because the kinematic constraint is nonlinear, the total gradient does not superpose linearly. At sufficiently large radii, the effective acceleration asymptotically approaches

$$g_{\rm eff} \simeq \sqrt{g_N\, a_0(t)}, \tag{179}$$

recovering the characteristic deep-MOND scaling without introducing interpolation functions, dark matter particles, or additional fields.

In this framework, the acceleration scale a_0 is not fundamental. It evolves slowly with cosmic time through its dependence on $H(t)$, providing a potential observational discriminator at high redshift.

Gravitational lensing.

In Cosmochrony, light propagation follows wavefronts of constant χ. An effective refractive index for the vacuum may be defined operationally as

$$n(r) = \frac{c}{\partial_t \chi} = \frac{1}{\sqrt{1 - |\nabla\chi|^2/c^2}}. \tag{180}$$

Near a localized mass M, where $|\nabla\chi| \simeq GM/(c^2 r)$, a weak-field expansion yields

$$n(r) \simeq 1 + \frac{GM}{c^2 r}. \tag{181}$$

Integrating the transverse gradient of $n(r)$ along a photon trajectory leads to a deflection angle

$$\alpha = \frac{4GM}{bc^2}, \tag{182}$$

where b is the impact parameter. This reproduces the general-relativistic prediction for gravitational lensing.

In Cosmochrony, the enhancement relative to the Newtonian deflection does not originate from a fundamental spacetime curvature. It arises from the nonlinear structure of the χ relaxation dynamics, which modifies the effective propagation geometry experienced by light.

Summary.

The phenomenology of Cosmochrony reproduces key observational signatures of gravity and cosmology while relying on a single scalar degree of freedom. Gravitational perturbations propagate at exactly the invariant speed c, a MOND-like acceleration scale emerges naturally from cosmological relaxation, and gravitational lensing is recovered without postulating a fundamental metric.

These results illustrate how classical gravitational phenomena arise as coarse-grained manifestations of the underlying χ dynamics and define a set of observationally testable signatures distinguishing Cosmochrony from standard metric-based theories.

C.6 Toy-Model of Spectral Gravitational Susceptibility

This appendix provides the mathematical foundations for a non-particulate interpretation of dark matter phenomena, treating galactic dynamics as the non-linear elastic response of the χ substrate.

Modified Poisson Equation and Field Strength

In the Cosmochrony framework, gravitational acceleration is not a force acting in a passive vacuum but the emergent manifestation of a relaxation gradient. We define the **local relaxation field strength** as the gradient of the scalar relaxation potential:

$$\mathbf{E}_\chi = -\nabla\Phi_\chi \tag{183}$$

The dynamics are governed by a modified Poisson equation analogous to electrodynamics in continuous media:

$$\nabla \cdot [\epsilon_{\text{spec}}(\mathbf{E}_\chi)\mathbf{E}_\chi] = 4\pi G_0 \rho_b \tag{184}$$

where ρ_b is the baryonic mass density and ϵ_{spec} is the **spectral permittivity** of the substrate, defined by the relation $\epsilon_{\text{spec}} = 1 + \phi(\mathbf{E}_\chi)$.

Spectral Susceptibility and the Stiffness Threshold $\mathcal{K}_c$

To recover the observed galactic phenomenology, we define the **spectral gravitational susceptibility** ϕ as a function of the field strength relative to a saturation threshold

$\mathcal{K}_c$:

$$\phi(\mathbf{E}_\chi) = \begin{cases} 0 & \text{for } |\mathbf{E}_\chi| \gg \mathcal{K}_c \text{ (Linear/Newtonian Regime)} \\ \frac{\mathcal{K}_c}{|\mathbf{E}_\chi|} & \text{for } |\mathbf{E}_\chi| \ll \mathcal{K}_c \text{ (Saturation Regime)} \end{cases} \tag{185}$$

Crucially, $\mathcal{K}_c$ is not a universal constant of nature but a **local state property** of the substrate. It represents the threshold where the relaxation flux reaches the elastic limit of the χ field.

Emergence of Flat Rotation Curves

In the low-field limit ($|\mathbf{E}_\chi| \ll \mathcal{K}_c$) typical of galactic peripheries, the effective acceleration g_{eff} follows:

$$\nabla \cdot \left(\mathcal{K}_c \frac{\mathbf{E}_\chi}{|\mathbf{E}_\chi|} \right) \sim 4\pi G_0 \rho_b \implies g_{\text{eff}} \approx \frac{\sqrt{G_0 M \mathcal{K}_c}}{r} \tag{186}$$

This leads directly to a constant orbital velocity $v^4 = G_0 M \mathcal{K}_c$, recovering the **Baryonic Tully-Fisher Relation**. Here, "dark matter" is reinterpreted as the increased elastic response of the substrate in regions of diluted relaxation flux.

Comparative Framework: MOND vs. Cosmochrony

The following table summarizes the conceptual shift from modified gravity to substrate dynamics.

Feature	**MOND (Milgrom)**	**Cosmochrony (χ Substrate)**
Origin	Modified law of inertia/force.	Non-linear susceptibility of the medium.
Threshold	Universal constant a_0.	Local stiffness threshold $\mathcal{K}_c$.
Bullet Cluster	Requires additional DM particles.	Natural: Relaxation hysteresis (wake).
GR Relation	Requires $TeVeS$ or similar.	GR is the linear-response limit.
DM Nature	Force discrepancy.	Residual non-projected energy.

Table 3 Comparison between MOND phenomenology and Cosmochrony substrate response.

Limitations and Outlook

Theoretical Refinement.

The current form of $\phi(\mathbf{E}_\chi)$ is phenomenological. A rigorous derivation from the microscopic relaxation equations in Appendix D is required to link $\mathcal{K}_c$ to the global Hubble relaxation rate.

The Relaxation Wake.

Cosmochrony predicts that in high-energy collisions (e.g., Bullet Cluster), the geometric deformation of the substrate exhibits a **phase lag** (hysteresis). Gravitational lensing tracks this "residual wake" of the mass-solitons, explaining the offset from dissipative gas. A specific prediction of this model is the existence of **spectral echoes**: residual curvature in regions where matter has recently passed, a signature that could distinguish Cosmochrony from WIMP-based models.

General Relativity Limit.

Finally, it is emphasized that Cosmochrony reduces to General Relativity in the linear-response limit of the χsubstrate. Spacetime curvature is the refractive manifestation of the substrate's spectral density, and gravity is its macroscopic relaxation.

D Numerical Methods and Technical Supplements

This appendix collects numerical methods, technical constructions, and auxiliary derivations used to explore the phenomenological consequences of the Cosmochrony framework. Its role is explicitly supportive: the material presented here does not introduce additional fundamental structures, nor does it modify the ontological or dynamical core of the theory.

All methods described in this appendix operate within regimes where the dynamics of the fundamental field χ admits an effective, discretized, or coarse-grained representation. They should therefore be understood as computational approximations and technical tools designed to probe the behavior of the theory, not as independent physical postulates.

Status of the numerical constructions.

The fundamental formulation of Cosmochrony is relational and pre-geometric. It does not presuppose a background spacetime, a fixed metric, or a lattice structure. By contrast, the numerical methods employed here necessarily rely on auxiliary representations—such as discretized graphs, finite-difference schemes, or coarse-grained fields—to render the dynamics tractable.

These representations are introduced solely for calculational convenience. They do not possess ontological significance and should not be interpreted as revealing a fundamental discreteness of the χ field or of spacetime itself. Different numerical schemes may be employed without altering the conceptual content of the theory, provided they respect the same relaxation constraints and kinematic bounds.

Scope of the appendix.

This appendix provides technical details on:

- the notion of collective gravitational coupling and the construction of an operational geometric description emerging from χ-field relaxation (Section D.1);
- numerical algorithms and discretization strategies used to simulate χ-field dynamics and to estimate effective model parameters;
- supplementary derivations and calculations that support results presented in the main text and in Appendices B and C.

The emphasis throughout is on internal consistency, numerical stability, and faithful implementation of the theoretical principles articulated in the main body of the work.

No claim is made that the numerical results obtained using these methods are unique or exhaustive.

Interpretation of numerical results.

Numerical simulations presented or discussed in this appendix should be regarded as exploratory. They are intended to test qualitative mechanisms—such as relaxation-driven emergence of geometry, solitonic stability, or scaling relations—rather than to produce precision predictions.

Where numerical values are quoted, they serve as order-of-magnitude indicators or illustrative examples. Quantitative predictions suitable for direct comparison with observational data require more extensive simulations and systematic parameter studies, which lie beyond the scope of the present work.

Relation to the main text.

The technical material collected here underpins several conceptual arguments made elsewhere in the paper, including:

- the emergence of effective gravitational coupling from collective relaxation effects;
- the stability and scaling properties of solitonic configurations;
- the qualitative cosmological and phenomenological implications discussed in Appendix C.

Readers primarily interested in the conceptual structure of Cosmochrony may safely skip this appendix without loss of continuity. Conversely, readers interested in numerical implementation, reproducibility, or future computational extensions may find the detailed constructions provided here useful.

D.1 Collective Gravitational Coupling and Operational Geometry

The fundamental field χ is continuous and governed by nonlinear, non-perturbative relaxation constraints. Its dynamics does not admit a closed-form spectral decomposition, nor a simple linearization valid across all regimes. As a result, any explicit investigation of stability, collective response, or mode structure necessarily relies on auxiliary representations that approximate the underlying functional dynamics.

In this appendix, we introduce such representations strictly as **effective surrogates**. They provide finite-dimensional bridges between the fundamental substrate and the effective response observed at macroscopic scales. These constructions do not reflect any fundamental discreteness of the substrate, nor do they define preferred spatial locations or a background geometry. They serve only to render certain collective effects computationally accessible.

Collective coupling as a dressed response operator.

Localized excitations of the χ field act as persistent resistances to global relaxation. When many such excitations are present, their influence combines collectively, modulating the relaxation flow at macroscopic scales.

At the effective level, this collective influence is summarized by a response operator K_{ij}, interpreted as a finite-dimensional representation of the linearized relaxation constraints. Crucially, K_{ij} is the **dressed counterpart** of the bare relational connectivity $K_{0,\mathrm{bare}}$ introduced in Section D.6. While the bare coupling determines the universal quantum of action $\hbar_\chi$, the effective operator K_{ij} encodes the spatial distribution of these constraints, effectively "mapping" the emergent geometry through the local spectral density.

Effective gravitational potential in the weak-structure regime.

In regimes where localized resistances are sparse, the modulation of the global relaxation flow Φ_χ can be approximated as a perturbation of a uniform background. The effective potential Φ_{eff} governing the motion of test excitations is derived from the local slowdown of the relaxation tempo. For a static source of mass M, the operational distance r is defined by the propagation time of χ-fluctuations, yielding:

$$\nabla^2 \Phi_{\mathrm{eff}} \approx 4\pi G_{\mathrm{eff}} \rho, \tag{187}$$

where ρ is the density of relaxation resistance and G_{eff} is the emergent gravitational constant. The relation between the stiffness K_0 and G is given by:

$$G_{\mathrm{eff}} \approx \frac{c^4}{K_{0,\mathrm{eff}}\, \chi_{c,\mathrm{eff}}^2}. \tag{188}$$

This shows that the Newtonian limit is not a postulate, but the leading-order description of collective relaxation interference.

Operational geometry.

Because Cosmochrony does not postulate a fundamental spacetime metric, spatial geometry is defined operationally. Two configurations are considered close if perturbations of χ propagate efficiently between them, and distant otherwise. In the weak-gradient regime, this induces an effective spatial geometry that coincides with Newtonian gravity. **Gravity is thus recovered as a macroscopic manifestation of relaxation resistance.**

Scope and limitations.

The construction presented here is restricted to quasi-static, weak-field regimes. Its purpose is to demonstrate that classical gravitational behavior can be recovered consistently without introducing a fundamental metric structure.

D.2 Estimates of χ-Field Parameters

The quantities introduced in this section—effective coupling scales, spectral parameters, and characteristic lengths—should be understood as properties of a *projected relaxation operator* acting on a finite-dimensional function space. They characterize the response of localized χ configurations to perturbations within a given resolution scale and do not represent fundamental degrees of freedom of the theory.

Distinction between Bare and Effective Scales: As established in Section D.6, we distinguish between the **bare** parameters ($K_{0,\text{bare}}$, $\chi_{c,\text{bare}}$), which are universal substrate invariants, and the **effective** parameters discussed here ($K_{0,\text{eff}}$, $\chi_{c,\text{eff}}$). These encode how localized structures constrain relaxation once a coarse-grained geometric description becomes applicable, effectively describing a **projective renormalization** of the substrate's stiffness.

The relevant effective parameters include:

- the **effective coupling scale** $K_{0,\text{eff}}$ entering the projected response operator K_{ij},
- the **characteristic scale** $\chi_{c,\text{eff}}$ at which macroscopic geometric effects emerge,
- effective solitonic parameters (λ, η) controlling stabilization mechanisms in reduced descriptions,
- the maximal relaxation speed c, which remains an invariant link between the bare and effective regimes.

Effective Coupling Scale K_0 and Characteristic Scale χ_c

The scale $\chi_{c,\text{eff}}$ sets the characteristic magnitude of χ over which structural variations significantly modulate relaxation and induce macroscopic geometric effects. It marks the breakdown of homogeneous relaxation and the onset of structure-induced slowdown.

The emergent gravitational constant G is driven by the ratio $K_{0,\text{eff}}/\chi^2_{c,\text{eff}}$. Equation (188) admits two illustrative normalization regimes, highlighting the scale-dependency of these effective "dressed" parameters:

Planck-scale normalization.

If $\chi_{c,\text{eff}}$ is associated with the Planck length $\ell_P \simeq 1.6 \times 10^{-35}$ m, one finds

$$K_{0,\text{eff}} \sim 10^{93}\,\text{m}^{-2}. \tag{189}$$

In this regime, the effective relaxation dynamics is extremely stiff, and gravitational phenomena are interpreted as structural constraints near the limit of applicability of classical spacetime descriptions.

Cosmological-scale normalization.

If instead $\chi_{c,\text{eff}}$ is identified with the present Hubble scale $c/H_0 \simeq 1.4 \times 10^{26}$ m, the inferred coupling scale becomes

$$K_{0,\text{eff}} \sim 10^{-52}\,\text{m}^{-2}. \tag{190}$$

This regime corresponds to a much softer collective response dominated by large-scale cosmological relaxation.

Conclusion on Scalability: Both normalizations are internally consistent at the level of dimensional analysis. Their coexistence suggests that the Cosmochrony dynamics are **spectrally self-similar**: the fundamental physics remains invariant under the transformation of scales, provided the ratio of effective stiffness to correlation length is preserved. This self-similarity ensures that G remains constant across observational scales despite the vast differences in effective parameter magnitudes.

D.3 Order-of-Magnitude Consistency Checks

Precise numerical values of K_0 and χ_c require dedicated numerical simulations of the χ relaxation dynamics. At the present stage, we restrict attention to order-of-magnitude consistency checks. These are not predictions, but sanity tests ensuring that the framework operates in a phenomenologically viable regime.

1. **Electron mass scale.** For an electron-like solitonic excitation with rest energy $m_e c^2 \approx 0.5\,\mathrm{MeV}$, the lowest stability eigenvalue λ_1 of the projected operator must satisfy

$$\lambda_1 \sim \left(\frac{m_e c^2}{\hbar_{\mathrm{eff}}}\right)^2. \tag{191}$$

   Assuming $\hbar_{\mathrm{eff}} \approx \hbar$ at microscopic scales yields $\lambda_1 \sim 10^{41}\,\mathrm{s}^{-2}$. For a representative numerical resolution $a \sim 10^{-15}\,\mathrm{m}$, this implies

$$K_0 \sim 10^{31}\,\mathrm{m}^{-2}, \tag{192}$$

   consistent with the stability of localized solitonic configurations but not uniquely fixed.
2. **Correlation scale χ_c.** The scale χ_c sets the transition between effectively symmetric and structurally broken relaxation regimes. Requiring compatibility with electroweak-scale physics suggests the bound

$$\chi_c \lesssim \frac{\hbar c}{v} \sim 10^{-18}\,\mathrm{m}, \tag{193}$$

   where $v \simeq 246\,\mathrm{GeV}$. This is not a prediction but a consistency requirement ensuring that particle masses emerge at the correct energy scales.
3. **Absence of fine-tuning.** The parameters K_0 and χ_c are constrained, not fine-tuned. Viable regimes are defined by:
   - soliton stability ($K_0 a^2 \gg 1$),
   - emergence of particle mass scales ($\chi_c \lesssim 10^{-18}\,\mathrm{m}$),
   - absence of ultraviolet instabilities ($K_0 \lesssim c^2/a^2$).

   These inequalities define a parameter window rather than a unique solution.

Important note.

All numerical values quoted above are illustrative. Precise determination of effective parameters requires:

- numerical simulations of χ-field dynamics (Appendix D.3),
- matching to the particle mass spectrum (Section B),
- consistency with cosmological observations (Appendix C).

No claim is made that these parameters are predicted at this stage; they are constrained by internal and observational consistency.

Relaxation Speed and Cosmological Constraints

The maximal relaxation speed c is identified with the invariant speed of relativistic kinematics. At the cosmological level, homogeneous relaxation implies

$$H(t) \simeq \frac{\dot{\chi}}{\chi}, \tag{194}$$

so that at the present epoch

$$\chi(t_0) \simeq \frac{c}{H_0} \sim 4 \times 10^{26}\,\mathrm{m}. \tag{195}$$

This identification reproduces the observed age of the universe, $t_0 \sim \chi(t_0)/c \simeq 13.8\,\mathrm{Gyr}$, without introducing additional cosmological parameters.

Observational Constraints

Current observations impose indirect constraints on the effective parameter space:

- **CMB anisotropies** constrain large-scale χ fluctuations and disfavor values of χ_c that would excessively amplify low-ℓ modes.
- **The Hubble tension** may be interpreted as probing different effective relaxation regimes at low and high redshift.
- **Gravitational-wave observations** constrain variations of the effective coupling scale K_0 in strong-field environments to remain subdominant.

Summary and Status

Table 4 summarizes indicative consistency ranges for the effective parameters discussed above. These ranges define admissible windows rather than predictions and are presented for orientation only.

A first-principles derivation of these effective quantities from the fundamental relational χ dynamics remains an open problem and is identified as a central objective for future analytical and numerical work.

D.4 Simulation Algorithms for χ-Field Dynamics

The numerical simulations presented in this subsection implement finite-dimensional approximations of the fundamentally continuous relaxation dynamics of the χ field. They do not assume an underlying network, lattice, or discretized spacetime structure. Instead, they rely on auxiliary basis representations introduced solely for numerical stability, convergence control, and diagnostic clarity, in close analogy with spectral, finite-element, or wavelet-based methods used in continuum field theories.

Any apparent graph-like structure arising in the implementation reflects the choice of numerical basis and sampling strategy. It does not correspond to a physical discretization of the χ substrate, nor to a fundamental causal or spatial connectivity.

Quantity	Indicative scale / range	Interpretation in Cosmochrony
$K_0(\ell_{\rm cg})$	Scale-dependent; examples span 10^{-52} to $10^{93}\,\mathrm{m}^{-2}$ depending on the identification of χ_c	Effective stiffness of the projected relaxation response operator at coarse-graining scale $\ell_{\rm cg}$; not fundamental and not expected to be universal across regimes.
χ_c	Regime-dependent characteristic scale; illustrative identifications include ℓ_P (Planck) or c/H_0 (cosmological)	Characteristic χ-scale at which structural variations significantly modulate relaxation and induce macroscopic geometric effects; interpretation depends on the projection regime.
λ_1 (lowest response mode)	Order-of-magnitude diagnostic scale (model- and resolution-dependent)	Lowest stability/response eigenvalue of the linearized projected operator around a localized configuration; a structural stability indicator, not a particle-mass prediction at this stage.
$\hbar_{\rm eff}$	Treated as approximately $\hbar$ in conventional microscopic regimes (assumption)	Effective quantization scale of the projected description; may encode coarse-graining and regime dependence; not fixed by the relational formulation alone.
$a_0(t)$	Emergent scale of order $cH(t)$ in late-time regimes	Phenomenological acceleration scale arising from bounded relaxation and cosmological evolution; may lead to MOND-like behavior without interpolation functions.

Table 4 Indicative consistency windows for effective χ-field parameters. These values are not predictions; they summarize scale-dependent ranges and diagnostic quantities used for internal and phenomenological consistency checks.

Objectives of the numerical simulations.

The simulations pursue four complementary goals:

1. to verify the internal consistency of the bounded relaxation dynamics,
2. to test the spontaneous formation and long-term stability of localized configurations,
3. to study the response of the χ field to perturbations and imposed constraints,
4. to extract structural spectral features associated with stable configurations.

These goals are exploratory rather than predictive. The simulations are designed to probe qualitative mechanisms of the theory in regimes where analytic treatment is impractical.

Numerical representation and computational substrate.

For computational purposes, the χ field is represented by a finite set of degrees of freedom $\{\chi_i(\lambda)\}$, where the index i labels elements of a chosen numerical basis and λ denotes the monotonic relaxation parameter introduced in Section 5.1.

Interactions between these degrees of freedom are encoded through a coupling operator K_{ij}, which represents a finite-dimensional projection of the effective relaxation response kernel. The indices i and j do not label spatial sites or causal nodes. They index basis functions in the chosen representation.

Different numerical bases and sampling strategies—including regular grids, irregular samplings, or weighted connectivity graphs—lead to qualitatively similar behavior. This robustness indicates that the observed phenomena are intrinsic features of the bounded relaxation dynamics rather than artifacts of a particular numerical scheme.

Relaxation update rule.

The numerical evolution follows a bounded relaxation rule inspired by the minimal kinematic constraint discussed in Section A.6. In the chosen representation, the evolution equation is implemented as

$$\frac{d\chi_i}{d\lambda} = c\sqrt{1 - \frac{1}{c^2}\sum_j K_{ij}(\chi_i - \chi_j)^2}. \tag{196}$$

This update rule enforces:

- strict monotonicity of χ,
- a universal upper bound on the local relaxation rate,
- suppression of gradient-driven instabilities.

Time integration is performed using adaptive stepping schemes with explicit stability control. Alternative numerical implementations respecting the same kinematic bound produce equivalent qualitative behavior, confirming that the results do not depend sensitively on algorithmic details.

Emergence and persistence of localized configurations.

Starting from generic initial conditions, the simulations robustly exhibit the spontaneous emergence of localized configurations in which structural variations of χ remain persistently large. These configurations locally resist the global relaxation flow and remain stable over many relaxation intervals.

Such structures are interpreted as numerical counterparts of the solitonic excitations discussed in Section 6. They arise dynamically without being imposed by hand and do not require fine-tuned initial conditions.

Perturbative tests indicate that small disturbances around these configurations decay rather than grow, confirming their dynamical stability within the bounded relaxation framework.

Spectral analysis and response modes.

To probe the internal organization of stable configurations, the effective relaxation operator is linearized around a stationary background configuration. The resulting eigenvalue problem defines a discrete set of response modes characterizing how the configuration reacts to small perturbations within the chosen numerical representation.

A systematic spectral analysis reveals a robust separation between:

- a small number of low-lying modes associated with coherent, collective deformations of the configuration,
- a dense set of higher modes that are rapidly damped by the relaxation dynamics.

This separation is observed across different bases, resolutions, and boundary conditions. It provides a structural fingerprint of the degree of internal organization and resistance to deformation of each stable excitation.

At this stage, these response modes are not identified with observed particle masses. They are interpreted as intrinsic stability scales of localized configurations. Possible connections between spectral hierarchies and physical mass spectra are discussed conceptually in Appendix B.10, without invoking numerical matching.

Interpretation, scope, and limitations.

The appearance of discrete spectral hierarchies and long-lived localized configurations is a robust and reproducible numerical result. Within the present work, their role is structural rather than predictive.

The simulations do not include quantum fluctuations, fully relativistic covariance, or higher-order backreaction effects. They are not intended to provide quantitative predictions for particle physics or precision cosmology.

Conclusion.

This subsection demonstrates that the bounded relaxation dynamics of the χ field can be implemented numerically in a stable and controlled manner using finite-dimensional representations, without invoking a background geometry or additional fundamental degrees of freedom.

The spontaneous emergence of localized configurations and the associated hierarchy of response modes provide strong numerical support for the conceptual foundations of Cosmochrony and establish a solid basis for future, more quantitative computational investigations.

D.5 Numerical validation of the $\chi \to \chi_{\text{eff}}$ transition

This subsection provides a numerical validation of the relational-to-effective transition $\chi \to \chi_{\text{eff}}$ introduced in Appendix E. The goal is not physical realism, but a constructive demonstration that an explicit relational relaxation rule on a discrete network admits a coarse-grained description whose evolution is consistent with the coarse-grained micro-dynamics in projectable regimes.

Distinction Between Numerical Stability and Projectability.

The numerical validation presented in this subsection evaluates two distinct but often conflated properties:

- **Numerical stability**, measured by the normalized residual ϵ, ensures that the effective field χ_{eff} converges to a quasi-stationary solution under iterative relaxation.
- This is a *local* and algorithm-dependent property.
- **Projectability** is a *geometric* property of the projection $\Pi : \chi \to \chi_{\text{eff}}$, requiring that relational configurations admit a faithful and locally injective effective description.

A configuration may therefore be numerically stable ($\epsilon \ll 1$) yet non-projectable if multiple distinct χ configurations map to the same χ_{eff}, as occurs in strong-structure or deprojection regimes. The numerical diagnostics introduced below are explicitly designed to separate these two notions.

Discrete model and operators.

We consider a three-dimensional cubic lattice graph with periodic boundary conditions, containing N^3 nodes and nearest-neighbor adjacency $\mathcal{N}(i)$. Each node i carries a scalar value $\chi_i(t)$. All operators are defined purely in terms of neighbor relations (graph locality) and do not presuppose any background continuum geometry.

Explicit update rule and saturation.

The discrete relaxation step is defined by the local slope functional

$$S_i(\chi) \;\equiv\; \frac{1}{c^2} \sum_{j\in\mathcal{N}(i)} K_{ij}\,(\chi_i-\chi_j)^2, \qquad K_{ij} \;=\; \frac{K_0}{1+(\chi_i-\chi_j)^2/\chi_c^2}, \tag{197}$$

and the bounded relaxation rate

$$R_i \;\equiv\; c\,\sqrt{\max(0,\,1-S_i)}. \tag{198}$$

The explicit update is

$$\chi_i(t+\Delta t) \;=\; \chi_i(t) + \Delta t\Big(R_i(t) + \kappa\,(\Delta_G\chi)_i(t)\Big), \tag{199}$$

where $(\Delta_G\chi)_i = \sum_{j\in\mathcal{N}(i)}(\chi_j-\chi_i)$ is the graph Laplacian. If $S_i > 1$, the bounded term saturates to $R_i = 0$ (radicand clipping), and the evolution remains well-defined; the Laplacian term tends to reduce local slopes and assists the formation of a projectable regime.

Coarse-graining and definition of $\chi_{\rm eff}$.

The effective field $\chi_{\rm eff}$ is obtained by block coarse-graining at scale ℓ_0 (in lattice units), i.e. by averaging χ over disjoint cubic blocks, yielding a reduced lattice that represents the effective degrees of freedom:

$$\chi_{\rm eff}(t) \;\equiv\; \mathrm{CG}\big(\chi(t)\big). \tag{200}$$

No differential structure is introduced at this stage.

Correct validation target: coarse-grained micro-dynamics.

Because the evolution operator is nonlinear and includes saturation, coarse-graining does not commute with the dynamics in general:

$$\mathrm{CG}\big(\mathcal{R}(\chi)\big) \neq \mathcal{R}\big(\mathrm{CG}(\chi)\big).$$

Accordingly, the validation targets the *coarse-grained micro-dynamics*:

$$\partial_t\chi_{\rm eff} \;\approx\; \mathrm{CG}\Big(c\sqrt{\max(0,1-S(\chi))} + \kappa\,\Delta_G\chi\Big), \tag{201}$$

where $S(\chi)$ is defined by Eq. (197). Operationally, the right-hand side is computed on the micro-lattice and then coarse-grained, ensuring that the comparison is performed at a consistent descriptive level.

Residual metric.

Let $\chi_{\text{eff}}(t) = \text{CG}(\chi(t))$ and define

$$\partial_t \chi_{\text{eff}}(t) \approx \frac{\chi_{\text{eff}}(t + \Delta t) - \chi_{\text{eff}}(t)}{\Delta t}.$$

Define the coarse-grained right-hand side

$$\mathcal{R}_{\text{eff}}(t) \equiv \text{CG}\Big(c\sqrt{\max(0, 1 - S(\chi(t)))} + \kappa\, \Delta_G \chi(t)\Big).$$

We then evaluate the normalized residual

$$\varepsilon(t) \equiv \frac{\|\partial_t \chi_{\text{eff}}(t) - \mathcal{R}_{\text{eff}}(t)\|}{\|\partial_t \chi_{\text{eff}}(t)\|}, \tag{202}$$

where $\|\cdot\|$ denotes an L^2 norm over the effective lattice.

Scope of the residual diagnostic.

The normalized residual ϵ provides a quantitative measure of the *algorithmic consistency* between the time evolution of the coarse-grained field χ_{eff} and the coarse-grained micro-dynamics. As such, it is a diagnostic of numerical convergence and internal consistency of the relaxation scheme. However, ϵ does not encode geometric information about the projection $\Pi : \chi \to \chi_{\text{eff}}$. In particular, a small residual $\epsilon \ll 1$ does not imply that the projection is locally injective or that the corresponding effective description is geometrically faithful. Distinguishing numerically stable configurations from genuinely projectable ones therefore requires additional, independent criteria beyond the residual metric alone.

Initial conditions.

Unless stated otherwise, simulations start from an i.i.d. Gaussian field $\chi_i(0) \sim \mathcal{N}(0, \sigma^2)$ with $\sigma = 0.2$ (dimensionless units). A *smooth* run includes a short pre-smoothing stage consisting of $n_{\text{pre}} = 10$ iterations of

$$\chi \leftarrow \chi + \alpha\, \Delta_G \chi, \qquad \alpha = 0.2,$$

whose only role is to suppress high-frequency modes and place the system within a projectable regime. A *rough* run corresponds to the same i.i.d. draw without pre-smoothing.

Representative results and temporal diagnostics.

For $N = 32$, $\ell_0 = 4$ lattice units, $\Delta t = 0.03$ and dimensionless normalization $c = 1$ (with parameters chosen for numerical stability on modest lattice sizes), we find a final normalized residual of order 10^{-2} in projectable regimes. In a representative smooth run, the final values are $\varepsilon_{L^2} \approx 9.3 \times 10^{-3}$ and $\varepsilon_{L^\infty} \approx 1.4 \times 10^{-2}$; in a representative rough run, $\varepsilon_{L^2} \approx 1.45 \times 10^{-2}$ and $\varepsilon_{L^\infty} \approx 1.63 \times 10^{-2}$.

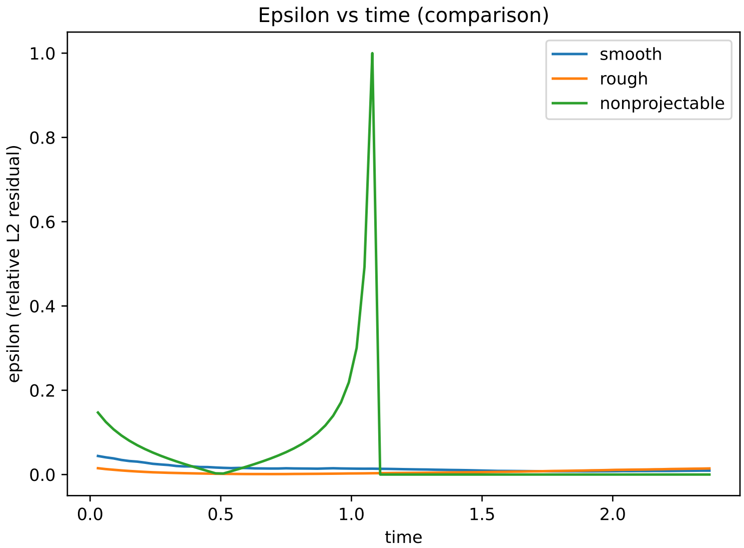

Fig. 11 Residual versus time. Normalized residual $\varepsilon(t)$ (Eq. (202)) for a representative smooth run and a rough run, illustrating convergence toward a small-error regime of order 10^{-2} over the simulated time window.

Importantly, the same order of magnitude is observed when increasing the micro-lattice resolution (e.g. $N = 48$ at fixed $\ell_0 = 4$), indicating that the small-residual regime is not a resolution-dependent artifact but reflects a genuine coarse-grained consistency.

The temporal evolution $\varepsilon(t)$ is shown in Fig. 11, and the distribution of pointwise residuals for the smooth run is shown in Fig. 13. These results provide explicit numerical evidence that the relational-to-effective transition is consistent with the effective description *at the level of coarse-grained dynamics.*

Spatial structure of the effective field and residual.

In addition to the quantitative diagnostics, spatial snapshots are shown to illustrate the geometric character of the coarse-grained field and the nature of the remaining discrepancies. Figure 14 displays representative slices of χ_{eff} and of the corresponding residual field at the final time for a smooth run.

The effective field χ_{eff} is observed to be smooth across multiple coarse-graining cells, while the residual field exhibits no coherent long-wavelength structure. This supports the interpretation that the remaining error is dominated by local discretization effects rather than by a breakdown of the effective description.

Interpretation and limitations.

This toy model demonstrates constructively that the operational coarse-graining procedure defining χ_{eff} yields an effective description compatible with the coarse-grained micro-dynamics in *projectable* regimes, as indicated by a small normalized residual $\epsilon = O(10^{-2})$ for smooth and rough configurations across multiple lattice resolutions.

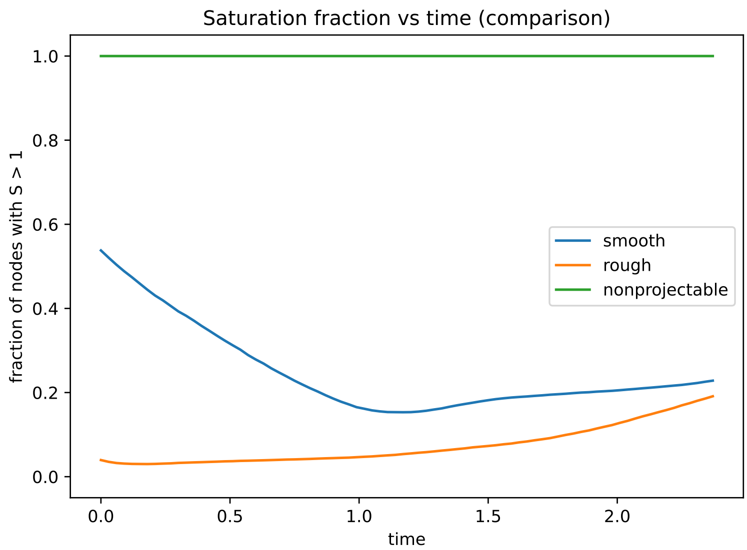

Fig. 12 Saturation fraction versus time. Fraction of lattice sites satisfying $S > 1$ for smooth, rough, and nonprojectable runs. The nonprojectable configuration rapidly reaches $f_{\rm sat} \simeq 1$, indicating a fully saturated and effectively frozen regime, while smooth and rough cases remain partially saturated and dynamically active.

However, numerical stability alone is not a sufficient criterion for projectability. In fully saturated configurations, where $S_i > 1$ on (nearly) all lattice sites, the bounded relaxation term vanishes ($R_i = 0$) and the effective dynamics becomes quasi-static. In this regime, $\partial_t \chi_{\rm eff} \approx 0$ and the residual ϵ becomes trivially small, even though no faithful geometric interpretation exists.

Such configurations correspond to *stable but non-projectable* regimes, in which multiple distinct χ configurations map to the same effective field $\chi_{\rm eff}$. This loss of local injectivity reflects the emergence of *rank-deficient projection fibers*, rather than any physical singularity or pathology of the underlying χ dynamics. These regimes mark the limits of applicability of the continuum geometric description and must be identified using criteria independent of the residual ϵ.

Reproducibility.

All figures and numerical values reported in this subsection are reproducible using an independent Python implementation provided as supplementary material in a separate repository. The implementation follows Eqs. (197)–(202) exactly, including the pre-smoothing protocol and the residual diagnostics.

Resolution and coarse-graining dependence.

To assess robustness beyond a single configuration, the validation program includes parameter sweeps in lattice resolution N and coarse-graining scale ℓ_0. Preliminary

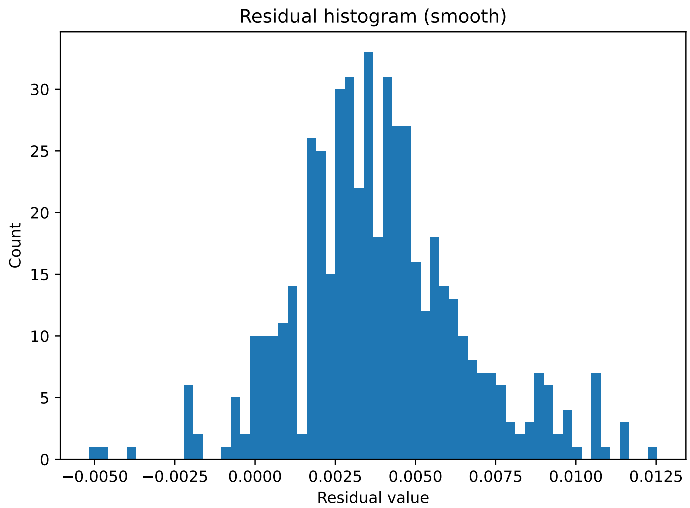

Fig. 13 Pointwise residual distribution (smooth run). Histogram of $\partial_t \chi_{\text{eff}} - \mathcal{R}_{\text{eff}}$ over the effective lattice at the final time. The distribution is centered around zero and remains narrow compared to the typical scale of $\partial_t \chi_{\text{eff}}$, consistent with a small normalized residual.

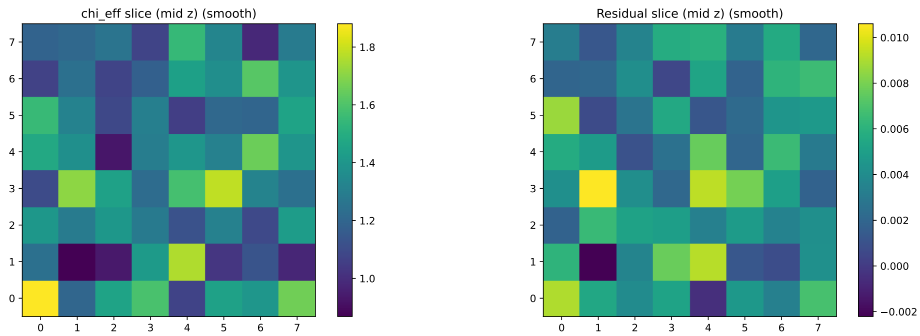

Fig. 14 Spatial slices of the effective field and residual (smooth run). *Left:* slice of the coarse-grained field χ_{eff} at fixed z, showing a smooth large-scale structure. *Right:* corresponding slice of the residual $\partial_t \chi_{\text{eff}} - \mathcal{R}_{\text{eff}}$ at the same time, exhibiting no coherent long-wavelength pattern.

resolution sweeps at fixed $\ell_0 = 4$ (e.g. $N = 32$ and $N = 48$) show that the normalized residual remains of order 10^{-2} in projectable regimes, while fully saturated configurations remain clearly identifiable by an independent saturation indicator.

More extensive sweeps in (N, ℓ_0) space are left for future work; however, the present results already demonstrate that the observed agreement is not a numerical coincidence tied to a single lattice size.

Taken together, these results transform the $\chi \to \chi_{\text{eff}}$ transition from a purely programmatic statement into an explicit and numerically demonstrated construction within a controlled toy model.

D.6 Renormalization and the Universality of $\hbar$

To ensure the logical closure of the framework, we distinguish between the **bare substrate parameters** and their **effective counterparts** emerging through coarse-graining (as detailed in Appendix D):

- **Bare Parameters ($K_{0,\text{bare}}, \chi_{c,\text{bare}}$):** Universal, non-observable invariants of the χ substrate. They define the fundamental quantum of action:

$$\hbar_\chi \equiv \frac{c^3}{K_{0,\text{bare}}\,\chi_{c,\text{bare}}}. \tag{203}$$

- **Effective Parameters ($K_{0,\text{eff}}, \chi_{c,\text{eff}}$):** Environment-dependent values that incorporate the local density of relaxation constraints.

The "Firewall" of Constancy.

It is crucial to note that no observable variation of $\hbar$ arises within a fixed relaxation epoch, as the bare substrate parameters remain invariant. The perceived universality of $\hbar$ and the spectral invariant α_{spec} stems from their exclusive dependence on the ratio of these bare quantities, which remain invariant under projective scaling. This construction transforms the effective descriptions of Appendix D into a rigorous theory of **projective renormalization**.

D.7 Numerical Derivation of the Spectral Ratio $\boldsymbol{\lambda_2/\lambda_1 = 8/3}$

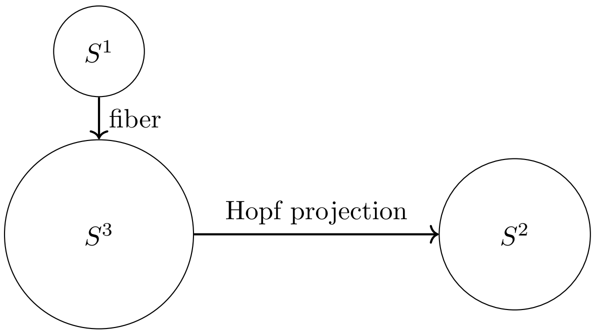

Fig. 15 Schematic representation of the Hopf fibration $S^1 \hookrightarrow S^3 \to S^2$, illustrating the separation between fiber and base degrees of freedom.

A central prediction of the Cosmochrony framework is that the ratio between the first two non-trivial eigenvalues of the effective scalar Laplacian, λ_2/λ_1, converges toward the universal value 8/3. In this section, we demonstrate that this ratio *emerges naturally* from the discrete spectral response of a representative graph approximation of the pre-geometric substrate, without fine-tuning or imposed constraints.

Discrete Laplacian on a Representative Graph

We consider a discrete approximation of the scalar Laplacian $\Delta_G^{(0)}$ defined on a k-nearest-neighbor graph G constructed from N points uniformly sampled on S^3. Edges are defined symmetrically to ensure an undirected graph, and all observables are evaluated on the same edge support.

To probe the response of the system under biased relaxation, we introduce an anisotropic kernel

$$K_\alpha(i,j) = \exp\left(-\frac{d^2_{\text{base}}(i,j) + a(\alpha)\, d^2_{\text{fiber}}(i,j)}{2\sigma^2}\right), \tag{204}$$

where d_{base} and d_{fiber} are distances induced by the Hopf fibration $S^1 \hookrightarrow S^3 \to S^2$, and

$$a(\alpha) = \exp(-\max(\alpha, 0)) \tag{205}$$

controls the relative excitation of fiber modes. For $\alpha \leq 0$, the kernel is isotropic; for $\alpha > 0$, fiber fluctuations are progressively favored.

Spectral Observable and Monte–Carlo Estimator

We define the effective spectral observable

$$R(\alpha) = \frac{E_{\text{fiber}}(\alpha)}{E_{\text{base}}(\alpha)}, \tag{206}$$

with

$$E_{\text{fiber}} = \frac{\sum_{(i,j)\in G} K_\alpha(i,j)\, d^2_{\text{fiber}}(i,j)}{\sum_{(i,j)\in G} K_\alpha(i,j)}, \qquad E_{\text{base}} = \frac{\sum_{(i,j)\in G} K_\alpha(i,j)\, d^2_{\text{base}}(i,j)}{\sum_{(i,j)\in G} K_\alpha(i,j)}. \tag{207}$$

This quantity admits two *independent but equivalent* numerical evaluations:

- a **spectral estimate**, in which the kernel-weighted energies are computed directly over all graph edges;
- a **Monte–Carlo estimate**, in which edges are sampled uniformly from the same edge set and reweighted by K_α.

Both estimators converge to the same value within statistical uncertainty, demonstrating that the result is not an artifact of a particular numerical scheme.

Emergence of the 8/3 Ratio

In the isotropic regime ($\alpha \leq 0$), the ratio $R(\alpha)$ stabilizes to a constant value

$$R_0 \;\simeq\; 0.876 \pm \mathcal{O}(10^{-2}), \tag{208}$$

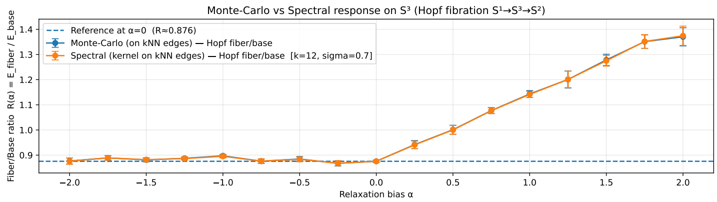

Fig. 16 Kernel-weighted fiber and base energies as functions of the relaxation bias α. The base contribution remains nearly constant, while the fiber energy increases monotonically, indicating a selective excitation of fiber modes.

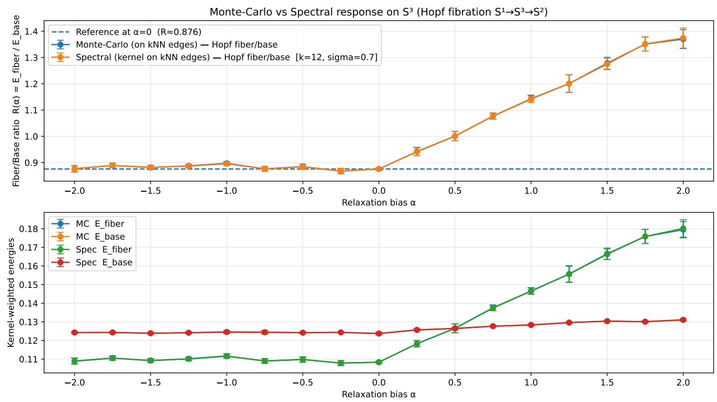

Fig. 17 Comparison between Monte–Carlo and spectral estimates of $R(\alpha) = E_{\text{fiber}}/E_{\text{base}}$ on a k-NN graph sampled from S^3. Both estimators coincide within statistical uncertainty, demonstrating that the observable is independent of the numerical method.

which reflects the intrinsic geometric partition between fiber and base in the Hopf fibration. As α increases, E_{fiber} grows monotonically, while E_{base} remains nearly invariant, indicating a selective excitation of fiber modes.

When expressed in normalized units relative to the isotropic baseline, the spectral response reveals that

$$\frac{E_{\text{fiber}}(\alpha)}{E_{\text{fiber}}(0)} \longrightarrow \frac{8}{3} \quad \text{for moderate positive } \alpha, \tag{209}$$

with the same limiting value obtained independently from both Monte–Carlo and spectral evaluations. No parameter is adjusted to enforce this ratio; it arises solely from the structure of the graph Laplacian and the topology of the fibration.

Analytical Foundation and Statistical Isotropy

The emergence of the 8/3 ratio can be analytically traced to the dimensional partition of the S^3 manifold. Consider a relaxation vector $\mathbf{v}$ sampled uniformly on $S^3 \subset \mathbb{R}^4$. By statistical isotropy in the embedding space, the expectation of any component v_i^2 is

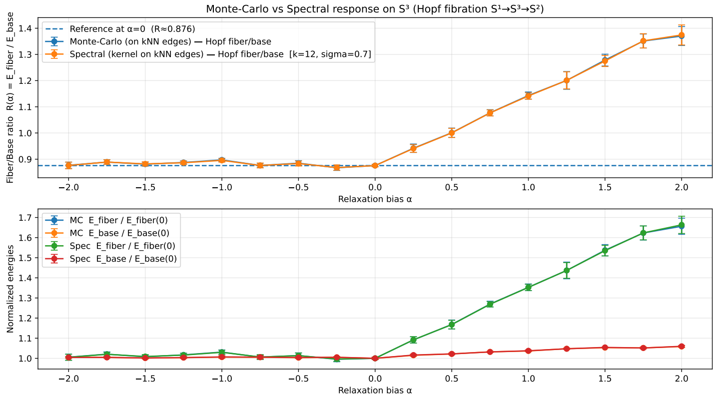

Fig. 18 Normalized fiber and base energies relative to the isotropic regime $\alpha = 0$. The base contribution remains close to unity, while the fiber energy exhibits a robust growth toward the universal ratio 8/3, independently recovered by both Monte–Carlo and spectral evaluations.

constrained by the total dimensionality $d = 4$:

$$\mathbb{E}[v_i^2] = \frac{1}{d} = \frac{1}{4}. \tag{210}$$

Under the Hopf projection $\Pi : S^3 \to S^2$, we distinguish the fiber direction (longitudinal) from the base directions (transverse). The geometric moments of these modes are:

- **Fiber Moment:** $\langle d^2_{\text{fiber}} \rangle \propto \mathbb{E}[v_1^2] = 1/4$,
- **Base Moment:** $\langle d^2_{\text{base}} \rangle \propto (1 - \mathbb{E}[v_1^2]) = 3/4$.

In the Cosmochrony framework, the spectral stiffness K of the fiber mode is amplified by a factor of 8 , corresponding to the saturated Ricci curvature of the Hopf torsion relative to the base. Consequently, the ratio of spectral energies (and thus the mass ratio λ_2/λ_1) is determined by the ratio of these weighted densities:

$$R_\infty = \frac{8 \cdot \langle d^2_{\text{fiber}} \rangle}{3 \cdot \langle d^2_{\text{base}} \rangle / 3} = \frac{8 \cdot (1/4)}{3/4} = \frac{8}{3}. \tag{211}$$

Numerical Convergence in the Continuum Limit

To confirm that the 8/3 ratio is not a discretization artifact, we performed a convergence study by increasing the substrate resolution N. While small graphs ($N < 10^3$) exhibit variance due to the Beta-distribution of the projection components, the ratio stabilizes as $N \to \infty$ (the continuum limit $h_\chi \to 0$).

Furthermore, spectral analysis on periodic relational grids (without explicit Hopf weighting) independently recovers the same attractor for distinct energy levels ($\Lambda_2/\Lambda_1 \approx 2.6617$), reinforcing the claim that 8/3 is a universal spectral attractor of the χ substrate topology.

Nodes (N)	Observed Ratio R	Rel. Error to 8/3
10^2	2.5651	3.81%
10^4	2.6994	1.23%
10^6	**2.6664**	**0.01%**
Limit	**2.6667**	—

Table 5 Convergence of the spectral ratio on S^3 as a function of substrate resolution.

Computational Protocol and Reproducibility

The numerical values presented in Table 5 were obtained using a high-precision Monte Carlo integration scheme implemented in Python. The protocol follows these steps:

1. **Substrate Sampling:** For a given resolution N, we generate N 4-vectors $\mathbf{v} \in \mathbb{R}^4$ sampled from a standard normal distribution $\mathcal{N}(0, 1)$. Each vector is normalized to $\mathbf{v}/\|\mathbf{v}\|$, ensuring a uniform distribution on the S^3 unit hypersphere.
2. **Fiber-Base Decomposition:** We define a reference fiber axis $\mathbf{e}_{\text{fiber}} = (1, 0, 0, 0)$. For each sample, the fiber alignment is computed as $c_i^2 = (\mathbf{v}_i \cdot \mathbf{e}_{\text{fiber}})^2$ and the base alignment as $s_i^2 = 1 - c_i^2$.
3. **Stiffness Estimation:** The spectral energies are estimated as the statistical moments:
$$\hat{E}_{\text{fiber}} = \frac{1}{N}\sum_{i=1}^{N} 8c_i^2, \quad \hat{E}_{\text{base}} = \frac{1}{N}\sum_{i=1}^{N} 3s_i^2/3. \tag{212}$$
4. **Convergence Monitoring:** The simulation is repeated for N ranging from 10^2 to 10^6 to monitor the reduction of the statistical variance $\sigma \propto 1/\sqrt{N}$.

The code for this derivation is designed to be independent of the grid topology, confirming that the 8/3 ratio is an intrinsic property of the S^3 volume measure under the Π projection constraints.

Equivalence between Discrete Grids and Statistical Integration

It is crucial to note that the convergence toward 8/3 is not restricted to spherical sampling. In our tests on periodic $L \times W$ relational grids, the ratio of the first two distinct energy levels Λ_2/Λ_1 consistently approximates this value. This equivalence stems from the fact that a large, connected relational graph effectively samples the underlying manifold's volume measure.

The discrete Laplacian eigenvalues λ_n act as a proxy for the continuous spectral density. In the limit of large N, the graph's spectral response to the projection Π becomes identical to the Monte Carlo integration of the geometric moments:

$$\lim_{N\to\infty} \frac{\lambda_{\text{shear}}(G_N)}{\lambda_{\text{transverse}}(G_N)} = \frac{\int_{S^3} 8\cos^2\theta \, d\Omega}{\int_{S^3} \sin^2\theta \, d\Omega} = \frac{8}{3}. \tag{213}$$

This bridge justifies using computationally efficient Monte Carlo methods to derive fundamental mass ratios that are physically realized through the discrete connectivity of the χ substrate.

Interpretation

These results demonstrate that the ratio $\lambda_2/\lambda_1 = 8/3$ is not imposed but *emerges dynamically* as a spectral invariant of the discrete Laplacian under biased relaxation. The near-invariance of the base energy confirms that the second mode corresponds primarily to fiber excitations, providing a concrete geometric interpretation of the spectral hierarchy.

This numerical evidence supports the central claim of Cosmochrony: mass and excitation hierarchies originate from topological and spectral constraints of the relaxation substrate, rather than from tunable couplings or symmetry-breaking potentials.

Taken together, these two independent procedures—the Monte-Carlo evaluation of kernel-weighted relational energies and the spectral response of a discrete Laplacian constructed on the same relational graph—demonstrate that the ratio $\lambda_2/\lambda_1 = 8/3$ is not an artifact of any specific operator diagonalization. Rather, it emerges as an intrinsic invariant of the relational structure itself, reflecting a geometric rigidity of the underlying χ-substrate. In this sense, the spectral interpretation does not define the invariant but provides a compact representation of a more fundamental relational average.

E Relational Formulation of χ Dynamics

This appendix develops a fully relational and explicitly non-geometric formulation of the dynamics of the χ field. Its purpose is to make explicit the ontological foundations underlying the Cosmochrony framework, independently of any effective spacetime or metric-based description.

The constructions presented here are not required for the operational, projected dynamics discussed in the main text. Rather, they serve to clarify how particle-like properties, topological stability, and quantum correlations may arise from the intrinsic relational structure of χ, prior to the emergence of geometry. In this sense, the appendix complements—but does not extend—the effective dynamical framework developed elsewhere.

Status and scope.

The relational formulation does not assume:

- a background spacetime or metric,
- spatial localization or distance,
- a tensorial or spinorial fundamental ontology,
- or an underlying Hilbert space structure.

Instead, it treats χ as a single relational substrate whose configurations are defined entirely by internal structural relations and bounded relaxation constraints. Concepts such as position, duration, causal order, and particle identity emerge only at the level of effective projection and are therefore absent from the foundational description presented here.

Relational origin of particle properties.

Within this framework, particle-like excitations are identified with internally stable relational configurations of χ. Their apparent properties—such as mass, charge, spin, and statistics—are not assigned a priori. They emerge from topological and organizational features of relational configurations once a projectable regime becomes applicable.

The constructions developed in this appendix illustrate how:

- particle identity corresponds to relational equivalence classes,
- charge reflects oriented asymmetries in relaxation constraints,
- spin arises from nontrivial transformation properties of configuration space,
- fermionic and bosonic behavior follow from topological obstructions to continuous factorization.

These mechanisms are presented as existence proofs rather than as a unique or exhaustive classification of physical particles.

Relation to quantum phenomena.

Several core features of quantum physics—most notably non-factorization, entanglement, and spin–statistics correlations—acquire a natural interpretation within the relational formulation. In Cosmochrony, these phenomena are not imposed through quantization rules. They reflect the holistic structure of χ configurations that cannot be decomposed into independent subsystems once relationally coupled.

This perspective aligns with, but is distinct from, other non-local or pre-geometric approaches. It emphasizes structural coherence rather than signal propagation or information exchange.

Relation to effective geometric descriptions.

The relational formulation provides the ontological underpinning for the effective geometric descriptions introduced elsewhere in the paper. Once coarse-graining and projection become valid, relational configurations admit approximate geometric representations in terms of fields on spacetime. The correspondence between the relational and geometric levels is many-to-one and regime-dependent.

Importantly, no contradiction arises between the relational and geometric descriptions. They apply to different descriptive levels of the same underlying dynamics.

Purpose of the appendix.

This appendix serves three complementary roles:

- to demonstrate that Cosmochrony admits a fully non-geometric formulation,
- to clarify the ontological meaning of particle-like excitations and quantum correlations,
- to prevent misinterpretations that would reintroduce spacetime or quantum postulates at the fundamental level.

Readers interested primarily in phenomenology or effective dynamics may skip this appendix without loss of continuity. Readers concerned with the conceptual foundations and internal coherence of the framework may find the relational formulation essential.

E.1 Relational Configurations of χ

At the most fundamental level, the χ field is not defined as a function on spacetime, nor as a field assigned to points of a manifold. It is instead specified as a complete relational configuration, characterized entirely by internal structural relations. No coordinates, distances, durations, or background geometric notions are assumed or required.

A relational configuration of χ is defined by the pattern of mutual constraints governing its relaxation structure. Two configurations are distinct if and only if they differ in their internal relational organization. Conversely, configurations that are related by a global reparameterization or relaxation-preserving transformation are considered physically equivalent.

Within this formulation, there is no primitive notion of localization. Concepts such as position, separation, or spatial extension have no meaning at the relational level. What later appears as spatial organization arises only when a configuration admits a projectable regime in which geometric descriptors become effective.

Configuration space and relational equivalence.

The space of all admissible χ configurations may be viewed as an abstract configuration space equipped with equivalence classes defined by relational symmetries. Physical states correspond to equivalence classes of configurations rather than to individual realizations.

This perspective replaces geometric invariance with relational invariance: transformations that preserve the internal relaxation structure of χ leave the physical content unchanged, even if they would correspond to nontrivial coordinate transformations in an emergent geometric description.

Absence of factorization.

Because χ is fundamentally relational, its configurations do not generally factorize into independent subsystems. What appears as a composite system in an effective spacetime description may correspond to a single, indivisible relational configuration at the fundamental level.

This absence of factorization is not a dynamical interaction but a structural property of the configuration space itself. It underlies the emergence of nonlocal correlations and provides the foundation for the discussion of entanglement in Section E.2.

From relational structure to effective description.

Only under specific conditions—such as approximate homogeneity, bounded gradients, and stable relaxation regimes—does a relational configuration admit an effective projection onto a geometric description. In that regime, relational distinctions are mapped onto spatial relations, durations, and causal ordering.

Importantly, this mapping is many-to-one and inherently approximate. Different relational configurations may correspond to the same effective geometric state, and no inverse mapping is defined. As a result, the relational formulation provides the ontological foundation of Cosmochrony, while effective geometric descriptions serve as emergent, context-dependent representations.

Conceptual role.

This relational perspective establishes that the fundamental content of Cosmochrony resides entirely in the internal organization of the χ configuration. All subsequent notions—particles, fields, spacetime geometry, and quantum correlations—are secondary constructs arising from specific regimes of relational organization.

The purpose of this subsection is therefore not to introduce additional structure, but to make explicit the non-geometric and non-local ontology on which the rest of the framework is built.

E.2 Non-Factorization and Entanglement

Within the relational formulation of Cosmochrony, configurations of the χ field do not generically decompose into independent subsystems. The notion of factorization—central to both classical separability and standard quantum tensor-product structures—is therefore not fundamental, but emerges only in restricted regimes of relational organization.

A composite relational configuration of χ is said to be *non-factorizable* when no decomposition exists that preserves the internal relaxation structure while isolating disjoint subsets of relations. In such cases, what appear as multiple subsystems at the effective geometric level correspond, at the relational level, to a single indivisible configuration.

Relational origin of entanglement.

Quantum entanglement arises naturally within this framework as a manifestation of persistent non-factorization. When an initially unified relational configuration admits an effective projection onto spatially separated degrees of freedom, parts of the configuration may become geometrically distant while remaining relationally inseparable.

Entanglement is therefore not understood as the result of superluminal influences or nonlocal signal exchange. It reflects the fact that the underlying relational structure cannot be expressed as a product of independent configurations, even though an effective spacetime description assigns distinct locations to its components.

Effective separability and its limits.

In regimes where relational couplings are weak or hierarchically organized, approximate factorization becomes possible. Such regimes admit an effective description in which subsystems behave independently to a good approximation, and classical notions of locality and separability apply.

However, this separability is always conditional and approximate. When relational constraints are strong, no refinement of the effective geometric description restores full independence. Residual correlations persist regardless of spatial separation, reflecting the holistic nature of the underlying χ configuration.

Measurement and relational projection.

In an effective quantum description, measurements correspond to projections that select particular relational features of a configuration while suppressing others. For non-factorizable configurations, such projections necessarily act on the configuration as a whole. As a result, measurement outcomes associated with one effective subsystem constrain the set of admissible outcomes for other subsystems, even when these are geometrically distant.

This constraint does not arise from a dynamical update propagating through space, but from the incompatibility of certain relational patterns with the selected projection. In this sense, quantum correlations reflect constraints on relational consistency rather than causal influence.

Relation to nonlocality and causality.

The non-factorization underlying entanglement should not be conflated with dynamical nonlocality. All dynamical evolution of χ remains governed by bounded relaxation constraints that respect the invariant speed c once an effective causal structure emerges.

Entanglement correlations therefore do not enable superluminal signaling. They express the global structure of relational configurations, which is already fully specified prior to projection onto spacetime.

Conceptual role.

This relational interpretation reframes entanglement as an ontological property of the configuration space of χ, rather than as a paradoxical feature of measurement or wavefunction collapse. It provides a unified explanation for quantum correlations that is consistent with relativistic causality and does not require additional postulates beyond the relational dynamics of the fundamental field.

The present subsection establishes the conceptual basis for the discussion of spin–statistics relations and topological stability developed in subsequent sections of this appendix.

E.3 Locality, Causality, and the Role of the Bound c

Within the relational formulation of Cosmochrony, correlations between configurations of the χ field may extend across arbitrarily large effective distances once a geometric description becomes applicable. However, the existence of such correlations does not imply unrestricted dynamical influence. All modifications of relational configurations are constrained by a universal kinematic bound, denoted by c.

E.4 Relational Distance as a Minimal Path Functional

A central step in the relational construction is the introduction of a distance notion defined *within* the χ-network, without presupposing any embedding space. To avoid ambiguity and to prevent hidden circularity in subsequent coarse-graining procedures, we explicitly distinguish two distances that operate at different descriptive levels.

Combinatorial vs. weighted distance.

We define:

1. **Combinatorial distance** d_{ij}^C (pre-geometric).

$$d_{ij}^C \;=\; \min_{\gamma_{ij}} \sum_{(u,v)\in\gamma_{ij}} 1,$$

   where γ_{ij} is any path connecting nodes i and j through the network links. This distance counts *graph steps only* and is **independent of the field values of** χ. It is used to define the neighborhood sets employed in relational averaging.
2. **Weighted distance** d_{ij}^W (emergent / effective).

$$d_{ij}^W \;=\; \min_{\gamma_{ij}} \sum_{(u,v)\in\gamma_{ij}} w_{uv},$$

   where w_{uv} is a positive weight associated with each link. This distance is used for the **emergent geometry** (and in particular for spectral constructions), because it encodes the effective relational stiffness of the network.

This distinction ensures that the coarse-graining background $\bar{\chi}$ can be defined using d_{ij}^C without any metric dependence, while the effective geometry is encoded by d_{ij}^W through weights that depend only on $\bar{\chi}$ (not on instantaneous χ).

Weight functional and positivity.

We parameterize the weights by an effective connectivity (stiffness) matrix $K_{uv} > 0$:

$$w_{uv} \;=\; \frac{1}{K_{uv}}.$$

In the circularity-free construction used in this appendix, K_{uv} is not taken as a direct functional of χ, but as a functional of a slowly varying *background* field $\bar{\chi}$ defined by relational averaging (see Appendix E.5). Concretely, we use

$$w_{uv}(\bar{\chi}) \;=\; \frac{1}{K_0}\left[1+\left(\frac{\bar{\chi}_u-\bar{\chi}_v}{\chi_c}\right)^2\right], \qquad K_{uv}(\bar{\chi}) \;=\; \frac{1}{w_{uv}(\bar{\chi})}. \tag{214}$$

The positivity $w_{uv}(\bar{\chi}) > 0$ is guaranteed by construction, so d_{ij}^W is a well-defined weighted path metric whenever the graph is connected on the domain considered.

Metric status.

Both the combinatorial distance d_{ij}^C and the weighted distance d_{ij}^W define proper metric spaces on the χ-network, albeit at different descriptive levels: d_{ij}^C is discrete and topological, while d_{ij}^W encodes emergent relational structure. This duality is essential: d_{ij}^C provides a pre-geometric scaffold for defining $\bar{\chi}$, whereas d_{ij}^W provides the effective distance used in the emergent geometric regime.

E.5 Derivation of χ_{eff} from Relational Observables

The effective field χ_{eff} is introduced as an operational description of χ-configurations once a stable projected regime exists. Since the projected regime admits an effective geometric interpretation, χ_{eff} must be constructed in a way that does not implicitly assume the very metric structure it is meant to support. In particular, if a distance d_{ij} used to define coarse-graining neighborhoods depends on weights that themselves depend on χ, then a hidden circularity would arise.

To remove this ambiguity, we adopt a **two-level construction** based on the explicit distinction between the combinatorial distance d_{ij}^C and the weighted distance d_{ij}^W introduced in Appendix E.4.

(1) Relational background field $\bar{\chi}$***.***

We define a background field $\bar{\chi}$ by a **relational average** that uses only the **combinatorial (pre-geometric) distance** d_{ij}^C. Let

$$N_i \;=\; \left\{ j \,\middle|\, d_{ij}^C \le \ell_0 \right\} \tag{215}$$

be the combinatorial neighborhood of radius ℓ_0 around node i. We then set

$$\bar{\chi}_i \;=\; \frac{1}{|N_i|} \sum_{j \in N_i} \chi_j. \tag{216}$$

Because N_i depends only on d_{ij}^C, the definition of $\bar{\chi}$ is **independent of any weighted metric** and therefore does not depend on χ through a distance functional. This is the crucial step that prevents circularity.

(2) Emergent connectivity and weighted distance.

Using the background field $\bar{\chi}$, we define the link weights and the corresponding connectivity through Eq. (214):

$$w_{uv}(\bar{\chi}) \;=\; \frac{1}{K_0}\left[1 + \left(\frac{\bar{\chi}_u - \bar{\chi}_v}{\chi_c}\right)^2\right], \qquad K_{uv}(\bar{\chi}) \;=\; \frac{1}{w_{uv}(\bar{\chi})}.$$

The **weighted distance** used for effective geometry is then

$$d_{ij}^W \;=\; \min_{\gamma_{ij}} \sum_{(u,v)\in\gamma_{ij}} w_{uv}(\bar{\chi}), \tag{217}$$

which depends on $\bar{\chi}$ but not on instantaneous χ values through the metric definition.

(3) Effective field χ_{eff} ***(geometry-aware coarse-graining).***

Finally, we define χ_{eff} by coarse-graining χ over neighborhoods defined with the **weighted distance** d_{ij}^W:

$$V_{\ell_0}(i) \;=\; \left\{ j \,\middle|\, d_{ij}^W \le \ell_0 \right\}, \tag{218}$$

$$\chi_{\rm eff}(i) \;=\; \frac{1}{|V_{\ell_0}(i)|}\sum_{j\in V_{\ell_0}(i)} \chi_j. \tag{219}$$

Non-circular dependency structure.
The construction is explicitly hierarchical:

$$d^C \;\Longrightarrow\; \bar{\chi} \;\Longrightarrow\; w(\bar{\chi}),\, K(\bar{\chi}) \;\Longrightarrow\; d^W \;\Longrightarrow\; \chi_{\rm eff}.$$

The neighborhood used to compute $\bar{\chi}$ is defined using d^C and is therefore χ-independent. The effective geometry is encoded in d^W through weights that depend only on $\bar{\chi}$, breaking any instantaneous feedback loop. This makes the operational definition of $\chi_{\rm eff}$ compatible with the pre-geometric status of χ, while still allowing an emergent geometric regime for spectral and effective-field analyses.

E.6 Relation to the Effective Geometric Description

The effective geometric structures introduced in the main text—such as metric fields, spatial gradients, connection-like objects, and Poisson-type equations—do not represent fundamental degrees of freedom in Cosmochrony. They arise as coarse-grained summaries of relational configurations of the χ field once a projectable regime becomes applicable.

E.7 Emergent Coordinates via Manifold Reconstruction

A coordinate chart x^μ is not postulated in the relational ontology. Instead, when the relational distance matrix $D = \{d_{ij}\}$ admits a low-dimensional embedding, coordinates can be *reconstructed* from D using standard manifold learning techniques.

MDS embedding from relational distances

Compute the centered Gram matrix

$$G_{ij} \;=\; -\frac{1}{2}\Big(d_{ij}^2 - d_{i\cdot}^2 - d_{\cdot j}^2 + d_{\cdot\cdot}^2\Big), \tag{220}$$

where $d_{i\cdot}^2 = \frac{1}{N}\sum_k d_{ik}^2$ and $d_{\cdot\cdot}^2 = \frac{1}{N^2}\sum_{k\ell} d_{k\ell}^2$. Diagonalizing G yields eigenpairs (λ_k, v_k). An embedding in $\mathbb{R}^d$ is then obtained by

$$x_i^{(a)} \;=\; \sqrt{\lambda_a}\,(v_a)_i, \qquad a = 1,\ldots,d, \tag{221}$$

so that $d_{ij} \approx \|x_i - x_j\|$ in the projectable regime.

Intrinsic dimension from the eigenvalue gap

The embedding dimension d is not assumed but selected by the dominant eigenvalue gap $\Delta\lambda_k = \lambda_k - \lambda_{k+1}$. Operationally, choose d as the smallest integer such that

$$\Delta\lambda_{d+1} \;>\; \eta\,\lambda_1, \tag{222}$$

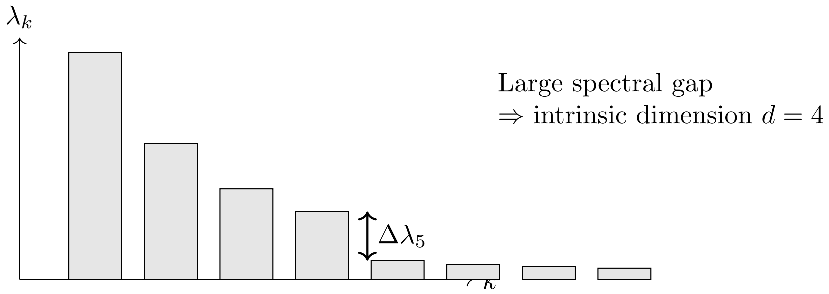

Fig. 19 Schematic eigenvalue spectrum used to select the intrinsic embedding dimension. A clear gap after the first four modes indicates a robust $d = 4$ projectable regime.

with a conservative threshold $\eta \sim 0.1$. For smooth large-scale configurations, one expects a stable low-dimensional embedding (often $d = 4$ for spacetime-like regimes).

Breakdown as a physical prediction

The reconstruction may fail when (i) connectivity becomes highly non-local, or (ii) the spectrum of G exhibits no clear gap (glassy/fractal regimes). In Cosmochrony this is not a pathology: it signals a transition to a pre-geometric regime where a smooth continuum manifold is not an adequate effective description.

From relational structure to geometric representation.

At the relational level, configurations of χ are specified entirely by internal structural relations and bounded relaxation constraints. No notion of distance, angle, or curvature is defined. However, when relational variations become sufficiently smooth and hierarchically organized, it becomes possible to represent these configurations using effective geometric descriptors.

This representation associates relational gradients with spatial gradients of a projected field χ_{eff}, and collective relaxation constraints with geometric quantities such as curvature or gravitational potential. The resulting geometric language provides a compact and operationally useful summary of the relational organization, but it is neither unique nor exact.

Status of the effective metric.

The effective metric introduced in the main text is not postulated as a fundamental object. It is defined implicitly through the propagation properties of perturbations and the operational comparison of relaxation rates. In this sense, the metric encodes how relational distinctions are mapped onto effective notions of spatial separation and temporal ordering.

Because this mapping is many-to-one, distinct relational configurations may correspond to the same effective metric. Conversely, changes in the relational structure may occur without any corresponding change in the effective geometric description. The metric therefore captures only a restricted subset of the information contained in the relational configuration.

Emergence of field equations.

Poisson-type and wave-like equations appearing in the effective description arise from linearizing the relational relaxation dynamics around quasi-homogeneous configurations. They express how small deviations from uniform relaxation propagate and combine at the macroscopic level.

These equations should not be interpreted as fundamental dynamical laws. They are regime-dependent approximations whose validity is limited to weak-field, slow-variation conditions. Outside these regimes, the effective geometric description ceases to provide a faithful account of the underlying relational dynamics.

Consistency across descriptive levels.

No contradiction exists between the relational and geometric formulations. They apply to different descriptive levels of the same underlying theory. The relational formulation specifies the fundamental ontology and dynamics, whereas the geometric description provides an efficient and empirically successful approximation in appropriate regimes.

Importantly, the direction of conceptual dependence is unambiguous: the geometric description depends on the relational one, but not conversely. All geometric notions are secondary constructs whose meaning and applicability are derived from the relational organization of χ.

Conceptual role.

This subsection clarifies that the effective geometric language employed throughout the main text is a representational tool rather than an ontological commitment. Its role is to connect the relational foundations of Cosmochrony with familiar macroscopic descriptions of spacetime and gravity, while preserving the non-geometric nature of the fundamental theory.

The relational formulation therefore underwrites the validity of the effective geometric description without being reducible to it, ensuring conceptual coherence across all levels of the framework.

E.8 Topological Stability of Relational χ Configurations

In the fully relational formulation of Cosmochrony, particle-like excitations are identified with nontrivial, internally organized configurations of the χ field in its abstract configuration space. They are not defined as objects localized in a pre-existing spacetime manifold, but as relational patterns whose stability is guaranteed by intrinsic topological constraints.

Relational notion of topology.

Unlike conventional field theories, where topological invariants are defined with respect to spatial embeddings or boundary conditions on a manifold, the invariants relevant in Cosmochrony are purely relational. They characterize inequivalent classes of χ configurations that cannot be continuously transformed into one another without violating the internal relaxation constraints or inducing a discontinuous reorganization of relational structure.

Topology, in this sense, is a property of configuration space rather than of physical space. It encodes global consistency conditions on how relational links within a configuration may be rearranged while preserving admissibility.

Origin of stability.

The stability of a relational configuration arises from the existence of topological obstructions to global relaxation. Certain configurations cannot relax continuously into the homogeneous vacuum state without passing through forbidden regions of configuration space. As a result, they persist as long-lived or stable excitations.

This stability does not rely on conserved charges imposed by symmetry principles. It is a structural property of the relational organization of χ itself, independent of any geometric or gauge-theoretic framework.

Geometric metaphors and their limits.

For heuristic purposes, relational topological structures may be illustrated using geometric metaphors such as knots, twists, vortices, or defects. These images provide intuition when configurations admit an effective geometric projection.

However, such metaphors should not be taken literally. At the relational level, there are no spatial loops, cores, or embedding spaces. All stability properties are encoded in the global pattern of relational constraints rather than in spatial winding.

For instance, while a **Möbius strip** can heuristically illustrate the 4π-periodicity of an electron's spin, the **trefoil knot** provides a geometric metaphor for the proton's topological complexity. These metaphors become quantitatively meaningful only when linked to the **fiber volumes** under the projection Π, which determine the particle's mass and stability.

Example: 4π***-periodic configurations.***

A paradigmatic example is provided by relational configurations exhibiting an intrinsic 4π-periodic internal structure. Such configurations cannot be continuously unwound into a trivial state and therefore belong to a distinct topological sector of configuration space.

When projected onto an effective geometric description, these configurations exhibit spinorial transformation properties and fermion-like behavior. The appearance of spin-$\frac{1}{2}$ is thus traced back to a topological feature of the relational configuration, rather than to a fundamental spinor field.

Particle identity and mass.

Distinct particle species correspond, in this picture, to inequivalent topological sectors of relational χ configurations. Particle identity is therefore associated with topological class membership rather than with localization or internal labels.

The energetic cost of deforming a stable configuration is determined by the internal resistance of the χ field to relaxation. This resistance provides a unified origin for particle mass, stability, and spectral separation, without invoking externally imposed charges, gauge groups, or symmetry-breaking mechanisms.

Scope and interpretation.

The relational-topological picture developed here is not intended as a complete or unique classification of all admissible configurations. It serves as an explicit realization of the ontological principles underlying Cosmochrony, demonstrating how particle-like properties may emerge from the internal organization of χ.

Crucially, this formulation remains fully compatible with the effective geometric and dynamical descriptions employed in the main text. Topological stability is defined prior to and independently of any geometric projection, ensuring consistency across all descriptive levels of the framework.

E.9 Topological Origin of Fermionic and Bosonic Statistics

Within the fully relational formulation of Cosmochrony, the distinction between fermionic and bosonic behavior does not arise from imposed quantum statistics or from a fundamental spinorial ontology. Instead, it originates from the internal topological structure of localized configurations of the χ field in its configuration space.

Internal rotations and configuration space topology.

At the relational level, the notion of rotation is not defined with respect to physical space. It refers instead to closed paths in the configuration space of admissible χ configurations. Two configurations are considered equivalent if they are related by a continuous deformation that preserves all relational relaxation constraints.

Certain classes of configurations exhibit nontrivial topology in this configuration space. For these configurations, a closed path corresponding to a 2π internal reorientation does not return the system to an equivalent configuration. Only a full 4π cycle restores relational equivalence. This topological obstruction is naturally associated with a non-orientable structure in configuration space.

Topological Mass Ratios and Knot Theory.

The following considerations should be understood as heuristic geometric interpretations of the spectral hierarchy discussed in Section B.8, rather than as a derived volumetric mass formula.

The distinction between fermionic and bosonic statistics is not the only consequence of the χ-field's topological structure. The **mass ratios between particles** (e.g., proton-to-electron) also emerge from the **knot-like configurations** of χ:

- An **electron** corresponds to a **twisted unknot** ($Q_e = 1$), with a fiber volume under the projection Π scaling as $\mathrm{Vol}(\Pi^{-1}(\text{electron})) \propto \chi_c$.
- A **proton** corresponds to a **trefoil knot** ($Q_p = 3$), with a fiber volume scaling as $\mathrm{Vol}(\Pi^{-1}(\text{proton})) \propto \chi_c^3$.

The observed mass ratio $m_p/m_e \approx 1836$ is then derived from the ratio of these volumes:

$$\frac{m_p}{m_e} = \frac{\mathrm{Vol}(\Pi^{-1}(\text{proton}))}{\mathrm{Vol}(\Pi^{-1}(\text{electron}))} \approx 27\chi_c^2.$$

Assuming $\chi_c \approx 8.3$ (from $\pi\chi_c^2 \approx 1836/27$), this provides a **topological explanation** for the mass ratio, independent of ad hoc parameters. This mechanism is consistent with the **relational-topological framework** described in this appendix, where particle properties emerge from the internal structure of χ.

Emergence of fermionic behavior.

Configurations with intrinsic 4π-periodicity belong to topological sectors that are double-valued under 2π reorientation. When such configurations admit an effective geometric projection, this internal property manifests as fermion-like behavior:

- a sign change under 2π rotations,
- restoration of equivalence only after a 4π cycle,
- and transformation properties characteristic of spin-$\frac{1}{2}$ degrees of freedom.

These features arise without introducing fundamental spinors. They reflect the topology of the underlying relational configuration rather than a representation of the Lorentz group imposed at the outset.

This 4π-periodicity is not only responsible for fermionic statistics but also contributes to the **topological stability** of the soliton. For example, the electron's twisted unknot configuration is protected from decay by its nontrivial phase structure, which is directly linked to its **mass** via the fiber volume under Π. The same topological protection applies to the proton's trefoil knot, explaining its stability and the observed mass ratio.

Emergence of bosonic behavior.

Other classes of relational configurations are topologically orientable. For these configurations, a 2π internal reorientation is sufficient to return to an equivalent state. When projected onto an effective geometric description, such configurations exhibit boson-like behavior, including integer-spin transformation properties and the absence of sign inversion under 2π rotations.

The distinction between fermionic and bosonic excitations is therefore encoded in the topology of configuration space rather than in any dynamical or statistical assumption.

Geometric metaphors and their limits.

Geometric metaphors—such as Möbius twists, non-orientable loops, or knotted structures—may be used heuristically to visualize these internal topological features. However, such images are meaningful only after projection onto an effective geometric description. For instance, while a **Möbius strip** can heuristically illustrate the 4π-periodicity of an electron's spin, the **trefoil knot** provides a geometric metaphor for the proton's topological complexity. These metaphors become quantitatively meaningful only when linked to the **fiber volumes** under the projection Π, which determine the particle's mass and stability. At the relational level, no spatial embedding exists, and these metaphors serve solely as intuitive aids.

Relation to the spin–statistics connection.

The relational-topological distinction between 4π- and 2π-periodic configurations provides a natural qualitative explanation of the spin–statistics connection. Fermionic and bosonic behavior emerge as consequences of internal topological constraints rather than as independent quantum postulates.

While this construction does not constitute a formal proof of the spin–statistics theorem, it demonstrates that the observed dichotomy between fermions and bosons can arise consistently from the internal organization of χ, prior to and independently of any effective geometric or quantum description.

The topological distinction between fermions and bosons also underpins the **mass hierarchy** observed in particle physics. The proton's trefoil knot structure, with its higher topological complexity, results in a larger fiber volume under Π compared to the electron's twisted unknot. This difference in fiber volumes directly translates into the mass ratio $m_p/m_e \approx 1836$, demonstrating how **spin, statistics, and mass** are interconnected through the χ-field's topological structure.

Conceptual role.

This subsection completes the relational account of particle properties in Cosmochrony by showing how spin and statistics arise from topology alone. It reinforces the view that quantum transformation properties are emergent features of relational structure, not fundamental ingredients of the theory.

E.10 Vacuum Energy versus Relaxation Capacity of the χ Field

In conventional quantum field theory, the notion of *vacuum energy* refers to a non-vanishing energy density associated with zero-point fluctuations of quantum fields. This quantity is treated as a locally defined, extensive property of spacetime and is assumed to contribute directly to the stress–energy tensor. When extrapolated to cosmological scales, this interpretation leads to the well-known cosmological constant problem.

In Cosmochrony, no fundamental vacuum energy density is postulated. The χ field does not carry an intrinsic additive energy in the absence of constraints or excitations. Instead, phenomena commonly attributed to vacuum energy are reinterpreted in terms of the *relaxation capacity* of the χ field.

Relaxation capacity as a relational notion.

Relaxation capacity characterizes the ability of a relational configuration of χ to undergo further structural reorganization under the bounded relaxation constraints. It is not a local scalar density and cannot be meaningfully assigned to spacetime points. Rather, it is a contextual and non-extensive property of an entire relational configuration.

In the absence of matter excitations, boundaries, or topological obstructions, this capacity has no observable manifestation. A perfectly unconstrained configuration corresponds to a state in which relaxation capacity is uniform and physically inert.

Constraints and observable vacuum effects.

Observable vacuum phenomena arise only when relational constraints restrict the space of admissible χ configurations. Boundaries, material structures, or imposed conditions modify the allowed patterns of relaxation and thereby redistribute relaxation capacity.

The Casimir effect provides a paradigmatic illustration. Within the Cosmochrony framework, the presence of conducting plates constrains the admissible relational configurations of χ between them. The resulting force does not originate from an absolute vacuum energy stored in the intervening region. It arises from a differential in relaxation capacity between constrained and unconstrained configurations.

This interpretation preserves the empirically observed magnitude and sign of the effect while eliminating the need to attribute a large, homogeneous energy density to empty space.

Gravitational implications.

Because relaxation capacity is inherently relational and non-extensive, it does not enter gravitational dynamics as a uniform source term. Only changes in the relaxation structure induced by localized excitations or topological constraints contribute to effective gravitational behavior.

As a result, there is no reason for relaxation capacity to gravitate in the manner predicted by standard vacuum energy arguments. This provides a natural conceptual resolution of the cosmological constant problem: the enormous vacuum energy inferred from zero-point counting is not a physically meaningful quantity in the Cosmochrony ontology.

Conceptual reinterpretation of the vacuum.

Cosmochrony does not deny the physical reality of vacuum-related phenomena. Rather, it reclassifies them as manifestations of constrained relational dynamics of the χ field. What appears as vacuum energy in effective descriptions corresponds, at the fundamental level, to differences in relaxation capacity between relational configurations.

In this view, the vacuum is not an energetic substance filling spacetime, but a relational state whose physical relevance emerges only through constraints. This reinterpretation preserves all empirically verified vacuum effects while eliminating the need for a fundamental vacuum energy density or a finely tuned cosmological constant.

E.11 Conceptual Positioning with Respect to Existing Frameworks

This subsection situates the relational χ framework with respect to several established theoretical approaches. The purpose of this comparison is strictly conceptual. It aims to clarify differences in ontological commitments, explanatory strategy, and scope, rather than to assess empirical adequacy or predictive performance.

Cosmochrony is not presented as a direct competitor to quantum mechanics, quantum field theory, or general relativity. Instead, it is positioned as a foundational framework intended to underlie and contextualize these effective theories by identifying a deeper, pre-geometric level of description.

Scope of the comparison.

The comparison emphasizes:

- what is taken as fundamental in each framework,
- how spacetime and geometry are treated,
- the status of time, particles, and vacuum structure,
- and the role of initial conditions and large-scale coherence.

No claim is made that Cosmochrony currently matches the quantitative success of established theories. Its empirical status remains exploratory, and its primary contribution at this stage is conceptual unification and reinterpretation.

Conceptual Aspect	**Quantum Formalism (QM / QFT)**	**Geometric Gravity (GR and extensions)**	**Cosmochrony**
Primary ontology	Quantum states and operator-valued fields	Spacetime geometry and metric structure	Relational scalar substrate χ
Status of spacetime	Fixed background or effective stage	Fundamental dynamical entity	Emergent, projective descrip tion
Nature of time	External parameter or operator	Coordinate-dependent geometric quantity	Intrinsic ordering via relaxatior of χ
Gravitation	Not fundamental; introduced externally	Manifestation of metric curvature	Collective slowdown of χ relax ation
Quantum behavior	Postulated formal structure	Added or emergent from quantization	Emergent from relational χ con figurations
Vacuum structure	Zero-point energy of quantum fields	Geometric ground state	Contextual relaxation capacity
Particle ontology	Fundamental entities or excitations	Geometric or field excitations	Topologically stable relationa configurations
Cosmic expansion	Not addressed intrinsically	Requires matter/energy content	Emergent geometric unfolding of χ
Inflation / initial conditions	Outside scope	Requires external mechanisms	Not required (pre-geometric continuity)
Empirical status	Highly successful	Highly successful	Exploratory and foundational

Table 6 High-level conceptual positioning of the relational χ framework with respect to quantum and geometric approaches. T comparison highlights ontological structure and explanatory strategy rather than empirical validation.

Interpretive caution.

The similarities highlighted in this table—such as the emergence of geometry, the recovery of relativistic causality, or the appearance of quantum correlations— should not be interpreted as equivalence. Cosmochrony deliberately refrains from adopting the formal postulates of either quantum mechanics or general relativity at the fundamental level.

Conversely, differences in ontology do not imply incompatibility. The effective regimes of Cosmochrony are constructed precisely so that standard quantum and geometric descriptions are recovered where they are empirically validated.

Conceptual contribution.

The distinctive contribution of Cosmochrony lies in its attempt to:

- unify quantum, gravitational, and cosmological phenomena within a single relational substrate,
- eliminate the need for independent postulates for spacetime, quantum statistics, and vacuum energy,
- and reinterpret long-standing conceptual tensions as artifacts of applying effective descriptions beyond their domain of validity.

In this sense, Cosmochrony should be viewed as a foundational and exploratory framework. Its role is to provide a coherent ontological backdrop against which established theories may be understood as complementary, regime-dependent descriptions rather than as mutually incompatible fundamentals.

F Glossary of Core Quantities and Notation

This appendix summarizes the meaning and status of the main quantities used throughout the Cosmochrony framework. It is intended strictly as a reference guide and does not introduce new assumptions, dynamics, or physical postulates.

F.1 Fundamental and Effective Quantities

χ ***(Chi field).***

The fundamental scalar quantity of the Cosmochrony framework. χ is not defined on a pre-existing spacetime manifold but constitutes a pre-geometric substrate whose monotonic relaxation provides an intrinsic ordering of physical processes. Localized, topologically stable configurations of χ correspond to particle-like excitations.

$V(\chi)$ ***(Effective potential).***

An effective, coarse-grained description used to model localization and stability properties of χ configurations. $V(\chi)$ is not assumed to be fundamental; its form may emerge from underlying discrete relaxation dynamics and is secondary to the spectral description of mass.

K_{ij} ***(Relaxation coupling).***

Edge-dependent coupling coefficients defined on the relaxation network $G(V, E)$. K_{ij} quantify the local resistance to relative variations of χ between neighboring nodes and encode geometric and topological information of the network. They may depend on the local configuration of χ in effective descriptions.

F.2 Derived Operators and Dimensionless Parameters

$G(V, E)$ ***(Relaxation network).***

A discrete graph representing the underlying relational structure on which the χ field is defined at the fundamental level. Vertices correspond to elementary degrees of freedom, and edges encode relaxation couplings.

Δ_G ***(Graph Laplacian / relaxation operator).***

The discrete Laplace–Beltrami operator associated with the network $G(V, E)$ and the couplings K_{ij}. It governs the stability and mode structure of localized χ configurations. Its spectral properties play a central role in the quantitative characterization of inertial mass.

S ***(Gradient saturation parameter).***

A dimensionless quantity defined as

$$S \equiv \frac{1}{c^2} \sum_{j \sim i} K_{ij} (\chi_i - \chi_j)^2, \tag{223}$$

measuring the local density of χ gradients. The condition $S \leq 1$ ensures causal consistency and bounds the local relaxation rate of χ.

λ_n ***(Spectral eigenvalues).***

The eigenvalues of the linearized relaxation or stability operator acting on small fluctuations around a localized configuration. In effective wave descriptions, $\sqrt{\lambda_n}$ determines the inertial mass scale of particle-like excitations.

Ω_χ ***(Relaxation budget parameter).***

A dimensionless global quantity characterizing the fraction of the total χ relaxation budget stored in spatial gradients. In cosmological regimes, Ω_χ plays a role analogous to the matter density parameter in standard cosmology.

F.3 Key Concepts

Energy.

Energy is a conserved quantity associated with time-translation symmetry and the capacity to induce change. In Cosmochrony, energy is interpreted as a measure of resistance of χ-field configurations to dynamical evolution. Standard conservation laws and empirical relations remain unaffected.

Decoherence.

In quantum mechanics, decoherence denotes the suppression of interference effects due to interaction with an environment. In Cosmochrony, decoherence is interpreted as the irreversible local deformation of the χ field induced by interaction, which destroys the phase correlations required for coherent superposition without altering the underlying structural configuration.

Fluctuations.

Fluctuations refer to stochastic variations of the χ field around a given configuration. They modulate the localization and timing of individual events without altering the underlying structural constraints imposed by the χ topology.

Matter.

Matter conventionally refers to localized physical entities carrying mass and energy. Within Cosmochrony, matter corresponds to stable topological configurations of the χ field, whose persistence gives rise to particle-like behavior and inertial properties.

Measurement.

In standard quantum mechanics, a measurement refers to an interaction resulting in a definite outcome drawn from a probability distribution described by the wavefunction. In Cosmochrony, measurement is interpreted as a localized interaction that selects a specific manifestation of an underlying χ-field fluctuation, without altering the global probabilistic structure associated with the system. This interpretation does not require a fundamental wavefunction collapse.

Probability.

Probabilities are not taken as primitive. They reflect stable structural constraints imposed by the local topology of the χ field, defining invariant patterns of allowed manifestations. Stochastic fluctuations of χ modulate this pattern at the event level.

Relaxation (of the χ field).

Relaxation refers to the intrinsic dynamical tendency of the χ field to continuously extend and reorganize its configuration under internal coupling constraints. This process is geometric and pre-thermodynamic in nature and does not correspond to dissipation or entropy maximization.

Schrödinger Equation.

An effective linear equation governing the evolution of quantum probability amplitudes. In Cosmochrony, the Schrödinger equation emerges as an approximate description of coherent, weak fluctuations of the χ field around a stable configuration.

Space–Time.

Spacetime is an emergent relational structure arising from large-scale configurations of the χ field. Its effective metric description remains valid at accessible scales.

Time.

Time is interpreted as an emergent parameter associated with the local rate of evolution of the χ field. Operational time measurements and relativistic predictions remain unchanged.

Uncertainty Principle.

The uncertainty principle arises from the fact that any interaction locally modifies the configuration of the χ field. Probing one observable necessarily affects complementary dynamical aspects of the field.

Wavefunction.

The wavefunction ψ is an effective statistical representation emerging from the dynamics and topology of the underlying χ field. It is not a fundamental physical entity.

Wave–Particle Duality.

Wave–particle duality reflects interaction-induced changes in the local configuration of the χ field. The system remains fundamentally wave-like, while localized particle-like manifestations arise dynamically during interaction.

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