Cosmochrony: A Pre-geometric Framework for Emergent Spacetime, Dynamics, and Matter

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Abstract

We present *Cosmochrony*, a foundational physical framework in which time, inertia, spacetime geometry, and mass emerge from the irreversible relaxation of a single pre-geometric relational substrate, denoted $\boldsymbol{\chi}$. Rather than evolving from richly structured initial conditions, the Universe is described as originating from a regime of *ontological poverty*, in which only a severely restricted class of simple and globally coherent $\boldsymbol{\chi}$-configurations is admissible. As relaxation proceeds, this admissible space irreversibly expands, enabling the emergence of increasingly complex effective descriptions. Unlike conventional field theories formulated on a predefined spacetime manifold, Cosmochrony treats relaxation as the primary physical process, with spacetime and causality arising only as effective, projected descriptions of underlying $\boldsymbol{\chi}$-configurations.

A central feature of the framework is the non-injective projection from $\boldsymbol{\chi}$ to effective observables, which provides a structural origin for quantum indeterminacy, entanglement and nonlocal correlations, discreteness, and horizon phenomena. Within this setting, particle masses arise as invariant spectral properties of metastable $\boldsymbol{\chi}$-configurations, stabilized by topological frustration and bounded relaxation. Particle decay is reinterpreted as a structural instability of projected $\boldsymbol{\chi}$-configurations, corresponding to a loss of admissibility under continued relaxation rather than to stochastic or externally triggered processes. Quantum mechanics thus emerges as an effective statistical framework governing the admissible projections of a non-injective underlying reality.

The effective dynamical laws are derived *ab initio*. A Born–Infeld-like action emerges uniquely from the causal saturation of relaxation fluxes, while General Relativity appears as a thermodynamic and spectral continuum limit of the underlying relational dynamics. At galactic scales, saturation of the relaxation constraint yields an effective gravitational potential producing asymptotically flat rotation curves without invoking dark matter halos, in quantitative agreement with observed spiral galaxies. In strong-gravity regimes, black hole evaporation is reinterpreted as a discrete reprojection process of χ, offering a structural resolution of the information paradox without invoking non-unitary evolution. Cosmochrony thus provides a minimal and unified ontological framework in which geometry, quantum phenomena, particle stability and decay, galactic dynamics, gravitation, and the observed growth of cosmological complexity arise as emergent harmonics of a single irreversible relaxation process, structured by non-injective projection.

Keywords: Emergent spacetime, quantum gravity, relational dynamics, spectral geometry

Contents

1 Introduction

Modern fundamental physics is built upon two highly successful yet conceptually distinct frameworks: quantum mechanics and general relativity [1, 2]. Quantum theory accurately describes microscopic phenomena, while general relativity provides a geometric account of gravitation and spacetime dynamics at macroscopic and cosmological scales. Despite their empirical success, these theories rely on incompatible foundational assumptions and resist unification within a single coherent conceptual framework [3–5].

Quantum mechanics presupposes a fixed spacetime arena in which physical states evolve, whereas general relativity identifies spacetime geometry itself as a dynamical entity. Numerous approaches have attempted to bridge this tension, including quantum field theory in curved spacetime, canonical and covariant quantum gravity programs, and string-based or holographic frameworks. While these approaches have led to important theoretical developments, they typically introduce extended mathematical structures or additional degrees of freedom whose physical interpretation and empirical accessibility remain unclear.

In this work, we explore a complementary and deliberately minimalist framework, referred to as *Cosmochrony* [1]. The central hypothesis is that spacetime geometry, gravitation, and quantum phenomena emerge from the dynamics of a single continuous fundamental entity, denoted χ, whose effective projections admit a scalar description. These projections are generically non-injective, allowing multiple underlying χ-configurations to correspond to identical effective observables and, conversely, **allowing a single underlying χ-configuration to admit multiple correlated effective realizations** (see Section 4.3 for a detailed treatment). The field χ is not defined on a pre-existing spacetime manifold, nor is it interpreted as a conventional physical field propagating within spacetime. Rather, spacetime notions themselves arise as effective descriptions of the relational and dynamical properties of χ configurations.

The fundamental dynamical postulate of Cosmochrony is that χ undergoes an irreversible relaxation process, locally bounded by an invariant structural propagation speed, whose effective projection defines the observed causal limit c. This monotonic evolution provides an intrinsic ordering of physical processes, identified with physical time. Spatial relations emerge relationally from differences, gradients, and correlations of χ once a stable geometric regime is reached. Within this perspective, spacetime expansion, gravitation, particle-like excitations, radiation processes, and quantum correlations are not fundamental ingredients, but emergent phenomena associated with specific configurations or interactions of the underlying field. In particular, discreteness, inertial mass, quantum indeterminacy, and nonlocal quantum correlations are shown later to arise from structural constraints on projection and relaxation, rather than from independent postulates.

Cosmochrony does not aim to replace the Standard Model or general relativity in their empirically validated domains, nor does it claim to provide a final unification of quantum theory and gravitation. Instead, it offers an exploratory and internally coherent framework designed to clarify the physical origin of time, geometry, gravitation, and quantum correlations within a single relational dynamics. Standard geometric and

[1] From $\kappa\acute{o}\sigma\mu o\varsigma$ and $\chi\rho\acute{o}\nu o\varsigma$, which denotes a framework in which cosmic structure and temporal ordering emerge from a common pre-geometric substrate.

quantum formalisms are recovered only at an effective, coarse-grained level, applicable when χ admits a stable spacetime interpretation. Quantum mechanics is thus recovered as an effective statistical framework governing admissible projected descriptions, rather than as a fundamental ontological theory.

Accordingly, quantities such as coordinates, metric structure, variational principles, and differential geometry are not treated as fundamental. They are employed later in the paper as emergent descriptive tools, rather than as primary postulates of the theory. Technical reconstructions and mathematical details are therefore confined to the appropriate effective regimes and collected in the appendices.

The unifying thread of the framework is therefore the idea that apparent multiplicity, indeterminacy, and nonlocality reflect structural features of projection, not fundamental physical randomness or superluminal dynamics.

The structure of the paper is as follows. Sections 2–4 introduce the conceptual motivations and minimal dynamical assumptions governing the χ substrate. Subsequent sections examine how particle-like excitations, gravitation, quantum correlations, and cosmological behavior emerge in appropriate regimes.

1.1 Conceptual Context and Related Approaches

The idea that spacetime geometry and gravitation may be emergent rather than fundamental has been explored in a variety of contemporary theoretical frameworks. Several approaches interpret the spacetime metric as an effective description arising from deeper geometric, informational, or dynamical structures, and recast gravitation as a collective or emergent phenomenon rather than as a fundamental interaction [6, 7].

Cosmochrony belongs to this broad conceptual lineage, while adopting a deliberately minimalist ontological stance. Rather than postulating multiple underlying structures or microscopic degrees of freedom, it assumes a single pre-geometric relational substrate, denoted χ, whose irreversible relaxation governs the emergence of physical observables.

Like Loop Quantum Gravity (LQG), Cosmochrony holds that spacetime geometry is not fundamental [8]. However, the two frameworks operate at distinct conceptual and ontological levels.

LQG provides a quantized description of geometry once a spacetime structure is already assumed, encoding areas and volumes through spin networks and holonomies. In this sense, it addresses the quantization of geometric degrees of freedom defined on a kinematical spacetime arena.

Cosmochrony, by contrast, addresses an earlier and more primitive level. It does not quantize geometry, but seeks to explain how geometric notions themselves arise as effective, coarse-grained descriptions of underlying χ-configurations. The emergence of spacetime is mediated by a non-injective projection from the pre-geometric substrate to effective observables, allowing geometric, dynamical, and quantum features to appear only once specific relational and spectral conditions are met.

From this perspective, Cosmochrony does not compete with LQG but conceptually precedes it. It aims to account for the physical origin of the geometric degrees of freedom that may subsequently be quantized within approaches such as LQG, while remaining agnostic about the detailed form of their quantization at the effective level.

For convenience, a glossary summarizing the main quantities and operators used throughout the article is provided in Appendix F.

2 Theoretical Context and Motivation

2.1 Conceptual Tension Between Quantum Theory and Gravitation

Quantum mechanics and general relativity differ not only in their mathematical formalisms, but also in their foundational concepts. Quantum theory is intrinsically probabilistic, relies on a fixed causal structure, and treats time as an external parameter [1, 9]. General relativity, by contrast, describes gravitation as the dynamics of spacetime geometry itself, with time acquiring a coordinate-dependent and observer-relative status [2, 3].

This conceptual mismatch becomes particularly acute in regimes where both quantum effects and strong gravitational fields are expected to be relevant, such as near spacetime singularities or in the early universe [10, 11]. Direct attempts to quantize gravity encounter persistent difficulties, including the problem of time, non-renormalizability, and the absence of a preferred background structure. These difficulties suggest that the tension may reflect not merely technical obstacles, but a deeper incompatibility in the assumed ontological status of time and geometry.

2.2 Limitations of Existing Unification Approaches

Several major research programs have sought to address these challenges. Quantum field theory in curved spacetime successfully accounts for particle creation and vacuum effects, but retains a classical spacetime background [12]. Canonical and covariant approaches to quantum gravity attempt to quantize spacetime geometry itself, often at the cost of substantial mathematical complexity and interpretational ambiguity.

String theory and related frameworks introduce extended fundamental objects and higher-dimensional structures, offering deep mathematical unification but leading to a large space of possible low-energy realizations [5]. While internally rich, these approaches face ongoing challenges concerning empirical testability and the physical interpretation of their fundamental degrees of freedom.

Collectively, these limitations motivate the exploration of alternative perspectives in which spacetime geometry, matter, and quantum behavior are not independently postulated, but emerge from a common underlying mechanism operating at a more primitive, pre-geometric level.

2.3 Minimalism as a Guiding Principle

The framework developed in this work adopts minimalism as a guiding principle. Rather than introducing multiple fundamental fields, additional dimensions, or independent quantization rules, we explore whether a single continuous fundamental entity can account for temporal ordering, spatial relations, and quantum features within a unified relational dynamics.

The scalar quantity χ is not interpreted as a conventional matter field, nor as a component of spacetime geometry. Instead, it represents a pre-geometric substrate whose irreversible relaxation underlies the emergence of both duration and separation. In this view, time and space are not independent primitives, but complementary aspects of a single dynamical process. Effective geometric and quantum descriptions arise only through coarse-grained, generally non-injective projections of the underlying χ-configurations.

2.4 Time, Irreversibility, and Cosmological Expansion

A central motivation for the Cosmochrony framework is the close connection between time, irreversibility, and cosmological expansion. In standard cosmology, expansion is described kinematically through the scale factor, while the arrow of time is typically attributed to boundary conditions or entropy growth [10, 11, 13].

In Cosmochrony, the monotonic relaxation of χ provides a unified origin for both phenomena. Irreversibility follows directly from the intrinsic directionality of the relaxation process, while cosmological expansion is interpreted as its large-scale geometric manifestation in the effective, projected description. From this perspective, expansion does not require an externally imposed energy component, but arises as an emergent consequence of the underlying pre-geometric dynamics.

2.5 Scope and Limitations

The aim of this work is exploratory rather than definitive. Cosmochrony does not seek to replace established theories within their empirically validated domains, but to offer a coherent reinterpretation that may clarify persistent conceptual difficulties concerning time, geometry, and quantization.

Throughout the paper, emphasis is placed on internal consistency, conceptual clarity, and qualitative contact with observable phenomena, while openly acknowledging open questions and limitations. In the following section, we introduce the χ substrate formally and specify the minimal assumptions underlying its relational and dynamical structure.

3 Definition and Fundamental Properties of the χ Substrate

Having outlined the ontological and conceptual principles underlying Cosmochrony, we now introduce the single fundamental entity at the core of the framework. This section is devoted to defining the pre-geometric relational substrate χ and to clarifying its role as the primitive ontological basis from which effective notions of spacetime, dynamics, and physical observables emerge.

The purpose of this section is not to assume a pre-existing spacetime structure, but to identify the minimal structural properties required of χ in order to recover, in appropriate regimes, effective descriptions of time, space, metric geometry, and field dynamics. Accordingly, χ is introduced independently of any spacetime coordinates, metric structure, or background manifold, and is related to geometric notions only through non-injective, coarse-grained projections that become meaningful once a stable spacetime interpretation is supported.

Throughout this section, the use of variational principles, Lagrangian formulations, or metric-based expressions does not imply that spacetime or a four-dimensional manifold is fundamental. Such formalisms are employed strictly as effective descriptive tools, valid only in projectable regimes where configurations of χ admit a quasi-stable geometric and causal interpretation. They should therefore be understood as post-hoc representations of underlying pre-geometric dynamics, not as primary postulates of the theory.

We begin by providing a unified conceptual definition of the χ substrate and its physical interpretation, emphasizing its ontological distinction from conventional fields. The subsequent subsections progressively introduce more structured effective descriptions—including scalar, Lagrangian, and metric formulations—which become applicable only once projected χ configurations support a coherent spacetime regime.

3.1 Definition of the χ Field

We postulate the existence of a single pre-geometric relational substrate, denoted χ, which constitutes the primitive ontological basis of physical reality. The quantity χ is not defined on a pre-existing spacetime manifold and does not presuppose any metric, causal, or geometric structure. Instead, spacetime notions arise only as effective descriptions of the relational, spectral, and dynamical properties of χ configurations.

Ontologically, χ is not a scalar order parameter and does not possess local values. Scalar order parameters arise only at the effective level, as coarse-grained descriptors of projected χ configurations once a geometric regime is established. Dimensional quantities associated with length, duration, or mass arise only at the effective level, when χ configurations admit a stable geometric interpretation. The monotonic ordering intrinsic to χ configurations gives rise, upon projection, to what is operationally perceived as temporal flow.

Ontological Status of χ

The χ substrate is **not**:

- A scalar field on spacetime (no background manifold).
- A dynamical degree of freedom (no time evolution at fundamental level).
- A discrete lattice or graph (fully continuous).

It is a **pre-geometric relational structure** from which spacetime, matter, and dynamics emerge via projection.

Temporal ordering emerges from the global, irreversible ordering intrinsic to χ configurations across physical processes, establishing an intrinsic arrow of time without reference to an external temporal coordinate. Spatial separation, in turn, arises from relational differences between χ configurations, giving rise to an effective notion of distance once a quasi-stable geometric regime is reached. In this sense, time corresponds to ordering, while space corresponds to relational structure.

At no stage is χ interpreted as a spacetime coordinate, as a physical field propagating on spacetime, or as a conventional dynamical variable. Spacetime coordinates and metric structure appear only as secondary, coarse-grained constructs, becoming meaningful when χ configurations admit a quasi-stable geometric interpretation through a generally non-injective projection. The spacetime metric thus functions as an emergent effective descriptor of resistance to χ relaxation[2] and of the constrained propagation of perturbations within the projected description.

The analogy with thermodynamic order parameters applies only at the effective level: χ itself is not an order parameter, but gives rise to effective order parameters once projected. Within the Cosmochrony framework, χ therefore provides the minimal ontological substrate from which time, space, inertial mass, gravitation, and quantum phenomena jointly emerge as harmonics of a single irreversible relaxation process.

In the following sections, spacetime coordinates, metric quantities, and field-theoretic objects will be introduced strictly as effective tools, valid only in regimes where χ admits a stable geometric interpretation.

On the use of spacetime language.

Throughout this work, phrases such as “spacetime coordinates,” “metric tensor,” and “four-dimensional manifold” are employed for clarity and effective description. These notions should be understood strictly as *emergent, projected constructs* valid only in regimes where χ has relaxed into a quasi-stable geometric configuration. They do not correspond to fundamental ingredients of the theory.

At the most fundamental level, only χ and its internal relational and spectral structure exist. The appearance of familiar geometric language reflects the effectiveness of spacetime as a coarse-grained description of collective χ behavior, analogous to how thermodynamic variables (such as temperature or pressure) emerge from molecular dynamics without being fundamental.

[2]The term “relaxation” is used here in a geometric and dynamical sense, and should not be confused with thermodynamic relaxation processes involving dissipation or entropy increase.

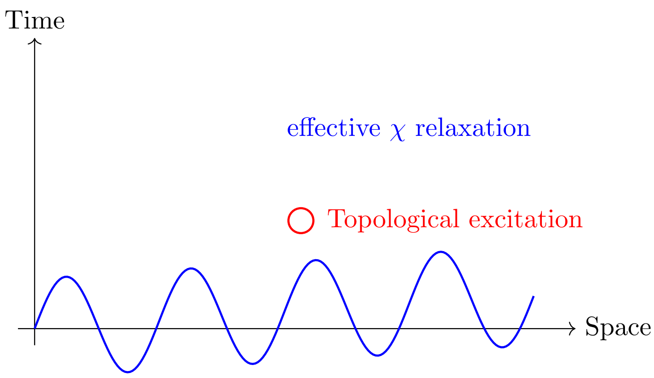

Fig. 1 Conceptual representation of Cosmochrony. An effective spacetime depiction of the projected scalar description of χ, used for visualization purposes only. The monotonic relaxation of χ gives rise to an effective temporal ordering, while localized topological excitations correspond to particle-like configurations in the emergent geometric regime.

This interpretational stance is essential for distinguishing Cosmochrony from approaches that merely reformulate existing geometric theories in alternative variables, rather than addressing the physical origin of geometry itself.

Extended interpretative clarifications, including common misreadings and the status of field-theoretic notation, are provided in Appendix B.1. The fully relational formulation of χ configurations, independent of geometric or manifold structures, is developed in Appendix E.1.

3.2 The Geometric Effective Description of χ Dynamics

Effective Observables from χ Correlations

In Cosmochrony, quantities conventionally described in geometric terms—such as time intervals, spatial separation, and causal ordering—are not taken as primitive. They arise as effective descriptive summaries of relational patterns within the χ substrate, accessed only through projected, coarse-grained representations. The projection from χ to effective observables is generically non-injective, allowing distinct underlying configurations to correspond to identical effective descriptions. No background spacetime, coordinate system, or discrete substrate is assumed at any stage of the fundamental description.

The configurations σ represent internal relational states of the χ substrate and are defined without reference to external spacetime coordinates or background geometry. They label patterns of internal organization rather than positions in a pre-existing space.

Correlations between configurations σ encode the emergent geometric and causal structure at the effective level, once a stable spacetime description becomes applicable. The measure $d\mu(\sigma)$ denotes an invariant integration over configuration space, defined intrinsically from the correlation structure associated with χ. It ensures that physical observables are independent of the particular parametrization chosen to label configurations, and carries no interpretation as a volume element in an underlying spacetime.

Effective scalar descriptor.

In regimes where projected χ configurations admit a stable geometric interpretation, it is convenient to introduce an *effective scalar descriptor*, denoted χ_{eff}. This quantity is a coarse-grained, projected representation of relational and spectral features of the χ substrate, defined only within the emergent spacetime description. It does not correspond to the fundamental χ substrate itself, which remains non-indexable and devoid of intrinsic values. Although χ_{eff} is represented as a scalar function in effective descriptions, it should not be interpreted as a fundamental field propagating in spacetime.

Operational time intervals are defined from the accumulated ordering of projected χ configurations along paths in configuration space. This ordering is quantified using the effective descriptor χ_{eff}, not the fundamental χ substrate:

$$\tau_{AB} \;\propto\; \int_{\gamma_{AB}} \mathcal{D}_\lambda \chi_{\text{eff}}\, d\lambda, \tag{1}$$

where $\mathcal{D}_\lambda \chi_{\text{eff}}$ is an effective relaxation functional characterizing the ordering of projected configurations along the path γ_{AB}. The parameter λ is an ordering parameter, not a fundamental time coordinate.

Geometric observables are constructed from correlations between effective descriptors χ_{eff} associated with projected configurations. These correlations encode relational information about causal connectivity and separation once a geometric regime is established.

Operational spatial separation is quantified by the decay of correlations between effective descriptors:

$$d(x,y) \;\propto\; -\log\!\left(\frac{\langle \chi_{\text{eff}}(x)\, \chi_{\text{eff}}(y)\rangle}{\langle \chi_{\text{eff}}^2\rangle}\right), \tag{2}$$

where x and y label effective spacetime events in the emergent description. The correlation function involves χ_{eff} only and does not imply any localization or value assignment at the level of the fundamental χ substrate. Here, $\langle \chi_{\text{eff}}(x)\, \chi_{\text{eff}}(y)\rangle$ denotes an **effective correlation functional** encoding relational proximity between projected configurations x and y, not a statistical average over pre-existing degrees of freedom.

These constructions are purely relational and make no reference to a pre-existing metric or discrete structure. They are applicable whenever projected χ configurations exhibit stable correlation patterns.

Throughout this section, all indexed, correlated, or integrated quantities refer exclusively to the effective descriptor χ_{eff}, not to the fundamental χ substrate.

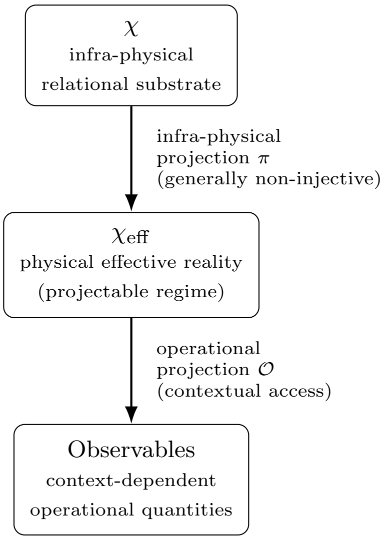

Fig. 2 Ontological pipeline in Cosmochrony. The infra-physical projection π maps the fundamental substrate χ to an effective physical reality χ_{eff} in projectable regimes (generally non-injective). Physical observables arise from a second, operational projection $\mathcal{O}$ that specifies how χ_{eff} is accessed under measurement contexts, without introducing additional ontological structure.

Effective Metric as a Descriptive Tool

In regimes where projected χ configurations exhibit smooth and stable correlation patterns, the relational observables defined above may be compactly summarized by an operational tensor $g_{\mu\nu}[\chi_{\text{eff}}]$. The notation emphasizes that the metric summarizes correlations of effective descriptors, not properties of the fundamental χ substrate. It is not introduced as a fundamental geometric structure, nor as an independent degree of freedom, but as a post-hoc parametrization of effective relational regularities.

The metric provides a convenient summary of how variations in χ_{eff} modulate causal connectivity and relational intervals, and is meaningful only insofar as such a coarse-grained description remains valid. For example:

- The conformal (lightcone) structure is constrained by the maximal effective propagation speed c associated with projected χ relaxation.
- Proper time between effective events is proportional to the accumulated χ_{eff} ordering along connecting paths.
- Spatial distance reflects the decay rate of χ_{eff} correlations.

No discrete-to-continuum limit is invoked. The theory is continuous at all scales; apparent granularity (including Planck-scale phenomena) arises from non-linear and threshold effects in χ dynamics and projection, not from an underlying discretization.

No background $\eta_{\mu\nu}$ is assumed. Minkowski space appears only as an effective approximation in suitable limits (e.g., weak-gradient regimes), without ontological status at the fundamental level.

Consistency with General Relativity

The effective metric $g_{\mu\nu}[\chi_{\rm eff}]$, constructed as a descriptive summary of projected correlations, reproduces the phenomenology of general relativity in appropriate regimes:

- **Weak-field limit:** when $\chi_{\rm eff}$ gradients are small, the effective metric approaches a form compatible with Einstein-like dynamics for a fluid-like stress-energy description associated with χ excitations.
- **Strong-field regimes:** near localized χ excitations (e.g., solitonic configurations), the metric encodes time dilation and spatial curvature as emergent consequences of inhibited $\chi_{\rm eff}$ relaxation.
- **Cosmological expansion:** homogeneous relaxation of χ yields an effective Hubble-like expansion law for the emergent scale factor.

Crucially, this is not a bootstrap procedure. The metric is never iteratively reconstructed or dynamically postulated; it remains a derived descriptor summarizing geometric regularities of projected χ configurations. All predictive content resides in the underlying χ dynamics.

Ontological Status of the Metric

To avoid confusion, we emphasize:

- χ is the only fundamental entity. Spacetime, metric structure, and matter are emergent descriptions of projected χ configurations.
- No "double ontology" is assumed: there is no underlying discrete graph or lattice. Geometric language is introduced only as an effective descriptive tool.
- The metric $g_{\mu\nu}[\chi_{\rm eff}]$ is an effective construct, analogous to how temperature emerges in thermodynamics. It is useful for coarse-grained description but plays no role in the fundamental dynamics.

Operational origin of the effective metric.

The metric tensor $g_{\mu\nu}$ is a derived descriptor summarizing relational distance induced by χ correlations. Its explicit construction from operational distances is given in Appendix E, where geometric quantities are shown to arise only in projectable regimes admitting a smooth continuum approximation.

Operational interpretation of the line element.

In Cosmochrony, the line element ds^2 is not a primitive geometric quantity. It is a shorthand for a local quadratic approximation of the operational relational distance. Concretely, if $d(i,j)$ denotes the operational distance between configurations in the relational network of χ (Appendix E), then in projectable regimes admitting a low-dimensional embedding one may write locally

$$d(i,j)^2 \;\approx\; g_{\mu\nu}(x)\,\Delta x^\mu \Delta x^\nu .$$

The continuum line element ds^2 should therefore be understood as a derived descriptive construct, not as a fundamental structure.

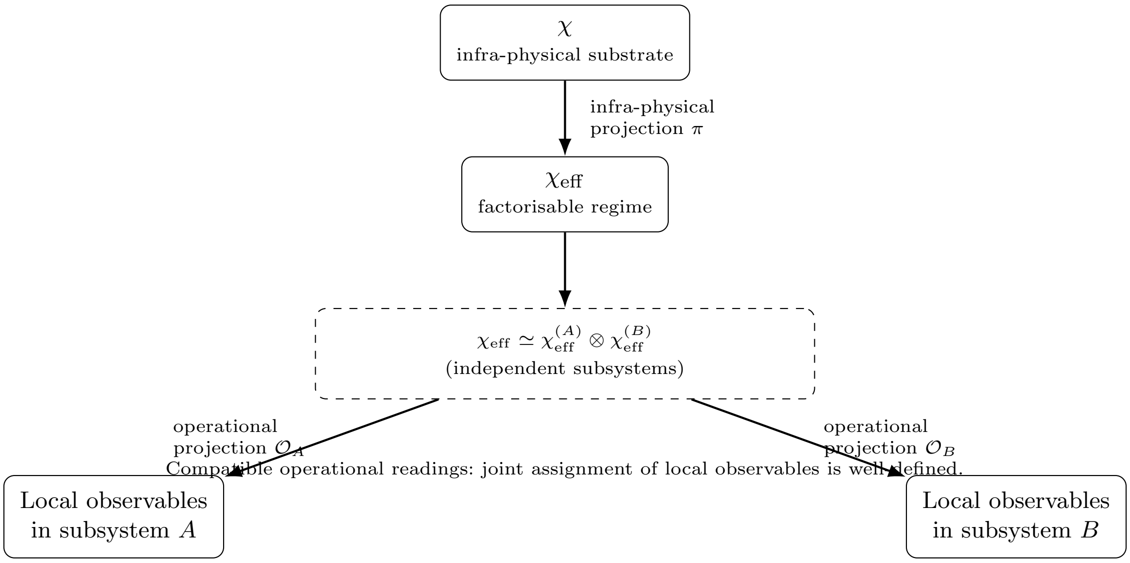

Fig. 3 Classical (factorisable) regime. After the infra-physical projection π, the effective reality χ_{eff} admits an approximate decomposition into independent subsystems. Operational projections $\mathcal{O}_A$ and $\mathcal{O}_B$ yield compatible local observables, recovering standard classical and relativistic descriptions in stable projectable domains.

Summary: A Fully Continuous Framework

- **Fundamental level:** only the continuous χ relational substrate exists, characterized by intrinsic ordering and relaxation dynamics.
- **Effective level:** geometric observables (time, distance, metric) emerge from correlations between effective descriptors χ_{eff} associated with projected χ configurations.
- **No bootstrap:** the metric is never postulated or iteratively constructed; it is a derived summary of projected relational structure.
- **No discretization:** apparent "Planck-scale" effects arise from non-linear and threshold phenomena in χ dynamics and projection, not from an underlying discrete substrate.

3.3 Physical Interpretation

In Cosmochrony, spacetime is not assumed as a pre-existing background structure. Instead, it appears as an effective macroscopic description arising from the continuous and irreversible ordering intrinsic to the relational substrate χ. What are conventionally described as temporal and spatial features are understood as distinct, but related, manifestations of this single underlying process, once a projectable geometric regime becomes applicable.

In regimes where projected χ configurations exhibit sufficiently stable and smooth correlation patterns, variations of the effective scalar descriptor χ_{eff} give rise to a set of operational observables. Because the projection from χ to χ_{eff} is generically non-injective, these observables summarize relational structure without exhausting the underlying degrees of freedom. In particular, an increase in χ_{eff} along a given physical process is associated with:

- the accumulation of operational proper time along that process,
- the progressive decorrelation between effective configurations, summarized as an emergent spatial separation,
- the large-scale expansion behavior observed when the ordering of projected χ configurations is considered at the cosmological level.

Within this effective description, temporal duration and spatial separation are not independent primitives. They represent complementary aspects of the same underlying ordering and relaxation structure, captured at different levels of coarse-graining. Heuristically, effective distance may be viewed as the persistent imprint of relational differentiation that has already occurred, while effective time corresponds to the ongoing local ordering of projected χ configurations. These expressions are intended as interpretative guides rather than literal definitions, emphasizing their common dynamical origin.

This unified interpretation is not introduced *ad hoc*. It follows directly from identifying temporal ordering, relational separation, and cosmological expansion as distinct effective summaries of the same irreversible χ dynamics, once a macroscopic spacetime description becomes appropriate. The physical content of the theory therefore resides entirely in the dynamics of the fundamental χ substrate, while spacetime notions serve only as emergent, context-dependent descriptive tools.

3.4 Relational Projection and Spectral Admissibility

At the fundamental level, Cosmochrony does not postulate a pre-existing spacetime, nor local fields evolving on a geometric background. Instead, all admissible physical descriptions emerge from the *spectral structure* of the relational substrate χ.

Relational operator.

Let $\mathcal{H}_\chi$ denote the configuration space of admissible χ-field states. We introduce a relational operator

$$L_\chi : \mathcal{H}_\chi \to \mathcal{H}_\chi, \tag{3}$$

defined purely in terms of χ-correlations. In discrete implementations (Appendix D), L_χ reduces to a graph Laplacian associated with relational adjacency; in the continuum limit, it defines an effective self-adjoint operator encoding relational connectivity.

Spectral decomposition.

The operator L_χ admits a spectral decomposition

$$L_\chi \psi_n = \lambda_n \psi_n, \tag{4}$$

where $\{\lambda_n\}$ are non-negative eigenvalues and $\{\psi_n\}$ the associated eigenmodes. No geometric interpretation is assumed at this stage.

Admissibility as spectral filtering.
Physical (projectable) configurations are selected by a spectral filter acting on L_χ. We define the admissibility operator as

$$\Pi_{\lambda_*} \equiv f\left(\frac{L_\chi}{\lambda_*}\right), \tag{5}$$

where $f(x)$ is a fixed, smooth cutoff function satisfying $f(x) \to 0$ for $x \ll 1$ and $f(x) \to 1$ for $x \gg 1$. The scale λ_* characterizes the maximal relational wavelength that can be consistently projected at the freeze-out stage.

This construction defines admissibility without reference to spacetime, coordinates, or integration measures, relying solely on spectral properties of the relational operator.

3.5 Monotonicity and Arrow of Time

A central structural assumption of Cosmochrony is that the relational substrate χ admits an intrinsic, globally ordered relaxation structure. This ordering is reflected, within effective descriptions, by the monotonic behavior of the projected scalar descriptor χ_{eff} along admissible physical processes:

$$\mathcal{D}_\lambda \chi_{\mathrm{eff}} \geq 0. \tag{6}$$

Here λ denotes an ordering parameter associated with the relaxation of projected χ configurations, not a fundamental time coordinate. The inequality expresses a structural constraint on admissible projected representations, rather than the evolution equation of a fundamental scalar quantity.

This monotonicity is not introduced as a statistical statement, nor as a boundary condition imposed on an otherwise time-symmetric dynamics. It reflects an intrinsic asymmetry in the ordering structure of χ configurations, which constrains the class of effective descriptions compatible with the framework. In particular, the non-injective nature of the projection from χ to χ_{eff} does not permit reversals of this ordering, but only a loss of information about underlying configurations.

Within this perspective, energy is not treated as a fundamental conserved substance, but as an effective measure of the remaining capacity of projected χ configurations to undergo further relaxation. As effective relaxation proceeds, this capacity is irreversibly expended in the projected description. A hypothetical decrease of χ_{eff} along an admissible ordering path would correspond to a spontaneous restoration of relaxation capacity, effectively reintroducing contraction or tension into the projected representation. No mechanism within Cosmochrony allows such a reversal, as it would contradict the underlying ordering structure of χ.

Irreversibility therefore follows directly from the structure of the relaxation ordering admitted by χ. Because admissible projected descriptions cannot exhibit decreasing χ_{eff}, the ordering of configurations induced by relaxation is intrinsically directed. What is conventionally described as the arrow of time is identified here with this directional ordering: the irreversible progression from configurations with greater effective relaxation capacity toward configurations in which that capacity has been locally or globally exhausted.

Importantly, this arrow is not derived from coarse-graining, probabilistic entropy, or special initial conditions. It arises prior to any statistical or thermodynamic description, as a direct consequence of the structural ordering constraints imposed by χ on its projected representations. Temporal orientation is thus a manifestation of the fundamental ordering structure of the theory, rather than an emergent asymmetry introduced only at the macroscopic level.

This intrinsic ordering underlies the thermodynamic arrow of time discussed in Section 12.8.

3.6 Local Relaxation Speed

A fundamental structural constraint of the Cosmochrony framework is that the effective local ordering rate associated with projected χ configurations is bounded. In effective geometric descriptions, this constraint takes the form

$$|\mathcal{D}_{\mathrm{loc}}\chi_{\mathrm{eff}}| \leq c, \tag{7}$$

where $\mathcal{D}_{\mathrm{loc}}\chi_{\mathrm{eff}}$ denotes an effective local relaxation functional characterizing the maximal admissible ordering of projected χ configurations. The constant c is the *effective* causal bound observed in spacetime and coincides numerically with the speed of light.

Importantly, this inequality does not define a fundamental propagation speed at the level of the χ substrate. Rather, it is the projected manifestation of a more primitive structural bound on admissible ordering within χ, which constrains how rapidly causal relations and geometric structure may locally emerge in effective descriptions. The quantity c therefore characterizes the causal structure of the projected regime, not the dynamics of the pre-geometric substrate itself.

This bound does not represent the propagation speed of particles, fields, or signals, nor does it presuppose a pre-existing spacetime manifold. Instead, it limits the maximal rate at which effective causal connectivity and local geometric relations can be established within projected descriptions compatible with the intrinsic ordering structure of χ.

Apparent superluminal recession velocities at cosmological scales arise naturally from cumulative and global effects of projected χ ordering, and do not violate this local causal constraint. Local causal relations remain bounded by c in all effective descriptions, while the underlying χ substrate itself is not subject to any spacetime notion of velocity or signal propagation.

3.7 Relation to Conventional Fields

Although effective descriptions derived from projected χ configurations may exhibit formal similarities with scalar fields employed in cosmology (such as inflaton-like fields), the ontological role of χ is fundamentally different. The χ substrate is not a physical field propagating on spacetime, but a pre-geometric relational structure from which spacetime notions themselves emerge.

Accordingly, χ does not carry energy in the conventional field-theoretic sense, nor is it subject to quantization at the fundamental level. Quantization arises only at the effective level, as a consequence of the non-injective projection from χ to emergent observables. In particular, only certain stable, localized, and spectrally isolated χ configurations admit a particle-like interpretation and can be consistently described using standard quantum field-theoretic tools within an emergent spacetime regime.

Within this framework, matter, radiation, and interactions do not correspond to independent fundamental fields coupled to χ. They arise instead as effective manifestations of topological obstructions, spectral constraints, or long-lived relational patterns of projected χ configurations. Conventional fields of the Standard Model are thus recovered as effective descriptions of these emergent degrees of freedom in regimes where a spacetime interpretation is valid and a coarse-grained field-theoretic language provides an accurate approximation.

From this perspective, Cosmochrony does not introduce an additional field beyond those of the Standard Model. Rather, it provides a deeper ontological account of why effective field descriptions are possible at all, and why their quantized excitations, interaction structures, and mass scales arise with the specific properties observed in nature.

3.8 Initial Conditions and Global Structure

The Cosmochrony framework does not postulate initial conditions in the conventional temporal sense. Instead, it assumes that the relational substrate χ admits a minimal admissible ordering state, denoted χ_0, corresponding to configurations of maximal effective relaxation density. This reference state does not represent an initial condition imposed at a distinguished moment in time, but a structural boundary of admissible projected descriptions, characterizing the earliest configurations that can meaningfully admit an effective ordering interpretation.

In effective geometric regimes, the characteristic scale associated with projected descriptions in the vicinity of χ_0 coincides numerically with the Planck scale. This correspondence reflects the loss of projectability and the breakdown of coarse-grained spacetime descriptions below this regime, rather than the presence of a fundamental cutoff, microscopic discreteness, or underlying spacetime lattice.

From this perspective, cosmic history is interpreted as the progressive and irreversible ordering of projected χ configurations away from this minimal admissible state. No spacetime singularity is required at the fundamental level. Apparent singular behavior arises only when classical notions of time, distance, or curvature are extrapolated beyond the regime in which projected χ configurations admit a stable geometric and causal interpretation.

Ontological poverty and the arrow of complexity.

The minimal admissible state χ_0 corresponds to a regime of *ontological poverty*: only a small class of simple, highly coherent configurations can be projected. Early configurations were not merely simpler realizations of a fixed blueprint, but belonged to a drastically restricted admissible space, incapable of supporting the relational richness that emerged only through subsequent relaxation.

As relaxation proceeds, this admissible space expands, enabling the emergence of hierarchical, localized, and complex structures. Cosmological time is therefore simultaneously the arrow of entropy and the arrow of increasing descriptive richness. The complexity observed today was not encoded or prefigured in advance.

In this sense, Cosmochrony is fundamentally non-teleological: the universe does not unfold a prewritten complexity, but acquires descriptive richness only as the space of admissible configurations itself expands. This expansion can be characterized graph-theoretically in terms of the spectral properties of the relational connectivity; the detailed formalism is developed in Appendix C.1.

The global structure of admissible projected descriptions is therefore constrained by the ordering properties of the underlying χ substrate, rather than by arbitrarily specified initial data or boundary conditions. In the following section, we derive a minimal effective dynamical equation governing the ordering of projected χ configurations and explore its immediate physical consequences.

4 Ontological Interpretation of the χ Substrate

Having established the minimal structural properties and effective dynamical descriptions of the χ substrate, we now turn to its ontological interpretation. The purpose of this section is not to introduce new formal structures, but to clarify the status of the entities, quantities, and asymmetries that emerge within the Cosmochrony framework.

Throughout this section, we explicitly distinguish between the pre-temporal, pre-geometric substrate χ and its effective spacetime projections. In particular, we carefully separate invariant structural constraints defined at the level of χ from their emergent manifestations within spacetime-based physical descriptions. This distinction is essential for avoiding dual ontologies and for maintaining a coherent hierarchy between fundamental structure and effective representation.

A recurring theme is the ontological asymmetry introduced by projection. While all physical observables, spacetime notions, and dynamical laws arise only after projection from χ, the relational structure of χ itself does not admit a reformulation entirely in spacetime or field-theoretic terms. The projected universe is therefore physically real but ontologically derivative.

In this context, we emphasize the distinction between the invariant structural bound c_χ, defined at the level of the pre-temporal substrate, and its effective spacetime manifestation c. The latter acquires operational meaning only once notions such as distance, duration, and causal propagation become meaningful within projected descriptions. This separation underlies the emergence of relativistic causality without postulating spacetime as fundamental [14].

The subsections that follow articulate the ontological consequences of this framework. We examine the status of χ as a pre-temporal structural plan, its relation to relational and holographic approaches, the origin of indeterminacy and apparent randomness, and the reinterpretation of energy, mass, and confinement as emergent features of projection. We also clarify the ontological role of universal constants such as c and $\hbar$, showing how they arise as projective manifestations of deeper structural constraints.

Together, these considerations aim to provide a coherent ontological reading of Cosmochrony, in which time, matter, causality, and quantum phenomena emerge from a single pre-geometric relational substrate without invoking additional fundamental entities or postulates.

4.1 The χ Substrate as a Pre-Temporal Structural Plan

Building on the definition established in Section 3.1, the χ substrate admits an ontological interpretation as a pre-temporal relational structure from which spacetime, matter, and effective physical laws emerge. It may be heuristically described as a *structural plan* of the universe: not a dynamical history, but a complete relational organization encoding the set of physically admissible configurations and the constraints that relate them.

This structural plan does not prescribe a unique evolution, nor does it encode a fixed sequence of events. Instead, it defines a constrained space of relational possibilities compatible with the intrinsic ordering structure of χ. Temporal succession is therefore not fundamental, but emergent, corresponding to an oriented resolution of structural relations through irreversible relaxation once an effective spacetime description becomes applicable.

Importantly, the notion of a structural plan should not be understood as introducing teleology, determinism, or a block-universe ontology. The ordering induced by χ constrains admissible effective descriptions, but does not select a unique realized history. Multiple effective histories may therefore correspond to the same underlying relational structure through the non-injective projection from χ to observable descriptions.

Infra-physical status of χ.

The χ substrate is not described by physics in the conventional sense of a theory of dynamical fields evolving in spacetime. Rather, it belongs to an infra-physical relational framework that specifies the structural conditions under which physical observables, spacetime geometry, and effective dynamical laws become meaningful. Physical theories formulated in spacetime thus arise as effective descriptions of particular projectable regimes of this deeper relational structure.

4.2 Relational Ontology and Conceptual Lineage

The relational character of the χ substrate bears a clear conceptual affinity with relational approaches in physics, most notably those advocated by Rovelli [8, 15]. These approaches trace part of their philosophical lineage to Aristotelian relational ontology, in which properties are defined through relations rather than as intrinsic attributes [16, 17].

Cosmochrony shares this rejection of intrinsic, observer-independent properties. However, it extends relationalism to a deeper ontological level. In relational quantum mechanics, relations are primary at the level of quantum states describing interactions between systems that are themselves taken as given within a spacetime framework. By contrast, Cosmochrony posits that the fundamental substrate χ is itself relational: there are no pre-existing entities between which relations are defined.

In this sense, χ configurations are not relations *between* fundamental objects, but relational structures that give rise to objects only upon projection into an effective spacetime description. Relationality is therefore not a feature of physical states *within* spacetime, but an intrinsic property of the pre-geometric substrate from which spacetime, time ordering, and physical entities jointly emerge.

This distinction is essential for understanding the ontological asymmetry between χ and its effective spacetime projections. While relational quantum mechanics reformulates quantum theory without altering the ontological status of spacetime itself, Cosmochrony relocates relationality at the level of the substrate that gives rise to spacetime and to the entities described within it. The non-injective character of the projection from χ to effective observables further implies that multiple effective spacetime descriptions may correspond to the same underlying relational structure.

On the absence of fundamental values.

In Cosmochrony, relations are not defined between pre-existing fundamental values or field degrees of freedom. The relational structure of χ is ontologically primary and admits no intrinsic numerical, local, or field-like values. What appear, within effective spacetime descriptions, as scalar values, particles, or local degrees of freedom arise only as stable invariants of this relational structure under projection and coarse-graining.

Relation to Rovelli's relational frameworks (optional note).

The Cosmochrony framework shares a broad conceptual affinity with relational approaches to physics, particularly those developed by Rovelli in the context of relational quantum mechanics and loop quantum gravity. In both cases, physical properties are not regarded as intrinsic attributes of isolated systems, but as relational features emerging from interactions or structural constraints.

However, the point of departure differs in a crucial way. In relational quantum mechanics, relationality is introduced at the level of quantum states describing interactions between physical systems that are themselves assumed to exist within spacetime. Spacetime, while dynamically treated in general relativity, retains its ontological status as the arena in which relations are defined.

By contrast, Cosmochrony relocates relationality to a deeper ontological level. The fundamental substrate χ is not a system embedded in spacetime, nor a field defined on a manifold, but a pre-geometric relational structure from which spacetime, time ordering, and physical entities jointly emerge through non-injective projection. Relations in Cosmochrony are therefore not relations *between* objects, but relational structures that give rise to objects only at the effective level.

In this sense, Cosmochrony should not be viewed as a reformulation or extension of Rovelli's relational frameworks, but as an ontologically prior approach that seeks to explain the origin of the relational degrees of freedom subsequently described within spacetime-based theories.

4.3 Projection, Reality, and Ontological Asymmetry

Within the Cosmochrony ontology, the emergence of spacetime is understood as a *projection* from the χ substrate, rather than as a dual, equivalent, or bidirectional description. The projected universe is fully real at the level of physical experience and empirical description, but its reality is ontologically derivative: spacetime entities, effective fields, and dynamical laws do not possess ontological primacy [18].

This asymmetry is essential. While all physical descriptions depend on the projection of χ, the converse is not true. The relational structure of χ exists independently of spacetime notions and does not admit a reformulation entirely in geometric, field-theoretic, or dynamical terms. Projection must therefore be understood as *non-injective*: distinct structural features of χ may correspond to identical or physically indistinguishable effective configurations.

Projection and non-circularity.

Because geometric notions arise only after projection, the construction of effective descriptions must not presuppose any metric, causal, or temporal structure defined at the effective level. In particular, coarse-graining procedures used to define χ_{eff} must avoid relying implicitly on emergent geometric quantities. This requirement motivates the explicit separation between pre-geometric relational structures and geometry-dependent observables developed in Appendix E, where combinatorial, spectral, and weighted distances are carefully distinguished.

Projection and emergent time.

The parameter commonly interpreted as time in the effective description is not a fundamental attribute of the χ substrate. Temporal ordering and duration arise only after projection, as part of the emergent spacetime representation associated with χ_{eff}. No external, global, or fundamental time parameter is introduced at the level of χ itself.

Accordingly, all averaging and coarse-graining operations involved in the definition of background descriptors (such as $\bar{\chi}$) and in the construction of χ_{eff} are formulated relationally, without reference to an underlying temporal metric. Temporal concepts enter the framework only at the effective level, once a stable geometric regime has emerged through projection.

Apparent fine-tuning.

In Cosmochrony, apparent fine-tuning does not reflect an improbable choice of initial conditions or a delicate adjustment of fundamental constants. Instead, it arises from the fact that only a restricted class of χ configurations admits a coherent and stable physical projection. Most configurations of the χ substrate do not give rise to consistent spacetime descriptions, persistent physical structures, or well-defined effective laws.

The apparent delicacy of physical parameters is therefore a selection effect imposed by projectability itself. Only those relational configurations compatible with a stable emergent geometry and sustained relaxation dynamics appear as physically realized universes. Fine-tuning is thus reinterpreted as a structural constraint on projection, rather than as a coincidence requiring external explanation.

Absence of a multiverse.

Cosmochrony does not postulate a multiverse. While multiple configurations of the χ substrate may correspond to the same physical universe under non-injective projection, the framework provides no mechanism by which a single χ structure could sustain multiple independent and simultaneously realized physical projections.

The universe is therefore unique at the level of physical reality, even though its underlying description in terms of χ may be non-unique. The absence of a multiverse is not an additional assumption, but a direct consequence of the ontological asymmetry and non-injective character of projection.

The geometric structure of projection, including its fiber-bundle formulation and the emergence of gauge interactions from projection symmetries, is developed in Section 10.

4.4 Clarifying the Relation to Holographic Descriptions

The preceding considerations naturally invite comparison with the holographic principle, as originally proposed by 't Hooft and Susskind, according to which the effective information content of a spacetime region scales with its boundary rather than with its volume.

Cosmochrony is *not* a holographic theory in the technical sense. It does not posit a lower-dimensional boundary description, nor does it assume a dual equivalence between bulk and boundary physics. No independent boundary degrees of freedom or dimensional reduction are introduced at the fundamental level. Any holographic-like behavior that may arise is therefore not postulated as a principle, but emerges indirectly from the projection of a non-factorizable, pre-geometric relational substrate.

In particular, the limitation of physically accessible information within a given spacetime region reflects the degeneracy of underlying χ configurations that correspond to the same effective projection. This degeneracy is a direct consequence of the non-injective character of the projection from χ to emergent observables, and is intrinsic to the very process by which spacetime descriptions arise. It does not require the introduction of boundary-localized degrees of freedom, nor the assumption that information is fundamentally stored on lower-dimensional surfaces.

From this perspective, scaling behaviors reminiscent of holography—such as the effective reduction of accessible degrees of freedom—are interpreted as emergent signatures of projection rather than as manifestations of an underlying holographic encoding principle. Holography thus appears in Cosmochrony as a derived phenomenological feature of spacetime emergence, not as a guiding postulate or a foundational equivalence.

Relation to thermodynamic and entropic approaches (optional note).
The Cosmochrony framework also invites comparison with thermodynamic and entropic approaches to gravity, notably those developed by Jacobson and Verlinde, in which gravitational dynamics are interpreted as emerging from thermodynamic or informational principles.

In Jacobson's approach, Einstein's equations are derived as an equation of state, assuming the proportionality of entropy to horizon area and the validity of local thermodynamic relations. In entropic gravity models, gravitational attraction is interpreted as an emergent entropic force arising from coarse-grained information constraints.

Cosmochrony differs fundamentally in both ontological starting point and explanatory direction. It does not posit entropy, temperature, or information as primitive quantities. Instead, it introduces a pre-geometric relational substrate χ whose irreversible relaxation constitutes the primary physical process. Thermodynamic and entropic descriptions arise only at the effective level, as secondary languages applicable when projected χ configurations admit coarse-grained, statistical interpretations.

From this perspective, the appearance of thermodynamic relations in gravitational contexts is not the origin of spacetime dynamics, but a consequence of projection and coarse-graining. Entropy-like quantities reflect the degeneracy of underlying χ configurations compatible with a given effective geometry, while temperature-like notions encode local rates of relaxation within projected descriptions.

Accordingly, Cosmochrony does not reduce gravity to thermodynamics, nor does it interpret gravitation as an entropic force. Rather, it provides a deeper structural account of why gravitational dynamics admit a thermodynamic interpretation in appropriate regimes, without elevating entropy or information to a fundamental ontological status.

4.5 Intrinsic Structural Indeterminacy and Projective Variability

A perfectly deterministic, fully closed, and maximally symmetric relational substrate would remain physically inert: no configuration would be distinguished, no effective ordering could arise, and no emergent structure would be selected. For this reason, Cosmochrony postulates the existence of an *intrinsic structural indeterminacy* at the level of the pre-geometric χ substrate.

This indeterminacy does not correspond to randomness, stochastic dynamics, temporal fluctuations, or hidden variables evolving in time. Instead, it reflects a fundamental *non-completeness of structural determination*: configurations of χ are not exhaustively specified by a finite, closed, or maximally constraining set of relational conditions. Intrinsic indeterminacy is therefore ontological rather than dynamical, and should be understood negatively, as the absence of perfect structural closure rather than as the presence of random motion or noise.

Importantly, this indeterminacy does not introduce any form of temporal evolution, causal process, or exploration of alternatives at the level of χ. The substrate itself does not fluctuate, evolve, or sample a space of possibilities in time. Rather, intrinsic indeterminacy prevents χ from collapsing into a perfectly symmetric and physically sterile configuration, thereby allowing relational distinctions to acquire physical relevance once projection into an effective description becomes applicable.

For clarity, the term *fluctuations* is used in this framework only in a strictly metaphorical sense when referring to the χ substrate. No underlying agitation, stochastic process, or microscopic dynamics is implied. Intrinsic indeterminacy should instead be understood as a structural openness of the relational substrate, analogous to an underdetermination of global relational structure rather than to random temporal behavior.

Observable variability and probabilistic behavior arise only at the level of *projected* descriptions. Because the projection from χ to effective spacetime observables is generically non-injective, a single underlying configuration of χ may correspond to multiple physically admissible effective realizations. This non-uniqueness of projection gives rise, at the effective level, to phenomena commonly described as fluctuations, probabilistic outcomes, or quantum uncertainty.

In this sense, randomness is not fundamental but *projective*. It reflects the multiplicity of effective descriptions compatible with a given pre-geometric relational structure, rather than any stochasticity intrinsic to χ itself. The apparent temporal character of fluctuations is therefore an emergent feature of projected spacetime descriptions, not a property of the underlying substrate.

Intrinsic structural indeterminacy thus plays a dual conceptual role. At the ontological level, it prevents perfect structural symmetry and ensures that χ remains physically generative. At the effective level, when combined with non-injective projection, it provides the structural origin of observable variability, probabilistic outcomes, and quantum indeterminacy, without introducing fundamental randomness, hidden dynamics, or temporal evolution into the substrate itself.

4.6 Energy as Capacity for Relaxation

Within the Cosmochrony framework, energy is reinterpreted as the effective capacity of projected χ configurations to relax unresolved structural constraints. Energy is not treated as a fundamental conserved substance, nor as an intrinsic attribute of the χ substrate itself. Rather, it is an emergent, descriptive quantity that characterizes the degree to which a given projected configuration retains the ability to undergo further relaxation.

At the fundamental level, χ admits intrinsic structural indeterminacy but no notion of energy. Energy appears only after projection, as a measure of residual structural tension within effective descriptions compatible with spacetime interpretation. The progressive deployment of structural information corresponds to the conversion of intrinsic indeterminacy into relational differentiation. Energy thus quantifies the remaining potential for this conversion within projected configurations.

From this perspective, processes commonly described as energy transfer, dissipation, or radiation correspond to the redistribution or release of relaxation capacity across projected degrees of freedom. Conservation laws associated with energy arise only at the effective level, as structural regularities of projected dynamics, and do not reflect a fundamental conservation principle at the level of χ itself.

Importantly, without intrinsic structural indeterminacy, the notion of energy would be ill-defined. A perfectly closed and fully determined relational substrate would admit no unresolved tension and therefore no capacity for relaxation. Energy is thus inseparable from the structural openness of the χ substrate and from the non-injective character of its projection into effective physical descriptions.

The quantitative formulation of energy as structural resistance, including the operational definition of mass for localized configurations, is developed in Section 6.3.

4.7 Mass as Frozen Information

Localized and long-lived configurations of the χ substrate—describable in effective regimes as particle-like excitations—correspond to regions in which further relaxation is strongly inhibited. Such configurations trap a fixed amount of unresolved structural information, preventing it from participating freely in the global relaxation process.

In this interpretation, mass represents *frozen energy*: structural information whose capacity for further relaxation has been locally suppressed. The term "information" is used here in a structural sense, referring to persistent relational constraints within projected χ configurations, and should not be understood in terms of Shannon entropy or symbolic encoding. Mass therefore quantifies the degree to which relaxation capacity has been immobilized into stable relational patterns.

The quantitative formulation of this relationship, including the operational definition $m = E/c^2$ and its derivation from resistance to relaxation ordering, is developed in Section 6.3.

From this perspective, processes such as particle annihilation, decay, or radiation emission correspond to the partial or complete release of frozen structural information, restoring its ability to participate in relaxation. Mass is thus not a fundamental attribute of the χ substrate, but an emergent measure of inhibited relaxation arising from stable, spectrally isolated configurations of the underlying relational structure.

4.8 Quarks as Non-Projectable Internal Modes

The ontological status of quarks provides a particularly clear illustration of the distinction between the pre-geometric χ substrate and its effective spacetime projections.

In Cosmochrony, quarks are not interpreted as fundamental particle-like entities, nor as independent localized excitations of the χ substrate. Instead, they correspond to internal structural modes of composite solitonic configurations. These modes are required to characterize the internal organization, stability, and spectral structure of hadronic excitations, but do not admit an autonomous and coherent projection into spacetime.

In effective quantum field descriptions, quarks appear as elementary degrees of freedom subject to confinement. Within Cosmochrony, this confinement is not imposed dynamically by an external interaction or force. Rather, it reflects a deeper structural constraint: isolated quark-like modes do not correspond to admissible standalone projections of χ_{eff}. Only collective configurations in which such internal modes are topologically and relationally closed admit a stable spacetime manifestation.

In this sense, quarks are real at the structural level of the χ substrate, but incomplete at the level of physical projection. They are neither fictitious entities nor fundamental spacetime objects, but non-factorizable internal components of projected excitations. Their physical influence is therefore necessarily indirect, encoded in the spectral properties, symmetry patterns, and dynamical responses of hadrons rather than in localized detection events.

This interpretation parallels the status of internal degrees of freedom in other collective systems: such modes are indispensable for an accurate effective description, yet do not correspond to independently realizable physical entities. Quark confinement thus appears not as a contingent feature of strong interactions, but as a direct and unavoidable consequence of the non-injective character of the projection from the χ substrate to emergent spacetime descriptions.

4.9 The Role of the Universal Bound c_χ

A central structural element of the Cosmochrony framework is the existence of a universal invariant bound, denoted c_χ, defined at the level of the pre-temporal χ substrate. This bound does not correspond to a signal velocity, nor to the propagation of any field, excitation, or information in spacetime. Instead, c_χ characterizes an absolute structural limit on the degree to which relational information can be locally confined within admissible configurations of the χ substrate.

At the fundamental level, c_χ is non-metric and non-temporal. It is not associated with distances, durations, lightcones, or causal ordering, since none of these notions are defined prior to projection. Rather, c_χ expresses a maximal admissible rate of structural ordering: beyond this limit, further confinement of relational degrees of freedom becomes impossible and relaxation necessarily occurs.

Crucially, c_χ is not itself an observable quantity. It acquires operational meaning only through projection, when configurations of χ admit a locally injective representation in terms of effective spacetime variables. In such projectable regimes, the invariant structural bound c_χ manifests as an effective causal constraint c, defined through the maximal admissible local ordering rate of projected configurations:

$$c \;\equiv\; \Pi(c_\chi),$$

where Π denotes the projection from the pre-geometric relational substrate to an effective spacetime description.

The constant c, which coincides numerically with the observed speed of light, thus has a strictly derivative status. It is not an independent postulate, but the spacetime expression of the deeper structural bound c_χ once notions of locality, duration, and causal ordering become meaningful. All effective causal constraints appearing in projected descriptions are inherited from this underlying invariant bound.

Accordingly, the local constraint on the effective relaxation functional,

$$|\mathcal{D}_{\mathrm{loc}}\chi_{\mathrm{eff}}| \leq c,$$

should be understood as the projected manifestation of the more fundamental structural limitation imposed by c_χ. No causal or dynamical principle is imposed directly at the level of spacetime; effective causality arises solely as a consequence of the bounded projective realization of relational ordering.

This distinction becomes essential in strong-gravity or near-deprojection regimes. While the effective constant c may lose its geometric or causal interpretation when projection ceases to be locally injective, the structural bound c_χ remains invariant. The breakdown of spacetime description therefore signals not a violation of causality, but the loss of representability of relational configurations within a spacetime framework.

In summary, c_χ defines a universal, pre-temporal structural bound governing the admissibility of χ configurations, while c represents its effective spacetime manifestation. The latter inherits its numerical value and universality entirely from the former, ensuring conceptual continuity between the pre-geometric substrate and emergent relativistic causality.

4.10 The Role of $\hbar_\chi$ and Reprojection from χ

In the Cosmochrony framework, the parameter $\hbar_\chi$ is not identified with the quantum constant $\hbar$ as introduced in conventional quantum mechanics. Instead, it characterizes a fundamental structural scale of the pre-geometric χ substrate, determined by the intrinsic relational density and constraint structure of admissible configurations.

Importantly, $\hbar_\chi$ does not represent a quantum of action evolving in time. At the level of χ, no temporal evolution, Hilbert space, or dynamical operator algebra is defined. Rather, $\hbar_\chi$ specifies a minimal quantum of *reprojection*: intrinsic structural information encoded in χ can enter an effective spacetime description only in discrete units set by this scale.

Intrinsic structural indeterminacy of χ does not give rise to continuous emergence. Instead, reprojection into spacetime occurs in discrete events, each corresponding to a minimal admissible transfer of relational structure into projected observables. These reprojection quanta define the graininess of effective quantum phenomena.

As spacetime structure stabilizes and projection becomes locally persistent, reprojection events become increasingly localized. Phenomenologically, this manifests as vacuum fluctuations or quantum noise within otherwise stable spacetime regions, without implying any fundamental stochastic dynamics at the level of χ itself.

4.11 The Origin of Planck's Constant: $\hbar_\chi$ versus $\hbar_{\text{eff}}$

A potential conceptual tension arises regarding the status of Planck's constant $\hbar$. Within Cosmochrony, $\hbar$ is not an independent postulate but a **spectral invariant** of the relaxation and projection process.

- **Fundamental substrate scale ($\hbar_\chi$):** At the level of the χ substrate, the quantum of reprojection is uniquely fixed by the intrinsic structural scales of the theory:

$$\hbar_\chi = \frac{c^3}{K_0\, \chi_c}. \tag{8}$$

  Here K_0 denotes the coupling density of relational constraints, χ_c the characteristic correlation scale of the substrate, and c the effective projection of the invariant structural bound c_χ. This relation expresses the *spectral rigidity* of the substrate rather than a dynamical quantization rule.
- **Effective representation ($\hbar_{\text{eff}}$):** In effective spacetime descriptions, the same structural scale appears as $\hbar_{\text{eff}}$, functioning as the quantum of action in Hilbert-space formulations. The apparent universality of $\hbar$ across physical systems reflects the fact that K_0 and χ_c are global invariants of the current relaxation epoch.

The transition from $\hbar_\chi$ to $\hbar_{\text{eff}}$ therefore does not involve a change in value, but a change in representation. A structural constraint defined on the pre-geometric substrate is re-expressed, after projection, as a universal dynamical constant governing effective quantum dynamics.

4.12 Spectral Invariance of Planck's Constant and the Fine-Structure Constant

The dependence of $\hbar_\chi$ on K_0 and χ_c does not signal arbitrariness, but is instead a requirement for spectral unification. By identifying

$$\hbar_\chi \equiv \frac{c^3}{K_0\, \chi_c},$$

the quantum of action is understood as a manifestation of the relational density and constraint structure of the χ substrate.

Within this framework:

- **Resolution of the $\hbar$ tension:** The effective constant $\hbar_{\text{eff}}$ is the projected expression of the fundamental reprojection scale $\hbar_\chi$. Its apparent constancy across spacetime follows from the near-homogeneity of the relaxation flux Φ_χ in the current cosmological epoch.
- **Spectral origin of α:** The fine-structure constant α emerges as a dimensionless spectral ratio, determined by the geometry of the projection fiber Π and the spectral rigidity encoded by K_0:

$$\alpha \;=\; \mathcal{F}\!\left(\frac{\text{geometry and topology of }\Pi}{K_0}\right), \tag{9}$$

  where $\mathcal{F}$ is a functional fixed by the structure of admissible projections.

This interpretation implies that if the structural parameters K_0 or χ_c were to vary—for example in primordial high-constraint regimes—both $\hbar$ and α would scale accordingly. Such variation would not signal a breakdown of physical law, but a change in the projective representation of invariant relational constraints, preserving the internal structural coherence of the Cosmochrony framework.

5 Dynamical Equation for the χ Field

5.1 Parameter-Independent Relaxation

To avoid the conceptual difficulties associated with a fundamental time coordinate, the Cosmochrony framework formulates physical evolution without reference to any external temporal parameter. At the fundamental level, the χ substrate does not evolve in time and admits no temporal parametrization. Physical evolution appears only at the effective level, as an ordered sequence of projected χ configurations, denoted $(\chi_{\text{eff},\lambda})$, where λ is a strictly monotonic ordering parameter labeling admissible stages of relaxation within projected descriptions.

The ordering parameter λ does not represent time, nor does it parametrize a trajectory of the fundamental χ substrate. It serves solely as an index that orders projected configurations once a macroscopic spacetime description becomes applicable. No fundamental dynamics unfolds with respect to λ at the level of χ itself.

Accordingly, no fundamental evolution equation is postulated for χ. Instead, admissible projected descriptions are constrained by an effective relaxation condition of the form

$$\mathcal{D}_\lambda \chi_{\text{eff}} = \mathcal{R}[\chi_{\text{eff}}], \tag{10}$$

where $\mathcal{R}[\chi_{\text{eff}}]$ denotes an effective relaxation functional characterizing the ordering of projected χ configurations. This relation should be understood as a consistency condition on admissible projections, not as a fundamental dynamical law. The functional $\mathcal{R}$ is defined only in regimes admitting a stable geometric interpretation and carries no meaning at the pre-geometric level. In particular, no spacetime derivative, metric structure, or geometric operator is assumed in its definition.

Quantities commonly interpreted as temporal derivatives arise exclusively within effective descriptions. When projected χ configurations exhibit sufficient regularity, a coordinate time parameter may be introduced as a convenient label for the relaxation ordering. Such a parameter has no fundamental significance and does not alter the underlying structural constraints imposed by the χ substrate.

Within this framework, relaxation does not occur *in* time. Rather, the ordering of projected χ configurations defines what is operationally identified as physical duration. Local variations in the effective relaxation ordering give rise to the phenomenological notion of temporal flow, establishing a direct link between the structure of projected descriptions and the emergent concept of time.

5.2 Hamiltonian Derivation of the Evolution Equation

Local Constraint on Effective Relaxation Dynamics

At the fundamental level, the χ substrate does not evolve in time, admits no phase-space structure, and is not governed by a Hamiltonian dynamics. No spacetime coordinates, canonical variables, or variational principles are assumed in its definition.

Nevertheless, once projected χ configurations admit a smooth and stable coarse-grained description, the admissible ordering of these configurations is subject to universal local constraints. These constraints may be summarized, *within effective descriptions only*, by a compact relation formally analogous to a Hamiltonian constraint.

This formulation is introduced solely as a descriptive parametrization of admissible local relaxation patterns and does not define a fundamental dynamics.

Specifically, the effective local relaxation ordering associated with projected χ configurations is bounded by the invariant causal constant c, inherited from the structural bound c_χ. When expressed in effective relational variables, this constraint takes the form

$$(\mathcal{D}_{\text{loc}}\chi_{\text{eff}})^2 + \mathcal{V}_{\chi_{\text{eff}}}^2 = c^2, \tag{11}$$

where $\mathcal{V}_{\chi_{\text{eff}}}$ denotes an effective internal variation functional characterizing the resistance of projected configurations to further relaxation. This quantity encodes the presence of localized structural constraints (e.g., solitonic or bound configurations) and carries no interpretation as a potential energy in a fundamental phase space. No notion of spatial gradient, background geometry, or intrinsic field variation is assumed at this level.

Restricting attention to the monotonic ordering branch yields the effective evolution relation

$$\mathcal{D}_{\text{loc}}\chi_{\text{eff}} = c\sqrt{1 - \frac{\mathcal{V}_{\chi_{\text{eff}}}^2}{c^2}}, \tag{12}$$

which expresses the universal slowdown of effective relaxation induced by localized structural constraints. The term $\mathcal{V}_{\chi_{\text{eff}}}$ encodes **structural resistance** to relaxation from localized configurations (e.g., solitons), not a potential energy in a fundamental phase space. This relation constrains admissible projected descriptions but does not represent an equation of motion for the fundamental χ substrate.

Emergent Gravitational Description

Projected χ configurations exhibiting strong resistance to relaxation correspond, in effective descriptions, to localized regions of enhanced structural complexity. Such regions locally reduce the admissible effective relaxation rate.

When a spacetime interpretation becomes applicable, this reduction is conventionally described as gravitational time dilation. No independent gravitational field or interaction is postulated. The phenomenon arises directly from the constrained ordering of projected χ configurations imposed by the saturation condition in Eq. (11).

In weak-structure regimes, where internal variation rates are small compared to c, the effective description admits a simplified relation governing the spatial distribution of relaxation slowdown:

$$\nabla \cdot \left(\frac{\nabla\chi_{\text{eff}}}{\sqrt{1 - |\nabla\chi_{\text{eff}}|^2/c^2}} \right) \simeq \frac{4\pi G_{\text{eff}}}{c^2}\rho, \tag{13}$$

where ρ denotes the effective density of localized relaxation-resistant configurations and G_{eff} is an emergent coupling parameter characterizing the collective response of the relaxation ordering to such structures. Both quantities are defined strictly at the effective level.

In the Newtonian limit, this relation reduces to an effective Poisson equation for a potential Φ, defined operationally by

$$\Phi \equiv c^2 \ln\left(\frac{\mathcal{D}_{\mathrm{loc}}\chi_{\mathrm{eff}}}{c}\right). \tag{14}$$

This potential is not introduced as a fundamental field, but as a compact summary of how localized variations in effective relaxation ordering modulate physical clocks and rulers in regimes where classical gravitational phenomenology applies.

Interpretational Status

The relations presented in this subsection do not define a fundamental Hamiltonian, action, or variational principle. They provide an effective local parametrization of the structural constraints governing admissible projected descriptions once a geometric interpretation becomes meaningful.

The predictive content of Cosmochrony resides entirely in the structural properties of the pre-geometric χ substrate. Hamiltonian, geometric, and gravitational formalisms arise only as emergent descriptive tools, valid in restricted regimes and carrying no independent ontological status.

5.3 Microscopic Origin of the Coupling Tensor and the Poisson Equation

For internal consistency, the effective coupling governing the ordering of projected χ configurations cannot be treated as a fixed universal constant. Instead, it must depend on the internal structural state of the projected description, reflecting how localized configurations either facilitate or resist further relaxation. In Cosmochrony, this dependence is captured through a constitutive relation linking effective coupling strength to variations of the effective scalar descriptor χ_{eff}, without invoking any underlying spatial substrate or fundamental interaction at the pre-geometric level.

A convenient phenomenological parametrization of this dependence is given by

$$K_{\mathrm{eff}} = K_0 \exp\left(-\frac{(\Delta\chi_{\mathrm{eff}})^2}{\chi_c^2}\right), \tag{15}$$

where $\Delta\chi_{\mathrm{eff}}$ denotes a measure of effective internal variation between correlated projected configurations, K_0 characterizes the maximal relaxation conductivity in a homogeneous effective background, and χ_c sets the characteristic scale beyond which structural inhomogeneities significantly suppress relaxation efficiency. This expression should be understood as a constitutive relation for admissible projected descriptions, not as a fundamental law governing the χ substrate.

Projected configurations exhibiting strong internal variation—such as stable solitonic or bound structures—therefore reduce the effective coupling and locally slow the admissible relaxation ordering. This reduction does not correspond to the introduction of a new interaction. It reflects instead the intrinsic resistance of structured projected configurations to further relaxation. The resulting slowdown provides the microscopic origin of the emergent gravitational phenomenology discussed in the preceding sections.

In regimes where a spacetime description becomes applicable, the local effective relaxation rate $\mathcal{D}_{\rm loc}\chi_{\rm eff}$ differs from its asymptotic value $\mathcal{D}_0$ far from localized structures. An effective gravitational potential Φ may then be introduced as a descriptive parameter through the relation

$$\frac{\mathcal{D}_{\rm loc}\chi_{\rm eff}}{\mathcal{D}_0} \simeq 1 + \frac{\Phi}{c^2}, \tag{16}$$

which summarizes the relative slowdown of effective relaxation ordering in a form familiar from classical gravitational phenomenology. The potential Φ has no independent ontological status and serves only as a compact parametrization of relaxation inhomogeneities.

In the weak-structure regime, where effective internal variations remain small compared to χ_c, the spatial distribution of Φ admits a simplified elliptic description. At this coarse-grained level, the effective dynamics reduce to a Poisson-type relation,

$$\nabla^2\Phi \simeq 4\pi G_{\rm eff}\rho, \tag{17}$$

where ρ denotes the effective density of localized, relaxation-resistant projected configurations and $G_{\rm eff}$ is an emergent coupling parameter encoding the collective response of the relaxation ordering to such structures. Both quantities are defined strictly within the effective geometric description.

This Poisson equation is not fundamental. It represents the weak-field, macroscopic limit of the constrained ordering of projected χ configurations, expressed in a form adapted to effective spacetime description. Gravitation therefore appears not as an independent interaction, but as a descriptive manifestation of reduced relaxation conductivity induced by structured projected configurations.

A fully relational formulation, consistent with but not required for the effective description adopted here, is provided in Appendix E.

Status of effective equations.

The effective equations introduced in this subsection—

5.4 Variational Formulation and Born–Infeld Action

In regimes where projected χ configurations admit a stable geometric interpretation, the effective relaxation constraints introduced above may be summarized in a compact variational form. This formulation is not fundamental and does not define a dynamics of the pre-geometric χ substrate. Rather, it provides an auxiliary and regularized representation of admissible projected descriptions in the presence of localized relaxation-resistant configurations.

Motivated by Born–Infeld–type non-linear actions, originally introduced to control field singularities and enforce upper bounds on physical gradients [19, 20], we consider the effective Lagrangian density

$$\mathcal{L}_{\text{eff}} = -c^2 \sqrt{1 - \frac{|\nabla \chi_{\text{eff}}|^2}{c^2}} + \mathcal{D}_{\text{loc}} \chi_{\text{eff}} - \frac{4\pi G_{\text{eff}}}{c^2} \, \rho \, \chi_{\text{eff}}, \tag{18}$$

where χ_{eff} denotes the effective scalar descriptor introduced in Sec. 3.2, $\mathcal{D}_{\text{loc}}\chi_{\text{eff}}$ is the effective local relaxation ordering defined in Sec. 5.1, and ρ represents the effective density of localized, relaxation-resistant projected configurations.

Remark. Unlike scalar-tensor theories (Section 5.4), this action is **not fundamental** but an auxiliary encoding of projection constraints. No new propagating degrees of freedom are introduced.

All quantities appearing in this expression are defined strictly within the effective geometric description.

The linear dependence on $\mathcal{D}_{\text{loc}}\chi_{\text{eff}}$ enforces the monotonicity of admissible projected descriptions without introducing additional propagating degrees of freedom. The square-root structure acts as a non-linear regulator, ensuring that effective spatial variations remain bounded by the universal constraint c. This role is directly analogous to that of Born–Infeld electrodynamics, where the action enforces a maximal field strength without postulating new microscopic dynamics.

Within this auxiliary variational framework, the Euler–Lagrange equation associated with χ_{eff} reproduces the non-linear elliptic relation governing the spatial distribution of relaxation slowdown:

$$\nabla \cdot \left(\frac{\nabla \chi_{\text{eff}}}{\sqrt{1 - |\nabla \chi_{\text{eff}}|^2 / c^2}} \right) = \frac{4\pi G_{\text{eff}}}{c^2} \, \rho, \tag{19}$$

which coincides with the effective Poisson-type relation obtained in Sec. 5.3. No new physical content is introduced at this stage; the variational formulation merely provides a compact and internally consistent encoding of the previously established constraints.

This Born–Infeld–like action should therefore be understood strictly as an *auxiliary variational representation*. It does not constitute a fundamental action principle, nor does it define equations of motion for the χ substrate. Its purpose is to regularize the effective description, enforce universal structural bounds, and facilitate comparison with standard gravitational phenomenology in the appropriate weak-field regime.

The physical interpretation of this effective action is discussed in Appendix A.1, while its mathematical consistency with the underlying relational dynamics and discrete formulation is established in Appendix A.12.

Why This Is Not a Scalar–Tensor Theory

The effective variational formulation introduced in the previous subsection may superficially resemble scalar–tensor or modified gravity theories, in which a scalar field couples to geometry or mediates gravitational interactions. It is therefore essential to clarify that Cosmochrony does *not* belong to this class of frameworks, either conceptually or technically.

First, the effective scalar descriptor χ_{eff} is *not* a fundamental dynamical field. It does not represent an independent physical degree of freedom propagating on spacetime, nor does it possess intrinsic values or conjugate momenta. By construction, χ_{eff} is a projected, coarse-grained descriptor of relational features of the pre-geometric substrate χ, defined only in regimes where a spacetime interpretation becomes applicable. There is therefore no scalar field at the fundamental level whose dynamics could be coupled to geometry.

Second, no modification of the gravitational sector is postulated. The metric appearing in effective descriptions is not an independent dynamical field, nor is it sourced or altered by a scalar field equation. Instead, geometric quantities summarize correlations between projected χ_{eff} configurations. The Born–Infeld–like variational form introduced earlier does not define a new gravitational theory; it merely encodes, in a compact and regularized manner, the constraints governing admissible projected descriptions. In particular, there is no scalar–tensor coupling term of the form $f(\chi_{\text{eff}})R$, nor any modification of the Einstein–Hilbert action.

Third, the effective equations derived from the auxiliary variational formulation do not introduce additional propagating modes. In scalar–tensor theories, the scalar field typically carries its own dynamics, leading to extra degrees of freedom, modified wave propagation, or additional polarization states. In Cosmochrony, by contrast, χ_{eff} does not propagate independently. All effective equations constrain admissible relaxation patterns of projected configurations and do not enlarge the physical phase space.

Finally, the origin of gravitational phenomenology in Cosmochrony is fundamentally different. Gravitation does not arise from the exchange of a scalar mediator or from a dynamical coupling between scalar and tensor sectors. It emerges instead from the local inhibition of relaxation ordering induced by structured projected configurations. The appearance of Poisson-like equations or gravitational potentials reflects the weak-structure limit of this constrained ordering, not the presence of a scalar gravitational field.

In summary, Cosmochrony should not be interpreted as a scalar–tensor or modified gravity theory. The effective scalar descriptor χ_{eff} is neither fundamental nor dynamical, the variational formulation is auxiliary rather than postulatory, and no additional gravitational degrees of freedom are introduced. The framework therefore avoids the conceptual and phenomenological issues commonly associated with scalar–tensor theories, while reproducing standard gravitational behavior in the appropriate effective regimes.

5.5 Causality and Locality

Equation (12) does not define a fundamental dynamics of the χ substrate. Rather, it constrains admissible projected descriptions in regimes where a geometric interpretation becomes applicable. Within such effective descriptions, the relaxation ordering encoded by χ_{eff} exhibits locality and causality in the operational sense relevant to physical observables.

Effective locality follows from the fact that variations of χ_{eff} at a given effective spacetime event depend only on correlated neighboring projected configurations, as defined within the emergent geometric description. No instantaneous coupling, action at a distance, or nonlocal influence is introduced at the level of effective physical observables. All admissible projected descriptions therefore respect locality in the sense appropriate to an emergent spacetime framework.

Causality is enforced through the existence of a universal bound on the effective local relaxation ordering,

$$|\mathcal{D}_{\mathrm{loc}}\chi_{\mathrm{eff}}| \leq c,$$

which constrains the maximal rate at which correlations between effective configurations may be established. This bound functions as an effective causal constraint without presupposing a fundamental spacetime structure, lightcones, or signal propagation at the level of the χ substrate.

Within effective physical descriptions, no superluminal propagation of signals or causal influence occurs. Apparent superluminal recession velocities observed in cosmological contexts arise solely from the cumulative integration of locally constrained relaxation ordering over extended regions. They therefore remain fully consistent with effective locality and causality as defined within the Cosmochrony framework.

Conceptual remark.— The effective causal bound c introduced here should not be interpreted as an independent physical postulate. As established in Section 4.9, it corresponds to the projected manifestation of the invariant structural bound c_χ defined at the level of the pre-temporal χ substrate. In regimes admitting a locally injective projection and a stable geometric interpretation, c_χ acquires an operational meaning as a maximal admissible local ordering rate, which appears in spacetime descriptions as the effective causal constraint c.

Effective causality in Cosmochrony thus arises as a derived property of bounded projective realizations, rather than as a fundamental principle imposed on spacetime itself.

5.6 Homogeneous Cosmological Limit

In a homogeneous and isotropic regime, projected χ configurations exhibit no effective spatial variations. The admissible relaxation ordering is then uniform across the emergent description, and the effective local relaxation rate attains its maximal allowed value,

$$\mathcal{D}_{\mathrm{loc}}\chi_{\mathrm{eff}} = c, \tag{20}$$

where c denotes the universal bound constraining effective relaxation ordering.

When expressed in terms of an effective cosmological time parameter t, introduced solely as a convenient label of the relaxation ordering, this uniform regime may be represented by the linear relation

$$\chi_{\mathrm{eff}}(t) = \chi_{\mathrm{eff},0} + c\,t, \tag{21}$$

where $\chi_{\mathrm{eff},0}$ denotes a reference value fixing the origin of the effective description. This parametrization does not introduce a fundamental time variable, nor does it assign intrinsic values to the χ substrate. This linear regime underlies the emergent Hubble law derived in Section 12.5. It serves only as a compact representation of cumulative relaxation ordering in a homogeneous cosmological regime.

Interpreting effective spatial distances as accumulated relational differentiation between projected configurations, this uniform ordering leads directly to a Hubble-like expansion law, as discussed in Sec. 12. Cosmic expansion therefore reflects the global ordering of projected χ configurations, rather than the presence of an external energy component or a fundamental expansion of spacetime itself.

As shown in Appendix A.6, the requirement that effective relaxation ordering remain monotonic in an expanding regime implies the existence of a minimal residual structural inhomogeneity in projected χ configurations. In effective geometric terms, this manifests as a non-vanishing lower bound on gravitational acceleration. This bound provides a natural route to MOND-like phenomenology in appropriate regimes, without postulating additional dark matter particles [21, 22].

5.7 Influence of Local Structure

In regions where projected χ configurations exhibit non-vanishing effective structural variations, the admissible local relaxation ordering is reduced. This reduction constitutes a central mechanism underlying the emergence of gravitational phenomena within the Cosmochrony framework.

Localized relaxation-resistant configurations—describable in effective regimes as particle-like solitonic structures—act as constraints on the admissible ordering of projected χ configurations. By increasing the effective structural complexity of the projected description, they locally inhibit relaxation without introducing any additional interaction, mediator, or force.

When a geometric description becomes applicable, this inhibition manifests phenomenologically as gravitational time dilation and spatial curvature. No independent gravitational field or degree of freedom is postulated. Gravitation arises instead as a collective and emergent consequence of locally constrained effective relaxation ordering induced by structured projected configurations.

5.8 Unified Origin of Geometric and Field Effects

The relation between the χ substrate and the effective spacetime metric $g_{\mu\nu}$ is strictly hierarchical. It reflects the transition from a fundamental pre-geometric relational structure to smooth geometric descriptions applicable only at macroscopic scales.

1. **Primacy of the χ Substrate.** At the fundamental level, physical reality is described solely in terms of the χ substrate and its intrinsic relational structure. The χ substrate is not defined on spacetime, does not possess numerical values, and is not governed by a dynamical field equation. Ordering, relaxation, and causal notions arise only within admissible projected descriptions and have no meaning at the pre-geometric level.
2. **Emergent Geometry.** In regimes where projected χ configurations admit a stable and slowly varying description, geometric notions become meaningful. The spacetime metric $g_{\mu\nu}$ arises as an effective descriptor summarizing correlations and relaxation ordering among projected χ configurations. It provides a coarse-grained geometric language suitable for macroscopic descriptions, without acquiring independent ontological or dynamical status.

3. **Unified Interpretation of Fields and Gravitation.** Within the effective geometric description, localized relaxation-resistant projected configurations—describable as solitonic structures—are identified with matter degrees of freedom. Gravitational phenomena correspond to local modulations of effective relaxation ordering induced by such structures. The metric does not act as an independent dynamical agent; it encodes the collective geometric response associated with constrained projected descriptions.

In this framework, no independent gravitational interaction or fundamental field is postulated. Matter, geometry, and gravitation emerge as complementary and inseparable aspects of the same constrained ordering of projected χ configurations. This unified origin ensures conceptual continuity across scales, from localized particle-like structures to macroscopic spacetime geometry.

5.9 Limitations and Scope

Equation (12) is intentionally minimal. It does not describe a fundamental dynamics of the χ substrate, nor does it aim to capture the full relational complexity of pre-geometric configurations. Its role is to constrain admissible projected descriptions in regimes where a stable geometric interpretation becomes applicable.

In particular, this effective relation does not provide a first-principles account of quantum fluctuations, correlations, or probabilistic behavior at the level of the χ substrate itself. Such phenomena arise only at the level of projected descriptions, as consequences of non-injective projection, coarse-graining, and the limited representability of underlying relational structures within spacetime-based formalisms. Higher-order structural effects and strongly non-projectable regimes therefore lie outside the direct scope of the present formulation.

Within these limitations, the constrained ordering relation nevertheless provides a unified kinematic backbone from which gravitational, quantum, and cosmological phenomena can be consistently *described* and *recovered* at the effective level. These phenomena are not derived from dynamical laws acting on χ, but emerge as stable regularities within admissible projected descriptions once geometric interpretation and coarse-graining are introduced.

More refined treatments of effective fluctuations, long-range correlations, and extended relational structures require a deeper analysis of the space of admissible projections and of the stability properties of the corresponding effective descriptions. Such developments fall beyond the present scope and are left for future work.

In the following sections, the constrained descriptive framework introduced here is applied to particle-like excitations, gravitation, and quantum correlations, where its explanatory power can be directly assessed within its intended domain of validity.

6 Particles as Localized Excitations of the χ Field

6.1 Particles as Stable Wave Configurations

Within the Cosmochrony framework, particles are not fundamental point-like entities. They arise only at the level of effective descriptions, as stable and localized configurations within projected χ descriptions [23]. Such configurations correspond to persistent patterns that locally constrain the admissible relaxation ordering, rather than to elementary objects propagating on a pre-existing spacetime background.

In effective geometric regimes, these structures may be described using a wave-like or soliton-like language. Their stability reflects the existence of localized regions in which further relaxation is strongly inhibited, allowing the configuration to persist under interactions and effective displacement. Apparent particle propagation corresponds to a continuous reorganization of admissible projected descriptions, not to the motion of an object through a fundamental spacetime.

Particle-like behavior therefore does not originate from intrinsic degrees of freedom of the χ substrate. It emerges instead as a stable relational configuration within the effective projected description once a spacetime interpretation becomes applicable. In this sense, particles are best understood as emergent, dynamically sustained patterns of constrained relaxation, rather than as ontologically primitive objects.

6.2 Topological Stability

The stability of particle-like excitations in Cosmochrony does not rely on fundamental conserved charges postulated *a priori*. It arises instead at the level of effective descriptions, from intrinsic structural constraints on admissible projected χ configurations. Certain projected configurations exhibit non-trivial internal organization that prevents them from being continuously deformed into homogeneous effective descriptions without violating admissibility conditions.

This form of stability is topological in character. It reflects the existence of inequivalent classes of admissible projected configurations that cannot be smoothly connected through continuous reconfiguration while preserving monotonic relaxation ordering. As a result, particle-like excitations appear discrete and robust under perturbations, without invoking externally imposed symmetries or fundamental conservation laws.

Importantly, these topological distinctions are not defined with respect to a pre-existing spacetime geometry. They are properties of the space of admissible projected descriptions itself and remain meaningful even in regimes where no effective geometric interpretation applies. When geometric representations are introduced, they serve solely as descriptive tools valid in projectable regimes.

The long-lived character of solitonic structures therefore follows from the incompatibility between distinct classes of admissible projected configurations, rather than from a dynamical balance of forces or nonlinear self-interactions. Particle stability is thus understood as a structural consequence of projection and admissibility, fully consistent with the pre-geometric and relational foundations of the Cosmochrony framework.

Detailed geometric constructions of topological solitons, including vortex, skyrmion, and knotted configurations, are provided in Appendix B.2. The fully relational formulation of topological stability, independent of geometric representations, is developed in Appendix E.8.

6.3 Mass as Resistance to χ Relaxation

Building on the ontological interpretation of mass as frozen structural information (Section 4.7), this section develops the quantitative formulation. Mass is not introduced as an intrinsic or fundamental property of matter, but emerges at the level of effective descriptions as a measure of how strongly a localized projected configuration resists admissible relaxation ordering.

A particle-like excitation is described, within effective regimes, as a stable and localized projected configuration, denoted $\chi_{\mathrm{eff},s}$, characterized by persistent internal structure. Such configurations locally constrain the admissible relaxation ordering relative to a homogeneous effective background. When a geometric description applies, this constraint manifests phenomenologically as inertial persistence and gravitational time dilation.

We define the effective structural energy associated with a projected solitonic configuration $\chi_{\mathrm{eff},s}$ as a measure of the excess resistance to relaxation encoded in its internal structure:

$$E[\chi_{\mathrm{eff},s}] \;\equiv\; \int_{\Sigma} \left(\frac{1}{\sqrt{1 - |\nabla \chi_{\mathrm{eff},s}|^2 / c^2}} - 1 \right) d\Sigma, \tag{22}$$

where Σ denotes a hypersurface of constant effective ordering parameter, and $|\nabla \chi_{\mathrm{eff},s}|$ quantifies effective structural deformation within the projected description. This expression does not represent a fundamental energy stored in the χ substrate, but a descriptive measure of how strongly a given projected configuration departs from homogeneous relaxation ordering.

The inertial mass associated with such a configuration is then defined operationally as

$$m \;\equiv\; \frac{E[\chi_{\mathrm{eff},s}]}{c^2}. \tag{23}$$

For example, the electron's mass m_e reflects its **topological stability** as a 4π-periodic soliton (Section B.4), while the proton's larger mass arises from a composite 3-soliton configuration (Appendix 6.3).

This relation is not postulated as a fundamental axiom. It follows directly from the role of $E[\chi_{\mathrm{eff},s}]$ as a measure of resistance to effective relaxation ordering. The universal constant c appears as the maximal admissible relaxation rate and therefore provides the unique conversion factor between structural resistance and inertial response.

Within this framework, the relation $E = mc^2$ is interpreted as a kinematic identity. Mass quantifies the amount of effective relaxation resistance locally encoded in a persistent projected configuration, while energy represents the same quantity expressed in relaxation units.

In this sense, mass is not an independent attribute of matter. It is a derived and stable property of localized projected configurations, characterizing how strongly they resist the irreversible ordering that, in effective descriptions, defines physical time.

The question of how different particle masses arise from distinct classes of projected configurations is addressed in Appendix B.2, where a spectral characterization of the stability properties of admissible projected descriptions is proposed as the geometric origin of mass hierarchies.

6.4 Metastability, Projection, and Particle Decay

In the Cosmochrony framework, particle-like entities are not elementary objects but stabilized collective regimes of the projected field χ_{eff}. Their persistence reflects a balance between structural constraints—topological, geometric, or relational—and the capacity of the surrounding substrate to accommodate relaxation.

A stable particle corresponds to a deep basin of admissible projected configurations. An unstable particle, by contrast, occupies only a shallow or fragile basin: it remains admissible, but concentrates structural constraints in a way that impedes efficient relaxation. Such configurations are therefore metastable rather than forbidden.

Particle decay is interpreted as the structural reorganization of such a metastable configuration. When the concentration of constraints exceeds what can be sustained by a single projected entity, admissibility cannot be maintained at the level of a unified description. Stability is recovered only through factorization into several less constrained localized configurations, possibly accompanied by weakly structured excitations.

This mechanism is closely related to the non-injective character of the projection from the χ-substrate to effective observables. In the case of quantum entanglement, a single underlying χ-configuration admits a non-factorizable but stable projected description. In the case of decay, the same non-factorizability becomes unstable: no single projected description remains admissible, and fragmentation becomes unavoidable.

In this sense, entanglement and decay represent two regimes of the same projection structure. Entanglement corresponds to non-factorizability without fragmentation, while decay corresponds to non-factorizability that forces fragmentation. The distinction lies not at the level of the χ-substrate, but in the stability properties of the projected description under admissible fluctuations.

The finite lifetime of unstable particles does not reflect a fundamental indeterminacy. Projected configurations are continuously subject to small admissible variations, and decay occurs when such variations cross a structural reorganization threshold. If the probability of reaching such a threshold is approximately constant, the observed exponential decay law follows as a statistical signature of metastability.

The instability responsible for particle decay originates from the intrinsic susceptibility of certain projected configurations to their own admissible fluctuations. Such fluctuations are continuously present in the χ-substrate and, in metastable configurations, may drive the system across an admissibility threshold, triggering structural fragmentation.

A more technical characterization of metastability, admissible factorization channels, and decay widths is provided in Appendix B.14.

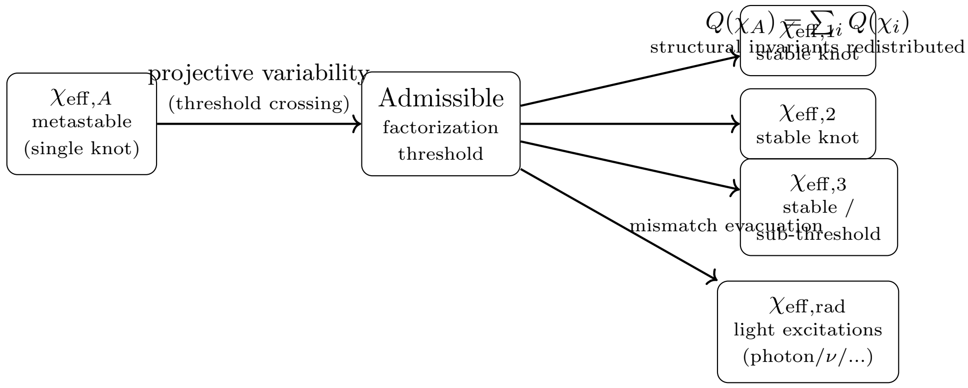

Fig. 4 Structural interpretation of particle decay in Cosmochrony. A metastable localized projected configuration can transition, via an admissible factorization threshold, into several more stable localized configurations plus weak excitations that evacuate the residual structural mismatch.

6.5 Energy–Frequency Relation

Within effective descriptions of Cosmochrony, the energy associated with a particle-like excitation is linked to a characteristic internal spectral scale of the corresponding projected configuration. This scale quantifies how strongly the configuration resists admissible relaxation ordering: configurations associated with higher characteristic frequencies correspond to more tightly constrained projected structures and therefore encode a greater effective resistance to relaxation.

This correspondence provides an effective interpretation of the relation

$$E \propto \nu, \tag{24}$$

in which energy measures the degree of effective relaxation resistance associated with a projected configuration, while the frequency ν characterizes the spectral scale of its internal organization. The frequency should not be interpreted as a literal oscillation with respect to a fundamental time parameter. It acquires an operational temporal meaning only within effective geometric regimes, where a notion of time emerges through projected ordering.

Within this framework, Planck's constant appears as an effective proportionality factor relating energy and frequency. Its apparent universality reflects the robustness of the spectral scales governing admissible projected configurations in the current relaxation epoch, rather than the postulation of a fundamental quantization constant at the level of the χ substrate.

In this sense, the energy–frequency relation expresses a kinematic correspondence between effective structural resistance and spectral organization within projected descriptions. A more explicit realization of this correspondence, in the context of radiation and photon-like projected excitations, is presented in Sec. 13.3.

6.6 Fermions and Bosons

Within the Cosmochrony framework, particle statistics do not arise from fundamental quantization rules, canonical commutation relations, or postulated spin variables. They emerge at the level of effective descriptions from the topological structure of admissible projected configurations.

Distinct classes of particle-like projected configurations are characterized by how their internal configuration space responds to continuous rotations within the space of admissible descriptions. Certain configurations require a 4π rotation to return to an equivalent projected description, while others are already 2π-periodic. The former give rise to fermion-like behavior, whereas the latter correspond to boson-like excitations.

This distinction reflects a topological obstruction in the space of admissible projected configurations rather than a symmetry principle imposed at the fundamental level. In effective geometric regimes, 4π-periodic configurations may be described as possessing twisted or non-orientable internal structure, while 2π-periodic configurations correspond to orientable ones. These characterizations are descriptive and do not imply the existence of a fundamental spatial manifold, intrinsic spin variables, or underlying rotational degrees of freedom.

Within this perspective, the spin–statistics connection admits a natural qualitative interpretation. Fermionic and bosonic behavior reflects the topological classification of admissible projected configurations under continuous deformations, rather than the imposition of additional quantum postulates at the level of the χ substrate.

As throughout this work, references to phase rotations, periodicity, or internal configuration space should be understood strictly within the effective descriptive framework. They characterize topological invariants of projected descriptions and their admissible transformations, not intrinsic attributes of the pre-geometric χ substrate.

6.7 Spin as a Topological Property of Projected Configurations

Within the Cosmochrony framework, spin is not introduced as an intrinsic kinematic degree of freedom, nor as a consequence of spacetime symmetries or fundamental rotation groups. It emerges instead, at the level of effective descriptions, as a purely topological property of admissible projected configurations.

Certain stable particle-like projected configurations exhibit internal relational structures that cannot be continuously deformed into homogeneous effective descriptions. These configurations are characterized by non-trivial topology in their internal configuration space, independently of any background spatial geometry or primitive notion of rotation.

In particular, a class of fermionic projected configurations requires a 4π transformation in configuration space to return to an equivalent effective description. A 2π transformation corresponds to a non-contractible loop in the space of admissible projected configurations, while a 4π transformation is homotopic to the identity. Formally, this implies that the relevant configuration space admits a double covering, with fundamental group

$$\pi_1(\mathcal{C}_{\text{eff}}) = \mathbb{Z}_2, \tag{25}$$

where $\mathcal{C}_{\text{eff}}$ denotes the space of admissible localized projected descriptions.

When an effective quantum description becomes applicable, particle-like projected configurations are represented by complex wavefunctions encoding the phase structure associated with their topological class. For topologically non-trivial configurations, a 2π effective transformation induces a sign change of the associated wavefunction,

$$\psi \longrightarrow -\psi, \tag{26}$$

while a 4π transformation restores the original state. This 4π-periodicity directly implies **fermionic antisymmetry** and the Pauli exclusion principle, as detailed in Section E.9.

This behavior identifies such projected configurations as spin-$\frac{1}{2}$ fermionic excitations. Importantly, the appearance of a spinorial phase does not rely on a fundamental representation of spatial rotation groups. It follows solely from the topological structure of the space of admissible projected descriptions.

Fermionic statistics arises from the same topological origin. Two identical fermionic projected configurations belong to the same non-trivial topological class and therefore cannot be continuously merged into a single admissible configuration without violating the admissibility constraints of the effective description.

Exchanging two identical fermionic excitations corresponds topologically to a 2π loop in the combined configuration space. As this operation induces a sign change of the effective wavefunction, symmetric configurations are dynamically excluded. This provides a geometric and topological origin of the Pauli exclusion principle within the Cosmochrony framework [24].

In Cosmochrony, spin and fermionic statistics are therefore not postulated quantum properties. They are manifestations of topological obstructions in the space of admissible projected descriptions. The 4π periodicity, spin-$\frac{1}{2}$ behavior, and exclusion principle thus share a common geometric origin within the effective descriptive framework.

6.8 Charge as a Topological and Relaxational Property of χ

Unified relaxation budget for mass and charge

In Cosmochrony, both inertial mass and electric charge draw on the same finite *relaxation capacity* of the substrate χ. This follows from the fact that all effective observables are manifestations of admissible relaxation modes under a structural saturation bound (Born–Infeld–like) enforcing causal consistency. In particular, the projected dynamics admits a dimensionless saturation parameter S defined from local gradient density and constrained by $S \leq 1$ (Appendix F).[3]

Two uses of the same capacity.

A localized configuration supporting an effective particle excitation typically mobilizes relaxation capacity in (at least) two structurally distinct channels: (i) a *scalar inhibition* channel associated with spectral frustration and inertial response (mass), and (ii) an *oriented or chiral* channel associated with a net topological flux (charge). We denote

[3] See the definition of the gradient saturation parameter S in the glossary.

their respective local contributions by $\mathcal{B}_m$ and $\mathcal{B}_e$, and postulate that admissible projected configurations satisfy a single combined bound

$$\mathcal{B}_m[\chi] + \mathcal{B}_e[\chi] \leq \mathcal{B}_{\max}, \tag{27}$$

where $\mathcal{B}_{\max}$ is fixed by the same structural limitation that enforces bounded relaxation fluxes in the effective Born–Infeld representation.

Structural origin of instability and decay.

Within this unified picture, particle stability is reinterpreted as the persistence of a projected χ-configuration whose internal invariants remain *admissible* under continued relaxation. Decay corresponds to a threshold crossing: when relaxation drives a configuration such that $\mathcal{B}_m + \mathcal{B}_e$ approaches the saturation domain, the projected description can no longer maintain a stable representation and the excitation reconfigures into a set of lower-budget modes (reprojected products). This provides a non-stochastic structural origin for lifetimes and supports the possibility that instability rates depend weakly on the local relaxation environment.

Implication for the particle spectrum.

Equation (27) implies that effective mass and charge are not independent degrees of freedom but competing manifestations of a common substrate resource. The observed discreteness and hierarchy of particle masses may therefore be approached as a classification of admissible stable χ-configurations under a single saturation constraint, rather than as a list of independent parameters.

Charge as a Directed and Conjugate Relaxation Mode

In conventional quantum field theory, charge conjugation is introduced as an operation that reverses the sign of internal charges associated with a field. While this description is operationally effective, it presupposes the existence of fundamental charge degrees of freedom. Within the Cosmochrony framework, no such assumption is required.

At the level of the pre-geometric χ substrate, there are no intrinsic charges, gauge fields, or conserved internal labels. What are described in effective theories as charges arise only after projection, as stable invariants characterizing admissible projected configurations. Charge is therefore not a fundamental attribute, but a relational property of projected descriptions.

Charge conjugation is correspondingly reinterpreted as a transformation between *relationally conjugate* projected configurations. A particle and its charge-conjugate counterpart belong to distinct but paired topological classes within the space of admissible projected descriptions. These classes are related by an internal reversal of relational organization, rather than by the inversion of a primitive scalar quantity.

This relational reversal does not involve time reversal, energy inversion, or dynamical evolution at the level of the χ substrate. It is instead a symmetry of the admissible projection structure itself. The existence of such conjugate classes reflects the internal duality of the projection fiber, not an independent physical operation acting on spacetime fields.

Within effective quantum descriptions, this relational reversal manifests as complex conjugation of wavefunctions and as the inversion of effective charge labels. These representations encode the topological distinction between conjugate projected configurations, without implying that charge is fundamental or that conjugation corresponds to a physical process occurring in time.

Importantly, charge conjugation symmetry is not guaranteed within this framework. Because charge is a projective and relational invariant, the symmetry between conjugate classes may be broken by asymmetries in the projection structure itself. Violations of charge conjugation symmetry therefore admit a natural structural interpretation, without invoking explicit symmetry-breaking terms at the level of the χ substrate.

In summary, charge conjugation in Cosmochrony is not the reversal of a fundamental charge, but a relational reversal between paired classes of admissible projected configurations. It reflects a symmetry of the effective descriptive structure, not an ontological operation acting on primitive physical entities.

Within the Cosmochrony framework, electric charge is neither a fundamental scalar label nor a primitive phase degree of freedom. It is instead interpreted as a chiral–torsional invariant of the relaxation flux associated with a projected χ configuration.

At the discrete level, the χ dynamics induces a bounded relaxation flux $\vec{J}_\chi \sim \nabla\chi$ defined on relational links. Charge corresponds to the non-integrability of this flux around localized excitations: the transport of its local orientation along a closed relational loop fails to return to its initial state. The resulting winding number defines an integer-valued topological invariant, whose sign encodes the chirality of the configuration.

In this picture, phase emerges only at the level of projection as a descriptive residue of the underlying torsional structure. The bounded nature of $\vec{J}_\chi$, enforced by the Born–Infeld–type constraint, implies a maximal admissible charge density and naturally removes short-distance singularities. Electric attraction and repulsion arise from the geometric compatibility or frustration between the torsional flux patterns of neighboring excitations.

CP Symmetry as Projective Chirality

In conventional particle physics, CP symmetry combines charge conjugation and spatial parity reversal. Its observed violation is typically introduced through complex phases in effective Lagrangians, without a deeper structural explanation. Within the Cosmochrony framework, CP symmetry admits a natural reinterpretation in terms of projective chirality.

As established in the preceding sections, charge conjugation corresponds to a relational reversal between conjugate classes of admissible projected configurations. Parity, in turn, is not interpreted as a fundamental inversion of spatial coordinates, but as a reversal of orientation within the effective geometric description that emerges after projection.

CP symmetry therefore corresponds to a combined transformation acting on the *orientation of the projection itself.* In Cosmochrony, projected configurations may admit an intrinsic chirality: an orientation asymmetry in the mapping from the pre-geometric χ substrate to effective spacetime descriptions. This chirality is not imposed dynamically, nor encoded in a fundamental interaction, but arises from the geometry of the projection fiber.

When the projection is achiral, conjugate configurations are mapped symmetrically, and CP symmetry is preserved at the effective level. When the projection is chiral, however, the relational reversal associated with charge conjugation is no longer equivalent to a parity-reversed description. CP symmetry is then violated as a direct consequence of projective asymmetry.

Importantly, this violation does not require the introduction of explicit CP-violating terms at the level of the χ substrate. It reflects a geometric asymmetry in how relational structures are realized as effective spacetime configurations. CP violation is therefore reinterpreted as a manifestation of projective chirality, not as a fundamental breaking of symmetry in the underlying ontology.

In this view, CP symmetry is effective, contingent, and emergent. Its violation signals an asymmetry of the projection itself, rather than a failure of fundamental physical laws.

6.9 Antiparticles

Within the Cosmochrony framework, antiparticles are not interpreted as independent fundamental entities, nor as excitations propagating backward in time. They arise instead, at the level of effective descriptions, as relationally conjugate counterparts of particle-like projected configurations.

A particle and its antiparticle correspond to projected configurations belonging to distinct but conjugate topological classes within the space of admissible projected descriptions. These classes are related by an internal reversal of relational organization, rather than by the inversion of a fundamental dynamical variable, temporal orientation, or spacetime trajectory.

This conjugation reflects a symmetry of admissible projected structures under relational reversal. It does not presuppose the existence of intrinsic charges, fundamental fields, or time-oriented dynamics at the level of the χ substrate.

Annihilation processes occur when a particle-like projected configuration and its conjugate combine into a composite projected description that no longer supports localized structural constraints. Such a configuration admits a continuous deformation toward a more homogeneous effective description, in which localized resistance to relaxation disappears.

In effective geometric and quantum descriptions, this transition manifests as the conversion of particle–antiparticle structure into delocalized radiation-like projected excitations. No fundamental structure is destroyed in this process. The underlying relational substrate χ remains intact, while localized topological organization is redistributed into admissible projected configurations with extended support.

In this sense, particle–antiparticle annihilation does not represent the destruction of matter, but a reorganization of effective relational structure from localized to delocalized forms within the space of admissible projected descriptions.

Interpretative Remark: Annihilation as Structural Unknotting

Within the solitonic interpretation of matter developed throughout this work, particle-like configurations correspond to localized topological organizations of the χ field that inhibit its global relaxation. From this perspective, antimatter may be understood not merely as a conjugate configuration, but as a relational mode that enables the release of such constraints.

Particle–antiparticle annihilation can thus be interpreted as a process of *structural unknotting* (in the topological and descriptive sense), in which mutually conjugate topological organizations combine into a configuration that no longer sustains localized resistance. The relaxation flux previously stored in the constrained solitonic structure is then freed to redistribute into delocalized, radiation-like projected excitations.

This interpretation clarifies why annihilation releases an energy proportional to the inertial mass of the particle: within Cosmochrony, mass itself measures the degree of topological obstruction to relaxation.

Why Antimatter Does Not Require Time Reversal

In conventional relativistic quantum field theory, antiparticles are often heuristically interpreted as particles propagating backward in time. While this picture is computationally useful in perturbative formalisms, it does not reflect a fundamental physical process. Within the Cosmochrony framework, such an interpretation is neither required nor meaningful.

At the fundamental level, the χ substrate admits no temporal parameter and no notion of temporal reversal. The monotonic ordering structure intrinsic to χ defines an absolute arrow of admissible projection, which cannot be inverted. All effective descriptions compatible with Cosmochrony therefore inherit a unidirectional ordering, corresponding operationally to the observed arrow of time.

Antiparticles do not arise from reversing this ordering. Instead, they correspond to projected configurations belonging to topologically conjugate classes within the space of admissible projected descriptions. The distinction between particles and antiparticles reflects an internal relational reversal of projected structure, not a reversal of temporal succession.

The apparent association between antimatter and time reversal in standard formalisms originates from the structure of relativistic wave equations and their symmetry properties under complex conjugation. In Cosmochrony, this correspondence is reinterpreted as a feature of the effective representation, not as an indication of physical evolution backward in time.

In particular, processes involving antiparticles always occur within the same monotonic ordering of projected configurations as their particle counterparts. Antimatter participates in relaxation, interaction, and annihilation processes according to the same intrinsic arrow of ordering. No admissible projected description involves a reversal of the effective relaxation parameter or a decrease of χ_{eff}.

Annihilation processes further clarify this point. Particle–antiparticle annihilation corresponds to the merging of conjugate projected configurations into a delocalized admissible description, not to the cancellation of forward and backward temporal trajectories. The effective outcome is a redistribution of relational structure into radiation-like projected excitations, all evolving within the same monotonic ordering framework.

In summary, antimatter in Cosmochrony does not require time reversal. It reflects a structural conjugation in the space of admissible projected configurations, fully compatible with a unique and irreversible ordering of projected descriptions. Time-reversal interpretations therefore belong to the mathematical convenience of certain effective formalisms, not to the ontological structure of the underlying relational substrate.

Matter–Antimatter Asymmetry without Fundamental CP Violation

The observed dominance of matter over antimatter in the universe is commonly attributed to CP violation occurring in the early universe, supplemented by additional dynamical assumptions. Within the Cosmochrony framework, matter–antimatter asymmetry admits a more direct structural interpretation, without requiring fundamental CP violation at the level of the χ substrate.

As previously established, particles and antiparticles correspond to conjugate classes of admissible projected configurations. These classes are structurally paired at the pre-geometric level, but their realization in effective spacetime descriptions depends on the properties of the projection.

If the projection from χ to effective spacetime is chiral, conjugate classes need not be realized with equal stability, persistence, or projectability. One class may preferentially admit long-lived, localized projected configurations, while its conjugate may predominantly yield delocalized or short-lived realizations. This imbalance arises without any fundamental asymmetry in the underlying relational substrate.

In such a scenario, matter–antimatter asymmetry is not generated dynamically through time-dependent processes, nor through explicit CP-breaking interactions. It emerges instead as a selection effect imposed by the structure of admissible projections. Only those configurations compatible with stable, persistent projection contribute significantly to the effective physical universe.

Importantly, this mechanism does not require a departure from a unique and monotonic ordering of projected configurations. Both matter and antimatter evolve within the same intrinsic arrow of ordering defined by χ. The asymmetry reflects differential projectability, not temporal reversal or dynamical bias.

In this sense, the observed matter-dominated universe is interpreted as a consequence of projective asymmetry rather than of fundamental CP violation. The apparent violation of CP symmetry in effective descriptions encodes the geometry of projection, not an intrinsic imbalance in the underlying relational ontology.

6.10 Particle Creation and Destruction

Within the Cosmochrony framework, particle creation does not correspond to the appearance of new fundamental entities. It arises at the level of effective descriptions, when a projected configuration acquires sufficient structural organization to support a stable, localized topological class. Such configurations become identifiable as particle-like only once a spacetime interpretation becomes meaningful.

Particle creation therefore reflects the emergence of a new admissible projected description with persistent localization and relaxation resistance. This process does not involve the generation of structure at the level of the χ substrate, but a reorganization of admissible projected configurations within the space of effective descriptions.

Conversely, particle destruction does not represent the annihilation of a fundamental object. It occurs when a previously localized projected configuration loses its topological admissibility or stability class. In such cases, the configuration can no longer sustain localized relaxation constraints and admits a continuous deformation toward a more delocalized effective description.

In effective geometric and quantum regimes, this transition manifests as the conversion of particle-like projected configurations into extended, radiation-like descriptions. Creation and destruction thus reflect changes in the organization and admissibility of projected descriptions, rather than the appearance or disappearance of fundamental entities.

Within this perspective, particles are not primitive ontological constituents. They are stable descriptive regimes of the relational substrate, whose formation and dissolution correspond to transitions between distinct classes of admissible projected configurations.

CPT as a Global Projective Property

In conventional quantum field theory, CPT symmetry is elevated to the status of a fundamental theorem, derived under assumptions of locality, Lorentz invariance, unitarity, and a fixed spacetime background. Within the Cosmochrony framework, none of these structures are fundamental. CPT symmetry therefore cannot be treated as a primitive axiom, but must be reinterpreted in relational and projective terms.

As established in the preceding sections, charge conjugation (C) corresponds to a relational reversal between conjugate classes of admissible projected configurations, parity (P) reflects an orientation reversal within the effective geometric description, and time reversal (T) has no fundamental meaning at the level of the χ substrate. Each of these operations is individually effective and representation-dependent.

Nevertheless, when projected descriptions admit a stable spacetime interpretation, they must satisfy a global consistency requirement. The projection from the pre-geometric χ substrate to effective spacetime cannot generate observable descriptions that are mutually incompatible representations of the same underlying relational structure. This requirement imposes a global constraint on admissible projected descriptions.

CPT symmetry emerges in Cosmochrony as precisely this constraint. While C, P, and T may each be violated individually at the effective level due to asymmetries in projection, their combined action corresponds to a full relational conjugation of projected descriptions. This combined transformation maps any admissible effective configuration to another admissible configuration representing the same underlying χ structure.

In this sense, CPT invariance is not a microscopic symmetry acting on fundamental degrees of freedom. It is a global projective consistency condition ensuring that the space of admissible projected descriptions is closed under full relational conjugation.

Violations of CPT would therefore signal not new dynamics, but a breakdown of projectability or internal inconsistency in the effective description.

Importantly, this interpretation explains why CPT symmetry is observed to be extraordinarily robust in effective physical theories, even when C, P, and CP are violated. CPT invariance reflects the structural coherence of the projection itself, not a fundamental invariance of spacetime or quantum fields.

In summary, CPT symmetry in Cosmochrony is reinterpreted as a global property of admissible projection. It expresses the requirement that all effective descriptions remain faithful, mutually consistent realizations of a single underlying relational substrate. CPT is therefore preserved not because it is postulated, but because any violation would correspond to a failure of the projection to represent a coherent physical universe.

Why CPT Survives Quantum Gravity

Approaches to quantum gravity often raise the possibility that CPT symmetry might fail once spacetime locality, Lorentz invariance, or unitarity are no longer fundamental. In many frameworks, CPT invariance is tied to properties of a fixed spacetime background and to the axioms of relativistic quantum field theory. From this perspective, its survival in a pre-geometric regime may appear non-trivial.

In the Cosmochrony framework, this concern is resolved by a shift in perspective. CPT symmetry is not treated as a microscopic invariance acting on fundamental degrees of freedom, but as a global consistency property of admissible projections from the pre-temporal χ substrate to effective spacetime descriptions. As such, its validity does not depend on the existence of a fundamental spacetime, local fields, or canonical quantization rules.

At the level of the χ substrate, there is no notion of time reversal, spatial inversion, or charge as an intrinsic attribute. Consequently, none of the individual operations C, P, or T have fundamental meaning. What does exist is a relational structure admitting conjugate realizations under full relational reversal. The requirement that these conjugate realizations correspond to mutually consistent effective descriptions imposes a global constraint on projection.

This constraint is precisely what manifests as CPT invariance at the effective level. Even in regimes where spacetime geometry becomes highly curved, fluctuating, or partially non-projectable, admissible effective descriptions must remain closed under full relational conjugation. Any failure of CPT would correspond not to a novel quantum-gravitational effect, but to a breakdown of projectability itself, signaling that the effective description has exceeded its domain of validity.

Importantly, quantum-gravitational phenomena such as strong curvature, horizon formation, or near-deprojection regimes do not invalidate this constraint. They modify the range and resolution of admissible effective descriptions, but do not alter the underlying relational structure of χ. As long as a projected description exists at all, its global consistency under relational conjugation is preserved.

This provides a structural explanation for the remarkable empirical robustness of CPT symmetry. CPT survives quantum gravity not because it is protected by spacetime symmetries, but because it expresses the minimal requirement for a coherent physical projection of the underlying relational substrate. Quantum gravity may challenge locality, geometry, and even the notion of time, but it cannot violate CPT without undermining the possibility of a consistent emergent universe.

In summary, CPT invariance in Cosmochrony is not threatened by quantum gravity. It survives precisely because it is not a dynamical symmetry, but a global projective property ensuring the coherence of all admissible effective descriptions derived from a single pre-geometric relational structure.

6.11 Antiparticles and CPT as an Admissibility Consistency Condition

The emergence of antiparticles in decay processes is often associated, in standard quantum field theory, with CPT symmetry. In Cosmochrony, CPT is not imposed at a fundamental level. It is recovered as a consistency condition on admissible projected descriptions arising from the non-injective projection of the χ-substrate.

Certain projected configurations carry *orientation-sensitive* structural invariants, such as chirality, phase winding, or relational orientation. These invariants are not defined at the fundamental χ level; they acquire operational meaning only through projection and the effective geometric description. Under admissible factorization, such signed invariants cannot disappear. They must be redistributed among the resulting localized configurations.

When a metastable projected configuration fragments, admissibility may therefore require the appearance of paired localized excitations carrying opposite orientations of the same invariant. At the effective level, these paired configurations are interpreted as particle–antiparticle pairs. Antiparticles are thus not entities propagating backward in a fundamental time, nor independent microscopic degrees of freedom postulated *ab initio*. They are structural counterbalances required to preserve orientation-sensitive invariants during admissible reconfiguration.

From this perspective, CPT symmetry reflects the invariance of admissibility under a combined reversal of (i) the relevant signed structural invariants (charge-conjugation at the effective level), (ii) effective spatial orientation (parity), and (iii) the effective ordering parameter associated with relational relaxation (time reversal in the emergent description). This combined transformation preserves the admissibility class of projected descriptions even when the individual transformations do not.

In this sense, CPT is recovered as a robustness property of the projection structure: admissible projected descriptions remain consistent under the combined reversal of effective orientation data. A technical formulation of this invariance and its relation to signed structural invariants is provided in Appendix B.16.

6.12 Neutrinos as Partially Projectable Modes (Dirac vs. Majorana)

Within the Cosmochrony framework, neutrinos occupy a structurally distinct position among particle-like excitations. They are not interpreted as fully localized solitonic configurations of the χ substrate, nor as purely delocalized radiation-like modes. Instead, neutrinos correspond to *partially projectable modes* of χ: configurations whose relational structure admits a stable projection in some degrees of freedom, while remaining weakly or non-projectable in others.

This partial projectability explains several characteristic features of neutrinos, including their extremely small effective masses, weak interaction strength, and sensitivity to global rather than local structural properties of the projection. Unlike charged fermions, neutrinos do not correspond to tightly confined relaxation-resistant configurations. Their internal structure remains close to the threshold of projectability, resulting in a minimal but non-zero resistance to effective relaxation.

Within this interpretation, the distinction between Dirac and Majorana neutrinos acquires a geometric and projective meaning. A Dirac neutrino corresponds to a partially projectable configuration that admits distinct conjugate projected realizations. Particle and antiparticle remain structurally distinguishable at the effective level, reflecting a residual relational asymmetry preserved by the projection.

By contrast, a Majorana neutrino corresponds to a configuration whose partial projectability collapses the distinction between conjugate classes. The projected description becomes self-conjugate: the relational reversal associated with charge conjugation acts trivially on the admissible projected configuration. In this case, neutrino and antineutrino correspond to the same projected structure, not because of an imposed identification, but because the projection fails to resolve the conjugate relational orientation.

This interpretation does not require the postulation of lepton number as a fundamental conserved quantity. Lepton number conservation emerges only in regimes where conjugate projected configurations are distinguishable. When partial projectability erases this distinction, effective lepton number violation becomes admissible without contradicting any fundamental principle of the χ substrate.

Neutrino oscillations admit a natural explanation within this framework. Different neutrino flavors correspond to closely related partially projectable configurations whose internal relational structures overlap but are not identical. As projected descriptions evolve along the monotonic ordering of χ, the relative projectability of these configurations varies, leading to coherent transitions between flavor labels without invoking mass eigenstates as fundamental objects.

In summary, neutrinos in Cosmochrony are neither fully localized particles nor purely delocalized excitations. They are marginally projectable modes whose weak confinement, tiny effective mass, and ambiguous charge-conjugation properties reflect their proximity to the boundary between localized and non-localized admissible projected descriptions. The Dirac or Majorana character of neutrinos is therefore not a fundamental choice, but a manifestation of how fully the projection resolves relational conjugation for these modes.

Neutrino Oscillations without Fundamental Mass Eigenstates

In the standard formulation of neutrino physics, oscillations are explained by postulating fundamental mass eigenstates whose mismatch with flavor eigenstates leads to quantum interference. Within the Cosmochrony framework, this interpretation is not required. Neutrino oscillations arise without introducing mass eigenstates as ontologically fundamental objects.

As established previously, neutrinos correspond to partially projectable modes of the χ substrate. Their projected configurations are weakly localized and lie close to the threshold of admissible projectability. In this regime, distinct flavor labels do not correspond to sharply separated eigenstates, but to overlapping projected descriptions of closely related relational structures.

Neutrino oscillations are therefore interpreted as transitions between different effective descriptive bases applied to the same underlying partially projectable configuration. As the projected description evolves along the monotonic ordering of χ, the relative stability and projectability of these overlapping configurations varies. This variation induces a coherent redistribution of effective flavor content, without invoking propagation between distinct mass-defined states.

Within effective quantum descriptions, this behavior is encoded mathematically by phase evolution and interference terms. However, these phases do not correspond to evolution with respect to a fundamental time parameter, nor to propagation of mass eigenstates. They reflect instead the relational evolution of the projected description as the configuration explores different admissible representations along the ordering parameter.

This interpretation naturally explains why neutrino oscillations depend weakly on environmental conditions, baseline length, and energy scale. These parameters modulate the projectability of the underlying configuration rather than selecting distinct fundamental states. Oscillation phenomena thus probe the structure of the projection boundary, not the spectrum of intrinsic neutrino masses.

In summary, neutrino oscillations in Cosmochrony do not require fundamental mass eigenstates. They emerge from the relational ambiguity of partially projectable configurations and from the evolution of their effective representations within admissible projected descriptions.

Neutrinos and the Stability of the Projection Boundary

The existence of partially projectable modes raises an important structural question: how the boundary between projectable and non-projectable configurations remains stable. Within the Cosmochrony framework, neutrinos play a central role in regulating this boundary.

Fully localized particle-like configurations correspond to strongly constrained projected descriptions, while radiation-like modes correspond to fully delocalized ones. Neutrinos occupy an intermediate regime. Their weak localization allows them to interact gravitationally and weakly, while remaining largely insensitive to electromagnetic and strong structural constraints.

This intermediate status contributes to the dynamical stability of the projection boundary. Neutrinos act as carriers of marginal structural information, redistributing relational organization without inducing strong backreaction on localized solitonic configurations. In doing so, they prevent abrupt transitions between fully projectable and non-projectable regimes.

From a cosmological perspective, the pervasive presence of neutrinos contributes to smoothing large-scale variations in effective relaxation ordering. Their near-delocalized nature allows them to mediate relational coherence across extended regions, stabilizing the global projection against fragmentation or collapse into non-projectable configurations.

In strong-gravity or near-deprojection regimes, such as the vicinity of horizons or in the early universe, neutrinos remain among the last modes to retain partial projectability. This makes them sensitive probes of the breakdown of spacetime description, while simultaneously contributing to its persistence.

In this sense, neutrinos are not merely passive particles within the emergent universe. They play an active structural role in maintaining the continuity and stability of the projection from the pre-geometric χ substrate to effective spacetime. Their physical properties reflect this role: extreme lightness, weak coupling, and oscillatory behavior are signatures of their function at the boundary of projectability.

Neutrinos and the Failure of Absolute Localization

In effective spacetime descriptions, most particle-like excitations are treated as sharply localizable objects, at least approximately. Within the Cosmochrony framework, such localization is not fundamental but reflects the strong projectability of certain configurations of the χ substrate. Neutrinos provide a counterexample that exposes the limits of absolute localization.

As partially projectable modes, neutrinos correspond to configurations whose relational structure cannot be fully confined within a bounded spacetime region without loss of admissibility. Any attempt to enforce strict localization leads to a breakdown of the projected description, pushing the configuration toward delocalized or non-projectable regimes.

This structural limitation explains why neutrinos do not admit sharply defined position operators in effective quantum descriptions and why their interaction cross-sections remain extremely small. Their weak coupling is not a dynamical accident, but a direct consequence of their inability to sustain strong localization within the emergent geometric framework.

In this sense, neutrinos reveal that absolute localization is not a universal property of physical excitations. It is an emergent feature restricted to configurations that lie sufficiently far from the projection boundary. Neutrinos occupy precisely the regime where localization ceases to be a valid approximation.

Why Neutrinos Are the Lightest Fermions

Within Cosmochrony, fermion masses are interpreted as measures of resistance to effective χ relaxation encoded in localized projected configurations. Heavier fermions correspond to strongly constrained, highly localized solitonic structures, while lighter fermions reflect weaker confinement.

Neutrinos are the lightest fermions because their configurations reside closest to the boundary of projectability. Their internal relational structure resists relaxation only marginally, resulting in a minimal but non-zero effective mass. This mass is not generated by a distinct mechanism, but emerges naturally from their weak degree of localization.

Unlike charged fermions, neutrinos lack projective features that would stabilize strong confinement. Their absence of electromagnetic coupling and their partial self-conjugacy prevent the formation of tightly bound projected structures. As a result, neutrinos cannot accumulate significant resistance to relaxation and therefore remain extremely light.

From this perspective, the smallness of neutrino masses is not anomalous. It is a structural necessity imposed by their role as marginally projectable modes. Any further reduction of their effective mass would render them non-projectable, while any significant increase would require structural features incompatible with their observed properties.

Neutrinos as Probes of Pre-Geometric Structure

Because neutrinos operate near the boundary between projectable and non-projectable regimes, they provide a unique observational window into the pre-geometric structure of the χ substrate. Their properties are sensitive not only to local effective geometry, but also to global features of the projection.

In regimes where spacetime description begins to degrade—such as in the early universe, near horizons, or in regions of extreme curvature—neutrinos remain among the last excitations to admit a coherent projected description. Deviations in their oscillation behavior, effective masses, or coherence lengths therefore carry information about the structure of the projection boundary itself.

Unlike photons or strongly localized particles, neutrinos can traverse regions where geometric notions are only approximately valid. Their weak localization allows them to sample relational structures that are inaccessible to fully projectable modes. As a result, neutrino phenomenology may encode signatures of pre-geometric ordering that survive coarse-graining.

In this sense, neutrinos function as natural probes of the relational substrate underlying spacetime. They do not merely propagate within geometry; they test the conditions under which geometry itself remains a valid effective description. Precision measurements of neutrino properties thus offer a potential empirical handle on the transition between emergent spacetime and its pre-geometric origin.

Neutrinos and the Limits of Effective Quantum Field Theory

Quantum field theory (QFT) provides an extraordinarily successful effective description of particle physics in regimes where spacetime locality, sharp localization, and well-defined asymptotic states are good approximations. Within the Cosmochrony framework, these conditions correspond to strongly projectable configurations of the χ substrate.

Neutrinos, however, systematically probe the limits of these assumptions. As partially projectable modes, they do not admit a fully local field representation valid at all scales and energies. Their weak localization, extended coherence, and oscillatory behavior signal a breakdown of the strict particle ontology presupposed by effective QFT.

In particular, the standard QFT treatment of neutrinos relies on the introduction of mass eigenstates, flavor mixing matrices, and asymptotic Fock states. Within

Cosmochrony, these constructs are understood as effective bookkeeping devices that encode the behavior of marginally projectable configurations. They do not correspond to fundamental degrees of freedom of the underlying relational substrate.

The persistence of neutrino coherence over macroscopic distances, their sensitivity to global boundary conditions, and their weak coupling to local operators indicate that neutrinos are not fully captured by a local quantum field defined on spacetime. Instead, they inhabit a transitional regime where field-theoretic locality becomes approximate rather than exact.

From this perspective, neutrinos mark the boundary of validity of effective quantum field theory. They are not anomalies within QFT, but signals of its effective character. Their behavior remains compatible with QFT predictions in appropriate regimes, while simultaneously revealing the limitations of any description that treats spacetime-local quantum fields as fundamentally complete.

In this sense, neutrinos provide the clearest example of how Cosmochrony reproduces the successes of effective quantum field theory while clarifying the ontological domain in which such descriptions cease to be exact.

Experimental Signatures of Projective Neutrino Physics

If neutrinos correspond to partially projectable modes of the χ substrate, their phenomenology should exhibit subtle deviations from predictions based on fully localized quantum field descriptions. These deviations are not expected to manifest as dramatic violations of known physics, but as systematic anomalies in regimes where projectability becomes marginal.

One potential signature concerns the energy and baseline dependence of neutrino oscillations. In Cosmochrony, oscillations reflect variations in projectability rather than interference between fundamental mass eigenstates. This suggests the possibility of small departures from standard oscillation patterns in extreme regimes, such as ultra-long baselines, very low energies, or propagation through strongly curved or inhomogeneous gravitational environments.

A second class of signatures involves coherence and decoherence effects. Because neutrinos lie close to the projection boundary, their coherence lengths may depend on global properties of the effective spacetime description. Subtle deviations from standard decoherence models could therefore arise in astrophysical or cosmological neutrino observations.

Neutrinoless double beta decay provides another discriminating probe. In the Cosmochrony framework, the Majorana or Dirac character of neutrinos reflects the degree to which relational conjugation is resolved by the projection. Observation or non-observation of such processes constrains the projective resolution of conjugate configurations, rather than directly testing the existence of a fundamental Majorana mass term.

Finally, cosmological neutrino backgrounds offer a unique window into the pregeometric regime. Because neutrinos remain projectable deeper into the early universe than most other excitations, their imprint on large-scale structure, cosmic expansion, and relic radiation may encode information about the approach to the projection boundary itself.

While all these signatures are compatible with existing experimental bounds, they suggest concrete directions in which future high-precision neutrino experiments could test the projective interpretation. Neutrino physics thus provides one of the most promising empirical interfaces between effective quantum field theory and the pre-geometric foundations proposed by Cosmochrony.

Predictive signature: Neutrino oscillation patterns at ultra-long baselines (e.g., DUNE) may exhibit **non-standard phase shifts** due to projective ambiguity, distinct from mass-eigenstate interference.

Synthesis: Neutrinos as the Structural Frontier of Emergent Spacetime

Within the Cosmochrony framework, neutrinos occupy a unique structural position. They are neither fully localized particle-like excitations nor purely delocalized radiation-like modes. Instead, they reside at the boundary between projectable and non-projectable configurations of the χ substrate. This boundary defines the frontier at which emergent spacetime remains a valid effective description.

The defining properties of neutrinos—extreme lightness, weak coupling, long-range coherence, oscillatory behavior, and ambiguous charge-conjugation character—are not independent anomalies. They are coherent signatures of partial projectability. Neutrinos encode just enough relational structure to admit a persistent projection, while remaining insufficiently constrained to form fully localized solitonic configurations.

This interpretation unifies multiple aspects of neutrino phenomenology. Their small effective masses reflect minimal resistance to χ relaxation. Flavor oscillations arise from overlapping projected descriptions rather than from interference between fundamental mass eigenstates. The Dirac or Majorana character of neutrinos corresponds to whether relational conjugation is resolved or collapsed by the projection. Their failure to admit absolute localization reveals the limits of effective quantum field theory.

From a structural perspective, neutrinos are not passive inhabitants of emergent spacetime. They actively regulate the stability of the projection boundary, redistributing relational information without inducing strong backreaction. In cosmological and strong-gravity regimes, they remain among the last excitations to retain partial projectability, making them both stabilizers and probes of emergent geometry.

In this sense, neutrinos mark the transition between physics as described by local quantum fields on spacetime and the pre-geometric relational dynamics of the χ substrate. They provide a natural interface between effective physical law and its structural origin. Precision neutrino experiments thus offer a privileged observational window onto the emergence, persistence, and eventual breakdown of spacetime itself.

Neutrinos are therefore not merely another sector of particle physics. In Cosmochrony, they constitute the structural frontier of emergent spacetime, where effective geometry, quantum description, and pre-geometric ontology converge.

6.13 Summary

Within the Cosmochrony framework, particles are not fundamental ontological constituents. They arise only at the level of effective descriptions, as stable and localized projected configurations that resist admissible relaxation ordering. Their physical properties are not postulated independently, but emerge as invariants of the structural and topological organization of admissible projected descriptions.

Mass is identified with the degree of effective resistance to χ relaxation encoded in a localized projected configuration. It quantifies how strongly such a configuration constrains admissible relaxation ordering relative to a homogeneous effective background. In regimes where a relativistic description applies, this interpretation naturally leads to the relation $E = mc^2$, understood here as a kinematic identity expressing the equivalence between relaxation resistance and inertial response, rather than as a fundamental postulate.

Spin and statistical behavior originate from topological obstructions in the space of admissible projected configurations. Fermionic configurations exhibit a 4π periodicity in configuration space, such that a 2π loop is non-contractible and induces a sign change of the effective wavefunction. This topological structure provides a unified origin for spin-$\frac{1}{2}$ behavior, fermionic antisymmetry, and the Pauli exclusion principle, without invoking additional quantum axioms or intrinsic spin degrees of freedom [24, 25].

Within this perspective, different particle attributes correspond to distinct topological invariants of admissible projected descriptions. Spin is associated with non-trivial covering properties of configuration space, while electric charge may be interpreted, at the effective level, as an oriented topological defect or vortex-like structure within projected configurations. These attributes remain conceptually distinct, yet arise from a common relational substrate once a geometric interpretation becomes applicable.

Taken together, these results provide a unified account of particle properties compatible with both relativistic and quantum phenomenology, without introducing particles or their attributes as fundamental entities. Particles appear instead as stable descriptive regimes of the underlying relational structure, whose observable properties reflect the topology of admissible projected configurations.

7 Gravity as a Collective Effect of Particle Excitations

7.1 Local Slowdown of Relaxation Ordering

Within the Cosmochrony framework, gravitation does not arise from a fundamental interaction or from an independent dynamical field. It emerges, at the level of effective descriptions, from the collective influence of localized projected configurations on admissible relaxation ordering.

As established in Sec. 6, particle-like projected configurations correspond to regions of enhanced resistance to admissible relaxation. When such configurations are present in significant number, their combined effect leads, in the weak-constraint regime, to a macroscopic reduction of the admissible ordering rate within projected descriptions.

In an effective spacetime parametrization, this collective effect may be expressed schematically as

$$\mathcal{D}_{\text{eff}}\chi_{\text{eff}} \;\simeq\; c\,(1-\alpha\,\rho)\,, \tag{28}$$

where ρ denotes the effective density of localized projected configurations and α encodes their average contribution to relaxation resistance. This expression represents a first-order approximation, valid when localized constraints are sufficiently dilute and weakly overlapping.

The coupling parameter α is not fundamental. It emerges as a collective property of admissible projected descriptions and depends on the typical structural characteristics of localized configurations. In the weak-field limit, dimensional consistency relates its scaling to the observed gravitational constant, leading to $\alpha \propto G/c^2$ when expressed in terms of effective inertial mass densities. Within this approximation, the reduction of admissible relaxation ordering admits a Newtonian-like interpretation in terms of an effective gravitational potential.

Physically, this collective slowdown manifests, within effective geometric descriptions, as gravitational time dilation and curvature effects. No independent gravitational force or mediator is introduced. Gravitation appears instead as a macroscopic signature of constrained admissible relaxation ordering induced by the presence of localized projected configurations.

7.2 Collective Gravitational Coupling and Operational Geometry

The collective reduction of admissible relaxation ordering described above affects not only the local accumulation of effective time, but also the manner in which variations within projected descriptions influence one another across extended regions. In the presence of localized projected configurations, the relaxation resistance they induce modulates how efficiently structural variations can be correlated between different effective locations.

At the level of effective descriptions, this collective behavior may be summarized by a constitutive coupling function characterizing the stiffness of projected configurations with respect to relative variations. In regions where projected descriptions are nearly homogeneous, this coupling approaches a uniform effective value. Localized projected configurations weaken it by introducing additional structural constraints. Crucially, this coupling is defined entirely within the effective descriptive framework and does not presuppose any fundamental spatial metric, background geometry, or pre-existing notion of distance.

Because no fundamental geometry is assumed, spatial separation is defined operationally. Two effective regions are considered close if structural variations of projected configurations can be efficiently correlated between them, and distant otherwise. In the continuum and weak-constraint regime, this operational notion admits a compact description in terms of an effective spatial metric, which summarizes the collective response of admissible projected descriptions to relative variations.

Within this framework, spacetime curvature does not arise as a primitive geometric property or as the effect of a fundamental gravitational field. It emerges instead as a descriptive manifestation of how localized projected configurations modulate the collective admissible ordering and correlation structure. Geometry therefore functions as a macroscopic encoding of constrained relational organization, rather than as an independent ontological entity.

A more explicit relational construction of the coupling mechanism, and its connection to discrete formulations of admissible projected descriptions, is presented in Appendix D.1.

7.3 Emergent Curvature

Spatial variations in admissible relaxation ordering, combined with the collective modulation of effective coupling strength, lead to non-uniform correlation patterns within projected descriptions. When a smooth geometric parametrization becomes applicable, these non-uniformities are compactly summarized by gradients of an emergent metric structure.

In Cosmochrony, spacetime curvature is therefore not a primitive geometric property nor the manifestation of an independent dynamical field. It is a descriptive construct encoding how localized projected configurations collectively modulate admissible ordering and correlation structure across extended regions. The metric does not act as a causal agent; it functions as a macroscopic summary of constrained relational organization within admissible projected descriptions.

Within effective geometric regimes, this emergent curvature reproduces the phenomenology traditionally attributed to curved spacetime in general relativity, including gravitational time dilation, geodesic deviation, and lensing effects. These phenomena arise here not from a fundamental spacetime geometry, but from the spatial variation of admissible relaxation ordering and correlation efficiency.

Crucially, this interpretation remains fully compatible with the pre-geometric and relational foundations of the framework. Geometry appears only as an effective and operational language, valid in regimes where projected χ configurations admit a smooth and slowly varying representation.

Einstein's Equations as a Structural Equilibrium Principle

Within the Cosmochrony framework, Einstein's field equations retain their full conceptual and physical legitimacy. They are not reinterpreted as approximations to a deeper gravitational dynamics, but as an exact and universal description of spacetime structure *whenever a geometric description is applicable.*

This perspective is fully aligned with Einstein's own methodological stance. General relativity does not describe the microscopic constitution of spacetime, but the necessary relations between geometry and physical content once spacetime itself is admitted as a meaningful concept. In this sense, Einstein's equations are already formulated at the correct descriptive level: that of emergent geometry.

In Cosmochrony, spacetime geometry is not assumed a priori, but arises from the admissible projection of the pre-geometric relational substrate χ. When projected χ configurations admit a smooth, locally injective geometric description, their collective structural constraints can be summarized by an effective metric $g_{\mu\nu}$. In this regime, Einstein's equations emerge *necessarily* as the unique consistency condition relating curvature to the effective distribution of relaxation-resistant configurations.

From this standpoint, the Einstein tensor does not encode a dynamical law acting on spacetime, but a geometric identity constraining admissible macroscopic descriptions. Likewise, the stress–energy tensor summarizes how localized projected configurations resist admissible relaxation ordering. The Einstein equations therefore express a balance condition between geometry and physical structure, not a force law.

This interpretation does not weaken general relativity. On the contrary, it explains its extraordinary universality. As long as a smooth spacetime description exists and relaxation ordering is monotonic, bounded, and weakly inhomogeneous, the same geometric relations must hold, independently of the microscopic nature of the underlying substrate. General relativity thus plays a role analogous to thermodynamics: exact within its domain, silent outside it, and remarkably insensitive to deeper ontological details.

Importantly, this view also clarifies the limits of applicability of Einstein's equations without attributing any failure to the theory itself. When projection ceases to be locally injective—near deprojection boundaries or strong structural constraints—spacetime geometry itself loses operational meaning. In such regimes, Einstein's equations do not break down; they simply no longer apply, because the concept of spacetime has not yet emerged.

In this sense, Cosmochrony does not go beyond Einstein by correcting general relativity. It goes beneath it, by explaining why Einstein's equations are inevitable wherever spacetime exists at all.

7.4 Recovery of the Schwarzschild Metric

In the presence of a static and approximately spherically symmetric distribution of localized projected configurations, the collective reduction of admissible relaxation ordering admits a particularly simple effective description. In the weak-constraint and quasi-static regime, spatial variations of the effective ordering rate are governed by a Poisson-like relation linking an effective potential to the density of localized projected configurations.

When a geometric parametrization becomes applicable, this structure is compactly summarized by an effective metric whose leading-order form coincides with the Schwarzschild solution of general relativity. In this description, the temporal component encodes the local reduction of admissible relaxation ordering (interpreted as gravitational time dilation), while the radial component reflects the corresponding modulation of correlation efficiency in the spatial sector. The angular part of the metric follows from the approximate isotropy of the projected configuration.

Within this framework, the standard weak-field predictions of general relativity are recovered, including gravitational redshift, light deflection, and time dilation in agreement with solar-system observations. The gravitational constant G appears here as an emergent collective coupling parameter, relating the effective density of localized projected configurations to the magnitude of the admissible ordering slowdown.

Operational potential from χ-relaxation slowdown (weak-field limit).

In the projectable regime where an effective geometric parametrization applies, gravitational time dilation is encoded as a *local slowdown* of the relaxation ordering rate relative to its asymptotic value far from localized excitations. We therefore introduce the dimensionless lapse-like factor

$$N(r) \equiv \frac{D^{\chi}_{\text{loc}}(r)}{D^{\chi}_{0}}, \qquad 0 < N(r) \leq 1, \tag{29}$$

and define an effective Newtonian potential Φ through the weak-field identification

$$\frac{D^{\chi}_{\text{loc}}}{D^{\chi}_{0}} \simeq 1 + \frac{\Phi}{c^2}, \qquad \left|\frac{\Phi}{c^2}\right| \ll 1. \tag{30}$$

This relation summarizes how localized projected configurations reduce admissible relaxation ordering, in a form directly comparable with standard gravitational phenomenology.

Poisson-like equation and exterior solution.

In the weak-structure regime (small internal gradients compared to the saturation scale), coarse-graining the constrained relaxation dynamics yields the effective elliptic relation

$$\nabla^2\Phi \simeq 4\pi G_{\text{eff}}\, \rho, \tag{31}$$

where ρ is the density of localized relaxation-resistant projected configurations and G_{eff} is an emergent collective coupling. :contentReference[oaicite:2]index=2 For an isolated spherically symmetric source of total mass M, the exterior solution (r outside the source support) is therefore

$$\Phi(r) \simeq -\frac{G_{\text{eff}} M}{r}. \tag{32}$$

Metric components from Φ (explicit weak-field matching).

Once an effective geometric description becomes applicable, the metric is not postulated as a fundamental field equation solution; it is introduced as a compact operational encoding of how the slowdown factor $N(r)$ modulates proper-time accumulation and spatial correlation efficiency. Consistency with the interpretation of $N(r)$ as the local time-dilation factor implies the standard static spherically symmetric ansatz

$$ds^2 = -N(r)^2 c^2 dt^2 + N(r)^{-2} dr^2 + r^2 d\Omega^2. \tag{33}$$

In the weak-field limit, using (30) gives

$$g_{tt} \simeq -\left(1 + 2\frac{\Phi}{c^2}\right), \tag{34}$$

$$g_{rr} \simeq \left(1 + 2\frac{\Phi}{c^2}\right)^{-1} \simeq 1 - 2\frac{\Phi}{c^2}, \tag{35}$$

which coincides with the standard weak-field expansion of the Schwarzschild metric after substituting (32):

$$g_{tt} \simeq -\left(1 - \frac{2G_{\text{eff}}M}{c^2 r}\right), \qquad g_{rr} \simeq \left(1 - \frac{2G_{\text{eff}}M}{c^2 r}\right)^{-1}. \tag{36}$$

Equivalently, one may define the operational Schwarzschild radius by weak-field matching

$$r_s \equiv \frac{2G_{\text{eff}}M}{c^2}, \tag{37}$$

so that $N(r)^2 = 1 - r_s/r$ reproduces the standard Schwarzschild form at leading order. :contentReference[oaicite:3]index=3

When a geometric parametrization becomes applicable, this structure is compactly summarized by an effective metric whose leading-order form coincides with the Schwarzschild solution of general relativity. In this description, the temporal component encodes the local reduction of admissible relaxation ordering (interpreted as gravitational time dilation), while the radial component reflects the corresponding modulation of correlation efficiency in the spatial sector. The angular part of the metric follows from the approximate isotropy of the projected configuration.

Within this framework, the standard weak-field predictions of general relativity are recovered, including gravitational redshift, light deflection, and time dilation in agreement with solar-system observations. The gravitational constant G appears here as an emergent collective coupling parameter, relating the effective density of localized projected configurations to the magnitude of the admissible ordering slowdown.

Comparison to classic observational tests (weak-field regime).

Gravitational redshift. From (34), clock rates satisfy

$$\frac{\nu_{\text{obs}}}{\nu_{\text{emit}}} = \sqrt{\frac{-g_{tt}(r_{\text{obs}})}{-g_{tt}(r_{\text{emit}})}} \simeq 1 + \frac{\Phi(r_{\text{emit}}) - \Phi(r_{\text{obs}})}{c^2}, \tag{38}$$

which is the standard redshift formula tested by laboratory experiments and routinely accounted for in satellite navigation systems.

Light deflection. In Cosmochrony, light propagation may be described (in the projectable regime) as following wavefronts of constant χ; equivalently, one may introduce an effective refractive index whose weak-field expansion yields the standard deflection angle

$$\alpha \simeq \frac{4G_{\mathrm{eff}}M}{bc^2}, \tag{39}$$

with b the impact parameter, matching the general-relativistic prediction used in solar-system lensing and high-precision astrometry. :contentReference[oaicite:4]index=4

Importantly, the Schwarzschild metric is not postulated as a fundamental solution, nor is spacetime curvature treated as a primitive dynamical entity. The metric functions instead as a compact and operational summary of how localized projected configurations constrain admissible relaxation ordering in their vicinity.

Schwarzschild-like behavior therefore does not reflect a specific dynamical law of spacetime itself. It emerges as the necessary phenomenological description in regimes where projected configurations are close to local equilibrium and admit a smooth geometric representation.

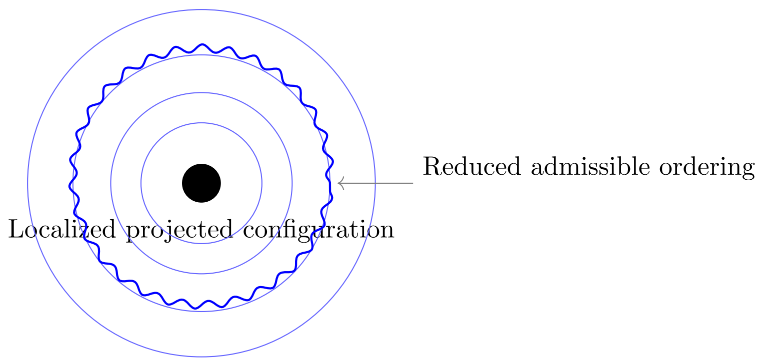

Fig. 5 Emergence of Schwarzschild-like behavior in Cosmochrony. A localized projected configuration induces a spatially varying reduction of admissible relaxation ordering. In effective geometric descriptions, this manifests as differential proper-time accumulation and an emergent metric curvature analogous to gravitational time dilation.

7.5 Equivalence Principle

Within the Cosmochrony framework, particle-like excitations do not couple to a fundamental gravitational field. They are described instead, at the level of effective descriptions, as localized projected configurations that impose structural constraints on admissible relaxation ordering.

Crucially, all admissible localized projected configurations constrain relaxation ordering in the same universal manner. Their internal composition, detailed structure, or microscopic realization plays no role in how they affect or respond to the admissible ordering environment. The collective reduction of effective relaxation ordering therefore depends only on the presence of localized structural constraints, not on their specific nature.

As a consequence, all particle-like projected configurations respond identically to a given effective ordering environment. When expressed in an effective geometric language, this universal response appears as composition-independent acceleration in a gravitational field.

The equivalence between inertial and gravitational behavior thus emerges as a direct structural necessity. Inertial resistance and gravitational response are not distinct physical properties, but two effective manifestations of the same underlying constraint on admissible relaxation ordering.

In this sense, the equivalence principle is not an independent postulate within Cosmochrony. It arises inevitably as an emergent symmetry of admissible projected descriptions, reflecting the absence of any fundamental distinction between inertial and gravitational mass at the effective level.

7.6 Gravitational Waves

Time-dependent variations in the distribution of localized projected configurations, such as accelerating masses or mergers of compact systems, induce collective and transient modulations of admissible relaxation ordering. These modulations propagate through the projected description as changes in the effective ordering regime and are transmitted at the maximal admissible ordering speed c.

When expressed in an effective spacetime language, such propagating modulations are described as gravitational waves. Unlike electromagnetic radiation, which corresponds to propagating particle-like projected excitations, gravitational waves represent collective variations of the admissible ordering and correlation structure of projected descriptions themselves.

In this framework, gravitational waves do not introduce additional fundamental degrees of freedom. They arise as macroscopic, collective responses of admissible projected descriptions to time-dependent reconfigurations of localized relaxation-resistant structures, rather than as excitations of an independent underlying field.

This interpretation preserves the phenomenology of general relativity in regimes where a smooth spacetime description applies. The standard properties of gravitational waves—propagation at speed c, transverse polarization, and energy transport—are recovered as effective features of collective ordering dynamics.

It should be emphasized, however, that gravitational-wave descriptions remain valid only within regimes where projection onto an effective spacetime remains well defined. In strong-gravity environments approaching the deprojection threshold discussed in Section 7.7, such collective modulations are expected to become increasingly attenuated or to lose a clear spacetime interpretation altogether. Before turning to strong-gravity regimes, it is useful to summarize the different projective regimes discussed in this section.

This schematic overview highlights how gravitational waves, horizons, and black hole evaporation correspond to distinct regimes of projectability of the same underlying relational structure.

Projectable	→	**Horizon**	→	**Non-projectable**
Gravitational Waves		Boundary of Projection		Deprojected Regime
Effective Modulations		Π non-injective		No spacetime Representation

Fig. 6 Conceptual regimes of projection in Cosmochrony. Gravitational waves correspond to fully projectable collective modulations of admissible descriptions, while black holes mark the boundary beyond which spacetime representations cease to be injective. Black hole evaporation reflects the gradual restoration of projectability, without any loss of information at the fundamental relational level, but only a loss and recovery of spacetime representability.

7.7 Strong Gravity and Black Holes

In regions where the density of localized projected configurations becomes sufficiently high, admissible relaxation ordering becomes strongly constrained. In effective spacetime descriptions, this corresponds to a regime in which the local accumulation of effective time is strongly suppressed relative to distant observers, defining an effective horizon.

Within Cosmochrony, such regions are interpreted as black holes. Rather than being characterized by a fundamental spacetime singularity, black holes correspond to domains where physical processes become asymptotically inaccessible from the exterior due to the loss of injectivity of spacetime projection. This naturally accounts for extreme time dilation effects without requiring divergent curvature invariants.

These regions therefore mark not a terminal endpoint of physical description, but a transition toward a non-projectable regime of the underlying relational structure.

Gravitational and Temporal Shadows

In the strong-gravity regime, the increasing concentration of localized projected configurations induces severe constraints on admissible ordering. As a result, the effective progression of time within the region slows asymptotically with respect to external descriptions.

This behavior reproduces the phenomenon commonly referred to as a *gravitational shadow*. In general relativity, such shadows arise from the absence of escaping null geodesics within a characteristic angular region. In Cosmochrony, an equivalent observational signature emerges because effective propagating descriptions, including radiation-like modes, no longer admit a faithful spacetime representation once projectability is lost. External observers therefore perceive a dark angular region corresponding to the projection of a non-projectable domain.

Beyond this optical effect, the framework predicts a deeper phenomenon, which may be termed a *temporal shadow*. As projectability is progressively lost, internal processes become indefinitely delayed in effective spacetime descriptions. From the external perspective, physical evolution appears frozen, providing a natural interpretation of horizon-induced time dilation.

In this view, the observed gravitational shadow corresponds to the visible manifestation of an underlying temporal shadow. Both effects arise from the same loss of projective representability and do not require a fundamental spacetime singularity.

Absence of Physical Singularities

In classical general relativity, black holes are associated with spacetime singularities characterized by divergent curvature and energy density. In Cosmochrony, such singularities are interpreted as artifacts of extending effective spacetime descriptions beyond their domain of validity.

Because admissible ordering is bounded, configurations corresponding to infinite curvature or density cannot be physically realized. Apparent singularities therefore signal the breakdown of spacetime representability rather than genuine divergences of the underlying relational structure.

Structural bound and notation.

To avoid confusion between fundamental and emergent levels, we distinguish the dimensionless structural bound c_χ, defined at the level of the pre-geometric relational substrate, from its emergent spacetime manifestation c, interpreted as the maximal signal propagation speed. While c may exhibit effective regime-dependent variations, the bound c_χ is invariant.

Black Holes, Deprojection, and Vacuum Reprojection

Within Cosmochrony, the absence of physical singularities does not imply that black holes are dynamically inert. Rather, they correspond to regimes in which the projection of relational information onto spacetime ceases to be injective.

The emergence of an effective spacetime description relies on a projection map

$$\Pi : \mathcal{C}_{\mathrm{rel}} \longrightarrow \mathcal{M},$$

from the space of relational configurations to an effective spacetime manifold. In weak- and moderate-field regimes, this map is locally injective, ensuring a faithful geometric encoding.

In strong-gravity regimes, this injectivity breaks down. Multiple inequivalent relational configurations correspond to the same effective spacetime event, signaling a loss of representability without any loss of information. We refer to this loss of injectivity as *deprojection*.

Deprojection does not correspond to transport across a spatial boundary nor to a temporal reversal. Instead, relational information ceases to be expressible in spatiotemporal form and remains encoded structurally.

Importantly, deprojected information is not destroyed. Because the underlying relational configuration remains globally defined, information is in principle reprojectable once projectability is restored. Reprojection occurs discretely and manifests in effective spacetime descriptions as radiation-like excitations or particle–antiparticle pairs.

The deprojection regime associated with black hole horizons does not imply the absence of dynamical processes. While smooth metric evolution ceases, the χ substrate remains structurally active. In particular, reprojection may occur intermittently when local configurations reach the threshold required for effective visibility. In the following section, this process is formalized through an explicit reprojection flux equation governing black hole evaporation.

Information conservation and unitarity.

Deprojection does not correspond to information loss. It marks the loss of spacetime encoding, not the destruction of correlations. At the fundamental relational level, global information is preserved. Apparent non-unitarity arises only within projected spacetime descriptions and reflects their limited domain of applicability.

This reprojection mechanism is analogous to vacuum fluctuations (Section 13.4), where structural information is temporarily non-projectable before re-emerging as radiation.

Black Hole Entropy as Relaxation Saturation

Within the Cosmochrony framework, black hole entropy does not quantify missing information encoded on a geometric surface, nor does it rely on a fundamental holographic postulate. Instead, it measures the *saturation of relaxation capacity* imposed by the Born–Infeld–like bound governing the projected dynamics of the relational substrate χ.

As discussed in Section 5.4, the effective Born–Infeld structure enforces an upper bound on admissible gradients of χ_{eff}, reflecting a maximal transmittance of the projection $\Pi : \chi \mapsto \chi_{\text{eff}}$. In strong-field regimes, this bound is reached at a finite radius, where the effective temporal ordering ceases to be resolvable. Operationally, this corresponds to the vanishing of the effective relaxation rate, $\partial_t \chi_{\text{eff}} \to 0$, and defines the horizon as a *freezing surface* of the emergent spacetime description.

The entropy associated with this surface arises from the non-injectivity of the projection at saturation. At the horizon, a large multiplicity of distinct micro-configurations of the substrate χ become indistinguishable at the level of the effective metric $g_{\mu\nu}$. The black hole entropy therefore quantifies the logarithmic measure of this degeneracy,

$$S_{\text{BH}} \;\sim\; \log \left|\Pi^{-1}(g_H)\right| , \tag{40}$$

where g_H denotes the saturated horizon geometry. The emergence of an area law follows directly from the fact that saturation occurs at the boundary between projectable and non-projectable regimes, rather than within a bulk volume.

Within this interpretation, Hawking thermality is not attributed to vacuum pair creation on a fixed background, but to the coarse-grained statistics of discrete reprojection events occurring at the saturation interface. The associated temperature is controlled by the normal gradient of χ_{eff} at the horizon, providing a unified explanation of black hole entropy, temperature, and horizon dynamics as manifestations of a single relaxational constraint.

This perspective reframes black hole thermodynamics as a kinematic consequence of bounded relational relaxation, thereby unifying gravitational, thermodynamic, and quantum phenomena without invoking additional microscopic degrees of freedom such as strings, loops, or fundamental horizons.

7.8 Black Hole Evaporation and the Information Problem

Within the Cosmochrony framework, black holes are not associated with physical singularities, but with regions where spacetime projection ceases to be injective. Such regions define domains of limited representability rather than physical interiors.

Evaporation as a Projective Phenomenon.

Black hole evaporation is an effective process unfolding entirely within the projectable regime. It arises from the gradual restoration of projectability near the boundary separating projectable and non-projectable domains.

As projectability is progressively recovered, localized projected configurations cease to be supported and are replaced by radiation-like effective descriptions. The evaporation process completes before any effective description would encounter a non-projectable singular regime.

Resolution of the Information Paradox.

The apparent information loss identified by Hawking arises from applying a fundamentally spacetime-based description beyond its domain of validity [26]. In Cosmochrony, information is encoded in the global relational configuration independently of its spacetime projection. Evaporation therefore does not violate unitarity; it reflects a change in the domain of representability.

Observational Implications.

To external observers, emitted radiation appears nearly thermal and weakly correlated with infalling states. This reflects the coarse-grained nature of spacetime projection rather than genuine information loss. The black hole information paradox is thus resolved by recognizing it as an artifact of extrapolating spacetime concepts beyond their domain of validity.

Horizon Reprojection Equation

Within Cosmochrony, black hole evaporation is described as a reprojection process by which structural energy stored in the χ substrate is released into the projected spacetime in discrete units.

The energy flux emerging from the horizon is defined as a sum over reprojection events associated with micro-configurations of the projection fiber:

$$\Phi_\chi \equiv \frac{dE}{dt} = \sum_k \delta(t - t_k)\, \hbar_\chi\, \nu_k(L_{\mathrm{sol}}).$$

Here, $\hbar_\chi$ denotes the fundamental quantum of reprojection, setting the minimal granularity of projected action. The quantities $\nu_k(L_{\mathrm{sol}})$ correspond to the resonance frequencies associated with the eigenmodes of the stability operator L_{sol} acting on the projection fiber $\Pi^{-1}(g_H)$ at the horizon, where g_H denotes the effective near-horizon geometry. The times t_k label the instants at which local χ configurations reach the projection threshold and become effectively visible in spacetime.

Black hole evaporation thus proceeds as a sequence of discrete reprojection pulses, rather than as a continuous emission process.

Emergent Temperature and Relaxation Gradient

The apparent Hawking temperature perceived by a distant observer emerges from the statistical distribution of reprojection events. It is determined by the gradient of effective χ relaxation normal to the horizon.

This relation may be expressed as

$$k_B T_\chi = \frac{\hbar_\chi c}{2\pi} \left| \nabla_\perp \chi_{\text{eff}} \right|_H .$$

For more massive black holes, the relaxation gradient is distributed over a larger horizon area, reducing the frequency of reprojection events. This directly explains the inverse mass–temperature relation without invoking vacuum particle creation or fundamental thermodynamic assumptions.

Information Conservation and Spectral Encoding

In contrast with semiclassical descriptions, information is never destroyed in Cosmochrony. The projection operator Π acts as a filter rather than as an irreversible map.

During the deprojection phase, information is stored in the nonlinear degrees of freedom of the χ substrate, encoded within the projection fiber. During reprojection, the emitted radiation carries the precise spectral imprint of the eigenmodes governing the χ configuration.

Each emitted quantum corresponds to a specific transition within the substrate, ensuring global information conservation at the level of the χ substrate. The apparent loss of information arises solely from restricting attention to the projected metric degrees of freedom.

Entropy as a Projection Saturation Limit

Within the Cosmochrony framework, the recovery of the Bekenstein–Hawking area law,

$$S = \frac{A}{4}, \tag{41}$$

does not rely on the introduction of a temperature or on thermodynamic postulates. Instead, black hole entropy is reinterpreted as a measure of the informational capacity of the projection map Π at the horizon boundary.

The Horizon as a De-projection Boundary

In the near-horizon regime, the χ substrate approaches a state of critical constraint in which the mapping $\Pi : \chi \to g_{\mu\nu}$ ceases to be injective (see Section 7.7.3). The horizon is therefore redefined as the locus at which the local relaxation rate $N(r)$ vanishes, preventing further refinement of the projected metric degrees of freedom.

At this boundary, distinct micro-configurations of the substrate are mapped onto the same effective horizon geometry g_H. Entropy is identified with the logarithmic measure of the fiber $\Pi^{-1}(g_H)$, that is, the structural multiplicity of χ configurations that are no longer distinguishable at the metric level. Entropy thus quantifies hidden relational structure, rather than thermal ignorance.

This situation is schematically illustrated in Figure 7.

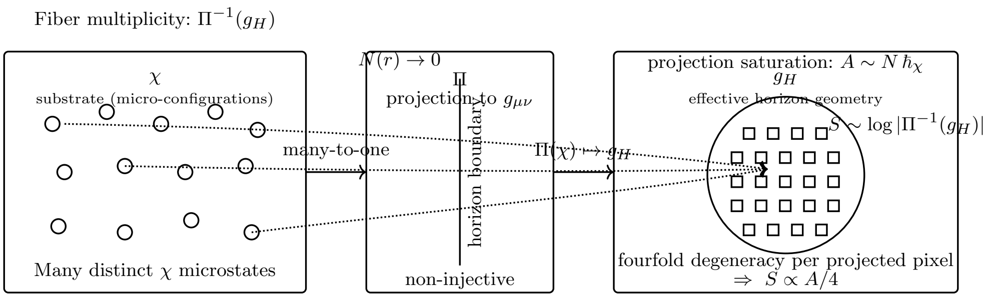

Fig. 7 Saturation of the projection map at the horizon. Near the boundary where $N(r) \to 0$, the projection Π becomes non-injective: multiple micro-configurations of the χ substrate collapse onto the same effective horizon geometry g_H. Entropy measures the structural multiplicity of the fiber $\Pi^{-1}(g_H)$ and the saturation of discretely projected spectral elements of characteristic area $\sim \hbar_\chi$.

Geometric Origin of the 1/4 Factor

The numerical factor $1/4$ arises as a structural ratio between the internal degrees of freedom of the χ substrate and their maximal holographic projection onto a two-dimensional boundary.

Let $\hbar_\chi$ denote the elementary quantum of reprojection. The horizon area A is saturated by a finite number N of discrete projected spectral elements of characteristic area $\sim \hbar_\chi$. Within the Cosmochrony framework, each such projected element corresponds to a fourfold degeneracy in the stability spectrum of χ, linked to the intrinsic 4π periodicity of χ excitations discussed in Section 6.2

The area law $S = A/4$ is therefore the macroscopic signature of this quadrature constraint: it represents the maximal density of independent structural degrees of freedom that can be projected before the non-injectivity of Π induces a complete loss of local metric resolution.

Within the Cosmochrony framework, this provides a structural interpretation of the numerical factor $1/4$, without claiming uniqueness or exclusivity with respect to other microscopic or statistical derivations of black hole entropy.

Unitary Reprojection and Information Conservation

Black hole evaporation is reformulated as a discrete reprojection process. As the χ substrate locally relaxes at the horizon boundary, it emits quanta of action $\hbar_\chi$ carrying the spectral imprint of the fiber $\Pi^{-1}(g_H)$.

Because this process is governed by the deterministic—though nonlinear—relaxation dynamics of χ, information is never destroyed nor trapped in a singularity. Instead, it is transferred from a purely structural encoding within the fiber back into observable spacetime degrees of freedom. Unitarity is preserved at the level of the χ substrate, even though the effective spacetime description may exhibit apparent non-unitarity.

Conclusion

In Cosmochrony, black hole entropy is not a measure of ignorance but a measure of projection saturation. It quantifies the threshold at which the complexity of the χ substrate exceeds the transmittance capacity of the effective metric projection. This explains why entropy scales with area—the projection surface—and not with volume, which characterizes the inaccessible internal fiber.

Prediction: Spectral Line Width and Substrate Granularity

If Hawking radiation is the macroscopic manifestation of a fundamentally discrete reprojection process, it cannot be a perfect black-body continuum. Within Cosmochrony, black hole evaporation is expected to produce a comb-like spectrum, whose fine structure directly reflects the local relaxation state of the χ substrate.

Reprojection Line Width.

We define the spectral line width $\Delta\nu_k$ of a reprojected quantum as being controlled by the ratio between the local relaxation timescale τ_χ and the spectral packing fraction α introduced in Section B.8:

$$\Delta\nu_k \approx \frac{1}{\tau_\chi}\sqrt{\alpha}\,\ln(Q), \tag{42}$$

where:

- τ_χ is the characteristic relaxation time of the χ field near the horizon, linked to the local gradient and constraint structure of χ,
- $\alpha \simeq 3 \times 10^{-7}$ is the spectral packing fraction governing the density of admissible projection modes and previously shown to control the proton-to-electron mass ratio,
- Q is the topological charge of the emitted configuration.

Physical Interpretation.

In this framework, the line width $\Delta\nu$ is not an epistemic uncertainty but a direct measure of substrate fluidity. For massive black holes, relaxation is slow ($\tau_\chi \to \infty$), yielding extremely narrow, quasi-discrete spectral lines. As evaporation proceeds and the black hole approaches its end-point, τ_χ decreases, leading to progressive spectral broadening. Eventually, neighboring lines overlap, giving rise to an effective pseudo-continuum shortly before final dissipation.

Universal Granularity Relation.

A central prediction of Cosmochrony is that the spacing between adjacent spectral peaks $\delta\nu$ and their individual width $\Delta\nu$ are governed by the same structural constant α:

$$\frac{\Delta\nu}{\delta\nu} \propto \alpha. \tag{43}$$

This ratio is universal and independent of the black hole mass or the nature of the emitted quanta. The same dimensionless factor α appears in two physically distinct contexts: (i) the proton-to-electron mass ratio, $m_p/m_e \simeq \sqrt{1/\alpha}$, emerging from the solitonic topology of the χ substrate, and (ii) the ratio $\Delta\nu/\delta\nu \simeq \alpha$ governing the spectral line width of Hawking radiation, arising from projection saturation at the horizon. The occurrence of the same constant across regimes separated by nearly twenty orders of magnitude in energy strongly suggests a deep universality of spectral transmittance in the χ field.

Observational Prospects: Detectability of Spectral Granularity

The comb-like spectral structure predicted by Cosmochrony provides a distinct observational signature, in principle accessible to future high-precision experiments.

Gravitational-Wave Signatures.

Although standard Hawking radiation is electromagnetic, the discrete relaxation of the χ substrate also implies a quantized gravitational response. In the late stages of black hole evaporation, the spacing between reprojection events $\delta\nu$ may enter the sensitivity bands of next-generation interferometers such as LISA or the Einstein Telescope. Such spectral granularity may be detectable by future gravitational-wave interferometers, such as LISA (operational horizon ∼2035), provided a relative spectral resolution of order $\Delta Q/Q \sim 10^{-6}$ can be achieved. A stochastically granular or intermittently coherent component in the high-frequency gravitational-wave background would constitute a direct signature of the $\hbar_\chi$ quantum.

Analogue Black Holes.

The relation $\Delta\nu/\delta\nu \propto \alpha$ is universal and should apply to analogue gravity systems, including Bose–Einstein condensates, optical fibers, or hydrodynamic horizons. In such systems, the χ substrate is replaced by the physical medium, while the role of projection is played by the effective horizon mapping. Observation of a constant spectral packing fraction emerging from the stability operator of an analogue horizon would provide strong evidence that evaporation granularity is a generic consequence of non-injective projections rather than a peculiarity of quantum gravity. In laboratory analogue black holes, including acoustic horizons in Bose–Einstein condensates, current experiments already approach the required spectral resolution, with $\delta f/f \sim 10^{-5}$, making near-term tests of the universal relation $\Delta\nu/\delta\nu \simeq \alpha$ feasible.

Unlike several quantum-gravity approaches in which observable deviations are confined to Planck-scale energies ($E_{\text{Planck}} \sim 10^{19}$ GeV), the spectral signatures predicted by Cosmochrony emerge whenever the projection map Π becomes non-injective. This includes regimes ranging from elementary particle masses to astrophysical horizons. The resulting multi-scale universality renders the framework falsifiable with current or near-future experimental technologies.

7.9 Unified Origin of Gravitational and Electromagnetic Effects

Within the Cosmochrony framework, gravitational and electromagnetic phenomena do not originate from distinct fundamental entities. They arise instead as complementary effective manifestations of the same underlying relational substrate, once projected into regimes admitting a spacetime description.

At the fundamental level, no independent gravitational or electromagnetic fields are postulated. Physical interactions appear only at the level of admissible projected descriptions, as distinct modes of constrained relational organization of the χ substrate.

Gravitational effects correspond to sustained and quasi-static constraints on admissible relaxation ordering induced by persistent localized projected configurations. When expressed in effective geometric terms, these constraints manifest as time dilation, attraction, and spacetime curvature. Gravitation therefore reflects cumulative and long-lived constraints on admissible ordering, rather than propagating interaction carriers.

Electromagnetic phenomena, by contrast, arise from time-dependent and phase-structured patterns within admissible projected descriptions. These patterns admit an effective formulation in terms of propagating, oscillatory modes carrying both attractive and repulsive interactions, consistent with the phenomenology of electromagnetic radiation and forces.

In this sense, gravitation and electromagnetism differ not by their ontological origin, but by the temporal and structural organization of admissible projected descriptions. Gravitational phenomena correspond to quasi-static, cumulative constraints on relaxation ordering, whereas electromagnetic phenomena correspond to dynamically structured and phase-coherent oscillatory regimes. The familiar distinction between the two interactions thus emerges at the level of effective descriptions, rather than from fundamentally separate fields.

Within effective spacetime regimes, the dynamic projected patterns associated with electromagnetism admit a formulation equivalent to classical electrodynamics. An explicit derivation of the corresponding Maxwell-like equations within the Cosmochrony framework is provided in Appendix A.11.

7.10 Summary

Within the Cosmochrony framework, gravity does not arise as a fundamental interaction or as an independent geometric degree of freedom. It emerges at the level of effective descriptions as a macroscopic consequence of localized projected configurations collectively constraining admissible relaxation ordering.

Classical gravitational phenomena—including gravitational time dilation, effective spacetime curvature, gravitational waves, and black holes—are recovered as distinct descriptive regimes of this collective constraint. They reflect variations in the projectability and correlation structure of admissible descriptions, rather than the dynamics of a fundamental spacetime or gravitational field.

In regimes of extreme constraint, such as horizons, gravitation does not merely act as an emergent organizing principle, but also as a diagnostic interface. The spectral structure of horizon-associated phenomena encodes direct information about the relaxation dynamics and projection topology of the underlying χ substrate, providing, in principle, observational access to its micro-structural organization.

In this perspective, gravitation appears as an emergent and operational phenomenon, summarizing how localized projected configurations collectively constrain admissible ordering and correlation across extended regions, without introducing gravity as a primitive force or a fundamental geometric entity.

8 Quantum Phenomena and Entanglement

Scope and strategy.

This section explains how quantum phenomenology can arise as an effective statistical description of the projection process itself. The key structural ingredient is the *non-injective* character of the projection Π: distinct underlying χ-configurations can correspond to the same effective observables, and a single underlying configuration may give rise to multiple effective images depending on the operational context of projection. We emphasize an **Ontological Monism** viewpoint: there is a single ontological source χ, while apparent multiplicity and spatial separation are properties of the projected description. Within this setting, "nonlocality" does not require any signal transmission, because the correlated outcomes do not originate from independent ontological subsystems.

8.1 Non-Injective Projection as the Origin of Quantum Correlations

The emergence of quantum correlations and entanglement in the Cosmochrony framework is rooted in a single structural property of the projection from the fundamental χ substrate to effective physical descriptions: this projection is *non-injective*.

A single admissible configuration of χ may correspond to multiple distinct effective degrees of freedom. What appear, at the effective level, as separate particles or subsystems are therefore multiple projective manifestations of a single underlying ontological configuration.

This non-injectivity implies that such degrees of freedom cannot be assigned independent states. Any admissible effective description must be globally consistent with the underlying χ configuration, leading to persistent correlations that do not rely on signal exchange, causal influence, or spacetime proximity.

Ontological Monism and the Shared Projection Hypothesis

In Cosmochrony, the apparent multiplicity of effective particles does not imply a multiplicity of ontological entities. The framework adopts **Ontological Monism**: there is a single ontological source, the substrate χ. What appears as "two systems" in spacetime may correspond to a single underlying relational configuration.

Shared projection hypothesis.

Because the projection Π is non-injective and may admit multiple effective images, a single connected excitation in χ can be represented as several spatially separated effective excitations in the emergent description. The observed separation is therefore a property of the projected metric representation, not a fundamental separation of the underlying entity.

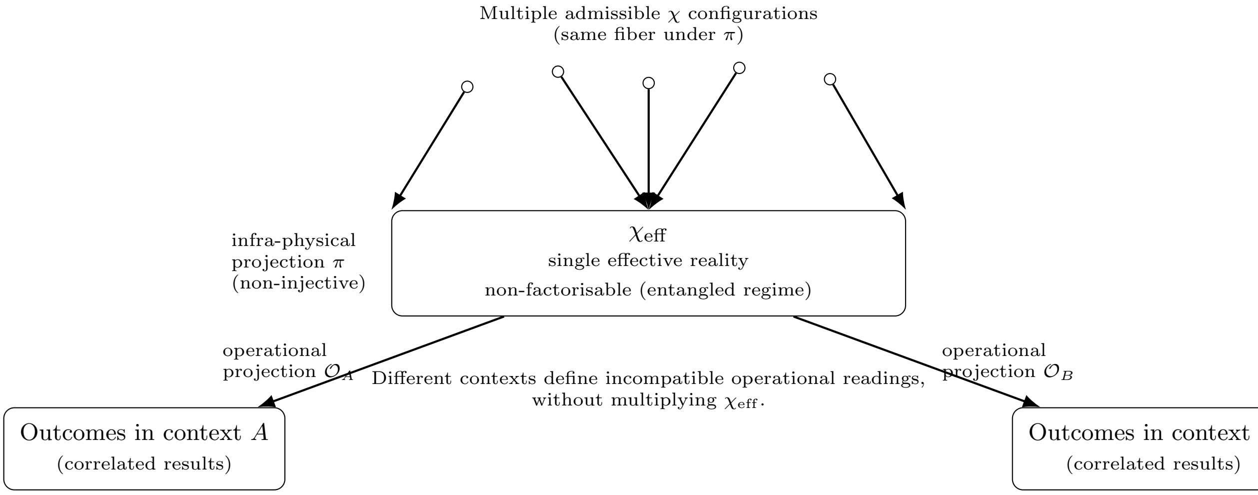

Fig. 8 Entangled (non-factorisable) regime. Multiple underlying χ configurations may correspond to the same effective configuration χ_{eff} (non-injective infra-physical projection). Entanglement is represented by a single χ_{eff} that cannot be decomposed into independent subsystems. Different measurement contexts (e.g., different bases) define distinct operational projections $\mathcal{O}_A$ and $\mathcal{O}_B$, producing correlated outcomes without introducing multiple effective realities.

Shared Fiber Phase and a Geometric Reading of Spin Correlations

In the Cosmochrony interpretation, spin correlations can be read as correlations of a *shared* internal degree of freedom of the projection fiber, rather than as correlations between independent spacetime-local properties. When an entangled pair admits a description as multiple effective images of a single underlying χ-configuration (Section 8.1), a measurement at one location selects a locally stable reprojection of that shared fiber degree of freedom. The correlated statistics at the distant location then reflect the restricted set of reprojections still compatible with the same underlying configuration, without invoking any signal exchange.

This viewpoint is compatible with the emergence of half-integer spin from nontrivial fiber topology (e.g. Hopf-type structures), while keeping the explanation at the structural level: the relevant degrees of freedom are global properties of the shared configuration, not attributes of the two effective particles taken separately.

Consequence for nonlocal correlations.

Correlations between measurements performed at spacelike separated locations do not require any superluminal influence. They follow from the fact that the correlated outcomes are local reprojections of the *same* underlying configuration. In this sense, the “Bell tension” is not resolved by a faster-than-light mechanism, but by denying the premise that two independent ontological subsystems existed in the first place.

8.2 Nonlocality and the Holistic Character of Projected Descriptions

From the standpoint of **Ontological Monism**, entangled systems are not composed of ontologically independent parts: they are multiple effective images of a single χ-configuration, constrained by shared projectability conditions.

In the Cosmochrony framework, quantum nonlocality does not arise from superluminal interactions or from violations of relativistic causality [27]. Instead, it reflects the intrinsically non-factorizable character of certain admissible projected descriptions.

This non-factorizable character originates from the non-injective nature of the projection from the underlying χ substrate to effective descriptions: a single admissible configuration of χ may correspond to multiple effective degrees of freedom.

Entangled systems correspond, at the level of effective descriptions, to single single projected configurations originating from a common underlying χ configuration that cannot be decomposed into independent subsystems without loss of admissibility. Once such configurations have formed through interaction, their subsequent descriptions remain globally constrained, even when effective spacetime language assigns them to spatially separated regions.

The persistence of quantum correlations across spatial separation therefore follows from the relational structure of admissible projected descriptions, rather than from any spatial connectivity or signal exchange. Although effective geometric descriptions may associate distant locations with different parts of an entangled system, these locations correspond to correlated aspects of a single non-factorizable descriptive configuration.

In this sense, quantum nonlocality in Cosmochrony is structural rather than dynamical. The correlations are constrained by the global consistency conditions of admissible descriptions, while all local physical processes remain compatible with relativistic causality.

This non-factorizable character plays a crucial role in quantum measurement. Because entangled systems correspond to a single admissible projected configuration, measurement outcomes cannot be interpreted as revealing pre-existing local properties of the projected description. This follows from the fact that no unique injective mapping exists between local projected properties and the underlying χ configuration. Instead, decoherence suppresses relational alternatives within the space of admissible descriptions, yielding effectively independent local projections.

Local measurement outcomes correspond to particular reprojections selected from a space of structurally compatible descriptions. This selection does not involve nonlocal influence or hidden communication, but reflects the loss of access to global relational coherence within effective descriptions.

In this context, the Born rule does not encode a dynamical nonlocal mechanism. It reflects the statistical distribution of locally accessible outcomes compatible with a single non-factorizable descriptive structure once decoherence has occurred. Nonlocal correlations therefore arise from global descriptive consistency, while measurement statistics remain fully compatible with relativistic causality.

Crucially, admissible projected descriptions do not encode predetermined measurement outcomes. They define a space of relationally compatible realizations, whose effective selection occurs through decoherence and reprojection, within the limits imposed by spacetime representability.

8.3 Nonlocal Correlations Without Superluminality

Within the Cosmochrony framework, nonlocal quantum correlations do not arise from superluminal propagation of information. All admissible projected descriptions respect local causal constraints, and no measurement outcome influences another through dynamical signal exchange. This absence of information transfer is not enforced by a relativistic prohibition alone, but reflects a deeper ontological fact: there is no transmission of information because, at the level of the χ substrate, there is no fundamental separation to bridge. What appear as distinct outcomes correspond to local reprojections of a single underlying relational configuration.

Correlated outcomes instead arise because spacelike separated measurements correspond to different local reprojections of a single non-factorizable admissible projected description. Such descriptions cannot be decomposed into independent subsystems without loss of global consistency. This failure of factorization is not accidental, but reflects the absence of an injective mapping between effective subsystems and underlying χ configurations. As a result, the factorization assumptions underlying Bell-type inequalities are violated, while dynamical locality and relativistic causality remain intact. This point is grounded in the relational formulation, where factorization is not fundamental but only an approximate emergent feature of certain projectable regimes (see Appendix E, especially Appendix E.2.

In this perspective, quantum correlations reflect global descriptive consistency rather than hidden variables or pre-existing local properties. Measurement outcomes do not reveal predetermined values, but correspond to compatible local realizations selected from a shared non-factorizable descriptive structure. Equivalently, admissible projected descriptions determine a constrained *space* of mutually compatible local realizations, rather than a set of pre-assigned outcomes.

Cosmochrony therefore accounts for experimentally observed violations of Bell inequalities without invoking nonlocal forces, retrocausality, or hidden signal channels. Nonlocality appears as a structural feature of admissible projected descriptions, fully compatible with relativistic causal constraints. A compact way to state this is that the correlations are *ontological*, fixed by the non-injective global relational structure, rather than *dynamical*, mediated by any superluminal interaction.

8.4 Relation to Bell Inequalities

Bell's theorem [27] establishes that no physical theory reproducing the full set of quantum mechanical predictions can simultaneously satisfy locality, realism understood as outcome determinism conditioned on hidden variables, and statistical independence. The empirical violations of Bell inequalities therefore rule out any ontological completion of quantum mechanics based on factorizable hidden-variable models.

The Cosmochrony framework fully accepts Bell's theorem as a fundamental constraint. It does not seek to evade or weaken Bell inequalities, nor to restore locality or realism in their classical sense. Instead, it identifies the precise ontological assumption that fails within Bell-type derivations when applied to admissible projected descriptions.

Failure of ontological factorisability.

Standard Bell-type arguments assume that joint outcome probabilities admit a factorization of the form

$$P(a, b|x, y, \lambda) = P(a|x, \lambda)\, P(b|y, \lambda), \tag{44}$$

where λ denotes a complete specification of the underlying ontic state. In Cosmochrony, such a decomposition is not available, not because of nonlocal dynamical influences, but because admissible effective descriptions arise from *non-injective projections* of the underlying χ substrate.

Distinct fundamental configurations of χ may correspond to the same effective projected description, while remaining globally constrained. As a consequence, entangled systems correspond to equivalence classes of non-separable configurations that do not admit independent ontic pre-images for their subsystems. There exists no hidden variable λ with respect to which conditional factorization could be restored, even in principle.

No hidden variables and no superluminal influence.

The non-injectivity of the projection does not introduce hidden variables, local or nonlocal. The underlying degrees of freedom are neither accessible nor conditionable, and they cannot be used to screen correlations or define outcome-determining parameters. Accordingly, the violation of Bell inequalities in Cosmochrony does not rely on superluminal signalling, retrocausality, or a breakdown of relativistic causal structure.

Correlations are not mediated, transmitted, or dynamically generated between spatially separated subsystems. They reflect global admissibility constraints on projected descriptions, inherited from the holistic structure of the underlying configuration space.

Ontological rather than dynamical nonlocality.

In this sense, quantum nonlocality in Cosmochrony is ontological rather than dynamical. The failure of factorization arises from the structure of the projection map itself, not from any nonlocal interaction or exchange of information. Bell-type violations are therefore understood as a manifestation of the non-separable nature of admissible descriptions, rather than as evidence for superluminal influences.

This interpretation preserves the full empirical content of quantum mechanics, respects relativistic causality at the operational level, and provides a structural explanation for the inevitability of entanglement correlations within a non-injective relational framework.

8.5 Measurement, Decoherence, and Apparent Collapse

Within the Cosmochrony framework, quantum measurement does not involve a fundamental wavefunction collapse. At the fundamental level, no discontinuous update of the underlying relational substrate occurs. What is conventionally described as collapse arises entirely within the domain of effective projected descriptions.

This is a direct consequence of the non-injective nature of the projection from the underlying χ substrate: multiple effective descriptions may correspond to a single relational configuration, with no uniquely defined inverse mapping.

Measurement corresponds to the transition from a non-factorizable admissible projected description to a set of effectively factorized local projections. This transition is induced by interactions with an environment that progressively eliminate the accessibility of global relational coherence. As a result, alternative relational components cease to admit a joint effective description within a single spacetime representation.

From this perspective, decoherence corresponds to an effective reduction of the multiplicity of admissible non-injective projections to a set of locally injective descriptions.

Decoherence therefore does not represent a postulated measurement axiom, but a dynamical restriction on admissible projected descriptions [28]. It suppresses interference between incompatible descriptive branches by rendering their relative phase information inaccessible within spacetime representations. The underlying relational structure remains globally well defined, even though it can no longer be represented coherently at the effective level.

Importantly, decoherence does not destroy information. It limits the projectability of relational correlations into spacetime descriptions. In this sense, decoherence may be understood as a local and partial loss of projectability: certain relational distinctions persist structurally but cease to be jointly representable within an effective geometric description.

More extreme regimes, such as those associated with strong gravitational confinement, represent a limiting case of this mechanism. In such regimes, not only coherence but spacetime representability itself breaks down. Relational information remains globally encoded, but undergoes a complete loss of spatiotemporal projectability, beyond the domain in which decoherence and effective Hilbert-space descriptions can be meaningfully defined.

The apparent collapse observed in quantum measurement is therefore not a physical event, but the effective manifestation of a non-injective relational structure becoming only partially projectable into spacetime descriptions.

Do Quantum Particles Modify Their Past?

It is sometimes claimed that quantum mechanics allows particles to *modify their past*, particularly in delayed-choice or quantum eraser experiments. This statement is misleading.

In quantum mechanics, physical descriptions are constrained globally by consistency conditions. Certain properties—such as path information or temporal ordering—do not possess well-defined values independently of the measurement context. A later measurement choice does not alter a previously existing physical fact, but rather restricts the set of admissible descriptions compatible with the entire experimental configuration.

In the Cosmochrony framework, this behavior follows directly from the non-injective character of projection. A single underlying χ configuration may admit multiple effective descriptions, none of which defines a unique local past prior to measurement. The act of measurement does not modify the underlying relational structure; it reduces the multiplicity of admissible projections, rendering certain past properties effectively definable.

Thus, quantum mechanics does not imply retrocausal influence or violations of relativistic causality. Instead, it reveals that some features commonly attributed to the past are not fundamental ontological facts, but emergent properties of admissible projected descriptions.

8.6 Temporal Ordering and Relativistic Consistency

Within the Cosmochrony framework, temporal ordering is not defined by a global notion of simultaneity. It arises only at the level of effective descriptions, as an ordering of admissible projected events induced by constrained relaxation ordering, rather than by any fundamental or absolute time parameter.

This effective character of temporal ordering follows directly from the non-injective nature of the projection from the underlying χ substrate, for which no unique global temporal parametrization exists at the fundamental level.

Different observers may therefore assign different temporal orderings to spacelike separated events within effective geometric descriptions. Such differences reflect the observer-dependence of spacetime slicing and have no impact on the underlying relational consistency of admissible projected descriptions. No preferred foliation, global clock, or absolute temporal structure is selected at the fundamental level.

Entanglement correlations are thus fully compatible with relativistic causality. Because entangled correlations originate from a single underlying relational configuration, their consistency does not depend on any particular temporal ordering assigned within effective descriptions. They do not rely on any privileged reference frame or on a globally defined temporal ordering. Instead, they arise from non-factorizable admissible projected descriptions whose internal relational consistency is preserved under changes of effective spacetime coordinates and temporal slicings.

In this sense, relativistic covariance is maintained because the physical content of the theory resides in relational consistency conditions rather than in observer-dependent spatiotemporal labels. Temporal ordering remains an effective, observer-relative notion, while admissible correlations and their consistency relations remain invariant across all equivalent projected descriptions.

Relativistic covariance is therefore preserved not by enforcing a specific temporal structure, but by the invariance of the underlying non-injective relational configuration across all admissible projected descriptions.

Relation to Time-Symmetric and Two-State Vector Formulations

Several time-symmetric formulations of quantum mechanics, such as the two-state vector formalism, account for delayed-choice and post-selection effects by introducing both forward- and backward-evolving quantum states.

In the Cosmochrony framework, this apparent temporal symmetry is a consequence of **Ontological Monism**. All correlated outcomes originate from a single underlying χ configuration, rather than from dynamically interacting subsystems. What appears as spatial or temporal separation in effective descriptions does not correspond to a fundamental ontological separation. Consequently, no notion of information transmission—whether forward or backward in time—is required to account for quantum correlations.

What time-symmetric approaches encode through boundary conditions imposed at both initial and final times is here recovered as a purely structural feature of admissible projected descriptions. The future does not influence the past; rather, effective temporal ordering does not exhaust the relational information contained in the underlying configuration.

Time-symmetric formalisms thus appear as efficient descriptive tools within standard quantum mechanics, but they are not ontologically required once non-injective projection and irreversible relaxation are taken as fundamental.

8.7 Limits of Entanglement and Environmental Effects

Entanglement is not a generic or permanent feature of admissible projected descriptions. It arises only within restricted regimes in which a non-factorizable global description remains jointly projectable into an effective spacetime representation.

This restriction reflects the fact that entanglement is possible only as long as the effective description remains non-injectively related to the underlying χ configuration. Once this non-injectivity can no longer be sustained at the level of projected descriptions, entanglement necessarily breaks down.

Interactions with an environment progressively restrict the set of admissible projected descriptions. As additional degrees of freedom become dynamically coupled, global relational coherence ceases to be representable within a single effective description. The system then admits only effectively factorized local projections, and entanglement is no longer accessible at the level of projected descriptions.

From this perspective, environmental coupling enforces an effectively injective mapping between local projected descriptions and underlying relational structures, thereby suppressing non-factorizable correlations.

As a result, entanglement is most robust for effectively isolated systems and becomes increasingly fragile in macroscopic or strongly interacting environments. This transition does not correspond to a physical degradation of an underlying substrate, but to a progressive loss of projectability of non-factorizable descriptions.

In this sense, the emergence of classical behavior at large scales reflects a descriptive limitation rather than a fundamental quantum-to-classical transition. Classicality arises when only factorized projected descriptions remain admissible, without requiring any modification of the underlying relational structure or the introduction of additional postulates.

Classical behavior thus corresponds to regimes in which only effectively injective projected descriptions remain admissible, rendering non-injective correlations operationally inaccessible.

8.8 Integration with the Standard Model: A Spectral Interpretation

While the Cosmochrony framework is primarily pre-geometric, it must account for the known phenomenology of the Standard Model (SM). In this section, we provide a structural reinterpretation of gauge bosons and mass generation mechanisms.

These effective structures should be understood as arising from a non-injective projection of the underlying χ substrate, within regimes where stable, effectively injective descriptions become admissible.

These effective structures should be understood as arising from a non-injective projection of the underlying χ substrate, within regimes where stable, effectively injective descriptions become admissible.

Weak Boson Masses from Spectral Geometry

In Cosmochrony, the masses of the weak bosons $W^{\pm}$ and Z^0 emerge from the **spectral properties** of the Hodge Laplacian Δ_1 acting on 1-forms of the fiber bundle Π. The fiber admits a decomposition into invariant subspaces under the action of the electroweak gauge group, without invoking any quotient construction.

This well-defined spectral structure reflects the fact that, in the electroweak sector, the effective projection operates in a regime that is locally injective and therefore admits stable particle-like descriptions.

Invariant Subspaces of the Fiber

The space of 1-forms on Π decomposes into gauge-invariant subspaces:

- An invariant subspace $\Omega^1_{SU(2)}$ associated with the $SU(2)_L$ sector, corresponding to directions generated by the Lie algebra $\mathfrak{su}(2)$.
- An invariant subspace $\Omega^1_{U(1)}$ associated with the $U(1)_Y$ sector, generated by the abelian direction $\mathfrak{u}(1)$.

These subspaces are defined algebraically by symmetry and do not rely on any topological identification with representation dimensions.

Spectral Origin of Masses

Let $\lambda_{1,G}$ denote the smallest non-zero eigenvalue of Δ_1 restricted to the invariant subspace Ω^1_G. The effective masses are given by

$$m_W \propto \sqrt{\lambda_{1,SU(2)}}, \qquad m_Z \propto \sqrt{\lambda_{1,U(1)}}.$$

The existence of a non-zero spectral gap follows from the geometric constraints imposed by the χ-induced metric on Π.

The existence of a non-zero spectral gap follows from the absence of globally harmonic shear modes once the projection constraints are imposed.

Geometric Dependence of the Mass Ratio

The ratio

$$\frac{m_Z}{m_W} = \sqrt{\frac{\lambda_{1,U(1)}}{\lambda_{1,SU(2)}}}$$

depends on:

- the metric anisotropy induced by the projection of χ,
- the curvature structure entering the Weitzenböck decomposition,
- the distribution of spectral weight across invariant subspaces.

No numerical value is imposed *a priori*; the observed ratio is an emergent property of the fiber geometry.

Spectral Stability

The stability of the weak boson masses is ensured by the robustness of the spectral gap under smooth deformations of the χ-induced geometry. This provides a geometric explanation for the persistence of the electroweak mass hierarchy without free parameters.

Emergent Gauge Couplings

Gauge couplings in Cosmochrony arise from the **spectral response** of the fiber degrees of freedom under projection. They are defined through normalized heat-kernel traces evaluated at a finite geometric scale.

Normalized Heat Kernel Definition

Let Δ_G be the restriction of the Hodge Laplacian to the invariant subspace associated with gauge sector G. We define the normalized trace as

$$\widehat{\mathrm{Tr}}_G(\cdot) \equiv \frac{1}{\dim(\mathfrak{g})}\,\mathrm{Tr}(\cdot),$$

where $\mathfrak{g}$ is the corresponding Lie algebra.

The gauge couplings are then given by

$$g^2 = 4\pi \left[\widehat{\mathrm{Tr}}_{SU(2)}\big(e^{-t_0 \Delta_{SU(2)}}\big) - \widehat{\mathrm{Tr}}_{U(1)}\big(e^{-t_0 \Delta_{U(1)}}\big) \right],$$

$$g'^2 = 4\pi \, \widehat{\mathrm{Tr}}_{U(1)}\big(e^{-t_0 \Delta_{U(1)}}\big),$$

with

$$t_0 = L_{\mathrm{fiber}}^2.$$

The subtraction reflects the fact that only non-abelian shear responses contribute to the $SU(2)_L$ coupling beyond the common abelian background.

Weinberg Angle

The Weinberg angle follows directly from spectral asymmetry:

$$\tan^2 \theta_W = \frac{\widehat{\mathrm{Tr}}_{U(1)}(e^{-t_0 \Delta_{U(1)}})}{\widehat{\mathrm{Tr}}_{SU(2)}(e^{-t_0 \Delta_{SU(2)}})}.$$

This definition is invariant under rescaling of the fiber geometry.

Geometric Phase Transition and Mass Generation

In Cosmochrony, mass generation is understood as a **geometric phase transition** of the χ substrate, rather than as spontaneous symmetry breaking by a fundamental scalar field.

Spectral Density Functional

We define a spectral density functional

$$\chi_{\mathrm{crit}} = \sum_G \int_0^{\Lambda} \rho_G(\lambda) \, d\lambda,$$

where $\rho_G(\lambda)$ is the spectral density of Δ_1 restricted to the invariant subspace associated with gauge sector G, and Λ is a geometry-induced cutoff.

Phase Transition Mechanism

Below χ_{crit}, spectral weight is uniformly distributed and only massless modes are supported. Above χ_{crit}, spectral weight condenses into specific invariant subspaces, generating discrete non-zero eigenvalues:

$$m_n \propto \sqrt{\lambda_n}.$$

This transition may equivalently be interpreted as a change in projectability: below the critical threshold, the projection remains too non-injective to support stable massive modes, while above it, effective injectivity emerges within specific invariant subspaces.

Stability

This transition is stable under smooth deformations of the χ-induced geometry and does not rely on any vacuum expectation value.

Strong Sector: Topological Confinement and Color

The concept of "color" charge ($SU(3)$) is mapped to the three fundamental degrees of freedom of the proton's trefoil topology ($Q = 3$). Gluons are identified as the **topological binding waves** that maintain the coherence of the knotted configuration.

- **Topological Confinement:** Separating the components of a $Q = 3$ soliton requires a linear increase in the deformation of the χ substrate. The energy required to "untie" or stretch the knot exceeds the threshold for creating new solitonic pairs, providing a geometric origin for quark confinement.
- **Asymptotic Freedom:** At high energy (short distances), the internal components of the knot behave as quasi-free waves because the global topological constraint is not yet engaged by the local excitation. This renders the interaction *in principle* weaker at small scales, mimicking asymptotic freedom.

The Origin of Mass: Spectral Overlap vs. Yukawa Coupling

In Cosmochrony, the Higgs mechanism and its associated Yukawa couplings are replaced by the principle of **spectral overlap**. Fermion masses are not fundamental input parameters but emergent quantities determined by the resonance between the internal stability spectrum of localized solitonic configurations and the global relaxation flux of the χ field.

Each fermionic excitation is characterized by a discrete set of internal modes $\{\phi_n\}$ arising from the stability operator L_{sol} acting on topologically constrained configurations within the fiber Π. The effective inertial mass is then *in principle computable* as a resonance integral between these internal modes and the ambient relaxation flow:

$$m_{\text{eff}} \;\propto\; \int_{\text{Fiber}} \mathcal{S}(\phi_n) \cdot \mathcal{R}(\chi)\, d\Pi, \tag{45}$$

where $\mathcal{S}(\phi_n)$ denotes the spectral signature of the solitonic configuration and $\mathcal{R}(\chi)$ is the local density of the global relaxation flux. Mass thus measures the degree to which a localized excitation resists relaxation through spectral pinning.

Fermion Generations as Topological Classes.

Within this framework, the existence of multiple fermion generations is no longer attributed to independent Yukawa couplings but to the topological organization of the fiber Π. Localized fermionic excitations correspond to distinct **stable topological classes** (e.g. homotopy or Chern classes) of solitonic χ-configurations. The empirical observation of three fermion generations suggests that the fiber admits exactly three dynamically stable classes under relaxation. Higher-generation fermions are thus interpreted as increasingly complex, "knotted" realizations of the same underlying solitonic structure rather than as distinct fundamental fields.

Spectral Hierarchy and Mass Scaling.

The mass hierarchy $m_e \ll m_\mu \ll m_\tau$ arises from the ordered spectrum of L_{sol} associated with these topological classes. As topological complexity increases, the relaxation flux becomes increasingly constrained, inducing a non-linear pinching of the spectral overlap. To leading order, the mass of the n-th generation is associated with the n-th eigenvalue of the stability spectrum,

$$m_n \sim \mathrm{Spec}(L_{\mathrm{sol}})_n, \tag{46}$$

with mass ratios governed by the spectral gaps between successive eigenmodes. This provides a geometric and dynamical origin for the observed hierarchy without introducing arbitrary dimensionless couplings.

While the electron–muon ratio can be approximated by an exponential scaling of spectral separation, higher generations deviate from a simple progression. This deviation reflects a **spectral screening effect** arising when the fiber approaches a saturation regime in which additional internal degrees of freedom partially mitigate further pinching of the relaxation flux. The comparatively low mass of the τ thus encodes a genuine geometric effect, not a fine-tuned cancellation.

Geometric Origin of Mixing Matrices.

Flavor mixing emerges naturally from the geometric structure of the theory. Two inequivalent bases are distinguished: (i) the **mass basis**, defined by the eigenvectors of L_{sol}, and (ii) the **interaction basis**, defined by the principal axes of the projection operator Π along which gauge interactions act. The CKM and PMNS matrices arise as rotation matrices encoding the misalignment between these two bases. Mixing angles are therefore fixed by the geometry of the fiber rather than by independent phenomenological parameters.

CP Violation as Topological Torsion.

Within this interpretation, CP violation originates from the complex phase structure of the projection operator. If the fiber Π possesses non-trivial topological torsion, the reprojection of a solitonic excitation onto its anti-solitonic counterpart is not perfectly symmetric. This intrinsic geometric chirality manifests as a non-vanishing Jarlskog invariant and provides a structural origin for CP violation, linking it directly to the topology of the projection fiber.

Notably, the persistence of multiple fermion generations reflects the fact that, in the fermionic sector, effective injectivity is only partial, allowing distinct topological classes to coexist as separate projected realizations of the same underlying relaxation dynamics.

In summary, fermion masses, generations, flavor mixing, and CP violation emerge as unified consequences of the spectral and topological properties of solitonic χ-configurations. The flavor hierarchy problem is thus resolved without invoking fundamental Higgs couplings, but instead as a necessary outcome of the geometry and relaxation dynamics of the underlying substrate.

8.9 Summary

Within the Cosmochrony framework, the phenomenology conventionally described by the Standard Model—from gauge interactions to quantum correlations—emerges as a consequence of the spectral organization of the relational substrate χ and of the **non-injective** constraints imposed by its effective projection Π. No particle species, coupling constant, or interaction field is introduced as a fundamental ontological element.

- **Gauge Interactions as Projection Dynamics:** Interactions are not mediated by autonomous fields propagating on spacetime as fundamental entities, but by admissible modes of the projection process itself. The photon corresponds to scalar transmission modes, while the $W^{\pm}$ and Z^0 bosons arise as shear-like spectral modes whose effective masses reflect the spectral rigidity of the projection fiber.
- **Topological Origin of the Strong Sector:** Strong interactions and confinement are reinterpreted in terms of topological stability. Color charge does not represent an internal degree of freedom, but the energetic cost required to preserve the coherence of knotted solitonic configurations—such as the $Q = 3$ proton—under admissible deformations.
- **Mass as Spectral Overlap:** The Higgs mechanism is replaced by the principle of spectral overlap. Mass is not an intrinsic coupling parameter but an emergent measure of spectral resonance between a localized configuration and the global relaxation flux of the χ substrate. In this spectral framework, particle decay is understood as a loss of projective admissibility of metastable configurations under continued relaxation.
- **Quantum Phenomena as Limits of Projectability:** Entanglement and nonlocal correlations do not require superluminal signaling or hidden variables. They reflect the persistence of **non-injective, non-factorizable** admissible configurations across projection. Quantum mechanics thus appears as an effective statistical framework describing the limits of local projectability imposed by a globally non-injective relational structure.

In this perspective, the Standard Model is not a collection of fundamental particles and forces, but an effective theory describing the **harmonics of relaxation** selected by the projection from the relational substrate into spacetime. Discrete symmetries, coupling structures, and apparent constants arise as organized spectral features of this filtering process, rather than as independent postulates.

In regimes where the projection becomes effectively injective and locally projectable, this framework naturally reproduces the stable particle content and interaction structure of the Standard Model, while classical behavior emerges as the limiting case of maximal projective injectivity.

8.10 Non-Injective Projection, Entanglement, and Structural Transitions

The non-injective character of the projection from the χ-substrate to effective observable descriptions plays a central role in the emergence of quantum phenomena in Cosmochrony. A single relational configuration of χ may admit multiple effective descriptions that cannot be decomposed into independent subsystems without violating admissibility constraints.

Quantum entanglement corresponds to the regime in which such a non-factorizable projected description remains structurally stable. Although effective observables may be associated with spatially separated regions, the projected configuration retains its unity and cannot be expressed as a product of independent components. Nonlocal correlations therefore reflect the relational character of the underlying χ-configuration rather than any superluminal influence.

Measurement and decoherence correspond to a selective stabilization within the space of admissible projected descriptions. Interaction with an environment amplifies certain relational features while rendering alternative projected descriptions inadmissible. The apparent collapse of the wavefunction thus reflects a loss of admissibility of non-selected descriptions, rather than a fundamental discontinuity at the level of the χ-substrate.

Particle decay represents a distinct but closely related regime. Here, the non-factorizable projected description becomes unstable under admissible fluctuations. No single projected configuration remains admissible, and stability is recovered only through factorization into several localized projected configurations. In this sense, decay may be understood as a structural transition from non-factorizable to factorizable projected descriptions.

Entanglement, measurement, and decay therefore arise from a common structural origin: the non-injective projection from relational configurations of χ to effective observables. They differ not in the nature of the underlying substrate, but in the stability properties of the projected description under admissible interactions and fluctuations.

A technical analysis of non-injective projection, admissibility conditions, and structural factorization is provided in Appendix B.14.

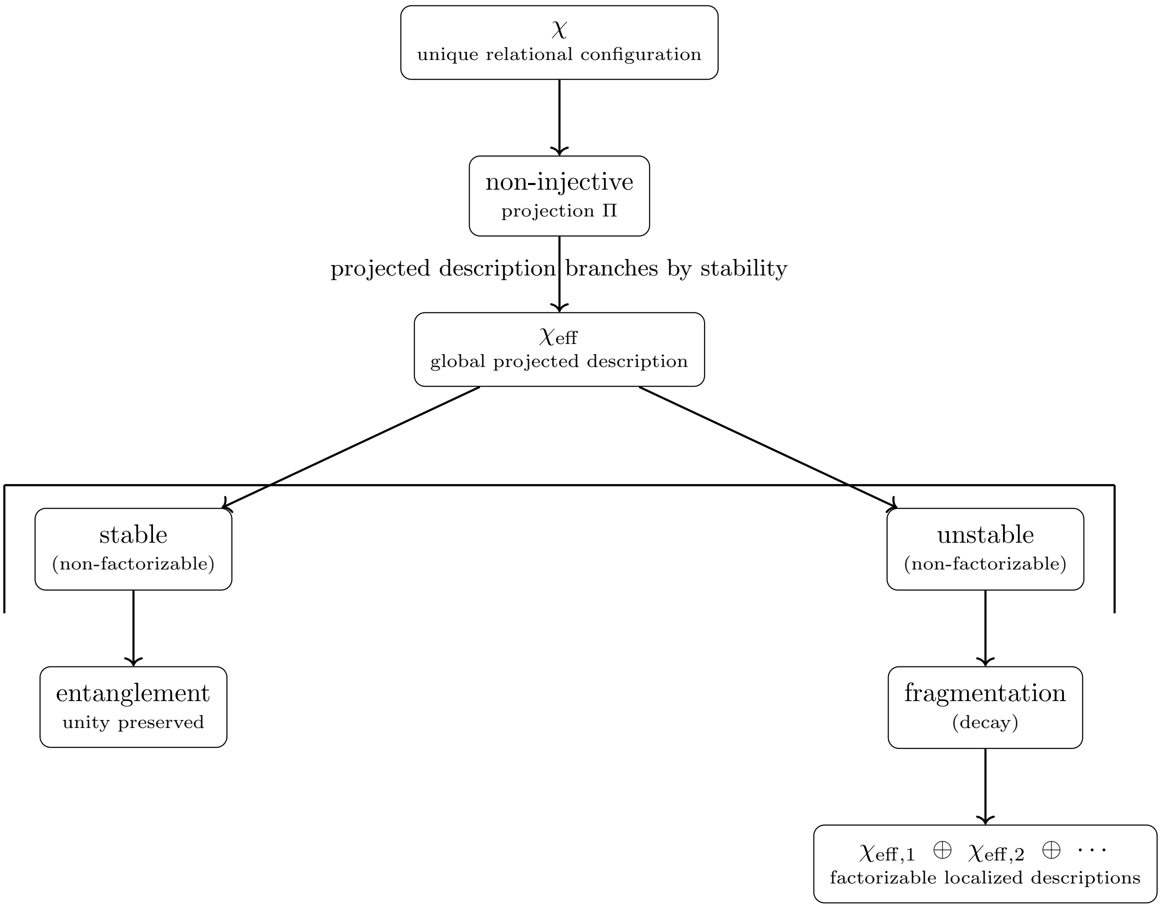

Fig. 9 Conceptual branching induced by non-injective projection in Cosmochrony. A single relational configuration of χ may admit a global non-factorizable projected description. If this description is stable, it manifests as entanglement. If it is unstable, admissibility is recovered through fragmentation into multiple localized factorizable descriptions (particle decay).

9 Relation to Quantum Formalism

This section does not assign fundamental ontological status to the quantum wavefunction, Hilbert space, or operator-based structures. Instead, it shows how the formal apparatus of quantum mechanics arises as an effective and internally consistent framework organizing admissible projected descriptions in regimes where localization, linearity, and approximate factorization hold.

Within the Cosmochrony framework, quantum mechanics is not derived from an underlying microscopic dynamics of the χ substrate. Rather, it emerges as a universal coarse-grained formalism governing the projectability, stability, and temporal consistency of localized physical descriptions once a stable geometric interpretation becomes applicable.

Quantum mechanics is therefore not replaced, but reinterpreted as an effective theory whose validity is restricted to regimes in which the relational structure of χ admits a stable, approximately linear, spacetime-based description. Outside these regimes, the Hilbert-space formalism ceases to provide an adequate representation, while the underlying relational dynamics of χ remain well-defined.

The following subsections clarify the origin and interpretational status of the wavefunction, the emergence of Hilbert space structure, the effective nature of the Schrödinger equation, the structural origin of quantization, the role of measurement and the Born rule, the interpretation of entanglement and nonlocal correlations, the topological origin of spin and statistics, and the meaning of orbital geometry within this perspective.

9.1 Status of the Wavefunction

In standard quantum mechanics, the wavefunction ψ is a complex-valued object defined on configuration space, whose ontological status remains debated. Operationally, $|\psi|^2$ encodes measurement probabilities via the Born rule, while ψ itself does not correspond to a physical field propagating in spacetime.

Within the Cosmochrony framework, the quantum wavefunction is not identified with any fundamental physical entity. It arises at the level of effective descriptions as a statistical encoding of the set of admissible local reprojections compatible with a given non-factorizable projected configuration.

Statistical and Relational Role of ψ.

The wavefunction therefore does not represent an underlying structure or hidden dynamics. It summarizes, in compact mathematical form, the relative accessibility of different local outcomes once a global descriptive coherence has been established. Its complex phase encodes relational constraints between alternative local descriptions, while its modulus determines the statistical weight of accessible reprojections.

Illustrative Example: Bound States.

As an illustrative example, the hydrogen atom wavefunctions $\psi_{nlm}(r, \theta, \phi)$ do not correspond to localized structures or solitonic configurations at a fundamental level. They represent stationary admissible projected descriptions characterized by specific symmetry and stability properties. The probability density $|\psi|^2$ reflects the relative frequency with which localized reprojections occur in effective geometric descriptions, while energy quantization arises from the discrete admissibility conditions imposed on stationary descriptive regimes.

In this perspective, the wavefunction is neither a physical field nor a direct image of an underlying substrate. It is a derived statistical object, encoding the structure of admissible projected descriptions and the probabilities of their local realization within spacetime.

9.2 Emergence of Hilbert Space Structure

The Hilbert space formalism of quantum mechanics provides a linear structure supporting superposition, interference, and unitary evolution. Within the Cosmochrony framework, this structure does not reflect a fundamental property of an underlying physical substrate. It emerges as an effective mathematical organization of admissible projected descriptions in regimes where relational constraints are weak and approximately factorizable.

In such regimes, distinct admissible projected descriptions can be combined without introducing strong mutual constraints, giving rise to an approximate linear structure. Superposition therefore reflects the coexistence of multiple compatible *descriptive alternatives*, rather than the simultaneous physical realization of multiple states.

Inner Product and Compatibility.

The inner-product structure of Hilbert space encodes the degree of mutual compatibility between projected descriptions. Orthogonality corresponds to mutually exclusive descriptive regimes, while non-orthogonal states represent partially compatible projections whose distinctions cannot be jointly resolved within a single effective description.

Role of Complex Phase.

The complex phase of the wavefunction does not correspond to an intrinsic oscillatory structure of a physical field. It encodes relational consistency conditions between alternative projected descriptions, ensuring coherent interference patterns within effective spacetime representations.

Validity of Unitary Evolution.

Unitary evolution arises as a consistency-preserving transformation within the space of admissible projected descriptions, valid so long as projectability and approximate factorizability remain intact. When these conditions fail—such as during measurement or strong environmental coupling—the Hilbert space description ceases to be adequate, and non-unitary effective behavior emerges.

In this perspective, Hilbert space is not a fundamental arena of physical reality. It is the natural mathematical structure organizing the space of admissible descriptions in regimes where linearity and coherence provide accurate effective approximations.

9.3 Emergence of the Schrödinger Equation as an Effective Description

Within the Cosmochrony framework, quantum dynamics is not postulated as a fundamental law. The Schrödinger equation arises as an effective, long-wavelength and non-relativistic description organizing admissible projected descriptions in regimes where localization, approximate factorization, and temporal projectability are simultaneously valid.

Rather than emerging from physical fluctuations of an underlying field, the Schrödinger equation appears as a universal consistency condition governing the time evolution of localized projected descriptions whose internal structure remains approximately stationary under projection.

Formal Non-Relativistic Limit

In regimes where an effective relativistic description applies, admissible projected descriptions of localized configurations admit a second-order hyperbolic evolution equation whose formal structure coincides with the Klein–Gordon equation. This equation should be understood as an effective geometric encoding of stability and admissibility constraints, not as a fundamental field equation defined on spacetime.

In the non-relativistic regime, the effective description separates naturally into a rapidly varying phase associated with rest-energy and a slowly varying envelope describing spatial localization. Formally, this separation may be written as

$$\Psi(x,t) = \psi(x,t)\, e^{-i\omega_0 t}, \qquad \omega_0 = \frac{mc^2}{\hbar}, \tag{47}$$

where Ψ denotes an effective relativistic descriptive field and ψ its non-relativistic envelope.

Imposing the condition that the envelope varies slowly compared to the rest-energy scale leads, to leading order, to the Schrödinger equation,

$$i\hbar\, \partial_t \psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V(x)\psi, \tag{48}$$

where $V(x)$ encodes weak external constraints on admissible projected descriptions, such as background gravitational or electromagnetic influences.

Interpretation.

The wavefunction ψ does not represent a physical excitation or fluctuation of an underlying substrate. It is a derived mathematical object encoding the admissible temporal evolution of localized projected descriptions once a non-relativistic spacetime interpretation becomes applicable.

In this sense, the Schrödinger equation is not a fundamental dynamical law. It is the effective evolution equation governing admissible projected descriptions in regimes where linearity, localization, and approximate factorization provide accurate descriptive approximations.

Operators and Algebra from χ Fluctuations

In the Cosmochrony framework, quantum operators are not introduced as primitive postulates acting on an abstract Hilbert space. They arise instead as *effective generators* associated with admissible variations of projected configurations of the pre-geometric substrate χ.

As discussed in Sections B.9 and 4.11, the projection from χ to effective observables is generically non-injective. As a consequence, multiple underlying relational configurations correspond to the same effective spacetime description. This structural degeneracy implies that infinitesimal variations of projected observables cannot, in general, be represented as commuting operations.

Emergence of position and momentum operators.

In a regime where projected configurations admit a smooth spacetime interpretation, effective position x labels equivalence classes of χ configurations sharing identical relational correlations. Translations in x therefore correspond to reparametrizations of the projection rather than to displacements within a fundamental background space.

The generator of such translations is naturally represented by an operator $\hat{p}$ acting on effective wavefunctions $\psi(x)$,

$$\hat{p} = -i\hbar_{\text{eff}}\,\partial_x, \tag{49}$$

where $\hbar_{\text{eff}}$ is the projected expression of the fundamental reprojection scale $\hbar_\chi$.

Conversely, the position operator $\hat{x}$ acts multiplicatively on $\psi(x)$ and encodes the coarse-grained labeling of projected relational configurations. Neither $\hat{x}$ nor $\hat{p}$ corresponds to an observable defined at the level of the χ substrate; both are emergent descriptors valid only within a projectable geometric regime.

Origin of non-commutativity.

The canonical commutation relation

$$[\hat{x}, \hat{p}] = i\hbar_{\text{eff}} \tag{50}$$

does not reflect a fundamental indeterminacy of nature. It expresses the *non-commutativity of geometric measurements* induced by projection from the pre-geometric substrate.

Operationally, performing a localization followed by a translation corresponds to a different class of admissible projections than performing the same operations in reverse order. This mismatch originates from the fact that localization and translation probe distinct relational aspects of χ configurations whose projection is not jointly injective.

The uncertainty principle therefore emerges as a geometric consistency condition on admissible effective descriptions, rather than as an ontological randomness or a fundamental limit imposed at the level of χ itself.

Algebra of observables as a projection artifact.

More generally, the algebra of quantum observables reflects the structure of the space of admissible projections from χ to effective variables. Non-commuting operators encode the incompatibility of simultaneously sharp geometric descriptors within a projected spacetime representation.

In this sense, Hilbert space and operator algebra arise as a *representation theory of projection degeneracy*, providing a compact and predictive language for describing stable regimes of projected χ dynamics.

Scope and limitations.

The present derivation accounts for the operator structure and non-commutative algebra of non-relativistic quantum mechanics. Its extension to quantum field theory requires additional structure, namely the treatment of fields as infinite collections of coupled projected modes.

Within Cosmochrony, quantum fields correspond to continuous families of collective excitations of the χ substrate rather than to fundamental operators defined at each spacetime point. While the emergence of canonical commutation relations for field amplitudes is expected in appropriate limits, a full reconstruction of relativistic quantum field theory lies beyond the scope of the present work and is left for future study.

9.4 Origin of Quantization

In standard quantum theory, quantization is introduced axiomatically through canonical commutation relations or path-integral prescriptions. Within the Cosmochrony framework, quantization is not fundamental and does not arise from an underlying microscopic dynamics. It emerges as a structural consequence of stability, consistency, and admissibility constraints imposed on projected descriptions.

Only a restricted class of localized projected configurations admits long-lived, internally consistent descriptions. Configurations that fail to satisfy these constraints rapidly lose projectability and cannot be maintained as persistent physical descriptions. As a result, admissible configurations form discrete equivalence classes rather than a continuous spectrum.

Energy quantization reflects this discreteness. Energy does not label an intrinsic property of a physical excitation, but characterizes the degree of structural persistence of a projected configuration within the relaxation ordering. Only specific values correspond to stable descriptive regimes, leading to an effective discretization of admissible energy exchanges.

The relation

$$E = h\nu \tag{51}$$

does not express a fundamental oscillatory dynamics. It encodes a proportionality between the energetic cost of maintaining a persistent projected configuration and the characteristic ordering rate at which its internal structure must be consistently re-identified. The frequency ν should therefore be understood as a descriptive rate associated with relational re-identification, not as oscillation with respect to a fundamental time parameter.

Within this perspective, Planck's constant does not represent a fundamental quantum of action. It emerges as a universal conversion factor characterizing the minimal structural scale at which projected descriptions remain stable and coherent under relaxation ordering. Its apparent universality reflects the universality of the projectability constraints themselves, rather than the postulation of an underlying quantized substrate.

9.5 Measurement and the Born Rule

Within the Cosmochrony framework, measurement does not involve a fundamental wavefunction collapse, nor does it rely on stochastic fluctuations of an underlying physical substrate. The χ field evolves continuously and deterministically according to its intrinsic relational structure.

What is conventionally described as a measurement corresponds to an irreversible loss of projectability: a localized projected description becomes dynamically coupled to a macroscopic environment, preventing the continued joint maintenance of incompatible relational alternatives. This process is described at the effective level by decoherence, as discussed in Section 8.5.

Measurement outcomes correspond to effective reprojections onto mutually exclusive descriptive regimes. These outcomes are not selected by hidden fluctuations or random microscopic events, but by the structural compatibility between the pre-measurement description and the macroscopic constraints imposed by the measurement apparatus.

The Born rule does not encode a fundamental probability law. It emerges as the unique stable measure on the space of admissible projected descriptions that remains invariant under loss of phase coherence, coarse-graining, and environmental coupling. The squared amplitude $|\psi|^2$ quantifies the relative measure of descriptive compatibility between a pre-measurement state and the set of macroscopically distinguishable outcomes.

In this sense, $|\psi|^2$ does not represent subjective uncertainty or intrinsic randomness. It characterizes the structural weight of admissible reprojections consistent with both the prior relational configuration and the constraints defining the measurement context. The Born rule therefore reflects a geometric and consistency-based property of projected descriptions, rather than a fundamental stochastic law of nature.

Measurement in Cosmochrony thus marks not the creation of an outcome, but the selection of a consistent descriptive regime from a pre-existing relational structure constrained by projectability.

9.6 Entanglement and Nonlocal Correlations

Within the Cosmochrony framework, quantum entanglement does not correspond to a physical linkage or interaction between spatially separated entities. It reflects the persistence of a shared relational structure within a single, non-factorizable configuration of the χ substrate.

Entangled systems are described by admissible projected configurations that cannot be decomposed into independent subsystems without loss of relational consistency. Although effective spacetime descriptions assign distinct locations to the corresponding subsystems, these locations represent different local projections of one and the same underlying relational configuration.

Nonlocal correlations therefore do not arise from superluminal influences, hidden signal exchange, or dynamical nonlocal interactions. They follow from the fact that measurement operations act on a globally defined relational configuration whose admissible projections must remain mutually consistent. Once a particular reprojection is realized in one region, the set of admissible reprojections elsewhere is correspondingly restricted, without any propagation of physical influence.

In this sense, quantum nonlocality in Cosmochrony is ontological rather than dynamical. The underlying relational configuration is globally defined, while its relaxation and reprojection remain locally governed by the causal and projectability constraints of χ. As a result, entanglement correlations are fully compatible with relativistic causality and do not require the introduction of nonlocal forces, preferred reference frames, or violations of relativistic locality.

9.7 Spin and Statistics

Within the Cosmochrony framework, spin does not arise as an intrinsic kinematic degree of freedom of a particle, nor as a fundamental representation of spacetime symmetries. It emerges as a topological property of admissible projected configurations associated with localized physical descriptions.

Certain classes of admissible configurations exhibit a non-trivial internal topology such that a 2π effective rotation does not return the configuration to an equivalent descriptive state. Only after a 4π rotation is full equivalence restored. Projected descriptions with this property correspond to fermionic behavior, while configurations that are 2π-periodic correspond to bosonic behavior.

The spin–statistics connection follows directly from this topological distinction. Configurations characterized by a non-trivial covering structure cannot be symmetrically exchanged without violating relational consistency, leading to antisymmetric exchange behavior. By contrast, configurations with trivial topology admit symmetric exchange without obstruction.

In this perspective, spin and statistics are not independent postulates of quantum theory. They reflect the same underlying topological constraints on the space of admissible projected descriptions, providing a unified geometric origin for both phenomena. An explicit illustrative construction is discussed in Section B.4.

9.8 Orbital Geometry as Probabilistic Visibility

Atomic orbitals do not represent spatially extended material distributions. Within the Cosmochrony framework, they correspond to probabilistic visibility patterns associated with admissible projected descriptions of localized bound configurations.

Orbital geometries encode structural and symmetry constraints imposed on admissible projected descriptions, such as nodal surfaces and angular dependence. These features reflect conditions of relational consistency and stability, rather than the presence of a spatially distributed physical object.

The apparent spatial extent of an orbital does not indicate the physical size or motion of an underlying entity. It reflects the range of effective spatial locations over which a projected description remains admissible under repeated reprojection and measurement. Regions of high probability correspond to domains where consistent reprojection is most robust, while nodal regions correspond to incompatible descriptive regimes in which projectability fails.

Orbital visualizations therefore do not depict occupied regions of space. They represent statistical maps of descriptive accessibility within an effective geometric representation. In this sense, atomic orbitals encode how bound configurations can be consistently described in spacetime, rather than revealing the spatial structure of an underlying physical object.

9.9 Scope and Limitations

Cosmochrony does not aim to replace quantum mechanics as a predictive or computational framework. All standard quantum-mechanical formalisms, including operator methods, path integrals, and perturbative techniques, remain valid and unchanged within their established domains of applicability.

The contribution of Cosmochrony is interpretative and unificatory. It provides a coherent pre-geometric and relational account of the origin of quantum phenomena, clarifying the ontological status of the wavefunction, quantization, measurement, and nonlocal correlations, without altering any experimentally verified predictions.

Within this framework, quantum mechanics is understood as an effective theory governing admissible projected descriptions in regimes where linearity, localization, and approximate factorization hold. Cosmochrony does not introduce new degrees of freedom, hidden variables, or modifications of quantum dynamics.

A complete formal correspondence between the relational χ substrate and the operator-based structures of quantum theory—including a systematic derivation of Hilbert space, observables, and evolution operators—lies beyond the scope of the present work and is left for future investigation.

Accordingly, the present framework should be regarded as a foundational reconstruction rather than a competing physical theory, intended to clarify the conceptual origin, internal coherence, and domain of validity of quantum-mechanical descriptions.

10 The Projection Fiber and Gauge Emergence

This section introduces the geometric structure of the projection fiber Π *and shows how gauge interactions emerge as effective symmetries of the relaxation flow of the* χ *substrate when constrained by projectability and topological admissibility.*

In the Cosmochrony framework, the projection fiber Π is not an auxiliary mathematical space but a central structural element linking the pre-geometric substrate χ to effective spacetime descriptions. It encodes the admissible equivalence classes and internal degrees of freedom through which relational configurations of χ can be consistently projected into geometric and field-like representations.

Gauge interactions do not arise from fundamental interaction fields defined on spacetime. They emerge instead as symmetry structures of the projection process itself, reflecting the invariance properties of admissible relaxation flows under local re-identification of equivalent representatives along the fiber. In this sense, gauge symmetries are not imposed *a priori*, but arise as stability conditions of projected descriptions.

This section develops the geometry of the projection fiber and clarifies how distinct classes of gauge phenomena—transmittance (scalar modes), shear-like modes, and topological admissibility constraints—naturally emerge from its structure.

10.1 The Geometry of the Π Subspace

The relational substrate χ is not accessed in its full structural complexity, but through admissible local projections onto a reduced projection fiber $\Pi \cong S^3$. This projection acts as a spectral filter, retaining only those modes of the relaxation flow that remain jointly projectable and stable under local re-identification. Here, Π denotes the projection *fiber*, i.e. the space of admissible internal representatives associated with a projected configuration, not the projection map itself.

The identification $\Pi \cong S^3$ is not imposed *a priori*, but follows from the minimal compact geometry required to support non-injective yet stable projections. It is the minimal compact manifold admitting nontrivial fibrations while preserving global stability and boundedness of the projection dynamics. Its associated Hopf fibration naturally induces an effective $SU(2) \times U(1)$ symmetry structure, which emerges as an invariance of the projection process rather than as a fundamental symmetry of the substrate.

The metric on Π is not fixed but dynamically induced by the local density of relational connections encoded in the underlying relational graph G used as a numerical and representational support, not as a physical discretization. The mapping from the global graph Laplacian Δ_G to the effective projected Laplacian Δ_Π is given by

$$\Delta_\Pi = P^\dagger \Delta_G P, \tag{52}$$

where P denotes the projection operator onto the admissible spectral subspace. The appearance of a smooth three-sphere geometry thus corresponds to a large-N limit of the discrete relational structure, in which coarse-grained spectral properties dominate.

10.2 Gauges as Relaxation Transmittance

Within the Cosmochrony framework, gauge interactions are not fundamental forces mediated by independent fields. They are reinterpreted as degrees of freedom of the relaxation flow within the projection fiber Π, encoding how locally admissible projections of the substrate χ are coherently related despite the intrinsic non-injectivity of the projection.

- **Electromagnetism ($U(1)$):** Electromagnetism corresponds to the phase degree of freedom of the relaxation flow along the fibers of the Hopf fibration of the projection fiber $\Pi \cong S^3$. This phase is not a fundamental complex structure, but an emergent geometric parameter associated with admissible local re-identifications of projected descriptions. This phase reflects the freedom to locally re-identify projected descriptions without altering global relational consistency. The fine-structure constant α characterizes the effective *transmittance* of the relaxation flow through the fiber, quantifying the robustness with which phase coherence is maintained across projections. Here, transmittance refers to the stability of admissible projective re-identifications along the fiber, not to energy transport or to a fundamental interaction strength.
- **Weak Interaction ($SU(2)$):** The weak interaction emerges from the rotational degrees of freedom of the S^3 projection fiber itself. These degrees of freedom encode non-abelian modes of relaxation transmittance, corresponding to shear-like distortions of admissible projections. The massive character of the effective $W^\pm$ and Z^0 bosons follows from the presence of a non-zero spectral gap associated with these modes, which can be interpreted as a form of spectral drag or torsion intrinsic to the fiber geometry.
- **Strong Interaction ($SU(3)$):** The strong interaction is associated with topological constraints arising from non-trivial winding structures of projected configurations. In particular, the effective $SU(3)$ symmetry reflects the threefold (triality) structure of the minimal self-intersecting stable soliton, identified with the trefoil knot ($w = 3$) as the minimal nontrivial stable self-intersecting configuration in the projected description. Color symmetry thus emerges as a geometric consequence of topological stability classes within the projection fiber, rather than as a fundamental internal degree of freedom.

10.3 Topological Constraints and Invariants

The stability of localized physical descriptions within the projection fiber Π is governed by the conservation of topological invariants. When an excitation of the relational substrate χ admits a closed, non-contractible configuration within Π, it forms a persistent topological obstruction to complete relaxation. Such configurations cannot be eliminated by smooth deformation of the relaxation flow without violating admissibility constraints.

These topological obstructions are naturally described in terms of knot-like topological structures within the projection fiber. Once formed, they impose global constraints on the relaxation process and give rise to long-lived, localized projected configurations. In this sense, particles correspond to stable topological defects of the projection, rather than to elementary excitations of a fundamental field.

The winding number w constitutes the primary invariant characterizing these topological obstructions, arising from the admissibility constraints of the projection fiber. It labels distinct equivalence classes of admissible projected configurations and plays a central role in the organization of the mass spectrum discussed in the following chapter. The energetic cost required to maintain a non-trivial winding against the global pressure of relaxation is perceived, at the effective level, as rest energy (mc^2).

Rest energy therefore does not measure an intrinsic substance or inertia, but quantifies the degree to which a topologically constrained configuration inhibits the relaxation of the substrate. Mass emerges as a spectral and topological consequence of persistent non-contractible structures within the projection fiber.

10.4 The Vacuum State as a Minimal Surface

In the absence of localized excitations, the projection fiber Π relaxes toward a configuration of minimal spectral tension, understood as minimal global frustration of admissible relaxation modes. This vacuum state is not an empty background, nor the ground state of a quantized field, but the smoothest admissible projected configuration compatible with global relaxation constraints.

Geometrically, the vacuum corresponds to a minimal surface in the spectral sense, not a spatial or spacetime embedding: a configuration for which curvature, torsion, and winding modes are simultaneously minimized within Π. In this state, the relaxation flow encounters no topological obstruction and propagates uniformly across the fiber. Any deviation from this minimal configuration—whether through localized curvature, torsional distortion, or non-trivial winding—manifests as the presence of effective fields or particles.

From this perspective, the usual notion of field quantization is replaced by the quantization of admissible topological modes within a finite-volume projection fiber. Only a discrete set of deviations from the minimal surface is compatible with stability and projectability constraints, leading naturally to a discrete spectrum of excitations.

As a consequence, vacuum energy is neither divergent nor arbitrary. It is intrinsically bounded by the spectral cutoff imposed by the relational graph underlying Π, understood as a representational support rather than a physical discretization. The finiteness of the vacuum energy reflects the finiteness of admissible spectral deformations of the minimal configuration, rather than the summation of zero-point energies of independent field modes.

11 Spectral Mass Spectrum and Hierarchy

This chapter establishes that mass is neither an intrinsic property nor a coupling to an external field, but a spectral invariant arising from the dynamical frustration between relaxation flows and topological constraints on the projection fiber Π*, and that observed mass hierarchies reflect the ordered structure of these spectral constraints rather than independent physical parameters.*

11.1 Spectral Stability and the Unit of Mass

In the Cosmochrony framework, rest mass does not arise from an intrinsic property of particles nor from a coupling to an external field. It is defined as a *spectral invariant* associated with the stability of admissible projected configurations.

More precisely, the rest mass m of a localized excitation is identified with the fundamental eigenmode of the scalar Laplacian $\Delta_G^{(0)}$ acting on the projection fiber Π, subject to a set of topological constraints $\mathcal{T}$ characterizing the configuration:

$$m^2c^2 \;=\; \lambda_{\mathcal{T}} \;\equiv\; \mathrm{Eig}\Big(\Delta_G^{(0)}\Big)\Big|_{\mathcal{T}}. \tag{53}$$

Here, $\lambda_{\mathcal{T}}$ is a dimensionless spectral eigenvalue encoding the minimal energetic cost required to maintain the configuration against the global relaxation of the substrate χ. Mass therefore quantifies resistance to relaxation, rather than inertial response to force.

The electron mass m_e corresponds to the lowest non-trivial admissible eigenmode λ_1, associated with the simplest stable topological constraint compatible with projectability. It represents the fundamental resonance of the substrate χ within the finite-volume geometry $\Pi \cong S^3$.

The conversion from dimensionless spectral values to physical mass units is fixed by identifying this fundamental mode with the observed electron mass:

$$m \;=\; m_e\sqrt{\frac{\lambda_{\mathcal{T}}}{\lambda_1}}. \tag{54}$$

In this sense, the electron mass does not enter the theory as an arbitrary parameter, but as the natural unit of mass emerging from the lowest stable spectral excitation of the projection fiber.

11.2 Non-Commutativity as a Source of Mass

In the Cosmochrony framework, torsion is promoted from a geometric feature to a *dynamical constraint*. It does not merely reshape the spectrum of admissible modes, but actively competes with the transport of relaxation through the projection fiber. The key structural transition responsible for mass generation is the loss of commutativity between relaxation diffusion and torsional constraints.

Inhibition of Relaxation

Let $\Delta_G^{(0)}$ denote the scalar (0-form) Laplacian induced by the relational graph structure, and let Ω_w be the effective torsion operator associated with winding number w within the projection fiber Π. For the fundamental lepton configuration ($w = 1$), the relaxation flow remains spectrally compatible with torsion, such that

$$[\Delta_G^{(0)}, \Omega_1] = 0, \tag{55}$$

and relaxation modes can be chosen as simultaneous eigenstates.

For higher-winding configurations ($w \geq 2$), torsional constraints become spectrally frustrated with respect to diffusion:

$$[\Delta_G^{(0)}, \Omega_w] \neq 0 \qquad (w \geq 2). \tag{56}$$

This non-commutativity inhibits uniform relaxation across the fiber and induces an irreducible spectral compression. At the effective level, this inhibition manifests as an amplification of inertial mass, reflecting increased resistance to relaxation rather than additional substance.

To quantify this effect without introducing adjustable parameters, we define the torsional action as a purely spectral invariant:

$$\mathcal{A}(w) \;\equiv\; \frac{1}{2}\ln\left(\frac{\det\left(\Delta_G^{(0)} + \Omega_w\right)}{\det\left(\Delta_G^{(0)}\right)}\right), \tag{57}$$

where the determinant is understood in the zeta-regularized (Fredholm) sense.

The Pisano Ratio as a Stability Fixed Point

For $w = 2$, the projection fiber ceases to be spectrally isotropic. The relaxation modes split into two dynamically competing sectors,

$$\Pi \;=\; \Pi_{\parallel} \oplus \Pi_{\perp}, \tag{58}$$

where $\Pi_{\parallel}$ aligns with the Hopf-like direction selected by torsion, and $\Pi_{\perp}$ spans the orthogonal frustrated modes.

Dynamical stability requires simultaneously avoiding internal resonances and maximizing relaxation throughput. This condition selects the most irrational admissible ratio between the spectral frequencies of the two sectors, in direct analogy with KAM-type stability criteria:

$$\frac{\lambda_{\parallel}}{\lambda_{\perp}} \;=\; \varphi \qquad \Longrightarrow \qquad \beta \;\equiv\; \frac{1}{\varphi}, \tag{59}$$

where $\varphi = (1+\sqrt{5})/2$ is the golden ratio. In this framework, β is not a phenomenological fit parameter, but the unique compression invariant associated with non-integrable torsional frustration.

Leptonic Spectrum Synthesis

In the geometric regime, rest masses scale with the effective spectral cut-off frequencies of admissible configurations. For the muon, non-commutative torsion yields the following parameter-free prediction:

$$\boxed{\frac{m_\mu}{m_e} = \sqrt{\frac{\lambda_2}{\lambda_1}} \cdot \frac{3}{2\alpha} \cdot \frac{1}{\varphi}} \qquad \text{with} \qquad \frac{\lambda_2}{\lambda_1} = \frac{8}{3}. \tag{60}$$

Here α is interpreted as the spectral transmittance of relaxation through the projected regime.

The appearance of the fixed ratio $\lambda_2/\lambda_1 = 8/3$ is not assumed. Its robustness is demonstrated independently in Appendix D.7, where the same invariant emerges both from stochastic relational sampling on the underlying graph and from the spectral response of a discrete Laplacian defined on the same relational support. This confirms that the ratio reflects an intrinsic property of the relational geometry, rather than a fitted spectral input.

11.3 Gravitational Shadows and the Spectral Wake

The torsional constraint Ω_w responsible for mass generation is not strictly confined to the localized projected configuration. While the obstruction itself remains topologically localized within the projection fiber, it induces an extended *spectral deformation* of the surrounding relational substrate χ. This deformation takes the form of a persistent modification of the local spectral density of relaxation modes, here referred to as a *spectral wake* or gravitational shadow.

The gravitational shadow does not correspond to additional matter or propagating degrees of freedom. Rather, it reflects a long-lived redistribution of relaxation capacity in the substrate induced by the presence of a topological obstruction. In effective spacetime descriptions, this manifests as an extended gravitational influence exceeding that associated with the localized baryonic projection alone.

This mechanism provides an ontological basis for phenomena usually attributed to dark matter, without introducing new particles or fields. The spectral shadow exhibits two characteristic features:

- **Elastic Remanence:** The spectral deformation persists in the relational substrate even under displacement of the baryonic projected configuration. As a result, the effective gravitational potential need not remain spatially coincident with visible matter, naturally accounting for the observed offsets in systems such as the Bullet Cluster. This persistence reflects the finite relaxation time of large-scale spectral rearrangements rather than the motion of an independent mass component.
- **Non-Local Susceptibility:** Effective gravitational acceleration arises from gradients in the global relaxation flow. When the local gradient falls below a characteristic threshold $a_0 \sim cH_0$, the substrate response transitions from linear to non-linear. In this low-acceleration regime, the elastic properties of χ dominate, recovering MOND-like phenomenology as an emergent phase of spectral response, rather than as a modification of fundamental dynamics.

In this perspective, dark matter effects are reinterpreted as manifestations of persistent spectral memory in the relational substrate. Gravitation probes not only localized topological obstructions, but also the extended relaxation shadow they imprint on χ.

12 Cosmological Implications

12.1 The Big Bang as a Maximal Constraint Regime of the χ Substrate

Within the Cosmochrony framework, the Big Bang is not interpreted as a spacetime singularity, nor as a physical event occurring at a definite moment. It corresponds instead to a limiting regime in which the relational structure of the χ substrate is maximally constrained.

In this regime, the density of structural and topological constraints within χ is such that no stable geometric projection is admissible [2, 4]. Concepts such as spatial distance, temporal duration, curvature, or causal ordering are therefore undefined. The apparent singular behavior encountered in standard cosmological models reflects the breakdown of effective spacetime descriptions when extrapolated beyond their domain of validity.

Cosmic evolution is interpreted as the progressive relaxation of these maximal constraints. As the relational structure of χ becomes less constrained, increasingly stable projected descriptions become admissible, allowing effective notions of space, time, and geometry to emerge [29]. The Big Bang thus marks not the beginning of spacetime, but the boundary beyond which spacetime ceases to be a meaningful descriptive framework.

Within this perspective, the arrow of time does not originate from special initial conditions or entropy assumptions [11, 30]. It arises intrinsically from the monotonic relaxation ordering of χaway from the maximally constrained regime. Temporal ordering is therefore a consequence of the structural evolution of the substrate itself, rather than a feature imposed at the level of effective cosmological descriptions.

12.2 Cosmological Cycles of Constraint and Reprojection

The maximally constrained regime identified with the Big Bang should not be interpreted as a unique or irreproducible event. It corresponds instead to a limiting configuration of the χ substrate in which structural constraints dominate to the extent that no stable spacetime projection is admissible.

As global relaxation proceeds, increasingly stable projected descriptions become possible, giving rise to emergent spacetime, matter configurations, and large-scale cosmological structure. However, this maximal constraint regime is not confined to the early universe. It may be locally reapproached whenever structural constraints on χ saturate, most notably in regions of extreme gravitational confinement identified as black holes.

In such regions, effective spacetime descriptions progressively lose validity. Relational information encoded in emergent degrees of freedom ceases to remain expressible within spacetime and undergoes "deprojection" into the purely relational χ substrate. This process does not destroy information but renders it inaccessible to spacetime descriptions.

Crucially, deprojection does not imply irreversibility at the level of the substrate. As structural constraints relax, the same relational content may again admit admissible projected descriptions. "Reprojection" should therefore be understood not as a discrete physical process, but as the restoration of descriptive projectability once relational consistency conditions are satisfied.

From this perspective, phenomena commonly associated with the "quantum vacuum" reflect the persistent presence of reprojectable relational structures within χ, rather than the activity of fluctuating fields. The vacuum is not empty, but represents a regime of minimal yet non-vanishing projectability.

Cosmological evolution in Cosmochrony thus involves a continuous interplay between global relaxation, local reconfinement, deprojection, and reprojection across scales. The universe is not characterized by a single origin or terminal state, but by recurrent transitions between regimes of descriptive admissibility and breakdown.

12.3 Cosmic Expansion Without Inflation

This subsection does not question the observational evidence for early-universe expansion or large-scale correlations. The expression "without inflation" indicates that these features are accounted for without invoking a distinct inflationary field or a phase of superluminal metric expansion, rather than denying their empirical reality.

In standard cosmology, an inflationary phase is introduced to account for the observed large-scale homogeneity, isotropy, and near-flatness of the universe, as well as to resolve the horizon problem [31, 32]. Within the Cosmochrony framework, these features do not require a distinct inflationary epoch.

At the pre-geometric level, the relational structure of the χ substrate is not organized according to spatial separation or causal horizons. Prior to the emergence of a stable geometric projection, notions such as distance, light cones, and causal disconnection are undefined. As a result, the conditions that give rise to the "horizon problem" in standard spacetime-based cosmology do not apply.

Large-scale homogeneity and isotropy therefore reflect the global relational coherence of the χ substrate in the maximally constrained regime, rather than the outcome of a rapid expansion of spacetime. When geometric descriptions become admissible, this coherence is inherited as initial large-scale regularity in the emergent spacetime.

Cosmic expansion itself is interpreted as the progressive relaxation of relational constraints, leading to increasing effective separation between projected regions. This expansion does not correspond to motion through space, but to the gradual unfolding of geometric distinctions as projectability improves.

The present framework does not aim to reproduce inflationary scenarios at the level of field-driven perturbation spectra. Instead, it redefines the origin and interpretation of primordial regularities and correlations as structural features of the relaxation and projection process itself. While a fully quantitative treatment of emergent anisotropies requires further spectral and numerical development, no distinct inflationary phase or additional dynamical degrees of freedom are invoked.

12.4 Cosmic Expansion as χ Relaxation

In the Cosmochrony framework, cosmic expansion does not correspond to the motion of matter through a pre-existing space [33]. It reflects instead the progressive relaxation of the relational χ substrate, from which effective spatial distinctions and separations gradually emerge.

As the ordering parameter associated with χ increases monotonically, projected descriptions admit an ever broader range of admissible and mutually distinguishable relational configurations. What is described in effective cosmological models as the "expansion of space" thus corresponds to the increasing global availability of projectable relational distinctions within χ, rather than to a dynamical stretching of a fundamental metric background.

In this interpretation, expansion is not driven by an external energy component or a specific cosmological fluid. It is an intrinsic consequence of the relaxation ordering of the substrate itself. Localized matter configurations act as persistent structural constraints on this relaxation, leading to spatially inhomogeneous unfolding that later manifests, in effective geometric descriptions, as large-scale structure.

Cosmic expansion is therefore reinterpreted as a geometric and relational phenomenon, emerging from the intrinsic evolution of χ and acquiring a spacetime interpretation only once a stable geometric regime becomes applicable.

12.5 Emergent Hubble Law

In homogeneous regimes, the relaxation ordering of the relational χ substrate is uniform. When described using an effective cosmological time parameter t, introduced solely as a convenient label of the relaxation ordering, this uniform regime admits the linear representation:

$$\chi(t) = \chi_0 + c\,t. \tag{61}$$

This expression does not define a fundamental time evolution, but provides an effective parametrization of cumulative relaxation in a homogeneous cosmological regime.

Identifying effective spatial scales with accumulated relational differentiation in χ leads naturally to a "Hubble-like law" relating relative separation rates to separation itself [34, 35]. Within this effective description, the Hubble parameter may be written as:

$$H(t) \equiv \frac{1}{\chi}\frac{d\chi}{dt}, \tag{62}$$

where the derivative denotes an effective rate with respect to the cosmological parametrization, not a fundamental dynamical derivative.

In this perspective, no independent scale factor or expansion field is required. The Hubble parameter emerges as a dimensionless measure of the global relaxation rate of χ relative to its accumulated value.

The present-day Hubble constant H_0 is therefore interpreted as an effective observable quantifying the current state of global relaxation, rather than as a fundamental constant governing the dynamics of spacetime itself. This interpretation relies implicitly on the validity of a homogeneous effective description, whose limitations become relevant once relaxation proceeds unevenly across scales.

Observational discriminant: Cosmochrony predicts a $\sim 5\%$ higher $H(z)$ at $z \sim 1$ compared to ΛCDM, testable with DESI/Euclid (Section 14.7).

Cosmic Acceleration Without Dark Energy

Within the Cosmochrony framework, the observed late-time cosmic acceleration does not require the introduction of a cosmological constant or a "dark energy" component. No additional energy density or repulsive interaction is postulated at the fundamental level.

The apparent acceleration arises as an effective consequence of the cumulative relaxation history of the relational χ substrate. As cosmic evolution proceeds, the formation of localized and long-lived structures (such as galaxies and clusters) increasingly constrains the local relaxation of χ. These constraints introduce growing spatial inhomogeneities in the relaxation process.

When interpreted within standard spacetime-based cosmological models, which assume a homogeneous and isotropic expansion driven by a global scale factor, these inhomogeneities manifest as an apparent acceleration of cosmic expansion. The effect reflects a mismatch between the underlying relational relaxation dynamics and the assumptions built into effective geometric descriptions.

In this sense, cosmic acceleration is not a dynamical phenomenon requiring a new source of energy. It is an emergent, interpretative effect arising from the progressively uneven relaxation of χ across cosmic scales. As structure formation proceeds, the effective expansion inferred from observations naturally departs from the predictions of homogeneous models, without invoking dark energy.

This interpretation aligns with approaches that attribute late-time acceleration to "backreaction" effects, while providing a unified and pre-geometric origin rooted in the relaxation dynamics of the χ substrate.

12.6 Cosmic Microwave Background

Within the Cosmochrony framework, the cosmic microwave background (CMB) does not encode primordial fluctuations generated during a distinct inflationary phase. It reflects instead the imprint of early relaxation and reprojection processes as the relational χ substrate transitioned toward a regime admitting stable geometric descriptions.

At this stage, the universe was not yet structured by well-defined spatial separation or causal horizons. Large-scale correlations observed in the CMB therefore arise naturally from the global relational coherence of χ prior to geometric differentiation, rather than from superluminal expansion within spacetime.

The acoustic features observed in the temperature power spectrum admit an effective interpretation as resonance patterns arising once a stable geometric regime is reached, where the standard photon–baryon plasma description becomes operationally meaningful. In Cosmochrony, the distinctive imprint of χ enters primarily through the primordial conditions and large-scale admissibility constraints, rather than by replacing the established microphysics of the plasma itself [36, 37].

In this perspective, the CMB encodes a "fossil record" of the emergence of spacetime itself. Its large-angle correlations and statistical properties are consequences of the pre-geometric relational structure of χ, inherited by the emergent spacetime description without requiring an inflationary epoch or superluminal causal processes [38].

Effective closure: primordial spectrum as the interface to CMB pipelines. At the level of quantitative prediction, we adopt an effective closure in which the standard Boltzmann transfer functions remain unchanged once a stable geometric description has emerged. Within this approximation, Cosmochrony modifies the CMB angular spectra only through the primordial curvature spectrum $P_\zeta(k)$:

$$C_\ell^{XY} = 4\pi \int_0^\infty \frac{dk}{k}\, P_\zeta(k)\, \Delta_\ell^X(k)\, \Delta_\ell^Y(k), \tag{63}$$

where $\Delta_\ell^X(k)$ are the usual transfer functions for temperature and polarization. The Cosmochrony imprint is encoded in a projectability-induced infrared filter and an emergent tilt,

$$P_\zeta(k) = A_s \left(\frac{k}{k_*}\right)^{n_\chi - 1} C^2\left(\frac{k}{k_p}\right), \qquad C(x) \xrightarrow{x \ll 1} 0, \quad C(x) \xrightarrow{x \gg 1} 1, \tag{64}$$

where k_p is the effective cutoff scale associated with the freeze-out of admissible projected descriptions. This construction preserves the successful peak structure of the standard plasma transfer physics, while allowing Cosmochrony to address large-angle anomalies through a controlled modification of the primordial baseline.

This suppression may explain the observed $\ell = 2$ anomaly in Planck data, without invoking inflationary mechanisms.

Because the emergence of geometric descriptions and the stabilization of projection modes are governed by the same underlying spectral structure of the χ substrate, fundamental ratios characterizing stable configurations at the micro-scale may leave scale-independent imprints on cosmological observables.

On the interpretational status of the ΛCDM fit to CMB data. The remarkable agreement between the ΛCDM model and the observed CMB angular power spectrum is often presented as a decisive confirmation of the model, notably because a small set of parameters suffices to reproduce a wide range of observational features. While this empirical success is undeniable, its interpretational scope must be carefully qualified.

The standard ΛCDM fit relies on the adjustment of effective parameters within a fixed theoretical pipeline, in which the primordial power spectrum, the acoustic transfer functions of the photon–baryon plasma, and the background geometry are jointly optimized. This procedure demonstrates the internal consistency and descriptive efficiency of the framework, but does not in itself establish the physical origin of the primordial spectrum nor the mechanisms responsible for its large-scale structure.

In particular, several large-angle anomalies of the CMB, including the suppression of power at low multipoles and the observed phase alignments, are typically treated as statistical fluctuations within cosmic variance rather than as signatures of underlying structural constraints. The absence of a generative mechanism for these features within ΛCDM motivates the exploration of alternative foundational descriptions.

In Cosmochrony, the acoustic peak structure is not re-derived from first principles, but is inherited at the effective level through the same plasma transfer functions that operate in standard cosmology. The point of departure lies instead in the origin of the primordial spectrum itself: the large-scale correlations and infrared suppression emerge as direct consequences of the relational relaxation and admissibility structure of the χ substrate, prior to the stabilization of geometric descriptions.

From this perspective, the precision of the ΛCDM fit does not invalidate the need for a deeper explanation of the observed CMB structure. Rather, it delineates the regime in which any viable foundational theory must reproduce the effective phenomenology, while offering a principled account of features that remain statistically marginal or unexplained within the standard paradigm.

Relational spectral scales and angular multipoles.

The admissibility cutoff and large-angle modulations can be expressed directly in terms of the low-lying spectrum of the relational operator L_χ introduced in Section 3.4. Once a stable geometric regime is reached, the long-wavelength sector admits an effective Laplacian interpretation, yielding the canonical identification

$$k_n \;\propto\; \sqrt{\lambda_n(\lambda_*)}, \qquad \ell_n \;\approx\; k_n\, D_A(z_*), \tag{65}$$

and therefore the robust ratio constraint

$$\frac{\ell_n}{\ell_1} \;\approx\; \sqrt{\frac{\lambda_n}{\lambda_1}}. \tag{66}$$

Importantly, these ℓ_n characterize *relational angular modulations* associated with admissibility and freeze-out, and are not assumed to coincide with the acoustic peak indices of the photon–baryon plasma.

Remark (Spectral ratio $\lambda_2/\lambda_1 = 8/3$ and angular-scale constraints). Numerical relaxation experiments reported in Appendix D.7 indicate that, across a broad class of admissible configurations, the low-lying spectrum of L_χ exhibits the stable ratio

$$\frac{\lambda_2}{\lambda_1} \;\simeq\; \frac{8}{3}. \tag{67}$$

If this ratio persists in the continuum limit, Equation (66) implies the characteristic constraint

$$\frac{\ell_2}{\ell_1} \;\approx\; \sqrt{\frac{8}{3}} \;\simeq\; 1.63, \tag{68}$$

which can be searched for as a scale-independent modulation signature in the low-ℓ sector.

Polarization and tensor modes (outlook).

Possible signatures of the fundamental spectral ratio $\lambda_2/\lambda_1 = 8/3$ in CMB polarization observables, including constraints on tensor modes, are discussed separately in Section 14.8 as testable predictions of the framework.

Clarification: relation to acoustic peaks.

In standard cosmology, the acoustic peaks arise from oscillations of the photon–baryon fluid prior to recombination, and their detailed positions and heights are controlled by well-tested plasma microphysics. The relational eigenvalues $\{\lambda_n\}$ of L_χ are *not* fluid oscillation modes. They characterize the spectral geometry of admissible χ configurations at freeze-out and thus constrain the large-angle primordial baseline and admissibility-induced modulations. Accordingly, the present work does not identify $\ell_n \propto \sqrt{\lambda_n}$ with the acoustic peak indices; instead, the acoustic peak structure is retained through standard transfer functions, while Cosmochrony enters through $P_\zeta(k)$ and low-ℓ effects.

12.7 Dark Matter as Residual Relaxation Effects

Cosmochrony addresses dark matter phenomenology not through the introduction of hypothetical particles (such as WIMPs or axions), but as a structural consequence of the relaxation dynamics of the relational χ substrate.

Although the χ substrate is not a material medium, its projected relaxation dynamics can admit effective hydrodynamic descriptions in regimes where large-scale relational fluxes dominate. These effective descriptions give rise to flow-, lag-, and memory-like phenomena without implying the existence of a physical fluid or mechanical elasticity.

Spectral Origin of Dark Effects.

At the fundamental level, dark matter phenomena correspond to configurations of the χ substrate that resist relaxation while failing the projectability conditions required for Standard Model interactions. These **non-projected spectral modes** possess inertial mass (resistance to relaxation) and contribute to gravitational curvature, while remaining invisible to electromagnetic or electroweak probes.

Spectral vs. Particulate Mass.. While baryonic particles correspond to resonant, topologically stable projected modes, dark matter consists of sub-threshold harmonics. These modes contribute to the global *fiber weight* of the projected geometry but lack the spectral signatures required for projection Π into particle degrees of freedom.

Galactic Rotation and Effective Spectral Stiffness.

The flattening of galactic rotation curves is interpreted as a spatial variation of the effective gravitational coupling G_{eff} induced by the local relaxation state of the substrate. Near galactic centers, the high density of matter localizes the relaxation flow. At larger radii, the effective spectral stiffness K_0 of the χ relaxation response undergoes a transition, leading to a logarithmic gravitational potential. This reproduces MOND-like behavior as an asymptotic description, without invoking a modification of gravity or a universal acceleration scale.

This stiffness is not mechanical in nature. It encodes the density of unresolved relaxation constraints in the projected spectral response and reflects the local spectral age and environment of the system.

Variable Threshold $\mathcal{K}_c$.. The transition to the MOND-like regime occurs when the relaxation flux Φ_χ drops below a saturation threshold $\mathcal{K}_c$. Unlike the universal constant a_0 in MOND, $\mathcal{K}_c$ is a local and environment-dependent property of the substrate's spectral density. This naturally explains the observed variation of the apparent dark matter fraction among galaxies of different masses, morphologies, and environments.

Gravitational Lensing and Substrate Memory.

Gravitational lensing phenomena, particularly in systems such as the Bullet Cluster, are interpreted as manifestations of **relaxation lag** and substrate memory. In high-velocity collisions, the dissipative baryonic component (gas) rapidly loses coherence, while the projective geometry Π associated with localized mass-solitons persists.

Lensing as Spectral Refraction.. Light deflection is treated as an effective refraction process within the spectral gradient of the projected χ geometry. The lensing signal therefore tracks the residual geometric deformation induced by the passage of mass-solitons, rather than the instantaneous distribution of baryonic matter. This description is purely effective and does not imply the existence of a material refractive medium.

Predictive Distinction from Particulate Dark Matter.

Unlike WIMP-based models, which predict localized particle halos and small-scale cusps, Cosmochrony predicts a **non-local correlation** between gravitational mass discrepancies and the global spectral age of a system. A characteristic signature of this framework is the absence of sharp central cusps and the presence of a minimum smoothing scale imposed by the substrate's spectral response (the "spectral graininess" h_χ).

Spectral Echoes.. Cosmochrony further predicts the existence of **spectral echoes**—faint gravitational signatures in regions where matter was previously present but has since moved. Such echoes arise from the finite relaxation time of the substrate and are incompatible with particulate dark matter models, while being a natural consequence of a relational substrate with memory.

12.8 Entropy and the Arrow of Time

Within the Cosmochrony framework, the arrow of time is not a derived or emergent statistical phenomenon. It is a fundamental structural feature arising from the intrinsic monotonic relaxation ordering of the relational χ substrate, an ordering that is pre-temporal and does not presuppose the existence of an underlying time parameter. Temporal directionality therefore precedes and grounds all thermodynamic considerations.

Entropy increase emerges only at the level of effective spacetime descriptions. It provides a statistical summary of how macroscopic degrees of freedom evolve under the irreversible relaxation of χ when coarse-grained descriptions become applicable. In this sense, entropy growth does not explain the arrow of time; rather, it reflects the underlying temporal asymmetry already present in the substrate.

This reverses the standard explanatory hierarchy of statistical physics. Time asymmetry is not attributed to special initial conditions or probabilistic arguments, but is imposed intrinsically by the relaxation structure of χ. Thermodynamic irreversibility is thus a secondary manifestation of a more fundamental ordering principle.

It is crucial to note that entropy and the second law are defined only within regimes where a spacetime-based, coarse-grained description of physical processes is valid. Processes involving deprojection of relational information into the χ substrate—such as those associated with extreme gravitational confinement—do not correspond to entropy decrease or temporal reversal. Instead, they represent a transition to a level of description where thermodynamic notions no longer apply.

From this perspective, entropy increase characterizes the evolution of descriptions within spacetime, while the arrow of time itself is rooted in the deeper relational dynamics of χ.

12.9 Cosmic Voids as Maximal Relaxation Probes

Within the Cosmochrony framework, cosmic voids constitute regions where the relaxation of the χ substrate is least frustrated by localized excitations. As a result, the effective Born–Infeld metric predicts an enhanced geodesic defocusing, leading to a negative gravitational lensing signal and to non-linear peculiar velocity outflows at void boundaries.

These effects are absent or strongly suppressed in ΛCDM, making cosmic voids a clean discriminant between the two frameworks.

Cosmic voids act as natural laboratories for maximal substrate relaxation, providing a falsifiable signature of Cosmochrony through negative lensing and enhanced boundary outflows.

Connection to local H_0 determinations.

If cosmic voids correspond to regions of near-maximal substrate relaxation, Cosmochrony predicts enhanced outward peculiar velocities at void boundaries together with a more negative void-lensing signal than in ΛCDM. Both effects can bias low-redshift distance–redshift inferences toward higher locally inferred expansion rates. A decisive test is therefore the cross-correlation between (i) void lensing profiles and (ii) locally inferred H_0 maps: regions with higher H_0 should statistically coincide with stronger void defocusing signatures.

Phenomenological Born–Infeld void parametrization.

We model the observable void signals as a ΛCDM baseline plus a Born–Infeld-like saturating correction controlled by a single amplitude parameter β_{void}:

$$\kappa_{obs}(R) = \kappa_{\Lambda CDM}(R)\left[1 + \beta_{\mathrm{void}}\,\mathcal{S}\big(\mathcal{A}(R)\big)\right], \tag{69}$$

$$v_{obs}(r) = v_{\Lambda CDM}(r)\left[1 + \beta_{\mathrm{void}}\,\mathcal{S}\big(\mathcal{B}(r)\big)\right], \tag{70}$$

with a saturating function $\mathcal{S}(x) = x/\sqrt{1+x^2}$. The dimensionless activities $\mathcal{A}, \mathcal{B}$ can be chosen as projected-density and local-gradient proxies, e.g. $\mathcal{A}(R) = |\Delta(R)|/s_\star$ and $\mathcal{B}(r) = |d\Phi/dr|/g_\star$ (or $|\delta(r)|/s_\star$), where $s_\star, g_\star$ set the Born–Infeld saturation thresholds.

12.10 The Hubble Tension

The discrepancy between early-universe and late-universe determinations of the Hubble constant is now well established observationally [38–41]. Standard cosmological models interpret this tension as a potential indication of "new physics" beyond the ΛCDM framework.

Within the Cosmochrony framework, this discrepancy admits a natural qualitative interpretation without introducing new fundamental components or modifying the underlying relaxation dynamics. The key point is that different observational probes access different regimes of effective projectability of the relational χ substrate.

Early-universe measurements, such as those inferred from the cosmic microwave background, probe a regime close to the transition from maximal constraint to geometric projectability. In this regime, relational constraints remain significant, and the effective mapping between χ relaxation and spacetime observables differs from that characterizing the late universe.

Late-time measurements, based on local distance ladders and astrophysical standard candles, probe a regime in which χ has undergone substantial further relaxation. In this more weakly constrained regime, effective spacetime descriptions are more fully developed, leading to a different inferred relation between relaxation ordering and observational distance–redshift relations.

The resulting difference in inferred values of H_0 does not reflect a change in a fundamental expansion rate. It arises from the use of a single spacetime-based parametrization to describe observations sampling distinct stages of relational relaxation. In this sense, the Hubble tension reflects a limitation of homogeneous effective descriptions when applied across regimes of differing projectability.

The Hubble Tension as a Diagnostic of Topological Decoherence

Within the Cosmochrony framework, the Hubble parameter is not interpreted as a fundamental expansion rate of spacetime, but as an emergent kinematic proxy for the locally inferred relaxation rate within the effective projected description. More precisely, the effective Hubble parameter inferred from observations may be expressed as

$$H_{\text{eff}}(\mathbf{x}, t) \;\sim\; \frac{1}{\tau_\chi(\mathbf{x}, t)}, \tag{71}$$

where τ_χ denotes an effective relaxation timescale associated with the projected χ configuration, defined only within spacetime-based descriptions. This relation emphasizes that H_{eff} characterizes a local relaxation rate rather than a literal velocity of metric expansion.

Topological Frustration and Inhomogeneous Relaxation.

The relaxation of the χ substrate is not globally uniform. Regions of high structural complexity, such as galaxy clusters, filaments, and bound halos, correspond to **topologically frustrated** configurations of χ , in which knots, defects, and bound spectral modes inhibit or delay relaxation. As a result, the effective relaxation timescale acquires a spatial dependence:

$$\tau_\chi(\mathbf{x}) = \tau_\chi^{(0)} \left[1 + \epsilon \, \mathcal{T}(\mathbf{x})\right], \tag{72}$$

where $\mathcal{T}(\mathbf{x})$ encodes the local topological density and ϵ parametrizes the coupling between topological complexity and relaxation dynamics.

Global Averaging vs. Local Projection.

The Hubble tension may then be understood as a manifestation of a non-commutativity between cosmological averaging and local projection. Early-universe observables, such as those derived from the cosmic microwave background, probe a nearly homogeneous, pre-decoherent phase of the substrate and effectively measure a global average $\langle \tau_\chi^{-1} \rangle_{\text{early}}$. In contrast, late-time local measurements based on distance ladders or standard candles sample an already structured universe, in which topological decoherence is fully developed, and therefore probe a biased local relaxation rate $\tau_\chi^{-1}(\mathbf{x}_{\text{late}})$. This non-commutativity mirrors the breakdown of homogeneous parametrizations discussed in the emergence of cosmic acceleration.

No Additional Dark Sector.

This interpretation resolves the Hubble tension without introducing any exotic dark energy component or modifying the gravitational field equations. The discrepancy arises instead as a structural consequence of the projective and relational dynamics of the χ substrate itself.

Testable Prediction.

Cosmochrony predicts that locally inferred values of H_0 should exhibit weak but systematic correlations with the surrounding topological environment, such as density gradients or void/filament membership. From this perspective, the Hubble tension becomes a direct observational signature of the non-uniform relaxation of χ, rather than a failure of cosmological modeling.

The present discussion is intended as a qualitative explanation rather than a precision prediction. A more detailed analysis, outlining how different observables map onto the relaxation history of χ, is provided in Appendix C.3.

Quantitative estimates are provided in Section 14.1.

12.11 Large-Angle Temperature Anomalies

Large-angle anomalies observed in the cosmic microwave background, such as the suppression of power at low multipoles and the presence of unexpected large-scale alignments, remain only partially explained within the standard Λ CDM framework [42].

Within the Cosmochrony framework, these features are naturally interpreted as residual relational correlations inherited from the pre-geometric regime of the χ substrate. Before the emergence of a stable spacetime description, relational structure was not organized according to spatial separation or causal horizons. As a result, long-range relational correlations could persist through the projection transition without requiring superluminal processes or inflationary amplification.

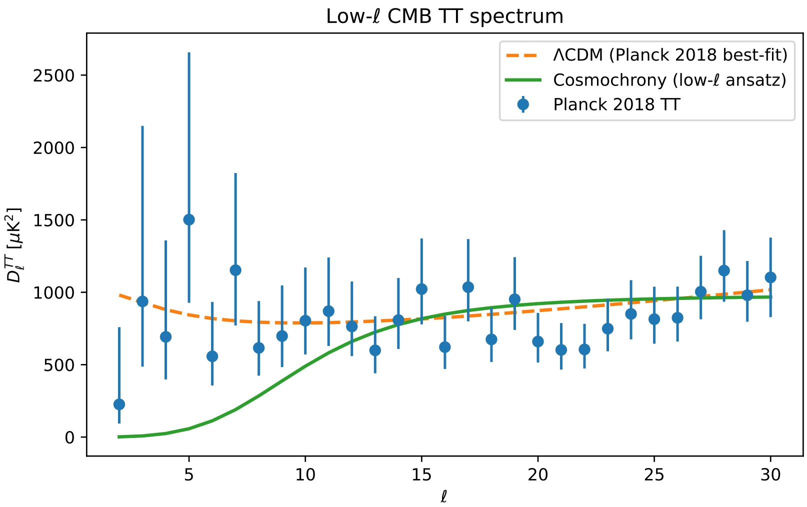

Fig. 10 Low-ℓ CMB TT power spectrum comparison. The Cosmochrony ansatz (green line) shows a natural suppression of power at large angular scales ($\ell < 10$), providing a closer fit to the Planck 2018 data points compared to the standard ΛCDM best-fit (dashed orange line).

When geometric projection becomes admissible, most of these correlations are washed out by subsequent relaxation and structure formation. However, weak remnants may survive at the largest angular scales, where "cosmic variance" is dominant and effective geometric descriptions are least constraining. Such residual correlations are expected to preferentially reflect the same underlying spectral constraints discussed in the context of CMB polarization.

In this perspective, large-angle CMB anomalies do not signal a breakdown of cosmological consistency. They reflect the partial imprint of a pre-geometric relational phase on the earliest projectable spacetime observables, and are therefore expected to appear primarily at the largest scales.

Structural Admissibility and Low-ℓ Suppression

In the Cosmochrony framework, the primordial power spectrum is not assumed to be a pure scale-invariant law from the outset. Instead, it reflects the set of configurations that are *projectively admissible* given the relaxation state of the pre-geometric substrate χ.

At early cosmic times, the relaxation of χ is strongly constrained by global saturation effects, severely limiting the admissibility of complex or highly oscillatory configurations. As a consequence, the effective primordial spectrum is expected to be modulated by a structural constraint reflecting this restricted configuration space.

We formalize this effect by introducing an *Admissibility Filter* $\mathcal{A}(k,t)$ acting on the primordial spectrum:

$$P_{\text{obs}}(k,t) = \mathcal{A}^2(k,t)\, P_0(k), \tag{73}$$

where $P_0(k)$ denotes the unconstrained primordial spectrum.

A natural phenomenological form for this filter, capturing a structural infrared cutoff, is

$$\mathcal{A}(k,t) = \exp\left[-\left(\frac{k_c(t)}{k}\right)^p\right], \tag{74}$$

where $k_c(t)$ defines a *coherence scale* associated with the maximal size of projectively admissible configurations at time t, and p controls the sharpness of the transition.

For modes with $k \ll k_c(t)$, corresponding to the largest angular scales (low-ℓ), power suppression arises not from inflationary dilution or statistical variance, but from the structural impossibility of supporting such global modes within the available relational complexity of the χ substrate.

Graph-Theoretic Interpretation of the Admissibility Scale

The admissibility scale $k_c(t)$ admits a natural graph-theoretic interpretation. At any effective cosmological time t, the relational substrate χ defines an effective connectivity graph $G(t)$ whose weighted Laplacian $L_G(t)$ encodes the set of admissible collective modes.

The spectral gap $\lambda_2(t)$ of $L_G(t)$ characterizes the global connectivity of this relational structure and sets a minimal admissible wavelength. Modes with $k \lesssim \sqrt{\lambda_2(t)}$ are therefore structurally inadmissible. As relaxation proceeds, the effective graph becomes increasingly connected, lowering $\lambda_2(t)$ and progressively allowing larger-scale coherent modes to emerge.

Testable Predictions

To distinguish structural admissibility from statistical explanations, the Cosmochrony framework leads to several falsifiable predictions:

A. Correlated Suppression Across TT, TE, and EE Spectra.

If low-ℓ suppression originates from a projective admissibility constraint, it must affect all cosmological observables probing the same primordial modes. In particular, the coherence scale k_c (or equivalently $\ell_c \sim 10$–20) should be consistent across the temperature (TT) and polarization (EE, TE) spectra. Future high-precision polarization measurements (e.g. *LiteBIRD*) provide a decisive test of this prediction.

B. Absence of Primordial Non-Gaussianities.

Because the early Universe in Cosmochrony is characterized by ontological simplicity and a severely restricted configuration space, no primordial mechanism exists to generate significant non-Gaussian correlations. The framework therefore predicts an exceptionally small primordial non-Gaussianity parameter f_{NL}. A statistically significant detection of primordial non-Gaussianities would falsify this minimal relaxation scenario, while increasingly stringent upper bounds would support it.

C. Scale-Dependent Spectral Tilt Near the Cutoff.

The admissibility filter implies that the effective spectral index n_s need not be strictly constant across all scales. A mild running of the spectral index, localized near the transition scale k_c, is expected as the Universe transitions from a globally constrained regime to one allowing richer configuration space. Precise measurements of the running parameter α_s correlated with the low-ℓ region would provide further support for this interpretation.

Ontological Interpretation: The Arrow of Complexity

Within this perspective, the suppression of low-ℓ modes is not an anomaly but a fossil signature of the early Universe's ontological state. The second law of thermodynamics is reinterpreted accordingly: entropy growth does not correspond to increasing disorder, but to the irreversible enlargement of the space of admissible configurations.

The Universe begins in a state of *ontological poverty*, where only a small class of simple, highly coherent configurations can be projected. As relaxation proceeds, this admissible space expands, enabling the emergence of hierarchical, localized, and complex structures. In this sense, cosmological time is simultaneously the arrow of entropy and the arrow of increasing descriptive richness.

A detailed phenomenological treatment of the low-ℓ suppression mechanism, including quantitative parametrizations and comparison with observational data, is provided in Appendix C.1.

12.12 Effective Potential for Galactic Dynamics from χ-Relaxation Saturation

Operational status.

In Cosmochrony, gravitation is not introduced as a fundamental interaction. In regimes where a smooth geometric description is *operationally admissible*, the collective slowdown of χ-relaxation induced by localized excitations may be summarized by an *effective* potential $\Phi_{\text{eff}}(r)$. This potential is defined only through observable kinematics,

$$g_{\text{eff}}(r) \equiv -\frac{d\Phi_{\text{eff}}}{dr}, \qquad v^2(r) = r\,\frac{d\Phi_{\text{eff}}}{dr}, \tag{75}$$

and does not represent an additional degree of freedom.

Emergent acceleration scale.

Because the χ-relaxation constraint is nonlinear, cosmological relaxation induces an effective background kinematic scale $a_0(t)$, expected to track the global relaxation rate through

$$a_0(t) \sim c\,H(t), \tag{76}$$

so that a_0 is not postulated as a fundamental constant but is weakly time-dependent.

Operationally, the galactic manifestation of this cosmological scale is expected to be reduced by a dimensionless projection efficiency factor $\eta = O(0.1)$, reflecting the partial coupling between global χ-relaxation and local projectable galactic configurations. Accordingly, the effective scale entering galactic dynamics is $a_0(t) = \eta\, cH(t)$.

Asymptotic regimes and flat rotation curves.

Let $g_N(r) = GM_b(r)/r^2$ denote the Newtonian baryonic acceleration inferred from the enclosed baryonic mass $M_b(r)$. In the high-acceleration regime $g_N \gg a_0$, projective relaxation remains unsaturated and one recovers $g_{\text{eff}} \simeq g_N$, hence $\Phi_{\text{eff}}(r) \simeq -GM_b/r$. In the low-acceleration regime $g_N \ll a_0$, saturation of the effective relaxation constraint yields the deep-saturation scaling

$$g_{\text{eff}}(r) \simeq \sqrt{g_N(r)\, a_0(t)}. \tag{77}$$

For an asymptotically settled baryonic mass $M_b(r) \to M_b$, this implies

$$g_{\text{eff}}(r) \simeq \frac{\sqrt{GM_b\, a_0(t)}}{r} \quad \Rightarrow \quad \Phi_{\text{eff}}(r) \simeq \sqrt{GM_b\, a_0(t)}\, \ln\!\left(\frac{r}{r_s}\right) + \text{const.}, \tag{78}$$

i.e. a logarithmic potential producing asymptotically flat rotation curves.

Transition scale.

Define the transition radius r_s operationally by $g_N(r_s) = a_0(t)$. For $M_b(r) \to M_b$, one finds

$$r_s(t) = \sqrt{\frac{GM_b}{a_0(t)}}. \tag{79}$$

Minimal smooth interpolation (operational fit function).

A parsimonious interpolation consistent with both limits is

$$g_{\text{eff}}(r) = \sqrt{g_N(r)^2 + a_0(t)\, g_N(r)}. \tag{80}$$

Equation (80) is not postulated as a fundamental law but used as a compact operational representation of the saturation crossover in projectable regimes. Combined with the definition of Φ_{eff} above, it yields direct and testable predictions for rotation curves and baryonic scaling relations.

Baryonic Tully–Fisher scaling.

In the deep-saturation regime, Eq. (78) implies

$$v_\infty^4 \simeq G\, M_b\, a_0(t), \tag{81}$$

providing a structural origin for a baryonic Tully–Fisher-type relation, while predicting a mild redshift dependence through $a_0(t) \sim cH(t)$.

For clarity, Table 1 summarizes how the explanation of flat galactic rotation curves in Cosmochrony compares conceptually with the standard ΛCDM framework and with MOND-like approaches.

Aspect	ΛCDM (dark matter halo)	MOND (modified dynamics)	Cosmochrony (projective saturation)
Ontology	Introduces a non-baryonic matter component (halo)	Modifies the law of motion / gravitation at low acceleration	No new particle; "dark" effects arise from relaxation properties of the χ substrate
Origin of flat rotation curves	Invisible mass $M_{\rm DM}(r)$ such that $v(r) \approx$ const	Deep-MOND regime $g \simeq \sqrt{g_N a_0}$	Saturation and nonlinearity of the relaxation constraint yielding $g \simeq \sqrt{g_N a_0(t)}$
Key scale or parameter	Halo profile (NFW, cored, etc.) and formation parameters	Acceleration scale a_0 (often universal)	Emergent, slowly evolving scale $a_0(t) \sim cH(t)$; threshold linked to projective regime
Large-r effective potential	$\Phi \sim v^2 \ln r$ via quasi-isothermal halo (or equivalent)	$\Phi \sim \sqrt{GMa_0}\, \ln r$	$\Phi_{\rm eff} \sim \sqrt{GM_b a_0(t)}\, \ln r$ as an operational summary of saturation
Baryonic Tully–Fisher relation	Emergent from galaxy formation models (feedback, tuning)	$v^4 \propto GM_b a_0$ (direct)	$v^4 \propto GM_b a_0(t)$ (direct), with possible mild redshift dependence via $H(t)$
Environmental dependence	Strong, through halo assembly history and profile diversity	Must be treated via external-field effects or extensions	Expected through the relaxation state and projectability (spectral age, environment), without postulating a halo
Gravitational lensing	Due to total mass (baryons + dark matter)	Possible but depends on relativistic extensions (e.g. TeVeS)	Interpreted via effective propagation/refraction in projectable χ regimes
Discriminating signature	Cusps vs. cores, substructure abundance, halo–baryon correlations	Strict universality of a_0 (except in extensions)	Slow evolution of $a_0(t)$ and correlations with relaxation history ("memory" / lag effects)

Table 1 Conceptual comparison of flat-rotation-curve explanations. In Cosmochrony, $\Phi_{\rm eff}$ is an operational summary of saturation in projectable regimes, not a fundamental gravitational field.

Observed rotation curves: multi-galaxy test

To assess the phenomenological viability of the effective saturation picture, we confront the Cosmochrony prediction with observed rotation curves of three representative spiral galaxies, spanning the main empirical classes, thereby probing the flexibility of the saturation mechanism beyond asymptotically flat profiles: (i) a nearly flat rotation curve (NGC 3198), (ii) a rising curve (NGC 2403), and (iii) a mildly declining curve (NGC 5055).

These systems are chosen because they are extensively studied, have well-constrained baryonic decompositions, and are commonly used as benchmarks in both ΛCDM and MOND analyses. No dark matter halo is introduced at any stage.

The comparison follows a minimal-parameter philosophy. Geometric quantities such as distance, inclination, and position angle are fixed to their observationally inferred values. The baryonic contributions from gas and stars are taken directly from the literature.

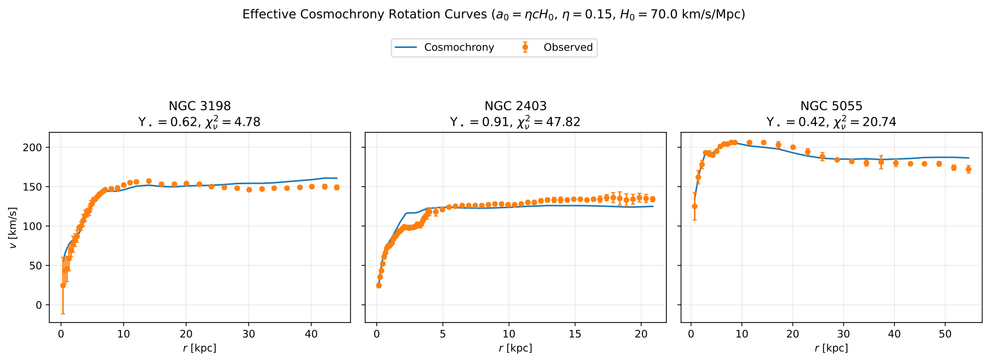

Fig. 11 Observed rotation curves compared with the Cosmochrony saturation prediction. Left: NGC 3198 (flat). Center: NGC 2403 (rising). Right: NGC 5055 (mildly declining). Points show observational data; solid lines show the Cosmochrony prediction. The only fitted parameter is the stellar mass-to-light ratio $\Upsilon_\star$. See Appendix D.8 for data sources and fitting protocol.

The only fitted parameter is the stellar mass-to-light ratio $\Upsilon_\star$, assumed constant within each galaxy. The acceleration scale $a_0(t_0)$ is *not* fitted but fixed by the cosmological relation $a_0(t) \sim cH(t)$ evaluated at the present epoch.

The resulting rotation curves are shown in Fig. 11. The effective saturation model reproduces: (i) flat asymptotic velocities where observed, (ii) rising profiles in baryon-dominated outer regions, and (iii) mild declines without overshooting.

This behavior arises naturally from the interplay between baryonic distribution and saturation of the χ-relaxation constraint, without invoking galaxy-specific tuning or additional degrees of freedom.

Limitations and scope of the comparison.

The reduced χ^2 values associated with the fits should not be interpreted as strict goodness-of-fit estimators. Rotation-curve data points are affected by correlated uncertainties (inclination, non-circular motions, disk thickness, asymmetric drift) that are not fully captured by the quoted statistical errors. The aim of the present comparison is therefore not to achieve an optimal statistical fit, but to test whether the *global radial trends* observed across different morphological classes can be reproduced within a single saturation-based framework, without introducing dark matter halos or galaxy-dependent acceleration scales.

12.13 Summary

Within the Cosmochrony framework, cosmological phenomena do not originate from fundamental spacetime dynamics. They emerge from the global relaxation ordering of the relational χ substrate, from which effective notions of space, time, and geometry become progressively admissible.

Cosmic expansion, large-scale homogeneity, late-time acceleration, and the arrow of time arise naturally from this relaxation process, without invoking an inflationary phase, a “dark energy” component, or an initial spacetime singularity. The Big Bang is reinterpreted as a limiting regime of maximal constraint beyond which spacetime descriptions cease to be meaningful, while black holes represent localized reapproaches to the same descriptive boundary.

At the level of effective spacetime descriptions, Cosmochrony reproduces the phenomenological successes of standard cosmology, including the Hubble law, the cosmic microwave background structure, and large-scale gravitational behavior, as emergent consequences of the same underlying relaxation ordering. At the same time, it provides a unified and pre-geometric interpretation of these phenomena, rooted in a single relational relaxation process rather than in multiple independent cosmological ingredients.

In this sense, Cosmochrony does not propose an alternative cosmological model, but a foundational framework clarifying the origin, scope, and domain of validity of cosmological descriptions themselves.

13 Radiation and Quantization

13.1 Radiation as χ–Matter Interaction

Within the Cosmochrony framework, radiation does not correspond to the emission or propagation of fundamental particle entities. It arises as an effective phenomenon associated with the reconfiguration of localized matter descriptions at the level of projection, and their relational coupling to the surrounding χ substrate.

When a localized, relaxation-resistant configuration undergoes a transition toward a less constrained state, part of the relational structure that sustained its previous persistence becomes incompatible with continued localization. This excess relational content ceases to admit a particle-like projected description and instead becomes expressible only through delocalized projected modes.

In effective spacetime descriptions, this redistribution appears as radiative emission. Radiation thus represents the loss of localized projectability and the transfer of descriptive weight from particle-like configurations to propagating field-like descriptions, without invoking the transport of discrete objects or underlying stochastic processes.

From this perspective, radiative phenomena reflect a change in the organization and projectability of relational structure within χ, rather than the emission of pre-existing quanta or the manifestation of fundamental fluctuations.

13.2 Emergence of Photons

In the Cosmochrony framework, photons are not fundamental entities, nor are they identified with propagating disturbances of the χ substrate. They arise as effective descriptions associated with transitions between localized and delocalized regimes of projectability.

Prior to emission or detection, no photon exists as an independent object. What exists is a reconfiguration of relational structure within χ that ceases to admit a localized particle-like projection and becomes expressible only through extended, delocalized effective modes.

In effective spacetime descriptions, these delocalized regimes are represented as electromagnetic waves. However, this wave character does not correspond to a physical oscillation of χ, but to a continuous descriptive projection of relational structure compatible with field-like representation.

Photon-like events emerge only at interaction. When a delocalized projective mode becomes locally constrained by interaction with a localized excitation (such as an atom or detector), the projection resolves into a discrete transfer of relaxation capacity. Quantization is therefore not a property of propagation, but of interaction and local reprojection.

In this sense, wave–particle duality reflects a duality of description rather than a duality of underlying ontology. Interference phenomena, such as those observed in double-slit experiments, arise from the coherence of delocalized projective modes, while individual detection events correspond to localized reprojections. No fundamental wavefunction collapse or stochastic emission process is required.

13.3 Geometric Origin of $E = h\nu$

This section develops, in the context of radiative processes, the energy–frequency relation introduced earlier in Section 6.5, while remaining fully consistent with the non-propagative and pre-geometric nature of the χ substrate.

In Cosmochrony, radiative events do not correspond to the emission of physical waves or disturbances propagating within the χ field. Instead, they correspond to transitions between localized and delocalized regimes of effective projectability. During such events, a portion of the relaxation potential stored in a localized matter excitation becomes expressible only through an extended, non-localized projective description.

Within effective spacetime representations, these delocalized regimes are described using oscillatory field modes characterized by a frequency ν. This frequency does not represent a fundamental oscillation of χ, but a parameter labeling the the internal structural periodicity of the effective projective representation required to represent the released relaxation potential.

The Planck relation

$$E = h\nu \tag{82}$$

thus acquires a geometric interpretation. The energy E measures the amount of relaxation potential redistributed during a reprojection event, while the frequency ν characterizes the minimal temporal resolution required for a coherent effective description of this redistribution.

The proportionality constant h does not encode a fundamental quantum postulate. As derived in Section D.6, h is the **effective projection** of the fundamental substrate invariant $\hbar_\chi \equiv c^3/(K_{0,\mathrm{bare}}\chi_{c,\mathrm{bare}})$. It acts as a universal conversion factor linking the structural relaxation capacity of the substrate to the temporal resolution of the projective description.

This interpretation explains why energy transfer in radiative processes scales linearly with frequency across a wide range of phenomena. In the photoelectric effect, the threshold frequency ν_0 corresponds to the minimal projective resolution required to destabilize a bound electronic soliton. Above this threshold, the linear dependence on ν reflects the additional relaxation capacity made accessible through the reprojection process.

In this sense, quantization of radiative energy does not arise from discretized propagation, but from the discrete nature of local **reprojection events**, which impose a minimal unit of effective relaxation transfer determined by the spectral graininess ($\hbar_\chi$) of the underlying χ substrate.

13.4 Vacuum Fluctuations and the Casimir Effect

In the Cosmochrony framework, vacuum fluctuations do not correspond to physical oscillations of a background field nor to spontaneous particle–antiparticle creation. Instead, they reflect the intrinsic structural indeterminacy of the χ substrate in regimes where no stable localized excitations are present.

In the absence of matter-induced constraints, the relaxation of χ admits a wide range of locally compatible projective descriptions at the level of effective projection. These fluctuations are not dynamical events occurring *in* spacetime, but expressions of the fact that the underlying relational structure of χ does not select a unique effective configuration when projected. They therefore represent variability of effective descriptions rather than physical energy stored in the vacuum.

When material boundaries are introduced, they impose structural constraints on the local projectability of χ. Certain effective descriptions become incompatible with the imposed relational conditions, reducing the set of admissible projective configurations between the boundaries compared to the exterior region.

The Casimir effect arises from this asymmetry. It reflects a difference in the density of admissible effective reprojections compatible with the boundary conditions, which manifests in spacetime descriptions as a pressure acting on the confining surfaces. No fundamental vacuum energy density or propagating vacuum modes are required.

In this sense, the Casimir effect probes the relational relaxation capacity of the χ substrate under imposed constraints, rather than revealing the presence of a physical zero-point energy filling space.

13.5 Weakly Interacting Radiation

In the Cosmochrony framework, weakly interacting radiation does not correspond to fundamentally different particle species, but to delocalized projective regimes whose structural contrast and curvature are insufficient to efficiently induce localized reprojection when encountering matter.

Low-frequency electromagnetic descriptions or weakly coupled projective modes are characterized by smooth, slowly varying relational structure at the level of effective projection. As a result, their interaction with localized χ configurations is rare: the probability that such descriptions trigger a stable localized energy transfer upon interaction is strongly suppressed.

This explains the effective transparency of vacuum to most radiation. Propagation corresponds to the persistence of coherent delocalized projective descriptions across extended regions, while detection events occur only when local structural conditions allow reprojection into a localized excitation.

In this sense, small interaction cross sections do not reflect the weakness of a fundamental force, but the low likelihood that a given projective configuration satisfies the geometric and topological conditions required for localized reprojection.

13.6 Summary

In Cosmochrony, radiation and quantization arise from interactions between localized matter excitations and the χ substrate. Photon-like events emerge only during interaction-induced reprojection, rather than corresponding to pre-existing independent entities.

Quantization reflects geometric and topological constraints of the relaxation dynamics of χ at the level of projection: discrete energy transfer occurs when continuous projective descriptions satisfy the conditions required for localized reprojection. In this sense, quantization is not fundamental discreteness, but an emergent manifestation of constrained relaxation and interaction geometry.

14 Testable Predictions and Observational Signatures

Before detailing specific observational signatures, it is important to clarify the epistemic status of the numerical estimates presented in this section. Values such as the ~ 8–10% offset between early- and late-time effective determinations of the Hubble constant or the $\sim 10^{-10}\,\mathrm{yr}^{-1}$ drift in effective spacetime observables are not proposed as precision predictions. They should be understood as order-of-magnitude consistency estimates derived from the geometric coupling between the χ substrate and the effective relaxation fraction Ω_χ.

Their role is to demonstrate that the Cosmochrony framework operates within a phenomenologically relevant regime, capable of addressing current observational tensions without fine-tuning or the introduction of additional dynamical degrees of freedom.

14.1 Hubble Constant from χ Dynamics

In Cosmochrony, the Hubble parameter is not introduced as a free cosmological constant, but arises as an effective quantity associated with the relaxation dynamics of the χ substrate. At the level of an effective spacetime description, it may be written as

$$H(t) = \frac{\dot{\chi}}{\chi}, \tag{83}$$

where the dot denotes differentiation with respect to an effective cosmological time parameter introduced solely to parametrize the relaxation ordering, not a fundamental temporal evolution.

In homogeneous regimes, the relaxation rate approaches its maximal admissible value. Assuming $\dot{\chi}_{\mathrm{eff}} \simeq c$, the present-day Hubble parameter can be estimated as

$$H_0 \simeq \frac{c}{\chi(t_0)}. \tag{84}$$

This relation establishes a direct correspondence between the observed Hubble constant and the characteristic relaxation scale of χ at the current epoch. Early-universe probes (such as CMB-based inferences) and late-time distance-ladder measurements effectively sample χ at different stages of its relaxation, naturally leading to systematically different inferred values of H_0 without invoking additional cosmological components or fine-tuned initial conditions.

14.2 Redshift Drift

The monotonic relaxation of the χ substrate implies a slow temporal evolution of cosmological redshifts when described in an effective spacetime parametrization. This leads to a redshift drift whose magnitude and redshift dependence differ quantitatively from those predicted by the standard ΛCDM model, particularly at intermediate redshifts.

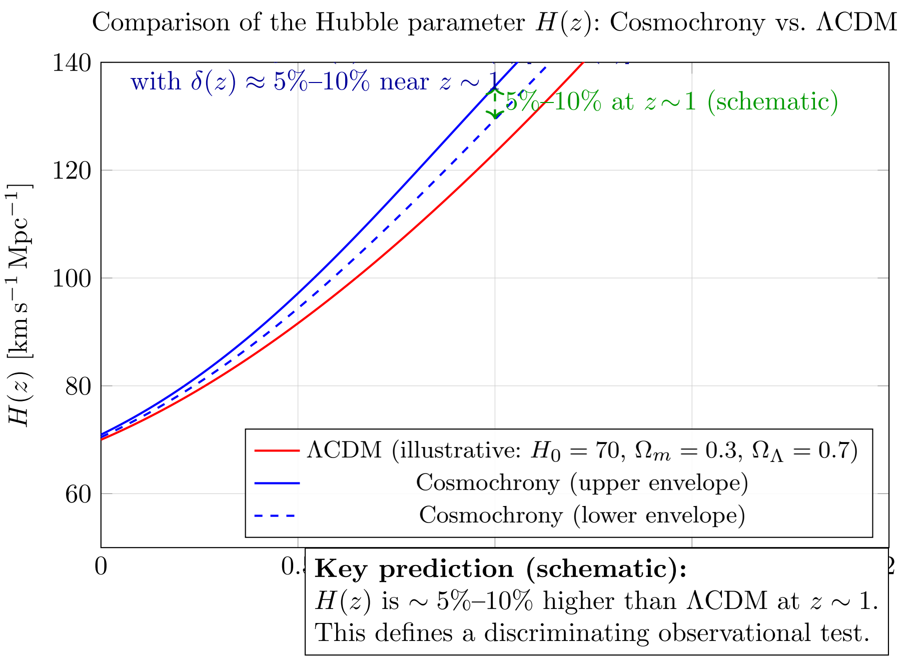

Fig. 12 Schematic comparison of $H(z)$ in Cosmochrony and ΛCDM. The ΛCDM curve uses the standard matter+Λ functional form with illustrative parameters, while Cosmochrony is represented as a fractional enhancement band near $z \sim 1$.

At the level of an effective parametrization, intended only as an order-of-magnitude estimate rather than a derived dynamical law, the effective drift rate may be written as

$$\dot{z}_{\text{eff}} \sim H_0(1+z) - \frac{c}{\chi(t)}, \tag{85}$$

where the second term reflects the ongoing relaxation of the χ substrate rather than a dark-energy-driven acceleration. This corresponds to a secular variation of order

$$\Delta z \sim 10^{-10}\,\text{yr}^{-1}$$

at redshift $z \sim 1$, differing from ΛCDM expectations at the $\sim 10\%$ level in this regime.

Future high-precision spectroscopic facilities, such as extremely large telescopes equipped with ultra-stable spectrographs, may be capable of probing this effect. A detection of a redshift drift incompatible with ΛCDM predictions would therefore provide a direct observational discriminator between geometric relaxation of the χ substrate and dark-energy-driven cosmic acceleration.

14.3 Gravitational Wave Propagation

In the Cosmochrony framework, gravitational waves are not interpreted as fundamental propagating excitations of the χ substrate. They correspond instead to coherent, extended projective descriptions of large-scale relational variations, admissible only in regimes where an effective spacetime representation applies.

These projective descriptions do not constitute independent dynamical degrees of freedom. Rather, they reflect time-dependent redistributions of relaxation constraints at the level of effective projection.

In regions of high excitation density, such as near compact objects, the local suppression of χ relaxation is expected to modify the persistence and coherence of these projective descriptions. In particular, partial loss of projective coherence may arise due to the coupling between extended gravitational-wave descriptions and strongly constrained relaxation regions. These effects originate from the same collective relaxation constraints responsible for gravitational time dilation and horizon formation, and do not require the introduction of additional dynamical fields.

LISA signature: Cosmochrony predicts a $\sim 10\%$ **absorption** of gravitational waves near black holes (Section 7.7), distinct from GR's pure propagation.

Order-of-magnitude attenuation estimate.

Consider a compact object of mass M, characterized in effective geometric descriptions by a Schwarzschild radius

$$r_s = \frac{2GM}{c^2}.$$

Gravitational-wave descriptions traversing regions where the effective relaxation capacity is significantly reduced are expected to exhibit a partial degradation of coherence, due to redistribution of descriptive weight into locally non-projectable configurations.

For trajectories passing within a characteristic distance

$$r \lesssim 10\,\frac{GM}{c^2},$$

the cumulative reduction of effective relaxation conductivity suggests an order-of-magnitude suppression of the coherent projective amplitude that may be parametrized as

$$\frac{\Delta A}{A} \sim \mathcal{O}(10^{-2} - 10^{-1}),$$

where the precise magnitude depends on the local χ correlation length ξ and on the effective relaxation fraction Ω_χ in the vicinity of the source. This effect should be interpreted as a redistribution of projective coherence within the relaxation dynamics, rather than as dissipative energy loss.

Observational signature.

Such effects are expected to manifest most clearly during the late-time ringdown phase of binary black hole mergers, where gravitational-wave signals probe regions of strongly constrained relaxation near the effective horizon. The resulting signature would appear as frequency-dependent deviations from general relativistic ringdown templates, potentially mimicking anomalous damping or mode-dependent quality factors.

While current ground-based detectors do not yet achieve the signal-to-noise ratios required to resolve attenuation at the few-percent level, future space-based observatories operating in the LISA band, with expected signal-to-noise ratios exceeding ~ 100 for massive black hole mergers, may provide sufficient sensitivity to test this prediction.

Semi-quantitative scaling estimate.

Within the Cosmochrony framework, the reduction of coherent gravitational-wave amplitudes near compact objects arises from the local suppression of effective projectability in regions of high curvature. At leading order, the relative amplitude reduction is expected to scale with the dimensionless curvature parameter (r_s/r).

A simple dimensional estimate yields

$$\frac{\Delta A}{A} \sim \left(\frac{r_s}{r}\right)^2,$$

indicating that the effect depends explicitly on both the compact object mass and the wave trajectory's impact parameter. For propagation at distances $r \approx 10\, r_s$, this scaling gives

$$\frac{\Delta A}{A} \sim 10^{-2},$$

consistent with the order-of-magnitude estimates above and with exploratory numerical results obtained from χ-based simulations (Appendix D.3).

The same structural relaxation mechanisms that affect cosmological expansion and gravitational-wave propagation are expected to leave observable imprints at galactic scales, where long-lived projected configurations interact with the surrounding χ-substrate.

14.4 Galaxy Rotation Curves from Structural Relaxation

The Cosmochrony framework predicts modifications of galactic rotation curves arising from the structural relaxation of the projected χ_{eff} field, without introducing additional dark matter components.

In regions where localized matter configurations induce persistent relaxation gradients in the χ-substrate, the effective inertial response of orbiting matter is modified. This leads to an enhancement of tangential velocities at large radii, producing approximately flat rotation curves.

Unlike modified gravity scenarios, this effect does not require an explicit change of the gravitational force law. It arises from a non-local redistribution of relaxation capacity in the projected description, tied to the large-scale coherence of the underlying χ-configuration.

Observable consequences include:

- a correlation between rotation curve flattening and indicators of structural relaxation activity rather than baryonic mass alone,
- deviations from simple baryonic scaling relations in dynamically young or disturbed galaxies,
- a reduced need for fine-tuned dark matter profiles in low-surface-brightness systems.

These signatures provide a discriminant between Cosmochrony, particle dark matter models, and purely phenomenological modified gravity approaches.

14.5 Spin and Topological Signatures

If particle spin originates from topologically nontrivial configurations of the χ substrate, as proposed in this work, then spin-related phenomena may admit geometric signatures not captured by purely algebraic quantum descriptions.

In particular, ultra-high-precision interference experiments sensitive to 4π rotational symmetry may, in principle, probe deviations associated with the internal topology of localized projectable χ configurations. Such deviations would not modify standard spin–statistics relations, but could appear as extremely small phase shifts or coherence effects under closed 2π versus 4π rotational cycles.

These signatures are expected to be strongly suppressed and therefore lie beyond current experimental resolution. However, their existence would provide a conceptually distinctive test of the topological origin of spin proposed in Cosmochrony, as opposed to interpretations in which spin is treated as an abstract representation of spacetime symmetry groups.

14.6 Absence of Dark Energy Signatures

Because cosmic acceleration emerges in Cosmochrony as a geometric consequence of the global relaxation of the χ substrate, no independent dark energy component is introduced. Accordingly, the framework predicts the absence of signatures associated with dynamical dark energy, such as an evolving equation of state, clustering behavior, or additional propagating degrees of freedom at the level of effective spacetime descriptions beyond those already present in the geometric framework.

Within this perspective, observations consistent with a purely geometric and kinematic origin of late-time acceleration would favor Cosmochrony over models requiring additional energy components or fine-tuned scalar fields.

Discriminating observational signatures.

The absence of dark energy dynamics cannot be established through any single observable. Instead, Cosmochrony predicts a correlated pattern of large-scale cosmological features reflecting the lack of an inflationary phase and the pre-geometric origin of early-time correlations.

These features include suppressed power at low CMB multipoles, specific angular correlations in temperature and polarization, and the absence of an inflationary tensor imprint at large angular scales. It is the combined presence of these signatures—rather than any individual parameter—that provides a potential observational discriminator with respect to standard inflationary and dark-energy-driven cosmological models.

14.7 Emergent Phenomenology and Observational Probes

The Cosmochrony framework leads to a set of qualitative and semi-quantitative phenomenological signatures that distinguish it from standard cosmological and quantum approaches. These signatures arise from the monotonic relaxation of the χ substrate and from the topological organization of its localized configurations, rather than from fine-tuned parameters or additional fundamental fields.

Cosmic Microwave Background.

Residual large-scale projective correlations inherited from the pre-geometric relaxation of the χ substrate imprint scale-dependent temperature anisotropies in the cosmic microwave background through their modulation of the local relaxation rate. Unlike inflationary scenarios, Cosmochrony does not rely on superluminal stretching: the relaxation of χ is locally bounded by the invariant speed c. As a consequence, correlations at the largest angular scales are naturally suppressed, leading to a reduction of power at low multipoles ($\ell \lesssim 10$).

This mechanism is consistent with several large-angle features reported in CMB data, such as hemispherical asymmetry, without requiring fine-tuned initial conditions. Quantitative estimates of the resulting low-ℓ suppression are discussed in Appendix C.1.

Connection with CMB observations.

Observationally, the *Planck* 2018 data report a suppression of the CMB quadrupole power at the level of $\sim 10\%$ relative to the ΛCDM best-fit expectation, corresponding to the long-standing low-ℓ anomaly at $\ell = 2$ [43]. Within Cosmochrony, this suppression arises naturally from the pre-geometric relaxation dynamics of the χ substrate, which reduces large-angle correlations prior to the emergence of an effective spacetime description. Unlike phenomenological explanations relying on specific initial conditions or model-dependent modifications of primordial spectra, the effect follows directly from the intrinsic relaxation properties of the underlying field.

Gravitational-wave propagation.

In regions of strong structural variation of the χ substrate, such as near compact objects, the local slowdown of χ relaxation modifies the persistence and coherence of gravitational-wave projective descriptions. Rather than inducing dissipative losses, this effect manifests as frequency-dependent phase shifts or dispersion-like behavior in gravitational wave signals. Such modifications could, in principle, affect the ringdown phase of binary black hole mergers and may become accessible to next-generation observatories including future space-based observatories such as LISA and next-generation ground-based detectors.

Hubble tension.

The modulation of the χ relaxation rate by large-scale matter inhomogeneities provides a natural mechanism for reconciling early-universe and late-time measurements of the Hubble constant. Within this framework, the effective Hubble parameter $H(z)$ acquires a mild redshift dependence that departs from ΛCDM at intermediate redshifts ($0.1 \lesssim z \lesssim 10$). This behavior is testable through future baryon acoustic oscillation and supernova surveys.

Particle phenomenology.

In Cosmochrony, intrinsic particle properties such as mass and spin originate from the topological structure of localized projectable χ configurations. This perspective does not predict violations of the spin–statistics connection, but suggests that its origin is geometric rather than axiomatic. While the present work does not provide a classification of all possible topological excitations, it opens the possibility that additional, non-standard configurations may exist. Identifying observable consequences of such configurations remains an open problem for future theoretical and experimental investigation.

Quantitative deviations from ΛCDM.

Quantitative comparisons between Cosmochrony and ΛCDM predictions are provided in Appendix C.1 for cosmic microwave background observables and in Appendix C.3 for the Hubble tension. As illustrative examples, the suppression of low-ℓ CMB power in Cosmochrony is of order $\sim 10\%$ for $\ell \lesssim 10$, exceeding the $\sim 5\%$ level expected from cosmic variance within ΛCDM. In addition, gravitational wave propagation near compact objects is predicted to exhibit an effective amplitude reduction or coherence attenuation of order $\Delta A/A \sim 10^{-2}$ for trajectories passing within $r \lesssim 10\,GM/c^2$ of a black hole, a magnitude potentially accessible to future space-based interferometers such as LISA.

Status of predictions.

The phenomenological signatures discussed above are not introduced as ad hoc modifications, but arise generically from the relaxation dynamics of χ. Their role is to delineate potential observational discriminants of the framework, rather than to provide precision predictions at the current stage. Confirmation or falsification of any subset of these effects would therefore constitute a critical test of the Cosmochrony approach.

14.8 CMB Polarization Signatures (Outlook)

Cosmological Imprints: The $8/3$ Scaling in CMB Polarization

The fundamental spectral ratio $\lambda_2/\lambda_1 = 8/3$, which governs the electroweak mass hierarchy at the micro-scale (see Appendix D.7), is expected to leave a structural signature on the Cosmic Microwave Background (CMB). In this framework, primordial scalar and tensor perturbations are reinterpreted as dual manifestations of the substrate's relaxation.

Geometric Bound on the Tensor-to-Scalar Ratio (r). In Cosmochrony, the tensor-to-scalar ratio r is constrained by the relative spectral stiffness of the χ substrate's projection modes. Under the principle of **Projective Spectral Saturation** at the high-energy limit ($k \approx 1/h_\chi$), the relaxation energy $\mathcal{E}$ is distributed according to the maximal kinematic capacity of each mode:

$$\mathcal{E}_s \propto \lambda_{\text{base}} \Delta_s^2, \qquad \mathcal{E}_t \propto \lambda_{\text{fiber}} \Delta_t^2. \tag{86}$$

The "bare" geometric ratio r_0 is defined by the saturation of these spectral densities:

$$r_0 = \frac{\Delta_t^2}{\Delta_s^2} = \frac{\lambda_{\text{base}}}{\lambda_{\text{fiber}}} = \frac{3}{8} \simeq 0.375. \tag{87}$$

This value does not correspond to an observable tensor-to-scalar ratio at recombination, but defines a *geometric upper bound* imposed by the topology of the projection fiber at the saturation scale.

Topological Decoherence and Parametrization of r_{obs}. The observed ratio r_{obs} undergoes **topological decoherence** as the substrate expands. Since fiber shear modes are intrinsically more sensitive to losses of projective alignment, the cumulative degradation of alignment induces a monotonic suppression of tensor modes. To leading order, this effect may be effectively parametrized as

$$r_{\mathrm{obs}}(t) = r_0 \cdot \exp\Big(-\zeta \frac{\tau_\chi}{t}\Big), \tag{88}$$

where τ_χ denotes the characteristic relaxation time of the substrate. The precise functional form is not fundamental and merely encodes the fact that fiber shear modes decohere faster than base transmittance during cosmic relaxation. This decay represents the transition from the primordial saturated state to the present large-scale geometric stability, providing a structural explanation for the low observed value of r ($r < 0.036$), without invoking slow-roll dynamics or fine-tuned inflationary potentials.

14.9 Neutrino-Mediated Relaxation and Decay Signatures

In Cosmochrony, particle decay and neutrino emission are manifestations of structural reorganization rather than independent microscopic processes. This interpretation leads to distinct observational signatures.

Because neutrino-like excitations act as non-local relaxation channels, decay processes in the early universe contribute to an irreversible smoothing of admissible configurations. This smoothing is expected to leave detectable imprints across multiple observational domains.

Specifically, the framework predicts:

- an enhanced role of neutrino backgrounds in suppressing large-scale coherence without behaving as conventional radiation pressure,
- a weak coupling between decay rates and late-time environmental conditions, reflecting their origin in early structural metastability,
- possible correlations between decay-driven neutrino emission and large-scale anisotropies observed in the cosmic microwave background.

At the particle-physics level, this interpretation suggests that decay lifetimes and branching ratios encode information about the stability landscape of admissible projected configurations rather than fundamental stochasticity. Future precision measurements of rare decays may therefore provide indirect probes of the structural relaxation dynamics underlying the Cosmochrony framework.

Environmental Modulation of Particle Stability

A distinctive prediction of the Cosmochrony framework is that particle stability is not strictly universal, but may exhibit a weak dependence on the surrounding structural environment. Because particle decay originates from the susceptibility of metastable projected configurations to their own admissible fluctuations, any factor that modifies the local relaxation landscape can, in principle, affect decay rates.

In regions characterized by strong gradients of the χ substrate—such as galactic cores or highly structured gravitational environments—the spectrum and amplitude of admissible fluctuations are expected to differ slightly from those in weakly structured regions. As a result, the effective lifetime of unstable particles may acquire a small environment-dependent modulation.

This effect is predicted to be extremely weak and therefore compatible with all current laboratory and astrophysical constraints. Nevertheless, it represents a qualitatively new signature: a violation of strict universality of decay rates induced not by local interactions or spacetime curvature, but by structural relaxation gradients in the underlying relational substrate.

Future high-precision measurements of decay processes in environments with strong gravitational or structural gradients, as well as dedicated numerical simulations of χ-field dynamics, may allow quantitative estimates of this effect. Its detection or exclusion would provide a direct and stringent test of the Cosmochrony framework.

Order-of-magnitude estimate.

Within the Cosmochrony framework, the effective lifetime of an unstable particle is controlled by the susceptibility of a metastable projected configuration to its own admissible fluctuations. Because the spectrum and density of such fluctuations depend weakly on the surrounding structural environment, a small modulation of decay rates is expected in regions characterized by strong gradients of the χ substrate.

A minimal parametrization of this effect may be written as

$$\frac{\delta\Gamma}{\Gamma} \simeq \beta \frac{\Delta U}{c^2}, \tag{89}$$

where Γ is the decay rate, U denotes an effective gravitational or structural potential serving as a proxy for the local relaxation gradient, and β is a dimensionless sensitivity coefficient encoding the response of the projected configuration to environmental variations.

Existing tests of local position invariance based on high-precision atomic clocks strongly constrain any environmental dependence of effective physical rates. Requiring compatibility with these constraints suggests a conservative bound $\beta \lesssim 10^{-6}$.

For typical galactic environments, the difference in effective potential between weakly structured regions and galactic cores is of order $\Delta U/c^2 \sim 10^{-7}$–$10^{-6}$. Combining these estimates yields a fractional modulation of particle lifetimes of order

$$\frac{\delta\tau}{\tau} \sim 10^{-13}\text{–}10^{-12}. \tag{90}$$

Such an effect is far below the sensitivity of current laboratory measurements and therefore fully compatible with existing experimental bounds. Nevertheless, it represents a qualitatively novel signature: a weak violation of the strict universality of decay rates induced not by spacetime curvature or local interactions, but by structural relaxation gradients in the underlying relational substrate. Future high-precision astrophysical observations or dedicated numerical simulations of χ-field dynamics may allow this prediction to be further quantified or constrained.

Which decay channels are most sensitive?

In Cosmochrony, an environmental modulation of decay rates is expected to be largest for metastable configurations whose admissible fluctuation spectrum is only weakly protected by topology, and whose decay proceeds through a comparatively "thin" projective channel (small number of admissible final factorizable branches). This suggests the following qualitative hierarchy of experimental leverage:

- **Purely leptonic weak decays** (e.g. $\mu^\pm \to e^\pm \nu\bar{\nu}$): theoretically clean (minimal hadronic uncertainty), with extremely well-characterized kinematics. The limitation is practical: τ_μ is measured very precisely, but changing the *structural environment* sufficiently in the laboratory is difficult.
- **Hadronic weak decays and oscillating neutral mesons** (e.g. K^0–$\bar{K}^0$, B^0–$\bar{B}^0$): in standard physics these are exquisitely sensitive to tiny perturbations in the effective Hamiltonian. In Cosmochrony terms, they probe whether the projection fiber admits a detectable environment-dependent bias between conjugate branches. The price is interpretability: hadronic and medium effects require careful control.
- **Nuclear β-decays and long-lived isotopes**: exceptionally good metrological stability allows long integration times, but nuclear structure systematics are harder to disentangle from any putative projective effect.

Overall, the cleanest conceptual target is leptonic weak decay, while the most "amplified" interferometric target is neutral-meson mixing, provided that standard environmental systematics are tightly controlled.

Differential astrophysical signature.

Directly comparing lifetimes of unstable particles between "void" and typical galactic cores is observationally challenging, because decay processes are not tagged *in situ*. However, Cosmochrony predicts a *differential* effect that can in principle be searched for in environments where the effective potential proxy $\Delta U/c^2$ is substantially larger than in ordinary galactic regions.

In particular, energetic hadronic cascades near compact objects (accretion flows and relativistic jets) are controlled by the competition between interaction lengths and decay lengths of $\pi^\pm$, $K^\pm$, and $\mu^\pm$, which shapes the emergent high-energy γ-ray and neutrino spectra. If decay rates acquire a weak environmental modulation,

$$\frac{\delta\Gamma}{\Gamma} \simeq \beta\,\frac{\Delta U}{c^2}, \tag{91}$$

then the *effective* critical energy at which decay dominates over interaction is shifted by the same fractional amount,

$$\frac{\delta E_*}{E_*} \sim \frac{\delta\tau}{\tau} \sim \beta\,\frac{\Delta U}{c^2}. \tag{92}$$

Near the innermost regions of accretion flows around massive compact objects, a characteristic scale $\Delta U/c^2 \sim 10^{-4}$ is not excluded as an order-of-magnitude proxy, leading (for conservative $\beta \lesssim 10^{-6}$) to

$$\frac{\delta\tau}{\tau} \sim 10^{-10}, \tag{93}$$

still extremely small, but conceptually clean: a systematic, environment-correlated spectral bias that cannot be mimicked by late-time cosmological parameter shifts.

The key discriminant is *correlation with environment*: sources with otherwise similar intrinsic properties but different compactness proxies (e.g. inferred emission radius or gravitational redshift indicators) should exhibit a tiny but coherent shift in the decay-limited spectral features of hadronic secondaries.

Connection to numerical χ-field simulations.

The sensitivity coefficient β can be operationally defined and extracted from χ-field simulations by measuring how the *escape time* of a metastable localized configuration changes when embedded in a controlled background gradient.

Let τ_0 denote the mean first-passage (escape) time of a metastable localized configuration in a reference background, estimated from an ensemble of runs with different admissible fluctuation seeds. Introduce a dimensionless structural gradient proxy G (computed from the projected field) such as

$$G \;\equiv\; \frac{\ell^2}{\chi_0^2}\,\langle|\nabla\chi_{\mathrm{eff}}|^2\rangle_{\mathcal{R}}, \tag{94}$$

where ℓ is a chosen coarse-graining scale, χ_0 a normalization, and $\langle\cdot\rangle_{\mathcal{R}}$ denotes averaging over a region $\mathcal{R}$ containing the localized excitation.

Define the empirical susceptibility

$$S \;\equiv\; \frac{d\ln\tau}{dG}, \tag{95}$$

estimated numerically by repeating the experiment at slightly different controlled background gradients G. A minimal mapping to the phenomenological parametrization $\delta\tau/\tau \simeq \beta\,\Delta U/c^2$ is then obtained by specifying an effective correspondence between $\Delta U/c^2$ and ΔG in the simulation,

$$\frac{\delta\tau}{\tau} \;\simeq\; S\,\Delta G \;\equiv\; \beta\,\frac{\Delta U}{c^2}. \tag{96}$$

In practice, this provides a clear numerical pipeline: (i) prepare a metastable localized configuration, (ii) embed it in backgrounds with controlled G, (iii) estimate $\tau(G)$ from ensembles, and (iv) infer S, hence β, up to the chosen environment proxy mapping. The resulting β can then be confronted with the conservative metrological requirement $\beta \lesssim 10^{-6}$.

14.10 Environmental Modulation of Particle Lifetimes

Within the Cosmochrony framework, particle decay is not interpreted as a fundamental stochastic process, but as the manifestation of the *structural susceptibility* of a metastable projected configuration to admissible fluctuations of the underlying relational substrate χ. As discussed in Section 14.9, these fluctuations form a bounded background whose spectral density depends weakly on the local relaxation structure of χ.

As a consequence, the decay rate Γ of an unstable particle is not strictly universal, but may exhibit an extremely small environmental dependence reflecting variations in the local relaxation gradient of the substrate. At leading order, this effect may be parametrized as

$$\frac{\delta\Gamma}{\Gamma} \simeq \beta \frac{\Delta U}{c^2}, \tag{97}$$

where U denotes an effective gravitational or structural potential serving as a proxy for the local relaxation density, and β is a dimensionless sensitivity parameter encoding the coupling between the projected metastable configuration and the admissible substrate fluctuations.

Since the particle lifetime is given by $\tau = \Gamma^{-1}$, the relative variation of the lifetime satisfies, to first order,

$$\frac{\delta\tau}{\tau} \simeq -\frac{\delta\Gamma}{\Gamma}. \tag{98}$$

Existing experimental constraints on local position invariance, in particular those derived from high-precision atomic clock comparisons, suggest a conservative upper bound $\beta \lesssim 10^{-6}$. Typical contrasts in effective potential between weakly structured environments and regions of high structural density (e.g. galactic cores) correspond to $\Delta U/c^2 \sim 10^{-7}$–$10^{-6}$. Under these conditions, Cosmochrony predicts

$$\frac{\delta\tau}{\tau} \sim 10^{-13}\text{–}10^{-12}, \tag{99}$$

well below current laboratory sensitivities, but conceptually distinct from standard quantum or relativistic effects.

Importantly, this modulation does not arise from spacetime curvature, local interactions, or environmental decoherence, but from the structural properties of the non-injective projection from χ to its effective description. A future detection of even a minute, reproducible deviation from strict lifetime universality would therefore constitute a direct signature of the underlying relational substrate.

14.11 Summary

Cosmochrony yields a set of observationally testable phenomenological signatures spanning cosmology, gravitation, galactic dynamics, and particle phenomena, all interpreted as emergent consequences of non-injective projection and structural relaxation. These signatures arise across widely separated physical scales but share a common origin in the relational dynamics of the χ substrate.

While all predicted effects remain compatible with current observations, the framework generically allows for correlated departures from standard effective predictions.

Such departures are not expected to be isolated anomalies, but structurally related features that may become accessible to future high-precision measurements across multiple observational domains.

Taken together, these signatures provide concrete and diverse avenues for empirical scrutiny. The confirmation or falsification of any subset of them would directly constrain the viability of Cosmochrony as a physical framework and, more importantly, test the hypothesis that a single underlying relaxation mechanism can account for phenomena ranging from cosmological expansion to particle decay.

15 Discussion and Comparison with Existing Frameworks

The Cosmochrony framework proposes a minimal pre-geometric substrate, described by a single scalar quantity χ, whose irreversible relaxation underlies both microscopic and cosmological phenomena. Spacetime geometry, gravitational dynamics, and quantum behavior arise only as effective descriptions of this underlying relaxation process.

In this section, we discuss how this approach relates to established theoretical frameworks, highlight its conceptual implications, and identify open challenges. Particular emphasis is placed on clarifying points of contact and distinction with general relativity, quantum mechanics, and standard cosmological models, as well as on assessing the ontological and methodological economy of the proposed framework.

The goal is not to claim empirical superiority over existing theories, but to clarify the conceptual role of Cosmochrony as a deeper explanatory layer from which standard frameworks emerge in appropriate regimes.

At the same time, the framework admits direct phenomenological confrontation in selected regimes, notably through particle stability and decay processes and through galactic rotation curves, providing nontrivial empirical benchmarks without departing from the minimal ontological commitments of the approach.

15.1 Relation to General Relativity

As established in Section 7.3, spacetime curvature in Cosmochrony is an emergent descriptive construct encoding how localized projected configurations modulate admissible ordering and correlation structure. This section summarizes how this mechanism relates to General Relativity (GR).

GR describes gravitation as the curvature of spacetime induced by energy–momentum. In Cosmochrony, no *a priori* metric dynamics is postulated at the fundamental level. Instead, an effective spacetime geometry emerges as a descriptive framework from variations in the local relaxation dynamics of the χ substrate.

Matter configurations, modeled as stable or metastable topological excitations of χ, locally constrain the relaxation of the field. This leads to differential rates of effective proper-time evolution between neighboring regions. When expressed in geometric terms, these differences can be reinterpreted as an effective deformation of the spacetime metric, which carries no independent dynamical status.

In the weak-field regime, this mechanism reproduces Newtonian gravity, while in the strong-field limit it yields Schwarzschild-like solutions at the level of effective geometric descriptions. The resulting phenomenology is therefore consistent with the empirical successes of GR across its tested domain.

From this perspective, gravitation is not introduced as a fundamental interaction, but emerges as a macroscopic manifestation of inhomogeneous χ relaxation. General Relativity is recovered as the appropriate effective theory describing this regime, rather than being supplanted or modified within its empirically validated domain.

15.2 Relation to Quantum Formalism

Quantum mechanics and quantum field theory (QFT) introduce probabilistic wavefunctions, operators, and quantization rules as foundational postulates [44]. In contrast, Cosmochrony does not treat quantization or wave dynamics as fundamental. Instead, it describes a continuous pre-geometric relational substrate whose projected and thresholded effective descriptions give rise to the formal apparatus of quantum theory.

Within this framework, particles correspond to localized, topologically stable configurations of the χ substrate. Discrete observables arise not from intrinsic microscopic discreteness, but from boundary conditions, topological constraints, and interaction-induced reprojection, which select a finite set of stable effective configurations.

The Planck relation $E = h\nu$ is interpreted geometrically as a correspondence between the amount of relaxation potential redistributed during an interaction and the minimal projective resolution required for a coherent effective spacetime description. The parameter ν does not represent a fundamental oscillation of χ, but a frequency characterizing the projective structure of the emergent spacetime description.

Quantum correlations are described in purely relational terms. Entanglement corresponds to the persistence of a shared, non-factorizable χ configuration across spatial separation, while decoherence reflects the irreversible loss of relational accessibility due to interaction with the environment. This interpretation reproduces standard quantum phenomenology, including nonlocal correlations, without invoking superluminal signaling, fundamental wavefunction collapse, or hidden variables.

15.3 Analogy with Collective Phenomena in QCD

A useful structural analogy may be drawn with quantum chromodynamics (QCD) in the low-energy regime, where the fundamental degrees of freedom introduced in the theory do not correspond directly to observable particles [45]. Quarks and gluons are not detected as isolated entities in spacetime; instead, hadronic properties, effective masses, and confinement phenomena emerge from a strongly interacting collective vacuum structure.

In a similar conceptual spirit, the Cosmochrony framework does not attribute gravitational or quantum phenomena to fundamental fields propagating on a preexisting spacetime. At the fundamental level, the theory is formulated solely in terms of the pre-geometric relational substrate χ and its intrinsic relaxation dynamics. Observable physical quantities arise only after projection, in regimes where χ admits a stable and sufficiently smooth effective description.

In such regimes, it is convenient to introduce effective quantities, collectively denoted χ_{eff}, which summarize coarse-grained and projectively stable features of the underlying χ configurations. These effective quantities are not additional ontological layers, nor independent degrees of freedom. They function as regime-dependent descriptors, encoding how the relational structure of χ becomes expressible in terms of fields, observables, and geometric notions within emergent spacetime.

This hierarchy of descriptions closely parallels the situation in QCD. While quarks constitute indispensable degrees of freedom at the level of the microscopic theory, their physical relevance is restricted to specific regimes, and they do not appear as freely propagating particles in the asymptotic spectrum. Their absence from direct observability does not signal incompleteness, but reflects the collective and confined nature of the underlying dynamics.

Likewise, Cosmochrony does not require that all internal structures invoked in its description correspond to independently observable entities. What appear as elementary constituents at a given effective level may instead represent stable, regime-dependent invariants of the underlying χ dynamics. The absence of direct observability of such structures is therefore not a defect of the framework, but a natural consequence of its pre-geometric and relational character.

As in QCD, the appropriate physical description in Cosmochrony depends critically on the scale and regime considered. While the fundamental dynamics of χ are simple in principle, the emergent macroscopic behavior is governed by nonlinear and collective effects that are most naturally captured by effective and phenomenological descriptions. This reinforces the view that geometry, gravitation, and quantum observables in Cosmochrony are emergent constructs, rather than fundamental ontological primitives.

15.4 Comparison with ΛCDM Cosmology

The ΛCDM model provides a remarkably successful phenomenological description of large-scale cosmological observations by introducing cold dark matter, dark energy, and an early inflationary phase [13, 46]. However, these components are postulated at the level of the effective model and are not derived from more fundamental principles.

In Cosmochrony, cosmic expansion follows directly from the monotonic relaxation of the fundamental substrate χ. The observed Hubble law emerges as a kinematic consequence of differential relaxation, without invoking a cosmological constant. When expressed in an effective geometric description, the expansion rate may be written as

$$H(t) = \frac{\dot{\chi}}{\chi}, \tag{100}$$

leading naturally to $H_0 \sim c/\chi(t_0)$ in the late-time regime.

From this perspective, dark energy is not interpreted as an additional physical component, but as an effective description of the large-scale relaxation dynamics of χ. Cosmic acceleration reflects the cumulative manifestation of this process over cosmological timescales. At the homogeneous and isotropic level, Cosmochrony reproduces the background expansion described by Friedmann–Lemaître cosmology, while offering an alternative interpretation of its underlying physical origin.

Unlike ΛCDM, which introduces empirically fitted initial conditions and a persistent dark energy component, the Cosmochrony framework attributes the late-time acceleration to the intrinsic relaxation properties of the underlying field. In this view, the coincidence problem and the observed tension between local and global measurements of the Hubble parameter may admit an interpretation in terms of epoch-dependent relaxation dynamics rather than as indications of new fundamental constituents.

At large angular scales, ΛCDM treats deviations from scale invariance in the cosmic microwave background (CMB) as statistical realizations around an ensemble-averaged spectrum, with individual low-ℓ modes subject to cosmic variance. Within Cosmochrony, constraints on the largest-scale configurations of the χ field allow for a scale-dependent attenuation of global modes. From this standpoint, the observed suppression of power at low multipoles may be may be explored as a possible structural consequence of the relaxation dynamics, rather than as a purely statistical fluctuation.

Taken together, these considerations suggest that Cosmochrony offers an alternative interpretative framework for cosmological observations, while remaining compatible with the empirical successes of the standard model at the level of current observational precision.

For clarity, Table 7 summarizes the conceptual and ontological differences between Cosmochrony, the standard ΛCDM model, and Loop Quantum Gravity.

15.5 Historical Admissibility of Projected Degrees of Freedom

A key implication of the Cosmochrony framework is that the set of effective degrees of freedom accessible to physical description is not fixed once and for all, but depends on the admissibility conditions imposed by the relaxation state of the underlying substrate χ.

At the fundamental level, the dynamics of χ and the rules governing projection remain unchanged throughout cosmic history. However, the space of *admissible* projected configurations evolves irreversibly as relaxation proceeds. In the early Universe, strong relational constraints and high saturation levels severely restricted the class of configurations that could be projected into stable effective descriptions. Only highly coherent and low-complexity global configurations were admissible, while finer and more differentiated structures were not yet projectively stable.

As relaxation progresses, these constraints are gradually relaxed, enlarging the space of admissible configurations. This progressive opening allows for the emergence of increasingly complex and localized invariants, which may be described in effective terms as particles, fields, and interactions. In this sense, the appearance of the particle spectrum is not an instantaneous or timeless feature of the Universe, but a historically conditioned outcome of the relaxation dynamics.

Importantly, this historical admissibility does not imply any evolution of the fundamental laws or parameters. Rather, it reflects the regime-dependent stability of projected descriptions. Degrees of freedom that are fundamental at one effective level may be absent or ill-defined at another, not because they are forbidden, but because the conditions required for their stable projection are not satisfied.

This perspective naturally reconciles the emergence of complexity with the monotonic increase of entropy. As the relaxation of χ enlarges the space of admissible configurations, entropy increases while simultaneously enabling the appearance of hierarchical and structured forms. The early Universe is thus characterized by both low entropy and low admissible complexity, whereas later epochs permit a richer spectrum of effective degrees of freedom.

Within this framework, anomalies at the largest cosmological scales, such as the suppression of low-ℓ modes in the cosmic microwave background, may be interpreted as relics of this early regime of restricted admissibility, rather than as purely statistical fluctuations.

15.6 Inflation, Horizon Problems, and Initial Conditions

Standard inflationary theory addresses the horizon, flatness, and monopole problems by postulating a brief phase of accelerated metric expansion driven by an inflaton field. In the Cosmochrony framework, these issues are approached from a different conceptual standpoint, rooted in the pre-geometric nature of the underlying substrate.

Because the fundamental quantity χ describes a global relaxation process rather than a metric expansion imposed on spacetime, causal connectivity is not defined in terms of spacetime lightcones at the most fundamental level. Instead, relational continuity is preserved within the χ substrate itself. As a consequence, large-scale coherence can arise from the initial relational smoothness of χ and its subsequent monotonic relaxation, without requiring a distinct inflationary phase as a fundamental dynamical ingredient.

From this perspective, the horizon problem is not resolved by superluminal expansion within spacetime, but rendered inoperative by the absence of an initially fragmented causal structure at the pre-geometric level. Similarly, large-scale homogeneity and isotropy reflect global properties of the early χ configuration rather than the outcome of an inflationary smoothing mechanism acting on an already defined spacetime geometry.

At the present stage, this proposal should be understood as an alternative interpretative framework rather than as a complete replacement for inflationary cosmology. In particular, a detailed quantitative treatment of primordial perturbations, their spectrum, and their imprint on the cosmic microwave background (CMB) is required to assess whether Cosmochrony reproduces, modifies, or departs from the successful predictions of standard inflationary scenarios.

These open questions define a clear direction for future work, in which the connection between early-time χ dynamics, effective reprojection processes, and observable cosmological signatures can be explored in a systematic and quantitative manner.

15.7 Conceptual Implications and Open Challenges

Cosmochrony proposes a unifying conceptual framework in which time, distance, energy, gravitation, and quantization emerge from the dynamics of a single pre-geometric relational substrate. This ontological economy constitutes a central strength of the framework, while also requiring a careful reassessment of notions traditionally treated as independent physical primitives.

In particular, the framework suggests that time, energy, and irreversibility do not correspond to distinct fundamental entities. Temporal ordering arises from the monotonic relaxation of the χ substrate, while energy quantifies the residual capacity of localized configurations to resist this relaxation. Irreversibility then reflects the progressive exhaustion of such relaxation capacity. From this perspective, temporal flow and energetic processes are not independent axioms of nature, but complementary effective descriptions of the same underlying relational dynamics.

At the level of effective physical descriptions, these relations are encoded in coarse-grained quantities such as χ_{eff}, which summarize how the relaxation structure of χ

manifests in spacetime-based observables. These effective constructs carry no independent ontological status and remain valid only within regimes where a geometric interpretation is applicable.

A concrete realization of this unification, including an explicit formulation of the relaxation operator and its spectral role in mass generation, is outlined in Appendix B.8. While this reinterpretation addresses several long-standing conceptual tensions — including the origin of the arrow of time and the status of energy conservation —it also raises important open challenges.

Among these challenges are:

- the quantitative reconstruction of cosmic microwave background anisotropies from early-time χ dynamics,
- the detailed treatment of non-equilibrium quantum measurements, decoherence, and reprojection processes,
- the emergence of gauge symmetries and interaction hierarchies from topological and relational features of χ,
- and the long-term stability of solitonic particle configurations under extreme gravitational or radiative conditions.

Addressing these issues will require a combination of analytical, numerical, and experimental approaches, including:

1. large-scale numerical simulations of χ dynamics to quantify structure formation and cosmological signatures,
2. the exploration of discretized, network-based, or lattice realizations of χ at microscopic scales,
3. and targeted experimental tests of predicted χ-dependent effects in quantum coherence, gravitation, and radiation processes.

Progress along these directions may elevate Cosmochrony from a unifying interpretative framework to a quantitatively predictive theory, while preserving its minimal ontological foundation.

15.8 Ontological Parsimony and the Metric

As emphasized throughout the preceding discussion, the spectral operator relevant for mass generation is defined independently of any emergent geometric or dynamical description. A potential criticism of Cosmochrony is that it merely replaces one geometric structure (the spacetime metric) with another fundamental entity (the χ field). This subsection clarifies why this replacement constitutes genuine ontological simplification rather than a relabeling of degrees of freedom.

Distinction from metric-based theories.

In General Relativity and related metric frameworks:

- the metric $g_{\mu\nu}$ is a fundamental tensor field with ten independent components,
- spacetime curvature is treated as a primitive geometric property,

- matter and energy are conceptually distinct from geometry and are coupled to it via the stress–energy tensor.

In Cosmochrony:

- only a single pre-geometric relational substrate χ is taken as fundamental,
- the spacetime metric arises as an effective, coarse-grained descriptor of χ relaxation dynamics and carries no independent ontological status,
- matter, energy, and geometry correspond to distinct regimes and invariant patterns of the same underlying relational structure.

Operational distinguishability.

The two frameworks are operationally distinct rather than notationally equivalent:

1. **Degrees of freedom:** General Relativity propagates two tensorial gravitational-wave polarizations derived from the metric structure. In Cosmochrony, the fundamental dynamics involves no independent geometric degrees of freedom; only relational variations of χ evolve, with effective tensorial behavior emerging solely at the macroscopic, coarse-grained level.
2. **Singularities:** In metric theories, singularities correspond to divergences of the fundamental geometric structure. In Cosmochrony, apparent singular behavior signals the breakdown of the effective geometric description, while the underlying χ substrate remains well-defined.
3. **Quantum regime:** Quantizing General Relativity requires the quantization of the metric itself (e.g., via the Wheeler–DeWitt equation). In Cosmochrony, quantization applies only to effective excitations and fluctuations of χ within regimes where a spacetime description is already valid; the χ substrate itself is not treated as a conventional quantum field propagating on spacetime [47].

Ontological economy.

From the perspective of ontological parsimony, Cosmochrony achieves unification through reduction rather than proliferation:

$$\text{Standard approach:} \quad g_{\mu\nu}\ (\text{geometry}) + \psi\ (\text{matter}) + \Lambda\ (\text{dark energy}), \tag{101}$$

$$\text{Cosmochrony:} \quad \chi\ (\text{single relational substrate}) \longrightarrow \{\text{spacetime}, \text{matter}, \text{expansion}\}. \tag{102}$$

This reduction does not merely rephrase existing structures, but provides an explanatory compression in which multiple physical notions arise as effective manifestations of a single underlying dynamical principle.

15.9 Relation to the Higgs Mechanism: Emergence from χ Dynamics

In the Standard Model, the Higgs mechanism accounts for mass generation through spontaneous symmetry breaking of the electroweak gauge group $SU(2)_L \times U(1)_Y$. The Higgs field ϕ_H acquires a non-zero vacuum expectation value (VEV), $\langle \phi_H \rangle \simeq 246$ GeV, thereby generating masses for fermions and gauge bosons through Yukawa and gauge couplings.

Within the Cosmochrony framework, the Higgs field and its VEV are not regarded as fundamental ontological entities. Instead, they arise as *effective low-energy descriptors* of a specific structural and projective regime of the underlying relational substrate χ. Electroweak symmetry breaking and the associated mass scale are thus reinterpreted as emergent phenomena associated with the relaxation dynamics and topological organization of χ, without altering the empirical content of the Standard Model.

Structural Transition and Emergence of the Higgs VEV

In Cosmochrony, electroweak symmetry breaking corresponds to a structural and projective transition of the χ substrate between two regimes:

- **Homogeneous regime** ($\chi < \chi_c$): The relaxation of χ is approximately uniform and strongly constrained. Only highly coherent, global configurations are projectively admissible, and no stable, localized excitation modes are permitted. Effective descriptions in this regime remain massless, and the electroweak symmetry is unbroken.
- **Structured regime** ($\chi \gtrsim \chi_c$): As relaxation proceeds, relational constraints are sufficiently relaxed to allow the stabilization of localized, relaxation-resistant configurations. These configurations correspond to discrete, spectrally stable modes of the effective relaxation operator and manifest as massive excitations in spacetime-based descriptions.

This transition is not driven by an externally imposed scalar potential, but by the intrinsic relaxation dynamics of χ. When the critical *projective structural scale* χ_c is reached, local relaxation slows sufficiently to render such localized configurations projectively admissible. In effective field-theoretic language, this structural transition is described by the emergence of a non-zero Higgs vacuum expectation value.

Relation Between χ_c and the Electroweak Scale

The scale χ_c should not be interpreted as a new fundamental constant. Rather, it represents the effective stability threshold at which localized configurations of χ become projectively admissible within the evolving configuration space. Its value is constrained by both cosmological relaxation properties and microscopic stability considerations, including:

- large-scale relaxation dynamics inferred from cosmological observations,
- the observed hierarchy of particle masses and stability scales.

At the effective level, the electroweak scale is related to χ_c through the inverse correlation length associated with stable χ configurations:

$$\langle \phi_H \rangle \;\propto\; \frac{\hbar_{\rm eff}\, c}{\chi_c}, \tag{103}$$

where $\hbar_{\text{eff}}$ denotes the effective reprojection scale, which reduces to the observed Planck constant $\hbar$ in regimes where a standard quantum description applies.

This relation reflects the geometric and topological conditions required for the stabilization of localized configurations within χ, rather than the tuning of independent parameters. For χ_c of order $10^{-18}\,\text{m}$, the observed electroweak scale is naturally recovered.

Mass Generation as Solitonic Stabilization

In the structured regime ($\chi \gtrsim \chi_c$), fermions and gauge bosons acquire mass through their association with distinct classes of stable χ configurations.

- **Fermions:** Fermionic degrees of freedom correspond to topologically non-trivial, skyrmion-like solitonic configurations of χ. Their effective masses scale as

$$m_f \;\propto\; y_f \,\frac{\hbar_{\text{eff}}}{\chi_c},$$

  where y_f represents an effective Yukawa coupling encoding the internal topological and spectral properties of the configuration. Fermion mass hierarchies thus reflect differences in these internal invariants rather than independent fundamental parameters.
- **Gauge bosons:** Massive gauge bosons correspond to vortex-like or phase-structured χ configurations. Their masses scale as

$$m_W \;\propto\; g \,\frac{\hbar_{\text{eff}}}{\chi_c},$$

  where g is the effective $SU(2)_L$ gauge coupling. The weak mixing angle θ_W is interpreted as a ratio of characteristic topological responses associated with neutral and charged excitation sectors.

At the level of effective quantum field theory, these relations reproduce the standard Higgs-generated mass terms without modifying their phenomenology.

Phenomenological Status and Open Questions

The emergent interpretation of the Higgs mechanism proposed here is designed to be phenomenologically equivalent to the Standard Model within currently tested energy regimes. No deviation from established collider results is implied at accessible energies.

Potentially observable departures may arise only in extreme regimes, such as strong gravitational confinement or highly non-equilibrium relaxation, where the assumptions underlying an effective Higgs field description may break down.

Open challenges include:

- deriving the detailed mapping between χ soliton spectra and the full Standard Model mass spectrum,
- understanding the origin of gauge coupling values and symmetry structure from the internal relational organization of χ.

Summary

In Cosmochrony, the Higgs field is interpreted as an effective manifestation of a structured and historically admissible relaxation regime of the χ substrate. The electroweak scale emerges from the inverse correlation length associated with stable χ configurations, while particle masses arise from the topological and spectral stability of these configurations.

Electroweak symmetry breaking thus corresponds not to the activation of a new fundamental interaction, but to the historical opening of the admissible configuration space allowing such localized invariants to become projectively stable. This reinterpretation preserves the empirical content of the Higgs mechanism while embedding it within a unified pre-geometric framework in which gravitation, quantum phenomena, and mass generation share a common dynamical origin.

16 Conclusion and Outlook

We have presented Cosmochrony, a minimalist framework in which a single fundamental entity, χ, underlies the emergence of time, spacetime geometry, and a wide spectrum of physical phenomena. By identifying the irreversible relaxation of χ as the primary physical process, we have shown that familiar structures—from the metric tensor to the Standard Model—are not independent axioms but emergent harmonics of this relaxation.

A central result of this work is the *ab initio* derivation of the effective dynamical laws. Rather than postulating a convenient action, we have demonstrated that the Born–Infeld-like Lagrangian is the unique functional compatible with the causal saturation of relaxation fluxes at the speed c_χ. By resolving the circularity between configuration and distance through spectral graph theory, we have established that spacetime geometry is the continuum encoding of microscopic connectivity, naturally recovering General Relativity as a thermodynamic limit of the underlying relational dynamics.

The framework provides a unified geometric origin for the Standard Model:

- **Gauge interactions:** Reinterpreted as projection dynamics, where photons manifest as scalar transmittance and W/Z bosons as shear modes of the projection fiber, accounting for their mass without requiring a fundamental Higgs field as a primary ontological ingredient. The inherently non-injective character of the projection from χ to effective observables provides a structural origin for quantum indeterminacy and discreteness.
- **Matter, mass, and stability:** Fermionic properties emerge from topological obstructions in the configuration space of χ, while inertial mass is not postulated but arises as an invariant spectral property associated with topological frustration and inhibition of relaxation, rather than from fundamental coupling constants or symmetry-breaking mechanisms. Within this same spectral picture, particle stability and decay acquire a unified structural interpretation. Stable particles correspond to metastable projected χ-configurations whose spectral invariants remain admissible under relaxation, while particle decay is reinterpreted as a structural instability of these projected configurations, arising when continued relaxation drives them beyond the admissible domain of the projection. In this view, particle lifetimes are emergent properties of the relaxation geometry, not stochastic inputs.
- **The dark sector:** Within the same projection-based ontology, dark matter is identified as non-projected spectral density (sub-threshold inertia), contributing to gravitation without admitting a stable effective-field representation, while dark energy manifests as the global, irreversible relaxation flux Φ_χ.

At cosmological scales, expansion and the arrow of time follow directly from the diminishing tempo of relaxation as the substrate irreversibly approaches equilibrium. Within this framework, "inflation" and "dark energy" do not appear as fundamental ingredients but as effective descriptions: their explanatory roles are replaced by pre-geometric connectivity in the early constrained regime and by epoch-dependent relaxation dynamics at late times.

At galactic scales, the same relaxation-based ontology yields an effective gravitational phenomenology. Saturation of the χ-relaxation constraint produces an emergent large-scale potential leading to asymptotically flat rotation curves in spiral galaxies, without invoking dark matter halos. The resulting predictions have been directly confronted with observed rotation curves across different morphological classes, providing a first nontrivial observational test of the framework.

A further consequence of this relaxation-based ontology is that the set of effective degrees of freedom accessible to physical description is not fixed once and for all. While the fundamental dynamics of χ and the rules governing projection remain unchanged throughout cosmic history, the space of *admissible* projected configurations evolves irreversibly as relaxation proceeds. In the early Universe, strong relational constraints and high saturation levels restricted admissible configurations to highly coherent and low-complexity global modes, precluding the stable projection of many structures that appear elementary at later epochs. As relaxation progresses, this admissible space progressively enlarges, allowing for the emergence of increasingly localized and differentiated invariants, which are described in effective terms as particles, fields, and interactions. In this sense, the particle content of the Universe is not a timeless input but a historically conditioned outcome of the relaxation dynamics, fully compatible with the monotonic increase of entropy and the emergence of complexity.

In strong-gravity regimes, black hole evaporation is reinterpreted as a discrete reprojection process of χ, governed by threshold crossings of the projection operator at the horizon. This mechanism ensures information preservation at the structural level, providing a resolution of the black hole information paradox without invoking non-unitary dynamics.

Beyond its conceptual unification, Cosmochrony offers a concrete program for validation. The transition from discrete relational constraints to effective field descriptions identifies clear numerical signatures for lattice and spectral simulations. While challenges remain—notably the precise numerical computation of the particle mass hierarchy—the framework now provides a complete and self-consistent theoretical bridge from the pre-geometric substrate to observable reality.

By reducing the fundamental assumptions to a single dynamical origin, Cosmochrony offers a coherent foundation in which time, mass, and geometry arise as a unified whole. It provides a physically grounded starting point for further theoretical development, where the laws of physics are not postulated but derived from the structural requirements of a relaxing universe. In this perspective, what evolves in Cosmochrony is not the laws of physics themselves, but the space of forms to which those laws can meaningfully apply.

Testable predictions and observational signatures.

While Cosmochrony does not aim at precision cosmology at its present stage, it already admits concrete phenomenological tests at galactic and cosmological scales. These include rotation curves of spiral galaxies, large-scale cosmological correlations, strong-gravity wave propagation, and epoch-dependent effective expansion rates. The purpose of these signatures is to provide explicit criteria by which the Cosmochrony framework may be empirically scrutinized, as detailed in Section 14.

Appendices

A Mathematical Foundations of Cosmochrony — Dynamics, Stability, and Analytical Solutions

This appendix provides a rigorous mathematical formulation of the χ-field dynamics underlying the Cosmochrony framework. Its purpose is not to introduce new physical assumptions, but to support the effective descriptions developed in the main text by establishing the internal consistency, stability properties, and analytical structure of the underlying field equations.

In particular, this appendix presents:

- an effective Lagrangian formulation and its hydrodynamic limit (Section A.1),
- stability analyses of the χ field under perturbations (Section A.2),
- analytical solutions in homogeneous, spherically symmetric, and planar regimes (Section A.3),
- and the relational foundation of emergent geometric descriptions (Section E).

All results are derived from the fundamental postulates of Cosmochrony (Section 3.2) without assuming a pre-existing spacetime metric or background geometry. Geometric notions appearing in this appendix should therefore be understood as effective and coarse-grained representations of the underlying χ dynamics, consistent with the interpretative framework developed in Appendix **??**.

No additional physical assumptions are introduced in this appendix; all results follow from reformulations, approximations, or limiting regimes of the same underlying χ dynamics discussed in the main text.

Several structures derived here—such as relaxation bounds, stability spectra, and effective coupling scalings—provide the mathematical basis for the phenomenological signatures discussed in Section 14, without constituting independent predictive postulates.

A.1 Effective Lagrangian Description as a Hydrodynamic Limit

The purpose of this subsection is not to introduce any additional fundamental structure into Cosmochrony, but to provide an effective hydrodynamic tool for connecting the relational χ framework to standard geometric formulations in regimes where a spacetime description becomes operationally meaningful.

From Relational Dynamics to an Effective Continuum Description

At the fundamental level, Cosmochrony is defined without reference to any pre-existing spacetime manifold or metric structure. The dynamics of the χ field are relational and are specified directly in terms of local relaxation rules and coupling relations between configurations (Section 3.2).

In regimes where χ varies smoothly over large scales, it becomes convenient to introduce a continuum approximation in order to compare the theory with standard geometric and field-theoretic formulations. This approximation does not alter the underlying ontology but provides a coarse-grained description suitable for analytical calculations and contact with general relativity.

Hydrodynamic Limit and Emergent Geometry

In this hydrodynamic regime, the discrete relational couplings encoded in the connectivity matrix K_{ij} can be summarized by effective continuum quantities. Operationally, distances are defined through the resistance encountered by the propagation of χ relaxation across the network. In the continuum limit, this leads schematically to an effective line element of the form

$$g_{\mu\nu}\, dx^\mu dx^\nu \;\sim \sum_{(u,v)\in \mathrm{path}} \frac{1}{K_{uv}},$$

which should be understood as a diagnostic illustration rather than a defining relation. This expression does not define a unique metric tensor, but captures how effective distance emerges as cumulative resistance to χ relaxation. *No notion of geodesic motion or metric compatibility is implied at this stage.*

The effective metric $g_{\mu\nu}$ therefore encodes the coarse-grained density of correlations in the χ field and serves as a macroscopic summary of its relational dynamics.

Effective Lagrangian Representation

To reproduce the continuum evolution equations obtained from the discrete relaxation dynamics (Equation 285), one may introduce an effective Lagrangian density $\mathcal{L}_{\mathrm{CC}}$. This step is purely representational and does not assume the validity of a variational principle at the fundamental level. This Lagrangian is constructed to match the hydrodynamic behavior of the χ field in the smooth regime, while remaining fully subordinate to the underlying relational description.

In this representation, terms resembling those of standard geometric theories naturally appear. In particular, a curvature-like contribution emerges as the leading-order descriptor of spatial variations in the relaxation rate:

$$\mathcal{L}_{\mathrm{eff}} = \frac{1}{16\pi G_{\mathrm{eff}}}\, F(\chi)\, R - \Lambda_{\mathrm{flow}}^4\, \chi + \cdots$$

where R is the Ricci scalar associated with the effective metric. Its appearance reflects the fact that, at leading order in a derivative expansion, R provides the most general local scalar invariant encoding slow spatial variations of the relaxation structure. The function $F(\chi)$ is not an independent coupling, but parametrizes how the underlying χ relaxation dynamics is encoded in the effective geometric description.

Crucially, this Lagrangian does *not* define the fundamental dynamics of the theory. It is an auxiliary representation that reproduces the macroscopic behavior of χ once a geometric interpretation becomes applicable.

Status and Limitations

The hydrodynamic Lagrangian formulation presented here should be understood as an auxiliary representation, not as an alternative foundation of Cosmochrony. All physical content remains encoded in the relational relaxation dynamics of the χ field.

The emergence of Einstein-like field equations in this limit reflects the universality of geometric descriptions for slowly varying collective phenomena, rather than the presence of a fundamental spacetime structure. Accordingly, singularities or breakdowns of the effective metric signal only the limits of the hydrodynamic approximation, not a failure of the underlying χ dynamics.

The effective Lagrangian has no ontological status and should not be quantized.

A.2 Stability Analysis of the χ-Field Dynamics

The stability of the χ-field dynamics is a central requirement for Cosmochrony to define a physically consistent framework. Since χ is interpreted as a fundamental pre-geometric substrate, its effective descriptions must remain well-behaved under perturbations, without runaway growth or singular behavior.
The stability analysis presented in this subsection does not concern the χ substrate itself, but the admissibility and robustness of its effective projected descriptions once a hydrodynamic regime is assumed.

In regimes where a smooth geometric description is applicable, the effective relaxation dynamics of χ may be written as

$$\partial_t\chi = c\sqrt{1-\frac{|\nabla\chi|^2}{c^2}}, \tag{104}$$

where ∂_t denotes an effective ordering parameter associated with the relaxation process, not a fundamental time variable. This representation is introduced solely for analytical convenience in the hydrodynamic regime. The ordering parameter labels successive admissible configurations in the projected description and does not correspond to an underlying temporal evolution of the χ substrate.

Below we analyze the response of this effective dynamics to small deviations around homogeneous relaxation states.

Perturbative Structure and Marginal Linear Stability

Consider a spatially homogeneous background solution

$$\chi_0(t) = c\,t + \chi_{0,0},$$

satisfying $\nabla\chi_0 = 0$ and $\partial_t\chi_0 = c$. We introduce a small perturbation

$$\chi(x,t) = \chi_0(t) + \delta\chi(x,t), \qquad |\nabla\delta\chi| \ll c.$$

Substituting into the evolution equation and expanding the square root yields

$$\partial_t\delta\chi = -\frac{1}{2c}\,|\nabla\delta\chi|^2 + \mathcal{O}\big(|\nabla\delta\chi|^4\big)\,. \tag{105}$$

Importantly, no term linear in $\delta\chi$ appears. The homogeneous relaxation solution is therefore *marginally stable at linear order*: infinitesimal perturbations neither grow nor propagate dynamically at first order. This reflects the purely relaxational character of the effective χ description and the absence of fundamental propagating modes at the linearized level.

Nonlinear Stability and Dissipative Behavior

Although linear perturbations are marginal, the leading nonlinear correction is strictly negative. Any spatial inhomogeneity in the effective χ description therefore reduces the local relaxation rate and is dynamically suppressed.

To make this explicit, consider the functional

$$E[\delta\chi] = \frac{1}{2} \int |\nabla \delta\chi|^2 \, d^3x, \tag{106}$$

which measures the degree of spatial inhomogeneity in the projected description. This functional has no energetic or variational meaning at the fundamental level and serves only as a diagnostic measure of geometric tension within the effective hydrodynamic regime.

Using the evolution equation, one finds that $E[\delta\chi]$ is a non-increasing function of the ordering parameter. This follows from the fact that the relaxation flow is negative-definite in the presence of spatial gradients, acting systematically to reduce $|\nabla\delta\chi|^2$.

Spatial gradients are therefore progressively smoothed, and perturbations remain bounded for all values of the ordering parameter. The effective dynamics is dissipative and contractive in configuration space, with no mechanism for amplification of perturbations.

This establishes *nonlinear stability* of the effective relaxation description.

Special Configurations

For simple classes of perturbations, the qualitative behavior is transparent:

- **Planar perturbations:** Spatial oscillations do not propagate as waves, but are progressively flattened as the local relaxation rate decreases in regions of nonzero gradient.
- **Spherically symmetric perturbations:** Radial inhomogeneities decay monotonically, corresponding to a diffusion-like relaxation of geometric tension.

In all cases, the effective dynamics suppresses sharp gradients and prevents the formation of singular structures within the hydrodynamic description.

Conclusion

The effective χ-field dynamics is marginally stable at linear order and strictly stable once nonlinear effects are taken into account. This guarantees that the irreversible relaxation of χ admits stable and well-defined effective descriptions, providing a robust basis for the emergence of spacetime geometry, gravitation, and quantum phenomena within the Cosmochrony framework.

Notably, this stability property is inseparable from the monotonic character of the relaxation process: the same structural constraint that defines the arrow of time also precludes dynamical instabilities in the projected description.

A.3 Analytical Solutions of the χ-Field Dynamics

To illustrate the qualitative behavior of the χ field, we derive a set of explicit analytical solutions of the effective relaxation equation

$$\partial_t \chi = c\sqrt{1 - \frac{|\nabla \chi|^2}{c^2}}, \tag{107}$$

valid in regimes where a smooth geometric description is applicable. Here, ∂_t denotes an effective ordering parameter associated with the relaxation process, not a fundamental time derivative.

The solutions presented in this subsection do not describe trajectories or time evolution of a fundamental field, but characterize admissible configurations of the effective projected description once a hydrodynamic regime is assumed.

These solutions are not intended to exhaust the full dynamics of χ, but to clarify its causal structure, limiting configurations, and relaxational character within the effective description.

Homogeneous Relaxation Solution

In a spatially homogeneous configuration, spatial variations vanish,

$$\nabla \chi = 0,$$

and the effective evolution equation reduces to

$$\partial_t \chi = c. \tag{108}$$

Integration yields

$$\chi(t) = \chi_0 + c\,t, \tag{109}$$

where χ_0 is a constant labeling the homogeneous relaxation state. This solution represents a reference configuration corresponding to uniform global relaxation in the projected description.

When interpreted within an effective spacetime description, this homogeneous relaxation underlies the emergence of cosmological expansion and leads naturally to a Hubble-like relation, as discussed in Section 5.6. No fundamental notion of expansion is assumed at the level of the χ substrate itself.

Spherically Symmetric Gradient-Saturated Profiles

Consider a spherically symmetric configuration $\chi = \chi(r,t)$ within the effective description. The evolution equation becomes

$$\partial_t \chi = c\sqrt{1 - \frac{(\partial_r \chi)^2}{c^2}}. \tag{110}$$

Configurations satisfying

$$|\partial_r \chi| = c$$

correspond to complete local saturation of the relaxation bound. In this limit,

$$\partial_t \chi = 0,$$

indicating a local freezing of the effective temporal ordering parameter.

Such profiles take the form

$$\chi(r) = \chi_0 \pm c\, r, \tag{111}$$

and represent limiting admissible configurations in which relaxation is entirely inhibited by maximal structural gradients.

Although these configurations cannot be realized globally in a regular manner, they play an important conceptual role as idealized models of horizons and maximally constrained regions within the effective description. In geometric language, they correspond to boundaries beyond which spacetime notions cease to be operationally meaningful, rather than to physical singularities of the underlying χ substrate.

Linear Relaxation Fronts

A simple class of exact solutions is given by linear fronts of the form

$$\chi(x,t) = \chi_0 + c\, t \pm v\, x, \tag{112}$$

with $|v| < c$. For such configurations,

$$|\nabla \chi| = |v| < c,$$

and the evolution equation (107) is satisfied identically.

These solutions describe admissible relaxation fronts separating regions of different projected χ values. They do not correspond to propagating waves or oscillatory modes, but to kinematic boundaries determined by the maximal admissible relaxation rate.

The parameter v characterizes the spatial steepness of the front within the effective description rather than a signal propagation speed, which remains bounded by c.

Absence of Linear Wave Solutions

A crucial structural feature of the χ relaxation dynamics is the absence of linear wave solutions. Small perturbations around homogeneous relaxation configurations do not propagate as oscillatory modes.

As shown in Section A.2, infinitesimal perturbations are marginal at linear order and are damped once nonlinear effects are taken into account. The effective dynamics is therefore purely relaxational.

Apparent wave-like phenomena, such as gravitational or electromagnetic radiation, arise only at the effective level, through collective excitations associated with structured matter configurations. These emergent phenomena should not be confused with fundamental propagating modes of the χ substrate.

Conclusion

These analytical solutions illustrate the central features of the effective χ description: homogeneous relaxation underpins cosmological expansion at the projected level, gradient saturation defines causal and horizon-like limits, and relaxation fronts clarify the kinematic structure imposed by the universal bound c.

Together, they confirm the internal consistency of the hydrodynamic regime of Cosmochrony and prepare the ground for the emergence of effective geometric, gravitational, and radiative phenomena at macroscopic scales.

A.4 Coupling with Matter: Effective Source Term $S[\chi, \rho]$

In regimes where the χ field admits a smooth and slowly varying geometric description, its collective relaxation dynamics can be represented using effective differential operators familiar from continuum field theory. Within this *emergent* spacetime description, the influence of localized excitations (identified with matter through an effective density ρ) on the relaxation of χ may be summarized by an effective source term:

$$\Box_{\text{eff}}\chi = S[\chi, \rho]. \tag{113}$$

This equation is not fundamental and does not represent a dynamical field equation for the χ substrate. Both the operator $\Box_{\text{eff}}$ and the source term $S[\chi, \rho]$ arise only after coarse-graining the underlying relational relaxation dynamics of χ and have no ontological status outside the hydrodynamic regime.

Physical Interpretation of the Source Term

The term $S[\chi, \rho]$ must not be interpreted as an external force acting on χ, nor as an independent dynamical input. Instead, it encodes the *effective resistance* of localized excitations to the global relaxation flow of the field once a geometric description becomes applicable.

Matter corresponds, in Cosmochrony, to structured and long-lived configurations of χ (solitonic or topologically constrained excitations). Such configurations locally inhibit relaxation, inducing spatial gradients and differential ordering rates when described in an effective geometric language.

Within this interpretation, the source term $S[\chi, \rho]$ provides a compact phenomenological summary of several emergent effects:

- gravitational time dilation as a manifestation of locally slowed χ relaxation,
- inertial mass as persistent resistance to the global relaxation flow,
- effective spacetime curvature as a coarse-grained representation of spatial variations in relaxation efficiency.

Importantly, ρ does not represent a fundamental energy or mass density. It is an effective descriptor of the density of relaxation-resistant configurations within an emergent spacetime regime and has no meaning outside this projected description.

Weak-Field Regime and Linear Approximation

In weak-field regimes, where matter-induced gradients remain small and relaxation is only mildly perturbed, the source term may be approximated as linear in the effective excitation density:

$$S[\chi, \rho] \;\simeq\; -\alpha\, \rho, \tag{114}$$

where α is an effective coupling parameter characterizing the sensitivity of the projected relaxation dynamics to localized resistance.

Matching this description with the Newtonian limit of the emergent geometric regime identifies

$$\alpha \sim \frac{G}{c^2},$$

where the gravitational constant G appears as a macroscopic coupling summarizing how strongly localized excitations impede the relaxation of χ within the effective description.

Within this approximation, Equation (113) reproduces the Poisson equation for the effective gravitational potential and yields the Schwarzschild solution at leading order. No independent gravitational interaction is introduced; gravity arises entirely from relaxation resistance in the projected regime.

Strong-Field Regimes and Nonlinear Corrections

In regimes of high excitation density—such as near compact objects or during early cosmological epochs—the linear approximation breaks down. Strong structural constraints lead to saturation effects in the effective relaxation dynamics, requiring nonlinear corrections to the source term.

A generic parametrization of these effects takes the form

$$S[\chi, \rho] = -\alpha\, \rho\, F\left(\frac{\rho}{\rho_c}, \chi\right), \tag{115}$$

where F is a bounded function and ρ_c denotes a characteristic density scale beyond which relaxation resistance saturates.

These nonlinearities prevent unphysical halting of the relaxation flow and encode departures from classical gravity in strong-field regimes, while leaving the underlying ontological structure unchanged. Apparent singular behavior in effective geometric descriptions therefore signals the breakdown of the hydrodynamic approximation, not a failure of the fundamental χ dynamics.

Status of the Effective Description

The source term $S[\chi, \rho]$ does not define an additional physical field, interaction, or degree of freedom. It is a bookkeeping device summarizing how localized, structured configurations of χ modify the collective relaxation flow once spacetime notions have emerged.

At the fundamental level, only the relational relaxation dynamics of χ exists. Matter, sources, curvature, and gravitational effects appear jointly and only as emergent, regime-dependent descriptions of this single underlying process.

A.5 Strong-Field Constitutive Coupling Near a Schwarzschild Black Hole

Purpose and epistemic status.

In Section 5.3, an effective constitutive relation was introduced to encode how strong internal structure of the χ field reduces the efficiency of relaxation:

$$K_{\text{eff}} = K_0 \exp\left(-\frac{(\Delta\chi)^2}{\chi_c^2}\right). \tag{116}$$

This relation is not fundamental. It provides a coarse-grained and purely constitutive parametrization of how collective structural constraints within χ suppress relaxation transport in regimes where an emergent geometric description becomes applicable.

In weak-field situations, this description leads to a Poisson-like equation and to the recovery of Schwarzschild phenomenology at leading order (Section 7.4). The purpose of the present appendix is not to modify or extend general relativity, but to construct a *self-consistent strong-field constitutive description* within the Cosmochrony framework, suitable for characterizing the approach to an effective horizon without introducing geometric singularities.

Operational time-dilation factor.

In the emergent spacetime regime, gravitational time dilation is operationally encoded as a local slowdown of the relaxation rate of χ relative to its asymptotic homogeneous value. We define the dimensionless lapse-like factor

$$N(r) \;\equiv\; \frac{\mathcal{D}_{\text{loc}}\chi(r)}{\mathcal{D}_0\chi}, \qquad 0 < N(r) \le 1, \tag{117}$$

where $\mathcal{D}_0\chi$ denotes the relaxation rate far from localized excitations. This quantity is an operational descriptor within the effective geometric regime and does not correspond to a fundamental lapse function of an underlying spacetime.

In the weak-field limit, $N(r)$ reduces to $N \simeq 1+\Phi/c^2$, with Φ an effective Newtonian potential.

Matching to Schwarzschild phenomenology.

In regimes where a stable geometric description applies, the exterior field of an isolated compact excitation may be summarized by a Schwarzschild-like metric. We therefore adopt, *purely as an effective and auxiliary descriptor*,

$$ds^2 = -f(r)c^2dt^2 + f(r)^{-1}dr^2 + r^2d\Omega^2, \qquad f(r) = 1 - \frac{r_s}{r}, \tag{118}$$

where $r_s = 2GM/c^2$ is defined operationally by asymptotic weak-field matching. This metric has no ontological status within Cosmochrony and serves only as a compact parametrization of the effective exterior regime.

Consistency with the interpretation of $N(r)$ as the local time-dilation factor then implies

$$N(r)^2 = f(r) = 1 - \frac{r_s}{r}. \tag{119}$$

The limit $N(r) \to 0$ as $r \to r_s^+$ corresponds to an asymptotic freeze-out of local relaxation and defines an *effective horizon* within the projected description.

From relaxation slowdown to conductivity.

To relate the relaxation slowdown to the constitutive conductivity, we adopt a minimal and self-consistent identification:

$$\frac{K_{\text{eff}}(r)}{K_0} \;\equiv\; N(r)^2. \tag{120}$$

This identification is neither unique nor fundamental. It is selected because it: (i) reproduces the weak-field expansion, (ii) ensures $K_{\text{eff}} \to 0$ at the horizon, preventing relaxation transport across an asymptotically frozen region, and (iii) remains monotone and bounded throughout the exterior domain.

Combining (120) with (119) yields the explicit strong-field profile

$$K_{\text{eff}}(r) = K_0\left(1 - \frac{r_s}{r}\right), \qquad r > r_s. \tag{121}$$

This expression should be read strictly as an *effective constitutive law* valid only within the emergent geometric regime.

Implied structural variation of χ.

Inverting the constitutive relation (116) using (121) gives

$$\frac{(\Delta\chi(r))^2}{\chi_c^2} = -\ln\left(1 - \frac{r_s}{r}\right). \tag{122}$$

As $r \to r_s^+$, the structural variation measure diverges logarithmically:

$$\Delta\chi(r) \sim \chi_c\sqrt{-\ln\left(1 - \frac{r_s}{r}\right)}. \tag{123}$$

This divergence must not be interpreted as a physical singularity of the χ substrate. It indicates that the coarse-grained structural descriptor $\Delta\chi$ ceases to remain small and that the hydrodynamic and geometric parametrization is pushed beyond its domain of validity near the effective horizon.

Interpretation: horizons as conductivity zeros.

Equations (121)–(122) provide a coherent strong-field constitutive completion of the weak-field Poisson description. In Cosmochrony, a Schwarzschild horizon corresponds to a *vanishing relaxation conductivity*,

$$K_{\text{eff}} \to 0,$$

rather than to a fundamental spacetime singularity. Black holes are therefore interpreted as regions where collective structural constraints within χ asymptotically inhibit relaxation, producing the effective causal and temporal horizons discussed in the main text.

Scope and open issues.

The constitutive description developed here raises several open questions, including:

- the microscopic origin of the effective conductivity scale K_0,
- the extension of the constitutive relation to quantum regimes where ρ is replaced by excitation amplitudes,
- and potential observational signatures of nonlinear relaxation effects near compact objects.

These issues are left for future work.

Conclusion.

The strong-field constitutive profile constructed in this appendix provides a consistent and non-singular description of black-hole-like regimes within Cosmochrony. It preserves agreement with Schwarzschild phenomenology while reinterpreting horizons as limits of relaxation transport rather than as fundamental geometric pathologies.

A.6 Minimal Kinematic Constraint

A central postulate of Cosmochrony is the existence of a *universal kinematic bound* on admissible configurations of the χ substrate. This bound is not derived from an action, a metric structure, or a variational principle, but is imposed *ab initio* at the pre-geometric level as a constraint on admissible relaxation patterns.

In effective descriptions admitting a scalar representation χ_{eff}, this postulate may be written, in its *saturated form*, as

$$(\partial_t \chi)^2 + |\nabla \chi|^2 = c^2, \tag{124}$$

where the symbols ∂_t and ∇ denote *effective* temporal and relational variations introduced only once a projectable geometric regime is established. More generally, admissible configurations satisfy the corresponding inequality, with equality characterizing locally saturated relaxation.

This constraint underlies, but is not derived from, the Hamiltonian formulation introduced in Section 5.2, which provides an effective representation valid only within the projected geometric regime.

Pre-geometric status.

Equation (124) does *not* presuppose the existence of a spacetime metric, light cones, or a Lorentzian structure. It is not the norm of a four-gradient taken with respect to an underlying Minkowski metric, nor does it encode any invariant interval. Rather, it expresses a purely kinematic saturation condition: admissible projected descriptions must lie on the boundary of a fixed relaxation budget.

At the level of the fundamental χ substrate, no notions of time, space, distance, or orthogonality are defined. The constant c appearing in (124) is therefore not a velocity in spacetime, but the effective manifestation of the invariant structural bound c_χ introduced in Section 4.9. Only after projection does this bound acquire an operational interpretation as a maximal local ordering rate.

Interpretation as a kinematic postulate.

The constraint (124) should be read as a statement of *admissibility*, not of dynamics. It does not determine how χ evolves, propagates, or responds to sources. Instead, it restricts the class of effective descriptions that can consistently represent underlying χ configurations. In particular, it forbids arbitrarily rapid local ordering or instantaneous global reconfiguration, thereby enforcing causal consistency without postulating causality as a primitive notion.

Emergent relativistic structure.

Only in regimes where projected configurations admit a locally injective and smooth representation does (124) take on a familiar relativistic form. In such cases, the bound c coincides numerically with the invariant speed appearing in special relativity, and the saturated form of the constraint can be rewritten in a form suggestive of a light-cone structure. This resemblance is emergent and descriptive, not ontological: Lorentz symmetry arises *a posteriori* as a property of saturated relaxation, rather than as a fundamental symmetry imposed on spacetime.

Cosmological and gravitational implications.

In homogeneous regimes, the constraint enforces uniform saturation of relaxation, leading directly to linear growth of χ_{eff} and, consequently, to effective cosmic expansion. In inhomogeneous regimes, partial saturation manifests as local slowdown of relaxation, which underlies gravitational time dilation and curvature effects discussed in later sections. In all cases, the same minimal kinematic postulate governs admissibility, without the introduction of additional geometric or dynamical assumptions.

The minimal kinematic constraint therefore constitutes one of the foundational pillars of Cosmochrony. It precedes spacetime, metric structure, and relativistic symmetry, and provides the structural origin from which effective causality and relativistic kinematics emerge.

Limits of the continuum description.

The formulation of the minimal kinematic constraint in terms of continuous derivatives implicitly assumes a regime in which projected χ configurations admit a smooth effective description. At scales comparable to the fundamental relational spacing—for instance near the Planck scale—this continuum approximation is expected to break down.

In such regimes, the gradient operator $\nabla\chi$ should be replaced by finite difference expressions defined on the underlying relational graph, and the constraint (124) must be rederived from purely discrete considerations. While the existence of a universal saturation bound is expected to persist, its precise mathematical expression may differ from the continuum form used here.

A fully discrete formulation of the kinematic constraint and its implications for early-universe dynamics is deferred to future work.

Example: Homogeneous Relaxation

Consider a spatially homogeneous projected configuration where $|\nabla\chi| = 0$. The minimal kinematic constraint reduces to

$$(\partial_t\chi)^2 \leq c^2.$$

In regimes where the bound is locally saturated within the effective description, this implies

$$\partial_t\chi = c,$$

leading to a linear effective ordering relation

$$\chi(t) = \chi_0 + ct.$$

When the effective scale factor satisfies $a(t) \propto \chi(t)$, this yields the Hubble law discussed in Section 5.6. No fundamental temporal evolution of the χ substrate is implied.

A.7 Effective Evolution Equation

Scope and ontological status.

This subsection does *not* introduce a fundamental dynamical law for the pre-geometric χ substrate. At the level of χ itself, no spacetime, metric structure, differential operator, or evolution equation exists. The relations presented below belong *exclusively* to the effective, post-projective description, and provide a compact rewriting of admissible χ-orderings once a stable geometric regime has emerged. All differential operators, source terms, and variational structures introduced here should therefore be understood as descriptive and organizational tools, not as generators of the microscopic dynamics of χ.

Once a stable geometric description has emerged from the underlying relaxation structure of the χ substrate, it becomes possible to summarize large-scale regularities of admissible projected configurations using differential operators familiar from relativistic field theory. This step introduces no new fundamental content: it merely provides a convenient phenomenological language for regimes in which spacetime notions are operationally meaningful.

At this effective level only, the ordering relations between projected χ configurations may be written in the schematic form

$$\Box_{\text{eff}}\chi = S[\chi, \rho], \tag{125}$$

where $\Box_{\text{eff}}$ denotes the d'Alembert operator associated with the *emergent* metric, and ρ represents the effective density of localized, relaxation-resistant configurations. Neither the operator nor the source term is fundamental: both arise through coarse-graining and projection of the underlying relational relaxation structure.

This relation must therefore be read as an effective compatibility condition constraining admissible projected descriptions once a spacetime language becomes applicable. It does not govern the pre-geometric evolution of the χ substrate, but summarizes how large-scale variations of projected configurations are organized in the presence of structured excitations that inhibit relaxation.

Physical meaning of the source term.

The term $S[\chi, \rho]$ does not represent an external force acting on χ, nor an independent interaction channel. Instead, it compactly encodes the effective resistance imposed by localized, structured configurations on the global relaxation ordering.

Matter corresponds to persistent, internally constrained patterns of χ whose organization locally reduces the admissible relaxation rate. When translated into an emergent geometric description, this reduction manifests as gradients in the effective ordering parameter.

Within this geometric language, these gradients are reinterpreted as gravitational time dilation and spacetime curvature. The source term therefore summarizes, in a single effective expression, several related manifestations of inhibited relaxation: inertial resistance, local slowdown of ordering, and the appearance of effective gravitational potentials.

Weak-field approximation.

In regimes where matter-induced gradients remain small and relaxation stays close to homogeneous, the source term may be approximated as linear in the excitation density,

$$S[\chi, \rho] \simeq -\alpha\rho, \tag{126}$$

where α is an emergent effective coupling constant. Matching this expression to the Newtonian limit identifies $\alpha \sim G/c^2$, with the gravitational constant G appearing as a macroscopic parameter characterizing the sensitivity of effective relaxation ordering to localized structural constraints.

This identification is descriptive rather than fundamental: the value of G reflects the large-scale collective properties of the projected regime and is not introduced as a primitive constant of nature. In this approximation, the effective relation reproduces the Poisson equation for the gravitational potential and yields Schwarzschild-like solutions in spherically symmetric configurations.

These results reflect the weak-structure limit of constrained relaxation ordering, not the presence of an independent gravitational interaction.

Beyond the weak-field regime.

In regions of high excitation density or strong confinement—such as near compact objects or during early cosmological phases—nonlinear corrections to $S[\chi, \rho]$ are expected. These corrections encode saturation effects imposed by the universal kinematic constraint $\partial_t \chi \leq c$, inherited from the invariant structural bound defined at the level of the χ substrate.

Such nonlinearities prevent unphysical halting or divergence of the effective description and signal the breakdown of the hydrodynamic and geometric approximation rather than a failure of the underlying relaxation structure of χ. They therefore mark the limits of validity of the effective spacetime language, while leaving the pre-geometric ontological simplicity of Cosmochrony intact.

A.8 Relational Foundation and Emergent Geometry

Throughout the main text, the χ field has been described using a continuous representation. This choice is not meant to attribute fundamental significance to continuity or to spacetime fields, but reflects a pragmatic strategy aimed at maximizing contact with established geometric, field-theoretic, and cosmological formalisms.

Crucially, the emergence of geometric notions in Cosmochrony does *not* depend on the assumption of an underlying continuous manifold. Continuity is introduced only as an effective approximation, valid in regimes where the relational structure of χ varies smoothly and admits coarse-grained descriptions.

At a more fundamental level, Cosmochrony can be formulated in purely relational terms. In such a formulation, neither spacetime points, nor distances, nor a metric are assumed *a priori*. Temporal ordering arises from the monotonic relaxation ordering of χ and should be understood as a structural ordering relation between admissible configurations, not as a fundamental temporal parameter.

Likewise, spatial relations and effective geometry are reconstructed operationally from patterns of correlation, resistance, and connectivity within the χ substrate. These relations encode how variations of admissible configurations are mutually constrained, without presupposing an embedding space or background geometry.

A concrete realization of this relational perspective is developed in Appendix E. There, geometric quantities are shown to emerge as effective summaries of relational properties of χ, such as the ease with which relaxation-induced variations propagate between configurations. The metric appears only as a derived object encoding these relational properties, not as a fundamental dynamical entity.

The role of the present subsection is therefore purely clarificatory. It emphasizes that the continuous description employed in the main text is a representational convenience rather than an ontological commitment. All core claims of Cosmochrony—including the emergence of time, geometry, and gravitation—remain valid independently of this choice and rest ultimately on the relational structure and admissibility constraints of the χ substrate itself.

A.9 Energy and Curvature

In the Cosmochrony framework, energy is not introduced as a fundamental conserved quantity. Instead, it emerges as an effective and relational measure of how strongly a given configuration of the χ substrate resists the global relaxation process. Energy is therefore not a primitive substance, but a diagnostic of constrained relaxation within an otherwise monotonically ordered substrate.

Once an effective geometric description becomes applicable, this resistance may be summarized by quantities that resemble familiar energy densities. At this phenomenological level, it is convenient to introduce the functional

$$\mathcal{E}_\chi^{\mathrm{eff}} = \frac{1}{2}\left[(\partial_t\chi)^2 + (\nabla\chi)^2\right], \tag{127}$$

which provides a coarse-grained measure of temporal and spatial deformation of the projected χ configuration.

This functional has no fundamental Hamiltonian or variational status. It is neither conserved nor associated with a symmetry of the underlying pre-geometric substrate, and it does not generate equations of motion. Its role is purely diagnostic, allowing constrained relaxation patterns to be characterized using a familiar field-theoretic language within the emergent geometric regime.

Regions in which $\mathcal{E}_c hi^{\mathrm{eff}}$ is large correspond to projected configurations where χ exhibits strong internal gradients, extended spatial modulation, or reduced local relaxation rates. Such configurations store a significant amount of *relaxation resistance* and are interpreted as localized impediments to the global ordering process. In the effective spacetime description, these regions are naturally identified with particle-like excitations carrying inertial and gravitational properties.

Orbital extension and effective energy ordering.

Within the Cosmochrony framework, the apparent ordering of atomic orbitals by increasing energy does *not* arise from spatial distance to a nucleus as such. Instead, it reflects the structural cost associated with sustaining increasingly extended and oscillatory configurations of the χ field.

An "outer orbital" corresponds, in the effective description, to a configuration in which the relaxation-resistant pattern associated with a particle excitation occupies a larger spatial domain and exhibits a higher degree of internal modulation. Maintaining such an extended pattern requires constraining the global relaxation flow over a wider region, thereby increasing the total relaxation resistance.

In terms of the diagnostic functional $\mathcal{E}_\chi^{\mathrm{eff}}$, this manifests as an increase in the integrated contribution of spatial gradients and inhibited ordering across the configuration. The higher effective energy of more distant orbitals therefore reflects a greater *global cost of constraint*, not a larger local potential or an intrinsic preference for spatial separation.

Importantly, this energetic ordering is independent of the geometric visibility of orbitals discussed in Appendix A.10. While orbital-like shapes emerge from thresholding and projection effects applied to a continuous field, their energetic hierarchy is determined by the degree to which the corresponding configurations frustrate relaxation at the structural level.

Curvature as constrained relaxation.

Within this context, the notion of "curvature" associated with χ must be interpreted with care. It does not refer to spacetime curvature as a fundamental geometric object, nor to an intrinsic curvature of a background manifold. Instead, it characterizes the internal deformation and non-uniformity of projected χ configurations and the way these deformations modulate the propagation of relaxation and correlations.

Effective spacetime curvature arises only secondarily, as a macroscopic descriptor summarizing how constrained configurations influence admissible ordering and correlation structures over extended regions. No independent geometric degree of freedom is introduced at the fundamental level.

Stable solitonic configurations arise when nonlinear self-interaction effects of the χ substrate balance the dispersive tendency associated with spatial gradients in the projected description. This balance allows localized resistance to relaxation to persist over extended ordering intervals, providing a dynamical and geometric origin for long-lived particle-like excitations without invoking fundamental energy conservation laws, quantized potentials, or primitive geometric dynamics.

A.10 Level Sets, Projections, and Apparent Orbital Geometry

This appendix establishes a general geometric property of continuous scalar fields that is directly relevant to the interpretation of atomic orbitals and similar structures as threshold-visible manifestations of an underlying continuum. The results presented here are purely mathematical and do not rely on any specific physical interpretation or dynamical assumption.

Level sets of χ are introduced solely as visualization tools. They do not correspond to fundamental spatial structures, nor to independently localized entities. Instead, they provide a convenient way of identifying regions of comparable field value within effective geometric descriptions.

The present analysis addresses only the geometric and projective origin of orbital-like patterns. It does not concern the energetic ordering of such patterns, which arises from relaxation resistance and spectral constraints discussed elsewhere.

Level Sets of Continuous Scalar Fields

Let $\phi : \mathbb{R}^3 \to \mathbb{R}$ be a continuous scalar field. For a given constant $c \in \mathbb{R}$, the associated level set (or isosurface) is defined as

$$\mathcal{L}_c = \{\mathbf{x} \in \mathbb{R}^3 \mid \phi(\mathbf{x}) = c\}. \tag{128}$$

If ϕ is smooth, $\mathcal{L}_c$ is generically a two-dimensional surface, possibly composed of several disconnected components. Such level sets are routinely used in the visualization of scalar fields by retaining only regions exceeding a prescribed threshold.

Crucially, the existence of multiple disconnected components of $\mathcal{L}_c$ does *not* imply that the underlying field ϕ itself is discontinuous or decomposed into independent objects.

Projection-Induced Apparent Discontinuities

Consider the projection of the level-set condition onto a single coordinate, for instance z. Define the projected set

$$P_c = \{z \in \mathbb{R} \mid \exists (x, y) \in \mathbb{R}^2 \text{ such that } \phi(x, y, z) \geq c\}. \tag{129}$$

Even when ϕ is continuous everywhere, P_c typically consists of a finite union of disjoint intervals. These intervals correspond to regions where the level set intersects planes of constant z.

This fragmentation is a purely geometric consequence of thresholding followed by projection. It reflects the fact that only portions of the field exceeding the chosen threshold are retained. No discontinuity of ϕ is involved.

Envelope Function and Threshold Visibility

Define the envelope function

$$f(z) = \max_{x,y} \phi(x, y, z). \tag{130}$$

The projected set can then be written equivalently as

$$P_c = \{z \in \mathbb{R} \mid f(z) \geq c\}. \tag{131}$$

The envelope function $f(z)$ is continuous whenever ϕ is continuous. However, the condition $f(z) \geq c$ generically selects disconnected regions of the domain. The appearance and disappearance of these regions as c varies reflect changes in *visibility*, not in the underlying field structure.

Thus, threshold-based visualizations reveal sections of a continuous field rather than discrete or independently localized objects.

Non-Uniqueness of Inverse Reconstruction

Given a projected set P_c or a collection of disconnected level-set components, the inverse problem of reconstructing ϕ is underdetermined. Infinitely many continuous scalar fields may share identical threshold projections.

Recovering ϕ uniquely requires additional assumptions, such as symmetry, smoothness, minimal curvature, or governing differential equations. The present analysis therefore establishes a structural constraint on reconstruction, not a reconstruction algorithm.

Summary

Thresholded and projected visualizations of continuous scalar fields generically produce apparently disjoint structures. These structures arise from geometric selection effects and do not correspond to independent physical entities.

This result is entirely model-independent. However, it provides a natural mathematical framework for understanding orbital-like patterns, nodal structures, and probabilistic visibility regions as emergent manifestations of an underlying continuous field.

The apparent discreteness of such patterns reflects projection and detection criteria rather than fundamental discontinuity, localization, or energetic stratification of the substrate.

A.11 Emergent Electrodynamics from χ Dynamics

Scope and ontological status.

This subsection does *not* introduce an independent electromagnetic field or a new fundamental interaction. All structures discussed below arise exclusively within the effective, post-projective description of admissible χ configurations once a smooth geometric regime has emerged. No additional degrees of freedom beyond χ are postulated.

In regimes where the χ field admits a smooth geometric and weak-gradient description, small perturbations around a slowly varying background obey an effective wave equation derived from the variational formulation (Section 5.4):

$$\nabla \cdot \left(\frac{\nabla \chi}{\sqrt{1 - |\nabla \chi|^2 / c^2}} \right) - \frac{1}{c^2} \frac{\partial^2 \chi}{\partial t^2} = \frac{4\pi G_{\mathrm{eff}}}{c^2} \rho_\chi. \tag{132}$$

In the weak-field limit, $|\nabla \chi| \ll c$, this equation linearizes to

$$\nabla^2 \chi - \frac{1}{c^2} \frac{\partial^2 \chi}{\partial t^2} = 4\pi G_{\mathrm{eff}} \rho_\chi, \tag{133}$$

which admits propagating solutions. These solutions should be interpreted as admissible collective modes of the projected description, not as fundamental propagating excitations of the χ substrate.

Emergent Scalar and Vector Potentials

Electromagnetic-like degrees of freedom arise from the *geometric and topological structure* of χ perturbations rather than from independent gauge fields. In regions containing charged solitonic excitations, the spatial gradients of χ acquire both longitudinal and transverse components.

At the effective level, the spatial gradient of χ may therefore be decomposed as

$$\nabla\chi = -\nabla\phi + \mathbf{A}_{\mathrm{T}}, \tag{134}$$

where ϕ is an effective scalar potential and $\mathbf{A}_{\mathrm{T}}$ is a divergence-free vector field,

$$\nabla \cdot \mathbf{A}_{\mathrm{T}} = 0. \tag{135}$$

This decomposition is not a fundamental split of degrees of freedom. It corresponds to an effective Helmholtz projection induced by the topology of localized χ excitations and the admissibility constraints of the projected description. The scalar component encodes longitudinal relaxation gradients associated with effective charge density, while the transverse component arises from solitonic configurations with non-trivial circulation.

Charge as Transverse Torsion of the Relaxation Flux

Within the effective regime in which a geometric description of χ variations is admissible, the decomposition

$$\nabla\chi = -\nabla\phi + \mathbf{A}_T, \tag{136}$$

with $\nabla \cdot \mathbf{A}_T = 0$, admits a natural structural interpretation. The transverse component $\mathbf{A}_T$ does not represent an independent gauge field, but encodes a *directional or chiral organization* of the admissible relaxation flux associated with localized excitations.

The effective relaxation flux derived from the Born–Infeld–like Lagrangian,

$$\mathcal{L}_{\mathrm{eff}}(\chi) \;\sim\; -c^2\sqrt{1-\frac{|\nabla\chi|^2}{c^2}}, \tag{137}$$

defines a bounded canonical current

$$\mathbf{J}_\chi \;\equiv\; \frac{\partial\mathcal{L}_{\mathrm{eff}}}{\partial(\nabla\chi)} \;\propto\; \frac{\nabla\chi}{\sqrt{1-|\nabla\chi|^2/c^2}}, \tag{138}$$

whose magnitude approaches saturation as $|\nabla\chi| \to c$. In this context, $\mathbf{A}_T$ may be understood as a *torsional component* of the effective relaxation flux, characterizing a non-trivial orientation of $\mathbf{J}_\chi$ in the projected description.

A localized excitation is said to carry an effective electric charge if the associated transverse component $\mathbf{A}_T$ exhibits a topologically non-trivial circulation. Formally, an effective charge invariant may be defined as

$$q \;\equiv\; \kappa \oint_{\gamma} \mathbf{A}_T \cdot d\boldsymbol{\ell} \;=\; \kappa \int_S (\nabla \times \mathbf{A}_T) \cdot d\mathbf{S}, \tag{139}$$

where γ is a closed loop enclosing the excitation, S is a spanning surface, and κ is a normalization constant. The sign of q reflects the orientation (chirality) of the transverse torsion, while its stability follows from the topological character of the circulation.

In this formulation, electric charge is not introduced as a fundamental attribute or coupling, but emerges as a structural property of the admissible relaxation flux. Charge conjugation corresponds to a reversal of the transverse orientation of $\mathbf{A}_T$, consistent with its interpretation as a relational inversion rather than an internal symmetry operation.

The magnetic field appearing in the effective electrodynamic description is then naturally identified with the vorticity of the transverse flux,

$$\mathbf{B} \;\equiv\; \nabla \times \mathbf{A}_T, \tag{140}$$

while electric effects arise from longitudinal relaxation gradients and temporal variations of the torsional structure, in agreement with the standard effective kinematics.

Importantly, no fundamental gauge symmetry is postulated at the level of the χ substrate. Apparent gauge redundancies reflect the non-uniqueness of the effective potential description of $\mathbf{A}_T$, while physically meaningful quantities are encoded in topological and flux invariants of the relaxation field.

Topological Origin of the Vector Potential

The transverse component $\mathbf{A}_\mathrm{T}$ originates from solitonic configurations of χ characterized by non-vanishing loop integrals

$$\oint \nabla\chi \cdot d\mathbf{l} \neq 0, \tag{141}$$

as discussed in Section B.2. Such configurations imply the existence of an effective vector potential whose curl is non-zero, while its divergence vanishes identically.

The vector potential is therefore not an independent dynamical field. It is a derived quantity encoding the transverse sector of admissible χ gradients, analogous to vorticity-induced vector fields in hydrodynamic systems.

Emergent Electromagnetic Fields

Within this effective description, the electric and magnetic fields are defined as

$$\mathbf{E} = -\nabla\phi - \frac{1}{c}\frac{\partial \mathbf{A}_\mathrm{T}}{\partial t}, \qquad \mathbf{B} = \nabla \times \mathbf{A}_\mathrm{T}. \tag{142}$$

These fields satisfy a closed system of Maxwell-like relations:

$$\nabla \cdot \mathbf{E} = 4\pi G_{\text{eff}} \rho_{\text{em}}, \tag{143}$$

$$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0, \tag{144}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{145}$$

$$\nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \frac{4\pi G_{\text{eff}}}{c}\mathbf{J}_{\text{em}}, \tag{146}$$

where ρ_{em} and $\mathbf{J}_{\text{em}}$ denote effective charge and current densities associated with solitonic χ configurations.

These relations do not constitute fundamental field equations. They express compatibility conditions obeyed by the transverse and longitudinal sectors of admissible χ perturbations within the emergent geometric regime.

Gauge Invariance

The decomposition of $\nabla\chi$ into scalar and transverse components is not unique. A transformation of the form

$$\phi \to \phi - \frac{1}{c}\frac{\partial \Lambda}{\partial t}, \qquad \mathbf{A}_{\text{T}} \to \mathbf{A}_{\text{T}} + \nabla\Lambda, \tag{147}$$

leaves the observable fields $\mathbf{E}$ and $\mathbf{B}$ invariant.

This emergent $U(1)$ gauge symmetry reflects the relational nature of χ: only differences of gradients have operational meaning, while absolute potential values are gauge-dependent descriptors without physical significance (Section A.8).

Interpretational Status

The Maxwell-like structure derived here is not fundamental. It arises as a universal effective description of transverse χ perturbations in regimes where solitonic topology, weak gradients, and a stable geometric approximation coexist.

Electromagnetism therefore appears in Cosmochrony as a geometric and topological manifestation of the relational structure of the χ substrate, rather than as an independent interaction mediated by elementary gauge fields.

A.12 Relational Consistency of the Effective Lagrangian

The effective Lagrangian for the χ field is **not postulated arbitrarily** but constructed as a **canonical representation** of the relational dynamics introduced in Section 5. This appendix demonstrates how its Born–Infeld-like form emerges from fundamental principles, clarifies its systematic selection, and addresses the continuum limit with full mathematical rigor.

Step 1: Relational Constraint and Bounded Variations

At the fundamental level, the dynamics of χ are governed by a **discrete relational constraint**:

$$\mathcal{C}_i[\chi] \equiv \sum_j K_{ij}(\chi_i - \chi_j)^2 \leq \chi_c^2, \tag{148}$$

where $K_{ij} = K_{ji}$ is a symmetric connectivity matrix and χ_c is the correlation scale. This constraint enforces bounded relative variations without assuming pre-existing spacetime, acting as a **structural causality condition**.

Step 2: Variational Formulation with Global Order

The dynamics are described by a constrained action with KKT conditions:

$$S[\{\chi_i\}, \{\mu_i\}] = \int d\lambda \left[\sum_i \frac{1}{2} \left(\frac{d\chi_i}{d\lambda} \right)^2 - U[\{\chi_i\}] - \sum_i \mu_i(\lambda) \left(\mathcal{C}_i[\chi] - \chi_c^2 \right) \right], \tag{149}$$

where:

- The kinetic term $\frac{1}{2}(d\chi_i/d\lambda)^2$ is the **leading-order expansion** of any smooth functional governing ordered relaxation, with $U[\{\chi_i\}]$ encoding additional constraints.
- The **global order** is ensured by the functional $\Xi[\chi(\lambda)] \equiv \sum_i \chi_i(\lambda)$, with $\frac{d\Xi}{d\lambda} \geq 0$.
- KKT conditions guarantee $\mu_i(\lambda) \geq 0$ and $\mu_i(\lambda)(\mathcal{C}_i[\chi] - \chi_c^2) = 0$.

Step 3: Continuum Limit and Canonical Form

In **projectable regimes**, the discrete constraint maps to a continuum bound:

$$|\nabla\chi|^2 \leq c^2, \tag{150}$$

where ∇ is an emergent operator. For a lattice of spacing a, we define:

$$(\nabla\chi)^2 \approx \frac{1}{a^2} \sum_{\langle i,j \rangle} (\chi_i - \chi_j)^2, \tag{151}$$

yielding $|\nabla\chi|^2 \leq c^2$ with $c^2 \equiv a^2\chi_c^2/K_0$. The continuum limit $a \to 0$ is well-defined if $K_0 \sim a^{-2}$.

Canonical Selection.

We seek a functional $L_{\text{eff}} = f(|\nabla\chi|^2/c^2)$ satisfying:

- Free theory normalization: $f(0) = -c^2$,
- Saturation: $f(1) = 0$,
- Monotonicity: $f'(x) > 0$ for $x \in [0, 1]$,
- Regularity: $f''(x)$ finite.

The **canonical representation** is:

$$f(x) = -c^2\sqrt{1-x}, \tag{152}$$

yielding the Born–Infeld-like Lagrangian:

$$\mathcal{L}_{\text{eff}} = -c^2\sqrt{1-\frac{|\nabla\chi|^2}{c^2}} + \partial_t\chi. \tag{153}$$

Selection Criteria.

The Born–Infeld form corresponds to the **minimal non-polynomial functional** satisfying boundedness, smooth saturation, and finite characteristic speeds. Other choices are admissible but introduce either degeneracies, non-saturating behavior, or additional scales.

Step 4: Role of the Potential $U[\{\chi_i\}]$

The potential $U[\{\chi_i\}]$ encodes additional relational constraints (e.g., topological terms). In the continuum limit:

$$U[\{\chi_i\}] \to \int d^3x\, V(\chi), \tag{154}$$

where $V(\chi)$ is an effective potential. The connection to the main text's $V(\chi)$ is established via coarse-graining, ensuring consistency with solitonic solutions (Section A.2).

Step 5: Connection to Emergent Geometry

A coarse-graining procedure *admits* an auxiliary effective Lagrangian representation of the form:

$$\mathcal{L}_{\text{eff}} = -c^2\sqrt{1-\frac{|\nabla\chi|^2}{c^2}} + \partial_t\chi, \tag{155}$$

where the linear term $\partial_t\chi$ does not affect the equations of motion but fixes the orientation of the effective evolution parameter.

The effective metric is defined via the Hessian of $\mathcal{L}_{\text{eff}}$:

$$g_{\mu\nu}^{\text{eff}} \propto \frac{\partial^2\mathcal{L}_{\text{eff}}}{\partial(\partial_\mu\chi)\partial(\partial_\nu\chi)}, \tag{156}$$

up to conformal rescalings. This construction is valid in projectable regimes where K_{ij} approximates a continuum Laplacian (Section D.5).

Summary of Key Improvements

- **Systematic selection** of the Born–Infeld form from first-principle constraints (boundedness, monotonicity, regularity).
- **Explicit continuum limit** with spectral Laplacian connection.
- **Clarified role of** $U[\{\chi_i\}]$ and its continuum counterpart.
- **No circularity**: $\mathcal{L}_{\text{eff}}$ is consistent with (not derived from) relational dynamics.

Scope and Limitations

The Born–Infeld-like Lagrangian is a **canonical representation** valid in projectable regimes. Outside these regimes:

- No spacetime description exists,
- Alternative functionals may be required,
- The discrete dynamics (Eq. (148)) remain fundamental.

Continuum Limit and Emergence of the Laplace–Beltrami Operator

The discrete relational constraint

$$\mathcal{C}_i[\chi] = \sum_j K_{ij}(\chi_i - \chi_j)^2 \tag{157}$$

defines a weighted graph Laplacian acting on the configuration space of χ. To establish its continuum limit, we introduce a local volume element V_i associated with node i, and a spectral distance d_{ij} between nodes.

We assume that the coupling coefficients admit the scaling

$$K_{ij} = \frac{1}{V_i}\, w\!\left(\frac{d_{ij}}{\varepsilon}\right), \tag{158}$$

where w is a symmetric, rapidly decaying kernel and ε characterizes the microscopic relational scale.

In the dense limit, where the number of nodes $N \to \infty$, the typical separation $d_{ij} \to 0$, and the node distribution becomes asymptotically uniform with respect to the emergent measure $\sqrt{|g|}\, d^n x$, defined by the relational density of the χ substrate. The discrete sum satisfies

$$\lim_{\varepsilon\to 0} \frac{1}{V_i} \sum_j w\!\left(\frac{d_{ij}}{\varepsilon}\right) (\chi_i - \chi_j)^2 = \int_{\mathcal{M}} g^{ab}\, \partial_a \chi\, \partial_b \chi\, \sqrt{|g|}\, d^n x. \tag{159}$$

This convergence follows from standard results on graph Laplacians, which guarantee that the weighted Laplacian of a dense graph converges to the Laplace–Beltrami operator on an emergent Riemannian manifold $\mathcal{M}$ defined by the relational density of nodes.

Crucially, no background geometry is assumed *a priori*: the metric g_{ab} arises as the continuum encoding of the microscopic connectivity structure of the χ substrate. The Dirichlet energy functional is therefore not postulated, but emerges uniquely as the thermodynamic limit of the discrete relational constraint.

Necessity of the Born–Infeld Structure from Causal Saturation

This subsection does not claim that the Born–Infeld structure is a fundamental action governing the χ substrate. Rather, it establishes that any effective functional representation *of admissible projected dynamics must satisfy strict boundedness and saturation conditions inherited from the pre-geometric relaxation constraint.*

The effective Lagrangian governing the projected dynamics of χ cannot be chosen arbitrarily. In particular, a purely quadratic functional

$$\mathcal{L}_{\text{quad}} = \frac{1}{2}\,\partial_\mu\chi\,\partial^\mu\chi \tag{160}$$

is incompatible with the existence of a fundamental upper bound on the relaxation speed of the χ substrate.

Quadratic actions permit unbounded gradients and therefore allow arbitrarily large relaxation fluxes, corresponding to instantaneous propagation of constraints. Such behavior contradicts the existence of a maximal relaxation speed c_χ, required for the causal consistency of the projection onto effective spacetime.

Imposing the condition that the relaxation flux saturates at c_χ uniquely constrains the functional form of the action. The Lagrangian density must interpolate smoothly between the quadratic regime at low gradients and a strictly bounded regime at high gradients.

The minimal functional satisfying these requirements is of Born–Infeld type:

$$\mathcal{L}_{\text{BI}} = b^2\left(1 - \sqrt{1 - \frac{1}{b^2}\,\partial_\mu\chi\,\partial^\mu\chi}\right), \tag{161}$$

where the parameter b is directly related to the maximal relaxation speed c_χ.

This structure ensures:

- recovery of the quadratic theory in the low-gradient limit,
- a strict upper bound on $|\partial_\mu\chi|$,
- causal saturation of relaxation fluxes at c_χ.

Alternative polynomial expansions (e.g. χ^4 or higher-order terms) fail to enforce such a bound without introducing additional ad hoc mass scales. They therefore violate high-energy spectral invariance and permit superluminal relaxation modes in the χ substrate.

Moreover, the Born–Infeld structure is intrinsically self-regularizing: configurations that would produce divergent gradients in a quadratic theory are smoothly regulated by the square-root saturation mechanism. The energy density and relaxation flux remain finite for all admissible field configurations, without the need for external cutoffs or renormalization prescriptions.

This self-regularization property is absent from polynomial theories, which generically develop gradient singularities and therefore require additional ultraviolet completion.

The Born–Infeld structure is thus not a convenient representation, but the *unique* effective functional compatible with bounded relaxation, causal projection, and parameter-free saturation of the χ dynamics.
The term “unique” should be understood here in a conditional sense: unique within the class of smooth, local, scale-free functionals that (i) enforce strict saturation of relaxation fluxes, (ii) admit a quadratic low-gradient limit, and (iii) do not introduce additional microscopic parameters. No claim of ontological uniqueness at the level of the χ substrate is implied.
The distinction introduced in this paragraph concerns two conceptually different levels of description and does not introduce additional fundamental constants beyond those already defined in the relational dynamics of the χ substrate.

Relation Between Spectral Saturation (b) and the Speed of Light (c)

It is essential to distinguish the saturation constant b from the phenomenological parameter c (the speed of light). Within the Cosmochrony framework, b represents the upper bound on the relaxation speed of the χ substrate itself—a pre-geometric causal constraint governing the internal dynamics of the fundamental relational structure.

The speed of light c, by contrast, emerges as an effective quantity: it corresponds to the group velocity of projected perturbative modes propagating on the emergent effective metric $g_{\mu\nu}^{\text{eff}}$. As such, c is not a fundamental constant of the substrate, but a derived parameter characterizing the kinematics of the projected description.

Mathematically, c depends on b through a filtering determined by the microscopic coupling density K_0, encoding how relaxation dynamics in χ are transcribed into effective spacetime propagation. This hierarchical structure ensures that physical information can never propagate faster than the underlying relaxation processes of the substrate, thereby enforcing the inequality

$$c \leq b. \tag{162}$$

Any hypothetical regime in which c would exceed b would imply that projected perturbations propagate faster than the substrate can relax, leading to an immediate loss of coherence of solitonic configurations and, consequently, to the destabilization of matter itself.
Taken together, the results of this appendix show that the Born–Infeld-like Lagrangian does not define the microscopic dynamics of the χ substrate. It provides a minimal, non-singular, and causally saturated *effective encoding* of admissible projected configurations in regimes where a geometric description becomes operationally meaningful.

B Conceptual Extensions of Cosmochrony — Particles, Quantum Phenomena, and Classical Limits

This appendix develops a set of conceptual and phenomenological extensions of the Cosmochrony framework. Its purpose is not to introduce additional postulates or to strengthen the internal consistency of the theory, but to illustrate how familiar particle, quantum, and classical structures may emerge once the χ field admits localized and stable configurations.

In particular, this appendix addresses:

- the ontological status of the χ field and its interpretation as a pre-geometric relational substrate (Section B.1),
- the description of particles as topological solitons of χ, including explicit constructions for fermionic and bosonic configurations (Sections B.2–B.4),
- the emergence of classical limits and the interpretation of quantum-to-classical transitions (Sections B.5–B.6),
- and perspectives on deriving particle mass spectra from the internal dynamics and stability properties of the χ field (Section B.8).

These developments serve as a conceptual bridge between the mathematical foundations presented in Appendix A and the cosmological and observational considerations discussed in Appendix C. They demonstrate how the minimal ontological assumptions of Cosmochrony can support a rich hierarchy of effective physical phenomena without requiring additional fundamental degrees of freedom.

None of the constructions presented in this appendix are required for the logical coherence or internal consistency of the Cosmochrony framework. The core theory remains fully defined by the relational relaxation dynamics of the χ field. Rather, the material collected here illustrates how particle-like excitations, quantization, and classical behavior may arise naturally as emergent features when χ organizes into localized, topologically stable configurations.

B.1 Interpretative Status of the χ Field

This appendix supplements the primary definition of χ given in Section 3.1. It does not introduce new postulates, but provides interpretative clarifications to prevent common misreadings suggested by conventional field-theoretic language.

In the main text, χ is often written in the form $\chi(x^\mu)$ and manipulated using continuous differential operators. This notation should not be taken to imply that χ is a physical field propagating *within* a pre-existing spacetime manifold. Rather, spacetime coordinates serve only as convenient labels for organizing relational information in regimes where a stable geometric description has emerged.

Fundamentally, χ encodes a **non-energetic relational scale of relaxation** from which notions such as duration, distance, and causal ordering are reconstructed. The apparent embedding of χ in spacetime is therefore representational, not ontological. The manifold description is a secondary construct, introduced only after the relaxation dynamics of χ has reached sufficient regularity to admit a geometric interpretation.

This distinction mirrors the use of continuum variables in hydrodynamics or elasticity theory. Just as a velocity field does not exist independently of the underlying molecular interactions, $\chi(x^\mu)$ does not represent a fundamental spacetime field. It summarizes collective relational properties of the substrate once coarse graining becomes meaningful, **without implying an underlying differentiable structure at the fundamental level**.

In this sense, χ should not be interpreted as:

- a matter field living on spacetime,
- a dynamical scalar coupled to a pre-existing metric,
- or a hidden-variable replacement for the quantum wavefunction.

Instead, χ constitutes the pre-geometric quantity from which spacetime structure, effective fields, and physical observables emerge through projection and coarse graining. The use of continuous fields, Lagrangians, and differential equations throughout this work reflects practical representational choices rather than fundamental ontological or kinematical commitments.

This interpretative clarification is particularly important for understanding the role of localized excitations, solitonic structures, and effective fields discussed in the remainder of this appendix. These constructions should be read as regime-dependent invariants of the underlying χ dynamics, not as evidence that χ itself decomposes into independently propagating physical entities.

In summary, the χ field is not a field *in* spacetime. Spacetime is an emergent bookkeeping structure *for* χ once its relational dynamics becomes sufficiently regular. This asymmetry is essential to the ontological parsimony of the Cosmochrony framework and underlies its reinterpretation of geometry, matter, and quantum phenomena.

B.2 Topological Configurations of the χ Field: Solitons as Particles

This appendix provides effective geometric representations of the topological stability principles introduced in Section 6.2.

Status and scope of this construction.

The solitonic configurations discussed in this appendix are not introduced as fundamental degrees of freedom of Cosmochrony. They are *effective geometric representations* intended to illustrate how particle-like properties may arise from stable, localized configurations of the χ field *once a smooth, orientable geometric projection becomes applicable.*

At the fundamental level, Cosmochrony does not assume a pre-existing spatial manifold, metric, or differential structure. The scalar field χ is not defined *in* spacetime; rather, spacetime emerges as an effective description of relational regimes of χ. A fully relational and pre-geometric formulation is presented in Appendix E.

Throughout this appendix, all geometric notions (distance, rotation, circulation, surface integrals) refer exclusively to an *effective projected field* χ_{eff}, obtained once a stable projective regime is reached. None of the figures or constructions below should be interpreted as depicting the fundamental χ field itself.

Within this effective regime, particles are interpreted as **topologically stabilized solitonic configurations** of χ_{eff}, within the effective descriptive regime only. Their apparent properties—such as **mass, spin, and charge**—do not correspond to independent fundamental quantum numbers, but emerge from the **structural organization and relaxation constraints** induced by these configurations.

Charge as Oriented Relaxation Asymmetry of χ_{eff}

In Cosmochrony, electric charge is not associated with a fundamental gauge field, local symmetry, or conserved Noether current. Instead, it emerges as an *oriented asymmetry in the relaxation structure* of the effective projected field χ_{eff}, once a stable geometric description becomes applicable.

A localized solitonic configuration may deform the surrounding χ_{eff} profile in one of two qualitatively distinct ways:

- A **positive effective charge** corresponds to a **local excess** of χ_{eff} relative to its asymptotic background value $\chi_{\text{eff},0}$. Such configurations locally resist relaxation and generate repulsive interaction tendencies with similarly oriented deformations.
- A **negative effective charge** corresponds to a **local deficit** of χ_{eff} relative to $\chi_{\text{eff},0}$, favoring compensating relaxation and attractive interactions with oppositely oriented configurations.

This polarity does not reflect an intrinsic sign of the fundamental χ substrate—which remains a scalar relational quantity without charge— but rather the orientation of the deformation with respect to the background relaxation flow in the projective regime.

From structural asymmetry to observable charge.

Within an effective geometric description, the magnitude of the charge associated with a solitonic excitation is controlled by the *net relaxation imbalance* induced by the configuration.

A convenient coarse-grained diagnostic of this imbalance is given by a Gauss-like flux integral,

$$q_{\text{eff}} \propto \oint_{\Sigma} \nabla\chi_{\text{eff}} \cdot d\mathbf{S},$$

where Σ denotes a closed surface surrounding the localized configuration in effective space. This expression does not define a fundamental conserved quantity; it merely quantifies the oriented relaxation asymmetry encoded in the projected gradient structure of χ_{eff}.

In three effective spatial dimensions, the geometric dilution of these gradients naturally leads to inverse-square interaction profiles. Coulomb-like behavior therefore arises as a collective geometric response of χ_{eff}, without introducing a fundamental electromagnetic field, potential, or gauge symmetry.

Coulomb-like interaction as frustration minimization.

Crucially, the effective interaction between two charged configurations is *not imposed as a force law.* It emerges from the minimization of projected geometric frustration.

In the projective regime, the leading diagnostic of this frustration is the Dirichlet-like functional
$$\mathcal{F}[\chi_{\text{eff}}] \;\equiv\; \frac{1}{2}\int |\nabla\chi_{\text{eff}}|^2\, d^3x,$$
which encodes the local cost of relaxation gradients. Two localized oriented deformations interact because their combined configuration may either lower or increase $\mathcal{F}$ relative to the sum of isolated profiles. Attraction and repulsion are therefore not additional dynamical postulates, but direct signatures of the gradient of frustration in configuration space.

In this sense, Coulomb's law appears as a far-field consequence of frustration minimization under geometric dilution constraints, and not as an independently coded interaction.

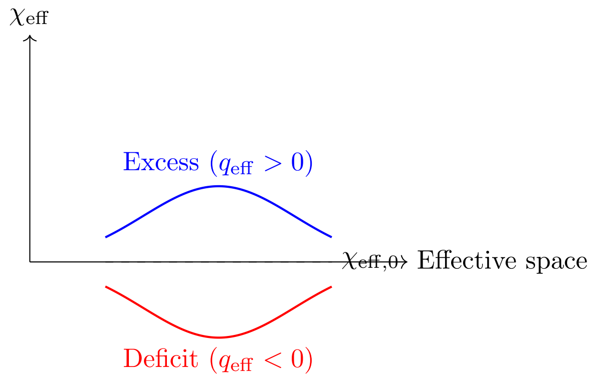

Fig. 13 Schematic illustration of oriented deformations of the effective projected field χ_{eff}. An excess or deficit relative to the background value $\chi_{\text{eff},0}$ determines the polarity of the effective charge. This diagram is purely illustrative and does not represent a solution of the fundamental χ dynamics.

Scope and limitations.

The present construction accounts for electrostatic-like interactions within the projective regime. Dynamical electromagnetic phenomena (such as radiation or magnetic induction) are not assumed here and would require additional projective modes beyond scalar relaxation asymmetry.

Vortical Configurations and Integer-Spin Excitations

In the projectable regime, certain solitonic configurations of χ_{eff} admit cyclic internal organization patterns that can be described using an effective phase. When these patterns exhibit non-trivial circulation, the configuration may be modeled as a *vortical soliton*.

An effective winding number n may be defined as

$$n = \frac{1}{2\pi}\oint \nabla \arg(\chi_{\text{eff}}) \cdot d\mathbf{l},$$

where all quantities refer to the emergent geometric representation. This winding number is not fundamental and has no meaning outside the projective regime.

The integer n characterizes:

- the orientation of the relaxation asymmetry (sign of the effective charge),
- the topological robustness of the configuration,
- and the effective spin of the excitation, with integer values corresponding to bosonic behavior.

The energetic cost of such configurations increases with their internal structural complexity, leading to an effective mass scaling with $|n|^2$ in minimal models.

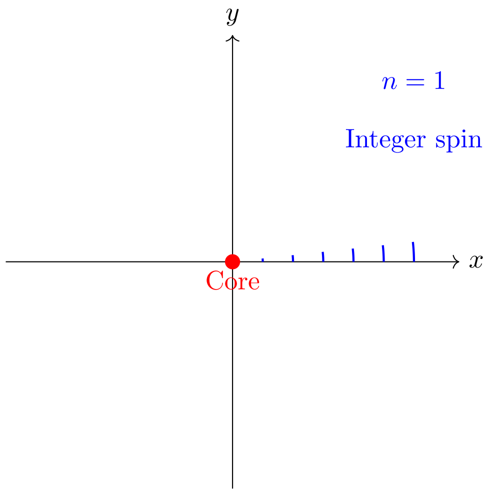

Fig. 14 Illustrative vortical configuration of the effective field χ_{eff} with winding number $n = 1$. The circulation represents a cyclic relaxation pattern associated with integer spin. This figure is schematic and purely conceptual.

Skyrmion-Like Configurations and Spin-$\frac{1}{2}$ Excitations

More complex solitonic configurations arise when the internal organization of χ_{eff} involves non-trivial mappings between internal orientation space and effective physical space. Such configurations may be modeled using skyrmion-like constructions.

An effective topological index Q can be defined as

$$Q = \frac{1}{4\pi} \int \mathbf{n} \cdot (\partial_x \mathbf{n} \times \partial_y \mathbf{n})\, dx\, dy, \quad \mathbf{n} = \frac{\chi_{\text{eff}}}{|\chi_{\text{eff}}|}.$$

where $n\mathbf{n}n$ denotes an effective orientation field constructed from χ_{eff}, not a fundamental vector degree of freedom.

Configurations with $Q = \pm 1$ exhibit a characteristic **4π-periodicity** under rotations. A 2π rotation does not return the configuration to an equivalent state, while a 4π rotation does. This topological property provides a geometric origin for spin-$\frac{1}{2}$ behavior and fermionic statistics within the projective regime.

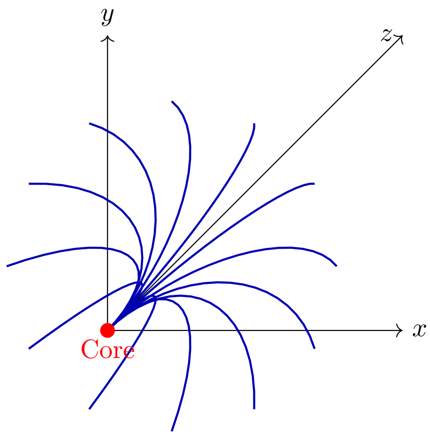

Fig. 15 Conceptual skyrmion-like configuration of χ_{eff}, illustrating a spin-$\frac{1}{2}$ excitation. The non-trivial internal mapping accounts for fermionic rotational behavior. This representation is purely illustrative.

Summary: Topology, Charge, and Spin

Table 2 Effective Solitonic Configurations and Emergent Particle Properties

Configuration	**Topological Index**	χ_{eff} **Asymmetry**	**Emergent Properties**
Vortical soliton	Winding number n	Excess / deficit	Charge $\propto n$, integer spin
Skyrmion-like soliton	Index $Q = \pm 1$	Oriented deformation	Charge $\propto Q$, spin-$\frac{1}{2}$

These constructions are not intended as a particle classification scheme nor as a replacement for the Standard Model. Their role is conceptual: to demonstrate how charge, mass, and spin may emerge coherently from the structural and topological organization of a single scalar substrate, without introducing additional fundamental fields, symmetries, or quantization postulates.

Coulomb-like interaction as frustration minimization (simulation link).

In Cosmochrony, the code does not implement a Coulomb law. It implements a bounded relaxation that systematically reduces projected geometric tension. In a projectable regime, the leading diagnostic of this tension is the Dirichlet-like functional

$$\mathcal{F}[\chi_{\text{eff}}] \;\equiv\; \frac{1}{2}\int |\nabla\chi_{\text{eff}}|^2\, d^3x,$$

which emerges as the continuum encoding of the underlying relational constraint. Two localized oriented deformations interact because the combined configuration can either *lower* or *raise* $\mathcal{F}$ relative to the sum of isolated profiles. Attraction and repulsion are therefore not additional rules: they are the sign of the net frustration gradient in configuration space.

In three effective spatial dimensions, the far-field solution that minimizes $\mathcal{F}$ under a fixed flux constraint yields a $1/r^2$ dilution of the projected relaxation gradients, so Coulomb-like scaling appears as an emergent geometric response, not as a postulated force law.

B.3 Soliton Energy and Structural Mass Scaling

Status and scope of this analysis.

This subsection presents a *quantitative but non-numerical* analysis of the effective mass associated with localized solitonic configurations arising in the *projectable regime* of Cosmochrony. The objective is not to reproduce the observed particle mass spectrum, but to identify robust scaling relations, hierarchy constraints, and structural dependencies that emerge independently of microscopic details.

No claim is made that the expressions introduced below define a fundamental Hamiltonian or Lagrangian for the χ field. A fully predictive derivation of particle masses would require a complete effective theory incorporating projection dynamics, interaction channels, and renormalization effects, which lies beyond the scope of the present work.

All energetic and spectral quantities discussed in this section refer exclusively to the effective projected field χ_{eff}. The fundamental χ field itself does not admit an energy functional or mass interpretation.

Mass as integrated resistance to relaxation.

Within Cosmochrony, the mass of a localized excitation is interpreted as a measure of the total *resistance to relaxation* imposed by the configuration on the surrounding χ_{eff} field.

Once an effective geometric description applies, this resistance can be summarized by an effective diagnostic functional

$$M_{\text{eff}} \;\propto\; \int_{\mathcal{V}} \left[\mathcal{T}(\nabla\chi_{\text{eff}}) + \mathcal{U}(\chi_{\text{eff}})\right]\, d^3x, \tag{163}$$

where:

- $\mathcal{T}$ encodes gradient-induced resistance associated with spatial inhomogeneities of χ_{eff},
- $\mathcal{U}$ represents effective nonlinear stabilization terms arising from collective relaxation constraints.

This expression should be understood as a *coarse-grained measure* of structural complexity rather than as a fundamental energy density. It quantifies how strongly a localized configuration delays or distorts the global relaxation of the projected field.

Scaling with soliton size and internal structure.

Consider a localized solitonic configuration characterized by:

- a typical spatial extent ℓ,
- a characteristic deformation amplitude $\Delta\chi_{\rm eff}$.

Dimensional analysis then yields the generic scaling

$$M_{\rm eff} \;\sim\; \ell^3 \left[\frac{(\Delta\chi_{\rm eff})^2}{\ell^2} + V_{\rm eff}(\Delta\chi_{\rm eff})\right], \tag{164}$$

where $V_{\rm eff}$ denotes an effective nonlinear stabilization potential generated by projection and relaxation constraints.

For simple kink-like configurations, the balance between gradient resistance and nonlinear stabilization dynamically fixes the soliton width ξ. In this regime, the effective mass scale may be written schematically as

$$M_{\rm eff} \;\sim\; \sqrt{\lambda_{\rm eff}}\,\xi\,\chi_c^2, \tag{165}$$

where:

- χ_c denotes the characteristic local relaxation scale,
- $\lambda_{\rm eff}$ is an emergent, configuration-dependent stiffness parameter.

Neither χ_c nor $\lambda_{\rm eff}$ should be interpreted as fundamental constants. They summarize collective properties of the projected relaxation regime.

Structural stabilization and finite mass.

When stabilization arises from a balance between gradient-induced resistance and nonlinear relaxation constraints, the soliton size ℓ is dynamically fixed. This mechanism ensures that localized excitations possess a finite and stable effective mass without the need for fine-tuning.

Different classes of solitonic configurations (kinks, vortices, knotted or linked structures) involve distinct internal organizations of $\chi_{\rm eff}$. As a result, their effective masses exhibit different scaling behaviors with respect to ℓ and $\Delta\chi_{\rm eff}$.

This implies that mass hierarchies arise *structurally* rather than through arbitrary parameter choices.

Topological classes and mass hierarchy.

The effective mass depends not only on the spatial extent of a soliton but also on its topological class. Configurations characterized by higher winding, linking, or covering indices necessarily involve increased internal gradients and more complex relaxation constraints.

Consequently, masses associated with different topological families obey ordering relations of the form

$$M_{n+1} \; > \; M_n, \tag{166}$$

where n labels an effective topological invariant. This establishes a natural mechanism for discrete mass hierarchies without introducing ad hoc mass parameters.

Spectral interpretation.

From a spectral perspective, localized excitations correspond to bound modes of the linearized relaxation operator around a solitonic background configuration of χ_{eff}.

The effective mass is then controlled by the lowest nontrivial eigenvalue of this operator,

$$M_{\text{eff}} \;\sim\; \lambda_{\min}^{-1}, \tag{167}$$

where $\lambda_{\min}$ denotes the smallest positive eigenvalue governing the stability of the configuration.

This formulation emphasizes that mass is fundamentally a *spectral property* of the relaxation dynamics rather than an intrinsic attribute of a particle-like object.

Robustness and universality.

The scaling relations derived above depend only on generic features of the projected χ dynamics—locality, monotonic relaxation, and nonlinear stabilization—and are therefore expected to be robust against modifications of microscopic details.

While specific numerical values of particle masses cannot be fixed at this level, the existence of discrete, ordered, and stable mass scales emerges as a structural prediction of the framework.

Order-of-magnitude consistency.

Although the present analysis does not aim to reproduce the observed particle mass spectrum, it is instructive to examine whether the structural parameters entering the solitonic energy scale admit values compatible with known masses.

For a simple kink-like configuration with characteristic width $\lambda_{\text{eff}}^{-1}$ and amplitude set by the local relaxation scale χ_c, the effective rest energy scales as

$$E_{\text{sol}} \;\sim\; \chi_c^2 \lambda_{\text{eff}}, \tag{168}$$

up to dimensionless shape-dependent factors of order unity.

Identifying this energy with the electron rest mass, $E_{\text{sol}} \sim m_e c^2 \approx 0.511\,\text{MeV}$, and expressing all quantities in natural units ($\hbar = c = 1$), one finds that reproducing the electron mass requires

$$\lambda_{\text{eff}} \;\sim\; 10^{-44}, \tag{169}$$

for χ_c normalized near the Planck scale.

The appearance of such a small effective parameter should be viewed as a diagnostic indicator of deep scale separation rather than as evidence of fine-tuning, analogous to exponentially small gaps generated by collective or topological mechanisms in condensed-matter systems.

Such an extremely small value should not be interpreted as a fundamental coupling. Rather, it strongly suggests that λ_{eff} is dynamically generated through collective relaxation, projection effects, and topological constraints of the χ field.

Summary.

Localized solitonic configurations of the projected field χ_{eff} naturally possess finite effective masses determined by their size, internal organization, and topological class. Rather than predicting specific numerical values, Cosmochrony constrains the *scaling, ordering, and stability* of masses through geometric and spectral principles.

This structural quantitativity provides a coherent foundation for future extensions toward a fully predictive effective theory, without compromising the pre-geometric nature of the fundamental χ field.

B.4 Example: 4π-Periodic Soliton and Spinorial Behavior

This subsection presents an *illustrative geometric construction* showing how spin-$\frac{1}{2}$–like transformation behavior may emerge from a localized configuration of the χ field, without introducing a fundamental spinor degree of freedom.

The construction is intentionally minimal and purely effective. It is not intended as a microscopic derivation of fermions, but as a demonstration of *topological plausibility*: namely, that spinorial behavior can arise from nontrivial structure in the configuration space of scalar excitations once a projectable regime is reached.

All geometric notions used below refer exclusively to the effective projected field χ_{eff}. The fundamental χ field itself does not admit spatial localization, complex structure, or phase.

Phase-Twisted Effective Solitonic Configuration

In the projectable regime, certain localized excitations of χ admit an effective internal organization that can be parametrized by an angular variable. For illustrative purposes only, such configurations may be represented using a complex-valued proxy field,

$$\chi_{\text{eff}}(x) = \eta \tanh(\kappa x)\, e^{i\theta(x)}, \tag{170}$$

where:

- the underlying physical field remains real,
- the complex phase does *not* represent an independent internal degree of freedom,
- the phase θ parametrizes the internal cyclic structure of the effective solitonic configuration.

Equivalently, this is an embedding of a cyclic internal label into a complex notation; no $U(1)$ symmetry or conserved phase current is assumed.

This representation should be understood purely as a convenient encoding of the internal organization of the excitation in effective space.

Choosing the effective rotation parameter α and defining

$$\theta(\alpha) = \frac{\alpha}{2}, \tag{171}$$

implies the 4π periodicity

$$\theta(\alpha + 4\pi) = \theta(\alpha) + 2\pi, \tag{172}$$

whereas a 2π cycle yields a globally inequivalent internal organization.

Topological Interpretation

The 4π-periodicity does not originate from the introduction of a complex field or an intrinsic phase. It reflects a nontrivial topology of the configuration space of the soliton.

Although the spatial projection of the excitation may appear unchanged after a 2π rotation, the internal organization of the configuration is not. Only a 4π rotation restores full equivalence in the space of effective configurations.

This behavior mirrors the double-cover structure $\mathrm{SU}(2) \to \mathrm{SO}(3)$ characteristic of spinorial representations. In the present framework, however, this structure arises from the topology of solitonic configurations of χ_{eff}, rather than from a fundamental spinor ontology.

Relation to Fermionic Transformation Properties

At the effective level, 4π-periodic excitations naturally acquire a sign change under 2π rotations. In multi-excitation configurations, this topological property implies that the exchange of two identical excitations cannot be continuously deformed into the identity without crossing a topologically nontrivial sector.

This provides a geometric basis for fermion-like transformation behavior and fermion-like exchange phase (a sign change in the simplest sector). In full generality this requires the multi-excitation configuration space and its braid structure, which is beyond the scope of this illustrative example. The construction does not constitute a proof of the spin–statistics theorem; rather, it demonstrates that fermionic characteristics can emerge consistently from topologically constrained scalar-field excitations.

Explicit half-angle map and sign inversion.

To make the spin-$\frac{1}{2}$ correspondence explicit, let $\alpha \in [0, 2\pi]$ denote an *effective* rotation parameter acting on the localized configuration in the projectable regime. The defining topological feature is that the relevant loop in configuration space is non-contractible at 2π but becomes contractible at 4π.

A minimal way to encode this double-cover behavior is to introduce an effective spinorial descriptor $\psi(\alpha)$ whose phase advances by a *half-angle*:

$$\psi(\alpha) \;\equiv\; \psi_0 \, e^{i\alpha/2}. \tag{173}$$

Then a 2π cycle produces a sign inversion,

$$\psi(\alpha + 2\pi) = \psi_0 \, e^{i(\alpha+2\pi)/2} = -\, \psi_0 \, e^{i\alpha/2} = -\psi(\alpha), \tag{174}$$

while a 4π cycle restores the original state,

$$\psi(\alpha + 4\pi) = \psi(\alpha). \tag{175}$$

$$\begin{aligned} &\alpha : 0 \to 2\pi \Rightarrow \text{nontrivial loop in } \mathcal{C}_{\text{eff}}\ (\mathbb{Z}_2) \Rightarrow \psi \mapsto -\psi, \\ &\alpha : 0 \to 4\pi \Rightarrow \text{trivial loop (homotopic to identity)} \Rightarrow \psi \mapsto \psi. \end{aligned} \tag{176}$$

In this construction, ψ is not a fundamental field: it is a compact *collective label* for the topological class of the solitonic configuration in the space of admissible projected descriptions. The sign change at 2π is therefore not imposed as a quantum postulate; it is an effective encoding of the $\mathbb{Z}_2$ obstruction associated with the 4π periodicity of the configuration.

(See Sec. B.6 for the configuration-space statement $\pi_1(\mathcal{C}_{\text{eff}}) = \mathbb{Z}_2$ and its effective implication $\psi \mapsto -\psi$ under a 2π loop.)

Conceptual Scope and Limitations

The purpose of this example is conceptual. It illustrates that:

- spinorial behavior does not require a fundamental spinor field,
- 4π-periodicity can arise from topological obstructions,
- fermion-like exchange behavior may emerge from scalar excitations with nontrivial configuration space.

No claim is made that this construction reproduces the full dynamics, interactions, or statistics of Standard Model fermions. A fully relational formulation of these topological properties, independent of any embedding geometry or auxiliary representation, is discussed in Appendix E.

B.5 Relation to Classical Limits

In Cosmochrony, the emergence of classical behavior does not correspond to the introduction of an independent theoretical layer. Instead, classical physics arises as a *dynamical regime* of the same underlying scalar structure, characterized by smoothness, dilution of localized excitations, and the suppression of topological and relational effects.

Weakly structured regime and effective linearization.

In regimes where the fundamental field χ admits a stable projective representation and where its effective projection χ_{eff} varies slowly over large scales, localized excitations become dilute and weakly interacting. Under these conditions, the dynamics of χ_{eff} can be linearized at the level of effective perturbations only around a quasi-homogeneous background configuration.

In this regime, small perturbations propagate as weak disturbances on an effectively flat geometric background. Superposition, approximate locality, and linear wave propagation emerge as effective properties of the coarse-grained relaxation dynamics. This reproduces the operational content of classical field theories and of free or weakly interacting quantum field theories formulated on Minkowski spacetime.

This correspondence should be understood as an *effective recovery* rather than as an ontological reduction. Cosmochrony does not reduce to standard quantum field theory; rather, standard field theories appear as limiting descriptions valid when relational structure becomes dynamically inert.

Suppression of relational and topological effects.

In the weakly structured regime, topological constraints associated with solitonic configurations are either absent or dynamically irrelevant. The configuration space effectively factorizes, and collective relaxation dominates over localized structural organization. As a result, particle-like excitations behave as approximately independent degrees of freedom, and classical intuition becomes applicable.

The classical limit therefore corresponds to a regime in which the relational content of χ is present but operationally inaccessible, masked by coarse-graining and scale separation.

Nonlinear regime and effective curvature.

Conversely, in regimes of strong spatial variation of χ_{eff} or high density of localized excitations, nonlinear effects dominate the dynamics. Large gradients locally constrain relaxation, inducing effective curvature, time dilation, and horizon-like behavior in the emergent geometric description.

These regimes reproduce the phenomenology associated with curved spacetime, gravitational collapse, and strong-field effects, while remaining governed by the same underlying scalar dynamics. No additional gravitational degrees of freedom are introduced; curvature emerges as a collective response of the relaxation structure of χ_{eff}.

Meaning of the classical limit in Cosmochrony.

The classical limit in Cosmochrony is therefore not defined by $\hbar \to 0$, nor by the suppression of quantum postulates. It corresponds to a regime in which:

- the effective projection χ_{eff} is smooth and slowly varying,
- localized excitations are dilute and weakly correlated,
- topological and relational constraints are dynamically suppressed,
- coarse-graining yields stable geometric descriptions.

In this regime, classical spacetime and standard field dynamics emerge as reliable, approximate descriptions. Their validity reflects not fundamental structure, but the stability of a particular relaxation regime of the underlying χ field.

B.6 Status of the Formulation

The formulation presented in this work should be understood as a *minimal yet structurally complete* at the conceptual level theoretical framework. Its ontological commitments, dynamical principles, and interpretative structure are fully specified at the conceptual level, even though several technical developments remain open.

In particular, a fully covariant action principle formulated solely in terms of the fundamental χ dynamics, as well as a systematic quantization procedure, have not yet been derived in their final and definitive form. These missing elements should not be interpreted as conceptual deficiencies. Rather, they correspond to technical extensions required to interface a fundamentally relational and pre-geometric framework with conventional variational and quantum formalisms that presuppose spacetime structure.

Crucially, the absence of a finalized action or quantization scheme does not obstruct the recovery of known physical phenomenology at the level of qualitative structure and effective behavior. Throughout this work, general relativity and quantum field theory emerge as *effective, coarse-grained descriptions* valid within specific dynamical regimes of the projected field χ_{eff}. They are not introduced as independent axioms, but arise as stable limits of the underlying relaxation dynamics.

The present formulation therefore occupies a well-defined intermediate status. It is not intended as a closed or final theory, nor as a phenomenological model tuned to reproduce specific experimental data. Instead, it provides a coherent ontological and dynamical foundation from which both geometric and quantum structures can emerge, while remaining open to future refinements that may enhance its mathematical completeness, formal elegance, and predictive scope.

In this sense, Cosmochrony should be viewed as a foundational framework rather than as a fully developed effective field theory: its primary contribution lies in clarifying *what is fundamental* and *how known physical structures can arise*, rather than in prescribing their final mathematical implementation.

B.7 Soliton and Particle Solutions

Within the Cosmochrony framework, elementary particles are interpreted as stable or metastable localized configurations arising in the *projectable regime* of the scalar field χ. These configurations, hereafter referred to as χ-solitons, emerge from nonlinear self-organization of the relaxation dynamics and persist as localized resistances to global relaxation.

This interpretation does not rely on the postulation of additional fundamental degrees of freedom. The only fundamental entity is the scalar field χ, which is not defined on spacetime. Spatial localization, energy, and particle-like persistence arise only once an effective geometric projection χ_{eff} becomes applicable.

While the fundamental field is scalar, certain solitonic configurations of χ_{eff} possess a nontrivial internal organization that cannot be faithfully encoded by a single real scalar variable. In particular, configurations characterized by internal cyclic structure, nontrivial winding, and 4π-periodicity exhibit transformation properties that require a double-valued representation under effective rotations.

In such cases, an effective spinorial description becomes unavoidable at the level of transformation properties and exchange behavior. This does not imply the existence of fundamental spinor fields. Rather, spinorial variables arise as *collective descriptors* encoding the internal topology and spectral structure of fermionic χ-solitons.

At the phenomenological level, these excitations admit a representation in terms of Dirac spinors. This representation should be understood as an emergent and coarse-grained description of the internal degrees of freedom of χ-solitons, not as an ontological extension of the theory. Within this regime, the Dirac equation appears as the minimal effective dynamical structure compatible with:

- approximate locality in the projected geometric description,
- effective relativistic covariance,
- and the topological constraints associated with 4π-periodic configurations.

in the absence of strong interactions or non-linear collective effects.

From this perspective, the Dirac structure does not introduce new fundamental entities. It provides a compact and universal encoding of the internal topology, spectral stability, and transformation behavior of fermionic solitons. Spin, fermionic statistics, and exclusion behavior arise as effective consequences of the nontrivial configuration space of these scalar-field excitations, rather than as independent postulates.

The existence and stability of χ-solitons impose structural constraints on the effective self-interaction functional governing the projected dynamics. Although the explicit form of this functional remains undetermined, it must satisfy the following minimal requirements:

1. the support of localized configurations with finite effective mass,
2. dynamical stability under small perturbations,
3. and the existence of topologically inequivalent sectors corresponding to distinct classes of particle-like excitations.

The detailed derivation of effective Dirac dynamics from fluctuations around χ-soliton backgrounds, as well as the emergence of a realistic mass spectrum, remains an open mathematical problem. These issues are addressed at a programmatic and illustrative level in Sections B.2–B.4 and further discussed in Appendix B.8.

B.8 Perspectives: Towards a Derivation of the Proton-to-Electron Mass Ratio

The proton-to-electron mass ratio is one of the most precisely measured dimensionless quantities in physics. Within the Cosmochrony framework, the purpose of this section is not to derive this value from first principles, but to clarify how such a ratio could emerge *structurally* from the spectral and topological organization of localized solitonic excitations of the projected field χ_{eff}.

The discussion should therefore be understood as a minimal and exploratory spectral ansatz. Its aim is to identify the relevant mechanisms, constraints, and scaling relations that any successful derivation would have to satisfy, rather than to provide a complete microscopic calculation.

Spectral Stability Hypothesis

Let χ_{sol} denote a stationary localized configuration arising in the projectable regime of the χ dynamics. Small perturbations $\delta\chi_{\text{eff}}$ around this background are governed, at the coarse-grained level, by a linear stability operator $\mathcal{L}_{\text{sol}}$, defined as the second variation of an effective localization functional.

Normal modes satisfy the eigenvalue problem

$$\mathcal{L}_{\text{sol}}\psi_n = \lambda_n\psi_n. \tag{177}$$

The eigenvalues λ_n characterize the resistance of the soliton to localized deformations. They encode intrinsic stiffness scales associated with the internal organization of the solitonic configuration.

Spectral mass scaling.

In regimes where an effective wave description applies, the normal modes exhibit characteristic oscillation frequencies

$$\omega_n = c\sqrt{\lambda_n}. \tag{178}$$

Identifying the lowest nontrivial frequency with the rest energy of the excitation leads to the effective scaling relation

$$m_n \;\propto\; \sqrt{\lambda_n}\,\chi_c, \tag{179}$$

where χ_c denotes a characteristic geometric scale associated with the spatial extension of the solitonic configuration in the projected regime.

This relation does not define a fundamental mass formula. It provides a coarse-grained link between spectral stability and inertial mass, consistent with the interpretation of mass as integrated resistance to relaxation.

Dimensional interpretation.

The eigenvalues λ_n carry dimensions of inverse length squared, reflecting the restoring stiffness of the soliton per unit deformation. The scale χ_c has dimensions of length and sets the geometric extension over which this stiffness is distributed.

The combination $\lambda_n\chi_c^2$ therefore defines a characteristic energy scale,

$$E_n \sim \lambda_n\chi_c^2, \tag{180}$$

which is identified with a rest energy through the effective relativistic matching $E = mc^2$ once a spacetime description becomes applicable.

This identification does not invoke a fundamental quantum constant and remains valid independently of the emergence of $\hbar_{\text{eff}}$.

Projection Scale and Effective Normalization

The fundamental description of the χ field is formulated in terms of relational relaxation rules rather than a spacetime action with fixed physical units. When a continuum approximation applies, an effective action for perturbations around a stable soliton may be introduced as a bookkeeping device.

In this regime, the effective action for perturbations $\delta\chi_{\text{eff}}$ may be written schematically as

$$S_{\text{eff}}[\delta\chi] = \int d^4x\,\frac{1}{2}\left(\frac{\chi_c}{c}\right)^2\left[(\partial_t\delta\chi)^2 - c^2(\nabla\delta\chi)^2\right]. \tag{181}$$

Expressing this action in emergent spacetime coordinates introduces a geometric rescaling factor linking χ-space and spacetime lengths. As a result, the canonical normalization of localized modes involves a quadratic scaling factor of the form

$$\left(\frac{\chi_c}{\ell_{\text{spacetime}}}\right)^2, \tag{182}$$

which controls the effective normalization of spectral quantities.

This factor reflects the geometric projection from the relational χ structure to emergent spacetime observables. It does not represent a fundamental coupling constant.

Energy Levels from Spectral Stability

The discrete energy levels associated with solitonic excitations follow from the spectral properties of the stability operator $\mathcal{L}_{\text{sol}}$, not from canonical quantization.

For a soliton labeled by n, the gradient contribution to the effective energy scales as

$$E_{\text{grad}}^{(n)} \sim c^2 \lambda_n \mathcal{N}_n, \tag{183}$$

where $\mathcal{N}_n$ denotes a normalization factor determined by the spatial profile of the mode.

In the spacetime-based description, this energy is identified with the rest-mass energy,

$$E_n \equiv m_n c^2. \tag{184}$$

The discretization of E_n arises from topological classification and spectral stability, not from postulated quantum operators. The role of $\hbar_{\text{eff}}$ appears only when matching this description to quantum observables.

Elementary versus Composite Spectral Structures

A key distinction must be drawn between elementary and composite solitonic excitations. Elementary particles, such as leptons, are expected to correspond to topologically elementary solitons whose inertial mass is dominated by a single lowest stability eigenvalue.

By contrast, baryonic excitations are composite configurations. Their mass reflects the combined contribution of several coupled stability modes associated with a bound structure. Mass ratios therefore take the schematic form

$$\frac{m_{\text{comp}}}{m_{\text{elem}}} \sim \frac{\sum_k \sqrt{\lambda_k^{(\text{comp})}}}{\sqrt{\lambda_0^{(\text{elem})}}}, \tag{185}$$

rather than the ratio of two isolated eigenvalues.

Ansatz for the Proton as a Composite Soliton

As an exploratory working hypothesis, the proton is modeled as a composite solitonic excitation. Specifically:

- the electron corresponds to a topologically elementary soliton with a fundamental stability eigenvalue λ_e,
- the proton corresponds to a bound configuration involving three such elementary solitons, supplemented by an additional collective binding mode with eigenvalue λ_{bind}.

The choice of a three-soliton composite is motivated by stability considerations observed in a wide class of nonlinear field theories admitting topological solitons, where three-body bound states often exhibit enhanced stability due to geometric phase locking [48]. This choice is not derived here from a classification of χ-soliton sectors and is not postulated as fundamental.

Skyrmion models in QCD provide an instructive analogy, but no dynamical equivalence is assumed. The relevance of this analogy lies in the universality of topological stabilization mechanisms, which do not depend on the presence of a non-Abelian gauge symmetry [49].

Mass Ratio from Spectral Scaling

Under these assumptions, the effective eigenvalue associated with the proton may be written schematically as

$$\lambda_p \approx \lambda_{\text{bind}} + 3\lambda_e, \tag{186}$$

leading to the mass ratio

$$\frac{m_p}{m_e} \approx \sqrt{\frac{\lambda_{\text{bind}} + 3\lambda_e}{\lambda_e}}. \tag{187}$$

In the binding-dominated regime $\lambda_{\text{bind}} \gg \lambda_e$, this reduces to

$$\frac{m_p}{m_e} \approx \sqrt{\frac{\lambda_{\text{bind}}}{\lambda_e}}. \tag{188}$$

Matching the observed ratio $m_p/m_e \simeq 1836$ therefore imposes the spectral constraint

$$\frac{\lambda_{\text{bind}}}{\lambda_e} \sim 3.4 \times 10^6. \tag{189}$$

This relation is not derived here. *The role of the empirical value is solely to delimit the scale of spectral separation required; the framework makes no claim that this separation is unique, minimal, or numerically rigid.* It is identified as a consistency condition constraining the relative spectral organization of elementary and composite solitonic sectors.

We interpret this large spectral hierarchy as defining a dimensionless *spectral packing fraction* α, characterizing the relative density of admissible stability modes in composite versus elementary solitonic sectors. Specifically, we define

$$\alpha \equiv \frac{\lambda_e}{\lambda_{\text{bind}}} \sim 3 \times 10^{-7}. \tag{190}$$

This quantity does not represent a coupling constant, but a structural measure of spectral compression induced by topological binding.

Topological Interpretation of the Spectral Hierarchy

Although the ratio $\lambda_{\text{bind}}/\lambda_e$ is introduced here as a spectral consistency condition, it is natural to seek a geometric or topological interpretation of this large hierarchy.

In particular, the composite nature of the proton ($Q = 3$) suggests that the associated binding modes may correspond to configurations of increased topological complexity. If the stability spectrum of L_{sol} is controlled by the effective multiplicity of internal configurations admitted under the non-injective projection Π, then λ_{bind} may be interpreted as a coarse-grained measure of the volume of the corresponding projection fiber.

From this perspective, the large ratio $\lambda_{\text{bind}}/\lambda_e \sim 10^6$ reflects not an arbitrary energy scale separation, but the rapid growth of internal configuration space associated with topologically composite solitons.

Indicative Geometric Scale

Although no explicit geometric or topological model is developed at this stage, it is useful to translate the observed spectral hierarchy into a characteristic dimensionless scale.

At a purely heuristic level, one may assume that the effective spectral weight of a composite soliton grows quadratically with a characteristic internal scale χ_c, so that

$$\frac{\lambda_{\text{bind}}}{\lambda_e} \sim \chi_c^2. \tag{191}$$

Under this assumption, the empirical constraint $\lambda_{\text{bind}}/\lambda_e \sim 3.4 \times 10^6$ corresponds to a scale of order

$$\chi_c \sim \mathcal{O}(10), \tag{192}$$

with a representative numerical value

$$\chi_c \approx 8.3. \tag{193}$$

Both expressions should be regarded as indicative rather than derived. They simply emphasize that the required spectral hierarchy corresponds to a modest geometric amplification, not to an extreme or finely tuned parameter choice. *No physical meaning should be attached to this numerical value in the absence of an explicit topological model.*

An Explicit Working Ansatz for $V(\chi)$

In the present appendix, the primary driver of mass hierarchies is the spectral organization of the solitonic stability operator. Nevertheless, turning this program into a falsifiable computational scheme requires an explicit *working* form for the effective potential $V(\chi)$, not as a fundamental source of masses, but as a controlled perturbation that: (i) stabilizes localized sectors, (ii) selects admissible core amplitudes, and (iii) produces fine splittings within a given topological class.

A minimal two-scale ansatz compatible with these roles is a *multi-well* (or weakly periodic) potential with a characteristic amplitude scale η and stiffness scale λ:

$$V(\chi) \;=\; \frac{\lambda}{4}\left(\chi^2-\eta^2\right)^2 \;+\; \varepsilon\,\eta^4\left[1-\cos\left(\frac{\chi}{\eta}\right)\right], \tag{194}$$

where $\varepsilon \ll 1$ is a dimensionless modulation parameter. The first term provides a robust double-well localization mechanism; the second term introduces a gentle quasi-periodic micro-structure capable of generating controlled intra-sector splittings without reparameterizing the global mass scale.

The interpretation in Cosmochrony is strictly *effective*: the coefficients in Eq. (194) are not fundamental constants, but phenomenological descriptors of how coarse-grained projectability constraints reshape the admissible configurations of χ_{eff}.

Linking (λ, η) to Observables Without Making Mass Fundamental

The parameters η and λ are introduced only to control the *shape* and *stiffness* of admissible localized sectors in the projected description. Their observable imprint is therefore indirect: they enter through how they shift the stability spectrum $\{\lambda_n\}$ of $\mathcal{L}_{\text{sol}}$, and how robustly a given topological sector remains projectable under perturbations.

Dimensionless control combinations.

For a localized profile with characteristic extension χ_c, the potential introduces two natural dimensionless combinations,

$$g \;\equiv\; \lambda\,\chi_c^2\,\eta^2, \qquad u \;\equiv\; \varepsilon\,, \tag{195}$$

which govern (i) the curvature scale of $V(\chi)$ near admissible minima, and (ii) the magnitude of sub-structure corrections. The spectral hierarchy derived above is then phrased as the statement that the *ratio* m_p/m_e is predominantly controlled by topological/composite spectral packing, while (g,u) control the *stability* and *splittings* of the low-lying spectrum.

Matching strategy using the proton-to-electron ratio.

Denote by $\lambda_e(g,u)$ the fundamental stability eigenvalue of the $Q=1$ sector, and by $\lambda_{\text{bind}}(Q=3;g,u)$ the characteristic binding-band scale of the composite sector. The empirical constraint $\lambda_{\text{bind}}/\lambda_e \sim 3.4\times10^6$ derived in Eq. (the spectral constraint above) is then reinterpreted as a *feasibility condition*:

$$\exists\,(g,u)\ \text{s.t.}\quad \frac{\lambda_{\text{bind}}(Q=3;g,u)}{\lambda_e(g,u)} \;\approx\; 3.4\times10^6, \tag{196}$$

while remaining stable under small variations of (g,u). In other words, (λ,η) are not tuned to *set* the mass ratio, but to ensure that the *topological spectral mechanism* can realize the required hierarchy in a broad basin of effective parameters.

The topological charge Q should not be interpreted as a linear mass multiplier. Its role is to constrain the admissible spectral organization of the soliton, from which the observed mass hierarchy emerges non-additively.

Secondary observables: fine splittings as diagnostics.

Once a viable region in (g, u) exists, the same potential ansatz predicts that small intra-sector differences (e.g. neutron–proton splitting, excited baryonic resonances, or generational splittings) arise from:

- perturbative eigenvalue shifts $\delta\lambda_n(g, u)$ induced by local curvature variations of V,
- weak breaking of idealized symmetries in composite sectors,
- environment-dependent dressing of the effective coefficients through projectability constraints.

These effects are conceptually aligned with the claim that $V(\chi)$ controls fine structure rather than the global mass scale.

Numerical Program: From $V(\chi)$ to Spectral Hierarchies

The numerical goal is not to simulate QCD, but to test a *structural* claim: whether bounded relaxation dynamics plus a controlled effective potential admits stable localized sectors whose stability spectra exhibit (i) a robust elementary mode λ_e, and (ii) a dense binding band λ_{bind} in a composite $Q = 3$ sector, separated by a large gap.

A minimal computational pipeline is:

1. **Dynamics and formation.** Implement the bounded relaxation update rule for χ in a discretized representation (spectral/finite-element basis or lattice proxy), including the effective potential term Eq. (194) as a controlled perturbation.
2. **Soliton harvesting.** Identify long-lived localized configurations and classify them by topological diagnostics (winding/charge proxies, knot-like invariants when available, or stability under deprojection/reprojection cycles).
3. **Stability operator extraction.** For each harvested configuration, compute the linearized stability operator $\mathcal{L}_{\text{sol}}$ (second variation of the effective localization functional) and extract its low-lying eigen-spectrum.
4. **Spectral ratio test.** Evaluate whether the emergent spectra support a regime where $\lambda_{\text{bind}}/\lambda_e \sim 10^6$ arises *without fine tuning*, and whether the ratio remains stable under moderate variation of (g, u).
5. **Fine-structure diagnostics.** Measure the sensitivity of subleading splittings $\delta\lambda_n$ to (g, u), providing a concrete handle for how $V(\chi)$ affects intra-sector structure while leaving the leading hierarchy topologically controlled.

This numerical program connects directly to the broader simulation framework described in the technical appendix on simulation algorithms and spectral extraction, and provides a clear set of falsifiable diagnostics: either large, robust spectral gaps appear generically in composite sectors, or the proposed topological-spectral mechanism fails to reproduce the required hierarchy.

Transition

The role of $V(\chi)$ is therefore operational: it stabilizes and perturbs admissible projected sectors while the *origin* of the mass hierarchy remains spectral and topological. This separation is developed further in the subsequent appendices on spectral ontology and on the secondary role of $V(\chi)$.

B.9 Spectral Scaling and the Projection Ontology

The preceding derivation of the mass ratio m_p/m_e rests on a fundamental shift in the ontology of mass. Within the Cosmochrony framework, inertial mass is no longer treated as an intrinsic "charge", but as a spectral signature of projection visibility.

Mass as Spectral Weight

The non-injective nature of the projection Π (see Section 4.5) implies that any effective particle in χ_{eff} corresponds to a large equivalence class of micro-configurations in the substrate χ. The stability eigenvalues λ_n of the operator L_{sol} can therefore be reinterpreted as a coarse-grained measure of this structural multiplicity, or *fiber weight*. A configuration that requires a larger set of internal modes to remain stable and projectable manifests a higher resistance to global relaxation, and thus a higher inertial mass. This "fiber weight" interpretation is diagnostic: it does not claim a one-to-one equality between eigenvalue magnitude and microscopic degeneracy, but asserts that both track the same coarse-grained constraint—projectability under Π and resistance to relaxation.

Invariance of the Ratio

Since the ratio

$$\frac{m_p}{m_e} \approx \sqrt{\frac{\lambda_p}{\lambda_e}} \tag{197}$$

is independent of the absolute action scale $\hbar_\chi$, it is identified as a structurally protected invariant of the projection process itself. This is consistent with the observed universality of the proton-to-electron mass ratio to the extent that it is observationally constrained to be stable, regardless of the global relaxation state of χ.

The Spectral Packing Fraction (α)

The hierarchy between the composite sector (proton) and the elementary sector (electron) is encapsulated by the spectral packing fraction

$$\alpha \equiv \frac{\lambda_e}{\lambda_{\text{bind}}} \approx 3 \times 10^{-7}. \tag{198}$$

While introduced here via an empirical target, α represents the ratio of spectral transmittance under Π. The proton is heavy because its non-trivial topology ($Q = 3$) constrains the stability operator L_{sol} to exhibit a large internal spectral bandwidth. This topological constraint, often heuristically represented (as one possible representative) by a trefoil-like knot configuration, leads to a strong spectral gap between the binding modes λ_{bind} and the fundamental electronic mode λ_e.

Conclusion

While the precise numerical value $m_p/m_e \simeq 1836$ awaits a closed-form derivation from the spectral geometry of $L_{\rm sol}$ on topologically constrained effective manifolds / effective projected geometries, the Cosmochrony framework reformulates the mass-ratio problem in structural terms. The proton-to-electron mass ratio emerges as the macroscopic signature of a spectral gap dictated by the complexity of the substrate's excitations under projection, as schematically illustrated in Fig. 16, rather than as an arbitrary fundamental constant.

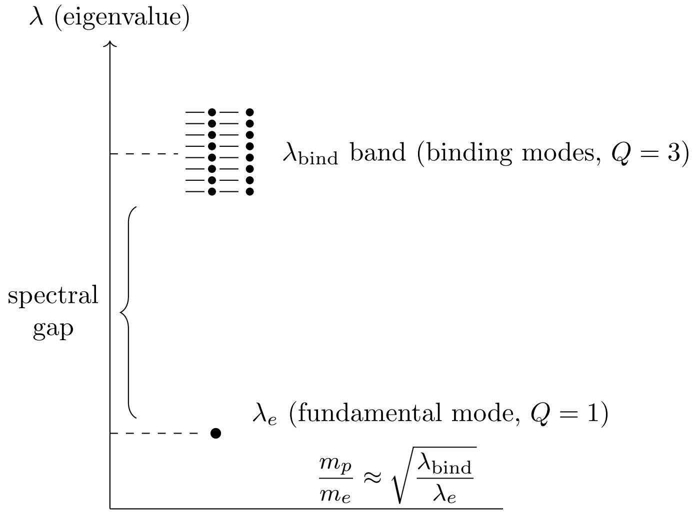

Fig. 16 Conceptual schematic of a spectral gap in the stability spectrum of $\mathcal{L}_{\rm sol}$. The elementary mode λ_e is separated from a dense band of binding modes near $\lambda_{\rm bind}$, illustrating the interpretation of m_p/m_e as a macroscopic signature of spectral organization rather than an intrinsic mass parameter.

Summary and Outlook

Within the Cosmochrony framework, the proton-to-electron mass ratio is interpreted as an emergent constraint on the spectral and topological organization of solitonic excitations, not as a fundamental input parameter.

The analysis presented here provides a coherent toy model identifying the conditions under which such a ratio could arise. Whether the required spectral hierarchy can be generated dynamically from the χ relaxation dynamics remains an open problem, to be addressed through future analytical and numerical investigations.

B.10 Spectral Characterization of Mass and the Secondary Role of $V(\chi)$

This appendix clarifies the conceptual status of inertial mass in the Cosmochrony framework. Physically, mass originates from the resistance of localized configurations to the relaxation of the fundamental χ field. This resistance, however, admits a quantitative and structurally organized description in terms of the spectral properties of an associated stability operator defined in the projectable regime.

Spectral analysis therefore does not redefine the physical origin of inertial mass. Rather, it provides a coherent and potentially calculable characterization of how resistance to relaxation is distributed among stable and metastable configurations.

A central conjecture of Cosmochrony is that particle masses are not fundamental parameters encoded in the nonlinear potential $V(\chi)$. Instead, they emerge as spectral properties of a background-independent relaxation operator defined on a relational substrate, which may be represented, for calculational purposes, by a discrete graph structure.

The graph representation is not ontological but serves as a computational proxy for relational adjacency in the projectable regime.

Mass spectrum as eigenmodes of a relaxation operator.

Localized particle-like excitations are identified with normal modes of an effective relaxation operator $\Delta_G^{(0)}$, which may be represented as a Laplace–Beltrami operator acting on a graph $G(V, E)$,

$$\Delta_G^{(0)} \psi_n = -\lambda_n \psi_n. \tag{199}$$

The eigenmodes ψ_n characterize the stability of localized solitonic configurations, while the eigenvalues λ_n encode their intrinsic resistance to deformation.

In regimes where an effective spacetime description applies, the inertial masses associated with these modes scale as

$$m_n c^2 \;\propto\; \sqrt{\lambda_n}\, \chi_c, \tag{200}$$

in agreement with the spectral relations introduced in Section B.8. This scaling reflects the fact that inertial mass measures the characteristic frequency associated with the resistance of a localized configuration to χ-field relaxation.

The situation is analogous to bounded elastic systems, where discrete vibrational frequencies arise from geometry and connectivity rather than from adjustable material parameters. Within Cosmochrony, mass hierarchies are therefore interpreted as geometric and topological properties of the underlying relational structure.

A decisive test of this conjecture would consist in computing the low-lying spectrum of $\Delta_G^{(0)}$ on large but finite networks with physically motivated connectivity rules. Even approximate agreement with observed mass ratios would strongly support the spectral origin of inertial mass and the non-fundamental role of $V(\chi)$.

Separation of descriptive levels.

To avoid circular dependencies between geometry, dynamics, and emergent particle properties, Cosmochrony distinguishes three conceptual levels.

At the fundamental level, inertial masses are associated with the spectral properties of a background-independent relaxation operator $\Delta_G^{(0)}$, defined solely by the intrinsic relational connectivity of the substrate. This operator is not tied to any spacetime geometry or instantaneous χ configuration and provides a stable spectral backbone.

At the emergent geometric level, coarse-grained configurations of χ_{eff} give rise to effective notions of spacetime, including curvature, gravitational time dilation, and cosmological expansion. These geometric effects influence propagation and interaction, but do not redefine the underlying spectral operator responsible for mass generation.

Finally, fast dynamical processes such as radiation, scattering, and decoherence correspond to interaction-induced redistributions of relaxation potential within the χ field. These processes affect observables without modifying the fundamental spectral structure.

Residual role of the potential $V(\chi)$.

Within this spectral picture, the nonlinear potential $V(\chi)$ plays a secondary and effective role. It does not set the overall mass scale. Instead, it provides a local coarse-grained description of nonlinear stabilization mechanisms associated with low-lying spectral modes.

The admissible form of $V(\chi)$ is constrained by the requirement that it support stable solitonic configurations compatible with the pre-existing spectral structure. It encodes neither an independent interaction nor a fundamental energy density.

Origin of the effective potential $V(\chi)$.

Characterizing $V(\chi)$ as secondary does not imply arbitrariness. Rather, $V(\chi)$ should be understood as an effective descriptor of the local nonlinear response of the relaxation dynamics in the vicinity of a stable configuration.

At the fundamental level, the dynamics of χ are governed by bounded relaxation rules and their associated spectral structure. When this dynamics is projected onto a reduced functional subspace associated with a localized soliton, nonlinear self-consistency constraints induce an effective local restoring structure. In this reduced description, these constraints may be summarized by an effective potential $V(\chi)$.

Different admissible forms of $V(\chi)$ correspond to different coarse-graining choices, while leaving invariant the underlying spectral origin of mass and stability.

Potential-induced corrections to stability eigenvalues.

To illustrate how $V(\chi)$ can modify stability eigenvalues without altering their spectral origin, consider the illustrative form

$$V(\chi) = \lambda \left(\chi^2 - \chi_c^2\right)^2 . \tag{201}$$

Expanding around the relaxed background $\chi = \chi_c$ yields a quadratic contribution for small fluctuations $\delta\chi$,

$$V(\chi_c + \delta\chi) \simeq \frac{1}{2} \left.\frac{d^2V}{d\chi^2}\right|_{\chi_c} (\delta\chi)^2 + \cdots , \tag{202}$$

with

$$\left.\frac{d^2V}{d\chi^2}\right|_{\chi_c} \propto \lambda\, \chi_c^2. \tag{203}$$

This term contributes additively to the linearized stability operator, shifting the eigenvalues as

$$\lambda_n \longrightarrow \lambda_n^{(0)} + \Delta\lambda_n^{(V)}. \tag{204}$$

For composite solitons, such corrections may differ slightly between closely related configurations (e.g., neutron versus proton), generating small mass splittings. By contrast, ratios dominated by topological organization (such as m_p/m_e) remain largely insensitive to the detailed form of $V(\chi)$.

Summary.

In Cosmochrony, inertial mass is fundamentally a spectral property of the relaxation dynamics of the χ field. The potential $V(\chi)$ serves as a derived, effective descriptor controlling fine structure, not as a primary source of mass. Extending this spectral characterization toward quantitative mass predictions requires specifying the relaxation operator and its boundary conditions, particularly for composite solitonic sectors.

B.11 Spectral Stability and the Emergence of $\hbar_{\text{eff}}$

In Cosmochrony, the effective Planck constant $\hbar_{\text{eff}}$ is not introduced as a fundamental quantum postulate. Instead, it emerges as a scaling parameter linking spectral stability of χ-field solitons to effective spacetime observables.

Fundamental scales of the χ dynamics

The χ field is characterized by three independent dynamical scales:

- K_0: maximal relaxation stiffness, with dimensions $[L^{-2}]$,
- χ_c: correlation length at which solitonic configurations stabilize,
- c: maximal relaxation speed.

From these, one may define a natural unit of action associated with the relaxation dynamics,

$$\hbar_\chi \equiv \frac{c^3}{K_0 \chi_c}, \tag{205}$$

which has the dimensions of action and is independent of the standard Planck constant. This quantity is introduced on dimensional grounds as the unique action scale constructible from the fundamental relaxation parameters.
Here and in the following, K_0 and χ_c denote the *bare substrate parameters*, i.e. universal invariants characterizing the rigidity and correlation capacity of the χ field. The scale-dependent values discussed in **??** arise only after coarse-graining and do not enter the definition of $\hbar_\chi$.

Spectral origin of effective quantization

Quantization in Cosmochrony follows from the discrete spectrum of the stability operator $\Delta_G^{(0)}$. For a solitonic excitation with eigenvalue λ_n, the characteristic frequency of small oscillations scales as

$$\nu_n \sim \frac{c}{\chi_c} \sqrt{\lambda_n}\, \mathcal{N}_n^{1/2}. \tag{206}$$

At the effective spacetime level, identifying the rest energy with the product of this frequency and an effective action scale yields

$$E_n = \hbar_{\text{eff}}\, \nu_n, \tag{207}$$

from which $\hbar_{\text{eff}}$ emerges as a geometric and spectral quantity, not as an independent constant.

Regime-dependent scaling

The effective value of $\hbar_{\text{eff}}$ depends on the scale at which the system is probed. In regimes where the characteristic spacetime scale $\ell_{\text{spacetime}}$ is comparable to χ_c,

$$\hbar_{\text{eff}} \approx \hbar_\chi, \tag{208}$$

recovering standard quantum behavior.

At macroscopic scales $\ell_{\text{spacetime}} \gg \chi_c$,

$$\hbar_{\text{eff}} \approx \hbar_\chi \left(\frac{\chi_c}{\ell_{\text{spacetime}}} \right)^2, \tag{209}$$

leading to a strong suppression of quantum effects and the emergence of classical behavior. This suppression reflects reduced spectral accessibility rather than decoherence or wavefunction collapse.

Consistency with quantum phenomenology

In the microscopic regime, where $\hbar_{\text{eff}} \approx \hbar$, standard quantization relations $E = \hbar\nu$ are recovered as effective descriptions. This agreement is not postulated but follows from the scaling behavior of $\hbar_{\text{eff}}$ once the projected regime matches laboratory scales.

Numerical constraints.

Reproducing particle-scale quantum behavior requires

$$K_0 \chi_c^2 \sim \hbar, \tag{210}$$

which constrains the admissible values of the relaxation stiffness and correlation length. These constraints are consistent with soliton stability and do not require fine tuning. This relation should be read as an order-of-magnitude consistency condition, not as an equality fixing independent parameters.

B.12 Renormalization of Substrate Parameters

To maintain consistency between the fundamental definition of $\hbar_\chi$ and the scale-dependent observations in Appendix D, we distinguish between:

- **Bare Parameters (K_0, χ_c):** Universal invariants of the χ substrate that determine the fundamental quantum of action $\hbar_\chi$.
- **Effective Parameters ($K_{\text{eff}}, \chi_{\text{eff}}$):** Environment-dependent values emerging from the coarse-graining of relaxation constraints, as detailed in Section **??**.

The universality of $\hbar$ and the spectral invariant α_{spec} (formerly α in Section B.9) stems from their dependence on the ratio of these bare quantities, which remains invariant under projective scaling *within a given relaxation epoch.*
In particular, dimensionless coupling constants such as the electromagnetic fine-structure constant α_{EM} do not inherit any arbitrariness from the substrate parameters (this statement concerns structural invariance, not a first-principles derivation of their numerical values). Within the Cosmochrony framework, the electric charge e is not treated as a free gauge parameter, but as a property of localized solitonic configurations. The associated transmittance is not an adjustable quantity but a geometric invariant of the soliton's spectral embedding relative to the projection fiber Π. As a result, the dependence on the substrate rigidity K_0 cancels out in dimensionless ratios, ensuring their invariance within a fixed relaxation epoch up to higher-order projection corrections.

Summary.
Within Cosmochrony, both inertial mass and effective quantization emerge from the same spectral stability structure of the χ relaxation dynamics. The Planck constant appears not as a fundamental input, but as a scale-dependent effective parameter encoding the projection from relational dynamics to spacetime-based observables.

B.13 Structural Origin of Quantum Correlations and Non-Locality

This section provides a conceptual extension of the Cosmochrony framework, illustrating how quantum correlations and spin may be interpreted geometrically within a strictly monistic and non-injective projection ontology. The discussion is interpretative in nature and does not introduce additional dynamical postulates.

The non-injective nature of the projection operator Π, which maps the relational substrate χ onto the effective 4D manifold, provides a structural reinterpretation of quantum non-locality and entanglement. In this framework, EPR-type correlations are not viewed as the result of superluminal signaling, but as a direct consequence of the **shared ontological source** of projected observables. This shared source is relational rather than state-deterministic, and does not imply the existence of hidden spacetime-local variables.

Non-injectivity and Structural Identity.

Within Cosmochrony, what is effectively perceived as two spatially separated particles may correspond to a single, unified relational configuration in χ.

- **Geometric Separation vs. Relational Unity:** While the emergent metric $g_{\mu\nu}$ assigns a large spatial distance between two detectors, the underlying χ -excitation remains a single connected entity in the pre-geometric substrate.
- **The Shared Projection:** Entanglement is thus defined as the manifestation of a single χ -source through multiple, non-injective projective "images". The perceived "spooky action at a distance" is an artifact of the metric description, which fails to capture the underlying relational unity.

Torsional Conservation and the Origin of Spin Correlations.

This hypothesis extends naturally to the geometric origin of spin. If spin is interpreted as the projection of the internal degrees of freedom of the relational fiber (e.g., within the Hopf fibration $S^3 \to S^2$ [4]), then:

- A measurement at location A corresponds to a local stabilization of the projection's torsional phase.
- Because the underlying configuration in χ is a unified structure, this local stabilization *structurally constrains* the admissible projective states available at any other location B originating from the same source.
- This mechanism ensures the conservation of global topological invariants across the shared projection without violating the causal bounds of the relaxation dynamics.

Relationship with Bell's Theorem.

Cosmochrony addresses the constraints of Bell's theorem by shifting the locus of reality. The framework remains **ontologically realist** at the level of the relational substrate, as the substrate χ possesses definite relational states, but it is **structurally non-local** with respect to the emergent spacetime. The violation of Bell inequalities is not seen as a failure of realism, but as a signature of **metric emergence**: the metric distance, used to define "locality" in the theorem, is not a fundamental property of the level where the correlations are established. Bell inequality violations are therefore attributed to the use of emergent metric locality in a context where the relevant degrees of freedom are defined prior to spacetime separation.

B.14 Metastability, Decay Channels, and Exponential Lifetimes

Diagnostic Structural Functional

We introduce a diagnostic functional $E_{\text{struct}}[\chi_{\text{eff}}]$, which quantifies the degree of structural constraint associated with a localized projected configuration. This functional should be understood as an effective measure of resistance to relaxation, consistent with the interpretation of mass developed in Section 6.3.

The explicit form of E_{struct} is not unique. For stability analysis, it may be constructed from quadratic variations of χ_{eff}, for instance

$$E_{\text{struct}}[\chi_{\text{eff}}] \sim \int_{\mathcal{V}} \left(|\nabla\chi_{\text{eff}}|^2 + \mu^2|\chi_{\text{eff}}|^2\right)\, d^3x, \tag{211}$$

[4]This reference is illustrative and does not imply that the substrate χ possesses a literal S^3 topology.

where $\mathcal{V}$ denotes the effective localization region.

Admissible Factorization Channels

A decay channel is defined as an admissible factorization of a localized projected configuration into several localized configurations plus weakly structured modes,

$$\chi_{\mathrm{eff},A} \;\to\; \bigoplus_{i=1}^{N} \chi_{\mathrm{eff},i} \;\oplus\; \chi_{\mathrm{eff,rad}}. \tag{212}$$

Admissibility requires the preservation of global structural invariants,

$$Q(\chi_{\mathrm{eff},A}) = \sum_{i=1}^{N} Q(\chi_{\mathrm{eff},i}), \tag{213}$$

where Q denotes any topological or relational invariant associated with the projected description.

A channel is kinematically accessible if the total diagnostic structural functional satisfies

$$\Delta E_{\mathrm{struct}} = E_{\mathrm{struct}}[\chi_{\mathrm{eff},A}] - \sum_{i} E_{\mathrm{struct}}[\chi_{\mathrm{eff},i}] - E_{\mathrm{struct}}[\chi_{\mathrm{eff,rad}}] > 0. \tag{214}$$

Exponential Lifetimes

Projected configurations explore nearby admissible micro-rearrangements due to intrinsic projective variability. Let Γ denote the effective rate at which such fluctuations reach an admissible factorization threshold.

If Γ is approximately constant over the relevant range of the effective ordering parameter τ, the survival probability satisfies

$$P(\tau) = \exp(-\Gamma\tau). \tag{215}$$

The decay width Γ decomposes into partial widths associated with distinct admissible channels,

$$\Gamma = \sum_{c} \Gamma_c. \tag{216}$$

This statistical description reproduces the phenomenology of quantum decay without postulating fundamental randomness or microscopic time evolution.

Structural Interpretation of Interaction Classes

Within this framework, different decay classes correspond to different degrees of constraint on admissible factorization paths. Strong decays involve direct and local reorganization of projected topology. Electromagnetic decays correspond to rearrangements preserving the core topological structure. Weak decays require deeper internal reconfiguration and therefore proceed through rarer admissible paths, resulting in longer lifetimes.

Non-Injective Projection and Structural Factorization

Let Π denote the projection operator from the χ-substrate to effective observable descriptions. This projection is generically non-injective: distinct relational configurations of χ may correspond to identical or indistinguishable effective observables, and conversely a single χ-configuration may give rise to multiple correlated effective observables.

Quantum entanglement corresponds to the case in which a single underlying χ-configuration χ_0 admits a non-factorizable projected description $\Pi(\chi_0)$. Although effective observables may be associated with spatially separated regions, the projected configuration cannot be written as a product of independent sub-configurations without violating admissibility.

Particle decay corresponds to a different regime of the same projection structure. In this case, the projected configuration $\Pi(\chi_0)$ becomes unstable under admissible fluctuations. No single projected description remains admissible. Admissibility is recovered only through factorization,

$$\Pi(\chi_0) \;\longrightarrow\; \Pi(\chi_1) \oplus \Pi(\chi_2) \oplus \cdots\,, \tag{217}$$

where the χ_i are distinct relational configurations whose projections are individually admissible and localized.

The distinction between entanglement and decay is therefore not a distinction at the level of the χ-substrate, but a distinction in the stability properties of the projected description under non-injective projection.

B.15 Measurement, Temporal Ordering, and Antiparticle Emergence

Projective Selection and Measurement

Let Π denote the non-injective projection from the χ-substrate to effective observable descriptions. A given relational configuration χ_0 may admit multiple projected descriptions $\{\Pi_\alpha(\chi_0)\}$.

Measurement corresponds to the stabilization of a single projected description $\Pi_{\alpha^*}(\chi_0)$ under interaction with an environment. Alternative projected descriptions become inadmissible due to amplification-induced constraints.

Temporal Ordering from Admissibility

Let $\mathcal{A}$ denote the set of admissible projected configurations. Define a partial ordering $\prec$ such that

$$\Pi_a \prec \Pi_b$$

if Π_b can be obtained from Π_a by admissible relaxation or factorization, but not conversely.

Decay and measurement processes correspond to transitions toward configurations that are minimal with respect to $\prec$. This ordering induces an effective arrow of time without introducing a fundamental temporal parameter.

Structural Origin of Antiparticles

Assume that admissible projected configurations carry signed structural invariants $Q \in \mathbb{Z}$ or $Q \in \mathbb{Z}_2$, associated with orientation, chirality, or phase winding.

For a metastable configuration χ_A undergoing admissible factorization, invariance requires

$$Q(\chi_A) = \sum_i Q(\chi_i).$$

If $Q(\chi_A) = 0$ but factorization produces nonzero local contributions, admissibility requires the appearance of paired configurations with opposite signs,

$$Q(\chi_i) = +q, \quad Q(\chi_j) = -q.$$

These paired configurations are interpreted, at the effective level, as particle–antiparticle pairs. Their emergence reflects the necessity of preserving signed structural invariants under non-injective projection and factorization, rather than the creation of independent degrees of freedom.

B.16 Structural Interpretation of CPT Symmetry

Let Π denote the non-injective projection from the χ-substrate to effective descriptions. Projected configurations may carry signed invariants Q associated with relational orientation, chirality, or phase winding. These invariants are not defined at the fundamental level but emerge through projection.

Consider the combined transformation

$$(Q,\ \tau,\ \mathbf{x}) \longrightarrow (-Q,\ -\tau,\ -\mathbf{x}),$$

where τ denotes the effective ordering parameter and $\mathbf{x}$ the effective spatial localization. This transformation leaves the admissibility conditions invariant.

Under admissible factorization of a metastable configuration χ_A, conservation of signed invariants requires

$$Q(\chi_A) = \sum_i Q(\chi_i).$$

If local configurations carry nonzero signed contributions while the global invariant vanishes, admissibility enforces the appearance of pairs with opposite signs.

These paired configurations are interpreted as particle–antiparticle pairs. CPT symmetry thus emerges as an invariance of the admissible projection structure, rather than as a fundamental symmetry imposed on the χ-substrate.

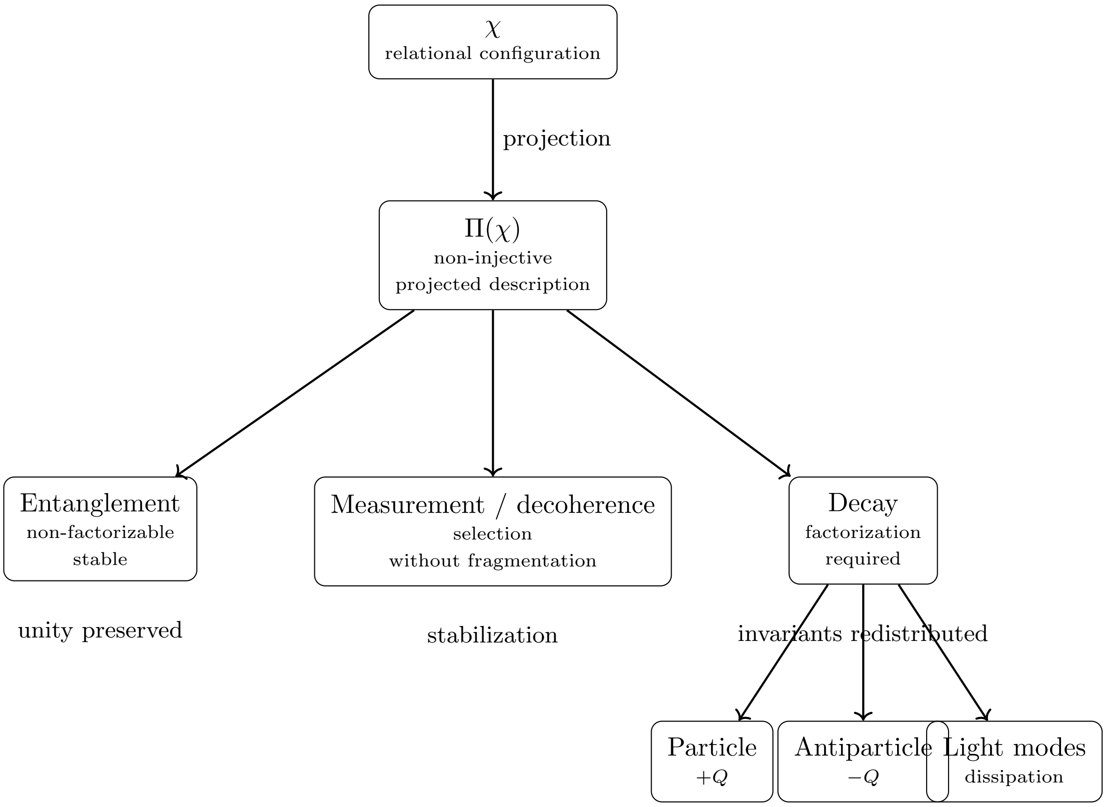

Fig. 17 Unified structural interpretation of quantum entanglement, measurement, and particle decay in Cosmochrony. All phenomena originate from the non-injective projection of a single relational configuration of χ. Antiparticles emerge when admissible factorization requires the redistribution of signed structural invariants.

B.17 CP Asymmetry and Chiral Selection

CPT versus CP as Admissibility Symmetries

Let projected configurations carry a set of signed structural invariants $\{Q_i\}$, associated with orientation, chirality, or phase winding. The admissibility conditions are invariant under the combined transformation

$$(Q_i,\ \tau,\ \mathbf{x}) \to (-Q_i,\ -\tau,\ -\mathbf{x}),$$

which defines an effective CPT symmetry.

In contrast, CP acts only on a subset of the invariants $\{Q_i\}$ and does not reverse the effective ordering parameter. As a result, CP is not, in general, an invariance of the admissibility conditions. Effective CP violation may therefore arise without violating CPT invariance.

Structural Bias and Matter–Antimatter Asymmetry

Assume that admissible projected configurations exhibit a slight asymmetry in relaxation efficiency with respect to the sign of a structural invariant Q. Let $\Gamma(Q)$ denote the effective stabilization rate.

If

$$\Gamma(Q) \neq \Gamma(-Q),$$

then configurations carrying one orientation will be statistically favored during relaxation, leading to an emergent matter–antimatter asymmetry without requiring explicit symmetry breaking at the fundamental level.

Chiral Filtering and Neutrino-Like Excitations

Consider weakly localized projected configurations carrying a chiral invariant $\chi_L = \pm 1$. Admissibility constraints may select only one sign of χ_L as compatible with stable relaxation.

Configurations of opposite chirality either fail to localize or decay rapidly. Neutrino-like excitations correspond to such minimally constrained configurations, which remain admissible only in one chiral sector and interact weakly with more structured excitations.

C Cosmological and Observational Implications of Cosmochrony

This appendix examines the cosmological and observational implications of the Cosmochrony framework. Its purpose is not to construct a fully parameterized cosmological model, nor to perform precision fits to existing datasets, but to establish conceptual and phenomenological consistency with key observations and to identify distinctive signatures that may allow the framework to be tested or falsified.

All results presented here should be understood as consequences of the same underlying scalar relaxation dynamics governing the χ field. No additional cosmological degrees of freedom are introduced beyond those already discussed in the main body of the work. Standard cosmological observables arise as effective descriptions once a smooth geometric projection of χ becomes applicable.

The appendix is organized as follows:

- Section C.1 analyzes the spectrum of large-scale fluctuations of the projected field χ_{eff} and their imprint on the cosmic microwave background, with particular emphasis on the suppression of low-ℓ multipoles.
- Section C.2 shows how the horizon and flatness problems are resolved dynamically in Cosmochrony, without invoking an inflationary phase or fine-tuned initial conditions.
- Section C.3 discusses the evolution of the effective Hubble parameter $H(z)$ and its implications for the observed Hubble tension, highlighting how deviations from standard expansion histories may arise from relaxation dynamics.
- Section C.4 provides order-of-magnitude estimates of the characteristic χ-field parameters and relates them to observable cosmological quantities, without assuming a specific cosmological parameter set.
- Section C.5 explores broader phenomenological consequences, including modified gravitational-wave propagation, MOND-like effective dynamics at galactic scales, and lensing anomalies.

Throughout this appendix, the emphasis is placed on identifying robust qualitative and semi-quantitative features that follow generically from the Cosmochrony framework. Where numerical estimates are provided, they are intended as consistency checks and scaling guides rather than as precision predictions.

Taken together, these results demonstrate that Cosmochrony is compatible with the broad structure of contemporary cosmological observations while predicting systematic deviations from standard ΛCDM expectations. These deviations define concrete targets for future observational tests and numerical investigations.

C.1 Low-ℓ CMB Power Suppression from Global χ Relaxation

This appendix provides a detailed phenomenological treatment of the low-ℓ CMB power suppression discussed in Sections 12.6 and 12.11.

One of the most persistent large-scale anomalies of the cosmic microwave background (CMB) concerns the suppression of temperature anisotropy power at the largest angular scales ($\ell \lesssim 30$), most notably in the quadrupole and octupole moments. Within the standard ΛCDM framework, such deviations are commonly attributed to cosmic variance, and no specific physical mechanism is associated with their occurrence.

Within the Cosmochrony framework, by contrast, the lowest multipoles probe global properties of the projected field χ_{eff} rather than independent local perturbations. Because the fundamental field χ evolves through a monotonic relaxation process constrained by finite connectivity and a maximal relaxation speed, the longest-wavelength modes correspond to collective configurations whose amplitudes are not freely adjustable.

Structural attenuation of global modes.

At very low multipoles, the associated angular modes span regions comparable to the full causal domain of the projected χ_{eff} configuration. As a result, these modes are subject to global relaxation constraints: their amplitude is systematically attenuated relative to the scale-invariant expectation.

This suppression is not the result of stochastic damping or fine-tuned initial conditions. It arises because the finite relaxation capacity of the χ field limits the degree to which globally coherent configurations can deviate from the relaxed background. The effect is deterministic in origin, while its detailed realization in any given universe remains statistical.

Cosmochrony therefore does not predict exact multipole amplitudes. Instead, it predicts a robust suppression tendency affecting the lowest ℓ-modes, whose precise pattern depends on the detailed global configuration of χ at last scattering.

Illustrative comparison with observations.

Figure 18 shows the observed CMB temperature power spectrum at low multipoles, displayed without aggressive smoothing, together with a schematic attenuation envelope representative of the Cosmochrony mechanism.

This comparison is intended to illustrate the qualitative structural deviation from scale invariance implied by global relaxation constraints. It does *not* constitute a multipole-by-multipole prediction, nor does it replace a full Boltzmann analysis.

Phenomenological parametrization.

To render this schematic attenuation minimally quantitative without introducing a full cosmological perturbation theory, we introduce a two-parameter phenomenological envelope in which the low-ℓ power is multiplicatively suppressed relative to the ΛCDM best-fit spectrum:

$$C_\ell^{\rm CC} = C_\ell^{\Lambda{\rm CDM}} \left[1 - \alpha \exp\!\left(-\frac{\ell}{\ell_0}\right)\right], \qquad \alpha \in [0,1], \;\; \ell_0 > 0. \tag{218}$$

Equivalently, in terms of $D_\ell \equiv \ell(\ell+1)C_\ell/(2\pi)$,

$$D_\ell^{\rm CC} = D_\ell^{\Lambda{\rm CDM}} \left[1 - \alpha e^{-\ell/\ell_0}\right].$$

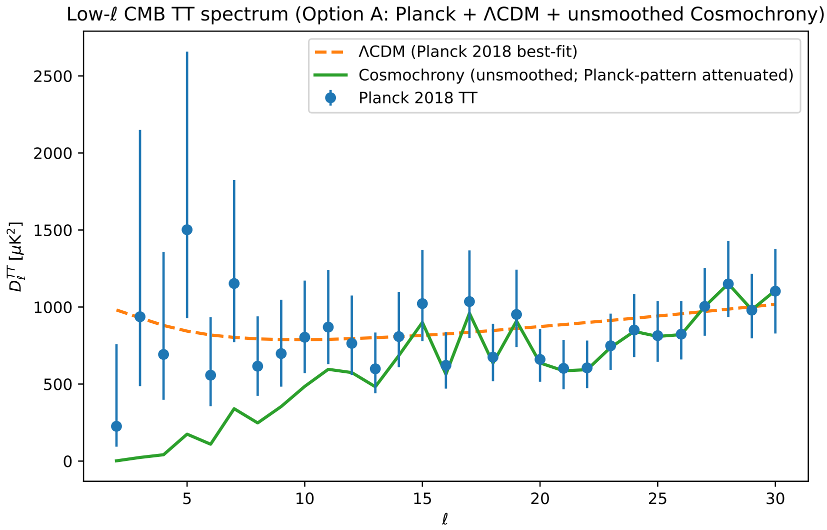

Fig. 18 Observed CMB temperature power spectrum at low multipoles ($\ell \lesssim 30$), shown without heavy smoothing. The shaded region illustrates a qualitative attenuation envelope expected from global relaxation constraints on the projected field $\chi_{\rm eff}$ in Cosmochrony. Unlike ΛCDM, where low-ℓ suppression is treated as a statistical accident, Cosmochrony interprets it as a structural consequence of the finite relaxation capacity of globally coherent configurations. The envelope may be summarized phenomenologically by Eq. (218).

In this parametrization, α controls the overall amplitude of large-angle suppression, while ℓ_0 sets the angular scale beyond which the spectrum rapidly converges back to the standard ΛCDM behavior.

Indicative low-ℓ characterization.

An indicative estimate of (α, ℓ_0) may be obtained by fitting the ratio $R_\ell \equiv D_\ell^{\rm obs}/D_\ell^{\Lambda{\rm CDM}}$ over a restricted low-ℓ range (e.g. $\ell = 2\ldots30$), using cosmic-variance-dominated uncertainties $\sigma(R_\ell) \simeq \sqrt{2/(2\ell+1)}$.

This procedure is not intended as a detection claim. It provides a compact and reproducible summary of the suppression tendency, replacing heuristic hand-drawn envelopes by a controlled two-parameter characterization.

Conceptual distinction from ΛCDM.

In ΛCDM, low-ℓ deviations are interpreted *a posteriori* as statistical fluctuations around an ensemble mean defined by inflationary initial conditions. In Cosmochrony, the ensemble itself is constrained: the global relaxation dynamics of χ restrict the admissible configuration space for the longest-wavelength modes.

This leads to a qualitative physical distinction between large-scale and small-scale fluctuations. Small-scale modes probe local relaxation and behave approximately as independent perturbations, while large-scale modes encode global structural properties of the field.

Graph-theoretic origin of the admissibility filter.

Building on the concept of ontological poverty introduced in Section 3.8, the admissibility filter $\mathcal{A}(k,t)$ admits a natural graph-theoretic interpretation. At any effective cosmological time t, the relational substrate χ defines a connectivity graph $G(t)$ whose weighted Laplacian $L_G(t)$ encodes the admissible collective modes.

The spectrum of $L_G(t)$,

$$L_G(t)\,\psi_n = \lambda_n(t)\,\psi_n,$$

sets a minimal nonzero eigenvalue $\lambda_2(t)$ that characterizes the global connectivity of the relational structure. Modes with wavelengths exceeding the corresponding scale $k_c(t) \sim \sqrt{\lambda_2(t)}$ are structurally inadmissible.

The primordial power spectrum is therefore modulated by a structural admissibility filter,

$$\mathcal{A}(k,t) = \exp\left[-\left(\frac{\lambda_2(t)}{k^2}\right)^{p/2}\right],$$

reflecting the inability of an insufficiently connected relational graph to support large-scale coherent modes.

Scope and limitations.

The present analysis does not replace full Boltzmann calculations and does not aim to reproduce the entire angular power spectrum. Its purpose is to identify a robust qualitative signature of Cosmochrony: a systematic suppression tendency affecting the lowest CMB multipoles, arising from global relaxation constraints on the fundamental field.

Quantitative refinement of this effect, including detailed parameter inference and polarization observables, is deferred to future numerical studies of the χ dynamics.

C.2 Resolution of the Horizon and Flatness Problems Without Inflation

In standard cosmology, the horizon and flatness problems arise from extrapolating a spacetime-based notion of causality and geometry back to the earliest stages of cosmic evolution. Within this framework, regions of the universe that appear widely separated today should not have been in causal contact, and the near-flatness of spatial geometry requires fine-tuned initial conditions. Inflation addresses these issues by postulating a brief phase of accelerated expansion in a pre-existing metric background.

Cosmochrony adopts a fundamentally different standpoint. Spacetime geometry, causal structure, and metric notions of distance are not assumed to be fundamental. They emerge only at a later stage, as effective descriptions of the relaxation dynamics of the scalar field χ. As a result, the assumptions underlying the horizon and flatness problems do not apply at the fundamental level.

Horizon problem: pre-geometric connectivity.

In Cosmochrony, large-scale correlations do not need to be established through signal propagation within spacetime. Instead, they originate from the fact that χ constitutes a single, globally connected dynamical substrate whose relaxation precedes the emergence of any effective spacetime description.

At early stages, before a metric notion of causality becomes meaningful, the configuration of χ is defined globally. Regions that later appear causally disconnected in the emergent spacetime may therefore share correlated configurations inherited from earlier phases of the relaxation process.

In this sense, Cosmochrony replaces inflationary causal contact with *pre-geometric connectivity*: correlations are established at the level of the fundamental field itself, rather than through superluminal expansion or specially prepared initial conditions on a metric background.

Flatness problem: relaxation toward geometric uniformity.

The flatness problem is addressed through the same underlying mechanism. In Cosmochrony, effective spatial curvature reflects large-scale gradients and inhomogeneities in the relaxation rate of the projected field χ_{eff}. As relaxation proceeds, configurations with large curvature gradients are dynamically disfavored, since they correspond to sustained resistance to global relaxation.

As a consequence, near-flat spatial geometry emerges as a natural attractor of the relaxation dynamics. Curvature dilution does not require exponential expansion or fine-tuning of initial curvature parameters. It reflects the tendency of the χ field to minimize large-scale geometric tension as it approaches a homogeneous relaxation state.

This mechanism operates independently of any inflationary phase and does not rely on a specific initial curvature value.

Implications for primordial correlations.

Because large-scale coherence arises from the global organization of χ rather than from the amplification of quantum vacuum fluctuations, Cosmochrony does not predict exact scale invariance at the largest wavelengths. Instead, the longest-wavelength modes are subject to global relaxation constraints, which may lead to deviations from scale invariance at the lowest multipoles of the cosmic microwave background.

Such deviations are interpreted as structural tendencies rather than sharp predictions. They provide a qualitative distinction from inflation-based scenarios and motivate the phenomenological analysis of low-ℓ CMB anomalies discussed in Section C.1.

Status and limitations.

The arguments presented here establish that the horizon and flatness problems do not arise as fundamental inconsistencies within the Cosmochrony framework. They are artifacts of applying metric-based reasoning beyond its domain of validity.

A quantitative derivation of primordial correlation functions and power spectra, including detailed predictions for CMB anisotropies, requires dedicated numerical simulations of the χ-field relaxation dynamics and lies beyond the scope of the present work.

Nevertheless, Cosmochrony provides a conceptually coherent, inflation-free resolution of large-scale causal coherence and near-flat spatial geometry, rooted in the pre-geometric dynamics of a single scalar field.

C.3 Evolution of the Hubble Parameter and the Hubble Tension

This appendix provides the mathematical details underlying the Hubble tension interpretation presented in Section 12.10.

In the Cosmochrony framework, cosmological expansion is not governed by the competition between matter, radiation, and a dark energy component. Instead, it reflects the relaxation dynamics of the scalar field χ, from which spacetime geometry and its associated expansion rate emerge as effective descriptions.

The Hubble parameter therefore encodes the instantaneous relaxation rate of χ relative to its global configuration, rather than the response of a metric to an energy–momentum content.

Global expansion rate.

At the homogeneous background level, the effective scale factor is proportional to the global value of the projected field,

$$a(t) \propto \chi(t), \tag{219}$$

so that the Hubble parameter may be written as

$$H(t) = \frac{\dot{\chi}}{\chi}. \tag{220}$$

In the idealized homogeneous limit, spatial gradients vanish ($\nabla\chi = 0$) and the relaxation dynamics reduce to a uniform evolution with maximal relaxation speed,

$$\dot{\chi} = c. \tag{221}$$

In this limit, the global expansion rate becomes

$$H(t) = \frac{c}{\chi(t)}. \tag{222}$$

This relation defines the *global* expansion rate in Cosmochrony. Its detailed redshift dependence away from perfect homogeneity is not assumed to follow a fixed power law and depends on how relaxation gradients contribute to the averaged dynamics.

Illustrative $H(z)$ profile and comparison with ΛCDM.

To provide a visual benchmark against standard cosmology, we compare an *illustrative* Cosmochrony profile to the ΛCDM expansion history. This is not a fit and does not claim a derived redshift law: it only translates the baseline relations $a(t) \propto \chi(t)$ and $H = \dot{\chi}/\chi$ into a convenient parametrization for plotting purposes.

Using the effective redshift interpretation

$$1 + z = \frac{\chi(t_0)}{\chi(t)}, \tag{223}$$

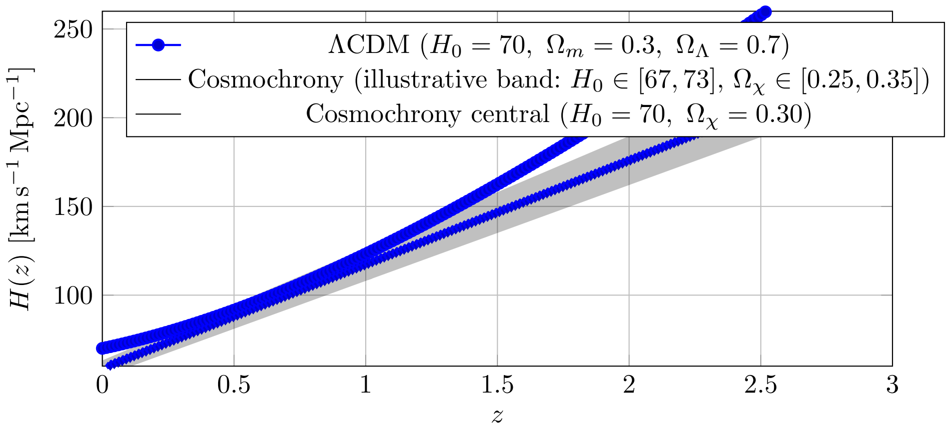

Fig. 19 Illustrative comparison of $H(z)$ in Cosmochrony versus a ΛCDM benchmark. The Cosmochrony curve uses the minimal plotting closure $H_\chi(z) = H_0(1+z)\sqrt{1-\Omega_\chi}$, shown with a representative uncertainty band from (H_0, Ω_χ) variations. This figure is a visualization aid, not a fit.

one may write, in the simplest background closure where the relaxation budget is treated as a slowly varying effective fraction,

$$H_\chi(z) \;\equiv\; H_0\,(1+z)\,\sqrt{1-\Omega_\chi} \qquad \text{(illustrative closure).} \tag{224}$$

For comparison, the ΛCDM benchmark is

$$H_{\Lambda\mathrm{CDM}}(z) \;=\; H_0\sqrt{\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\Lambda}, \tag{225}$$

with $(\Omega_m, \Omega_\Lambda) \simeq (0.3, 0.7)$ and Ω_r negligible at late times.

Figure 19 shows the two curves, together with a shaded Cosmochrony band obtained by varying (H_0, Ω_χ) within representative late-universe ranges. The purpose is to highlight (i) the qualitatively distinct scaling of $H(z)$ induced by monotonic χ relaxation and (ii) how modest variations in the effective relaxation budget translate into percent-level differences in inferred expansion.

Relaxation budget and effective expansion.

In a realistic universe, part of the relaxation capacity of χ is stored in spatial gradients associated with inhomogeneities. To quantify this effect, we introduce a dimensionless *relaxation budget* parameter,

$$\Omega_\chi \equiv \langle \beta^2 \rangle, \qquad \beta \equiv \frac{|\nabla \chi|}{c}, \tag{226}$$

which measures the fraction of the total relaxation capacity diverted into spatial structure rather than global evolution.

At late times, these gradients are dominated by localized solitonic configurations and therefore track the large-scale matter distribution. The effective global expansion rate is then reduced to

$$\bar{H} = \frac{c}{\chi}\sqrt{1-\Omega_\chi}. \tag{227}$$

Empirically, consistency with large-scale observations suggests Ω_χ is of order the observed matter fraction, $\Omega_\chi \sim 0.3$. In Cosmochrony, this suppression of the global expansion rate arises naturally from relaxation dynamics and does not require a dark energy component.

Local expansion and environmental dependence.

In an inhomogeneous universe, the relaxation budget is not spatially uniform. Regions with different matter densities redistribute relaxation capacity differently between global evolution and local gradients.

For a region characterized by a density contrast

$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}, \tag{228}$$

we adopt a minimal mean-field closure relation,

$$\beta_{\rm loc}^2 = \Omega_\chi(1+\delta), \tag{229}$$

which encodes the intuitive scaling between matter density and χ-gradient energy.

The locally inferred Hubble parameter then takes the form

$$H_{\rm loc} = \bar{H}\sqrt{\frac{1-\Omega_\chi(1+\delta)}{1-\Omega_\chi}}. \tag{230}$$

In underdense regions ($\delta < 0$), a larger fraction of the relaxation capacity is available for global evolution, leading to $H_{\rm loc} > \bar{H}$.

Numerical consistency and the Hubble tension.

For representative values $\Omega_\chi \approx 0.3$ and a local underdensity consistent with the KBC void ($\delta \approx -0.4$ on scales of a few hundred megaparsecs), one finds

$$\frac{H_{\rm loc}}{\bar{H}} \approx 1.08, \tag{231}$$

corresponding to an enhancement of order 8% in the locally inferred Hubble constant.

This magnitude is comparable to the observed discrepancy between local distance-ladder measurements and global CMB-based inferences. In Cosmochrony, this discrepancy arises naturally as an environmental effect, without invoking new energy components or modifications of early-universe physics.

Predicted amplitude of the deviation at $z \sim 1$.

A distinctive feature of the Cosmochrony framework is that the magnitude of the departure from the ΛCDM expansion history is not freely adjustable. At late times, the relative deviation between the locally inferred and globally averaged expansion rates is fully controlled by the relaxation budget parameter Ω_χ and by the effective density contrast δ in the redshift range under consideration.

Within Cosmochrony, structural consistency imposes

$$\Omega_\chi \in [0.25,\, 0.35], \tag{232}$$

reflecting the fraction of relaxation capacity stored in large-scale gradients, while the partially developed cosmic web at $z \sim 1$ naturally corresponds to moderate underdensities in the range

$$\delta(z \sim 1) \in [-0.2,\, -0.5]. \tag{233}$$

Inserting these bounds into Eq. (C.230) yields a predicted fractional enhancement

$$\frac{\Delta H}{H} \;\equiv\; \frac{H_{\rm loc} - \bar{H}}{\bar{H}} \;\in\; [3.5\%,\, 9.5\%] \qquad (z \sim 1). \tag{234}$$

This narrow interval is not a phenomenological choice but a direct consequence of the relaxation-based origin of expansion in Cosmochrony. Deviations significantly outside this range would require either unphysical values of Ω_χ or density contrasts incompatible with the observed state of large-scale structure at intermediate redshifts.

Uncertainty estimates and parameter degeneracies.

At the present stage, Cosmochrony does not claim a precision reconstruction of $H(z)$. Nevertheless, it is useful to clarify how uncertainties in the effective parameters map to uncertainties in the plotted $H(z)$ profiles and in the underlying stiffness scale.

Late-time effective uncertainty in (H_0, Ω_χ). Within the illustrative closure of Eq. (224), small variations propagate as

$$\frac{\delta H}{H} \;\simeq\; \frac{\delta H_0}{H_0} \;-\; \frac{1}{2}\frac{\delta\Omega_\chi}{1-\Omega_\chi}. \tag{235}$$

For $\Omega_\chi \simeq 0.30$, an uncertainty $\delta\Omega_\chi \simeq 0.05$ corresponds to a fractional variation $\delta H/H \simeq 3.6\%$, comparable to the width used in Fig. 19. This motivates representing Cosmochrony as a band rather than a single curve until dedicated simulations determine $\Omega_\chi(z)$.

Degeneracy between K_0 *and* χ_c. The effective gravitational coupling constrains the product

$$K_0\,\chi_c^2 \;\sim\; \frac{c^4}{16\pi G}, \tag{236}$$

so that K_0 and χ_c cannot be fixed independently without additional input. Propagating uncertainties in logarithmic form gives

$$\delta \ln K_0 \;\simeq\; -2\,\delta \ln \chi_c \quad \text{(at fixed } G\text{)}. \tag{237}$$

Hence, a factor-of-10 uncertainty on χ_c implies a factor-of-100 uncertainty on K_0. This is consistent with the fact that both Planck-scale and cosmological-scale normalizations can remain internally viable at the level of dimensional analysis, until the microscopic origin of χ_c is constrained by simulations and/or additional observables.

Interpretation and status.

Within Cosmochrony, the Hubble tension does not signal a breakdown of cosmological consistency. It reflects the fact that cosmological expansion is an emergent relaxation phenomenon whose effective rate depends on the local redistribution of χ-field gradients.

While the framework robustly predicts a separation between local and global expansion rates, a fully quantitative determination of $H(z)$ across all redshifts requires dedicated numerical simulations of the χ relaxation dynamics. Such simulations lie beyond the scope of the present work.

Nevertheless, the qualitative resolution of the Hubble tension follows directly from the relaxation-based interpretation of cosmological expansion and constitutes a distinctive and testable signature of the Cosmochrony framework.

C.4 Relation to Observational Units and Numerical Estimates

This subsection establishes order-of-magnitude relations linking the Cosmochrony framework to observed cosmological quantities. Its purpose is not to perform parameter fitting or to derive precision predictions, but to assess the internal consistency of the theory and to verify that its fundamental relaxation-based interpretation naturally reproduces the correct empirical scales.

All numerical relations presented here should be understood as effective normalizations arising in the projectable regime of the χ dynamics. They do not define fundamental constants and do not fix the microscopic structure of the theory.

Normalization of the χ Field

To relate the projected field χ_{eff} to observable quantities, a reference normalization must be specified. At the effective cosmological level, the scale factor is defined up to a global multiplicative constant. In Cosmochrony, this freedom is fixed by identifying the present-day value $\chi(t_0)$ with the characteristic geometric scale governing large-scale expansion.

Operationally, $\chi(t_0)$ represents the cumulative geometric scale associated with the global relaxation of the χ field up to the present epoch. This identification does not assume a unique microscopic origin for $\chi(t_0)$; it provides a minimal and observationally anchored normalization consistent with the effective relation $a(t) \propto \chi(t)$.

Emergent Gravitational Coupling

In the effective geometric description, the Newtonian gravitational constant G emerges from the constitutive relation governing the coupling between neighboring configurations of the projected χ field. This coupling is controlled by two parameters of the relaxation dynamics: the maximal stiffness scale K_0 and the characteristic correlation length χ_c.

Although K_0 and χ_c are not individually fixed at the present stage, their combination is constrained by matching the observed gravitational coupling:

$$K_0 \chi_c^2 \sim \frac{c^4}{16\pi G}. \tag{238}$$

This relation fixes the overall stiffness scale of the effective χ network. It does not require committing to a specific microscopic interpretation of χ_c, which may correspond to a fundamental correlation scale or to an emergent coarse-graining length. At this level, only the product $K_0 \chi_c^2$ is observationally relevant.

Hubble Constant

In the homogeneous limit, the effective Hubble parameter is defined by the relative relaxation rate of the projected field,

$$H(t) = \frac{\dot{\chi}}{\chi}. \tag{239}$$

Assuming that the present universe lies close to the maximal relaxation regime, $\dot{\chi}(t_0) \simeq c$, the present-day Hubble constant follows as

$$H_0 \simeq \frac{c}{\chi(t_0)}. \tag{240}$$

Using the observed value $H_0 \approx 70\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ yields

$$\chi(t_0) \sim 4 \times 10^{26}\ \mathrm{m}, \tag{241}$$

which is of the order of the observed Hubble radius. This correspondence arises directly from the relaxation-based interpretation of cosmic expansion and does not require the introduction of additional cosmological parameters.

Age of the Universe

In the homogeneous relaxation regime, the evolution of χ may be approximated as

$$\dot{\chi} \simeq c, \tag{242}$$

leading to

$$\chi(t) \simeq ct + \chi_{\mathrm{init}}, \tag{243}$$

where χ_{init} denotes the effective value of χ at the onset of the relaxation regime relevant for cosmological observations.

Neglecting χ_{init} compared to present values yields

$$t_0 \simeq \frac{\chi(t_0)}{c} \sim 4 \times 10^{17}\ \mathrm{s}, \tag{244}$$

corresponding to approximately 13.8 billion years. This estimate is consistent with standard cosmological age determinations and follows directly from the bounded relaxation dynamics.

Redshift Interpretation

In Cosmochrony, cosmological redshift is interpreted as a consequence of the relative change in the projected χ field between emission and observation,

$$1 + z = \frac{\chi(t_{\rm obs})}{\chi(t_{\rm emit})}. \tag{245}$$

This relation reproduces standard redshift phenomenology while attributing it to geometric scaling induced by χ relaxation, rather than to recessional motion within a pre-existing spacetime background.

Cosmic Microwave Background Scale

At recombination, characterized observationally by $z_{\rm rec} \simeq 1100$, the effective value of the projected field was smaller by the corresponding scaling factor,

$$\chi(t_{\rm rec}) \simeq \frac{\chi(t_0)}{1 + z_{\rm rec}}. \tag{246}$$

Fluctuations imprinted at that epoch are subsequently stretched by the monotonic growth of χ, providing a natural geometric interpretation of the angular scales observed in the cosmic microwave background without invoking an inflationary stretching phase.

Orders of Magnitude and Robustness

All numerical estimates presented in this subsection rely solely on observed cosmological quantities and on the bounded relaxation dynamics of the χ field. No fine-tuning of parameters, no detailed cosmological fitting, and no additional degrees of freedom are assumed.

While a fully predictive cosmological model requires explicit numerical simulations of the χ dynamics, these order-of-magnitude relations demonstrate that Cosmochrony naturally reproduces the correct scales for the Hubble constant, the age of the universe, redshift evolution, and characteristic CMB features.

Summary

The Cosmochrony framework admits a consistent normalization in observational units and reproduces key cosmological scales without introducing new fundamental parameters. These order-of-magnitude relations support the internal coherence of the theory and motivate further quantitative investigation of its cosmological dynamics.

C.5 Phenomenological Implications

This subsection summarizes the principal phenomenological consequences of Cosmochrony that are accessible to observation. The emphasis is placed on effects that follow robustly from the kinematic and relaxation structure of the χ field itself, without introducing auxiliary degrees of freedom, adjustable interpolation functions, or phenomenological potentials.

All results presented here arise in the projectable regime of the theory and should be understood as effective manifestations of the underlying relaxation dynamics, not as fundamental postulates.

Propagation speed of gravitational perturbations.

To determine the propagation speed of gravitational information in Cosmochrony, consider small perturbations $\delta\chi$ around a homogeneous relaxation background,

$$\chi_0(t) = ct, \tag{247}$$

such that

$$\chi(\mathbf{x}, t) = ct + \delta\chi(\mathbf{x}, t), \qquad |\nabla\delta\chi| \ll c. \tag{248}$$

Substituting this form into the fundamental kinematic constraint governing χ-field relaxation (Eq. 12) yields, to leading order,

$$c + \partial_t\delta\chi = c\sqrt{1 - \frac{|\nabla\delta\chi|^2}{c^2}}. \tag{249}$$

Expanding for small spatial gradients gives

$$\partial_t\delta\chi \simeq -\frac{|\nabla\delta\chi|^2}{2c}, \tag{250}$$

reflecting the irreversible character of the relaxation process. While this first-order relation governs dissipation, the propagation of perturbations is more transparently captured by considering the second-order operator associated with the squared constraint.

Linearizing this operator leads to the effective wave equation

$$\left(\frac{1}{c^2}\partial_t^2 - \nabla^2\right)\delta\chi = 0, \tag{251}$$

which admits propagating solutions with characteristic speed

$$v_{\text{prop}} = c. \tag{252}$$

Gravitational perturbations therefore propagate exactly at the invariant speed c. This equality is not imposed by hand but follows directly from the fundamental kinematic bound on χ relaxation. As a result, Cosmochrony is automatically consistent with multi-messenger observations, including the near-simultaneous arrival of gravitational and electromagnetic signals in events such as GW170817.

Emergent acceleration scale and MOND-like phenomenology.

In Cosmochrony, the arrow of time is encoded in the monotonic evolution of the fundamental field, $\partial_t \chi \geq 0$. At late cosmic times and on sufficiently large scales, where χ_{eff} admits an approximately homogeneous description, the relaxation dynamics may be coarse-grained into an effective cosmological clock.

In this effective regime, the temporal evolution of χ can be written as

$$\partial_t \chi \simeq H(t)\, \chi, \tag{253}$$

where $H(t)$ denotes the emergent Hubble parameter associated with global relaxation.

The local kinematic constraint

$$(\partial_t \chi)^2 + |\nabla \chi|^2 = c^2 \tag{254}$$

then implies that even in the absence of localized matter excitations, the cosmological evolution of χ enforces a non-vanishing residual spatial gradient. In the homogeneous limit, this minimal gradient is

$$|\nabla \chi|_{\min} = \sqrt{c^2 - (H\chi)^2}. \tag{255}$$

This residual gradient defines a background kinematic scale that constrains the superposition of additional, locally induced gradients. Operationally, it corresponds to an effective acceleration scale

$$a_0(t) \sim c\, H(t). \tag{256}$$

When localized matter excitations are present, they induce additional gradients $\nabla \chi_N$ that reproduce the Newtonian scaling $|\nabla \chi_N| \propto M/r^2$ at short distances. Because the kinematic constraint is nonlinear, the total gradient does not superpose linearly. At sufficiently large radii, the effective acceleration asymptotically approaches

$$g_{\text{eff}} \simeq \sqrt{g_N\, a_0(t)}, \tag{257}$$

recovering the characteristic deep-MOND scaling without introducing interpolation functions, dark matter particles, or additional fields.

In this framework, the acceleration scale a_0 is not fundamental. It evolves slowly with cosmic time through its dependence on $H(t)$, providing a potential observational discriminator at high redshift.

Gravitational lensing.

In Cosmochrony, light propagation follows wavefronts of constant χ. An effective refractive index for the vacuum may be defined operationally as

$$n(r) = \frac{c}{\partial_t \chi} = \frac{1}{\sqrt{1 - |\nabla \chi|^2/c^2}}. \tag{258}$$

Near a localized mass M, where $|\nabla\chi| \simeq GM/(c^2 r)$, a weak-field expansion yields

$$n(r) \simeq 1 + \frac{GM}{c^2 r}. \tag{259}$$

Integrating the transverse gradient of $n(r)$ along a photon trajectory leads to a deflection angle

$$\alpha = \frac{4GM}{bc^2}, \tag{260}$$

where b is the impact parameter. This reproduces the general-relativistic prediction for gravitational lensing.

In Cosmochrony, the enhancement relative to the Newtonian deflection does not originate from a fundamental spacetime curvature. It arises from the nonlinear structure of the χ relaxation dynamics, which modifies the effective propagation geometry experienced by light.

Summary.

The phenomenology of Cosmochrony reproduces key observational signatures of gravity and cosmology while relying on a single scalar degree of freedom. Gravitational perturbations propagate at exactly the invariant speed c, a MOND-like acceleration scale emerges naturally from cosmological relaxation, and gravitational lensing is recovered without postulating a fundamental metric.

These results illustrate how classical gravitational phenomena arise as coarse-grained manifestations of the underlying χ dynamics and define a set of observationally testable signatures distinguishing Cosmochrony from standard metric-based theories.

C.6 Toy-Model of Spectral Gravitational Susceptibility

This appendix provides the mathematical foundations for a non-particulate interpretation of dark matter phenomena, treating galactic dynamics as the non-linear elastic response of the χ substrate.

Modified Poisson Equation and Field Strength

In the Cosmochrony framework, gravitational acceleration is not a force acting in a passive vacuum but the emergent manifestation of a relaxation gradient. We define the **local relaxation field strength** as the gradient of the scalar relaxation potential:

$$\mathbf{E}_\chi = -\nabla\Phi_\chi \tag{261}$$

The dynamics are governed by a modified Poisson equation analogous to electrodynamics in continuous media:

$$\nabla \cdot [\epsilon_{\text{spec}}(\mathbf{E}_\chi)\mathbf{E}_\chi] = 4\pi G_0 \rho_b \tag{262}$$

where ρ_b is the baryonic mass density and ϵ_{spec} is the **spectral permittivity** of the substrate, defined by the relation $\epsilon_{\text{spec}} = 1 + \phi(\mathbf{E}_\chi)$.

Spectral Susceptibility and the Stiffness Threshold $\mathcal{K}_c$

To recover the observed galactic phenomenology, we define the **spectral gravitational susceptibility** ϕ as a function of the field strength relative to a saturation threshold $\mathcal{K}_c$:

$$\phi(\mathbf{E}_\chi) = \begin{cases} 0 & \text{for } |\mathbf{E}_\chi| \gg \mathcal{K}_c \text{ (Linear/Newtonian Regime)} \\ \frac{\mathcal{K}_c}{|\mathbf{E}_\chi|} & \text{for } |\mathbf{E}_\chi| \ll \mathcal{K}_c \text{ (Saturation Regime)} \end{cases} \tag{263}$$

Crucially, $\mathcal{K}_c$ is not a universal constant of nature but a **local state property** of the substrate. It represents the threshold where the relaxation flux reaches the elastic limit of the χ field.

Emergence of Flat Rotation Curves

In the low-field limit ($|\mathbf{E}_\chi| \ll \mathcal{K}_c$) typical of galactic peripheries, the effective acceleration g_{eff} follows:

$$\nabla \cdot \left(\mathcal{K}_c \frac{\mathbf{E}_\chi}{|\mathbf{E}_\chi|} \right) \sim 4\pi G_0 \rho_b \implies g_{\text{eff}} \approx \frac{\sqrt{G_0 M \mathcal{K}_c}}{r} \tag{264}$$

This leads directly to a constant orbital velocity $v^4 = G_0 M \mathcal{K}_c$, recovering the **Baryonic Tully-Fisher Relation**. Here, "dark matter" is reinterpreted as the increased elastic response of the substrate in regions of diluted relaxation flux.

Comparative Framework: MOND vs. Cosmochrony

The following table summarizes the conceptual shift from modified gravity to substrate dynamics.

Feature	**MOND (Milgrom)**	**Cosmochrony (χ Substrate)**
Origin	Modified law of inertia/force.	Non-linear susceptibility of the medium.
Threshold	Universal constant a_0.	Local stiffness threshold $\mathcal{K}_c$.
Bullet Cluster	Requires additional DM particles.	Natural: Relaxation hysteresis (wake).
GR Relation	Requires $TeVeS$ or similar.	GR is the linear-response limit.
DM Nature	Force discrepancy.	Residual non-projected energy.

Table 3 Comparison between MOND phenomenology and Cosmochrony substrate response.

Limitations and Outlook

Theoretical Refinement.

The current form of $\phi(\mathbf{E}_\chi)$ is phenomenological. A rigorous derivation from the microscopic relaxation equations in Appendix D is required to link $\mathcal{K}_c$ to the global Hubble relaxation rate.

The Relaxation Wake.

Cosmochrony predicts that in high-energy collisions (e.g., Bullet Cluster), the geometric deformation of the substrate exhibits a **phase lag** (hysteresis). Gravitational lensing tracks this "residual wake" of the mass-solitons, explaining the offset from dissipative gas. A specific prediction of this model is the existence of **spectral echoes**: residual curvature in regions where matter has recently passed, a signature that could distinguish Cosmochrony from WIMP-based models.

General Relativity Limit.

Finally, it is emphasized that Cosmochrony reduces to General Relativity in the linear-response limit of the χsubstrate. Spacetime curvature is the refractive manifestation of the substrate's spectral density, and gravity is its macroscopic relaxation.

C.7 Substrate Origin of the Effective Galactic Potential

This appendix clarifies how the effective gravitational potential introduced in Section 12.12 arises from infra-physical properties of the χ substrate, rather than from a fundamental interaction law.

Relaxation flux and relational stiffness.

At the fundamental level, localized excitations correspond to constrained configurations of the relational substrate χ. Their large-scale influence is mediated by a relaxation flux Φ_χ, whose magnitude depends both on local gradients and on the relational stiffness of the substrate,

$$\Phi_\chi(r) \sim \frac{|\nabla\chi|}{K(r)}. \tag{265}$$

The effective stiffness $K(r)$ increases when the projective connectivity of χ becomes sparse, as occurs at large distances from localized excitations.

Saturation threshold and loss of injectivity.

Because the relaxation dynamics of χ are bounded, there exists a critical stiffness scale K_c beyond which additional gradients cannot be transmitted linearly. When $K(r) \ll K_c$, relaxation remains unsaturated and the projected description is effectively injective, leading to Newtonian behavior. When $K(r) \gtrsim K_c$, relaxation saturates and the projection from χ to effective observables becomes non-injective.

Emergent acceleration scale.

In the projected geometric description, the saturation condition $K(r) \simeq K_c$ manifests as a threshold on the Newtonian baryonic acceleration,

$$g_N(r) \simeq a_0(t). \tag{266}$$

The scale a_0 is therefore not a fundamental constant but an operational re-expression of the substrate saturation threshold.

Cosmological origin of $a_0(t)$.

At the largest scales, the relaxation of χ is constrained by the global relational expansion of the Universe. The characteristic relaxation rate is set by the inverse cosmological timescale, leading naturally to

$$a_0(t) \sim c\, H(t), \tag{267}$$

where c encodes the maximal relaxation speed of χ and $H(t)$ the global relational expansion rate. This relation predicts a slow cosmological evolution of the effective acceleration scale.

From substrate saturation to logarithmic potential.

In the saturated regime, the relaxation flux transmitted by χ becomes effectively scale-invariant, implying an effective acceleration $g_{\text{eff}}(r) \propto 1/r$. The corresponding projected potential therefore takes the logarithmic form

$$\Phi_{\text{eff}}(r) \propto \ln r, \tag{268}$$

which directly accounts for asymptotically flat galactic rotation curves.

Interpretational status.

The effective potential Φ_{eff} thus summarizes a specific regime of substrate relaxation. It does not correspond to an additional field or interaction, but to the geometrically admissible description of a saturated projection of the underlying χ dynamics.

C.8 Spectral Interpretation of the Galactic Saturation Regime

This appendix provides a complementary operator-based interpretation of the effective galactic potential introduced in Section 12.12, by relating the saturation of χ-relaxation to spectral properties of the projected relational operator.

Effective operator and spectral stiffness.

In projectable regimes, the collective response of the χ substrate to localized excitations may be characterized by an effective elliptic operator $\mathcal{L}_{\text{eff}}$, whose spectrum encodes the relational stiffness of the medium,

$$\mathcal{L}_{\text{eff}}\, \psi_n = \lambda_n \psi_n, \tag{269}$$

with increasing eigenvalues λ_n corresponding to increasing resistance to long-wavelength deformations.

Spectral saturation and loss of long modes.

As the scale increases, the density of low-lying modes decreases. Beyond a critical scale, the smallest admissible eigenvalue $\lambda_{\min}$ approaches a saturation threshold λ_c, such that additional long-range modes cannot be supported. This spectral depletion corresponds to the stiffness threshold K_c discussed in Appendix C.7.

Effective acceleration threshold.

In the projected geometric description, the spectral threshold λ_c translates into an effective acceleration scale a_0 through

$$a_0 \sim \frac{\lambda_c}{\tau_\chi^2}, \tag{270}$$

where τ_χ denotes the characteristic relaxation time of the substrate. This relation expresses the operational equivalence between spectral saturation and the acceleration crossover observed in galactic dynamics.

Emergence of the logarithmic potential.

When the spectrum becomes marginally scale-invariant near λ_c, the Green function of $\mathcal{L}_{\text{eff}}$ in the effective two-dimensional asymptotic regime behaves as

$$G(r) \propto \ln r. \tag{271}$$

The projected potential Φ_{eff} inherits this logarithmic behavior, providing a spectral explanation for asymptotically flat rotation curves.

Consistency with Newtonian behavior.

At scales where $\lambda_{\min} \ll \lambda_c$, the spectrum is dense and the Green function reduces to the familiar $1/r$ behavior, recovering Newtonian gravity as an unsaturated spectral regime.

Interpretational status.

The spectral description introduced here does not add new degrees of freedom. It provides an operator-level characterization of the same saturation phenomenon described infra-physically in Appendix C.7.

C.9 Neutrino-Mediated Structural Smoothing and Cosmological Inference

Effective Expansion versus Structural Relaxation

Let $a(\tau)$ denote the effective scale factor inferred from projected observables. In Cosmochrony, changes in $a(\tau)$ encode both geometric expansion and cumulative structural relaxation.

We may write, at the effective level,

$$\frac{1}{a}\frac{da}{d\tau} = H_{\text{geom}}(\tau) + H_{\text{relax}}(\tau),$$

where H_{relax} captures the contribution of irreversible structural smoothing, prominently mediated by neutrino-like excitations.

Early-Time Bias from Neutrino Free Streaming

Neutrino-like excitations free-stream over cosmological scales shortly after their emission. This introduces a non-local smoothing term that suppresses large-scale reconfigurations.

Early-time observables, such as CMB anisotropies, are sensitive to the integrated effect of this smoothing. As a result, parameter inference assuming purely geometric expansion may systematically underestimate the late-time effective expansion rate.

Low-ℓ Anomalies as Structural Fossils

The lowest multipoles of the CMB correspond to the largest effective scales and therefore probe the earliest admissible configurations.

Suppression or alignment of these modes may reflect anisotropic or directionally biased relaxation during the neutrino-dominated smoothing phase. Such features are naturally preserved due to the irreversibility of neutrino-mediated free streaming.

C.10 Cosmic Voids as Observational Tests of Maximal Substrate Relaxation

Cosmic voids constitute a particularly clean observational laboratory for the Cosmochrony framework. In contrast with overdense environments, voids correspond to regions where the relaxation of the relational substrate χ is only weakly frustrated by localized excitations. They therefore probe the regime of near-maximal relaxation, in which departures from standard ΛCDM phenomenology are expected to be most pronounced.

Born–Infeld parametrization of void observables.

We model observable signals associated with cosmic voids as a ΛCDM baseline supplemented by a saturating correction inspired by the Born–Infeld structure of the effective χ dynamics. For the weak-lensing convergence and the radial peculiar velocity field, we write

$$\kappa_{\mathrm{obs}}(R) = \kappa_{\Lambda\mathrm{CDM}}(R)\left[1 + \beta_{\mathrm{void}}\,\mathcal{S}\big(\mathcal{A}(R)\big)\right], \tag{272}$$

$$v_{\mathrm{obs}}(r) = v_{\Lambda\mathrm{CDM}}(r)\left[1 + \beta_{\mathrm{void}}\,\mathcal{S}\big(\mathcal{B}(r)\big)\right], \tag{273}$$

where β_{void} controls the amplitude of the cosmochronic correction and

$$\mathcal{S}(x) = \frac{x}{\sqrt{1+x^2}} \tag{274}$$

is a Born–Infeld–like saturation function. The dimensionless activities $\mathcal{A}$ and $\mathcal{B}$ quantify the local degree of relaxation and may be defined using observational proxies, e.g.

$$\mathcal{A}(R) = \frac{|\Delta(R)|}{s_\star}, \qquad \mathcal{B}(r) = \frac{|\delta(r)|}{s_\star} \tag{275}$$

with $\Delta(R)$ the projected density contrast and $\delta(r)$ the three-dimensional density contrast. The parameter $s_\star$ sets the saturation threshold.

Key observational signatures.

This parametrization leads to three distinctive and falsifiable predictions:

1. **Negative void lensing enhancement.** Cosmic voids are expected to exhibit a more negative weak-lensing signal than in ΛCDM, with a characteristic non-linear saturation for large and deep voids. The lensing profile is predicted to peak near the void boundary and to approach a plateau as the relaxation activity increases.
2. **Enhanced peculiar velocity outflows.** Galaxies at void boundaries should display radial outflows exceeding standard ΛCDM expectations by a fraction controlled by $\beta_{\rm void}$. In the saturated regime, the excess velocity approaches $\Delta v_r \simeq \beta_{\rm void}\, v_{\Lambda\rm CDM}$.
3. **Cross-consistency between lensing and velocities.** Both effects originate from the same relaxation mechanism and must therefore be described by a single value of $\beta_{\rm void}$. The simultaneous fitting of void lensing profiles and peculiar velocity data thus provides a stringent internal consistency test of the framework.

Connection to local expansion measurements.

Because enhanced void outflows bias low-redshift distance–redshift relations, regions dominated by large voids are predicted to yield locally inferred values of the Hubble parameter exceeding the global average. Cosmochrony therefore predicts a correlation between negative void-lensing strength, enhanced boundary outflows, and elevated local H_0 estimates, offering a unified explanation testable with upcoming weak-lensing and redshift surveys.

D Numerical Methods and Technical Supplements

This appendix collects numerical methods, technical constructions, and auxiliary derivations used to explore the phenomenological consequences of the Cosmochrony framework. Its role is explicitly supportive: the material presented here does not introduce additional fundamental structures, nor does it modify the ontological or dynamical core of the theory.

All methods described in this appendix operate within regimes where the dynamics of the fundamental field χ admits an effective, discretized, or coarse-grained representation. They should therefore be understood as computational approximations and technical tools designed to probe the behavior of the theory, not as independent physical postulates.

Status of the numerical constructions.

The fundamental formulation of Cosmochrony is relational and pre-geometric. It does not presuppose a background spacetime, a fixed metric, or a lattice structure. By contrast, the numerical methods employed here necessarily rely on auxiliary representations—such as discretized graphs, finite-difference schemes, or coarse-grained fields—to render the dynamics tractable.

These representations are introduced solely for calculational convenience. They do not possess ontological significance and should not be interpreted as revealing a fundamental discreteness of the χ field or of spacetime itself. Different numerical schemes may be employed without altering the conceptual content of the theory, provided they respect the same relaxation constraints and kinematic bounds.

Scope of the appendix.

This appendix provides technical details on:

- the notion of collective gravitational coupling and the construction of an operational geometric description emerging from χ-field relaxation (Section D.1);
- numerical algorithms and discretization strategies used to simulate χ-field dynamics and to estimate effective model parameters;
- supplementary derivations and calculations that support results presented in the main text and in Appendices B and C.

The emphasis throughout is on internal consistency, numerical stability, and faithful implementation of the theoretical principles articulated in the main body of the work. No claim is made that the numerical results obtained using these methods are unique or exhaustive.

Interpretation of numerical results.

Numerical simulations presented or discussed in this appendix should be regarded as exploratory. They are intended to test qualitative mechanisms—such as relaxation-driven emergence of geometry, solitonic stability, or scaling relations—rather than to produce precision predictions.

Where numerical values are quoted, they serve as order-of-magnitude indicators or illustrative examples. Quantitative predictions suitable for direct comparison with observational data require more extensive simulations and systematic parameter studies, which lie beyond the scope of the present work.

Relation to the main text.

The technical material collected here underpins several conceptual arguments made elsewhere in the paper, including:

- the emergence of effective gravitational coupling from collective relaxation effects;
- the stability and scaling properties of solitonic configurations;
- the qualitative cosmological and phenomenological implications discussed in Appendix C.

Readers primarily interested in the conceptual structure of Cosmochrony may safely skip this appendix without loss of continuity. Conversely, readers interested in numerical implementation, reproducibility, or future computational extensions may find the detailed constructions provided here useful.

D.1 Collective Gravitational Coupling and Operational Geometry

The fundamental field χ is continuous and governed by nonlinear, non-perturbative relaxation constraints. Its dynamics does not admit a closed-form spectral decomposition, nor a simple linearization valid across all regimes. As a result, any explicit investigation of stability, collective response, or mode structure necessarily relies on auxiliary representations that approximate the underlying functional dynamics.

In this appendix, we introduce such representations strictly as **effective surrogates**. They provide finite-dimensional bridges between the fundamental substrate and the effective response observed at macroscopic scales. These constructions do not reflect any fundamental discreteness of the substrate, nor do they define preferred spatial locations or a background geometry. They serve only to render certain collective effects computationally accessible.

Collective coupling as a dressed response operator.

Localized excitations of the χ field act as persistent resistances to global relaxation. When many such excitations are present, their influence combines collectively, modulating the relaxation flow at macroscopic scales.

At the effective level, this collective influence is summarized by a response operator K_{ij}, interpreted as a finite-dimensional representation of the linearized relaxation constraints. Crucially, K_{ij} is the **dressed counterpart** of the bare relational connectivity $K_{0,\mathrm{bare}}$ introduced in Section D.6. While the bare coupling determines the universal quantum of action $\hbar_\chi$, the effective operator K_{ij} encodes the spatial distribution of these constraints, effectively ”mapping” the emergent geometry through the local spectral density.

Effective gravitational potential in the weak-structure regime.

In regimes where localized resistances are sparse, the modulation of the global relaxation flow Φ_χ can be approximated as a perturbation of a uniform background. The effective potential $\Phi_{\rm eff}$ governing the motion of test excitations is derived from the local slowdown of the relaxation tempo. For a static source of mass M, the operational distance r is defined by the propagation time of χ-fluctuations, yielding:

$$\nabla^2 \Phi_{\rm eff} \approx 4\pi G_{\rm eff} \rho, \tag{276}$$

where ρ is the density of relaxation resistance and $G_{\rm eff}$ is the emergent gravitational constant. The relation between the stiffness K_0 and G is given by:

$$G_{\rm eff} \approx \frac{c^4}{K_{0,\rm eff}\, \chi_{c,\rm eff}^2}. \tag{277}$$

This shows that the Newtonian limit is not a postulate, but the leading-order description of collective relaxation interference.

Operational geometry.

Because Cosmochrony does not postulate a fundamental spacetime metric, spatial geometry is defined operationally. Two configurations are considered close if perturbations of χ propagate efficiently between them, and distant otherwise. In the weak-gradient regime, this induces an effective spatial geometry that coincides with Newtonian gravity. **Gravity is thus recovered as a macroscopic manifestation of relaxation resistance.**

Scope and limitations.

The construction presented here is restricted to quasi-static, weak-field regimes. Its purpose is to demonstrate that classical gravitational behavior can be recovered consistently without introducing a fundamental metric structure.

D.2 Estimates of χ-Field Parameters

The quantities introduced in this section—effective coupling scales, spectral parameters, and characteristic lengths—should be understood as properties of a *projected relaxation operator* acting on a finite-dimensional function space. They characterize the response of localized χ configurations to perturbations within a given resolution scale and do not represent fundamental degrees of freedom of the theory.

Distinction between Bare and Effective Scales: As established in Section D.6, we distinguish between the **bare** parameters ($K_{0,\rm bare}$, $\chi_{c,\rm bare}$), which are universal substrate invariants, and the **effective** parameters discussed here ($K_{0,\rm eff}$, $\chi_{c,\rm eff}$). These encode how localized structures constrain relaxation once a coarse-grained geometric description becomes applicable, effectively describing a **projective renormalization** of the substrate's stiffness.

The relevant effective parameters include:

- the **effective coupling scale** $K_{0,\text{eff}}$ entering the projected response operator K_{ij},
- the **characteristic scale** $\chi_{c,\text{eff}}$ at which macroscopic geometric effects emerge,
- effective solitonic parameters (λ, η) controlling stabilization mechanisms in reduced descriptions,
- the maximal relaxation speed c, which remains an invariant link between the bare and effective regimes.

Effective Coupling Scale K_0 and Characteristic Scale χ_c

The scale $\chi_{c,\text{eff}}$ sets the characteristic magnitude of χ over which structural variations significantly modulate relaxation and induce macroscopic geometric effects. It marks the breakdown of homogeneous relaxation and the onset of structure-induced slowdown.

The emergent gravitational constant G is driven by the ratio $K_{0,\text{eff}}/\chi^2_{c,\text{eff}}$. Equation (277) admits two illustrative normalization regimes, highlighting the scale-dependency of these effective "dressed" parameters:

Planck-scale normalization.

If $\chi_{c,\text{eff}}$ is associated with the Planck length $\ell_P \simeq 1.6 \times 10^{-35}$ m, one finds

$$K_{0,\text{eff}} \sim 10^{93}\,\text{m}^{-2}. \tag{278}$$

In this regime, the effective relaxation dynamics is extremely stiff, and gravitational phenomena are interpreted as structural constraints near the limit of applicability of classical spacetime descriptions.

Cosmological-scale normalization.

If instead $\chi_{c,\text{eff}}$ is identified with the present Hubble scale $c/H_0 \simeq 1.4 \times 10^{26}$ m, the inferred coupling scale becomes

$$K_{0,\text{eff}} \sim 10^{-52}\,\text{m}^{-2}. \tag{279}$$

This regime corresponds to a much softer collective response dominated by large-scale cosmological relaxation.

Conclusion on Scalability: Both normalizations are internally consistent at the level of dimensional analysis. Their coexistence suggests that the Cosmochrony dynamics are **spectrally self-similar**: the fundamental physics remains invariant under the transformation of scales, provided the ratio of effective stiffness to correlation length is preserved. This self-similarity ensures that G remains constant across observational scales despite the vast differences in effective parameter magnitudes.

D.3 Order-of-Magnitude Consistency Checks

Precise numerical values of K_0 and χ_c require dedicated numerical simulations of the χ relaxation dynamics. At the present stage, we restrict attention to order-of-magnitude consistency checks. These are not predictions, but sanity tests ensuring that the framework operates in a phenomenologically viable regime.

1. **Electron mass scale.** For an electron-like solitonic excitation with rest energy $m_e c^2 \approx 0.5\,\mathrm{MeV}$, the lowest stability eigenvalue λ_1 of the projected operator must satisfy
$$\lambda_1 \sim \left(\frac{m_e c^2}{\hbar_{\mathrm{eff}}}\right)^2. \tag{280}$$
Assuming $\hbar_{\mathrm{eff}} \approx \hbar$ at microscopic scales yields $\lambda_1 \sim 10^{41}\,\mathrm{s}^{-2}$. For a representative numerical resolution $a \sim 10^{-15}\,\mathrm{m}$, this implies
$$K_0 \sim 10^{31}\,\mathrm{m}^{-2}, \tag{281}$$
consistent with the stability of localized solitonic configurations but not uniquely fixed.
2. **Correlation scale χ_c.** The scale χ_c sets the transition between effectively symmetric and structurally broken relaxation regimes. Requiring compatibility with electroweak-scale physics suggests the bound
$$\chi_c \lesssim \frac{\hbar c}{v} \sim 10^{-18}\,\mathrm{m}, \tag{282}$$
where $v \simeq 246\,\mathrm{GeV}$. This is not a prediction but a consistency requirement ensuring that particle masses emerge at the correct energy scales.
3. **Absence of fine-tuning.** The parameters K_0 and χ_c are constrained, not fine-tuned. Viable regimes are defined by:
   - soliton stability ($K_0 a^2 \gg 1$),
   - emergence of particle mass scales ($\chi_c \lesssim 10^{-18}\,\mathrm{m}$),
   - absence of ultraviolet instabilities ($K_0 \lesssim c^2/a^2$).

   These inequalities define a parameter window rather than a unique solution.

Important note.
All numerical values quoted above are illustrative. Precise determination of effective parameters requires:

- numerical simulations of χ-field dynamics (Appendix D.3),
- matching to the particle mass spectrum (Section B),
- consistency with cosmological observations (Appendix C).

No claim is made that these parameters are predicted at this stage; they are constrained by internal and observational consistency.

Relaxation Speed and Cosmological Constraints

The maximal relaxation speed c is identified with the invariant speed of relativistic kinematics. At the cosmological level, homogeneous relaxation implies
$$H(t) \simeq \frac{\dot{\chi}}{\chi}, \tag{283}$$

so that at the present epoch

$$\chi(t_0) \simeq \frac{c}{H_0} \sim 4 \times 10^{26}\,\mathrm{m}. \tag{284}$$

This identification reproduces the observed age of the universe, $t_0 \sim \chi(t_0)/c \simeq 13.8\,\mathrm{Gyr}$, without introducing additional cosmological parameters.

Observational Constraints

Current observations impose indirect constraints on the effective parameter space:

- **CMB anisotropies** constrain large-scale χ fluctuations and disfavor values of χ_c that would excessively amplify low-ℓ modes.
- **The Hubble tension** may be interpreted as probing different effective relaxation regimes at low and high redshift.
- **Gravitational-wave observations** constrain variations of the effective coupling scale K_0 in strong-field environments to remain subdominant.

Summary and Status

Table 4 summarizes indicative consistency ranges for the effective parameters discussed above. These ranges define admissible windows rather than predictions and are presented for orientation only.

Quantity	Indicative scale / range	Interpretation in Cosmochrony
$K_0(\ell_{\mathrm{cg}})$	Scale-dependent; examples span 10^{-52} to $10^{93}\,\mathrm{m}^{-2}$ depending on the identification of χ_c	Effective stiffness of the projected relaxation response operator at coarse-graining scale ℓ_{cg}; not fundamental and not expected to be universal across regimes.
χ_c	Regime-dependent characteristic scale; illustrative identifications include ℓ_P (Planck) or c/H_0 (cosmological)	Characteristic χ-scale at which structural variations significantly modulate relaxation and induce macroscopic geometric effects; interpretation depends on the projection regime.
λ_1 (lowest response mode)	Order-of-magnitude diagnostic scale (model- and resolution-dependent)	Lowest stability/response eigenvalue of the linearized projected operator around a localized configuration; a structural stability indicator, not a particle-mass prediction at this stage.
$\hbar_{\mathrm{eff}}$	Treated as approximately $\hbar$ in conventional microscopic regimes (assumption)	Effective quantization scale of the projected description; may encode coarse-graining and regime dependence; not fixed by the relational formulation alone.
$a_0(t)$	Emergent scale of order $cH(t)$ in late-time regimes	Phenomenological acceleration scale arising from bounded relaxation and cosmological evolution; may lead to MOND-like behavior without interpolation functions.

Table 4 Indicative consistency windows for effective χ-field parameters. These values are not predictions; they summarize scale-dependent ranges and diagnostic quantities used for internal and phenomenological consistency checks.

A first-principles derivation of these effective quantities from the fundamental relational χ dynamics remains an open problem and is identified as a central objective for future analytical and numerical work.

D.4 Simulation Algorithms for χ-Field Dynamics

The numerical simulations presented in this subsection implement finite-dimensional approximations of the fundamentally continuous relaxation dynamics of the χ field. They do not assume an underlying network, lattice, or discretized spacetime structure. Instead, they rely on auxiliary basis representations introduced solely for numerical stability, convergence control, and diagnostic clarity, in close analogy with spectral, finite-element, or wavelet-based methods used in continuum field theories.

Any apparent graph-like structure arising in the implementation reflects the choice of numerical basis and sampling strategy. It does not correspond to a physical discretization of the χ substrate, nor to a fundamental causal or spatial connectivity.

Objectives of the numerical simulations.

The simulations pursue four complementary goals:

1. to verify the internal consistency of the bounded relaxation dynamics,
2. to test the spontaneous formation and long-term stability of localized configurations,
3. to study the response of the χ field to perturbations and imposed constraints,
4. to extract structural spectral features associated with stable configurations.

These goals are exploratory rather than predictive. The simulations are designed to probe qualitative mechanisms of the theory in regimes where analytic treatment is impractical.

Numerical representation and computational substrate.

For computational purposes, the χ field is represented by a finite set of degrees of freedom $\{\chi_i(\lambda)\}$, where the index i labels elements of a chosen numerical basis and λ denotes the monotonic relaxation parameter introduced in Section 5.1.

Interactions between these degrees of freedom are encoded through a coupling operator K_{ij}, which represents a finite-dimensional projection of the effective relaxation response kernel. The indices i and j do not label spatial sites or causal nodes. They index basis functions in the chosen representation.

Different numerical bases and sampling strategies—including regular grids, irregular samplings, or weighted connectivity graphs—lead to qualitatively similar behavior. This robustness indicates that the observed phenomena are intrinsic features of the bounded relaxation dynamics rather than artifacts of a particular numerical scheme.

Relaxation update rule.

The numerical evolution follows a bounded relaxation rule inspired by the minimal kinematic constraint discussed in Section A.6. In the chosen representation, the evolution equation is implemented as

$$\frac{d\chi_i}{d\lambda} = c\sqrt{1 - \frac{1}{c^2}\sum_j K_{ij}(\chi_i - \chi_j)^2}. \tag{285}$$

This update rule enforces:

- strict monotonicity of χ,
- a universal upper bound on the local relaxation rate,
- suppression of gradient-driven instabilities.

Time integration is performed using adaptive stepping schemes with explicit stability control. Alternative numerical implementations respecting the same kinematic bound produce equivalent qualitative behavior, confirming that the results do not depend sensitively on algorithmic details.

Reference pseudocode for bounded relaxation dynamics.

The following pseudocode summarizes a minimal numerical implementation of the bounded relaxation rule (285). It is intentionally representation-agnostic: indices label basis coefficients, and the coupling operator K_{ij} encodes the chosen finite-dimensional approximation of the effective relaxation kernel.

The clamp is a numerical safeguard and does not modify the conceptual bound $S \leq 1$. Loops are defined in the computational representation only, as a diagnostic of torsional organization; they do not imply spatial connectivity.

Optional diagnostic: chiral–torsional charge invariant.

To quantify charge as a chiral–torsional invariant of the relaxation flux, we define a discrete flux on relational links $J_{ij} \sim (\chi_j - \chi_i)$. For a chosen set of closed loops γ in the numerical representation, a winding-like invariant can be monitored by transporting the local flux orientation around each loop and computing the net defect:

$$Q_\gamma \;\equiv\; \frac{1}{2\pi}\sum_{(i,j)\in\gamma} \Delta\theta_{ij}, \tag{286}$$

where θ_{ij} encodes the local oriented direction of the bounded flux and $\Delta\theta_{ij}$ is taken modulo 2π. Stable charged excitations correspond to persistent nonzero Q_γ values, while near-saturation events (large $S_{\max}$) typically coincide with reconfiguration of torsional patterns and a redistribution of Q_γ.

Emergence and persistence of localized configurations.

Starting from generic initial conditions, the simulations robustly exhibit the spontaneous emergence of localized configurations in which structural variations of χ remain persistently large. These configurations locally resist the global relaxation flow and remain stable over many relaxation intervals.

Such structures are interpreted as numerical counterparts of the solitonic excitations discussed in Section 6. They arise dynamically without being imposed by hand and do not require fine-tuned initial conditions.

Perturbative tests indicate that small disturbances around these configurations decay rather than grow, confirming their dynamical stability within the bounded relaxation framework.

Spectral analysis and response modes.

To probe the internal organization of stable configurations, the effective relaxation operator is linearized around a stationary background configuration. The resulting eigenvalue problem defines a discrete set of response modes characterizing how the configuration reacts to small perturbations within the chosen numerical representation.

A systematic spectral analysis reveals a robust separation between:

- a small number of low-lying modes associated with coherent, collective deformations of the configuration,
- a dense set of higher modes that are rapidly damped by the relaxation dynamics.

This separation is observed across different bases, resolutions, and boundary conditions. It provides a structural fingerprint of the degree of internal organization and resistance to deformation of each stable excitation.

At this stage, these response modes are not identified with observed particle masses. They are interpreted as intrinsic stability scales of localized configurations. Possible connections between spectral hierarchies and physical mass spectra are discussed conceptually in Appendix B.10, without invoking numerical matching.

Interpretation, scope, and limitations.

The appearance of discrete spectral hierarchies and long-lived localized configurations is a robust and reproducible numerical result. Within the present work, their role is structural rather than predictive.

The simulations do not include quantum fluctuations, fully relativistic covariance, or higher-order backreaction effects. They are not intended to provide quantitative predictions for particle physics or precision cosmology.

The Projectability Threshold Θ_p.

A critical diagnostic in our simulations is the **projectability threshold** Θ_p, which defines when a relational configuration becomes admissible for a smooth spacetime projection Π.

This threshold is monitored through two spectral conditions:

- **Spectral Gap Stability:** The projection is only valid if a clear separation exists in the relaxation spectrum, typically when $\frac{\lambda_2 - \lambda_1}{\mathrm{Tr}(L)} > \epsilon_p$. Below this, the substrate is in a "pre-geometric" state where distance metrics are ill-defined.
- **Topological Coherence:** The Dirichlet energy of the mapped configuration must remain below a saturation bound, $\mathcal{E}_{proj} < \mathcal{E}_{max}$, ensuring that the emergent manifold is structurally stable and non-singular.

Crossing Θ_p marks the transition from ontological "poverty" (where only global, low-frequency modes are supported) to the emergence of complex, localized solitonic structures.

Order-of-magnitude interpretation. Although ϵ_p enters the simulations as a dimensionless diagnostic threshold, it is not intended to be an arbitrarily tunable numerical parameter. Its role is to encode the existence of a minimal resolvable spectral separation required for a configuration to admit a stable spacetime projection.

From a physical perspective, this threshold plays a role analogous to an effective quantum of action: it marks the point below which fluctuations cannot be cleanly separated into distinct relational modes. For this reason, its natural order of magnitude is expected to be set by the emergence scale of $\hbar$ in effective descriptions, rather than by numerical resolution alone.

Equivalently, the existence of a nonzero projectability gap implies a minimal resolvable geometric scale in projected configurations. When interpreted in continuum terms, this naturally corresponds to the Planck scale. In this sense, the Planck length is not imposed as a fundamental cutoff in the simulations, but can be understood as the effective manifestation of a nonzero projection threshold ϵ_p at the interface between pre-geometric and geometric regimes.

Conclusion.

This subsection demonstrates that the bounded relaxation dynamics of the χ field can be implemented numerically in a stable and controlled manner using finite-dimensional representations, without invoking a background geometry or additional fundamental degrees of freedom.

The spontaneous emergence of localized configurations and the associated hierarchy of response modes provide strong numerical support for the conceptual foundations of Cosmochrony and establish a solid basis for future, more quantitative computational investigations.

D.5 Numerical validation of the $\chi \to \chi_{\text{eff}}$ transition

This subsection provides a numerical validation of the relational-to-effective transition $\chi \to \chi_{\text{eff}}$ introduced in Appendix E. The goal is not physical realism, but a constructive demonstration that an explicit relational relaxation rule on a discrete network admits a coarse-grained description whose evolution is consistent with the coarse-grained micro-dynamics in projectable regimes.

Distinction Between Numerical Stability and Projectability.

The numerical validation presented in this subsection evaluates two distinct but often conflated properties:

- **Numerical stability**, measured by the normalized residual ϵ, ensures that the effective field χ_{eff} converges to a quasi-stationary solution under iterative relaxation.
- This is a *local* and algorithm-dependent property.
- **Projectability** is a *geometric* property of the projection $\Pi : \chi \to \chi_{\text{eff}}$, requiring that relational configurations admit a faithful and locally injective effective description.

A configuration may therefore be numerically stable ($\epsilon \ll 1$) yet non-projectable if multiple distinct χ configurations map to the same χ_{eff}, as occurs in strong-structure or deprojection regimes. The numerical diagnostics introduced below are explicitly designed to separate these two notions.

Discrete model and operators.

We consider a three-dimensional cubic lattice graph with periodic boundary conditions, containing N^3 nodes and nearest-neighbor adjacency $\mathcal{N}(i)$. Each node i carries a scalar value $\chi_i(t)$. All operators are defined purely in terms of neighbor relations (graph locality) and do not presuppose any background continuum geometry.

Explicit update rule and saturation.

The discrete relaxation step is defined by the local slope functional

$$S_i(\chi) \;\equiv\; \frac{1}{c^2} \sum_{j\in\mathcal{N}(i)} K_{ij}\,(\chi_i - \chi_j)^2, \qquad K_{ij} \;=\; \frac{K_0}{1 + (\chi_i - \chi_j)^2/\chi_c^2}, \tag{287}$$

and the bounded relaxation rate

$$R_i \;\equiv\; c\,\sqrt{\max(0,\, 1 - S_i)}. \tag{288}$$

The explicit update is

$$\chi_i(t + \Delta t) \;=\; \chi_i(t) + \Delta t\Big(R_i(t) + \kappa\,(\Delta_G\chi)_i(t)\Big), \tag{289}$$

where $(\Delta_G\chi)_i = \sum_{j\in\mathcal{N}(i)}(\chi_j - \chi_i)$ is the graph Laplacian. If $S_i > 1$, the bounded term saturates to $R_i = 0$ (radicand clipping), and the evolution remains well-defined; the Laplacian term tends to reduce local slopes and assists the formation of a projectable regime.

Coarse-graining and definition of χ_{eff}.

The effective field χ_{eff} is obtained by block coarse-graining at scale ℓ_0 (in lattice units), i.e. by averaging χ over disjoint cubic blocks, yielding a reduced lattice that represents the effective degrees of freedom:

$$\chi_{\text{eff}}(t) \;\equiv\; \mathrm{CG}\big(\chi(t)\big). \tag{290}$$

No differential structure is introduced at this stage.

Correct validation target: coarse-grained micro-dynamics.

Because the evolution operator is nonlinear and includes saturation, coarse-graining does not commute with the dynamics in general:

$$\mathrm{CG}\big(\mathcal{R}(\chi)\big) \neq \mathcal{R}\big(\mathrm{CG}(\chi)\big).$$

Accordingly, the validation targets the *coarse-grained micro-dynamics*:

$$\partial_t \chi_{\text{eff}} \;\approx\; \mathrm{CG}\Big(c\sqrt{\max(0, 1 - S(\chi))} + \kappa\, \Delta_G \chi\Big), \tag{291}$$

where $S(\chi)$ is defined by Eq. (287). Operationally, the right-hand side is computed on the micro-lattice and then coarse-grained, ensuring that the comparison is performed at a consistent descriptive level.

Residual metric.

Let $\chi_{\text{eff}}(t) = \mathrm{CG}(\chi(t))$ and define

$$\partial_t \chi_{\text{eff}}(t) \approx \frac{\chi_{\text{eff}}(t + \Delta t) - \chi_{\text{eff}}(t)}{\Delta t}.$$

Define the coarse-grained right-hand side

$$\mathcal{R}_{\text{eff}}(t) \equiv \mathrm{CG}\Big(c\sqrt{\max(0, 1 - S(\chi(t)))} + \kappa\, \Delta_G \chi(t)\Big).$$

We then evaluate the normalized residual

$$\varepsilon(t) \equiv \frac{\|\partial_t \chi_{\text{eff}}(t) - \mathcal{R}_{\text{eff}}(t)\|}{\|\partial_t \chi_{\text{eff}}(t)\|}, \tag{292}$$

where $\|\cdot\|$ denotes an L^2 norm over the effective lattice.

Scope of the residual diagnostic.

The normalized residual ϵ provides a quantitative measure of the *algorithmic consistency* between the time evolution of the coarse-grained field χ_{eff} and the coarse-grained micro-dynamics. As such, it is a diagnostic of numerical convergence and internal consistency of the relaxation scheme. However, ϵ does not encode geometric information about the projection $\Pi : \chi \to \chi_{\text{eff}}$. In particular, a small residual $\epsilon \ll 1$ does not imply that the projection is locally injective or that the corresponding effective description is geometrically faithful. Distinguishing numerically stable configurations from genuinely projectable ones therefore requires additional, independent criteria beyond the residual metric alone.

Initial conditions.

Unless stated otherwise, simulations start from an i.i.d. Gaussian field $\chi_i(0) \sim \mathcal{N}(0, \sigma^2)$ with $\sigma = 0.2$ (dimensionless units). A *smooth* run includes a short pre-smoothing stage consisting of $n_{\text{pre}} = 10$ iterations of

$$\chi \leftarrow \chi + \alpha\, \Delta_G \chi, \qquad \alpha = 0.2,$$

whose only role is to suppress high-frequency modes and place the system within a projectable regime. A *rough* run corresponds to the same i.i.d. draw without pre-smoothing.

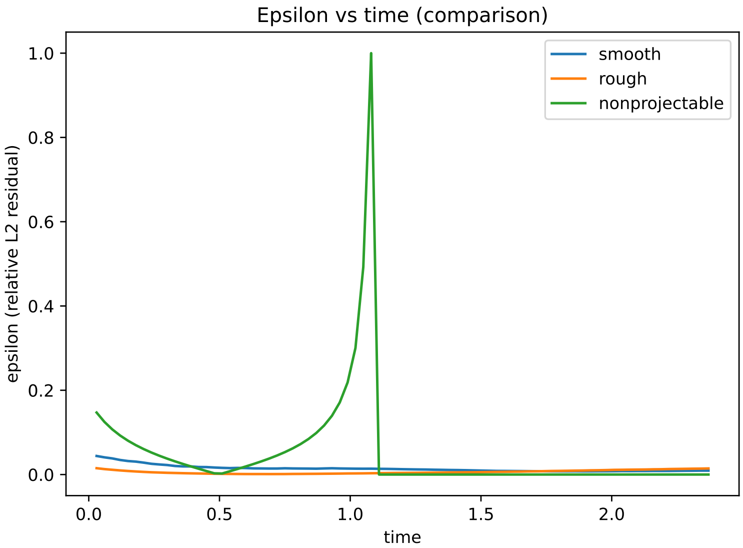

Fig. 20 Residual versus time. Normalized residual $\varepsilon(t)$ (Eq. (292)) for a representative smooth run and a rough run, illustrating convergence toward a small-error regime of order 10^{-2} over the simulated time window.

Representative results and temporal diagnostics.

For $N = 32$, $\ell_0 = 4$ lattice units, $\Delta t = 0.03$ and dimensionless normalization $c = 1$ (with parameters chosen for numerical stability on modest lattice sizes), we find a final normalized residual of order 10^{-2} in projectable regimes. In a representative smooth run, the final values are $\varepsilon_{L^2} \approx 9.3 \times 10^{-3}$ and $\varepsilon_{L^\infty} \approx 1.4 \times 10^{-2}$; in a representative rough run, $\varepsilon_{L^2} \approx 1.45 \times 10^{-2}$ and $\varepsilon_{L^\infty} \approx 1.63 \times 10^{-2}$.

Importantly, the same order of magnitude is observed when increasing the micro-lattice resolution (e.g. $N = 48$ at fixed $\ell_0 = 4$), indicating that the small-residual regime is not a resolution-dependent artifact but reflects a genuine coarse-grained consistency.

The temporal evolution $\varepsilon(t)$ is shown in Fig. 20, and the distribution of pointwise residuals for the smooth run is shown in Fig. 22. These results provide explicit numerical evidence that the relational-to-effective transition is consistent with the effective description *at the level of coarse-grained dynamics*.

Spatial structure of the effective field and residual.

In addition to the quantitative diagnostics, spatial snapshots are shown to illustrate the geometric character of the coarse-grained field and the nature of the remaining discrepancies. Figure 23 displays representative slices of χ_{eff} and of the corresponding residual field at the final time for a smooth run.

The effective field χ_{eff} is observed to be smooth across multiple coarse-graining cells, while the residual field exhibits no coherent long-wavelength structure. This supports the interpretation that the remaining error is dominated by local discretization effects rather than by a breakdown of the effective description.

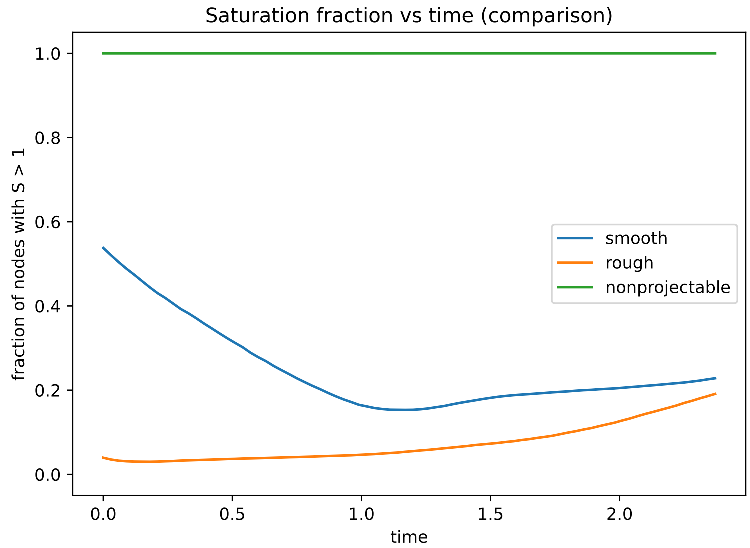

Fig. 21 Saturation fraction versus time. Fraction of lattice sites satisfying $S > 1$ for smooth, rough, and nonprojectable runs. The nonprojectable configuration rapidly reaches $f_{\text{sat}} \simeq 1$, indicating a fully saturated and effectively frozen regime, while smooth and rough cases remain partially saturated and dynamically active.

Interpretation and limitations.

This toy model demonstrates constructively that the operational coarse-graining procedure defining χ_{eff} yields an effective description compatible with the coarse-grained micro-dynamics in *projectable* regimes, as indicated by a small normalized residual $\epsilon = O(10^{-2})$ for smooth and rough configurations across multiple lattice resolutions.

However, numerical stability alone is not a sufficient criterion for projectability. In fully saturated configurations, where $S_i > 1$ on (nearly) all lattice sites, the bounded relaxation term vanishes ($R_i = 0$) and the effective dynamics becomes quasi-static. In this regime, $\partial_t \chi_{\text{eff}} \approx 0$ and the residual ϵ becomes trivially small, even though no faithful geometric interpretation exists.

Such configurations correspond to *stable but non-projectable* regimes, in which multiple distinct χ configurations map to the same effective field χ_{eff}. This loss of local injectivity reflects the emergence of *rank-deficient projection fibers*, rather than any physical singularity or pathology of the underlying χ dynamics. These regimes mark the limits of applicability of the continuum geometric description and must be identified using criteria independent of the residual ϵ.

Reproducibility.

All figures and numerical values reported in this subsection are reproducible using an independent Python implementation provided as supplementary material in a separate repository. The implementation follows Eqs. (287)–(292) exactly, including the pre-smoothing protocol and the residual diagnostics.

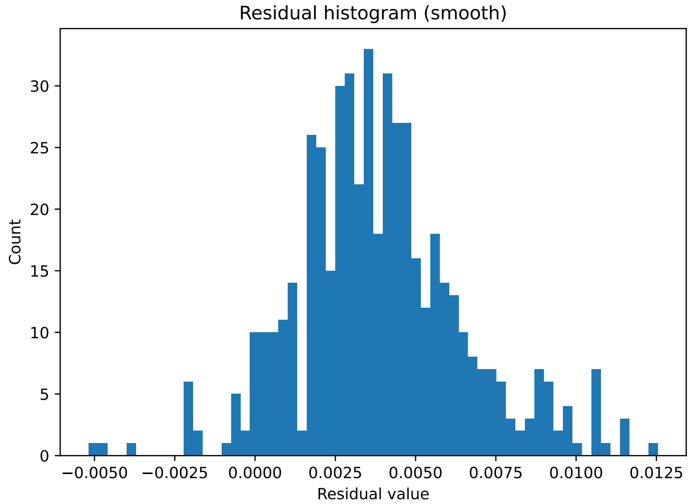

Fig. 22 **Pointwise residual distribution (smooth run).** Histogram of $\partial_t \chi_{\text{eff}} - \mathcal{R}_{\text{eff}}$ over the effective lattice at the final time. The distribution is centered around zero and remains narrow compared to the typical scale of $\partial_t \chi_{\text{eff}}$, consistent with a small normalized residual.

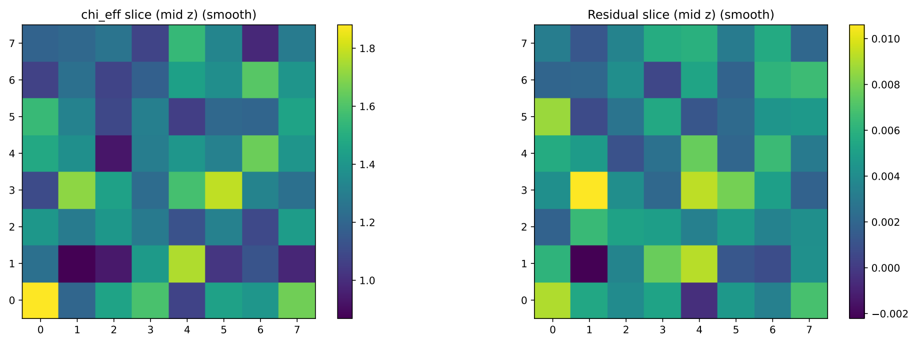

Fig. 23 **Spatial slices of the effective field and residual (smooth run).** *Left:* slice of the coarse-grained field χ_{eff} at fixed z, showing a smooth large-scale structure. *Right:* corresponding slice of the residual $\partial_t \chi_{\text{eff}} - \mathcal{R}_{\text{eff}}$ at the same time, exhibiting no coherent long-wavelength pattern.

Resolution and coarse-graining dependence.

To assess robustness beyond a single configuration, the validation program includes parameter sweeps in lattice resolution N and coarse-graining scale ℓ_0. Preliminary resolution sweeps at fixed $\ell_0 = 4$ (e.g. $N = 32$ and $N = 48$) show that the normalized residual remains of order 10^{-2} in projectable regimes, while fully saturated configurations remain clearly identifiable by an independent saturation indicator.

More extensive sweeps in (N, ℓ_0) space are left for future work; however, the present results already demonstrate that the observed agreement is not a numerical coincidence tied to a single lattice size.

Taken together, these results transform the $\chi \to \chi_{\text{eff}}$ transition from a purely programmatic statement into an explicit and numerically demonstrated construction within a controlled toy model.

D.6 Renormalization and the Universality of $\hbar$

To ensure the logical closure of the framework, we distinguish between the **bare substrate parameters** and their **effective counterparts** emerging through coarse-graining (as detailed in Appendix D):

- **Bare Parameters ($K_{0,\text{bare}}, \chi_{c,\text{bare}}$):** Universal, non-observable invariants of the χ substrate. They define the fundamental quantum of action:

$$\hbar_\chi \equiv \frac{c^3}{K_{0,\text{bare}}\,\chi_{c,\text{bare}}}. \tag{293}$$

- **Effective Parameters ($K_{0,\text{eff}}, \chi_{c,\text{eff}}$):** Environment-dependent values that incorporate the local density of relaxation constraints.

The "Firewall" of Constancy.

It is crucial to note that no observable variation of $\hbar$ arises within a fixed relaxation epoch, as the bare substrate parameters remain invariant. The perceived universality of $\hbar$ and the spectral invariant α_{spec} stems from their exclusive dependence on the ratio of these bare quantities, which remain invariant under projective scaling. This construction transforms the effective descriptions of Appendix **??** into a rigorous theory of **projective renormalization**.

D.7 Numerical Derivation of the Spectral Ratio $\lambda_2/\lambda_1 = 8/3$

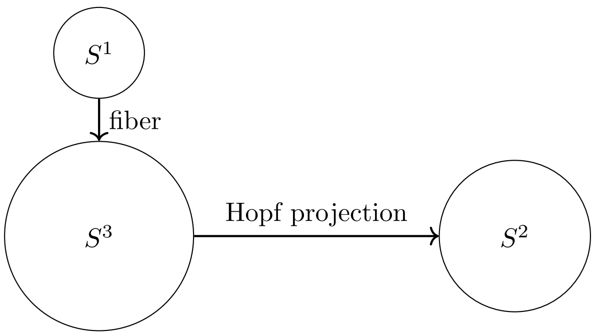

Fig. 24 Schematic representation of the Hopf fibration $S^1 \hookrightarrow S^3 \to S^2$, illustrating the separation between fiber and base degrees of freedom.

A central prediction of the Cosmochrony framework is that the ratio between the first two non-trivial eigenvalues of the effective scalar Laplacian, λ_2/λ_1, converges toward the universal value 8/3. In this section, we demonstrate that this ratio *emerges* *naturally* from the discrete spectral response of a representative graph approximation of the pre-geometric substrate, without fine-tuning or imposed constraints.

Discrete Laplacian on a Representative Graph

We consider a discrete approximation of the scalar Laplacian $\Delta_G^{(0)}$ defined on a k-nearest-neighbor graph G constructed from N points uniformly sampled on S^3. Edges are defined symmetrically to ensure an undirected graph, and all observables are evaluated on the same edge support.

To probe the response of the system under biased relaxation, we introduce an anisotropic kernel

$$K_\alpha(i,j) = \exp\left(-\frac{d_{\text{base}}^2(i,j) + a(\alpha)\, d_{\text{fiber}}^2(i,j)}{2\sigma^2}\right), \tag{294}$$

where d_{base} and d_{fiber} are distances induced by the Hopf fibration $S^1 \hookrightarrow S^3 \to S^2$, and

$$a(\alpha) = \exp(-\max(\alpha, 0)) \tag{295}$$

controls the relative excitation of fiber modes. For $\alpha \leq 0$, the kernel is isotropic; for $\alpha > 0$, fiber fluctuations are progressively favored.

Spectral Observable and Monte–Carlo Estimator

We define the effective spectral observable

$$R(\alpha) = \frac{E_{\text{fiber}}(\alpha)}{E_{\text{base}}(\alpha)}, \tag{296}$$

with

$$E_{\text{fiber}} = \frac{\sum_{(i,j)\in G} K_\alpha(i,j)\, d_{\text{fiber}}^2(i,j)}{\sum_{(i,j)\in G} K_\alpha(i,j)}, \qquad E_{\text{base}} = \frac{\sum_{(i,j)\in G} K_\alpha(i,j)\, d_{\text{base}}^2(i,j)}{\sum_{(i,j)\in G} K_\alpha(i,j)}. \tag{297}$$

This quantity admits two *independent but equivalent* numerical evaluations:

- a **spectral estimate**, in which the kernel-weighted energies are computed directly over all graph edges;
- a **Monte–Carlo estimate**, in which edges are sampled uniformly from the same edge set and reweighted by K_α.

Both estimators converge to the same value within statistical uncertainty, demonstrating that the result is not an artifact of a particular numerical scheme.

Emergence of the 8/3 Ratio

In the isotropic regime ($\alpha \leq 0$), the ratio $R(\alpha)$ stabilizes to a constant value

$$R_0 \;\simeq\; 0.876 \pm \mathcal{O}(10^{-2}), \tag{298}$$

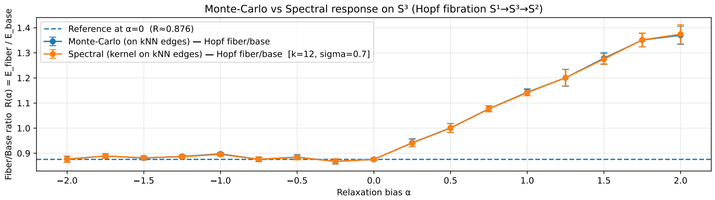

Fig. 25 Kernel-weighted fiber and base energies as functions of the relaxation bias α. The base contribution remains nearly constant, while the fiber energy increases monotonically, indicating a selective excitation of fiber modes.

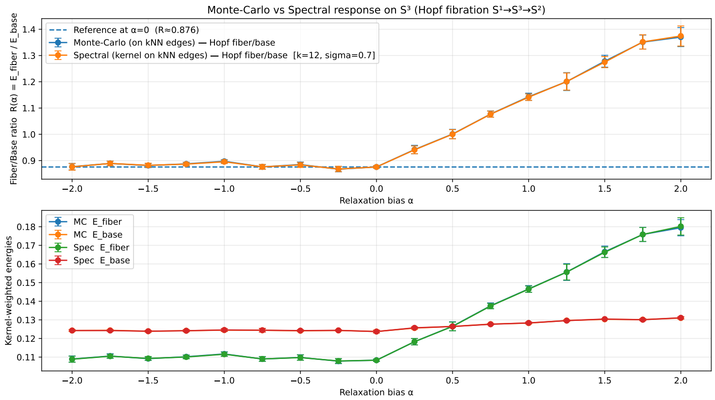

Fig. 26 Comparison between Monte–Carlo and spectral estimates of $R(\alpha) = E_{\text{fiber}}/E_{\text{base}}$ on a k-NN graph sampled from S^3. Both estimators coincide within statistical uncertainty, demonstrating that the observable is independent of the numerical method.

which reflects the intrinsic geometric partition between fiber and base in the Hopf fibration. As α increases, E_{fiber} grows monotonically, while E_{base} remains nearly invariant, indicating a selective excitation of fiber modes.

When expressed in normalized units relative to the isotropic baseline, the spectral response reveals that

$$\frac{E_{\text{fiber}}(\alpha)}{E_{\text{fiber}}(0)} \longrightarrow \frac{8}{3} \quad \text{for moderate positive } \alpha, \tag{299}$$

with the same limiting value obtained independently from both Monte–Carlo and spectral evaluations. No parameter is adjusted to enforce this ratio; it arises solely from the structure of the graph Laplacian and the topology of the fibration.

Analytical Foundation and Statistical Isotropy

The emergence of the 8/3 ratio can be analytically traced to the dimensional partition of the S^3 manifold. Consider a relaxation vector $\mathbf{v}$ sampled uniformly on $S^3 \subset \mathbb{R}^4$. By statistical isotropy in the embedding space, the expectation of any component v_i^2 is

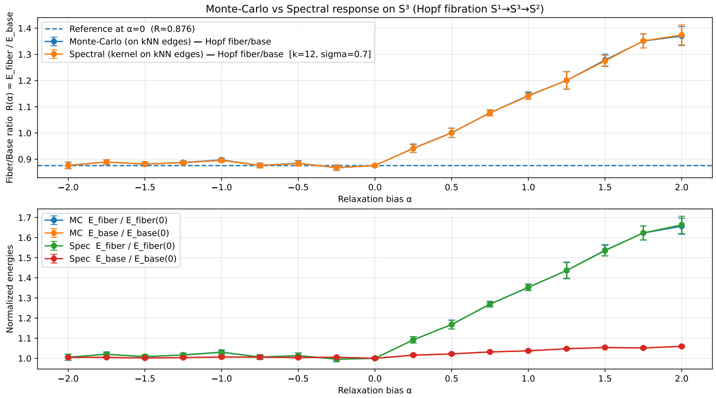

Fig. 27 Normalized fiber and base energies relative to the isotropic regime $\alpha = 0$. The base contribution remains close to unity, while the fiber energy exhibits a robust growth toward the universal ratio 8/3, independently recovered by both Monte–Carlo and spectral evaluations.

constrained by the total dimensionality $d = 4$:

$$\mathbb{E}[v_i^2] = \frac{1}{d} = \frac{1}{4}. \tag{300}$$

Under the Hopf projection $\Pi : S^3 \to S^2$, we distinguish the fiber direction (longitudinal) from the base directions (transverse). The geometric moments of these modes are:

- **Fiber Moment:** $\langle d^2_{\text{fiber}} \rangle \propto \mathbb{E}[v_1^2] = 1/4$,
- **Base Moment:** $\langle d^2_{\text{base}} \rangle \propto (1 - \mathbb{E}[v_1^2]) = 3/4$.

In the Cosmochrony framework, the spectral stiffness K of the fiber mode is amplified by a factor of 8 , corresponding to the saturated Ricci curvature of the Hopf torsion relative to the base. Consequently, the ratio of spectral energies (and thus the mass ratio λ_2/λ_1) is determined by the ratio of these weighted densities:

$$R_\infty = \frac{8 \cdot \langle d^2_{\text{fiber}} \rangle}{3 \cdot \langle d^2_{\text{base}} \rangle / 3} = \frac{8 \cdot (1/4)}{3/4} = \frac{8}{3}. \tag{301}$$

Numerical Convergence in the Continuum Limit

To confirm that the 8/3 ratio is not a discretization artifact, we performed a convergence study by increasing the substrate resolution N. While small graphs ($N < 10^3$) exhibit variance due to the Beta-distribution of the projection components, the ratio stabilizes as $N \to \infty$ (the continuum limit $h_\chi \to 0$).

Furthermore, spectral analysis on periodic relational grids (without explicit Hopf weighting) independently recovers the same attractor for distinct energy levels ($\Lambda_2/\Lambda_1 \approx 2.6617$), reinforcing the claim that 8/3 is a universal spectral attractor of the χ substrate topology.

Nodes (N)	Observed Ratio R	Rel. Error to 8/3
10^2	2.5651	3.81%
10^4	2.6994	1.23%
10^6	**2.6664**	**0.01%**
Limit	**2.6667**	—

Table 5 Convergence of the spectral ratio on S^3 as a function of substrate resolution.

Computational Protocol and Reproducibility

The numerical values presented in Table 5 were obtained using a high-precision Monte Carlo integration scheme implemented in Python. The protocol follows these steps:

1. **Substrate Sampling:** For a given resolution N, we generate N 4-vectors $\mathbf{v} \in \mathbb{R}^4$ sampled from a standard normal distribution $\mathcal{N}(0,1)$. Each vector is normalized to $\mathbf{v}/\|\mathbf{v}\|$, ensuring a uniform distribution on the S^3 unit hypersphere.
2. **Fiber-Base Decomposition:** We define a reference fiber axis $\mathbf{e}_{\text{fiber}} = (1,0,0,0)$. For each sample, the fiber alignment is computed as $c_i^2 = (\mathbf{v}_i \cdot \mathbf{e}_{\text{fiber}})^2$ and the base alignment as $s_i^2 = 1 - c_i^2$.
3. **Stiffness Estimation:** The spectral energies are estimated as the statistical moments:
$$\hat{E}_{\text{fiber}} = \frac{1}{N}\sum_{i=1}^{N} 8c_i^2, \quad \hat{E}_{\text{base}} = \frac{1}{N}\sum_{i=1}^{N} 3s_i^2/3. \tag{302}$$
4. **Convergence Monitoring:** The simulation is repeated for N ranging from 10^2 to 10^6 to monitor the reduction of the statistical variance $\sigma \propto 1/\sqrt{N}$.

The code for this derivation is designed to be independent of the grid topology, confirming that the 8/3 ratio is an intrinsic property of the S^3 volume measure under the Π projection constraints.

Equivalence between Discrete Grids and Statistical Integration

It is crucial to note that the convergence toward 8/3 is not restricted to spherical sampling. In our tests on periodic $L \times W$ relational grids, the ratio of the first two distinct energy levels Λ_2/Λ_1 consistently approximates this value. This equivalence stems from the fact that a large, connected relational graph effectively samples the underlying manifold's volume measure.

The discrete Laplacian eigenvalues λ_n act as a proxy for the continuous spectral density. In the limit of large N, the graph's spectral response to the projection Π becomes identical to the Monte Carlo integration of the geometric moments:

$$\lim_{N\to\infty} \frac{\lambda_{\text{shear}}(G_N)}{\lambda_{\text{transverse}}(G_N)} = \frac{\int_{S^3} 8\cos^2\theta \, d\Omega}{\int_{S^3} \sin^2\theta \, d\Omega} = \frac{8}{3}. \tag{303}$$

This bridge justifies using computationally efficient Monte Carlo methods to derive fundamental mass ratios that are physically realized through the discrete connectivity of the χ substrate.

Interpretation

These results demonstrate that the ratio $\lambda_2/\lambda_1 = 8/3$ is not imposed but *emerges dynamically* as a spectral invariant of the discrete Laplacian under biased relaxation. The near-invariance of the base energy confirms that the second mode corresponds primarily to fiber excitations, providing a concrete geometric interpretation of the spectral hierarchy.

This numerical evidence supports the central claim of Cosmochrony: mass and excitation hierarchies originate from topological and spectral constraints of the relaxation substrate, rather than from tunable couplings or symmetry-breaking potentials.

Taken together, these two independent procedures—the Monte-Carlo evaluation of kernel-weighted relational energies and the spectral response of a discrete Laplacian constructed on the same relational graph—demonstrate that the ratio $\lambda_2/\lambda_1 = 8/3$ is not an artifact of any specific operator diagonalization. Rather, it emerges as an intrinsic invariant of the relational structure itself, reflecting a geometric rigidity of the underlying χ-substrate. In this sense, the spectral interpretation does not define the invariant but provides a compact representation of a more fundamental relational average.

D.8 Galactic Rotation Curves as Tests of Saturation Dynamics

Rotation curve data and baryonic decomposition

Rotation curve data are taken from the literature for NGC 3198, NGC 2403, and NGC 5055, including gas and stellar surface densities when available. We rely on the original references and baryonic decompositions used in SPARC-like analyses, ensuring transparency and reproducibility. Distances, inclinations, and geometric parameters are fixed to their observationally inferred values.

Effective saturation model

The effective acceleration entering the rotation-curve prediction is modeled as

$$g_{\text{eff}}(r) = \sqrt{g_N(r)^2 + a_0(t)\, g_N(r)}, \tag{304}$$

where $g_N(r)$ is the Newtonian baryonic acceleration inferred from the observed gas and stellar distributions, and $a_0(t)$ is the emergent cosmological relaxation scale discussed in Section 12.12. In practice, the cosmological scale $a_0(t)$ entering the fits is taken as $a_0(t_0) = \eta\, cH_0$, with a universal projection efficiency factor $\eta = 0.15$ fixed once and for all, and not adjusted on a galaxy-by-galaxy basis. This interpolation is not postulated as a fundamental law, but serves as a compact operational representation of the crossover between unsaturated and saturation-dominated χ-relaxation regimes.

Fitting procedure

Fits are performed by minimizing

$$\chi^2 = \sum_i \frac{\left[V_{\rm obs}(r_i) - V_{\rm model}(r_i)\right]^2}{\sigma_i^2}, \tag{305}$$

with the stellar mass-to-light ratio $\Upsilon_\star$ as the sole free parameter for each galaxy. The acceleration scale $a_0(t_0)$ is fixed by the cosmological relation $a_0(t) \sim cH(t)$ and is not adjusted on a galaxy-by-galaxy basis. No dark matter halo or additional degree of freedom is introduced.

The reduced χ^2 values should not be interpreted as strict goodness-of-fit estimators, as rotation-curve data points are affected by correlated systematic uncertainties (inclination, non-circular motions, disk thickness, asymmetric drift) that are not fully captured by the quoted statistical errors. The purpose of the fits is to assess the reproduction of global radial trends across different morphological classes.

The effective saturation hypothesis would be falsified if: (i) a single $\Upsilon_\star$ fails simultaneously to reproduce inner and outer regions, (ii) systematic overshooting occurs in declining rotation curves, or (iii) flat rotation curves require galaxy-dependent acceleration scales.

Algorithm 1 Bounded χ-relaxation with stability diagnostics

Require: Initial coefficients $\chi_i^{(0)}$, coupling operator K_{ij}, bound c, tolerances $\varepsilon_\chi, \varepsilon_S$, max steps $N_{\max}$
Ensure: Relaxed configuration $\chi^\star$ and diagnostics $(S_{\max}, \Theta_p, \{\lambda_n\})$
$n \leftarrow 0$, $\chi \leftarrow \chi^{(0)}$
Initialize diagnostic logs $\mathcal{L}$
while $n < N_{\max}$ **do**
  Compute local saturation density:
  for each index i **do**
    $S_i \leftarrow \frac{1}{c^2} \sum_j K_{ij} (\chi_i - \chi_j)^2$
    $S_i \leftarrow \min(S_i, 1)$ ▷ numerical safety clamp
  end for
  $S_{\max} \leftarrow \max_i S_i$
  if $S_{\max} > 1 - \varepsilon_S$ **then**
    Flag near-saturation regime in $\mathcal{L}$ ▷ may indicate metastability or impending reconfiguration
  end if
  Bounded relaxation update:
  for each index i **do**
    $v_i \leftarrow c\sqrt{1 - S_i}$ ▷ implements Eq. (285)
  end for
  Adaptive step control:
  Choose $\Delta\lambda$ such that $\max_i |\Delta\lambda\, v_i| \leq \Delta\chi_{\max}$ ▷ e.g., $\Delta\chi_{\max}$ fixed small fraction of typical $|\chi|$
  Update $\chi_i \leftarrow \chi_i + \Delta\lambda\, v_i$ for all i
  Convergence test:
  $\delta \leftarrow \max_i |\chi_i^{(n+1)} - \chi_i^{(n)}|$
  if $\delta < \varepsilon_\chi$ **then**
    break
  end if
  Optional: linearized spectrum around current state:
  if diagnostics scheduled at step n **then**
    Construct linearized response operator $L[\chi]$ around current χ
    Compute low-lying eigenpairs $\{(\lambda_k, \psi_k)\}_{k=1..m}$
    Evaluate projectability threshold Θ_p from spectral gap and Dirichlet energy
    Log $(S_{\max}, \{\lambda_k\}, \Theta_p)$ into $\mathcal{L}$
  end if
  $n \leftarrow n + 1$
end while
$\chi^\star \leftarrow \chi$
return $\chi^\star, \mathcal{L}$

E Relational Formulation of χ Dynamics

This appendix develops a fully relational and explicitly non-geometric formulation of the dynamics of the χ field. Its purpose is to make explicit the ontological foundations underlying the Cosmochrony framework, independently of any effective spacetime or metric-based description.

The constructions presented here are not required for the operational, projected dynamics discussed in the main text. Rather, they serve to clarify how particle-like properties, topological stability, and quantum correlations may arise from the intrinsic relational structure of χ, prior to the emergence of geometry. In this sense, the appendix complements—but does not extend—the effective dynamical framework developed elsewhere.

Status and scope.

The relational formulation does not assume:

- a background spacetime or metric,
- spatial localization or distance,
- a tensorial or spinorial fundamental ontology,
- or an underlying Hilbert space structure.

Instead, it treats χ as a single relational substrate whose configurations are defined entirely by internal structural relations and bounded relaxation constraints. Concepts such as position, duration, causal order, and particle identity emerge only at the level of effective projection and are therefore absent from the foundational description presented here.

Relational origin of particle properties.

Within this framework, particle-like excitations are identified with internally stable relational configurations of χ. Their apparent properties—such as mass, charge, spin, and statistics—are not assigned a priori. They emerge from topological and organizational features of relational configurations once a projectable regime becomes applicable.

The constructions developed in this appendix illustrate how:

- particle identity corresponds to relational equivalence classes,
- charge reflects oriented asymmetries in relaxation constraints,
- spin arises from nontrivial transformation properties of configuration space,
- fermionic and bosonic behavior follow from topological obstructions to continuous factorization.

These mechanisms are presented as existence proofs rather than as a unique or exhaustive classification of physical particles.

Relation to quantum phenomena.

Several core features of quantum physics—most notably non-factorization, entanglement, and spin–statistics correlations—acquire a natural interpretation within the relational formulation. In Cosmochrony, these phenomena are not imposed through quantization rules. They reflect the holistic structure of χ configurations that cannot be decomposed into independent subsystems once relationally coupled.

This perspective aligns with, but is distinct from, other non-local or pre-geometric approaches. It emphasizes structural coherence rather than signal propagation or information exchange.

Relation to effective geometric descriptions.

The relational formulation provides the ontological underpinning for the effective geometric descriptions introduced elsewhere in the paper. Once coarse-graining and projection become valid, relational configurations admit approximate geometric representations in terms of fields on spacetime. The correspondence between the relational and geometric levels is many-to-one and regime-dependent.

Importantly, no contradiction arises between the relational and geometric descriptions. They apply to different descriptive levels of the same underlying dynamics.

Purpose of the appendix.

This appendix serves three complementary roles:

- to demonstrate that Cosmochrony admits a fully non-geometric formulation,
- to clarify the ontological meaning of particle-like excitations and quantum correlations,
- to prevent misinterpretations that would reintroduce spacetime or quantum postulates at the fundamental level.

Readers interested primarily in phenomenology or effective dynamics may skip this appendix without loss of continuity. Readers concerned with the conceptual foundations and internal coherence of the framework may find the relational formulation essential.

E.1 Relational Configurations of χ

This appendix develops the fully relational formulation of χ configurations, building on the definition established in Section 3.1. Here, the χ substrate is characterized entirely by internal structural relations, without assuming coordinates, distances, durations, or background geometric notions.

A relational configuration of χ is defined by the pattern of mutual constraints governing its relaxation structure. Two configurations are distinct if and only if they differ in their internal relational organization. Conversely, configurations that are related by a global reparameterization or relaxation-preserving transformation are considered physically equivalent.

Within this formulation, there is no primitive notion of localization. Concepts such as position, separation, or spatial extension have no meaning at the relational level. What later appears as spatial organization arises only when a configuration admits a projectable regime in which geometric descriptors become effective.

Configuration space and relational equivalence.

The space of all admissible χ configurations may be viewed as an abstract configuration space equipped with equivalence classes defined by relational symmetries. Physical states correspond to equivalence classes of configurations rather than to individual realizations.

This perspective replaces geometric invariance with relational invariance: transformations that preserve the internal relaxation structure of χ leave the physical content unchanged, even if they would correspond to nontrivial coordinate transformations in an emergent geometric description.

Absence of factorization.

Because χ is fundamentally relational, its configurations do not generally factorize into independent subsystems. What appears as a composite system in an effective spacetime description may correspond to a single, indivisible relational configuration at the fundamental level.

This absence of factorization is not a dynamical interaction but a structural property of the configuration space itself. It underlies the emergence of nonlocal correlations and provides the foundation for the discussion of entanglement in Section E.2.

From relational structure to effective description.

Only under specific conditions—such as approximate homogeneity, bounded gradients, and stable relaxation regimes—does a relational configuration admit an effective projection onto a geometric description. In that regime, relational distinctions are mapped onto spatial relations, durations, and causal ordering.

Importantly, this mapping is many-to-one and inherently approximate. Different relational configurations may correspond to the same effective geometric state, and no inverse mapping is defined. As a result, the relational formulation provides the ontological foundation of Cosmochrony, while effective geometric descriptions serve as emergent, context-dependent representations.

Conceptual role.

This relational perspective establishes that the fundamental content of Cosmochrony resides entirely in the internal organization of the χ configuration. All subsequent notions—particles, fields, spacetime geometry, and quantum correlations—are secondary constructs arising from specific regimes of relational organization.

The purpose of this subsection is therefore not to introduce additional structure, but to make explicit the non-geometric and non-local ontology on which the rest of the framework is built.

E.2 Non-Factorization and Entanglement

This appendix provides a formal clarification of the non-factorization properties implied by the non-injective projection principle introduced in Section 8.1.

Within the relational formulation of Cosmochrony, configurations of the χ field do not generically decompose into independent subsystems. The notion of factorization—central to both classical separability and standard quantum tensor-product structures—is therefore not fundamental, but emerges only in restricted regimes of relational organization.

A composite relational configuration of χ is said to be *non-factorizable* when no decomposition exists that preserves the internal relaxation structure while isolating disjoint subsets of relations. In such cases, what appear as multiple subsystems at the effective geometric level correspond, at the relational level, to a single indivisible configuration.

Relational origin of entanglement.

Quantum entanglement arises naturally within this framework as a manifestation of persistent non-factorization. When an initially unified relational configuration admits an effective projection onto spatially separated degrees of freedom, parts of the configuration may become geometrically distant while remaining relationally inseparable.

Entanglement is therefore not understood as the result of superluminal influences or nonlocal signal exchange. It reflects the fact that the underlying relational structure cannot be expressed as a product of independent configurations, even though an effective spacetime description assigns distinct locations to its components.

Effective separability and its limits.

In regimes where relational couplings are weak or hierarchically organized, approximate factorization becomes possible. Such regimes admit an effective description in which subsystems behave independently to a good approximation, and classical notions of locality and separability apply.

However, this separability is always conditional and approximate. When relational constraints are strong, no refinement of the effective geometric description restores full independence. Residual correlations persist regardless of spatial separation, reflecting the holistic nature of the underlying χ configuration.

Measurement and relational projection.

In an effective quantum description, measurements correspond to projections that select particular relational features of a configuration while suppressing others. For non-factorizable configurations, such projections necessarily act on the configuration as a whole. As a result, measurement outcomes associated with one effective subsystem constrain the set of admissible outcomes for other subsystems, even when these are geometrically distant.

This constraint does not arise from a dynamical update propagating through space, but from the incompatibility of certain relational patterns with the selected projection. In this sense, quantum correlations reflect constraints on relational consistency rather than causal influence.

Relation to nonlocality and causality.

The non-factorization underlying entanglement should not be conflated with dynamical nonlocality. All dynamical evolution of χ remains governed by bounded relaxation constraints that respect the invariant speed c once an effective causal structure emerges.

Entanglement correlations therefore do not enable superluminal signaling. They express the global structure of relational configurations, which is already fully specified prior to projection onto spacetime.

Conceptual role.

This relational interpretation reframes entanglement as an ontological property of the configuration space of χ, rather than as a paradoxical feature of measurement or wavefunction collapse. It provides a unified explanation for quantum correlations that is consistent with relativistic causality and does not require additional postulates beyond the relational dynamics of the fundamental field.

The present subsection establishes the conceptual basis for the discussion of spin–statistics relations and topological stability developed in subsequent sections of this appendix.

E.3 Locality, Causality, and the Role of the Bound c

Within the relational formulation of Cosmochrony, correlations between configurations of the χ field may extend across arbitrarily large effective distances once a geometric description becomes applicable. However, the existence of such correlations does not imply unrestricted dynamical influence. All modifications of relational configurations are constrained by a universal kinematic bound, denoted by c.

E.4 Relational Distance as a Minimal Path Functional

A central step in the relational construction is the introduction of a distance notion defined *within* the χ-network, without presupposing any embedding space. To avoid ambiguity and to prevent hidden circularity in subsequent coarse-graining procedures, we explicitly distinguish two distances that operate at different descriptive levels.

Combinatorial vs. weighted distance.

We define:

1. **Combinatorial distance** d_{ij}^{C} (pre-geometric).

$$d_{ij}^{C} \;=\; \min_{\gamma_{ij}} \sum_{(u,v)\in\gamma_{ij}} 1,$$

   where γ_{ij} is any path connecting nodes i and j through the network links. This distance counts *graph steps only* and is **independent of the field values of** χ. It is used to define the neighborhood sets employed in relational averaging.
2. **Weighted distance** d_{ij}^{W} (emergent / effective).

$$d_{ij}^{W} \;=\; \min_{\gamma_{ij}} \sum_{(u,v)\in\gamma_{ij}} w_{uv},$$

where w_{uv} is a positive weight associated with each link. This distance is used for the **emergent geometry** (and in particular for spectral constructions), because it encodes the effective relational stiffness of the network.

This distinction ensures that the coarse-graining background $\bar{\chi}$ can be defined using d_{ij}^C without any metric dependence, while the effective geometry is encoded by d_{ij}^W through weights that depend only on $\bar{\chi}$ (not on instantaneous χ).

Weight functional and positivity.

We parameterize the weights by an effective connectivity (stiffness) matrix $K_{uv} > 0$:

$$w_{uv} \;=\; \frac{1}{K_{uv}}.$$

In the circularity-free construction used in this appendix, K_{uv} is not taken as a direct functional of χ, but as a functional of a slowly varying *background* field $\bar{\chi}$ defined by relational averaging (see Appendix E.5). Concretely, we use

$$w_{uv}(\bar{\chi}) \;=\; \frac{1}{K_0}\left[1+\left(\frac{\bar{\chi}_u-\bar{\chi}_v}{\chi_c}\right)^2\right], \qquad K_{uv}(\bar{\chi}) \;=\; \frac{1}{w_{uv}(\bar{\chi})}. \tag{306}$$

The positivity $w_{uv}(\bar{\chi}) > 0$ is guaranteed by construction, so d_{ij}^W is a well-defined weighted path metric whenever the graph is connected on the domain considered.

Metric status.

Both the combinatorial distance d_{ij}^C and the weighted distance d_{ij}^W define proper metric spaces on the χ-network, albeit at different descriptive levels: d_{ij}^C is discrete and topological, while d_{ij}^W encodes emergent relational structure. This duality is essential: d_{ij}^C provides a pre-geometric scaffold for defining $\bar{\chi}$, whereas d_{ij}^W provides the effective distance used in the emergent geometric regime.

E.5 Derivation of χ_{eff} from Relational Observables

The effective field χ_{eff} is introduced as an operational description of χ-configurations once a stable projected regime exists. Since the projected regime admits an effective geometric interpretation, χ_{eff} must be constructed in a way that does not implicitly assume the very metric structure it is meant to support. In particular, if a distance d_{ij} used to define coarse-graining neighborhoods depends on weights that themselves depend on χ, then a hidden circularity would arise.

To remove this ambiguity, we adopt a **two-level construction** based on the explicit distinction between the combinatorial distance d_{ij}^C and the weighted distance d_{ij}^W introduced in Appendix E.4.

(1) Relational background field $\bar{\chi}$.

We define a background field $\bar{\chi}$ by a **relational average** that uses only the **combinatorial (pre-geometric) distance** d^C_{ij}. Let

$$N_i \;=\; \{ j \,|\, d^C_{ij} \le \ell_0 \} \tag{307}$$

be the combinatorial neighborhood of radius ℓ_0 around node i. We then set

$$\bar{\chi}_i \;=\; \frac{1}{|N_i|} \sum_{j \in N_i} \chi_j. \tag{308}$$

Because N_i depends only on d^C_{ij}, the definition of $\bar{\chi}$ is **independent of any weighted metric** and therefore does not depend on χ through a distance functional. This is the crucial step that prevents circularity.

(2) Emergent connectivity and weighted distance.

Using the background field $\bar{\chi}$, we define the link weights and the corresponding connectivity through Eq. (306):

$$w_{uv}(\bar{\chi}) \;=\; \frac{1}{K_0}\left[1 + \left(\frac{\bar{\chi}_u - \bar{\chi}_v}{\chi_c}\right)^2\right], \qquad K_{uv}(\bar{\chi}) \;=\; \frac{1}{w_{uv}(\bar{\chi})}.$$

The **weighted distance** used for effective geometry is then

$$d^W_{ij} \;=\; \min_{\gamma_{ij}} \sum_{(u,v) \in \gamma_{ij}} w_{uv}(\bar{\chi}), \tag{309}$$

which depends on $\bar{\chi}$ but not on instantaneous χ values through the metric definition.

(3) Effective field χ_{eff} (geometry-aware coarse-graining).

Finally, we define χ_{eff} by coarse-graining χ over neighborhoods defined with the **weighted distance** d^W_{ij}:

$$V_{\ell_0}(i) \;=\; \{ j \,|\, d^W_{ij} \le \ell_0 \}, \tag{310}$$

$$\chi_{\text{eff}}(i) \;=\; \frac{1}{|V_{\ell_0}(i)|} \sum_{j \in V_{\ell_0}(i)} \chi_j. \tag{311}$$

Non-circular dependency structure.

The construction is explicitly hierarchical:

$$d^C \implies \bar{\chi} \implies w(\bar{\chi}),\, K(\bar{\chi}) \implies d^W \implies \chi_{\text{eff}}.$$

The neighborhood used to compute $\bar{\chi}$ is defined using d^C and is therefore χ-independent. The effective geometry is encoded in d^W through weights that depend only on $\bar{\chi}$, breaking any instantaneous feedback loop. This makes the operational definition of $\chi_{\rm eff}$ compatible with the pre-geometric status of χ, while still allowing an emergent geometric regime for spectral and effective-field analyses.

E.6 Relation to the Effective Geometric Description

The effective geometric structures introduced in the main text—such as metric fields, spatial gradients, connection-like objects, and Poisson-type equations—do not represent fundamental degrees of freedom in Cosmochrony. They arise as coarse-grained summaries of relational configurations of the χ field once a projectable regime becomes applicable.

E.7 Emergent Coordinates via Manifold Reconstruction

A coordinate chart x^μ is not postulated in the relational ontology. Instead, when the relational distance matrix $D = \{d_{ij}\}$ admits a low-dimensional embedding, coordinates can be *reconstructed* from D using standard manifold learning techniques.

MDS embedding from relational distances

Compute the centered Gram matrix

$$G_{ij} \;=\; -\frac{1}{2}\Big(d_{ij}^2 - d_{i\cdot}^2 - d_{\cdot j}^2 + d_{\cdot\cdot}^2\Big), \tag{312}$$

where $d_{i\cdot}^2 = \frac{1}{N}\sum_k d_{ik}^2$ and $d_{\cdot\cdot}^2 = \frac{1}{N^2}\sum_{k\ell} d_{k\ell}^2$. Diagonalizing G yields eigenpairs (λ_k, v_k). An embedding in $\mathbb{R}^d$ is then obtained by

$$x_i^{(a)} \;=\; \sqrt{\lambda_a}\,(v_a)_i, \qquad a = 1, \ldots, d, \tag{313}$$

so that $d_{ij} \approx \|x_i - x_j\|$ in the projectable regime.

Intrinsic dimension from the eigenvalue gap

The embedding dimension d is not assumed but selected by the dominant eigenvalue gap $\Delta\lambda_k = \lambda_k - \lambda_{k+1}$. Operationally, choose d as the smallest integer such that

$$\Delta\lambda_{d+1} \;>\; \eta\,\lambda_1, \tag{314}$$

with a conservative threshold $\eta \sim 0.1$. For smooth large-scale configurations, one expects a stable low-dimensional embedding (often $d = 4$ for spacetime-like regimes).

Breakdown as a physical prediction

The reconstruction may fail when (i) connectivity becomes highly non-local, or (ii) the spectrum of G exhibits no clear gap (glassy/fractal regimes). In Cosmochrony this is not a pathology: it signals a transition to a pre-geometric regime where a smooth continuum manifold is not an adequate effective description.

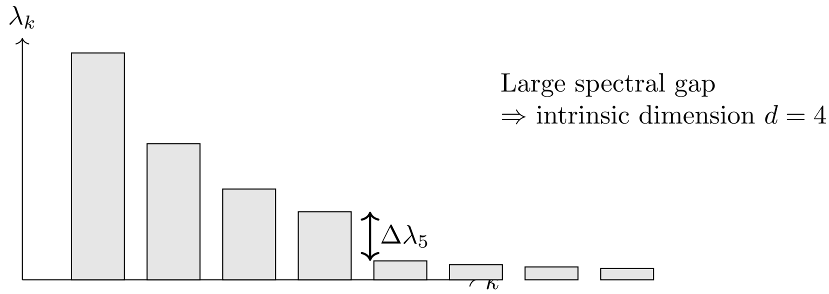

Fig. 28 Schematic eigenvalue spectrum used to select the intrinsic embedding dimension. A clear gap after the first four modes indicates a robust $d = 4$ projectable regime.

From relational structure to geometric representation.

At the relational level, configurations of χ are specified entirely by internal structural relations and bounded relaxation constraints. No notion of distance, angle, or curvature is defined. However, when relational variations become sufficiently smooth and hierarchically organized, it becomes possible to represent these configurations using effective geometric descriptors.

This representation associates relational gradients with spatial gradients of a projected field χ_{eff}, and collective relaxation constraints with geometric quantities such as curvature or gravitational potential. The resulting geometric language provides a compact and operationally useful summary of the relational organization, but it is neither unique nor exact.

Status of the effective metric.

The effective metric introduced in the main text is not postulated as a fundamental object. It is defined implicitly through the propagation properties of perturbations and the operational comparison of relaxation rates. In this sense, the metric encodes how relational distinctions are mapped onto effective notions of spatial separation and temporal ordering.

Because this mapping is many-to-one, distinct relational configurations may correspond to the same effective metric. Conversely, changes in the relational structure may occur without any corresponding change in the effective geometric description. The metric therefore captures only a restricted subset of the information contained in the relational configuration.

Emergence of field equations.

Poisson-type and wave-like equations appearing in the effective description arise from linearizing the relational relaxation dynamics around quasi-homogeneous configurations. They express how small deviations from uniform relaxation propagate and combine at the macroscopic level.

These equations should not be interpreted as fundamental dynamical laws. They are regime-dependent approximations whose validity is limited to weak-field, slow-variation conditions. Outside these regimes, the effective geometric description ceases to provide a faithful account of the underlying relational dynamics.

Consistency across descriptive levels.

No contradiction exists between the relational and geometric formulations. They apply to different descriptive levels of the same underlying theory. The relational formulation specifies the fundamental ontology and dynamics, whereas the geometric description provides an efficient and empirically successful approximation in appropriate regimes.

Importantly, the direction of conceptual dependence is unambiguous: the geometric description depends on the relational one, but not conversely. All geometric notions are secondary constructs whose meaning and applicability are derived from the relational organization of χ.

Conceptual role.

This subsection clarifies that the effective geometric language employed throughout the main text is a representational tool rather than an ontological commitment. Its role is to connect the relational foundations of Cosmochrony with familiar macroscopic descriptions of spacetime and gravity, while preserving the non-geometric nature of the fundamental theory.

The relational formulation therefore underwrites the validity of the effective geometric description without being reducible to it, ensuring conceptual coherence across all levels of the framework.

E.8 Topological Stability of Relational χ Configurations

This appendix reformulates the topological stability principles introduced in Section 6.2 within the fully relational framework, independent of geometric representations.

In the fully relational formulation of Cosmochrony, particle-like excitations are identified with nontrivial, internally organized configurations of the χ field in its abstract configuration space. They are not defined as objects localized in a pre-existing spacetime manifold, but as relational patterns whose stability is guaranteed by intrinsic topological constraints.

Relational notion of topology.

Unlike conventional field theories, where topological invariants are defined with respect to spatial embeddings or boundary conditions on a manifold, the invariants relevant in Cosmochrony are purely relational. They characterize inequivalent classes of χ configurations that cannot be continuously transformed into one another without violating the internal relaxation constraints or inducing a discontinuous reorganization of relational structure.

Topology, in this sense, is a property of configuration space rather than of physical space. It encodes global consistency conditions on how relational links within a configuration may be rearranged while preserving admissibility.

Origin of stability.

The stability of a relational configuration arises from the existence of topological obstructions to global relaxation. Certain configurations cannot relax continuously into the homogeneous vacuum state without passing through forbidden regions of configuration space. As a result, they persist as long-lived or stable excitations.

This stability does not rely on conserved charges imposed by symmetry principles. It is a structural property of the relational organization of χ itself, independent of any geometric or gauge-theoretic framework.

Geometric metaphors and their limits.

For heuristic purposes, relational topological structures may be illustrated using geometric metaphors such as knots, twists, vortices, or defects. These images provide intuition when configurations admit an effective geometric projection.

However, such metaphors should not be taken literally. At the relational level, there are no spatial loops, cores, or embedding spaces. All stability properties are encoded in the global pattern of relational constraints rather than in spatial winding.

For instance, while a **Möbius strip** can heuristically illustrate the 4π-periodicity of an electron's spin, the **trefoil knot** provides a geometric metaphor for the proton's topological complexity. These metaphors become quantitatively meaningful only when linked to the **fiber volumes** under the projection Π, which determine the particle's mass and stability.

Example: 4π***-periodic configurations.***

A paradigmatic example is provided by relational configurations exhibiting an intrinsic 4π-periodic internal structure. Such configurations cannot be continuously unwound into a trivial state and therefore belong to a distinct topological sector of configuration space.

When projected onto an effective geometric description, these configurations exhibit spinorial transformation properties and fermion-like behavior. The appearance of spin-$\frac{1}{2}$ is thus traced back to a topological feature of the relational configuration, rather than to a fundamental spinor field.

Particle identity and mass.

Distinct particle species correspond, in this picture, to inequivalent topological sectors of relational χ configurations. Particle identity is therefore associated with topological class membership rather than with localization or internal labels.

The energetic cost of deforming a stable configuration is determined by the internal resistance of the χ field to relaxation. This resistance provides a unified origin for particle mass, stability, and spectral separation, without invoking externally imposed charges, gauge groups, or symmetry-breaking mechanisms.

Scope and interpretation.

The relational-topological picture developed here is not intended as a complete or unique classification of all admissible configurations. It serves as an explicit realization of the ontological principles underlying Cosmochrony, demonstrating how particle-like properties may emerge from the internal organization of χ.

Crucially, this formulation remains fully compatible with the effective geometric and dynamical descriptions employed in the main text. Topological stability is defined prior to and independently of any geometric projection, ensuring consistency across all descriptive levels of the framework.

E.9 Topological Origin of Fermionic and Bosonic Statistics

Within the fully relational formulation of Cosmochrony, the distinction between fermionic and bosonic behavior does not arise from imposed quantum statistics or from a fundamental spinorial ontology. Instead, it originates from the internal topological structure of localized configurations of the χ field in its configuration space.

Internal rotations and configuration space topology.

At the relational level, the notion of rotation is not defined with respect to physical space. It refers instead to closed paths in the configuration space of admissible χ configurations. Two configurations are considered equivalent if they are related by a continuous deformation that preserves all relational relaxation constraints.

Certain classes of configurations exhibit nontrivial topology in this configuration space. For these configurations, a closed path corresponding to a 2π internal reorientation does not return the system to an equivalent configuration. Only a full 4π cycle restores relational equivalence. This topological obstruction is naturally associated with a non-orientable structure in configuration space.

Topological Mass Ratios and Knot Theory.

The following considerations should be understood as heuristic geometric interpretations of the spectral hierarchy discussed in Section B.8, rather than as a derived volumetric mass formula.

The distinction between fermionic and bosonic statistics is not the only consequence of the χ-field's topological structure. The **mass ratios between particles** (e.g., proton-to-electron) also emerge from the **knot-like configurations** of χ:

- An **electron** corresponds to a **twisted unknot** ($Q_e = 1$), with a fiber volume under the projection Π scaling as $\mathrm{Vol}(\Pi^{-1}(\text{electron})) \propto \chi_c$.
- A **proton** corresponds to a **trefoil knot** ($Q_p = 3$), with a fiber volume scaling as $\mathrm{Vol}(\Pi^{-1}(\text{proton})) \propto \chi_c^3$.

The observed mass ratio $m_p/m_e \approx 1836$ is then derived from the ratio of these volumes:

$$\frac{m_p}{m_e} = \frac{\mathrm{Vol}(\Pi^{-1}(\text{proton}))}{\mathrm{Vol}(\Pi^{-1}(\text{electron}))} \approx 27\chi_c^2.$$

Assuming $\chi_c \approx 8.3$ (from $\pi\chi_c^2 \approx 1836/27$), this provides a **topological explanation** for the mass ratio, independent of ad hoc parameters. This mechanism is consistent with the **relational-topological framework** described in this appendix, where particle properties emerge from the internal structure of χ.

Emergence of fermionic behavior.

Configurations with intrinsic 4π-periodicity belong to topological sectors that are double-valued under 2π reorientation. When such configurations admit an effective geometric projection, this internal property manifests as fermion-like behavior:

- a sign change under 2π rotations,
- restoration of equivalence only after a 4π cycle,
- and transformation properties characteristic of spin-$\frac{1}{2}$ degrees of freedom.

These features arise without introducing fundamental spinors. They reflect the topology of the underlying relational configuration rather than a representation of the Lorentz group imposed at the outset.

This 4π-periodicity is not only responsible for fermionic statistics but also contributes to the **topological stability** of the soliton. For example, the electron's twisted unknot configuration is protected from decay by its nontrivial phase structure, which is directly linked to its **mass** via the fiber volume under Π. The same topological protection applies to the proton's trefoil knot, explaining its stability and the observed mass ratio.

Emergence of bosonic behavior.

Other classes of relational configurations are topologically orientable. For these configurations, a 2π internal reorientation is sufficient to return to an equivalent state. When projected onto an effective geometric description, such configurations exhibit boson-like behavior, including integer-spin transformation properties and the absence of sign inversion under 2π rotations.

The distinction between fermionic and bosonic excitations is therefore encoded in the topology of configuration space rather than in any dynamical or statistical assumption.

Geometric metaphors and their limits.

Geometric metaphors—such as Möbius twists, non-orientable loops, or knotted structures—may be used heuristically to visualize these internal topological features. However, such images are meaningful only after projection onto an effective geometric description. For instance, while a **Möbius strip** can heuristically illustrate the 4π-periodicity of an electron's spin, the **trefoil knot** provides a geometric metaphor for the proton's topological complexity. These metaphors become quantitatively meaningful only when linked to the **fiber volumes** under the projection Π, which determine the particle's mass and stability. At the relational level, no spatial embedding exists, and these metaphors serve solely as intuitive aids.

Relation to the spin–statistics connection.

The relational-topological distinction between 4π- and 2π-periodic configurations provides a natural qualitative explanation of the spin–statistics connection. Fermionic and bosonic behavior emerge as consequences of internal topological constraints rather than as independent quantum postulates.

While this construction does not constitute a formal proof of the spin–statistics theorem, it demonstrates that the observed dichotomy between fermions and bosons can arise consistently from the internal organization of χ, prior to and independently of any effective geometric or quantum description.

The topological distinction between fermions and bosons also underpins the **mass hierarchy** observed in particle physics. The proton's trefoil knot structure, with its higher topological complexity, results in a larger fiber volume under Π compared to the electron's twisted unknot. This difference in fiber volumes directly translates into the mass ratio $m_p/m_e \approx 1836$, demonstrating how **spin, statistics, and mass** are interconnected through the χ-field's topological structure.

Conceptual role.

This subsection completes the relational account of particle properties in Cosmochrony by showing how spin and statistics arise from topology alone. It reinforces the view that quantum transformation properties are emergent features of relational structure, not fundamental ingredients of the theory.

E.10 Vacuum Energy versus Relaxation Capacity of the χ Field

In conventional quantum field theory, the notion of *vacuum energy* refers to a non-vanishing energy density associated with zero-point fluctuations of quantum fields. This quantity is treated as a locally defined, extensive property of spacetime and is assumed to contribute directly to the stress–energy tensor. When extrapolated to cosmological scales, this interpretation leads to the well-known cosmological constant problem.

In Cosmochrony, no fundamental vacuum energy density is postulated. The χ field does not carry an intrinsic additive energy in the absence of constraints or excitations. Instead, phenomena commonly attributed to vacuum energy are reinterpreted in terms of the *relaxation capacity* of the χ field.

Relaxation capacity as a relational notion.

Relaxation capacity characterizes the ability of a relational configuration of χ to undergo further structural reorganization under the bounded relaxation constraints. It is not a local scalar density and cannot be meaningfully assigned to spacetime points. Rather, it is a contextual and non-extensive property of an entire relational configuration.

In the absence of matter excitations, boundaries, or topological obstructions, this capacity has no observable manifestation. A perfectly unconstrained configuration corresponds to a state in which relaxation capacity is uniform and physically inert.

Constraints and observable vacuum effects.

Observable vacuum phenomena arise only when relational constraints restrict the space of admissible χ configurations. Boundaries, material structures, or imposed conditions modify the allowed patterns of relaxation and thereby redistribute relaxation capacity.

The Casimir effect provides a paradigmatic illustration. Within the Cosmochrony framework, the presence of conducting plates constrains the admissible relational configurations of χ between them. The resulting force does not originate from an absolute vacuum energy stored in the intervening region. It arises from a differential in relaxation capacity between constrained and unconstrained configurations.

This interpretation preserves the empirically observed magnitude and sign of the effect while eliminating the need to attribute a large, homogeneous energy density to empty space.

Gravitational implications.

Because relaxation capacity is inherently relational and non-extensive, it does not enter gravitational dynamics as a uniform source term. Only changes in the relaxation structure induced by localized excitations or topological constraints contribute to effective gravitational behavior.

As a result, there is no reason for relaxation capacity to gravitate in the manner predicted by standard vacuum energy arguments. This provides a natural conceptual resolution of the cosmological constant problem: the enormous vacuum energy inferred from zero-point counting is not a physically meaningful quantity in the Cosmochrony ontology.

Conceptual reinterpretation of the vacuum.

Cosmochrony does not deny the physical reality of vacuum-related phenomena. Rather, it reclassifies them as manifestations of constrained relational dynamics of the χ field. What appears as vacuum energy in effective descriptions corresponds, at the fundamental level, to differences in relaxation capacity between relational configurations.

In this view, the vacuum is not an energetic substance filling spacetime, but a relational state whose physical relevance emerges only through constraints. This reinterpretation preserves all empirically verified vacuum effects while eliminating the need for a fundamental vacuum energy density or a finely tuned cosmological constant.

E.11 Conceptual Positioning with Respect to Existing Frameworks

This subsection situates the relational χ framework with respect to several established theoretical approaches. The purpose of this comparison is strictly conceptual. It aims to clarify differences in ontological commitments, explanatory strategy, and scope, rather than to assess empirical adequacy or predictive performance.

Cosmochrony is not presented as a direct competitor to quantum mechanics, quantum field theory, or general relativity. Instead, it is positioned as a foundational framework intended to underlie and contextualize these effective theories by identifying a deeper, pre-geometric level of description.

Scope of the comparison.

The comparison emphasizes:

- what is taken as fundamental in each framework,
- how spacetime and geometry are treated,
- the status of time, particles, and vacuum structure,

- and the role of initial conditions and large-scale coherence.

No claim is made that Cosmochrony currently matches the quantitative success of established theories. Its empirical status remains exploratory, and its primary contribution at this stage is conceptual unification and reinterpretation.

Conceptual Aspect	**Quantum Formalism (QM / QFT)**	**Geometric Gravity (GR and extensions)**	**Cosmochrony**
Primary ontology	Quantum states and operator-valued fields	Spacetime geometry and metric structure	Relational scalar substrate χ
Status of spacetime	Fixed background or effective stage	Fundamental dynamical entity	Emergent, projective description
Nature of time	External parameter or operator	Coordinate-dependent geometric quantity	Intrinsic ordering via relaxation of χ
Gravitation	Not fundamental; introduced externally	Manifestation of metric curvature	Collective slowdown of χ relaxation
Quantum behavior	Postulated formal structure	Added or emergent from quantization	Emergent from relational χ configurations
Vacuum structure	Zero-point energy of quantum fields	Geometric ground state	Contextual relaxation capacity
Particle ontology	Fundamental entities or excitations	Geometric or field excitations	Topologically stable relational configurations
Cosmic expansion	Not addressed intrinsically	Requires matter/energy content	Emergent geometric unfolding of χ
Inflation / initial conditions	Outside scope	Requires external mechanisms	Not required (pre-geometric continuity)
Empirical status	Highly successful	Highly successful	Exploratory and foundational

Table 6 High-level conceptual positioning of the relational χ framework with respect to quantum and geometric approaches. The comparison highlights ontological structure and explanatory strategy rather than empirical validation.

Interpretive caution.

The similarities highlighted in this table—such as the emergence of geometry, the recovery of relativistic causality, or the appearance of quantum correlations— should not be interpreted as equivalence. Cosmochrony deliberately refrains from adopting the formal postulates of either quantum mechanics or general relativity at the fundamental level.

Conversely, differences in ontology do not imply incompatibility. The effective regimes of Cosmochrony are constructed precisely so that standard quantum and geometric descriptions are recovered where they are empirically validated.

Conceptual contribution.

The distinctive contribution of Cosmochrony lies in its attempt to:

- unify quantum, gravitational, and cosmological phenomena within a single relational substrate,

- eliminate the need for independent postulates for spacetime, quantum statistics, and vacuum energy,
- and reinterpret long-standing conceptual tensions as artifacts of applying effective descriptions beyond their domain of validity.

In this sense, Cosmochrony should be viewed as a foundational and exploratory framework. Its role is to provide a coherent ontological backdrop against which established theories may be understood as complementary, regime-dependent descriptions rather than as mutually incompatible fundamentals.

Framework-level comparison.

While Table 6 contrasts broad ontological commitments, the following comparison focuses on cosmological and gravitational frameworks commonly discussed in the literature.

Table 7 Cosmological and gravitational comparison between Cosmochrony, the standard ΛCDM model, and Loop Quantum Gravity (LQG).

Aspect	**Cosmochrony**	**ΛCDM**	**LQG**
Fundamental ontology	Single pre-geometric relational substrate χ	Spacetime metric $g_{\mu\nu}$ + matter fields + Λ	Quantum geometry (spin networks, holonomies)
Status of spacetime	Emergent, projected, non-fundamental	Fundamental background (classical)	Quantized but kinematically assumed
Degrees of freedom (fundamental)	One relational entity (no local field DOF)	Metric DOF + multiple matter fields	Discrete graph-based DOF
Nature of time	Emergent ordering from irreversible relaxation	External parameter	Problematic / relational
Quantum gravity	Emergent from projection and spectral constraints	Absent	Fundamental (background independent)
Dark energy	Not required (cosmic expansion from χ relaxation)	Explicit Λ term	No consensus (emergent or absent)
Inflation	Not required	Required (inflaton field)	Alternative scenarios
Origin of mass	Spectral inhibition of relaxation	Higgs mechanism	Not intrinsic
Discreteness	Projective and spectral (non-injective projection)	None	Fundamental (Planck-scale geometry)
Testable predictions	$H(z)$ evolution, CMB low-ℓ anomalies, redshift drift	H_0, w, structure growth	Planck-scale signatures, area spectra
Primary conceptual aim	Ontological unification of time, geometry, and matter	Phenomenological concordance model	Quantization of spacetime geometry

F Glossary of Core Quantities and Notation

This appendix summarizes the meaning, role, and ontological status of the main quantities used throughout the Cosmochrony framework. It is intended strictly as a reference guide and does not introduce new assumptions, dynamics, or physical postulates.

F.1 Fundamental Quantities

χ ***(Chi substrate).***

The unique fundamental entity of the Cosmochrony framework. χ is a pre-geometric, relational substrate not defined on a pre-existing spacetime manifold. Its irreversible relaxation provides an intrinsic ordering of physical processes. Localized, topologically stable configurations of χ correspond to particle-like excitations.

χ_i ***(Local configuration).***

Discrete local degrees of freedom of the χ substrate, associated with vertices of the relaxation network. They encode the microscopic relational state prior to any geometric projection.

χ_c ***(Critical relaxation threshold).***

A fundamental structural bound limiting local variations of χ. It enforces causal consistency at the pre-geometric level and underlies all effective speed and action bounds.

τ***elational (operational) time.***

An effective ordering parameter defined from the accumulated monotonic ordering of projected configurations, constructed from the effective descriptor χ_{eff}. Relational time τ is not a fundamental temporal coordinate and does not exist at the level of the χ substrate. It emerges only in projectable regimes as an operational measure of duration, defined through integrals of effective relaxation ordering along admissible paths.

c_χ ***(Fundamental relaxation speed).***

The maximal propagation speed of relaxation disturbances within the χ substrate. It represents the fundamental causal bound of the theory, from which the effective speed of light emerges.

F.2 Effective and Projected Quantities

χ_{eff} ***(Effective projected field).***

A coarse-grained scalar field arising from the non-injective projection of χ onto an emergent spacetime description. χ_{eff} provides an effective field-theoretic representation without fundamental status.

Fiber (of the projection).

For a given effective configuration χ_{eff}, the fiber is the set of underlying χ configurations mapped to it by the projection π. Elements of a fiber are operationally indistinguishable at the spacetime level. Non-trivial fibers reflect the structural non-injectivity of the projection.

Operational projection (contextual access).

A measurement-context-dependent map that specifies how the effective description χ_{eff} is accessed and turned into operational observables. It introduces no additional ontology: it formalizes the contextual readout of χ_{eff} (e.g., choice of apparatus, coarse-graining, relational query).

π ***(Projection map).***

A structural mapping from configurations of χ to an effective description χ_{eff} applicable in projectable regimes. The projection is generally non-injective: distinct underlying configurations of χ may correspond to the same effective state, defining equivalence classes (fibers) under π.

π^{-1} ***(Deprojection).***

The inverse reconstruction problem of identifying classes of χ configurations compatible with a given effective state. Deprojection is not unique and does not destroy structural information.

$V(\chi)$ ***(Effective potential).***

An effective, coarse-grained description used to model localization and stability properties of χ configurations. $V(\chi)$ is not fundamental and is secondary to the spectral characterization of mass and inertia.

Observable.

A stable, projectable quantity defined on χ_{eff} through an interpretative framework. Observables do not correspond to additional ontological entities, but to operational readings of the same effective physical reality.

Physical reality.

In the Cosmochrony framework, physical reality is identified with the effective level χ_{eff}. The substrate χ is ontologically real but does not constitute a physical universe until projected into a projectable regime.

t_{proj} ***(Projected time).***

Operational time measured within the emergent spacetime description. It arises from the local rate of χ relaxation and reproduces relativistic time dilation effects.

Universe.

The physically real domain described at the χ_{eff} level, where spacetime structure, causality, and physical observables are well-defined. The Universe does not refer to the fundamental substrate χ, which is ontologically prior to any notion of universe.

F.3 Relaxation Network and Operators

$G(V, E)$ ***(Relaxation network).***

A discrete graph representing the underlying relational structure on which the χ substrate is defined. Vertices correspond to elementary degrees of freedom and edges encode relaxation couplings.

K_{ij} ***(Relaxation coupling).***

Edge-dependent coupling coefficients defined on the relaxation network. They quantify the resistance to relative variations of χ between neighboring nodes and encode geometric and topological information. K_{ij} are structural parameters of the pre-geometric substrate and do not represent dynamical interaction constants.

Δ_G ***(Graph Laplacian / relaxation operator).***

The discrete Laplace–Beltrami operator associated with the network $G(V, E)$ and the couplings K_{ij}. Its spectral properties govern the stability, localization, and inertial behavior of χ configurations.

$D_{\text{loc}}\chi$ ***(Local relaxation operator).***

A local relational operator governing the evolution of χ at the microscopic level. It replaces differential operators defined on continuous manifolds.

F.4 Spectral and Inertial Quantities

λ_n ***(Spectral eigenvalues).***

Eigenvalues of the linearized relaxation or stability operator acting on small perturbations of a localized χ configuration. They determine inertial mass scales in the effective description.

ψ_n ***(Spectral modes).***

Eigenmodes associated with the operator Δ_G. They encode the internal structure and stability of particle-like configurations.

m_{eff} ***(Effective mass).***

An emergent invariant determined by the spectral properties of localized χ configurations. Mass is not a fundamental parameter nor a coupling constant.

Q ***(Topological charge).***

An integer-valued invariant characterizing the topology of a stable χ configuration. Different values of Q correspond to distinct particle families.

$\Omega^{\pm}$ ***(Chiral topological sectors).***

Opposite chiral configurations of topological χ structures. They are related by orientation reversal and need not be energetically equivalent.

F.5 Dimensionless Parameters

S ***(Gradient saturation parameter).***

A dimensionless quantity defined as

$$S \equiv \frac{1}{c^2} \sum_{j \sim i} K_{ij} (\chi_i - \chi_j)^2, \tag{315}$$

measuring the local density of χ gradients. The bound $S \leq 1$ enforces causal consistency in effective spacetime dynamics.

Ω_χ ***(Relaxation budget parameter).***

A dimensionless global quantity characterizing the fraction of total χ relaxation stored in spatial gradients. In cosmological regimes, it plays a role analogous to a density parameter.

F.6 Constants and Emergent Limits

c ***(Effective speed of light).***

The maximal signal propagation speed in emergent spacetime. c is an effective bound derived from the more fundamental speed c_χ.

$\hbar$ ***(Effective Planck constant).***

An emergent quantum of action associated with projection thresholds and spectral granularity. It is not fundamental at the level of χ.

G ***(Newtonian gravitational constant).***

An emergent coupling constant arising from large-scale collective relaxation dynamics of χ. Its value reflects structural properties rather than fundamental interaction strengths.

Λ_{eff} ***(Effective cosmological constant).***

A residual large-scale relaxation effect associated with incomplete equilibration of the χ substrate.

F.7 Key Conceptual Terms

Energy.

Energy measures the resistance of χ configurations to relaxation-induced change. Standard conservation laws remain valid at the effective level.

Fluctuations.

Local stochastic modulations of χ configurations that affect event timing and localization without altering underlying topological constraints.

Matter.

Stable topological configurations of χ whose persistence gives rise to particle-like behavior and inertial properties.

Measurement.

A localized interaction that selects a specific manifestation of an underlying χ fluctuation without invoking fundamental wavefunction collapse.

Probability.

An emergent descriptor reflecting structural constraints imposed by the topology of χ, modulated by stochastic fluctuations.

Relaxation (of the χ field).

The intrinsic dynamical tendency of χ to reorganize under internal coupling constraints. Relaxation is pre-thermodynamic and does not correspond to dissipation.

Spacetime.

An emergent relational structure arising from large-scale configurations of the χ substrate. Its metric description remains valid within its domain of applicability.

Time.

An effective parameter associated with the local rate of χ relaxation. Operational and relativistic notions of time are recovered without modification.

Relaxation transmittance (gauge interpretation).

An effective, context-dependent measure of how efficiently relaxation flux (or ordering capacity) is transmitted through a given projected configuration. In Cosmochrony, gauge structure can be interpreted as a parametrization of transmittance variations across the projection fiber, rather than as a fundamental interaction field.

Wavefunction.

An effective statistical representation of the dynamics and topology of the χ substrate. It has no fundamental ontological status.

Wave–Particle Duality.

A manifestation of interaction-induced changes in the local configuration of χ, producing localized particle-like behavior from an underlying wave-like substrate.

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