Computational Physics Homework # 202.

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When you hand in the homework, you should gather all your files into a single tarball file as follows.

- Use an unix command `tar -czf <file name>.tar.gz <file 1> <file 2>` $\cdots$.
- For undergraduate students, put a copy of a tarball <file name>.tar.gz into a directory:

  `/physics/upload/comp2023/<user-ID>`.
- For graduate students, put a copy of a tarball <file name>.tar.gz into a directory:

  `/physics/upload/acomp2023/<user-ID>`.
- You must use the GNU `make` command and `Makefile` to compile the code starting from the homework `hw101`.

Newton-Raphson Method

1. [One-dimensional Newton Method] In the lecture, you learned how to find a root for one-dimensional equation. Let us consider the following function: $f(x)$.

$$f(x) = \sin\left(10 \cdot \frac{x^2+1}{x^4+x^2+1} \cdot \exp[-\frac{1}{2}x^2]\right) \tag{1}$$

   (a) Make a plot of $f(x)$ as a function of x in the range of $0 \le x \le 5$. [HINT] You may use the gnuplot program in the Linux system.

(b) Find all the roots for the equation: $f(x) = 0$ for $x \geq 0$, using the Newton-Raphson method.
[HINT] The values of the roots should have a numerical precision better than 1.0×10^{-10}.

2. [Multi-dimensioanl Newton Method] In the class, you learned how to find a root of multi-dimensional equations. Let us consider the following equations in 4-dimensional space:

$$\begin{aligned} d(t+0) &= 0.2047 \times 10^{+3} \\ d(t+1) &= 0.1473 \times 10^{+3} \\ d(t+2) &= 0.1059 \times 10^{+3} \\ d(t+3) &= 0.7634 \times 10^{+2} \end{aligned}$$

Here, $t = 10$ and $L = 64$. The theoretical prediction goes that the data should behave as the following function:

$$\begin{aligned} f(t) &= Z_1[\exp(-m_1 t) + \exp(-m_1(L-t))] \\ &+ Z_2(-1)^t[\exp(-m_2 t) + \exp(-m_2(L-t))] \end{aligned}$$

(a) Using the multi-dimensional Newton-Raphson method, solve the above equations and obtain Z_1, m_1, Z_2, and m_2.
HINT: The domain of the parameters is

$$\begin{aligned} &0.5 \times 10^{+4} \leq Z_1 \leq 0.6 \times 10^{+4} \\ &0.1 \leq m_1 \leq 0.5 \\ &0.1 \times 10^{+8} \leq Z_2 \leq 0.5 \times 10^{+8} \\ &0.1 \leq m_2 \leq 5.0 \end{aligned}$$

(b) Discuss the precision of your results and their uncertainty.
※ REMARK: The parameter m_1 corresponds to the mass of pion with taste V_4. This data is obtained using the quark mass $am_x = am_y = 0.025$ on the MILC coarse lattice with $am_l/am_s = 0.01/0.05$. We use the Golterman sink operator of $P \times V_4$ and cubic wall sources.