Task 1

Let's consider first integral

$$I_1 = \int_0^{\infty} \sin(x^2 \sin(\frac{x^2+3}{x^4+1}) \exp(-x^2) \tag{1}$$

Integral function has one maximum. So to divide the interval into three pieces, I chose two infection points. This poinst approximatly are $x \approx 0.8$ and $x \approx 1.2$. I find it from equation (2)

$$\frac{d^2 f(x)}{dx^2} = 0 \tag{2}$$

So we have 3 intervals $x \in [0, 0.8] \bigcup [0.8, 1.2] \bigcup [1.2, 2]$

Let's find the value of the integral on each interval using the Simpson's method. We need the function values at each end and in the middle of the intervals. This data is printed in Table 1

Table 1: Function values

x	$f(x)$
0	0.0
0.4	0.008
0.8	0.176
1	0.290
1.2	0.235
1.6	0.076
2	0.018

Simpson's method is as follows:

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a) + 4f(a+h) + f(b)\right], \tag{3}$$

where $h = \frac{a+b}{2}$ is step of method.

Now we calculate values of integral for all intervals.

$$\int_0^{0.8} f(x)dx \approx 0.028$$

$$\int_{0.8}^{1.2} f(x)dx \approx 0.105$$

$$\int_{1.2}^{2} f(x)dx \approx 0.074 \quad (4)$$

Summing up the values we get

$$I_1 = \int_0^{\infty} f(x)dx \approx 0.207 \quad (5)$$

Let's consider second integral

$$I_2 = \int_0^{\infty} \sin(x^2 \cos(\frac{x^2+3}{x^4+1}) \exp(-x^2) \quad (6)$$

This integral function has many extremes. But absolute values of the extremes decrease significantly with increasing x. So to divide the interval into three pieces, I chose first minimum and first maximum points. This points approximatly are $x = 0.75$ and $x = 1.4$. I find it from equation (7)

$$\frac{df(x)}{dx} = 0 \quad (7)$$

So we have 3 intervals $x \in [0, 0.75] \bigcup [0.75, 1.4] \bigcup [1.4, 2]$

Let's find the value of the integral on each interval using the Simpson's method. We need the function values at each end and in the middle of the intervals. This data is printed in Table 2

Table 2: Function values

x	$f(x)$
0	0.0
0.375	-0.122
0.75	-0.278
1.075	-0.075
1.4	0.120
1.6	0.073
2	-0.009

Now we calculate values of integral for all intervals by formula (3).

$$\int_{0}^{0.75} f(x)dx \approx -0.096$$
$$\int_{0.75}^{1.4} f(x)dx \approx -0.049$$
$$\int_{1.4}^{2} f(x)dx \approx 0.027 \quad (8)$$

Summing up the values we get

$$I_2 = \int_{0}^{\infty} f(x)dx \approx -0.118 \quad (9)$$

Answer

$$I_1 = 0.207$$

$$I_2 = -0.118$$