Final Exam : Topics in Cosmology

1. Via the χ^2-statistics, fit the Sheth-Tormen analytic formula to the numerically obtained mass function from the MDPL2 Rockstar halo catalog by simultaneously adjusting the values of Ω_m and σ_8. For this fitting, you may fix the three characteristic parameters of the Sheth-Tormen formula at the original values given in Sheth & Tormen (1999), and fix the other cosmological parameters at the values of the Planck cosmology (provided in the MDPL2 website). Plot the 68% contour of $\chi^2(\Omega_m, \sigma_8)$ in the Ω_m-σ_8 plane, and show that the contour shape is indeed well approximated by the relation of $\sigma_8 \approx A\Omega_m^{\beta}$ with $A \approx 0.5$ and $\beta \approx 0.5$(30 pt.). Find the best-fit values of A and β (30 pt.).

2. Analyze the MDPL2 Rockstar halo catalog at $z = 2$ to determine the cumulative mass function, $N(> M)$, in the mass range of $M \geq 10^{14}\, h^{-1}\, M_\odot$. Fit the numerically obtained $N(> M)$ with the cumulative Sheth-Tormen mass function by adjusting the value of Ω_m with the above relation between Ω_m and σ_8. Find the best-fit value of Ω_m (20 pt.).

3. As you learned during the class, the linear growth rate, f, can be approximately expressed as $f = \Omega_m^{\alpha}$ where α depends on the dark energy equation of state, w. Using the approximate expression for $\alpha(w)$ along with the expression for the linear growth factor $D(z)$ in terms of $f(\alpha)$ for a wCDM case, all of which you learned during the class, perform the following.

   (a) Via the χ^2-statistics, fit the Sheth-Tormen analytic formula to the numerically obtained mass function from the MDPL2 Rockstar halo catalog at $z = 0.5$ in the mass range of $M \geq 10^{14}\, h^{-1}\, M_\odot$ by adjusting w and Ω_m. For this task, you may use the relation between Ω_m and σ_8 that you find in Problem 2. (20 pt.)

   (b) Plot the 68%, 95% and 99% contours of $\chi^2(\Omega_m, w)$ in the Ω_m-w plane. (10 pt.)

   (c) Fix the value of Ω_m and σ_8 at the Planck values, and parameterize w as $w = w_0 + w_a(1 - a)$. Then, do the same calculation as 3.(a) but by adjusting w_0 and w_a. Plot the 68%, 95% and 99% contours of $\chi^2(w_0, w_a)$. (15 pt.)

   (d) Can you figure out why the cluster mass function at fixed redshift is not good enough to precisely constrain both of w_a and w_0? (5 pt.)