Final Exam : Topics in Cosmology

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1. I obtained matter power spectrum from *CAMB* library as I did in the mid exam, but with different values of Ω_m in the range (0 - 1). I also investigated σ_8 in the range (0.6 - 1). Then, I divided each parameter (Ω_m, σ_8) space into 10^5 pixels and calculated χ^2 in the mass range $M \geq 10^{14} h^{-1} M_\odot$ with Poisson errors. I obtained $\Omega_m = 0.31$ and $\sigma_8 = 0.79$ as best fitting value. Then I gathered all the points in the 1σ area and fitted to $\sigma_8 \approx A \cdot \Omega_m^\beta$.

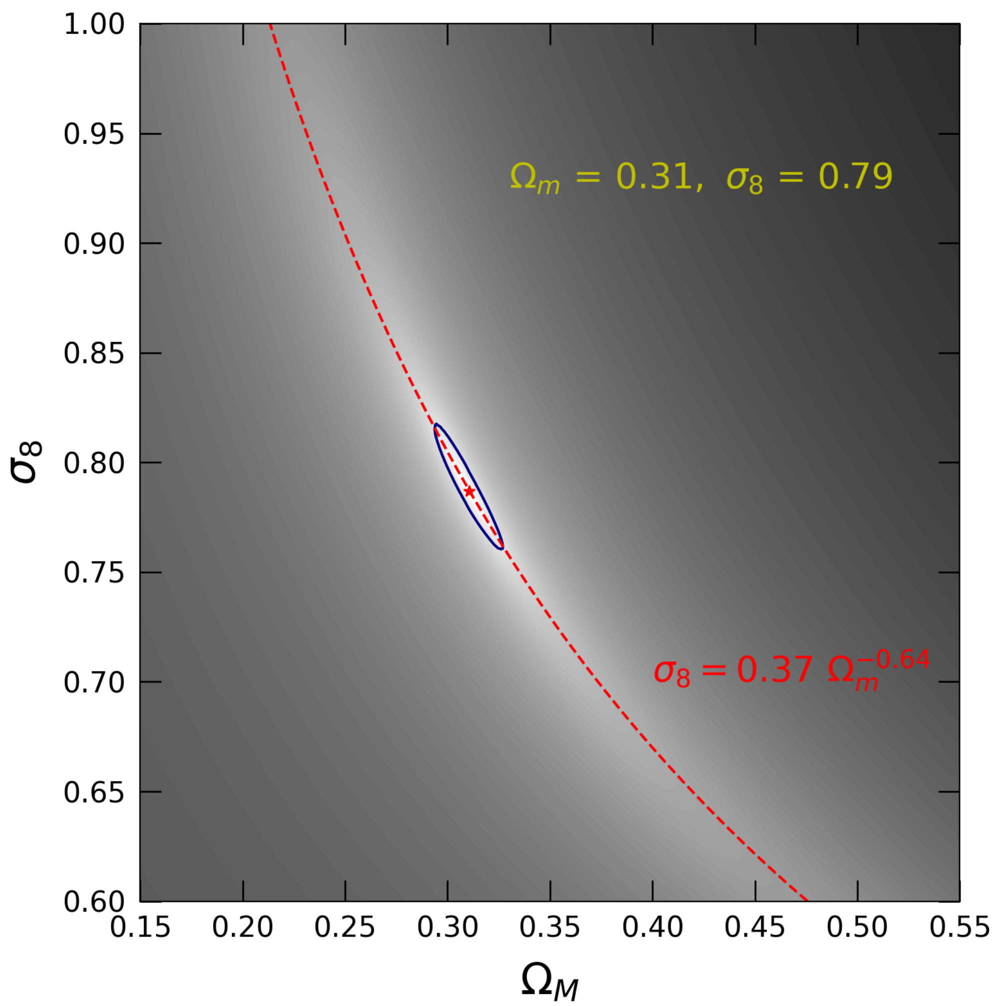


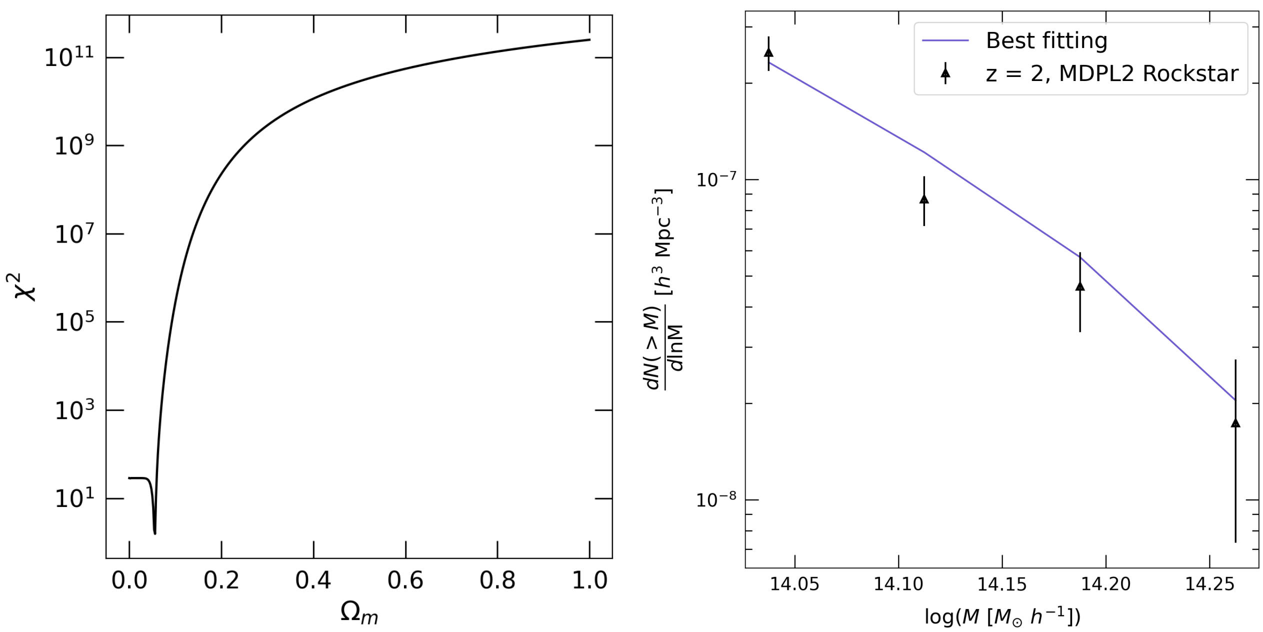
Figure 1: Problem 1. Colors in the background shows $\log(\chi^2)$, brighter color means lower χ^2 value. Red marker indicates χ^2_{min} and blue contour line shows 1σ confidence level.($\chi^2_{min} + 2.3$) Fitted red dashed line shows $\sigma_8 \approx 0.37 \cdot \Omega_m^{-0.64}$ relation.

2. I divided $z = 2$ Rockstar halo catalog into 5 bins and calculated cumulative mass function in the range $M \geq 10^{14}h^{-1}M_\odot$. I had to use only 5 bins because there were only 44 haloes in the catalog. I obtained matter power spectrum at $z = 2$ from *CAMB* just like problem 1. I used $\sigma_8 \approx 0.5 \cdot \Omega_m^{-0.5}$ relation to remove redundant parameter σ_8 and investigated χ^2 in the range $(\Omega_m : 0 - 1)$.

$$\frac{dN(z=2)}{d\ln M} = \sqrt{\frac{2}{\pi}} \cdot \frac{\overline{\rho}}{M} \cdot \left|\frac{d\ln\sigma}{d\ln M}\right| \cdot f_{ST}(\nu_c)$$

$$= \sqrt{\frac{2}{\pi}} \cdot \frac{1}{M} \cdot \left|\frac{d\ln\sigma(z=2)}{d\ln M}\right| \cdot f_{ST}\left(\frac{\delta_c}{\sigma(z=2)}\right) \cdot \Omega_m(z=2)\frac{3H^2(z=2)}{8\pi G}$$

I got $\Omega_m = 0.06$, which is wrong value. I am not sure why, but because the fitting looks okay, I think I made some mistakes with normalizing the power spectrum.



(a) χ^2

(b) Cumulative mass function and fitted mass function

Figure 2: Problem 2

3. (a) $f \equiv \dfrac{d\ln D}{d\ln a} = \Omega_m(a)^{\alpha(w)}$, $\alpha(w) \equiv \dfrac{3}{5 - \frac{w}{1-w}}$

$D(a) = a \cdot \exp\left[\int_a^1 1 - \Omega_m^{\alpha(w)}(a')\dfrac{da'}{a'}\right]$

$P(k,a) = P(k;\Omega_m,\sigma_8)\cdot D^2(a)$

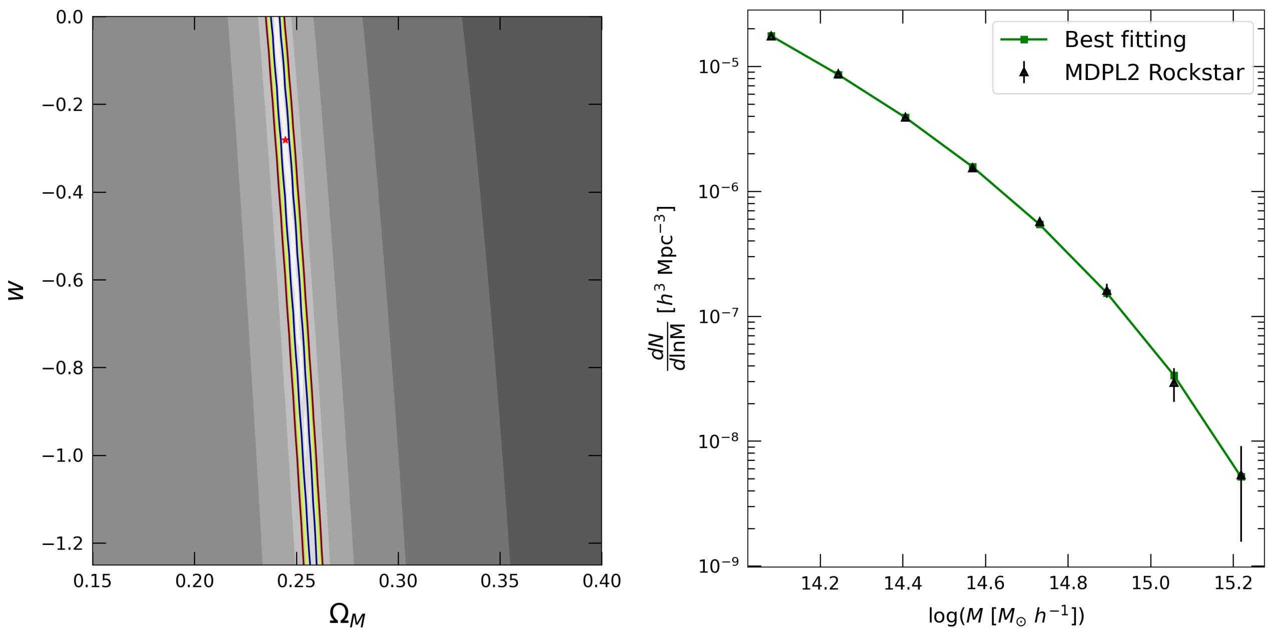
Using *CAMB* library and (Ω_m, σ_8) relation we have $P(k, z = 0)$ and we can calculate $P(k, z = 0.5)$ using $D(a)$.

$P(k, z = 0.5) \rightarrow \sigma(M, z = 0.5) \rightarrow \dfrac{dN(z = 0.5)}{dM}$

Ω_0 : (0 - 1) , w : (-2 - 0)

I got $\Omega_m = 0.24$ and $w = -0.28$ as best fitting value, which is also wrong.

(b) $\chi^2_{1\sigma} = \chi^2_{min} + 2.3$, $\chi^2_{2\sigma} = \chi^2_{min} + 6.17$, $\chi^2_{3\sigma} = \chi^2_{min} + 11.8$



(a) χ^2

(b) Mass function and fitted mass function

Figure 3: Problem 3

(c) Same with (a) but $w = w_0 + w_a \cdot (1-a)$ and fixing $\Omega_m = 0.31$ and $\sigma_8 = 0.8228$.

w_0 : (-2 - 0) , w_a : (-2 - 2)

I wasn't able to find fitting value even if I make parameter space much wider.