Topics in Cosmology: Midexam

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Note: The order of 68%, 95%, and 99% in all the figures is written reversely. I found out my blunder after all the calculation. Because it takes a lot of time to calculate again, I could not correct these mistakes. I apologize for the inconvenience.

Problem 1

In order to change σ_8 in CAMB code, the amplitude of power spectrum A_s has to be controlled by multiplying the square of σ^8 ratio to the initial A_s. With $0.1 \leq \Omega_m \leq 1$ and $0.1 \leq \sigma_8 \leq 2.5$ (100 points respectively), the power spectra are calculated. The halo mass function is obtained from Sheth-Tormen analytic formula (Eq. 1 and 2) with the calculated power spectra. The best-fit mass function and numerically derived mass function (Rockstar) are plotted in Figure 1.

$$\left|\frac{dF}{d\ln M}\right| = \left|\frac{d\ln\sigma^{-1}}{d\ln M}\right| f[\sigma(M)] \tag{1}$$

$$f(\sigma) = A(1+(\frac{\sigma^2}{a\delta_c^2})^{-p})\sqrt{\frac{2a}{\pi}}\frac{\delta_c}{\sigma}\exp(-\frac{a\delta_c}{2\sigma^2}) \tag{2}$$

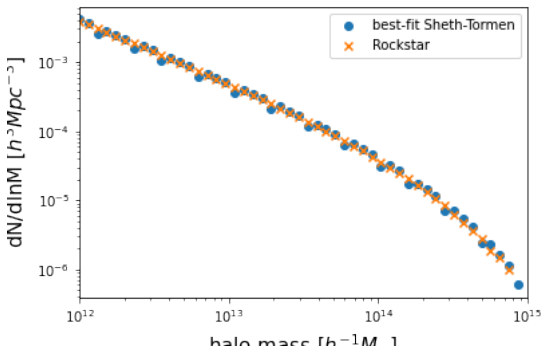

Figure 1: Halo mass function at z = 0 from Rockstar catalog(orange cross) and best-fit Sheth-Tormen(blue dots)

The the contours of $\chi^2(\Omega_m, \sigma_8)$ is shown in Figure 2. From the chi^2 statistics, the best-fit relation between Ω_m and σ_8 can be approximated as $\sigma_8 = (0.35 \pm 0.13)\omega_m^{-0.55\pm0.13}$. Considering the coarse grid size of (Ω_m, σ_8), which results in the sharp contour, the estimated Ω_m-σ_8 relation is slightly different from reference. However, the exponential form is consistent and the errors are within the range.

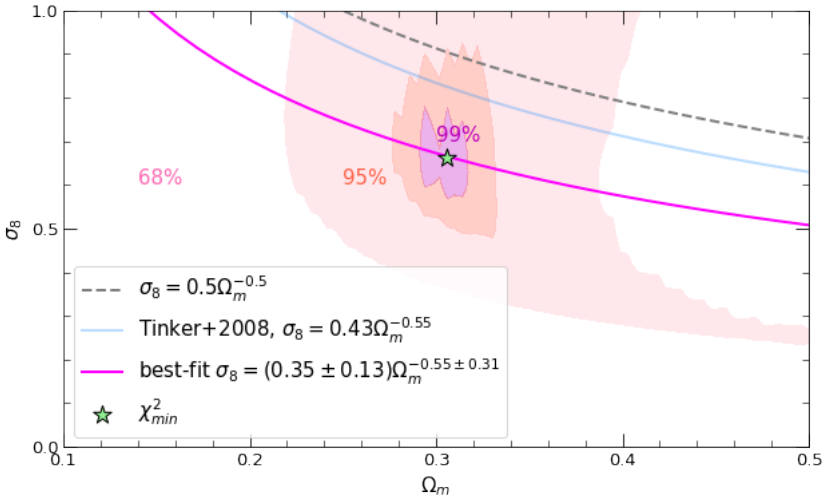

Figure 2: the 68%, 95%, and 99% contours of $\chi^2(\Omega_m - \sigma_8)$ in the $\Omega_m - \sigma_8$ plane

Problem 2

With $0.1 \leq \Omega_m \leq 1$ and corresponding σ_8 derived from $\sigma_8 = (0.35 \pm 0.13)\omega_m^{-0.55 \pm 0.13}$, the halo mass function is also calculated. Due to high redshift(z = 2), total number of halos massive than $M \geq 10^{14} h^{-1} M_\odot$ in MDPL2 Rockstar catalog is 44. Therefore, the halo mass range for fitting is limited to $10^{14} h^{-1} M_\odot \leq M \leq 10^{15} h^{-1} M_\odot$ (Figure 3) excluding the higher mass range.

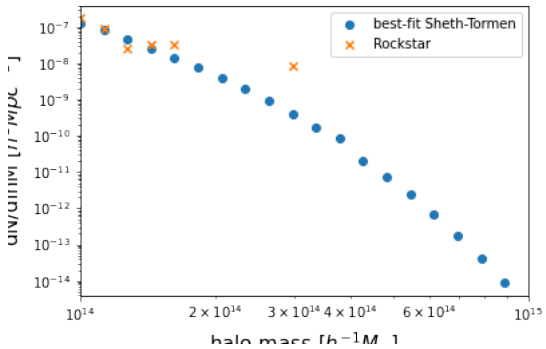

Figure 3: Halo mass function at z = 2 from Rockstar catalog(orange cross) and best-fit Sheth-Tormen(blue dots)

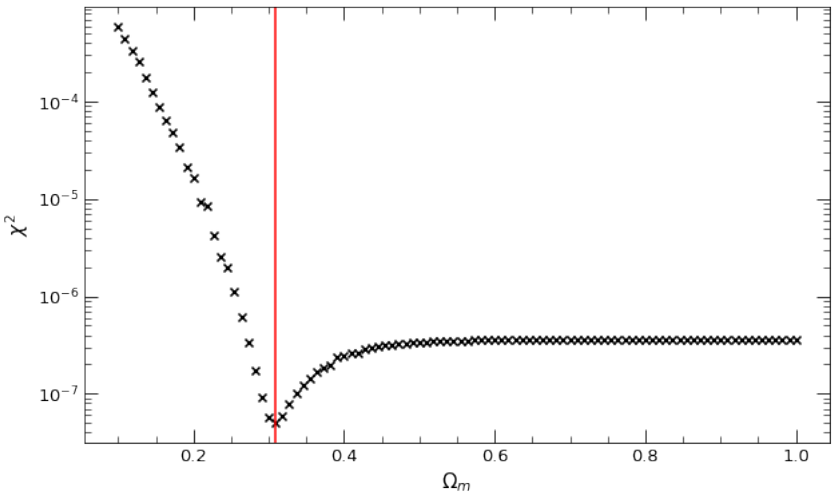

Figure 4: χ^2 distribution as a function of Ω_m

Figure 4 shows the χ^2 distribution as a function of Ω_m. The best-fit value of Ω_m is 0.31. This result is consistent with previous studies.

Problem 3

(a), (b)

Total 12094 halos are found in MDPL2 Rockstar catalog at z = 0.5. The Sheth-Tormen mass function is fitted to the numerical mass function from MDPL2 Rockstar catalog by adjusting ω (from 0.1 to 0.5) and Ω_m (from -4 to 0). The σ_8 derived from $\sigma_8 = (0.35 \pm 0.13)\omega_m^{-0.55\pm0.13}$ is also applied.

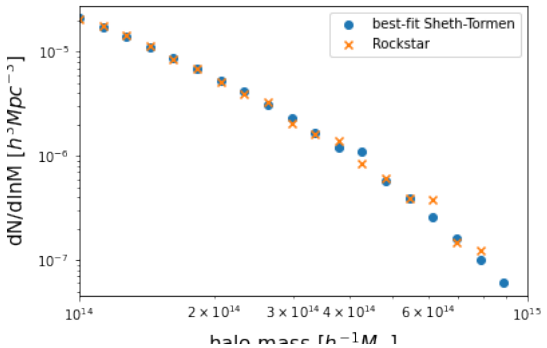

Figure 5: Halo mass function at z = 0.5 from Rockstar catalog(orange cross) and best-fit Sheth-Tormen(blue dots)

Figure 5 shows the halo mass function from Rockstar catalog and best-fit Sheth-Tormen. The 68%, 95%, and 99% contours of $\chi^2(\Omega_m, \omega)$ in the $\Omega_m - \omega$ plane are also shown in Figure 6. The best-fit values of (Ω_m, ω) are calculated as (0.33, -1.47). Similar to problem 1, sharp contour might result from coarse grid of (Ω_m, ω).

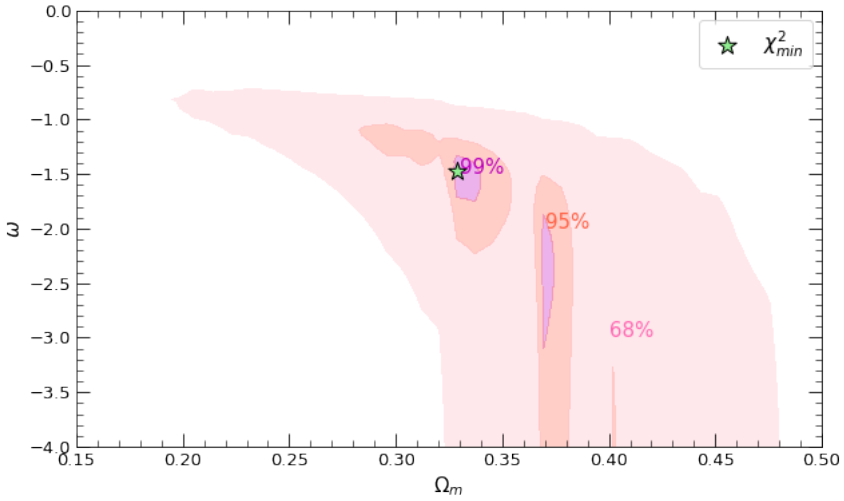

Figure 6: the 68%, 95%, and 99% contours of $\chi^2(\Omega_m, \omega)$ in the $\Omega_m - \omega$ plane

(c)

The dark energy model in CAMB has to be changed in order to apply flexible $\omega(\alpha)$ to the result. By adjusting ω_0 from -2 to 0 and ω_a from -2 to 2, the same calculation as problem 3 (a), (b) is done. Figure 7 shows the 68%, 95%, and 99% contours of $\chi^2(\omega_0, \omega_a)$.

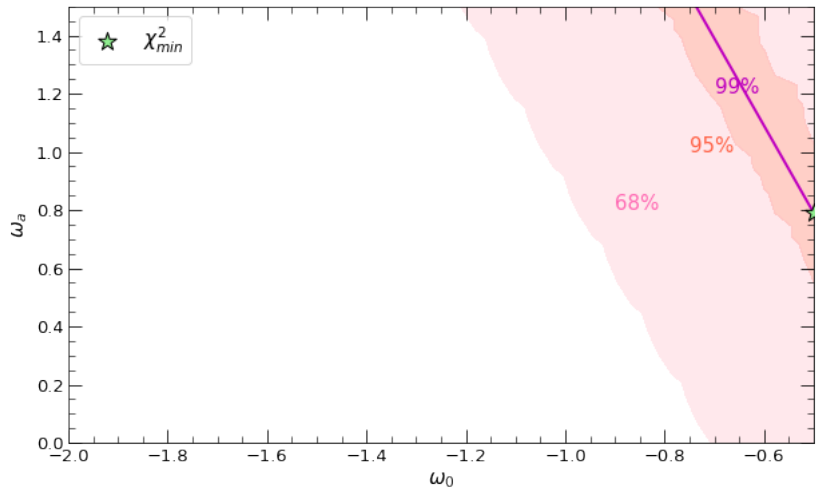

Figure 7: the 68%, 95%, and 99% contours of $\chi^2(\omega_0, \omega_a)$ in the $\omega_0 - \omega_a$ plane

(d)

The cluster mass function at fixed redshift is not good enough to precisely constrain both of ω_m and ω_0. The ω_a, dependent on time by definition, has to have the value near -1 in order to be consistent with observation at present epoch. However, the ω_a can have wide range of values(large uncertainties) as shown in Figure 7. Determining ω_a is kind of fine-tuned with boundary condition from present epoch.