Summary

MA Lei

October 18, 2012

1 Objectives

For LCDM, interacting models, and CPL, calculate

- ξ range for varying EoS while fixing $\Omega m0$
- ξ range for varying $\Omega m0$ or r, while fixing ω
- Does $\xi > 0$ means energy transfer to dark energy in this method?

2 Background

Deceleration parameter reads

$$q(z) = -1 + \frac{1+z}{H}\frac{\mathrm{d}H}{\mathrm{d}z} \tag{1}$$

For interaction models, the Friedmann equaitons,

$$\dot{\rho}_c + 3H\rho_c = Q_c \tag{2a}$$
$$\dot{\rho}_d + 3H(1+w)\rho_d = -Q_c \tag{2b}$$

$Q_c = \xi H \rho_c$ Background equations,

$$\Omega m = \Omega m0(1+z)^{3-\xi} \tag{3a}$$
$$\Omega d = (\Omega d0 + \frac{\xi}{3w+\xi}\Omega m0)(1+z)^{3(1+w)} + \frac{-\xi}{\xi+3w}\Omega m = \Omega\bar{d}0(1+z)^3 + \frac{-\xi}{\xi+3w}\Omega m \tag{3b}$$

$Q_c = \xi H \rho_d$

$$\Omega m = (\Omega m0 + \frac{\xi}{\xi+3w}\Omega d0)(1+z)^3 + \frac{-\xi}{\xi+3w}\Omega d = \omega\bar{m}0(1+z)^3 + \frac{-\xi}{\xi+3w}\Omega d \tag{4a}$$
$$\Omega d = \Omega d0(1+z)^{3(1+w)+\xi} \tag{4b}$$

Eqn 3 and eqn 4 shows that the coupling constant has two effects,

1. Change the amplitude of the evolution of matter or dark energy energy density.
2. Transfer energy between DE and DM.

2.1 Some definitions

1. For short

$$r = \frac{\Omega m0}{\Omega d0}$$

CPL EoS is

$$w = w0 + w1\frac{z}{1+z}$$

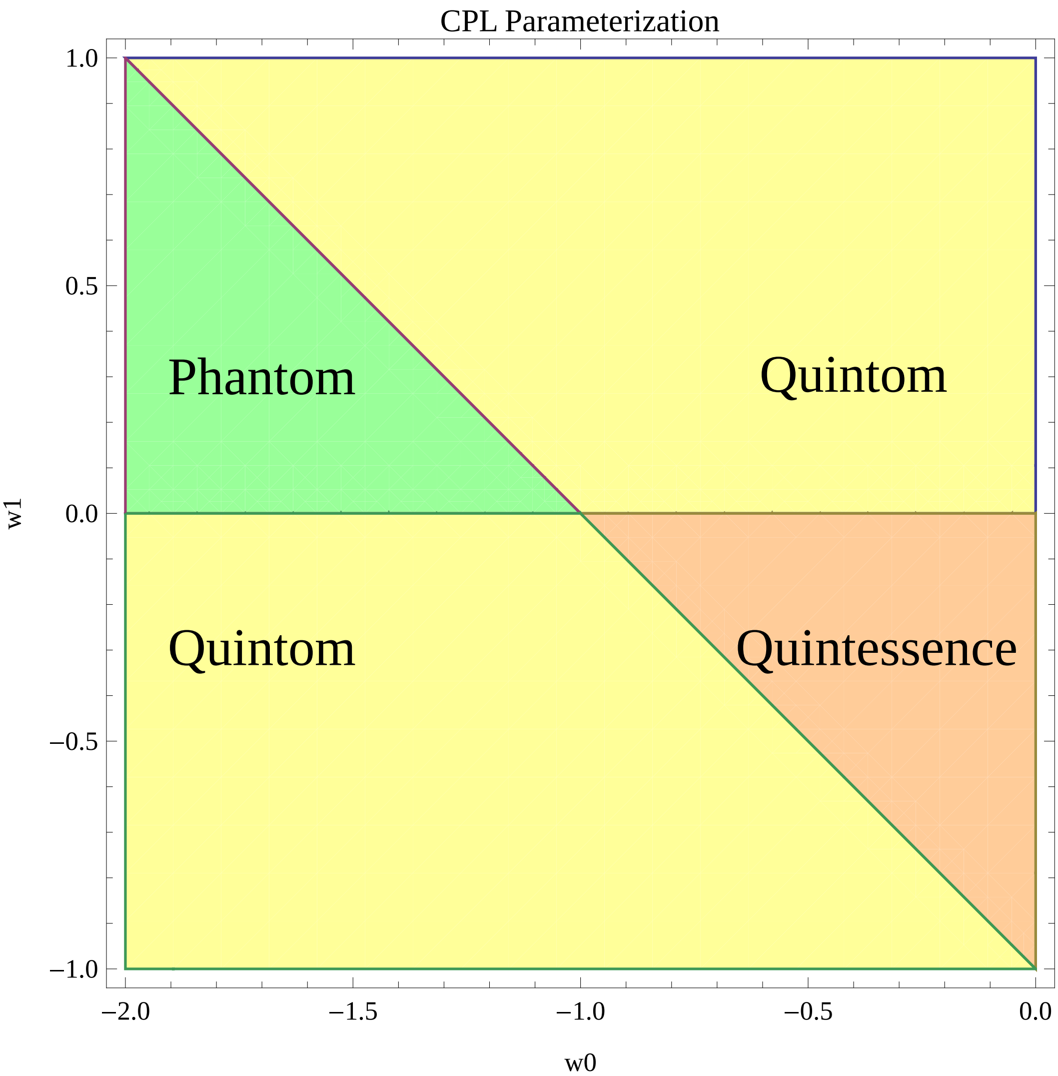


Figure 1: CPL classification

Classification Figure 1 shows how to category the dark energy models in CPL parametrization.

3 Data & Method

3.1 Data

LCDM Parameters From WMAP, $\Omega m0 = 0.265$

Constraints $\Omega m0 = 0.247(+0.013, -0.013)$; Transition redshift 0.426 (+0.082, -0.050).(arXiv:1205.4688 arXiv:astro-ph/0611572).

In ($\Omega m0$, Transition redshift) plane, allowed region is a rectangle centred at (0.274, 0.426) with two diagonal points (0.261, 0.376) and (0.287, 0.508).

CPL $\Omega m0 = 0.269(+0.017, -0.008)$, $w0 = -0.97(+0.12, -0.07)$, $w1 = 0.03(+0.26, -0.75)$

4 Results

Check the files in files folder.

4.1 $Q_c = \xi H \rho_c$

Results table

$Q_c=\xi\ H\ \rho_c$, constant ξ, constant w=-1: Results for ξ			
Ωm0/Ωd0\Transition	z_t=0.376	z_t=0.426	z_t=0.508
r=0.358	-1.25282	-0.965436	-0.617444
r=0.378	-1.15011	-0.875189	-0.542347
r=0.398	-1.05453	-0.791252	-0.472561

Figure 2: ICC Result table

Figure 3 shows that

ρ_c-**Dec-1** The universe decelerates faster at the early stage for smaller interaction constant ξ even they have the same matter fraction.

ρ_c-**Dec-2** For the same ξ, the deceleration converge ($q = (1 - \xi)/2$ with $3w + \xi < 0$) at early time.

Figure 4 shows

ρ_c-**Trans-1** Transition happens earlier when matter fraction is smaller. Matter is against DE's pressure.

ρ_c-**Trans-2** Transition is later when ξ is smaller.Energy transfers to DE when ξ is negative, then why later transition? [1]

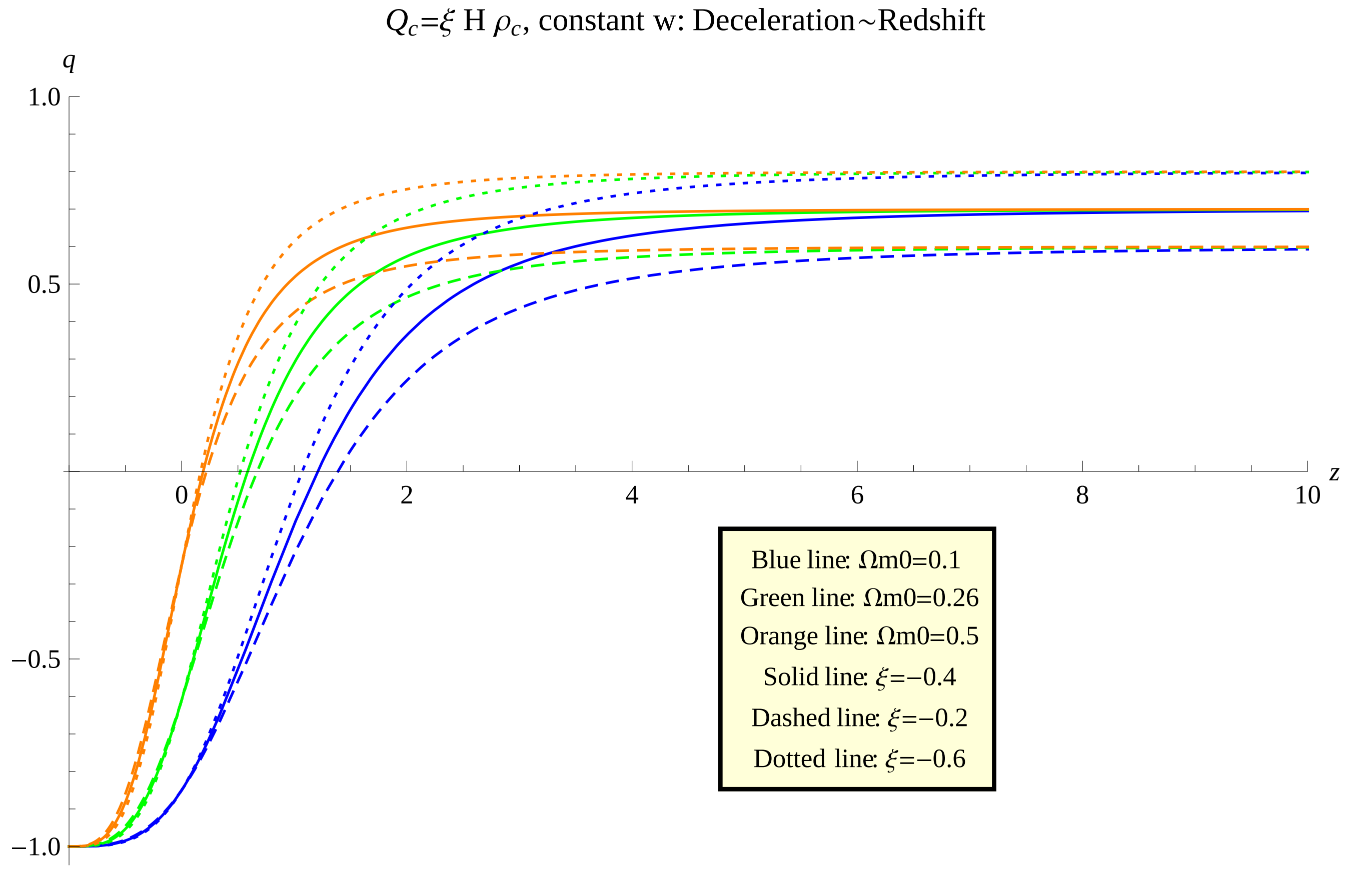


Figure 3: Deceleration parameter

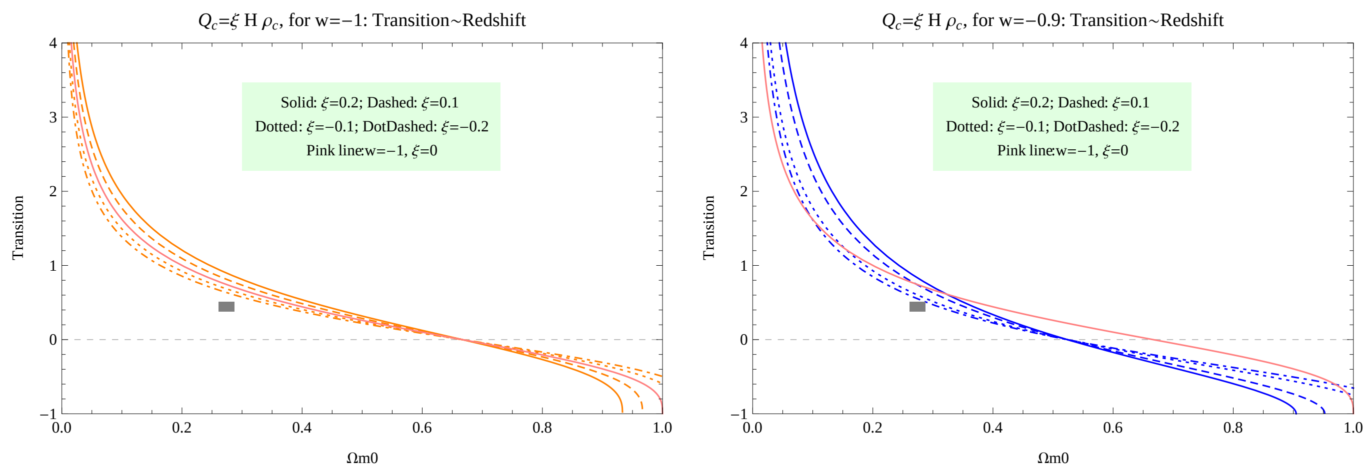


Figure 4: Transition redshift.

ξ results for Q_c=ξ H ρ_c (Fitting data: Data From, 2)			
w	Center	Lower	Upper
-1.183	-0.881565	-1.29687	-0.443589
-1.087	-0.88948	-1.29859	-0.459135
-0.991	-0.875238	-1.27522	-0.456176

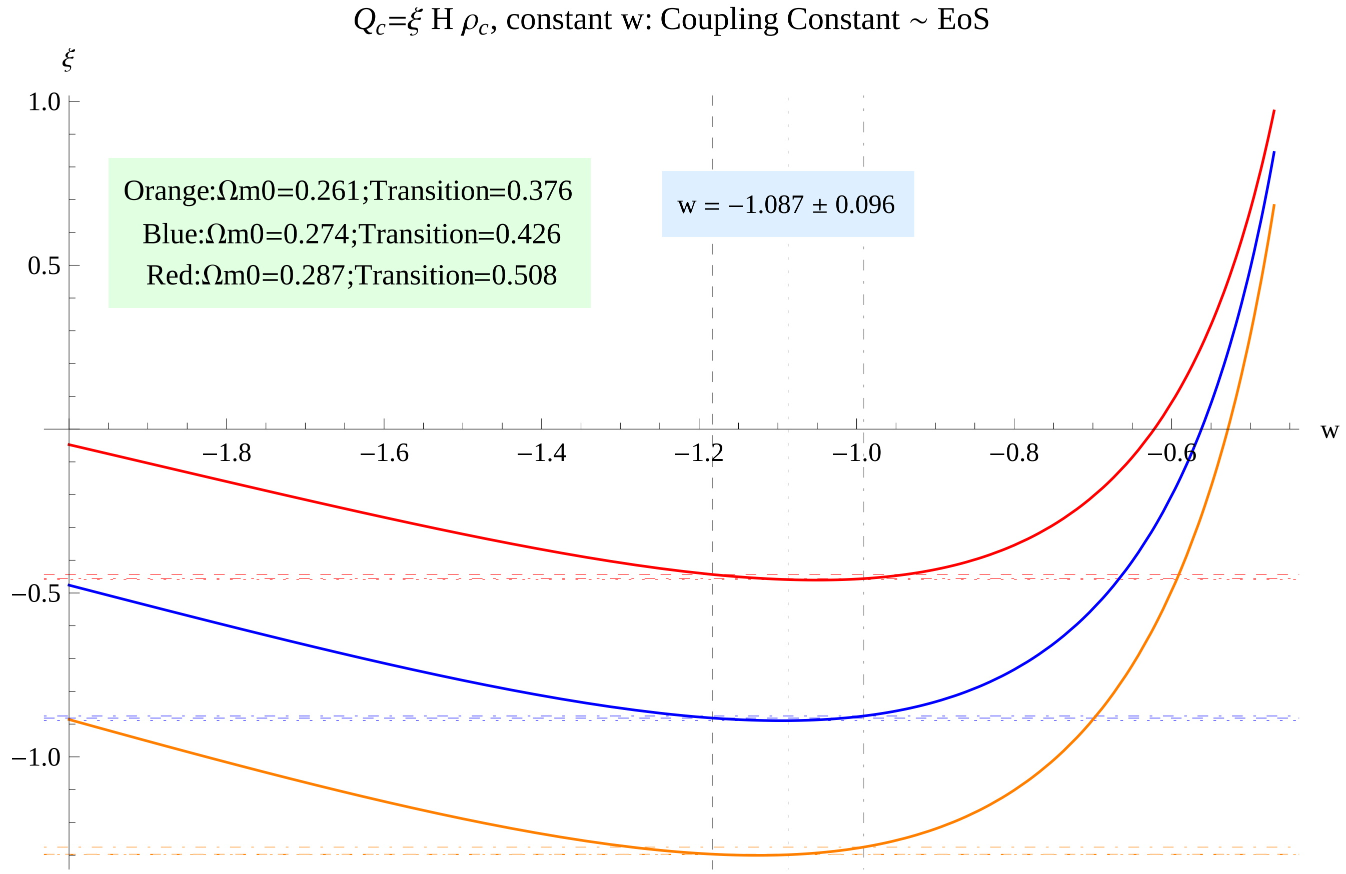


Figure 5: Interacting coefficient for $Q_c = \xi H \rho_c$

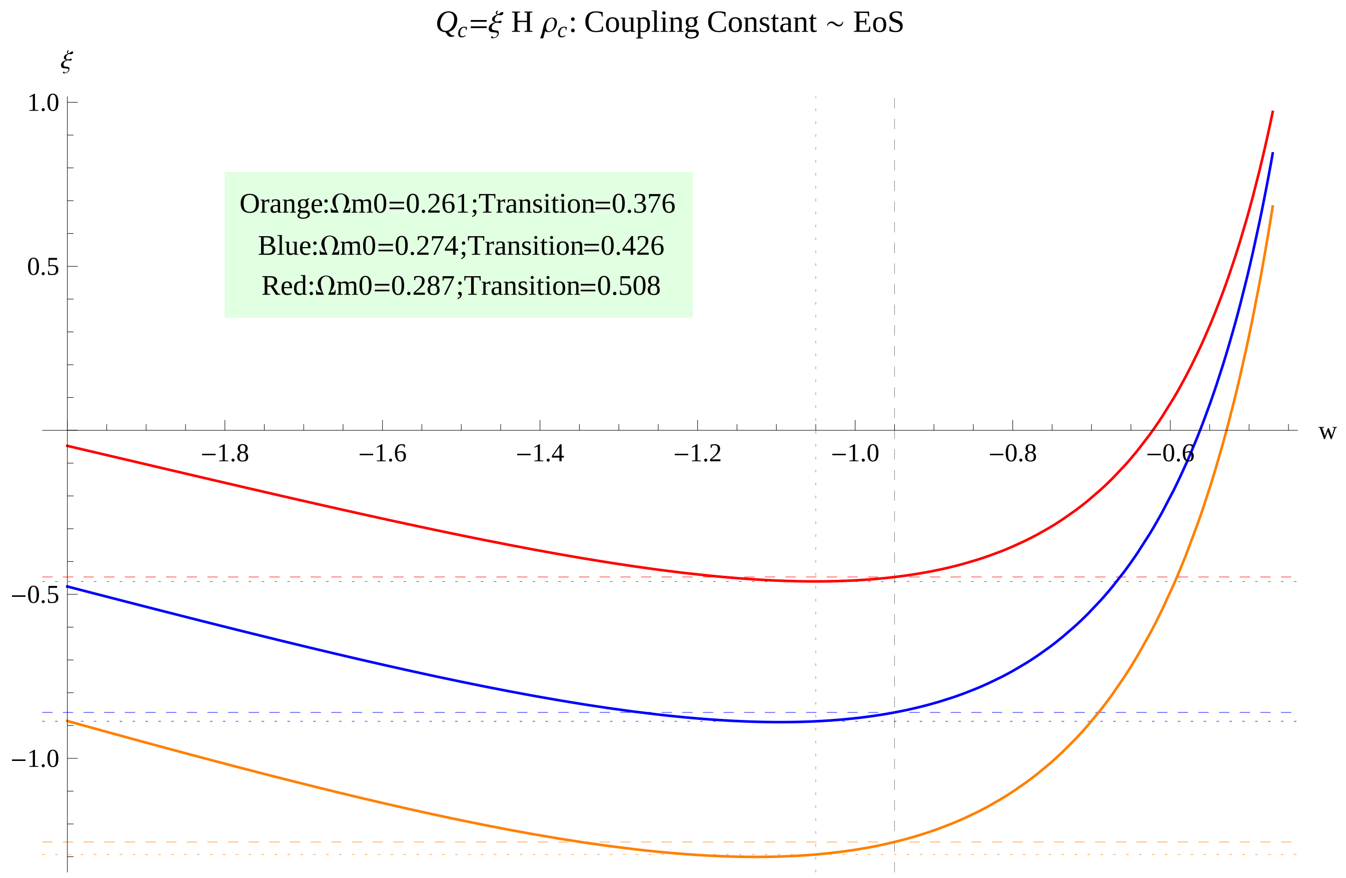


Figure 6: Interacting coefficient for $Q_c = \xi H \rho_c$

Figure 5 and figure 6 (vertical lines are $w = -1 \pm 0.05$) show the value of ξ for different EoS.

Results from this figure:

(EoS is -1 within 5w=-1 (-1.279,-0.457) center:-0.878 ; w=-1.05 (-1.293,-0.461) center:-0.887 ; w=-0.95 (-1.255,-0.447) center:-0.860 ;

ρ_c**-xiVSw-1** NOT monotonic. The result of ξ is greatly affected by $\Omega m0$. In correspondence with another

ρ_c**-xiVSw-2** For different $\Omega m0$, ξ values deviate greatly form each other at small w.

Figures 7 show how do we constrain ξ and how do EoS change our constrain results with the transition fixed.

ρ_c**-xiVS**$\Omega m0$**-1** The smaller, the more difference among ξ values of different EoS. Reason for this is less matter has less effect on the evolution thus the property of dark energy determines more about the transition.

ρ_c**-xiVS**$\Omega m0$**-2** Second figure shows

+ System with smaller w needs smaller coupling to achieve the same transition time, as expected.
+ So we give the result that $w \in (-0.58406, -0.3334)$ if we constrain $\Omega m0 = 0.2603$ and transition redshift 0.426.

4.2 $Q_c = \xi H \rho_c$, CPL

For a flat universe, choose the parameters w0=-1.02,w1=0.6, the region for interation cosntant ξ should be (-1.04,-0.21) with a center at -0.64, derived from the (transition redshift, $\Omega m0$) plane, while a result of (-1.01, -0.23) with a center at -0.63, derived from (transition redshift, $\Omega m0/$

First let's have a look at the deceleration parameter.

Figure 8 is right similar to constant EoS situation.

Figures 9 show

ρ_c**-ICCPL-TVS**$\Omega m0$**-1** Coupling acts on these model similar to constant EoS model.

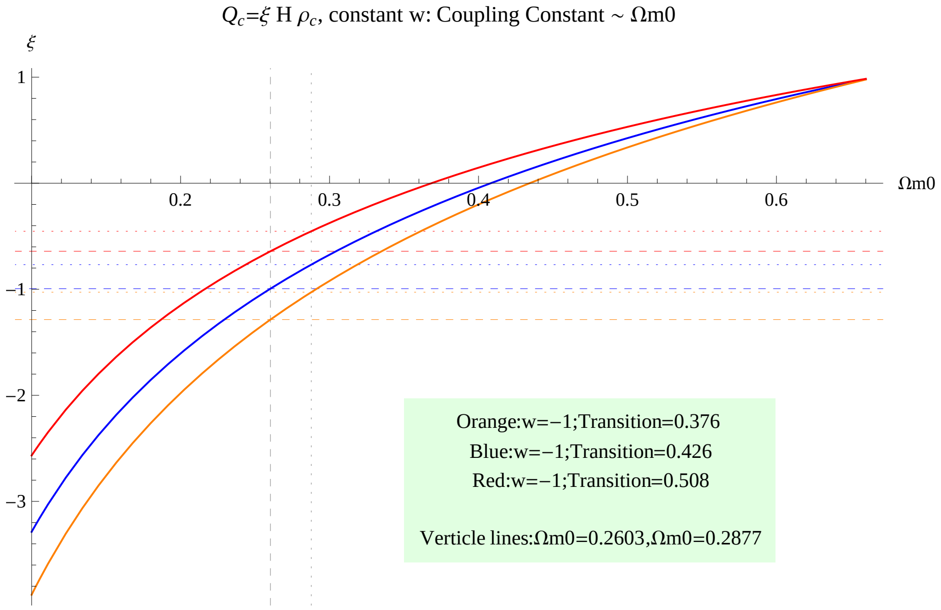
ρ_c**-ICCPL-TVS**$\Omega m0$**-2** The change in $w1$ has a similar effect with the change of ξ. Larger $w1$ corresponds to smaller ξ. The reason for this is both negative ξ and larger $w1$ enhances the energy density of dark energy (check using the CPL EoS).

The dots in figure 10 are the data set of $\xi = 0$. If we need $\xi < 0$, i.e., energy transfers from dark matter to dark energy, the allowed parameter space is the striped area.

For $\Omega m0 \in 0.274\ (1 \pm 0.05)$

Table of ξ for different $\Omega m0$~Transition combination

$\Omega m0$~Transition	0.426	0.376	0.508
0.2603	-0.994339	-1.28571	-0.641508
0.274	-0.877755	-1.15303	-0.544482
0.2877	-0.767582	-1.02756	-0.452892



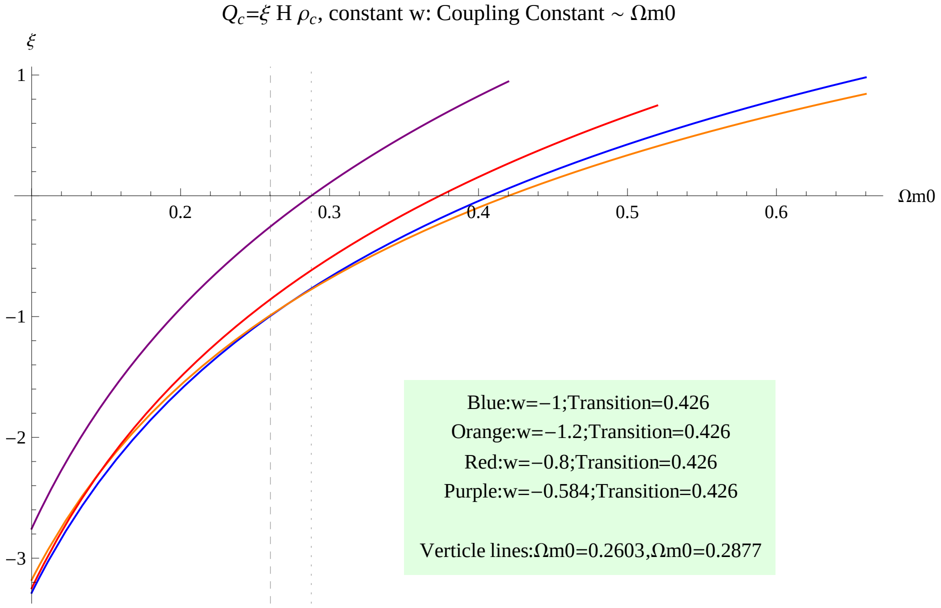


Figure 7: Interacting coefficient changing with $\Omega m0$ for $Q_c = \xi H \rho_c$

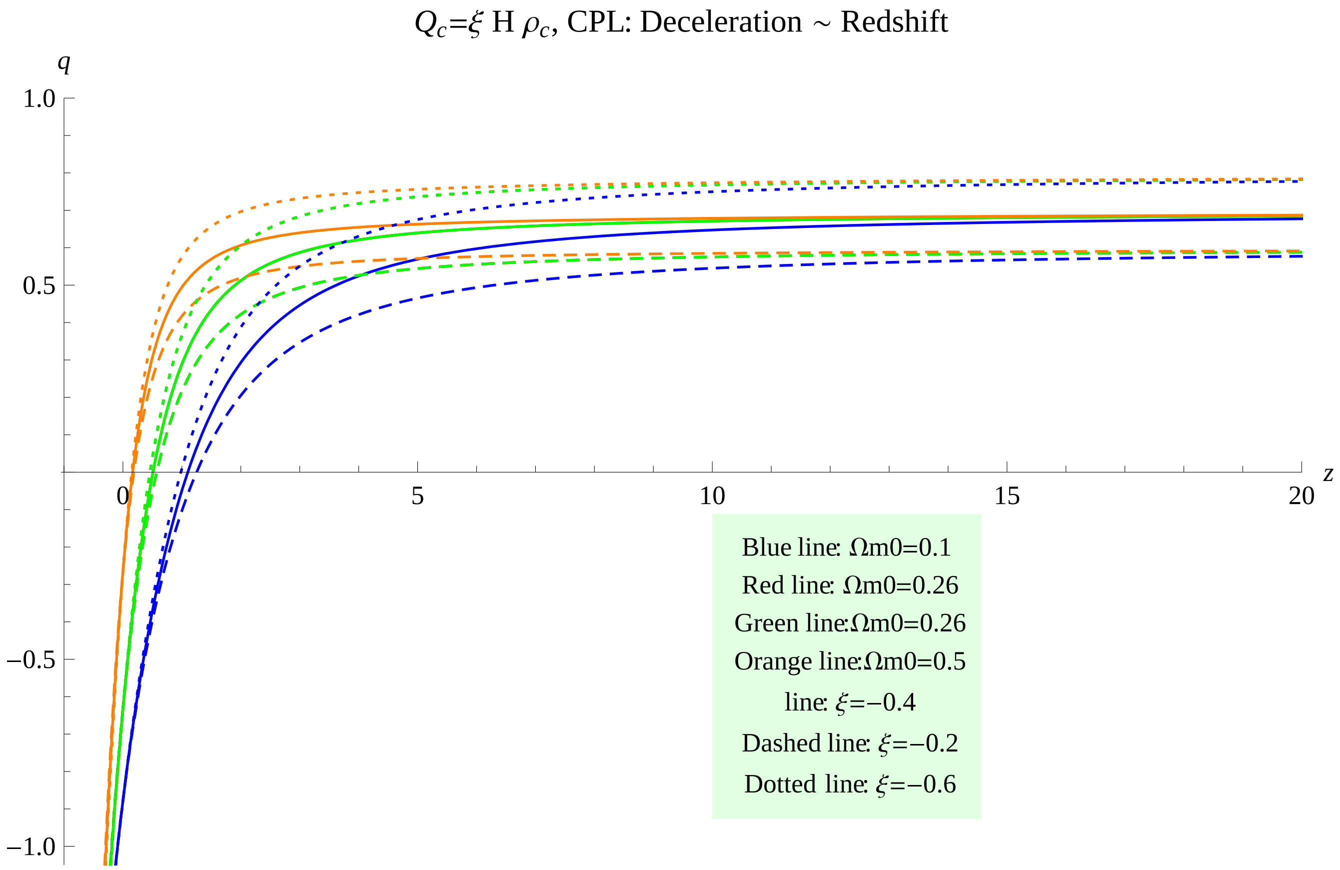


Figure 8: Deceleration parameters for ICCPL ($Q_c = \xi H \rho_c$ with CPL parametrized EoS)

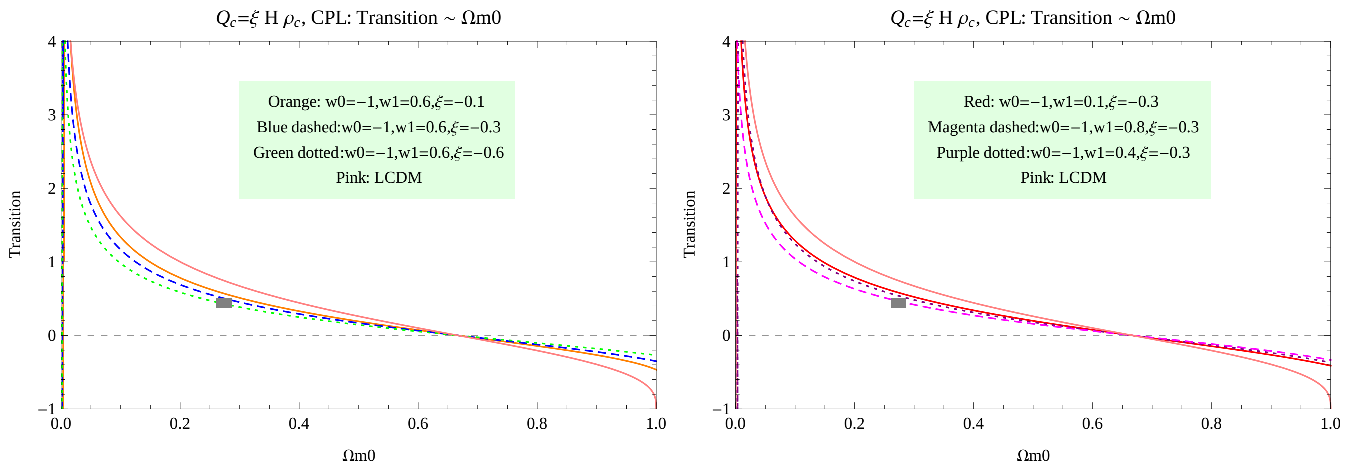


Figure 9: The effect of EoS parameters on Transition and $\Omega m0$

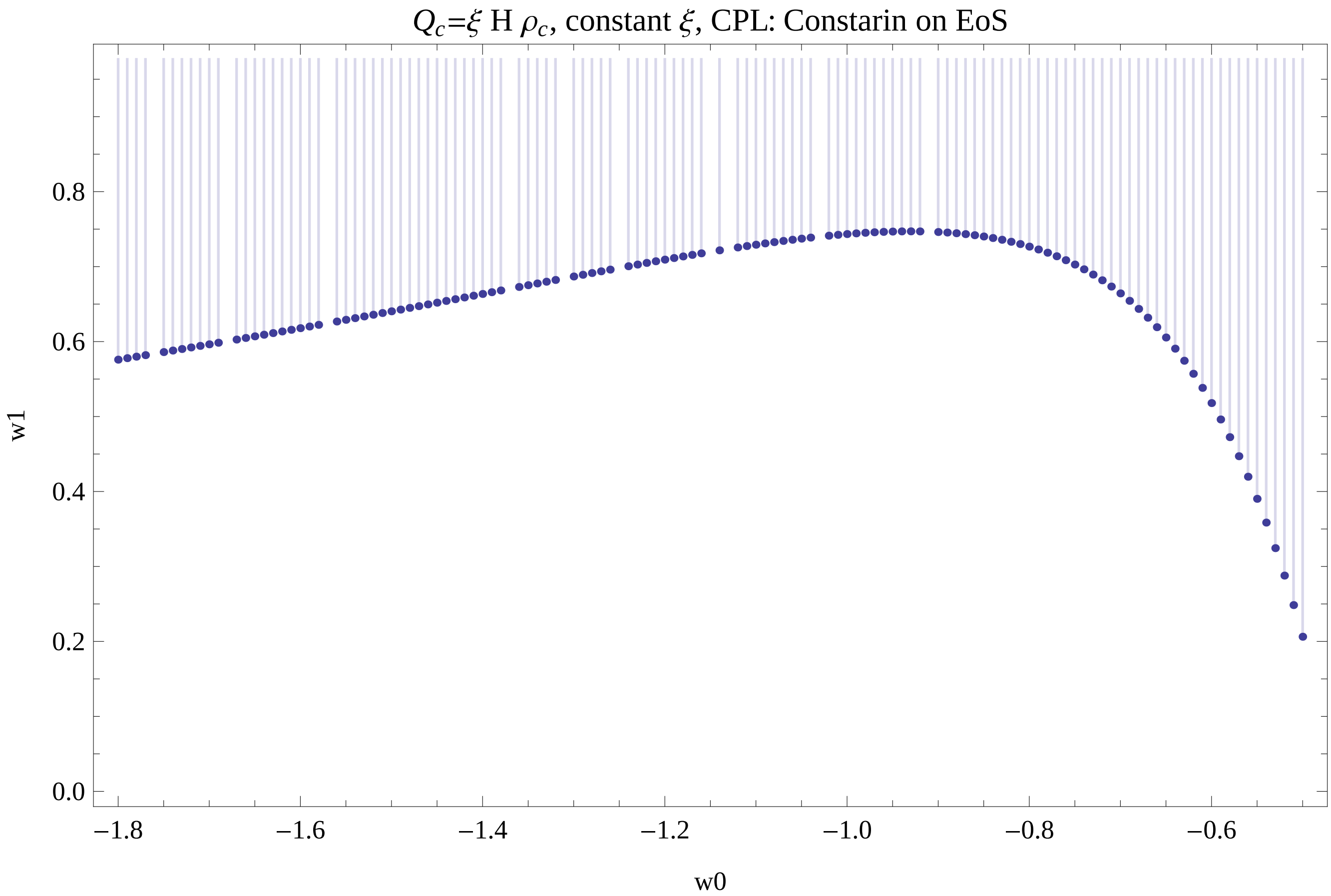


Figure 10: Lower bound in the W1 w0 parameter space

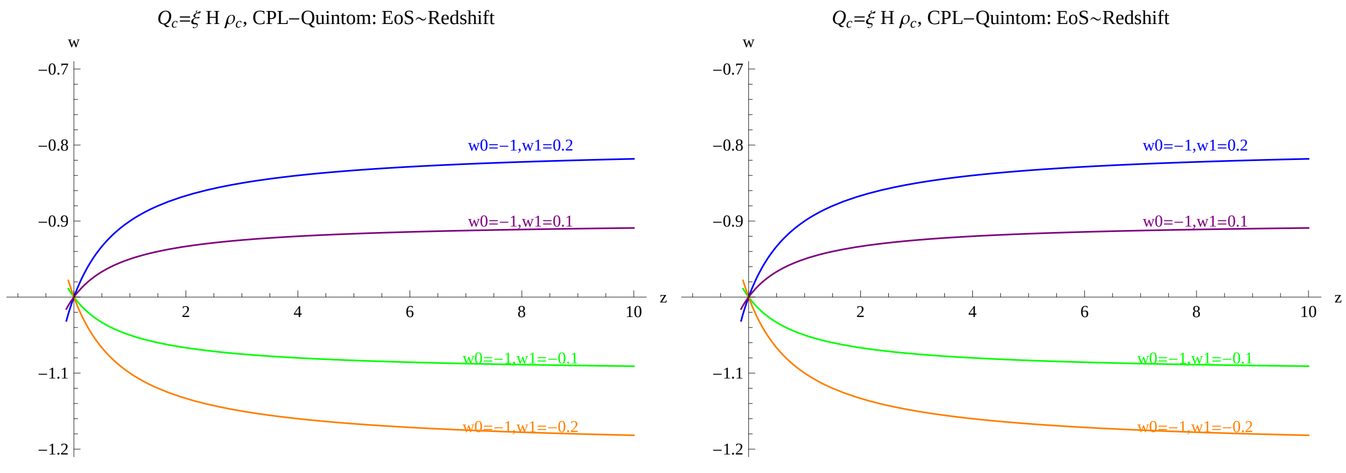


Figure 11: The EoS

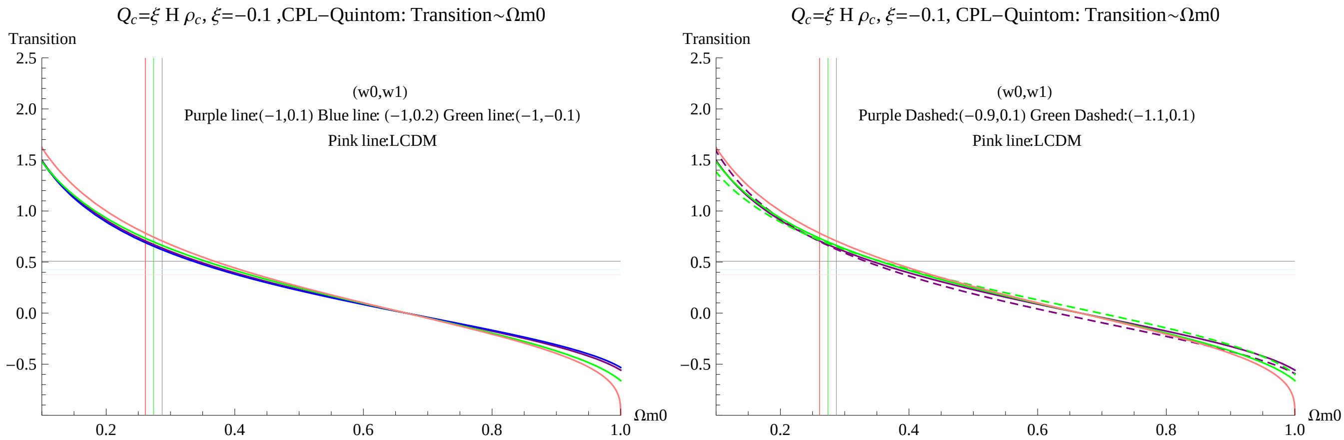


Figure 12: Transition vs $\Omega m0$

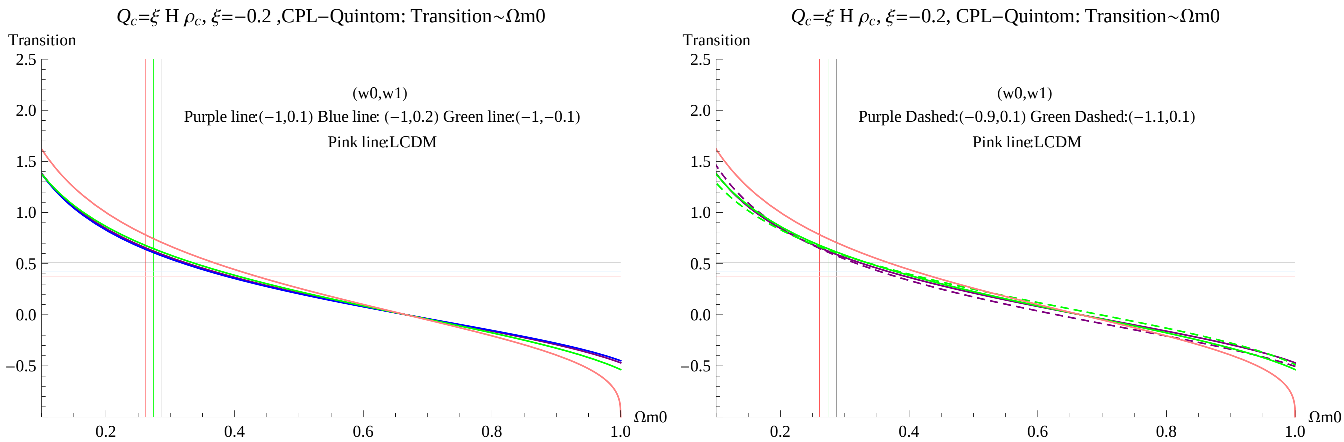


Figure 13: Transition vs $\Omega m0$

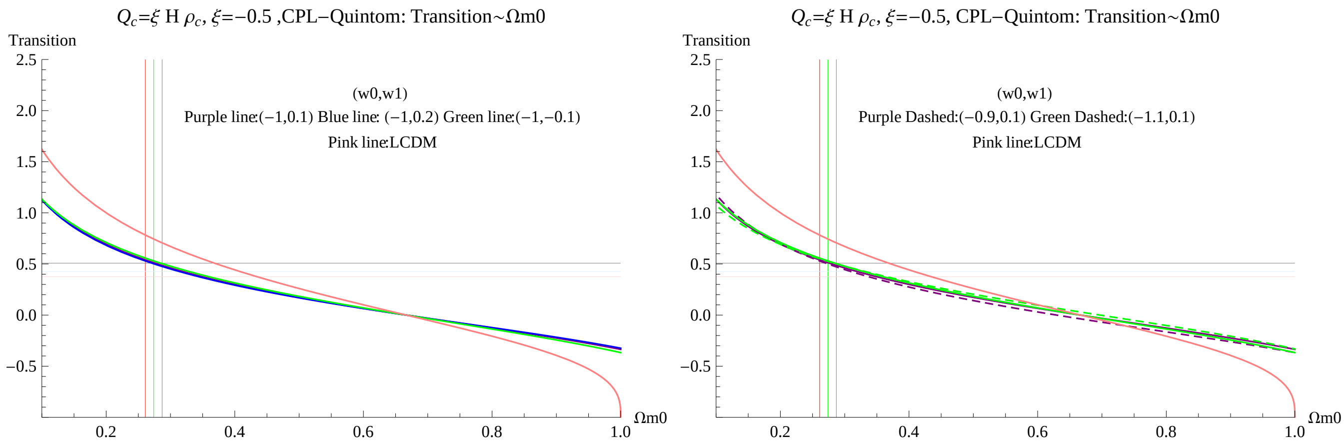


Figure 14: Transition vs $\Omega m0$

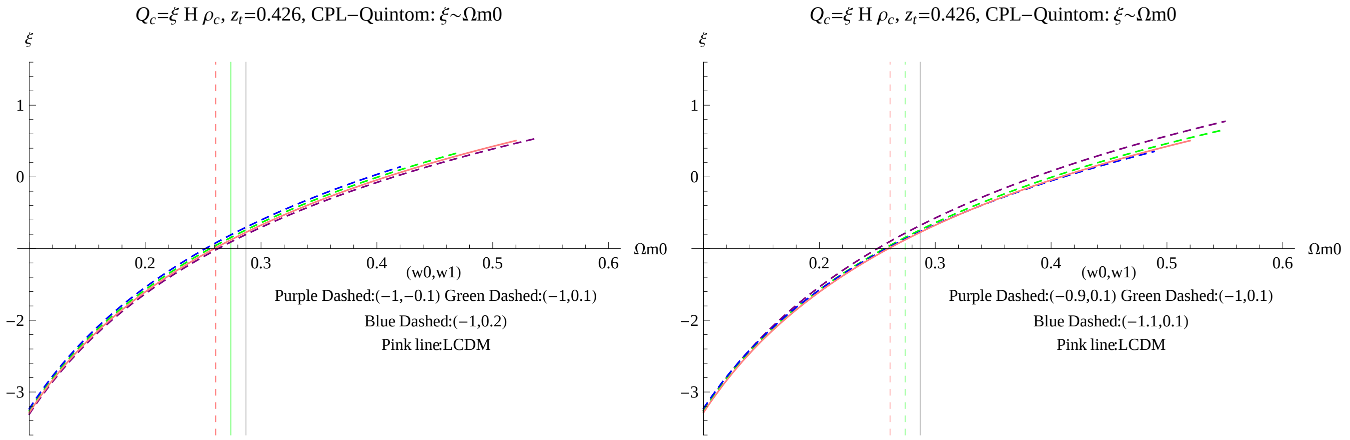


Figure 15: ξ vs $\Omega m0$

4.2.1 Quintom

(Figures 11, 12, 15, 13, 14)

The left figure in 12 indicates a possible stationary point.[2]

Comparing figure 14, figure 13 and figure 12, it seems that all lines cross point (0.66, 0), the reason of which, however, is because $w0 = -1$ and a small nearly zero $w1$ means a similar evolution with $Q_c = \xi H \rho_c$ with a constant EoS. [3]

(Other results are shown on the complete results files.)

0

4.2.2 Quintessence

(Figures 16, 17 and 18.)

There is a features in the right figure of 17. The difference between lines become smaller when $\Omega m0$ is small and at about 0.35. Reason for this is when $\Omega m0$ is smaller, the evolution is mainly determined by Ωd ([4])

4.2.3 Phantom

(Figures 19, 20, 21)

4.3 $Q_c = \xi H \rho_d$

(Figures 22, 23, 24, 25)

---

[1]Reasons below. All solutions of equation 2 have the same value at $z = 0$, i.e. now. Equation 2a tells us a positive ξ leads to smaller energy density of dark matter at early time of the universe, thus dark energy takes over quickly if the transition happens before today. (For more details, calculation are shown in supplement_ 08-10.pdf file.)

[2]Only possible because I can only partially prove there is a nearly stationary. This is on my *Cosmologia Notebook*.

[3]The lines do not intercept with each other at the same point actually.

[4]In the solutions of ρ_c and ρ_d, it is clear that $\Omega m0$ determines how much $w1$ will affect the result because the term $e^{3w1\frac{1}{1+z}}$ is small since $w1$ and $1/(1+z)$ are small and $\Omega m0$ is the amplitude of the other term in which $w1$ is in charge. Check *Cosmologia Notebook*.

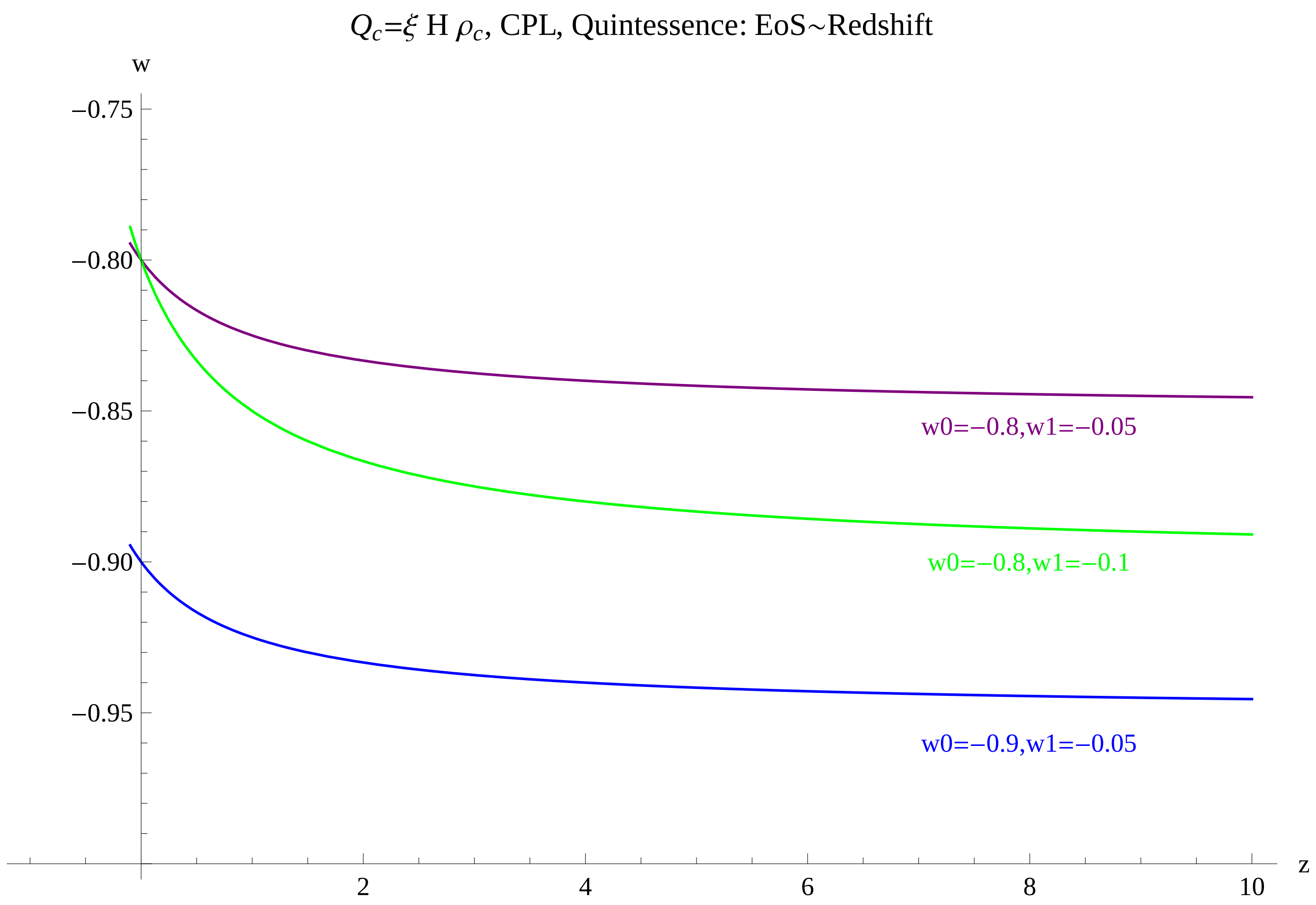


Figure 16: The EoS

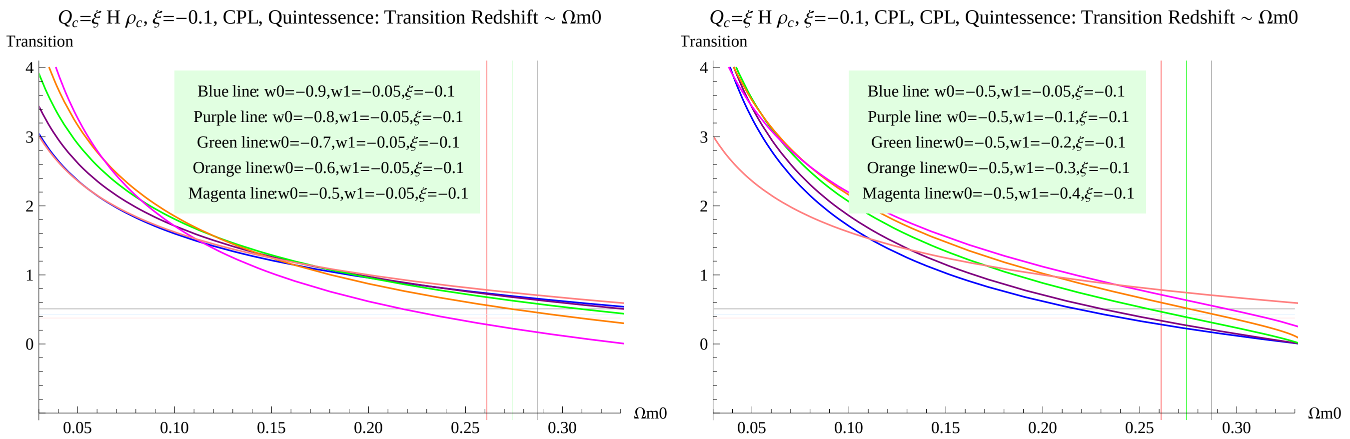


Figure 17: Transition vs $\Omega m0$

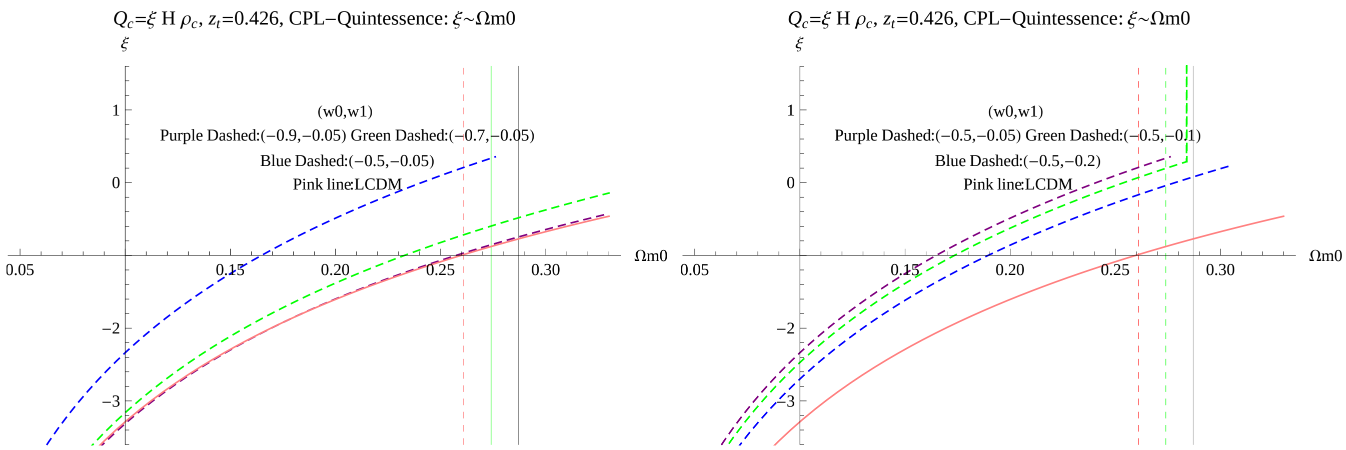


Figure 18: ξ vs $\Omega m0$

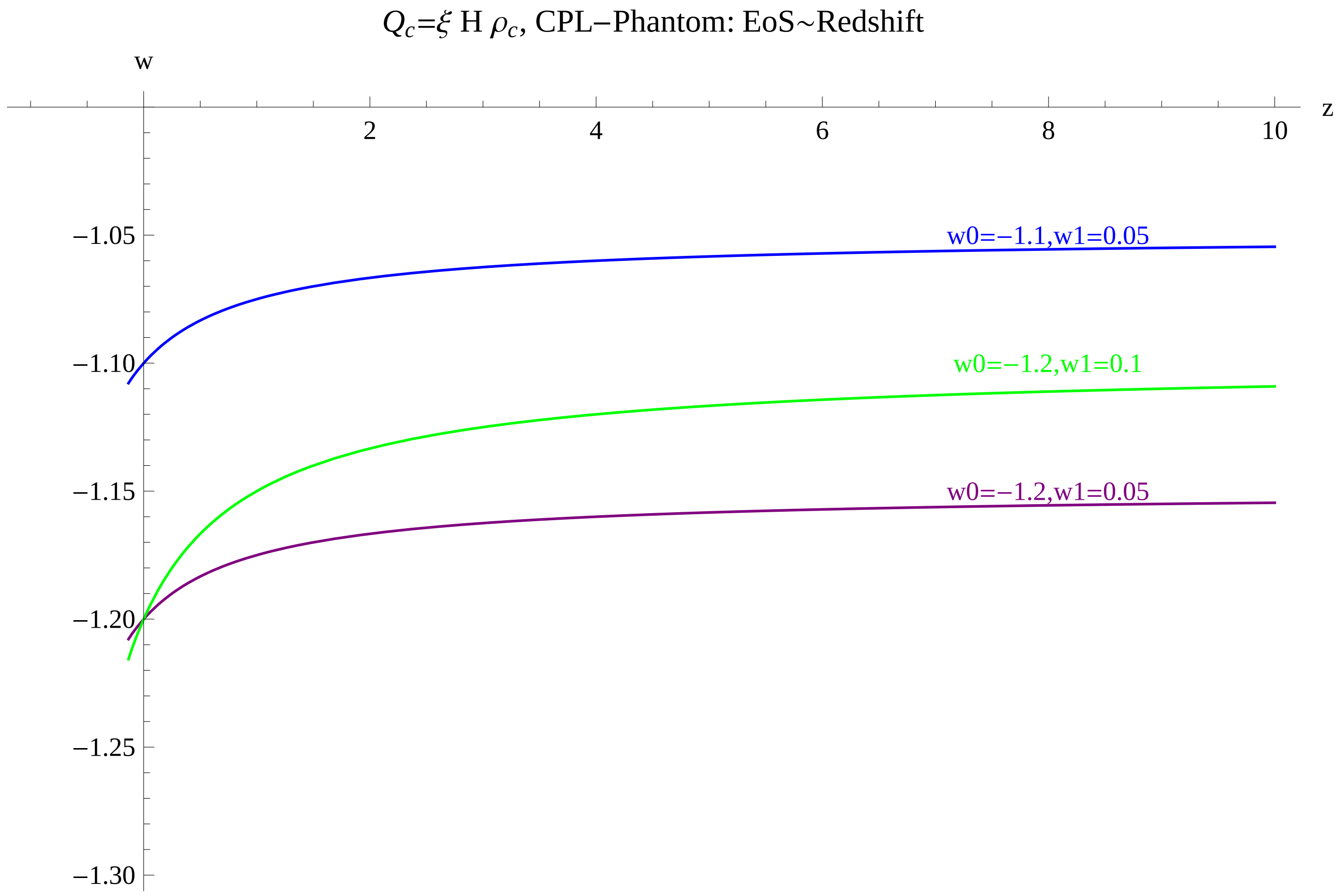


Figure 19: The EoS

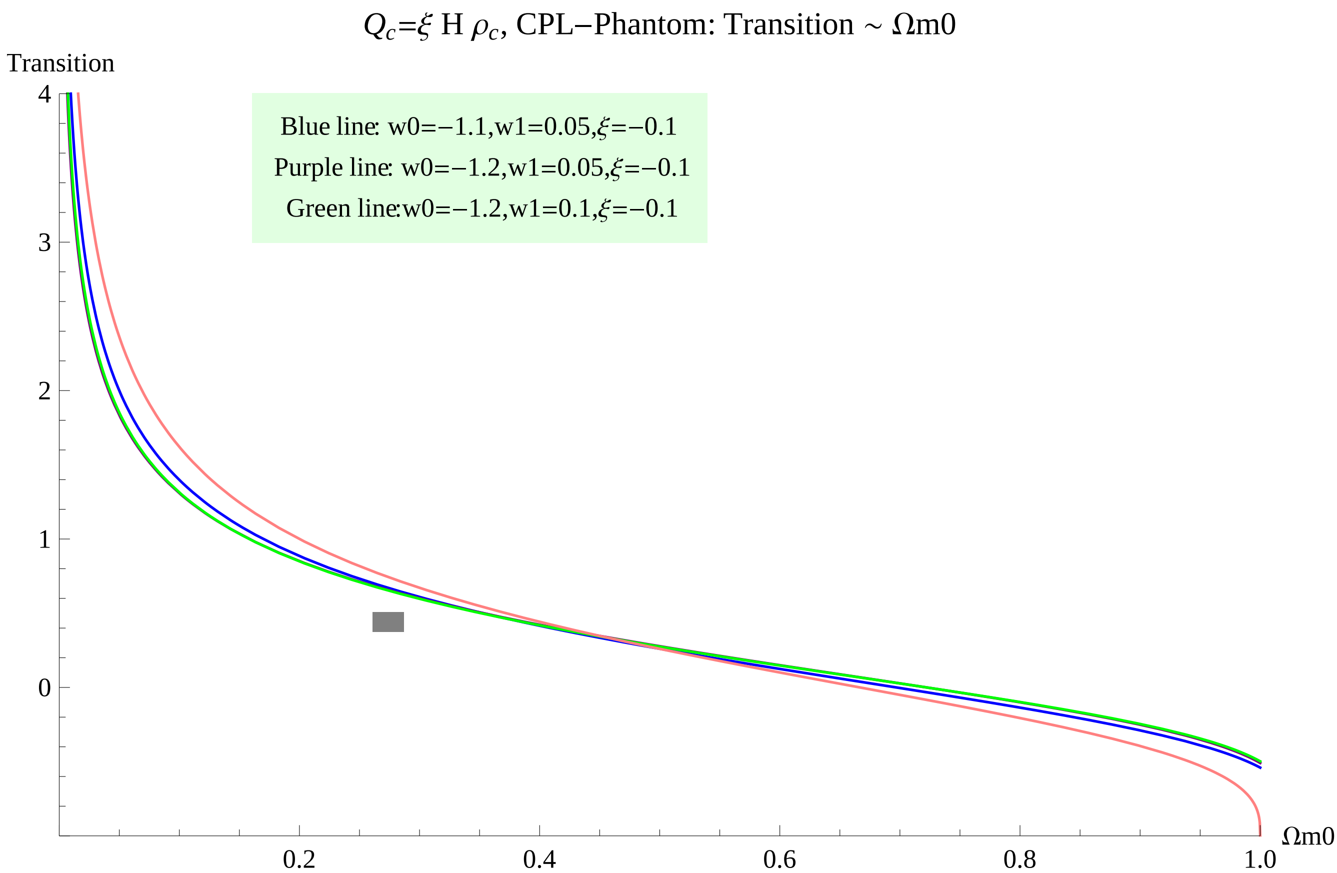


Figure 20: Transition vs $\Omega m0$

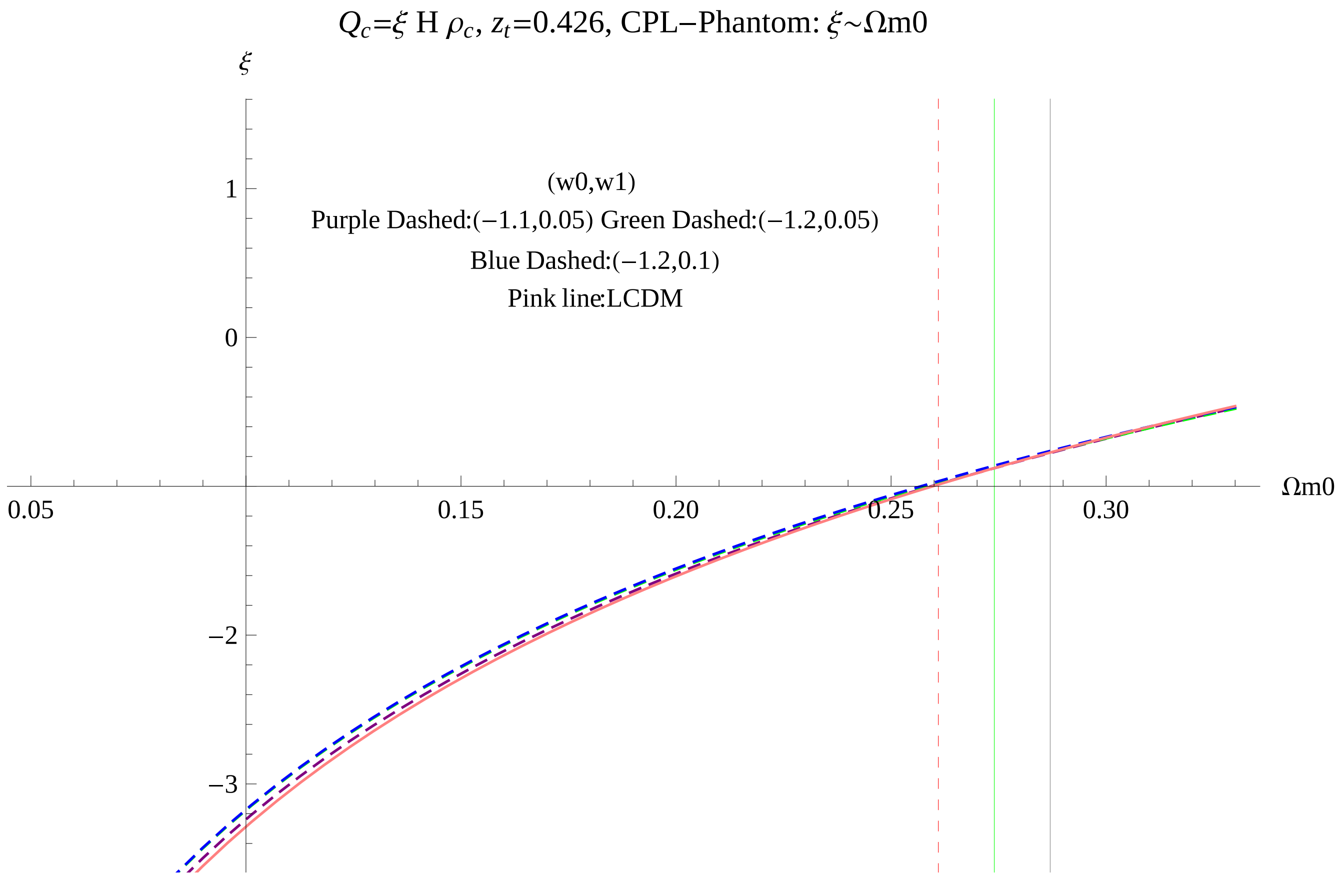


Figure 21: ξ vs $\Omega m0$

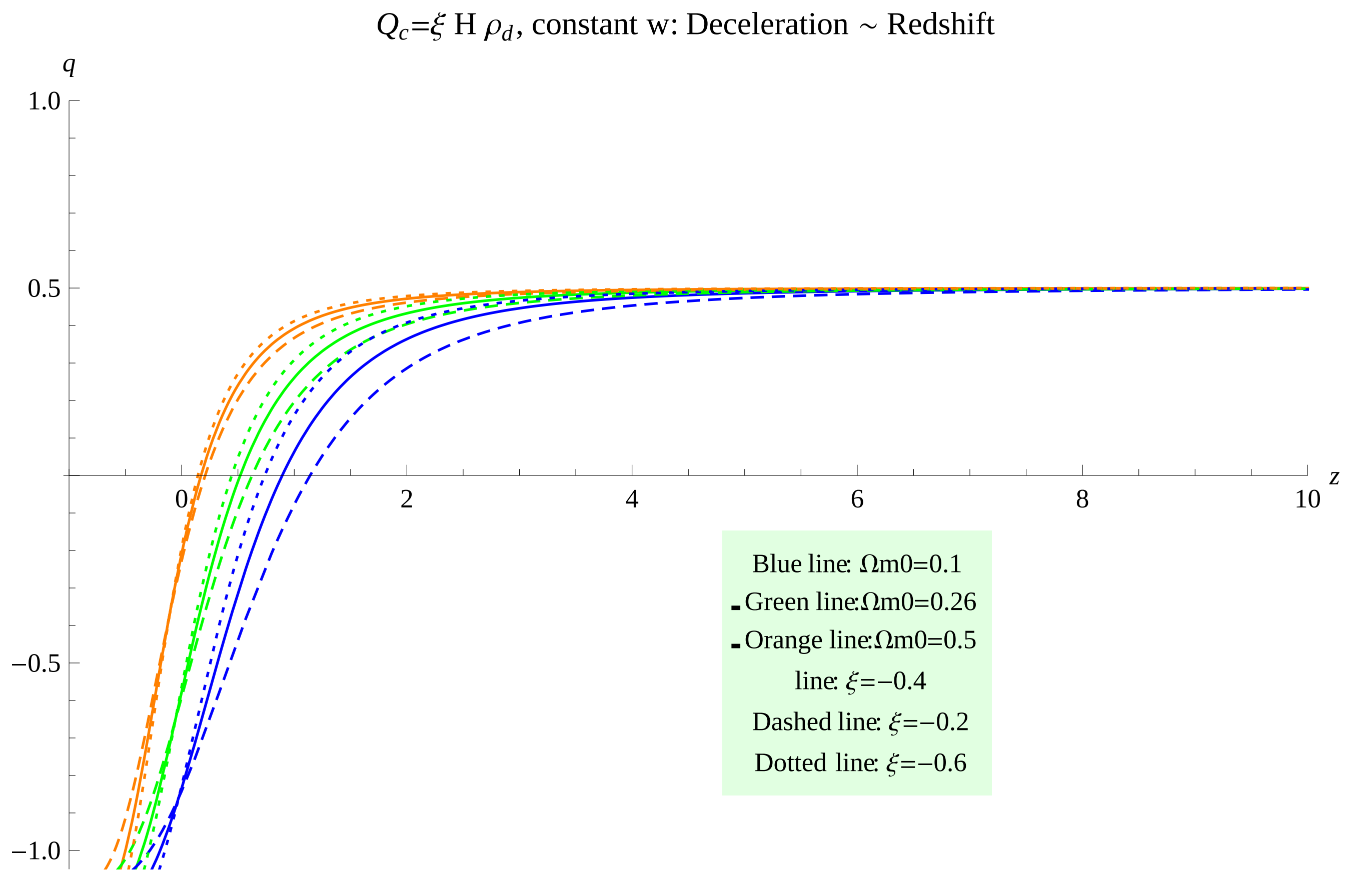


Figure 22: Deceleration parameter

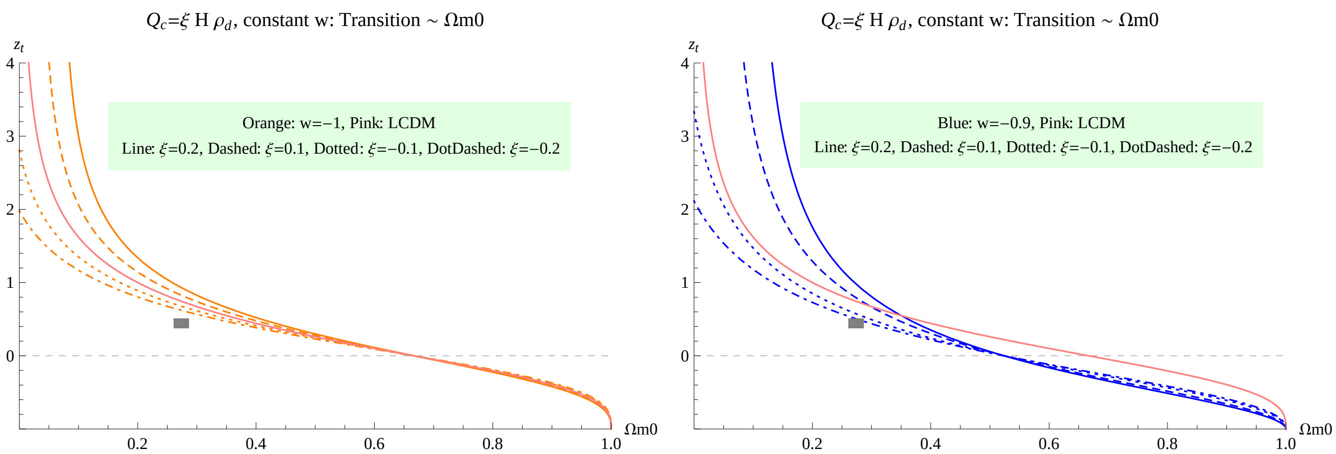


Figure 23: Transition vs $\Omega m0$. For a flat universe, choose the parameters w0=-1.02,w1=0.6, the region for interation cosntant ξ should be (-1.04,-0.21) with a center at -0.64, derived from the (transition redshift, $\Omega m0$) plane, while a result of (-1.01, -0.23) with a center at -0.63, derived from (transition redshift, $\Omega m0/\Omega d0$) plane.

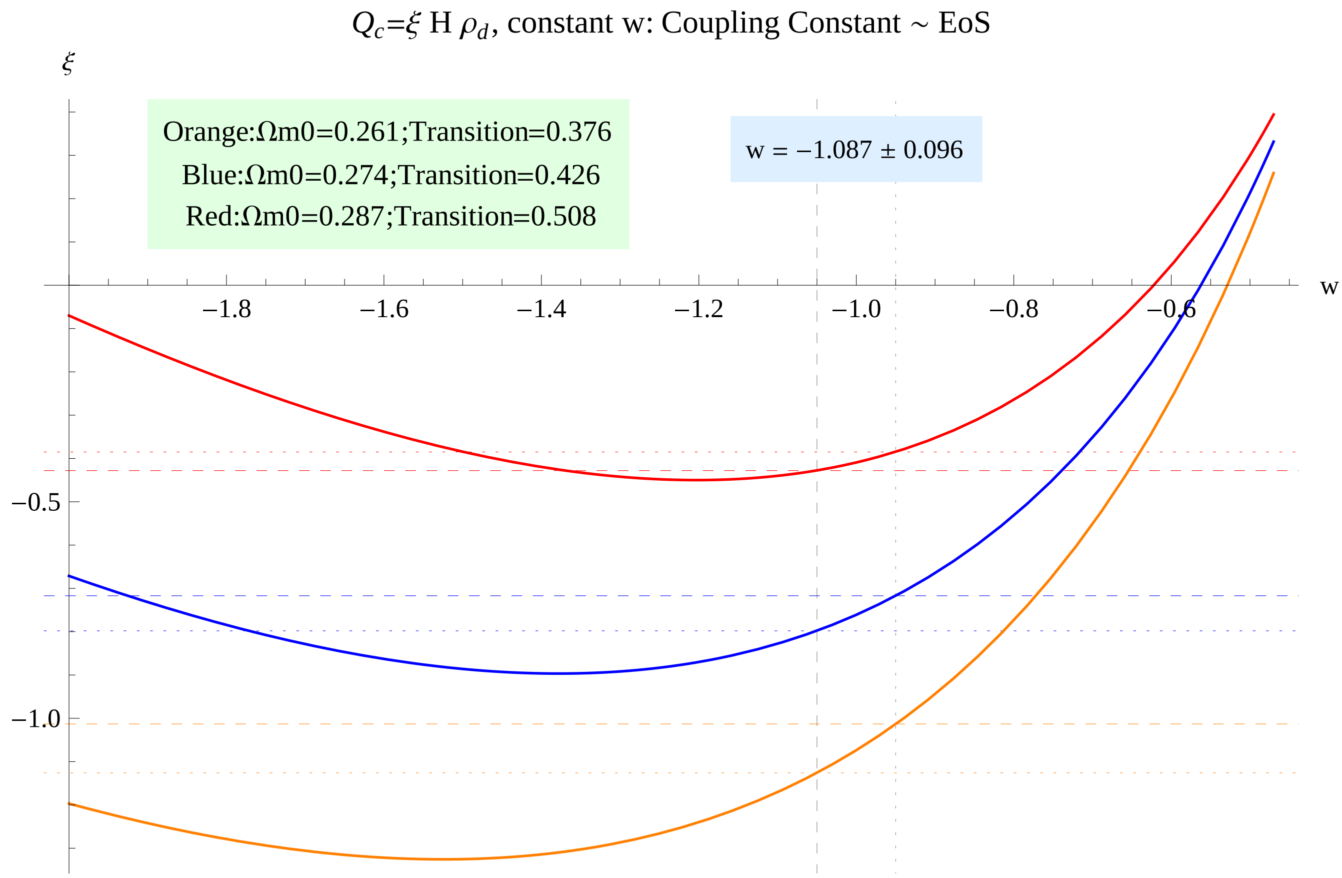


Figure 24: ξ VS w

Figures 23 is

4.4 I2CCPL

(Figures 27, 28)
Figure 27 shows the all the deceleration are the same at very early time.

4.4.1 Quintom

(Figures 29, 30)

4.4.2 Quintessence

(Figures 31, 32)

4.4.3 Phantom

(Figures 33, 34)
There is always a almost-stationary point.

$Q_c=\xi\ H\ \rho_d$,Constant w. (Data used:Data From, 2)			
w	Center	Lower	Upper
-1.183	-0.864289	-1.22984	-0.449552
-1.087	-0.820486	-1.15946	-0.437339
-0.991	-0.753634	-1.06346	-0.405262

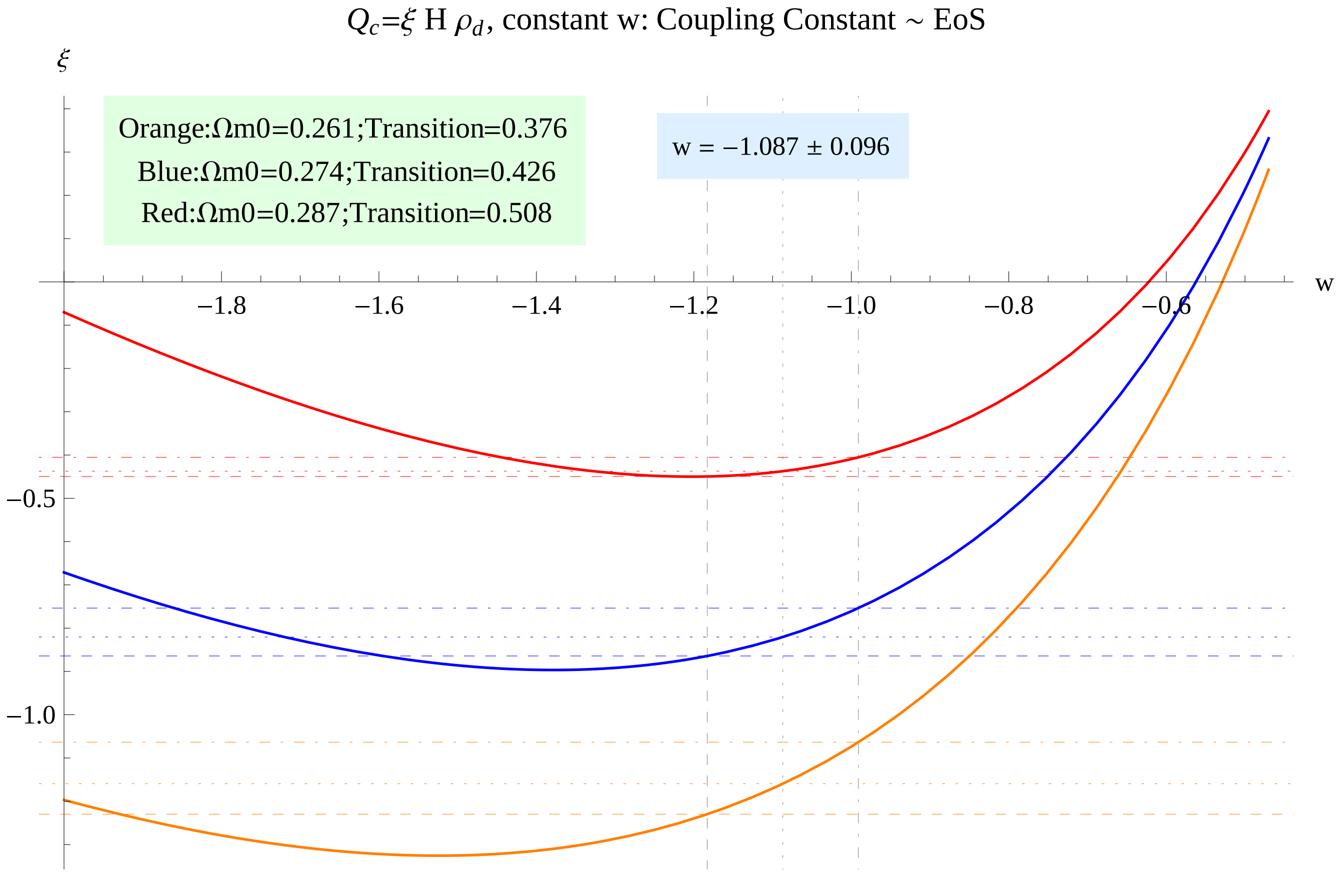


Figure 25: ξ VS w

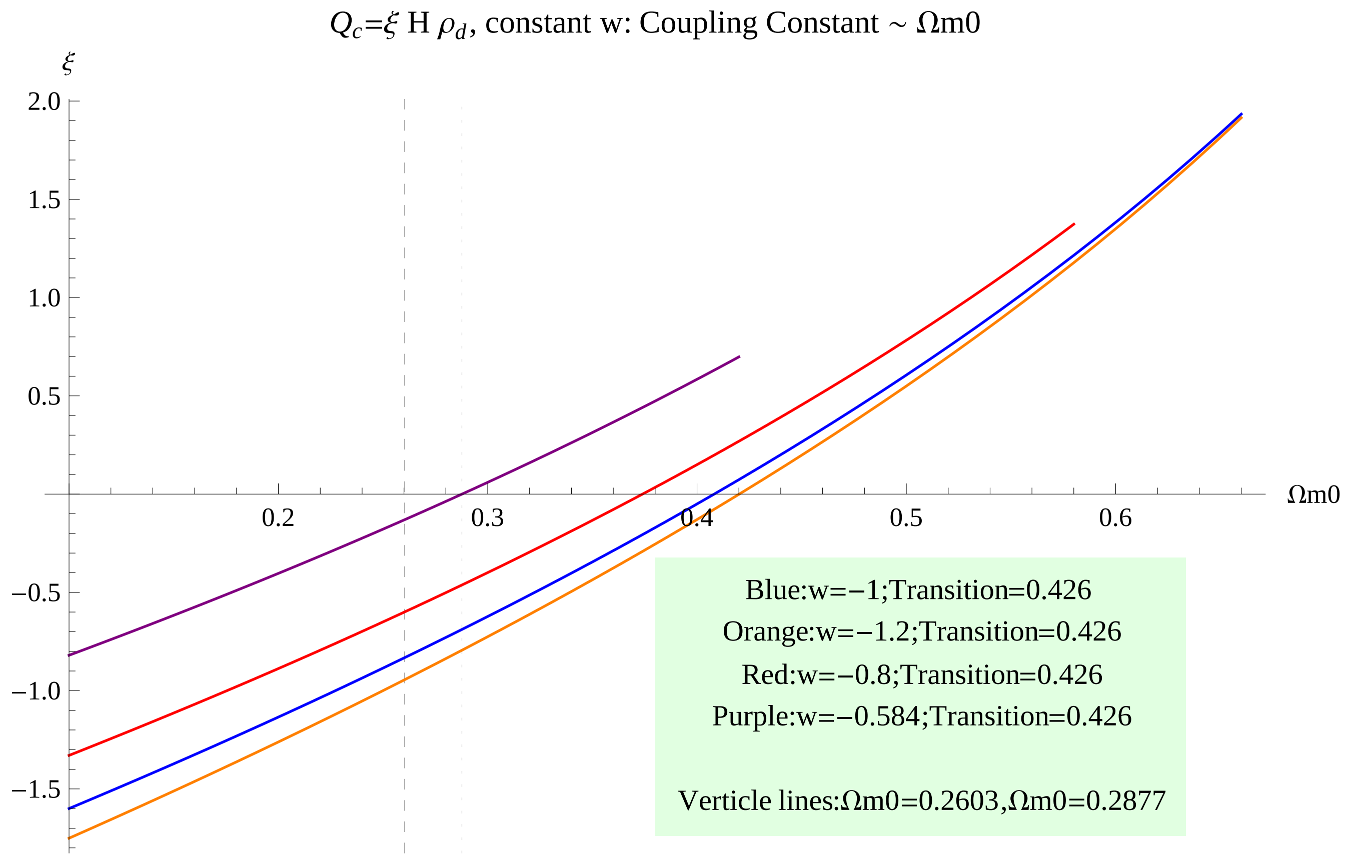


Figure 26: ξ VS $\Omega m0$

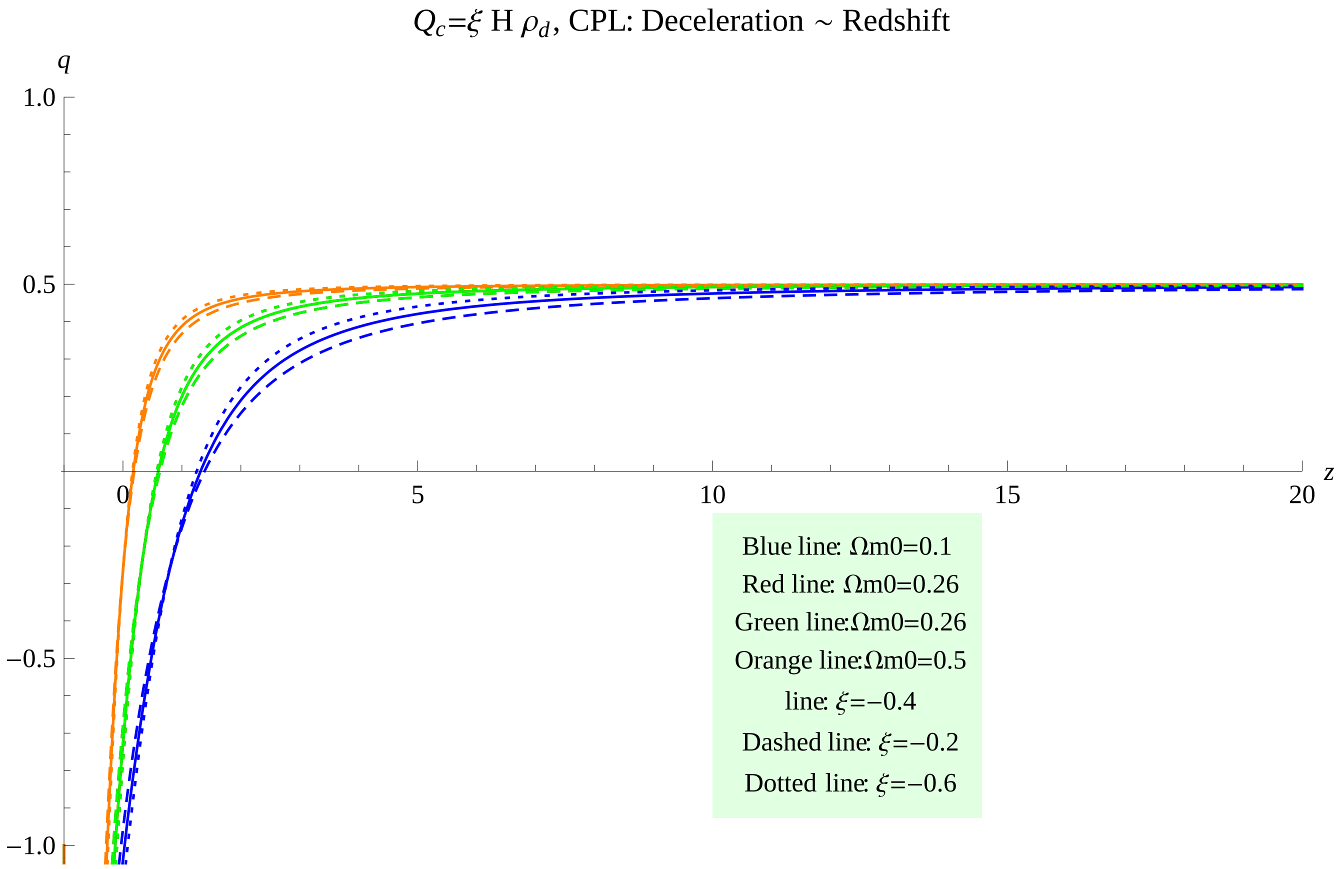


Figure 27: Deceleration parameter

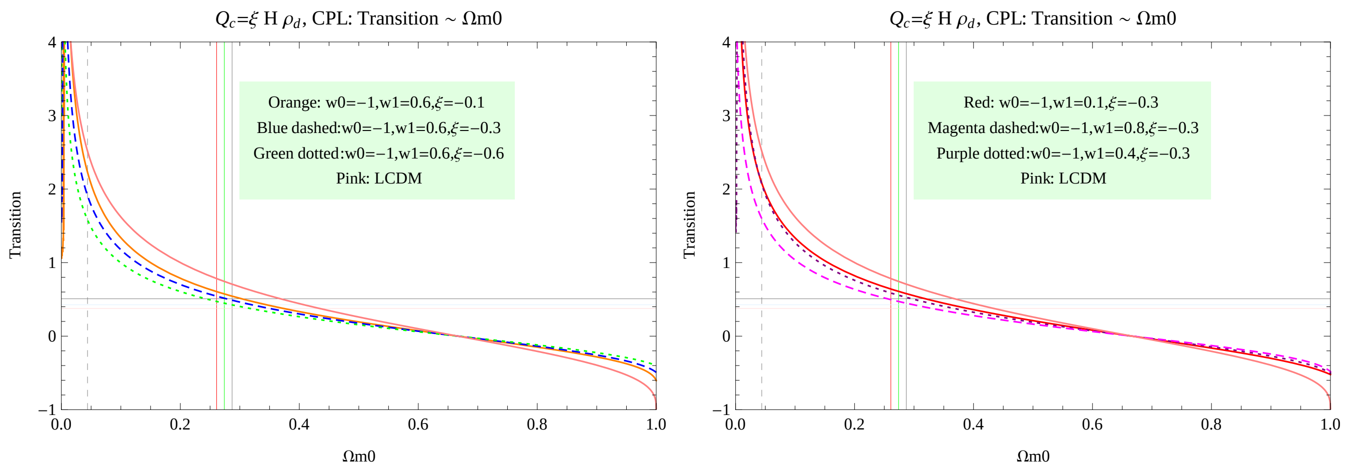


Figure 28: Transition VS $\Omega m0$

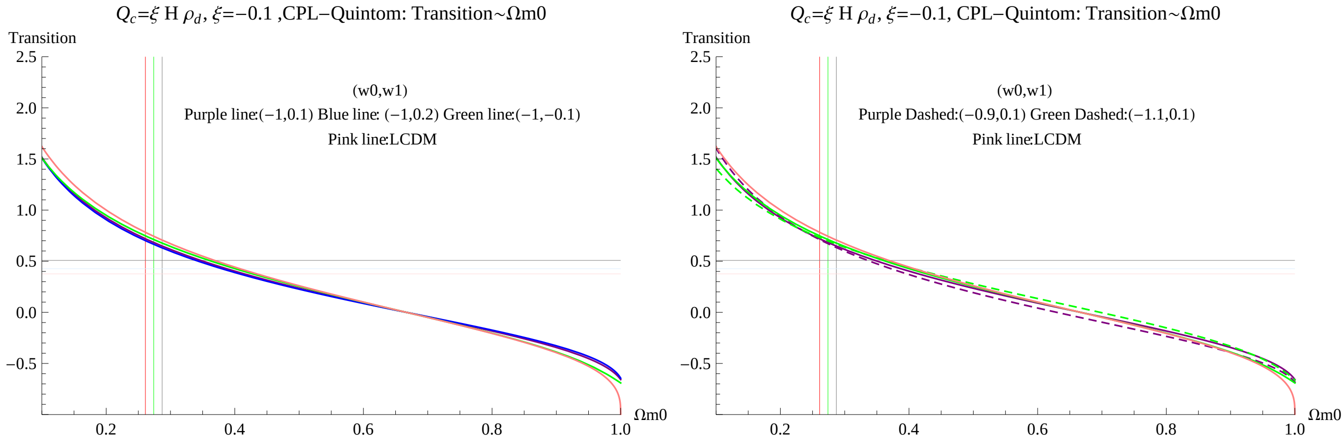


Figure 29: Transition VS $\Omega m0$

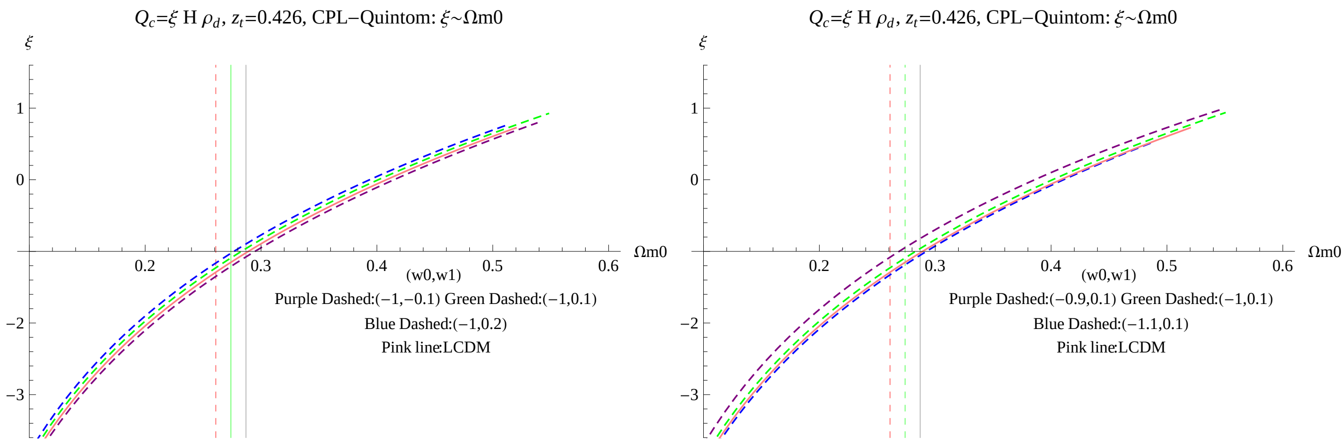


Figure 30: ξ VS $\Omega m0$

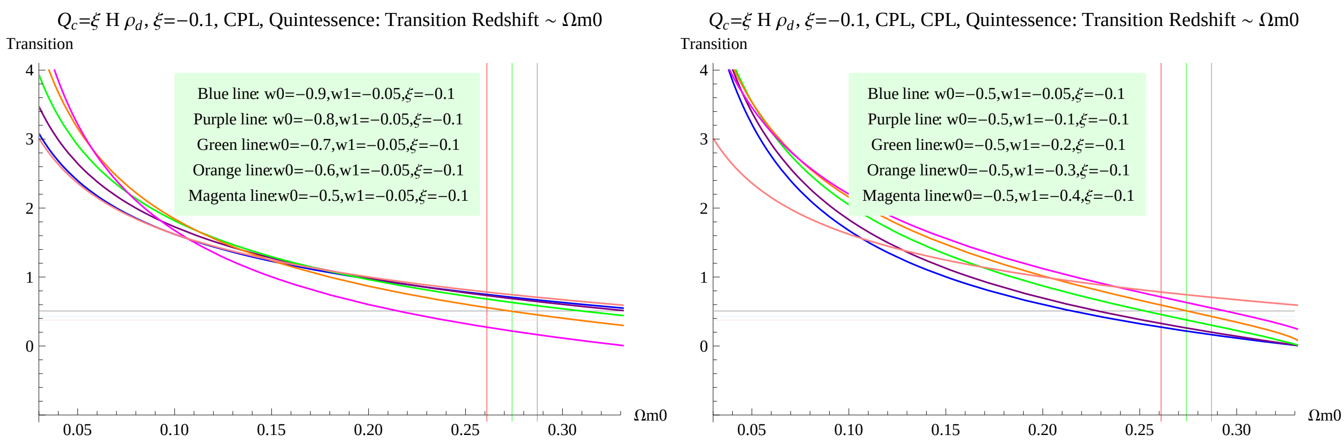


Figure 31: Transition VS $\Omega m0$

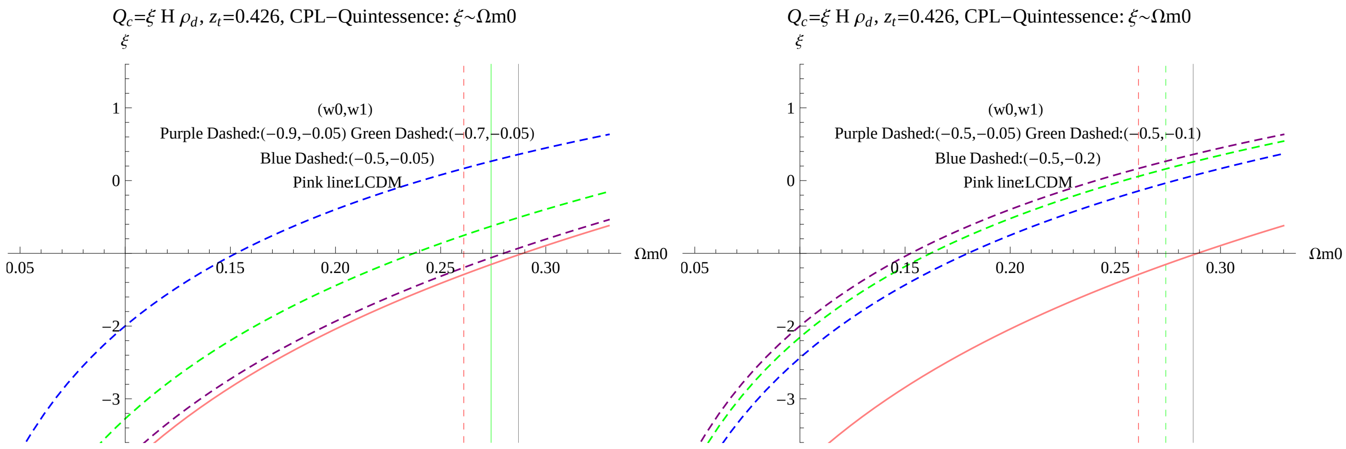


Figure 32: ξ VS $\Omega m0$

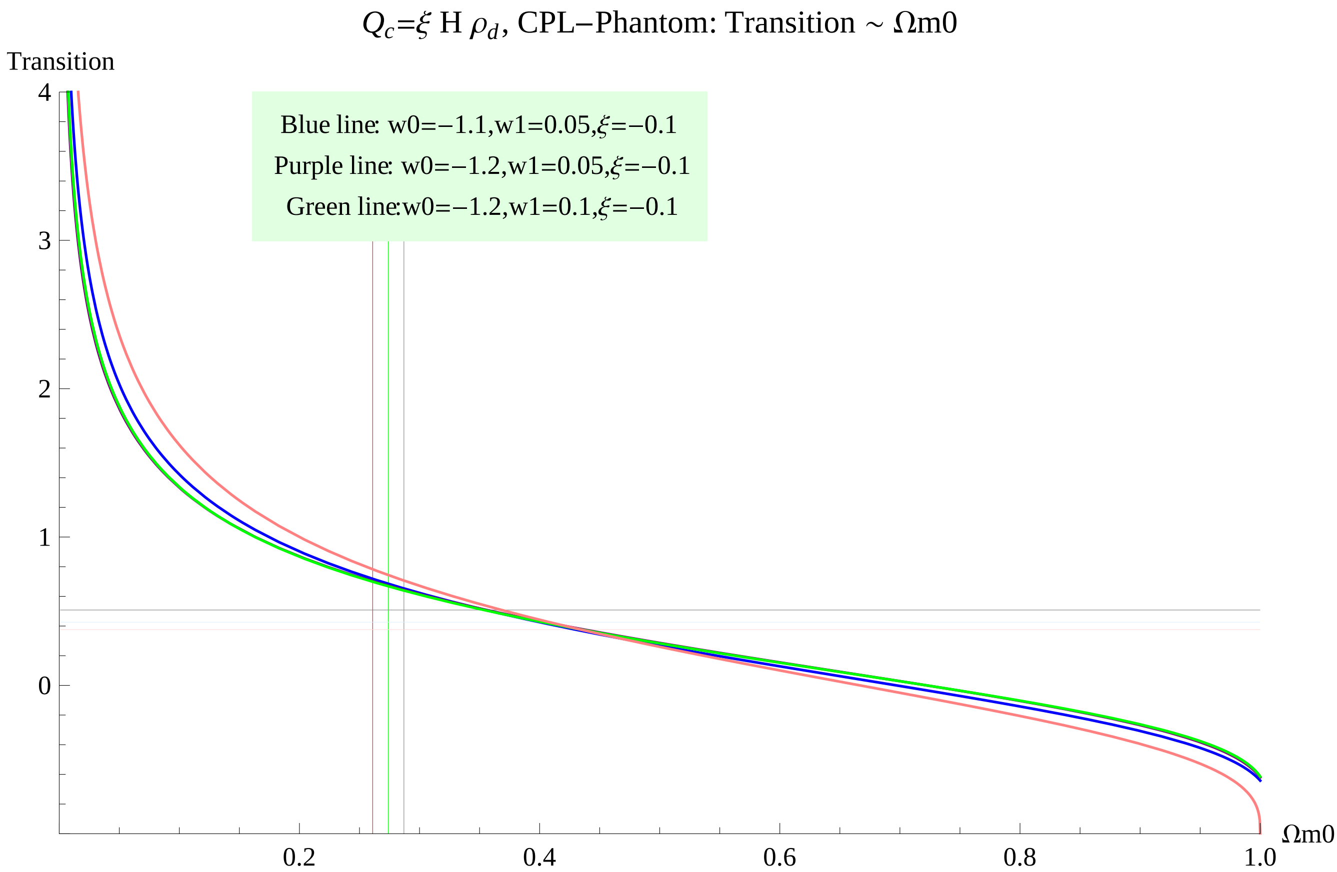


Figure 33: Transition VS $\Omega m0$

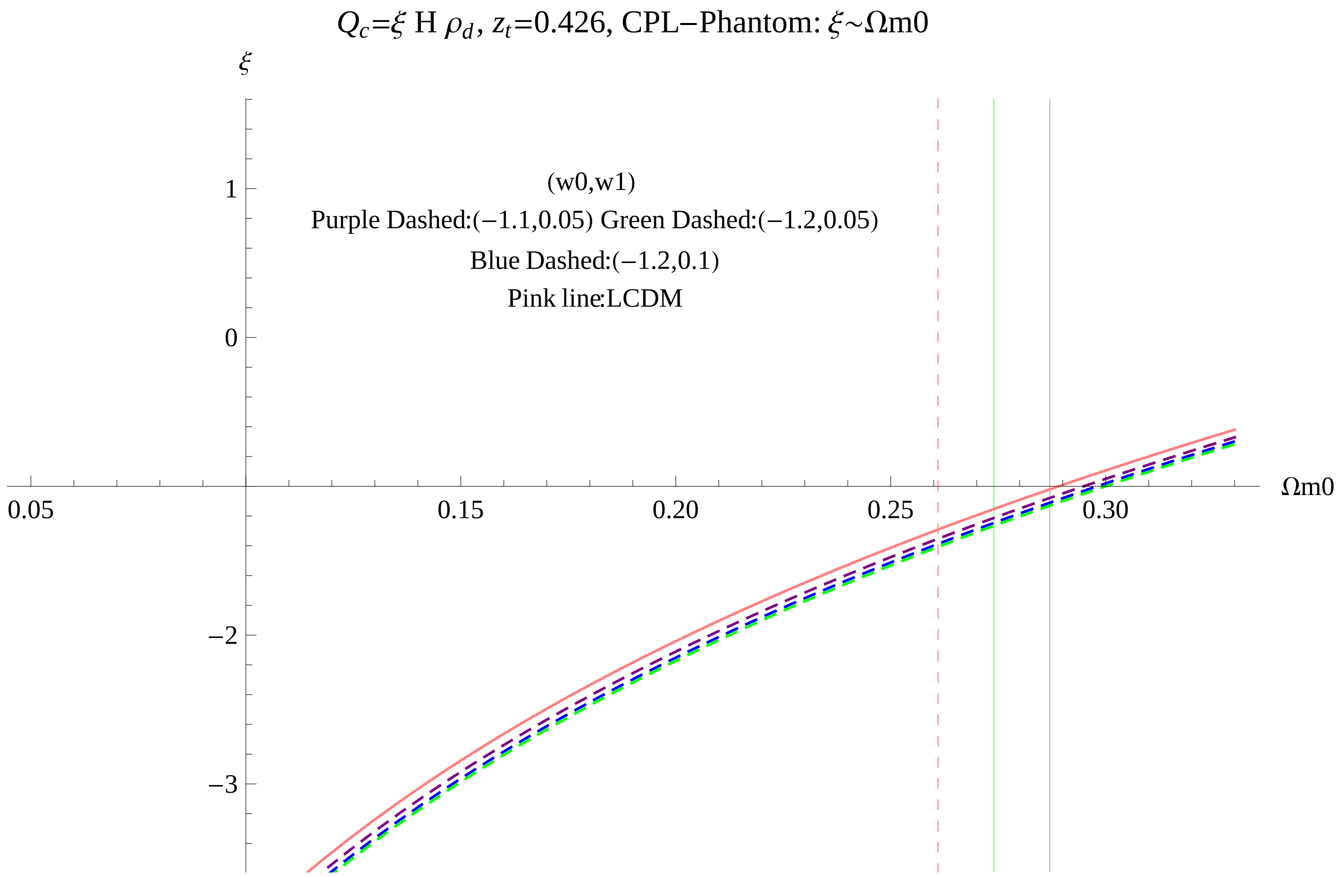
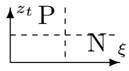
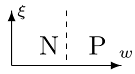
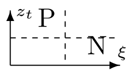


Figure 34: Transition VS $\Omega m0$

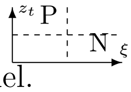
5 Summary

Table 1: Parameters in Different Models

Model	Parameter	Transition		Deceleration		ξ (fixed transition)	
		Behavior	Figures	How	Figures	How	Figures
ICC	ξ		4	N	3	-	-
	w	CC1	4				5, 6
	$\Omega m0$	N	4	P	3	P	7
ICCPL	ξ	CC2	9, 12, 13, 14, 17	N	8	-	-
	$w0$	CC3	9, 12, 13, 14, 17			P	15, 18, 21
	$w1$	CC	12, 13, 14, 17, 20			P	15, 18, 21
	$\Omega m0$	N	9, 12, 13, 14, 17, 20	P	8	P	15, 18, 21
I2CC	ξ		23	N	22	-	-
	w	CC4	23				24, 25
	$\Omega m0$	N	23	P	22	P	26
I2CCPL	ξ	CC	28, 29, 31, 33	N	27	-	-
	$w0$	CC	29, 31, 33			P	30, 32, 34
	$w1$	CC	28, 29, 31, 33			P	30, 32, 34
	$\Omega m0$	N	28, 29, 31, 33	P	27	P	30, 32, 34

Some notations in table 1:

- Models
  1. ICC: $Q_c = \xi H \rho_c$ with a constant EoS
  2. ICCPL: $Q_c = \xi H \rho_c$ with a CPL parametrized EoS
  3. I2CC: $Q_c = \xi H \rho_d$ with a constant EoS
  4. I2CCPL: $Q_c = \xi H \rho_d$ with a CPL parametrized EoS
- CCX means sort of complicated.
  1. CC1: Depends on $\Omega m0$. Small $\Omega m0$ is P, large $\Omega m0$ is N.

2. CC2: Complicated but in some proper region just like 

3. CC3: In some region it behaves likes P, as in ICC model.
4. CC4: Depends on $\Omega m0$. Small $\Omega m0$ is P, large $\Omega m0$ is N.