Some Comments

- **Physics in the interaction model**

  Model

  $$\dot{\rho_c} + 3H\rho_c = Q_c \tag{1}$$

  $$\dot{\rho_d} + 3H(1+w)\rho_d = Q_d \tag{2}$$

  Forms used

  1. $Q_c = \xi H \rho_c$, energy transfer: ξ>0, DE->DM; ξ<0, DM->DE
  2. $Q_c = \xi H \rho_d$, energy transfer: ξ>0, DE->DM; ξ<0, DM->DE

  However, the conservation of energy is always satisfied. So

  $Q_c + Q_d = 0$

- **Physics**

  1. $Q_c = \xi H \rho_c$

  Energy transfer is proportional to DM. In this case, the physics behind it is probabily a decay of DM. Ignoring the expansion effect, equation 1 is essentially the decay process of DM only when $Q_c > 0$. In an expotentional decay process, the decay rate is proportional to the total particle number (total energy) of the species, i.e., $Q_c = \Gamma \rho_c$ with Γ>0.

  From this point of view, it is more natural to choose ξ>0. But, this does NOT exclude ξ<0 of course.

  2. $Q_c = \xi H \rho_d$

  Similarly, it is better to choose ξ<0 in this condition. This does NOT exclude ξ<0 neither.

- **How to determine transition redshift.**
- **Some tips**
- **More about ICC and I2CC**
- **ICC**
- **I2CC**

  The following are the results of different EoS w for I2CC.

In[847]:=
```
Grid[{{tabξI2CCSum2d}, {tabξI2CCSum2c}, {tabξI2CCSum2b},
  {tabξI2CCSum2}, {tabξI2CCSum2e}, {tabξI2CCSum2f}, {tabξI2CCSum2g}}]
```

Out[847]=

For Ωm0∈{0.261,0.274,0.287},w=−0.7
Table of ξ for different Ωm0~Transition combination

Ωm0↘.Transition	0.426	0.376	0.508
0.261	−0.414982	−0.551355	−0.249743
0.274	−0.351859	−0.482765	−0.193228
0.287	−0.287785	−0.413122	−0.135889

For Ωm0∈{0.261,0.274,0.287},w=−0.8
Table of ξ for different Ωm0~Transition combination

Ωm0↘.Transition	0.426	0.376	0.508
0.261	−0.596208	−0.773551	−0.381502
0.274	−0.531577	−0.703492	−0.323418
0.287	−0.465997	−0.63238	−0.264512

For Ωm0∈{0.261,0.274,0.287},w=−0.9
Table of ξ for different Ωm0~Transition combination

Ωm0↘.Transition	0.426	0.376	0.508
0.261	−0.730803	−0.94401	−0.472902
0.274	−0.66466	−0.872481	−0.413236
0.287	−0.597567	−0.799899	−0.352751

For Ωm0∈{0.261,0.274,0.287},w=−1
Table of ξ for different Ωm0~Transition combination

Ωm0↘.Transition	0.426	0.376	0.508
0.261	−0.828666	−1.07368	−0.532564
0.274	−0.760999	−1.00068	−0.471298
0.287	−0.692386	−0.92662	−0.409217

For Ωm0∈{0.261,0.274,0.287},w=−1.1
Table of ξ for different Ωm0~Transition combination

Ωm0↘.Transition	0.426	0.376	0.508
0.261	−0.896881	−1.17039	−0.566664
0.274	−0.827676	−1.0959	−0.503776
0.287	−0.757528	−1.02037	−0.440076

For Ωm0∈{0.261,0.274,0.287},w=−1.2
Table of ξ for different Ωm0~Transition combination

Ωm0↘.Transition	0.426	0.376	0.508
0.261	−0.940694	−1.23995	−0.579778
0.274	−0.869934	−1.16396	−0.515241
0.287	−0.798235	−1.08693	−0.449899

For Ωm0∈{0.261,0.274,0.287},w=−1.3
Table of ξ for different Ωm0~Transition combination

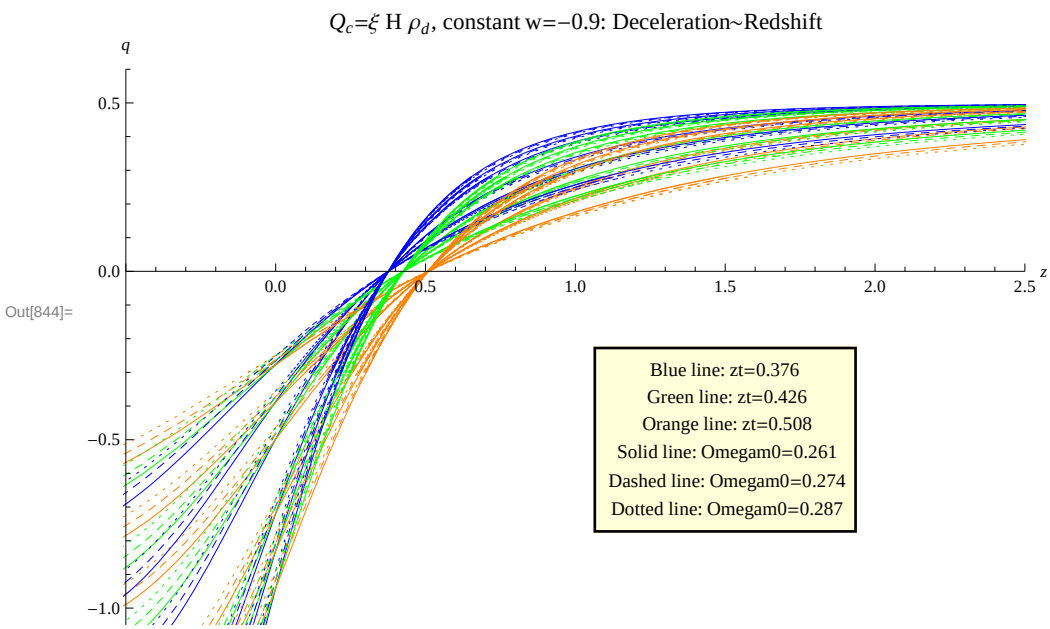
Ωm0↘.Transition	0.426	0.376	0.508
0.261	−0.964104	−1.28676	−0.575397
0.274	−0.891769	−1.20926	−0.509183
0.287	−0.8185	−1.13072	−0.44217

All lines with the same color intersect at the same point because all lines with the same color have the same transition redshift. This is a good check of the calcuation.

The smaller the EoS is, the steeper the lines are at about z~0.5.

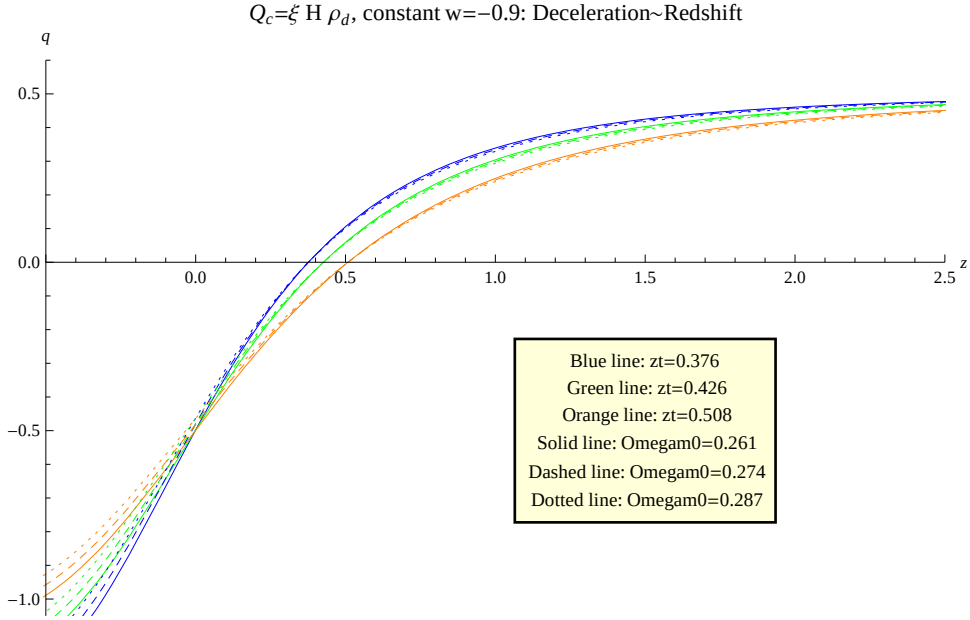
In[844]:=
```
Show[{pldecI2CCShowSum2b[[1]], pldecI2CCShowSum2c[[1]],
  pldecI2CCShowSum2d[[1]]}, {pldecI2CCShowSum2e[[1]],
  pldecI2CCShowSum2f[[1]], pldecI2CCShowSum2g[[1]]}]
```

Out[844]=

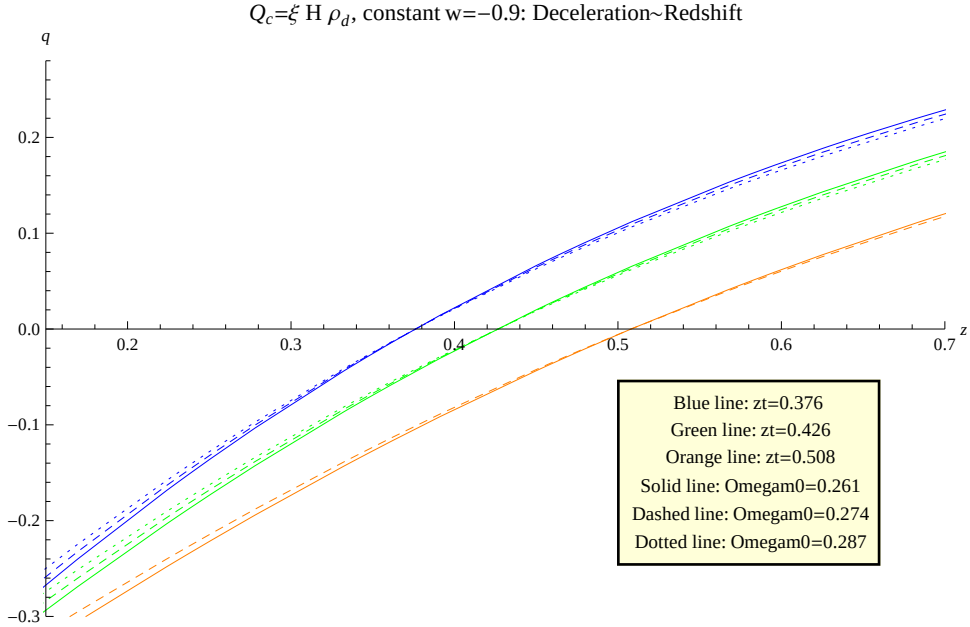


In[996]:=
```
Grid[{{pldecI2CCShowSum2b}, {pldecI2CCShowSum2c}, {pldecI2CCShowSum2d},
  {pldecI2CCShowSum2e}, {pldecI2CCShowSum2f}, {pldecI2CCShowSum2g}}]
```

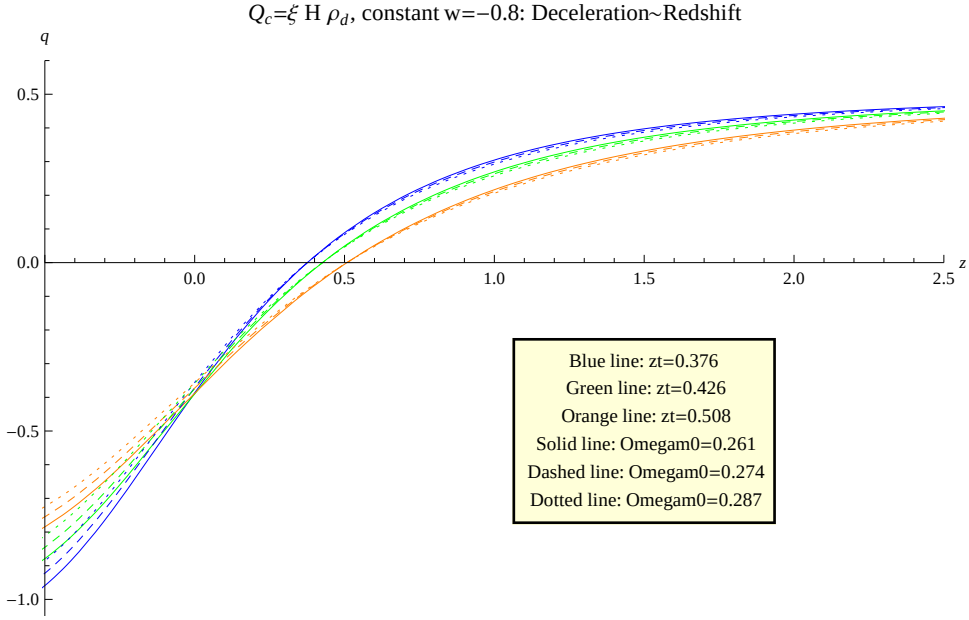
Q_c=ξ H ρ_d, constant w=−0.9: Deceleration~Redshift



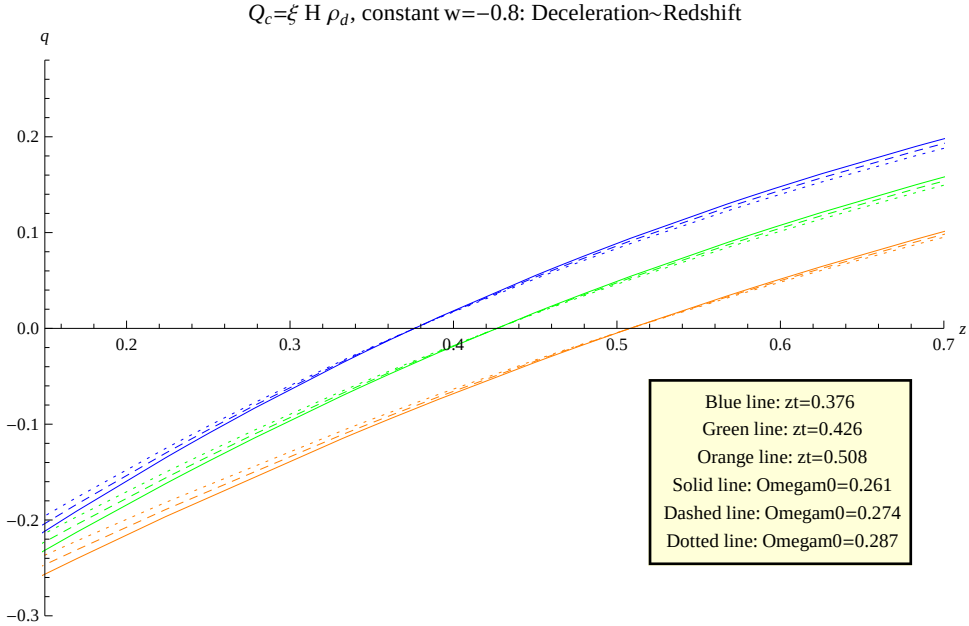
Q_c=ξ H ρ_d, constant w=−0.9: Deceleration~Redshift



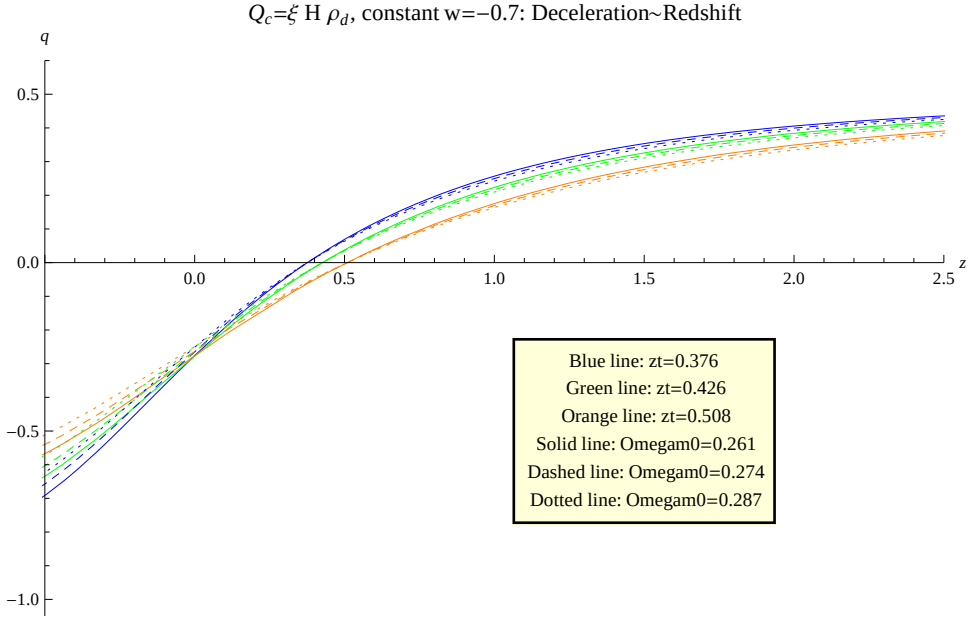
Q_c=ξ H ρ_d, constant w=−0.8: Deceleration~Redshift



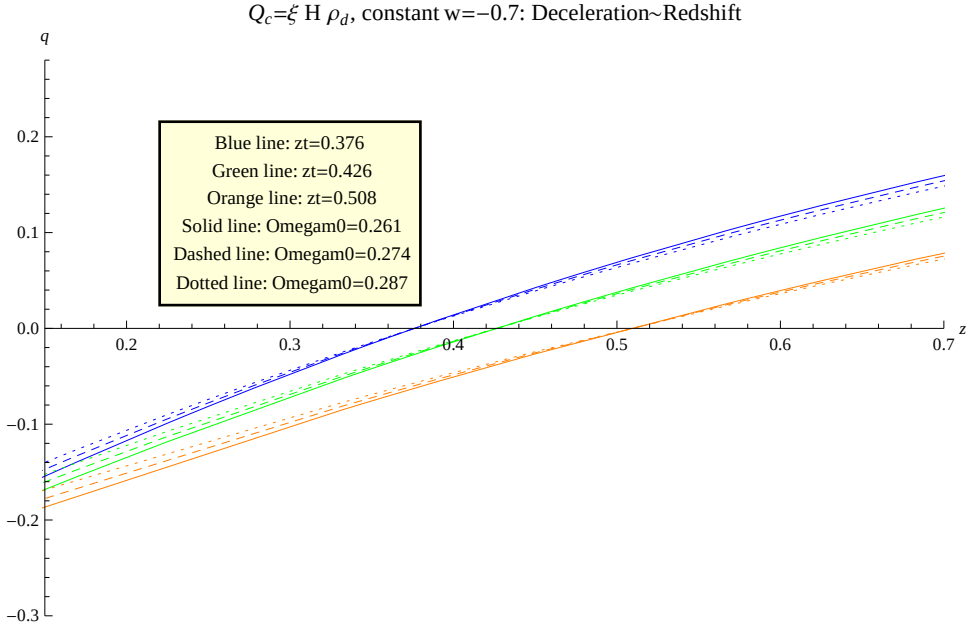
Q_c=ξ H ρ_d, constant w=−0.8: Deceleration~Redshift



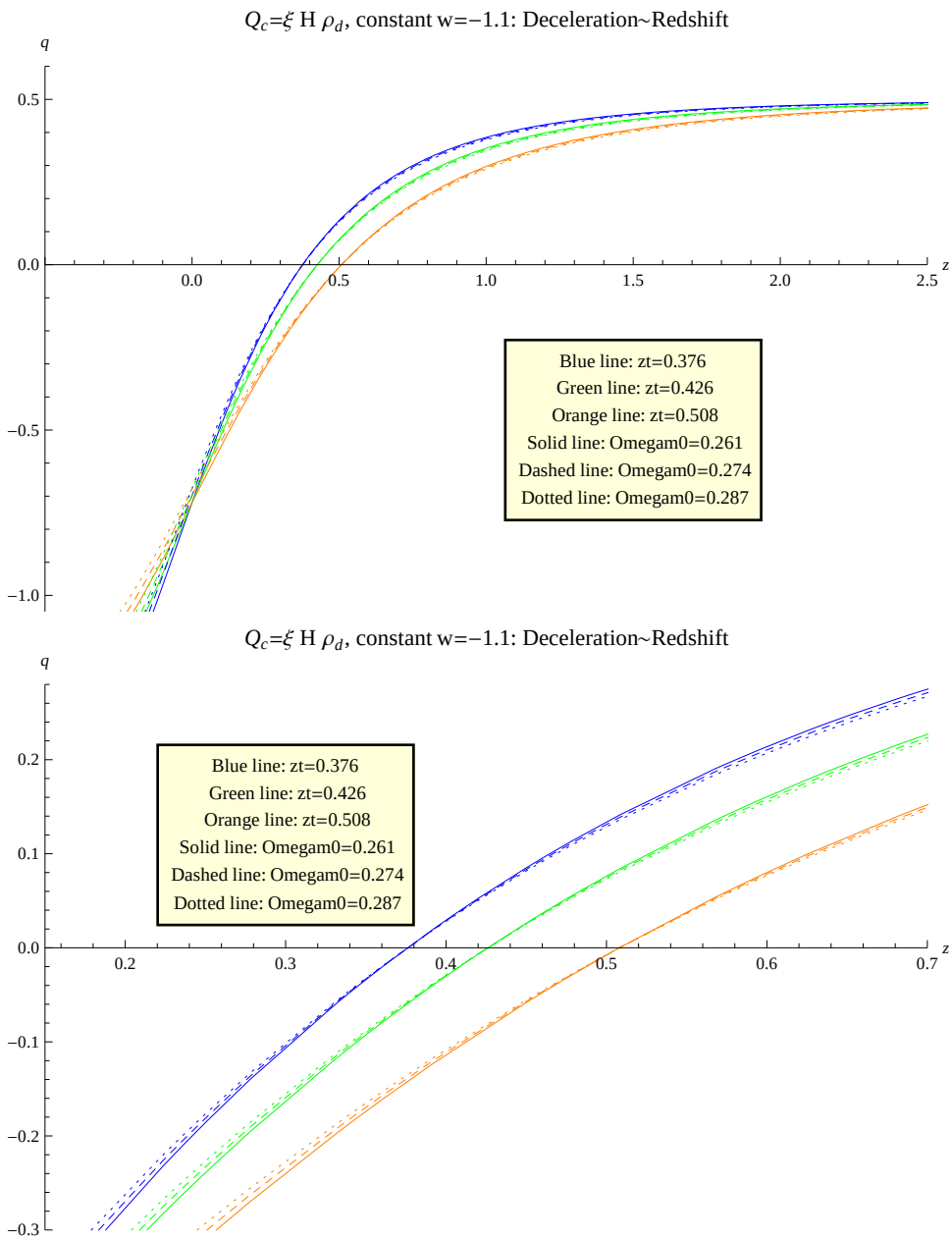
$Q_c=\xi$ H ρ_d, constant w=−0.7: Deceleration~Redshift



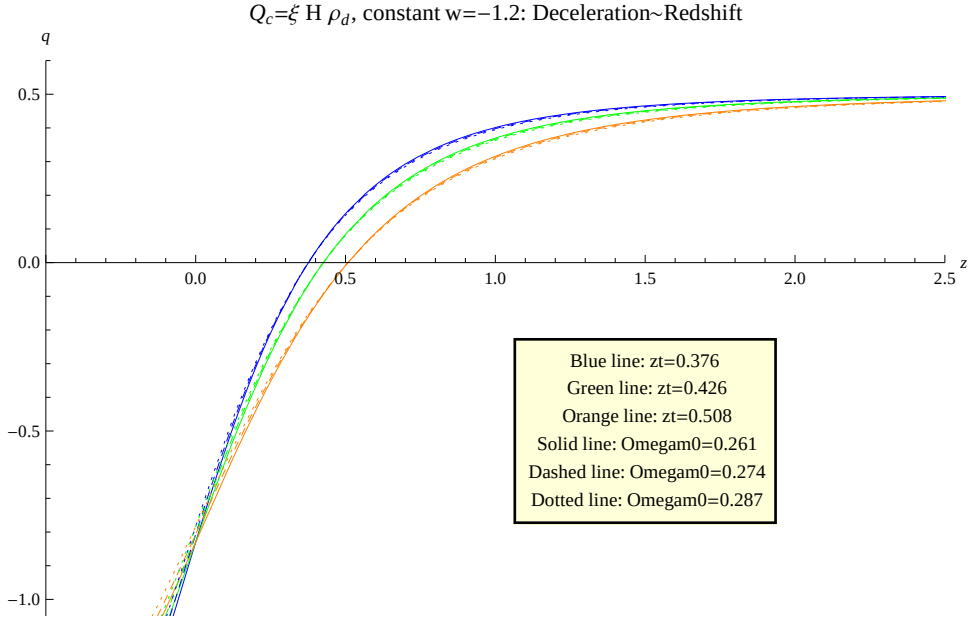
$Q_c=\xi$ H ρ_d, constant w=−0.7: Deceleration~Redshift



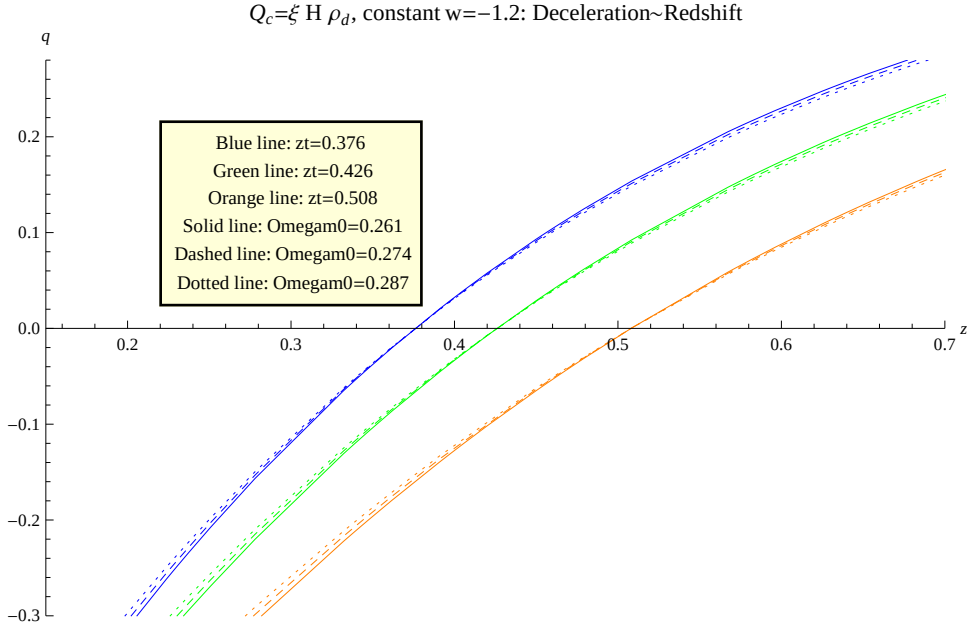
Out[996]=


$Q_c=\xi$ H ρ_d, constant w=−1.1: Deceleration~Redshift
q
z
0.5
0.0
−0.5
−1.0
0.0
0.5
1.0
1.5
2.0
2.5
Blue line: zt=0.376
Green line: zt=0.426
Orange line: zt=0.508
Solid line: Omegam0=0.261
Dashed line: Omegam0=0.274
Dotted line: Omegam0=0.287
$Q_c=\xi$ H ρ_d, constant w=−1.1: Deceleration~Redshift
q
z
0.2
0.1
0.0
−0.1
−0.2
−0.3
0.2
0.3
0.4
0.5
0.6
0.7
Blue line: zt=0.376
Green line: zt=0.426
Orange line: zt=0.508
Solid line: Omegam0=0.261
Dashed line: Omegam0=0.274
Dotted line: Omegam0=0.287

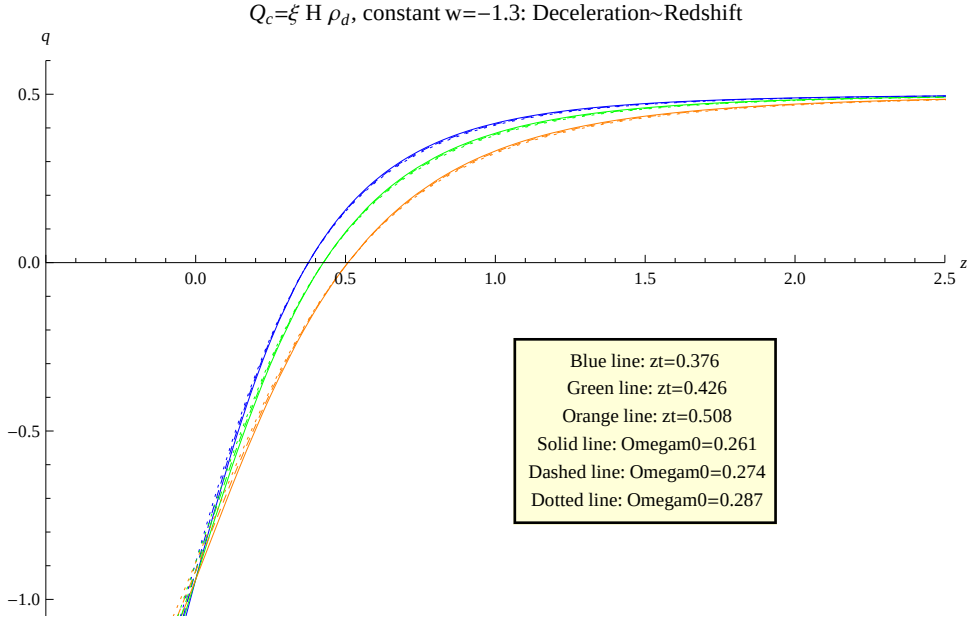
Q_c=ξ H ρ_d, constant w=−1.2: Deceleration~Redshift



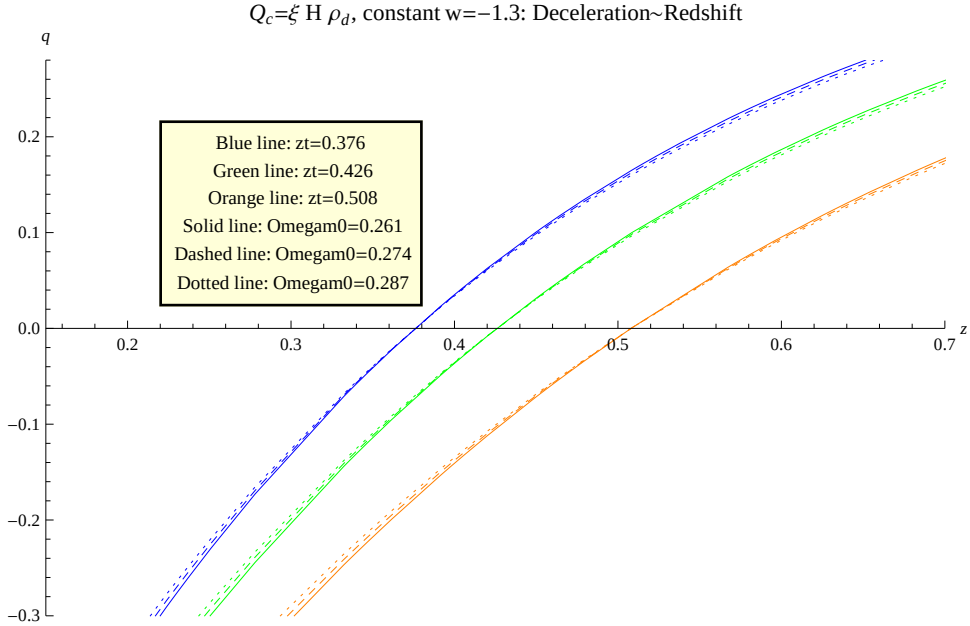
Q_c=ξ H ρ_d, constant w=−1.2: Deceleration~Redshift



$Q_c=\xi$ H ρ_d, constant w=−1.3: Deceleration~Redshift



$Q_c=\xi$ H ρ_d, constant w=−1.3: Deceleration~Redshift

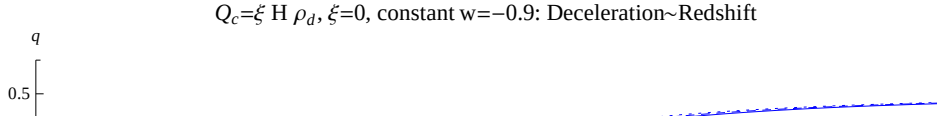


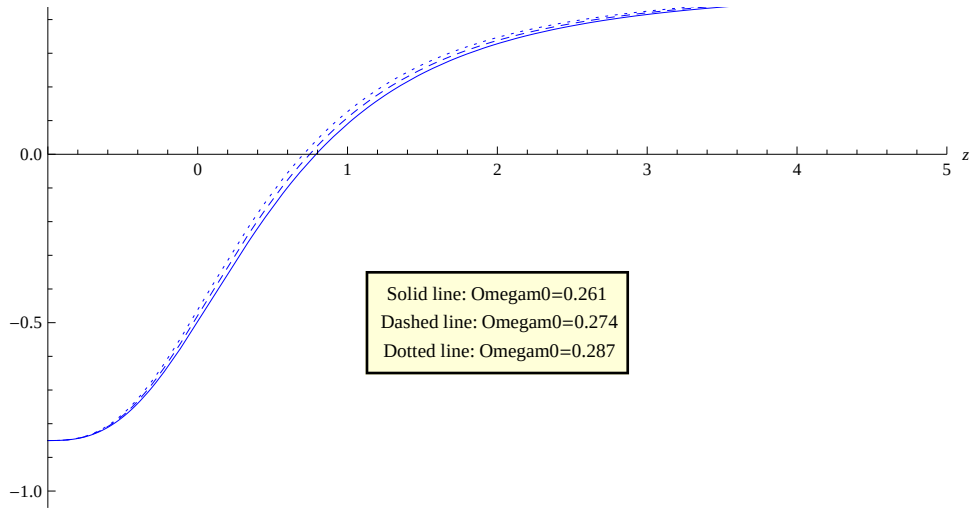
What about the redshift range when ξ=0. The results show redshift is quite sensitive with respect to EoS.

In[994]:=
```
Grid[{{ξI2CCSum2noint1}, {ξI2CCSum2noint2}, {ξI2CCSum2noint3}}]
```

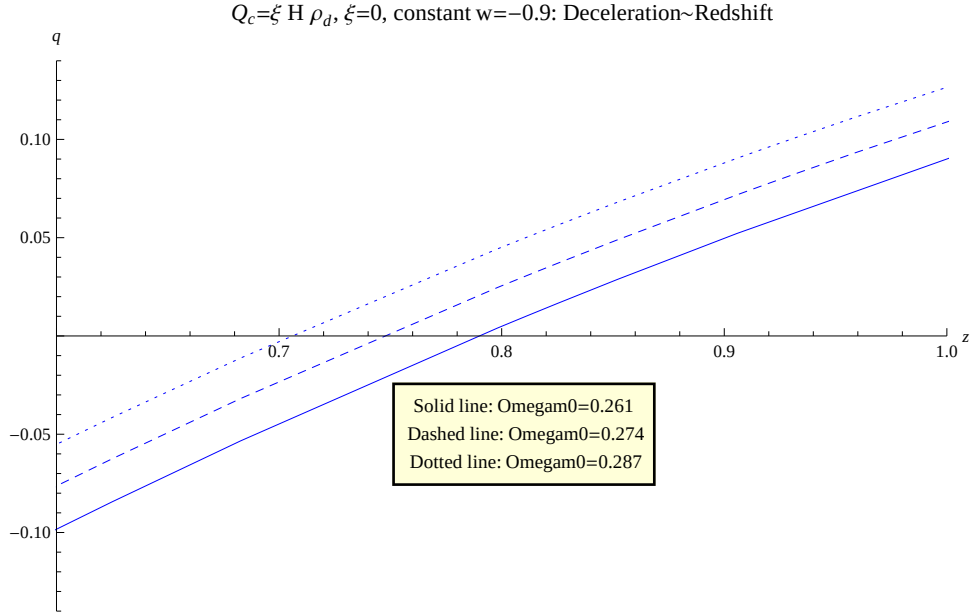
For Ωm0∈{0.261,0.274,0.287},w=−0.9 **Table of transition redshift for different ξ and Ωm0 combination**			
Ωm0	0.261	0.274	0.287
ξ=0	0.789618	0.746172	0.705001
Obs	0.508	0.426	0.376
△=obs−row1	−0.281618	−0.320172	−0.329001
$\frac{\triangle}{obs}=\left\lvert\frac{obs-row1}{obs}\right\rvert$	0.554366	0.751578	0.875003

$Q_c=\xi$ H ρ_d, ξ=0, constant w=−0.9: Deceleration~Redshift





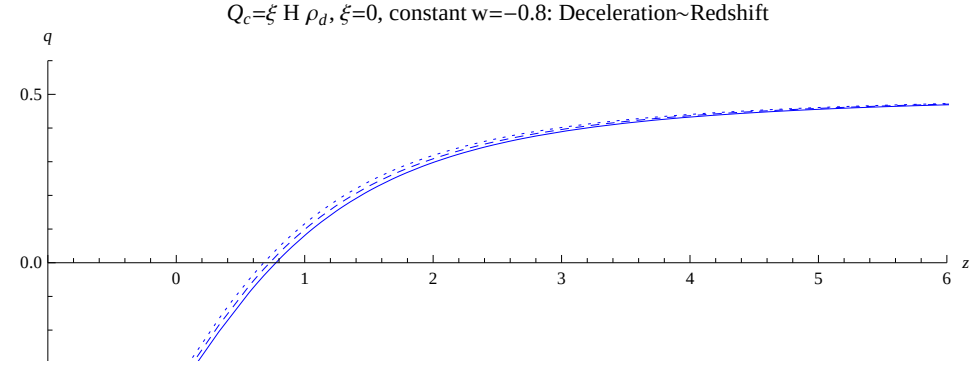
Q_c=ξ H ρ_d, ξ=0, constant w=−0.9: Deceleration~Redshift



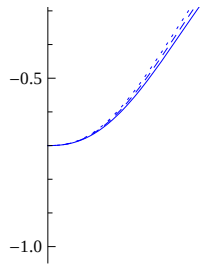
For Ωm0∈{0.261,0.274,0.287},w=−0.8
Table of transition redshift for different ξ and Ωm0 combination

ξ∴Ωm0	0.261	0.274	0.287
0	0.775095	0.726689	0.680957
Obs	0.508	0.426	0.376
△=obs−row1	−0.267095	−0.300689	−0.304957
$\frac{\triangle}{obs} = \left\lvert \frac{obs-row1}{obs} \right\rvert$	0.525778	0.705844	0.811055

Q_c=ξ H ρ_d, ξ=0, constant w=−0.8: Deceleration~Redshift

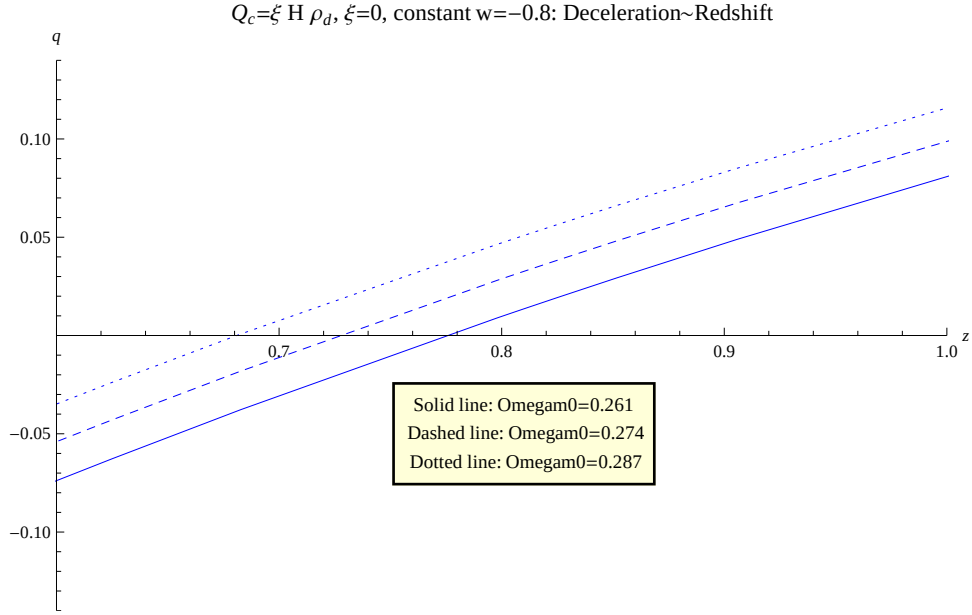


Out[994]=



Solid line: Omegam0=0.261
Dashed line: Omegam0=0.274
Dotted line: Omegam0=0.287

$Q_c=\xi$ H ρ_d, ξ=0, constant w=−0.8: Deceleration~Redshift

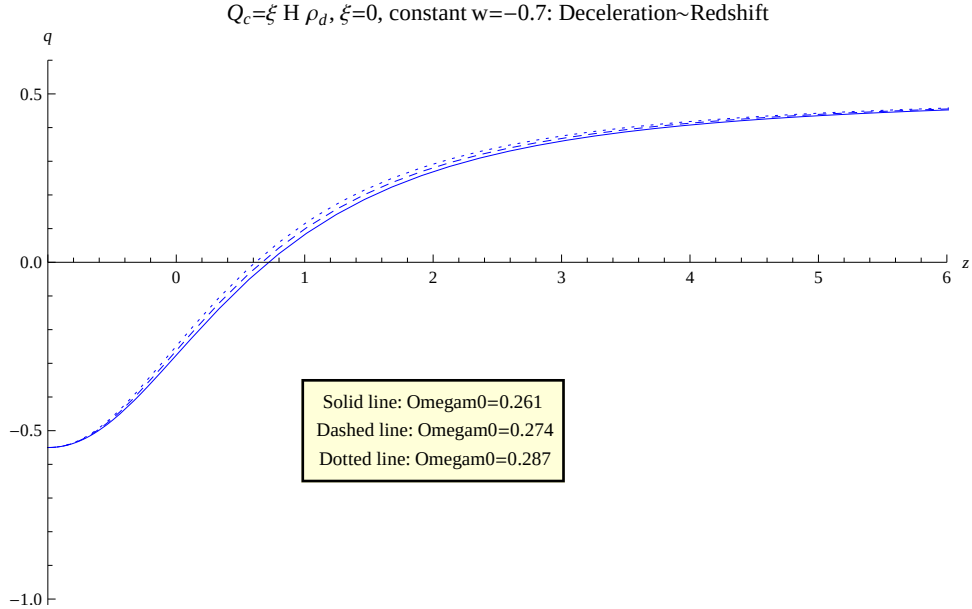


Solid line: Omegam0=0.261
Dashed line: Omegam0=0.274
Dotted line: Omegam0=0.287

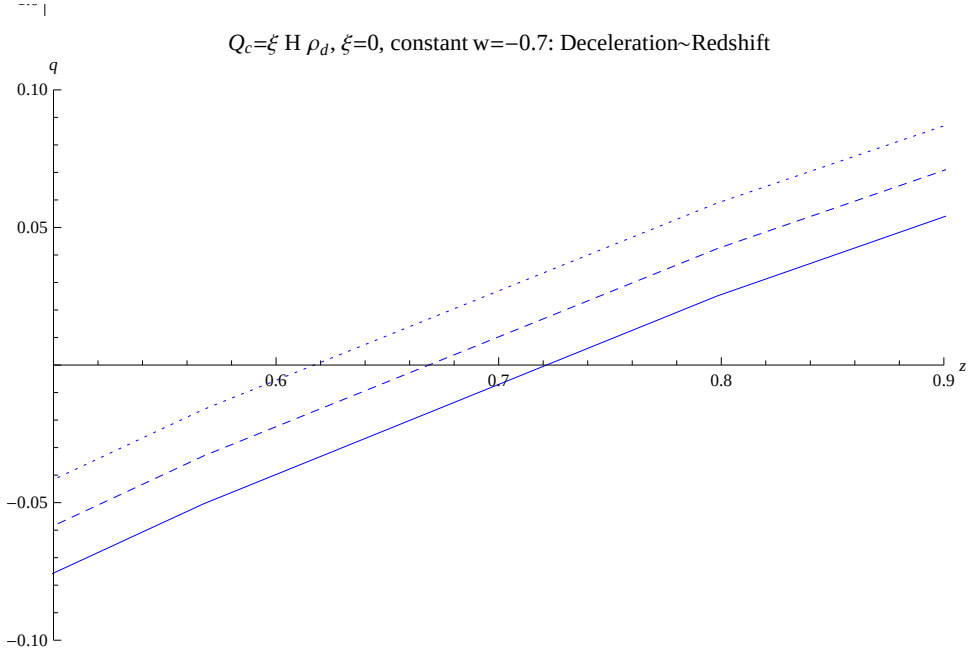
For Ωm0∈{0.261,0.274,0.287},w=−0.7
Table of transition redshift for different ξ and Ωm0 combination

ξ∴Ωm0	0.261	0.274	0.287
0	0.717714	0.664286	0.614005
Obs	0.508	0.426	0.376
△=obs−row1	−0.209714	−0.238286	−0.238005
$\frac{\triangle}{obs} = \left\lvert \frac{obs-row1}{obs} \right\rvert$	0.412822	0.559357	0.632992

$Q_c=\xi$ H ρ_d, ξ=0, constant w=−0.7: Deceleration~Redshift



Solid line: Omegam0=0.261
Dashed line: Omegam0=0.274
Dotted line: Omegam0=0.287


Q_c=ξ H ρ_d, ξ=0, constant w=−0.7: Deceleration~Redshift
q
0.10
0.05
−0.05
−0.10
0.6
0.7
0.8
0.9
z