INTRO

PRE

LCDM Model

Interacting Models

Summary

Supplementary

▪

1. Why does smaller ξ lead to earlier transition in Q=ξ H ρ_c model?

$\dot{\rho}_c + 3\,H\,\rho_c = \xi\,H\,\rho_c$, positive ξ means energy flow from DE to DM, negative ξ means DM to DE.

This can be explained with the equations in the file I sent last time. I'll retype them here.

Evolution of energy density for $Q_c = \xi\,H\,\rho_c$, constant ξ, constant w, and ξ≠-3w

$$\Omega m = \Omega m0\ (1+z)^{3-\xi} \tag{2}$$

$$\Omega d = \left(\Omega d0 + \frac{\xi}{3\,w+\xi}\,\Omega m0\right)(1+z)^{3\,(1+w)} + \frac{-\xi}{3\,w+\xi}\,\Omega m \equiv \Omega d0'\ (1+z)^{3\,(1+w)} + \frac{-\xi}{3\,w+\xi}\,\Omega m \tag{3}$$

Evolution of energy density for $Q_c = \xi\,H\,\rho_d$, constant ξ, constant w, and ξ≠-3w

$$\Omega m = \left(\Omega m0 + \frac{\xi}{\xi+3\,w}\,\Omega d0\right)(1+z)^{3} + \frac{-\xi}{\xi+3\,w}\,\Omega d \equiv \Omega m0'\ (1+z)^{3} + \frac{-\xi}{\xi+3\,w}\,\Omega d \tag{4}$$

$$\Omega d = \Omega d0\ (1+z)^{3\,(1+w)+\xi} \tag{5}$$

So in the two cases, coupling constant has two effects:

1. Amplifies the curve of deceleration parameter / energy density.
2. Energy flow between DE and DM.

These solotions have the same Ωd and Ωm values at z=0. That means energy flow of DM to DE would yeild a larger energy density of DM at z>0 than LCDM. That means there would be less DM for a larger ξ if the transition happens before z=0.

This is also the reason of that amplification effect mentioned above. A smaller ξ will decrease the factor $\left(\Omega d0 + \frac{\xi}{3\,w+\xi}\,\Omega m0\right)(1+z)^{3\,(1+w)}$ in Ωd. I will plot the evolution of Ωd in terms of redshift.

In[65]:=
```
Omegadfunctest[Ωd0_, Ωm0_, w_, ξ_, z_] :=
  (Ωd0 + ξ/(3 w + ξ) Ωm0) (1 + z)^(3 (1+w)) + (-ξ)/(3 w + ξ) (Ωm0 (1 + z)^(3-ξ));
Omegamfunctest[Ωm0_, ξ_, z_] := Ωm0 (1 + z)^(3-ξ);
```

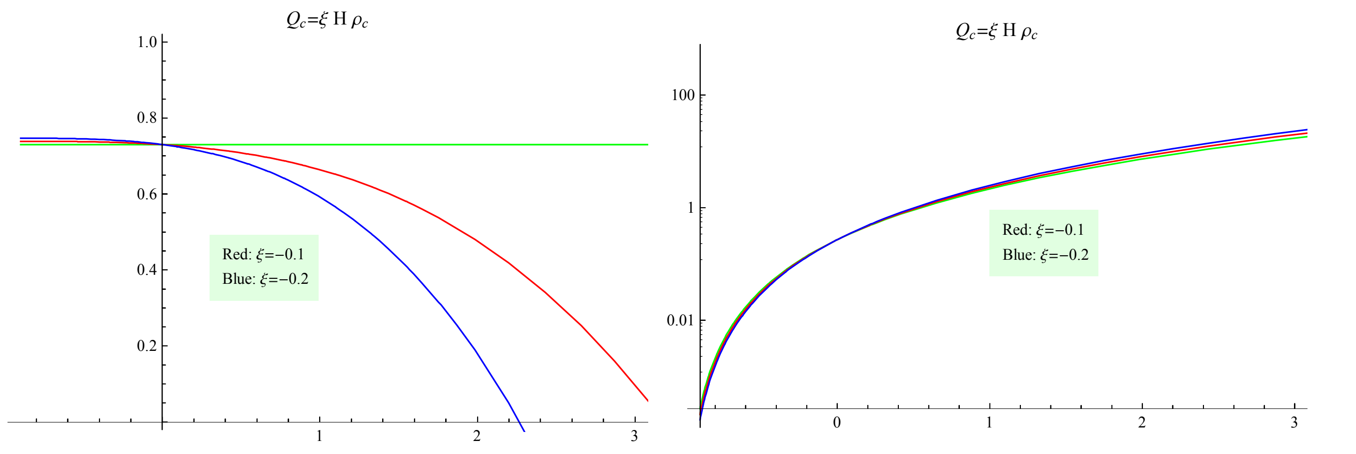
In[67]:=

```
Omegadplottest[Ωd0_, Ωm0_, w_, ξ_, color_] :=
  Plot[Omegadfunctest[Ωd0, Ωm0, w, ξ, z], {z, -0.9, 10}, PlotStyle → color];
Omegamplottest[Ωm0_, ξ_, color_] :=
  LogPlot[Omegamfunctest[Ωm0, ξ, z], {z, -0.9, 10}, PlotStyle → color];
```

In[86]:=

```
Grid[
 {{Show[Omegadplottest[0.73, 0.27, -1, 0, Green],
    Omegadplottest[0.73, 0.27, -1, -0.1, Red],
    Omegadplottest[0.73, 0.27, -1, -0.2, Blue], PlotRange → {{-0.9, 3}, {0, 1}},
    PlotLabel → "Qc=ξ H ρc",
    Epilog → Inset[Framed[Style["Red: ξ=-0.1\n Blue: ξ=-0.2", 10],
       Background → LightGreen, FrameStyle → None], {0.3, 0.5}, {Left, Top}],
    ImageSize → 400], Show[Omegamplottest[0.27, 0, Green],
    Omegamplottest[0.27, -0.1, Red], Omegamplottest[0.27, -0.2, Blue],
    PlotRange → {{-0.9, 3}, Automatic}, PlotLabel → "Qc=ξ H ρc",
    Epilog → Inset[Framed[Style["Red: ξ=-0.1\n Blue: ξ=-0.2", 10],
       Background → LightGreen, FrameStyle → None], {1, 0.01}, {Left, Top}],
    ImageSize → 400]}}]
```

Out[86]=

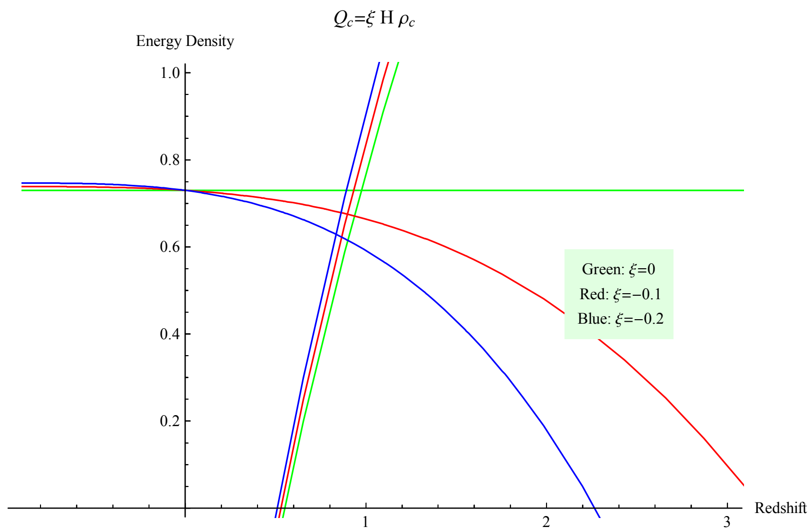


The following figure shows that if the transition happens before z=0, a larger.

In[87]:=

```
Grid[
 {{Show[Omegadplottest[0.73, 0.27, -1, 0, Green], Omegamplottest[0.27, 0, Green],
    Omegadplottest[0.73, 0.27, -1, -0.1, Red], Omegamplottest[0.27, -0.1, Red],
    Omegadplottest[0.73, 0.27, -1, -0.2, Blue], Omegamplottest[0.27, -0.2, Blue],
    PlotRange → {{-0.9, 3}, {0, 1}}, AxesLabel → {"Redshift", "Energy Density"},
    PlotLabel → "Qc=ξ H ρc",
    Epilog → Inset[Framed[Style["Green: ξ=0\n Red: ξ=-0.1\n Blue: ξ=-0.2", 10],
       Background → LightGreen, FrameStyle → None], {2.1, 0.6}, {Left, Top}],
    ImageSize → 500]}}]
```

Out[87]=

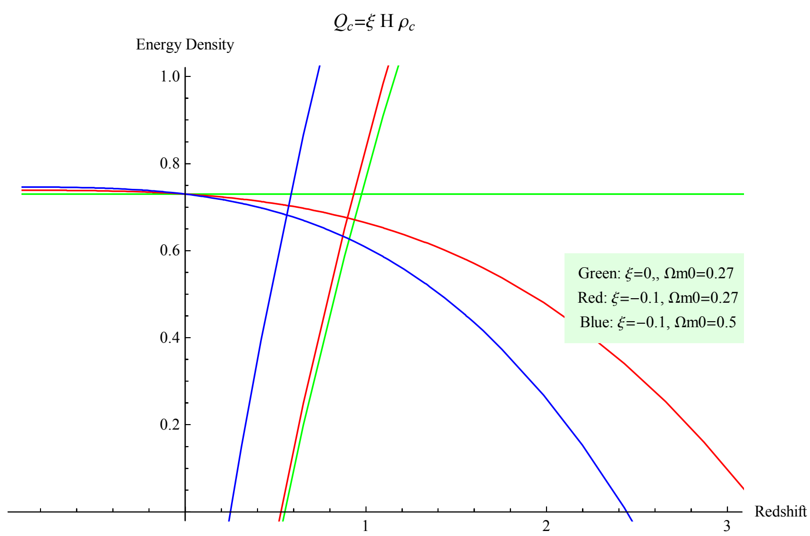


Ωm0 here is the energy density of matter today. If we have more matter today, the transition happens nearer to z=0. Since the energy density of DE and DM varies less with a smaller redshift, the effect of ξ would be reduced.

In[88]:=

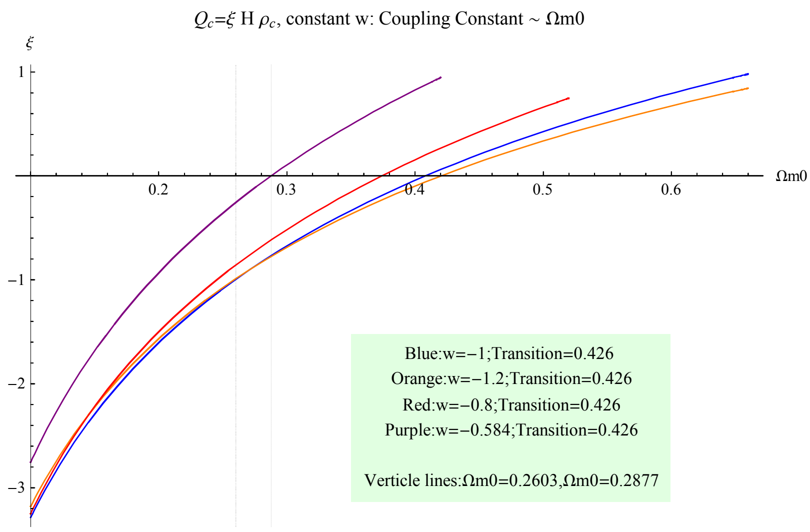
```
Grid[
 {{Show[Omegadplottest[0.73, 0.27, -1, 0, Green], Omegamplottest[0.27, 0, Green],
    Omegadplottest[0.73, 0.27, -1, -0.1, Red], Omegamplottest[0.27, -0.1, Red],
    Omegadplottest[0.73, 0.5, -1, -0.1, Blue], Omegamplottest[0.5, -0.1, Blue],
    PlotRange → {{-0.9, 3}, {0, 1}}, AxesLabel → {"Redshift", "Energy Density"},
    PlotLabel → "Qc=ξ H ρc",
    Epilog →
     Inset[
      Framed[
       Style[
        "Green: ξ=0,, Ωm0=0.27\n Red: ξ=-0.1, Ωm0=0.27\n Blue: ξ=-0.1, Ωm0=0.5",
        10], Background → LightGreen, FrameStyle → None], {2.1, 0.6}, {Left, Top}],
    ImageSize → 500]}}]
```

Out[88]=



Interacting model $Q_c = \xi H \rho_c$ with constant ξ and constant EoS w.

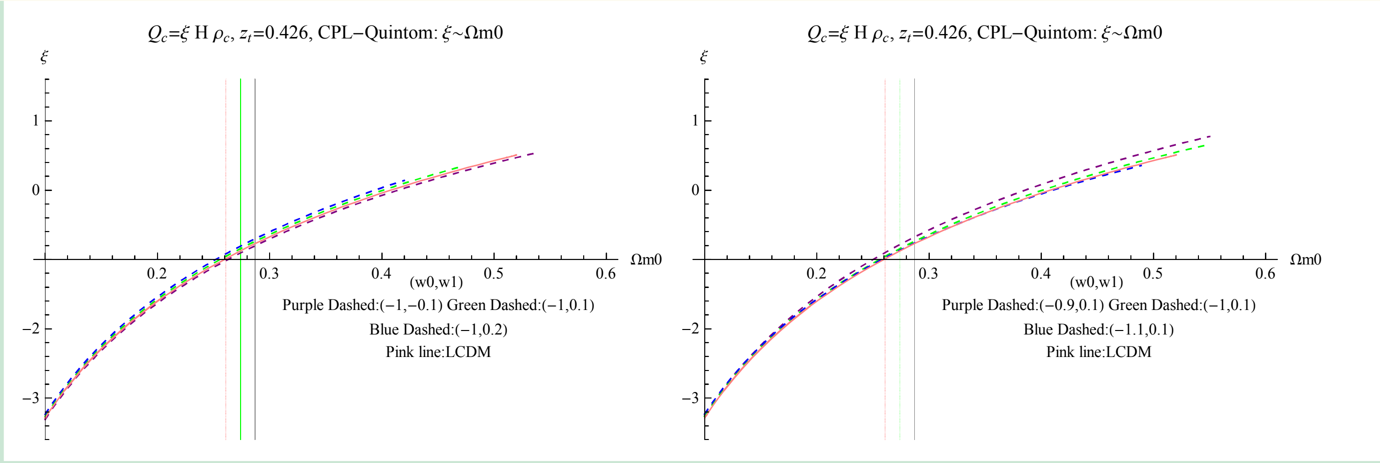
Out[1]=



EoS value when ξ=0	
⋱	Transition 0.426
Ωm0=0.2877	-0.58406

Result: w∈(-0.58406, -0.45064) if we constrain Ωm0∈(0.2603,2877) and transition redshift 0.426.

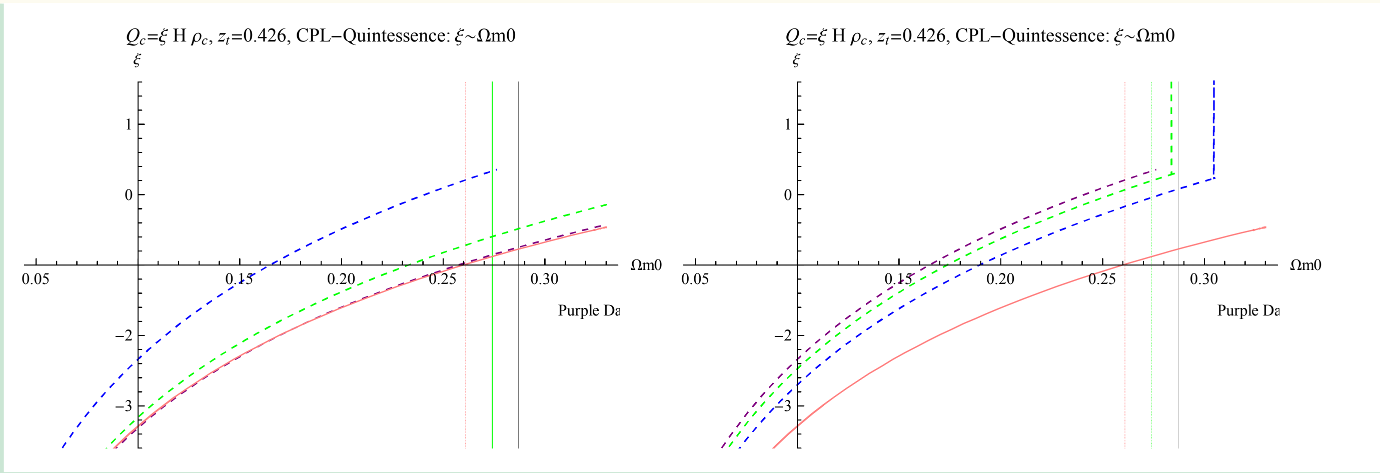
Interacting model $Q_c = \xi H \rho_c$ with constant ξ and CPL parameterized EoS $w = w0 + w1 \frac{z}{1+z}$.



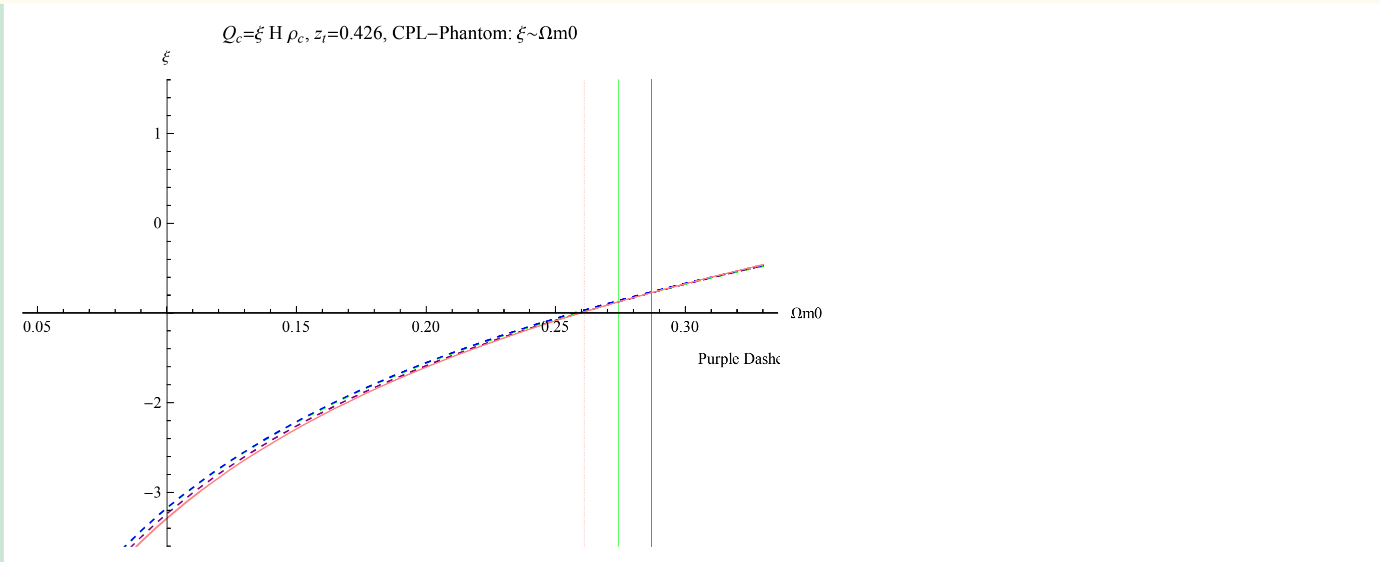
Caption of the following figures:

Figure on the left:
(w0,w1) value of the lines:
Purple Dashed:(-0.9,-0.05); Green Dashed:(-0.7,-0.05); Blue Dashed:(-0.5,-0.05); Pink line:LCDM

Figure on the right:
(w0,w1) value:
Purple Dashed:(-0.5,-0.05); Green Dashed:(-0.5,-0.1); Blue Dashed:(-0.5,-0.2); Pink line:LCDM

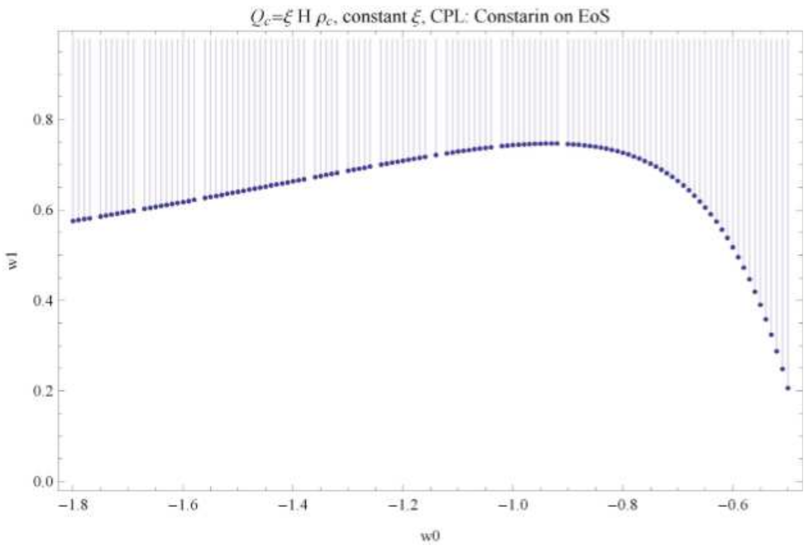


Caption for the following figure:
(w0, w1) value:
Purple Dashed : (-1.1, 0.05); Green Dashed : (-1.2, 0.05); Blue Dashed : (-1.2, 0.1) ; Pink line : LCDM

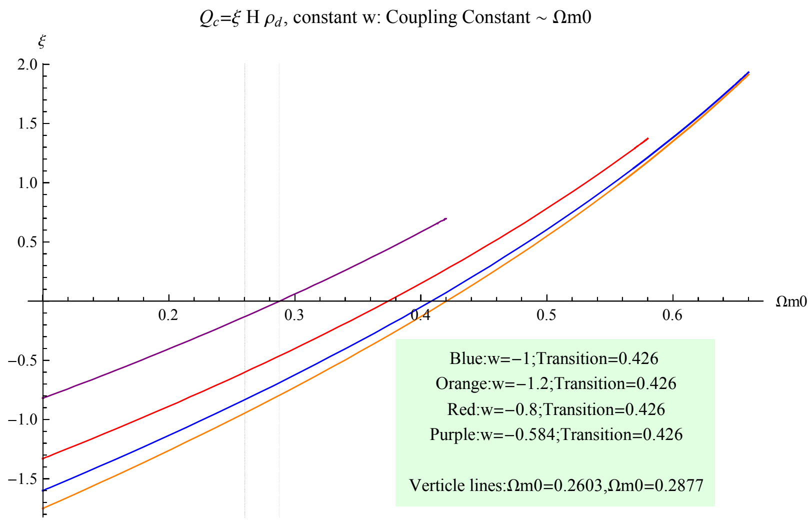


Lower boundary of the the allowed parameters. Since lower boundary is calculated when ξ=0, this remains the same for different models we investaged here.

There should be a upper boundary. But I haven’t calculated it yet. The upper boundary should vary for different models.



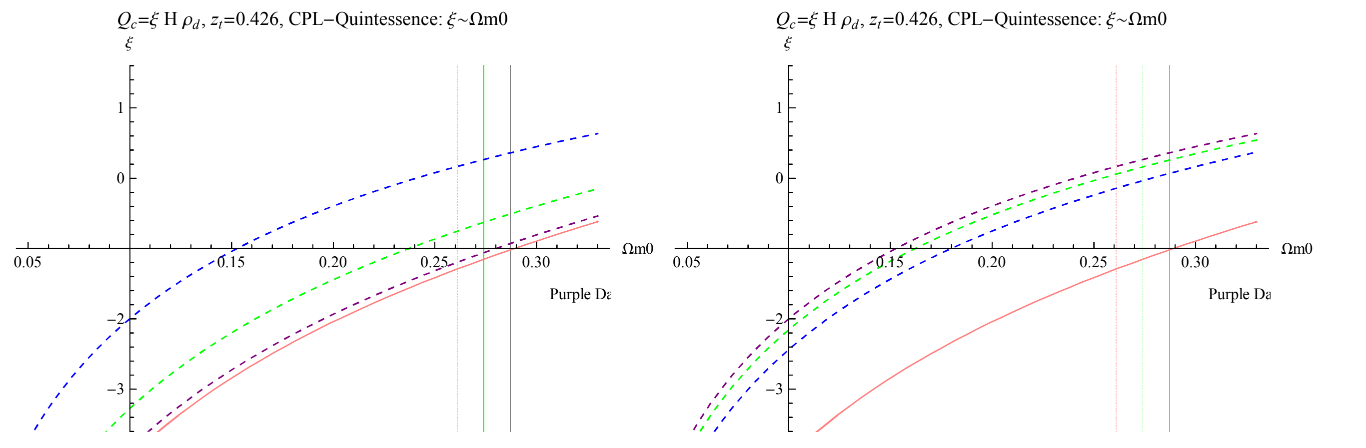
- **Interacting model $Q_c = \xi H \rho_d$ with constant ξ and constant EoS w.**



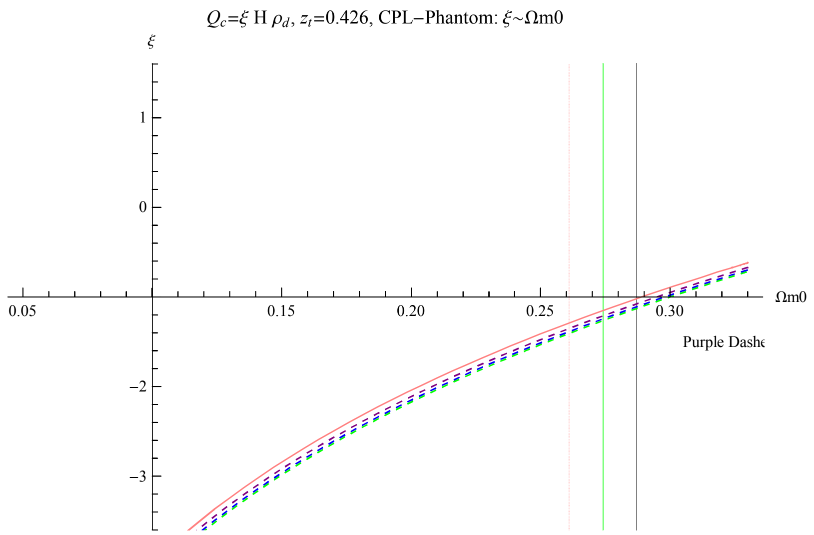
Q_c=ξ H ρ_d, constant w: EoS value when ξ=0	
⋱	Transition 0.426
Ωm0=0.2877	−

Figure on the left
(w0, w1) :
 Purple Dashed : (-0.9, -0.05); Green Dashed : (-0.7, -0.05);
Blue Dashed : (-0.5, -0.05); Pink line : LCDM

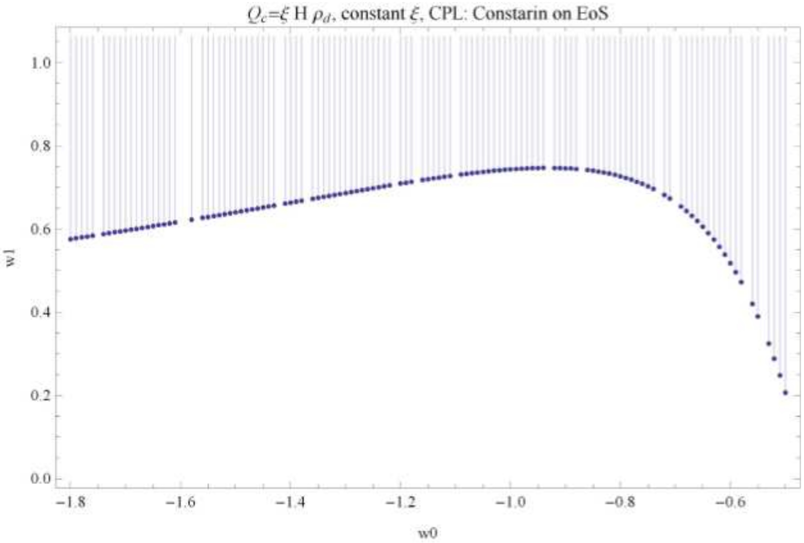
Figure on the right
(w0, w1) :
 Purple Dashed : (-0.5, -0.05); Green Dashed : (-0.5, -0.1);
Blue Dashed : (-0.5, -0.2); Pink line : LCDM



The following figure:
(w0,w1):
 Purple Dashed:(-1.1,0.05); Green Dashed:(-1.2,0.05); Blue Dashed:(-1.2,0.1); Pink line:LCDM



Here I recalculated the lower boundary in this model. This is exactly the same with the one I plotted previously.


Q_c=ξ H ρ_d, constant ξ, CPL: Constarin on EoS
w1
w0
1.0
0.8
0.6
0.4
0.2
0.0
−1.8
−1.6
−1.4
−1.2
−1.0
−0.8
−0.6