INTRO

PRE

LCDM Model

Interacting Models

Summary

- **Interacting models**
  - **List of what to make clear**
  - **BASIC**

Evolution of energy density for $Q_c = \xi H \rho_c$, constant ξ, constant w, and $\xi \neq$-3w

$$\Omega m = \Omega m0 \, (1 + z)^{3-\xi}$$

$$\Omega d = \left(\Omega d0 + \frac{\xi}{3 w + \xi} \Omega m0\right) (1 + z)^{3 (1+w)} + \frac{-\xi}{3 w + \xi} \Omega m \equiv \Omega d0' \, (1 + z)^{3 (1+w)} + \frac{-\xi}{3 w + \xi} \Omega m$$

Evolution of energy density for $Q_c = \xi H \rho_d$, constant ξ, constant w, and $\xi \neq$-3w

$$\Omega m = \left(\Omega m0 + \frac{\xi}{\xi + 3 w} \Omega d0\right) (1 + z)^{3} + \frac{-\xi}{\xi + 3 w} \Omega d \equiv \Omega m0' \, (1 + z)^{3} + \frac{-\xi}{\xi + 3 w} \Omega d$$

$$\Omega d = \Omega d0 \, (1 + z)^{3 (1+w) + \xi}$$

So in the two cases, coupling constant has two effects:

1. Amplifies the curve of deceleration parameter.
2. Energy flow between DE and DM.

- **Interacting model $Q_c = \xi H \rho_c$ with constant ξ and constant EoS w.**

Derived from (transition redshift, Ωm0) plane, the allowed region for coupling constant ξ is (-1.28, -0.46) with a center at -0.88, i.e., $-0.88^{+0.42}_{-0.40}$, taken the case that the universe is flat, and choose the EoS parameter {w=-1}.
Derived from the (transition redshift, $\frac{\Omega m0}{\Omega d0}$) plane, the allowed region of coupling constant ξ is (-1.25,- 0.47) with a center at -0.88, i.e., $-0.88^{+0.41}_{-0.37}$.
There is a bit difference between the two answers. One possible reason is the second method doesn’t assume a flat universe, while the first one supposes the universe is flat.

The full table of fitting results are shown below. The light purple element are the final resuls.

In[315]:=

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tabξFinaltICC
```

Out[315]=

$Q_c = \xi\ H\ \rho_c$, constant ξ, constant w=-1: Results for ξ			
Ωm0/Ωd0∴Transition	z_t=0.376	z_t=0.426	z_t=0.508
r=0.358	-1.25282	-0.965436	-0.617444
r=0.378	-1.15011	-0.875189	-0.542347
r=0.398	-1.05453	-0.791252	-0.472561

To check the consistancy of the two methods ((Transition, Ωm0) plane fitting and (Transition, Ωm0/Ωd0) plane fitting), we find out the fitting results of coupling constant ξ for a flat universe, i.e., $r = \frac{\Omega m0}{1-\Omega m0}$ in the (Transition, Ωm0) plane, applying the data from (Transition, Ωm0/Ωd0) plane. By solving out Ωm0, we get $\Omega m0 = \frac{r}{1+r}$ (this is a monotonic function) in this case. Thus if we use the constrain that r∈(0.358, 0.398) with a center value 0.378, the value of Ωm0 is (0.263623,0.284692), centered at 0.274311. Use this set of value of Ωm0 as the contrain, we have the fitting results in (Transition, Ωm0) plane, which is $-0.88^{+0.41}_{-0.37}$. This result is exactly the same as the result directly derived from (transition redshift, $\frac{\Omega m0}{\Omega d0}$) plane. The same has been done to $Q_c = \xi\, H\, \rho_d$ with ξ constant and w constant model, and the result is that the two methods are also consistant.

The plots of deceleration parameter are shown below. At the limit z->Infinity, the deceleration parameter is degenerate for different Ωm0 in this constant ξ and constant w model. Theoretically, this limit is determined by the interaction coupling constant ξ, which is $\frac{(1-\xi)}{2}$, with $3\,w + \xi < 0$.

In[316]:=

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pldecICCShowSum
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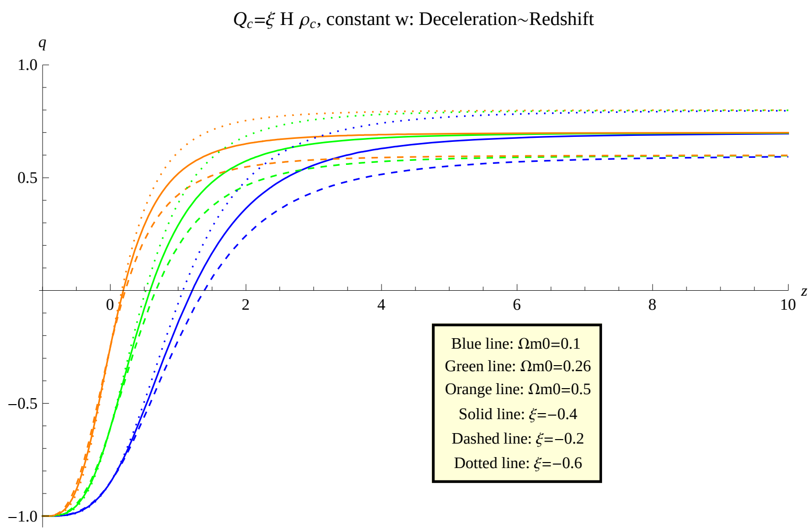
Out[316]=

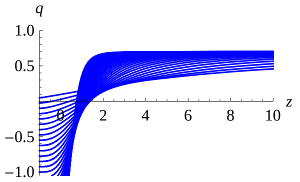
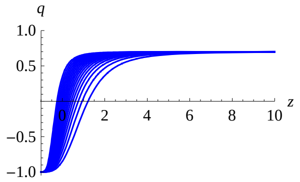
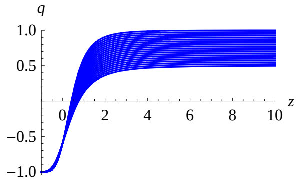


Check the effect of different parameters on deceleration parameter.
Interaction ξ changes the value of deceleration parameter at $z \to \infty$ limit. EoS changes the the whole shape. Matter fraction determines how fast q varies, but just in a small time scale.

In[317]:=

```
varyingICCShowSum
```

Out[317]=

$Q_c=\xi$ H ρ_c with constant ξ and w		
Varying EoS	Varying Ωm0	Varying ξ
		

A toy to play with is also provied. Slide the bars to view **the effects of different parameters on deceleration parameter**.

In[318]:=

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pldecICCManSum
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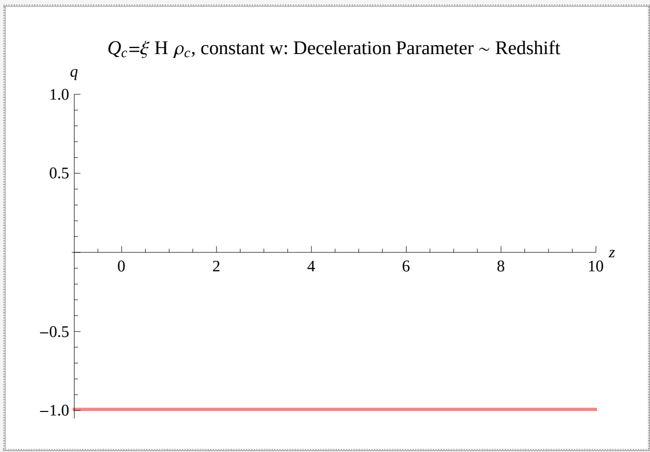
Out[318]=



Matter Fraction 0.26

Interaction -0.4

Equation of State -1

Pink is the deceleration parameter for LCDM.
Orange is for interacting model with Q=ξ H ρ_m

It can be inferred from the expression for Ωm and Ωd that if the transition happens before z=0, increasing couling ξ will bring forward the transition and if the it happens after z=0, increasing coupling ξ will delay the emergence of transition. The following figure shows this result.
Gray rectangle is the region given by Riess (References, Data From, 2).

Orange for w=-1
Blue for w=-0.9

Solid line: ξ=0.2
Dashed line: ξ=0.1
Dotted line: ξ=-0.1
DotDashed line: ξ=-0.2

Pink solid line:w=-1, ξ=0

In[319]:=

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plztrvsΩm0ICCSum
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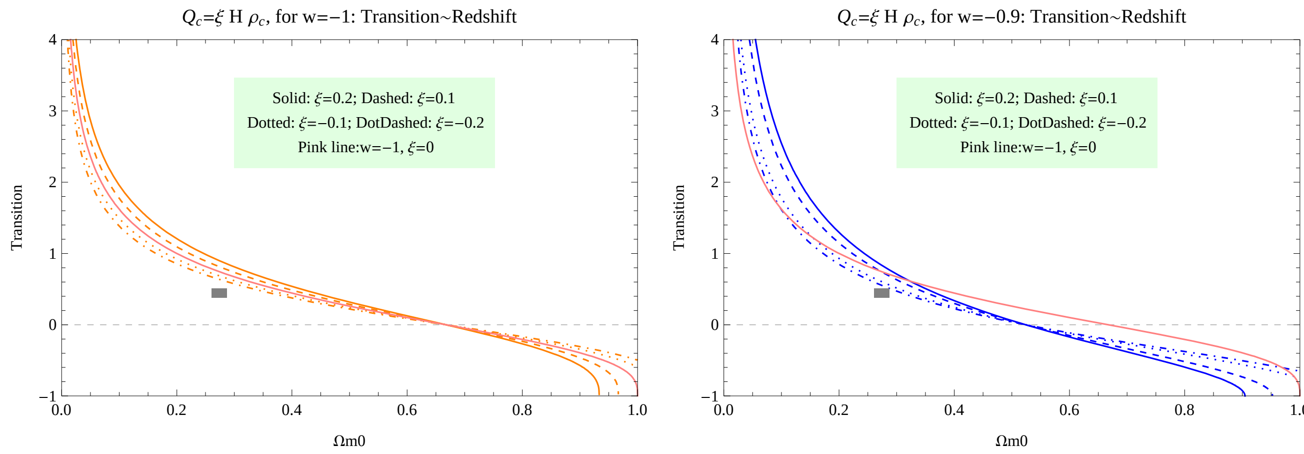
Out[319]=

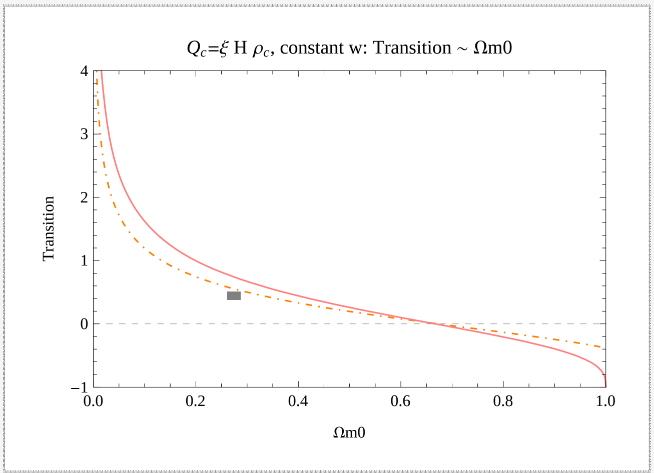


This can also be seen clearly from the following toy. Gray rectangle is the region given by Riess (References, Data From, 2) .

In[320]:=

```
plztrICCManSum
```

Out[320]=



Interaction

−0.4

Equation of state

−1.02

Pink is the transition redshift vs Ωm0 for LCDM.
Orange DotDashed line is interacting model,
for Q=ξ H ρ_m with constant w,
where ξ and w are both constants

The fitting results of coupling constant ξ can also be dynamic.

In[321]:=

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fitξICCManSum
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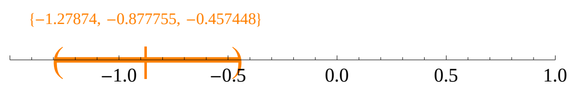
Out[321]=



Equation of State

−1

This is the fitting result from transition redshift data,
for $Q_c = \xi\ H\ \rho_c$ with w and ξ constant

The parenthesis shows the upper and lower value
while the verticle line show the center value.

The three numbers are left value, center value, right value respectively.

Slide to see how do the two parameters affect the coupling constant results.

For different constant EoS, the fitting results using $\Omega m0 \in (0.261, 0.287)$ with a center value 0.274 and Transition redshift $\in$ (0.376,0.508) with a center value 0.426. When EoS is very small, the line cross zero. But that is not so useful.

Some data points are derived using w = -1.087 ± 0.096 (from Reference, Data From, 3).

In[322]:=

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tabξvwExamICC
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Out[322]=

ξ results for $Q_c = \xi\ H\ \rho_c$ (Fitting data: Data From, 2)			
w	Center	Lower	Upper
−1.183	−0.881565	−1.29687	−0.443589
−1.087	−0.88948	−1.29859	−0.459135
−0.991	−0.875238	−1.27522	−0.456176

A plot showing these data points and the curves of ξ ~ w.

In[323]:=

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pltξvwExamICC
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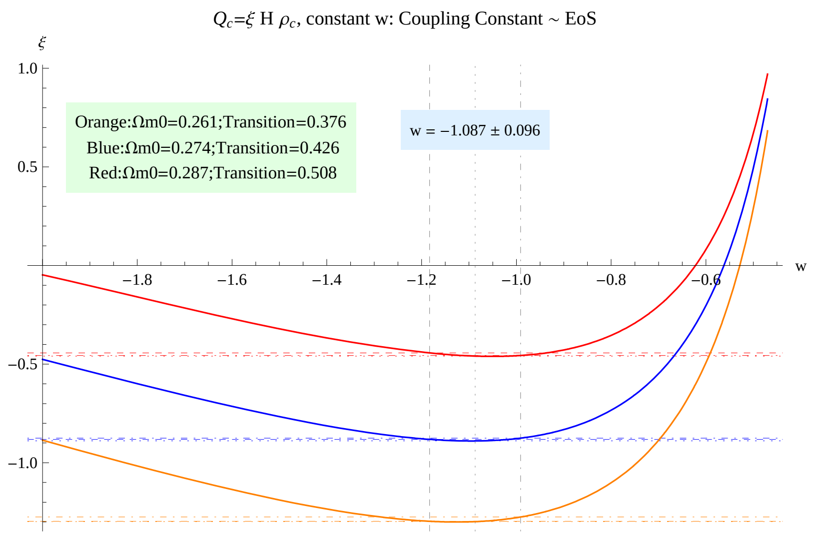
Out[323]=



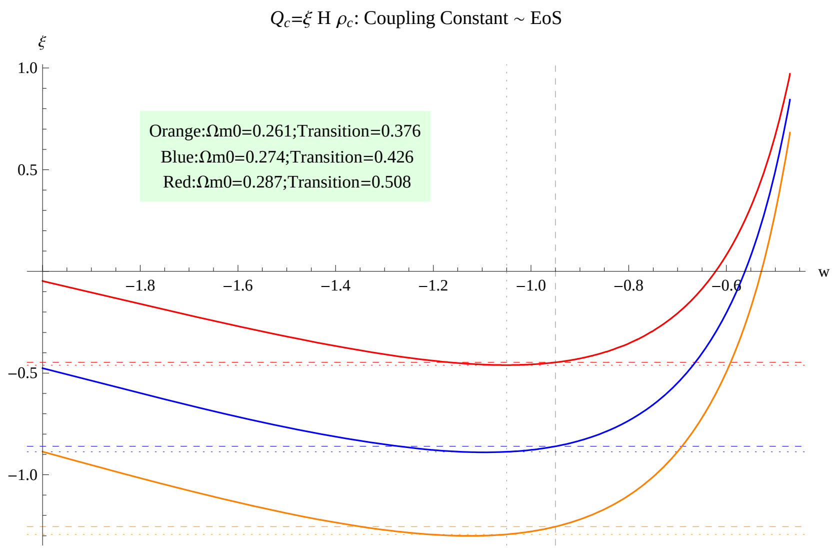
Or just casually use the following parameters.
(-1 within 5%: (-1.05,-0.95))
w=-1 (-1.279,-0.457) center:-0.878
w=-1.05 (-1.293,-0.461) center:-0.887
w=-0.95 (-1.255,-0.447) center:-0.860

The following graph show how do ξ changes with EoS. The grid lines are the results of $w = -1 \pm 0.05$. Two verticle lines are -1.05 and -0.95 respectively. Horizontal lines are their intersections with the ξ~w lines. EoS does not monotonically change ξ. And the minima of these line occurs at a larger w with an increasing Ωm0.

In[324]:=

```
pltξvwExam1ICC
```

Out[324]=



Now we assume we do not have the observed Ωm0 data, how do this Ωm0 change the result of ξ.
In other words, if the observed Ωm0 data float around some value, then how is the fitting result?
We also consider the curvature.
In the figure below, it seems that there is a point where three lines converge. This has something to do with the phenomena that

In[325]:=

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pltξvΩm0ICCManSum
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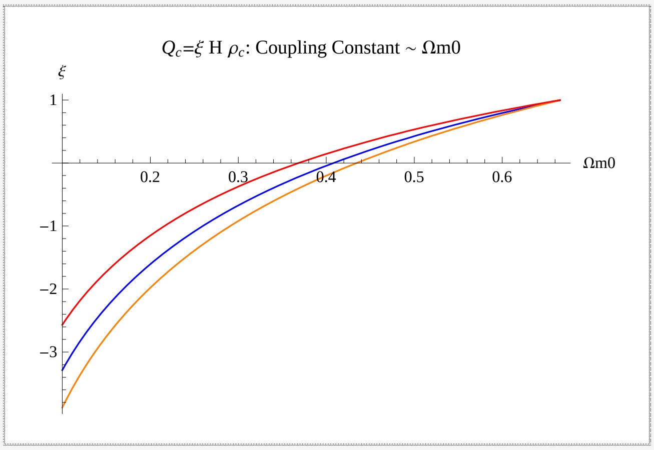
Out[325]=



Curvature

Equation of State

Blue line is the center of ξ

The other lines are the upper and lower value

Slide to check how do matter change this curve.

Some data for flat ΛCDM universe. The following data shows how Ωm0 change our results for ξ if we already have transition redshift data {0.426|0.376,0.508}.

If Ωm0 varies 5 percent from 0.274,

In[326]:=

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tabξICCSum
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Out[326]=

For Ωm0∈0.274 (1 ± 0.05)

Table of ξ for different Ωm0~Transition combination

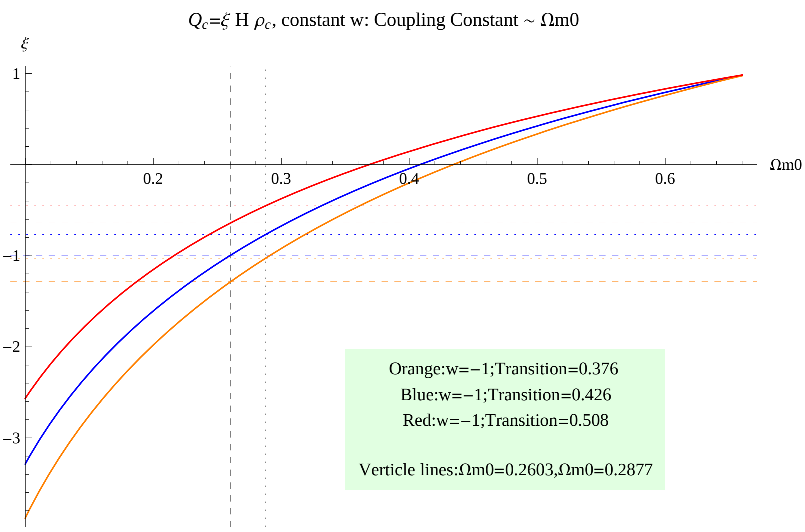
Ωm0∖Transition	0.426	0.376	0.508
0.2603	−0.994339	−1.28571	−0.641508
0.274	−0.877755	−1.15303	−0.544482
0.2877	−0.767582	−1.02756	−0.452892

Monotonic line.

In[327]:=

```
pltξvΩm0ICCSum
```

Out[327]=



Besides Ωm0, we can also find out the effects of Curvature, EoS. Assuming we have a constrain of Transition redshift (0.376,0.508) with a center at 0.426.

In[328]:=

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fitξ2ICCManSum
```
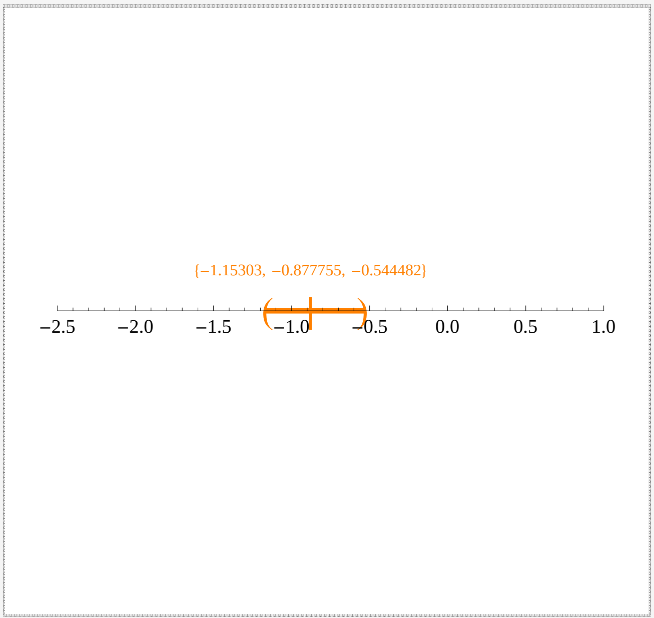

Out[328]=



Matter Fraction 0.274

Curvature 0

Equation of State −1

Model: $Q_c=\xi$ H ρ_c, constant w, constant ξ

This is the fitting result of ξ from only transition redshift data without the knowledge of Ωm0.

Slide to check how do EoS change this curve.

Interacting model $Q_c = \xi H \rho_c$ with constant ξ and CPL parameterized EoS $w = w0 + w1 \frac{z}{1+z}$.

For a flat universe, choose the parameters {w0=-1.02,w1=0.6}, the region for interation cosntant ξ should be (-1.04,-0.21) with a center at -0.64, i.e., $-0.64^{+0.42}_{-0.40}$,derived from the (transition redshift, Ωm0) plane, while a result of (-1.01, -0.23) with a center at -0.63, i.e., $-0.63^{+0.40}_{-0.38}$, derived from (transition redshift, $\frac{\Omega m0}{\Omega d0}$) plane.

Deceleration parameter is shown below. Behaves similiar to the constant ξ constant w situation.

In[329]:=

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pldecICCPLShowSum
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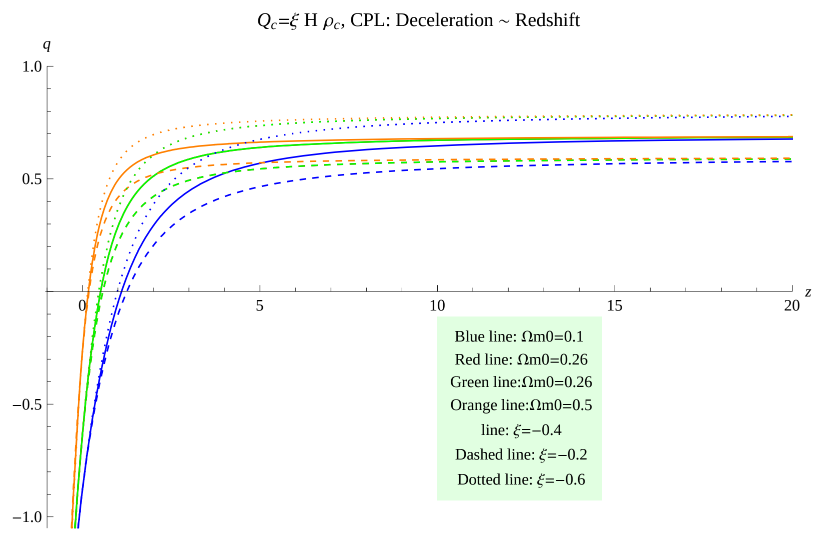
Out[329]=



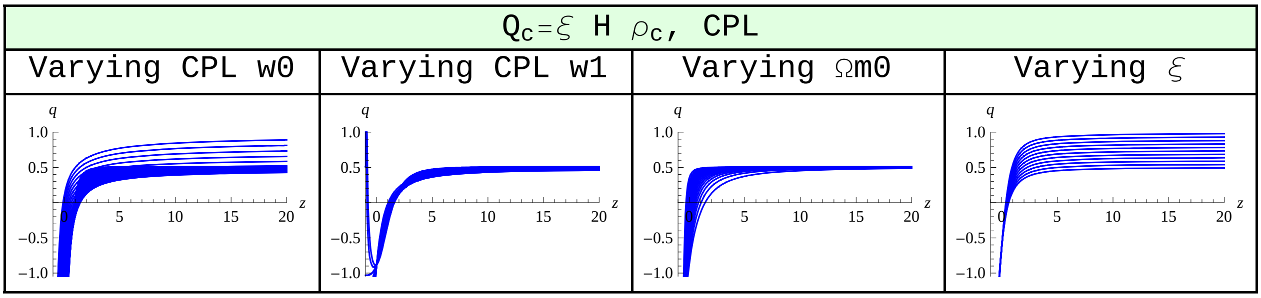
The following plots shows the effect of different parameters. Each plot shows how do the deceleration parameter vs redshift line change under uniformly distributed w0,w1,Ωm0 or ξ.
w0 moves the line up or down, but not monotonously;
w1 changes the late time behavior;

Ωm0 changes the slope;
ξ has moves the line up or down;

In[330]:=

```
varyingICCPLShowSum
```

Out[330]=



A toy to play with the q~z plot. When ξ=0, w0=-1, w1=0, the curve reduced to LCDM curve.

In[331]:=

```
pldecICCPLManSum
```

A plot shows how bad it is to use transition redshift to constrain interacting model. This is a CPL parameterized example. For $\xi\in$(-0.8,0), the line just stays near the allowed region constrained by Riess's results (References, Data From, 2).

In[332]:=

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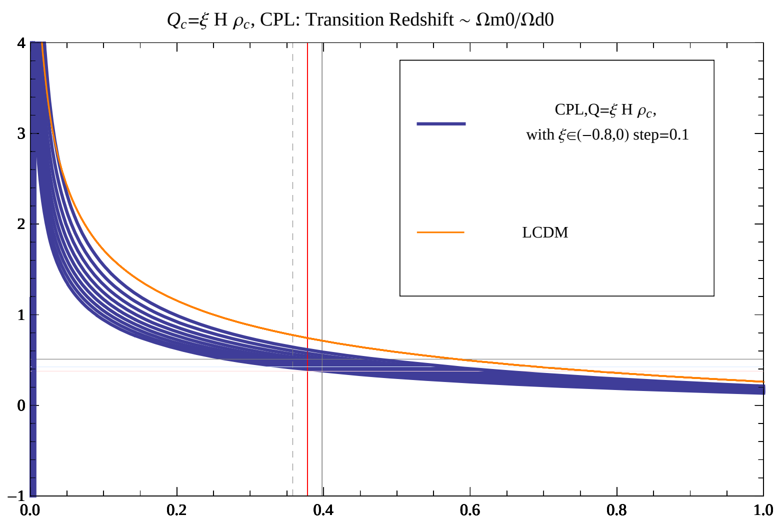
pltransrICCPLDense
```

Out[332]=



From the manipulate below, we find for w0=-1, there is a point on this transition ~ Ωm0 curve do not change with coupling constant ξ and w1. (Well, what's the use of that...)

In[333]:=

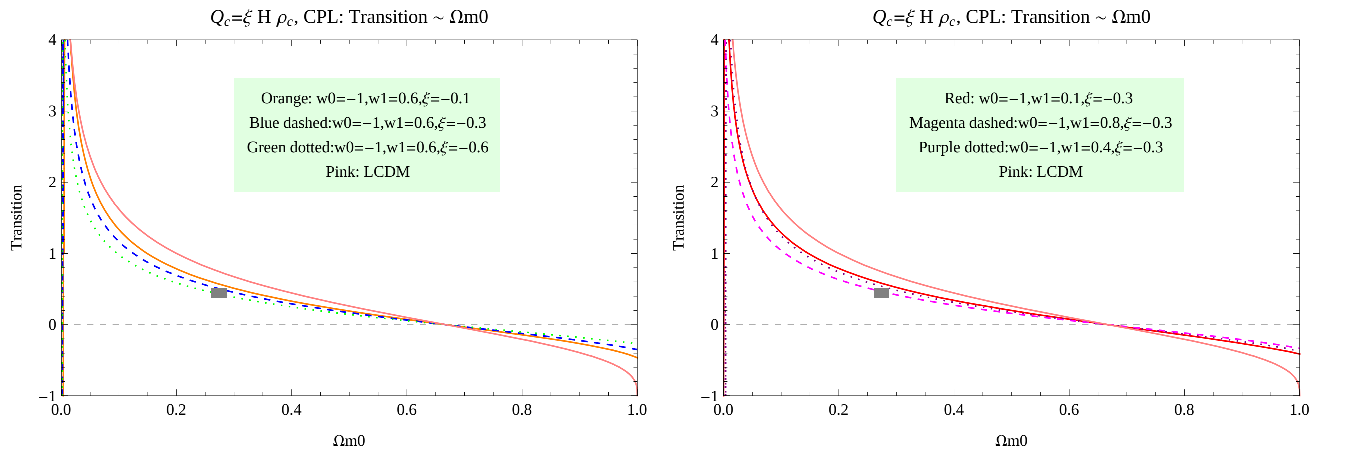
```
plztrICCPLManSum
```

An explicit proof of this statement.

In[334]:=

```
plztrExamICCPLSum
```

Out[334]=



A manipulate of the fitting results.

In[335]:=

```
fitξICCPLManSum
```

Category

For different w0 and w1 in its EoS equation,

In[336]:=

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plEoSPhaseICCPL
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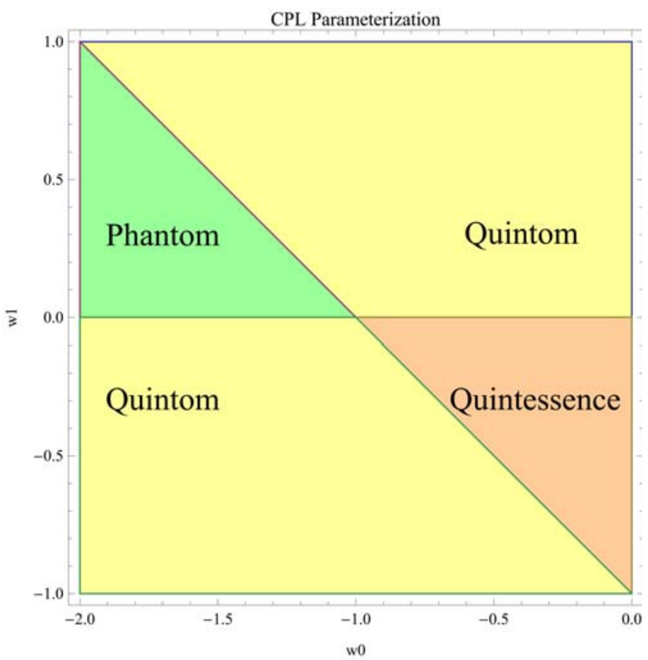
Out[336]=



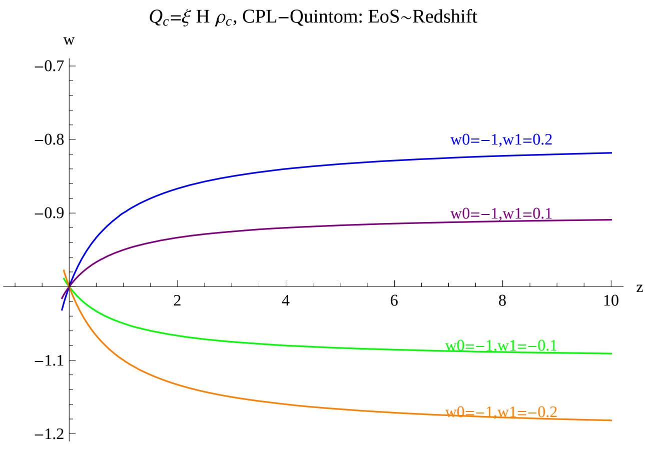
Quintom

Color illustrations for the following two figures.
“Purple line:(-1,0.1) Blue line: (-1,0.2) Green line:(-1,-0.1)\n Pink line:LCDM”
“Purple Dashed:(-0.9,0.1) Green Dashed:(-1.1,0.1)”

In[337]:=

```
plEoSICCPLQuintomSum
```

Out[337]=



Plots of Transition redshift vs Ωm0.
Legends are shown on the plots. Hard to distingush from each other.

In[338]:=

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plztrICCPLQuintomSum
```
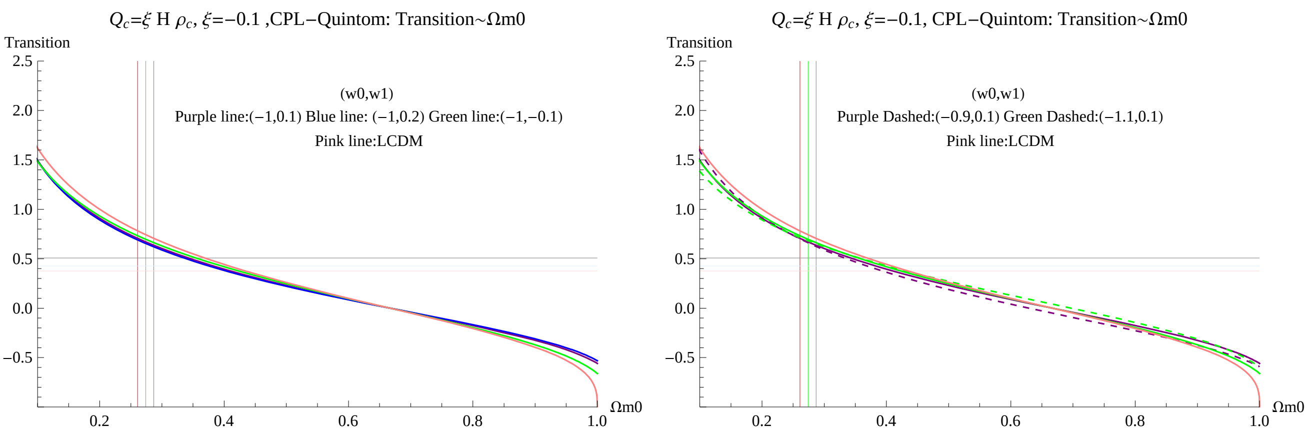

Out[338]=



For **different EoS**, the fitting results are different. The following table and plots show how do w0 and w1 change the results.

In[339]:=

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tabξvwExamICCPLQuintom
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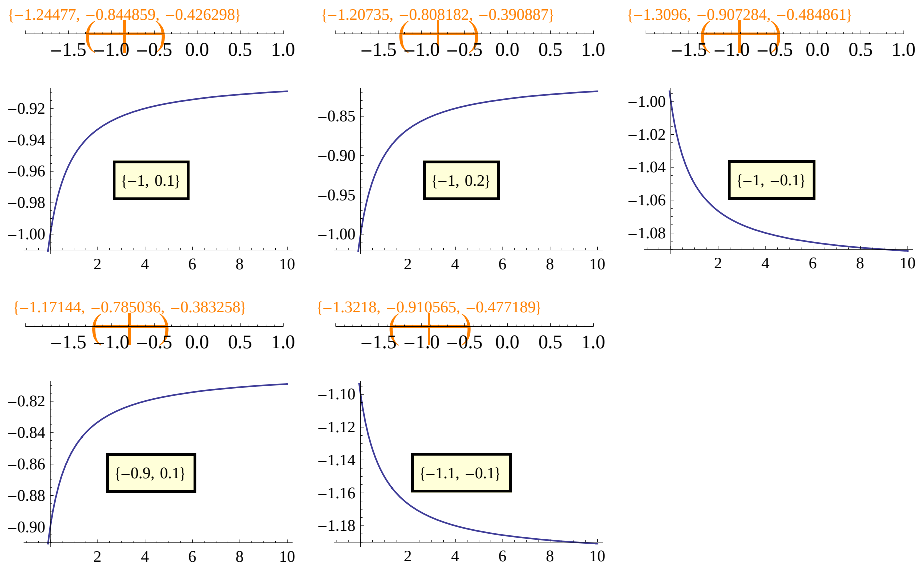
Out[339]=

ξ results for Q_c=ξ H ρ_d, CPL,Quintom.			
{w0,w1}	Center	Lower	Upper
{−1, −0.1}	−0.907284	−1.3096	−0.484861
{−1, 0}	−0.877755	−1.27874	−0.457448
{−1, 0.1}	−0.844859	−1.24477	−0.426298
{−0.9, 0.1}	−0.785036	−1.17144	−0.383258
{−1.1, −0.1}	−0.910565	−1.3218	−0.477189

In[340]:=

```
pltfitξICCPLQuintomSum
```

Out[340]=

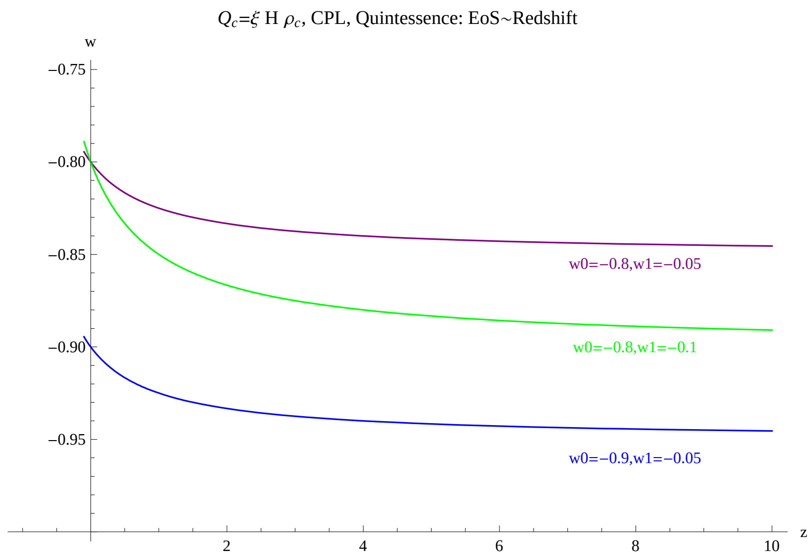


Quintessence

In[341]:=

```
plEoSICCPLQuintessenceSum
```

Out[341]=



Different from constant w results,

In[342]:=

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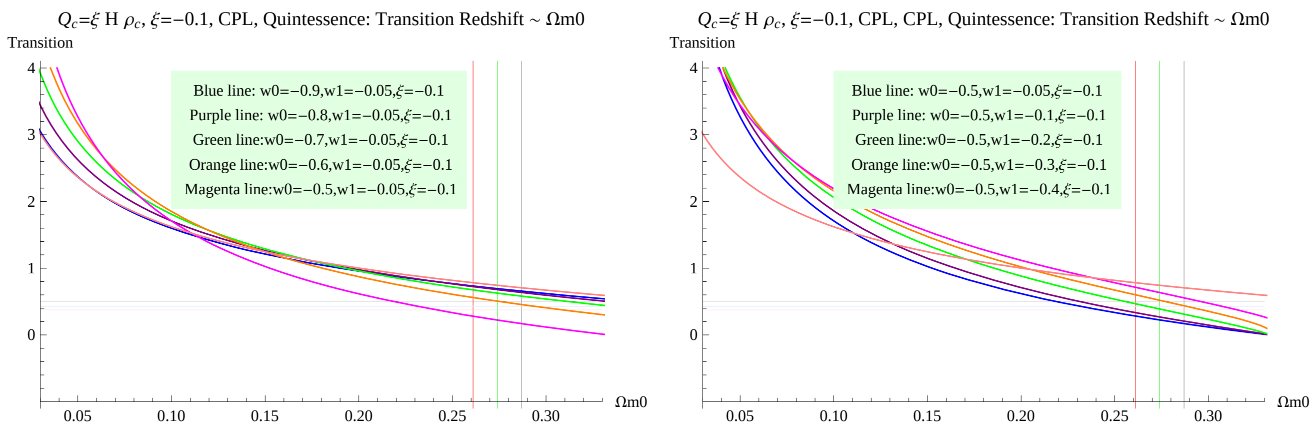
plztrICCPLQuintessenceSum
```

Out[342]=



Some ξ fitting results are shown below. This shows how do w0 and w1 change ξ results.

In[343]:=
```
tabξvwExamICCPLQuintessence
```

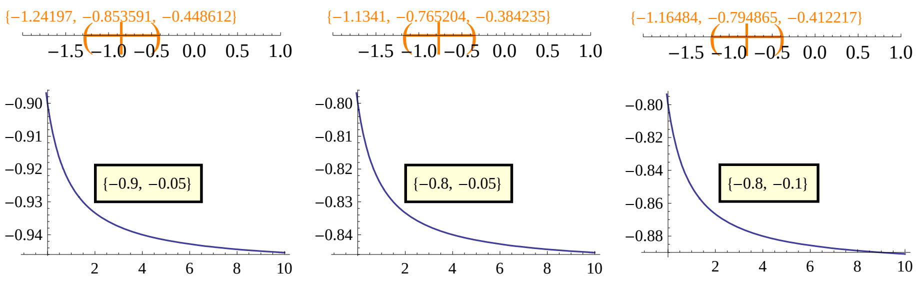
Out[343]=

ξ results for $Q_c = \xi\, H\, \rho_d$, CPL,Quintessence.

{w0,w1}	Center	Lower	Upper
{−0.9, −0.05}	−0.853591	−1.24197	−0.448612
{−0.8, −0.05}	−0.765204	−1.1341	−0.384235
{−0.8, −0.1}	−0.794865	−1.16484	−0.412217

In[344]:=
```
pltfitξICCPLQuintessenceSum
```

Out[344]=



Phantom

In[345]:=
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plEoSICCPLPhantomSum
```
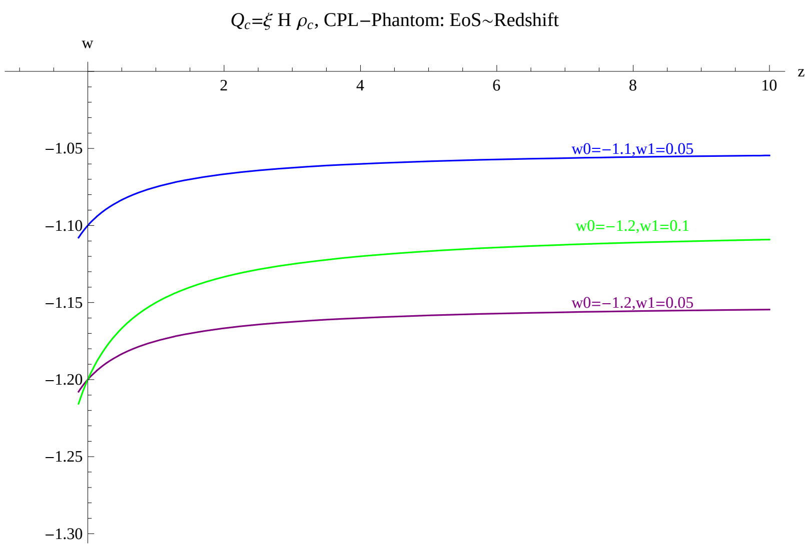
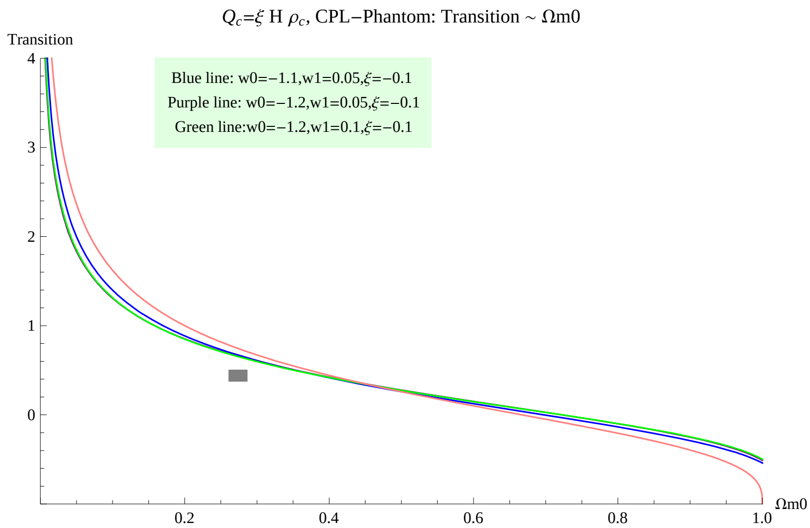

Out[345]=



In[346]:=

```
plztrICCPLPhantomSum
```

Out[346]=



In[347]:=

```
tabξvwExamICCPLPhantom
```

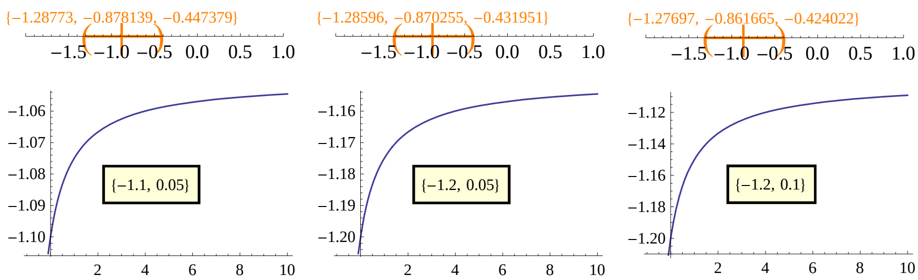
Out[347]=

ξ results for Q_c=ξ H ρ_d, CPL,Phantom.			
{w0,w1}	Center	Lower	Upper
{−1.1, 0.05}	−0.878139	−1.28773	−0.447379
{−1.2, 0.05}	−0.870255	−1.28596	−0.431951
{−1.2, 0.1}	−0.861665	−1.27697	−0.424022

In[348]:=

```
pltfitξICCPLPhantomSum
```

Out[348]=



Interacting model $Q_c = \xi H \rho_d$ with constant ξ and constant EoS w.

Derived from (transition redshift, Ωm0) plane, the allowed region for coupling constant ξ is (-1.06,-0.42) with a center at -0.76, i.e., $-0.76^{+0.34}_{-0.30}$, taken the case that the universe is flat, and choose the EoS parameter {w=-1}.
Derived from the (transition redshift, $\frac{\Omega m0}{\Omega d0}$) plane, the allowed region of coupling constant ξ is (-1.07,-0.41) with a center at -0.76, i.e., $-0.76^{+0.35}_{-0.31}$.

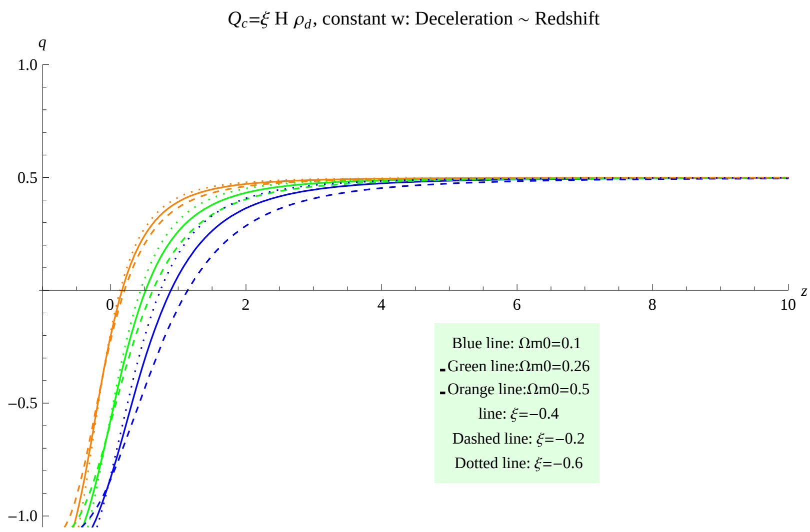
The plots of deceleration parameter are shown below. At the limit z->Infinity, the deceleration parameter ALL goes to $\frac{1}{2}$.

Theoretically, this limit is $\frac{1}{2}$ which is not related to any parameters, with $3\,w + \xi < 0$.

In[349]:=

```
pldecI2CCShowSum
```

Out[349]=

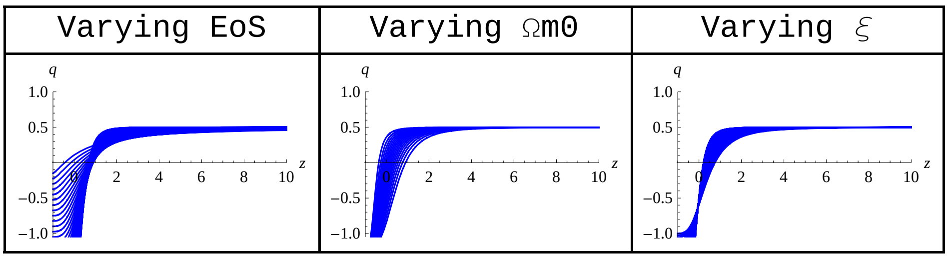


To check the effect of different parameters, another plot is shown.

In[350]:=

```
varyingI2CCShowSum
```

Out[350]=



A toy to play with deceleration vs z curve is also provied

In[351]:=

```
pldecI2CCManSum
```

If the transition happens before z=0, increasing couling ξ will delay the transition.
If the it happens after z=0, increasing coupling ξ will bring forward the emergence of transition.

The following figure shows this result.

Gray rectangle is the region given by Riess.

Orange for w=-1
Blue for w=-0.9

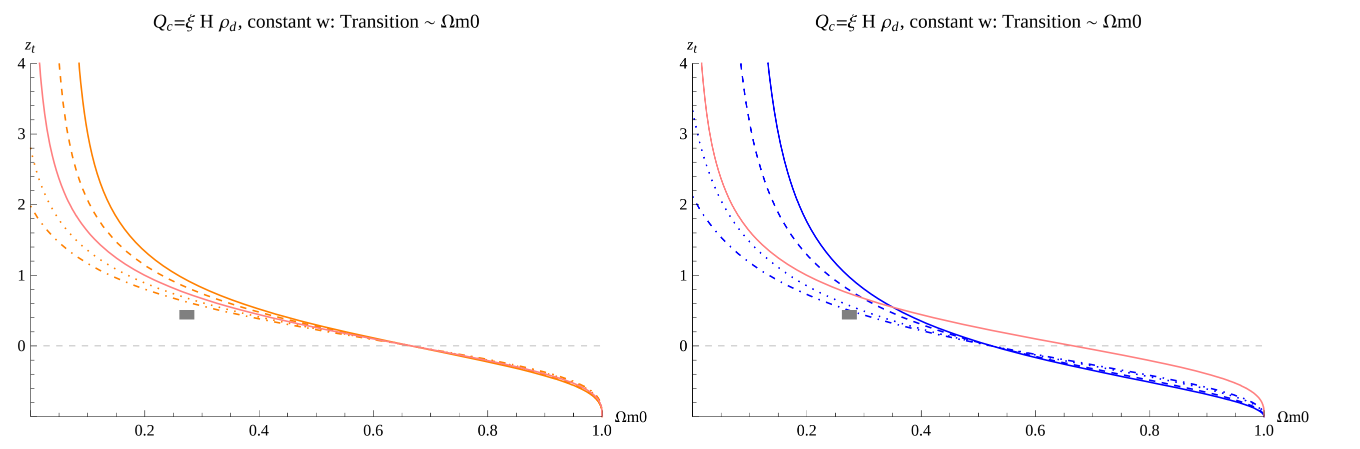
line: ξ=0.2
Dashed: ξ=0.1
Dotted: ξ=-0.1
DotDashed: ξ=-0.2

Pink line:w=1, ξ=0

In[352]:=

```
plztrvsΩm0I2CCSum
```

Out[352]=

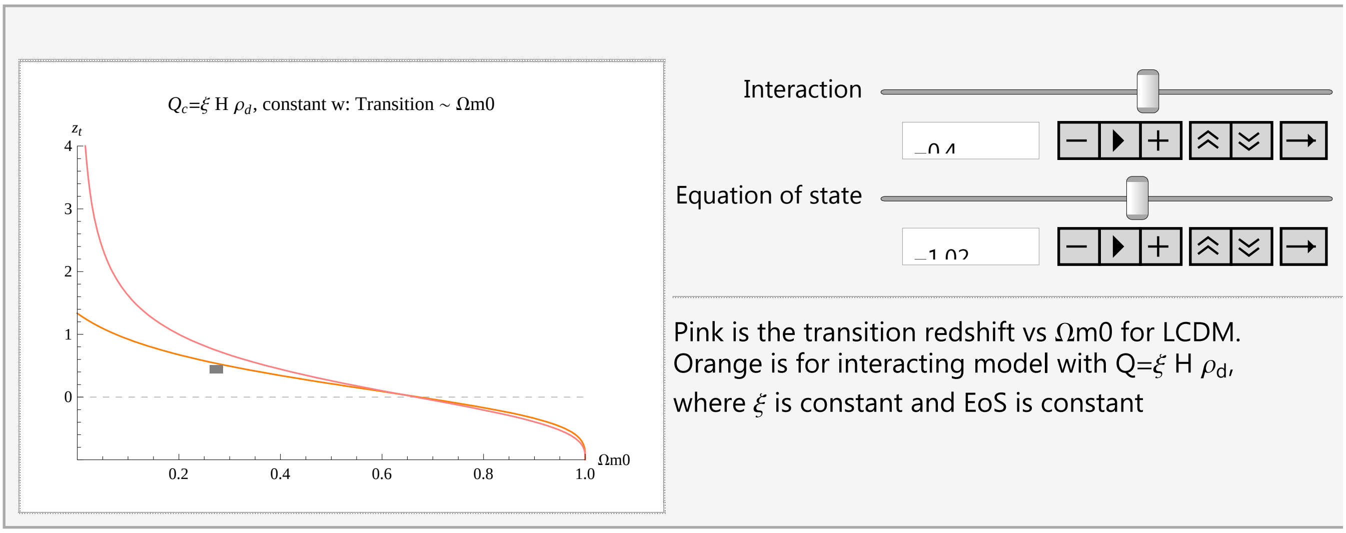


A toy of transition redshift. Gray rectangle is the allowed region of Ωm0~Transition redshift

In[353]:=

```
plztrI2CCManSum
```

Out[353]=

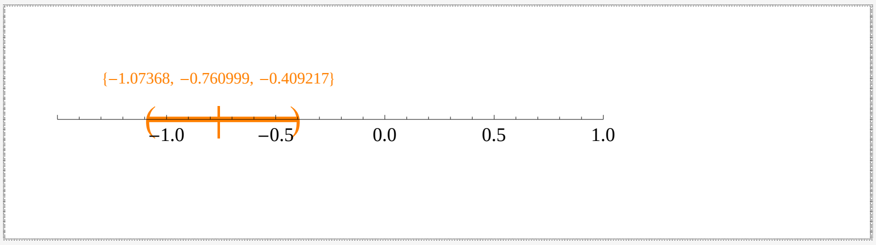


The fitting results of coupling constant ξ is

In[354]:=

```
fitξI2CCManSum
```

Out[354]=



Equation of State

−1

This is the fitting result from transition redshift data.

$Q_c = \xi$ H ρ_d

The parenthesis shows the upper and lower value
while the verticle line show the center value.

The three numbers are left value, center value, right value respectively.

Slide to see how do the two parameters affect the coupling constant results.

For different constant EoS, the fitting results using Ωm0∈(0.261,0.287) with a center value 0.274 and Transition redshift ∈ (0.376,0.508) with a center value 0.426. When EoS is very small, the line might cross zero. But that is not so useful.

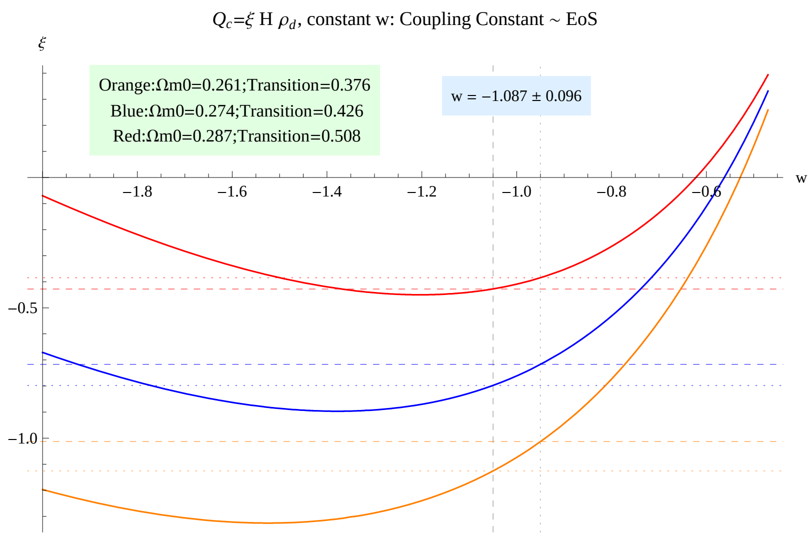
Some data: (-1 within 5%: (-1.05,-0.95))
w=-1 (-1.074,-0.409) center:-0.761
w=-1.05 (-1.126,-0.428) center:-0.798
w=-0.95 (-1.013,-0.385) center:-0.717

The following graph show how do ξ changes with EoS. The grid lines are the results of $w = -1 \pm 0.05$. Two verticle lines are -1.05 and -0.95 respectively. Horizontal lines are their intersections with the ξ~w lines.

In[355]:=

```
pltξvwExam1I2CC
```

Out[355]=



Or we can use some fitting results from WMAP etc. Take the example of w=-1.087±0.096.

In[356]:=

```
tabξvwExamI2CC
```

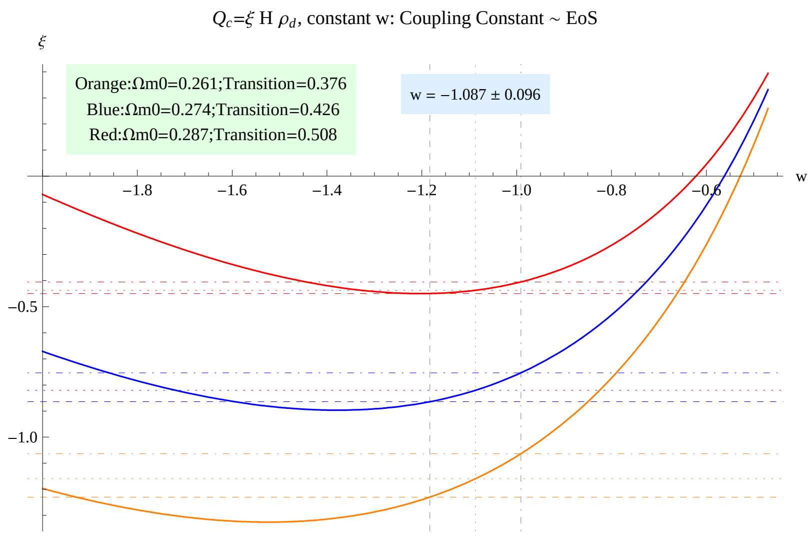
Out[356]=

$Q_c=\xi\ H\ \rho_d$, Constant w. (Data used:Data From, 2)			
w	Center	Lower	Upper
-1.183	-0.864289	-1.22984	-0.449552
-1.087	-0.820486	-1.15946	-0.437339
-0.991	-0.753634	-1.06346	-0.405262

In[357]:=

```
pltξvwExamI2CC
```

Out[357]=

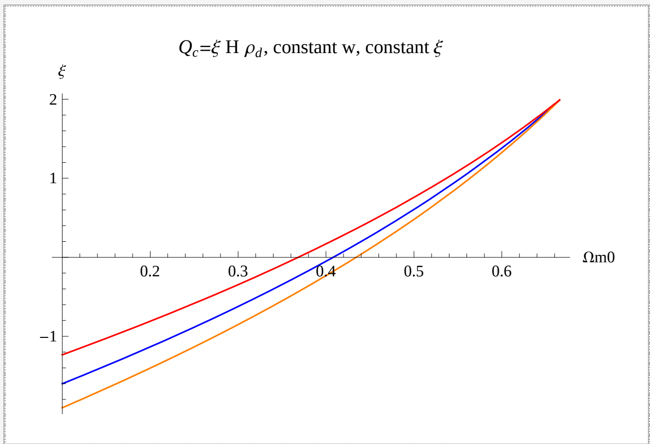


Now we assume we do not have the observed Ωm0 data, how do this Ωm0 change the result. In other words, if the observed Ωm0 data float around some value, then how is the fitting result? We also consider the curvature.

In[358]:=

```
pltξvΩm0I2CCManSum
```

Out[358]=



Curvature

0

Equation of State

1

Blue line is the center of ξ

The other lines are the upper and lower value

Slide to check how do matter change this curve.

If Ωm0 varies 0.05 percent from 0.274,

In[359]:=

```
tabξI2CCSum
```

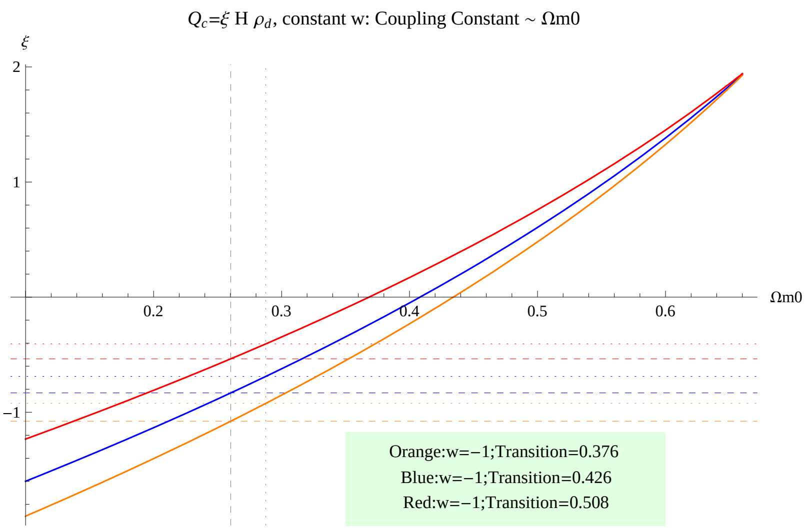
Out[359]=

For Ωm0∈0.274 (1 ± 0.05) Table of ξ for different Ωm0~Transition combination			
Ωm0⋱Transition	0.426	0.376	0.508
0.2603	−0.832284	−1.07758	−0.53584
0.274	−0.760999	−1.00068	−0.471298
0.2877	−0.688664	−0.922602	−0.40585

In[360]:=

```
pltξvΩm0I2CCSum
```

Out[360]=

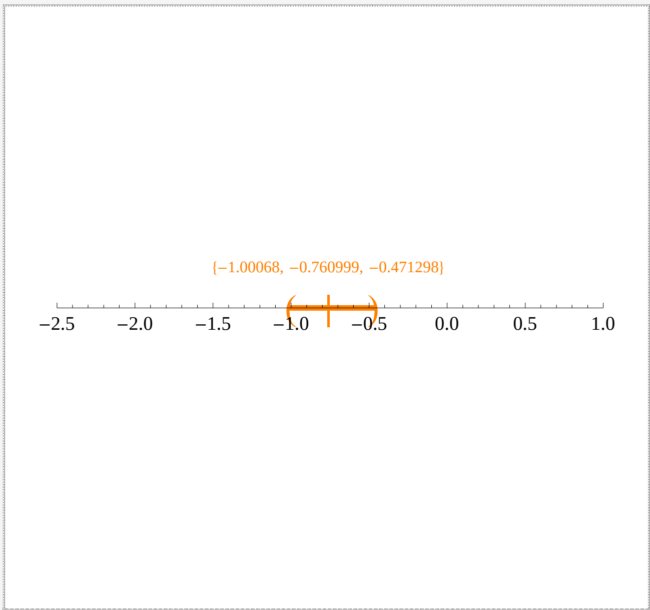


In addition, we can also find out the effects of Curvature, EoS. Assuming we have a constrain of Transition redshift (0.376,0.508) with a center at 0.426.

In[361]:=

```
fitξ2I2CCManSum
```

Out[361]=



Matter Fraction

0.274

Curvature

0

Equation of State

−1

Q_c=ξ H ρ_d, constant w, constant ζ.

This is the fitting result of ξ from only transition redshift data without the knowledge of Ωm0.

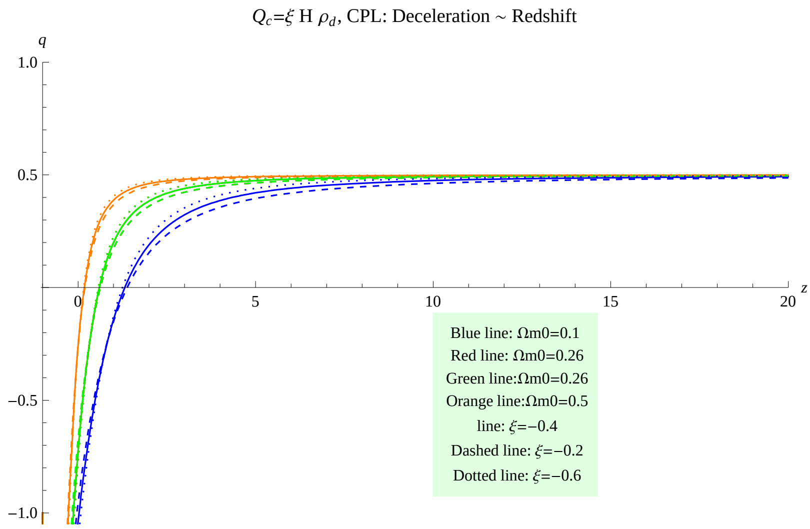
Slide to check how do EoS change this curve.

■ Interacting model $Q_c = \xi H \rho_d$ with constant ξ and CPL parameterization.

In[362]:=

```
pldecI2CCPLShowSum
```

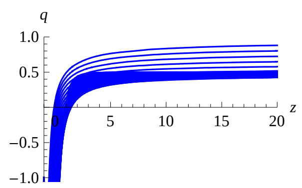
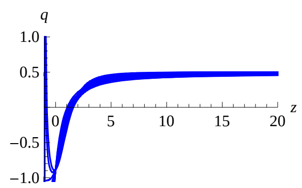
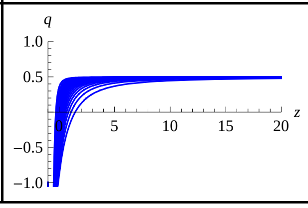
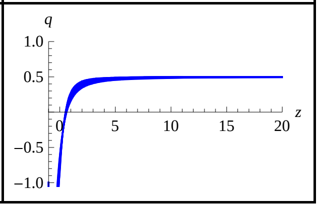
Out[362]=



In[363]:=

```
varyingI2CCPLShowSum
```

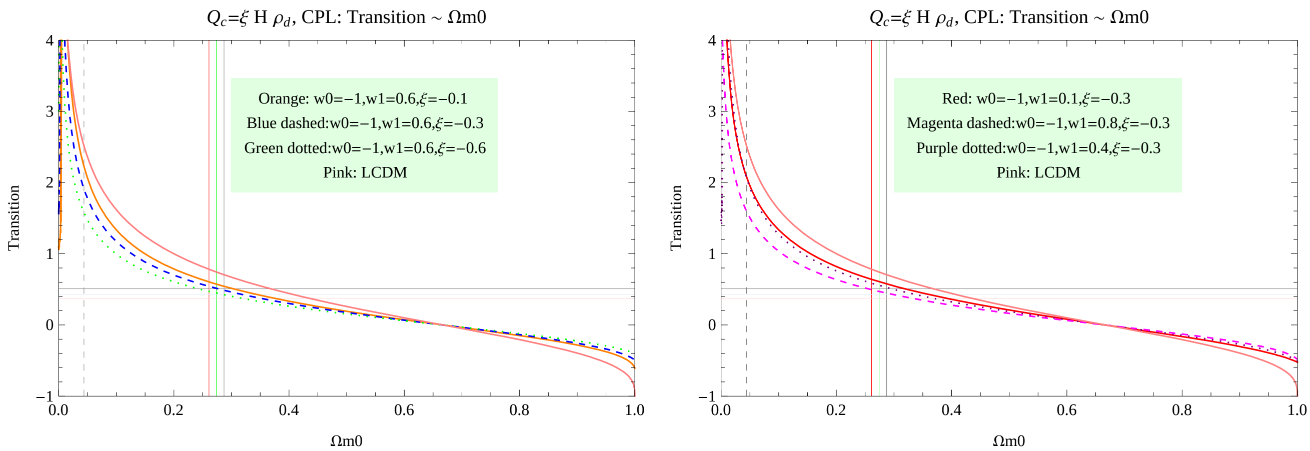
Out[363]=

Q_c=ξ H ρ_d, CPL			
Varying CPL w0	Varying CPL w1	Varying Ωm0	Varying ξ
			

In[364]:=

```
plztrExamI2CCPLSum
```

Out[364]=

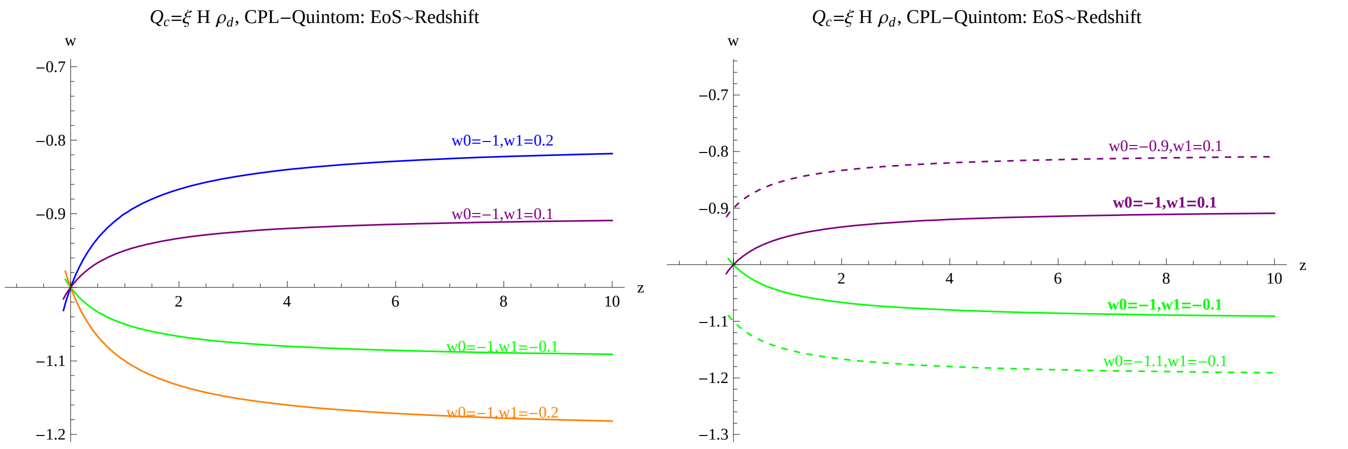


Quintom

In[365]:=

```
plEoSI2CCPLQuintomSum
```

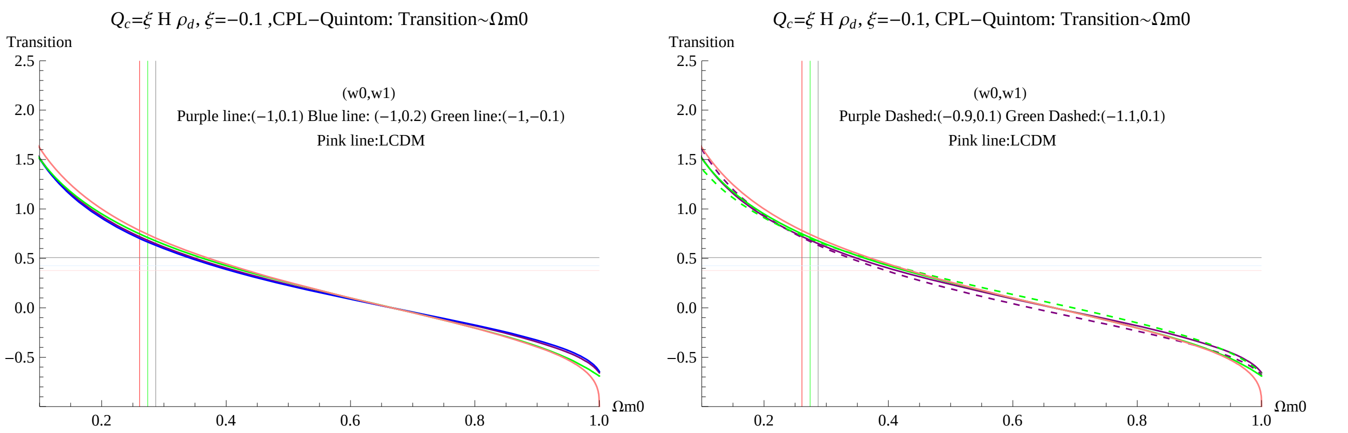
Out[365]=



In[366]:=

```
plztrI2CCPLQuintomSum
```

Out[366]=



In[367]:=

```
tabξvwExamI2CCPLQuintom
```

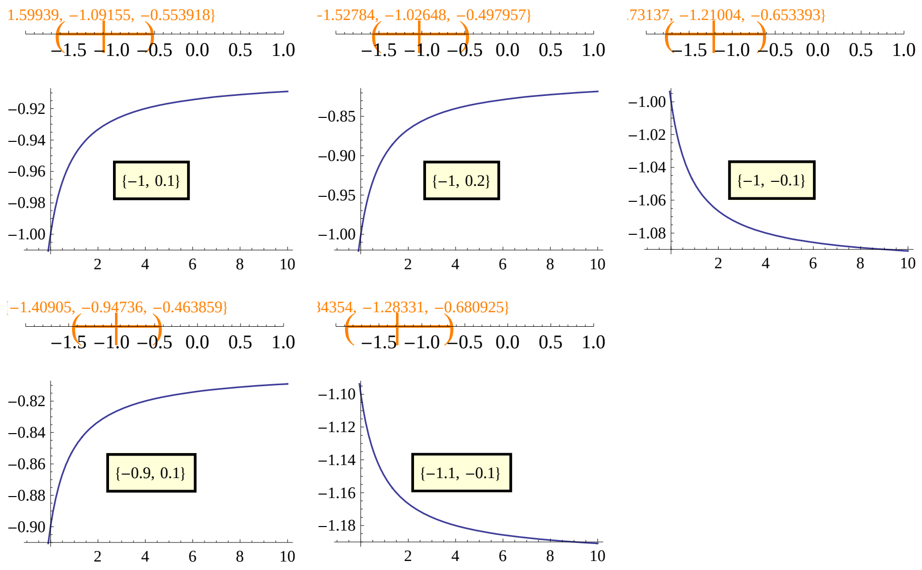
Out[367]=

ξ results for $Q_c=\xi\ H\ \rho_d$, CPL,Quintom.			
{w0,w1}	Center	Lower	Upper
{–1, –0.1}	–1.21004	–1.73137	–0.653393
{–1, 0}	–1.15265	–1.66715	–0.605615
{–1, 0.1}	–1.09155	–1.59939	–0.553918
{–0.9, 0.1}	–0.94736	–1.40905	–0.463859
{–1.1, –0.1}	–1.28331	–1.84354	–0.680925

In[368]:=

```
pltfitξI2CCPLQuintomSum
```

Out[368]=

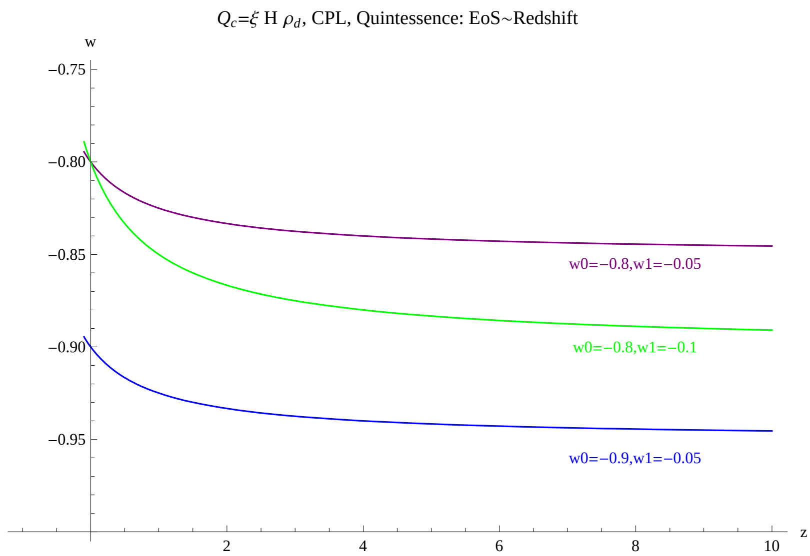


Quintessence

In[369]:=

```
plEoSI2CCPLQuintessenceSum
```

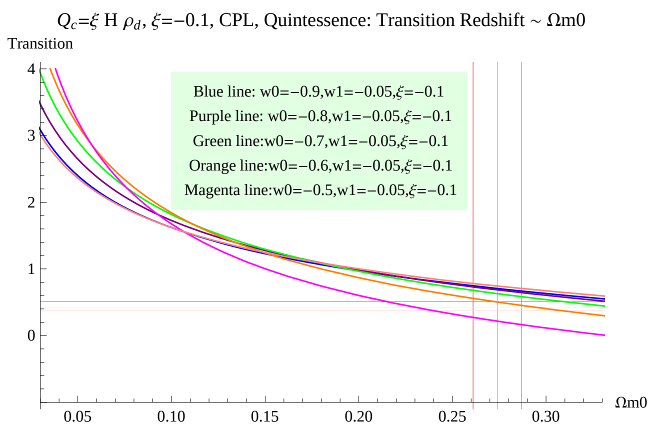
Out[369]=

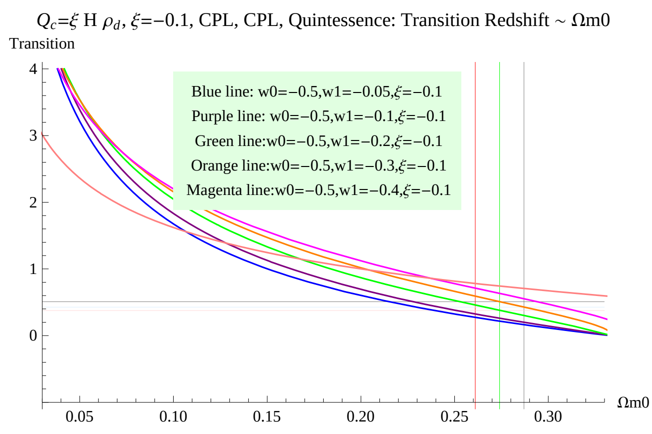


In[370]:=

```
plztrI2CCPLQuintessenceSum
```

Out[370]=





In[371]:=

```
tabξvwExamI2CCPLQuintessence
```

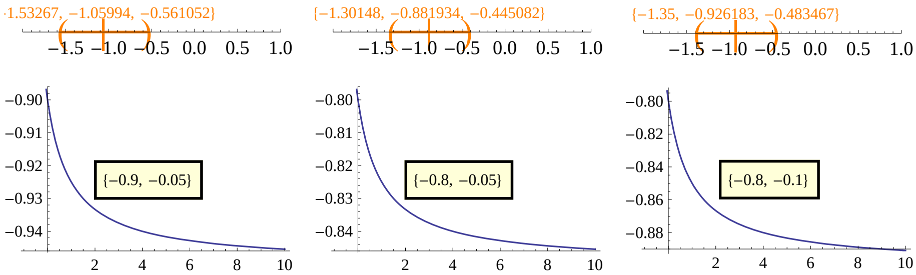
Out[371]=

ξ results for $Q_c = \xi\ H\ \rho_d$, CPL,Quintessence.			
{w0,w1}	Center	Lower	Upper
{−0.9, −0.05}	−1.05994	−1.53267	−0.561052
{−0.8, −0.05}	−0.881934	−1.30148	−0.445082
{−0.8, −0.1}	−0.926183	−1.35	−0.483467

In[372]:=

```
pltfitξI2CCPLQuintessenceSum
```

Out[372]=

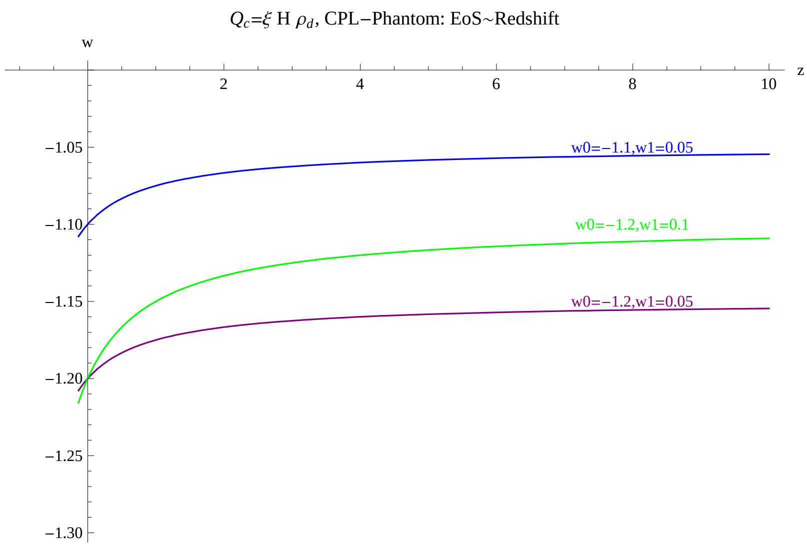


Phantom

In[373]:=

```
plEoSI2CCPLPhantomSum
```

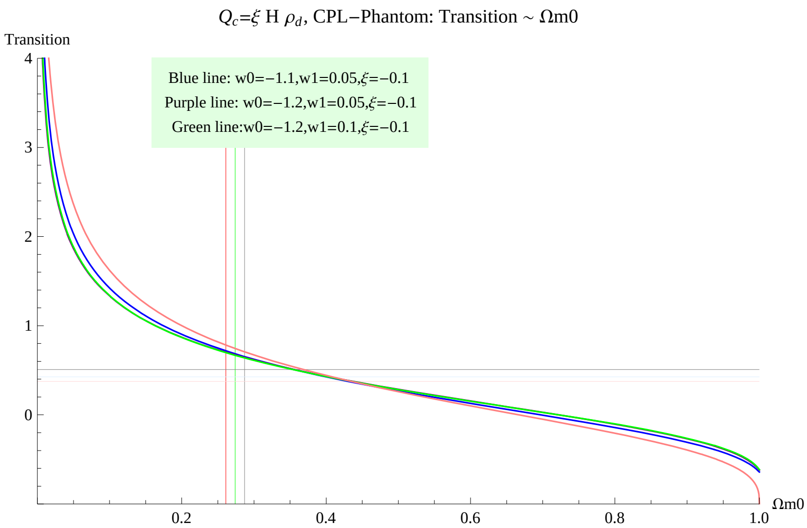
Out[373]=



In[374]:=

```
plztrI2CCPLPhantomSum
```

Out[374]=



In[375]:=

```
tabξvwExamI2CCPLPhantom
```

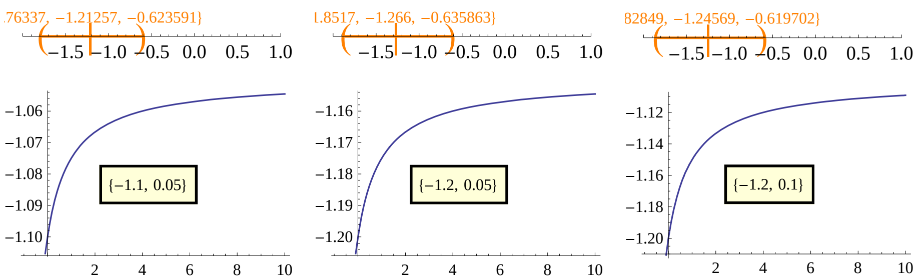
Out[375]=

ξ results for Q_c=ξ H ρ_d, CPL,Phantom.			
{w0,w1}	Center	Lower	Upper
{−1.1, 0.05}	−1.21257	−1.76337	−0.623591
{−1.2, 0.05}	−1.266	−1.8517	−0.635863
{−1.2, 0.1}	−1.24569	−1.82849	−0.619702

In[376]:=

```
pltfitξI2CCPLPhantomSum
```

Out[376]=



References

Data From

1. CPL data

Combining SN1a, BAO 3, WMAP5, H(z)   (From arXiv:0909.0596)

$\Omega m0 = 0.269^{+0.017}_{-0.008}$, $w0 = -0.97^{+0.12}_{-0.07}$, $w1 = 0.03^{+0.26}_{-0.75}$.

2. LCDM

From WMAP: Ωm0=0.265
arXiv:astro-ph/0611572, Riess et al   :
arXiv:1205.4688 : Ωm0=0.247 (+0.013, -0.013) and Transition 0.426 (+0.082, -0.050)

3. Equation of state

From arXiv:1202.0545v1
$w = -1.087 \pm 0.096$.