

# Title

MA Lei

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## 1 Objectives

For LCDM, interacting models, and CPL, calculate

- $\xi$  range for varying EoS while fixing  $\Omega m_0$
- $\xi$  range for varying  $\Omega m_0$  or  $r$ , while fixing  $\omega$
- Does  $\xi > 0$  means energy transfer to dark energy in this method?

## 2 Background

Deceleration parameter reads

$$q(z) = -1 + \frac{1+z}{H} \frac{dH}{dz} \quad (1)$$

For interaction models, the Friedmann equations,

$$\dot{\rho}_c + 3H\rho_c = Q_c \quad (2a)$$

$$\dot{\rho}_d + 3H(1+w)\rho_d = -Q_c \quad (2b)$$

$Q_c = \xi H \rho_c$  Background equations,

$$\Omega m = \Omega m_0 (1+z)^{3-\xi} \quad (3a)$$

$$\Omega d = (\Omega d_0 + \frac{\xi}{3w+\xi} \Omega m_0) (1+z)^{3(1+w)} + \frac{-\xi}{\xi+3w} \Omega m = \Omega \bar{d}_0 (1+z)^3 + \frac{-\xi}{\xi+3w} \Omega m \quad (3b)$$

$Q_c = \xi H \rho_d$

$$\Omega m = (\Omega m_0 + \frac{\xi}{\xi+3w} \Omega d_0) (1+z)^3 + \frac{-\xi}{\xi+3w} \Omega d = \omega \bar{m}_0 (1+z)^3 + \frac{-\xi}{\xi+3w} \Omega d \quad (4a)$$

$$\Omega d = \Omega d_0 (1+z)^{3(1+w)+\xi} \quad (4b)$$

Eqn 3 and eqn 4 shows that the coupling constant has two effects,

1. Change the amplitude of the evolution of matter or dark energy energy density.
2. Transfer energy between DE and DM.

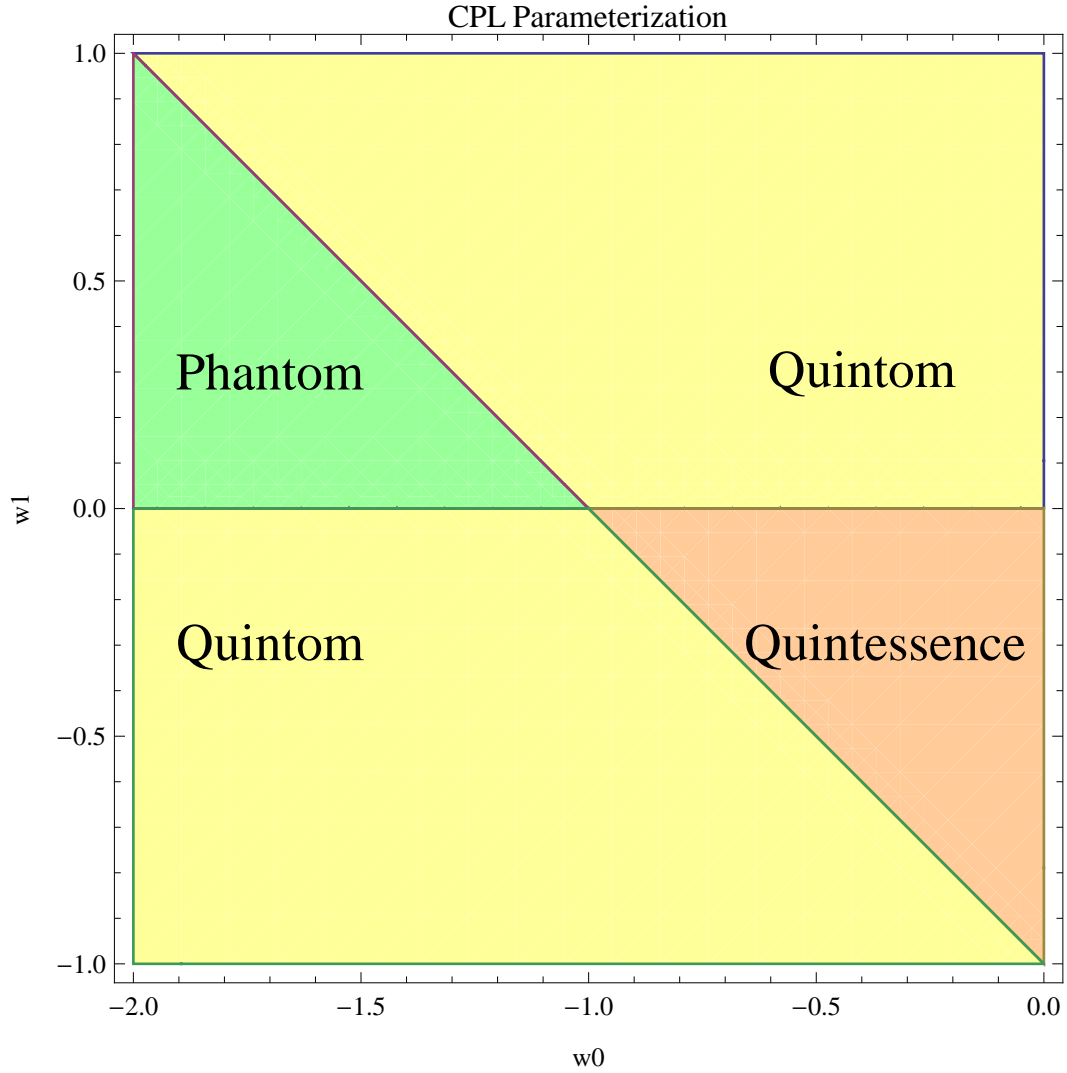
## 2.1 Some definitions

1. For short

$$r = \frac{\Omega_{m0}}{\Omega_{d0}}$$

**CPL** EoS is

$$w = w_0 + w_1 \frac{z}{1+z}$$



**Figure 1:** CPL classification

**Classification** Figure 1 shows how to category the dark energy models in CPL parametrization.

### 3 Data & Method

#### 3.1 Data

**LCDM Parameters** From WMAP,  $\Omega_m0 = 0.265$

**Constraints**  $\Omega_m0 = 0.247(+0.013, -0.013)$ ; Transition redshift  $0.426(+0.082, -0.050)$ .(arXiv:1205.4688, arXiv:astro-ph/0611572).

In  $(\Omega_m0, \text{Transition redshift})$  plane, allowed region is a rectangle centred at  $(0.274, 0.426)$  with two diagonal points  $(0.261, 0.376)$  and  $(0.287, 0.508)$ .

**CPL**  $\Omega_m0 = 0.269(+0.017, -0.008)$ ,  $w0 = -0.97(+0.12, -0.07)$ ,  $w1 = 0.03(+0.26, -0.75)$

### 4 Results

Check the files in files folder.

#### 4.1 $Q_c = \xi H \rho_c$

Results table

$Q_c = \xi H \rho_c$ , constant $\xi$ , constant $w = -1$ : Results for $\xi$			
$\Omega_m0/\Omega_d0 \backslash \text{Transition}$	$z_t = 0.376$	$z_t = 0.426$	$z_t = 0.508$
$r = 0.358$	-1.25282	-0.965436	-0.617444
$r = 0.378$	-1.15011	-0.875189	-0.542347
$r = 0.398$	-1.05453	-0.791252	-0.472561

**Figure 2:** ICC Result table

Figure 3 shows that

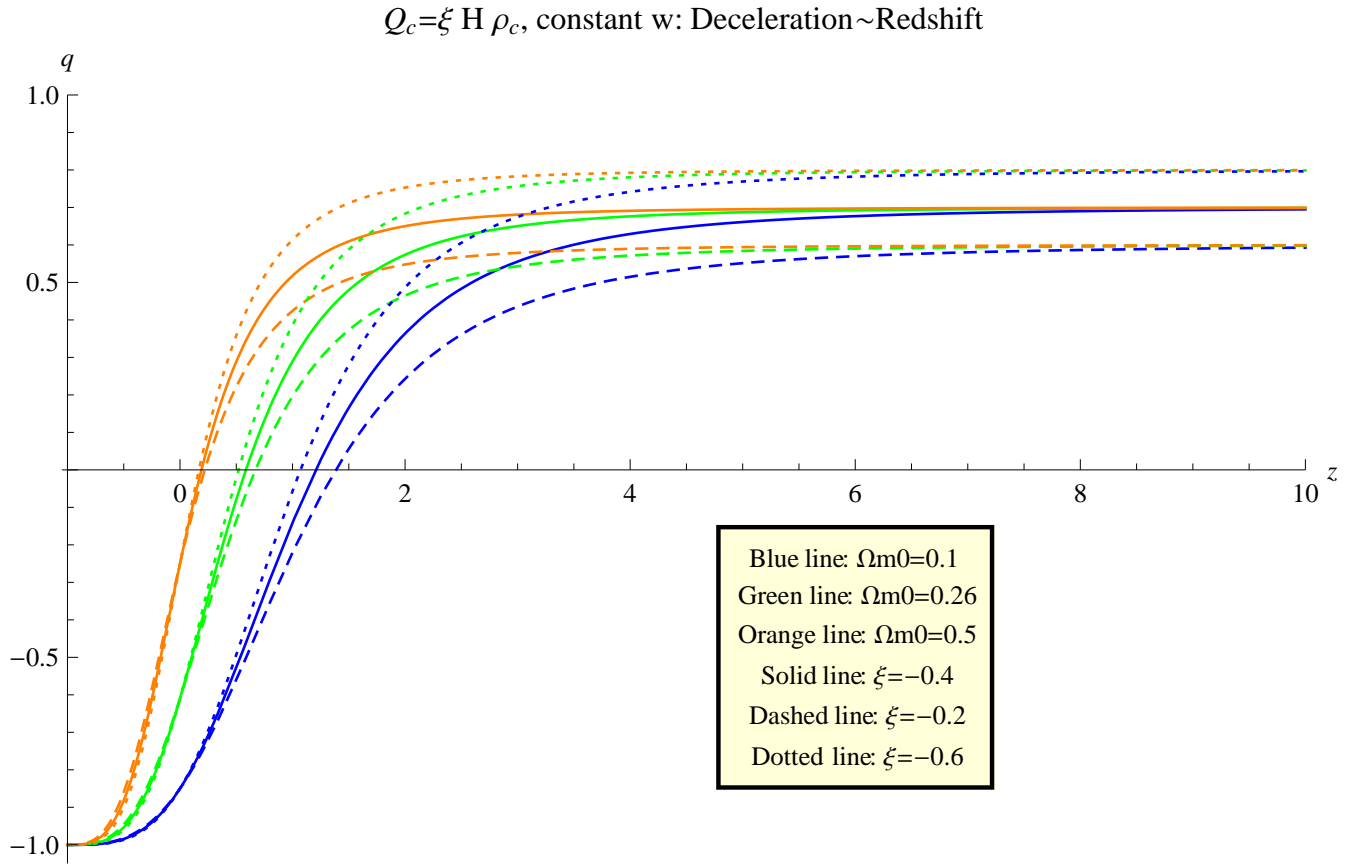
$\rho_c$ -Dec-1 The universe decelerates faster at the early stage for smaller interaction constant  $\xi$  even they have the same matter fraction.

$\rho_c$ -Dec-2 For the same  $\xi$ , the deceleration converge ( $q = (1 - \xi)/2$  with  $3w + \xi < 0$ ) at early time.

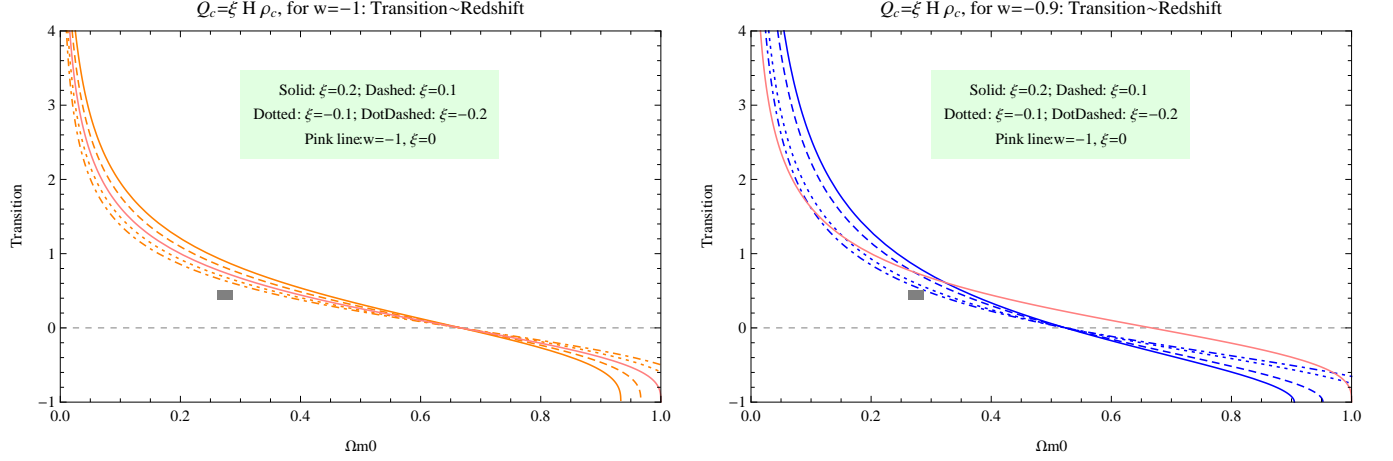
Figure 4 shows

$\rho_c$ -Trans-1 Transition happens earlier when matter fraction is smaller. Matter is against DE's pressure.

$\rho_c$ -Trans-2 Transition is later when  $\xi$  is smaller. Energy transfers to DE when  $\xi$  is negative, then why later transition? <sup>1</sup>



**Figure 3:** Deceleration parameter



**Figure 4:** Transition redshift.

Figure 5 and figure 6 (vertical lines are  $w = -1 \pm 0.05$ ) show the value of  $\xi$  for different EoS. Results from this figure:

(EoS is -1 within  $5w=-1$  (-1.279,-0.457) center:-0.878 ;  $w=-1.05$  (-1.293,-0.461) center:-0.887 ;  $w=-0.95$  (-1.255,-0.447) center:-0.860 ;

$\rho_c\text{-xiVS}w\text{-1}$  NOT monotonic. Thus it is not clear what is the importance of the value of  $w$ . The result of  $\xi$  is greatly affected by  $\Omega m0$ . In correspondence with another

$\rho_c\text{-xiVS}w\text{-2}$  For different  $\Omega m0$ ,  $\xi$  values deviate greatly from each other at small  $w$ .

Figures 7 show how do we constrain  $\xi$  and how do EoS change our constrain results with the transition fixed.

$\rho_c\text{-xiVS}\Omega m0\text{-1}$  The smaller, the more difference among  $\xi$  values of different EoS. Reason for this is less matter has less effect on the evolution thus the property of dark energy determines more about the transition.

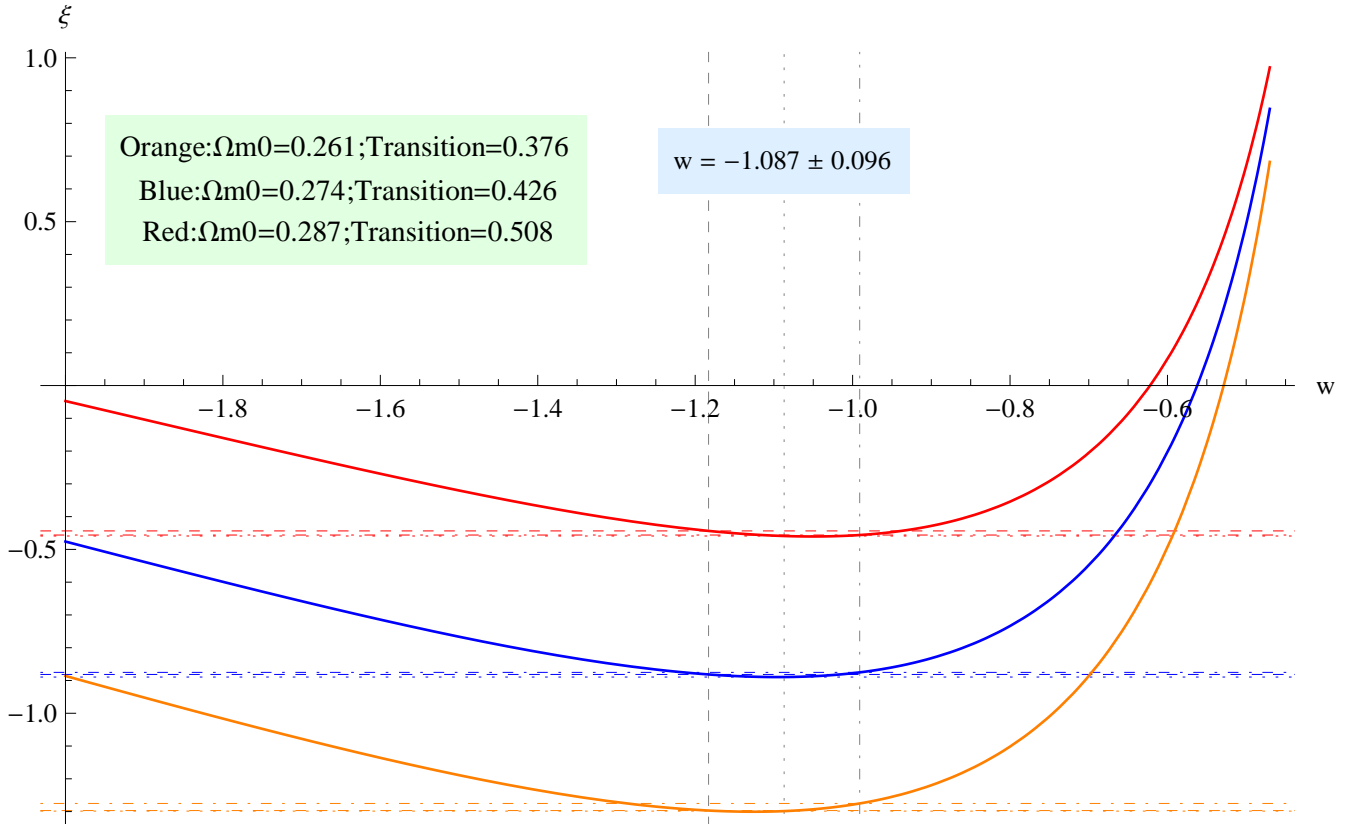
$\rho_c\text{-xiVS}\Omega m0\text{-2}$  Second figure shows

- + System with smaller  $w$  needs smaller coupling to achieve the same transition time, as expected.
- + So we give the result that  $w \in (-0.58406, -0.3334)$  if we constrain  $\Omega m0 = 0.2603$  and transition redshift 0.426.

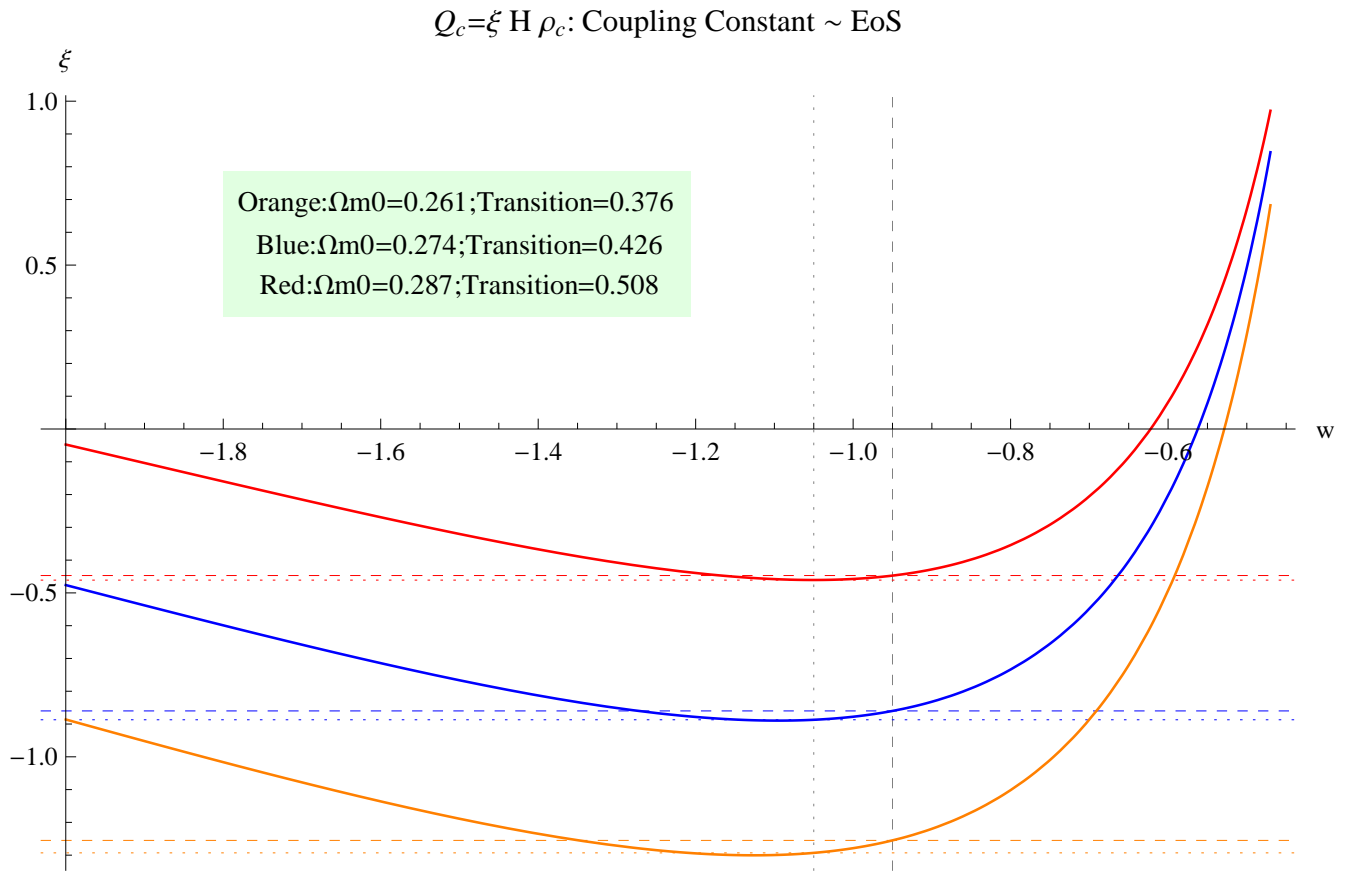
<sup>1</sup>Reasons below. All solutions of equation 2 have the same value at  $z = 0$ , i.e. now. Equation 2a tells us a positive  $\xi$  leads to smaller energy density of dark matter at early time of the universe, thus dark energy takes over quickly if the transition happens before today. (For more details, calculation are shown in supplement\_08-10.pdf file.)

$\xi$ results for $Q_c = \xi H \rho_c$ (Fitting data: Data From, 2)			
w	Center	Lower	Upper
-1.183	-0.881565	-1.29687	-0.443589
-1.087	-0.88948	-1.29859	-0.459135
-0.991	-0.875238	-1.27522	-0.456176

$Q_c = \xi H \rho_c$ , constant w: Coupling Constant  $\sim$  EoS



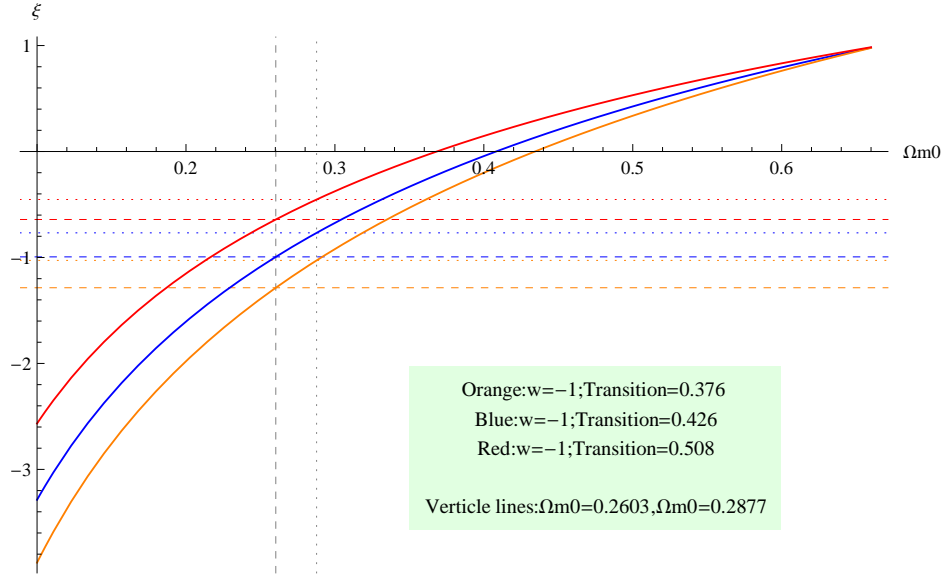
**Figure 5:** Interacting coefficient for  $Q_c = \xi H \rho_c$



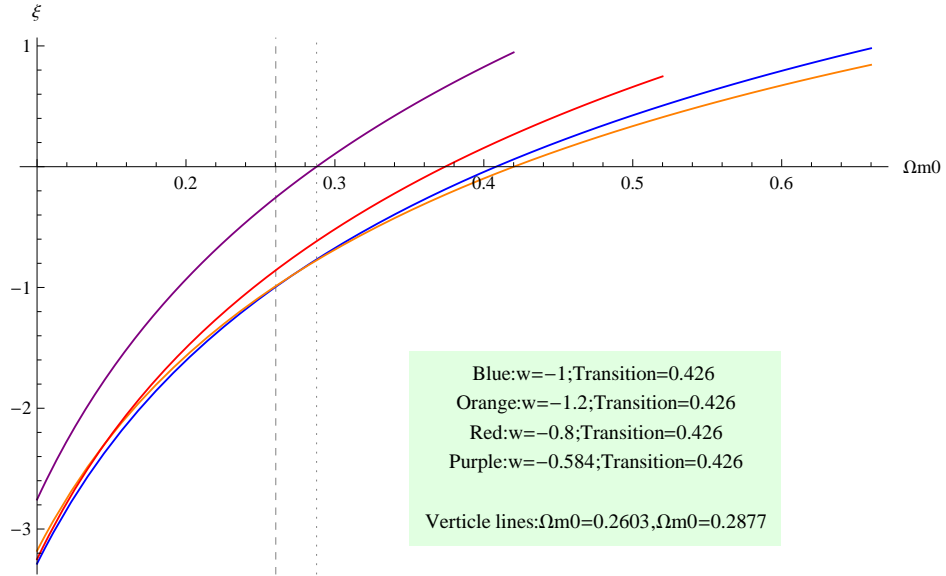
**Figure 6:** Interacting coefficient for  $Q_c = \xi H \rho_c$

For $\Omega_{m0} \in 0.274 (1 \pm 0.05)$			
Table of $\xi$ for different $\Omega_{m0}$ -Transition combination			
$\Omega_{m0}$ -Transition	0.426	0.376	0.508
0.2603	-0.994339	-1.28571	-0.641508
0.274	-0.877755	-1.15303	-0.544482
0.2877	-0.767582	-1.02756	-0.452892

$Q_c = \xi H \rho_c$ , constant  $w$ : Coupling Constant  $\sim \Omega_{m0}$



$Q_c = \xi H \rho_c$ , constant  $w$ : Coupling Constant  $\sim \Omega_{m0}$



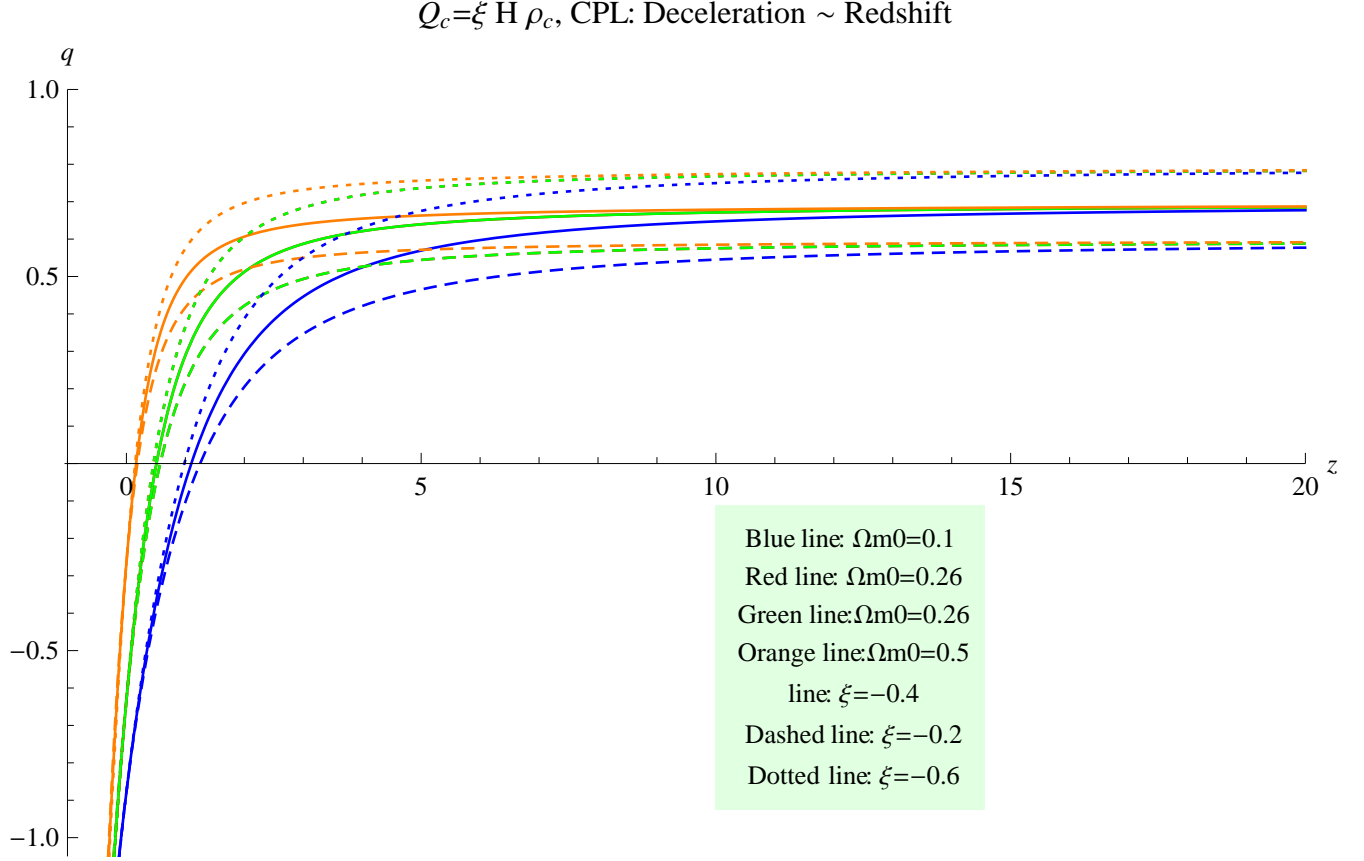
**Figure 7:** Interacting coefficient changing with  $\Omega_{m0}$  for  $Q_c = \xi H \rho_c$



## 4.2 $Q_c = \xi H \rho_c$ , CPL

For a flat universe, choose the parameters  $w_0=-1.02, w_1=0.6$ , the region for interaction constant  $\xi$  should be  $(-1.04, -0.21)$  with a center at  $-0.64$ , derived from the (transition redshift,  $\Omega_{m0}$ ) plane, while a result of  $(-1.01, -0.23)$  with a center at  $-0.63$ , derived from (transition redshift,  $\Omega_{m0}/$

First let's have a look at the deceleration parameter.



**Figure 8:** Deceleration parameters for ICCPL ( $Q_c = \xi H \rho_c$  with CPL parametrized EoS)

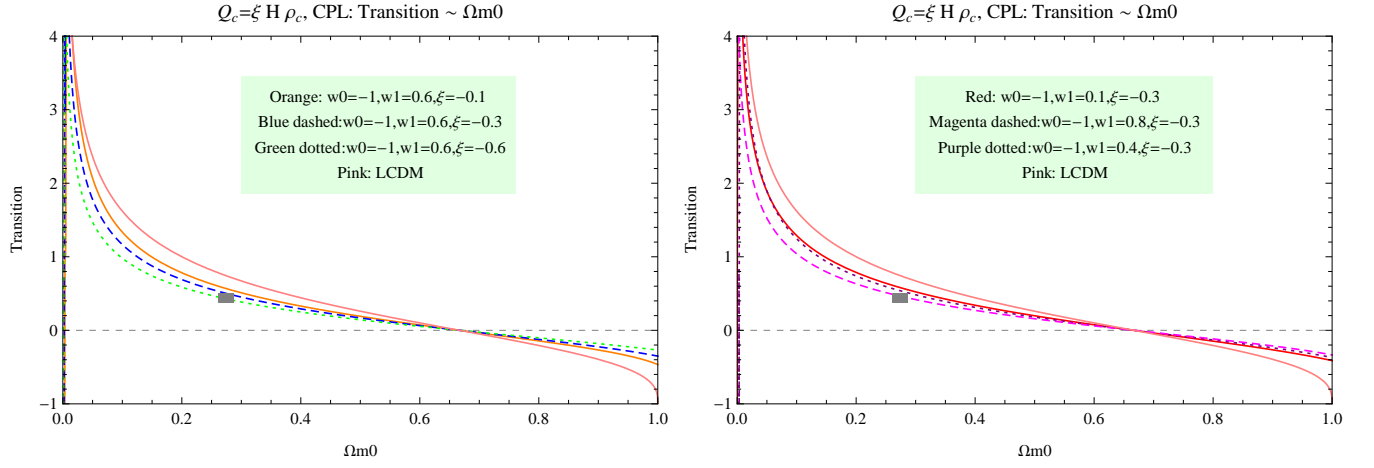
Figure 8 is right similar to constant EoS situation.

Figures 9 show

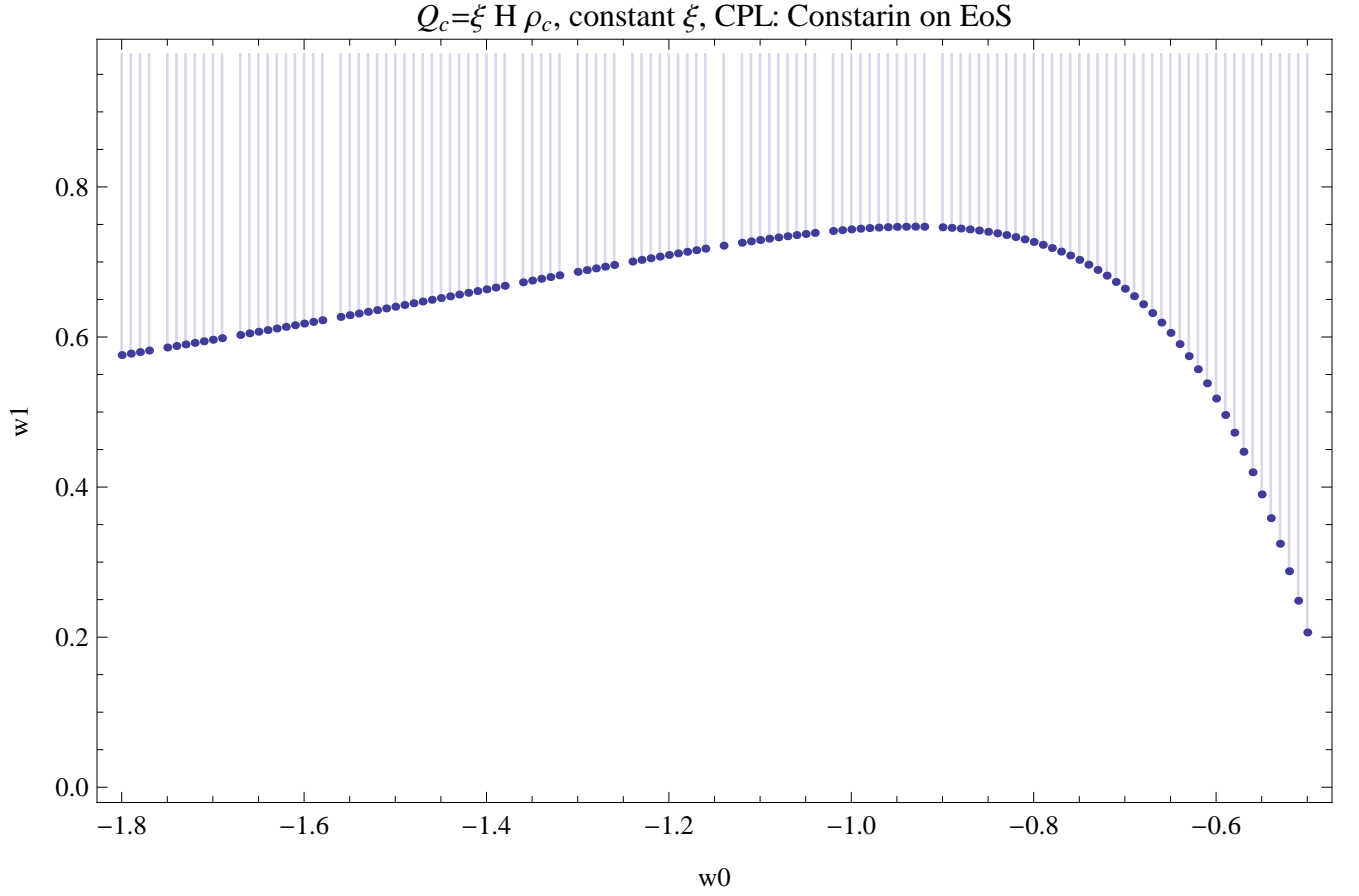
$\rho_{c\text{-ICCPL-TV}\Omega_{m0}=1}$  Coupling acts on these model similar to constant EoS model.

$\rho_{c\text{-ICCPL-TV}\Omega_{m0}=2}$  The change in  $w_1$  has a similar effect with the change of  $\xi$ . Larger  $w_1$  corresponds to smaller  $\xi$ . The reason for this is both negative  $\xi$  and larger  $w_1$  enhances the energy density of dark energy (check using the CPL EoS).

The dots in figure 10 are the data set of  $\xi = 0$ . If we need  $\xi < 0$ , i.e., energy transfers from dark matter to dark energy, the allowed parameter space is the striped area.



**Figure 9:** The effect of EoS parameters on Transition and  $\Omega m_0$



**Figure 10:** Lower bound in the W1  $w_0$  parameter space

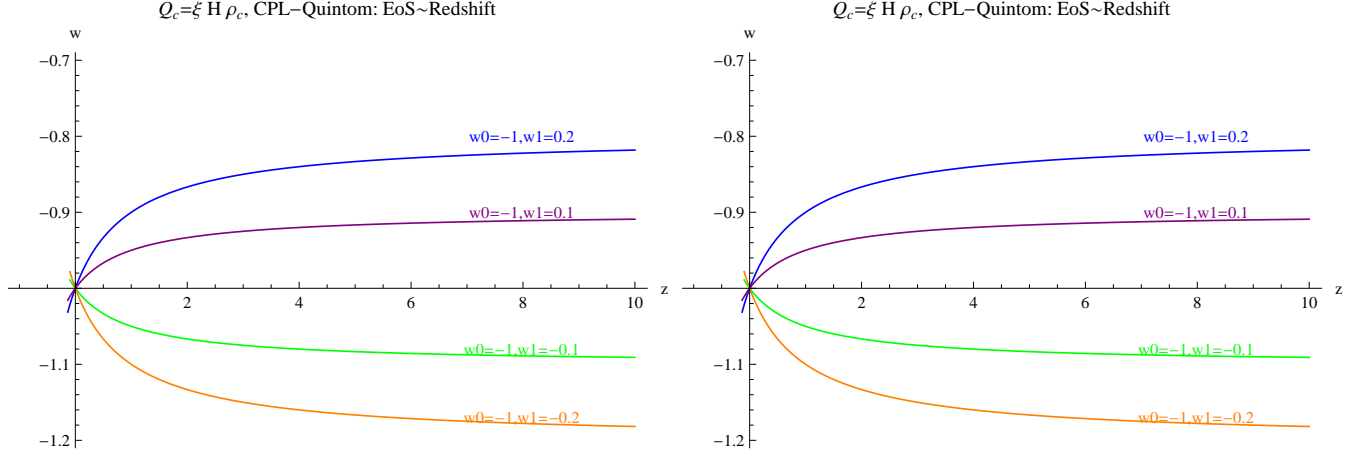


Figure 11: The EoS

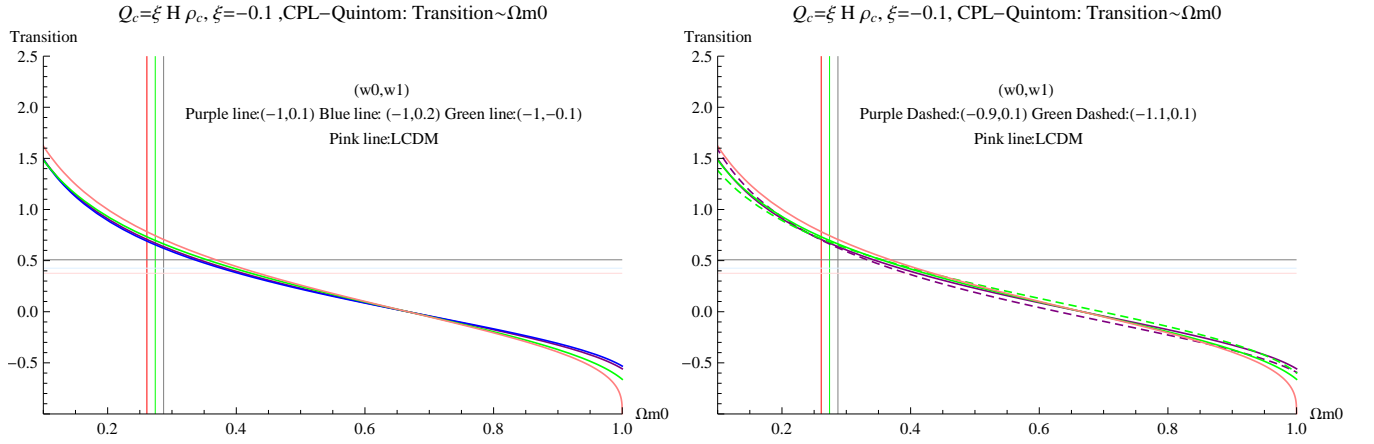
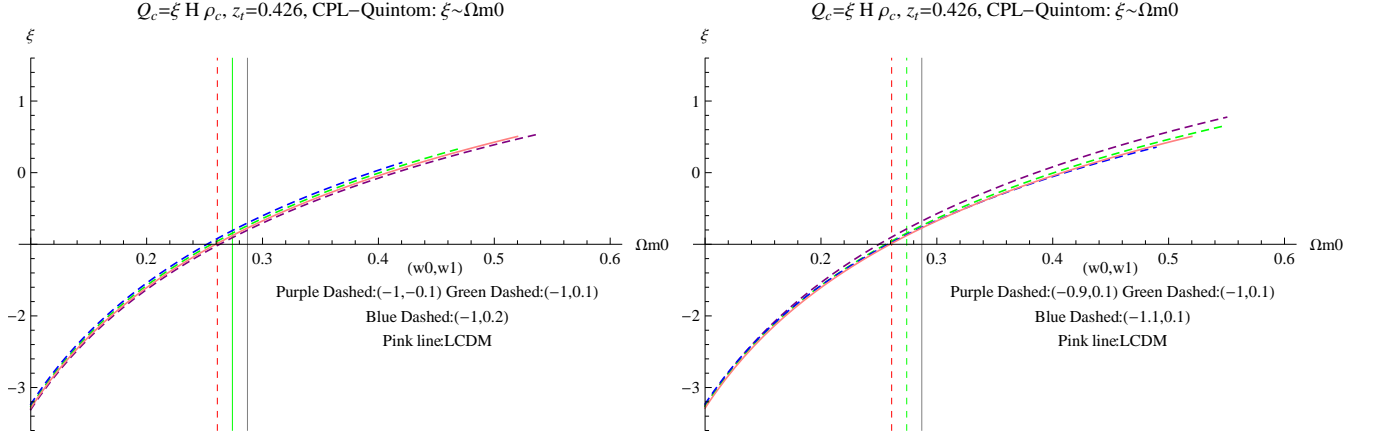


Figure 12: Transition vs  $\Omega_{m0}$



**Figure 13:**  $\xi$  vs  $\Omega m0$

#### 4.2.1 Quintom

(Figures 11, 12, 13)

The left figure in 12 indicates a possible stationary point.<sup>2</sup>  
(Other results are shown on the complete results files.)

#### 4.2.2 Quintessence

(Figures 14, 15 and 16. )

#### 4.2.3 Phantom

(Figures 17, 18, 19, ??)

### 4.3 $Q_c = \xi H \rho_d$

(Figures 20, 21, 22, 23)

## 4.4 I2CCPL

(Figures 25, 26)

Figure 25 shows the all the deceleration are the same at very early time.

#### 4.4.1 Quintom

(Figures 27, 28)

#### 4.4.2 Quintessence

(Figures 29, 30)

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<sup>2</sup>Only possible because I can only partially prove there is a nearly stationary. This is on my *Cosmologia Notebook*.

$Q_c = \xi H \rho_c$ , CPL, Quintessence: EoS ~ Redshift

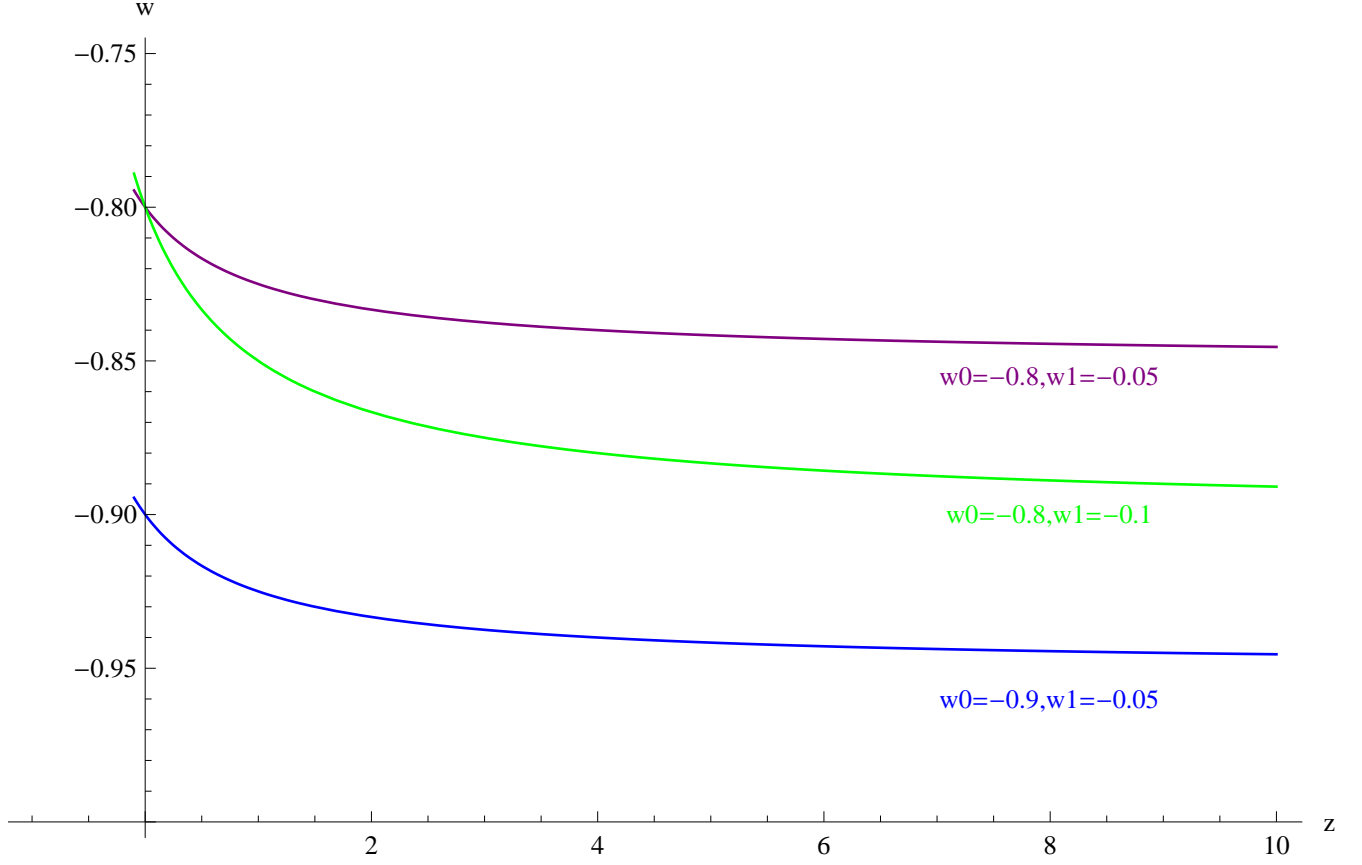
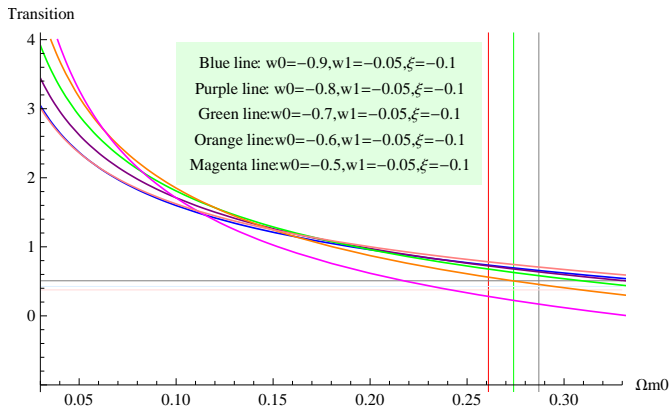


Figure 14: The EoS

$Q_c = \xi H \rho_c$ ,  $\xi = -0.1$ , CPL, Quintessence: Transition Redshift ~  $\Omega m_0$



$Q_c = \xi H \rho_c$ ,  $\xi = -0.1$ , CPL, CPL, Quintessence: Transition Redshift ~  $\Omega m_0$

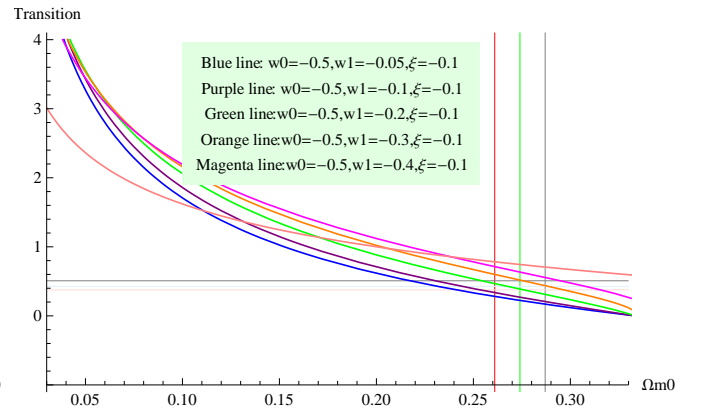
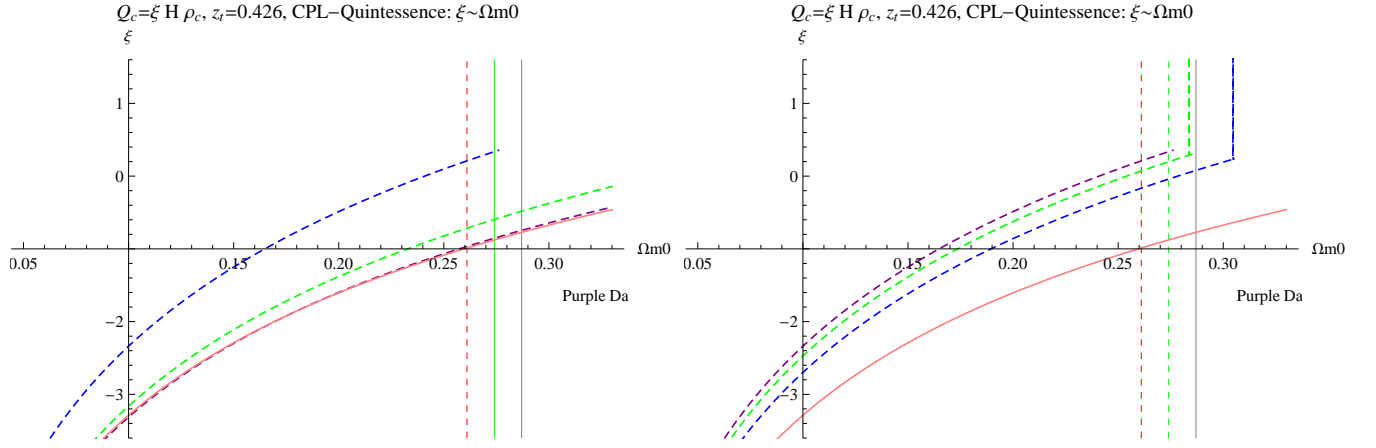
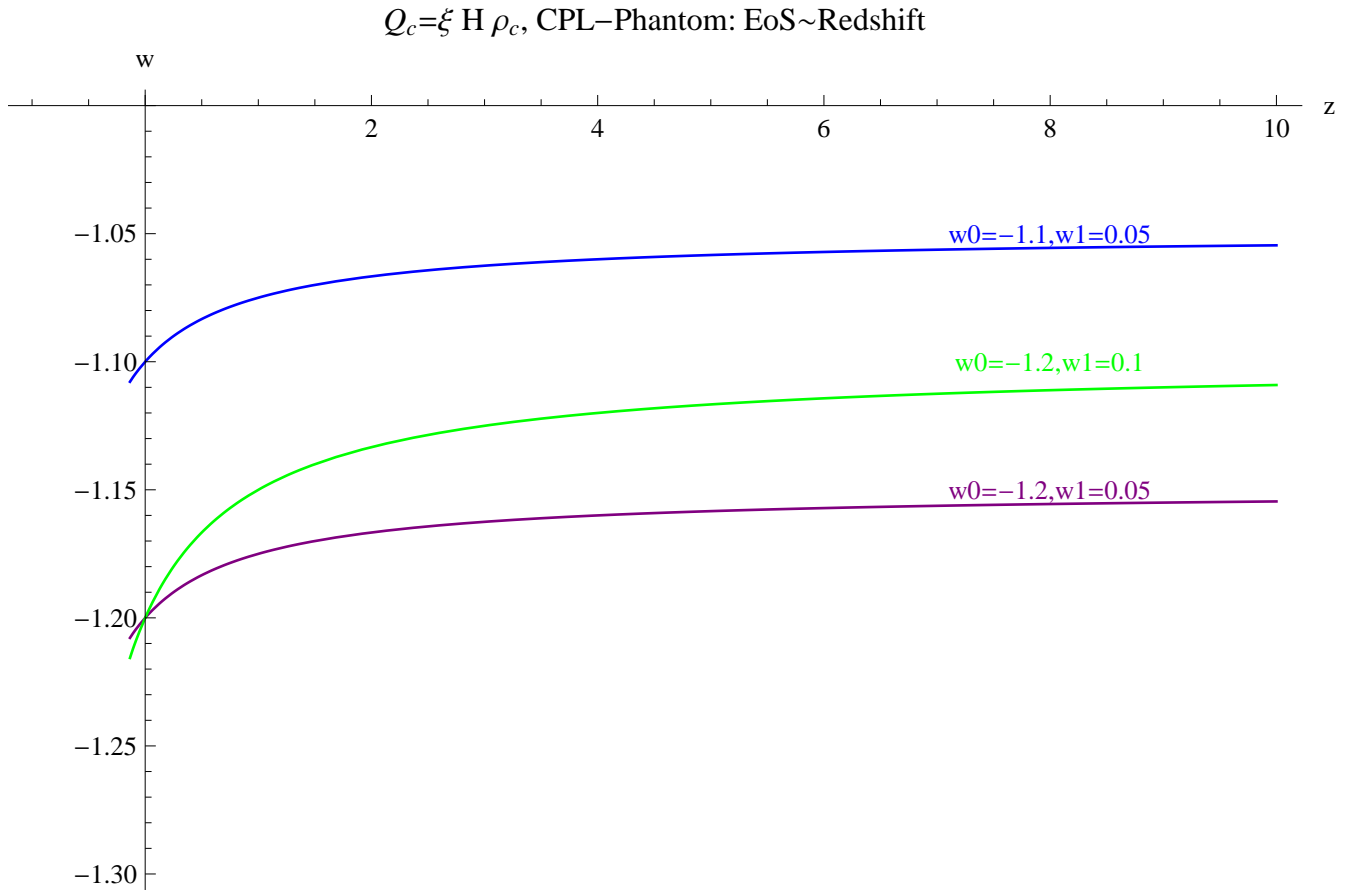


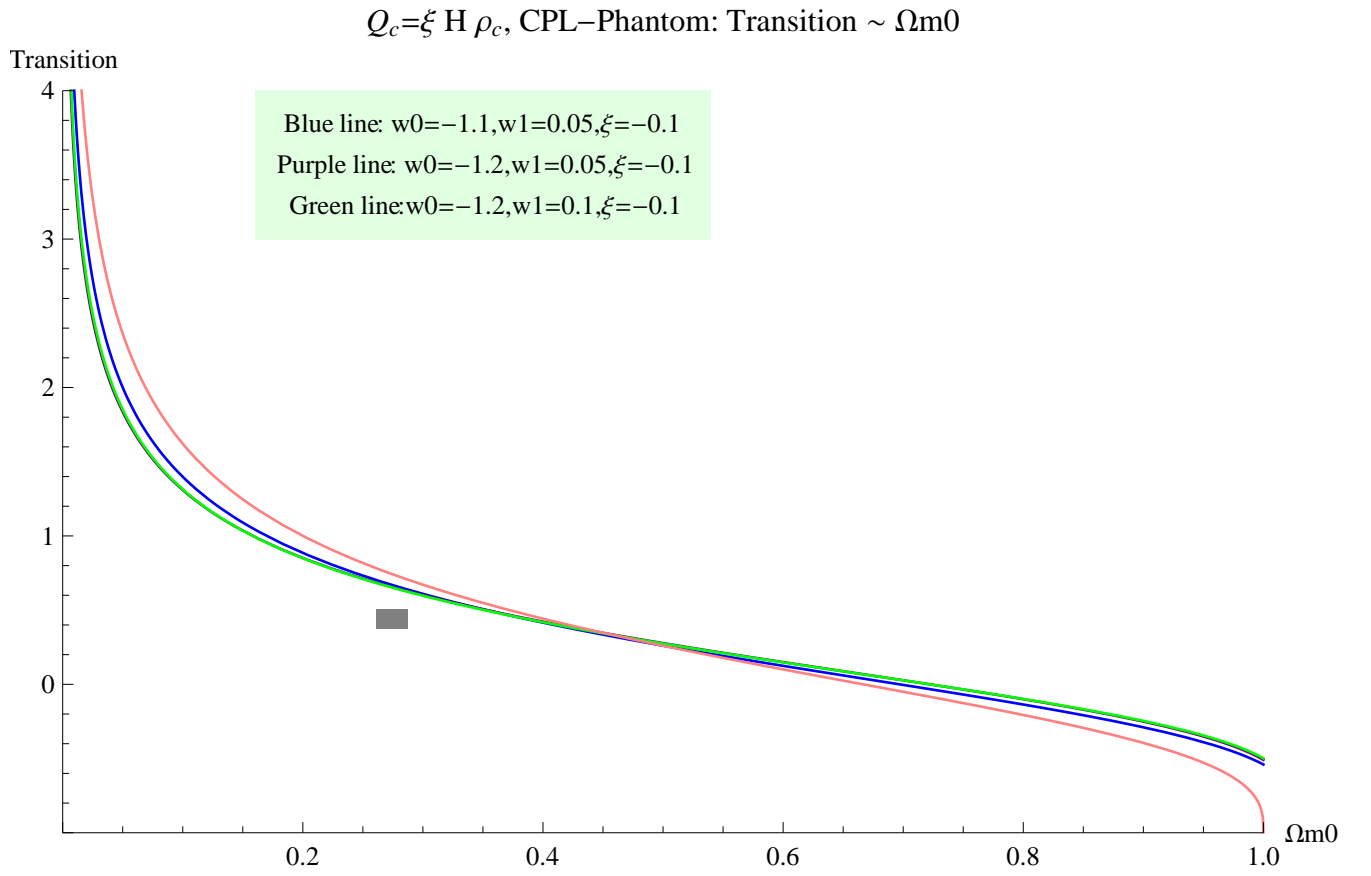
Figure 15: Transition vs  $\Omega m_0$



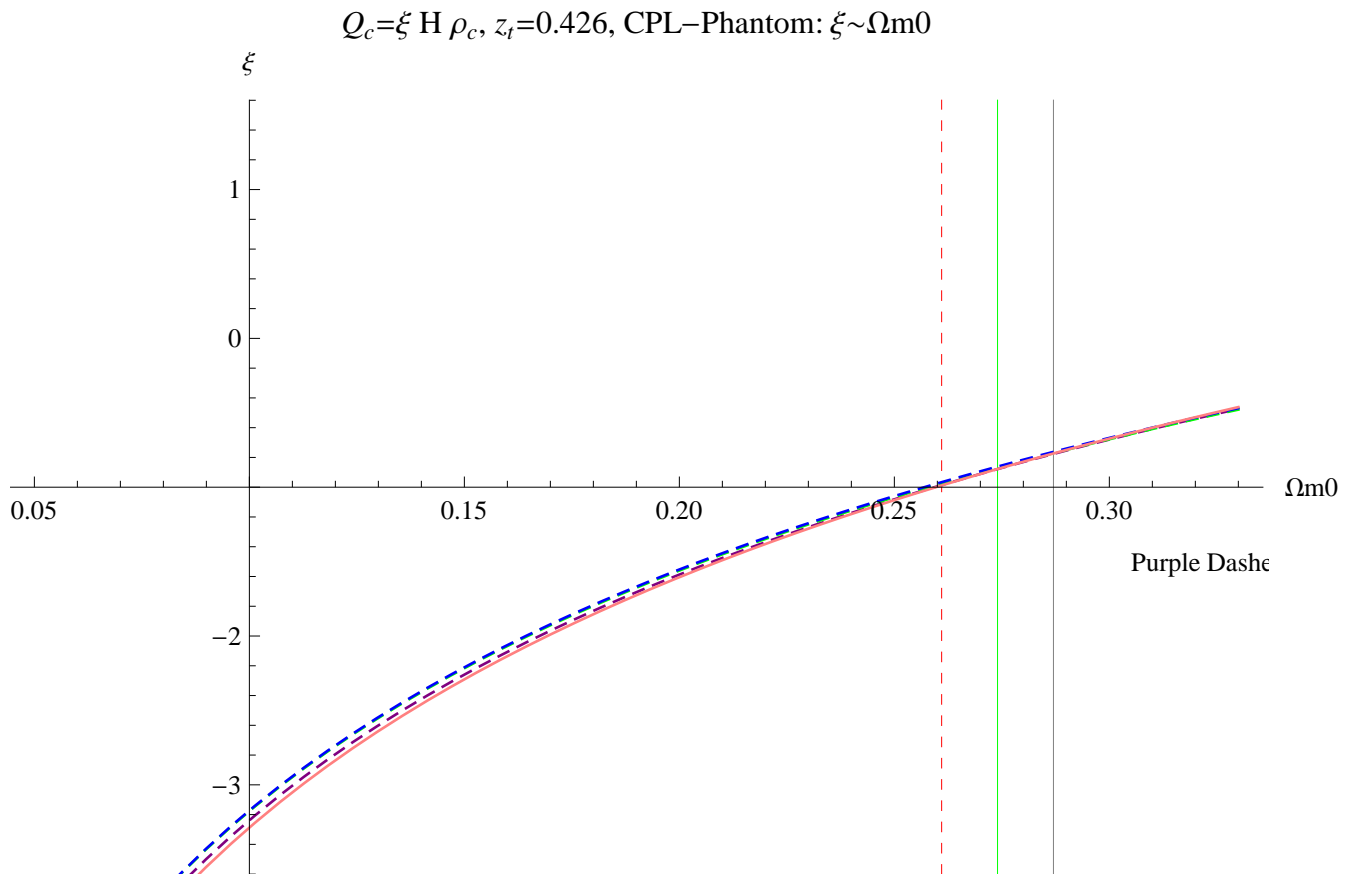
**Figure 16:**  $\xi$  vs  $\Omega_{m0}$



**Figure 17:** The EoS

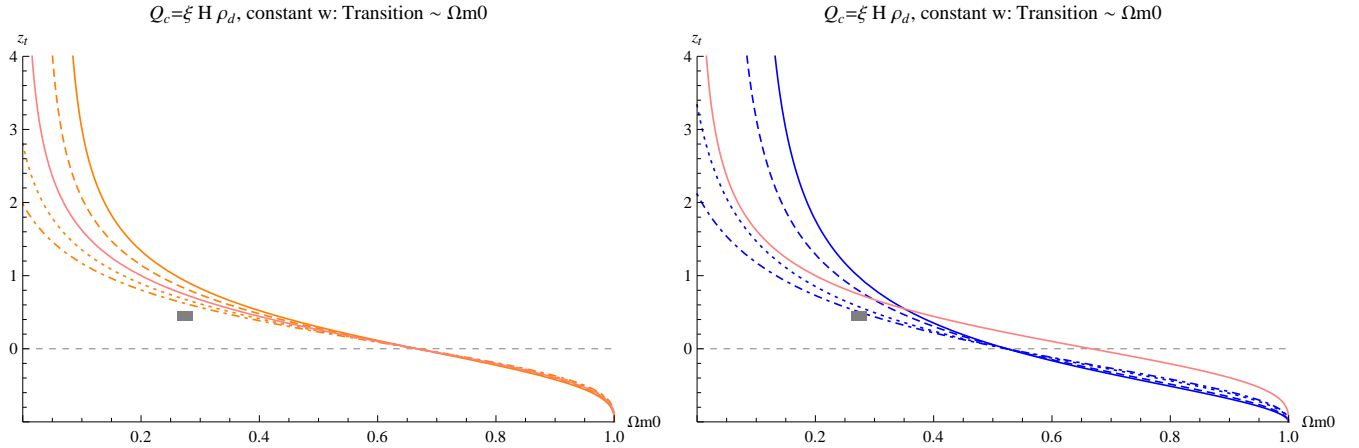
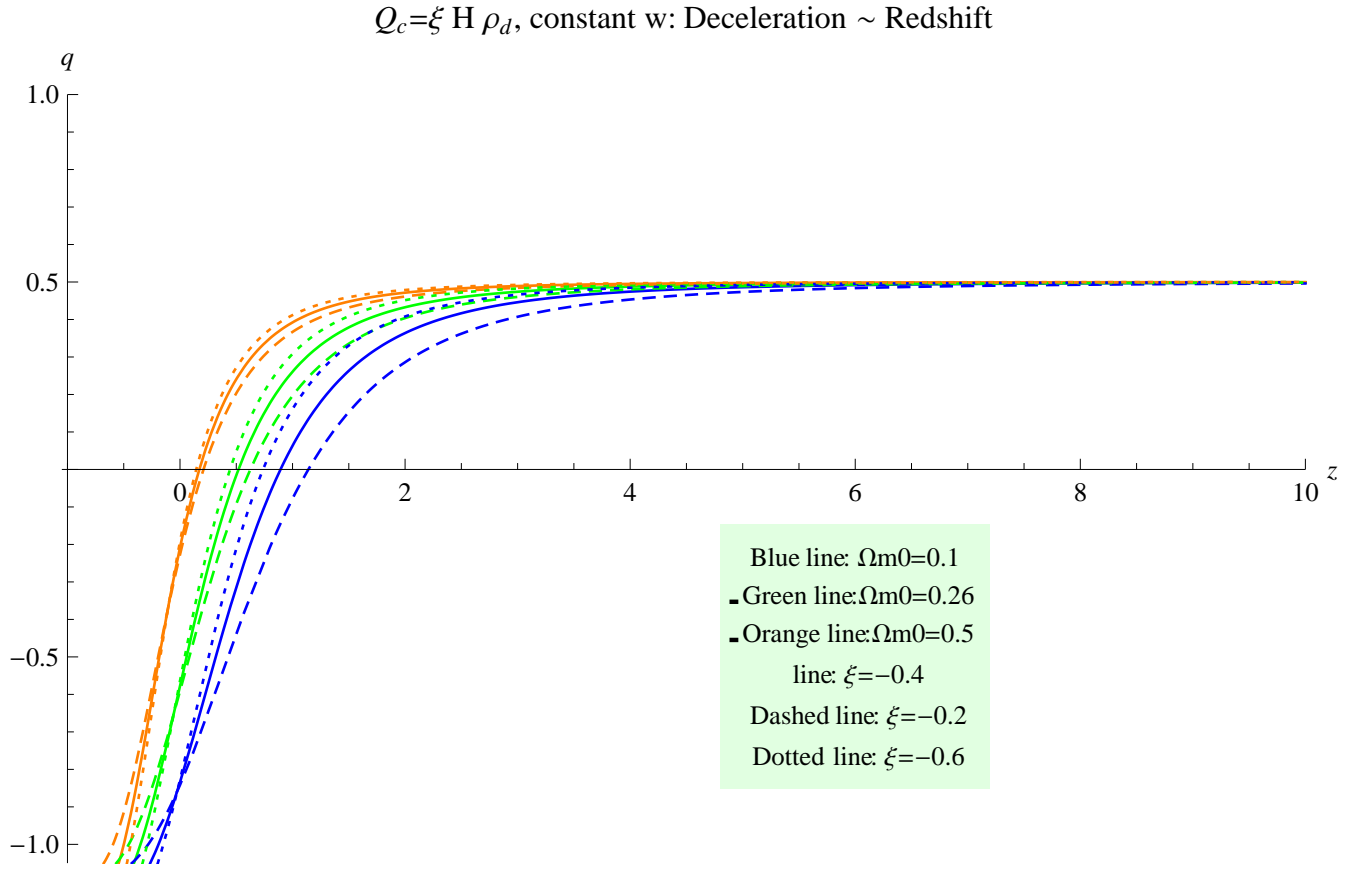


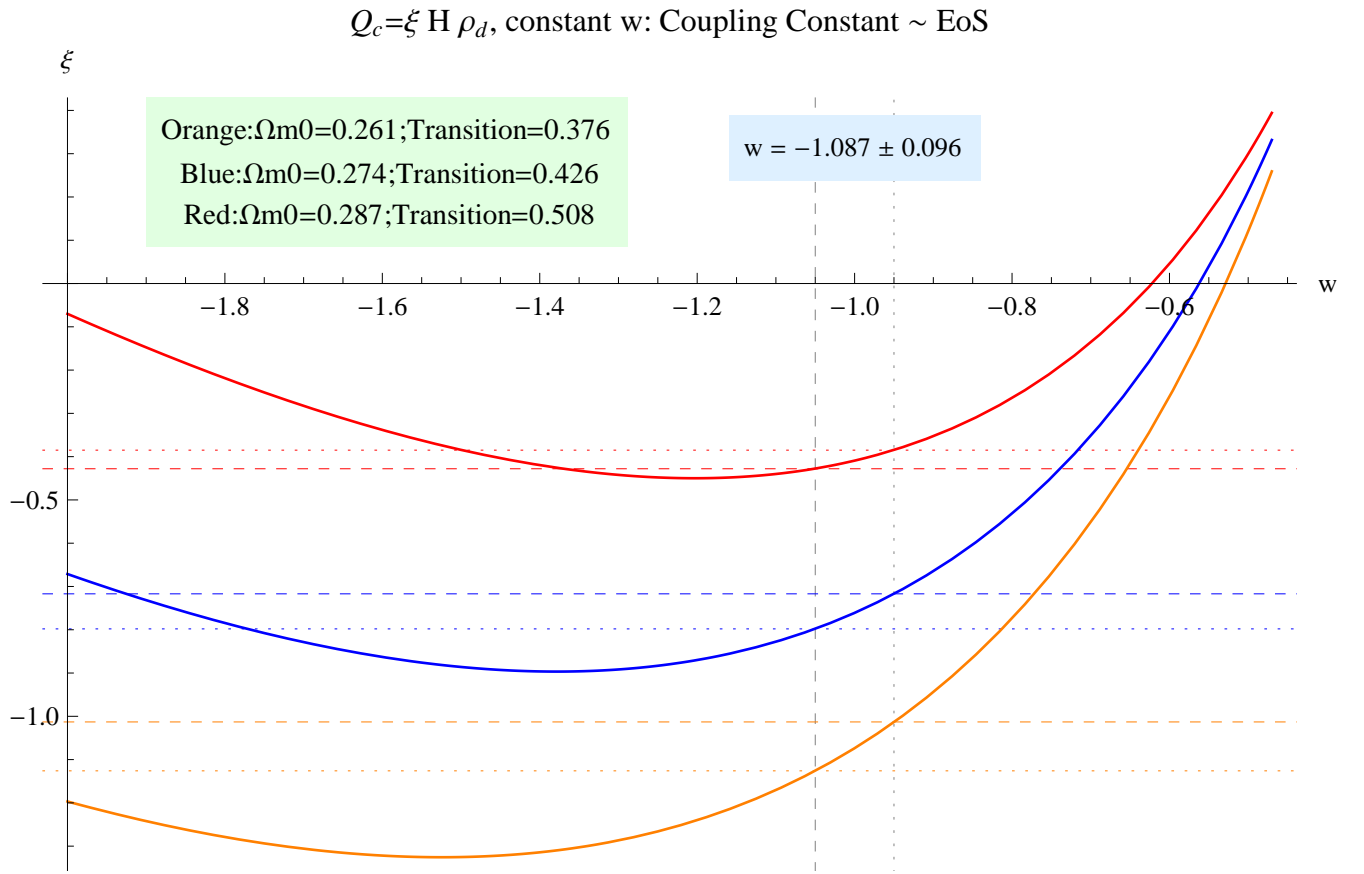
**Figure 18:** Transition vs  $\Omega_{m0}$



**Figure 19:**  $\xi$  vs  $\Omega m_0$



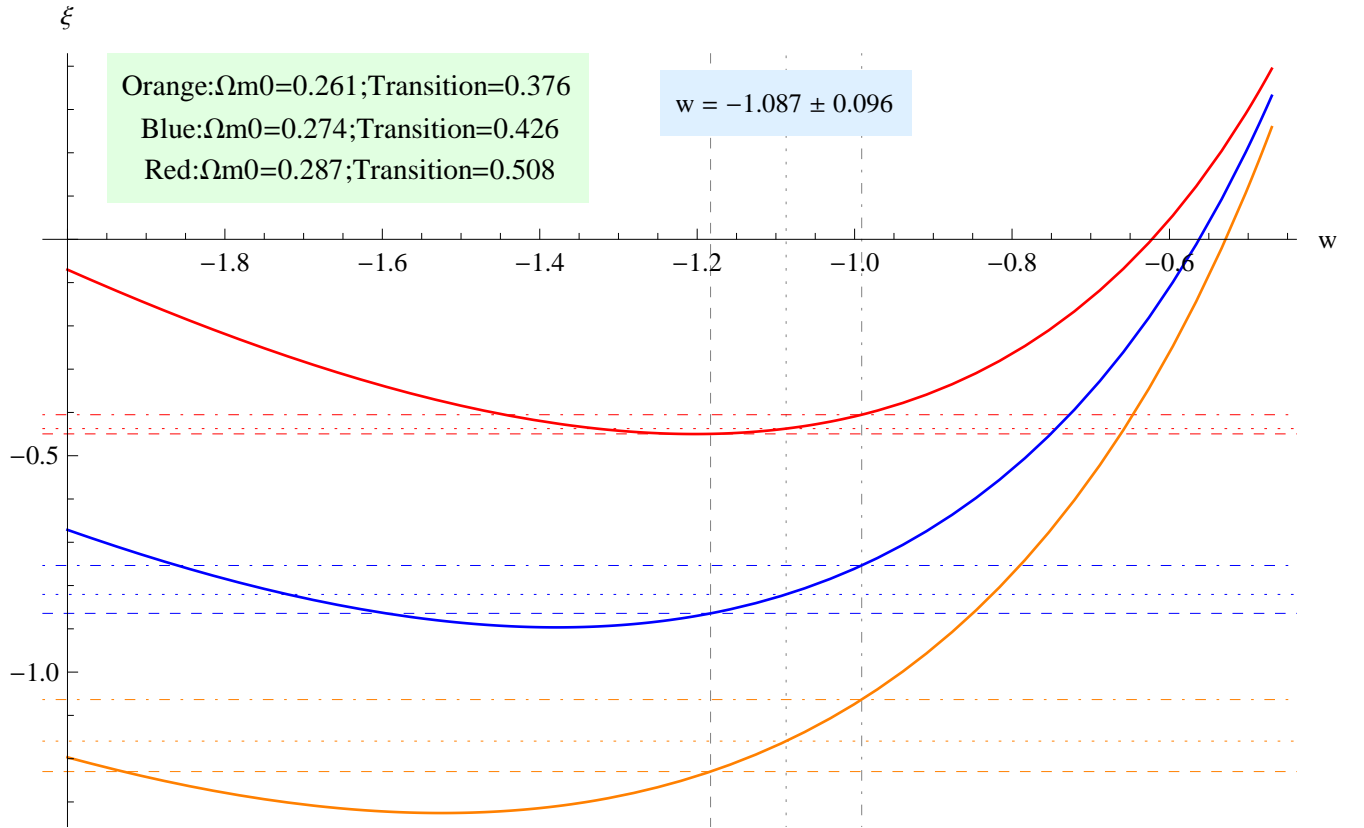




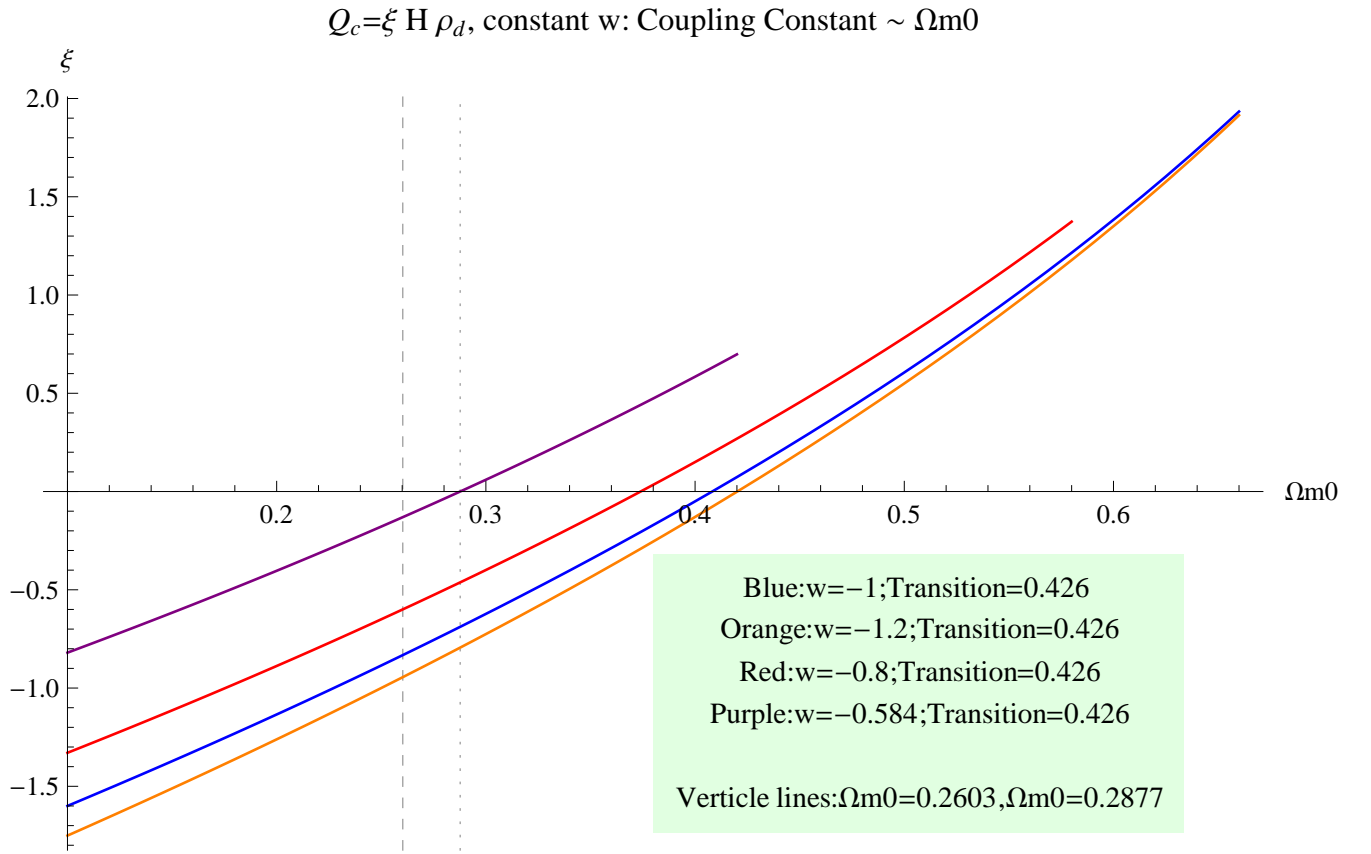
**Figure 22:**  $\xi$  VS  $w$

$Q_c = \xi H \rho_d, \text{Constant } w. \text{ (Data used: Data From, 2)}$			
w	Center	Lower	Upper
-1.183	-0.864289	-1.22984	-0.449552
-1.087	-0.820486	-1.15946	-0.437339
-0.991	-0.753634	-1.06346	-0.405262

$Q_c = \xi H \rho_d, \text{constant } w: \text{Coupling Constant} \sim \text{EoS}$

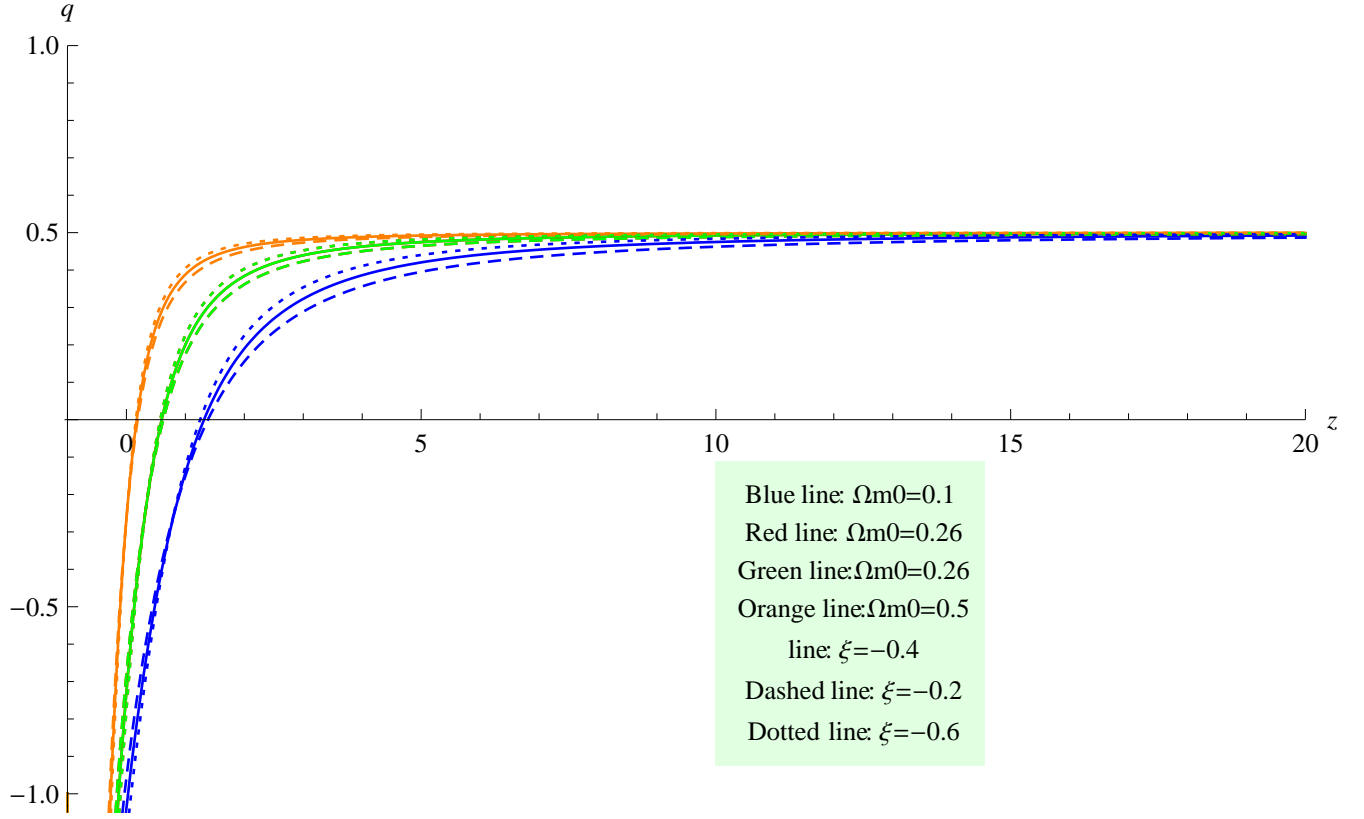


**Figure 23:**  $\xi$  VS  $w$

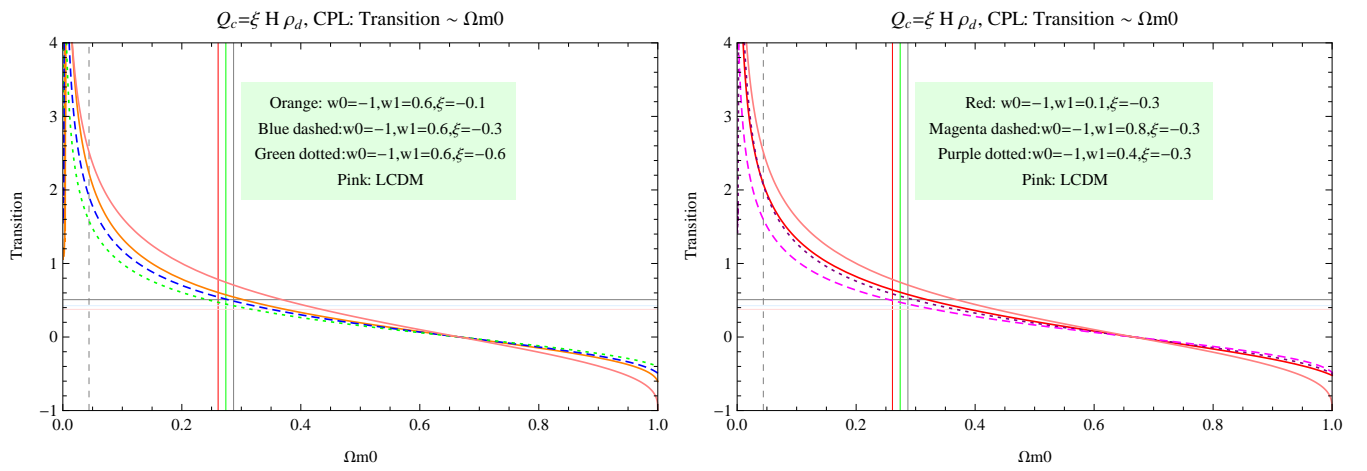


**Figure 24:**  $\xi$  VS  $\Omega_{m0}$

$Q_c = \xi H \rho_d$ , CPL: Deceleration  $\sim$  Redshift



**Figure 25:** Deceleration parameter



**Figure 26:** Transition VS  $\Omega_{m0}$

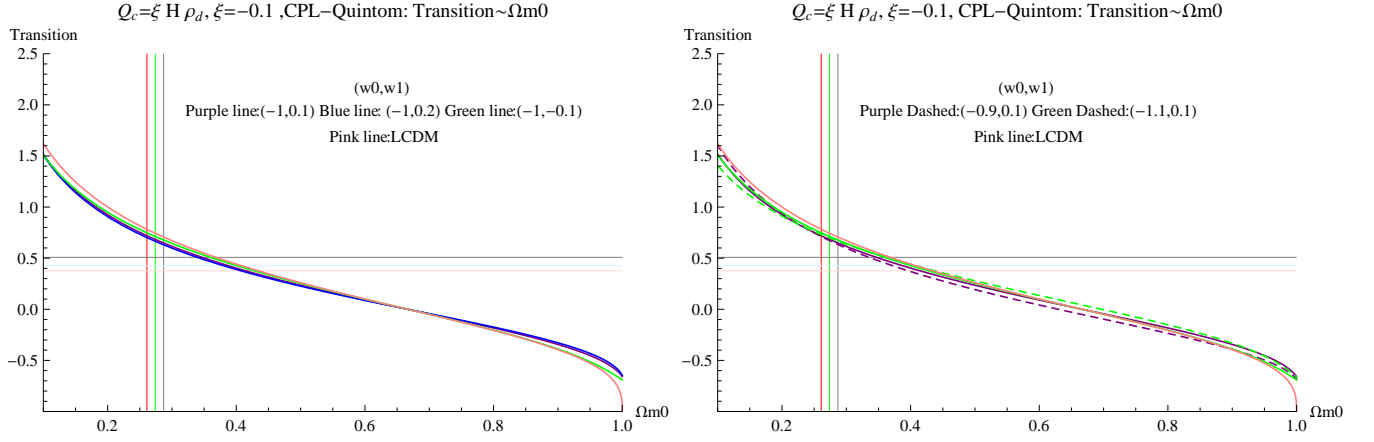


Figure 27: Transition VS  $\Omega_{m0}$

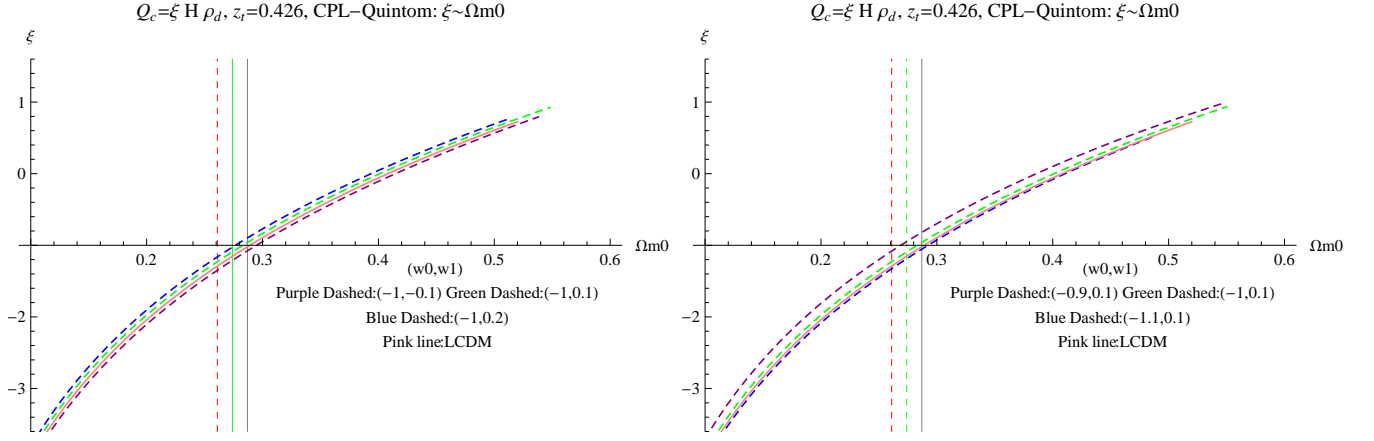


Figure 28:  $\xi$  VS  $\Omega_{m0}$

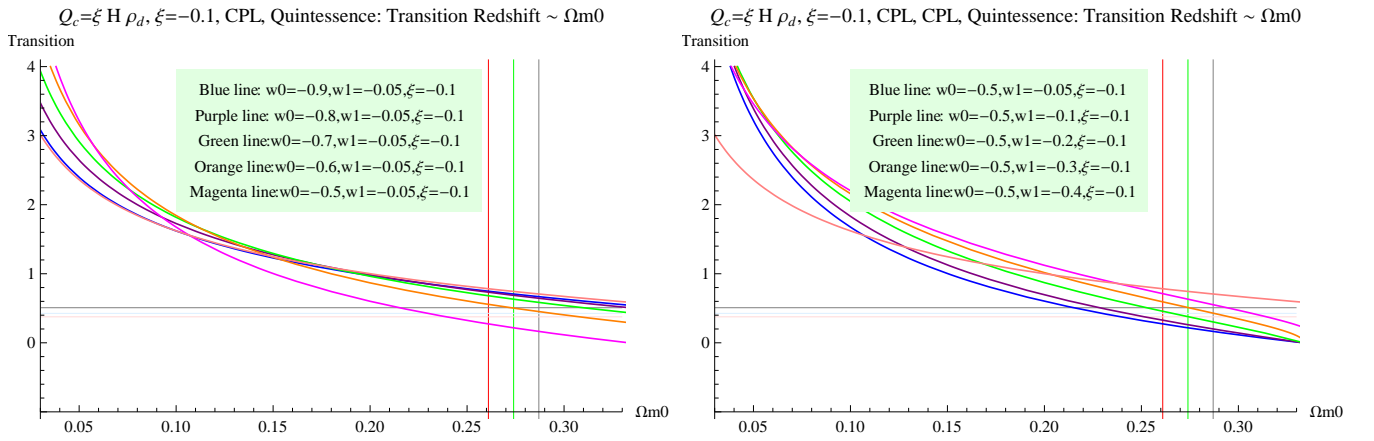


Figure 29: Transition VS  $\Omega_{m0}$

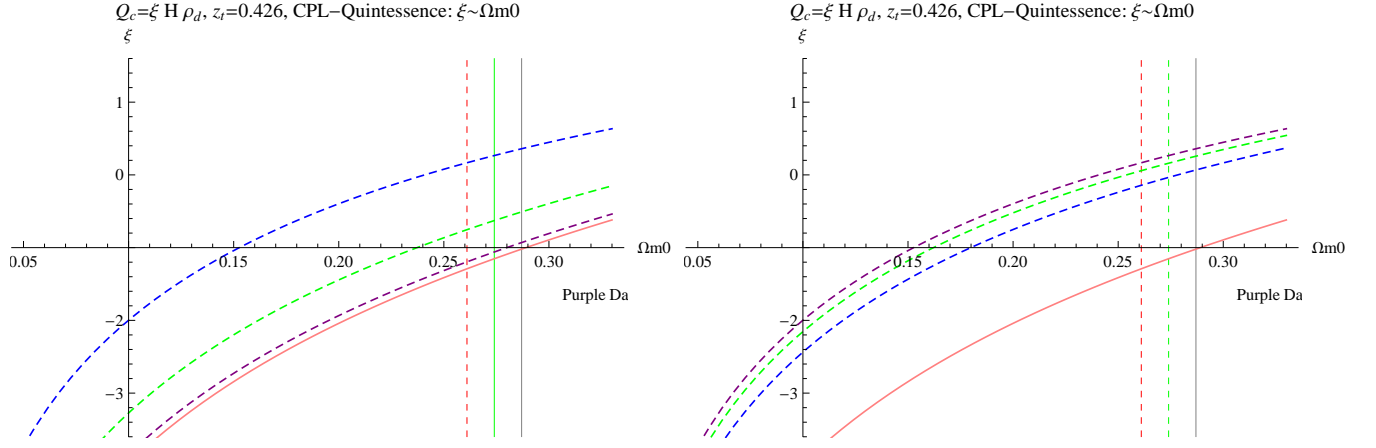
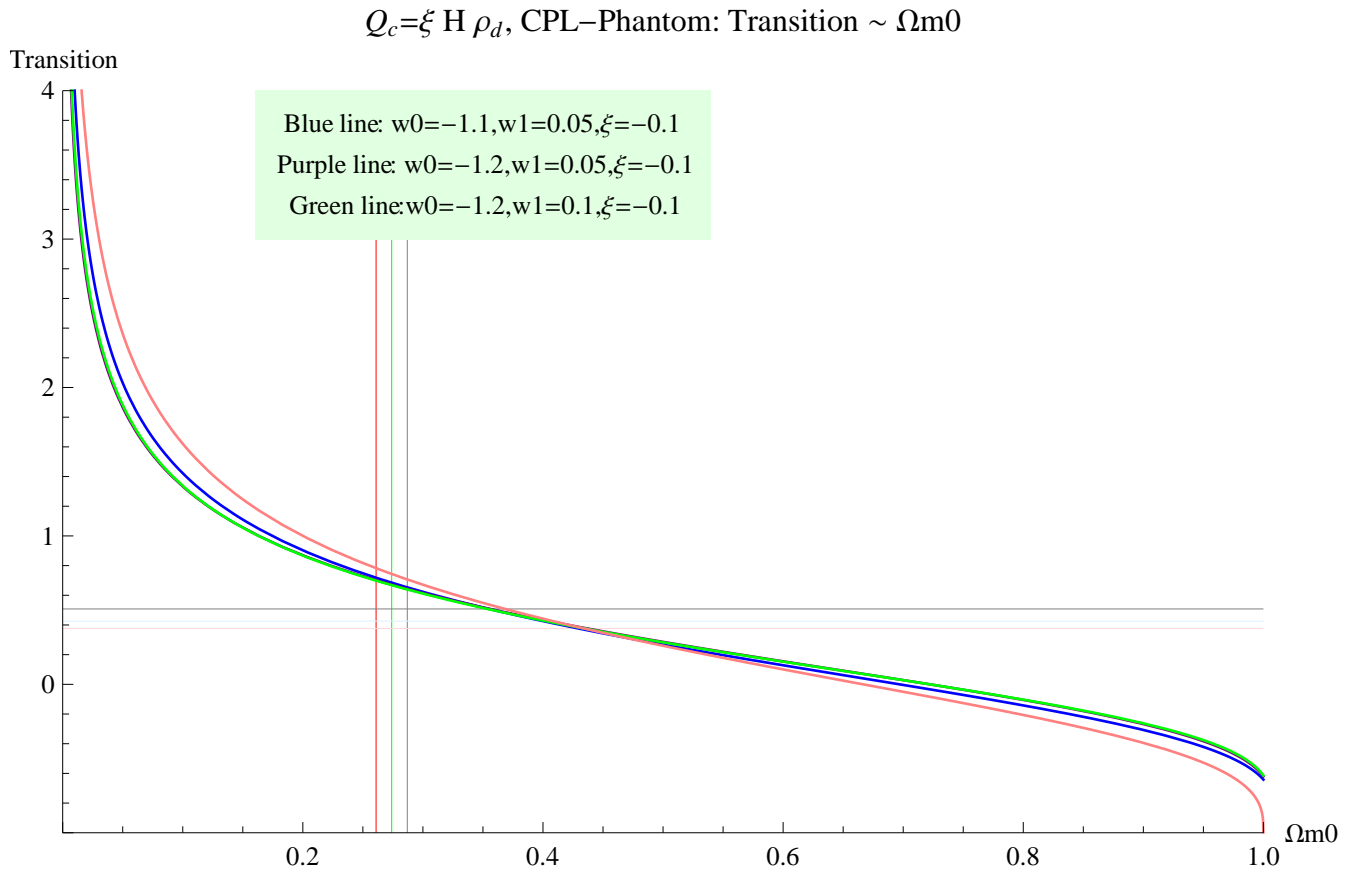


Figure 30:  $\xi$  VS  $\Omega_{m0}$

#### 4.4.3 Phantom

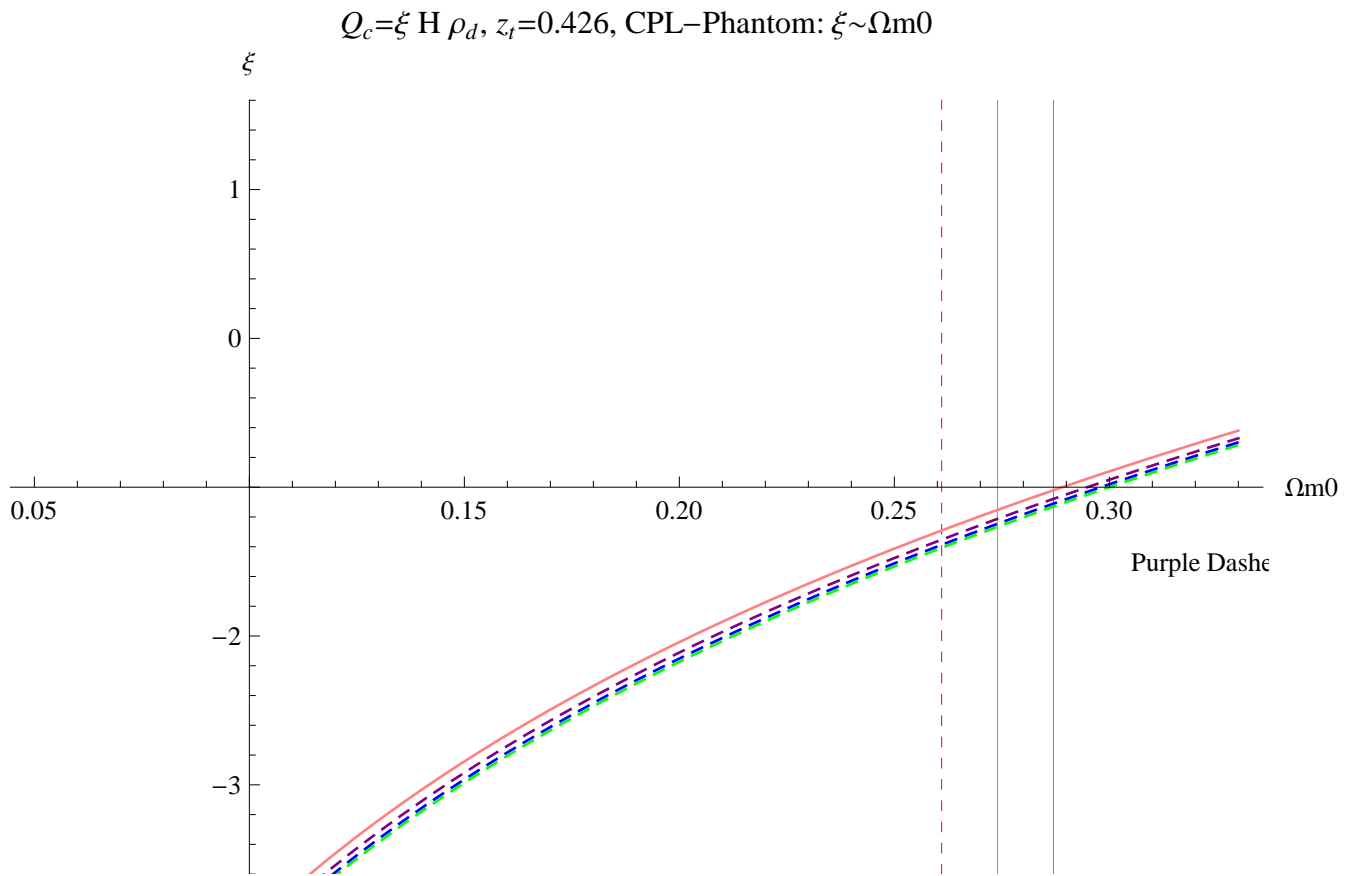
(Figures 31, 32)

There is always a almost-stationary point.



**Figure 31:** Transition VS  $\Omega_{m0}$





**Figure 32:** Transition VS  $\Omega_{m0}$