

Summary

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1 Objectives

For LCDM, interacting models, and CPL, calculate

- ξ range for varying EoS while fixing Ωm_0
- ξ range for varying Ωm_0 or r , while fixing ω
- Does $\xi > 0$ means energy transfer to dark energy in this method?

2 Background

Deceleration parameter reads

$$q(z) = -1 + \frac{1+z}{H} \frac{dH}{dz} \quad (1)$$

For interaction models, the Friedmann equations,

$$\dot{\rho}_c + 3H\rho_c = Q_c \quad (2a)$$

$$\dot{\rho}_d + 3H(1+w)\rho_d = -Q_c \quad (2b)$$

$Q_c = \xi H \rho_c$ Background equations,

$$\Omega m = \Omega m_0 (1+z)^{3-\xi} \quad (3a)$$

$$\Omega d = (\Omega d_0 + \frac{\xi}{3w+\xi} \Omega m_0) (1+z)^{3(1+w)} + \frac{-\xi}{\xi+3w} \Omega m = \Omega \bar{d}_0 (1+z)^3 + \frac{-\xi}{\xi+3w} \Omega m \quad (3b)$$

$Q_c = \xi H \rho_d$

$$\Omega m = (\Omega m_0 + \frac{\xi}{\xi+3w} \Omega d_0) (1+z)^3 + \frac{-\xi}{\xi+3w} \Omega d = \omega \bar{m}_0 (1+z)^3 + \frac{-\xi}{\xi+3w} \Omega d \quad (4a)$$

$$\Omega d = \Omega d_0 (1+z)^{3(1+w)+\xi} \quad (4b)$$

Eqn 3 and eqn 4 shows that the coupling constant has two effects,

1. Change the amplitude of the evolution of matter or dark energy energy density.
2. Transfer energy between DE and DM.

2.1 Some definitions

1. For short

$$r = \frac{\Omega m_0}{\Omega d_0}$$

CPL EoS is

$$w = w_0 + w_1 \frac{z}{1+z}$$

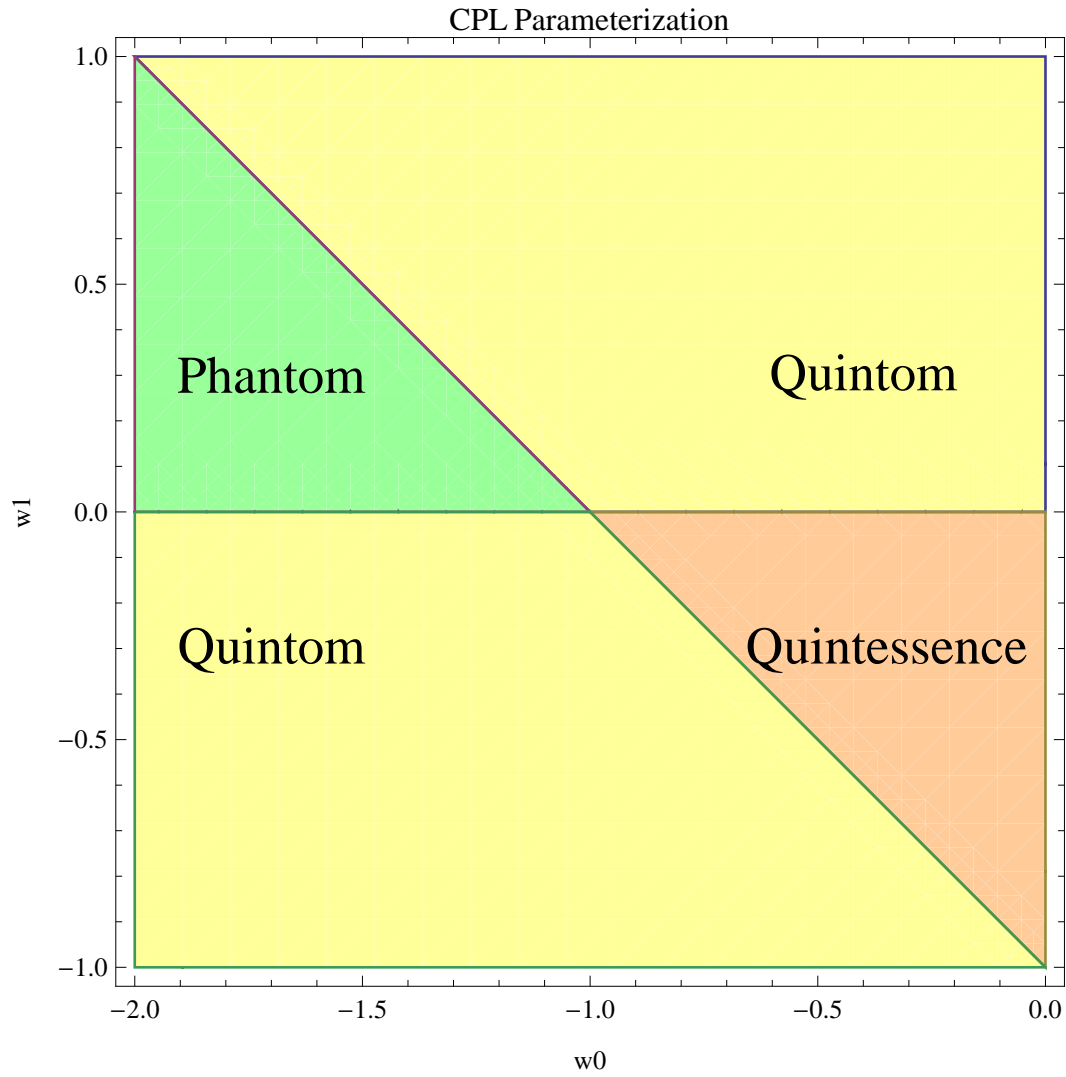


Figure 1: CPL classification

Classification Figure 1 shows how to category the dark energy models in CPL parametrization.

3 Data & Method

3.1 Data

ΛCDM Parameters From WMAP, $\Omega m0 = 0.265$

Constraints $\Omega m0 = 0.247(+0.013, -0.013)$; Transition redshift 0.426 (+0.082, -0.050).(arXiv: 1205.4688, arXiv: astro-ph/0611572).

In $(\Omega m0, \text{Transition redshift})$ plane, allowed region is a rectangle centred at (0.274, 0.426) with two diagonal points (0.261, 0.376) and (0.287, 0.508).

CPL $\Omega m0 = 0.269(+0.017, -0.008)$, $w0 = -0.97(+0.12, -0.07)$, $w1 = 0.03(+0.26, -0.75)$

4 Results

Check the files in files folder.

4.1 $Q_c = \xi H \rho_c$

Results table

$Q_c = \xi H \rho_c$, constant ξ , constant $w = -1$: Results for ξ			
$\Omega m0 / \Omega d0 \backslash \text{Transition}$	$z_t = 0.376$	$z_t = 0.426$	$z_t = 0.508$
$r = 0.358$	-1.25282	-0.965436	-0.617444
$r = 0.378$	-1.15011	-0.875189	-0.542347
$r = 0.398$	-1.05453	-0.791252	-0.472561

Figure 2: ICC Result table

Figure 3 shows that

$\rho_c\text{-Dec-1}$ The universe decelerates faster at the early stage for smaller interaction constant ξ even they have the same matter fraction.

$\rho_c\text{-Dec-2}$ For the same ξ , the deceleration converge ($q = (1 - \xi)/2$ with $3w + \xi < 0$) at early time.

Figure 4 shows

$\rho_c\text{-Trans-1}$ Transition happens earlier when matter fraction is smaller. Matter is against DE's pressure.

$\rho_c\text{-Trans-2}$ Transition is later when ξ is smaller. Energy transfers to DE when ξ is negative, then why later transition? ¹

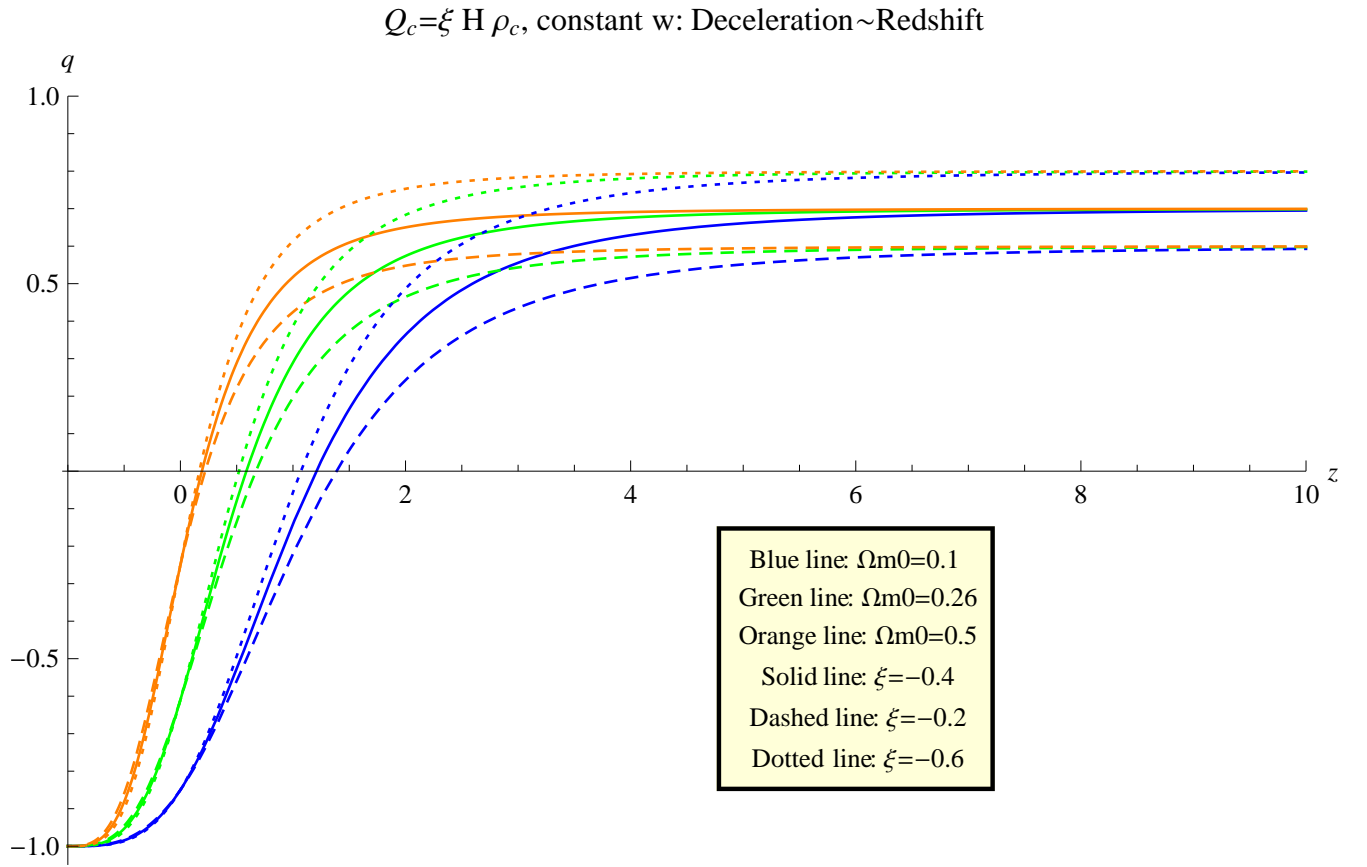


Figure 3: Deceleration parameter

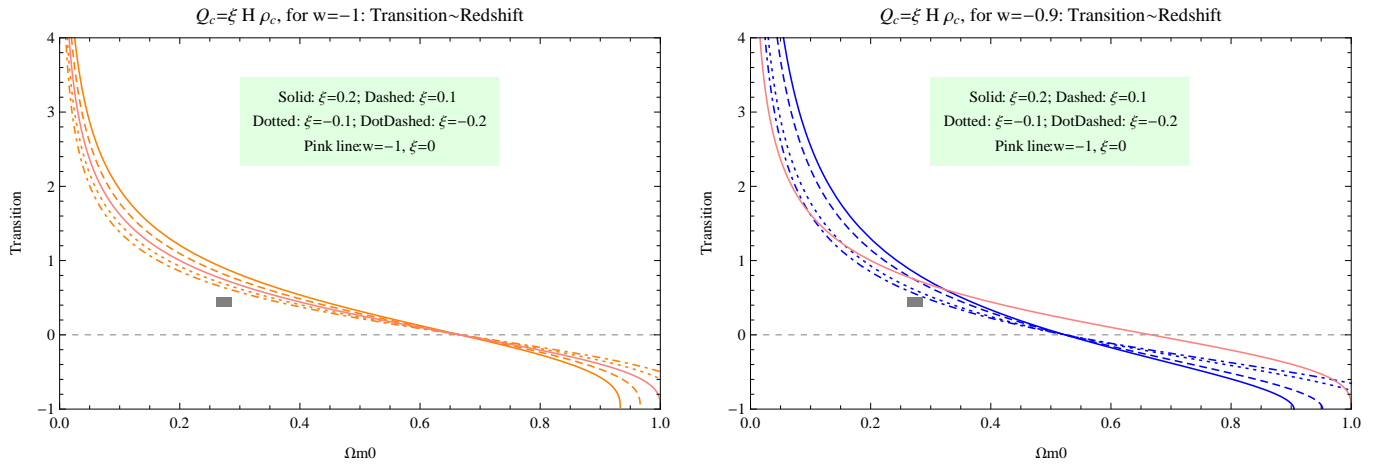


Figure 4: Transition redshift.

ξ results for $Q_c = \xi H \rho_c$ (Fitting data: Data From, 2)			
w	Center	Lower	Upper
-1.183	-0.881565	-1.29687	-0.443589
-1.087	-0.88948	-1.29859	-0.459135
-0.991	-0.875238	-1.27522	-0.456176

$Q_c = \xi H \rho_c$, constant w: Coupling Constant \sim EoS

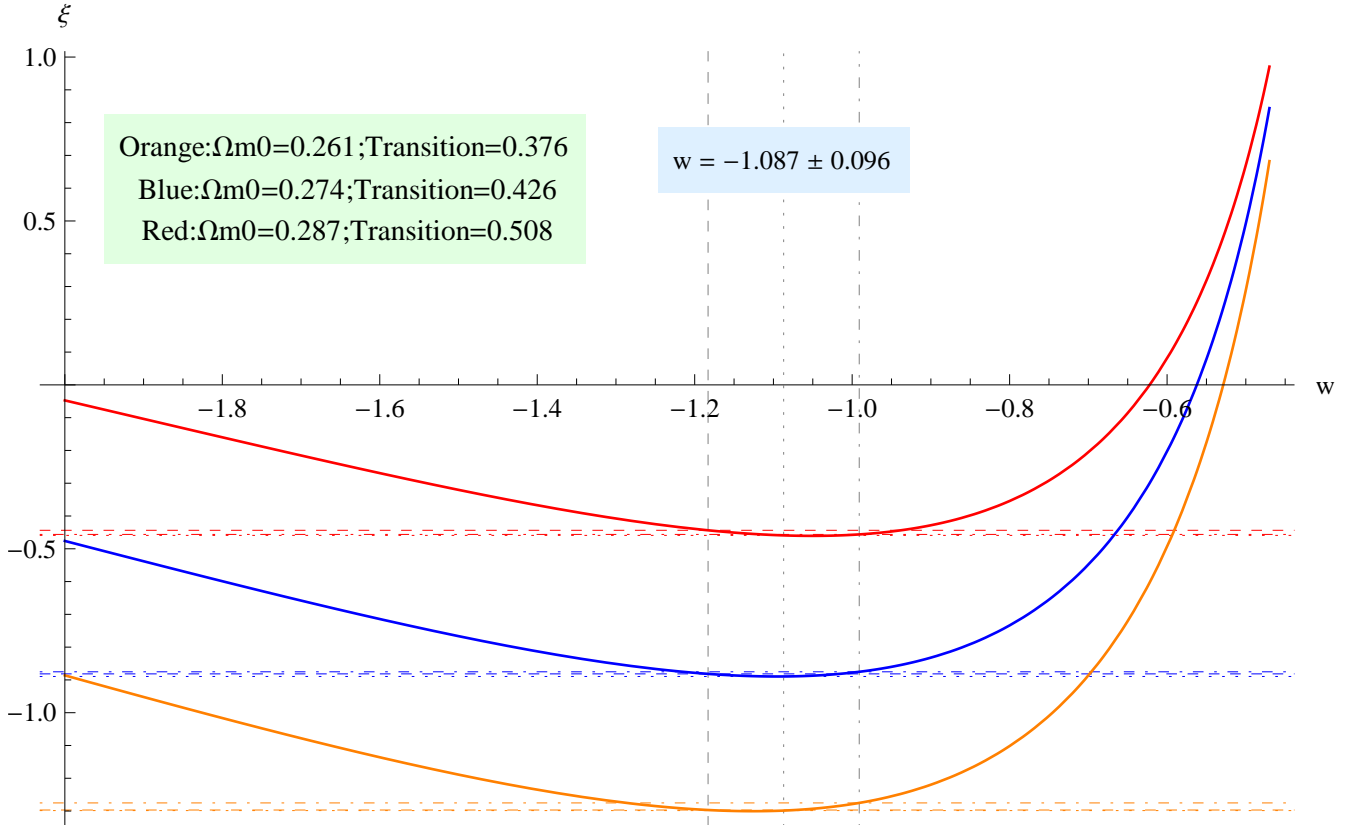


Figure 5: Interacting coefficient for $Q_c = \xi H \rho_c$

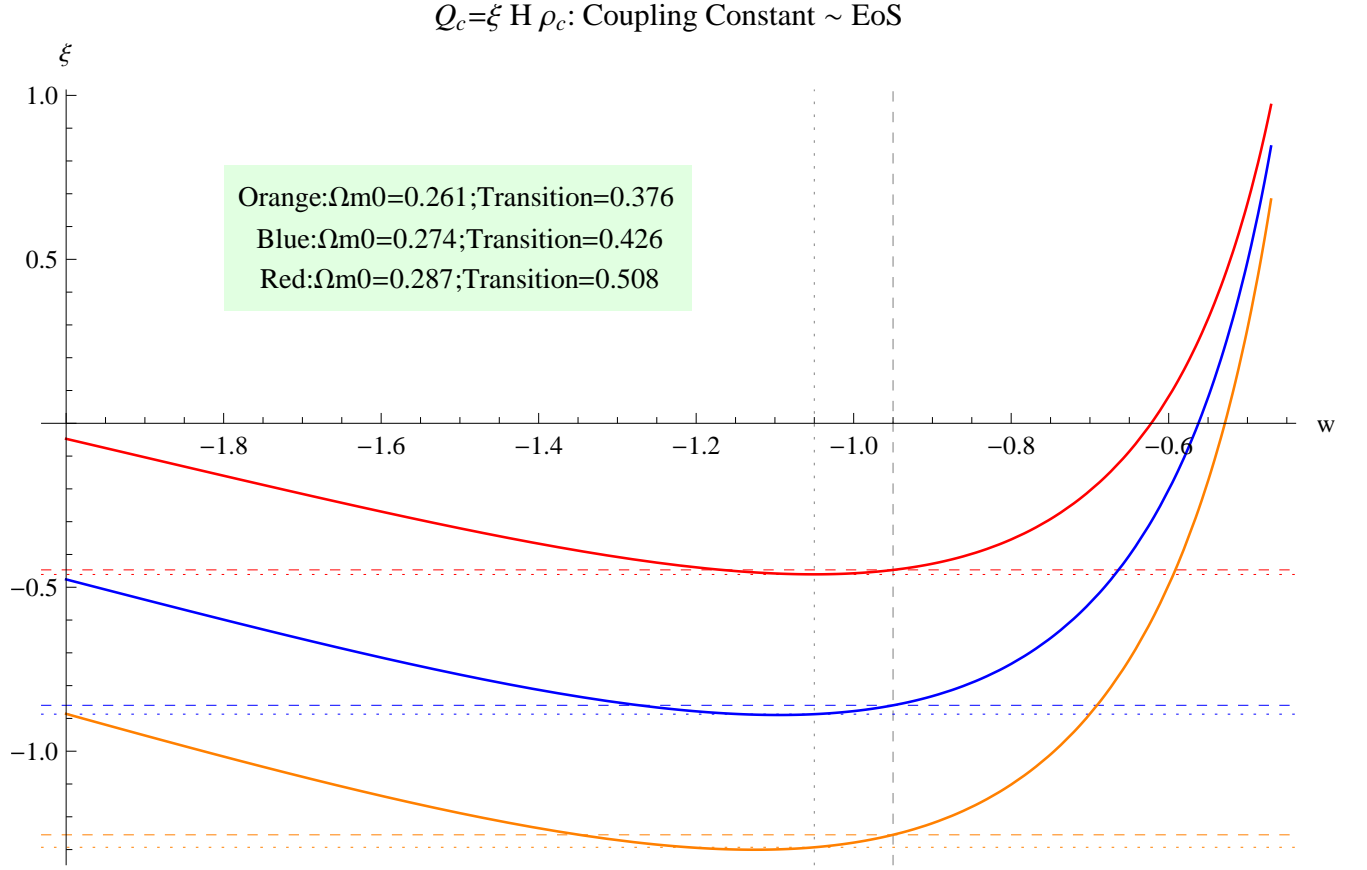


Figure 6: Interacting coefficient for $Q_c = \xi H \rho_c$

Figure 5 and figure 6 (vertical lines are $w = -1 \pm 0.05$) show the value of ξ for different EoS. Results from this figure:

(EoS is -1 within 5w=-1 (-1.279,-0.457) center:-0.878 ; w=-1.05 (-1.293,-0.461) center:-0.887 ; w=-0.95 (-1.255,-0.447) center:-0.860 ;

ρ_c -xiVS w -1 NOT monotonic. The result of ξ is greatly affected by Ωm_0 . In correspondence with another

ρ_c -xiVS w -2 For different Ωm_0 , ξ values deviate greatly from each other at small w .

Figures 7 show how do we constrain ξ and how do EoS change our constrain results with the transition fixed.

ρ_c -xiVS Ωm_0 -1 The smaller, the more difference among ξ values of different EoS. Reason for this is less matter has less effect on the evolution thus the property of dark energy determines more about the transition.

ρ_c -xiVS Ωm_0 -2 Second figure shows

- + System with smaller w needs smaller coupling to achieve the same transition time, as expected.
- + So we give the result that $w \in (-0.58406, -0.3334)$ if we constrain $\Omega m_0 = 0.2603$ and transition redshift 0.426.

4.2 $Q_c = \xi H \rho_c$, CPL

For a flat universe, choose the parameters $w_0=-1.02, w_1=0.6$, the region for interaction constant ξ should be (-1.04,-0.21) with a center at -0.64, derived from the (transition redshift, Ωm_0) plane, while a result of (-1.01, -0.23) with a center at -0.63, derived from (transition redshift, Ωm_0).

First let's have a look at the deceleration parameter.

Figure 8 is right similar to constant EoS situation.

Figures 9 show

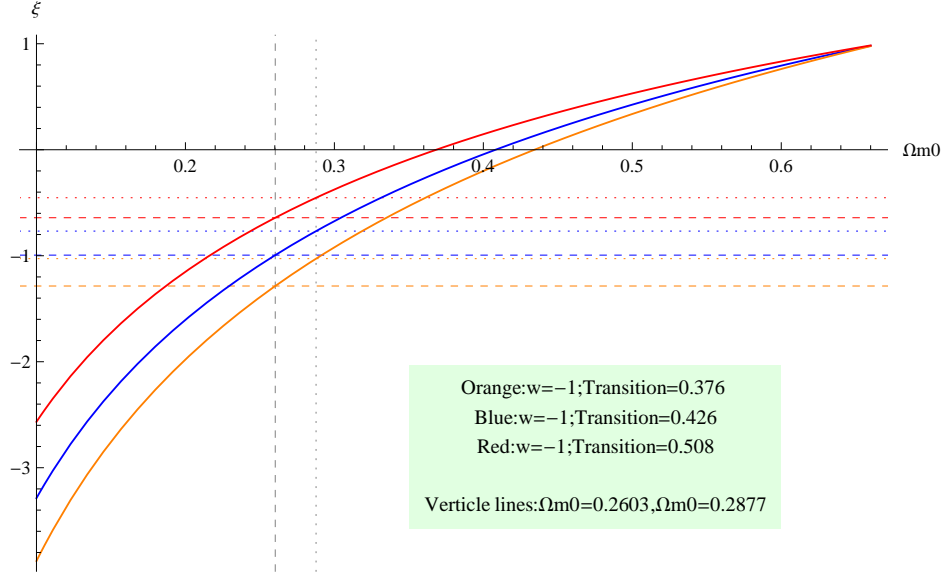
ρ_c -ICCPL-TV $S\Omega m_0$ -1 Coupling acts on these model similar to constant EoS model.

ρ_c -ICCPL-TV $S\Omega m_0$ -2 The change in w_1 has a similar effect with the change of ξ . Larger w_1 corresponds to smaller ξ . The reason for this is both negative ξ and larger w_1 enhances the energy density of dark energy (check using the CPL EoS).

The dots in figure 10 are the data set of $\xi = 0$. If we need $\xi < 0$, i.e., energy transfers from dark matter to dark energy, the allowed parameter space is the striped area.

For $\Omega_{m0} \in 0.274 (1 \pm 0.05)$			
Table of ξ for different Ω_{m0} -Transition combination			
Ω_{m0} :Transition	0.426	0.376	0.508
0.2603	-0.994339	-1.28571	-0.641508
0.274	-0.877755	-1.15303	-0.544482
0.2877	-0.767582	-1.02756	-0.452892

$Q_c = \xi H \rho_c$, constant w : Coupling Constant $\sim \Omega_{m0}$



$Q_c = \xi H \rho_c$, constant w : Coupling Constant $\sim \Omega_{m0}$

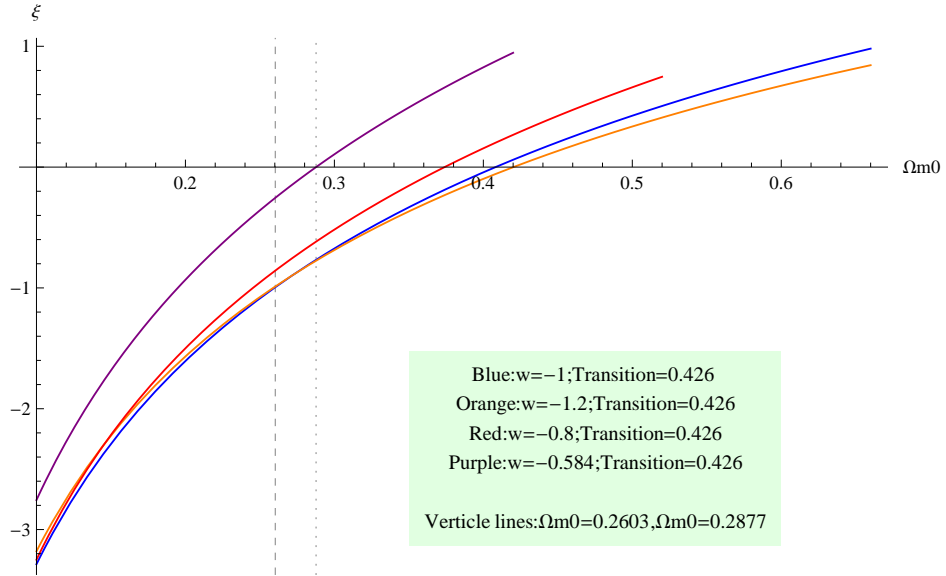


Figure 7: Interacting coefficient changing with Ω_{m0} for $Q_c = \xi H \rho_c$

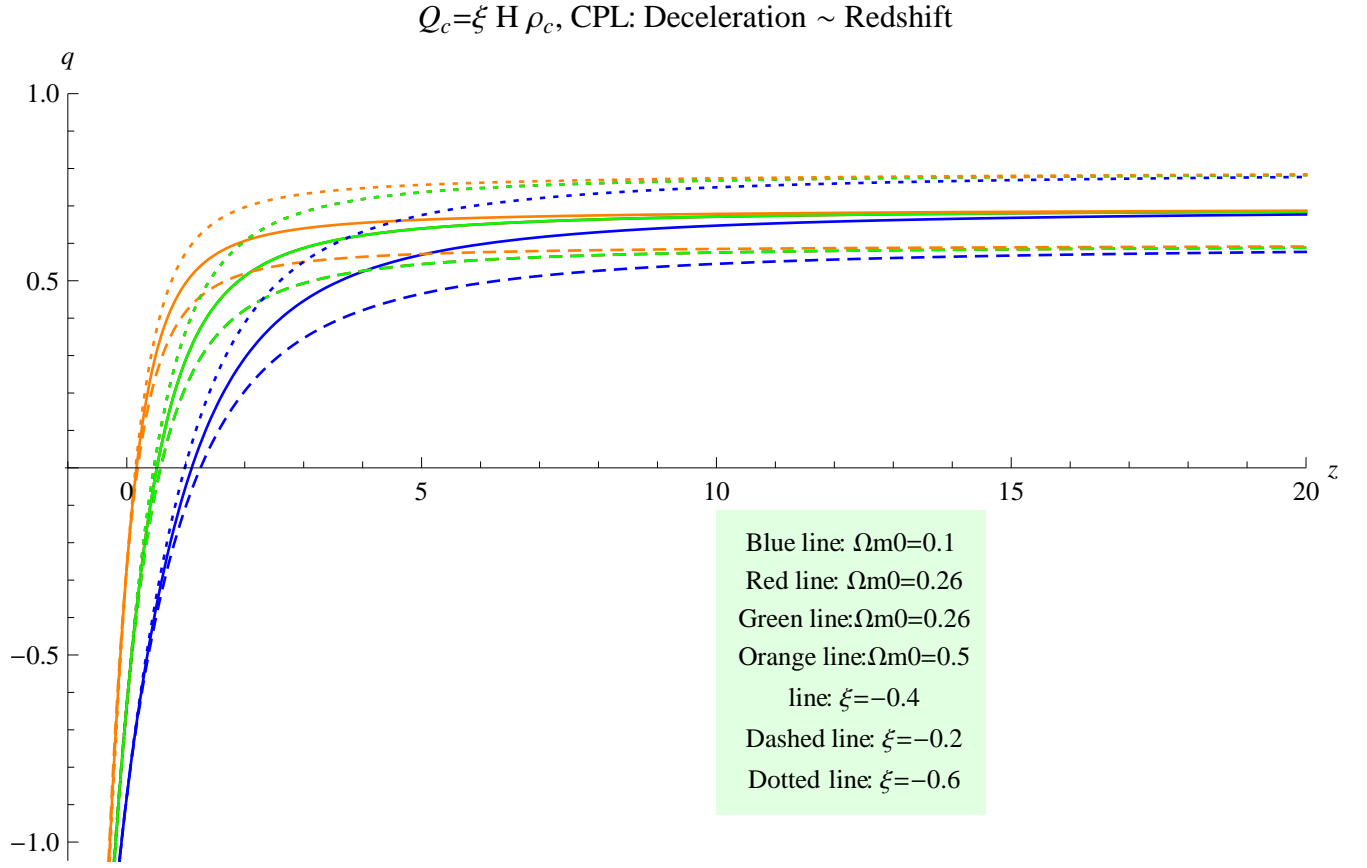


Figure 8: Deceleration parameters for ICCPL ($Q_c = \xi H \rho_c$ with CPL parametrized EoS)

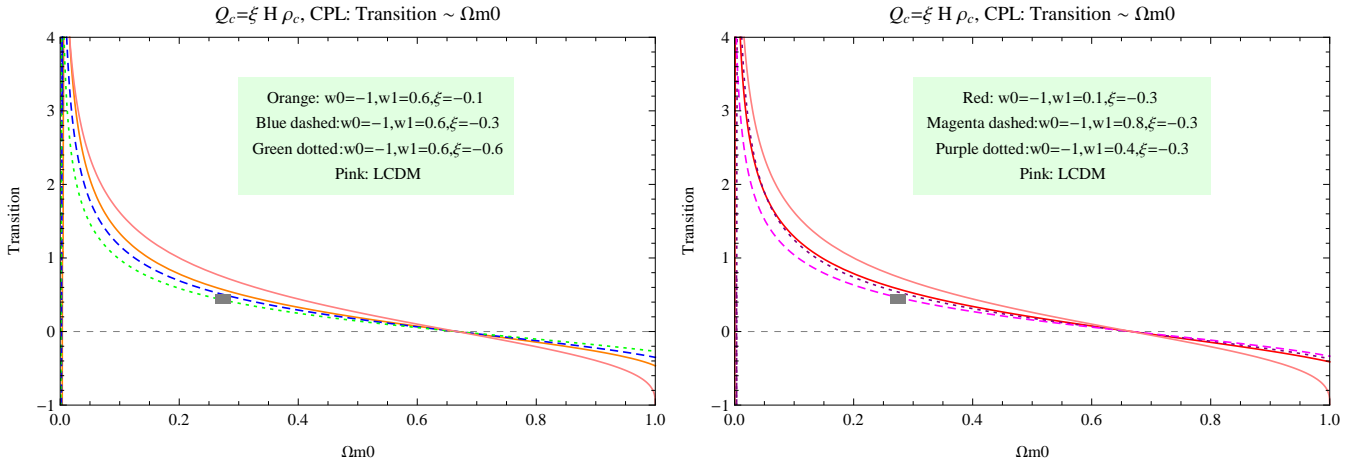


Figure 9: The effect of EoS parameters on Transition and Ω_{m0}

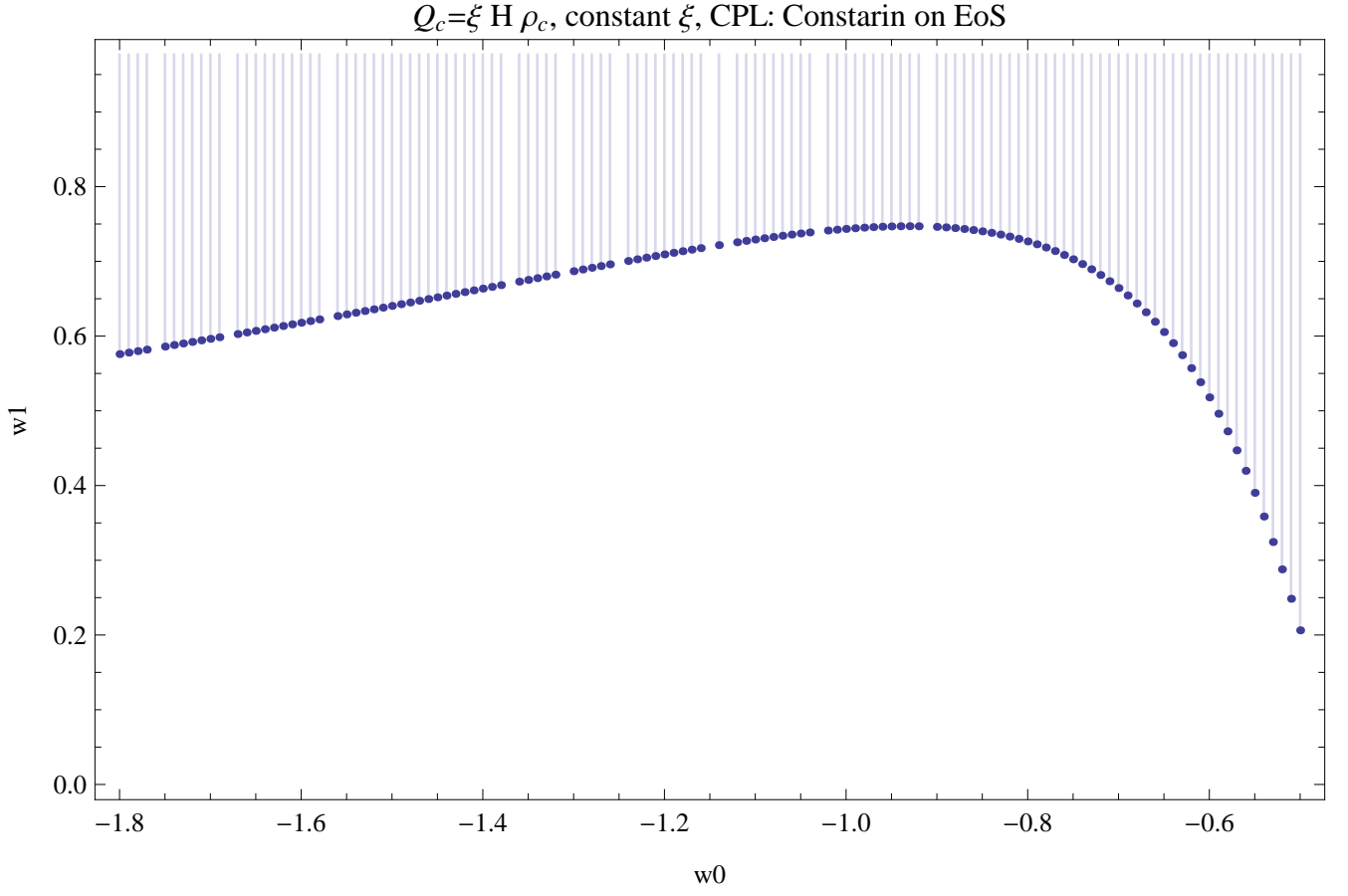


Figure 10: Lower bound in the w_1 w_0 parameter space

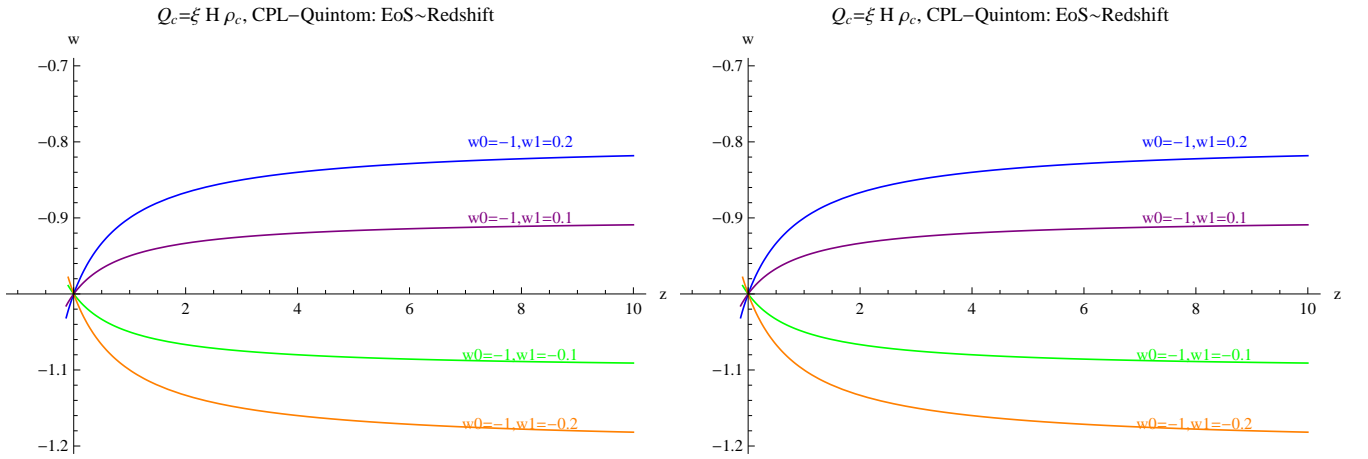


Figure 11: The EoS

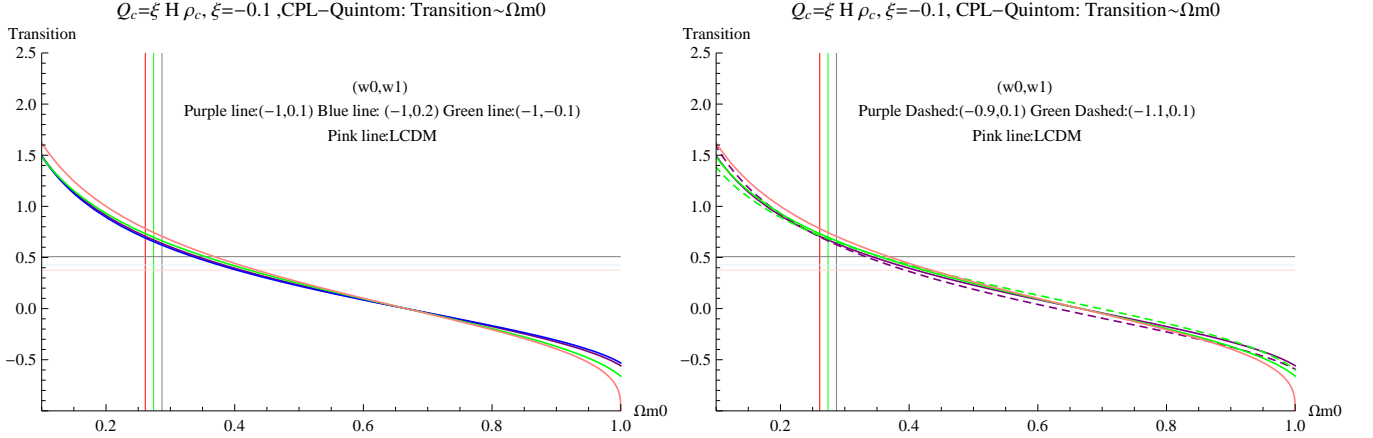


Figure 12: Transition vs Ω_{m0}

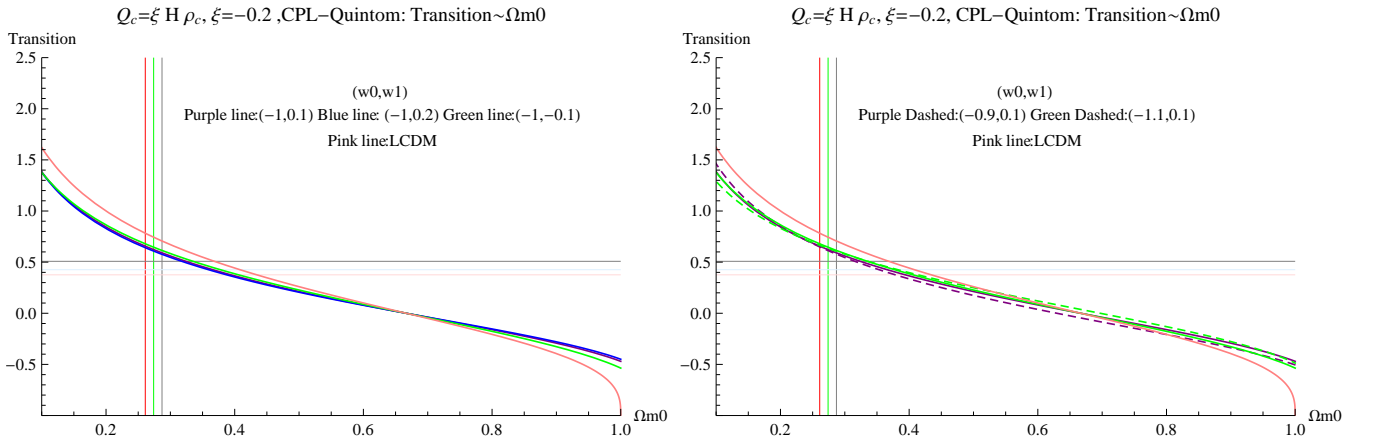


Figure 13: Transition vs Ω_{m0}

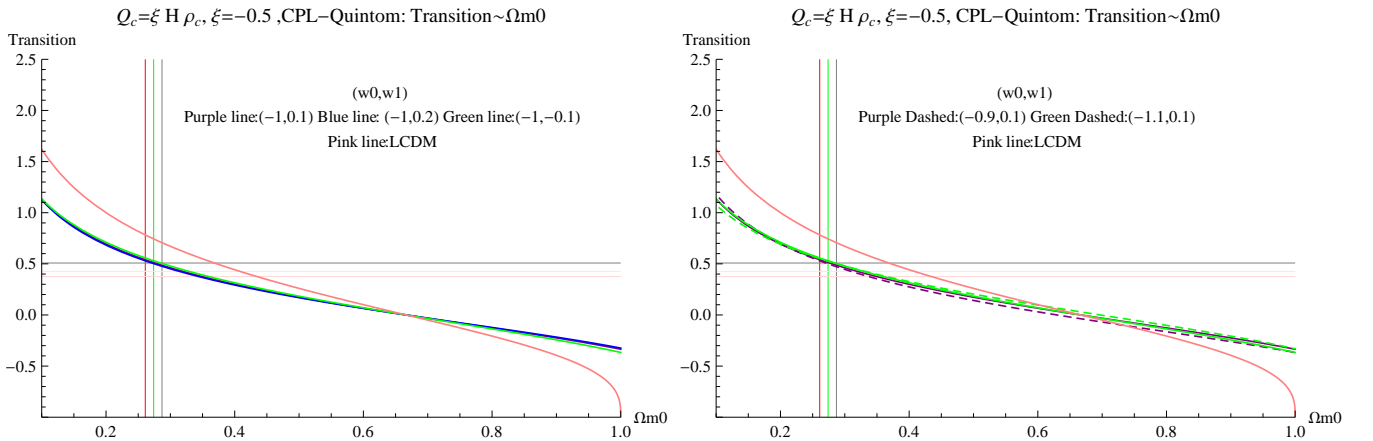


Figure 14: Transition vs Ω_{m0}

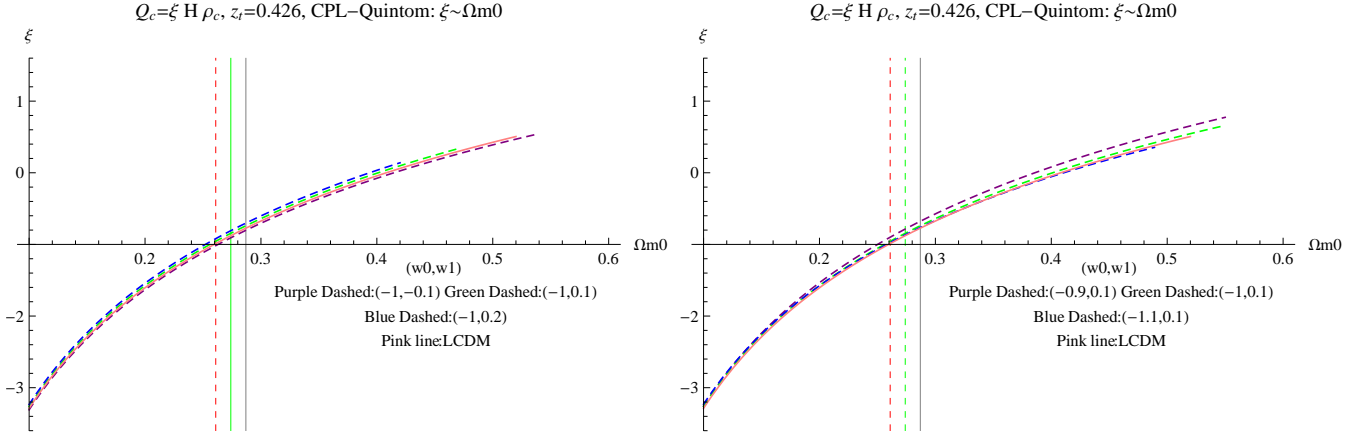


Figure 15: ξ vs $\Omega m0$

4.2.1 Quintom

(Figures 11, 12, 15, 13, 14)

The left figure in 12 indicates a possible stationary point.²

Comparing figure 14, figure 13 and figure 12, it seems that all lines cross point $(0.66, 0)$, the reason of which, however, is because $w0 = -1$ and a small nearly zero $w1$ means a similar evolution with $Q_c = \xi H \rho_c$ with a constant EoS.³

(Other results are shown on the complete results files.)

0

4.2.2 Quintessence

(Figures 16, 17 and 18.)

There is a features in the right figure of 17. The difference between lines become smaller when $\Omega m0$ is small and at about 0.35. Reason for this is when $\Omega m0$ is smaller, the evolution is mainly determined by Ωd (⁴)

4.2.3 Phantom

(Figures 19, 20, 21)

4.3 $Q_c = \xi H \rho_d$

(Figures 22, 23, 24, 25)

¹Reasons below. All solutions of equation 2 have the same value at $z = 0$, i.e. now. Equation 2a tells us a positive ξ leads to smaller energy density of dark matter at early time of the universe, thus dark energy takes over quickly if the transition happens before today. (For more details, calculation are shown in supplement_08-10.pdf file.)

²Only possible because I can only partially prove there is a nearly stationary. This is on my *Cosmologia Notebook*.

³The lines do not intercept with each other at the same point actually.

⁴In the solutions of ρ_c and ρ_d , it is clear that $\Omega m0$ determines how much $w1$ will affect the result because the term $e^{3w1 \frac{1}{1+z}}$ is small since $w1$ and $1/(1+z)$ are small and $\Omega m0$ is the amplitude of the other term in which $w1$ is in charge. Check *Cosmologia Notebook*.

$Q_c = \xi H \rho_c$, CPL, Quintessence: EoS ~ Redshift

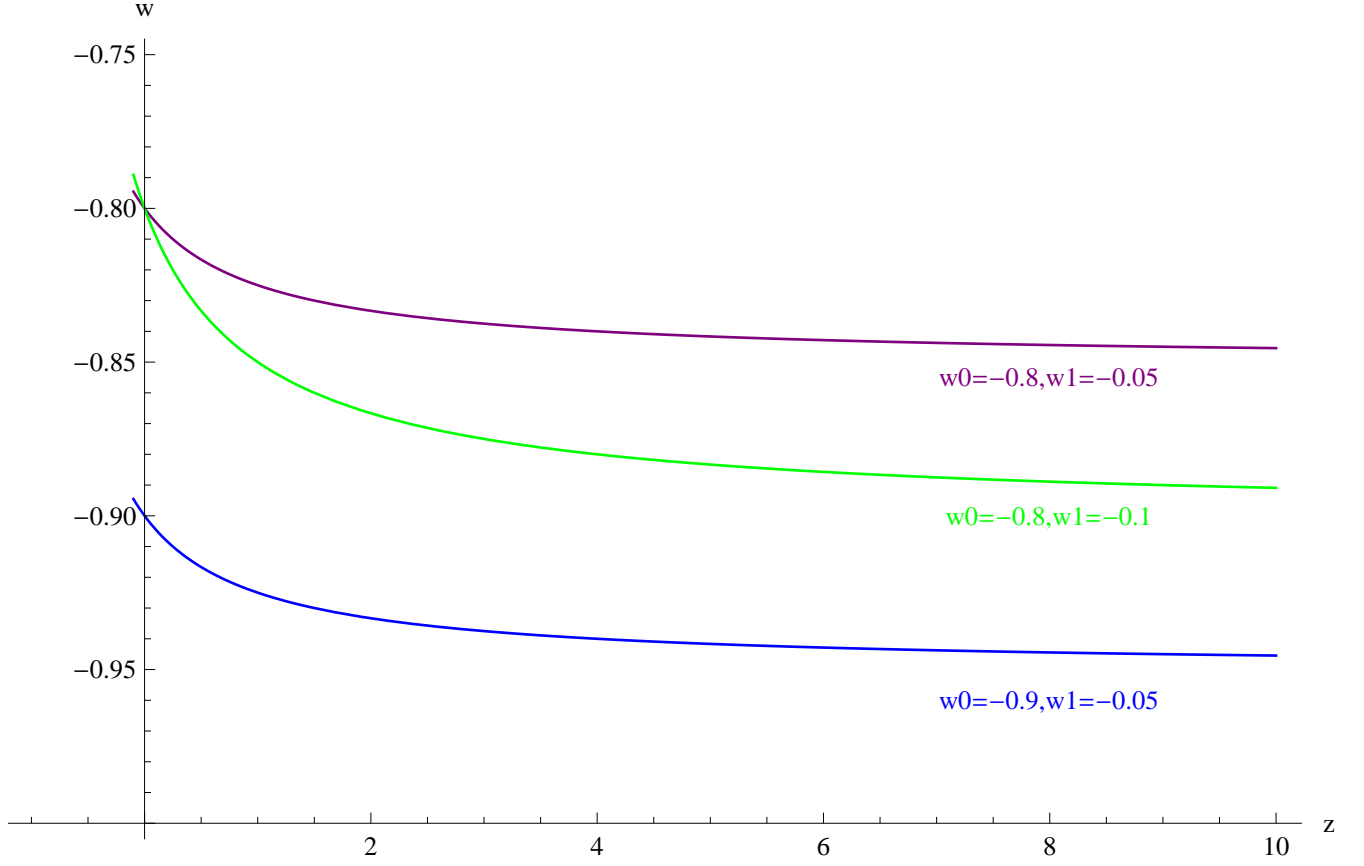
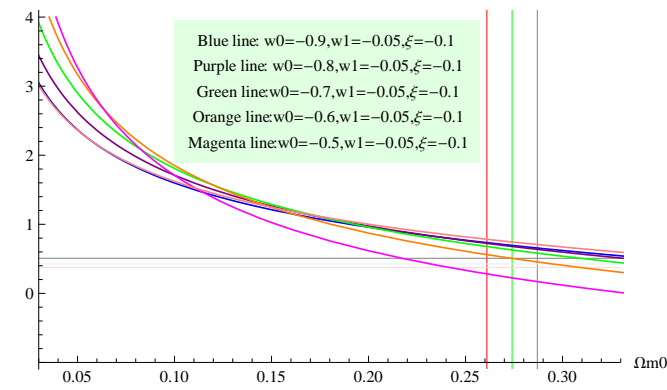


Figure 16: The EoS

$Q_c = \xi H \rho_c$, $\xi = -0.1$, CPL, Quintessence: Transition Redshift ~ Ωm_0



$Q_c = \xi H \rho_c$, $\xi = -0.1$, CPL, CPL, Quintessence: Transition Redshift ~ Ωm_0

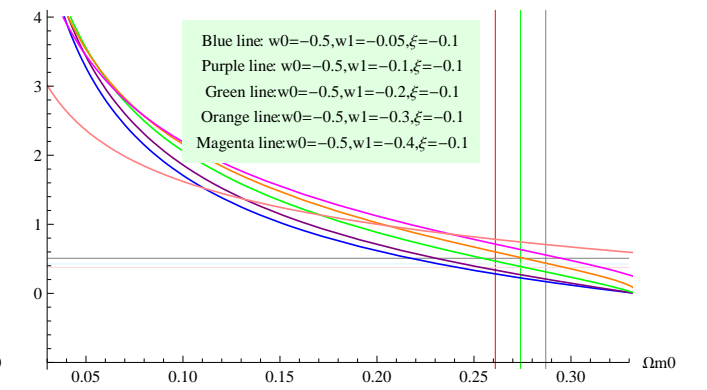


Figure 17: Transition vs Ωm_0

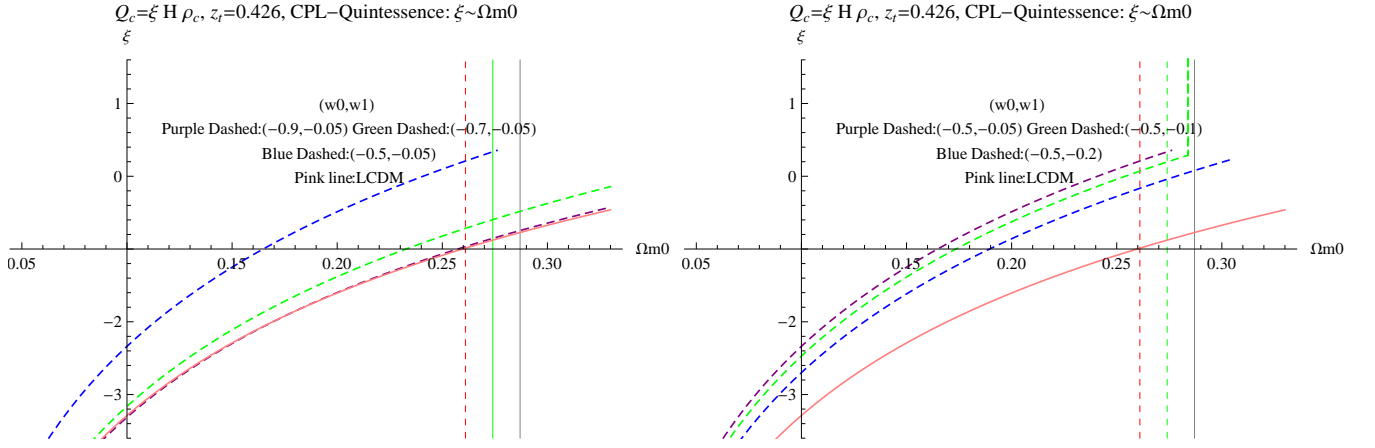


Figure 18: ξ vs Ω_{m0}

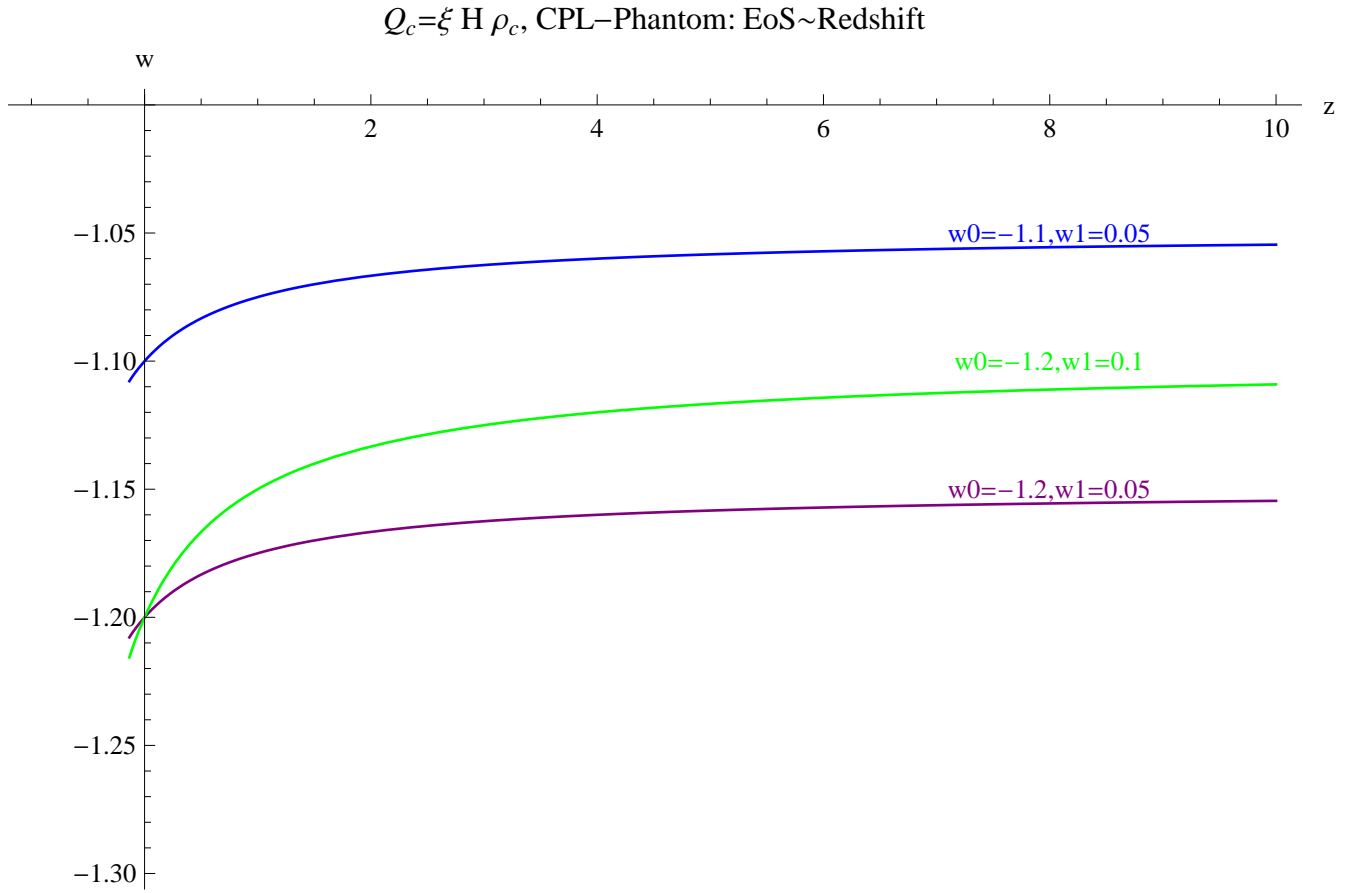


Figure 19: The EoS

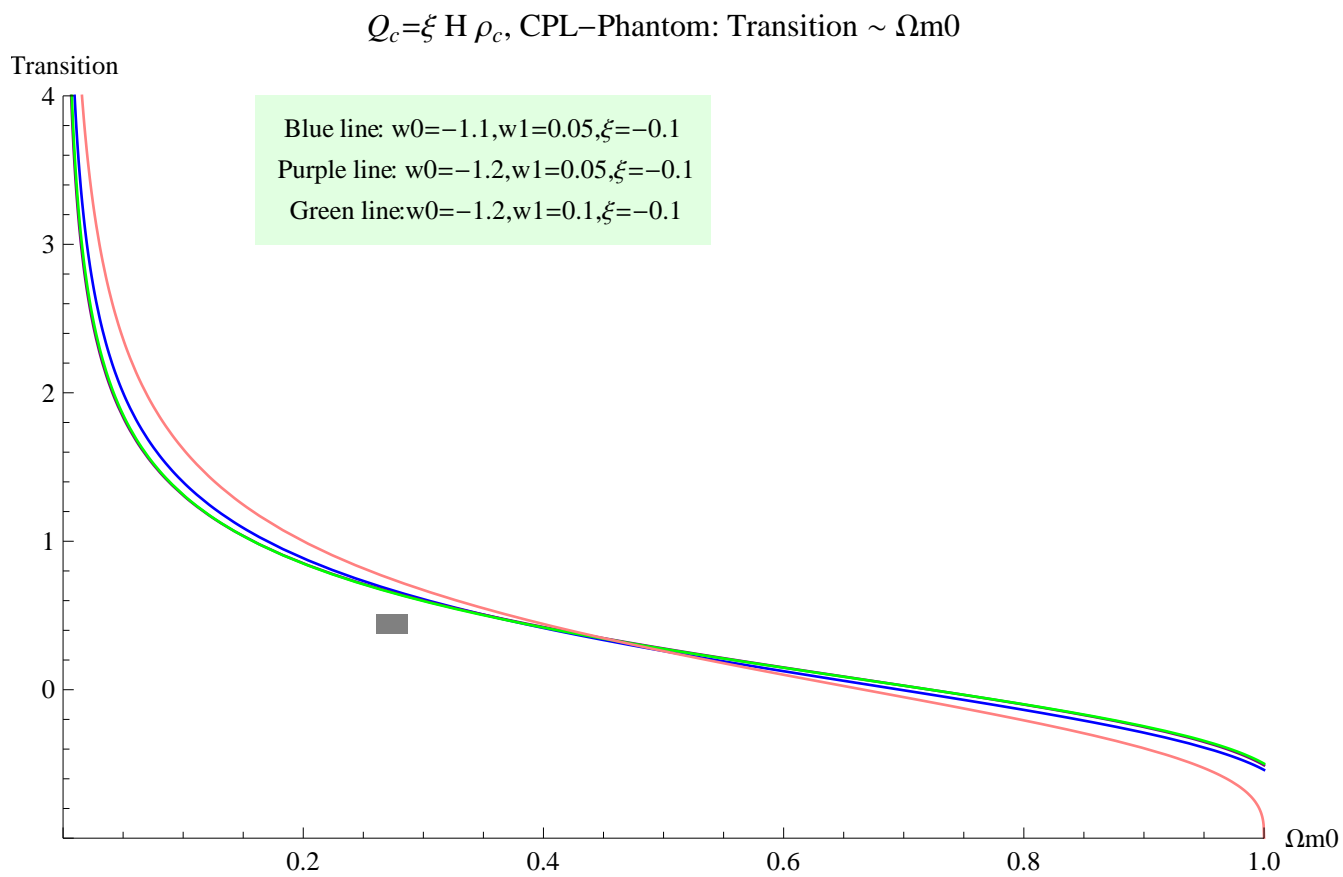


Figure 20: Transition vs Ω_{m0}

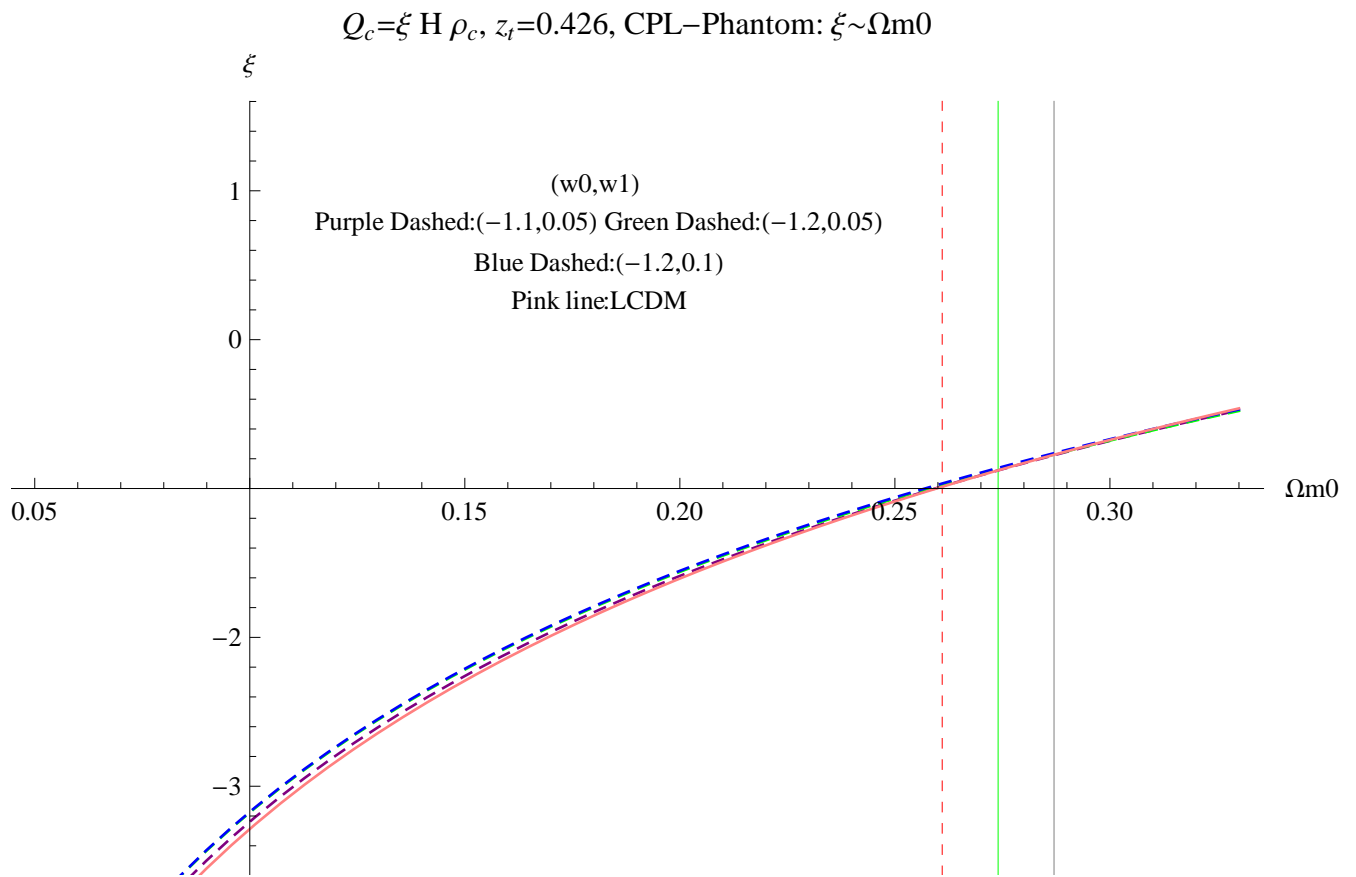


Figure 21: ξ vs Ωm_0

$Q_c = \xi H \rho_d$, constant w : Deceleration \sim Redshift

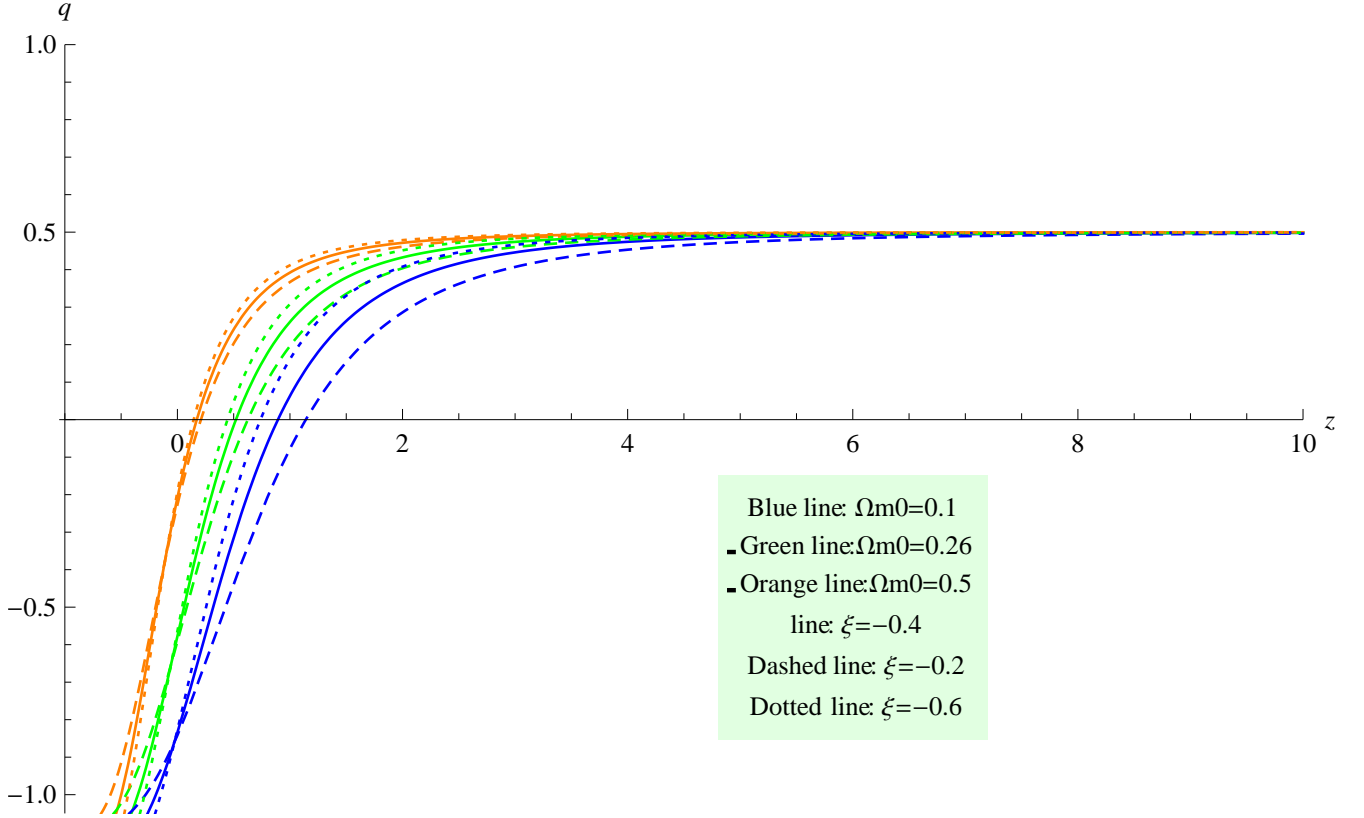


Figure 22: Deceleration parameter

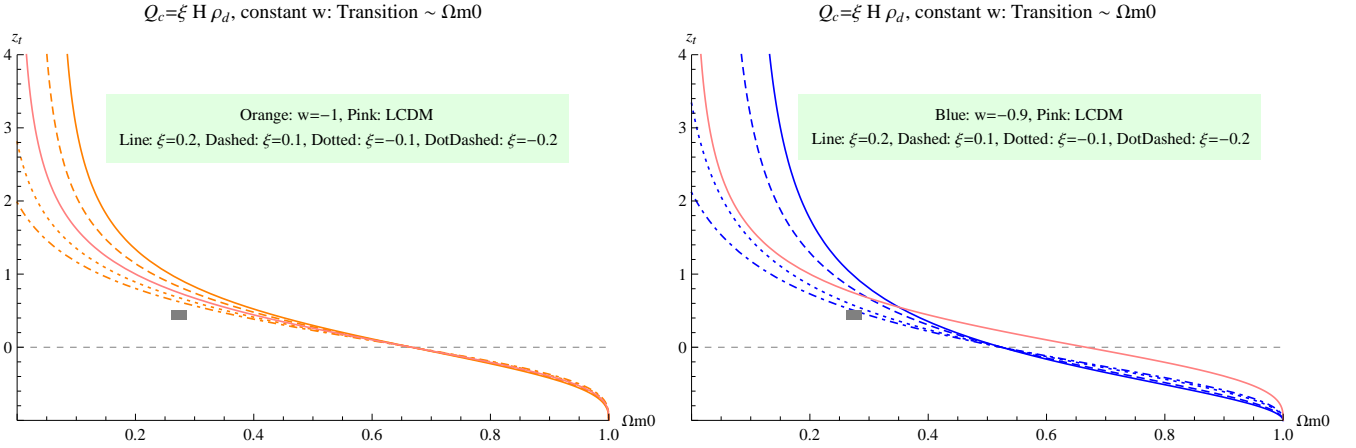


Figure 23: Transition vs Ωm_0 . For a flat universe, choose the parameters $w_0 = -1.02, w_1 = 0.6$, the region for interaction constant ξ should be $(-1.04, -0.21)$ with a center at -0.64 , derived from the (transition redshift, Ωm_0) plane, while a result of $(-1.01, -0.23)$ with a center at -0.63 , derived from (transition redshift, $\Omega m_0 / \Omega d_0$) plane.

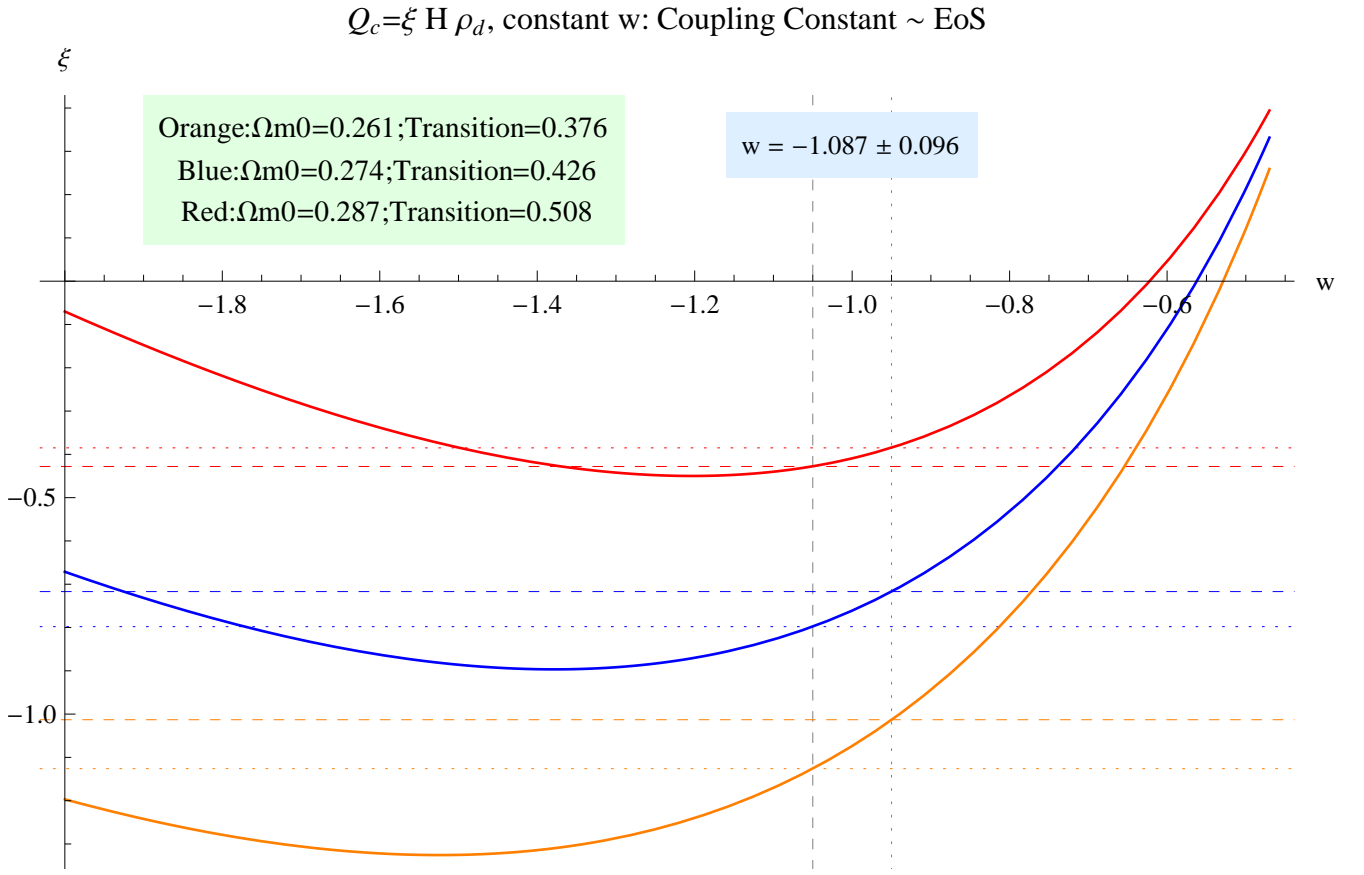


Figure 24: ξ VS w

Figures 23 is

4.4 I2CCPL

(Figures 27, 28)

Figure 27 shows the all the deceleration are the same at very early time.

4.4.1 Quintom

(Figures 29, 30)

4.4.2 Quintessence

(Figures 31, 32)

4.4.3 Phantom

(Figures 33, 34)

There is always a almost-stationary point.

$Q_c = \xi H \rho_d, \text{Constant } w. \text{ (Data used: Data From, 2)}$			
w	Center	Lower	Upper
-1.183	-0.864289	-1.22984	-0.449552
-1.087	-0.820486	-1.15946	-0.437339
-0.991	-0.753634	-1.06346	-0.405262

$Q_c = \xi H \rho_d, \text{constant } w: \text{Coupling Constant } \sim \text{EoS}$

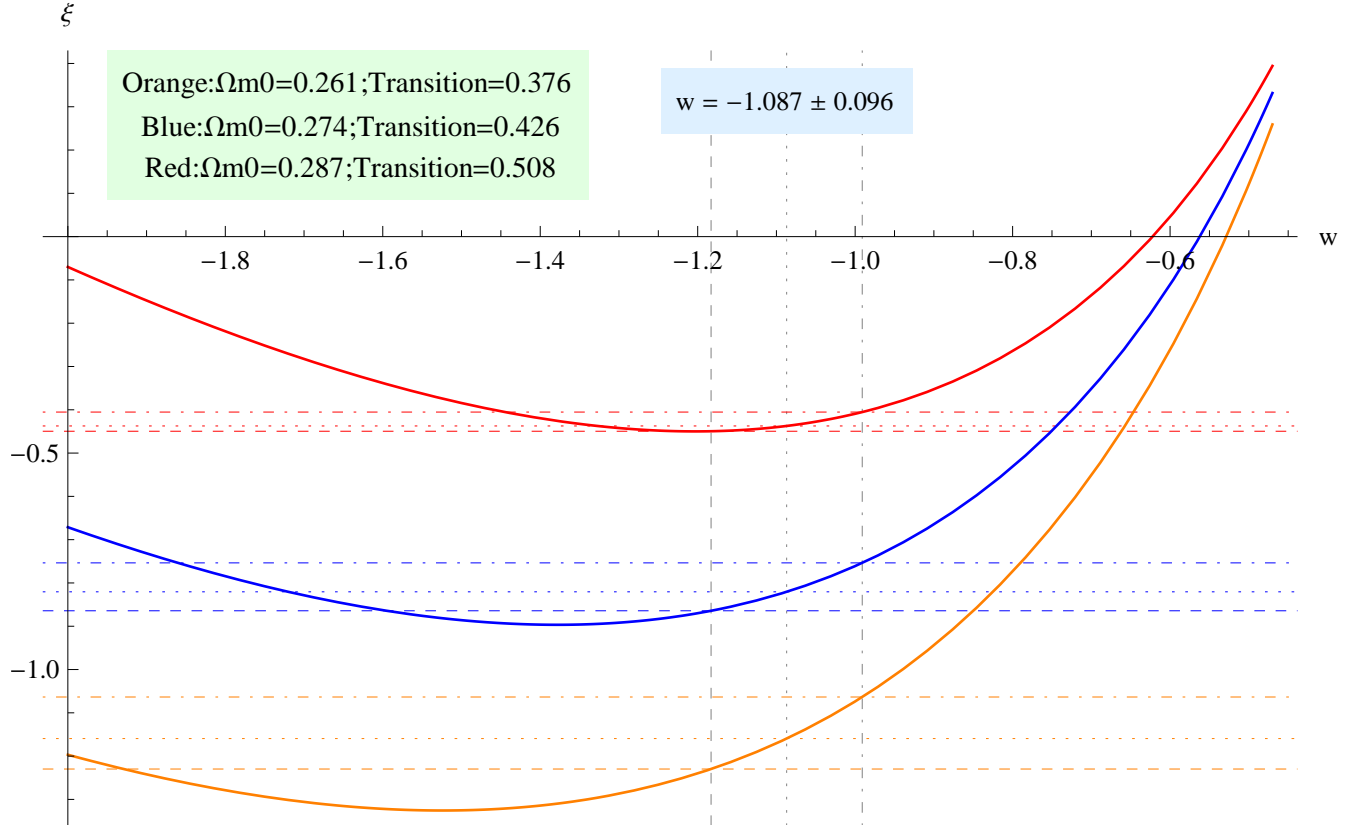


Figure 25: ξ VS w

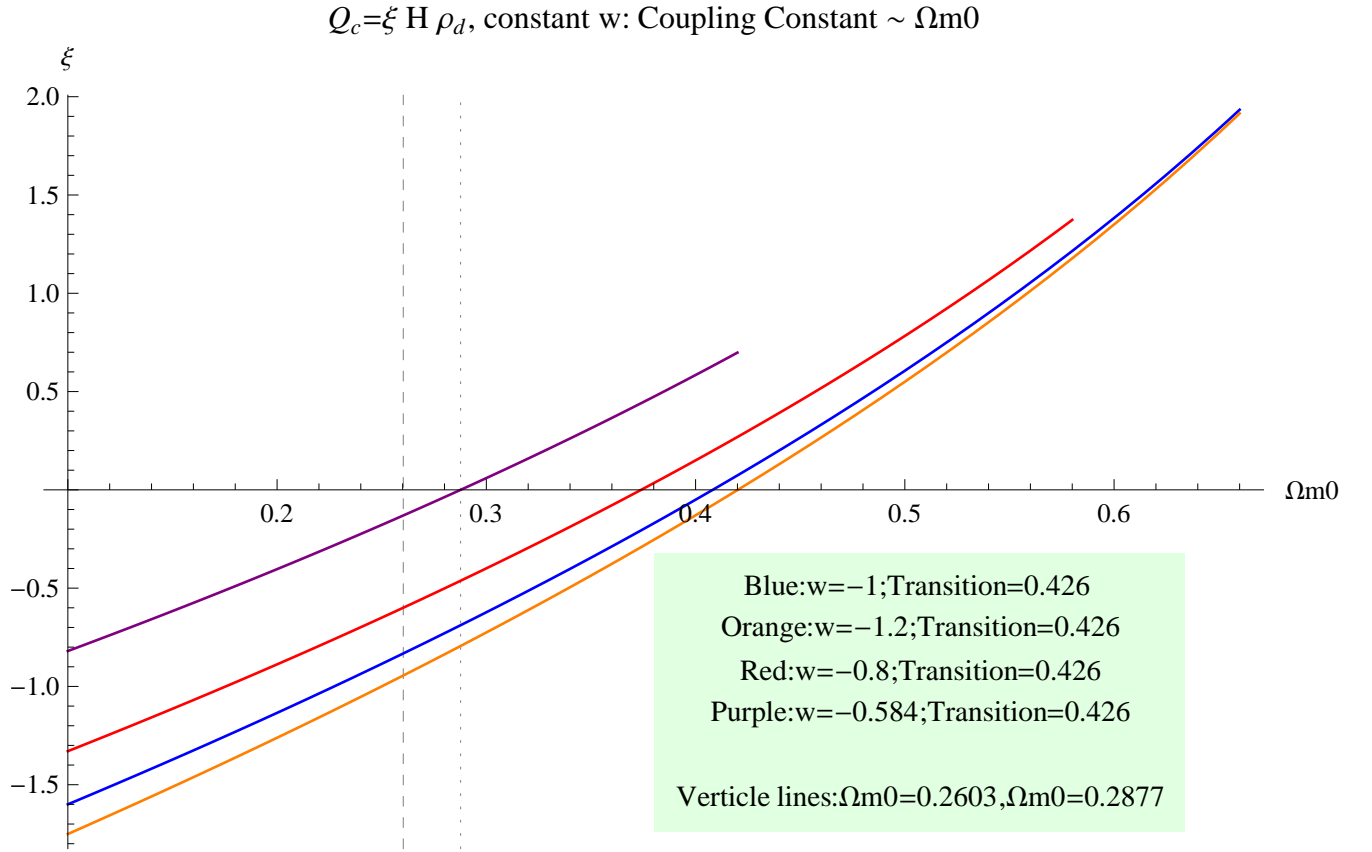


Figure 26: ξ VS Ω_{m0}

$Q_c = \xi H \rho_d$, CPL: Deceleration \sim Redshift

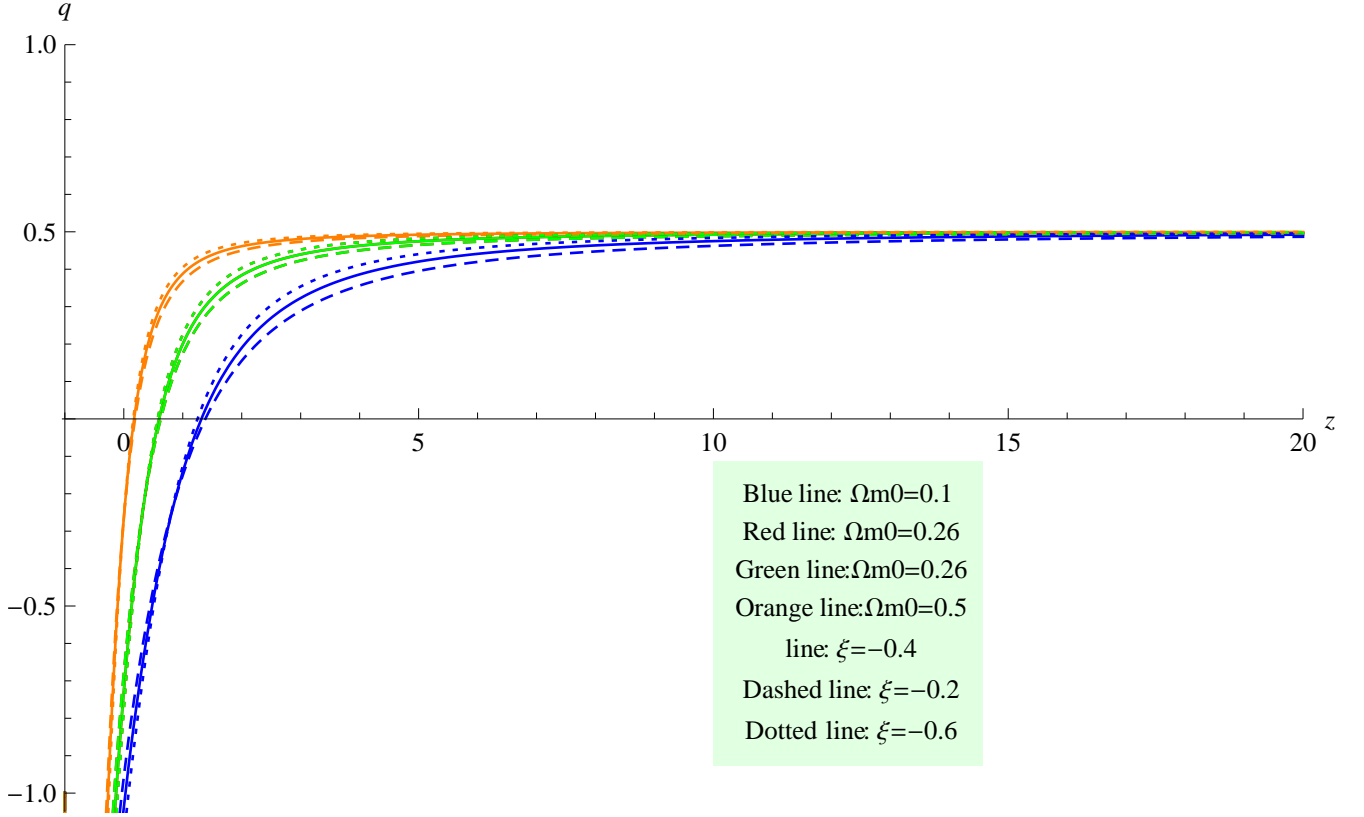


Figure 27: Deceleration parameter

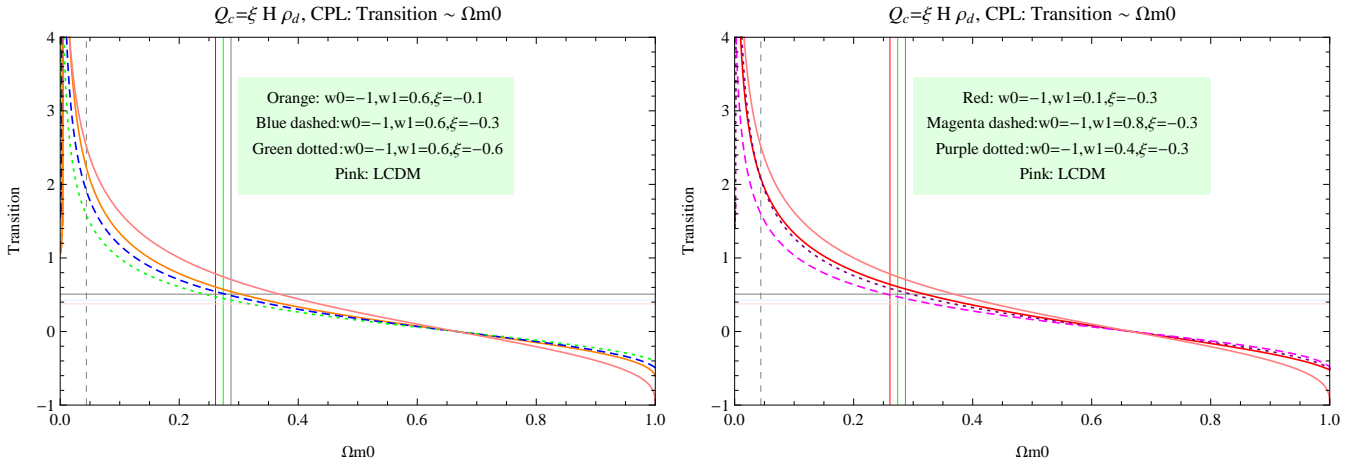


Figure 28: Transition VS Ω_{m0}

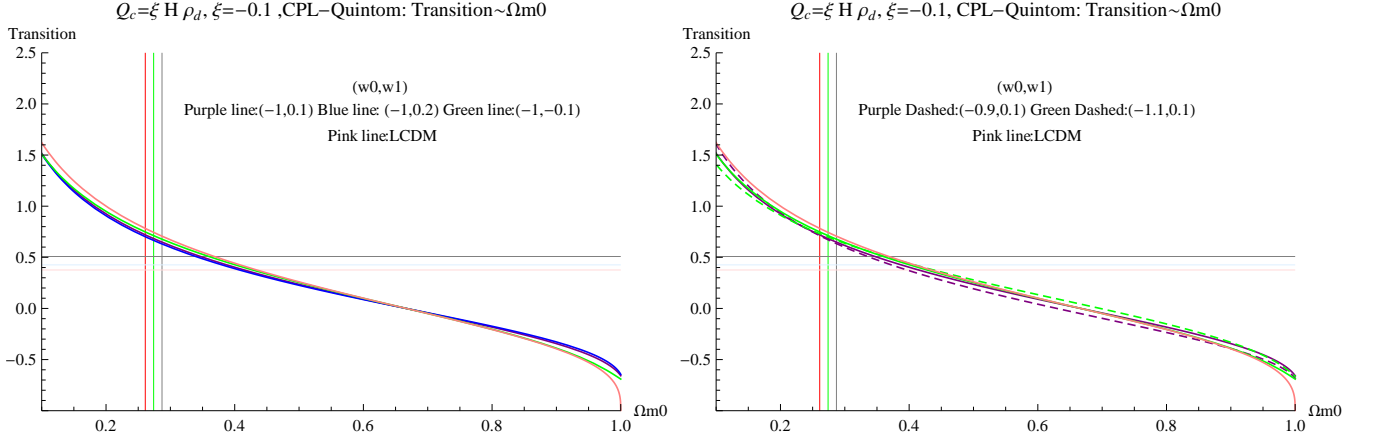


Figure 29: Transition VS Ω_{m0}

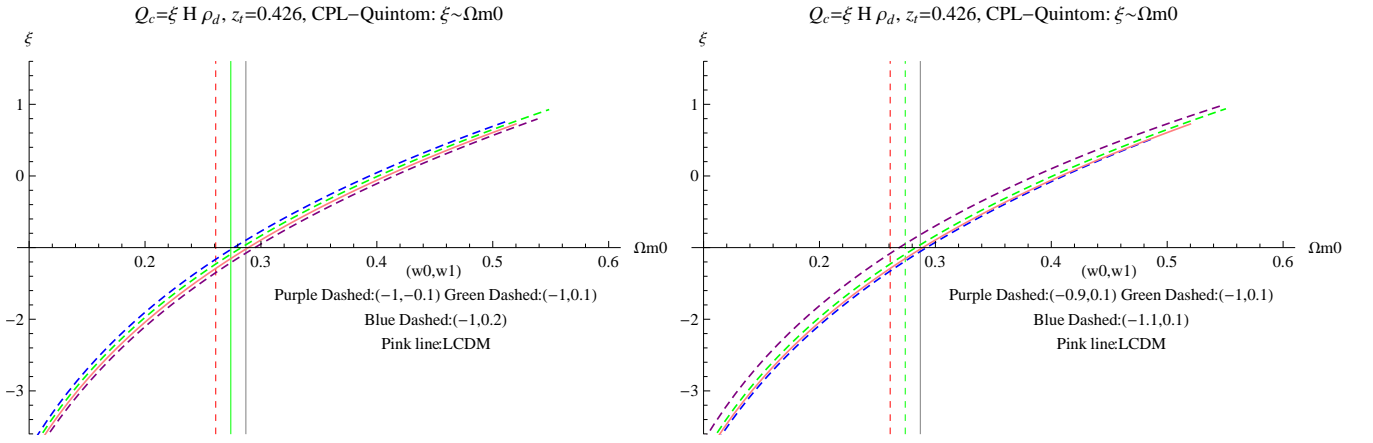


Figure 30: ξ VS Ω_{m0}

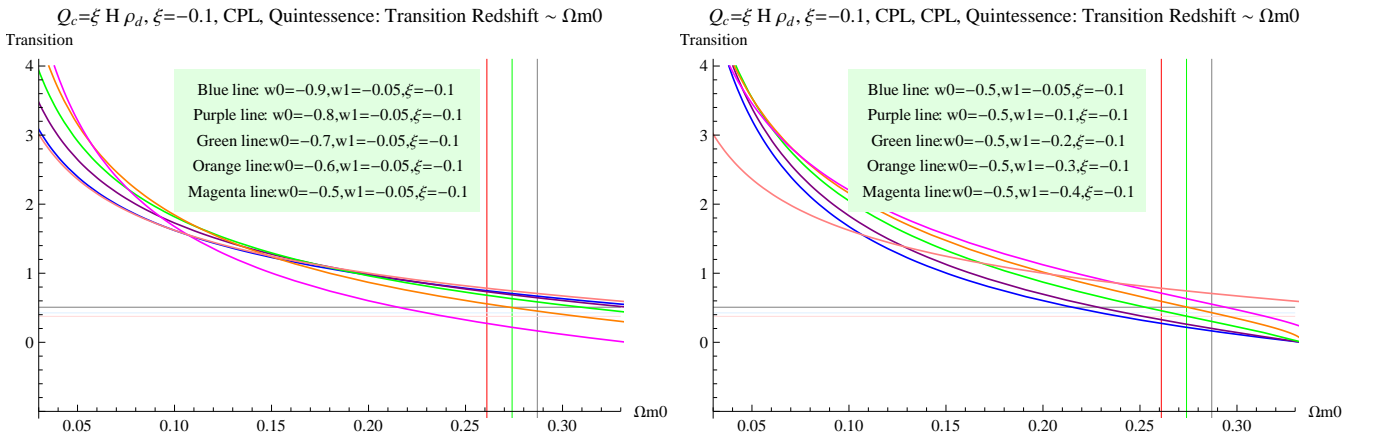


Figure 31: Transition VS Ω_{m0}

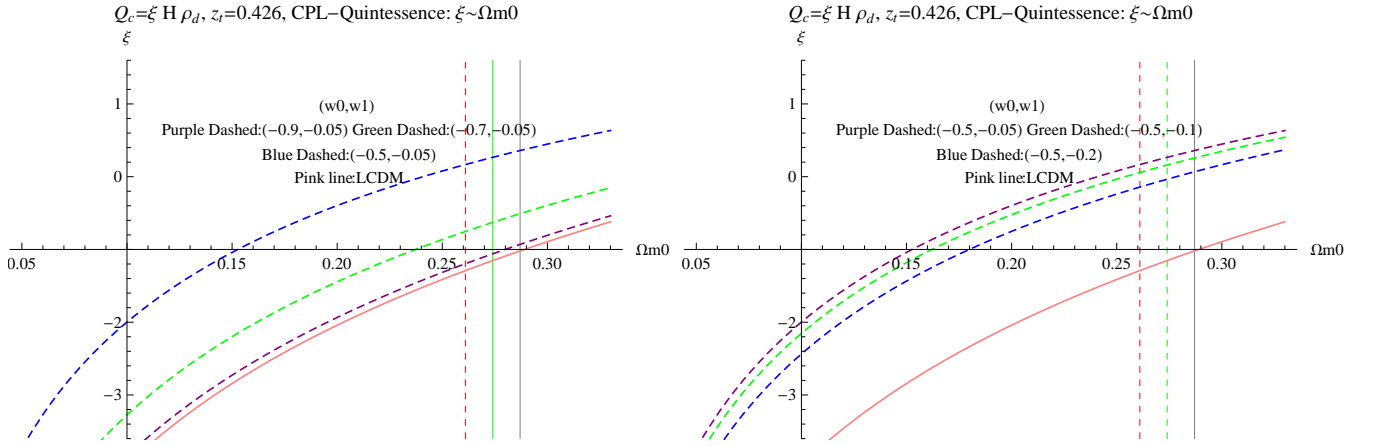


Figure 32: ξ VS Ω_{m0}

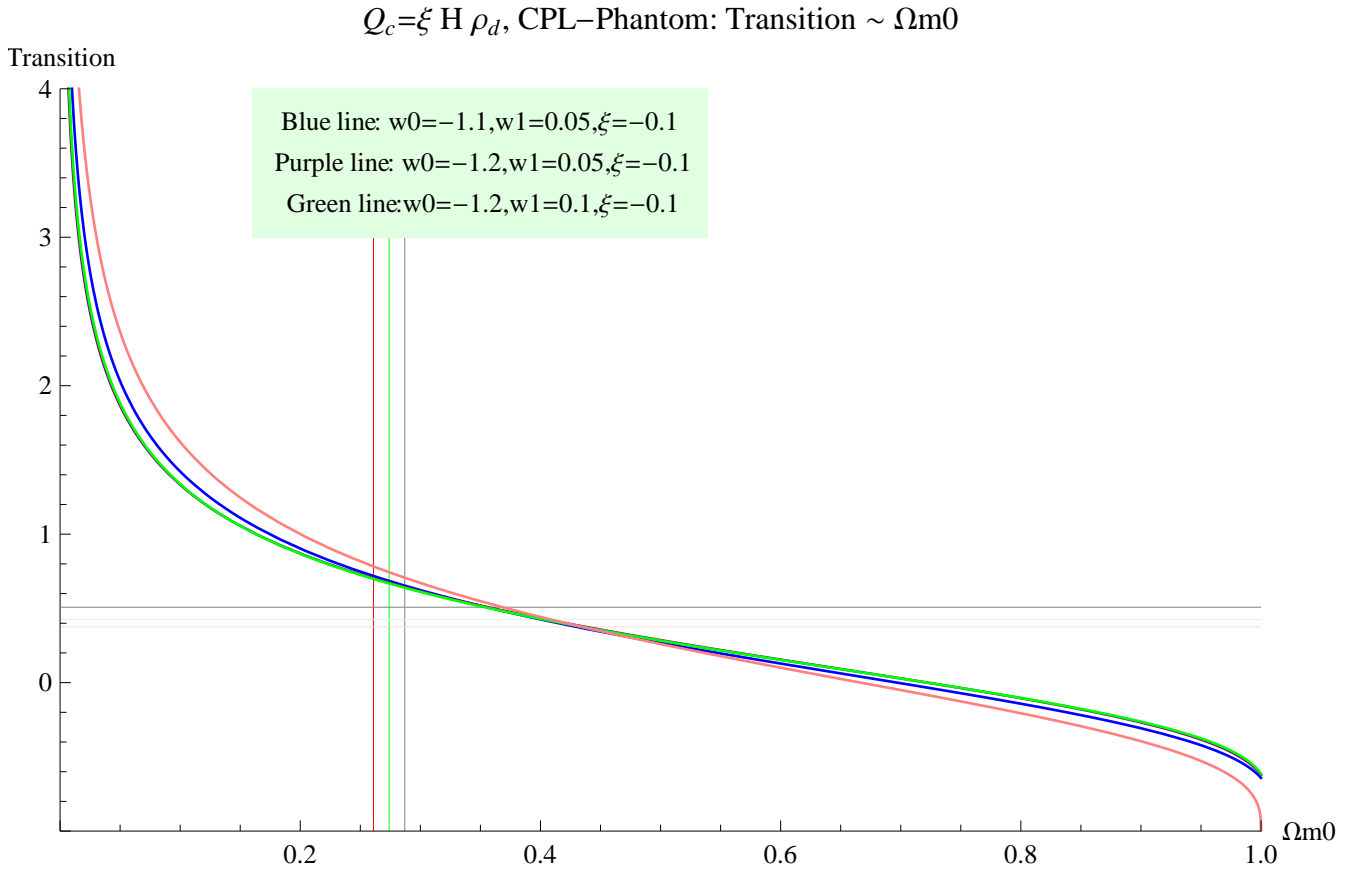


Figure 33: Transition VS Ω_{m0}

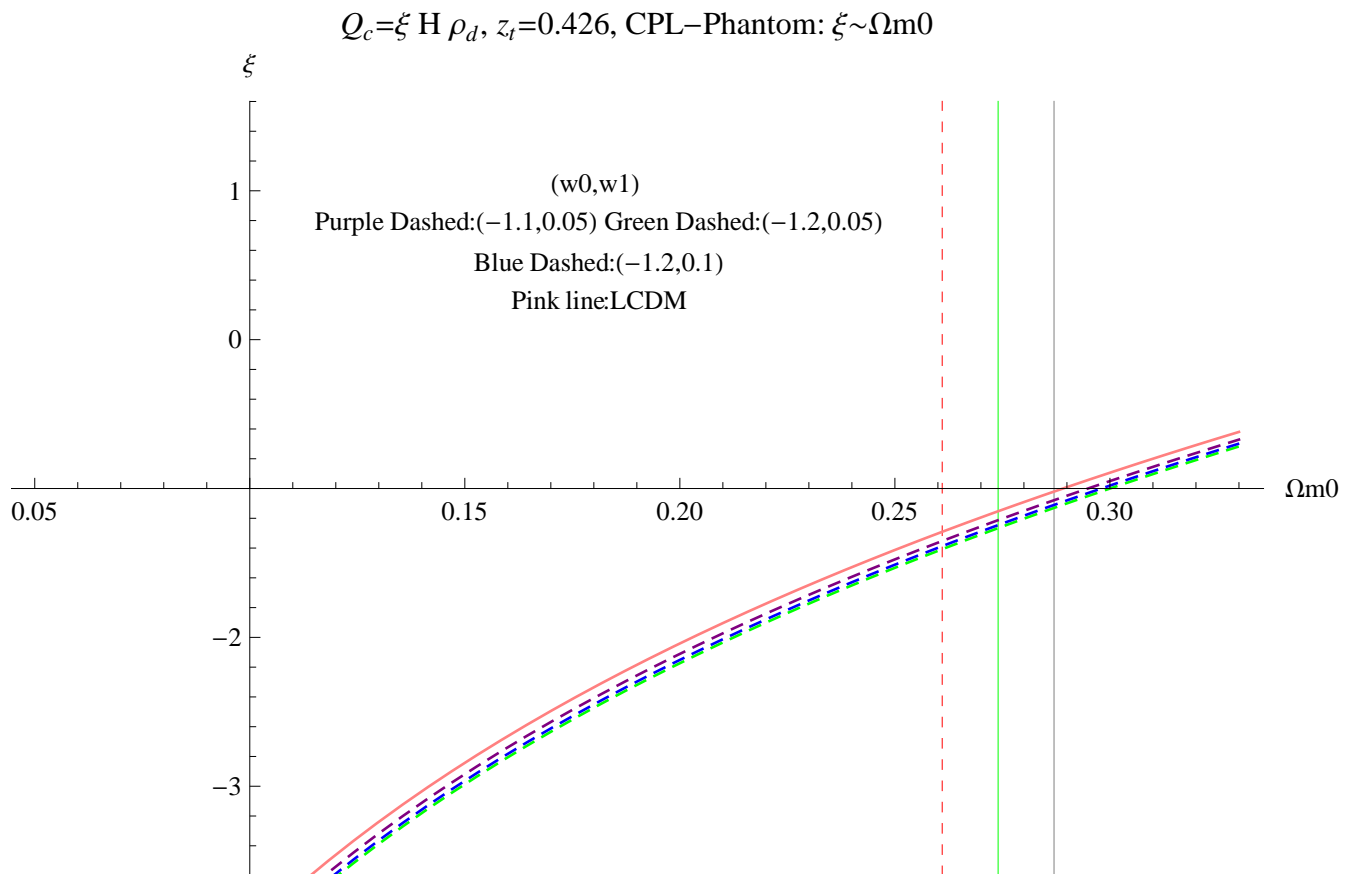
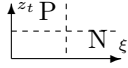

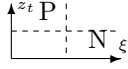
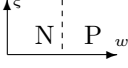


Figure 34: Transition VS Ω_{m0}

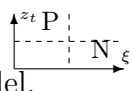
5 Summary

Table 1: Parameters in Different Models

Model	Parameter	Transition		Deceleration		ξ (fixed transition)	
		Behavior	Figures	Behavior	Figures	Behavior	Figures
ICC	ξ		4	N	3	-	-
	w	CC1	4				5, 6
	$\Omega m0$	N	4	P	3	P	7
ICCPL	ξ	CC2	9, 12, 13, 14, 17	N	8	-	-
	$w0$	CC3	9, 12, 13, 14, 17			P	15, 18, 21
	$w1$	CC	12, 13, 14, 17, 20			P	15, 18, 21
	$\Omega m0$	N	9, 12, 13, 14, 17, 20	P	8	P	15, 18, 21
I2CC	ξ		23	N	22	-	-
	w	CC4	23				24, 25
	$\Omega m0$	N	23	P	22	P	26
I2CCPL	ξ	CC	28, 29, 31, 33	N	27	-	-
	$w0$	CC	29, 31, 33			P	30, 32, 34
	$w1$	CC	28, 29, 31, 33			P	30, 32, 34
	$\Omega m0$	N	28, 29, 31, 33	P	27	P	30, 32, 34

Some notations in table 1:

- Models
 1. ICC: $Q_c = \xi H \rho_c$ with a constant EoS
 2. ICCPL: $Q_c = \xi H \rho_c$ with a CPL parametrized EoS
 3. I2CC: $Q_c = \xi H \rho_d$ with a constant EoS
 4. I2CCPL: $Q_c = \xi H \rho_d$ with a CPL parametrized EoS
- 1. N: negative correlated
 2. P: positive correlated

- CC means sort of complicated.
 1. CC1: Depends on Ωm_0 . Small Ωm_0 is P, large Ωm_0 is N.
 2. CC2: Complicated but in some proper region just like 
 3. CC3: In some region it behaves likes P, as in ICC model.
 4. CC4: Depends on Ωm_0 . Small Ωm_0 is P, large Ωm_0 is N.