Research Survival Handbook
(**Unfinished**)

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0.1 How to survive the calculations of Special Relativity

0.1.1 Important Relations

Metric in use

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1}$$

0.2 Quantities and Operations

d'Alembertian d'Alembert operator, or wave operator, is the Lapace operator in Minkowski space. [1]

$$\Box \equiv \partial_\mu \partial^\nu = \eta_{\mu\nu} \partial^\mu \partial^\nu \tag{2}$$

In the usual t,x,y,z natural orthonormal basis,

$$\begin{aligned} \Box &= -\partial_t^2 + \partial_x^2 + \partial_y^2 + \partial_z^2 && (3) \\ &= -\partial_t^2 + \Delta^2 && (4) \\ &= -\partial_t^2 + \nabla && (5) \end{aligned}$$

On wiki [2], they give some applications to it.

klein-Gordon equation $(\Box + m^2)\phi = 0$

wave equation for electromagnetic field in vacuum For the electromagnetic four-potential $\Box A^\mu = 0$[3]

wave equation for small vibrations $\Box_c u(t,x) = 0 \to u_{tt} - c^2 u_{xx} = 0$

[1] Actually, there are more general definations for Lapacian, which includes this d'Alembertian of course.

[2] wiki:D'Alembert_operator

[3] Gauge

0.3 Fields and Particles

0.3.1 Energy-Momentum Tensor for Particles

$$S_p \equiv -mc \int \int \mathrm{d}s\mathrm{d}\tau \sqrt{-\dot{x}^\mu g_{\mu\nu} \dot{x}^\nu} \delta^4(x^\mu - x^\mu(s)), \tag{6}$$

in which $x^\mu(s)$ is the trajectory of the particle. Then the energy density ρ corresponds to $m\delta^4(x^\mu - x^\mu(s))$.

The Largrange density

$$\mathcal{L} = -\int \mathrm{d}smc\sqrt{-\dot{x}^\mu g_{\mu\nu} \dot{x}^\nu} \delta^4(x^\mu - x^\mu(s)) \tag{7}$$

Energy-momentum density is $\mathcal{T}^{\mu\nu} = \sqrt{-g}T^{\mu\nu}$ is

$$\mathcal{T}^{\mu\nu} = -2\frac{\partial \mathcal{L}}{\partial g_{\mu\nu}} \tag{8}$$

Finally,

$$\begin{aligned} \mathcal{T}^{\mu\nu} &= \int \mathrm{d}s \frac{mc\dot{x}^\mu \dot{x}^\nu}{\sqrt{-\dot{x}^\mu g_{\mu\nu} \dot{x}^\nu}} \delta(t - t(s))\delta^3(\vec{x} - \vec{x}(t)) && (9) \\ &= m\dot{x}^\mu \dot{x}^\nu \frac{\mathrm{d}s}{\mathrm{d}t} \delta^3(\vec{x} - \vec{x}(s(t))) && (10) \end{aligned}$$

0.4 Theorems

0.4.1 Killing Vector Related

0.1. *ξ^a is Killing vector field, T^a is the tangent vector of geodesic line. Then $T^a \nabla_a (T^b \xi_b) = 0$, that is $T^b \xi_b$ is a constant on geodesics.*

0.5 Topics

0.5.1 Redshift

In geometrical optics limit, the angular frequency ω of a photon with a 4-vector K^a, measured by a observer with a 4-velocity Z^a, is $\omega = -K_a Z^a$.

0.5.2 Stationary vs Static

Stationay "A stationary spacetime admits a timelike Killing vector field. That a stationary spacetime is one in which you can find a family of observers who observe no changes in the gravitational field (or sources such as matter or electromagnetic fields) over time."

When we say a field is stationary, we only mean the field is time-independent.

Static "A static spacetime is a stationary spacetime in which the timelike Killing vector field has vanishing vorticity, or equivalently (by the Frobenius theorem) is hypersurface orthogonal. A static spacetime is one which admits a slicing into spacelike hypersurfaces which are everywhere orthogonal to the world lines of our 'bored observers'"

When we say a field is static, the field is both time-independent and symmetric in a time reversal process.