Research Survival Handbook
(**Unfinished**)

MA Lei

@ Interplanetary Immigration Agency

General Tips

- Pick moderate statembents. Extreme statements are usually wrong.

- Use Taylor expansion to deal with extreme cases, e.g. $h\nu \ll kT$, $e^{h\nu/kT} \approx 1 + \frac{h\nu}{kT}$.

- When knowing L^2 value, be careful to calculate l from $\hbar^2 l(l+1)$, two solutions.

- Conservation of momentum (including angular momentum) should be checked before conservation of energy.

- Be careful about dimension of the problem, e.g. in 3D, radial wave, $P = \int |\psi|^2 d\vec{r} = \int |\psi|^2 4\pi r^2 dr$

- Read underlined words carefully.

- Calculate T^4 carefully.

- Don't think too hard, the questions are easy enough to be solved in 2 minutes.

- Use method of elimination.

- Dimensional analysis is always useful.

- Usually order of magnitude calculation is good enough.

- In general, $F = -\nabla(\text{potential energy})$, but in E&M notice V stands for potential, not potential energy, so $F = -\nabla(q \cdot \text{potential})$

- Usually it is convenient to set $h = \hbar = c = \cdots = 1$, but if ans differs from choices, that's a signal we need to keep them.

- When you get stuck, take limits

- If some experimenter is involved in the question, it is usually a failed experiment.

- **Things to work on**

0.1 Thermodynamics and Statistical Mechanics

- PV diagram plots change in pressure wrt to volume for some process. The work done by the gas is the area **under the curve.**

- If the cyclic process moves clockwise around the loop, then W will be positive, and it represents a heat engine. If it moves counterclockwise, then W will be negative, and it represents a heat pump.

- The most basic definition of entropy is

$$dS \geq \frac{dQ}{T}$$

- Heat transfer

    - Conduction: rate

    $$H = \frac{\Delta Q}{\Delta t} = -kA\frac{T_2 - T_1}{L}, \qquad \frac{dQ}{dt} = -kA\frac{dT}{dx}$$

    where A is area, k is a constant.
    - Convection (probably not in GRE),

    $$H = \frac{\Delta Q}{\Delta T} = hA(T_s - T_\infty)$$

    where T_s is the surface temperature, h =convective heat-transfer coefficient. There are both natural and forced convections.

- Radiation

    $$\text{Power} = \epsilon\sigma AT^4$$

    ϵ =emissivity, $\epsilon \in [0, 1]$. Net loss= $\epsilon\sigma A(T_{\text{emission}}^4 - T_{\text{absorption}}^4)$

- Wien's displacement law: The absolute temperature of a blackbody and the peak wavelength of its radiation are inversely proportional:

    $$\lambda_{\text{max}}T = 2.898 \times 10^{-3} \text{ m·K}$$

- Ideal gas law

    $$PV = nRT = NkT$$

- Kinetic theory of gas

    $$P = \frac{1}{3}\rho v_{\text{rms}}^2 \qquad v_{\text{rms}} = \sqrt{\frac{3kT}{m}}, \qquad \bar{v} = \sqrt{\frac{8kT}{\pi m}}, \qquad v_{\text{most probable}} = v_m = \sqrt{\frac{2kT}{m}}$$

- Maxwell-Boltzmann distribution (less likely to be in GRE), number of molecules with energy between E and $E + dE$

    $$N(E)dE = \frac{2N}{\sqrt{\pi}(kT)^{3/2}}\sqrt{E}e^{-E/kT}dE$$

    $$f(v)d^3v = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT}d^3v$$

    $$P(v) = \sqrt{\frac{2}{\pi}}\left(\frac{m}{kT}\right)^{3/2} v^2 e^{-mv^2/2kT}$$

    (from which we can derive v_m)

- Mean free path of a gas molecule of radius b

$$l = \frac{1}{4\pi T b^2 (N/V)}$$

- Van der Waals equation of state

$$(P + an^2/V^2)(V - bn) = nRT$$

$$(P + aN^2/V^2)(V - Nb) = NkT$$

- Adiabatic process

$$PV^\gamma = \text{const}$$

For an ideal gas to expand adiabatically from $(P_1, V_1) \rightarrow (P_2, V_2)$, work done by the gas is

$$W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

derived from $W = \int_{V_1}^{V_2} P dV$.

- The greatest possible thermal efficiency of an engine operating between two heat reservoirs is that of a Carnot engine, one that operates in the Carnot cycle. Max efficiency is

$$y^\star = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}}$$

For the case of the refrigerator

$$\kappa = \frac{Q_{\text{cold}}}{W} \qquad \kappa_{\text{Carnot}} = \left(\frac{T_{\text{hot}}}{T_{\text{cold}}} - 1 \right)^{-1}$$

Carnot=adiabatic+isothermal, $dS = 0$. Otto=adiabatic+isobaric

$$y = 1 - \frac{T_d - T_a}{T_c - T_b}$$

- Dalton's Law

$$P = P_1 + P_2 = (n_1 + n_2)\frac{RT}{V}$$

- The critical isotherm is the line that just touches the critical liquid-vapor region

$$\left(\frac{dP}{dV} \right)_c = 0 \qquad \left(\frac{d^2 P}{dV^2} \right)_c = 0$$

with c the critical point. Equilibrium region is where pressure and chemical potential for the two states of matter equal, usually a pressure constant region in the $P - V$ diagram.

- In the Dulong-Petit law,

$$C_V = \frac{dE}{dT} = 3R$$

- Laws of thermodynamics

    - 0th: If two thermodynamic systems are each in thermal equilibrium with a third, then they are in thermal equilibrium with each other.
    - 1st: $\Delta U = Q - W$ (conservation of energy)
    - 2nd: Entropy increases/heat flows from hot to cold/heat cannot be completely converted into work.
    - 3rd: As $T \to 0$, $S \to$ constant minimum.

- Change in entropy for a system where specific heat and temperature are constant;

$$\Delta S = Nk \ln \frac{V}{V_0}$$

- Change in energy for an ideal gas:

$$\Delta U = C_V \Delta T$$

- Work done by ideal gas:

$$W = \int P dV = \begin{cases} NkT \ln \frac{V_2}{V_1} & \text{Isothermal} \\ P\Delta V & \text{Ideal gas, constant Pressure} \end{cases}$$

- Partition function:

$$Z = \sum_i e^{-\beta E_i} = \int dE\, \Omega(E) e^{-\beta E} = \int dE\, e^{-\beta A(E)}$$

where $A(E)$ is the Helmholtz free energy and $\Omega(E)$ is the degeneracy.

$$P(E_i) = \frac{e^{-\beta E_i}}{Z}$$

$$S = k \ln \Omega = -k \sum_i P_i \ln P_i$$

- Equipartition Theorem: (1) Classical canonical and (2) quadratic dependence: each particle has energy $\frac{1}{2}kT$ for each quadratic canonical degree of freedom.

- Internal energy

$$dU = TdS - PdV$$

Enthalpy

$$H = U + PV \qquad dH = TdS + VdP \qquad \text{isobaric}$$

Helmholtz

$$F = U - TS, \qquad dF = -SdT - PdV \qquad \text{isothermal}$$

Gibbs free energy

$$G = U - TS + PV, \qquad dG = -SdT + VdP$$

- Heat capacities:

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = T\left(\frac{\partial S}{\partial T}\right)_V$$

$$C_P = \left(\frac{\partial U}{\partial T}\right)_P + P\left(\frac{\partial V}{\partial T}\right)_P = T\left(\frac{\partial S}{\partial T}\right)_P = \left(\frac{\partial H}{\partial T}\right)_P$$

- Fun stuff:

$$\langle E \rangle = -\frac{\partial}{\partial \beta} \ln Z, \qquad F = -kT \ln Z$$

$$S = k \ln Z + \langle E \rangle / T, \qquad dS = \int \frac{dQ}{T}$$

Gibbs-Helmholtz equation.

$$U = F + TS = F - T\left(\frac{\partial F}{\partial T}\right)_V = -T^2 \left(\frac{\partial}{\partial T}\right)_V \left(\frac{F}{T}\right)$$

- Availability of system

$$A = U + P_0 V - T_0 S$$

In natural change, A cannot increase.

- Diatomic gas

$$U = \frac{5}{2}kT$$

- Maxwell Relations

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V = \frac{\partial^2 U}{\partial S \partial V}$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P = \frac{\partial^2 H}{\partial S \partial P}$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V = -\frac{\partial^2 A}{\partial T \partial V}$$

$$-\left(\frac{\partial S}{\partial P}\right)_T = \left(\frac{\partial V}{\partial T}\right)_P = \frac{\partial^2 G}{\partial T \partial P}$$

- For ideal gas in adiabatic process, $W = \Delta U = \frac{3}{2} N k \Delta T$

- Clockwise enclosed area in a $P - V$ diagram is the work done <u>by</u> the gas in a cycle.

- Chemical potential

$$\mu(T, V, N) = \left(\frac{\partial F}{\partial N}\right)_{T,V}$$

At equilibrium μ is uniform, F achieves minimum.

- $P_{\text{boson}} \propto T^{5/2}$, $P_{\text{classical}} \propto T$, $P_{\text{fermion}} \propto T_F$ (very big). $T_{\text{classical}} \gg T_{\text{boson}}$

- A thermodynamic system in maximal probability state is stable.

- Both Debye and Einstein assume $3N$ independent Harmonic oscillators for lattice. Einstein took a constant frequency

0.2 Quantum Mechanics

- Uncertainty principle

$$\Delta x \Delta p \geq \frac{\hbar}{2}, \qquad \Delta E \Delta t \geq \frac{\hbar}{2}$$

- Schrodinger equation

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V \Psi$$

- Commutator relation:

$$[AB, C] = ABC - CAB = ABC - ACB + ACB - CAB = A[B, C] + [A, C]B$$

- De Broglie

$$\lambda = \frac{h}{p} = \frac{hc}{E} = \frac{h}{\sqrt{2mkT}}$$

  (The last equality is thermal)

- A one-dimensional problem has no degenerate states.

- Heisenberg's uncertainty principle generalized:

$$\Delta A \Delta B \geq \frac{1}{2}|\langle [A, B] \rangle|$$

- Infinite square well

$$\psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}, \qquad E_n = \frac{n^2 \pi^2 \hbar^2}{2ma^2}, \qquad n \geq 1$$

  Delta-function well $V = -\alpha\delta(x)$. Only one bound state, many scattering states.

$$\psi(x) = \sqrt{\frac{m\alpha}{\hbar}} e^{-m\alpha|x|/\pi^2}, \qquad E = -\frac{m\alpha^2}{2\hbar^2}$$

  Shallow, narrow well, there is always at least one bound state.

- Selection rule

$$\Delta l = \pm 1, \qquad \Delta m_l = \pm 1 \text{ or } 0, \qquad \Delta j = \pm 1 \text{ or } 0$$

  Electric dipole radiation $\Leftrightarrow \Delta l = 0$. Magnetic dipole or electric quadrupole transitions are ``forbidden" but do occur occasionally.

- Stimulated and spontaneous emission rate $\propto |p|^2$ where

$$p \equiv q\langle \psi_b | z | \psi_a \rangle$$

  The lifetime of an excited state is $\tau = \left(\sum A_i \right)^{-1}$ where A_i are spontaneous emission rates.

- Time-independent first order perturbation

$$E_n^1 = E_n^0 + \langle \psi_n^0 | H' | \psi_n^0 \rangle, \qquad \psi_n^1 = \psi_n^0 + \sum_{m \neq n} \frac{\langle \psi_m^0 | H' | \psi_n^0 \rangle}{E_n^0 - E_m^0} \psi_m^0$$

- Quantum approximation of rotational energy

$$E_{\text{rot}} = \frac{\hbar^2 l(l+1)}{2I}$$

- Fermi energy

$$E_F = kT_F \simeq \frac{1}{2}mv^2$$

- Differential cross-section

$$\frac{d\sigma}{d\Omega} = \frac{\text{scattered flux/unit of solid angle}}{\text{incident flux/unit of surface}}$$

- Intrinsic magnetic moment

$$\vec{\mu} = \gamma\vec{S}, \qquad \gamma = \frac{eg}{2m}$$

  where g is the Lande g-factor. If m points up, $\vec{\mu}$ points down.

- Total cross section

$$\sigma = \int D(\theta)d\Omega, \qquad D(\theta) = \frac{d\sigma}{d\Omega}$$

- Stark effect is the electrical analog to the Zeeman effect.

- Born-Oppenheimer approximation: the assumption that the electronic motion and the nuclear motion in molecules can be separated, i.e.

$$\psi_{\text{molecule}} = \psi_e \psi_{\text{nuclei}}$$

- In Stern-Gerlach experiment, a beam of neutral silver atoms are sent through an inhomogeneous magnetic field. Classically, nothing happens as the atoms are neutral with Larmor precession, the beam would be deflected into a smear. But it actually deflects into $2s + 1$ beams, thus corroborating with the fact electrons are at spin $\frac{1}{2}$

- Know the basic spherical harmonics

$$Y_0^0 = \sqrt{\frac{1}{4\pi}}, \qquad Y_1^{\pm 1} = \mp\sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\phi}, \qquad Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

- Probability density current

$$\vec{J} = \frac{\hbar}{2mi}(\psi^*\nabla\psi - \psi\nabla\psi^*) = \Re\left(\psi^*\frac{\hbar}{im}\nabla\psi\right)$$

- Laser operates by going from lower state to high state (population inversion), then falls back on a metastable state in between (not all the way down due to selection rule).

- Neat identities:

$$\langle \mathcal{O} \rangle = \int \Psi^* \mathcal{O} \Psi dx, \qquad [f(x), p] = i\hbar \frac{\partial f}{\partial x}, \qquad p = -i\hbar \nabla$$

- Ehrenfest's Theorem: expectation values obey classical laws.

$$m \frac{d^2 \langle x \rangle}{dt^2} = \frac{d \langle p \rangle}{dt} = \left\langle -\frac{\partial V}{\partial x} \right\rangle$$

- If $V(x)$ is even, $\psi(x)$ can always be taken to be even or odd.

- More identities:

$$\langle H \rangle = \sum_{n=1}^{\infty} |c_n|^2 E_n, \qquad \delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx} dk$$

- Tunneling shows exponential decay.

- The ground state of even potential is even and has no nodes.

- In stationary states, all expectation values are independent of t.

- Harmonic oscillators:

$$H = \hbar\omega(a_- a_+ - \tfrac{1}{2}) = \hbar\omega(a_+ a_- + \tfrac{1}{2}), \qquad a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x \mp ip)$$

$$[a_- a_+] = 1, \qquad N \equiv a_+ a_-, \qquad N\psi_n = n\psi_n$$

$$a_+ \psi_n = \sqrt{n+1}\,\psi_{n+1}, \qquad a_- \psi_n = \sqrt{n}\,\psi_{n-1}$$

$$\psi_n = \frac{1}{\sqrt{n!}}(a_+)^n \psi_0, \qquad x = \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-), \qquad p = i\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-)$$

- Fourier transforms:

$$\Phi(p, t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} e^{-px/\hbar} \Psi(x, t) dx$$

$$\Psi(x, t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} e^{ipx/\hbar} \Phi(p, t) dp$$

- Operators changing in time:

$$\frac{d\langle Q \rangle}{dt} = \frac{i}{\hbar}\langle [H, Q] \rangle + \left\langle \frac{\partial Q}{\partial t} \right\rangle$$

- Virial theorem, in stationary state

$$2\langle T \rangle = \left\langle x \frac{dV}{dx} \right\rangle$$

- Hydrogen atom revisited:

$$E_n \propto \text{reduced mass}$$
$$\propto Z^2$$
$$\propto 1/n^2$$
$$= -\left[\frac{m}{2\hbar^2}\left(\frac{e^2}{2\pi\epsilon_0}\right)^2\right]\frac{1}{n^2} = \frac{E_1}{n^2}$$
$$E_n(Z) = Z^2 E_n$$
$$a(Z) = \frac{a}{Z}$$
$$R(Z) = Z^2 R$$

Bohr radius $a = 4\pi\epsilon_0\hbar^2/me^2 = 0.528 \times 10^{-10}$ meters.

$$\psi_{100}(r,\theta,\phi) = \frac{1}{\sqrt{\pi a^3}}e^{-r/a}$$

- Angular momentum

$$[L_i, L_j] = i\hbar L_k \epsilon_{ijk}$$

where $\epsilon_{ijk} = 1$ for even permutations, -1 for odd permutations, zero otherwise.

$$L_\pm = L_x \pm iL_y, \qquad [L^2, L_i] = 0$$
$$L^2 f_l^m = \hbar^2 l(l+1), \qquad L_z f_l^m = \hbar m f_l^m$$
$$L_\pm f_l^m = \hbar\sqrt{(l \mp m)(l \pm m + 1)}f_l^m = \hbar\sqrt{l(l+1) - m(m \pm 1)}f_l^m$$
$$[L_z, x] = i\hbar y, \qquad [L_z, p_x] = i\hbar p_y, \qquad [L_z, y] = -i\hbar x, \qquad [L_z, p_y] = -i\hbar p_x$$
$$L_z = \frac{\hbar}{i}\frac{\partial}{\partial\phi}$$

- Spin,

$$S^2 = \frac{3}{4}\hbar^2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\vec{S} = \frac{\hbar}{2}\vec{\sigma}$$

$$\chi_+^{(x)} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}, \qquad \chi_-^{(x)} = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$$

$$\langle S_x^2 \rangle = \langle S_y^2 \rangle = \langle S_z^2 \rangle = \frac{\hbar^2}{4}$$

- Clebsch-Gorden coefficients

$$|sm\rangle = \sum_{m_1+m_2=m} C^{s_1 s_2 s}_{m_1 m_2 m} |s_1 m_1\rangle |s_2 m_2\rangle$$

$$|s_1 m_1\rangle |s_2 m_2\rangle = \sum_s C^{s_1 s_2 s}_{m_1 m_2 m} |sm\rangle$$

- Continuity equation

$$\nabla \cdot \vec{J} = -\frac{\partial}{\partial t} |\psi|^2$$

$$\int_S \vec{J} \cdot d\vec{a} = -\frac{d}{dt} \int_V |\psi|^2 d^3\vec{r}$$

- Representation of angular momentum.

$$^{2s+1}\mathcal{L}_J$$

where $s =$ spin, $\mathcal{L} =$ orbital, $J =$ total. Hund's rule: (1) State with highest spin will have lowest energy given Pauli principle satisfied; (2) For given spin and anti-symmetrization highest $\mathcal{L}$ have lowest energy; (3) Lowest level has $J = |L - S|$, if more than half-filled $J = L + S$.

- Fermi gas

$$k_F = (3\rho\pi^2)^{1/3}, \qquad \rho = Nq/V, \qquad v_F = \sqrt{2E_F/m}$$

Degeneracy pressure

$$P \propto \rho^{5/3} m_e^{-1} m_p^{-5/3}$$

- Particle distributions

$$n(\epsilon) = \begin{cases} e^{-\beta(\epsilon-\mu)} & \text{Maxwell-Boltzmann} \\ (e^{\beta(\epsilon-\mu)} + 1)^{-1} & \text{Fermi-Dirac} \\ (e^{\beta(\epsilon-\mu)} - 1)^{-1} & \text{Bose-Einstein} \end{cases}$$

Blackbody density

$$\rho(\omega) = \frac{\hbar\omega^3}{\pi^2 c^3 (e^{\hbar\omega/kT} - 1)}$$

- Fine structure→spin-orbit coupling. Relativistic correction $\alpha = 1/137.056$. Then Lamb shift is from the electric field, then Hyperfine structure due to magnetic interaction between electrons and protons, then spin-spin coupling (21 cm line)

- Fine structure breaks degeneracy in l but still have j

- Fermi's golden rule is a way to calculate the transition rate (probability of transition per unit time) from one energy eigenstate of a quantum system into a continuum of energy eigenstates, due to a perturbation.

- Full shell and close to a full shell configuration are more difficult to ionize.

- Larmor precession:

$$\vec{\Gamma} = \vec{\mu} \times \vec{B} = \gamma \vec{J} \times \vec{B}$$

and we get $\omega = \gamma B$, where Γ is the torque, μ is the magnetic moment, and J is total angular momentum.

0.3 Atomic Physics

- $\Delta E = hf = \hbar\omega = hc/\lambda$. $hc = 12.4$ keV·Å $= 1240$ eV·nm, de Broglie wavelength $\lambda = h/p$.

- Emission due to transition from level n to level m

$$\frac{1}{\lambda} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

$m = 1$ Lyman series, $m = 2$ Balmer series.

$$R = 1.097 \times 10^7 \text{m}^{-1}, \qquad E_n = -\frac{13.6 \text{ eV}}{n^2}$$

- Hydrogen model extended, $Z =$number of protons, quantities scale as

$$E \sim Z^2, \qquad \lambda \sim \frac{1}{Z^2}$$

Reduced-mass correction to emission formula is

$$\frac{1}{\lambda} = \frac{RZ^2}{1 + m/M}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

where m is the mass of electron, M is the mass of the proton, $m/M = 1/1836$.

- Bohr postulate $L = mvr = n\hbar$

- Zeeman effect: splitting of a spectral line into several components in the presence of a static magnetic field.

- k series refers to the innermost shell (K, L, M, N) so transition to innermost shell.

$$E = -13.6(Z-1)^2\left(1 - \frac{1}{n_i^2}\right) \text{ eV}$$

where the $(Z-1)^2$ is a shielding approximation.

- Frank-Hertz Experiment: Electrons of a certain energy range can be scattered inelastically, and the energy lost by electrons is discrete.

- Spectroscopic notation is a standard way to write down the angular momentum quantum number of a state,

$$^{2s+1}L_j$$

where s is the total spin quantum number, $2s + 1$ is the number of spin states, L refers to the orbital angular momentum quantum number ℓ but is written as $S, P, D, F, \ldots$ for $\ell = 0, 1, 2, 3, \ldots$ and j is the total angular momentum quantum number. So for hydrogen we could have things like

$$^{2}P_{\frac{3}{2}}, {}^{2}P_{\frac{1}{2}}$$

(since $s = 1/2$ and $\ell = 1$, spin up versus spin down).

0.4   Special Relativity

- Energy:

$$E^2 = (pc)^2 + (mc^2)^2$$

For massless particles, $E = pc = h\nu$

- Relativistic Doppler Effect

$$\lambda = \sqrt{\frac{1 \pm \beta}{1 \mp \beta}} \lambda_0$$

$\beta = v/c$. Sign is determined by whether source is moving away or closer.

- Space-time interval

$$\Delta s^2 = c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

- Lorentz transformation

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

- Relativistic addition of velocities

$$u'_x = \frac{u_x + v}{1 + u_x v/c^2}, \qquad u'_y = \frac{u_y}{\gamma(1 + u_x v/c^2)}, \qquad u'_z = \frac{u_z}{\gamma(1 + u_x v/c^2)}, \qquad \gamma \equiv \frac{1}{\sqrt{1 - \beta^2}}$$

- Lorentz-Transformation of EM, parallel and perpendicular to direction o motion.

$$\vec{E}'_\parallel = \vec{E}_\parallel, \qquad \vec{E}'_\perp = \gamma(\vec{E}_\perp + \vec{v} \times \vec{B}_\perp)$$

$$\vec{B}'_\parallel = \vec{B}_\parallel, \qquad \vec{B}'_\perp = \gamma(\vec{B}_\perp - \vec{v} \times \vec{E}_\perp/c^2)$$

- Relativistic energy/momentum

$$E = \gamma mc^2, \qquad p = \gamma mv$$

- In every closed system, the total relativistic energy and momentum are conserved.

- Spacelike separation means two events can happen at the same time, which requires

$$\Delta s^2 = c^2\Delta t^2 - \Delta x^2 < 0$$

- Transverse Doppler shift:

$$f = \frac{f'}{\sqrt{1 - \beta^2}} \text{ or } f = f'\sqrt{1 - \beta^2}$$

- Four-vectors can be useful. We can define

$$\mathbf{P} = \left(\frac{E}{c}, \mathbf{p}\right)$$

and the dot product

$$\mathbf{P}^2 = \frac{E^2}{c^2} - p^2 = m^2c^2$$

to get

$$E^2 = m^2 + p^2.$$

Remember, this mass is invariant, so we can equate the **P** vector at different times.

0.5 Laboratory Methods

- If measurements are independent (or intervals in a Poisson process are independent) both expected value and variance increase linearly with time, so longer time can improve uncertainty, which is usually defined as

$$\frac{\sigma}{R} \propto \frac{1}{\sqrt{t}}$$

- In Poisson distribution, $\sigma = \sqrt{\bar{x}}$.

- Error analysis, estimating uncertainties. If you are sure the value is closer to 26 than to 25 or 27, then record best estimate 26 ± 0.5.

- Propagation of uncertainties for sum of random and independent variables

$$\delta x = \sqrt{\sum_i (\delta x_i)^2}$$

  If multiplication or divisions are involved, use fractional uncertainty:

$$\frac{\delta q}{|q|} = \sqrt{\sum_i \left(\frac{\delta x_i}{x_i}\right)^2}$$

- Experimental uncertainties can be revealed by repeating the measurements are called random errors; those that cannot be revealed in this way are called systematic errors.

- If the the uncertainties are different for different measurements, we have

$$\bar{x} = \frac{\sum(x_i/\sigma_i^2)}{\sum_i(1/\sigma_i)^2} \qquad \sigma_{\bar{x}}^2 = \frac{1}{\sum_i(1/\sigma_i^2)}$$

0.6   Specialized Topics

- Photoelectric effect.

$$E_{\text{photon}} = \phi + K_{\text{max}}$$

  (or the sum of the work function and the kinetic energy).

- Compton scattering:

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

  where m_e is the mass of the atom: $h/m_e c$ is the Compton wavelength of the electron, and λ' is the new wavelength.

- X-ray Bragg reflection

$$n\lambda = 2d\sin\theta$$

  (compare to **diffraction grating** $n\lambda = d\sin\theta$)

- 1.602×10^{-19}J$= e(1 \text{ V}) = 1$ eV.

- In solid-state physics, effective mass is

$$m^* = \frac{\hbar^2}{d^2 E/dk^2}$$

- Electronic filters: high pass means $\omega \to \infty$, $V_{\text{in}} = V_{\text{out}}$. Usually look at $I = V_{\text{in}}/Z$, $Z = R + i(X_L - X_C)$, $X_L = \omega L$, $X_C = 1/\omega C$.

- Band spectra is a term that refers to using EM waves to probe molecules.

- Solid state:

$$\text{primitive cell} = \frac{\text{unit cell}}{\text{\# of lattice points in a Bravais lattice}}$$

Simple cubic $\to$ 1 point, body-centered $\to$ 2 points, face-centered $\to$ 4 points.

- Resistivity of undoped semiconductor varies as $1/T$.

- Nuclear physics: binding energy is a form of potential energy, convention is to take it as positive. It's the energy needed to separate into different constituents. It is usually subtracted for other energy to tally total energy.

- Pair production refers to the creation of an elementary particle and its antiparticle. Usually need high energy (at least the total mass).

- At low energies, photoelectric-effect dominates Compton scattering.
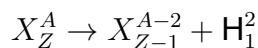
- Radioactivity: Beta decay

$$X_Z^A \to X_{Z+1}'^{A} + \beta_{-1}^0 + \nu$$

Alpha:

$$X_Z^A \to X_{Z-2}'^{A-4} + \text{He}_2^4$$
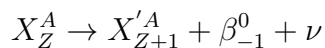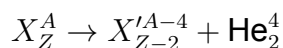
Gamma

$$X_Z^A \to X_Z^A + \gamma$$
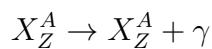
Deuteron decay (not natural)

$$X_Z^A \to X_{Z-1}^{A-2} + \text{H}_1^2$$

Radioactivity usually follows Poisson distribution.

- Coaxial cable terminated at an end with characteristic impedance in order to avoid reflection of signals from the terminated end of cable.

- Human eyes can only see things in motion up to ~ 25 Hz.

- In magnetic field, e are more likely to be emitted in a direction opposite to the spin direction of the decaying atom.

- Op-amp (operational amplifiers): if you only have two days to prepare for the GRE, this is not worth the effort, maximum one question on this. Read ``The Art of Electronics'' to check this out.

- The specific heat of a superconductor jumps to a lower value at the critical temperature (resistivity jumps too)

- Elementary particles: review the quarks, leptons, force carriers, generations, hadrons.

  - Family number conserved
  - Lepton number conserved
  - Strangeness is conserved (except for weak interactions)
  - Baryon number is conserved

- Internal conversion is a radioactive decay where an excited nucleus interacts with an electron in one of the lower electron shells, causing the electron to be emitted from the atom. It is not beta decay.