Harrison Zeldovich Prescription

Harrison(1970) & Zeldovich(1972)

1 Harrison Zeldovich Prescription

All perturbations that come into the horizon have the same amplitude.

$$\Delta(k_i, t_i) = \Delta(k_e, t_e) = \text{Const.} \tag{1}$$

Attention $k = 2\pi/L$ is the comoving wavenumber.

1.1 What does that mean? & Evolution of This Prescription.

One kind of understanding of the assumption is

1. The perturbations are generated to be the same value (comoving value).
2. When the perturbations are outside of the horizon, their comoving measurement do not evolve.

However, this might be wrong (and it is wrong [1]). This only gives us a intuitive inspiration. The only thing we know is 1.

Am I wrong?

The assumption firstly used is $\delta_0 = \beta_E N^{-n}$. Or equivalently, $\delta_0 \propto L^{-3n}$. Since Harrison proved that there is a throshold epoch where the initial perturbations are located, all the perturbations generated at that epoch can be discribed as $\delta_0 \sim L^{-3n} \sim k^{3n}$ and $n = \frac{2}{3}$. i.e., primiordial perturbations are $\delta_0 \sim L_0^{-2} \sim a(t_H)^2/L^2_{H-physical}$. [$t_0$ denotes the time when the perturbation are generated. t_H is the time of horizon crossing, i.e., $2ct_H = \lambda_H$.]

> During RD, $a(t)^2 = 2\sqrt{C}t$, then $\delta_H \sim a(t_H)^2/t_H^2 \sim a(t_H)^2/a(t_H)^4 \sim 1/t_H$. The amplitude of the perturbations are time dependent because the moment of horizon crossing t_H are different for different perturbations. So what are the mistakes here?

If $\delta_H \sim k_H^2$ remains the same for all perturbations and $P(k_H) \sim k_H$, from the defination of another measurement of perturbations

$$\Delta(k) = \frac{k^3}{2\pi^2} P(k), \tag{2}$$

we have $\Delta^2 = \delta_H^2$ when the perturbation comes into the horizon ($k = \lambda_H$), which indicates 1.

Generally speaking, one would choose the initial power spectrum to be power law form, i.e., $P(k) \propto k^n$, or evern more simple, scale free Zeldovich spectrum, $P(k) \propto k$. [2]

1.1.1 Primitive Spectrum

It is expected that Δ is scale free. The primitive spectrum given by inflation (or just a dimension analysis) is

$$P_\Phi(k) = \frac{50\pi^2}{9k^3} \left(\frac{k}{H_0}\right)^{\hat{n}-1} \delta_H^2 \left(\frac{\Omega_m}{D_1(a=1)}\right)^2 \tag{3}$$

When $\hat{n} = 1$, that is exact a scale invariant perturbation, i.e., $P(k) \sim k^{-3}$

THE PROBLEM IS, How do the orginal $\mu_0 \sim L_0^{-3n} \sim k_0^2$ evolve to $P(k_0) \sim k_0$?

$$P(k_0) \sim \mu_0^2/k_0^3 \sim k_0. \tag{4}$$

[1] Even the largest scale potential perturbations damp to 9/10 of their original value.

[2] `http://fisica.usac.edu.gt/public/curccaf_proc/borganihtml/node5.html`