Power Spctrum & Its Evolution

MA Lei

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Progress Note

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1 Power Spectra & Gauge

1.1 Defination

Convention 1.1 (Line Element)**.**

$$\mathrm{d}s^2 = -a^2(1+2AY)\mathrm{d}\tau^2 - a^2 BY_j \mathrm{d}t\mathrm{d}x^j + a^2(\gamma_{ij} + 2H_L Y\gamma_{ij} + 2H_T Y_{ij})\mathrm{d}x^i \mathrm{d}x^j$$

Defination 1.1 (Power Spectrum)**.** *Power spectrum in k space is defined as*

$$P(k) = \left\langle \left| D_g(\vec{k}) \right|^2 \right\rangle$$

in which $\Delta_g = \delta_k + 3(1+w)\mathcal{R}$ stands for the energy density contrast in flat slicing, i.e., $\mathcal{R} \equiv H_L + \frac{1}{3}H_T = 0$.[1]

Some dimensional analysis will be done in the following.

Energy density contrast in real space is defined to be dimension free, that is to say, energy density contrast in fourier space have the demension of $[L]^3$ since $[\delta(\vec{r})] \sim [\delta_k][\mathrm{d}^3k]$ within which quantities in square brackets stand for their dimension.

Problem is, quantities that are not dimension free will lead to "discrepancies and floating-point errors in computations"[2]. Then most people are glad to use the dimensionless power spectrum[3],

$$\Delta_k^2 \equiv \frac{4\pi k^3 P(k)}{(2\pi)^3} \tag{1}$$

When we are talking about matter power spectrum we always use a dimensionless one from now on. (CMBEASY plotted $P_k^{(s)} = \frac{k^3\langle|\delta|^2\rangle}{2\pi^2}$ as the sychronous gauge power sepctrum while $P_k^{(i)} = \frac{k^3\langle|\Delta_g|^2\rangle}{2\pi^2}$. Michael Doran called this the gauge ambiguities. In other words, Michael Doran use different definations for different gauge! He mentioned that "Had I plotted the synchronous gauge dnsity contrast infered from the gauge invariant one, the two curves would of course fall on top of each other.[4]" The invariant power spectrum $P_k^{(i)}$ reduces to sychronous gauge power spectrum $P_k^{(s)}$) when we consider power sepctrum for cdm under horizon after LSS. From this point of view, this defination is not so treachery and it is not bad to stick to Micheal Doran's defination.)

A calculation in *Cosmology (2008)* by Weinberg shows that under adiabatic initial condition and with spectral index $n_s = 1$,

$$P_k \equiv \Delta_k^2 = \mathrm{Const.} \cdot kT^2(k).$$

[1] I have no idea why Ruth Durrer define it this way. D_g is some kind of energy density contrast when the curvature $\mathcal{R}$ vanishes. Maybe this curvature independent defination is better than other curvature dependent energy contrast definations.

[2] http://www.ocf.berkeley.edu/ adriand/class/files/p228/11.pdf (Page 3).

[3] The role of $(2\pi)^3$ in Fourier transformation can be reversed. Then other possible defination of such a dimensionless power spectrum is to time a $(2\pi)^3$ factor on the following one.

[4] http//cosmocoffee.info/viewtopic.php?t=204&highlight=astroph+0503277

Trnasfer function $T(k) \sim \frac{1}{k^2}$ ($\rightarrow P_k \sim \frac{1}{k^3}$) when k is very large and $T(k) \sim$ Const. ($\rightarrow P_k \sim k$) when k is very small.[5]

To get a intuitive view of Weinberg's calcuation, we consider the following statements.

Given a HZ initial power spectrum $P_k = Ak$, the current power spectrum has the same value at small wave number (shown in Figure 1[6]).

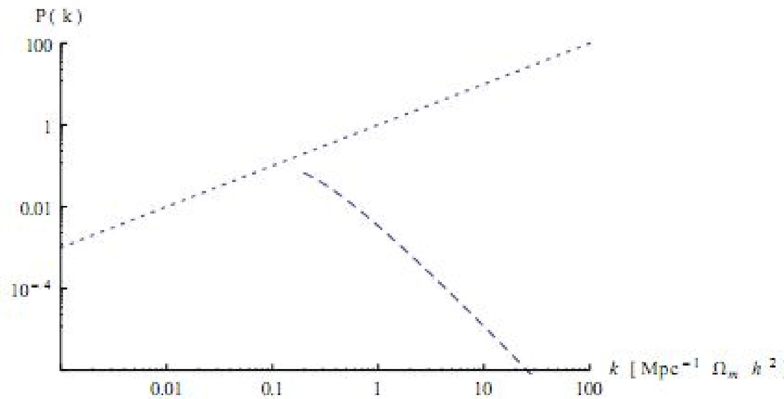

Figure 1: Current power spectrum keeps the initial value at small wave number.

As a reminder, $P(k)$ with $[Length]^3$ dimension can also be calculated easily by dividing P_k by k^3.

[5]Weinberg uses different notations. He uses q for comoving wave number while here we are using k for that. His calculation of this power spectrum is under sychronous gauge.

[6]`http://www.itp.uzh.ch/courses/phy513`