Power Spectra of f(R) Models

1.1 Conventions

- $' = \frac{\mathrm{d}}{\mathrm{d}a}$
- Lagrange density of gravity

$$\mathcal{L} = R + f(R) \tag{1}$$

- $f_R \equiv \frac{\mathrm{d}f}{\mathrm{d}R}$.
- $f_{RR} \equiv \frac{\mathrm{d}^2 f}{\mathrm{d}R^2}$.
- Ω_{i0} is the current fraction density of i component.
- Ω_i is the fraction density of i component at time a, i.e., $\Omega_i \equiv \Omega_i(a)$.
- Ω is the total density at time a, i.e., $\Omega \equiv \Omega(a)$.
- Dimension
  1. $[H] \sim \mathrm{Mpc}^{-1}$
  2. $[R] \sim \mathrm{Mpc}^{-2}$
  3. $[f(R)] \sim [R] \sim \mathrm{Mpc}^{-2}$

1.2 Models

- Hu&Sawicki Model

$$f_{HS} = -m^2 \frac{C_1(\frac{R}{m^2})^n}{C_2(\frac{R}{m^2})^n + 1} \tag{2}$$

Its series expansion at $m^2/R \sim 0$ is [1]

$$f_{HS} \approx -\frac{C_1}{C_2}{}_1 m^2 + \frac{C_1}{C_2^2} m^2 (\frac{m^2}{R})^n - \frac{C_1}{C_2^3} m^2 (\frac{m^2}{R})^{2n} + \frac{C_1}{C_2^4} m^2 (\frac{m^2}{R})^{3n} \tag{3}$$

[1] From LCDM model, this quantity is much smaller than 1 through out the history. Three orders are kept because its second derivative of R will be used.

- Starobinsky Model

$$f_S = \lambda R_s[(1+\frac{R^2}{R_s})^{-n} - 1] \tag{4}$$

Its series expansion at $R_s^2/R^2 \sim 0$ is

$$f_S \approx -\lambda R_S + \lambda R_S(\frac{R_S^2}{R^2})^n - \lambda R_S \cdot n(\frac{R_S^2}{R^2})^{n+1} + \lambda R_S \cdot \frac{1}{2}(n+n^2)(\frac{R_S^2}{R^2})^{n+2} \tag{5}$$

$$-\lambda R_S \cdot \frac{1}{6}(2n+3n^2+n^3)(\frac{R_S^2}{R^2})^{n+3} \tag{6}$$

- Tsujikawa Model

$$f_T = -\mu R_T \tanh(\frac{R}{R_T}) \tag{7}$$

Its approximation at $R_T/R << 1$ is

$$f_T \approx -\mu R_T + 2\mu R_T e^{-2\frac{R}{R_T}} - 2\mu R_T e^{-4\frac{R}{R_T}} + 2\mu R_T e^{-6\frac{R}{R_T}} - 2\mu R_T e^{-8\frac{R}{R_T}} + 2\mu R_T e^{-10\frac{R}{R_T}} \tag{8}$$

- Exponential Gravity Model

$$f_E = -\beta R_E(1 - e^{-R/R_E}) \tag{9}$$

1.3 Hu&Sawicki Model

1.3.1 Parameters and more

. $n = 1$

. $d_1 = 6\frac{\Omega_{x0}}{\Omega_{m0}}$

. $d_2 = -f_{R0}\left(\frac{12}{\Omega_{m0}} - 9\right)^2$

. $\Omega = \Omega_{m0}a^{-3} + \Omega_{r0}a^{-4}$

1.3.2 Background

Hubble function is just an rearrangement of Friedmann equationk, which is

$$H = \sqrt{\frac{H_0^2\Omega + \frac{1}{6}(f_R \cdot R - f)}{1 + f_R + 3af_{RR} \cdot R'}} \tag{10}$$

For Hu&Sawicki' model, the related terms are

* $R = 3H_0^2\Omega + 2d_1m^2$. [2]

[2] Wayne Hu and Ignacy Sawicki, Models of $f(R)$ cosmic acceleration that evade solar system tests, Phys. Rev. D 76, 064004 (2007).

* $f \approx -\frac{C_1}{C_2}m^2 + \frac{C_1}{C_2^2}m^2\frac{m^2}{R^2} - \frac{C_1}{C_2^3}m^2(\frac{m^2}{R})^2 + \frac{C_1}{C_2^4}(\frac{m^2}{R})^3$
* $f_R \approx -\frac{C_1}{C_2^2}(\frac{m^2}{R})^2 + 2\frac{C_1}{C_2^3}(\frac{m^2}{R})^3$
* $f_{RR} \approx 2\frac{C_1}{C_2^2}\frac{m^4}{R}$

Put them into Eq (10),and drop high order terms,

$$H = H_0\sqrt{\frac{\Omega + \frac{1}{6}d_1\Omega_{m0} - \frac{1}{3}d_2\Omega_{m0}\frac{1}{\hat{R}}a^{-3}}{1 - d_2\frac{\Omega_{m0}^2}{\hat{R}^2} - 27 \times 2d_2\frac{\Omega_{m0}^3}{\hat{R}^3}a^{-3}}}, \tag{11}$$

where $d_1 \equiv \frac{C_1}{C_2}$, $d2 \equiv \frac{C_1}{C_2^2}$, $\Omega = \Omega_{m0}a^{-3} + \Omega_{r0}a^{-4}$.

1.3.3 Perturbation Theory

Perturbation equation in Newtonian gauge, matter domination era and later

$$\delta_m'' + \left(\frac{H'}{H} + \frac{3}{a}\right)\delta_m' - \frac{3}{2}\bar{\chi}_{eff}\frac{H_0^2}{aH^2}\Omega_m\delta_m = 0, \tag{12}$$

where $\bar{\chi}_{eff} \equiv \frac{\chi_{eff}}{\chi} = \frac{1}{f_R+1}\frac{4+(\frac{f_R+1}{f_{RR}-R})\frac{a^2}{k^2}}{3[1+\frac{1}{3}(\frac{f_R+1}{R_{RR}}-R\frac{a^2}{k^2})]}$, $\chi_{eff}^2 \equiv 8\pi G_{eff}$ is corresponds to $\chi^2 \equiv 8\pi G$ in LCDM.

A numerical calculation shows $\bar{\chi}_{eff}$ doesn't change the solutions of the perturbation. I have checked both large k and small k at about $a \sim 10$, the result doesn't change at a four digit precision. Besides, I plotted out $\bar{\chi}_{eff}$ (figure 1) and it only starts change slightly at about $a = 0.2$. So I drop this $\bar{\chi}_{eff}$ term in the following numerical calculation.

1.3.4 Results

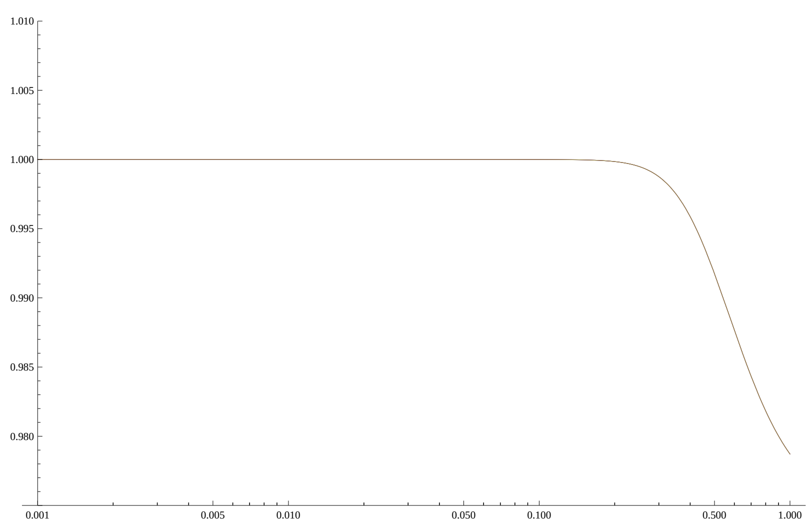

Figure 1: $\bar{\chi}_{eff}$ of Hu&Sawicki model.

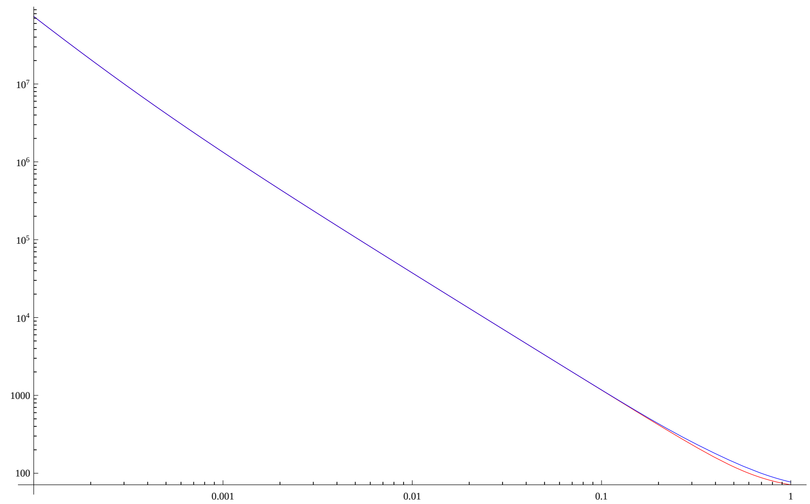

Figure 2: Hubble function of Hu&Sawicki model and LCDM. Upper line (blue) is the Hu&Sawicki model. Lower line (red) is for LCDM.

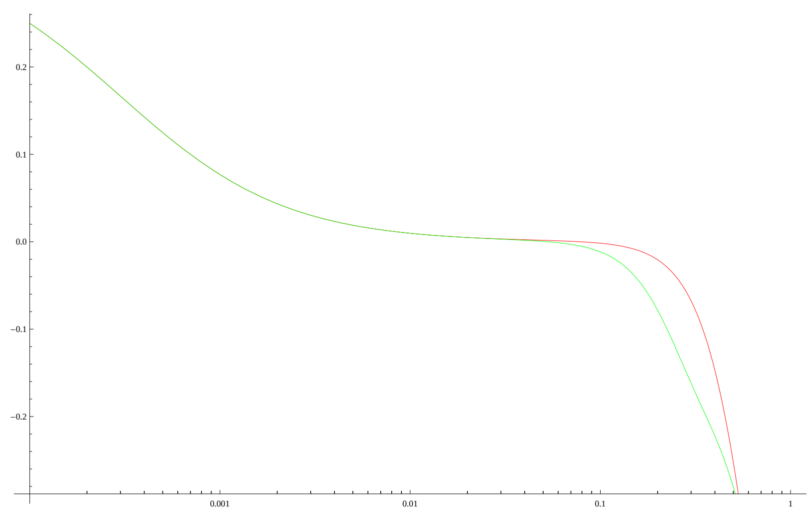

Figure 3: Effective EoS of Hu&Sawicki model. Red line is LCDM model while green line is Hu&Sawicki model.

Figure 2 shows the Hubble function.

Figure 3 is the effective equation of state of Hu&Sawicki model.

Figure 4 is the growth function of Hu&Sawicki model. (It is a pity that this figure does not completely show the growth function.)

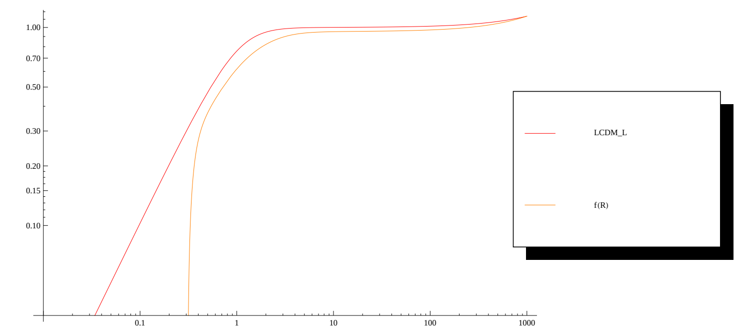

Figure 4: Growth function of Hu&Sawicki model.

Figure 5 is $Q = \frac{\text{Power of Hu\&Sawicki model}}{\text{Power of LCDM model}}$ factor. (I do not have the power spectra of LCDM in Newtonian gauge, so this is the furthest I can go.)

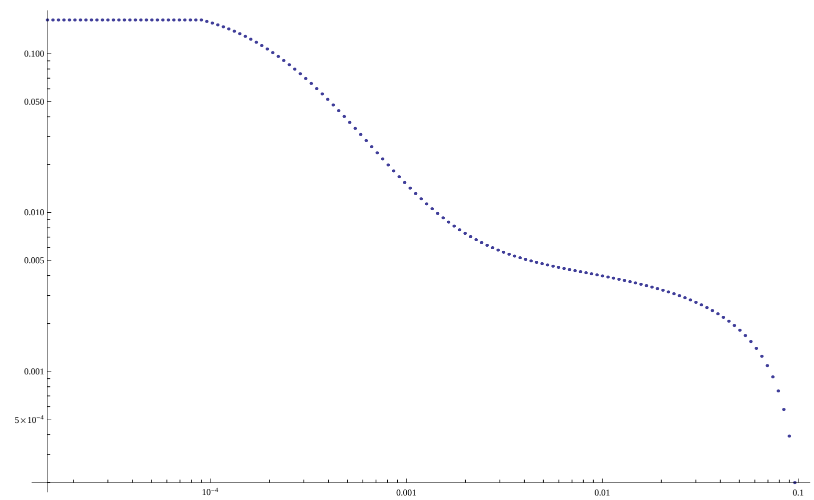

Figure 5: $Q = \frac{\text{Power of Hu\&Sawicki model}}{\text{Power of LCDM model}}$.

1.4 Starobinsky Model

1.4.1 Parameters

. $n = 1$

. $\lambda = 6\frac{\Omega_{x0}}{\Omega_{m0}}$

. $R_S = H_0^2\Omega_{m0}$

1.4.2 Background

* $f = -\lambda \frac{R_S}{c^2} + \lambda \frac{R_S}{c^2} \frac{R_S^2}{R^2} - \lambda \frac{R_S}{c^2} (\frac{R_S^2}{R^2})^2$

* $f_R = -2\lambda \frac{R_S^3}{R^3} + 4\lambda \frac{R_S^5}{R^5}$

* $f_{RR} = 6\lambda \frac{R_S^3}{R^4} c^2$

A similar method (using Eq (10)) shows the background of Starobinsky's model. (Since the equation is a bit complicated, I won't write it here. It is in a mathematica notebook.)

1.4.3 Perturbations

Similar to Hu&Sawicki's model, that $\bar{\chi}_{eff}$ can be substituted by 1. The fact is this model adds much less corrections to LCDM than Hu&Sawicki's.

1.4.4 Results

Figure 6 is $\bar{\chi}_{eff}$.

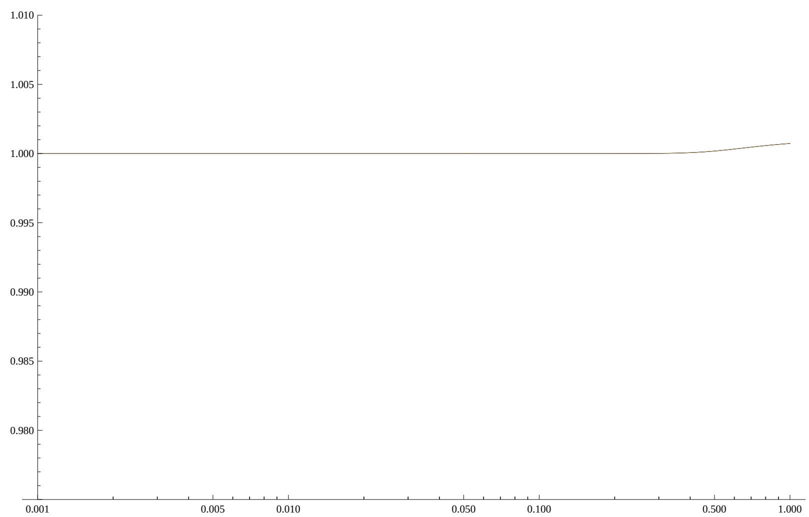

Figure 6: $\bar{\chi}_{eff}$ of Starobinsky' model.

Figure 7 is the Hubble function of Starobinsky' model. The two lines are the same. (Not really the same, the deviations are too small to be shown.)

Figure 8 is the effective equation of state.

Figure 9 is the growth function of Starobinsky's model.

Figure 10 is $Q = \frac{\text{Power of Hu\&Sawicki model}}{\text{Power of LCDM model}}$.

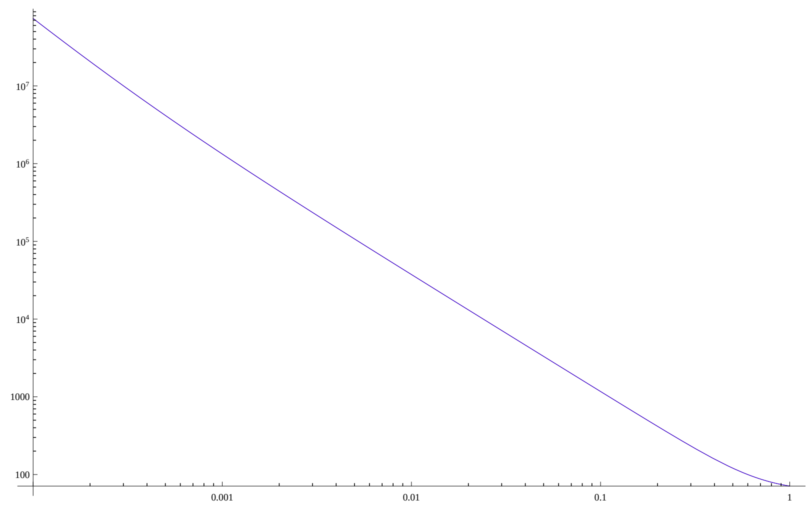

Figure 7: Hubble function of Starobinsky's model and that of LCDM. They are completely the same in this figure.

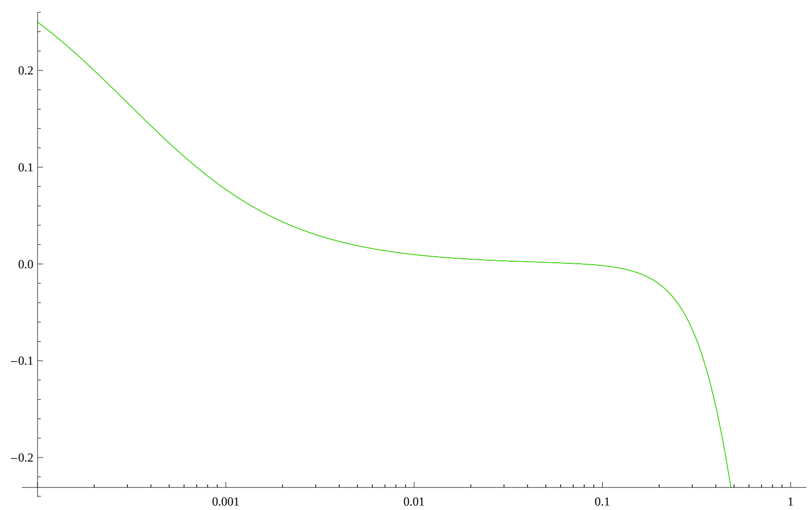

Figure 8: Effective equation of state are the same too.

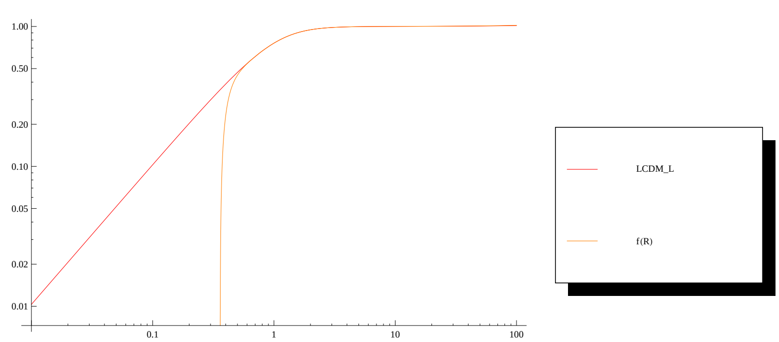

Figure 9: Growth function of Starobinsky's model.

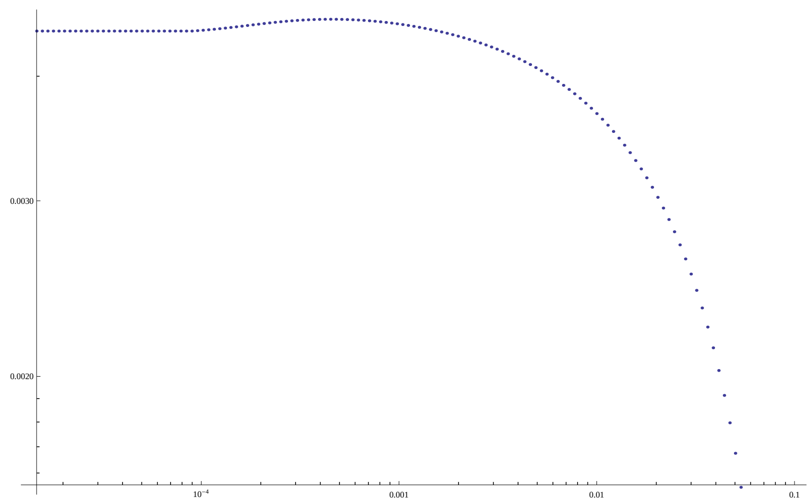

Figure 10: $Q = \frac{\text{Power of Hu\&Sawicki model}}{\text{Power of LCDM model}}$ of Starobinsky's model.