4.1.2 ΛCDM

Parameters

$$w = -1; \Omega_{\mathrm{DE0}} = 0.734; \Omega_{\mathrm{k0}} = 0; \Omega_{\mathrm{m0}} = 0.1334 \big/ \left(0.71^2\right); \Omega_{\mathrm{r0}} = 8.09 * 10^{-5};$$

Hubble distance:

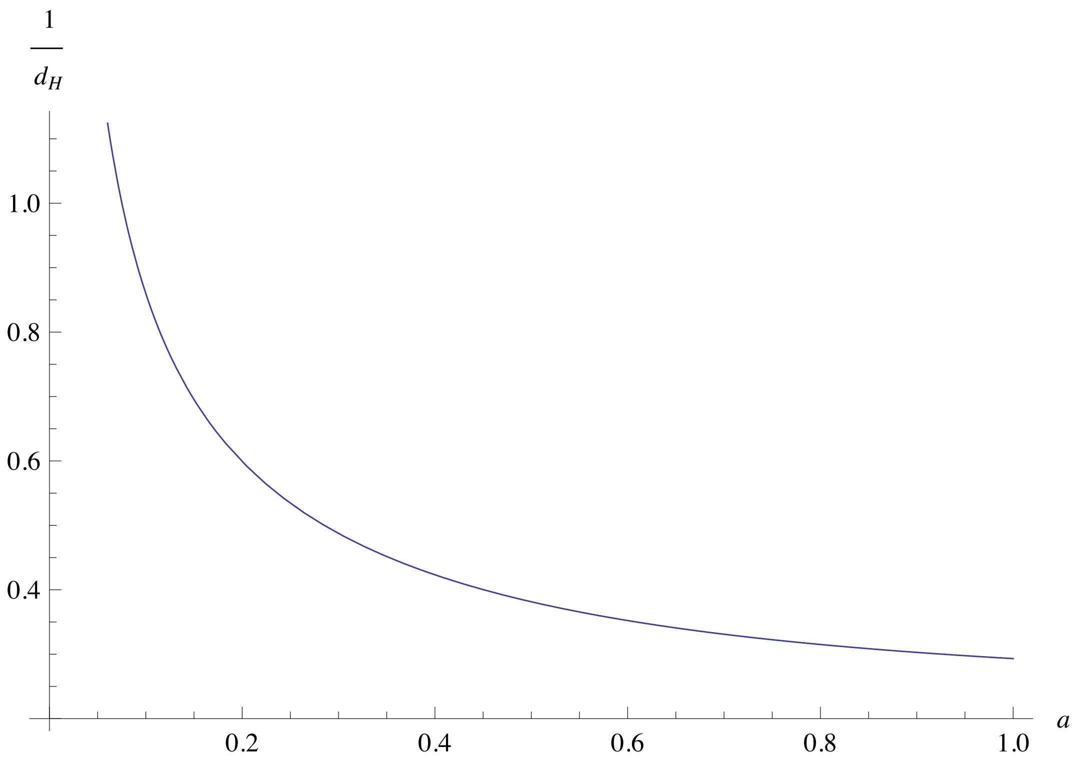


Figure 5: $\frac{1}{d_H}$ vs a

growth factor:
Growth vs k

4.1.3 LCDM and Dark Energy

Parameters are listed below.

$$\Omega_{\mathrm{DE0}} = 0.734; \Omega_{\mathrm{k0}} = 0; \Omega_{\mathrm{m0}} = 0.1334 \big/ \left(0.71^2\right); \Omega_{\mathrm{r0}} = 8.09 * 10^{-5}; \tag{45}$$
$$\Omega_{\mathrm{m0},s} = 1; \Omega_{\mathrm{r0},s} = 8.09 * 10^{-5}; \tag{46}$$
$$h = 0.71; H_0 = \frac{100h}{300000}; \tag{47}$$

Each color in the figures represents a model.

Color	**Model**
Red	sCDM
Orange	LCDM
Yellow	$w = -0.25$
Green	$w = -0.5$
Blue	$w = -0.75$

Figure 8 shows the differences of the evolution of the Hubble distance. All the data are normalised with the inverse of sCDM's Hubble distance. The shape of the lines can be explained by the fact that DE or

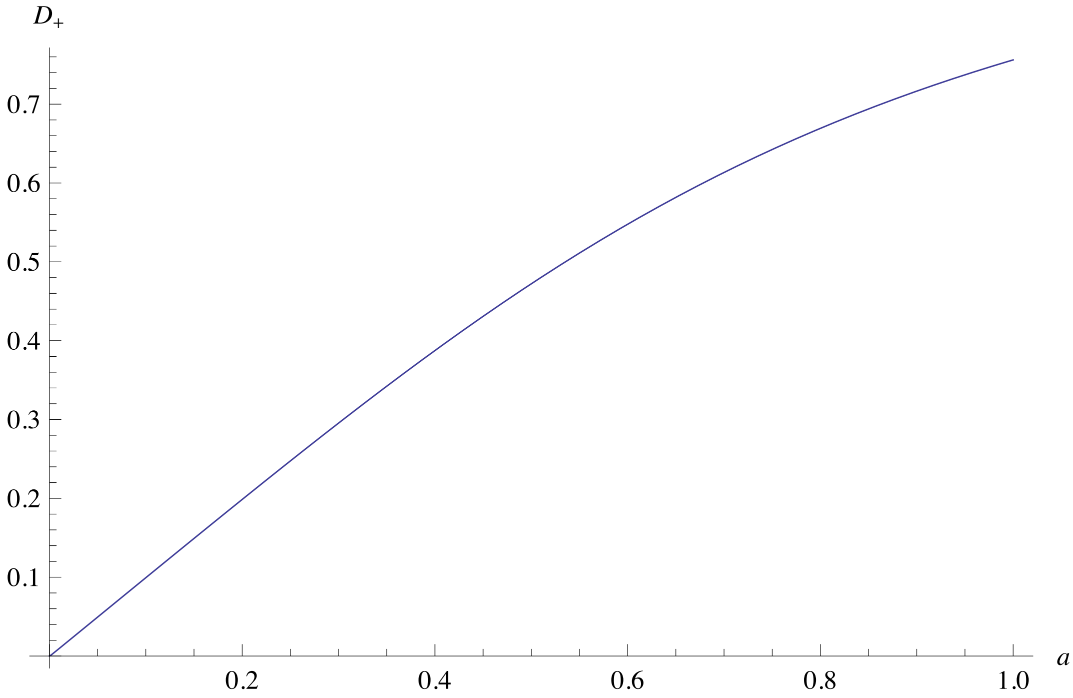


Figure 6: growth factor

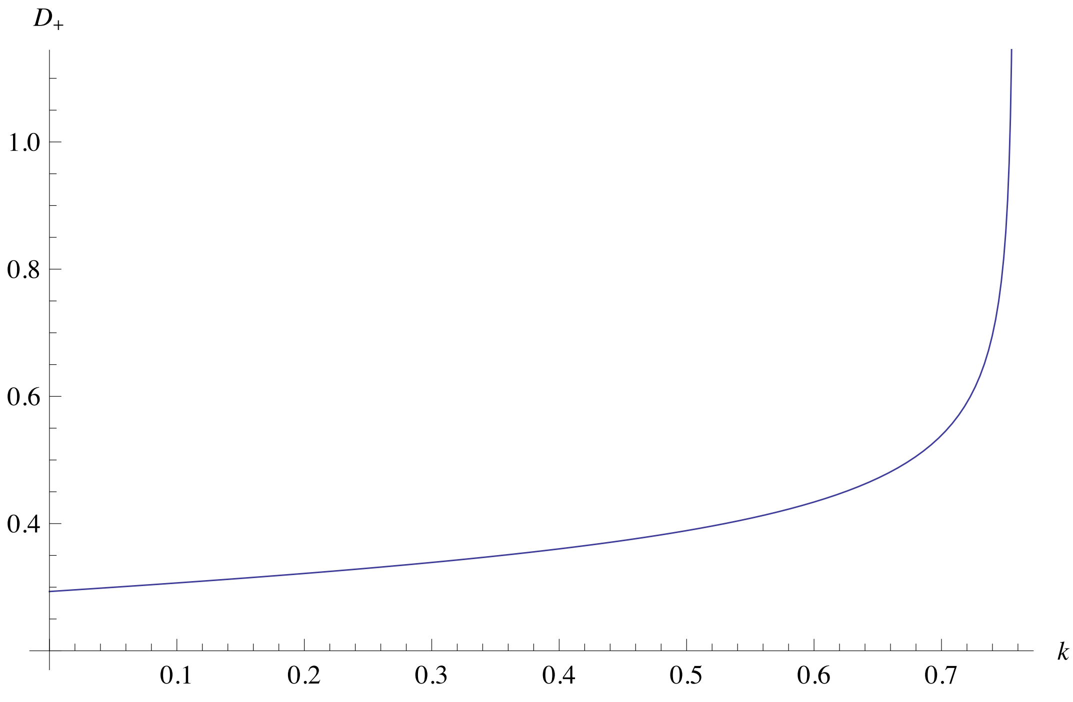


Figure 7: Growth

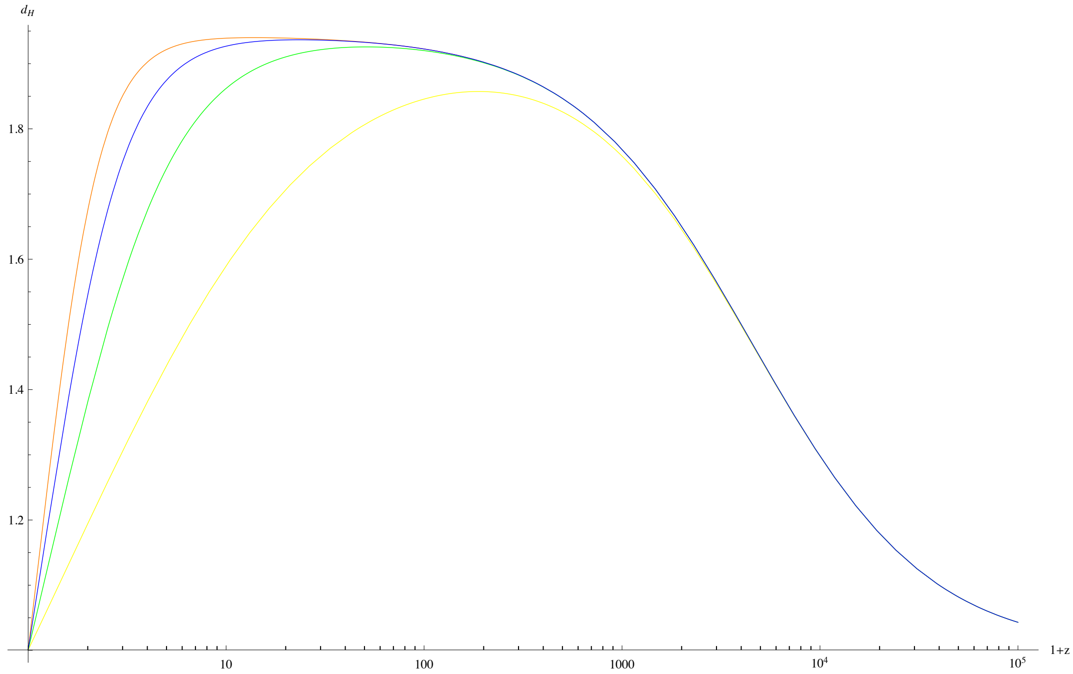


Figure 8: Hubble distances

Lambda only changes the background of the universe at late times after RD. The reason for the dropping down of the lines is that the Hubble functions should have the same value today ($1+z=1$). That is also the reason for the fact that they cross the same point at $1+z=1$. (Values of Hubble equations should converge at late times. So the part with $1+z<1$ is useless.)

Through figure 8 the three DE models fall between LCDM and sCDM which should be a straight line of value 1. Since the EoS of three DE models are exactly between 0 and -1, this result is quite reasonable. This figure also shows that the DE model with $w=-0.25$ obviously deviates from LCDM at an early age of $z\sim 1000$, while other models deviate after about $z\sim 50$.

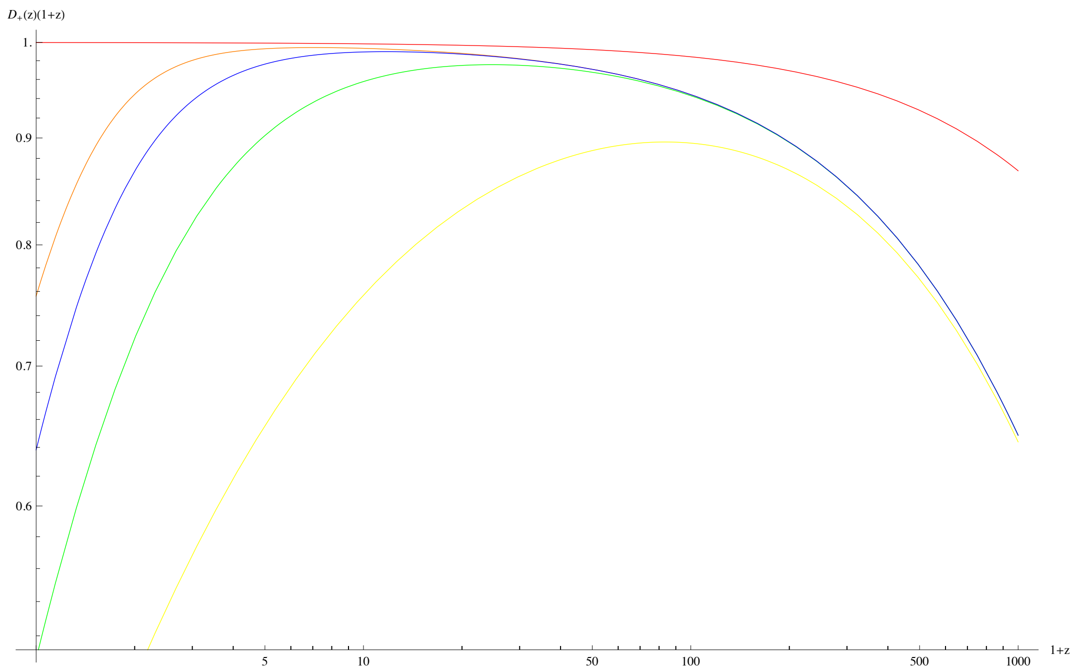


Figure 9: Growth factors vs $1+z$ of DE and ΛCDM

Figure 9 are the growth factors of the models. The going down lines are due to the late age effect of dark energy which suppresses the evolution of perturbations.

[*Why does the yellow line ($w = -0.25$) behave so strangely? Though we only use the part with $1 + z$ larger than 1, it is hard to imagine it crossing LCDM (while other lines crossing nothing).*]

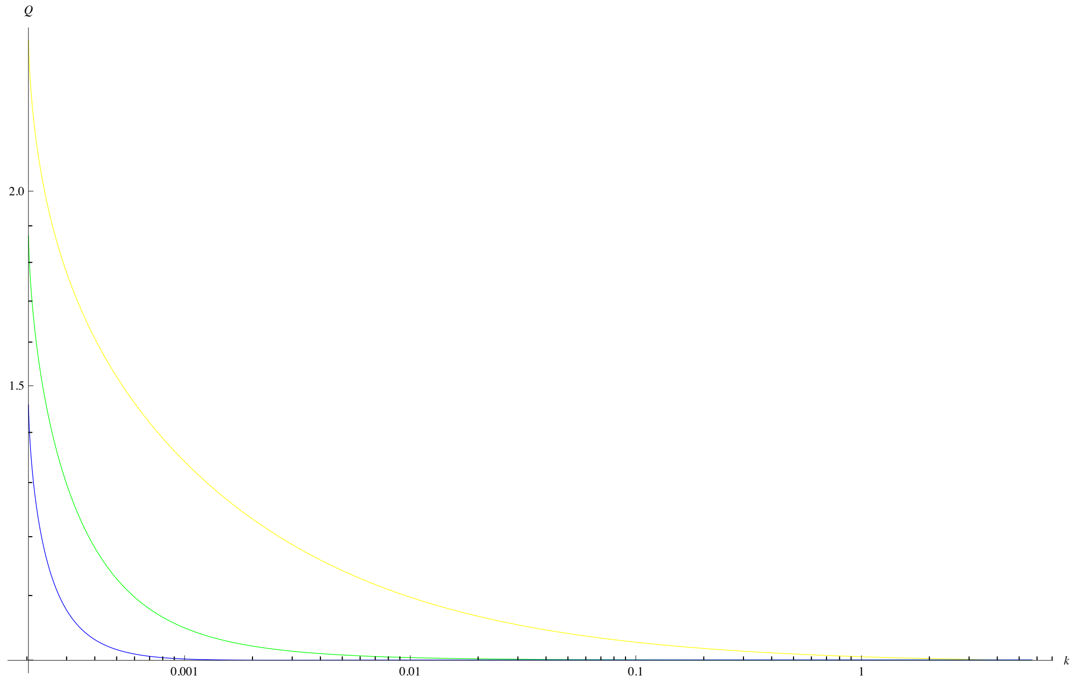


Figure 10: Q factors

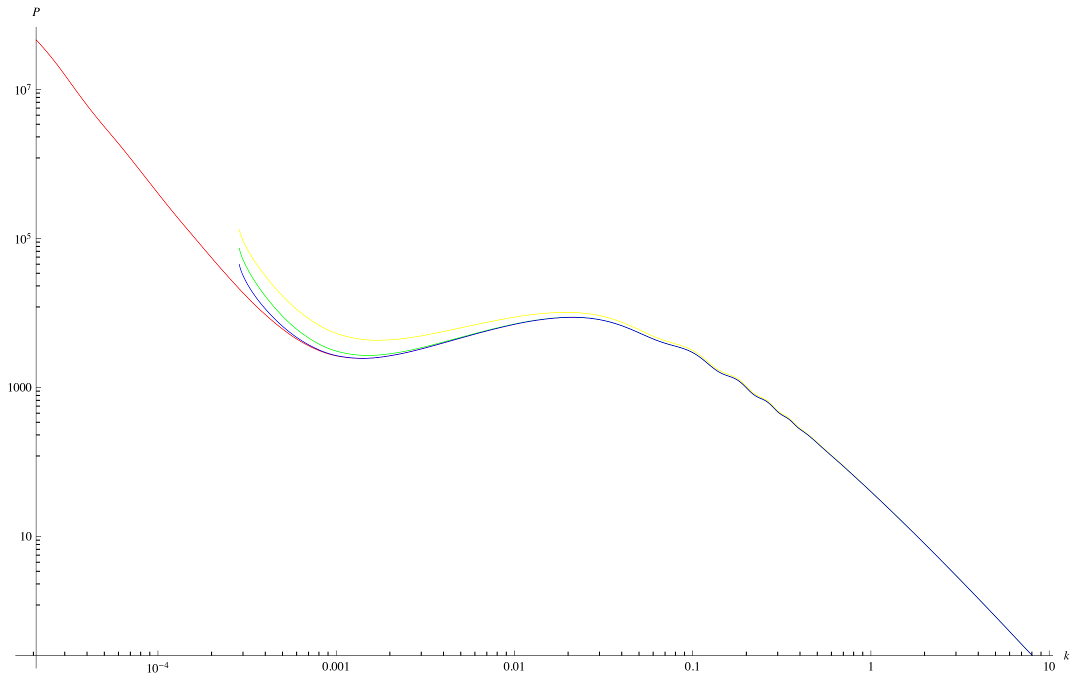


Figure 11: Power spectrum of LCDM and several DE models

Figure 11 shows the power spectrums, which are generated by the standard LCDM model. The figure gives the lines the right trend when change the EoS, just as right as the Hubble distance.

One thing that is really annoying is that as the EoS becomes more and more close to LCDM, it is harder and harder to distinguish the DE models with LCDM models. The blue line is a good example for this statement.

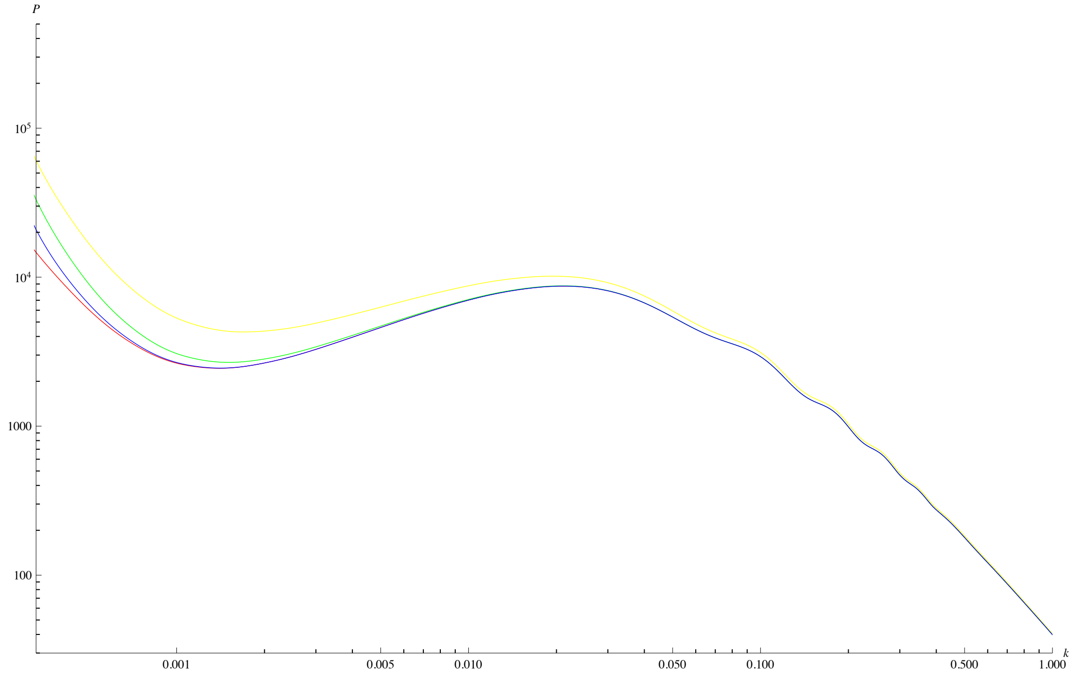


Figure 12: Power spectrum (within a range) of LCDM and several DE models

These results supports the point that this method works well for these time independent EoS DE models and it can be show the differences of the models in between the time interval we are working on.

4.2 CPL

4.2.1 What Is CPL Parameterisation?

Notes in *Cosmology Project Notebook*.
The EoS here is $w = w_0 + w_a(1 - a)$.

4.2.2 Generating Power Sepctrum

The figures in this subsubsection follows the following rules unless exceptions are stated:

- Red for sCDM;orange for LCDM;yellow for

Parameters table (other parameters are exactly the same with the previous calculation):

Color	**Model**
Red	sCDM
Orange	LCDM
Yellow	$w_a = -0.20$
Green	$w_a = -0.30$
Blue	$w_a = -0.32$
Cyan	$w_a = -0.34$
Purple	$w_a = -0.44$

In these CPL models, the parameters make sure that $w_0 + w_a = -1$ in order to generate a background analogous to LCDM.

It should be made clear that the range of validity is around $10^{-4} < k < 10$ or equivalently a range of $10^{-3} < a < 1$.

Figure 13 shows the Hubble distances. The $1 + z < 1$ part is also useless so I cut them off. The smaller the EoS is, the larger the Hubble distance is during late ages. The parameters chosen here are all fits to

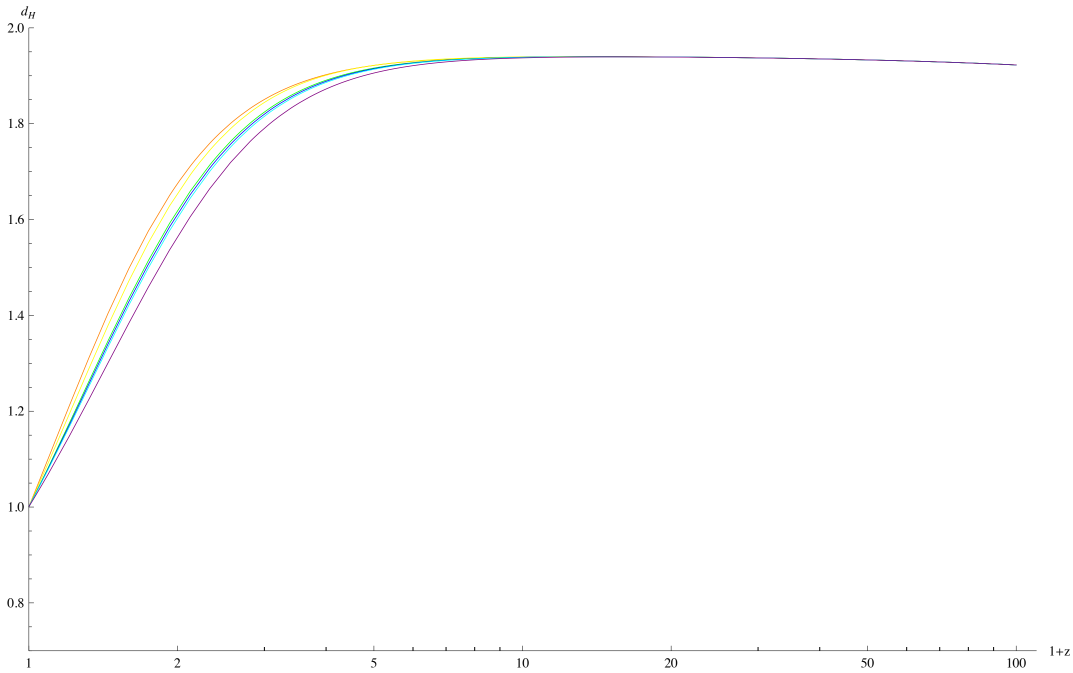


Figure 13: The Hubble distance of different models including sCDM, LCDM and five other CPL parameterised dark energy model

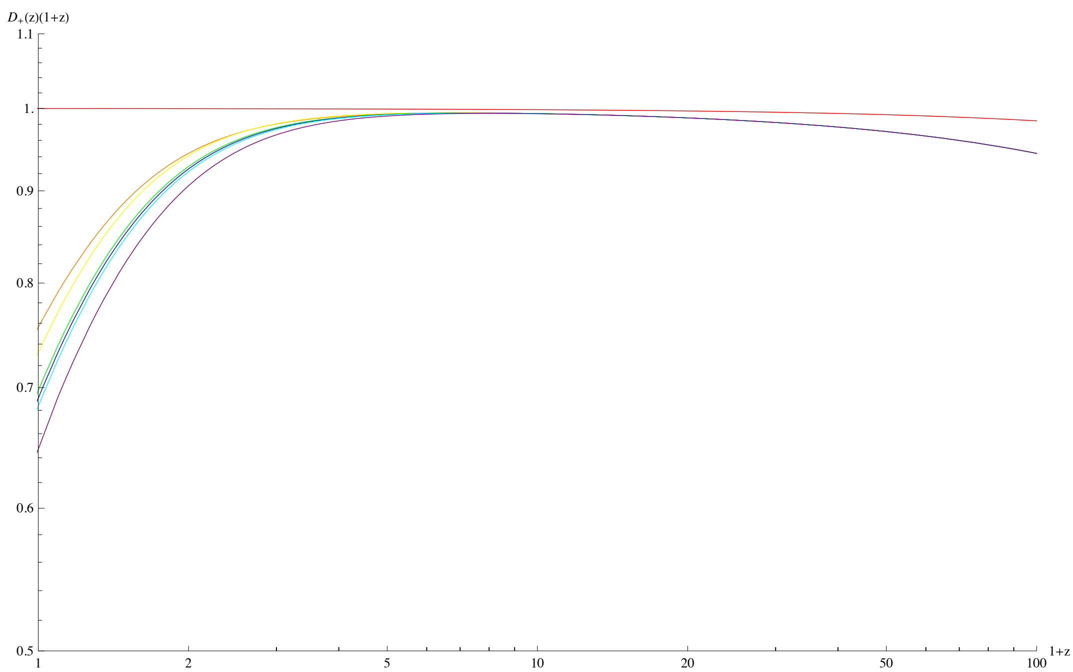


Figure 14: The growth factors of different models including sCDM, LCDM and five other CPL parameterised dark energy model

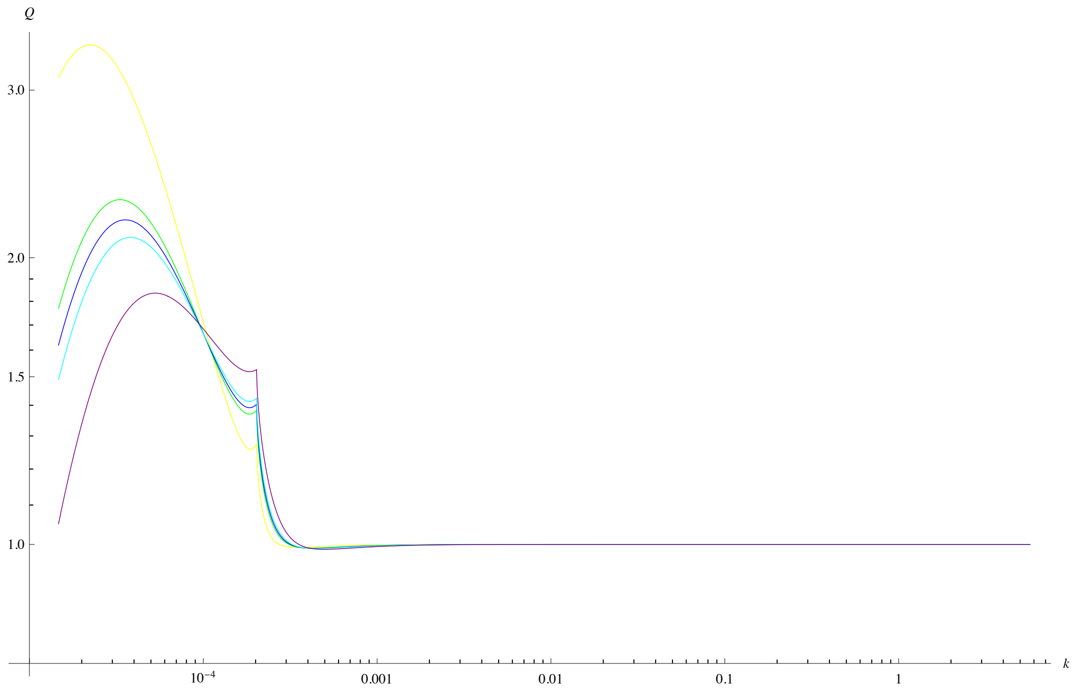


Figure 15: The Q factors of different models including sCDM, LCDM and five other CPL parameterised dark energy model

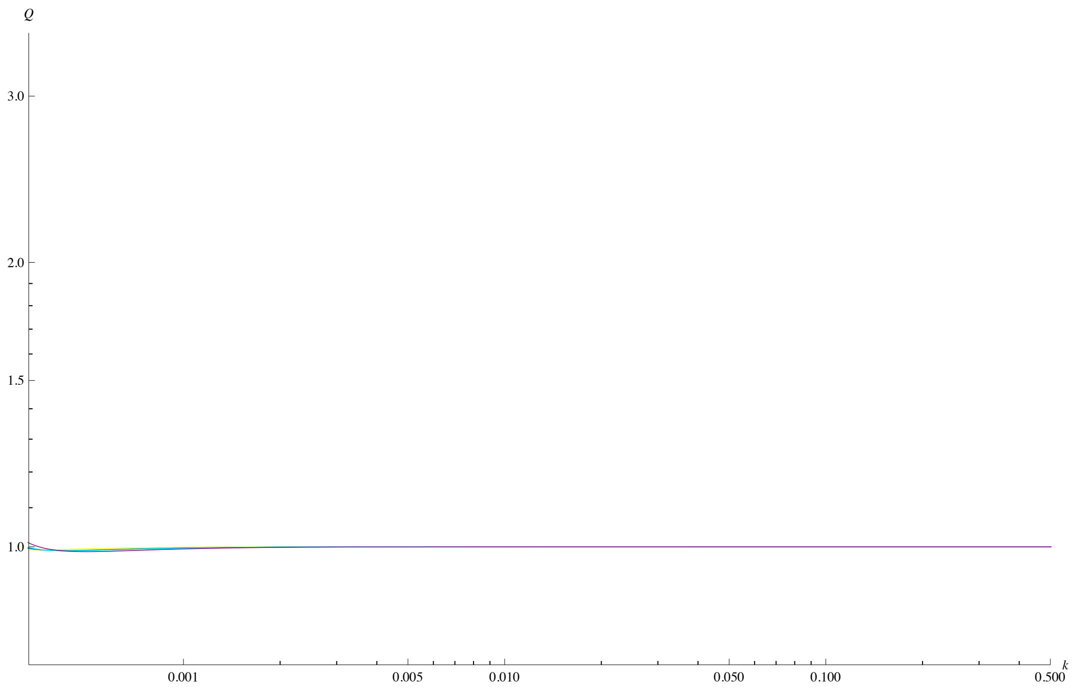


Figure 16: The Q factors (within a range) of different models including sCDM, LCDM and five other CPL parameterised dark energy model

LCDM well. So the difference between between them is small. However the differences are large enough to be noticed.

Figure 14 has peaks. These peaks comes from the $1 + z < 1$ part of the growth factor. These should be cut off latter.

The figures show the smaller the parameters w_a, the larger deviation from LCDM at late times.

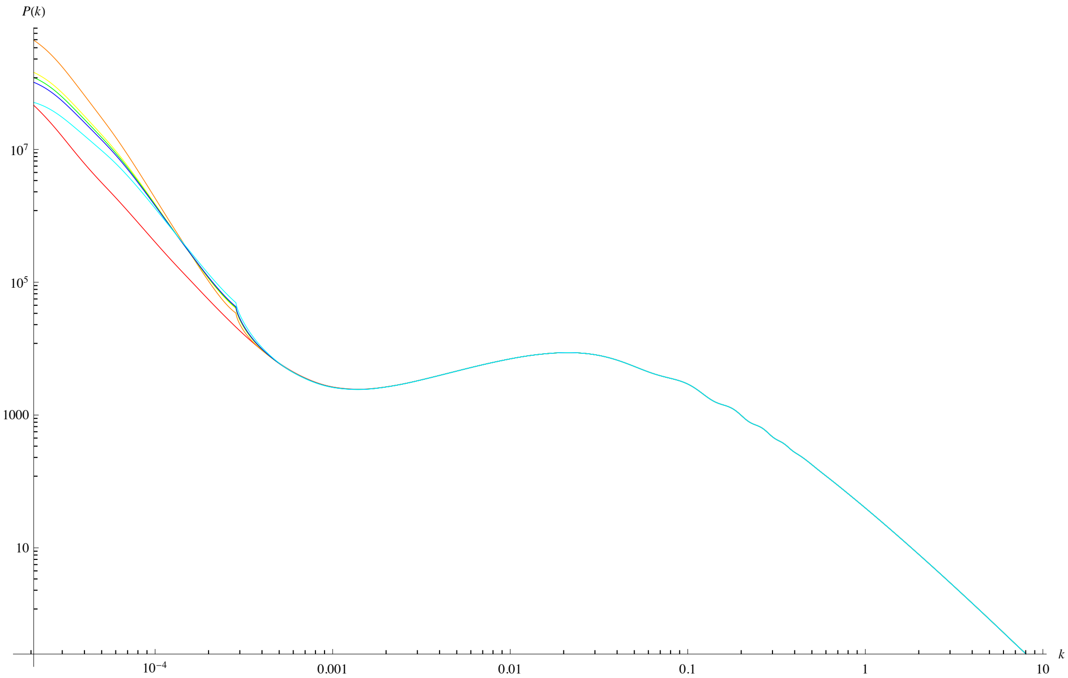


Figure 17: The power spectrums of different models including sCDM, LCDM and five other CPL parameterised dark energy model

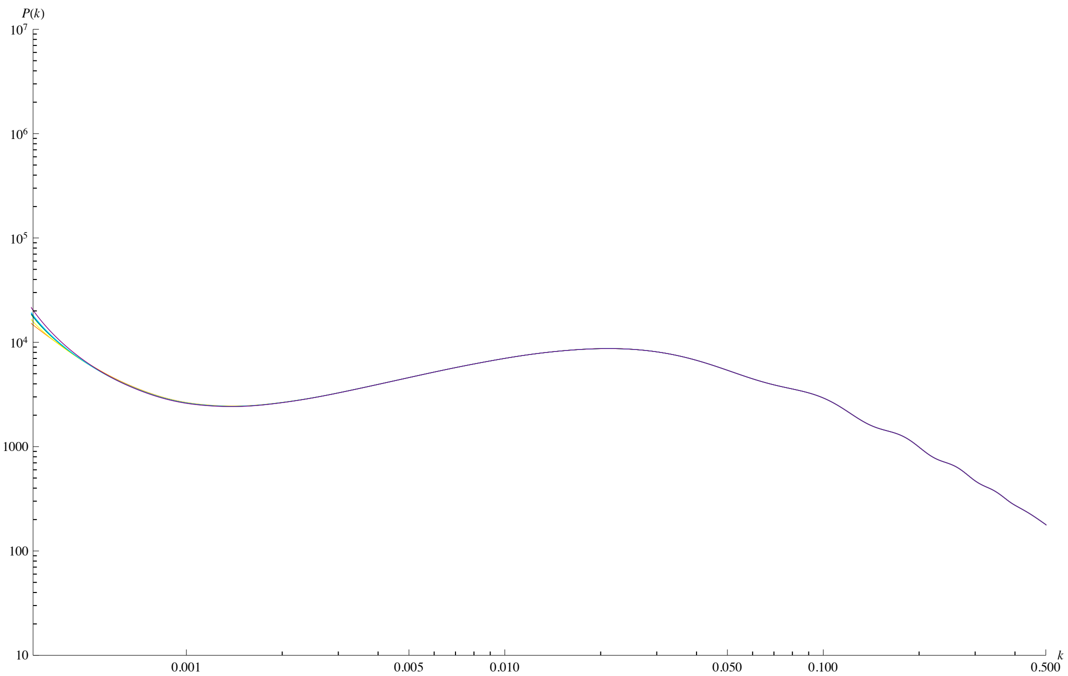


Figure 18: The power spectrums (within a range) of different models including sCDM, LCDM and five other CPL parameterised dark energy model

Figure 18 shows that the power spectrum can hardly be recognised until today. (I am making too small changes in the parameters. More references needed here in order make this clear.)

4.3 Supplements

In Fig1a of Fernando's, the dashed line is d_H,LCDM/d_H,CDM, thick line is d_H,DE/d_H,CDM.
But you told me that your lines are for d_H, not the ratio, right?
Besides your dashed line is d_H,DE/d_H,LCDM, while in Fig1a of Fernando's the thin black line is d_H,LCDM/d_H,DE.

z<0 means future, at present z=0. Please plot the ratio as Fernando did and use the same scale as he used in his figure for easy comparison.

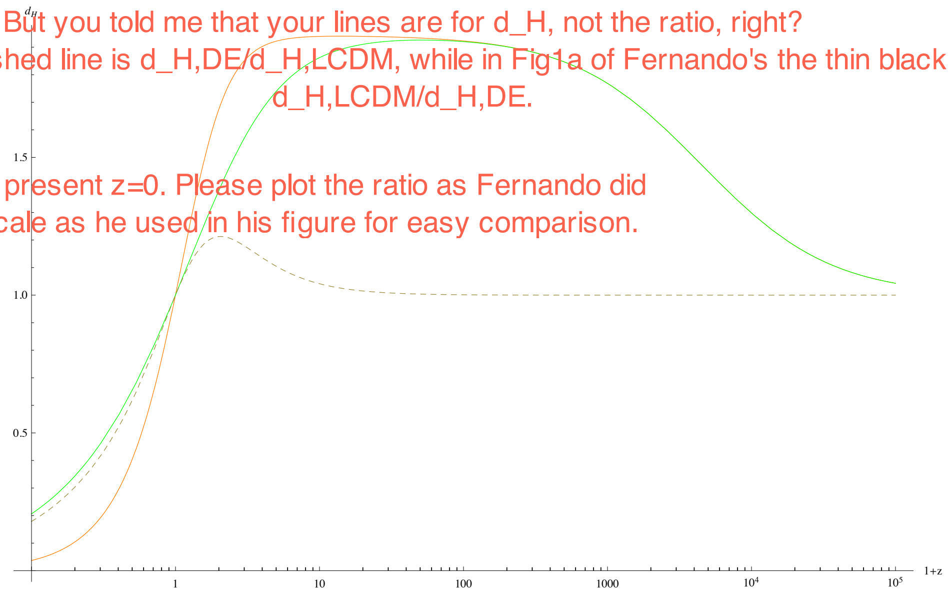


Figure 19: Hubble distance. Dashed line is the Hubble distance ratio of DE (with $w = -0.5$) and LCMD.

1. Figure 19 corresponds to Figure 1 a in Fernando's. I think $1 + z < 1$ is of nonsense because redshift z should be larger than 1 if we only care about the present and the past. So I only plotted the $1 + z > 1$. Here my figure is different from Fernando's at about $1 + z < 10$. All the lines converge at $z = 0$ in my plot because all the Hubble functions becomes the Hubble constant of today at that redshift. I have no idea why Fernando's plot do not. (And I have no idea why he plotted $z < 0$).
2. As said in the previous item, redshift less than zero is not so usefull.

z<0 can be useful for the future observation

3. I have already check the effect of Ω_{m0} and Ω_{de0}. The figure does not change very much.

please explain to me why you have the blow up for small k? while for Ferando's, they showed decay for small enough k?
Can you plot in the same scale as Fernando's Fig.2a,2b to show whether your results are consistent with Fernando's till k~10^-3?
You mean that values of \Omega_DE_0 and \Omega_m_0 will not change the result?
Please show this by comparison with Fernando's 2a,2b in the same scale by two comparisons
choosing \Omega_m0=0.25, \Omega_mo=0.3 and \Omega_DE correspondingly.

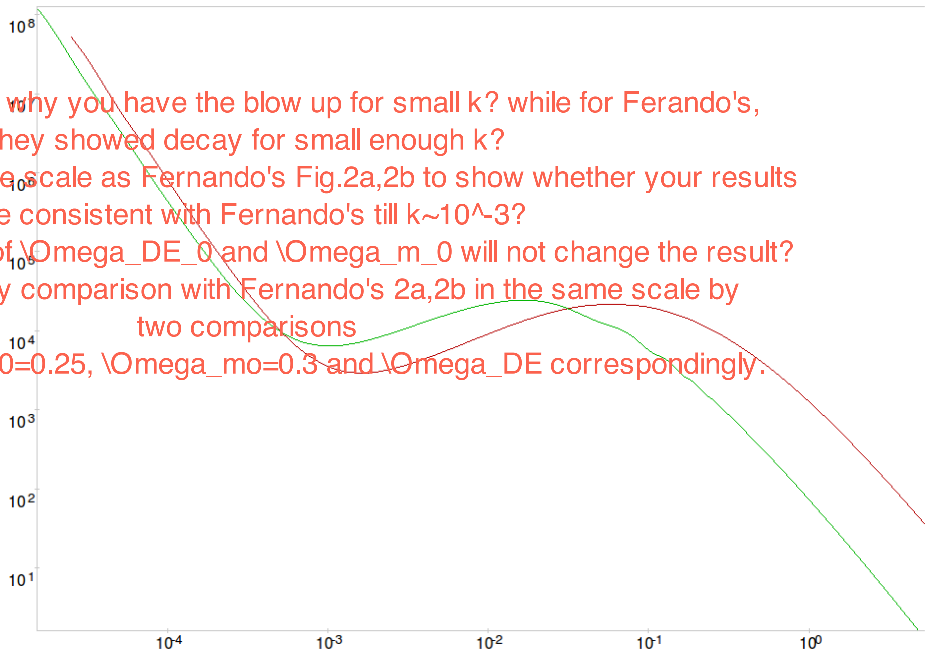


Figure 20: Power spectrum of dark energy model with $\Omega_{DE0} = 0$ and $\Omega_{DE0} = 0.7$. The red line is the sCDM model.

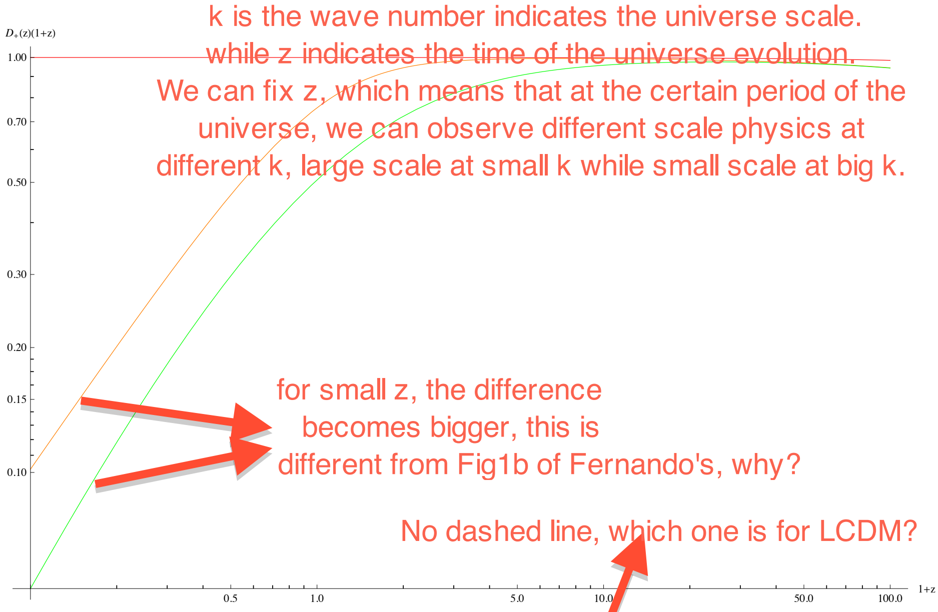
you need to choose different DE and DM abundances to do the comparison with Fig2a,Fig2b of Fernando's

That's why I did not plot figure varying in Ω_{DE0}.

4. I think there is no need to plot figures on different DE and DM abundances. Is there anything to be expected from plotting these figures?

   Small k corresponds to negative redshift. why small k~ negative z??

k is the wave number indicates the universe scale. ~~while z indicates the time of the universe evolution.~~ We can fix z, which means that at the certain period of the universe, we can observe different scale physics at different k, large scale at small k while small scale at big k.



for small z, the difference becomes bigger, this is different from Fig1b of Fernando's, why?

No dashed line, which one is for LCDM?

z<0 is not useless

Figure 21: Growth factors are given in this figure. Dashed line is the Hubble distance ratio of DE (with $w = -0.5$) and LCMD.

5. The growth factors in figure 21 is plotted within $1+z \sim [0.1, 1000]$. As argued in 1, the data is useless when $z < 0$ unless one is going to check the future evolution of the universe. **That is why I give two figures for one quantity sometimes: one for a larger range and one for a suitable range.** ???

6. The CPL EoS used here is $w = w_0 + w_a(1-a)$. The paramters chosen are listed in the table below.

Color	Model
Red	sCDM
Orange	LCDM
Yellow	$w_0 = -1$ & $w_a = 0.1$
Green	$w_0 = -1$ & $w_a = -0.1$
Blue	$w_0 = -0.9$ & $w_a = -0.1$
Cyan	$w_0 = -0.9$ & $w_a = 0.1$
Purple	$w_0 = -0.9$ & $w_a = -0.2$

The parameters chosen only have effects on small redshift. This is because the parameters all satisfy the condition that $w_0 \to -1$ and $w_a \to 0$.

By comparing yellow line ($w_0 = -1, w_a = 0.1$) and green line ($w_0 = -1, w_a = -0.1$), we can see positive w_a leads to a larger then LCDM Hubble distance when $w_0 = -1$. This is in accord with the series form of Hubble equation 65 since $w_0 = -1$ eliminates the first order term and the w_a determines whether the expression is larger then one. Other comparisons like green line and blue line, blue line and purple line in Figure 22 23 24 can also be interpreted by equation 65.

Growth factors are also following the expansion. Cyan line ($w_0 = -0.9, w_a = 0.1$) has smaller Hubble distances that others at late time, so the growth drops since growth factor is positively correlated to the size of the Hubble distance. The changes in growth factors when changing the parameters are exactly positively correlated to the size of Hubble distance changes. For example, the yellow line ($w_0 = -1, w_a = 0.1$) crosses the purple

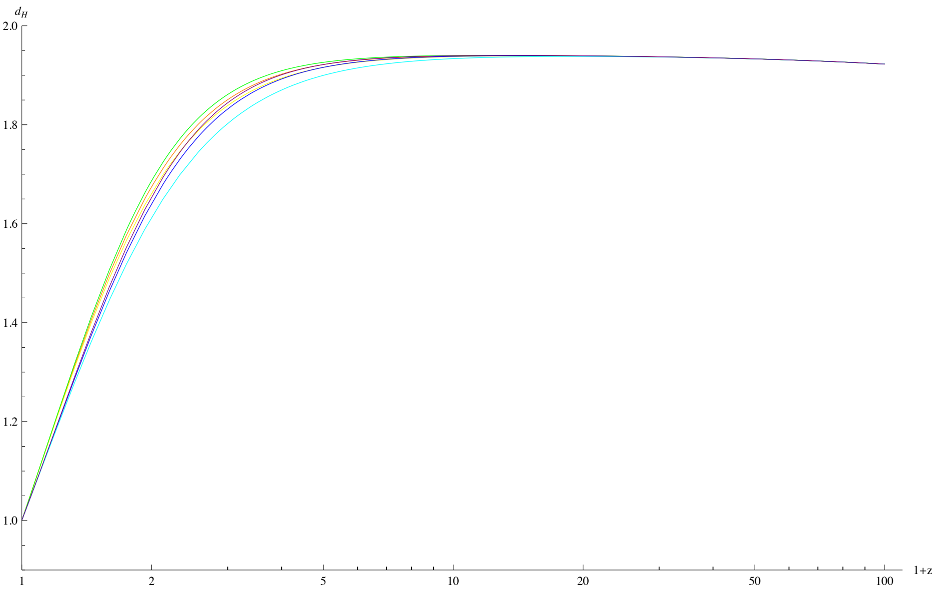


Figure 22: Hubble distance of CPL dark energy.

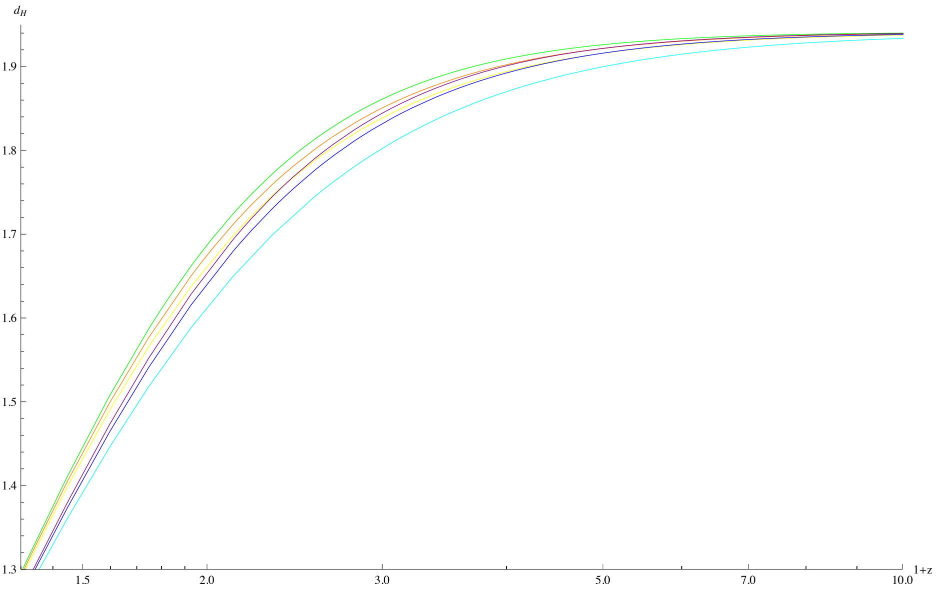


Figure 23: Hubble distance of CPL dark energy.

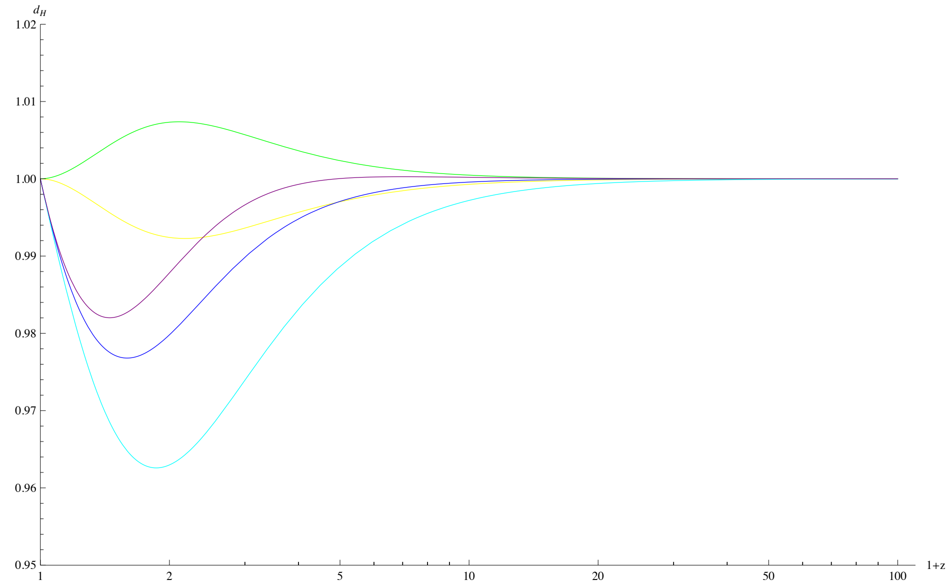
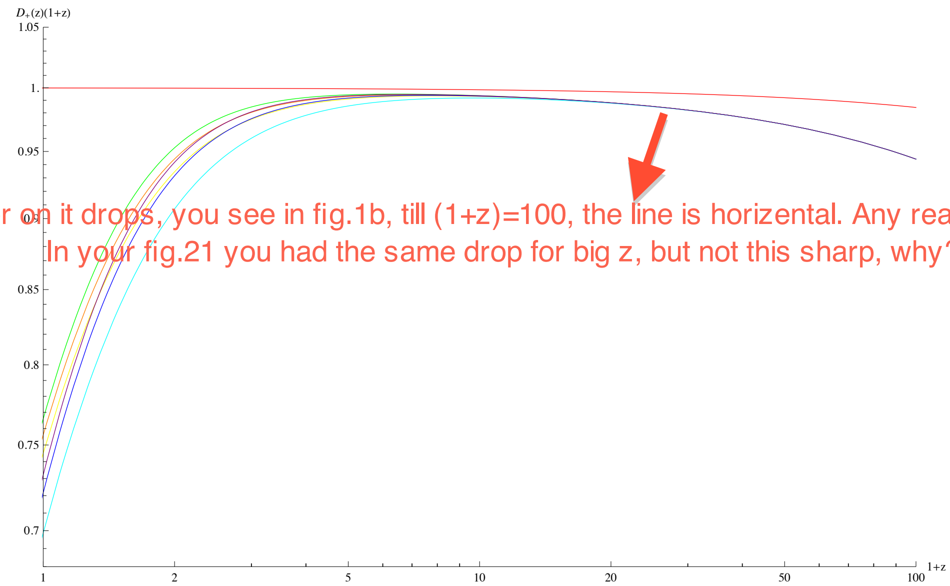


Figure 24: Hubble distance of CPL dark energy in units of LCDM Hubble distances at the corresponding value.



why later on it drops, you see in fig.1b, till (1+z)=100, the line is horizental. Any reason behind??
In your fig.21 you had the same drop for big z, but not this sharp, why??

Figure 25: Growth factors of CPL dark energy and LCDM in units of sCDM.

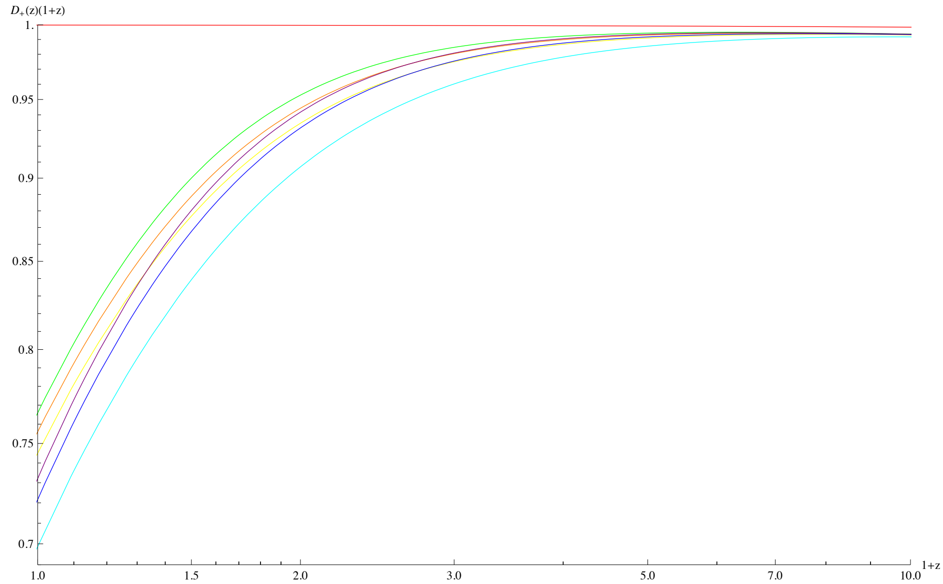


Figure 26: Growth factors of CPL dark energy and LCDM in units of sCDM.

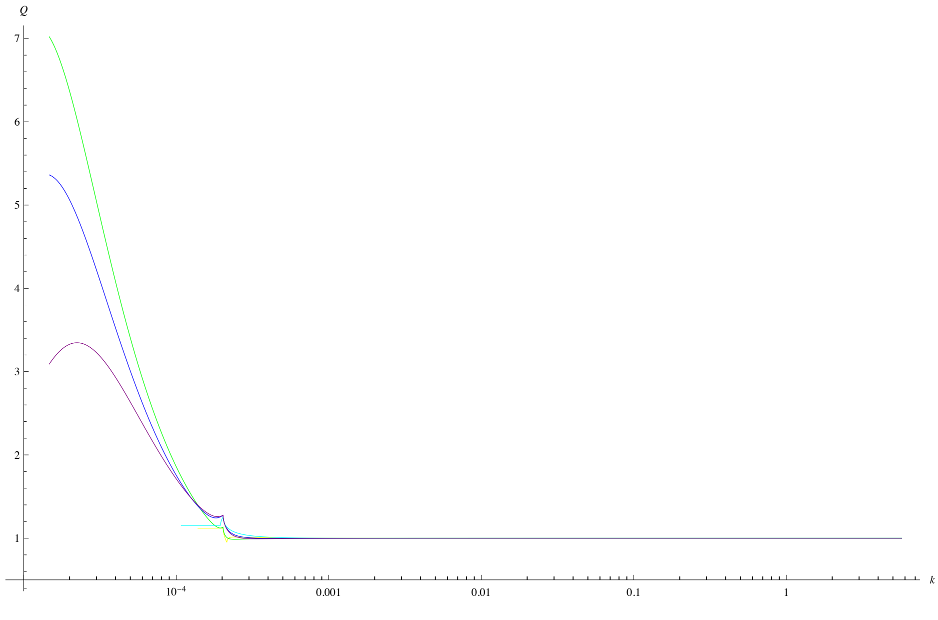


Figure 27: Q factors of CPL dark energy and LCDM in units of sCDM.

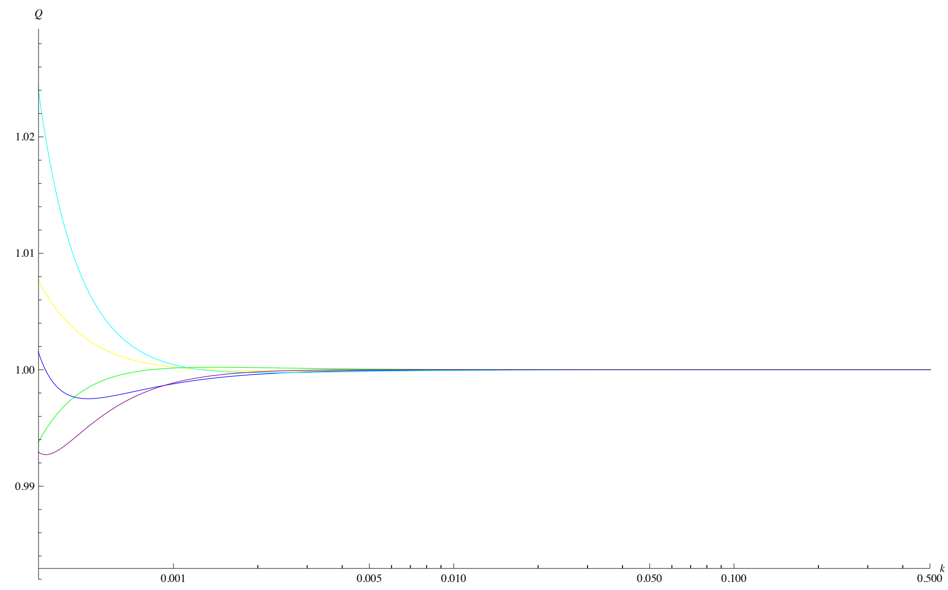


Figure 28: Q factors of CPL dark energy and LCDM in units of sCDM.

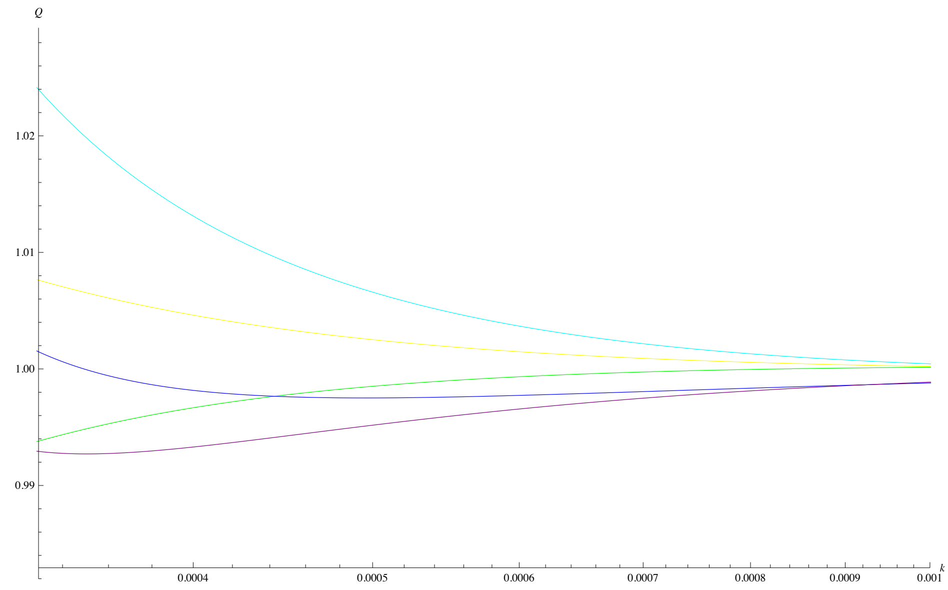


Figure 29: Q factors of CPL dark energy and LCDM in units of sCDM.

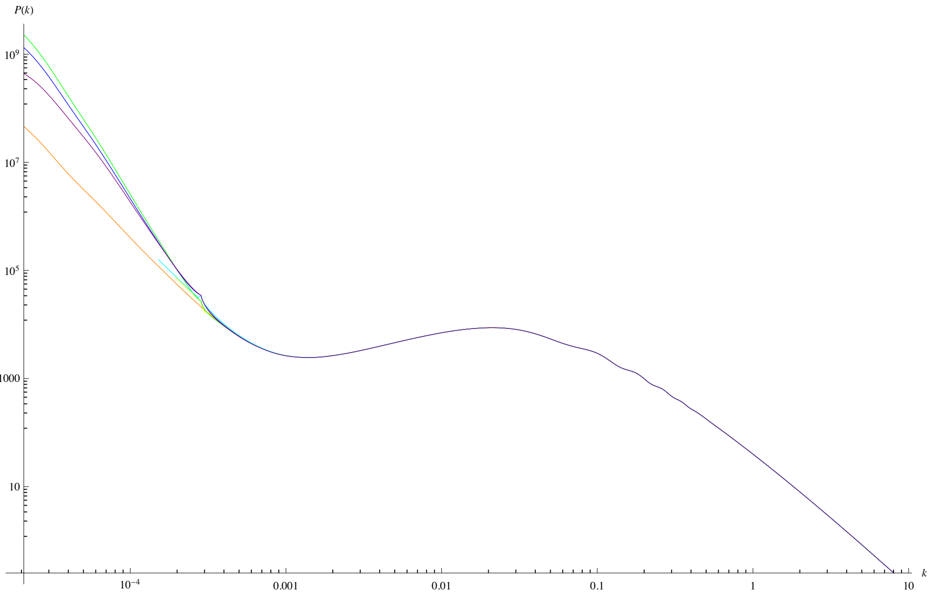


Figure 30: Power spectrum of CPL dark energy and LCDM in units of sCDM.

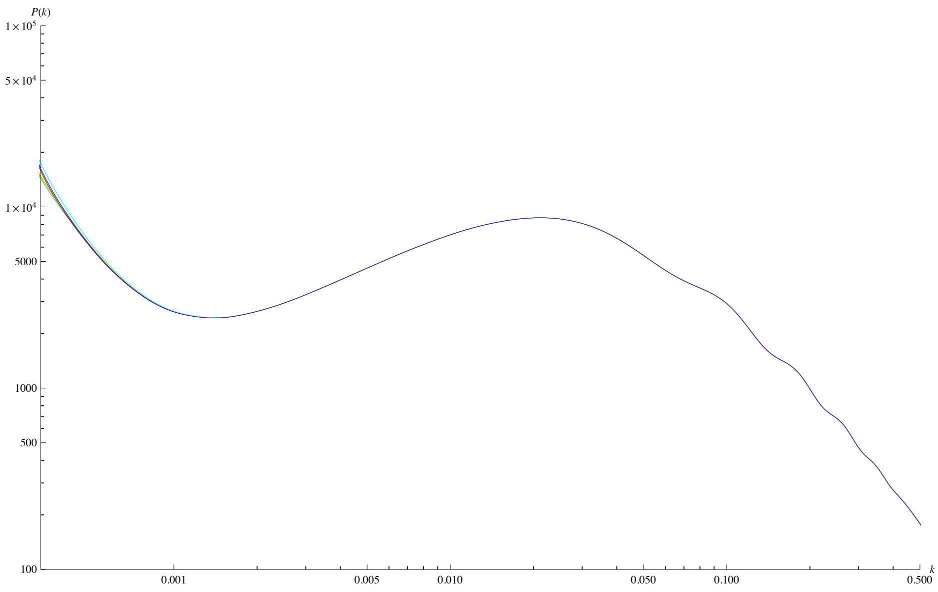


Figure 31: Power spectrum of CPL dark energy and LCDM in units of sCDM.

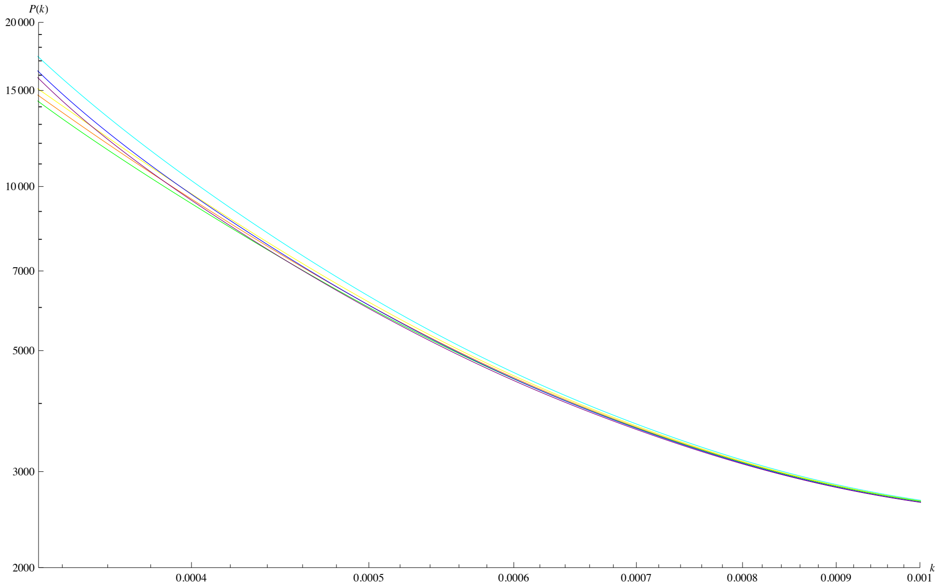


Figure 32: Power spectrum of CPL dark energy and LCDM in units of sCDM.

why you have the k~a relation?

line ($w_0 = -0.9, w_a = -0.2$) in figure 25 26 as they cross each other in the figure of Hubble distance.

Figure 27 has a region $k < 3.3 * 10^{-4}$ that is corresponding to scale factor $a > 1$. So I give another two plots figure 28 and 29. The same thing is done for power spectrum.

The power spectrum is consistent with the Hubble distance figure: larger Hubble distance generates weaker power.

again why??

7. Because we are using adiabatic initial conditions with a form $P \sim \frac{1}{k^3}$. At $a \sim 10^{-3}$ and $a \sim 1$, we have $k \sim 1$ and $k \sim 10^{-3}$ correspondingly. The growth factor with amplifies the initial perturbation by about 10^3 to get the current value $P \sim 10^6$ for $k \sim 1$ and the current value is 10^9 for initial perturbation at $a \sim 1$ (or $k \sim 10^{-3}$) since it has not evolved. This simple calculation show the shape of power spectrum is definitely going up at small k.

   item As stated previously, the inconsistent part is $z < 0$, so there is no need to worry about.

I do not understand this explanation!!

4.4 Including DM-DE perturbation

5 Todo List

- ~~Find all the growth factors if they are useful.~~
- ~~Calculate the fiducial power spectrum. (I tried to do some calculation. But it seemed the curves will never be exactly~~
- ~~The spectrum is normalised to be the same at $a \sim 0$. So that the σ_8 of different models are the same.~~
- Switch from adiabatic initial condition to isocurvature initial condition will change the expression of growth factors (changing the power of H), thus leading to different results.
- Introduce interacting between dark energy and dark matter.
- Other quantities in cosmology? Can one generalise this method to more calculations?

6 Appendix

6.1 Growth Factor Revisited

In this part, prime (') stands for the derivative with time t, over dot stands for the derivative with scale factor a.

Use Navier Stokes equations for matter (with pressure $p = 0$) as the basic equation set. [18]

Using $\delta_m = \frac{\rho_m - \bar{\rho}_m}{\bar{\rho}_m}$, the equations become

$$\begin{aligned} \frac{\partial \delta_m}{\partial t} + a^{-1}\vec{v}_m \cdot \delta_m &= -a^{-1}(1+\delta_m)\nabla \cdot \vec{v}_m & (48) \\ \frac{\partial \vec{v}_m}{\partial t} + a^{-1}(\vec{v}_m \cdot \nabla)\vec{v}_m &= -\frac{\nabla \Phi}{a} - H\vec{v}_m & (49) \\ a^{-2}\nabla^2\Phi &= 4\pi G\bar{\rho}_m\delta_m & (50) \end{aligned}$$

In these equations, Φ is the gravitational potential.

This is too complex to solve. So we only use the linear approximation,that is, no second or higher order of δ_m, $\vec{v}_m$, Φ appear, because we already use those as perturbations. Finally,

$$\begin{aligned} \frac{\partial \delta_m}{\partial t} &= -a^{-1}\nabla \cdot \vec{v}_m & (51) \\ \frac{\partial \vec{v}_m}{\partial t} &= -\frac{\nabla \Phi}{a} - H\vec{v}_m & (52) \\ a^{-2}\nabla^2\Phi &= 4\pi G\bar{\rho}_m\delta_m & (53) \end{aligned}$$

Easily, one can use the well known trick for this kind of equations to find the second derivative equation for δ_m, or the basic function for this problem,

$$\delta_m'' + 2H\delta_m' - 4\pi G\bar{\rho}_m\delta_m = 0 \tag{54}$$

However, we usually use scale factor a as a measurement of time. Thus we would transform it the following form

$$\ddot{\delta}_m + (\frac{\mathrm{d}\ln H}{\mathrm{d}a} + \frac{3}{a}) - \frac{3\Omega_{m0}H_0^2}{2a^5H^2}\delta_m = 0 \tag{55}$$

Almost each book of ODE will tell us how to solve such an equation.

After a little work, we will get two special solutions:

$$\begin{aligned} \delta_{m,1} &= H & (56) \\ \delta_{m,2} &= H\int_0^a \frac{1}{a'^3H(a')^3}\mathrm{d}a' & (57) \end{aligned}$$

General solution is

$$\delta_m = C_1H + CH\int_0^a \frac{1}{a'^3H(a')^3}\mathrm{d}a' \tag{58}$$

Since H is decaying, drop C_1H, then

$$\delta_m = CH\int_0^a \frac{1}{a'^3H(a')^3}\mathrm{d}a' \tag{59}$$

Apply initial condition $\delta_m(a_i) = \delta_i$

$$C = \frac{\delta_i}{H(a_i)}\frac{1}{\int_0^{a_i}\frac{1}{a'^3H(a')^3}}\mathrm{d}a' \tag{60}$$

To simplify these expressions, we now define $D_+ = \frac{5}{2}\Omega_{m0}H(a)\int_0^a \frac{1}{a'^3H(a')^3}\mathrm{d}a'$. (One might be confused with this defination at first. The constant multiplied the the original part is to ensure one condition: at matter domination,D_+ is a. This condition endow D_+ with more physics.) Then

$$C = \delta_i\frac{5\Omega_{m0}H_0^2}{2}\frac{1}{D_+(a_i)} \tag{61}$$

[18] *Cosmology Project* Notebook

With this, we have

$$\delta_m(a) = \delta_i \frac{D_+(a)}{D_+(a_i)} \tag{62}$$

Actually, this $D_+(a)$ is a growing mode of the equation, and we all call it growth factor.

6.2 CPL

6.2.1 Equation of State

$$w(a) = w_0 + w_a(1-a) = w_0 + w_a \frac{z}{1+z} \tag{63}$$

6.2.2 Hubble Function

By solving the Friedmann equation using the CPL parameterisation, the contribution to the background of CPL dark energy is $\Omega_{de0} a^{-3(1+w_0+w_a)} e^{-3w_a(1-a)}$.

Thus the Hubble function should be

$$H(a) = H_0 \sqrt{\Omega_{m0} a^{-3} + \Omega_{r0} a^{-4} + \Omega_{de0} a^{-3(1+w_0+w_a)} e^{-3w_a(1-a)}} \tag{64}$$

The Taylor series of dark energy contribution is

$$a^{-3(1+w_a+w_0)} e^{-3w_a(1-a)} = 1 + 3(1+w_0)(a-1) + \frac{3}{2}(4 + w_a + 7w_0 + 3w_0^2)(1-a)^2 + ... \tag{65}$$

To create a mimic of LCDM at low redshift, $w_0 \to 0$ should be satisfied. To make a more rigorous condition, w_a should be much smaller than 1 for this cancels the contribution of z^2 term once $w_0 = -1$.

In order to generate a background similar to LCDM, the parameters should carefully chosen according to the criteria that $w_0 \to 0$ and $w_a \to 0$.