- **I can't get a plot exactly the same as figure1(a).**

  I use the following parameters.

$$\Omega_{\text{DE0}} = 0.734; \Omega_{\text{k0}} = 0; \Omega_{\text{m0}} = 0.1334 \big/ \left(0.71^2\right); \Omega_{\text{r0}} = 8.09 * 10^{-5};$$
$$h = 0.71; H_0 = \frac{100h}{300000};$$

  The Hubble equations are

$$H_s[\text{a\_}] = \text{Hs0}\sqrt{\frac{\Omega_{\text{m0},s}}{a^3} + \frac{\Omega_{\text{r0},s}}{a^4}} \tag{1}$$
$$H_L[\text{a\_}] = \text{HL0}\sqrt{\Omega_{\text{DE0}} + \frac{\Omega_{\text{m0}}}{a^3} + \frac{\Omega_{\text{r0}}}{a^4}} \tag{2}$$
$$H_d[\text{a\_}] = \text{Hd0}\sqrt{\frac{\Omega_{\text{DE0}}}{a^{1.5}} + \frac{\Omega_{\text{m0}}}{a^3} + \frac{\Omega_{\text{r0}}}{a^4}} \tag{3}$$

  in which, subscripts ${}_{s}, {}_{L}, {}_{d}$ denote stand CDM, LCDM, DE(with $w = -0.5$) respectively.

  Using these equations and three points on figure2(a), we find

$$H_s\left[10^{-6}\right] = H_L\left[10^{-6}\right] \tag{4}$$
$$H_s[1] = 1.25 H_L[1] \tag{5}$$
$$H_s[0.3] = 1.9 H_L[0.3] \tag{6}$$

  After simplifying, they become

$$\text{Hs0}^2\left(1. \times 10^{18}\Omega_{\text{m0},s} + 1. \times 10^{24}\Omega_{\text{r0},s}\right) = 4.54612 \times 10^{12} \tag{7}$$
$$\text{Hs0}^2\left(\Omega_{\text{m0},s} + \Omega_{\text{r0},s}\right) = 8.740454543229165\text{*\^{}-8} \tag{8}$$
$$\text{Hs0}^2\left(37.037\Omega_{\text{m0},s} + 123.457\Omega_{\text{r0},s}\right) == 2.132220394599451\text{*\^{}-6} \tag{9}$$

  To see this more easily, we change the arguments, $Hs0 \to x, \Omega_{m0.s} \to x, \Omega_{r0,s} \to z$, the equations are

$$x^2\left(y + 10^6 z\right) = 4.5461211111111111 * 10^{-6} \tag{10}$$
$$x^2(y + z) = 8.740454543229165 * 10^{-8} \tag{11}$$
$$x^2(y + 3.33333z) = 5.7569950654185174 * 10^{-8} \tag{12}$$

  From equation 11 and 12, we can find that there won't be any positive solutions.

  The key point is that the last term in 12 should be larger than that of 11. This lead to the fact that the ratio value in figure2(a) at $1 + z \sim 3.3$ should be larger than 2.3 if the value at $1 + z = 1$ is 1.25, as the figure shows.

  This evalution indicates that the parameters of LCDM used to generate figure2(a) is very different from the one I am using, which is extracted from the WMAP7 year data. In principal, I can always fit all the values from this figure. Anyhow, this is stupid to do so and this is not easy because the radiation part is small.

- **As for figure2(a), here is the problem.**

  For $k < 0.0003$ modes, they are outside of the horizon today. So all the modes with $k < 0.0003$ has the same amplitude of matter density δ_m. Since the power is $P \sim \frac{\delta^2}{k^3}$, the line goes up when k runs to the smaller value.

  I have no idea with the descending of the line in figure2(a).

- **I think we shouldn't use the formulas (12) and (13) in their paper.**

  The assumption that the primordinal matter perturbation δ_i (or potential perturbation Φ_i) are the same is not a substantial point, since this δ_i gives us the freedom of renormalizing the power spectrum.

  It's better to start over. the matter perturbation at any a is

  $$\delta_m(a) = \delta_i \frac{1}{D_+(a_i)} D_+(a)$$

  where δ_i is the initial perturbation, D_+ is the growth factor, a_i is the scale factor at intial.

  Then the Q factor should be defined as

  $$Q = \frac{\delta_i^X D_+^X(a)/D_+^X(a_i)}{\delta_i^F D_+^F(a)/D_+^F(a_i)}$$

  Their work take the condition that $\delta_i^X = \delta_i^L$. However, this leaves us no parameters for the normalization of power spectrum. If we keep δ_i^X and δ_i^L, then we can change these to adjust the power spectrum.

  Their paper did not metion where the freedom of renormalization comes from.