INTRO

- This is a program for χ^2 fitting, using SN1a data.
- SN data are taken from http://supernova.lbl.gov/Union/
-
-
-

PRE

Settings

Include the packages needed.

```
Needs["ErrorBarPlots`"]
<< PhysicalConstants`
```

```
Needs["PlotLegends`"]
```

Set work directory. Modify it to your own directory before runing this program. DATA folder are placed in this directory. DATA folder should contain the SN data files.

```
SetDirectory["E:\\Nutshare\\Store\\Projects\\DataFitting"];
```

Conventions, parameters, etc

Line element: $ds^2 = -dt^2 + a[t]^2 \left(\frac{dr^2}{1-k\,r^2} + r^2\left(d\theta^2 + \mathrm{Sin}[\theta]^2\, d\phi^2\right)\right)$

Ωm0: matter fraction
Ωd0: dark energy/cosmological constant fraction
Ωr0: radiation fraction

Ωm0w: matter fraction given by WMAP
Ωd0w: dark energy/cosmological constant fraction given by WMAP
Ωr0w: radiation fraction given WMAP

```
Ωm0w = 0.27;
Ωd0w = 0.73;
Ωr0w = 8.09 * 10^-5;
```

Basic Equations in LCDM

H0: Hubble constant in unit of km/s/Mpc
c: speed of light

```
H0w = 71;
c = SpeedOfLight * Second/(1000 Meter)
gra = GravitationalConstant
```

$$\frac{149\,896\,229}{500}$$

$$\frac{6.67428\times 10^{-11}\ \text{Meter}^2\ \text{Newton}}{\text{Kilogram}^2}$$

LCDM Model

Basic Definations

hubble[Ωm0_,Ωd0_,Ωk0_,z_]: Hubble functions in LCDM.
H[z]: an example of hubble function with given parameters.

```
hubble[Ωm0_, Ωd0_, Ωk0_, z_] := H0w Sqrt[Ωm0 (1 + z)^3 + Ωd0 + Ωk0 (1 + z)^2];
H[z_] = hubble[Ωm0w, Ωd0w, 0, z];
```

q[Ωm0_,Ωd0_,Ωk0_,z_]:Deceleration parameter. DEF:
$q[\Omega m0\_, \Omega d0\_, \Omega k0\_, z\_] = \frac{-1}{H[z]}\ \frac{D[D[a[t],t],t]}{D[a[t],t]}$

```
q[Ωm0_, Ωd0_, Ωk0_, z_] =
  -1 + (1 + z)/hubble[Ωm0, Ωd0, Ωk0, z] D[hubble[Ωm0, Ωd0, Ωk0, z], z];
```

```
q[Ωm0, Ωd0, Ωk0, 0]
```

$$-1+\frac{2\,\Omega k0+3\,\Omega m0}{2\,(\Omega d0+\Omega k0+\Omega m0)}$$

$$-1+\frac{2\,\Omega k0+3\,\Omega m0}{2\,(\Omega d0+\Omega k0+\Omega m0)}$$

$$-1+\frac{2\,\Omega k0+3\,\Omega m0}{2\,(\Omega d0+\Omega k0+\Omega m0)}$$

```
Limit[q[Ωm0, Ωd0, Ωk0, z], z → Infinity]
```

$$\frac{1}{2}$$

(General) Friedmann equations are

$$\frac{3\,(a[t]'+k)}{a[t]^2} = 8\,\pi\,G\,\rho$$

$$2\,\frac{a[t]''}{a[t]} + \frac{a[t]'+k}{a[t]^2} = -8\,\pi\,G\,p$$

Since “zt” will be used as a parameter in most cases, we define “ztr” as a name for transition redshift, “ztrr” for transition redshift with a parameter r.
ztr[Ωm0_,Ωd0_,z_]: Transition redshift in LCDM with Ωk0 not zero with full parameters
ztrr[r_,z_]: with $r = \frac{\Omega m0}{\Omega d0}$

```
ztr[Ωm0_, Ωd0_] = (2 Ωd0/Ωm0)^(1/3) - 1;
ztrr[r_] = (2/r)^(1/3) - 1;
```

Regard “zt” as a paramter, solve out Ωd0 and Ωk0=1-Ωm0-Ωd0

$$\Omega d0 = \frac{1}{2}\,\Omega m0\,(1+zt)^3$$
$$\Omega k0 = 1 - \Omega m0 - \frac{1}{2}\,\Omega m0\,(1+zt)^3 \qquad (1)$$

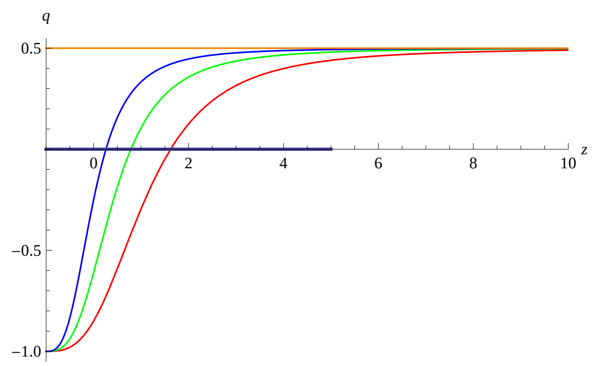
Transition redshift, deceleration parameter, theoretically.

Deceleration parameter

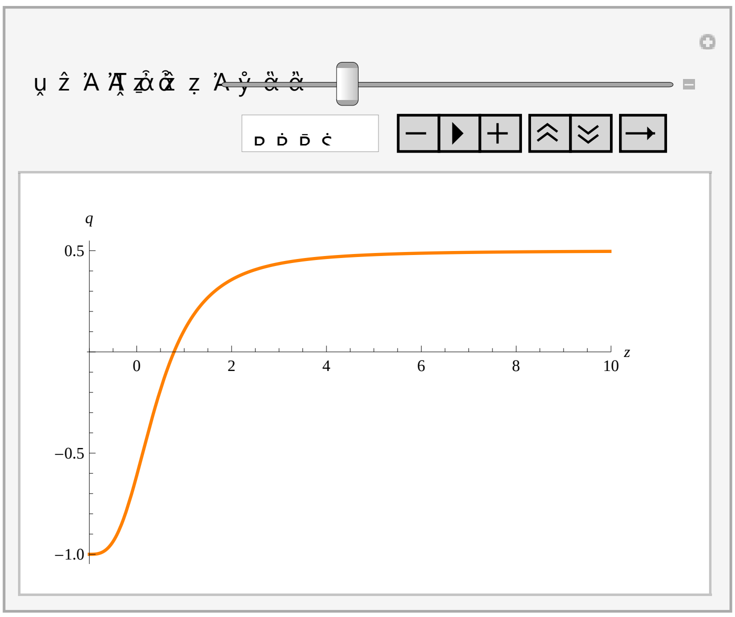
Deceleration parameter can be ploted with respect to redshift z.
Using flat FRW model. That is Ωk0=0.

```
pldec[Ωm0v_, color_] :=
 Plot[q[Ωm0v, 1 - Ωm0v, 0, z], {z, -1, 10},
  PlotRange → {{-1.05, 10}, {-1.05, 0.55}}, PlotStyle → color,
  AxesOrigin → {-1, 0}, AxesLabel → {z, q}]
```

```
Show[{pldec[0.1, Red], pldec[0.26, Green], pldec[0.5, Blue],
  pldec[1, Orange], Plot[0, {z, -1, 5}, PlotStyle → Thick]}]
```



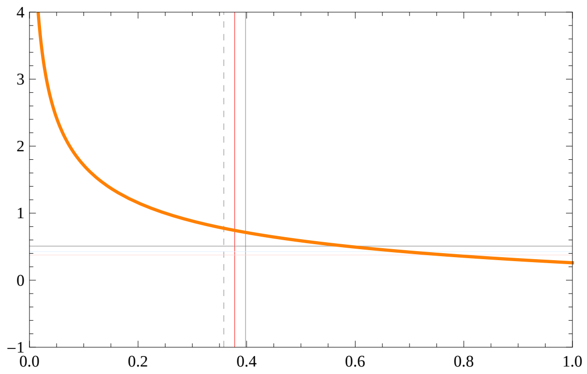
```
Manipulate[pldec[Ωm0v, {Orange, Thick}],
 {{Ωm0v, 0.26, "Matter Fraction"}, 0, 1, Appearance → "Open"},
 SaveDefinitions → True]
```



Plot deceleration parameter with respect to r. This is not related to Ωk0, i.e., the curvature of the spacetime won’t affect this result.

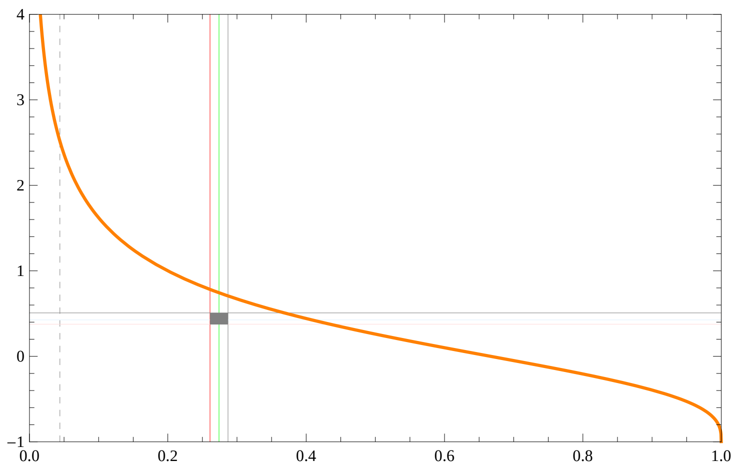
```
plztr[color_] := Plot[ztr[Ωm0, 1 - Ωm0], {Ωm0, 0, 1},
  PlotRange → {{0, 1}, {-1, 4}}, PlotStyle → color,
  AxesOrigin → {0, -1}]
```

```
pldecrr = Plot[ztrr[r], {r, 0, 1}, PlotRange → {{0, 1}, {-1, 4}},
  PlotStyle → {Thick, Orange}, AxesOrigin → {0, -1}, Frame → True,
  GridLines → {{{0.358, Dashed}, {0.378, Directive[Red]}, 0.398},
    {{0.376, LightRed}, {0.426, LightBlue}, {0.508, Gray}}}]
```



In a flat universe, Ωk0=0. Then Ωd0=1-Ωm0.

```
pldecr =
 Show[Plot[ztr[Ωm0, 1 - Ωm0], {Ωm0, 0, 1}, PlotRange → {{0, 1}, {-1, 4}},
   PlotStyle → {Thick, Orange}, AxesOrigin → {0, -1}, Frame → True,
   GridLines →
    {{{0.044, Dashed}, {0.261, Red}, {0.274, Green}, {0.287, Gray}},
     {{0.376, LightRed}, {0.426, LightBlue}, {0.508, Gray}}}],
  Graphics[
   {Gray, Rectangle[Scaled[{0.261, .2752}], Scaled[{0.287, .3016}]]}],
   Frame → True]]
```



Interacting Models

- **Take the form $Q_c = \xi H \rho_c$.**

Hubble function without curvature

```
hubbleICC[H0_, Ωd0_, Ωm0_, w_, ξ_, z_] := H0 √(Ωm + Ωd) ;
```

δ is constant + EoS w is constant

Definations

Fraction energy density

```
ΩmICC[Ωm0_, ξ_, z_] := Ωm0 (1 + z)^(3-ξ)
```

```
ΩdICC[Ωm0_, Ωd0_, w_, ξ_, z_] :=
 Ωd0 (1 + z)^(3 (1+w)) - ξ/(ξ + 3 w) Ωm0 (1 + z)^(3 (1+w)) ((1 + z)^(-ξ-3 w) - 1)
```

```
(*
(* These are manipulates that we can used to check the
 behavior of Energy fraction of different species.*)
Manipulate[Plot[ΩmICC[Ωm0,ξ,z],{z,-0.5,10}],{Ωm0,0.01,0.99},
  {ξ,-0.1,0.1}]
 Manipulate[Plot[ΩdICC[Ωm0,1-Ωm0,w,ξ,z],{z,-0.5,10}],
  {Ωm0,0.01,0.99},{w,-1.2,-0.8},{ξ,-0.1,0.1}]
*)
```

Hubble function

```
hubbleICC[H0ICC_, Ωm0ICC_, Ωd0ICC_, wICC_, ξICC_, z_] =
 H0ICC √(ΩmICC[Ωm0ICC, ξICC, z] + ΩdICC[Ωm0ICC, Ωd0ICC, wICC, ξICC, z])
```

$$\mathrm{H0ICC}\sqrt{\left((1+z)^{3(1+\mathrm{wICC})}\,\Omega\mathrm{d0ICC}+(1+z)^{3-\xi\mathrm{ICC}}\,\Omega\mathrm{m0ICC}-\frac{(1+z)^{3(1+\mathrm{wICC})}\left(-1+(1+z)^{-3\,\mathrm{wICC}-\xi\mathrm{ICC}}\right)\xi\mathrm{ICC}\,\Omega\mathrm{m0ICC}}{3\,\mathrm{wICC}+\xi\mathrm{ICC}}\right)}$$

Deceleration parameter

```
qICC[H0ICC_, Ωm0ICC_, Ωd0ICC_, wICC_, ξICC_, z_] =
  -1 + (1 + z)/hubbleICC[H0ICC, Ωm0ICC, Ωd0ICC, wICC, ξICC, z]
    D[hubbleICC[H0ICC, Ωm0ICC, Ωd0ICC, wICC, ξICC, z], z];
```

```
qICC[H0ICC, Ωm0ICC, Ωd0ICC, wICC, ξICC, z] // FullSimplify
```

$$\left(-3\,\mathrm{wICC}\,(-1+\xi\mathrm{ICC})\,\Omega\mathrm{m0ICC}+(1+3\,\mathrm{wICC})\,(1+z)^{3\,\mathrm{wICC}+\xi\mathrm{ICC}}\,(3\,\mathrm{wICC}\,\Omega\mathrm{d0ICC}+\xi\mathrm{ICC}\,(\Omega\mathrm{d0ICC}+\Omega\mathrm{m0ICC}))\right)\Big/\left(6\,\mathrm{wICC}\,\Omega\mathrm{m0ICC}+2\,(1+z)^{3\,\mathrm{wICC}+\xi\mathrm{ICC}}\,(3\,\mathrm{wICC}\,\Omega\mathrm{d0ICC}+\xi\mathrm{ICC}\,(\Omega\mathrm{d0ICC}+\Omega\mathrm{m0ICC}))\right)$$

```
Limit[qICC[H0ICC, Ωm0ICC, Ωd0ICC, wICC, ξICC, z] // FullSimplify,
 z → Infinity, Assumptions → (3 wICC + ξICC) < 0]
```

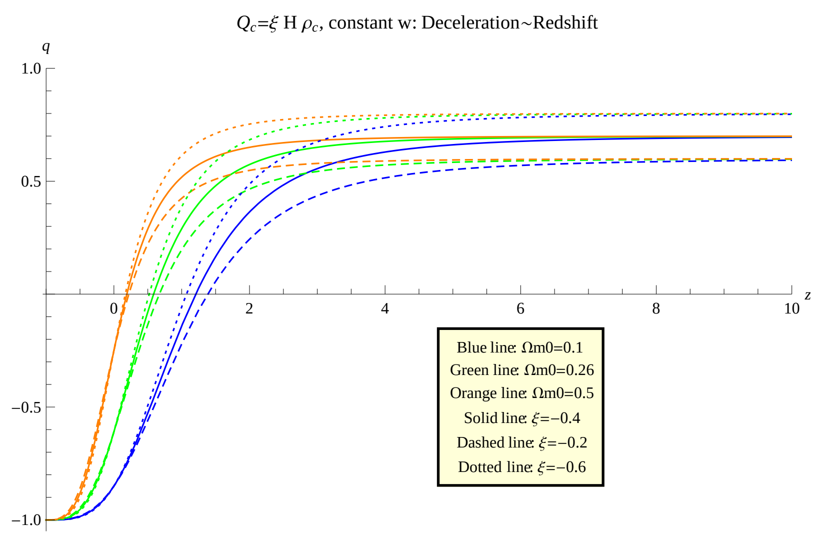
$$\frac{1-\xi\mathrm{ICC}}{2}$$

At z-> Infinity limit, qIIC->$\frac{(1-\xi)}{2}$, with 3 wICC+ξICC<0. This is different from LCDM model.

Plots, showcase, manipulate toys

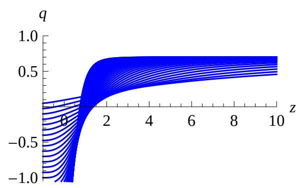
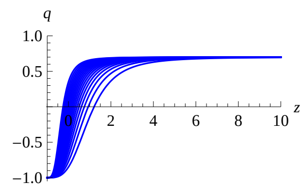
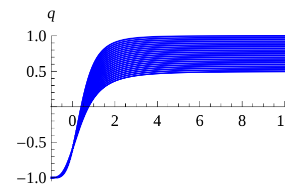
```
pldecICC[Ωm0ICC_, wICC_, ξICC_, color_] :=
  Plot[qICC[H0w, Ωm0ICC, 1 - Ωm0ICC, wICC, ξICC, z], {z, -1, 10},
   PlotRange → {{-1.05, 10}, {-1.05, 1}}, PlotStyle → color,
   AxesOrigin → {-1, 0}, AxesLabel → {z, q}];
```

```
pldecICCShowSum =
 Show[{pldecICC[0.1, -1, -0.4, Blue], pldecICC[0.26, -1, -0.4, Green],
   pldecICC[0.5, -1, -0.4, Orange],
   pldecICC[0.1, -1, -0.2, Directive[Blue, Dashed]],
   pldecICC[0.26, -1, -0.2, Directive[Green, Dashed]],
   pldecICC[0.5, -1, -0.2, Directive[Orange, Dashed]],
   pldecICC[0.1, -1, -0.6, Directive[Blue, Dotted]],
   pldecICC[0.26, -1, -0.6, Directive[Green, Dotted]],
   pldecICC[0.5, -1, -0.6, Directive[Orange, Dotted]]},
  Epilog →
   Inset[
    Framed[
     Style[
      "Blue line: Ωm0=0.1\n Green line: Ωm0=0.26\n Orange
        line: Ωm0=0.5\n Solid line: ξ=-0.4\n Dashed
        line: ξ=-0.2\n Dotted line: ξ=-0.6", 10],
     Background → LightYellow], {6, -0.5}],
  PlotLabel → "Qc=ξ H ρc, constant w: Deceleration~Redshift",
  ImageSize → 500]
```



z->Infinity is a degenerate limit. For constant ξ and constant w models, this limit is determined by the interaction strength ξ . This might be useful if more complicated models are investigated and no large deviations are shown. [Nota]

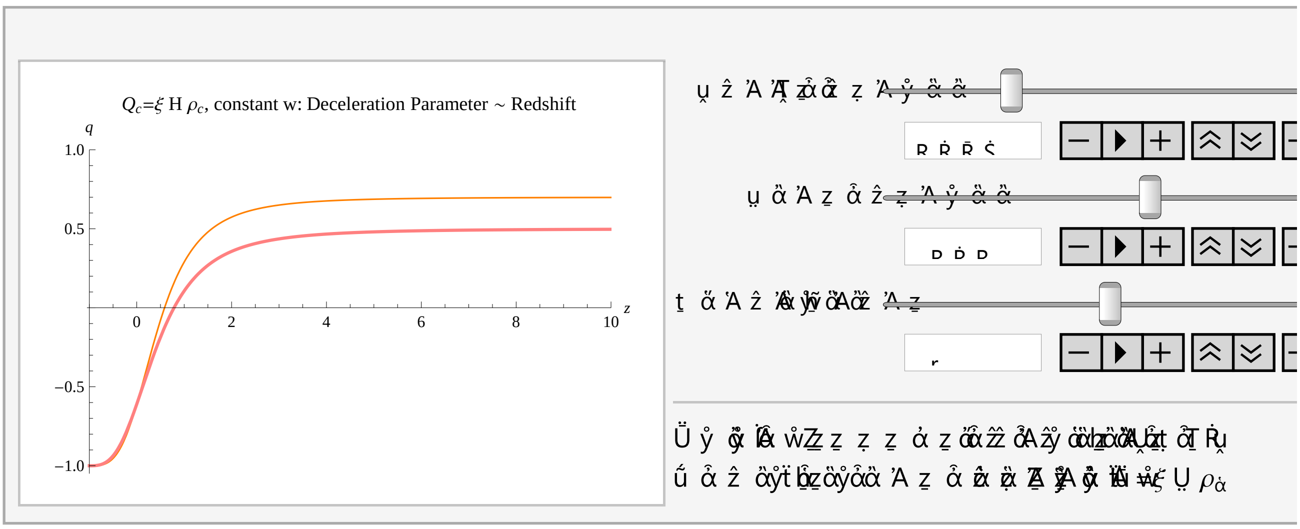
```
varyingICCShowSum =
  Grid[{{"Qc=ξ H ρc with constant ξ and w", SpanFromLeft},
    {"Varying EoS", "Varying Ωm0", "Varying ξ"},
    {Show[Table[pldecICC[0.1, wICC, -0.4, Blue],
       {wICC, -2, -0.3, 0.05}]],
     Show[Table[pldecICC[Ωm0ICC, -1, -0.4, Blue],
       {Ωm0ICC, 0.1, 0.9, 0.05}]],
     Show[Table[pldecICC[0.27, -1, ξICC, Blue],
       {ξICC, -1, 0, 0.05}]]}}, Frame → All,
   Background → {{None}, {LightGreen, LightGray, None}},
   Alignment → Center, ItemSize → 13]
```

$Q_c=\xi\ H\ \rho_c$ with constant ξ and w		
Varying EoS	Varying Ωm0	Varying ξ
		

Interaction ξ changes the limit, i.e., what value will it be at z->Infinity. EoS changes the the whole shape. Matter fraction also changes how fast q varies, but just in a small time scale.

Use movable slides to check how do the parameters affect the deceleration parameter. Just a toy.

```
pldecICCManSum =
 Manipulate[
  Show[{pldecICC[Ωm0ICC, wICC, ξICC, Orange],
    pldec[Ωm0ICC, {Pink, Thick}]},
   PlotLabel →
    "Qc=ξ H ρc, constant w: Deceleration Parameter ~ Redshift"],
  {{Ωm0ICC, 0.26, "Matter Fraction"}, 0, 1, Appearance → "Open"},
  {{ξICC, -0.4, "Interaction"}, -1, 0, Appearance → "Open"},
  {{wICC, -1, "Equation of State"}, -1.5, -0.5, Appearance → "Open"},
  Delimiter, Style["Pink is the deceleration parameter for LCDM.",
   Medium], Style["Orange is for interacting model with Q=ξ H ρm",
   Medium], ControlPlacement → {Right, Right, Right},
  SaveDefinitions → True]
```



Transition redshift definations and equations.

Find out the expression for transition redshift

```
(3 wICC + 1) ΩdICC[Ωm0ICC, Ωd0ICC, wICC, ξICC, z] +
   ΩmICC[Ωm0ICC, ξICC, z] == 0 // Simplify
```

$$(1+z)^{3-\xi ICC}\,\Omega m0ICC + (1+3\,wICC)\,(1+z)^{3+3\,wICC}\left(\Omega d0ICC + \frac{\left(1-(1+z)^{-3\,wICC-\xi ICC}\right)\xi ICC\,\Omega m0ICC}{3\,wICC+\xi ICC}\right) == 0$$

```
ztrICC[Ωm0ICC_, Ωd0ICC_, wICC_, ξICC_] =
```

$$-1+\left(\frac{\frac{-\xi ICC}{\xi ICC+3\,wICC}\,(1+3\,wICC)+1}{(1+3\,wICC)\left(\frac{-\xi ICC}{\xi ICC+3\,wICC}\,\Omega m0ICC-\Omega d0ICC\right)}\,\Omega m0ICC\right)^{\frac{1}{\xi ICC+3\,wICC}}$$

$$-1+\left(\frac{\left(1-\frac{(1+3\,wICC)\,\xi ICC}{3\,wICC+\xi ICC}\right)\Omega m0ICC}{(1+3\,wICC)\left(-\Omega d0ICC-\frac{\xi ICC\,\Omega m0ICC}{3\,wICC+\xi ICC}\right)}\right)^{\frac{1}{3\,wICC+\xi ICC}}$$

The first solution is trivil. So the second one is taken.

Define rICC = $\frac{\Omega m0ICC}{\Omega d0ICC}$

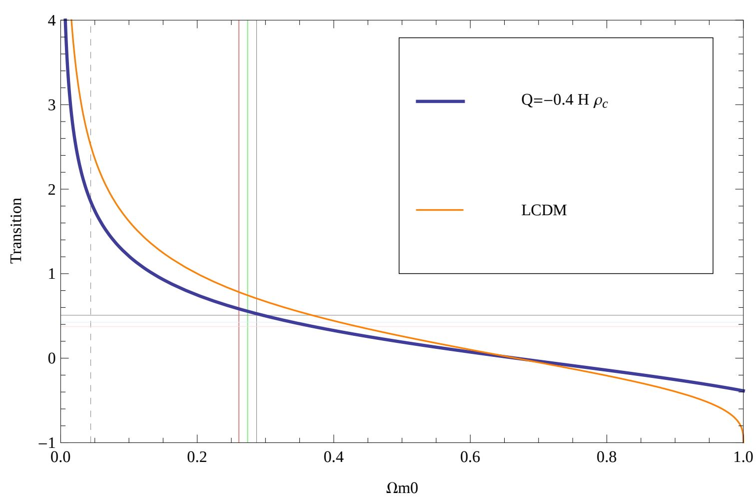
```
ztrrICC[rICC_, wICC_, ξICC_] =
```

$$-1+\left(\frac{\frac{-\xi ICC}{\xi ICC+3\,wICC}\,(1+3\,wICC)+1}{(1+3\,wICC)\left(\frac{-\xi ICC}{\xi ICC+3\,wICC}\,rICC-1\right)}\,rICC\right)^{\frac{1}{3\,wICC+\xi ICC}};$$

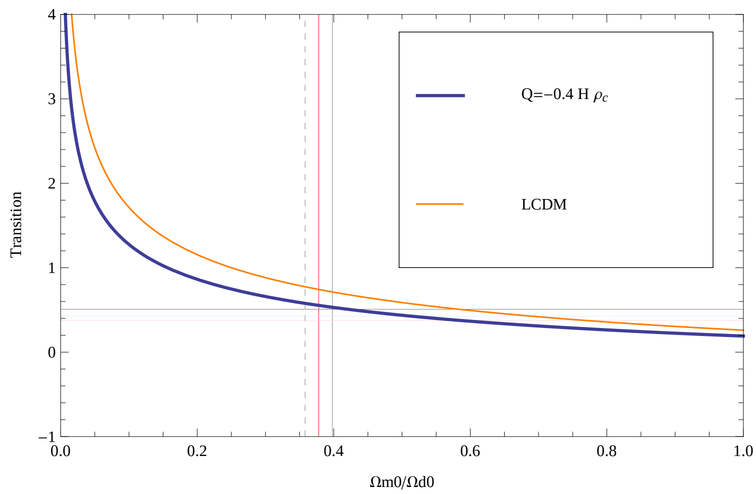
Visualiztion of transition redshift

Check the behavior of this transition redshift.

```
pldecrICC =
 Plot[{ztrICC[Ωm0ICC, 1 - Ωm0ICC, -1, -0.4], ztr[Ωm0ICC, 1 - Ωm0ICC]},
  {Ωm0ICC, 0, 1}, FrameLabel → {"Ωm0", "Transition"},
  PlotRange → {{0, 1}, {-1, 4}}, PlotStyle → {Thick, Orange},
  AxesOrigin → {0, -1}, Frame → True, PlotRangeClipping → False,
  GridLines →
   {{{0.044, Dashed}, {0.261, Red}, {0.274, Green}, {0.287, Gray}},
    {{0.376, LightRed}, {0.426, LightBlue}, {0.508, Gray}}},
  PlotLegend → {"Q=-0.4 H ρc", "LCDM"}, LegendPosition → {0.0, -0.05},
  LegendShadow → None, ImageSize → 500]
```

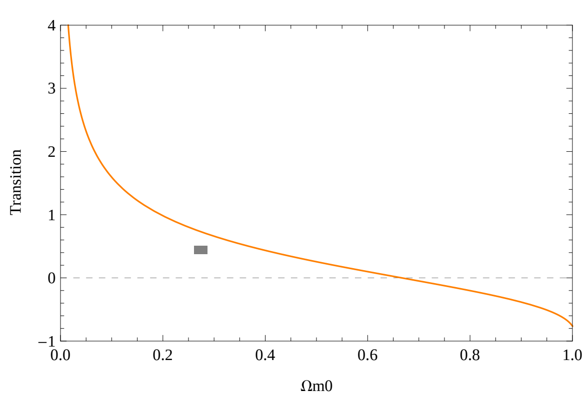


```
pldecrrICC = Plot[{ztrrICC[rICC, -1, -0.4], ztrr[rICC]},
  {rICC, 0, 1}, PlotRange → {{0, 1}, {-1, 4}},
  PlotStyle → {Thick, Orange}, AxesOrigin → {0, -1}, Frame → True,
  FrameLabel → {"Ωm0/Ωd0", "Transition"},
  GridLines → {{{0.358, Dashed}, {0.378, Directive[Red]}, 0.398},
    {{0.376, LightRed}, {0.426, LightBlue}, {0.508, Gray}}},
  PlotLegend → {"Q=-0.4 H ρc", "LCDM"}, LegendPosition → {0.0, -0.05},
  LegendShadow → None, ImageSize → 500]
```




```
plztrICC[wICC_, ξICC_, color_] :=
  Plot[ztrICC[Ωm0ICC, 1 - Ωm0ICC, wICC, ξICC], {Ωm0ICC, 0, 1},
   PlotRange → {{0, 1}, {-1, 4}}, PlotStyle → color,
   AxesOrigin → {0, -1}, Frame → True];
```

```
Show[plztrICC[-1, -0.02, Orange],
 Graphics[
  {Gray, Rectangle[Scaled[{0.261, .2752}], Scaled[{0.287, .3016}]]}],
  Frame → True], FrameLabel → {"Ωm0", "Transition"},
 GridLines → {{}, {{0, Dashed}}}]
```



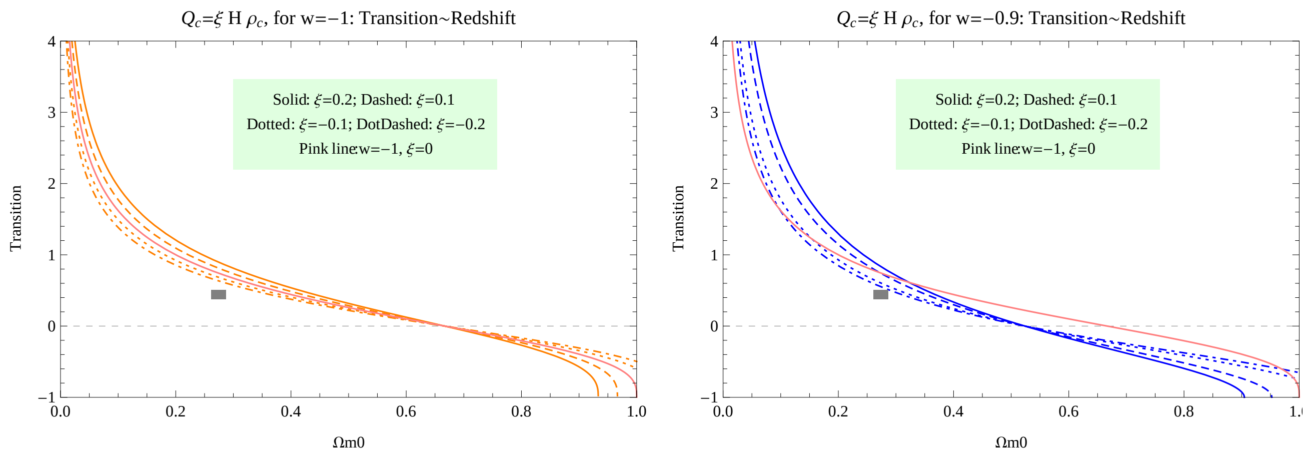
The following figure:

Orange for w=-1
Blue for w=-0.9

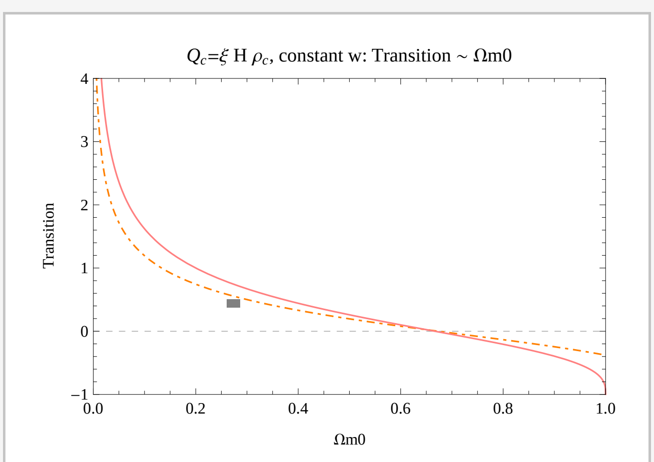
line: ξ=0.2
Dashed: ξ=0.1
Dotted: ξ=-0.1
DotDashed: ξ=-0.2

Pink line:w=-1, ξ=0

```
plztrvsΩm0ICCSum =
 Grid[
  {{Show[{plztrICC[-1, 0.2, Orange],
      plztrICC[-1, 0.1, {Orange, Dashed}],
      plztrICC[-1, -0.1, {Orange, Dotted}],
      plztrICC[-1, -0.2, {Orange, DotDashed}],
      Plot[ztr[Ωm0ICC, 1 - Ωm0ICC], {Ωm0ICC, 0, 1},
       PlotRange → {{0, 1}, {-1, 4}}, PlotStyle → Pink,
       AxesOrigin → {0, -1}]},
     Graphics[{Gray, Rectangle[Scaled[{0.261, .2752}],
        Scaled[{0.287, .3016}]]}, Frame → True],
     FrameLabel → {"Ωm0", "Transition"},
     GridLines → {{}, {{0, Dashed}}},
     Epilog →
      Inset[
       Framed[
        Style[
         "Solid: ξ=0.2; Dashed: ξ=0.1\n Dotted: ξ=-0.1;
           DotDashed: ξ=-0.2\n Pink line:w=-1, ξ=0", 10],
        Background → LightGreen, FrameStyle → None], {0.3, 3.5},
       {Left, Top}],
     PlotLabel → "Qc=ξ H ρc, for w=-1: Transition~Redshift",
     ImageSize → 400],
    Show[{plztrICC[-0.7, 0.2, Blue],
      plztrICC[-0.7, 0.1, {Blue, Dashed}],
      plztrICC[-0.7, -0.1, {Blue, Dotted}],
      plztrICC[-0.7, -0.2, {Blue, DotDashed}],
      Plot[ztr[Ωm0ICC, 1 - Ωm0ICC], {Ωm0ICC, 0, 1},
       PlotRange → {{0, 1}, {-1, 4}}, PlotStyle → Pink,
       AxesOrigin → {0, -1}]},
     Graphics[{Gray, Rectangle[Scaled[{0.261, .2752}],
        Scaled[{0.287, .3016}]]}, Frame → True],
     FrameLabel → {"Ωm0", "Transition"},
     GridLines → {{}, {{0, Dashed}}},
     Epilog →
      Inset[
       Framed[
        Style[
         "Solid: ξ=0.2; Dashed: ξ=0.1\n Dotted: ξ=-0.1;
           DotDashed: ξ=-0.2\n Pink line:w=-1, ξ=0", 10],
        Background → LightGreen, FrameStyle → None], {0.3, 3.5},
       {Left, Top}],
     PlotLabel → "Qc=ξ H ρc, for w=-0.9: Transition~Redshift",
     ImageSize → 400]
   }}]
```




```
plztrICCManSum =
 Manipulate[
  Show[{plztrICC[wICC, ξICC, {Orange, DotDashed}],
    Plot[ztr[Ωm0ICC, 1 - Ωm0ICC], {Ωm0ICC, 0, 1},
     PlotRange → {{0, 1}, {-1, 4}}, PlotStyle → Pink,
     AxesOrigin → {0, -1}]},
   Graphics[{Gray, Rectangle[Scaled[{0.261, .2752}],
      Scaled[{0.287, .3016}]]}], Frame → True],
   FrameLabel → {"Ωm0", "Transition"},
   GridLines → {{}, {{0, Dashed}}},
   PlotLabel → "Qc=ξ H ρc, constant w: Transition ~ Ωm0"],
  {{ξICC, -0.4, "Interaction"}, -1, 1, Appearance → "Open"},
  {{wICC, -1.02, "Equation of state"}, -2, -0.3, Appearance → "Open"},
  Delimiter,
  Style["Pink is the transition redshift vs Ωm0 for LCDM.", Medium],
  Style["Orange DotDashed line is interacting model,", Medium],
  Style[
   "for Q=ξ H ρm with constant w, \n where ξ and w are both
     constants", Medium], ControlPlacement → {Right, Right},
  SaveDefinitions → True]
```



Find out the allowed region of coupling constant.

To find out the region of ξ, set w=-1 and Ωd0=1-Ωm0. Let the ztr-Ωm0 line cross points (0.287,0.508) and (0.261, 0.376).

```
ztrICC[0.287, 1 - 0.287, -1, ξICC1] == 0.508
```

$$-1 + 0.1435^{\frac{1}{-3+\xi ICC1}} \left(-\frac{1 + \frac{2\,\xi ICC1}{-3+\xi ICC1}}{-0.713 - \frac{0.287\,\xi ICC1}{-3+\xi ICC1}}\right)^{\frac{1}{-3+\xi ICC1}} == 0.508$$

```
ξICCffunc[Ωm0ICC_, Ωd0ICC_, wICC_, data_] :=
 ξICC /. FindRoot[ztrICC[Ωm0ICC, Ωd0ICC, wICC, ξICC] == data,
   {ξICC, -0.6}]
```

```
ξICCf2[Ωm0ICC_, Ωd0ICC_, wICC_] :=
 ξICC /. FindRoot[ztrICC[Ωm0ICC, Ωd0ICC, wICC, ξICC] == 0.508,
   {ξICC, -0.6}]
```

```
ξICCf1[Ωm0ICC_, Ωd0ICC_, wICC_] :=
 ξICC /. FindRoot[ztrICC[Ωm0ICC, Ωd0ICC, wICC, ξICC] == 0.376,
   {ξICC, -0.6}]
```

Cross the Center of best fit. (0.274,0.426)

```
ξICCfc[Ωm0ICC_, Ωd0ICC_, wICC_] :=
 ξICC /. FindRoot[ztrICC[Ωm0ICC, Ωd0ICC, wICC, ξICC] == 0.426,
   {ξICC, -0.6}]
```

According to the data of transition redshift.

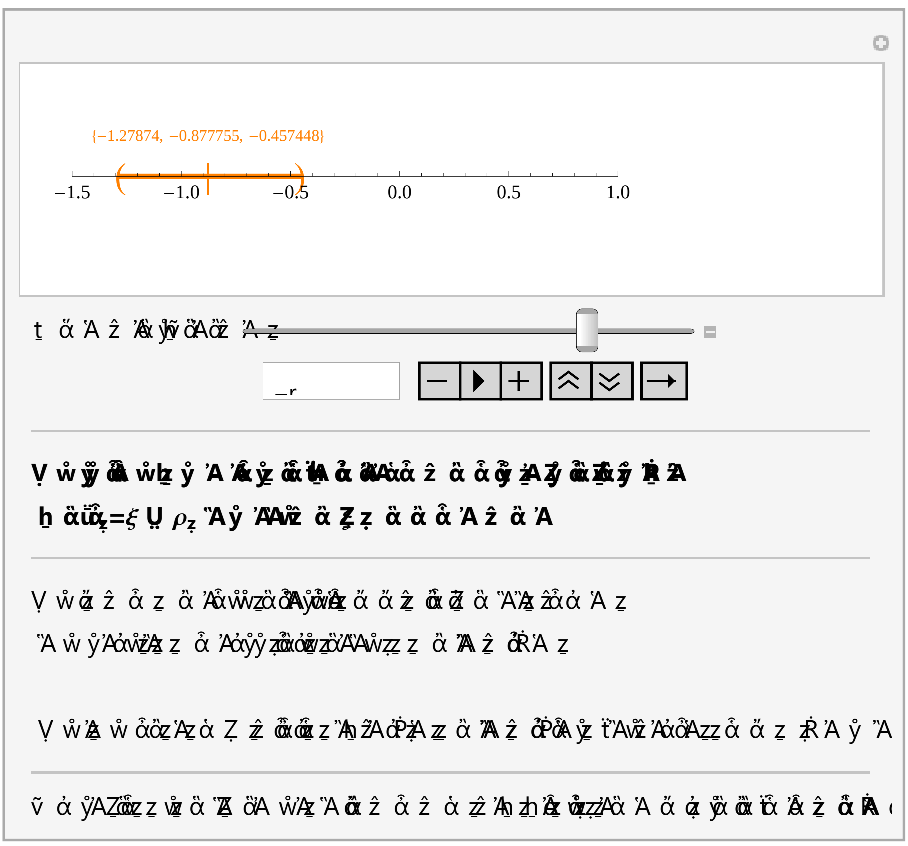
```
ξICCf2[wICC_] := ξICCffunc[0.287, 1 - 0.287, wICC, 0.508]
```

```
ξICCf1[wICC_] := ξICCffunc[0.261, 1 - 0.261, wICC, 0.376]
```

```
ξICCfc[wICC_] := ξICCffunc[0.274, 1 - 0.274, wICC, 0.426]
```

```
numPlot[ss1_, {s_, c_, e_}, ee_, {start_, end_}] :=
 Graphics[{Orange, Thickness[.01],
   Text[Style[ss1, Large, Orange], {s, 0}],
   Text[Style["|", Large, Orange], {c, 0}],
   Text[Style[ee, Large, Orange], {e, 0}], Line[{{s, 0}, {e, 0}}],
   Text[{Dynamic[s], Dynamic[c], Dynamic[e]}, {c, 0.5}]},
  Axes → {True, False}, AxesStyle → Directive[Thin, Black, 12],
  PlotRange → {{start, end}, {0, 0}}, AspectRatio → 0.3]
```

```
fitξICCManSum =
 Manipulate[numPlot["(", {ξICCf1[wICC], ξICCfc[wICC], ξICCf2[wICC]},
   ")", {-1.5, 1}], {{wICC, -1, "Equation of State"}, -3,
   -0.47, Appearance → "Open"}, Delimiter,
  Style[
   "This is the fitting result from transition redshift
     data,\n for Qc=ξ H ρc with w and ξ constant", Bold],
  Delimiter,
  "The parenthesis shows the upper and lower value \n while
    the verticle line show the center value.",
  Style[
   "\n The three numbers are left value, center value,
     right value respectively."], Delimiter, Delimiter,
  Style[
   "Slide to see how do the two parameters affect the
     coupling constant results."],
  ControlPlacement → {Bottom, Bottom}, SaveDefinitions → True]
```

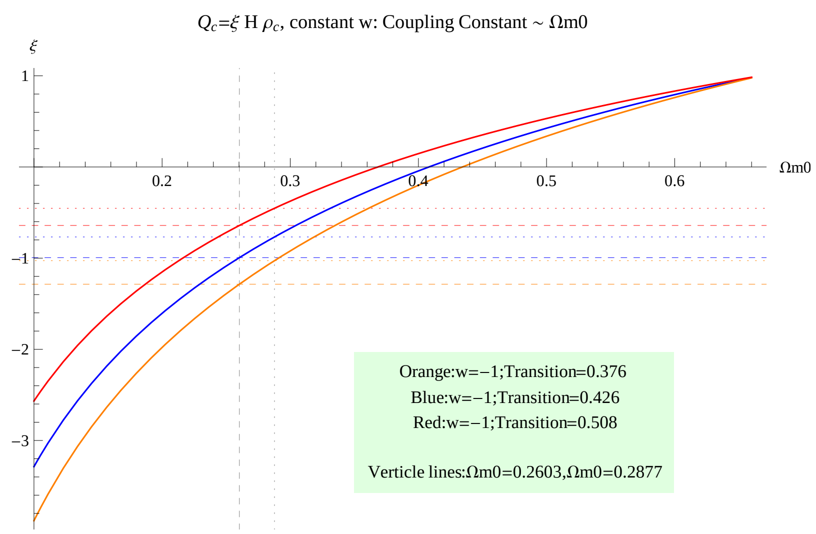


Explicit report of ξ~Ωm0 result.

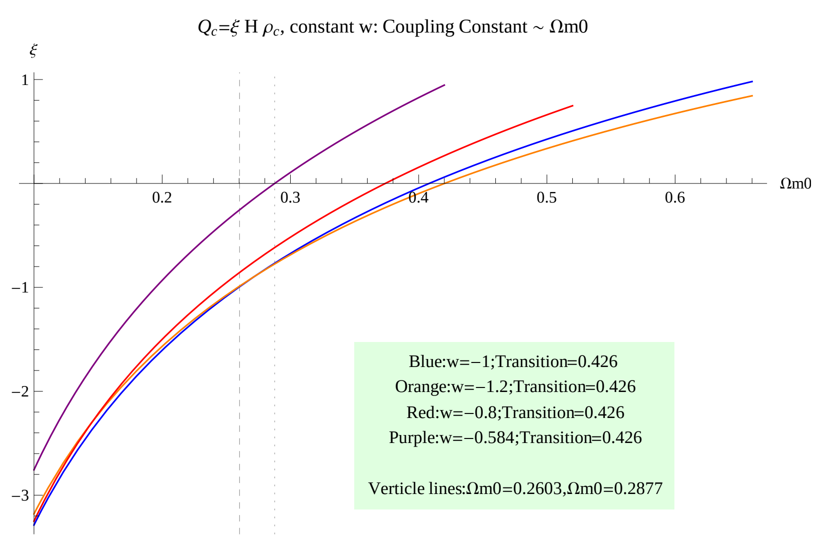
```
per = 0.05;
tabξICCSum =
 Grid[
  {{"For Ωm0∈0.274" (1 ± per)
     Style["Table of ξ for different Ωm0~Transition combination",
      Bold], SpanFromLeft, SpanFromLeft},
   {Style["Ωm0⋱Transition", Small, Bold], 0.426, 0.376, 0.508},
   {0.274 (1 - per), ξICCffunc[0.274 (1 - per), 1 - 0.274 (1 - per) - 0,
     -1, 0.426], ξICCffunc[0.274 (1 - per), 1 - 0.274 (1 - per) - 0,
     -1, 0.376], ξICCffunc[0.274 (1 - per), 1 - 0.274 (1 - per) - 0,
     -1, 0.508]},
   {0.274, ξICCffunc[0.274, 1 - 0.274 - 0, -1, 0.426],
    ξICCffunc[0.274, 1 - 0.274 - 0, -1, 0.376],
    ξICCffunc[0.274, 1 - 0.274 - 0, -1, 0.508]},
   {0.274 (1 + per), ξICCffunc[0.274 (1 + per), 1 - 0.274 (1 + per) - 0,
     -1, 0.426], ξICCffunc[0.274 (1 + per), 1 - 0.274 (1 + per) - 0,
     -1, 0.376], ξICCffunc[0.274 (1 + per), 1 - 0.274 (1 + per) - 0,
     -1, 0.508]}}, Frame → All,
  Background → {{LightGray, None}, {LightGreen, LightGray, None}},
  Alignment → Center, ItemSize → 8]
```

For Ωm0∈0.274 (1 ± 0.05) **Table of ξ for different Ωm0~Transition combination**			
Ωm0⋱Transition	0.426	0.376	0.508
0.2603	−0.994339	−1.28571	−0.641508
0.274	−0.877755	−1.15303	−0.544482
0.2877	−0.767582	−1.02756	−0.452892

```
pltξvΩm0ICCSum =
 Plot[{ξICCffunc[Ωm0ICC, 1 - Ωm0ICC - 0, -1, 0.426],
   ξICCffunc[Ωm0ICC, 1 - Ωm0ICC - 0, -1, 0.376],
   ξICCffunc[Ωm0ICC, 1 - Ωm0ICC - 0, -1, 0.508]}, {Ωm0ICC, 0.1, 0.66},
  PlotStyle → {Blue, Orange, Red}, AxesLabel → {"Ωm0", "ξ"},
  GridLines →
   {{{0.2603, Directive[Gray, Dashed]},
     {0.2877`, Directive[Gray, Dotted]}},
    {{-1.2857, Directive[Orange, Dashed]},
     {-0.9943, Directive[Blue, Dashed]},
     {-0.6415, Directive[Red, Dashed]},
     {-1.0276, Directive[Orange, Dotted]},
     {-0.7676, Directive[Blue, Dotted]},
     {-0.4529, Directive[Red, Dotted]}}},
  Epilog →
   Inset[
    Framed[
     Style[
      "Orange:w=-1;Transition=0.376\n Blue:w=-1;Transition=0.426\n
        Red:w=-1;Transition=0.508\n \n Verticle
        lines:Ωm0=0.2603,Ωm0=0.2877", 11],
     Background → LightGreen, FrameStyle → None], {0.35, -2},
    {Left, Top}],
  PlotLabel → "Qc=ξ H ρc, constant w: Coupling Constant ~ Ωm0",
  ImageSize → 500]
```



```
pltξvΩm0ICCSum2 =
 Show[
   {Plot[{ξICCffunc[Ωm0ICC, 1 - Ωm0ICC - 0, -1, 0.426],
      ξICCffunc[Ωm0ICC, 1 - Ωm0ICC - 0, -1.2, 0.426]},
     {Ωm0ICC, 0.1, 0.66}, PlotStyle → {Blue, Orange},
     AxesLabel → {"Ωm0", "ξ"}],
    Plot[ξICCffunc[Ωm0ICC, 1 - Ωm0ICC - 0, -0.8, 0.426],
     {Ωm0ICC, 0.1, 0.52}, PlotStyle → Red, AxesLabel → {"Ωm0", "ξ"}],
    Plot[ξICCffunc[Ωm0ICC, 1 - Ωm0ICC - 0,
      eosICC1 /.
       FindRoot[{ξICCffunc[0.2877, 1 - 0.2877 - 0, eosICC1, 0.426] ==
          0}, {eosICC1, -0.5}], 0.426], {Ωm0ICC, 0.1, 0.42},
     PlotStyle → Purple, AxesLabel → {"Ωm0", "ξ"}]},
   GridLines →
    {{{0.2603, Directive[Gray, Dashed]},
      {0.2877`, Directive[Gray, Dotted]}}, {}},
   Epilog →
    Inset[
     Framed[
      Style[
       "Blue:w=-1;Transition=0.426\n
         Orange:w=-1.2;Transition=0.426\n
         Red:w=-0.8;Transition=0.426\n
         Purple:w=-0.584;Transition=0.426\n \n
         Verticle lines:Ωm0=0.2603,Ωm0=0.2877", 11],
      Background → LightGreen, FrameStyle → None], {0.35, -1.5},
     {Left, Top}],
   PlotLabel → "Qc=ξ H ρc, constant w: Coupling Constant ~ Ωm0",
   ImageSize → 500] // Quiet
```



Different plot ranges are chosen because for very large Ωm0, it is impossible to choose a parameter so that the transition redshift is

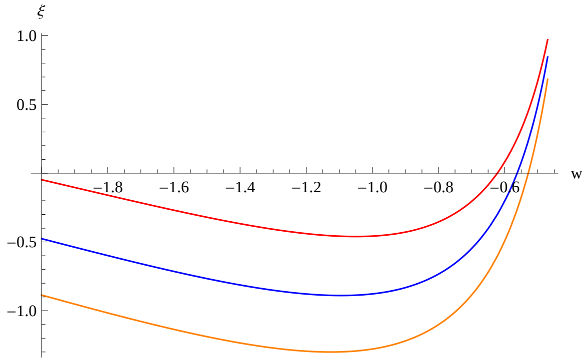
```
tabξvΩm0ICCSum21 =
 Grid[
   {{"Qc=ξ H ρc, constant w: ξ when transition redshift is 0.426",
     SpanFromLeft}, {"⋱", "w=-0.45064", "w=-0.45063"},
    {"Ωm0=0.2603", ξICCffunc[0.2603, 1 - 0.2603 - 0, -0.45064, 0.426],
     ξICCffunc[0.2603, 1 - 0.2603 - 0, -0.45063, 0.426]}},
   Frame → All,
   Background → {{LightGray, None}, {LightGreen, LightGray, None}},
   ItemSize → 13] // Quiet
tabξvΩm0ICCSum22 =
 Grid[{{"Qc=ξ H ρc, constant w:EoS value when ξ=0", SpanFromLeft},
    {"⋱", "Transition 0.426"},
    {"Ωm0=0.2877",
     eosICC1 /.
      FindRoot[{ξICCffunc[0.2877, 1 - 0.2877 - 0, eosICC1, 0.426] == 0},
       {eosICC1, -0.5}]}}, Frame → All, ItemSize → 13,
   Background → {{LightGray, None}, {LightGreen, LightGray, None}}] //
  Quiet
```

$Q_c=\xi$ H ρ_c, constant w: ξ when transition redshift is 0.426		
⋱	w=−0.45064	w=−0.45063
Ωm0=0.2603	0.999886	−3.64749

$Q_c=\xi$ H ρ_c, constant w:EoS value when ξ=0	
⋱	Transition 0.426
Ωm0=0.2877	−0.58406

So we give the result that w∈(-0.58406, -0.45064) if we constrain Ωm0=0.2603 and transition redshift 0.426.

```
pltξvwICC = Plot[{ξICCfc[wICC], ξICCf1[wICC], ξICCf2[wICC]},
  {wICC, -2, -0.47}, PlotStyle → {Blue, Orange, Red},
  AxesLabel → {"w", "ξ"}]
```



Choose the values from Reference 3. w = -1.087 ± 0.096,

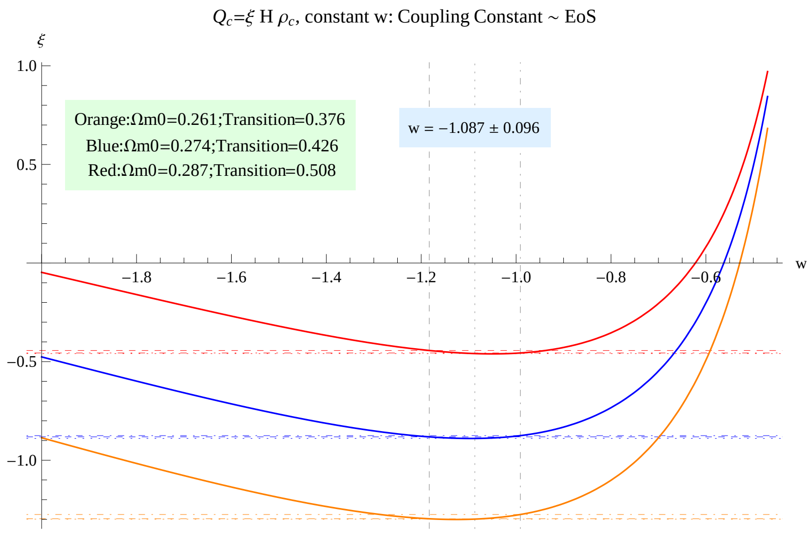
```
ξvwExamICC =
 {Block[{wICC = -1.087 - 0.096},
   {wICC, ξICCfc[wICC], ξICCf1[wICC], ξICCf2[wICC]}],
  Block[{wICC = -1.087}, {wICC, ξICCfc[wICC], ξICCf1[wICC],
    ξICCf2[wICC]}], Block[{wICC = -1.087 + 0.096},
   {wICC, ξICCfc[wICC], ξICCf1[wICC], ξICCf2[wICC]}]}
```

```
{{-1.183, -0.881565, -1.29687, -0.443589},
 {-1.087, -0.88948, -1.29859, -0.459135},
 {-0.991, -0.875238, -1.27522, -0.456176}}
```

```
tabξvwExamICC =
 Grid[Prepend[Prepend[ξvwExamICC, {"w", "Center", "Lower", "Upper"}],
   {"ξ results for Qc=ξ H ρc (Fitting data: Data From, 2)",
    SpanFromLeft}], Frame → All,
  Background → {{LightGray, None}, {LightGreen, LightGray, None}},
  Alignment → Center, ItemSize → 8]
```

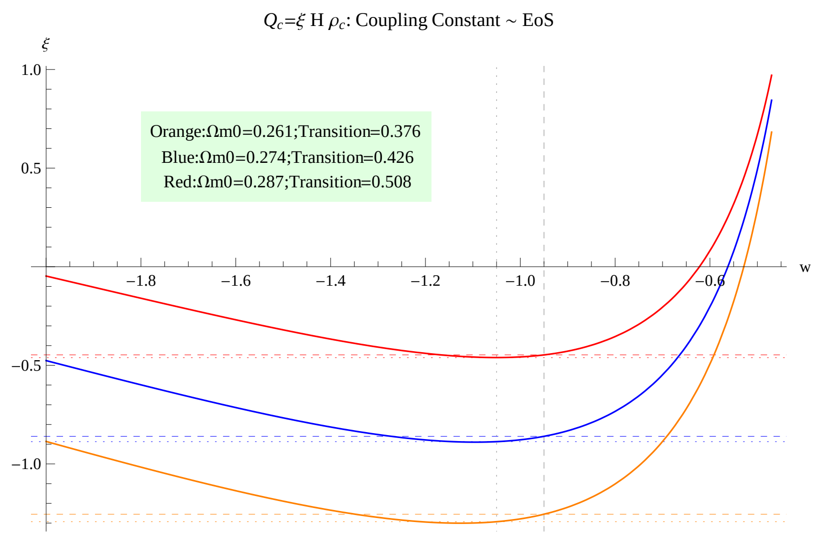
ξ results for Q_c=ξ H ρ_c (Fitting data: Data From, 2)			
w	Center	Lower	Upper
-1.183	-0.881565	-1.29687	-0.443589
-1.087	-0.88948	-1.29859	-0.459135
-0.991	-0.875238	-1.27522	-0.456176

```
pltξvwExamICC =
 Show[pltξvwICC,
  GridLines →
   {{{ξvwExamICC[[1, 1]], Directive[Gray, Dashed]},
     {ξvwExamICC[[2, 1]], Directive[Gray, Dotted]},
     {ξvwExamICC[[3, 1]], Directive[Gray, DotDashed]}},
    {{ξvwExamICC[[1, 3]], Directive[Orange, Dashed]},
     {ξvwExamICC[[1, 2]], Directive[Blue, Dashed]},
     {ξvwExamICC[[1, 4]], Directive[Red, Dashed]},
     {ξvwExamICC[[2, 3]], Directive[Orange, Dotted]},
     {ξvwExamICC[[2, 2]], Directive[Blue, Dotted]},
     {ξvwExamICC[[2, 4]], Directive[Red, Dotted]},
     {ξvwExamICC[[3, 3]], Directive[Orange, DotDashed]},
     {ξvwExamICC[[3, 2]], Directive[Blue, DotDashed]},
     {ξvwExamICC[[3, 4]], Directive[Red, DotDashed]}}},
  Epilog →
   {Inset[Framed[Style["w = -1.087 ± 0.096", 10],
      Background → LightBlue, FrameStyle → None], {-1.087, 0.8},
     {0, Top}],
    Inset[
     Framed[
      Style[
       "Orange:Ωm0=0.261;Transition=0.376\n
         Blue:Ωm0=0.274;Transition=0.426\n
         Red:Ωm0=0.287;Transition=0.508", 11],
      Background → LightGreen, FrameStyle → None], {-1.95, 0.84},
     {Left, Top}]},
  PlotLabel → "Qc=ξ H ρc, constant w: Coupling Constant ~ EoS",
  ImageSize → 500]
```



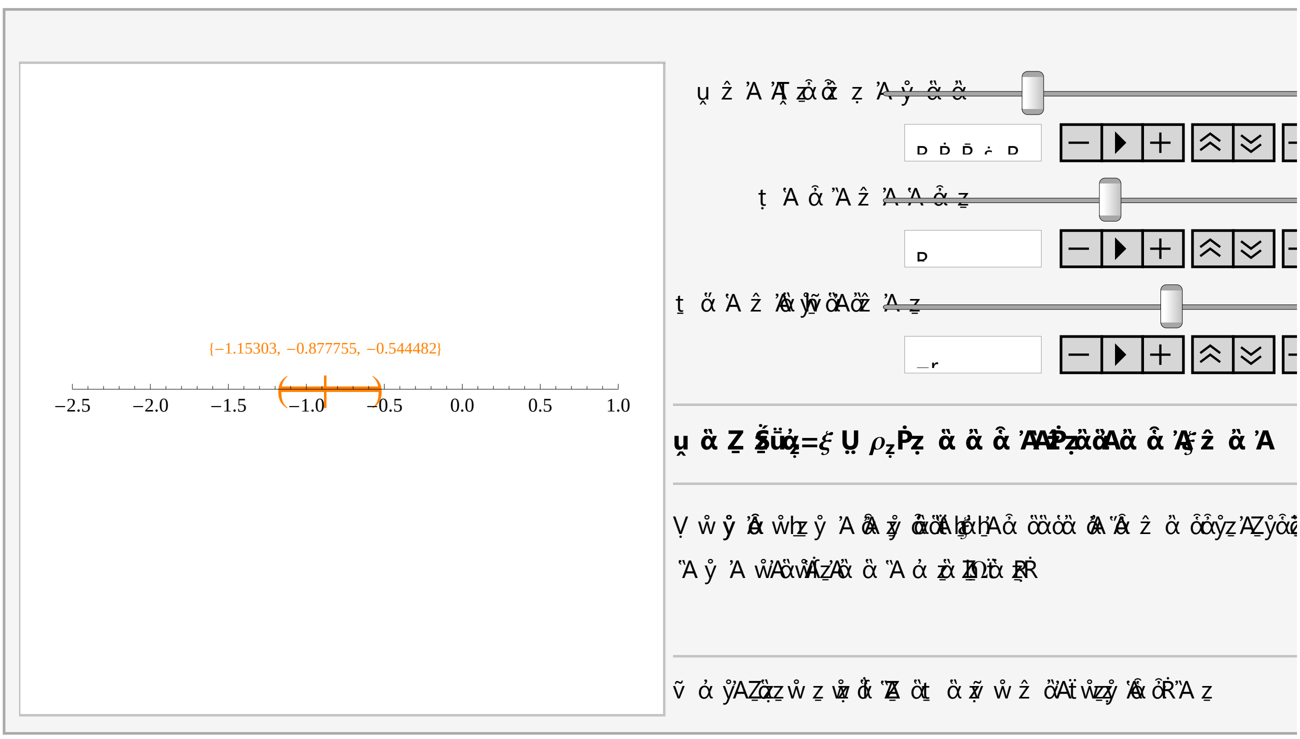
Or more casually, use w=-1±0.05

```
pltξvwExam1ICC =
 Show[pltξvwICC,
  GridLines →
   {{{-0.95, Directive[Gray, Dashed]},
     {-1.05, Directive[Gray, Dotted]}},
    {{-1.255, Directive[Orange, Dashed]},
     {-0.860, Directive[Blue, Dashed]},
     {-0.447, Directive[Red, Dashed]},
     {-1.293, Directive[Orange, Dotted]},
     {-0.887, Directive[Blue, Dotted]},
     {-0.461, Directive[Red, Dotted]}}},
  Epilog →
   Inset[
    Framed[
     Style[
      "Orange:Ωm0=0.261;Transition=0.376\n
        Blue:Ωm0=0.274;Transition=0.426\n
        Red:Ωm0=0.287;Transition=0.508", 11],
     Background → LightGreen, FrameStyle → None], {-1.8, 0.8},
    {Left, Top}], PlotLabel → "Qc=ξ H ρc: Coupling Constant ~ EoS"]
```



Now we assume we do not have the observed Ωm0 data, how do this Ωm0 change the result. In other words, if the observed Ωm0 data float around some value, then how is the fitting result? We also consider the curvature.

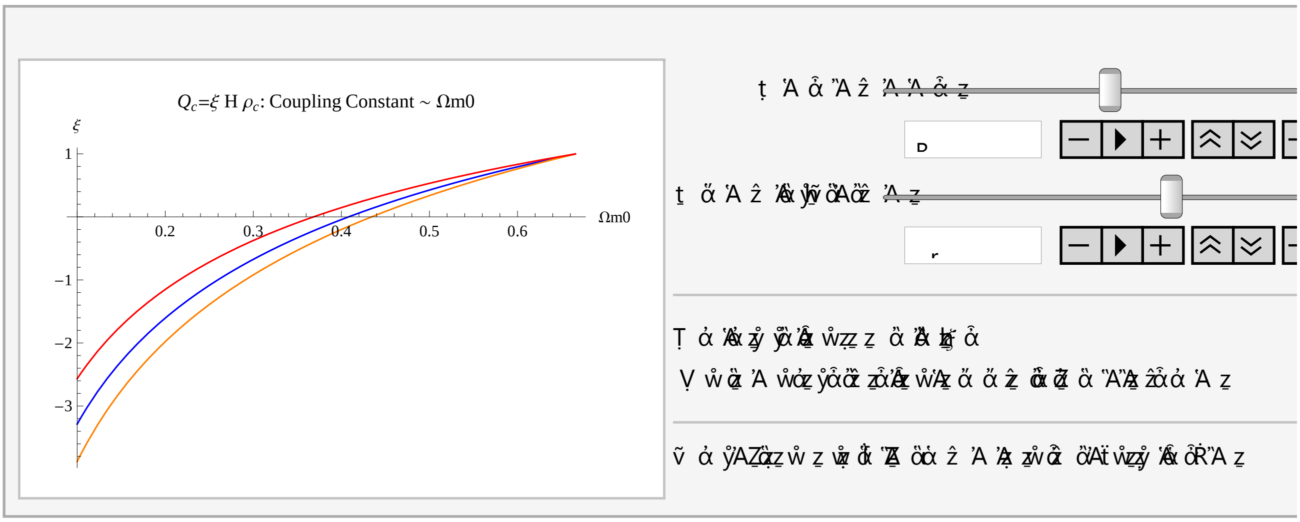
```
fitξ2ICCManSum =
 Manipulate[
  numPlot["(", {ξICCffunc[Ωm0ICC, 1 - Ωm0ICC - Ωk0ICC, wICC, 0.376],
     ξICCffunc[Ωm0ICC, 1 - Ωm0ICC - Ωk0ICC, wICC, 0.426],
     ξICCffunc[Ωm0ICC, 1 - Ωm0ICC - Ωk0ICC, wICC, 0.508]}, ")",
    {-2.5, 1}], {{Ωm0ICC, 0.274, "Matter Fraction"}, 0.1, 0.66,
    Appearance → "Open"},
   {{Ωk0ICC, 0, "Curvature"}, -0.1, 0.1, Appearance → "Open"},
   {{wICC, -1, "Equation of State"}, -2, -0.47, Appearance → "Open"},
  Delimiter, Style["Model: Qc=ξ H ρc, constant w, constant ξ",
    Bold, Medium], Delimiter,
  Style[
    "This is the fitting result of ξ from only transition
      redshift data.\n without the knowledge of Ωm0. "],
  Delimiter, Style["Slide to check how do EoS change this curve."],
  ControlPlacement → {Right}, SaveDefinitions → True]
```




```
{tmpe /. FindRoot[ztrICC[tmpe, 1 - tmpe, -1, 0.1] == 0, {tmpe, 0.1}],
 tmpe /. FindRoot[ztrICC[tmpe, 1 - tmpe, -1, 0.1] == 0, {tmpe, 0.2}]}
```

```
{0.666667, 0.666667}
```

```
pltξvΩm0ICCManSum =
 Manipulate[
  Plot[{ξICCffunc[Ωm0ICC, 1 - Ωm0ICC - Ωk0ICC, wICC, 0.426],
     ξICCffunc[Ωm0ICC, 1 - Ωm0ICC - Ωk0ICC, wICC, 0.376],
     ξICCffunc[Ωm0ICC, 1 - Ωm0ICC - Ωk0ICC, wICC, 0.508]},
   {Ωm0ICC, 0.1,
    (tmpe /. FindRoot[ztrICC[tmpe, 1 - tmpe - Ωk0ICC, wICC, 0.1] == 0,
        {tmpe, 0.1}]) - 0.001}, PlotStyle → {Blue, Orange, Red},
   AxesLabel → {"Ωm0", "ξ"},
   PlotLabel → "Qc=ξ H ρc: Coupling Constant ~ Ωm0"],
  {{Ωk0ICC, 0, "Curvature"}, -0.1, 0.1, Appearance → "Open"},
  {{wICC, -1, "Equation of State"}, -2, -0.47, Appearance → "Open"},
  Delimiter,
  Style[
   "Blue line is the center of ξ\n The other lines are the
     upper and lower value"], Delimiter,
  Style["Slide to check how do matter change this curve."],
  ControlPlacement → {Right}, SaveDefinitions → True]
```



To summarize, taken the case that the universe is flat, and choose the parameters to be {w=-1}, the region for interaction constant ξ should be (-0.46, -1.28) with a center at -0.88, i.e., $-0.88^{+0.42}_{-0.40}$.

```
ztrrICC[0.358, -1, ξICC] == 0.376
```

$$-1 + 0.179^{\frac{1}{-3+\xi ICC}} \left(-\frac{1 + \frac{2\,\xi ICC}{-3+\xi ICC}}{-1 - \frac{0.358\,\xi ICC}{-3+\xi ICC}}\right)^{\frac{1}{-3+\xi ICC}} == 0.376$$

```
ξICCrffunc[rICC_, wICC_, data_] :=
 ξICC /. FindRoot[ztrrICC[rICC, wICC, ξICC] == data, {ξICC, -0.6}]
```

```
ξICCrf2[wICC_] := ξICCrffunc[0.398, wICC, 0.508]
```

```
ξICCrf1[wICC_] := ξICCrffunc[0.358, wICC, 0.376]
```

Center

```
ξICCrfc[wICC_] := ξICCrffunc[0.378, wICC, 0.426]
```

A example is (w=-1)

```
{ξICCrf1[-1], ξICCrfc[-1], ξICCrf2[-1]}
```

```
{-1.25282, -0.875189, -0.472561}
```

To summarize, taken the case that the universe is flat, and choose the parameters to be {w=-1}, the region for interation cosntant ξ should be (-1.25, -0.47) with a center at -0.88, i.e., $-0.88^{+0.41}_{-0.37}$.

Or more detailed

```
tabξFinaltICC =
 Grid[
  Prepend[MapThread[Prepend,
    {Prepend[
      {{Item[ξICCrffunc[0.358, -1, 0.376], Background → LightPurple],
        ξICCrffunc[0.358, -1, 0.426], ξICCrffunc[0.358, -1, 0.508]},
       {ξICCrffunc[0.378, -1, 0.376],
        Item[ξICCrffunc[0.378, -1, 0.426], Background → LightPurple],
        ξICCrffunc[0.378, -1, 0.508]},
       {ξICCrffunc[0.398, -1, 0.376], ξICCrffunc[0.398, -1, 0.426],
        Item[ξICCrffunc[0.398, -1, 0.508],
         Background → LightPurple]}},
      {"zt=0.376", "zt=0.426", "zt=0.508"}],
     {Style["Ωm0/Ωd0⋱Transition", Small], "r=0.358", "r=0.378",
      "r=0.398"}}],
   {"Qc=ξ H ρc, constant ξ, constant w=-1: Results for ξ",
    SpanFromLeft}],
  Background → {{LightGray, None}, {LightGreen, LightGray, None}},
  Frame → All, ItemSize → 8]
```

Q_c=ξ H ρ_c, constant ξ, constant w=-1: Results for ξ			
Ωm0/Ωd0⋱Transition	z_t=0.376	z_t=0.426	z_t=0.508
r=0.358	-1.25282	-0.965436	-0.617444
r=0.378	-1.15011	-0.875189	-0.542347
r=0.398	-1.05453	-0.791252	-0.472561

This is a bit different from the result we got from Ωm0~ transition redshift plane. One possible reason is the second method doesn't assume a flat universe, while the first one supposes the universe is flat.

In arXiv:0801.4233, a CPL parameterization of EoS and three-year WMAP data, SN Ia data, BAO gives a result of ξ~-0.02

Check consistancy

The consistancy between (Transition, Ωm0) plane fitting and (Transition, Ωm0/Ωd0) plane fitting is checked.

For a flat universe, $r = \frac{\Omega m0}{1-\Omega m0}$. Solve out Ωm0, we get $\Omega m0 = \frac{r}{1+r}$(this is a monotonic function). Thus if we use the constrain that r∈(0.358, 0.398) with a center value 0.378, the value of Ωm0 is

```
{0.358/(1 + 0.358), 0.378/(1 + 0.378), 0.398/(1 + 0.398)}
```

```
{0.263623, 0.274311, 0.284692}
```

```
{ξICCffunc[0.263623, 1 - 0.263623, -1, 0.376],
 ξICCffunc[0.27431059506531`, 1 - 0.27431059506531`, -1, 0.426],
 ξICCffunc[0.28469241773962806`, 1 - 0.28469241773962806`, -1, 0.508]}
```

```
{-1.25282, -0.875189, -0.472561}
```

This result is exactly the same as the results from (Transition, r) plane.

δ is constant + EoS w CPL

Definations and functions.

CPL EoS $w = \text{w0} + \text{w1}\,\frac{z}{1+z}$

Useful relations.

```
hubblecmpintICCPL[w0_, w1_, ξICCPL_, z_] =
 Integrate[Exp[-3 w1/(1 + tmp)] (1 + tmp)^(-3 (w0+w1) - ξICCPL - 1), {tmp, 0, z},
  Assumptions → {tmp ∈ Reals && z ∈ Reals && z > -1}]
```

$$-\text{ExpIntegralE}[1-3\,(\text{w0}+\text{w1})-\xi\text{ICCPL},\,3\,\text{w1}] + \left(\frac{1}{1+z}\right)^{3\,(\text{w0}+\text{w1})+\xi\text{ICCPL}}\ \text{ExpIntegralE}\left[1-3\,(\text{w0}+\text{w1})-\xi\text{ICCPL},\,\frac{3\,\text{w1}}{1+z}\right]$$

```
hubblecmpintICCPLtest[w0_, w1_, ξICCPL_, z_?NumberQ] :=
 NIntegrate[Exp[-3 w1/(1 + tmp)] (1 + tmp)^(-3 (w0+w1) - ξICCPL - 1), {tmp, 0, z}]
```

```
ΩdICCPLtest[Ωd0_, Ωm0_, w0_, w1_, ξICCPL_, z_] :=
  Ωm0 (1 + z)^(3 (1+w0+w1)) Exp[3 w1/(1 + z)] ξICCPL
      hubblecmpintICCPLtest[w0, w1, ξICCPL, z] +
    Ωd0 (1 + z)^(3 (1+w0+w1)) Exp[3 w1/(1 + z) - 3 w1] // Simplify
```

```
hubblecmpintICCPLtest[-1, 0.6, -0.2, 10] // Timing
```

```
{0.078, 14.1439}
```

```
hubblecmpintICCPL[-1, 0.6, -0.2, 5] // Timing
```

```
{0., 4.68394 + 0. i}
```

```
Re[hubblecmpintICCPL[-1, 0.6, -0.2, 10] // Timing]
```

```
{0., 14.1439}
```

Fraction energy density

```
ΩmICCPL[Ωm0_, ξICCPL_, z_] := Ωm0 (1 + z)^(3-ξICCPL)
```

```
ΩdICCPL[Ωd0_, Ωm0_, w0_, w1_, ξICCPL_, z_] :=
  Ωm0 (1 + z)^(3 (1+w0+w1)) Exp[3 w1/(1 + z)] ξICCPL
      hubblecmpintICCPL[w0, w1, ξICCPL, z] +
    Ωd0 (1 + z)^(3 (1+w0+w1)) Exp[3 w1/(1 + z) - 3 w1] // Simplify
```

Hubble function

```
hubbleICCPL[H0ICCPL_, Ωd0ICCPL_, Ωm0ICCPL_, w0ICCPL_, w1ICCPL_,
   ξICCPL_, z_] :=
  H0ICCPL
   √(ΩmICCPL[Ωm0ICCPL, ξICCPL, z] +
      ΩdICCPL[Ωd0ICCPL, Ωm0ICCPL, w0ICCPL, w1ICCPL, ξICCPL, z])
```

```
hubbleDICCPL[H0ICCPL_, Ωd0ICCPL_, Ωm0ICCPL_, w0ICCPL_, w1ICCPL_,
    ξICCPL_, z_] =
   D[hubbleICCPL[H0ICCPL, Ωd0ICCPL, Ωm0ICCPL, w0ICCPL, w1ICCPL,
     ξICCPL, z], z];
```

Deceleration parameter

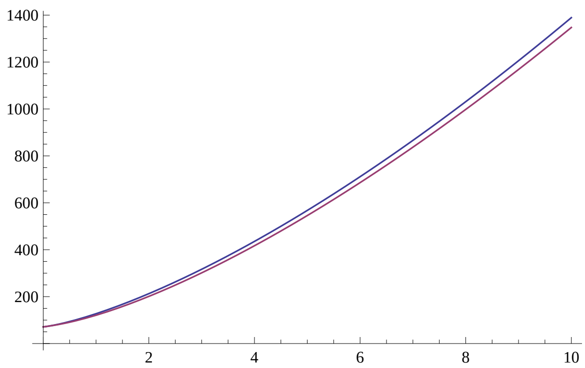
```
qICCPL[H0ICCPL_, Ωd0ICCPL_, Ωm0ICCPL_, w0ICCPL_, w1ICCPL_,
  ξICCPL_, z_] :=
 -1 +
  (1 + z) / hubbleICCPL[H0ICCPL, Ωd0ICCPL, Ωm0ICCPL, w0ICCPL,
     w1ICCPL, ξICCPL, z]
   hubbleDICCPL[H0ICCPL, Ωd0ICCPL, Ωm0ICCPL, w0ICCPL, w1ICCPL,
    ξICCPL, z]
```

Check the definations and functions.

```
Re[hubbleDICCPL[H0w, Ωd0w, Ωm0w, -1, 0.6, -0.02, 10]]
```

```
188.13
```

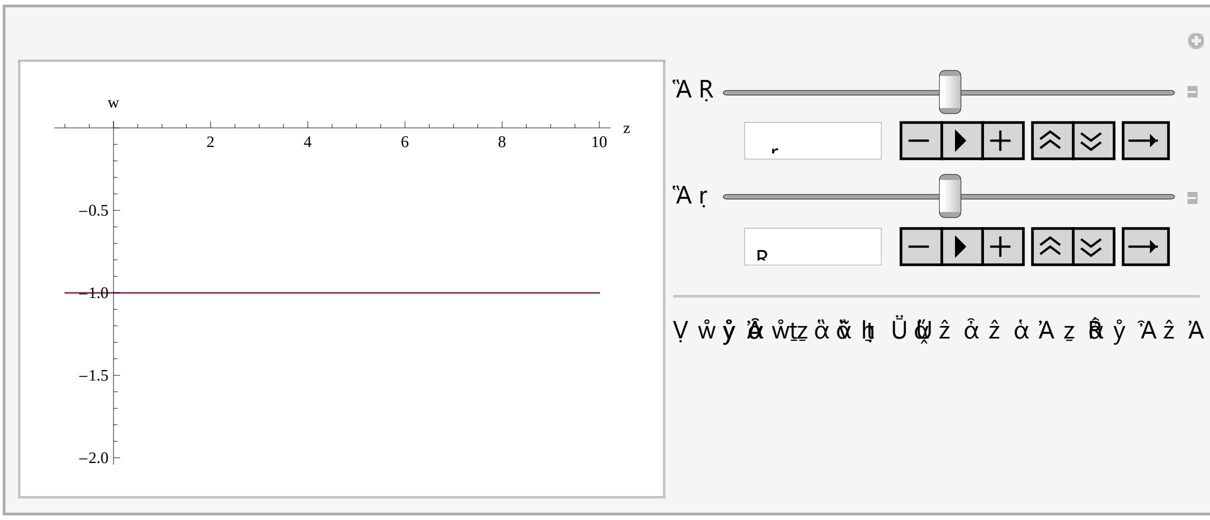
```
Plot[{hubbleICCPL[H0w, 0.73, 0.27, -1, 0.6, -0.02, z],
  hubble[Ωm0w, Ωd0w, 0, z]}, {z, 0, 10}]
```



Equation of State

The equation of state

```
plEoSICCPLMan =
 Manipulate[Plot[{w0 + w1 z/(1 + z), -1}, {z, -0.99, 10},
   AxesLabel → {"z", "w"}],
  {{w0, -1, "w0"}, -2, 0, Appearance → "Open"},
  {{w1, 0, "w1"}, -1, 1, Appearance → "Open"}, Delimiter,
  Style["This is the EoS of CPL paramterization."],
  ControlPlacement → {Right, Right}, SaveDefinitions → True]
```

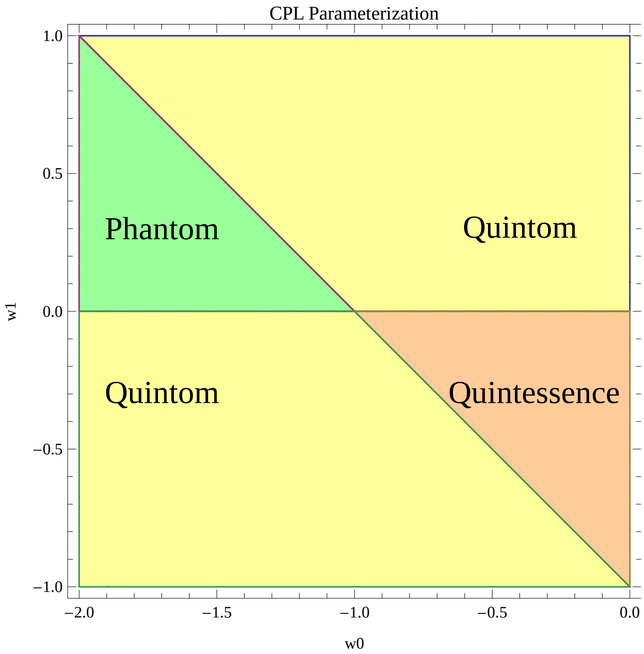


It is clear that the line is mono. Now we category it into phantom, quintessence and quintom. Limit of w at z->∞ is w0+w1.

1. w1>0, monotone-increasing function.
   1.1. w0+w1>-1 Quintom
   1.2. w0+w1=- 1 Then w0 must be -1 and w1=0.
   1.3. w0+w1<-1 Phantom
2. w1<0, monotone-decreasing function.
   2.1. w0+w1>-1 Quintessence
   2.2. w0+w1=-1 Then w0 must be -1 and w1=0.
   2.3. w0+w1<-1 Quintom

Thus we can draw a map of EoS on which we color out different categories.

```
plEoSPhaseICCPL =
 RegionPlot[{w0 + w1 > -1 && w1 > 0, w0 + w1 <= -1 && w1 > 0,
   w0 + w1 > -1 && w1 < 0, w0 + w1 <= -1 && w1 < 0}, {w0, -2, 0},
  {w1, -1, 1},
  PlotStyle → {Directive[Yellow, Opacity[0.4]],
    Directive[Green, Opacity[0.4]],
    Directive[Orange, Opacity[0.4]]},
  Epilog → {Inset[Style["Phantom", 20], {-1.7, 0.3}, {0, 0}],
    Inset[Style["Quintom", 20], {-1.7, -0.3}, {0, 0}],
    Inset[Style["Quintom", 20], {-0.4, 0.3}, {0, 0}],
    Inset[Style["Quintessence", 20], {-0.35, -0.3}, {0, 0}]},
  Frame → True, FrameLabel → {"w0", "w1"},
  PlotLabel → "CPL Parameterization", ImageSize → 400]
```

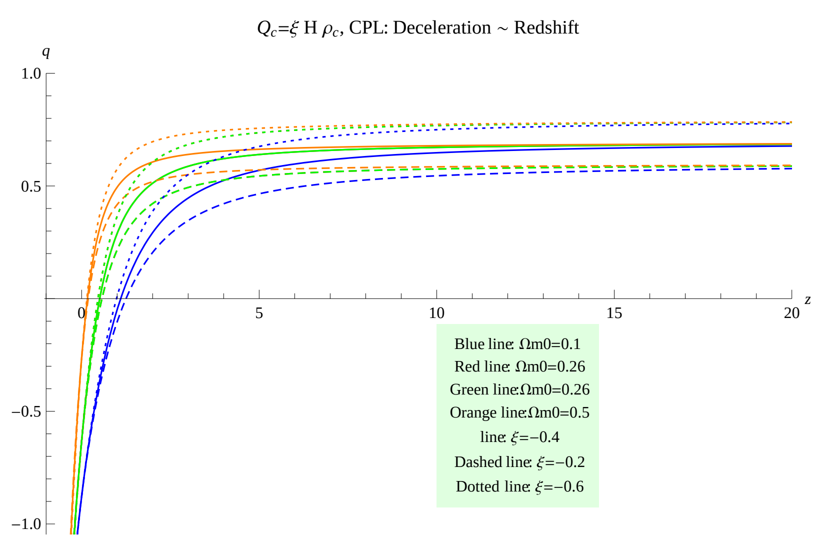


Plots, showcase, examinations

```
pldecICCPL[Ωm0ICCPL_, ξICCPL_, w0ICCPL_, w1ICCPL_, color_] :=
  Plot[Re[qICCPL[H0w, 1 - Ωm0ICCPL, Ωm0ICCPL, w0ICCPL, w1ICCPL,
      ξICCPL, z]], {z, -1, 20}, PlotRange → {{-1.05, 20}, {-1.05, 1}},
   PlotStyle → color, AxesOrigin → {-1, 0}, AxesLabel → {z, q}];
```

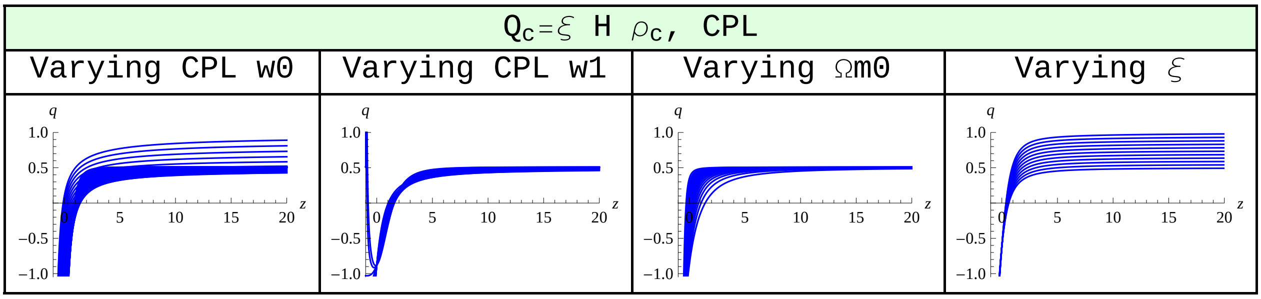
Check the effects of different parameters.

```
pldecICCPLShowSum =
 Show[{pldecICCPL[0.1, -0.4, -1.02, 0.60, Blue],
   pldecICCPL[0.26, -0.4, -1.02, 0.60, Red],
   pldecICCPL[0.26, -0.4, -1.02, 0.60, Green],
   pldecICCPL[0.5, -0.4, -1.02, 0.60, Orange],
   pldecICCPL[0.1, -0.2, -1.02, 0.60, Directive[Blue, Dashed]],
   pldecICCPL[0.26, -0.2, -1.02, 0.60, Directive[Red, Dashed]],
   pldecICCPL[0.26, -0.2, -1.02, 0.60, Directive[Green, Dashed]],
   pldecICCPL[0.5, -0.2, -1.02, 0.60, Directive[Orange, Dashed]],
   pldecICCPL[0.1, -0.6, -1.02, 0.60, Directive[Blue, Dotted]],
   pldecICCPL[0.26, -0.6, -1.02, 0.60, Directive[Red, Dotted]],
   pldecICCPL[0.26, -0.6, -1.02, 0.60, Directive[Green, Dotted]],
   pldecICCPL[0.5, -0.6, -1.02, 0.60, Directive[Orange, Dotted]]},
  Epilog →
   Inset[
    Framed[
     Style[
      "Blue line: Ωm0=0.1\n Red line: Ωm0=0.26\n Green
        line:Ωm0=0.26\n Orange line:Ωm0=0.5\n line:
        ξ=-0.4\n Dashed line: ξ=-0.2\n Dotted line: ξ=-0.6",
      10], Background → LightGreen, FrameStyle → None],
    {10, -0.1}, {Left, Top}],
  PlotLabel → "Qc=ξ H ρc, CPL: Deceleration ~ Redshift",
  ImageSize → 500]
```



z->Infinity is a degenerate limit. For constant ξ and constant w models, this limit is determined by the interaction strength ξ . This might be useful if more complicated models are investigated and no large deviations are shown. [Nota]

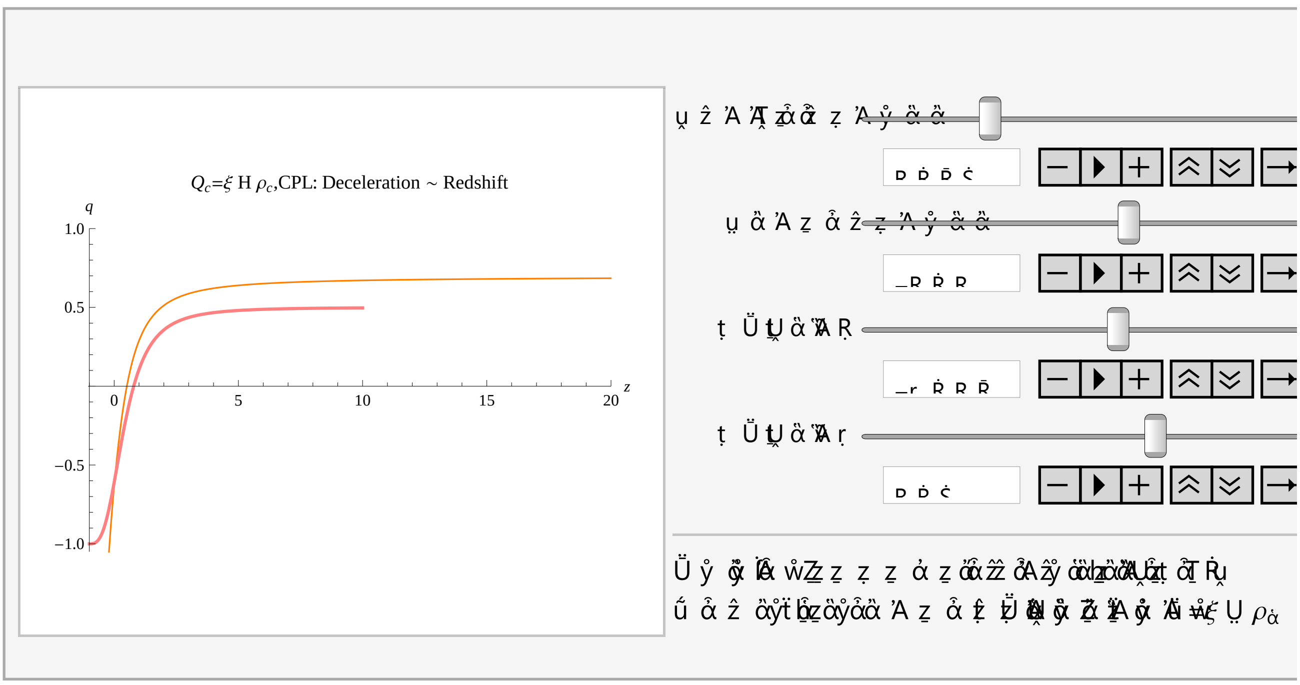
```
varyingICCPLShowSum =
 Grid[{{"Qc=ξ H ρc, CPL", SpanFromLeft},
   {"Varying CPL w0", "Varying CPL w1", "Varying Ωm0", "Varying ξ"},
   {Show[Table[pldecICCPL[0.1, -0.02, w0ICCPL, 0.6, Blue],
      {w0ICCPL, -2, -0.3, 0.05}]],
    Show[Table[pldecICCPL[0.1, -0.02, -1.02, w1ICCPL, Blue],
      {w1ICCPL, -0.2, 1, 0.1}]],
    Show[Table[pldecICCPL[Ωm0ICCPL, -0.02, -1.02, 0.6, Blue],
      {Ωm0ICCPL, 0.1, 0.9, 0.05}]],
    Show[Table[pldecICCPL[0.27, ξICCPL, -1.02, 0.6, Blue],
      {ξICCPL, -1, 0, 0.1}]]}}, Frame → All,
  Background → {{None}, {LightGreen, None}}]
```



Interaction ξ changes the limit, i.e., what value will it be at z->Infinity. EoS changes the the whole shape. Matter fraction also changes how fast q varies, but just in a small time scale.

Use movable slides to check how do the parameters affect the deceleration parameter. Just a toy.

```
pldecICCPLManSum =
 Manipulate[
  Show[{pldecICCPL[Ωm0ICCPL, ξICCPL, w0ICCPL, w1ICCPL, Orange],
    pldec[Ωm0ICCPL, {Pink, Thick}]},
   PlotLabel → "Qc=ξ H ρc,CPL: Deceleration ~ Redshift"],
  {{Ωm0ICCPL, 0.26, "Matter Fraction"}, 0, 1, Appearance → "Open"},
  {{ξICCPL, -0.4, "Interaction"}, -1, 0, Appearance → "Open"},
  {{w0ICCPL, -1.02, "CPL EoS w0"}, -2, -0.3, Appearance → "Open"},
  {{w1ICCPL, 0.6, "CPL EoS w1"}, -0.2, 1, Appearance → "Open"},
  Delimiter, Style["Pink is the deceleration parameter for LCDM.",
   Medium],
  Style["Orange is for interacting CPL model with Q=ξ H ρm",
   Medium], ControlPlacement → {Right, Right, Right},
  SaveDefinitions → True]
```



Transition redshift

Find out the expression for transition redshift

```
ztrICCPL[Ωm0ICCPL_, Ωd0ICCPL_, ξICCPL_, w0ICCPL_, w1ICCPL_] =
  Re[
   z /.
    FindRoot[
     (1 + 3 (w0ICCPL + w1ICCPL z/(1 + z)))
        ΩdICCPL[Ωd0ICCPL, Ωm0ICCPL, w0ICCPL, w1ICCPL, ξICCPL, z] +
       ΩmICCPL[Ωm0ICCPL, ξICCPL, z] == 0, {z, 3}]];
```

```
ztrICCPLtest2[Ωm0ICCPL_, Ωd0ICCPL_, ξICCPL_, w0ICCPL_, w1ICCPL_] =
  Re[
    z /.
      FindRoot[
        (1 + 3 (w0ICCPL + w1ICCPL z/(1 + z)))
          ΩdICCPL[Ωd0ICCPL, Ωm0ICCPL, w0ICCPL, w1ICCPL, ξICCPL, z] +
         ΩmICCPL[Ωm0ICCPL, ξICCPL, z] == 0, {z, 3}]];
```

[illegible]

[illegible]

[illegible]

[illegible]

The following is just used for test.

```
ztrICCPLtest[Ωm0ICCPL_, Ωd0ICCPL_, ξICCPL_, w0ICCPL_, w1ICCPL_] :=
 z /.
  FindRoot[
   (1 + 3 (w0ICCPL + w1ICCPL z/(1 + z)))
      ΩdICCPLtest[Ωd0ICCPL, Ωm0ICCPL, w0ICCPL, w1ICCPL, ξICCPL, z] +
     ΩmICCPL[Ωm0ICCPL, ξICCPL, z] == 0, {z, 3}]
ztrICCPLtest[0.274, 1 - 0.274, -0.02, -1, 0.6]
ztrICCPLtest[0.1, 1 - 0.1, -0.1, -0.9, -0.05]
ztrICCPL[0.1, 1 - 0.1, -0.1, -0.9, -0.05]
ztrICCPLtest2[0.1, 1 - 0.1, -0.1, -0.9, -0.05]
plottestfunctiontemp[Ωm0ICCPL_, Ωd0ICCPL_, ξICCPL_, w0ICCPL_,
  w1ICCPL_, z_] :=
 (1 + 3 (w0ICCPL + w1ICCPL z/(1 + z)))
   ΩdICCPLtest[Ωd0ICCPL, Ωm0ICCPL, w0ICCPL, w1ICCPL, ξICCPL, z] +
  ΩmICCPL[Ωm0ICCPL, ξICCPL, z]
```

```
0.600079
```

```
1.59781
```

```
1.59781
```

```
1.59781
```

```
(*
(*This is a test of how well are the findroot results.*)
Plot[{plottestfunctiontemp[temp,1-temp,-0.1,-0.9,-0.05,
   ztrICCPLtest2[temp,1-temp,-0.1,-0.9,-0.05]],
  plottestfunctiontemp[temp,1-temp,-0.1,-0.9,-0.05,
   ztrICCPLtest[temp,1-temp,-0.1,-0.9,-0.05]],
  plottestfunctiontemp[temp,1-temp,-0.1,-0.9,-0.05,
   ztrICCPL[temp,1-temp,-0.1,-0.9,-0.05]]},{temp,0,1},
 PlotStyle→{Red,Orange,Blue}]
*)
```

Define rICC = $\frac{\Omega m0ICC}{\Omega d0ICC}$

```
ΩmrICCPL[rICCPL_, ξICCPL_, z_] = rICCPL (1 + z)^(3-ξICCPL);
```

```
ΩdrICCPL[rICCPL_, w0ICCPL_, w1ICCPL_, ξICCPL_, z_] =
  rICCPL (1 + z)^(3 (1+w0ICCPL+w1ICCPL)) Exp[3 w1ICCPL/(1 + z)] ξICCPL
     hubblecmpintICCPL[w0ICCPL, w1ICCPL, ξICCPL, z] +
    (1 + z)^(3 (1+w0ICCPL+w1ICCPL)) Exp[3 w1ICCPL/(1 + z) - 3 w1ICCPL] // Simplify;
```

```
ztrrICCPL[rICCPL_, ξICCPL_, w0ICCPL_, w1ICCPL_] =
  Re[
   z /.
    FindRoot[
     (1 + 3 (w0ICCPL + w1ICCPL z/(1 + z)))
       ΩdrICCPL[rICCPL, w0ICCPL, w1ICCPL, ξICCPL, z] +
      ΩmrICCPL[rICCPL, ξICCPL, z] == 0, {z, 3}]];
```

[illegible]

Test this.

```
ztrrICCPL[0.5, -0.02, -1, 0.6]
ztrrICCPL[0.5, 0, -1, 0.6]
```

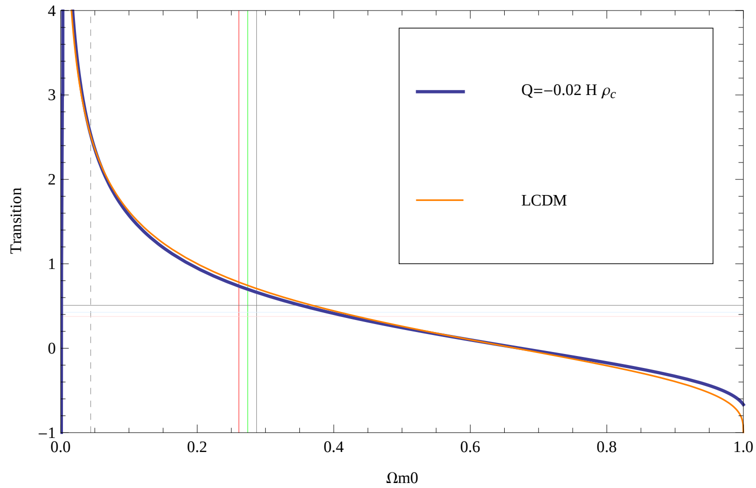
```
0.46757
```

```
0.474097
```

Visualization of transition redshift.

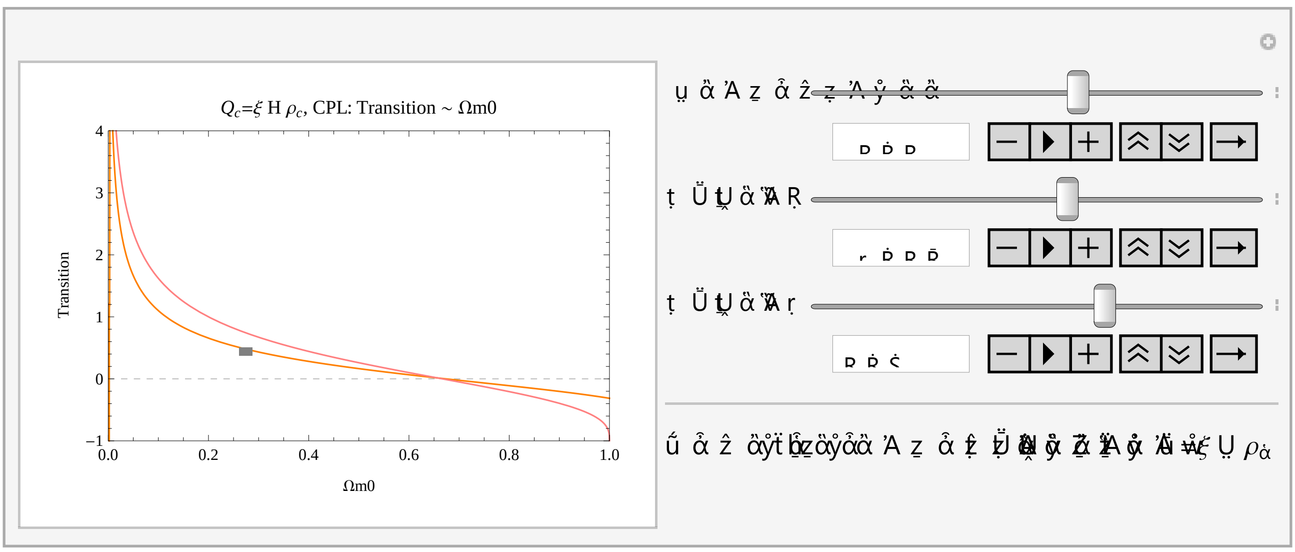
Check the behavior of this transition redshift.

```
pldecΩm0ICCPL =
 Plot[{ztrICCPL[Ωm0ICCPL, 1 - Ωm0ICCPL, -0.02, -1.02, 0.2],
   ztr[Ωm0ICCPL, 1 - Ωm0ICCPL]}, {Ωm0ICCPL, 0, 1},
  PlotRange → {{0, 1}, {-1, 4}}, PlotStyle → {Thick, Orange},
  AxesOrigin → {0, -1}, Frame → True,
  GridLines →
   {{{0.044, Dashed}, {0.261, Red}, {0.274, Green}, {0.287, Gray}},
    {{0.376, LightRed}, {0.426, LightBlue}, {0.508, Gray}}},
  PlotLegend → {"Q=-0.02 H ρc", "LCDM"},
  LegendPosition → {0.0, -0.05}, LegendShadow → None,
  ImageSize → 500, FrameLabel → {"Ωm0", "Transition"}]
```

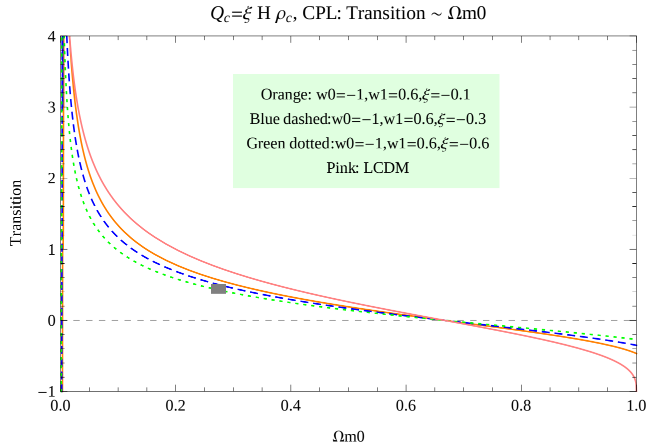


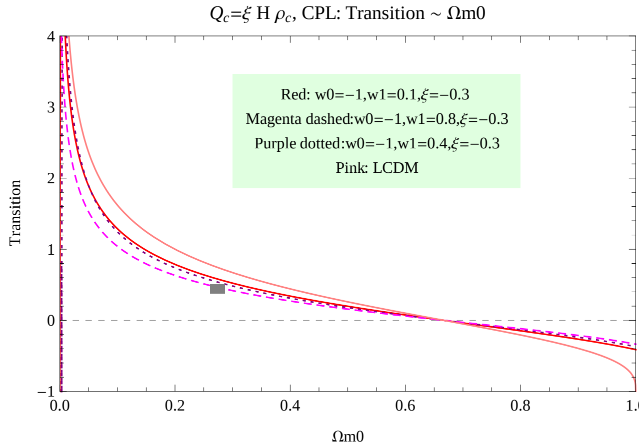

```
plztrICCPL[ξICCPL_, w0ICCPL_, w1ICCPL_, color_,
   {ranges_: 0, rangee_: 1}] :=
  Plot[ztrICCPL[Ωm0ICCPL, 1 - Ωm0ICCPL, ξICCPL, w0ICCPL, w1ICCPL],
   {Ωm0ICCPL, 0, 1}, PlotRange → {{ranges, rangee}, {-1, 4}},
   PlotStyle → color, AxesOrigin → {0, -1}, Frame → True,
   FrameLabel → {"Ωm0", "Transtion"}];
```

```
plztrICCPLManSum =
 Manipulate[
  Show[{plztrICCPL[ξICCPL, w0ICCPL, w1ICCPL, Orange, {0, 1}],
    Plot[ztr[Ωm0ICCPL, 1 - Ωm0ICCPL], {Ωm0ICCPL, 0, 1},
     PlotRange → {{0, 1}, {-1, 4}}, PlotStyle → Pink,
     AxesOrigin → {0, -1}]},
   Graphics[{Gray, Rectangle[Scaled[{0.261, .2752}],
      Scaled[{0.287, .3016}]]}], Frame → True],
   GridLines → {{}, {{0, Dashed}}},
   FrameLabel → {"Ωm0", "Transition"},
   PlotLabel → "Qc=ξ H ρc, CPL: Transition ~ Ωm0"],
  {{ξICCPL, -0.4, "Interaction"}, -1, 0, Appearance → "Open"},
  {{w0ICCPL, -1.02, "CPL EoS w0"}, -2, -0.3, Appearance → "Open"},
  {{w1ICCPL, 0.6, "CPL EoS w1"}, -0.2, 1, Appearance → "Open"},
  Delimiter,
  Style["Orange is for interacting CPL model with Q=ξ H ρm",
   Medium], ControlPlacement → {Right, Right, Right},
  SaveDefinitions → True]
```

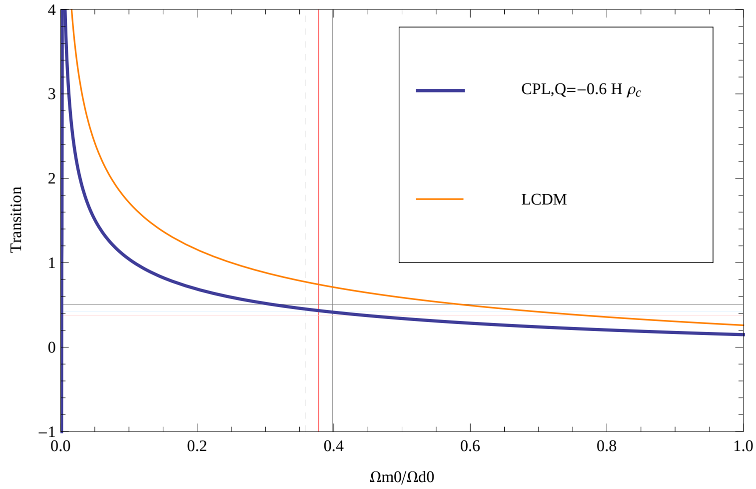


```
plztrExamICCPLSum =
 Grid[
  {{Show[{plztrICCPL[-0.1, -1, 0.6, Orange, {0, 1}],
      plztrICCPL[-0.3, -1, 0.6, {Blue, Dashed}, {0, 1}],
      plztrICCPL[-0.6, -1, 0.6, {Green, Dotted}, {0, 1}],
      Plot[ztr[Ωm0ICCPL, 1 - Ωm0ICCPL], {Ωm0ICCPL, 0, 1},
       PlotRange → {{0, 1}, {-1, 4}}, PlotStyle → Pink,
       AxesOrigin → {0, -1}]},
     Graphics[{Gray, Rectangle[Scaled[{0.261, .2752}],
        Scaled[{0.287, .3016}]]}, Frame → True],
     GridLines → {{}, {{0, Dashed}}},
     Epilog →
      Inset[
       Framed[
        Style[
         "Orange: w0=-1,w1=0.6,ξ=-0.1\n Blue
           dashed:w0=-1,w1=0.6,ξ=-0.3\n Green
           dotted:w0=-1,w1=0.6,ξ=-0.6\n Pink: LCDM", 10],
        Background → LightGreen, FrameStyle → None], {0.3, 3.5},
       {Left, Top}], FrameLabel → {"Ωm0", "Transition"},
     PlotLabel → "Qc=ξ H ρc, CPL: Transition ~ Ωm0",
     ImageSize → 400],
    Show[{plztrICCPL[-0.3, -1, 0.1, Red, {0, 1}],
      plztrICCPL[-0.3, -1, 0.4, {Purple, Dotted}, {0, 1}],
      plztrICCPL[-0.3, -1, 0.8, {Magenta, Dashed}, {0, 1}],
      Plot[ztr[Ωm0ICCPL, 1 - Ωm0ICCPL], {Ωm0ICCPL, 0, 1},
       PlotRange → {{0, 1}, {-1, 4}}, PlotStyle → Pink,
       AxesOrigin → {0, -1}]},
     Graphics[{Gray, Rectangle[Scaled[{0.261, .2752}],
        Scaled[{0.287, .3016}]]}, Frame → True],
     GridLines → {{}, {{0, Dashed}}},
     Epilog →
      Inset[
       Framed[
        Style[
         "Red: w0=-1,w1=0.1,ξ=-0.3\n Magenta
           dashed:w0=-1,w1=0.8,ξ=-0.3\n Purple
           dotted:w0=-1,w1=0.4,ξ=-0.3\n Pink: LCDM", 10],
        Background → LightGreen, FrameStyle → None], {0.3, 3.5},
       {Left, Top}], FrameLabel → {"Ωm0", "Transition"},
     PlotLabel → "Qc=ξ H ρc, CPL: Transition ~ Ωm0",
     ImageSize → 400]}}]
```

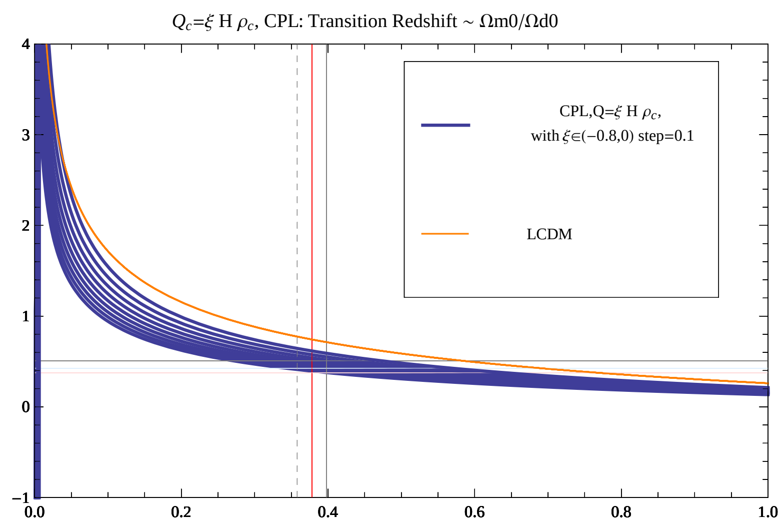





```
pldecrICCPL =
 Plot[{ztrrICCPL[rICCPL, -0.6, -1.02, 0.6], ztrr[rICCPL]},
  {rICCPL, 0, 1}, PlotRange → {{0, 1}, {-1, 4}},
  PlotStyle → {Thick, Orange}, AxesOrigin → {0, -1}, Frame → True,
  GridLines → {{{0.358, Dashed}, {0.378, Directive[Red]}, 0.398},
    {{0.376, LightRed}, {0.426, LightBlue}, {0.508, Gray}}},
  PlotLegend → {"CPL,Q=-0.6 H ρc", "LCDM"},
  LegendPosition → {0.0, -0.05}, LegendShadow → None,
  ImageSize → 500, FrameLabel → {"Ωm0/Ωd0", "Transition"}]
```



```
pltransrICCPLDense =
 Show[
  Table[Plot[{ztrrICCPL[rICCPL, ξICCPL, -1.02, 0.6], ztrr[rICCPL]},
    {rICCPL, 0, 1}, PlotRange → {{0, 1}, {-1, 4}},
    PlotStyle → {Thick, Orange}, AxesOrigin → {0, -1},
    Frame → True,
    GridLines → {{{0.358, Dashed}, {0.378, Directive[Red]}, 0.398},
      {{0.376, LightRed}, {0.426, LightBlue}, {0.508, Gray}}},
    PlotLegend → {"CPL,Q=ξ H ρc,\n with ξ∈(-0.8,0) step=0.1",
      "LCDM"}, LegendPosition → {0.0, -0.05}, LegendShadow → None],
   {ξICCPL, -0.8, 0, 0.1}], ImageSize → 500,
  FrameLabel → {"Ωm0/Ωd0", "Transition"},
  PlotLabel → "Qc=ξ H ρc, CPL: Transition Redshift ~ Ωm0/Ωd0",
  ImageSize → 500]
```



Find out allowed region of interactions.

To find out the region of ξ, set w=-1 and Ωd0=1-Ωm0. Let the ztr-Ωm0 line cross points (0.287,0.508) and (0.261, 0.376).

```
Off[FindRoot::nlnum]
Off[ReplaceAll::reps]
ztrICCPL[Ωm0ICCPL, Ωd0ICCPL, ξICCPL, w0ICCPL, w1ICCPL]
On[FindRoot::nlnum]
On[ReplaceAll::reps]
```

$$\mathrm{Re}\left[z\ /.\ \mathrm{FindRoot}\left[\left(1+3\left(\mathrm{w0ICCPL}+\frac{\mathrm{w1ICCPL}\ z}{1+z}\right)\right)\Omega\mathrm{dICCPL}[\Omega\mathrm{d0ICCPL},\Omega\mathrm{m0ICCPL},\mathrm{w0ICCPL},\mathrm{w1ICCPL},\xi\mathrm{ICCPL},z]+\Omega\mathrm{mICCPL}[\Omega\mathrm{m0ICCPL},\xi\mathrm{ICCPL},z]==0,\{z,3\}\right]\right]$$

```
(* ztrICCPL[0.287,1-0.287,ξICCPL1,-1.02,0.6]==0.508 *)
```

```
ξICCPLffunc[Ωm0ICCPL_, Ωd0ICCPL_, w0ICCPL_, w1ICCPL_, data_] :=
 ξICCPL /.
  FindRoot[ztrICCPL[Ωm0ICCPL, Ωd0ICCPL, ξICCPL, w0ICCPL, w1ICCPL] ==
    data, {ξICCPL, -0.6}]
```

```
ξICCPLf2[Ωm0ICCPL_, Ωd0ICCPL_, w0ICCPL_, w1ICCPL_] :=
 ξICCPL /.
  FindRoot[ztrICCPL[Ωm0ICCPL, Ωd0ICCPL, ξICCPL, w0ICCPL, w1ICCPL] ==
    0.508, {ξICCPL, -0.6}]
```

```
ξICCPLf1[Ωm0ICCPL_, Ωd0ICCPL_, w0ICCPL_, w1ICCPL_] :=
 ξICCPL /.
  FindRoot[ztrICCPL[Ωm0ICCPL, Ωd0ICCPL, ξICCPL, w0ICCPL, w1ICCPL] ==
    0.376, {ξICCPL, -0.6}]
```

Cross the Center of best fit. (0.274,0.426)

```
ξICCPLfc[Ωm0ICCPL_, Ωd0ICCPL_, w0ICCPL_, w1ICCPL_] :=
 ξICCPL /.
  FindRoot[ztrICCPL[Ωm0ICCPL, Ωd0ICCPL, ξICCPL, w0ICCPL, w1ICCPL] ==
    0.426, {ξICCPL, -0.6}]
```

According to the data of transition redshift.

```
ξICCPLf2[w0ICCPL_, w1ICCPL_] :=
 ξICCPLffunc[0.287, 1 - 0.287, w0ICCPL, w1ICCPL, 0.508]
```

```
ξICCPLf1[w0ICCPL_, w1ICCPL_] :=
 ξICCPLffunc[0.261, 1 - 0.261, w0ICCPL, w1ICCPL, 0.376]
```

```
ξICCPLfc[w0ICCPL_, w1ICCPL_] :=
 ξICCPLffunc[0.274, 1 - 0.274, w0ICCPL, w1ICCPL, 0.426]
```

An example

```
Off[FindRoot::nlnum]
Off[ReplaceAll::reps]
{ξICCPLf1[-1.02, 0.6], ξICCPLfc[-1.02, 0.6], ξICCPLf2[-1.02, 0.6]}
On[FindRoot::nlnum]
On[ReplaceAll::reps]
```

```
{-1.03882, -0.635668, -0.213119}
```

To summarize, taken the case that the universe is flat, and choose the parameters to be {w0=-1.02,w1=0.6}, the region for interaction constant ξ should be (-1.04,-0.21) with a center at -0.64, i.e., $-0.64^{+0.42}_{-0.40}$.

```
ξICCPLrffunc[rICCPL_, w0ICCPL_, w1ICCPL_, data_] :=
 ξICCPL /.
  FindRoot[ztrrICCPL[rICCPL, ξICCPL, w0ICCPL, w1ICCPL] == data,
   {ξICCPL, -0.6}]
```

```
ξICCPLrf2[rICCPL_, w0ICCPL_, w1ICCPL_] :=
 ξICCPL /.
  FindRoot[ztrrICCPL[rICCPL, ξICCPL, w0ICCPL, w1ICCPL] == 0.508,
   {ξICCPL, -0.6}]
```

```
ξICCPLrf1[rICCPL_, w0ICCPL_, w1ICCPL_] :=
 ξICCPL /.
  FindRoot[ztrrICCPL[rICCPL, ξICCPL, w0ICCPL, w1ICCPL] == 0.376,
   {ξICCPL, -0.6}]
```

Cross the Center of best fit. (0.358,0.426)

```
ξICCPLrfc[rICCPL_, w0ICCPL_, w1ICCPL_] :=
 ξICCPL /.
  FindRoot[ztrrICCPL[rICCPL, ξICCPL, w0ICCPL, w1ICCPL] == 0.426,
   {ξICCPL, -0.6}]
```

According to the data of transition redshift.

```
ξICCPLrf2[w0ICCPL_, w1ICCPL_] :=
 ξICCPLrffunc[0.398, w0ICCPL, w1ICCPL, 0.508]
```

```
ξICCPLrf1[w0ICCPL_, w1ICCPL_] :=
 ξICCPLrffunc[0.358, w0ICCPL, w1ICCPL, 0.376]
```

```
ξICCPLrfc[w0ICCPL_, w1ICCPL_] :=
 ξICCPLrffunc[0.378, w0ICCPL, w1ICCPL, 0.426]
```

An example

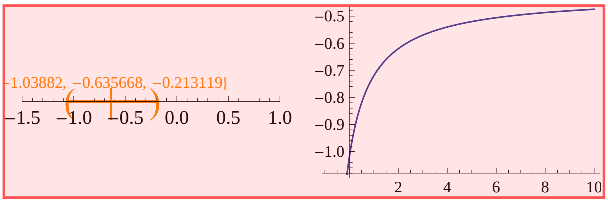
```
Off[FindRoot::nlnum]
Off[ReplaceAll::reps]
{ξICCPLrf1[-1.02, 0.6], ξICCPLrfc[-1.02, 0.6],
 ξICCPLrf2[-1.02, 0.6]}
On[FindRoot::nlnum]
On[ReplaceAll::reps]
```

```
{-1.01249, -0.633048, -0.228688}
```

To summarize, taken the case that the universe is flat, and choose the parameters to be {w0=-1.02,w1=0.6}, the region for interation cosntant ξ should be (-1.01, -0.23) with a center at -0.63, i.e., $-0.63^{+0.40}_{-0.38}$.

In arXiv:0801.4233, a CPL parameterization of EoS and three-year WMAP data, SN Ia data, BAO gives a result of ξ~-0.02
We can see from

```
Off[FindRoot::nlnum]
Off[ReplaceAll::reps]
fitξICCPLManSum =
 Manipulate[
  Grid[
   {{numPlot["(", {ξICCPLf1[w0ICCPL, w1ICCPL],
       ξICCPLfc[w0ICCPL, w1ICCPL], ξICCPLf2[w0ICCPL, w1ICCPL]},
      ")", {-1.5, 1}], Plot[w0ICCPL + w1ICCPL temp/(temp + 1),
      {temp, -0.9, 10}]}}],
  {{w0ICCPL, -1.02, "Equation of State w0"}, -2, -0.47,
   Appearance → "Open"},
  {{w1ICCPL, 0.6, "Equation of State w1"}, -1, 1,
   Appearance → "Open"}, Delimiter,
  Style[
   "This is the fitting result from transition redshift data.",
   Bold], Delimiter,
  "The parenthesis shows the upper and lower value \n while
    the verticle line show the center value.",
  Style[
   "\n The three numbers are left value, center value,
     right value respectively."], Delimiter, Delimiter,
  Style[
   "Slide to see how do the two parameters affect the
     coupling constant results."],
  ControlPlacement → {Bottom, Bottom}, SaveDefinitions → True]
```



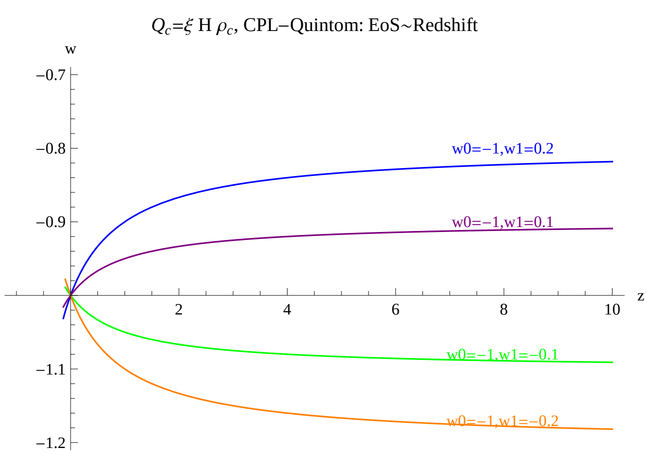

```
(*
On[FindRoot::nlnum]
 On[ReplaceAll::reps]
*)
```

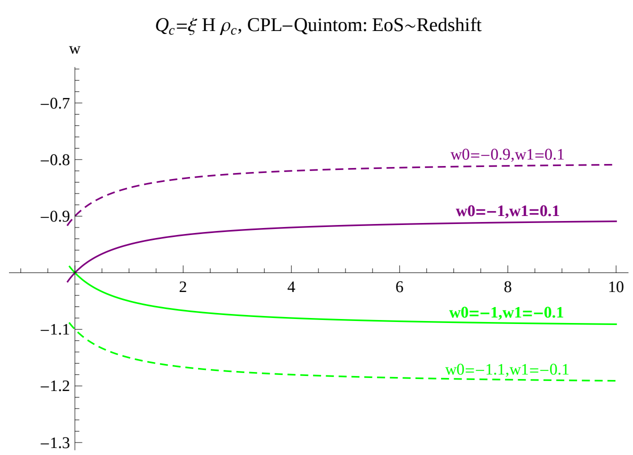
Quintom Case

The transition redshift. Six groups of parameters.
(-1, 0.1) (-1, 0.2) (-0.9,0.1)
(-1, -0.1) (-1, -0.2) (-1.1,-0.1)
In two plots,
(-1, 0.1) (-1, 0.2) (-1, -0.1) (-1, -0.2)
(-1, 0.1)   (-0.9,0.1) (-1, -0.1)   (-1.1,-0.1)

```
pltICCPLEoSfunc[w0ICCPL_, w1ICCPL_, color_] :=
  Plot[{w0ICCPL + w1ICCPL z/(1 + z)}, {z, -0.99, 10}, PlotStyle → color,
   AxesLabel → {"z", "w"}];
```

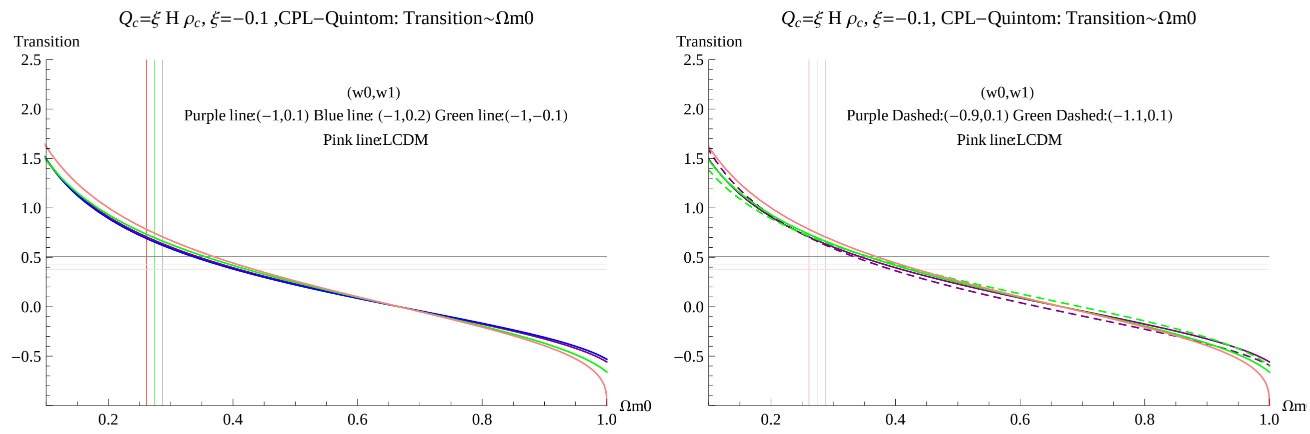
```
plEoSICCPLQuintomSum =
 Grid[
  {{Show[pltICCPLEoSfunc[-1, 0.2, Blue],
      pltICCPLEoSfunc[-1, 0.1, Purple],
      pltICCPLEoSfunc[-1, -0.1, Green],
      pltICCPLEoSfunc[-1, -0.2, Orange],
      PlotRange → {{-0.99, 10}, {-1.2, -0.7}}, AxesOrigin → {0, -1},
      ImageSize → 400,
      PlotLabel → "Qc=ξ H ρc, CPL-Quintom: EoS~Redshift",
      Epilog →
       {Inset[Framed[Style["w0=-1,w1=0.2", 10, Blue],
           Background → None, FrameStyle → None], {8, -0.80}, {0, 0}],
        Inset[Framed[Style["w0=-1,w1=0.1", 10, Purple],
           Background → None, FrameStyle → None], {8, -0.9}, {0, 0}],
        Inset[Framed[Style["w0=-1,w1=-0.1", 10, Green],
           Background → None, FrameStyle → None], {8, -1.08}, {0, 0}],
        Inset[Framed[Style["w0=-1,w1=-0.2", 10, Orange],
           Background → None, FrameStyle → None], {8, -1.17}, {0, 0}]}],
    Show[pltICCPLEoSfunc[-1, 0.1, Purple],
      pltICCPLEoSfunc[-0.9, 0.1, {Purple, Dashed}],
      pltICCPLEoSfunc[-1, -0.1, Green],
      pltICCPLEoSfunc[-1.1, -0.1, {Green, Dashed}],
      PlotRange → {{-1, 10}, {-1.3, -0.65}}, AxesOrigin → {0, -1},
      ImageSize → 400,
      PlotLabel → "Qc=ξ H ρc, CPL-Quintom: EoS~Redshift",
      Epilog →
       {Inset[Framed[Style["w0=-0.9,w1=0.1", 10, Purple],
           Background → None, FrameStyle → None], {8, -0.79}, {0, 0}],
        Inset[Framed[Style["w0=-1,w1=0.1", 10, Bold, Purple],
           Background → None, FrameStyle → None], {8, -0.89}, {0, 0}],
        Inset[Framed[Style["w0=-1,w1=-0.1", 10, Bold, Green],
           Background → None, FrameStyle → None], {8, -1.07}, {0, 0}],
        Inset[Framed[Style["w0=-1.1,w1=-0.1", 10, Green],
           Background → None, FrameStyle → None], {8, -1.17},
          {0, 0}]}]}}]
```





"Purple line:(-1,0.1) Blue line: (-1,0.2) Green line:(-1,-0.1)\n Pink line:LCDM"
"Purple Dashed:(-0.9,0.1) Green Dashed:(-1.1,0.1)"

```
plztrICCPLQuintomSum =
 Grid[
   {{Show[{plztrICCPL[-0.1, -1, 0.1, Purple, {0.1, 1}],
       plztrICCPL[-0.1, -1, 0.2, Blue, {0.1, 1}],
       plztrICCPL[-0.1, -1, -0.1, Green, {0.1, 1}],
       Plot[ztr[Ωm0ICCPL, 1 - Ωm0ICCPL], {Ωm0ICCPL, 0, 1},
        PlotRange → {{0.1, 1}, {-1, 3}}, PlotStyle → Pink,
        AxesOrigin → {0.1, -1}]},
      GridLines → {{{0.261, Red}, {0.274, Green}, {0.287, Gray}},
        {{0.376, LightRed}, {0.426, LightBlue}, {0.508, Gray}}},
      AxesOrigin → {0.1, -1}, PlotRange → {{0.1, 1}, {-1, 2.5}},
      Frame → False, AxesLabel → {"Ωm0", "Transition"},
      PlotLabel → "Qc=ξ H ρc, ξ=-0.1 ,CPL-Quintom: Transition~Ωm0",
      Epilog →
       Inset[
        Framed[
         Style[
          "(w0,w1)\n Purple line:(-1,0.1) Blue line:
            (-1,0.2) Green line:(-1,-0.1)\n
            Pink line:LCDM", 10], Background → None,
         FrameStyle → None], {0.3, 2.4}, {Left, Top}],
      ImageSize → 400],
     Show[{plztrICCPL[-0.1, -1, 0.1, Purple, {0.1, 1}],
       plztrICCPL[-0.1, -0.9, 0.1, {Purple, Dashed}, {0.1, 1}],
       plztrICCPL[-0.1, -1, -0.1, Green, {0.1, 1}],
       plztrICCPL[-0.1, -1.1, -0.1, {Green, Dashed}, {0.1, 1}],
       Plot[ztr[Ωm0ICCPL, 1 - Ωm0ICCPL], {Ωm0ICCPL, 0, 1},
        PlotRange → {{0.1, 1}, {-1, 3}}, PlotStyle → Pink,
        AxesOrigin → {0.1, -1}]},
      GridLines → {{{0.261, Red}, {0.274, Green}, {0.287, Gray}},
        {{0.376, LightRed}, {0.426, LightBlue}, {0.508, Gray}}},
      AxesOrigin → {0.1, -1}, PlotRange → {{0.1, 1}, {-1, 2.5}},
      Frame → False, AxesLabel → {"Ωm0", "Transition"},
      PlotLabel → "Qc=ξ H ρc, ξ=-0.1, CPL-Quintom: Transition~Ωm0",
      Epilog →
       Inset[
        Framed[
         Style[
          "(w0,w1)\n Purple Dashed:(-0.9,0.1) Green
            Dashed:(-1.1,0.1)\n Pink line:LCDM", 10],
         Background → None, FrameStyle → None], {0.3, 2.4},
        {Left, Top}], ImageSize → 400]}}] // Quiet
```



As for the effect of EoS, we can split it to w0 effect and w1 effect.

First construct the plot function.

```
(*
pltξvw0ICCPL[w1ICCPL_,w0ICCPLs_,w0ICCPLe_]:=
 Plot[{ξICCPLfc[w0ICCPL,w1ICCPL],ξICCPLf1[w0ICCPL,w1ICCPL],
   ξICCPLf2[w0ICCPL,w1ICCPL]},{w0ICCPL,w0ICCPLs,w0ICCPLe}]
*)
```

Choose the values from Reference 3. w0 = -1.087 ± 0.096,

```
ξvwExamICCPLQuintom =
 {Block[{w0ICCPL = -1, w1ICCPL = -0.1},
   {{w0ICCPL, w1ICCPL}, ξICCPLfc[w0ICCPL, w1ICCPL],
    ξICCPLf1[w0ICCPL, w1ICCPL], ξICCPLf2[w0ICCPL, w1ICCPL]}],
  Block[{w0ICCPL = -1, w1ICCPL = 0},
   {{w0ICCPL, w1ICCPL}, ξICCPLfc[w0ICCPL, w1ICCPL],
    ξICCPLf1[w0ICCPL, w1ICCPL], ξICCPLf2[w0ICCPL, w1ICCPL]}],
  Block[{w0ICCPL = -1, w1ICCPL = 0.1},
   {{w0ICCPL, w1ICCPL}, ξICCPLfc[w0ICCPL, w1ICCPL],
    ξICCPLf1[w0ICCPL, w1ICCPL], ξICCPLf2[w0ICCPL, w1ICCPL]}],
  Block[{w0ICCPL = -0.9, w1ICCPL = 0.1},
   {{w0ICCPL, w1ICCPL}, ξICCPLfc[w0ICCPL, w1ICCPL],
    ξICCPLf1[w0ICCPL, w1ICCPL], ξICCPLf2[w0ICCPL, w1ICCPL]}],
  Block[{w0ICCPL = -1.1, w1ICCPL = -0.1},
   {{w0ICCPL, w1ICCPL}, ξICCPLfc[w0ICCPL, w1ICCPL],
    ξICCPLf1[w0ICCPL, w1ICCPL], ξICCPLf2[w0ICCPL, w1ICCPL]}]}
```

```
{{{-1, -0.1}, -0.907284, -1.3096, -0.484861},
 {{-1, 0}, -0.877755, -1.27874, -0.457448},
 {{-1, 0.1}, -0.844859, -1.24477, -0.426298},
 {{-0.9, 0.1}, -0.785036, -1.17144, -0.383258},
 {{-1.1, -0.1}, -0.910565, -1.3218, -0.477189}}
```

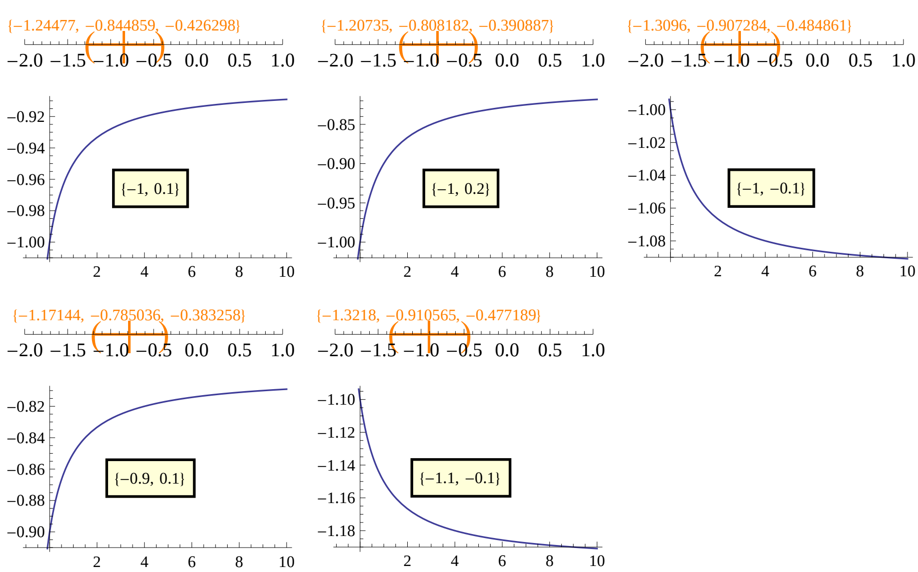
```
tabξvwExamICCPLQuintom =
 Grid[Prepend[Prepend[ξvwExamICCPLQuintom,
    {"{w0,w1}", "Center", "Lower", "Upper"}],
   {"ξ results for Qc=ξ H ρd, CPL,Quintom.", SpanFromLeft}],
  Frame → All,
  Background → {{LightGray, None}, {LightGreen, LightGray, None}},
  Alignment → Center, ItemSize → 8]
```

ξ results for $Q_c=\xi$ H ρ_d, CPL,Quintom.			
{w0,w1}	Center	Lower	Upper
{-1, -0.1}	-0.907284	-1.3096	-0.484861
{-1, 0}	-0.877755	-1.27874	-0.457448
{-1, 0.1}	-0.844859	-1.24477	-0.426298
{-0.9, 0.1}	-0.785036	-1.17144	-0.383258
{-1.1, -0.1}	-0.910565	-1.3218	-0.477189

```
(*
pltξvwExamICCPL=Show[pltξvwICCPL[????????],
  GridLines→
   {{{ξvwExamICC[[1,1]],Directive[Gray,Dashed]},
     {ξvwExamICC[[2,1]],Directive[Gray,Dotted]},
     {ξvwExamICC[[3,1]],Directive[Gray,DotDashed]}},
    {{ξvwExamICC[[1,3]],Directive[Orange,Dashed]},
     {ξvwExamICC[[1,2]],Directive[Blue,Dashed]},
     {ξvwExamICC[[1,4]],Directive[Red,Dashed]},
     {ξvwExamICC[[2,3]],Directive[Orange,Dotted]},
     {ξvwExamICC[[2,2]],Directive[Blue,Dotted]},
     {ξvwExamICC[[2,4]],Directive[Red,Dotted]},
     {ξvwExamICC[[3,3]],Directive[Orange,DotDashed]},
     {ξvwExamICC[[3,2]],Directive[Blue,DotDashed]},
     {ξvwExamICC[[3,4]],Directive[Red,DotDashed]}}},
  Epilog→
   {Inset[Framed[Style["w = -1.087 ± 0.096",10],
      Background→LightBlue,FrameStyle→None],{-1.087,0.8},{0,Top}],
    Inset[
     Framed[
      Style[
       "Orange:Ωm0=0.261;Transition=0.376\n
         Blue:Ωm0=0.274;Transition=0.426\n
         Red:Ωm0=0.287;Transition=0.508",11],
      Background→LightGreen,FrameStyle→None],{-1.95,0.84},
     {Left,Top}]},ImageSize→500]
*)
```

```
pltfitξICCPLfunc[w0ICCPL_, w1ICCPL_] :=
  Grid[
   {{numPlot["(", {ξICCPLf1[w0ICCPL, w1ICCPL],
        ξICCPLfc[w0ICCPL, w1ICCPL], ξICCPLf2[w0ICCPL, w1ICCPL]},
       ")", {-2, 1}]},
    {Plot[w0ICCPL + w1ICCPL temp/(temp + 1), {temp, -0.9, 10},
       Epilog → Inset[Framed[Style[{w0ICCPL, w1ICCPL}, 10],
          Background → LightYellow], {Center, Center},
         {Center, Center}]]}}];
```

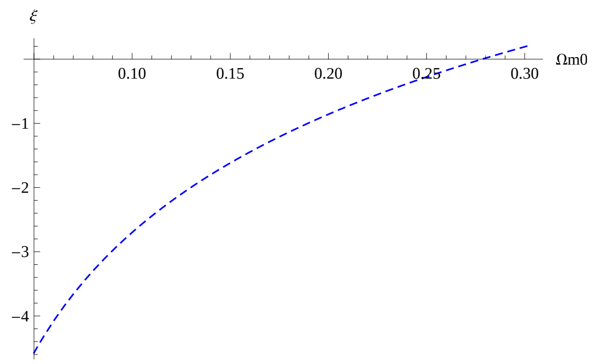
```
pltfitξICCPLQuintomSum =
 Grid[{{pltfitξICCPLfunc[-1, 0.1], pltfitξICCPLfunc[-1, 0.2],
    pltfitξICCPLfunc[-1, -0.1]},
   {pltfitξICCPLfunc[-0.9, 0.1], pltfitξICCPLfunc[-1.1, -0.1]}}]
```




```
pltfitξvΩm0QuintomSum[w0ICCPL_, w1ICCPL_, color_,
   {Ωm0start_, Ωm0end_}] :=
  Plot[ξICCPLffunc[Ωm0ICCPL, 1 - Ωm0ICCPL, w0ICCPL, w1ICCPL, 0.426],
   {Ωm0ICCPL, Ωm0start, Ωm0end}, PlotStyle → color,
   AxesLabel → {"Ωm0", "ξ"}];
```

Check sigularities. Why is that?

```
pltfitξvΩm0QuintomSum[-0.5, -0.2, {Blue, Dashed}, {0.05, 0.304}] //
 Quiet
```




```
ξICCPLffunc[0.46, 1 - 0.46, -0.5, -0.2, 0.426] // Quiet // Timing
```

{13.463, -94.3131}

```
ztrICCPL[0.46, 1 - 0.46, ξICCPL, -0.5, -0.2] == 0.426
```

$$\mathrm{Re}\left[z\ /.\ \mathrm{FindRoot}\left[\left(1+3\left(-0.5-\frac{0.2\,z}{1+z}\right)\right)\Omega\mathrm{dICCPL}[0.54, 0.46, -0.5, -0.2, \xi\mathrm{ICCPL}, z] + \Omega\mathrm{mICCPL}[0.46, \xi\mathrm{ICCPL}, z] == 0, \{z, 3\}\right]\right] == 0.426$$

```
(1 + 3 (w0ICCPL + w1ICCPL z/(1 + z)))
    ΩdICCPL[Ωd0ICCPL, Ωm0ICCPL, w0ICCPL, w1ICCPL, ξICCPL, z] +
   ΩmICCPL[Ωm0ICCPL, ξICCPL, z] == 0 // Simplify
```

$$(1+z)^{3-\xi\mathrm{ICCPL}}\,\Omega\mathrm{m0ICCPL} + e^{\frac{3\,\mathrm{w1ICCPL}}{1+z}}\,(1+z)^{3\,(1+\mathrm{w0ICCPL}+\mathrm{w1ICCPL})}\left(1+3\,\mathrm{w0ICCPL}+\frac{3\,\mathrm{w1ICCPL}\,z}{1+z}\right)\Big(e^{-3\,\mathrm{w1ICCPL}}\,\Omega\mathrm{d0ICCPL} - \xi\mathrm{ICCPL}\,\Omega\mathrm{m0ICCPL}\,\mathrm{ExpIntegralE}[1-3\,(\mathrm{w0ICCPL}+\mathrm{w1ICCPL})-\xi\mathrm{ICCPL}, 3\,\mathrm{w1ICCPL}] + \left(\frac{1}{1+z}\right)^{3\,\mathrm{w0ICCPL}+3\,\mathrm{w1ICCPL}+\xi\mathrm{ICCPL}}\xi\mathrm{ICCPL}\,\Omega\mathrm{m0ICCPL}\,\mathrm{ExpIntegralE}\left[1-3\,(\mathrm{w0ICCPL}+\mathrm{w1ICCPL})-\xi\mathrm{ICCPL}, \frac{3\,\mathrm{w1ICCPL}}{1+z}\right]\Big) == 0$$

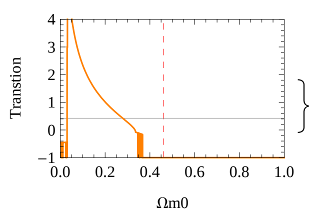
```
ztrICCPL[0.46, 1 - 0.46, 0, -0.5, x]
```

```
Re[
  z /. FindRoot[(1 + 3 (-0.5 + x z/(1 + z))) ΩdICCPL[0.54, 0.46, -0.5, x, 0, z] +
      ΩmICCPL[0.46, 0, z] == 0, {z, 3}]]
```

```
FindRoot[
 (1 + 3 (-0.5` + (-0.2 z)/(1 + z))) ΩdICCPL[0.54`, 0.46`, -0.5`, -0.2, 0, z] +
   ΩmICCPL[0.46`, 0, z] == 0, {z, -0.9}]
```

```
{z → -1.}
```

```
Show[plztrICCPL[0, -0.5, -0.2, Orange, {0, 1}],
   GridLines → {{{0.46, Directive[Red, Dashed]}}, {{0.426, Gray}}}] //
  Quiet // Timing
```

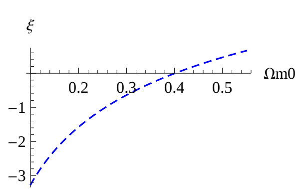
{2.808, 

}

```
ξICCPLffunc[0.2, 1 - 0.2, -1, 0.1, 0.426] // Timing // Quiet
```

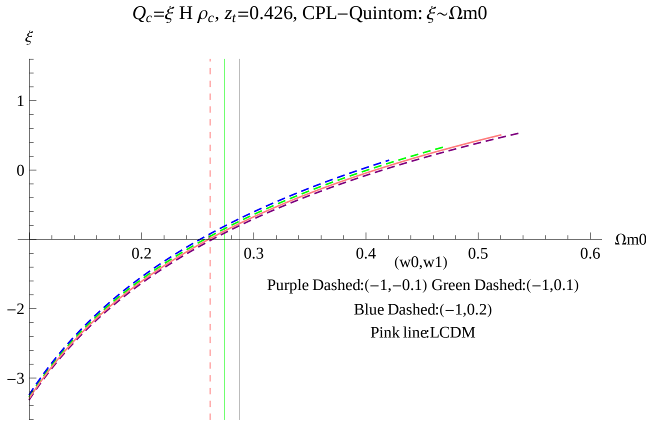
```
{0.499, -1.57483}
```

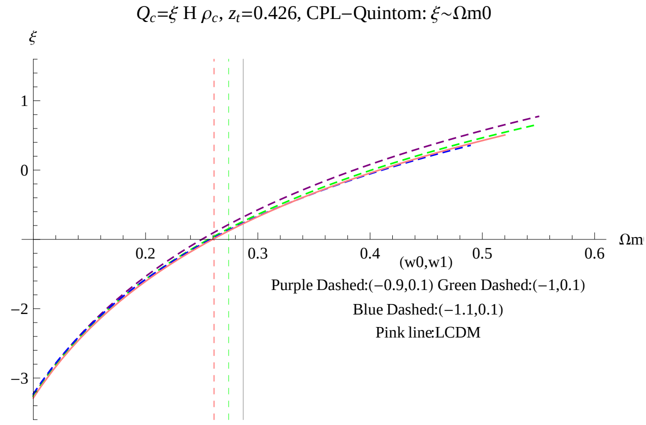
```
pltfitξvΩm0QuintomSum[-1, 0.1, {Blue, Dashed}, {0.1, 0.55}] // Quiet //
 Timing
```

{46.021, 

}

```
pltξvΩm0ICCPLQuintomSum =
 Grid[
   {{Show[pltfitξvΩm0QuintomSum[-1, -0.1, {Purple, Dashed},
       {0.1, 0.538}], pltfitξvΩm0QuintomSum[-1, 0.1,
       {Green, Dashed}, {0.1, 0.471}],
      pltfitξvΩm0QuintomSum[-1, 0.2, {Blue, Dashed}, {0.1, 0.42}],
      pltfitξvΩm0QuintomSum[-1, 0, Pink, {0.1, 0.52}],
      GridLines → {{{0.261, Directive[Red, Dashed]},
         {0.274, Green}, {0.287, Gray}}, {}}, AxesOrigin → {0.1, -1},
      PlotRange → {{0.1, 0.6}, {-3.5, 1.5}}, Frame → False,
      AxesLabel → {"Ωm0", "ξ"},
      PlotLabel → "Qc=ξ H ρc, zt=0.426, CPL-Quintom: ξ~Ωm0",
      Epilog →
       Inset[
        Framed[
         Style[
          "(w0,w1)\n Purple Dashed:(-1,-0.1) Green
            Dashed:(-1,0.1)\n Blue Dashed:(-1,0.2)\n
            Pink line:LCDM", 10], Background → None,
         FrameStyle → None], {0.3, -1}, {Left, Top}],
      ImageSize → 400],
     Show[pltfitξvΩm0QuintomSum[-0.9, 0.1, {Purple, Dashed},
       {0.1, 0.55}], pltfitξvΩm0QuintomSum[-1, 0.1,
       {Green, Dashed}, {0.1, 0.55}],
      pltfitξvΩm0QuintomSum[-1.1, 0.1, {Blue, Dashed}, {0.1, 0.489}],
      pltfitξvΩm0QuintomSum[-1, 0, Pink, {0.1, 0.52}],
      GridLines → {{{0.261, Directive[Red, Dashed]},
         {0.274, Directive[Green, Dashed]}, {0.287, Gray}}, {}},
      AxesOrigin → {0.1, -1}, PlotRange → {{0.1, 0.6}, {-3.5, 1.5}},
      Frame → False, AxesLabel → {"Ωm0", "ξ"},
      PlotLabel → "Qc=ξ H ρc, zt=0.426, CPL-Quintom: ξ~Ωm0",
      Epilog →
       Inset[
        Framed[
         Style[
          "(w0,w1)\n Purple Dashed:(-0.9,0.1) Green
            Dashed:(-1,0.1)\n Blue Dashed:(-1.1,0.1)\n
            Pink line:LCDM", 10], Background → None,
         FrameStyle → None], {0.3, -1}, {Left, Top}],
      ImageSize → 400]}}] // Quiet
```



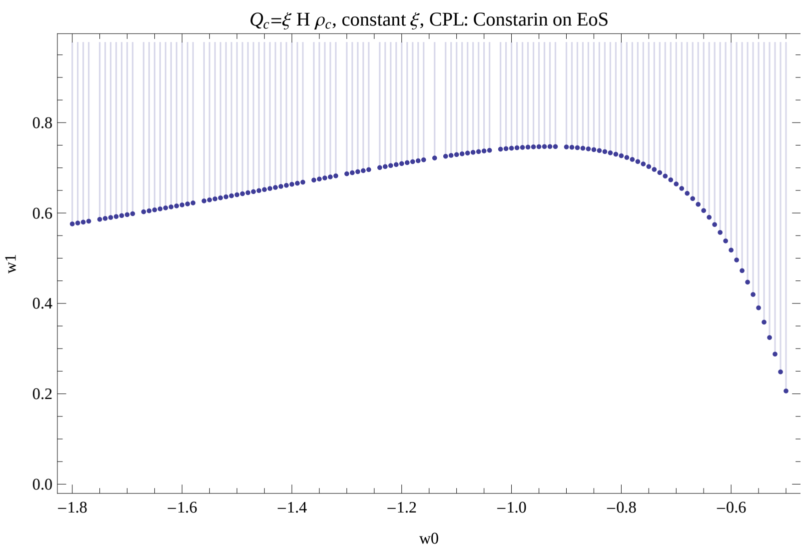



```
(*
tabw0vw1ICCPLfull=
    Table[{ztrICCPL[0.274,1-0.274,0,w0ICCPLtemp,w1ICCPLtemp],
      ξICCPLffunc[0.274,1-0.274,w0ICCPLtemp,w1ICCPLtemp,0.426],
      w0ICCPLtemp,w1ICCPLtemp},{w0ICCPLtemp,-1.8,-0.5,0.05},
     {w1ICCPLtemp,-1,1,0.01}];//Quiet//Timing
*)
```

```
tabw0vw1ICCPL2 =
    Table[{w0ICCPLtemp,
      eosICCPLw1 /.
       FindRoot[
        {ξICCffunc[0.274, 1 - 0.274 - 0, w0ICCPLtemp, eosICCPLw1] == 0},
        {eosICCPLw1, -0.5}]}, {w0ICCPLtemp, -1.8, -0.5, 0.01}]; //
  Quiet // Timing
```

```
{156.344, Null}
```

```
pltw0vw1ConsICCPL =
 ListPlot[tabw0vw1ICCPL2, FrameLabel → {"w0", "w1"}, Filling → Top,
  Frame → True,
  PlotLabel → "Qc=ξ H ρc, constant ξ, CPL: Constarin on EoS",
  ImageSize → 500]
```

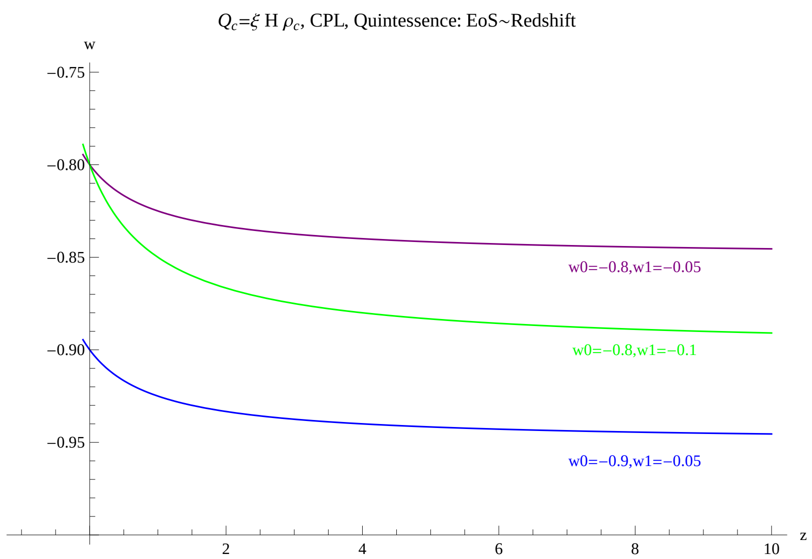



```
(*
tabw0vw1ICCPL1=
    Table[
     {eosICCPLw0/.
       FindRoot[
        {ξICCPLffunc[0.274,1-0.274,eosICCPLw0,w1ICCPLtemp,0.426]==
          0},{eosICCPLw0,-0.5}],w1ICCPLtemp},
     {w1ICCPLtemp,-0.5,0.5,0.05}];//Quiet//Timing
*)
```

Quintessence

Choose some Quintessence parameters.
(-0.9,-0.05) (-0.8,-0.05)(-0.8,-0.1)

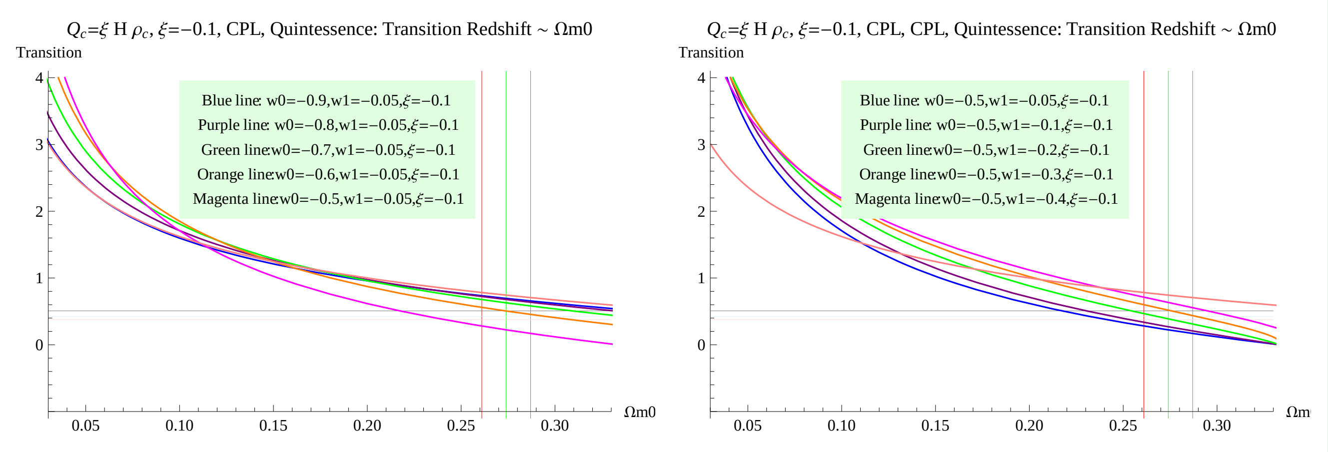
```
plEoSICCPLQuintessenceSum =
 Grid[
  {{Show[pltICCPLEoSfunc[-0.9, -0.05, Blue],
     pltICCPLEoSfunc[-0.8, -0.05, Purple],
     pltICCPLEoSfunc[-0.8, -0.1, Green],
     PlotRange → {{-0.99, 10}, {-1.0, -0.75}}, AxesOrigin → {0, -1},
     Epilog →
      {Inset[Framed[Style["w0=-0.9,w1=-0.05", 10, Blue],
          Background → None, FrameStyle → None], {8, -0.96}, {0, 0}],
       Inset[Framed[Style["w0=-0.8,w1=-0.05", 10, Purple],
          Background → None, FrameStyle → None], {8, -0.855}, {0, 0}],
       Inset[Framed[Style["w0=-0.8,w1=-0.1", 10, Green],
          Background → None, FrameStyle → None], {8, -0.9}, {0, 0}]},
     PlotLabel → "Qc=ξ H ρc, CPL, Quintessence: EoS~Redshift",
     ImageSize → 500]}}]
```




```
plztrICCPLQuintessenceSum =
 Grid[
  {{Show[{Plot[ztrICCPL[Ωm0ICCPL, 1 - Ωm0ICCPL, -0.1, -0.9, -0.05],
       {Ωm0ICCPL, 0.01, 1}, PlotRange → {{0, 1}, {-1, 4}},
       PlotStyle → Blue, AxesOrigin → {0, -1},
       PerformanceGoal → "Quality"],
      Plot[ztrICCPL[Ωm0ICCPL, 1 - Ωm0ICCPL, -0.1, -0.8, -0.05],
       {Ωm0ICCPL, 0.01, 1}, PlotRange → {{0, 1}, {-1, 4}},
       PlotStyle → Purple, AxesOrigin → {0, -1},
       PerformanceGoal → "Quality"],
      Plot[ztrICCPL[Ωm0ICCPL, 1 - Ωm0ICCPL, -0.1, -0.7, -0.05],
       {Ωm0ICCPL, 0.01, 0.94}, PlotRange → {{0, 1}, {-1, 4}},
       PlotStyle → Green, AxesOrigin → {0, -1},
       PerformanceGoal → "Quality"],
      Plot[ztrICCPL[Ωm0ICCPL, 1 - Ωm0ICCPL, -0.1, -0.6, -0.05],
       {Ωm0ICCPL, 0.01, 0.94}, PlotRange → {{0, 1}, {-1, 4}},
```

```
      PlotStyle → Orange, AxesOrigin → {0, -1},
      PerformanceGoal → "Quality"],
     Plot[ztrICCPL[Ωm0ICCPL, 1 - Ωm0ICCPL, -0.1, -0.5, -0.05],
      {Ωm0ICCPL, 0.01, 0.94}, PlotRange → {{0, 1}, {-1, 4}},
      PlotStyle → Magenta, AxesOrigin → {0, -1},
      PerformanceGoal → "Quality"],
     Plot[ztr[Ωm0ICCPL, 1 - Ωm0ICCPL], {Ωm0ICCPL, 0, 1},
      PlotRange → {{0.01, 0.94}, {-1, 4}}, PlotStyle → Pink,
      AxesOrigin → {0, -1}]},
    GridLines → {{{0.261, Red}, {0.274, Green}, {0.287, Gray}},
      {{0.376, LightRed}, {0.426, LightBlue}, {0.508, Gray}}},
    AxesOrigin → {0.03, -1}, PlotRange → {{0.03, 0.33}, Automatic},
    Frame → False, AxesLabel → {"Ωm0", "Transition"},
    Epilog →
     Inset[
      Framed[
       Style[
        "Blue line: w0=-0.9,w1=-0.05,ξ=-0.1\n Purple line:
          w0=-0.8,w1=-0.05,ξ=-0.1\n Green
          line:w0=-0.7,w1=-0.05,ξ=-0.1\n Orange
          line:w0=-0.6,w1=-0.05,ξ=-0.1\n Magenta
          line:w0=-0.5,w1=-0.05,ξ=-0.1", 10],
       Background → LightGreen, FrameStyle → None], {0.1, 4},
      {Left, Top}],
    PlotLabel →
     "Qc=ξ H ρc, ξ=-0.1, CPL, Quintessence: Transition
       Redshift ~ Ωm0", ImageSize → 400],
   Show[{Plot[ztrICCPL[Ωm0ICCPL, 1 - Ωm0ICCPL, -0.1, -0.5, -0.05],
      {Ωm0ICCPL, 0.01, 1}, PlotRange → {{0, 1}, {-1, 4}},
      PlotStyle → Blue, AxesOrigin → {0, -1},
      PerformanceGoal → "Quality"],
     Plot[ztrICCPL[Ωm0ICCPL, 1 - Ωm0ICCPL, -0.1, -0.5, -0.1],
      {Ωm0ICCPL, 0.01, 1}, PlotRange → {{0, 1}, {-1, 4}},
      PlotStyle → Purple, AxesOrigin → {0, -1},
      PerformanceGoal → "Quality"],
     Plot[ztrICCPL[Ωm0ICCPL, 1 - Ωm0ICCPL, -0.1, -0.5, -0.2],
      {Ωm0ICCPL, 0.01, 0.94}, PlotRange → {{0, 1}, {-1, 4}},
      PlotStyle → Green, AxesOrigin → {0, -1},
      PerformanceGoal → "Quality"],
     Plot[ztrICCPL[Ωm0ICCPL, 1 - Ωm0ICCPL, -0.1, -0.5, -0.3],
      {Ωm0ICCPL, 0.01, 0.94}, PlotRange → {{0, 1}, {-1, 4}},
      PlotStyle → Orange, AxesOrigin → {0, -1},
      PerformanceGoal → "Quality"],
     Plot[ztrICCPL[Ωm0ICCPL, 1 - Ωm0ICCPL, -0.1, -0.5, -0.4],
      {Ωm0ICCPL, 0.01, 0.94}, PlotRange → {{0, 1}, {-1, 4}},
      PlotStyle → Magenta, AxesOrigin → {0, -1},
      PerformanceGoal → "Quality"],
```

```
Plot[ztr[Ωm0ICCPL, 1 - Ωm0ICCPL], {Ωm0ICCPL, 0, 1},
  PlotRange → {{0.01, 0.94}, {-1, 4}}, PlotStyle → Pink,
  AxesOrigin → {0, -1}]},
GridLines → {{{0.261, Red}, {0.274, Green}, {0.287, Gray}},
  {{0.376, LightRed}, {0.426, LightBlue}, {0.508, Gray}}},
AxesOrigin → {0.03, -1}, PlotRange → {{0.03, 0.33}, Automatic},
Frame → False, AxesLabel → {"Ωm0", "Transition"},
Epilog →
 Inset[
  Framed[
   Style[
    "Blue line: w0=-0.5,w1=-0.05,ξ=-0.1\n Purple line:
      w0=-0.5,w1=-0.1,ξ=-0.1\n Green
      line:w0=-0.5,w1=-0.2,ξ=-0.1\n Orange
      line:w0=-0.5,w1=-0.3,ξ=-0.1\n Magenta
      line:w0=-0.5,w1=-0.4,ξ=-0.1", 10],
   Background → LightGreen, FrameStyle → None], {0.1, 4},
  {Left, Top}],
PlotLabel →
 "Qc=ξ H ρc, ξ=-0.1, CPL, CPL, Quintessence: Transition
   Redshift ~ Ωm0", ImageSize → 400]}}]
```



As for the effect of EoS, we can split it to w0 effect and w1 effect.

Choose some values of w0 and w1 causually.

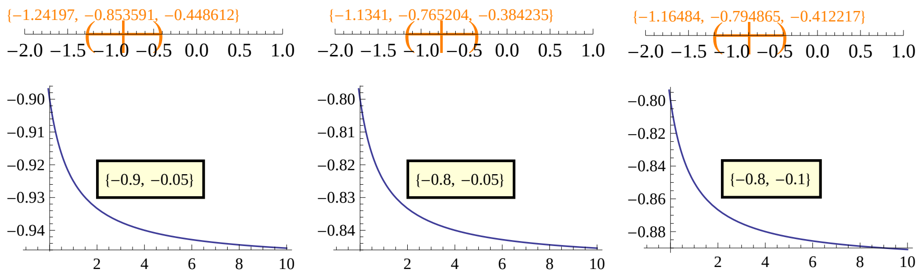
```
ξvwExamICCPLQuintessence =
 {Block[{w0ICCPL = -0.9, w1ICCPL = -0.05},
   {{w0ICCPL, w1ICCPL}, ξICCPLfc[w0ICCPL, w1ICCPL],
    ξICCPLf1[w0ICCPL, w1ICCPL], ξICCPLf2[w0ICCPL, w1ICCPL]}],
  Block[{w0ICCPL = -0.8, w1ICCPL = -0.05},
   {{w0ICCPL, w1ICCPL}, ξICCPLfc[w0ICCPL, w1ICCPL],
    ξICCPLf1[w0ICCPL, w1ICCPL], ξICCPLf2[w0ICCPL, w1ICCPL]}],
  Block[{w0ICCPL = -0.8, w1ICCPL = -0.1},
   {{w0ICCPL, w1ICCPL}, ξICCPLfc[w0ICCPL, w1ICCPL],
    ξICCPLf1[w0ICCPL, w1ICCPL], ξICCPLf2[w0ICCPL, w1ICCPL]}]}
```

```
{{{-0.9, -0.05}, -0.853591, -1.24197, -0.448612},
 {{-0.8, -0.05}, -0.765204, -1.1341, -0.384235},
 {{-0.8, -0.1}, -0.794865, -1.16484, -0.412217}}
```

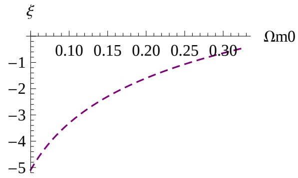
```
tabξvwExamICCPLQuintessence =
 Grid[Prepend[Prepend[ξvwExamICCPLQuintessence,
    {"{w0,w1}", "Center", "Lower", "Upper"}],
   {"ξ results for Qc=ξ H ρd, CPL,Quintessence.", SpanFromLeft}],
  Frame → All,
  Background → {{LightGray, None}, {LightGreen, LightGray, None}},
  Alignment → Center, ItemSize → 8]
```

ξ results for Q_c=ξ H ρ_d, CPL,Quintessence.			
{w0,w1}	Center	Lower	Upper
{-0.9, -0.05}	-0.853591	-1.24197	-0.448612
{-0.8, -0.05}	-0.765204	-1.1341	-0.384235
{-0.8, -0.1}	-0.794865	-1.16484	-0.412217

```
pltfitξICCPLQuintessenceSum =
 Grid[
  {{pltfitξICCPLfunc[-0.9, -0.05], pltfitξICCPLfunc[-0.8, -0.05],
    pltfitξICCPLfunc[-0.8, -0.1]}}]
```

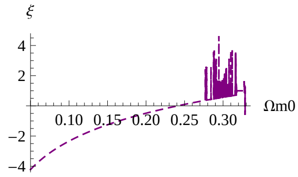


```
pltfitξvΩm0QuintomSum[-0.9, -0.05, {Purple, Dashed}, {0.05, 0.33}] //
  Timing // Quiet
```

{35.677, 

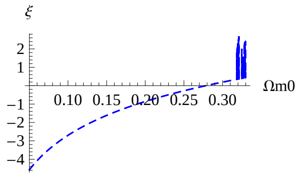
}

```
pltfitξvΩm0QuintomSum[-0.5, -0.05, {Purple, Dashed}, {0.05, 0.33}] //
  Timing // Quiet
```

{367.102, 

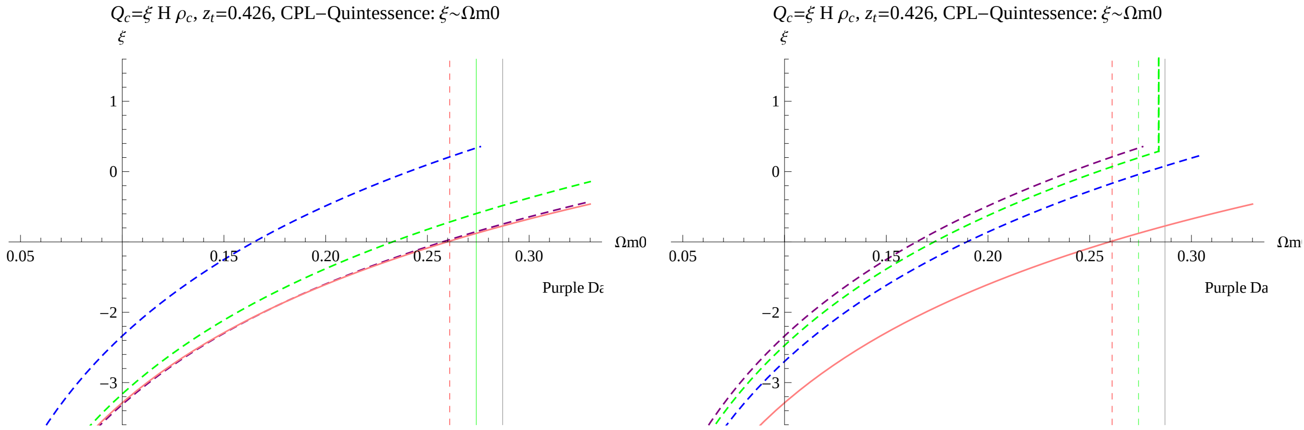
}

```
pltfitξvΩm0QuintomSum[-0.5, -0.2, {Blue, Dashed}, {0.05, 0.33}] //
  Timing // Quiet
```

{107.5, 

}

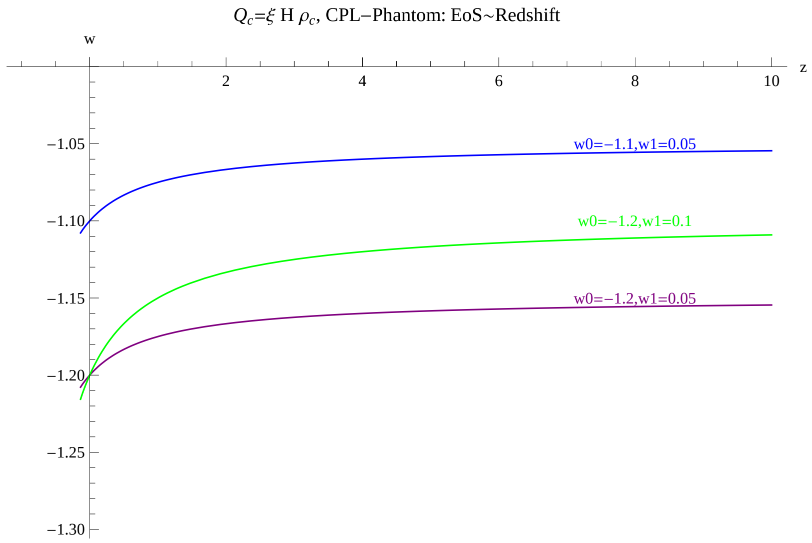
```
pltξvΩm0ICCPLQuintessenceSum =
 Grid[
   {{Show[pltfitξvΩm0QuintomSum[-0.9, -0.05, {Purple, Dashed},
       {0.05, 0.33}], pltfitξvΩm0QuintomSum[-0.7, -0.05,
       {Green, Dashed}, {0.05, 0.33}],
      pltfitξvΩm0QuintomSum[-0.5, -0.05, {Blue, Dashed},
       {0.05, 0.276}], pltfitξvΩm0QuintomSum[-1, 0, Pink,
       {0.05, 0.33}],
      GridLines → {{{0.261, Directive[Red, Dashed]},
         {0.274, Green}, {0.287, Gray}}, {}}, AxesOrigin → {0.1, -1},
      PlotRange → {{0.05, 0.33}, {-3.5, 1.5}}, Frame → False,
      AxesLabel → {"Ωm0", "ξ"},
      PlotLabel → "Qc=ξ H ρc, zt=0.426, CPL-Quintessence: ξ~Ωm0",
      Epilog →
       Inset[
        Framed[
         Style[
          "(w0,w1)\n Purple Dashed:(-0.9,-0.05) Green
            Dashed:(-0.7,-0.05)\n Blue
            Dashed:(-0.5,-0.05)\n Pink line:LCDM", 10],
         Background → None, FrameStyle → None], {0.3, -1},
        {Left, Top}], ImageSize → 400],
     Show[pltfitξvΩm0QuintomSum[-0.5, -0.05, {Purple, Dashed},
       {0.05, 0.276}], pltfitξvΩm0QuintomSum[-0.5, -0.1,
       {Green, Dashed}, {0.05, 0.284}],
      pltfitξvΩm0QuintomSum[-0.5, -0.2, {Blue, Dashed},
       {0.05, 0.304}], pltfitξvΩm0QuintomSum[-1, 0, Pink,
       {0.05, 0.33}],
      GridLines → {{{0.261, Directive[Red, Dashed]},
         {0.274, Directive[Green, Dashed]}, {0.287, Gray}}, {}},
      AxesOrigin → {0.1, -1}, PlotRange → {{0.05, 0.33}, {-3.5, 1.5}},
      Frame → False, AxesLabel → {"Ωm0", "ξ"},
      PlotLabel → "Qc=ξ H ρc, zt=0.426, CPL-Quintessence: ξ~Ωm0",
      Epilog →
       Inset[
        Framed[
         Style[
          "(w0,w1)\n Purple Dashed:(-0.5,-0.05) Green
            Dashed:(-0.5,-0.1)\n Blue
            Dashed:(-0.5,-0.2)\n Pink line:LCDM", 10],
         Background → None, FrameStyle → None], {0.3, -1},
        {Left, Top}], ImageSize → 400]}}] // Quiet
```



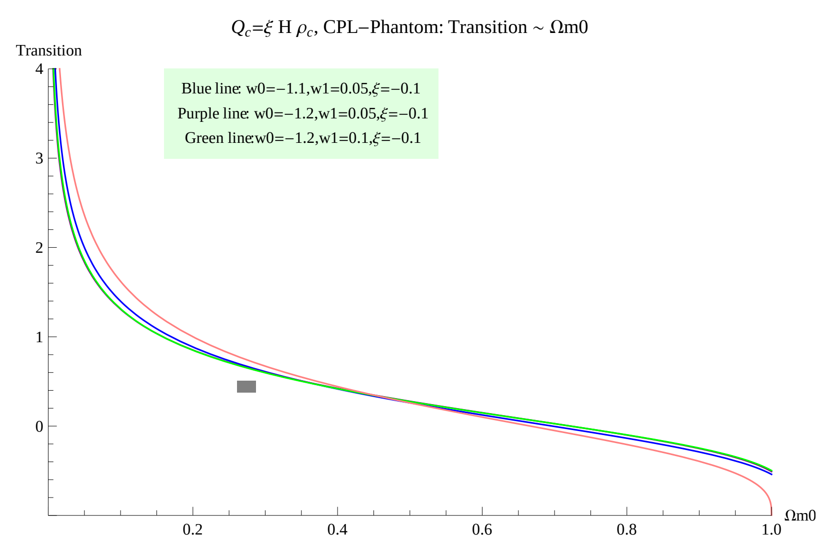
Phantom

Some parameters that give us a phantom mode
(-1.1,0.05) (-1.2,0.05) (-1.2,0.1)

```
plEoSICCPLPhantomSum =
 Grid[
  {{Show[pltICCPLEoSfunc[-1.1, 0.05, Blue],
     pltICCPLEoSfunc[-1.2, 0.05, Purple],
     pltICCPLEoSfunc[-1.2, 0.1, Green],
     PlotRange → {{-0.99, 10}, {-1.3, -1}}, AxesOrigin → {0, -1},
     Epilog →
      {Inset[Framed[Style["w0=-1.1,w1=0.05", 10, Blue],
         Background → None, FrameStyle → None], {8, -1.05}, {0, 0}],
       Inset[Framed[Style["w0=-1.2,w1=0.05", 10, Purple],
         Background → None, FrameStyle → None], {8, -1.15}, {0, 0}],
       Inset[Framed[Style["w0=-1.2,w1=0.1", 10, Green],
         Background → None, FrameStyle → None], {8, -1.1}, {0, 0}]},
     PlotLabel → "Qc=ξ H ρc, CPL-Phantom: EoS~Redshift",
     ImageSize → 500]}}]
```



```
plztrICCPLPhantomSum =
 Grid[
  {{Show[{plztrICCPL[-0.1, -1.1, 0.05, Blue, {0, 1}],
      plztrICCPL[-0.1, -1.2, 0.05, Purple, {0, 1}],
      plztrICCPL[-0.1, -1.2, 0.1, Green, {0, 1}],
      Plot[ztr[Ωm0ICCPL, 1 - Ωm0ICCPL], {Ωm0ICCPL, 0, 1},
       PlotRange → {{0, 1}, {-1, 4}}, PlotStyle → Pink,
       AxesOrigin → {0, -1}]},
     Graphics[{Gray, Rectangle[Scaled[{0.261, .2752}],
        Scaled[{0.287, .3016}]]}, Frame → True], AxesOrigin → {0, -1},
     Frame → False, AxesLabel → {"Ωm0", "Transition"},
     Epilog →
      Inset[
       Framed[
        Style[
         "Blue line: w0=-1.1,w1=0.05,ξ=-0.1\n Purple line:
           w0=-1.2,w1=0.05,ξ=-0.1\n Green
           line:w0=-1.2,w1=0.1,ξ=-0.1", 10],
        Background → LightGreen, FrameStyle → None], {0.35, 3.5},
       {0, 0}],
     PlotLabel → "Qc=ξ H ρc, CPL-Phantom: Transition ~ Ωm0",
     ImageSize → 500]}}]
```



As for the effect of EoS, we can split it to w0 effect and w1 effect.

Choose some values of w0 and w1 causually.

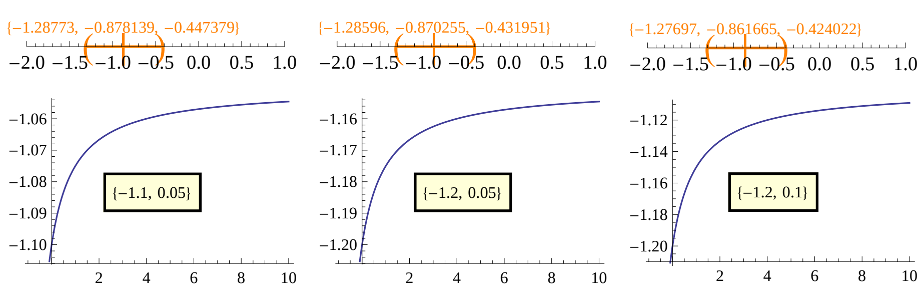
```
ξvwExamICCPLPhantom =
 {Block[{w0ICCPL = -1.1, w1ICCPL = 0.05},
   {{w0ICCPL, w1ICCPL}, ξICCPLfc[w0ICCPL, w1ICCPL],
    ξICCPLf1[w0ICCPL, w1ICCPL], ξICCPLf2[w0ICCPL, w1ICCPL]}],
  Block[{w0ICCPL = -1.2, w1ICCPL = 0.05},
   {{w0ICCPL, w1ICCPL}, ξICCPLfc[w0ICCPL, w1ICCPL],
    ξICCPLf1[w0ICCPL, w1ICCPL], ξICCPLf2[w0ICCPL, w1ICCPL]}],
  Block[{w0ICCPL = -1.2, w1ICCPL = 0.1},
   {{w0ICCPL, w1ICCPL}, ξICCPLfc[w0ICCPL, w1ICCPL],
    ξICCPLf1[w0ICCPL, w1ICCPL], ξICCPLf2[w0ICCPL, w1ICCPL]}]}
```

```
{{{-1.1, 0.05}, -0.878139, -1.28773, -0.447379},
 {{-1.2, 0.05}, -0.870255, -1.28596, -0.431951},
 {{-1.2, 0.1}, -0.861665, -1.27697, -0.424022}}
```

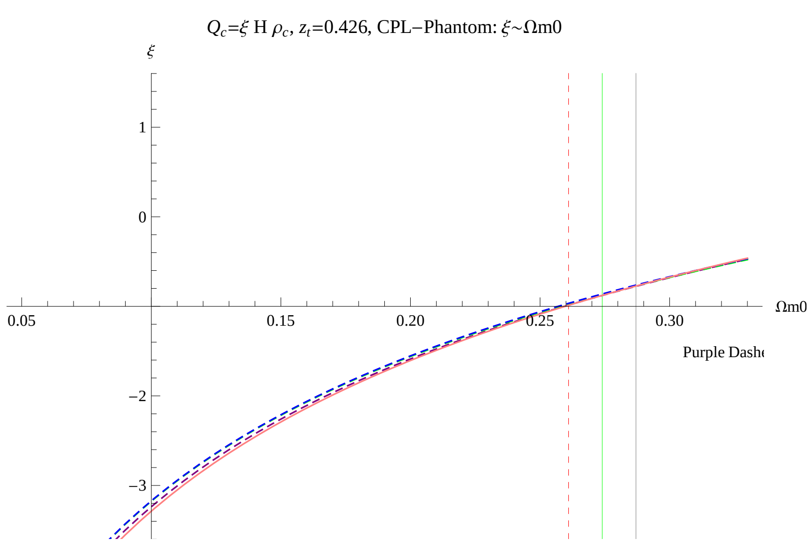
```
tabξvwExamICCPLPhantom =
 Grid[Prepend[Prepend[ξvwExamICCPLPhantom,
    {"{w0,w1}", "Center", "Lower", "Upper"}],
   {"ξ results for Qc=ξ H ρd, CPL,Phantom.", SpanFromLeft}],
  Frame → All,
  Background → {{LightGray, None}, {LightGreen, LightGray, None}},
  Alignment → Center, ItemSize → 8]
```

ξ results for $Q_c = \xi\ H\ \rho_d$, CPL,Phantom.			
{w0,w1}	Center	Lower	Upper
{-1.1, 0.05}	-0.878139	-1.28773	-0.447379
{-1.2, 0.05}	-0.870255	-1.28596	-0.431951
{-1.2, 0.1}	-0.861665	-1.27697	-0.424022

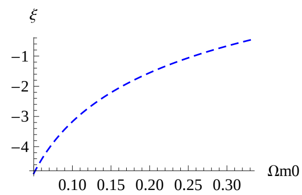
```
pltfitξICCPLPhantomSum =
 Grid[{{pltfitξICCPLfunc[-1.1, 0.05], pltfitξICCPLfunc[-1.2, 0.05],
    pltfitξICCPLfunc[-1.2, 0.1]}}]
```



```
pltξvΩm0ICCPLPhantomSum =
 Show[pltfitξvΩm0QuintomSum[-1.1, 0.05, {Purple, Dashed},
    {0.05, 0.33}], pltfitξvΩm0QuintomSum[-1.2, 0.05,
    {Green, Dashed}, {0.05, 0.33}],
   pltfitξvΩm0QuintomSum[-1.2, 0.1, {Blue, Dashed}, {0.05, 0.33}],
   pltfitξvΩm0QuintomSum[-1, 0, Pink, {0.05, 0.33}],
   GridLines →
    {{{0.261, Directive[Red, Dashed]}, {0.274, Green}, {0.287, Gray}},
     {}}, AxesOrigin → {0.1, -1},
   PlotRange → {{0.05, 0.33}, {-3.5, 1.5}}, Frame → False,
   AxesLabel → {"Ωm0", "ξ"},
   PlotLabel → "Qc=ξ H ρc, zt=0.426, CPL-Phantom: ξ~Ωm0",
   Epilog →
    Inset[
     Framed[
      Style[
       "(w0,w1)\n Purple Dashed:(-1.1,0.05) Green
         Dashed:(-1.2,0.05)\n Blue Dashed:(-1.2,0.1)\n
         Pink line:LCDM", 10], Background → None,
      FrameStyle → None], {0.3, -1}, {Left, Top}], ImageSize → 500] //
  Quiet
```




```
pltfitξvΩm0QuintomSum[-1.2, 0.1, {Blue, Dashed}, {0.05, 0.33}] //
  Timing // Quiet
```

{36.488,  }

Take the form $Q_c = \xi H \rho_d$. I2 model

Hubble function without curvature

```
hubbleI2CC[H0_, Ωd0_, Ωm0_, w_, ξ_, z_] := H0 √(Ωm + Ωd);
```

Conventions:
parameters : H0, Ωm0, Ωd0, w, ξ, z.

Constant ξ + Constant w. I2CC model

Definations

Fraction energy density

```
ΩmI2CC[Ωm0I2CC_, Ωd0I2CC_, wI2CC_, ξI2CC_, z_] :=
  (Ωm0I2CC + ξI2CC/(ξI2CC + 3 wI2CC)) (1 + z)^3 +
   (-ξI2CC)/(ξI2CC + 3 wI2CC) ΩdI2CC[Ωd0I2CC, wI2CC, ξI2CC, z]
```

```
ΩdI2CC[Ωd0I2CC_, wI2CC_, ξI2CC_, z_] := Ωd0I2CC (1 + z)^(3 (1+wI2CC)+ξI2CC)
```

Hubble function

```
hubbleI2CC[H0I2CC_, Ωm0I2CC_, Ωd0I2CC_, wI2CC_, ξI2CC_, z_] =
 H0I2CC
  √(ΩmI2CC[Ωm0I2CC, Ωd0I2CC, wI2CC, ξI2CC, z] +
     ΩdI2CC[Ωd0I2CC, wI2CC, ξI2CC, z])
```

$$H0I2CC\sqrt{\left((1+z)^{3(1+wI2CC)+\xi I2CC}\,\Omega d0I2CC - \frac{(1+z)^{3(1+wI2CC)+\xi I2CC}\,\xi I2CC\,\Omega d0I2CC}{3\,wI2CC+\xi I2CC} + (1+z)^3\left(\frac{\xi I2CC}{3\,wI2CC+\xi I2CC}+\Omega m0I2CC\right)\right)}$$

Deceleration parameter

```
qI2CC[H0I2CC_, Ωm0I2CC_, Ωd0I2CC_, wI2CC_, ξI2CC_, z_] =
  -1 + (1 + z)/hubbleI2CC[H0I2CC, Ωm0I2CC, Ωd0I2CC, wI2CC, ξI2CC, z]
    D[hubbleI2CC[H0I2CC, Ωm0I2CC, Ωd0I2CC, wI2CC, ξI2CC, z], z];
```

```
qI2CC[H0I2CC, Ωm0I2CC, Ωd0I2CC, wI2CC, ξI2CC, z] // FullSimplify
```

$$\left(\xi I2CC + 3\, wI2CC\, (1+z)^{3\, wI2CC+\xi I2CC}\, (1 + 3\, wI2CC + \xi I2CC)\, \Omega d0I2CC + 3\, wI2CC\, \Omega m0I2CC + \xi I2CC\, \Omega m0I2CC\right) \Big/ \left(2 \left(\xi I2CC\, (1 + \Omega m0I2CC) + 3\, wI2CC \left((1+z)^{3\, wI2CC+\xi I2CC}\, \Omega d0I2CC + \Omega m0I2CC\right)\right)\right)$$

```
Limit[qI2CC[H0I2CC, Ωm0I2CC, Ωd0I2CC, wI2CC, ξI2CC, z] // FullSimplify,
 z → Infinity, Assumptions → (3 wI2CC + ξI2CC) < 0]
```

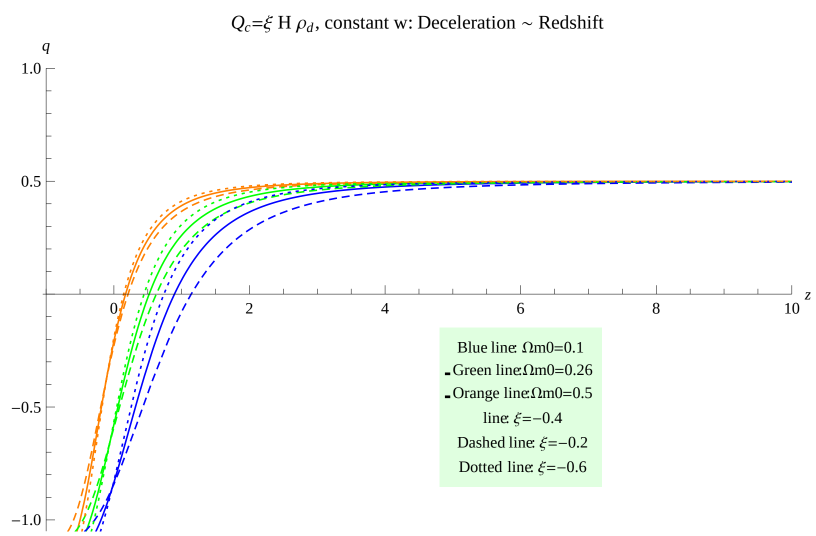
$$\frac{1}{2}$$

At z-> Infinity limit, deceleration parameter $\rightarrow \frac{1}{2}$, with 3 wICC+ξICC<0. This is different from LCDM model.

Plots, showcase, manipulate toys

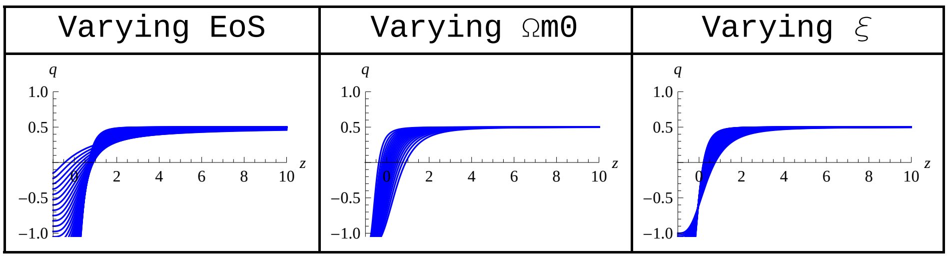
```
pldecI2CC[Ωm0I2CC_, wI2CC_, ξI2CC_, color_] :=
  Plot[qI2CC[H0w, Ωm0I2CC, 1 - Ωm0I2CC, wI2CC, ξI2CC, z],
   {z, -1, 10}, PlotRange → {{-1.05, 10}, {-1.05, 1}},
   PlotStyle → color, AxesOrigin → {-1, 0}, AxesLabel → {z, q}];
```

```
pldecI2CCShowSum =
 Show[{pldecI2CC[0.1, -1, -0.4, Blue],
   pldecI2CC[0.26, -1, -0.4, Green], pldecI2CC[0.5, -1, -0.4, Orange],
   pldecI2CC[0.1, -1, -0.2, Directive[Blue, Dashed]],
   pldecI2CC[0.26, -1, -0.2, Directive[Green, Dashed]],
   pldecI2CC[0.5, -1, -0.2, Directive[Orange, Dashed]],
   pldecI2CC[0.1, -1, -0.6, Directive[Blue, Dotted]],
   pldecI2CC[0.26, -1, -0.6, Directive[Green, Dotted]],
   pldecI2CC[0.5, -1, -0.6, Directive[Orange, Dotted]]},
  Epilog →
   Inset[
    Framed[
     Style[
      "Blue line: Ωm0=0.1\n Green line:Ωm0=0.26\n Orange
        line:Ωm0=0.5\n line: ξ=-0.4\n Dashed line:
        ξ=-0.2\n Dotted line: ξ=-0.6", 10],
     Background → LightGreen, FrameStyle → None], {6, -0.5}],
  PlotLabel → "Qc=ξ H ρd, constant w: Deceleration ~ Redshift",
  ImageSize → 500]
```



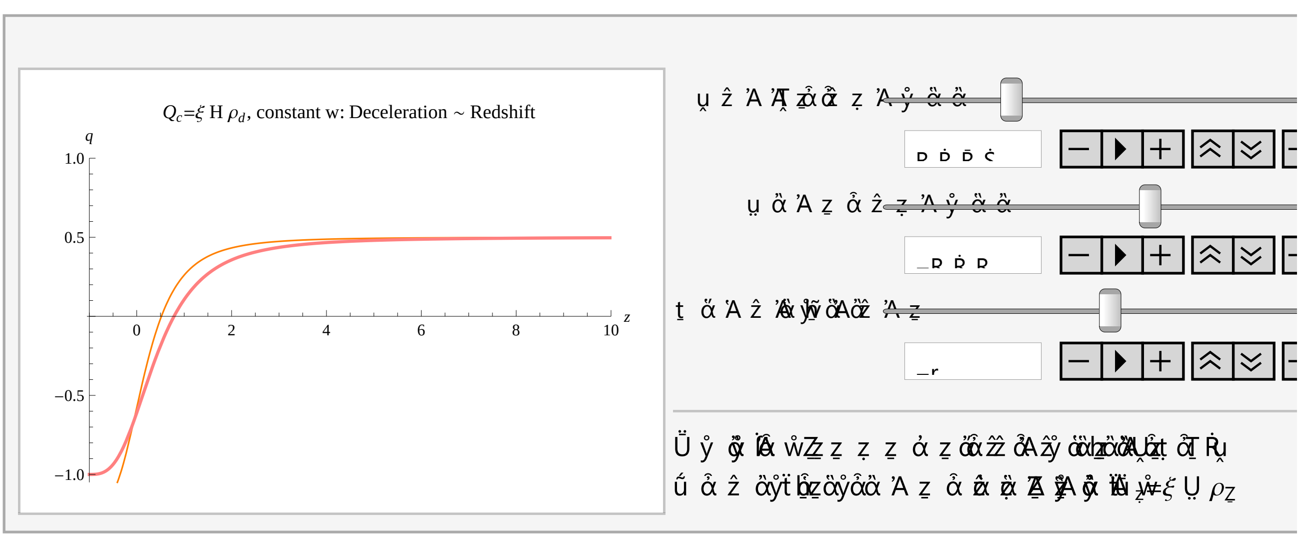
z->Infinity is a degenerate limit. For constant ξ and constant w models, this limit is determined by the interaction strength ξ. This might be useful if more complicated models are investigated and no large deviations are shown. [Nota]

```
varyingI2CCShowSum =
 Grid[{{"Varying EoS", "Varying Ωm0", "Varying ξ"},
   Table[
    {Show[Table[pldecI2CC[0.1, wI2CC, -0.4, Blue],
       {wI2CC, -2, -0.3, 0.05}]],
     Show[Table[pldecI2CC[Ωm0I2CC, -1, -0.4, Blue],
       {Ωm0I2CC, 0.1, 0.9, 0.05}]],
     Show[Table[pldecI2CC[0.27, -1, ξI2CC, Blue],
       {ξI2CC, -2, 0, 0.05}]]}]}, Frame → All]
```



Use movable slides to check how do the parameters affect the deceleration parameter. Just a toy.

```
pldecI2CCManSum =
 Manipulate[
  Show[{pldecI2CC[Ωm0I2CC, wI2CC, ξI2CC, Orange],
    pldec[Ωm0I2CC, {Pink, Thick}]},
   PlotLabel → "Qc=ξ H ρd, constant w: Deceleration ~ Redshift"],
  {{Ωm0I2CC, 0.26, "Matter Fraction"}, 0, 1, Appearance → "Open"},
  {{ξI2CC, -0.4, "Interaction"}, -1, 0, Appearance → "Open"},
  {{wI2CC, -1, "Equation of State"}, -1.5, -0.5, Appearance → "Open"},
  Delimiter, Style["Pink is the deceleration parameter for LCDM.",
   Medium], Style["Orange is for interacting model with Qc=ξ H ρd",
   Medium], ControlPlacement → {Right, Right, Right},
  SaveDefinitions → True]
```



Transition redshift definations and equations.

Find out the expression for transition redshift

```
ΩmI2CC[Ωm0I2CC_, Ωd0I2CC_, wI2CC_, ξI2CC_, z_] :=
  (Ωm0I2CC + ξI2CC/(ξI2CC + 3 wI2CC)) (1 + z)^3 +
   (-ξI2CC)/(ξI2CC + 3 wI2CC) ΩdI2CC[Ωd0I2CC, wI2CC, ξI2CC, z]
```

```
ΩdI2CC[Ωd0I2CC_, wI2CC_, ξI2CC_, z_] := Ωd0I2CC (1 + z)^(3 (1+wI2CC)+ξI2CC)
```

```
(3 wI2CC + 1) ΩdI2CC[Ωd0I2CC, wI2CC, ξI2CC, z] +
   ΩmI2CC[Ωm0I2CC, Ωd0I2CC, wI2CC, ξI2CC, z] == 0 // Simplify
```

$$\frac{1}{3\,\text{wI2CC}+\xi\text{I2CC}}(1+z)\left(\xi\text{I2CC}\left(1+3\,\text{wI2CC}\,(1+z)^{3\,\text{wI2CC}+\xi\text{I2CC}}\,\Omega\text{d0I2CC}+\Omega\text{m0I2CC}\right)+3\,\text{wI2CC}\left((1+3\,\text{wI2CC})\,(1+z)^{3\,\text{wI2CC}+\xi\text{I2CC}}\,\Omega\text{d0I2CC}+\Omega\text{m0I2CC}\right)\right)=0$$

```
ztrI2CC[Ωm0I2CC_, Ωd0I2CC_, wI2CC_, ξI2CC_] =
  -1 + ((3 wI2CC + ξI2CC) Ωm0I2CC + ξI2CC Ωd0I2CC)/(-3 wI2CC (3 wI2CC + ξI2CC + 1) Ωd0I2CC))^(1/(3 wI2CC+ξI2CC))
```

$$-1+3^{-\frac{1}{3\,\text{wI2CC}+\xi\text{I2CC}}}\left(-\frac{\xi\text{I2CC}\,\Omega\text{d0I2CC}+(3\,\text{wI2CC}+\xi\text{I2CC})\,\Omega\text{m0I2CC}}{\text{wI2CC}\,(1+3\,\text{wI2CC}+\xi\text{I2CC})\,\Omega\text{d0I2CC}}\right)^{\frac{1}{3\,\text{wI2CC}+\xi\text{I2CC}}}$$

```
ztrI2CC[0.27, 0.73, -1, -0.4]
```

0.540312

The first solution is trivil. So the second one is taken.

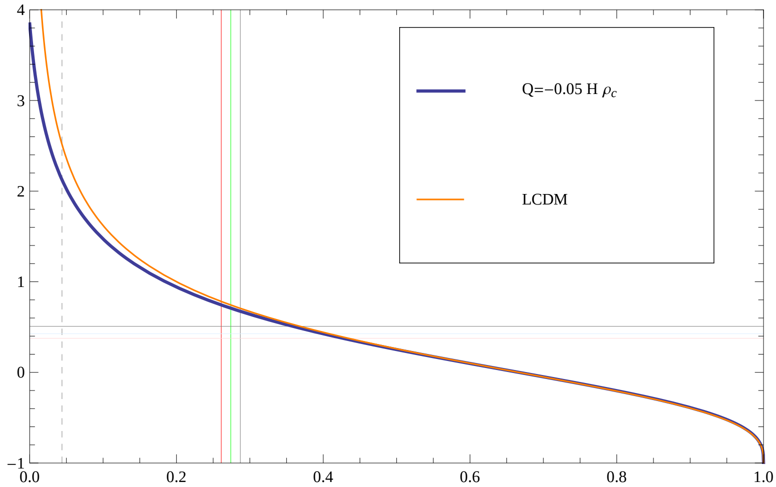
Define rICC = $\frac{\Omega\text{m0ICC}}{\Omega\text{d0ICC}}$

```
ztrrI2CC[rI2CC_, wI2CC_, ξI2CC_] =
  -1 + (((3 wI2CC + ξI2CC) rI2CC + ξI2CC)/(-3 wI2CC (3 wI2CC + ξI2CC + 1)))^(1/(3 wI2CC+ξI2CC));
```

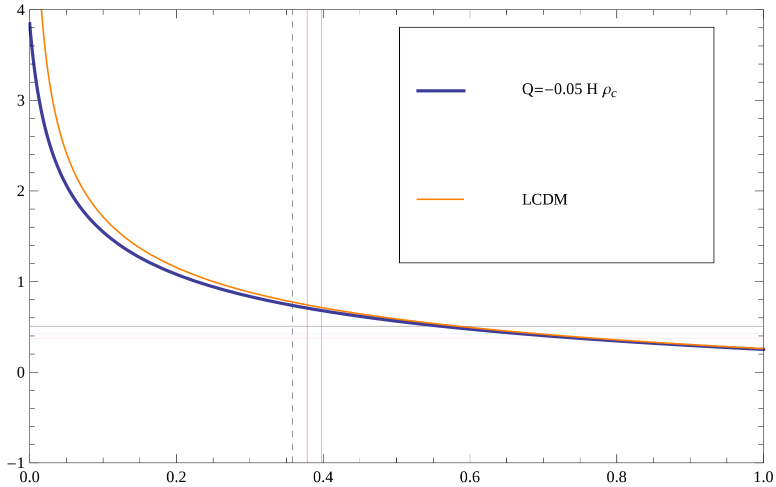
Visualiztion of transition redshift

Check the behavior of this transition redshift.

```
pldecrI2CC =
 Plot[{ztrI2CC[Ωm0I2CC, 1 - Ωm0I2CC, -1, -0.05],
   ztr[Ωm0I2CC, 1 - Ωm0I2CC]}, {Ωm0I2CC, 0, 1},
  PlotRange → {{0, 1}, {-1, 4}}, PlotStyle → {Thick, Orange},
  AxesOrigin → {0, -1}, Frame → True,
  GridLines →
   {{{0.044, Dashed}, {0.261, Red}, {0.274, Green}, {0.287, Gray}},
    {{0.376, LightRed}, {0.426, LightBlue}, {0.508, Gray}}},
  PlotLegend → {"Q=-0.05 H ρc", "LCDM"},
  LegendPosition → {0.0, -0.05}, LegendShadow → None, ImageSize → 500]
```




```
pldecrrI2CC = Plot[{ztrrI2CC[rI2CC, -1, -0.05], ztrr[rI2CC]},
  {rI2CC, 0, 1}, PlotRange → {{0, 1}, {-1, 4}},
  PlotStyle → {Thick, Orange}, AxesOrigin → {0, -1}, Frame → True,
  GridLines → {{{0.358, Dashed}, {0.378, Directive[Red]}, 0.398},
    {{0.376, LightRed}, {0.426, LightBlue}, {0.508, Gray}}},
  PlotLegend → {"Q=-0.05 H ρc", "LCDM"},
  LegendPosition → {0.0, -0.05}, LegendShadow → None, ImageSize → 500]
```



```
plztrI2CC[wI2CC_, ξI2CC_, color_] :=
  Plot[ztrI2CC[Ωm0I2CC, 1 - Ωm0I2CC, wI2CC, ξI2CC], {Ωm0I2CC, 0, 1},
   PlotRange → {{0, 1}, {-1, 4}}, PlotStyle → color,
   AxesOrigin → {0, -1}, AxesLabel → {"Ωm0", "zt"}];
```

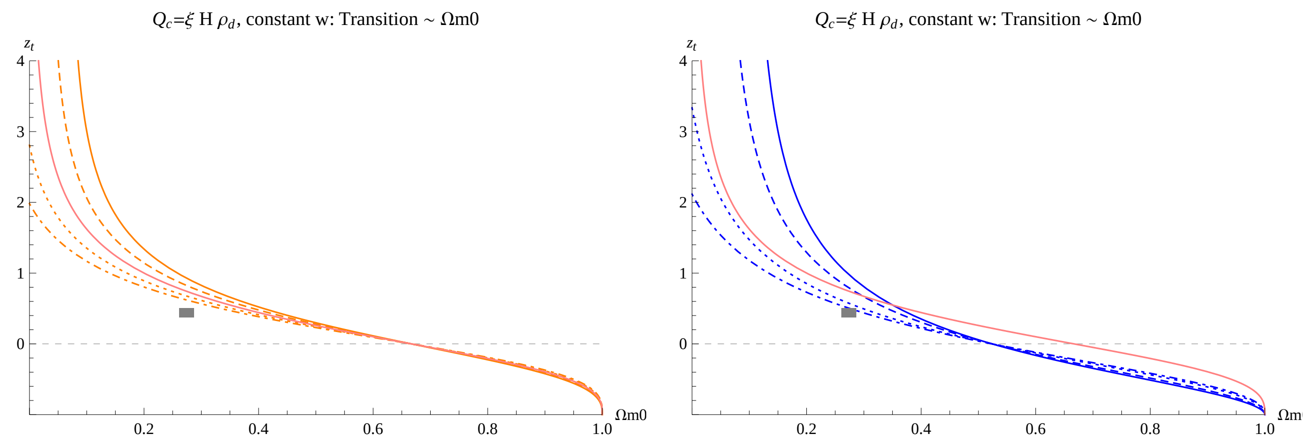
The following figure:

Orange for w=-1
Blue for w=-0.9

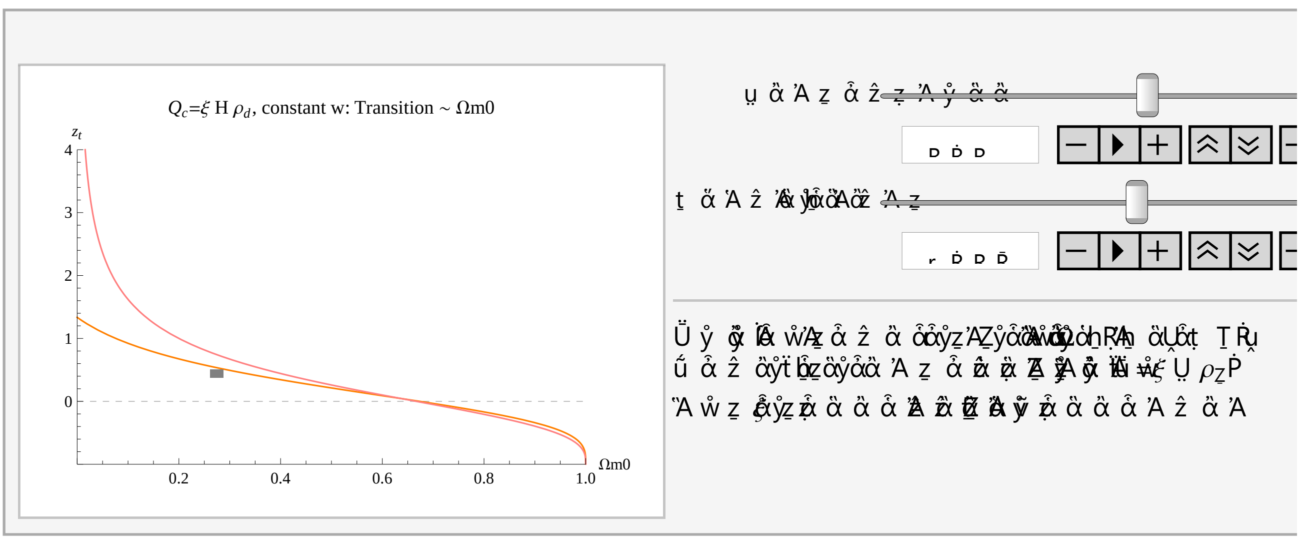
line: ξ=0.2
Dashed: ξ=0.1
Dotted: ξ=-0.1
DotDashed: ξ=-0.2

Pink line:w=1, ξ=0

```
plztrvsΩm0I2CCSum =
 Grid[
  {{Show[{plztrI2CC[-1, 0.2, Orange],
      plztrI2CC[-1, 0.1, {Orange, Dashed}],
      plztrI2CC[-1, -0.1, {Orange, Dotted}],
      plztrI2CC[-1, -0.2, {Orange, DotDashed}],
      Plot[ztr[Ωm0I2CC, 1 - Ωm0I2CC], {Ωm0I2CC, 0, 1},
       PlotRange → {{0, 1}, {-1, 4}}, PlotStyle → Pink,
       AxesOrigin → {0, -1}]},
     Graphics[{Gray, Rectangle[Scaled[{0.261, .2752}],
        Scaled[{0.287, .3016}]]}, Frame → True],
     FrameLabel → {"Ωm0", "Transition"},
     GridLines → {{}, {{0, Dashed}}},
     PlotLabel → "Qc=ξ H ρd, constant w: Transition ~ Ωm0",
     ImageSize → 400],
    Show[{plztrI2CC[-0.7, 0.2, Blue],
      plztrI2CC[-0.7, 0.1, {Blue, Dashed}],
      plztrI2CC[-0.7, -0.1, {Blue, Dotted}],
      plztrI2CC[-0.7, -0.2, {Blue, DotDashed}],
      Plot[ztr[Ωm0I2CC, 1 - Ωm0I2CC], {Ωm0I2CC, 0, 1},
       PlotRange → {{0, 1}, {-1, 4}}, PlotStyle → Pink,
       AxesOrigin → {0, -1}]},
     Graphics[{Gray, Rectangle[Scaled[{0.261, .2752}],
        Scaled[{0.287, .3016}]]}, Frame → True],
     FrameLabel → {"Ωm0", "Transition"},
     GridLines → {{}, {{0, Dashed}}},
     PlotLabel → "Qc=ξ H ρd, constant w: Transition ~ Ωm0",
     ImageSize → 400]
  }}]
```



```
plztrI2CCManSum =
 Manipulate[
  Show[{plztrI2CC[wI2CC, ξI2CC, Orange],
    Plot[ztr[Ωm0I2CC, 1 - Ωm0I2CC], {Ωm0I2CC, 0, 1},
     PlotRange → {{0, 1}, {-1, 4}}, PlotStyle → Pink,
     AxesOrigin → {0, -1}]},
   Graphics[{Gray, Rectangle[Scaled[{0.261, .2752}],
      Scaled[{0.287, .3016}]]}], Frame → True],
   FrameLabel → {"Ωm0", "Transition"},
   GridLines → {{}, {{0, Dashed}}},
   PlotLabel → "Qc=ξ H ρd, constant w: Transition ~ Ωm0"],
  {{ξI2CC, -0.4, "Interaction"}, -1, 0, Appearance → "Open"},
  {{wI2CC, -1.02, "Equation of state"}, -2, -0.3,
   Appearance → "Open"}, Delimiter,
  Style["Pink is the transition redshift vs Ωm0 for LCDM.", Medium],
  Style["Orange is for interacting model with Q=ξ H ρd,", Medium],
  Style["where ξ is constant and EoS is constant", Medium],
  ControlPlacement → {Right, Right, Right}, SaveDefinitions → True]
```



Find out the allowed region of coupling constant.

To find out the region of ξ, set w=-1 and Ωd0=1-Ωm0. Let the ztr-Ωm0 line cross points (0.287,0.508) and (0.261, 0.376).

```
ztrI2CC[0.287, 1 - 0.287, -1, ξI2CC1] == 0.508
```

$$-1 + 0.467508^{\frac{1}{-3+\xi I2CC1}} \left(\frac{0.287\,(-3 + \xi I2CC1) + 0.713\,\xi I2CC1}{-2 + \xi I2CC1}\right)^{\frac{1}{-3+\xi I2CC1}} == 0.508$$

```
ξI2CCffunc[Ωm0I2CC_, Ωd0I2CC_, wI2CC_, data_] :=
 ξI2CC /. FindRoot[ztrI2CC[Ωm0I2CC, Ωd0I2CC, wI2CC, ξI2CC] == data,
   {ξI2CC, -0.6}]
```

```
ξI2CCf2[Ωm0I2CC_, Ωd0I2CC_, wI2CC_] :=
 ξI2CC /. FindRoot[ztrI2CC[Ωm0I2CC, Ωd0I2CC, wI2CC, ξI2CC] == 0.508,
   {ξI2CC, -0.6}]
```

```
ξI2CCf1[Ωm0I2CC_, Ωd0I2CC_, wI2CC_] :=
 ξI2CC /. FindRoot[ztrI2CC[Ωm0I2CC, Ωd0I2CC, wI2CC, ξI2CC] == 0.376,
   {ξI2CC, -0.6}]
```

Cross the Center of best fit. (0.274,0.426)

```
ξI2CCfc[Ωm0I2CC_, Ωd0I2CC_, wI2CC_] :=
 ξI2CC /. FindRoot[ztrI2CC[Ωm0I2CC, Ωd0I2CC, wI2CC, ξI2CC] == 0.426,
   {ξI2CC, -0.6}]
```

According to the data of transition redshift.

```
ξI2CCf2[wI2CC_] := ξI2CCffunc[0.287, 1 - 0.287, wI2CC, 0.508]
```

```
ξI2CCf1[wI2CC_] := ξI2CCffunc[0.261, 1 - 0.261, wI2CC, 0.376]
```

```
ξI2CCfc[wI2CC_] := ξI2CCffunc[0.274, 1 - 0.274, wI2CC, 0.426]
```

An example

```
{ξI2CCf1[-1], ξI2CCfc[-1], ξI2CCf2[-1]}
```

```
{-1.07368, -0.760999, -0.409217}
```

To summarize, taken the case that the universe is flat, and choose the parameters to be {w=-1}, the region for interaction constant ξ should be (-1.07,-0.41) with a center at -0.76, i.e., $-0.76^{+0.35}_{-0.31}$.

```
ztrrI2CC[0.358, -1, ξI2CC] == 0.376
```

$$-1 + 3^{-\frac{1}{-3+\xi I2CC}} \left(\frac{0.358\,(-3+\xi I2CC) + \xi I2CC}{-2+\xi I2CC}\right)^{\frac{1}{-3+\xi I2CC}} == 0.376$$

```
ξI2CCrffunc[rI2CC_, wI2CC_, data_] :=
 ξI2CC /. FindRoot[ztrrI2CC[rI2CC, wI2CC, ξI2CC] == data,
   {ξI2CC, -0.6}]
```

```
ξI2CCrf2[wI2CC_] := ξI2CCrffunc[0.398, wI2CC, 0.508]
```

```
ξI2CCrf1[wI2CC_] := ξI2CCrffunc[0.358, wI2CC, 0.376]
```

Center

```
ξI2CCrfc[wI2CC_] := ξI2CCrffunc[0.378, wI2CC, 0.426]
```

A example is (w=-1)

```
{ξI2CCrf1[-1], ξI2CCrfc[-1], ξI2CCrf2[-1]}
```

```
{-1.05903, -0.759371, -0.420298}
```

To summarize, taken the case that the universe is flat, and choose the parameters to be {w=-1}, the region for interation cosntant ξ should be (-1.06,-0.42) with a center at -0.76, i.e., $-0.76^{+0.34}_{-0.30}$.

This is a bit different from the result we got from Ωm0~ transition redshift plane. One possible reason is the second method doesn't assume a flat universe, while the first one supposes the universe is flat.

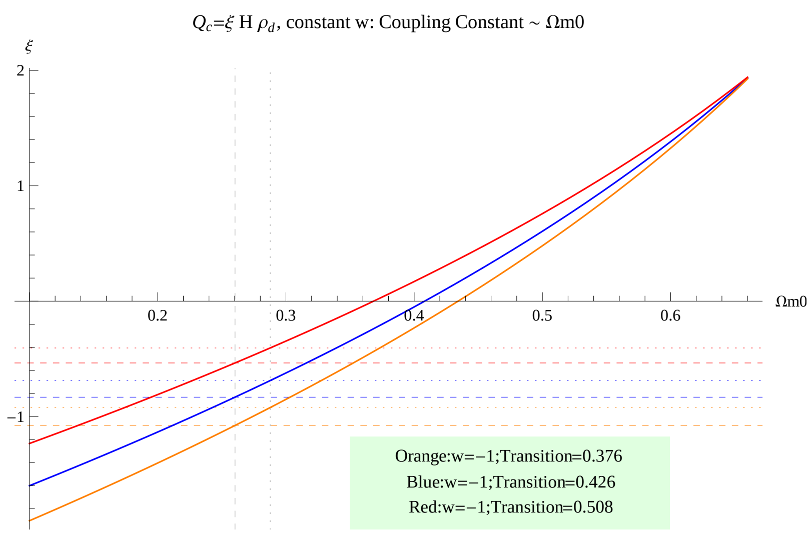
In arXiv:0801.4233, a CPL parameterization of EoS and three-year WMAP data, SN Ia data, BAO gives a result of ξ~-0.04

Explicit report of ξ~Ωm0 result.

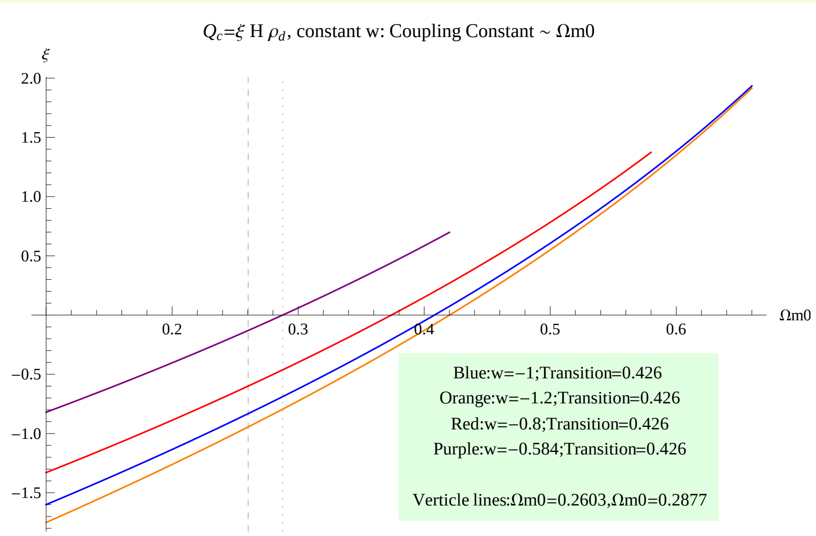
```
perI2CC = 0.05;
tabξI2CCSum =
 Grid[
  {{"For Ωm0∈0.274" (1 ± perI2CC)
     Style["Table of ξ for different Ωm0~Transition combination",
      Bold], SpanFromLeft, SpanFromLeft},
   {Style["Ωm0⋱Transition", Small, Bold], 0.426, 0.376, 0.508},
   {0.274 (1 - perI2CC), ξI2CCffunc[0.274 (1 - perI2CC),
     1 - 0.274 (1 - perI2CC) - 0, -1, 0.426],
    ξI2CCffunc[0.274 (1 - perI2CC), 1 - 0.274 (1 - perI2CC) - 0,
     -1, 0.376], ξI2CCffunc[0.274 (1 - perI2CC),
     1 - 0.274 (1 - perI2CC) - 0, -1, 0.508]},
   {0.274, ξI2CCffunc[0.274, 1 - 0.274 - 0, -1, 0.426],
    ξI2CCffunc[0.274, 1 - 0.274 - 0, -1, 0.376],
    ξI2CCffunc[0.274, 1 - 0.274 - 0, -1, 0.508]},
   {0.274 (1 + perI2CC), ξI2CCffunc[0.274 (1 + perI2CC),
     1 - 0.274 (1 + perI2CC) - 0, -1, 0.426],
    ξI2CCffunc[0.274 (1 + perI2CC), 1 - 0.274 (1 + perI2CC) - 0,
     -1, 0.376], ξI2CCffunc[0.274 (1 + perI2CC),
     1 - 0.274 (1 + perI2CC) - 0, -1, 0.508]}}, Frame → All,
  Background → {{LightGray, None}, {LightGreen, LightGray, None}},
  Alignment → Center, ItemSize → 8]
```

For Ωm0∈0.274 (1 ± 0.05) **Table of ξ for different Ωm0~Transition combination**			
Ωm0⋱Transition	0.426	0.376	0.508
0.2603	−0.832284	−1.07758	−0.53584
0.274	−0.760999	−1.00068	−0.471298
0.2877	−0.688664	−0.922602	−0.40585

```
pltξvΩm0I2CCSum =
 Plot[{ξI2CCffunc[Ωm0ICC, 1 - Ωm0ICC - 0, -1, 0.426],
   ξI2CCffunc[Ωm0ICC, 1 - Ωm0ICC - 0, -1, 0.376],
   ξI2CCffunc[Ωm0ICC, 1 - Ωm0ICC - 0, -1, 0.508]}, {Ωm0ICC, 0.1, 0.66},
  PlotStyle → {Blue, Orange, Red}, AxesLabel → {"Ωm0", "ξ"},
  GridLines →
   {{{0.2603, Directive[Gray, Dashed]},
     {0.2877`, Directive[Gray, Dotted]}},
    {{-1.0776, Directive[Orange, Dashed]},
     {-0.8323, Directive[Blue, Dashed]},
     {-0.5358, Directive[Red, Dashed]},
     {-0.9226, Directive[Orange, Dotted]},
     {-0.6887, Directive[Blue, Dotted]},
     {-0.4059, Directive[Red, Dotted]}}},
  Epilog →
   Inset[
    Framed[
     Style[
      "Orange:w=-1;Transition=0.376\n Blue:w=-1;Transition=0.426\n
        Red:w=-1;Transition=0.508\n \n Verticle
        lines:Ωm0=0.2603,Ωm0=0.2877", 11],
     Background → LightGreen, FrameStyle → None], {0.35, -1.15},
    {Left, Top}],
  PlotLabel → "Qc=ξ H ρd, constant w: Coupling Constant ~ Ωm0",
  ImageSize → 500]
```



```
pltξvΩm0I2CCSum2 =
 Show[
   {Plot[{ξI2CCffunc[Ωm0I2CC, 1 - Ωm0I2CC - 0, -1, 0.426],
      ξI2CCffunc[Ωm0I2CC, 1 - Ωm0I2CC - 0, -1.2, 0.426]},
     {Ωm0I2CC, 0.1, 0.66}, PlotStyle → {Blue, Orange},
     AxesLabel → {"Ωm0", "ξ"}],
    Plot[ξI2CCffunc[Ωm0I2CC, 1 - Ωm0I2CC - 0, -0.8, 0.426],
     {Ωm0I2CC, 0.1, 0.58}, PlotStyle → Red, AxesLabel → {"Ωm0", "ξ"}],
    Plot[ξI2CCffunc[Ωm0I2CC, 1 - Ωm0I2CC - 0,
      eosI2CC1 /.
       FindRoot[{ξI2CCffunc[0.2877, 1 - 0.2877 - 0, eosI2CC1, 0.426] ==
          0}, {eosI2CC1, -0.5}], 0.426], {Ωm0I2CC, 0.1, 0.42},
     PlotStyle → Purple, AxesLabel → {"Ωm0", "ξ"}]},
   GridLines →
    {{{0.2603, Directive[Gray, Dashed]},
      {0.2877`, Directive[Gray, Dotted]}}, {}},
   Epilog →
    Inset[
     Framed[
      Style[
       "Blue:w=-1;Transition=0.426\n
         Orange:w=-1.2;Transition=0.426\n
         Red:w=-0.8;Transition=0.426\n
         Purple:w=-0.584;Transition=0.426\n \n
         Verticle lines:Ωm0=0.2603,Ωm0=0.2877", 11],
      Background → LightGreen, FrameStyle → None], {0.38, -0.3},
     {Left, Top}],
   PlotLabel → "Qc=ξ H ρd, constant w: Coupling Constant ~ Ωm0",
   ImageSize → 500] // Quiet
```



Different plot ranges are chosen because for very large Ωm0, it is impossible to choose a parameter so that the transition redshift is

```
tabξvΩm0I2CCSum21 =
 Grid[
   {{"Qc=ξ H ρd, constant w: ξ when transition redshift is 0.426",
     SpanFromLeft}, {"⋱", "w=-0.4506", "w=-0.4507"},
    {"Ωm0=0.2603", ξI2CCffunc[0.2603, 1 - 0.2603 - 0, -0.4506, 0.426],
     ξI2CCffunc[0.2603, 1 - 0.2603 - 0, -0.4507, 0.426]}},
   Frame → All,
   Background → {{LightGray, None}, {LightGreen, LightGray, None}},
   ItemSize → 13] // Quiet
tabξvΩm0I2CCSum22 =
 Grid[{{"Qc=ξ H ρd, constant w: EoS value when ξ=0", SpanFromLeft},
    {"⋱", "Transition 0.426"},
    {"Ωm0=0.2877",
     eosI2CC1 /.
      FindRoot[{ξI2CCffunc[0.2877, 1 - 0.2877 - 0, eosI2CC1, 0.426] ==
          0}, {eosI2CC1, -0.5}]}}, Frame → All, ItemSize → 13,
   Background → {{LightGray, None}, {LightGreen, LightGray, None}}] //
  Quiet
```

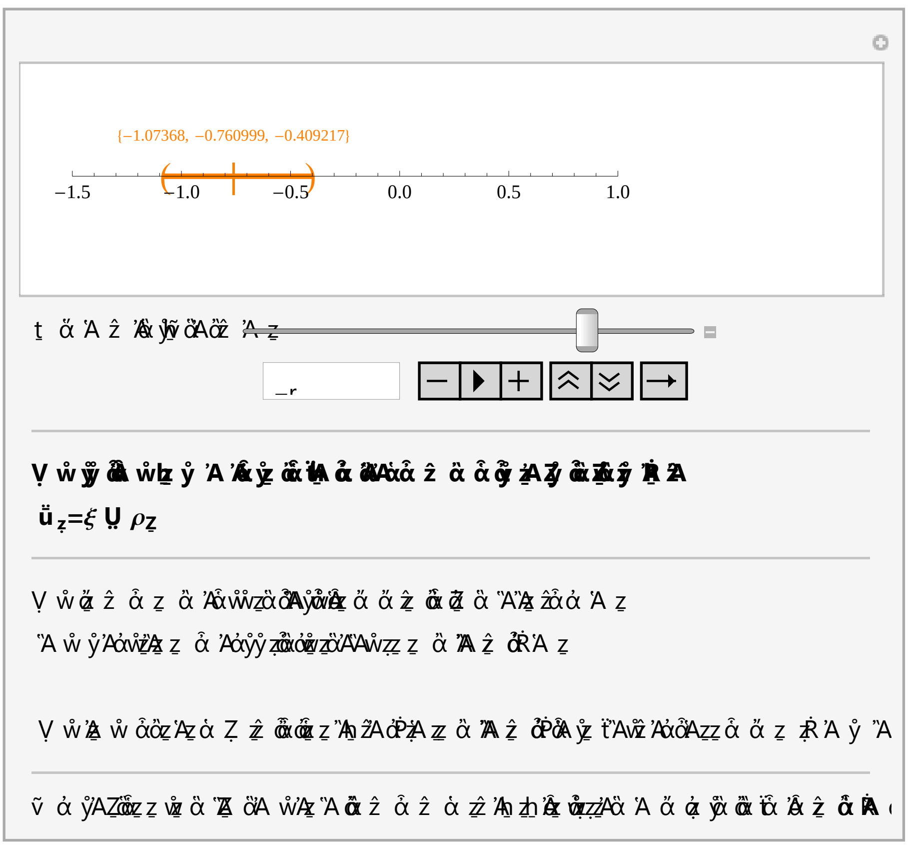
$Q_c = \xi$ H ρ_d, constant w: ξ when transition redshift is 0.426		
⋱	w=−0.4506	w=−0.4507
Ωm0=0.2603	−17.5369	0.351603

$Q_c = \xi$ H ρ_d, constant w: EoS value when ξ=0	
⋱	Transition 0.426
Ωm0=0.2877	−0.58406

So we give the result that w ∈ (-0.58406, -0.4507) if we constrain Ωm0 = 0.2603 and transition redshift 0.426.

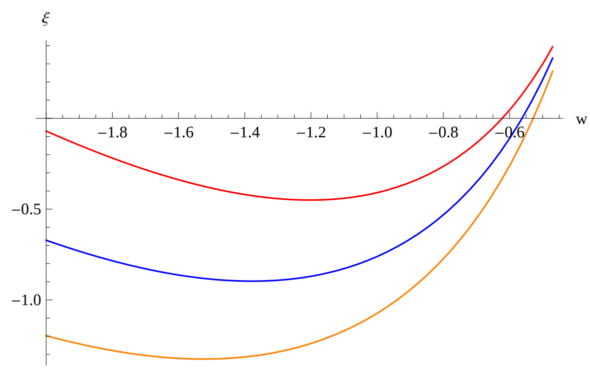
Fitting result of ξ with different EoS.

```
fitξI2CCManSum =
 Manipulate[
  numPlot["(", {ξI2CCf1[wI2CC], ξI2CCfc[wI2CC], ξI2CCf2[wI2CC]},
   ")", {-1.5, 1}], {{wI2CC, -1, "Equation of State"}, -3,
   -0.47, Appearance → "Open"}, Delimiter,
  Style[
   "This is the fitting result from transition redshift
     data.\n Qc=ξ H ρd", Bold], Delimiter,
  "The parenthesis shows the upper and lower value \n while
    the verticle line show the center value.",
  Style[
   "\n The three numbers are left value, center value,
     right value respectively."], Delimiter, Delimiter,
  Style[
   "Slide to see how do the two parameters affect the
     coupling constant results."],
  ControlPlacement → {Bottom, Bottom}, SaveDefinitions → True]
```



Coupling constant ξ vs EoS

```
pltξvwI2CC = Plot[{ξI2CCfc[wI2CC], ξI2CCf1[wI2CC], ξI2CCf2[wI2CC]},
  {wI2CC, -2, -0.47}, PlotStyle → {Blue, Orange, Red},
  AxesLabel → {"w", "ξ"}]
```



Choose the values from Reference 3. w = -1.087 ± 0.096,

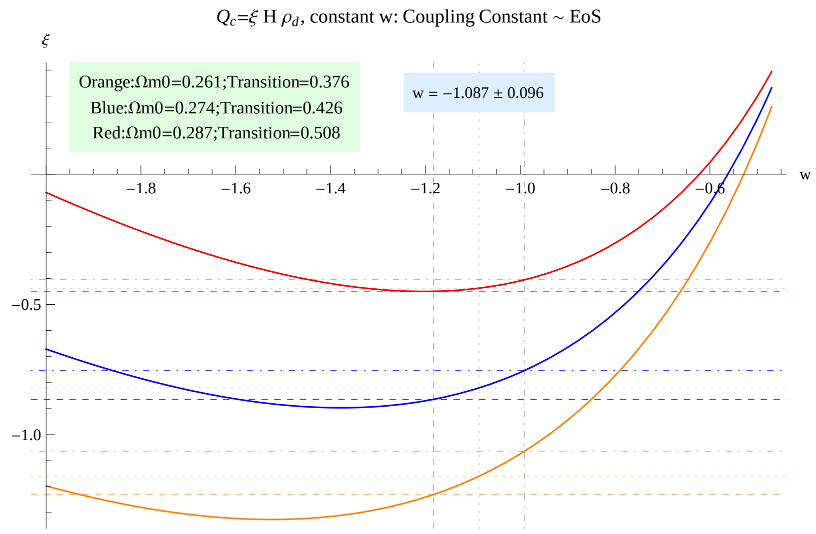
```
ξvwExamI2CC =
 {Block[{wI2CC = -1.087 - 0.096},
   {wI2CC, ξI2CCfc[wI2CC], ξI2CCf1[wI2CC], ξI2CCf2[wI2CC]}],
  Block[{wI2CC = -1.087},
   {wI2CC, ξI2CCfc[wI2CC], ξI2CCf1[wI2CC], ξI2CCf2[wI2CC]}],
  Block[{wI2CC = -1.087 + 0.096},
   {wI2CC, ξI2CCfc[wI2CC], ξI2CCf1[wI2CC], ξI2CCf2[wI2CC]}]}
```

```
{{-1.183, -0.864289, -1.22984, -0.449552},
 {-1.087, -0.820486, -1.15946, -0.437339},
 {-0.991, -0.753634, -1.06346, -0.405262}}
```

```
tabξvwExamI2CC =
 Grid[
  Prepend[Prepend[ξvwExamI2CC, {"w", "Center", "Lower", "Upper"}],
   {"Qc=ξ H ρd,Constant w. (Data used:Data From, 2)",
    SpanFromLeft}], Frame → All,
  Background → {{LightGray, None}, {LightGreen, LightGray, None}},
  Alignment → Center, ItemSize → 8]
```

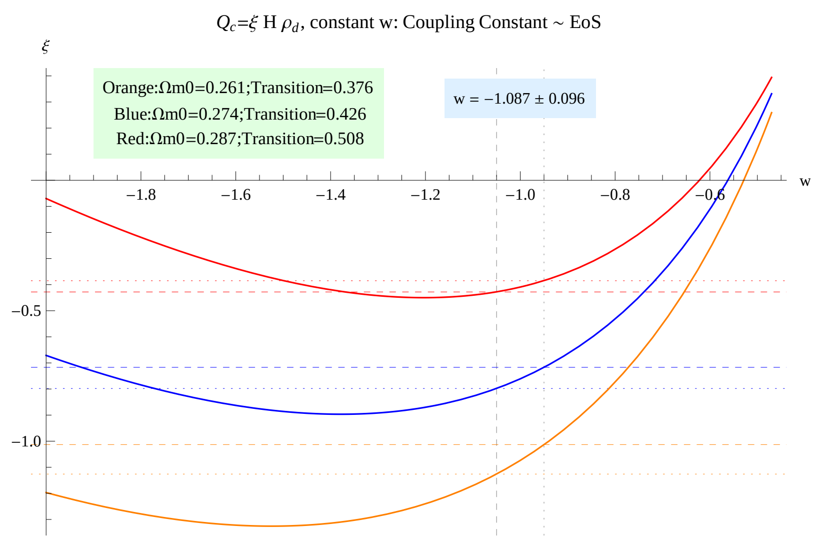
$Q_c=\xi\ H\ \rho_d$,Constant w. (Data used:Data From, 2)			
w	Center	Lower	Upper
−1.183	−0.864289	−1.22984	−0.449552
−1.087	−0.820486	−1.15946	−0.437339
−0.991	−0.753634	−1.06346	−0.405262

```
pltξvwExamI2CC =
 Show[pltξvwI2CC,
  GridLines →
   {{{ξvwExamI2CC[[1, 1]], Directive[Gray, Dashed]},
     {ξvwExamI2CC[[2, 1]], Directive[Gray, Dotted]},
     {ξvwExamI2CC[[3, 1]], Directive[Gray, DotDashed]}},
    {{ξvwExamI2CC[[1, 3]], Directive[Orange, Dashed]},
     {ξvwExamI2CC[[1, 2]], Directive[Blue, Dashed]},
     {ξvwExamI2CC[[1, 4]], Directive[Red, Dashed]},
     {ξvwExamI2CC[[2, 3]], Directive[Orange, Dotted]},
     {ξvwExamI2CC[[2, 2]], Directive[Blue, Dotted]},
     {ξvwExamI2CC[[2, 4]], Directive[Red, Dotted]},
     {ξvwExamI2CC[[3, 3]], Directive[Orange, DotDashed]},
     {ξvwExamI2CC[[3, 2]], Directive[Blue, DotDashed]},
     {ξvwExamI2CC[[3, 4]], Directive[Red, DotDashed]}}},
  Epilog →
   {Inset[Framed[Style["w = -1.087 ± 0.096", 10],
      Background → LightBlue, FrameStyle → None], {-1.087, 0.4},
     {0, Top}],
    Inset[
     Framed[
      Style[
       "Orange:Ωm0=0.261;Transition=0.376\n
         Blue:Ωm0=0.274;Transition=0.426\n
         Red:Ωm0=0.287;Transition=0.508", 11],
      Background → LightGreen, FrameStyle → None], {-1.95, 0.44},
     {Left, Top}]},
  PlotLabel → "Qc=ξ H ρd, constant w: Coupling Constant ~ EoS",
  ImageSize → 500]
```



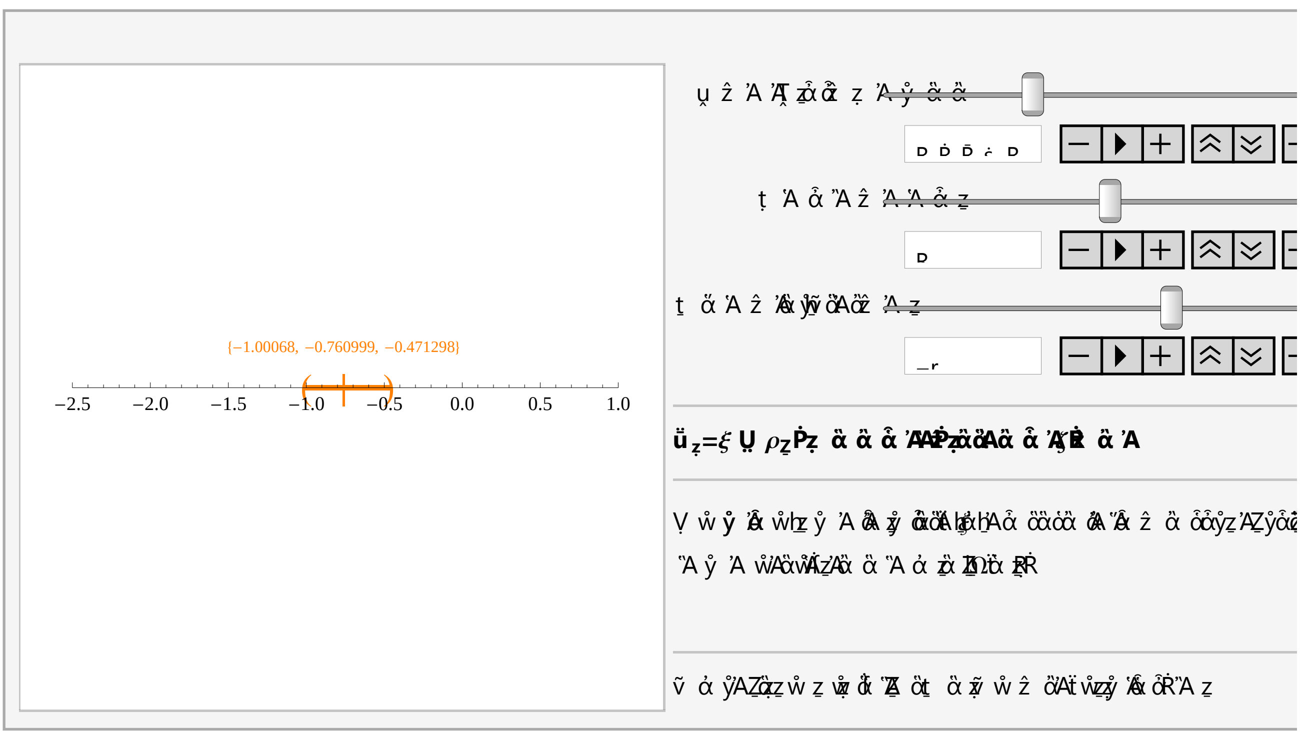
Or more casually, use w=-1±0.05

```
pltξvwExam1I2CC =
 Show[pltξvwI2CC,
  GridLines →
   {{{-1.05, Directive[Gray, Dashed]},
     {-0.95, Directive[Gray, Dotted]}},
    {{-1.013, Directive[Orange, Dashed]},
     {-0.717, Directive[Blue, Dashed]},
     {-0.428, Directive[Red, Dashed]},
     {-1.126, Directive[Orange, Dotted]},
     {-0.798, Directive[Blue, Dotted]},
     {-0.385, Directive[Red, Dotted]}}},
  Epilog →
   {Inset[Framed[Style["w = -1.087 ± 0.096", 10],
       Background → LightBlue, FrameStyle → None], {-1, 0.4},
     {0, Top}],
    Inset[
     Framed[
      Style[
       "Orange:Ωm0=0.261;Transition=0.376\n
         Blue:Ωm0=0.274;Transition=0.426\n
         Red:Ωm0=0.287;Transition=0.508", 11],
      Background → LightGreen, FrameStyle → None], {-1.9, 0.44},
     {Left, Top}]},
  PlotLabel → "Qc=ξ H ρd, constant w: Coupling Constant ~ EoS",
  ImageSize → 500]
```

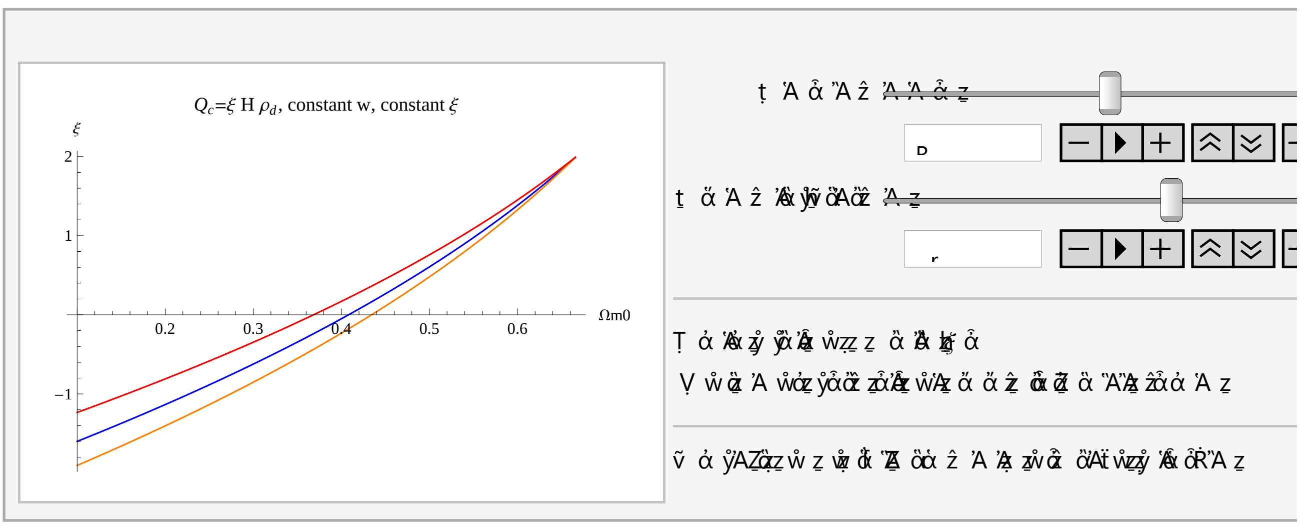


If we use transition redshift data only,

```
fitξ2I2CCManSum =
 Manipulate[
  numPlot["(",
   {ξI2CCffunc[Ωm0I2CC, 1 - Ωm0I2CC - Ωk0I2CC, wI2CC, 0.376],
    ξI2CCffunc[Ωm0I2CC, 1 - Ωm0I2CC - Ωk0I2CC, wI2CC, 0.426],
    ξI2CCffunc[Ωm0I2CC, 1 - Ωm0I2CC - Ωk0I2CC, wI2CC, 0.508]},
   ")", {-2.5, 1}], {{Ωm0I2CC, 0.274, "Matter Fraction"}, 0.1,
   0.66, Appearance → "Open"},
  {{Ωk0I2CC, 0, "Curvature"}, -0.1, 0.1, Appearance → "Open"},
  {{wI2CC, -1, "Equation of State"}, -2, -0.47, Appearance → "Open"},
  Delimiter, Style["Qc=ξ H ρd, constant w, constant ξ.", Bold],
  Delimiter,
  Style[
   "This is the fitting result of ξ from only transition
     redshift data.\n without the knowledge of Ωm0. "],
  Delimiter, Style["Slide to check how do EoS change this curve."],
  ControlPlacement → {Right}, SaveDefinitions → True]
```



```
pltξvΩm0I2CCManSum =
 Manipulate[
  Plot[{ξI2CCffunc[Ωm0I2CC, 1 - Ωm0I2CC - Ωk0I2CC, wI2CC, 0.426],
    ξI2CCffunc[Ωm0I2CC, 1 - Ωm0I2CC - Ωk0I2CC, wI2CC, 0.376],
    ξI2CCffunc[Ωm0I2CC, 1 - Ωm0I2CC - Ωk0I2CC, wI2CC, 0.508]},
   {Ωm0I2CC, 0.1,
    (tmpe /. FindRoot[ztrI2CC[tmpe, 1 - tmpe - Ωk0I2CC, wI2CC, 0.1] == 0,
        {tmpe, 0.1}]) - 0.001}, PlotStyle → {Blue, Orange, Red},
   AxesLabel → {"Ωm0", "ξ"},
   PlotLabel → "Qc=ξ H ρd, constant w, constant ξ"],
  {{Ωk0I2CC, 0, "Curvature"}, -0.1, 0.1, Appearance → "Open"},
  {{wI2CC, -1, "Equation of State"}, -2, -0.47, Appearance → "Open"},
  Delimiter,
  Style[
   "Blue line is the center of ξ\n The other lines are the
     upper and lower value"], Delimiter,
  Style["Slide to check how do matter change this curve."],
  ControlPlacement → {Right}, SaveDefinitions → True]
```



- Check consistancy

```
{ξI2CCffunc[0.263623, 1 - 0.263623, -1, 0.376],
 ξI2CCffunc[0.274310595065312`, 1 - 0.274310595065312`, -1, 0.426],
 ξI2CCffunc[0.28469241773962806`, 1 - 0.28469241773962806`,
  -1, 0.508]}
```

```
{-1.05903, -0.759371, -0.420298}
```

Constant ξ + CPL w. I2CPL model.

- Definations

CPL EoS $w = \text{w0} + \text{w1}\,\dfrac{z}{1+z}$

In the following calculation, some complex numbers will occur. But

Prepare

```
hubblecmpintI2CCPL[w0_, w1_, ξI2CCPL_, z_?NumberQ] :=
  Integrate[Exp[3 w1/(1 + tmp) - 3 w1] (1 + tmp)^(3 (w0+w1)+ξI2CCPL-1), {tmp, 0, z},
   Assumptions → {tmp ∈ Reals && z ∈ Reals && z > -1}]
```

```
hubblecmpintI2CCPLtest[w0_, w1_, ξI2CCPL_, z_?NumberQ] :=
  NIntegrate[Exp[3 w1/(1 + tmp) - 3 w1] (1 + tmp)^(3 (w0+w1)+ξI2CCPL-1), {tmp, 0, z}]
```

```
hubblecmpintI2CCPL[-1, 0.1, 0.1, 10] // Timing
Re[%]
hubblecmpintI2CCPLtest[-1, 0.1, 0.1, 10] // Timing
```

$\{0.921,\ 0.353978 - 2.63192\times 10^{-15}\ i\}$

{0.921, 0.353978}

{0.015, 0.353978}

Matter density

```
ΩmI2CCPL[Ωd0I2CCPL_, Ωm0I2CCPL_, w0I2CCPL_, w1I2CCPL_, ξI2CCPL_, z_] =
  Ωm0I2CCPL (1 + z)^3 - ξI2CCPL Ωm0I2CCPL (1 + z)^3
    hubblecmpintI2CCPL[w0I2CCPL, w1I2CCPL, ξI2CCPL, z];
```

```
ΩmI2CCPLtest[Ωd0I2CCPL_, Ωm0I2CCPL_, w0I2CCPL_, w1I2CCPL_,
   ξI2CCPL_, z_?NumberQ] =
  Ωm0I2CCPL (1 + z)^3 - ξI2CCPL Ωm0I2CCPL (1 + z)^3
    hubblecmpintI2CCPLtest[w0I2CCPL, w1I2CCPL, ξI2CCPL, z];
```

```
ΩdI2CCPL[Ωd0I2CCPL_, w0I2CCPL_, w1I2CCPL_, ξI2CCPL_, z_] :=
 Ωd0I2CCPL Exp[-3 z w1I2CCPL/(1 + z)] (1 + z)^(3 (1+w0I2CCPL+w1I2CCPL)+ξI2CCPL)
```

Hubble function.

```
hubbleI2CCPL[H0I2CCPL_, Ωd0I2CCPL_, Ωm0I2CCPL_, w0I2CCPL_,
   w1I2CCPL_, ξI2CCPL_, z_] =
  H0I2CCPL
   √(ΩmI2CCPL[Ωd0I2CCPL, Ωm0I2CCPL, w0I2CCPL, w1I2CCPL, ξI2CCPL, z] +
      ΩdI2CCPL[Ωd0I2CCPL, w0I2CCPL, w1I2CCPL, ξI2CCPL, z]);
```

```
hubbleI2CCPLtest[H0I2CCPL_, Ωd0I2CCPL_, Ωm0I2CCPL_, w0I2CCPL_,
   w1I2CCPL_, ξI2CCPL_, z_?NumberQ] =
  H0I2CCPL
   √(ΩmI2CCPLtest[Ωd0I2CCPL, Ωm0I2CCPL, w0I2CCPL, w1I2CCPL,
       ξI2CCPL, z] + ΩdI2CCPL[Ωd0I2CCPL, w0I2CCPL, w1I2CCPL,
       ξI2CCPL, z]);
```

```
hubbleDI2CCPL[H0I2CCPL_, Ωd0I2CCPL_, Ωm0I2CCPL_, w0I2CCPL_,
   w1I2CCPL_, ξI2CCPL_, z_] =
  D[hubbleI2CCPL[H0I2CCPL, Ωd0I2CCPL, Ωm0I2CCPL, w0I2CCPL,
    w1I2CCPL, ξI2CCPL, z], z];
```

```
hubbleDI2CCPLtest[H0I2CCPL_, Ωd0I2CCPL_, Ωm0I2CCPL_, w0I2CCPL_,
   w1I2CCPL_, ξI2CCPL_, z_] =
  (hubbleI2CCPLtest[H0I2CCPL, Ωd0I2CCPL, Ωm0I2CCPL, w0I2CCPL,
      w1I2CCPL, ξI2CCPL, z + 0.001] -
     hubbleI2CCPLtest[H0I2CCPL, Ωd0I2CCPL, Ωm0I2CCPL, w0I2CCPL,
      w1I2CCPL, ξI2CCPL, z - 0.001]) / 0.002;
```

```
hubbleI2CCPL[H0w, 0.7, 0.3, -1, 0.1, 0.1, 10] // Timing
Re[%]
hubbleI2CCPLtest[H0w, 0.7, 0.3, -1, 0.1, 0.1, 10] // Timing
hubbleDI2CCPL[H0w, 0.7, 0.3, -1, 0.1, 0.1, 10] // Timing
Re[%]
hubbleDI2CCPLtest[H0w, 0.7, 0.3, -1, 0.1, 0.1, 10] // Timing
```

$\{0.656, 1395.93 + 1.89755 \times 10^{-13}\, i\}$

{0.656, 1395.93}

{0.015, 1395.93}

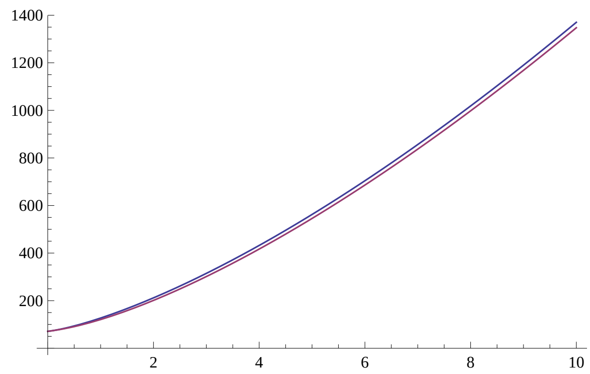
$\{44.336, 189.745 + 2.59585 \times 10^{-14}\, i\}$

{44.336, 189.745}

{0.031, 189.745}

Check whether this hubble function is reasonable.

```
Plot[{hubbleI2CCPLtest[H0w, 0.73, 0.27, -1, 0.6, -0.02, z],
  hubble[Ωm0w, Ωd0w, 0, z]}, {z, 0, 10}]
```



Deceleration parameter

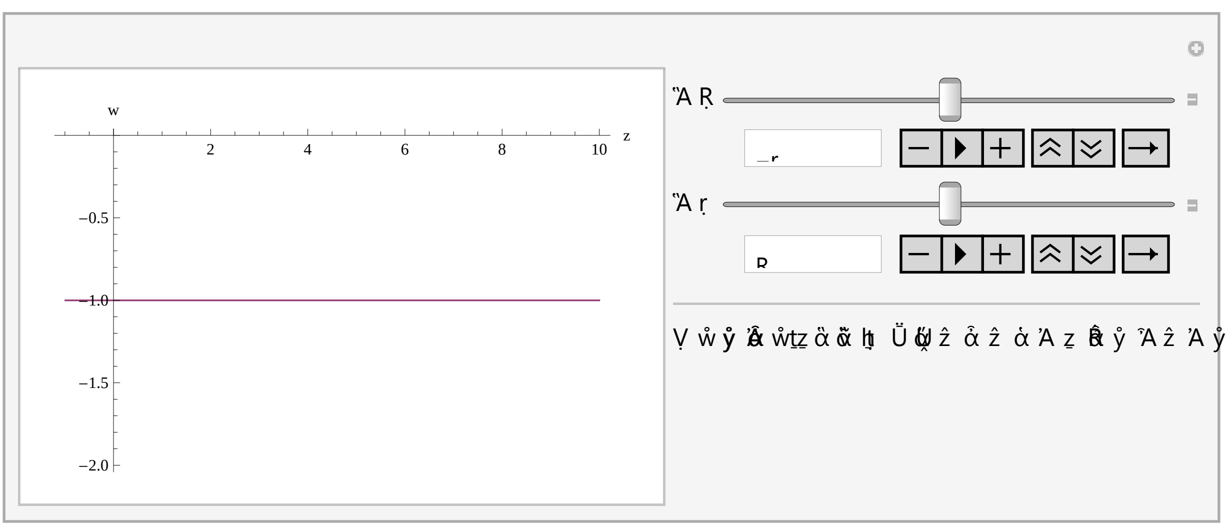
```
qI2CCPL[H0I2CCPL_, Ωd0I2CCPL_, Ωm0I2CCPL_, w0I2CCPL_, w1I2CCPL_,
   ξI2CCPL_, z_] =
  -1 +
   (1 + z) / hubbleI2CCPLtest[H0I2CCPL, Ωd0I2CCPL, Ωm0I2CCPL,
      w0I2CCPL, w1I2CCPL, ξI2CCPL, z]
    hubbleDI2CCPLtest[H0I2CCPL, Ωd0I2CCPL, Ωm0I2CCPL, w0I2CCPL,
     w1I2CCPL, ξI2CCPL, z];
```

```
qI2CCPL[H0w, 0.7, 0.3, -1, 0.1, 0.1, 10] // Timing
```

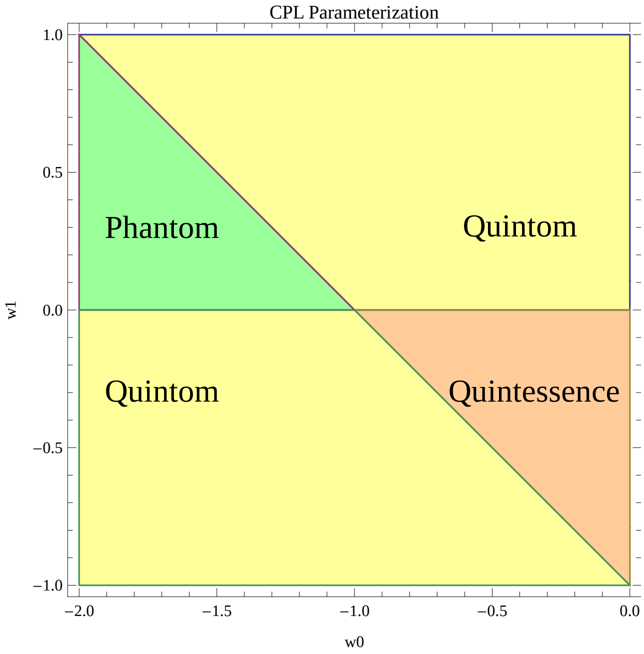
```
{0.031, 0.495197}
```

Equation of State

```
plEoSI2CCPLMan = plEoSICCPLMan
```



```
plEoSPhaseI2CCPL = plEoSPhaseICCPL
```

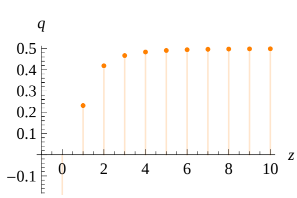


Plot deceleration parameter and

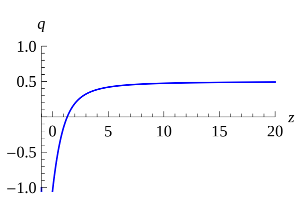
```
pldecI2CCPL[Ωm0I2CCPL_, ξI2CCPL_, w0I2CCPL_, w1I2CCPL_, color_] :=
  Plot[qI2CCPL[H0w, 1 - Ωm0I2CCPL, Ωm0I2CCPL, w0I2CCPL, w1I2CCPL,
    ξI2CCPL, z], {z, -1, 20}, PlotRange → {{-1.05, 20}, {-1.05, 1}},
   PlotStyle → color, AxesOrigin → {-1, 0}, AxesLabel → {z, q}];
```

```
discretepldecI2CCPL[Ωm0I2CCPL_, ξI2CCPL_, w0I2CCPL_, w1I2CCPL_,
  color_] :=
 DiscretePlot[qI2CCPL[H0w, 1 - Ωm0I2CCPL, Ωm0I2CCPL, w0I2CCPL,
   w1I2CCPL, ξI2CCPL, z], {z, -1, 10, 1}, PlotStyle → color,
  AxesOrigin → {-1, 0}, AxesLabel → {z, q}]
```

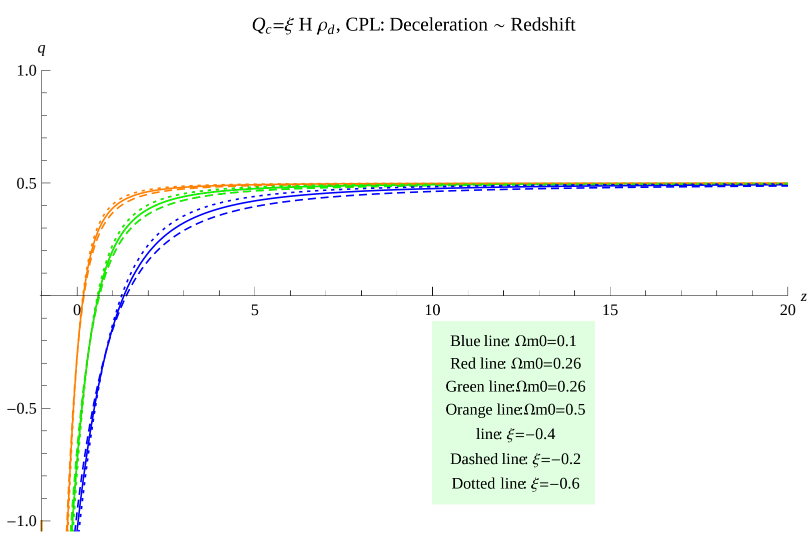
```
discretepldecI2CCPL[0.3, -0.4, -1, 0.1, Orange] // Timing //
 Quiet
```

{0.515,  }


```
pldecI2CCPL[0.1, -0.4, -1.02, 0.60, Blue] // Timing // Quiet
```

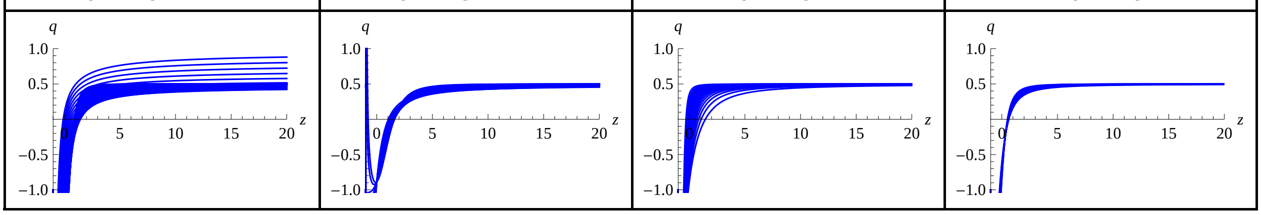
{9.391,  }

```
pldecI2CCPLShowSum =
 Show[{pldecI2CCPL[0.1, -0.4, -1.02, 0.60, Blue],
    pldecI2CCPL[0.26, -0.4, -1.02, 0.60, Red],
    pldecI2CCPL[0.26, -0.4, -1.02, 0.60, Green],
    pldecI2CCPL[0.5, -0.4, -1.02, 0.60, Orange],
    pldecI2CCPL[0.1, -0.2, -1.02, 0.60, Directive[Blue, Dashed]],
    pldecI2CCPL[0.26, -0.2, -1.02, 0.60, Directive[Red, Dashed]],
    pldecI2CCPL[0.26, -0.2, -1.02, 0.60, Directive[Green, Dashed]],
    pldecI2CCPL[0.5, -0.2, -1.02, 0.60, Directive[Orange, Dashed]],
    pldecI2CCPL[0.1, -0.6, -1.02, 0.60, Directive[Blue, Dotted]],
    pldecI2CCPL[0.26, -0.6, -1.02, 0.60, Directive[Red, Dotted]],
    pldecI2CCPL[0.26, -0.6, -1.02, 0.60, Directive[Green, Dotted]],
    pldecI2CCPL[0.5, -0.6, -1.02, 0.60, Directive[Orange, Dotted]]},
   Epilog →
    Inset[
     Framed[
      Style[
       "Blue line: Ωm0=0.1\n Red line: Ωm0=0.26\n Green
         line:Ωm0=0.26\n Orange line:Ωm0=0.5\n
         line: ξ=-0.4\n Dashed line: ξ=-0.2\n Dotted
         line: ξ=-0.6", 10], Background → LightGreen,
      FrameStyle → None], {10, -0.1}, {Left, Top}],
   PlotLabel → "Qc=ξ H ρd, CPL: Deceleration ~ Redshift",
   ImageSize → 500] // Quiet
```



```
varyingI2CCPLShowSum =
 Grid[{{"Qc=ξ H ρd, CPL", SpanFromLeft},
    {"Varying CPL w0", "Varying CPL w1", "Varying Ωm0",
     "Varying ξ"},
    {Show[Table[pldecI2CCPL[0.1, -0.02, w0I2CCPL, 0.6, Blue],
       {w0I2CCPL, -2, -0.3, 0.05}]],
     Show[Table[pldecI2CCPL[0.1, -0.02, -1.02, w1I2CCPL, Blue],
       {w1I2CCPL, -0.2, 1, 0.1}]],
     Show[Table[pldecI2CCPL[Ωm0I2CCPL, -0.02, -1.02, 0.6, Blue],
       {Ωm0I2CCPL, 0.1, 0.9, 0.05}]],
     Show[Table[pldecI2CCPL[0.27, ξI2CCPL, -1.02, 0.6, Blue],
       {ξI2CCPL, -1, 0, 0.1}]]}}, Frame → All,
   Background → {{None}, {LightGreen, None}}] // Quiet
```

$Q_c=\xi\ H\ \rho_d$, CPL			
Varying CPL w0	Varying CPL w1	Varying Ωm0	Varying ξ




```
(* Do not run the following if not needed. *)
(*
pldecI2CCPLManSum=
 Manipulate[
   Show[{pldecI2CCPL[Ωm0I2CCPL,ξI2CCPL,w0I2CCPL,w1I2CCPL,Orange],
     pldec[Ωm0I2CCPL,{Pink,Thick}]},
    PlotLabel→"Qc=ξ H ρd,CPL: Deceleration ~ Redshift"],
   {{Ωm0I2CCPL,0.26,"Matter Fraction"},0,1,Appearance→"Open"},
   {{ξI2CCPL,-0.4,"Interaction"},-1,0,Appearance→"Open"},
   {{w0I2CCPL,-1.02,"CPL EoS w0"},-2,-0.3,Appearance→"Open"},
   {{w1I2CCPL,0.6,"CPL EoS w1"},-0.2,1,Appearance→"Open"},
   Delimiter,Style["Pink is the deceleration parameter for LCDM.",
    Medium],
   Style["Orange is for interacting CPL model with Q=ξ H ρd",
    Medium],ControlPlacement→{Right,Right,Right},
   SaveDefinitions→True]//Quiet
*)
```

Transition Redshift

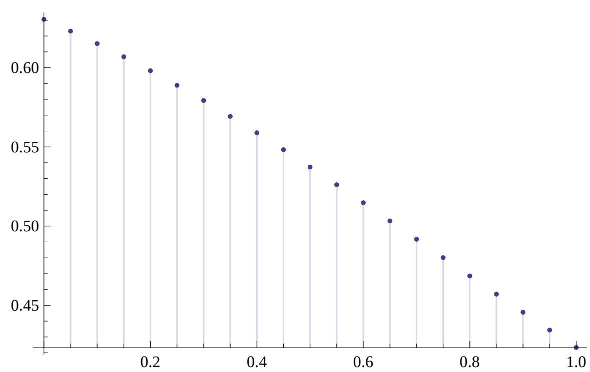
Find out the expression for transition redshift

```
ztrI2CCPL[Ωm0I2CCPL_, Ωd0I2CCPL_, ξI2CCPL_, w0I2CCPL_, w1I2CCPL_] :=
  z /.
   FindRoot[
    (1 + 3 (w0I2CCPL + w1I2CCPL z/(1 + z)))
       ΩdI2CCPL[Ωd0I2CCPL, w0I2CCPL, w1I2CCPL, ξI2CCPL, z] +
     ΩmI2CCPLtest[Ωd0I2CCPL, Ωm0I2CCPL, w0I2CCPL, w1I2CCPL,
       ξI2CCPL, z] == 0, {z, 3}];
```

```
ztrI2CCPL[0.3, 1 - 0.3, 0, -1, 0.1] // Timing
```

```
{0.203, 0.655139}
```

```
DiscretePlot[{ztrI2CCPL[0.3, 1 - 0.3, -0.1, -1, tmp]},
  {tmp, 0, 1, 0.05}] // Quiet
```



The following is just used for test.

```
(*
(*This is a test of how well are the findroot results.*)
Plot[{plottestfunctiontemp[temp,1-temp,-0.1,-0.9,-0.05,
    ztrICCPLtest2[temp,1-temp,-0.1,-0.9,-0.05]],
  plottestfunctiontemp[temp,1-temp,-0.1,-0.9,-0.05,
    ztrICCPLtest[temp,1-temp,-0.1,-0.9,-0.05]],
  plottestfunctiontemp[temp,1-temp,-0.1,-0.9,-0.05,
    ztrICCPL[temp,1-temp,-0.1,-0.9,-0.05]]},{temp,0,1},
 PlotStyle→{Red,Orange,Blue}]
*)
```

Define rICC = $\frac{\Omega m0ICC}{\Omega d0ICC}$

```
ΩmrI2CCPL[rI2CCPL_, w0I2CCPL_, w1I2CCPL_, ξI2CCPL_, z_] =
  rI2CCPL (1 + z)^3 - ξI2CCPL rI2CCPL (1 + z)^3
    Re[hubblecmpintI2CCPLtest[w0I2CCPL, w1I2CCPL, ξI2CCPL, z]];
```

```
ΩdrI2CCPL[rI2CCPL_, w0I2CCPL_, w1I2CCPL_, ξI2CCPL_, z_] :=
  Exp[-3 w1I2CCPL] Exp[3 w1I2CCPL/(1 + z)] (1 + z)^(3 (1+w0I2CCPL+w1I2CCPL) +ξI2CCPL);
```

```
ztrrI2CCPL[rI2CCPL_, ξI2CCPL_, w0I2CCPL_, w1I2CCPL_] :=
  z /.
   FindRoot[
    (1 + 3 (w0I2CCPL + w1I2CCPL z/(1 + z)))
       ΩdrI2CCPL[rI2CCPL, w0I2CCPL, w1I2CCPL, ξI2CCPL, z] +
      ΩmrI2CCPL[rI2CCPL, w0I2CCPL, w1I2CCPL, ξI2CCPL, z] == 0, {z, 3}];
```

Test this.

```
ztrrI2CCPL[0.5, -0.02, -1, 0.6] // Timing
```

```
{0.203, 0.468557}
```

Visualization

Check the behavior of this transition redshift.

```
ztrI2CCPL[0.1, 1 - 0.1, -0.02, -1, 0.2] // Timing
```

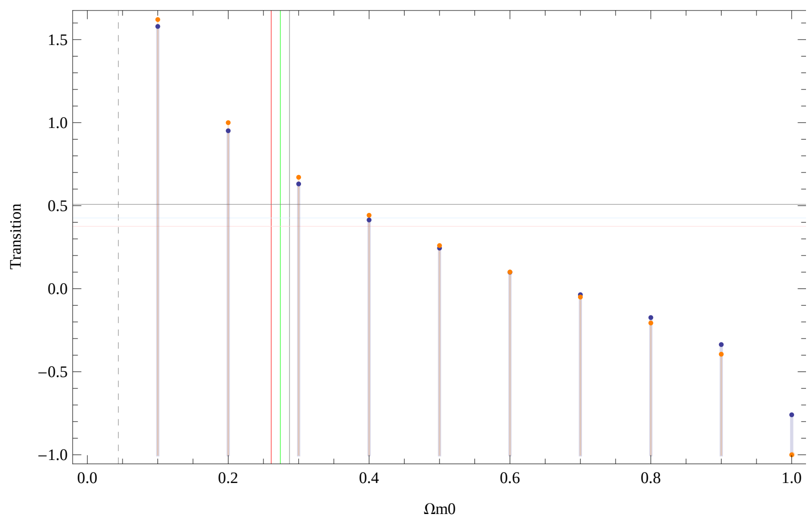
```
{0.171, 1.59839}
```

```
ztrI2CCPL[0.25, 1 - 0.25, -0.02, -1, 0.2] // Timing
ztrI2CCPL[0.25, 1 - 0.25, 0.02, -1, 0.2] // Timing
```

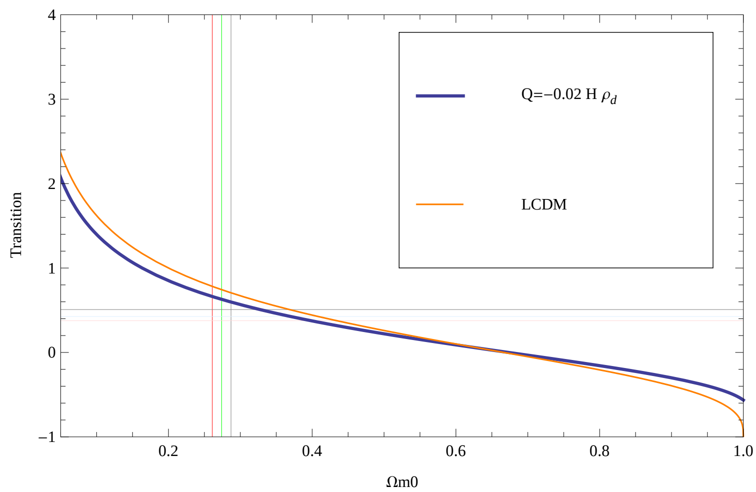
```
{0.172, 0.772156}
```

```
{0.218, 0.793023}
```

```
discretepldecΩm0I2CCPL =
  DiscretePlot[{ztrI2CCPL[Ωm0I2CCPL, 1 - Ωm0I2CCPL, -0.02, -1.02, 0.2],
     ztr[Ωm0I2CCPL, 1 - Ωm0I2CCPL]}, {Ωm0I2CCPL, 0, 1, 0.1},
    PlotStyle → {Thick, Orange}, AxesOrigin → {0, -1}, Frame → True,
    GridLines →
     {{{0.044, Dashed}, {0.261, Red}, {0.274, Green}, {0.287, Gray}},
      {{0.376, LightRed}, {0.426, LightBlue}, {0.508, Gray}}},
    ImageSize → 500, FrameLabel → {"Ωm0", "Transition"}] // Quiet
```



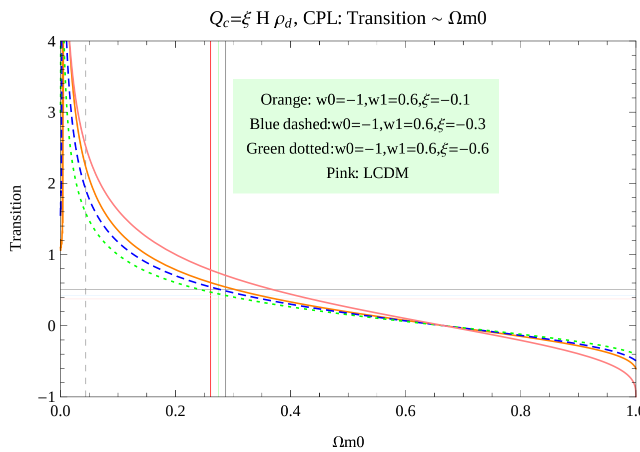

```
pldecΩm0I2CCPL =
  Plot[{ztrI2CCPL[Ωm0I2CCPL, 1 - Ωm0I2CCPL, -0.2, -1.02, 0.2],
     ztr[Ωm0I2CCPL, 1 - Ωm0I2CCPL]}, {Ωm0I2CCPL, 0, 1},
    PlotRange → {{0.05, 1}, {-1, 4}}, PlotStyle → {Thick, Orange},
    AxesOrigin → {0.05, -1}, Frame → True,
    GridLines →
     {{{0.044, Dashed}, {0.261, Red}, {0.274, Green}, {0.287, Gray}},
      {{0.376, LightRed}, {0.426, LightBlue}, {0.508, Gray}}},
    PlotLegend → {"Q=-0.02 H ρd", "LCDM"},
    LegendPosition → {0.0, -0.05}, LegendShadow → None,
    ImageSize → 500, FrameLabel → {"Ωm0", "Transition"}] // Quiet
```

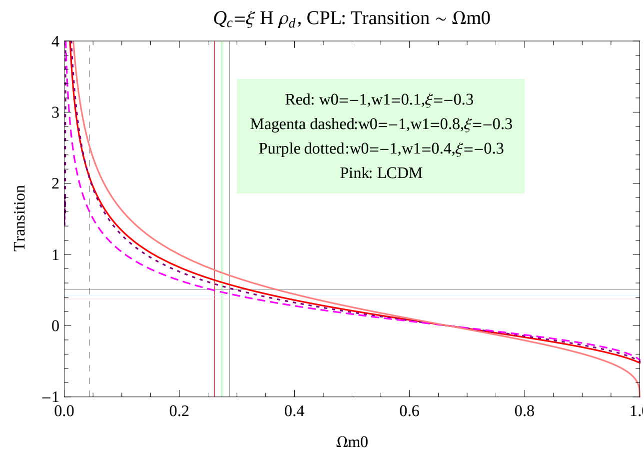


```
plztrI2CCPL[ξI2CCPL_, w0I2CCPL_, w1I2CCPL_, color_,
   {ranges_: 0, rangee_: 1}] :=
  Plot[ztrI2CCPL[Ωm0I2CCPL, 1 - Ωm0I2CCPL, ξI2CCPL, w0I2CCPL,
    w1I2CCPL], {Ωm0I2CCPL, 0, 1},
   PlotRange → {{ranges, rangee}, {-1, 4}}, PlotStyle → color,
   AxesOrigin → {0, -1}, Frame → True,
   FrameLabel → {"Ωm0", "Transtion"}];
```

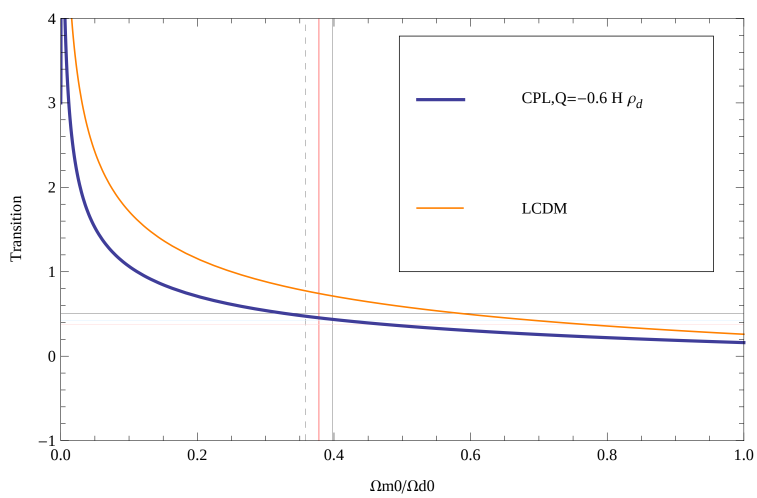
```
(* Do not evaluate this cell if not necessary. *)
(*
plztrI2CCPLManSum=
 Manipulate[
   Show[{plztrI2CCPL[ξI2CCPL,w0I2CCPL,w1I2CCPL,Orange,{0,1}],
     Plot[ztr[Ωm0I2CCPL,1-Ωm0I2CCPL],{Ωm0I2CCPL,0,1},
      PlotRange→{{0,1},{-1,4}},PlotStyle→Pink,AxesOrigin→{0,-1}]},
    Graphics[{Gray,Rectangle[Scaled[{0.261,.2752}],
       Scaled[{0.287,.3016}]]}],Frame→True],
    GridLines→{{},{{0,Dashed}}},FrameLabel→{"Ωm0","Transition"},
    PlotLabel→"Qc=ξ H ρd, CPL: Transition ~ Ωm0"],
   {{ξI2CCPL,-0.4,"Interaction"},-1,0,Appearance→"Open"},
   {{w0I2CCPL,-1.02,"CPL EoS w0"},-2,-0.3,Appearance→"Open"},
   {{w1I2CCPL,0.6,"CPL EoS w1"},-0.2,1,Appearance→"Open"},
   Delimiter,
   Style["Orange is for interacting CPL model with Q=ξ H ρm",
    Medium],ControlPlacement→{Right,Right,Right},
   SaveDefinitions→True]//Quiet
*)
```

```
plztrExamI2CCPLSum =
 Grid[
   {{Show[{plztrI2CCPL[-0.1, -1, 0.6, Orange, {0, 1}],
       plztrI2CCPL[-0.3, -1, 0.6, {Blue, Dashed}, {0, 1}],
       plztrI2CCPL[-0.6, -1, 0.6, {Green, Dotted}, {0, 1}],
       Plot[ztr[Ωm0I2CCPL, 1 - Ωm0I2CCPL], {Ωm0I2CCPL, 0, 1},
        PlotRange → {{0, 1}, {-1, 4}}, PlotStyle → Pink,
        AxesOrigin → {0, -1}]},
      GridLines → {{{0.044, Dashed}, {0.261, Red}, {0.274, Green},
         {0.287, Gray}}, {{0.376, LightRed}, {0.426, LightBlue},
         {0.508, Gray}}}, GridLines → {{}, {{0, Dashed}}},
      Epilog →
       Inset[
        Framed[
         Style[
          "Orange: w0=-1,w1=0.6,ξ=-0.1\n Blue
            dashed:w0=-1,w1=0.6,ξ=-0.3\n Green
            dotted:w0=-1,w1=0.6,ξ=-0.6\n Pink: LCDM",
          10], Background → LightGreen, FrameStyle → None],
        {0.3, 3.5}, {Left, Top}], FrameLabel → {"Ωm0", "Transition"},
      PlotLabel → "Qc=ξ H ρd, CPL: Transition ~ Ωm0",
      ImageSize → 400],
     Show[{plztrI2CCPL[-0.3, -1, 0.1, Red, {0, 1}],
       plztrI2CCPL[-0.3, -1, 0.4, {Purple, Dotted}, {0, 1}],
       plztrI2CCPL[-0.3, -1, 0.8, {Magenta, Dashed}, {0, 1}],
       Plot[ztr[Ωm0I2CCPL, 1 - Ωm0I2CCPL], {Ωm0I2CCPL, 0, 1},
        PlotRange → {{0, 1}, {-1, 4}}, PlotStyle → Pink,
        AxesOrigin → {0, -1}]},
      GridLines → {{{0.044, Dashed}, {0.261, Red}, {0.274, Green},
         {0.287, Gray}}, {{0.376, LightRed}, {0.426, LightBlue},
         {0.508, Gray}}}, GridLines → {{}, {{0, Dashed}}},
      Epilog →
       Inset[
        Framed[
         Style[
          "Red: w0=-1,w1=0.1,ξ=-0.3\n Magenta
            dashed:w0=-1,w1=0.8,ξ=-0.3\n Purple
            dotted:w0=-1,w1=0.4,ξ=-0.3\n Pink: LCDM",
          10], Background → LightGreen, FrameStyle → None],
        {0.3, 3.5}, {Left, Top}], FrameLabel → {"Ωm0", "Transition"},
      PlotLabel → "Qc=ξ H ρd, CPL: Transition ~ Ωm0",
      ImageSize → 400]}}] // Quiet
```

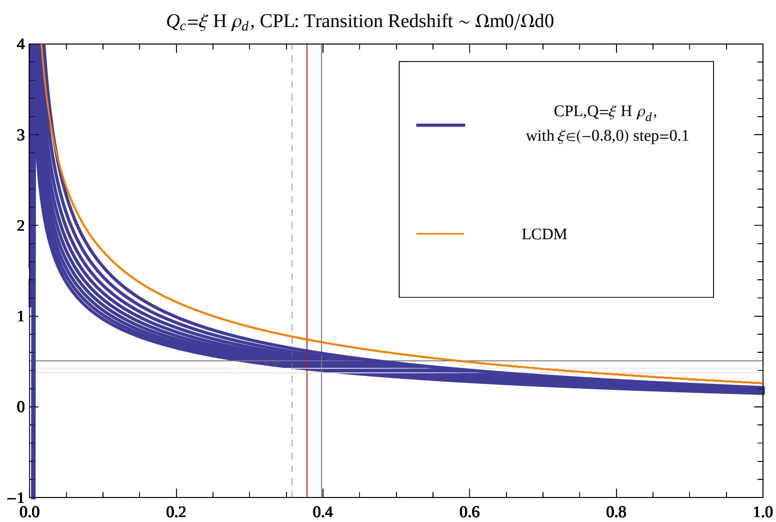





```
pldecrI2CCPL =
 Plot[{ztrrI2CCPL[rI2CCPL, -0.6, -1.02, 0.6], ztrr[rI2CCPL]},
   {rI2CCPL, 0, 1}, PlotRange → {{0, 1}, {-1, 4}},
   PlotStyle → {Thick, Orange}, AxesOrigin → {0, -1}, Frame → True,
   GridLines → {{{0.358, Dashed}, {0.378, Directive[Red]}, 0.398},
     {{0.376, LightRed}, {0.426, LightBlue}, {0.508, Gray}}},
   PlotLegend → {"CPL,Q=-0.6 H ρd", "LCDM"},
   LegendPosition → {0.0, -0.05}, LegendShadow → None,
   ImageSize → 500, FrameLabel → {"Ωm0/Ωd0", "Transition"}] // Quiet
```



```
pltransrI2CCPLDense =
 Show[
   Table[Plot[{ztrrI2CCPL[rI2CCPL, ξI2CCPL, -1.02, 0.6],
       ztrr[rI2CCPL]}, {rI2CCPL, 0, 1}, PlotRange → {{0, 1}, {-1, 4}},
     PlotStyle → {Thick, Orange}, AxesOrigin → {0, -1},
     Frame → True,
     GridLines → {{{0.358, Dashed}, {0.378, Directive[Red]}, 0.398},
       {{0.376, LightRed}, {0.426, LightBlue}, {0.508, Gray}}},
     PlotLegend → {"CPL,Q=ξ H ρd,\n with ξ∈(-0.8,0) step=0.1",
       "LCDM"}, LegendPosition → {0.0, -0.05}, LegendShadow → None],
    {ξI2CCPL, -0.8, 0, 0.1}], ImageSize → 500,
   FrameLabel → {"Ωm0/Ωd0", "Transition"},
   PlotLabel → "Qc=ξ H ρd, CPL: Transition Redshift ~ Ωm0/Ωd0",
   ImageSize → 500] // Quiet
```



- **Find out allowed region of coupling constant**

To find out the region of ξ, set w=-1 and Ωd0=1-Ωm0. Let the ztr-Ωm0 line cross points (0.287,0.508) and (0.261, 0.376).

```
Off[FindRoot::nlnum]
Off[ReplaceAll::reps]
ztrI2CCPL[Ωm0I2CCPL, Ωd0I2CCPL, ξI2CCPL, w0I2CCPL, w1I2CCPL]
On[FindRoot::nlnum]
On[ReplaceAll::reps]
```

[illegible]

[illegible]

[illegible]

[illegible]

```
z /.
 FindRoot[(1 + 3 (w0I2CCPL + (w1I2CCPL z)/(1 + z))) ΩdI2CCPL[Ωd0I2CCPL, w0I2CCPL,
      w1I2CCPL, ξI2CCPL, z] + ΩmI2CCPLtest[Ωd0I2CCPL,
     Ωm0I2CCPL, w0I2CCPL, w1I2CCPL, ξI2CCPL, z] == 0, {z, 3}]
```

```
(* ztrICCPL[0.287,1-0.287,ξICCPL1,-1.02,0.6]==0.508 *)
```

```
ξI2CCPLffunc[Ωm0I2CCPL_, Ωd0I2CCPL_, w0I2CCPL_, w1I2CCPL_, data_] :=
 ξI2CCPL /.
  FindRoot[
   ztrI2CCPL[Ωm0I2CCPL, Ωd0I2CCPL, ξI2CCPL, w0I2CCPL, w1I2CCPL] ==
    data, {ξI2CCPL, -0.6}]
```

```
ξI2CCPLf2[Ωm0I2CCPL_, Ωd0I2CCPL_, w0I2CCPL_, w1I2CCPL_] :=
 ξI2CCPL /.
  FindRoot[
   ztrI2CCPL[Ωm0I2CCPL, Ωd0I2CCPL, ξI2CCPL, w0I2CCPL, w1I2CCPL] ==
    0.508, {ξI2CCPL, -0.6}]
```

```
ξI2CCPLf1[Ωm0I2CCPL_, Ωd0I2CCPL_, w0I2CCPL_, w1I2CCPL_] :=
 ξI2CCPL /.
  FindRoot[
   ztrI2CCPL[Ωm0I2CCPL, Ωd0I2CCPL, ξI2CCPL, w0I2CCPL, w1I2CCPL] ==
    0.376, {ξI2CCPL, -0.6}]
```

Cross the Center of best fit. (0.274,0.426)

```
ξI2CCPLfc[Ωm0I2CCPL_, Ωd0I2CCPL_, w0I2CCPL_, w1I2CCPL_] :=
 ξI2CCPL /.
  FindRoot[
   ztrI2CCPL[Ωm0I2CCPL, Ωd0I2CCPL, ξI2CCPL, w0I2CCPL, w1I2CCPL] ==
    0.426, {ξI2CCPL, -0.6}]
```

According to the data of transition redshift.

```
ξI2CCPLf2[w0I2CCPL_, w1I2CCPL_] :=
 ξI2CCPLffunc[0.287, 1 - 0.287, w0I2CCPL, w1I2CCPL, 0.508]
```

```
ξI2CCPLf1[w0I2CCPL_, w1I2CCPL_] :=
 ξI2CCPLffunc[0.261, 1 - 0.261, w0I2CCPL, w1I2CCPL, 0.376]
```

```
ξI2CCPLfc[w0I2CCPL_, w1I2CCPL_] :=
 ξI2CCPLffunc[0.274, 1 - 0.274, w0I2CCPL, w1I2CCPL, 0.426]
```

An example

```
Off[FindRoot::nlnum]
Off[ReplaceAll::reps]
{ξI2CCPLf1[-1.02, 0.6], ξI2CCPLfc[-1.02, 0.6],
 ξI2CCPLf2[-1.02, 0.6]}
On[FindRoot::nlnum]
On[ReplaceAll::reps]
```

```
{-1.24538, -0.758365, -0.251702}
```

```
ξI2CCPLrffunc[rI2CCPL_, w0I2CCPL_, w1I2CCPL_, data_] :=
 ξI2CCPL /.
  FindRoot[ztrrI2CCPL[rI2CCPL, ξI2CCPL, w0I2CCPL, w1I2CCPL] == data,
   {ξI2CCPL, -0.6}]
```

```
ξI2CCPLrf2[rI2CCPL_, w0I2CCPL_, w1I2CCPL_] :=
 ξI2CCPL /.
  FindRoot[ztrrI2CCPL[rI2CCPL, ξI2CCPL, w0I2CCPL, w1I2CCPL] == 0.508,
   {ξI2CCPL, -0.6}]
```

```
ξI2CCPLrf1[rI2CCPL_, w0I2CCPL_, w1I2CCPL_] :=
 ξI2CCPL /.
  FindRoot[ztrrI2CCPL[rI2CCPL, ξI2CCPL, w0I2CCPL, w1I2CCPL] == 0.376,
   {ξI2CCPL, -0.6}]
```

Cross the Center of best fit. (0.358,0.426)

```
ξI2CCPLrfc[rI2CCPL_, w0I2CCPL_, w1I2CCPL_] :=
 ξI2CCPL /.
  FindRoot[ztrrI2CCPL[rI2CCPL, ξI2CCPL, w0I2CCPL, w1I2CCPL] == 0.426,
   {ξI2CCPL, -0.6}]
```

According to the data of transition redshift.

```
ξI2CCPLrf2[w0I2CCPL_, w1I2CCPL_] :=
 ξI2CCPLrffunc[0.398, w0I2CCPL, w1I2CCPL, 0.508]
```

```
ξI2CCPLrf1[w0I2CCPL_, w1I2CCPL_] :=
 ξI2CCPLrffunc[0.358, w0I2CCPL, w1I2CCPL, 0.376]
```

```
ξI2CCPLrfc[w0I2CCPL_, w1I2CCPL_] :=
 ξI2CCPLrffunc[0.378, w0I2CCPL, w1I2CCPL, 0.426]
```

An example

```
Off[FindRoot::nlnum]
Off[ReplaceAll::reps]
{ξI2CCPLrf1[-1.02, 0.6], ξI2CCPLrfc[-1.02, 0.6],
 ξI2CCPLrf2[-1.02, 0.6]}
On[FindRoot::nlnum]
On[ReplaceAll::reps]
```

```
{-1.21472, -0.755302, -0.269941}
```

```
(* Do not evaluate it unless it is necessary. *)
(*
Off[FindRoot::nlnum]
  Off[ReplaceAll::reps]
  fitξI2CCPLManSum=
 Manipulate[
  Grid[
   {{numPlot["(",{ξI2CCPLf1[w0I2CCPL,w1I2CCPL],
        ξI2CCPLfc[w0I2CCPL,w1I2CCPL],ξI2CCPLf2[w0I2CCPL,w1I2CCPL]},
       ")",{-1.5,1}],Plot[w0I2CCPL+w1I2CCPL temp/(temp+1),
       {temp,-0.9,10}]}}],
  {{w0I2CCPL,-1.02,"Equation of State w0"},-2,-0.47,
   Appearance->"Open"},
  {{w1I2CCPL,0.6,"Equation of State w1"},-1,1,Appearance->"Open"},
  Delimiter,
  Style[
   "This is the fitting result from transition redshift data.",
   Bold],Delimiter,
  "The parenthesis shows the upper and lower value \n while
    the verticle line show the center value.",
  Style[
   "\n The three numbers are left value, center value,
     right value respectively."],Delimiter,Delimiter,
  Style[
   "Slide to see how do the two parameters affect the
     coupling constant results."],
  ControlPlacement->{Bottom,Bottom},SaveDefinitions->True]
*)
```

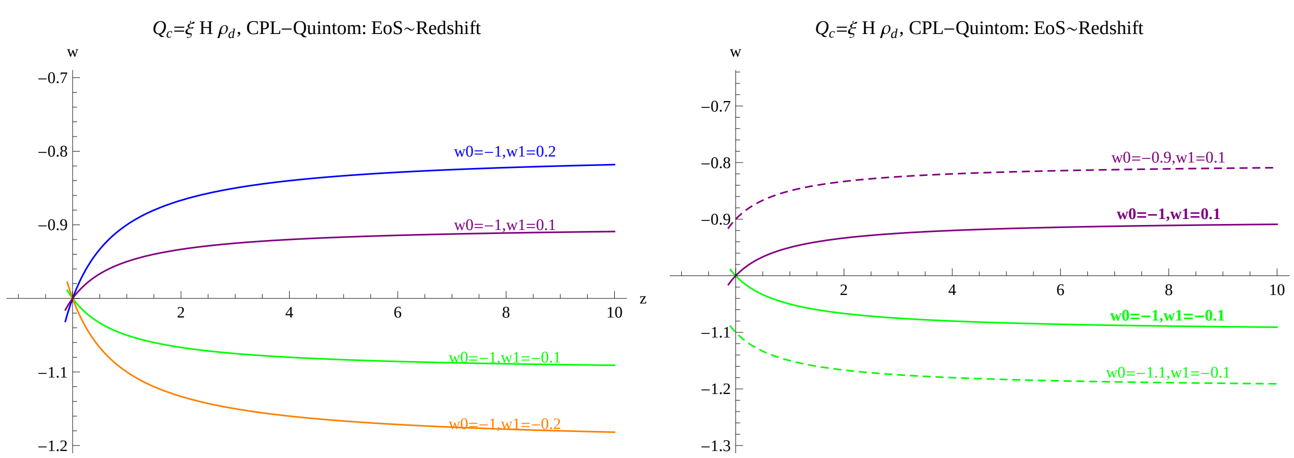
```
(*
On[FindRoot::nlnum]
 On[ReplaceAll::reps]
*)
```

Quintom Case

The transition redshift. Six groups of parameters.
(-1, 0.1) (-1, 0.2) (-0.9,0.1)
(-1, -0.1) (-1, -0.2) (-1.1,-0.1)
In two plots,
(-1, 0.1) (-1, 0.2) (-1, -0.1) (-1, -0.2)
(-1, 0.1)   (-0.9,0.1) (-1, -0.1)   (-1.1,-0.1)

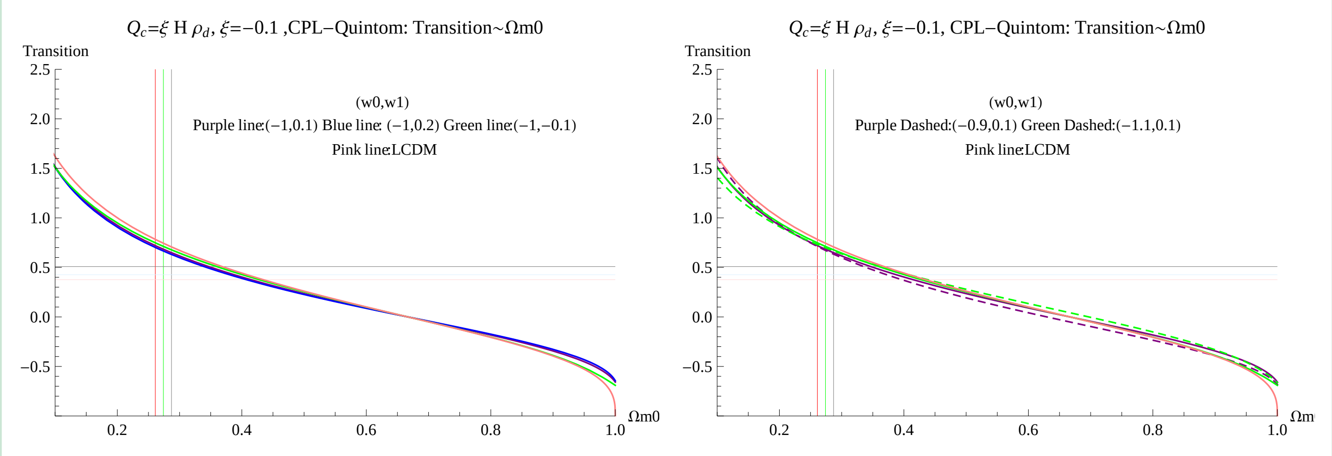
```
pltI2CCPLEoSfunc[w0I2CCPL_, w1I2CCPL_, color_] :=
  Plot[{w0I2CCPL + w1I2CCPL z/(1 + z)}, {z, -0.99, 10}, PlotStyle → color,
   AxesLabel → {"z", "w"}];
```

```
plEoSI2CCPLQuintomSum =
 Grid[
  {{Show[pltI2CCPLEoSfunc[-1, 0.2, Blue],
     pltI2CCPLEoSfunc[-1, 0.1, Purple],
     pltI2CCPLEoSfunc[-1, -0.1, Green],
     pltI2CCPLEoSfunc[-1, -0.2, Orange],
     PlotRange → {{-0.99, 10}, {-1.2, -0.7}}, AxesOrigin → {0, -1},
     ImageSize → 400,
     PlotLabel → "Qc=ξ H ρd, CPL-Quintom: EoS~Redshift",
     Epilog →
      {Inset[Framed[Style["w0=-1,w1=0.2", 10, Blue],
          Background → None, FrameStyle → None], {8, -0.80}, {0, 0}],
       Inset[Framed[Style["w0=-1,w1=0.1", 10, Purple],
          Background → None, FrameStyle → None], {8, -0.9}, {0, 0}],
       Inset[Framed[Style["w0=-1,w1=-0.1", 10, Green],
          Background → None, FrameStyle → None], {8, -1.08}, {0, 0}],
       Inset[Framed[Style["w0=-1,w1=-0.2", 10, Orange],
          Background → None, FrameStyle → None], {8, -1.17}, {0, 0}]}],
    Show[pltI2CCPLEoSfunc[-1, 0.1, Purple],
     pltI2CCPLEoSfunc[-0.9, 0.1, {Purple, Dashed}],
     pltI2CCPLEoSfunc[-1, -0.1, Green],
     pltI2CCPLEoSfunc[-1.1, -0.1, {Green, Dashed}],
     PlotRange → {{-1, 10}, {-1.3, -0.65}}, AxesOrigin → {0, -1},
     ImageSize → 400,
     PlotLabel → "Qc=ξ H ρd, CPL-Quintom: EoS~Redshift",
     Epilog →
      {Inset[Framed[Style["w0=-0.9,w1=0.1", 10, Purple],
          Background → None, FrameStyle → None], {8, -0.79}, {0, 0}],
       Inset[Framed[Style["w0=-1,w1=0.1", 10, Bold, Purple],
          Background → None, FrameStyle → None], {8, -0.89}, {0, 0}],
       Inset[Framed[Style["w0=-1,w1=-0.1", 10, Bold, Green],
          Background → None, FrameStyle → None], {8, -1.07}, {0, 0}],
       Inset[Framed[Style["w0=-1.1,w1=-0.1", 10, Green],
          Background → None, FrameStyle → None], {8, -1.17},
        {0, 0}]}]}}]
```



"Purple line:(-1,0.1) Blue line: (-1,0.2) Green line:(-1,-0.1)\n Pink line:LCDM"
"Purple Dashed:(-0.9,0.1) Green Dashed:(-1.1,0.1)"

```
plztrI2CCPLQuintomSum =
 Grid[
   {{Show[{plztrI2CCPL[-0.1, -1, 0.1, Purple, {0.1, 1}],
       plztrI2CCPL[-0.1, -1, 0.2, Blue, {0.1, 1}],
       plztrI2CCPL[-0.1, -1, -0.1, Green, {0.1, 1}],
       Plot[ztr[Ωm0I2CCPL, 1 - Ωm0I2CCPL], {Ωm0I2CCPL, 0, 1},
        PlotRange → {{0.1, 1}, {-1, 3}}, PlotStyle → Pink,
        AxesOrigin → {0.1, -1}]},
      GridLines → {{{0.261, Red}, {0.274, Green}, {0.287, Gray}},
        {{0.376, LightRed}, {0.426, LightBlue}, {0.508, Gray}}},
      AxesOrigin → {0.1, -1}, PlotRange → {{0.1, 1}, {-1, 2.5}},
      Frame → False, AxesLabel → {"Ωm0", "Transition"},
      PlotLabel → "Qc=ξ H ρd, ξ=-0.1 ,CPL-Quintom: Transition~Ωm0",
      Epilog →
       Inset[
        Framed[
         Style[
          "(w0,w1)\n Purple line:(-1,0.1) Blue line:
            (-1,0.2) Green line:(-1,-0.1)\n
            Pink line:LCDM", 10], Background → None,
         FrameStyle → None], {0.3, 2.4}, {Left, Top}],
      ImageSize → 400],
     Show[{plztrI2CCPL[-0.1, -1, 0.1, Purple, {0.1, 1}],
       plztrI2CCPL[-0.1, -0.9, 0.1, {Purple, Dashed}, {0.1, 1}],
       plztrI2CCPL[-0.1, -1, -0.1, Green, {0.1, 1}],
       plztrI2CCPL[-0.1, -1.1, -0.1, {Green, Dashed}, {0.1, 1}],
       Plot[ztr[Ωm0I2CCPL, 1 - Ωm0I2CCPL], {Ωm0I2CCPL, 0, 1},
        PlotRange → {{0.1, 1}, {-1, 3}}, PlotStyle → Pink,
        AxesOrigin → {0.1, -1}]},
      GridLines → {{{0.261, Red}, {0.274, Green}, {0.287, Gray}},
        {{0.376, LightRed}, {0.426, LightBlue}, {0.508, Gray}}},
      AxesOrigin → {0.1, -1}, PlotRange → {{0.1, 1}, {-1, 2.5}},
      Frame → False, AxesLabel → {"Ωm0", "Transition"},
      PlotLabel → "Qc=ξ H ρd, ξ=-0.1, CPL-Quintom: Transition~Ωm0",
      Epilog →
       Inset[
        Framed[
         Style[
          "(w0,w1)\n Purple Dashed:(-0.9,0.1) Green
            Dashed:(-1.1,0.1)\n Pink line:LCDM", 10],
         Background → None, FrameStyle → None], {0.3, 2.4},
        {Left, Top}], ImageSize → 400]}}] // Quiet
```



As for the effect of EoS, we can split it to w0 effect and w1 effect.

First construct the plot function.

```
(*
pltξvw0I2CCPL[w1I2CCPL_,w0I2CCPLs_,w0I2CCPLe_]:=
 Plot[{ξI2CCPLfc[w0I2CCPL,w1I2CCPL],ξI2CCPLf1[w0I2CCPL,w1I2CCPL],
   ξI2CCPLf2[w0I2CCPL,w1I2CCPL]},{w0I2CCPL,w0I2CCPLs,w0I2CCPLe}]
*)
```

Choose the values from Reference 3. w0 = -1.087 ± 0.096,

```
ξvwExamI2CCPLQuintom =
 {Block[{w0I2CCPL = -1, w1I2CCPL = -0.1},
   {{w0I2CCPL, w1I2CCPL}, ξI2CCPLfc[w0I2CCPL, w1I2CCPL],
    ξI2CCPLf1[w0I2CCPL, w1I2CCPL], ξI2CCPLf2[w0I2CCPL, w1I2CCPL]}],
  Block[{w0I2CCPL = -1, w1I2CCPL = 0},
   {{w0I2CCPL, w1I2CCPL}, ξI2CCPLfc[w0I2CCPL, w1I2CCPL],
    ξI2CCPLf1[w0I2CCPL, w1I2CCPL], ξI2CCPLf2[w0I2CCPL, w1I2CCPL]}],
  Block[{w0I2CCPL = -1, w1I2CCPL = 0.1},
   {{w0I2CCPL, w1I2CCPL}, ξI2CCPLfc[w0I2CCPL, w1I2CCPL],
    ξI2CCPLf1[w0I2CCPL, w1I2CCPL], ξI2CCPLf2[w0I2CCPL, w1I2CCPL]}],
  Block[{w0I2CCPL = -0.9, w1I2CCPL = 0.1},
   {{w0I2CCPL, w1I2CCPL}, ξI2CCPLfc[w0I2CCPL, w1I2CCPL],
    ξI2CCPLf1[w0I2CCPL, w1I2CCPL], ξI2CCPLf2[w0I2CCPL, w1I2CCPL]}],
  Block[{w0I2CCPL = -1.1, w1I2CCPL = -0.1},
   {{w0I2CCPL, w1I2CCPL}, ξI2CCPLfc[w0I2CCPL, w1I2CCPL],
    ξI2CCPLf1[w0I2CCPL, w1I2CCPL], ξI2CCPLf2[w0I2CCPL, w1I2CCPL]}]}
```

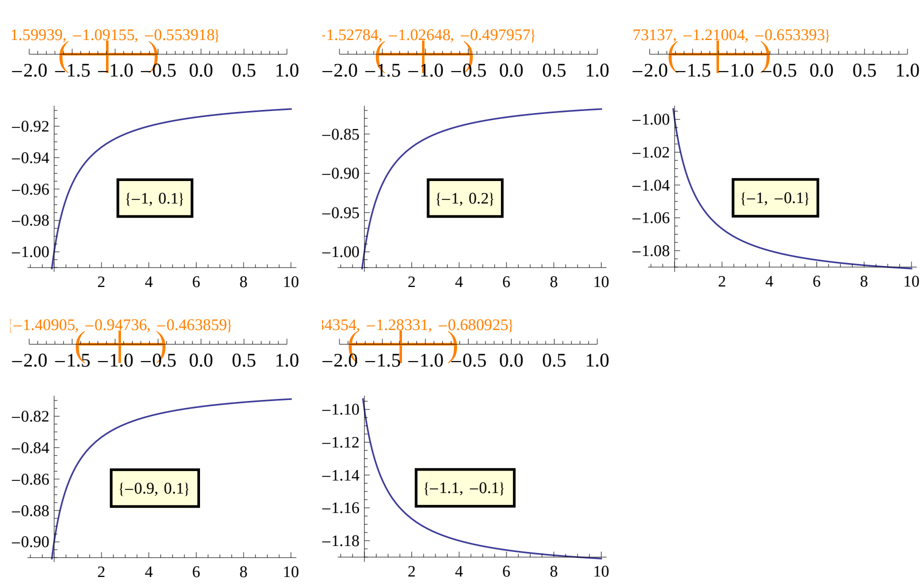
```
{{{-1, -0.1}, -1.21004, -1.73137, -0.653393},
 {{-1, 0}, -1.15265, -1.66715, -0.605615},
 {{-1, 0.1}, -1.09155, -1.59939, -0.553918},
 {{-0.9, 0.1}, -0.94736, -1.40905, -0.463859},
 {{-1.1, -0.1}, -1.28331, -1.84354, -0.680925}}
```

```
tabξvwExamI2CCPLQuintom =
 Grid[Prepend[Prepend[ξvwExamI2CCPLQuintom,
    {"{w0,w1}", "Center", "Lower", "Upper"}],
   {"ξ results for Qc=ξ H ρd, CPL,Quintom.", SpanFromLeft}],
  Frame → All,
  Background → {{LightGray, None}, {LightGreen, LightGray, None}},
  Alignment → Center, ItemSize → 8]
```

ξ results for Q_c=ξ H ρ_d, CPL,Quintom.			
{w0,w1}	Center	Lower	Upper
{-1, -0.1}	-1.21004	-1.73137	-0.653393
{-1, 0}	-1.15265	-1.66715	-0.605615
{-1, 0.1}	-1.09155	-1.59939	-0.553918
{-0.9, 0.1}	-0.94736	-1.40905	-0.463859
{-1.1, -0.1}	-1.28331	-1.84354	-0.680925

```
pltfitξI2CCPLfunc[w0I2CCPL_, w1I2CCPL_] :=
  Grid[
   {{numPlot["(", {ξI2CCPLf1[w0I2CCPL, w1I2CCPL],
       ξI2CCPLfc[w0I2CCPL, w1I2CCPL],
       ξI2CCPLf2[w0I2CCPL, w1I2CCPL]}, ")", {-2, 1}]},
    {Plot[w0I2CCPL + w1I2CCPL temp/(temp + 1), {temp, -0.9, 10},
      Epilog → Inset[Framed[Style[{w0I2CCPL, w1I2CCPL}, 10],
         Background → LightYellow], {Center, Center},
        {Center, Center}]]}}];
```

```
pltfitξI2CCPLQuintomSum =
 Grid[{{pltfitξI2CCPLfunc[-1, 0.1], pltfitξI2CCPLfunc[-1, 0.2],
     pltfitξI2CCPLfunc[-1, -0.1]},
    {pltfitξI2CCPLfunc[-0.9, 0.1], pltfitξI2CCPLfunc[-1.1, -0.1]}}]
```

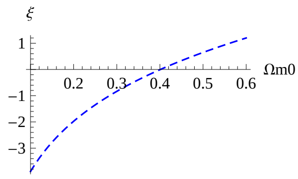



```
pltfitξvΩm0I2CCPLQuintomSum[w0I2CCPL_, w1I2CCPL_, color_,
    {Ωm0start_, Ωm0end_}] :=
  Plot[ξI2CCPLffunc[Ωm0I2CCPL, 1 - Ωm0I2CCPL, w0I2CCPL,
     w1I2CCPL, 0.426], {Ωm0I2CCPL, Ωm0start, Ωm0end},
    PlotStyle → color, AxesLabel → {"Ωm0", "ξ"}];
```

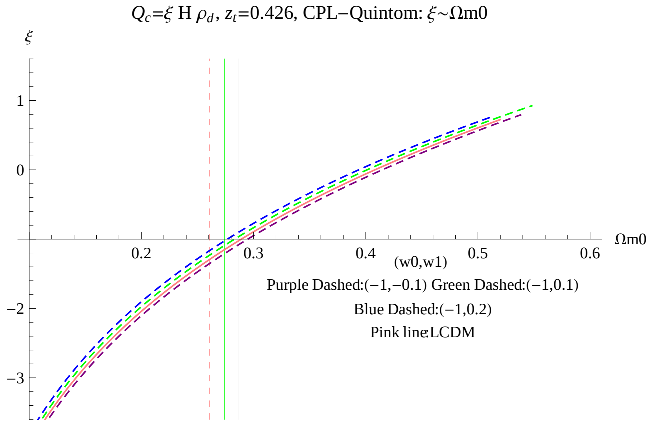
```
ξI2CCPLffunc[0.2, 1 - 0.2, -1, 0.1, 0.426] // Timing // Quiet
```

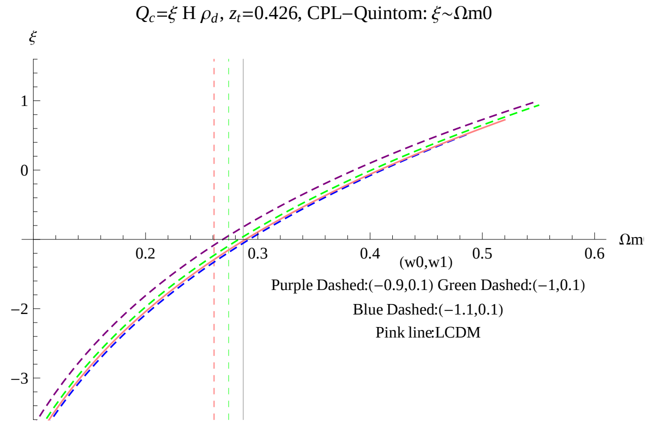
```
{3.682, -1.9741}
```

```
pltfitξvΩm0I2CCPLQuintomSum[-1, 0.1, {Blue, Dashed}, {0.1, 0.5}] //
  Quiet // Timing
```

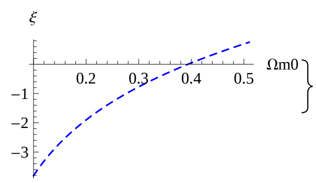
{581.915, }

```
pltξvΩm0I2CCPLQuintomSum =
 Grid[
   {{Show[pltfitξvΩm0I2CCPLQuintomSum[-1, -0.1, {Purple, Dashed},
       {0.1, 0.538}], pltfitξvΩm0I2CCPLQuintomSum[-1, 0.1,
       {Green, Dashed}, {0.1, 0.548}],
      pltfitξvΩm0I2CCPLQuintomSum[-1, 0.2, {Blue, Dashed},
       {0.1, 0.51}], pltfitξvΩm0I2CCPLQuintomSum[-1, 0, Pink,
       {0.1, 0.52}],
      GridLines → {{{0.261, Directive[Red, Dashed]},
         {0.274, Green}, {0.287, Gray}}, {}}, AxesOrigin → {0.1, -1},
      PlotRange → {{0.1, 0.6}, {-3.5, 1.5}}, Frame → False,
      AxesLabel → {"Ωm0", "ξ"},
      PlotLabel → "Qc=ξ H ρd, zt=0.426, CPL-Quintom: ξ~Ωm0",
      Epilog →
       Inset[
        Framed[
         Style[
          "(w0,w1)\n Purple Dashed:(-1,-0.1) Green
            Dashed:(-1,0.1)\n Blue Dashed:(-1,0.2)\n
            Pink line:LCDM", 10], Background → None,
         FrameStyle → None], {0.3, -1}, {Left, Top}],
      ImageSize → 400],
     Show[pltfitξvΩm0I2CCPLQuintomSum[-0.9, 0.1, {Purple, Dashed},
       {0.1, 0.55}], pltfitξvΩm0I2CCPLQuintomSum[-1, 0.1,
       {Green, Dashed}, {0.1, 0.55}],
      pltfitξvΩm0I2CCPLQuintomSum[-1.1, 0.1, {Blue, Dashed},
       {0.1, 0.489}], pltfitξvΩm0I2CCPLQuintomSum[-1, 0,
       Pink, {0.1, 0.52}],
      GridLines → {{{0.261, Directive[Red, Dashed]},
         {0.274, Directive[Green, Dashed]}, {0.287, Gray}}, {}},
      AxesOrigin → {0.1, -1}, PlotRange → {{0.1, 0.6}, {-3.5, 1.5}},
      Frame → False, AxesLabel → {"Ωm0", "ξ"},
      PlotLabel → "Qc=ξ H ρd, zt=0.426, CPL-Quintom: ξ~Ωm0",
      Epilog →
       Inset[
        Framed[
         Style[
          "(w0,w1)\n Purple Dashed:(-0.9,0.1) Green
            Dashed:(-1,0.1)\n Blue Dashed:(-1.1,0.1)\n
            Pink line:LCDM", 10], Background → None,
         FrameStyle → None], {0.3, -1}, {Left, Top}],
      ImageSize → 400]}}] // Quiet
```

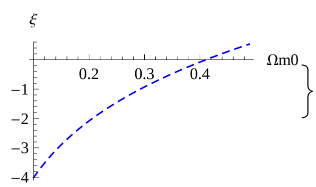





```
pltfitξvΩm0I2CCPLQuintomSum[-1, 0.2, {Blue, Dashed}, {0.1, 0.51}] //
  Timing // Quiet
```

{623.551,  }

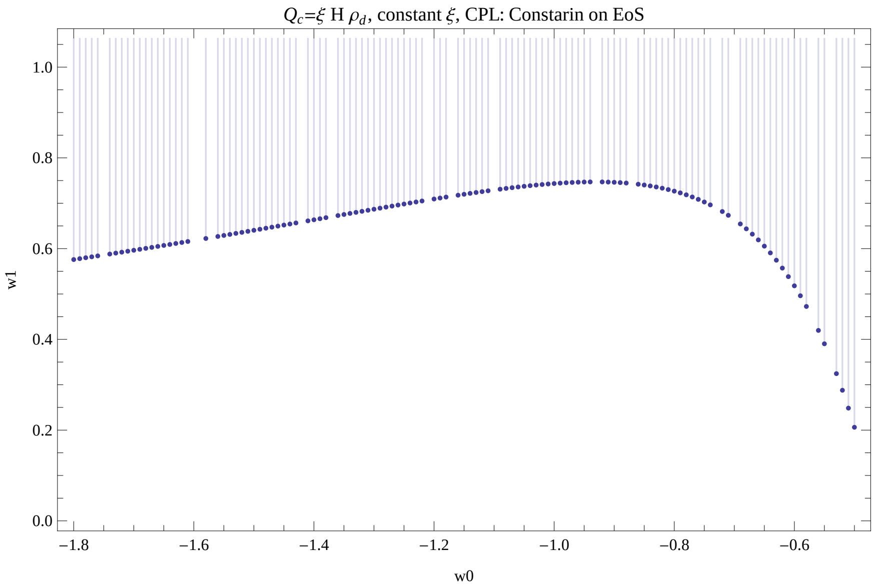

```
pltfitξvΩm0I2CCPLQuintomSum[-1.1, 0.1, {Blue, Dashed},
   {0.1, 0.489}] // Timing // Quiet
```

{398.13,  }


```
tabw0vw1I2CCPL2 =
    Table[{w0I2CCPLtemp,
      eosI2CCPLw1 /.
       FindRoot[
        {ξI2CCffunc[0.274, 1 - 0.274 - 0, w0I2CCPLtemp, eosI2CCPLw1] ==
          0}, {eosI2CCPLw1, -0.5}]},
     {w0I2CCPLtemp, -1.8, -0.5, 0.01}]; // Quiet // Timing
```

{178.029, Null}

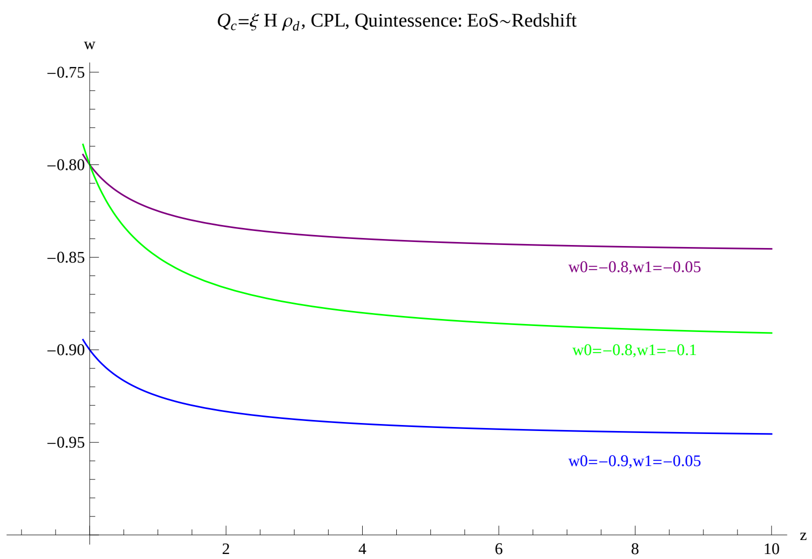
```
pltw0vw1ConsI2CCPL =
 ListPlot[tabw0vw1I2CCPL2, FrameLabel → {"w0", "w1"},
  Filling → Top, Frame → True,
  PlotLabel → "Qc=ξ H ρd, constant ξ, CPL: Constarin on EoS",
  ImageSize → 500]
```



Quintessence

Choose some Quintessence parameters.
(-0.9,-0.05) (-0.8,-0.05)(-0.8,-0.1)

```
plEoSI2CCPLQuintessenceSum =
 Grid[
  {{Show[pltI2CCPLEoSfunc[-0.9, -0.05, Blue],
     pltI2CCPLEoSfunc[-0.8, -0.05, Purple],
     pltI2CCPLEoSfunc[-0.8, -0.1, Green],
     PlotRange → {{-0.99, 10}, {-1.0, -0.75}}, AxesOrigin → {0, -1},
     Epilog →
      {Inset[Framed[Style["w0=-0.9,w1=-0.05", 10, Blue],
          Background → None, FrameStyle → None], {8, -0.96}, {0, 0}],
       Inset[Framed[Style["w0=-0.8,w1=-0.05", 10, Purple],
          Background → None, FrameStyle → None], {8, -0.855}, {0, 0}],
       Inset[Framed[Style["w0=-0.8,w1=-0.1", 10, Green],
          Background → None, FrameStyle → None], {8, -0.9}, {0, 0}]},
     PlotLabel → "Qc=ξ H ρd, CPL, Quintessence: EoS~Redshift",
     ImageSize → 500]}}]
```




```
plztrI2CCPLQuintessenceSum =
 Grid[
  {{Show[{Plot[ztrI2CCPL[Ωm0I2CCPL, 1 - Ωm0I2CCPL, -0.1, -0.9, -0.05],
        {Ωm0I2CCPL, 0.01, 1}, PlotRange → {{0, 1}, {-1, 4}},
        PlotStyle → Blue, AxesOrigin → {0, -1},
        PerformanceGoal → "Quality"],
      Plot[ztrI2CCPL[Ωm0I2CCPL, 1 - Ωm0I2CCPL, -0.1, -0.8, -0.05],
        {Ωm0I2CCPL, 0.01, 1}, PlotRange → {{0, 1}, {-1, 4}},
        PlotStyle → Purple, AxesOrigin → {0, -1},
        PerformanceGoal → "Quality"],
      Plot[ztrI2CCPL[Ωm0I2CCPL, 1 - Ωm0I2CCPL, -0.1, -0.7, -0.05],
        {Ωm0I2CCPL, 0.01, 0.94}, PlotRange → {{0, 1}, {-1, 4}},
        PlotStyle → Green, AxesOrigin → {0, -1},
        PerformanceGoal → "Quality"],
      Plot[ztrI2CCPL[Ωm0I2CCPL, 1 - Ωm0I2CCPL, -0.1, -0.6, -0.05],
        {Ωm0I2CCPL, 0.01, 0.94}, PlotRange → {{0, 1}, {-1, 4}},
```

```
      PlotStyle → Orange, AxesOrigin → {0, -1},
      PerformanceGoal → "Quality"],
     Plot[ztrI2CCPL[Ωm0I2CCPL, 1 - Ωm0I2CCPL, -0.1, -0.5, -0.05],
      {Ωm0I2CCPL, 0.01, 0.94}, PlotRange → {{0, 1}, {-1, 4}},
      PlotStyle → Magenta, AxesOrigin → {0, -1},
      PerformanceGoal → "Quality"],
     Plot[ztr[Ωm0I2CCPL, 1 - Ωm0I2CCPL], {Ωm0I2CCPL, 0, 1},
      PlotRange → {{0.01, 0.94}, {-1, 4}}, PlotStyle → Pink,
      AxesOrigin → {0, -1}]},
    GridLines → {{{0.261, Red}, {0.274, Green}, {0.287, Gray}},
      {{0.376, LightRed}, {0.426, LightBlue}, {0.508, Gray}}},
    AxesOrigin → {0.03, -1}, PlotRange → {{0.03, 0.33}, Automatic},
    Frame → False, AxesLabel → {"Ωm0", "Transition"},
    Epilog →
     Inset[
      Framed[
       Style[
        "Blue line: w0=-0.9,w1=-0.05,ξ=-0.1\n Purple line:
          w0=-0.8,w1=-0.05,ξ=-0.1\n Green
          line:w0=-0.7,w1=-0.05,ξ=-0.1\n Orange
          line:w0=-0.6,w1=-0.05,ξ=-0.1\n Magenta
          line:w0=-0.5,w1=-0.05,ξ=-0.1", 10],
       Background → LightGreen, FrameStyle → None], {0.1, 4},
      {Left, Top}],
    PlotLabel →
     "Qc=ξ H ρd, ξ=-0.1, CPL, Quintessence: Transition
       Redshift ~ Ωm0", ImageSize → 400],
   Show[{Plot[ztrI2CCPL[Ωm0I2CCPL, 1 - Ωm0I2CCPL, -0.1, -0.5, -0.05],
      {Ωm0I2CCPL, 0.01, 1}, PlotRange → {{0, 1}, {-1, 4}},
      PlotStyle → Blue, AxesOrigin → {0, -1},
      PerformanceGoal → "Quality"],
     Plot[ztrI2CCPL[Ωm0I2CCPL, 1 - Ωm0I2CCPL, -0.1, -0.5, -0.1],
      {Ωm0I2CCPL, 0.01, 1}, PlotRange → {{0, 1}, {-1, 4}},
      PlotStyle → Purple, AxesOrigin → {0, -1},
      PerformanceGoal → "Quality"],
     Plot[ztrI2CCPL[Ωm0I2CCPL, 1 - Ωm0I2CCPL, -0.1, -0.5, -0.2],
      {Ωm0I2CCPL, 0.01, 0.94}, PlotRange → {{0, 1}, {-1, 4}},
      PlotStyle → Green, AxesOrigin → {0, -1},
      PerformanceGoal → "Quality"],
     Plot[ztrI2CCPL[Ωm0I2CCPL, 1 - Ωm0I2CCPL, -0.1, -0.5, -0.3],
      {Ωm0I2CCPL, 0.01, 0.94}, PlotRange → {{0, 1}, {-1, 4}},
      PlotStyle → Orange, AxesOrigin → {0, -1},
      PerformanceGoal → "Quality"],
     Plot[ztrI2CCPL[Ωm0I2CCPL, 1 - Ωm0I2CCPL, -0.1, -0.5, -0.4],
      {Ωm0I2CCPL, 0.01, 0.94}, PlotRange → {{0, 1}, {-1, 4}},
      PlotStyle → Magenta, AxesOrigin → {0, -1},
      PerformanceGoal → "Quality"],
```

```
    Plot[ztr[Ωm0I2CCPL, 1 - Ωm0I2CCPL], {Ωm0I2CCPL, 0, 1},
     PlotRange → {{0.01, 0.94}, {-1, 4}}, PlotStyle → Pink,
     AxesOrigin → {0, -1}]},
   GridLines → {{{0.261, Red}, {0.274, Green}, {0.287, Gray}},
     {{0.376, LightRed}, {0.426, LightBlue}, {0.508, Gray}}},
   AxesOrigin → {0.03, -1}, PlotRange → {{0.03, 0.33}, Automatic},
   Frame → False, AxesLabel → {"Ωm0", "Transition"},
   Epilog →
    Inset[
     Framed[
      Style[
       "Blue line: w0=-0.5,w1=-0.05,ξ=-0.1\n Purple line:
         w0=-0.5,w1=-0.1,ξ=-0.1\n Green
         line:w0=-0.5,w1=-0.2,ξ=-0.1\n Orange
         line:w0=-0.5,w1=-0.3,ξ=-0.1\n Magenta
         line:w0=-0.5,w1=-0.4,ξ=-0.1", 10],
      Background → LightGreen, FrameStyle → None], {0.1, 4},
     {Left, Top}],
   PlotLabel →
    "Qc=ξ H ρd, ξ=-0.1, CPL, CPL, Quintessence: Transition
      Redshift ~ Ωm0", ImageSize → 400]}}]
```

```
$Aborted
```

As for the effect of EoS, we can split it to w0 effect and w1 effect.

Choose some values of w0 and w1 causually.

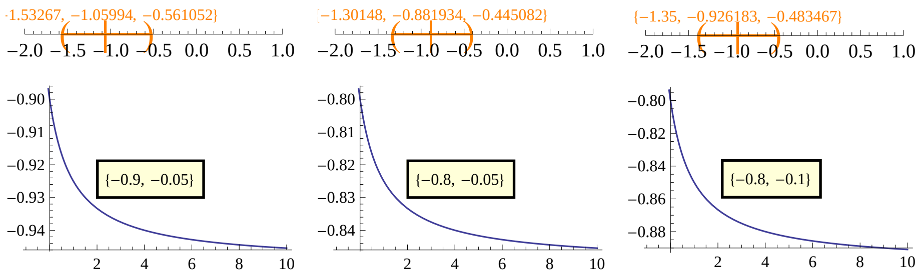
```
ξvwExamI2CCPLQuintessence =
 {Block[{w0I2CCPL = -0.9, w1I2CCPL = -0.05},
   {{w0I2CCPL, w1I2CCPL}, ξI2CCPLfc[w0I2CCPL, w1I2CCPL],
    ξI2CCPLf1[w0I2CCPL, w1I2CCPL], ξI2CCPLf2[w0I2CCPL, w1I2CCPL]}],
  Block[{w0I2CCPL = -0.8, w1I2CCPL = -0.05},
   {{w0I2CCPL, w1I2CCPL}, ξI2CCPLfc[w0I2CCPL, w1I2CCPL],
    ξI2CCPLf1[w0I2CCPL, w1I2CCPL], ξI2CCPLf2[w0I2CCPL, w1I2CCPL]}],
  Block[{w0I2CCPL = -0.8, w1I2CCPL = -0.1},
   {{w0I2CCPL, w1I2CCPL}, ξI2CCPLfc[w0I2CCPL, w1I2CCPL],
    ξI2CCPLf1[w0I2CCPL, w1I2CCPL], ξI2CCPLf2[w0I2CCPL, w1I2CCPL]}]}
```

```
{{{-0.9, -0.05}, -1.05994, -1.53267, -0.561052},
 {{-0.8, -0.05}, -0.881934, -1.30148, -0.445082},
 {{-0.8, -0.1}, -0.926183, -1.35, -0.483467}}
```

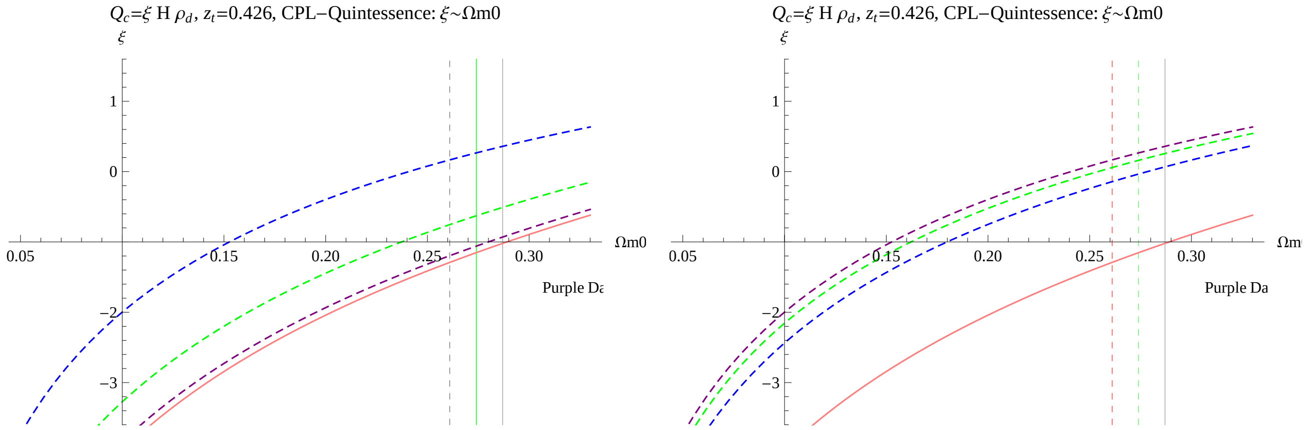
```
tabξvwExamI2CCPLQuintessence =
 Grid[Prepend[Prepend[ξvwExamI2CCPLQuintessence,
     {"{w0,w1}", "Center", "Lower", "Upper"}],
    {"ξ results for Qc=ξ H ρd, CPL,Quintessence.", SpanFromLeft}],
  Frame → All,
  Background → {{LightGray, None}, {LightGreen, LightGray, None}},
  Alignment → Center, ItemSize → 8]
```

ξ results for Q_c=ξ H ρ_d, CPL,Quintessence.			
{w0,w1}	Center	Lower	Upper
{-0.9, -0.05}	-1.05994	-1.53267	-0.561052
{-0.8, -0.05}	-0.881934	-1.30148	-0.445082
{-0.8, -0.1}	-0.926183	-1.35	-0.483467

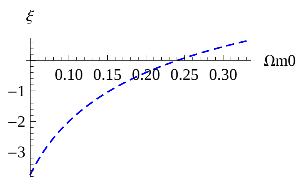
```
pltfitξI2CCPLQuintessenceSum =
 Grid[
  {{pltfitξI2CCPLfunc[-0.9, -0.05], pltfitξI2CCPLfunc[-0.8, -0.05],
    pltfitξI2CCPLfunc[-0.8, -0.1]}}]
```



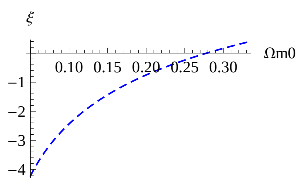
```
pltξvΩm0I2CCPLQuintessenceSum =
 Grid[
   {{Show[pltfitξvΩm0I2CCPLQuintomSum[-0.9, -0.05,
       {Purple, Dashed}, {0.05, 0.33}],
      pltfitξvΩm0I2CCPLQuintomSum[-0.7, -0.05, {Green, Dashed},
       {0.05, 0.33}], pltfitξvΩm0I2CCPLQuintomSum[-0.5,
       -0.05, {Blue, Dashed}, {0.05, 0.33}],
      pltfitξvΩm0I2CCPLQuintomSum[-1, 0, Pink, {0.05, 0.33}],
      GridLines → {{{0.261, Directive[Red, Dashed]},
         {0.274, Green}, {0.287, Gray}}, {}}, AxesOrigin → {0.1, -1},
      PlotRange → {{0.05, 0.33}, {-3.5, 1.5}}, Frame → False,
      AxesLabel → {"Ωm0", "ξ"},
      PlotLabel → "Qc=ξ H ρd, zt=0.426, CPL-Quintessence: ξ~Ωm0",
      Epilog →
       Inset[
        Framed[
         Style[
          "(w0,w1)\n Purple Dashed:(-0.9,-0.05) Green
            Dashed:(-0.7,-0.05)\n Blue
            Dashed:(-0.5,-0.05)\n Pink line:LCDM", 10],
         Background → None, FrameStyle → None], {0.3, -1},
        {Left, Top}], ImageSize → 400],
     Show[pltfitξvΩm0I2CCPLQuintomSum[-0.5, -0.05,
       {Purple, Dashed}, {0.05, 0.33}],
      pltfitξvΩm0I2CCPLQuintomSum[-0.5, -0.1, {Green, Dashed},
       {0.05, 0.33}], pltfitξvΩm0I2CCPLQuintomSum[-0.5, -0.2,
       {Blue, Dashed}, {0.05, 0.33}],
      pltfitξvΩm0I2CCPLQuintomSum[-1, 0, Pink, {0.05, 0.33}],
      GridLines → {{{0.261, Directive[Red, Dashed]},
         {0.274, Directive[Green, Dashed]}, {0.287, Gray}}, {}},
      AxesOrigin → {0.1, -1}, PlotRange → {{0.05, 0.33}, {-3.5, 1.5}},
      Frame → False, AxesLabel → {"Ωm0", "ξ"},
      PlotLabel → "Qc=ξ H ρd, zt=0.426, CPL-Quintessence: ξ~Ωm0",
      Epilog →
       Inset[
        Framed[
         Style[
          "(w0,w1)\n Purple Dashed:(-0.5,-0.05) Green
            Dashed:(-0.5,-0.1)\n Blue
            Dashed:(-0.5,-0.2)\n Pink line:LCDM", 10],
         Background → None, FrameStyle → None], {0.3, -1},
        {Left, Top}], ImageSize → 400]}}] // Quiet
```




```
pltfitξvΩm0I2CCPLQuintomSum[-0.5, -0.05, {Blue, Dashed},
    {0.05, 0.33}] // Timing // Quiet
```

{533.367, }

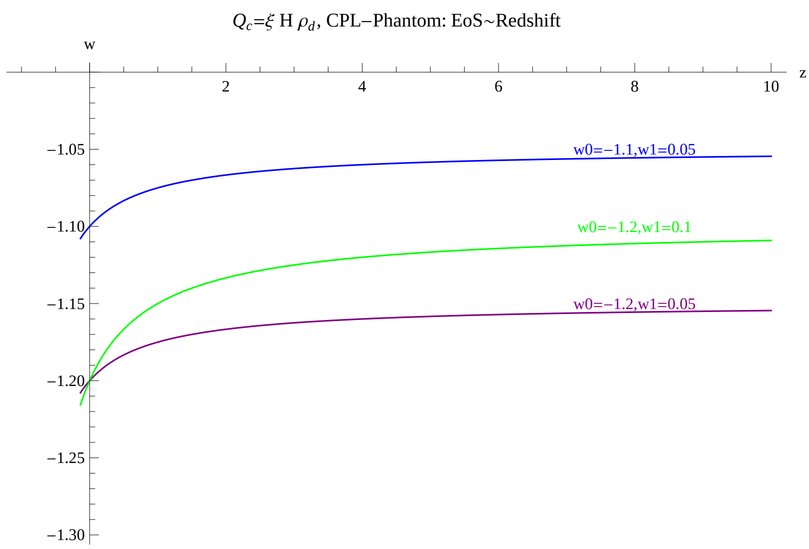

```
pltfitξvΩm0I2CCPLQuintomSum[-0.5, -0.2, {Blue, Dashed},
    {0.05, 0.33}] // Timing // Quiet
```

{438.27, }

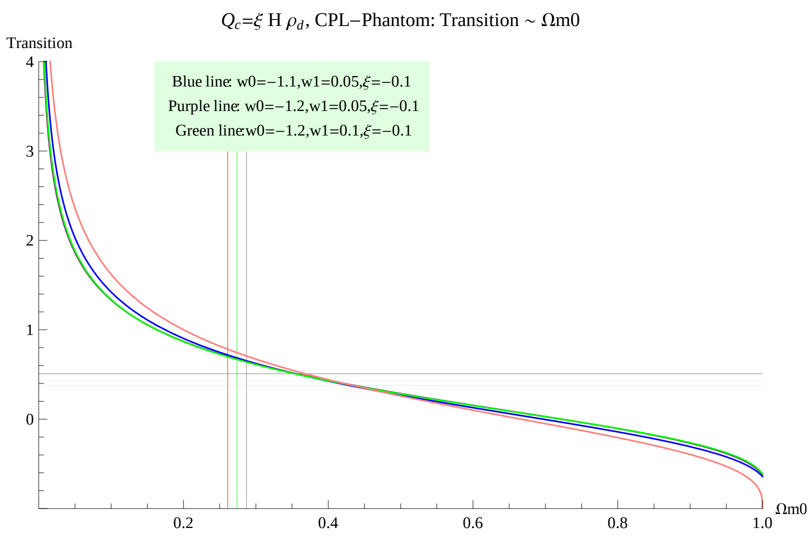
Phantom

Some parameters that give us a phantom mode
(-1.1,0.05) (-1.2,0.05) (-1.2,0.1)

```
plEoSI2CCPLPhantomSum =
 Grid[
  {{Show[pltI2CCPLEoSfunc[-1.1, 0.05, Blue],
     pltI2CCPLEoSfunc[-1.2, 0.05, Purple],
     pltI2CCPLEoSfunc[-1.2, 0.1, Green],
     PlotRange → {{-0.99, 10}, {-1.3, -1}}, AxesOrigin → {0, -1},
     Epilog →
      {Inset[Framed[Style["w0=-1.1,w1=0.05", 10, Blue],
          Background → None, FrameStyle → None], {8, -1.05}, {0, 0}],
       Inset[Framed[Style["w0=-1.2,w1=0.05", 10, Purple],
          Background → None, FrameStyle → None], {8, -1.15}, {0, 0}],
       Inset[Framed[Style["w0=-1.2,w1=0.1", 10, Green],
          Background → None, FrameStyle → None], {8, -1.1}, {0, 0}]},
     PlotLabel → "Qc=ξ H ρd, CPL-Phantom: EoS~Redshift",
     ImageSize → 500]}}]
```



```
plztrI2CCPLPhantomSum =
 Grid[
  {{Show[{plztrI2CCPL[-0.1, -1.1, 0.05, Blue, {0, 1}],
      plztrI2CCPL[-0.1, -1.2, 0.05, Purple, {0, 1}],
      plztrI2CCPL[-0.1, -1.2, 0.1, Green, {0, 1}],
      Plot[ztr[Ωm0I2CCPL, 1 - Ωm0I2CCPL], {Ωm0I2CCPL, 0, 1},
       PlotRange → {{0, 1}, {-1, 4}}, PlotStyle → Pink,
       AxesOrigin → {0, -1}]},
     GridLines → {{{0.261, Red}, {0.274, Green}, {0.287, Gray}},
       {{0.376, LightRed}, {0.426, LightBlue}, {0.508, Gray}}},
     AxesOrigin → {0, -1}, Frame → False,
     AxesLabel → {"Ωm0", "Transition"},
     Epilog →
      Inset[
       Framed[
        Style[
         "Blue line: w0=-1.1,w1=0.05,ξ=-0.1\n Purple line:
           w0=-1.2,w1=0.05,ξ=-0.1\n Green
           line:w0=-1.2,w1=0.1,ξ=-0.1", 10],
        Background → LightGreen, FrameStyle → None], {0.35, 3.5},
       {0, 0}],
     PlotLabel → "Qc=ξ H ρd, CPL-Phantom: Transition ~ Ωm0",
     ImageSize → 500]}}]
```



As for the effect of EoS, we can split it to w0 effect and w1 effect.

Choose some values of w0 and w1 causually.

```
ξvwExamI2CCPLPhantom =
 {Block[{w0I2CCPL = -1.1, w1I2CCPL = 0.05},
   {{w0I2CCPL, w1I2CCPL}, ξI2CCPLfc[w0I2CCPL, w1I2CCPL],
    ξI2CCPLf1[w0I2CCPL, w1I2CCPL], ξI2CCPLf2[w0I2CCPL, w1I2CCPL]}],
  Block[{w0I2CCPL = -1.2, w1I2CCPL = 0.05},
   {{w0I2CCPL, w1I2CCPL}, ξI2CCPLfc[w0I2CCPL, w1I2CCPL],
    ξI2CCPLf1[w0I2CCPL, w1I2CCPL], ξI2CCPLf2[w0I2CCPL, w1I2CCPL]}],
  Block[{w0I2CCPL = -1.2, w1I2CCPL = 0.1},
   {{w0I2CCPL, w1I2CCPL}, ξI2CCPLfc[w0I2CCPL, w1I2CCPL],
    ξI2CCPLf1[w0I2CCPL, w1I2CCPL], ξI2CCPLf2[w0I2CCPL, w1I2CCPL]}]}
```

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

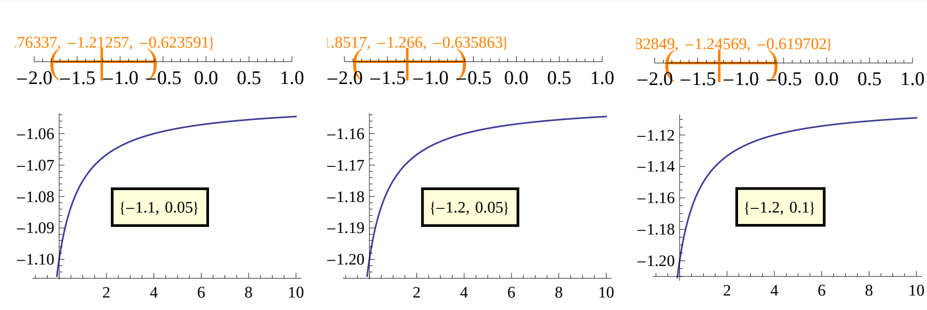
[illegible]

```
{{{-1.1, 0.05}, -1.21257, -1.76337, -0.623591},
 {{-1.2, 0.05}, -1.266, -1.8517, -0.635863},
 {{-1.2, 0.1}, -1.24569, -1.82849, -0.619702}}
```

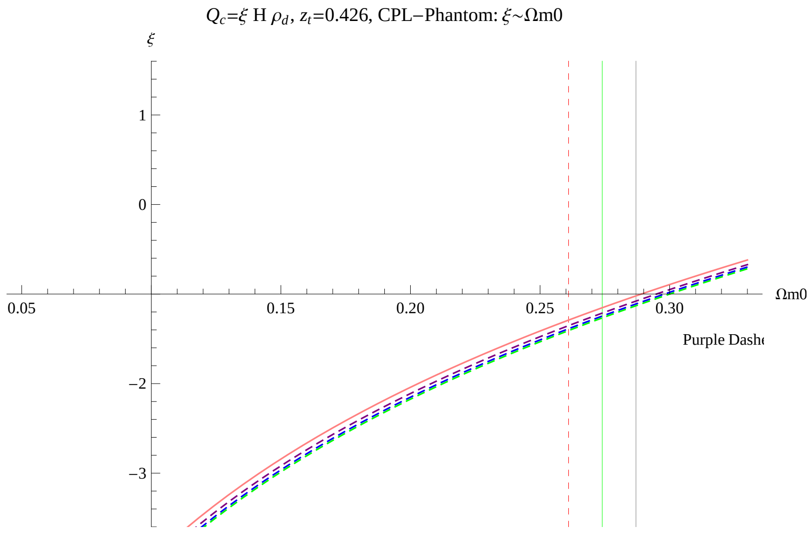
```
tabξvwExamI2CCPLPhantom =
 Grid[Prepend[Prepend[ξvwExamI2CCPLPhantom,
    {"{w0,w1}", "Center", "Lower", "Upper"}],
   {"ξ results for Qc=ξ H ρd, CPL,Phantom.", SpanFromLeft}],
  Frame → All,
  Background → {{LightGray, None}, {LightGreen, LightGray, None}},
  Alignment → Center, ItemSize → 8]
```

ξ results for Q_c=ξ H ρ_d, CPL,Phantom.			
{w0,w1}	Center	Lower	Upper
{−1.1, 0.05}	−1.21257	−1.76337	−0.623591
{−1.2, 0.05}	−1.266	−1.8517	−0.635863
{−1.2, 0.1}	−1.24569	−1.82849	−0.619702

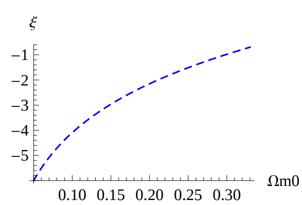
```
pltfitξI2CCPLPhantomSum =
 Grid[
  {{pltfitξI2CCPLfunc[-1.1, 0.05], pltfitξI2CCPLfunc[-1.2, 0.05],
    pltfitξI2CCPLfunc[-1.2, 0.1]}}]
```



```
pltξvΩm0I2CCPLPhantomSum =
 Show[pltfitξvΩm0I2CCPLQuintomSum[-1.1, 0.05, {Purple, Dashed},
    {0.05, 0.33}], pltfitξvΩm0I2CCPLQuintomSum[-1.2, 0.05,
    {Green, Dashed}, {0.05, 0.33}],
   pltfitξvΩm0I2CCPLQuintomSum[-1.2, 0.1, {Blue, Dashed},
    {0.05, 0.33}], pltfitξvΩm0I2CCPLQuintomSum[-1, 0, Pink,
    {0.05, 0.33}],
   GridLines →
    {{{0.261, Directive[Red, Dashed]}, {0.274, Green}, {0.287, Gray}},
     {}}, AxesOrigin → {0.1, -1},
   PlotRange → {{0.05, 0.33}, {-3.5, 1.5}}, Frame → False,
   AxesLabel → {"Ωm0", "ξ"},
   PlotLabel → "Qc=ξ H ρd, zt=0.426, CPL-Phantom: ξ~Ωm0",
   Epilog →
    Inset[
     Framed[
      Style[
       "(w0,w1)\n Purple Dashed:(-1.1,0.05) Green
         Dashed:(-1.2,0.05)\n Blue Dashed:(-1.2,0.1)\n
         Pink line:LCDM", 10], Background → None,
      FrameStyle → None], {0.3, -1}, {Left, Top}], ImageSize → 500] //
  Quiet
```




```
pltfitξvΩm0I2CCPLQuintomSum[-1.2, 0.1, {Blue, Dashed},
   {0.05, 0.33}] // Timing // Quiet
```

{269.335, }

Summary

Interacting models

List of what to make clear

1. [Quantitively] Fixed Ωm0, the allowed ξ with a changing of EoS.
2. [Quantitively] Fixed ω, the allowed ξ with a change Ωm0 or r.
3. [Quantitively] ξ is minus means energy transfer to dark matter, which delays the appearence of dark energy dominated era. What is the result of this method.
4. [Quantitively] CPL parameterization can be categorized into 3 cases. Check their effects on the fitting. How do different category change the ξ fitting results.
   - 4.1. Phantom
   - 4.2. Quintessence
   - 4.3. Crossing -1, Quintom
5. [Quantitively] The effect of Ωm0 in different CPL parameterizations.

BASIC

Evolution of energy density for $Q_c = \xi H \rho_c$, constant ξ, constant w, and $\xi \neq$-3w

$$\Omega m = \Omega m0\ (1+z)^{3-\xi}$$

$$\Omega d = \left(\Omega d0 + \frac{\xi}{3\,w+\xi}\,\Omega m0\right)(1+z)^{3\,(1+w)} + \frac{-\xi}{3\,w+\xi}\,\Omega m \equiv \Omega d0'\,(1+z)^{3\,(1+w)} + \frac{-\xi}{3\,w+\xi}\,\Omega m$$

Evolution of energy density for $Q_c = \xi H \rho_d$, constant ξ, constant w, and $\xi \neq$-3w

$$\Omega m = \left(\Omega m0 + \frac{\xi}{\xi+3\,w}\,\Omega d0\right)(1+z)^{3} + \frac{-\xi}{\xi+3\,w}\,\Omega d \equiv \Omega m0'\,(1+z)^{3} + \frac{-\xi}{\xi+3\,w}\,\Omega d$$

$$\Omega d = \Omega d0\ (1+z)^{3\,(1+w)+\xi}$$

So in the two cases, coupling constant has two effects:

1. Amplifies the curve of deceleration parameter.
2. Energy flow between DE and DM.

Interacting model $Q_c = \xi H \rho_c$ with constant ξ and constant EoS w.

Derived from (transition redshift, Ωm0) plane, the allowed region for coupling constant ξ is (-1.28, -0.46) with a center at -0.88, i.e., $-0.88^{+0.42}_{-0.40}$, taken the case that the universe is flat, and choose the EoS parameter {w=-1}.
Derived from the (transition redshift, $\frac{\Omega m0}{\Omega d0}$) plane, the allowed region of coupling constant ξ is (-1.25,- 0.47) with a center at -0.88, i.e., $-0.88^{+0.41}_{-0.37}$.
There is a bit difference between the two answers. One possible reason is the second method doesn't assume a flat universe, while the first one supposes the universe is flat.

The full table of fitting results are shown below. The light purple element are the final resuls.

tabξFinaltICC

$Q_c = \xi\ H\ \rho_c$, constant ξ, constant w=-1: Results for ξ			
Ωm0/Ωd0·.Transition	z_t=0.376	z_t=0.426	z_t=0.508
r=0.358	-1.25282	-0.965436	-0.617444
r=0.378	-1.15011	-0.875189	-0.542347
r=0.398	-1.05453	-0.791252	-0.472561

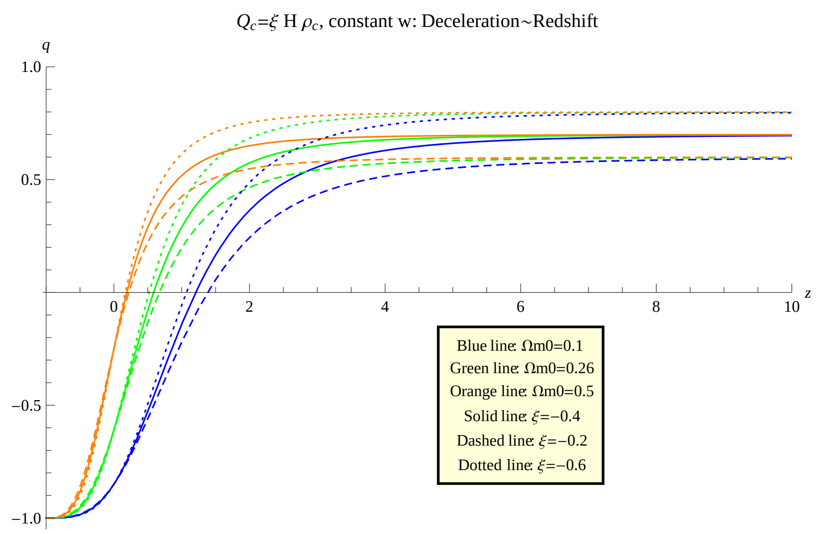
To check the consistancy of the two methods ((Transition, Ωm0) plane fitting and (Transition, Ωm0/Ωd0) plane fitting), we find out the fitting results of coupling constant ξ for a flat universe, i.e., $r = \frac{\Omega m0}{1-\Omega m0}$ in the (Transition, Ωm0) plane, applying the data from (Transition, Ωm0/Ωd0) plane.

By solving out Ωm0, we get $\Omega m0 = \frac{r}{1+r}$(this is a monotonic function) in this case. Thus if we use the constrain that r∈(0.358, 0.398) with a center value 0.378, the value of Ωm0 is (0.263623,0.284692), centered at 0.274311. Use this set of value of Ωm0 as the contrain, we have the fitting results in (Transition, Ωm0) plane, which is $-0.88^{+0.41}_{-0.37}$. This result is exactly the same as the result directly derived from (transition redshift, $\frac{\Omega m0}{\Omega d0}$) plane. The same has been done to $Q_c = \xi\, H\, \rho_d$ with ξ constant and w constant model, and the result is that the two methods are also consistant.

The plots of deceleration parameter are shown below. At the limit z->Infinity, the deceleration parameter is degenerate for different Ωm0 in this constant ξ and constant w model.

Theoretically, this limit is determined by the interaction coupling constant ξ, which is $\frac{(1-\xi)}{2}$, with $3\,w + \xi < 0$.

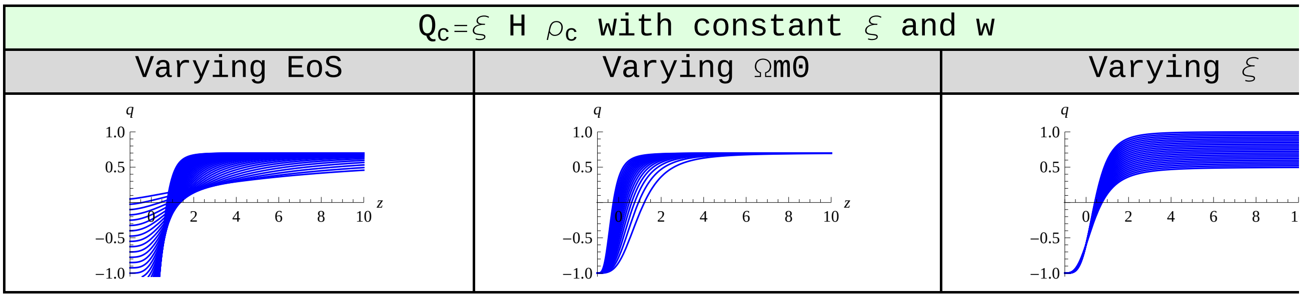
pldecICCShowSum



Check the effect of different parameters on deceleration parameter.

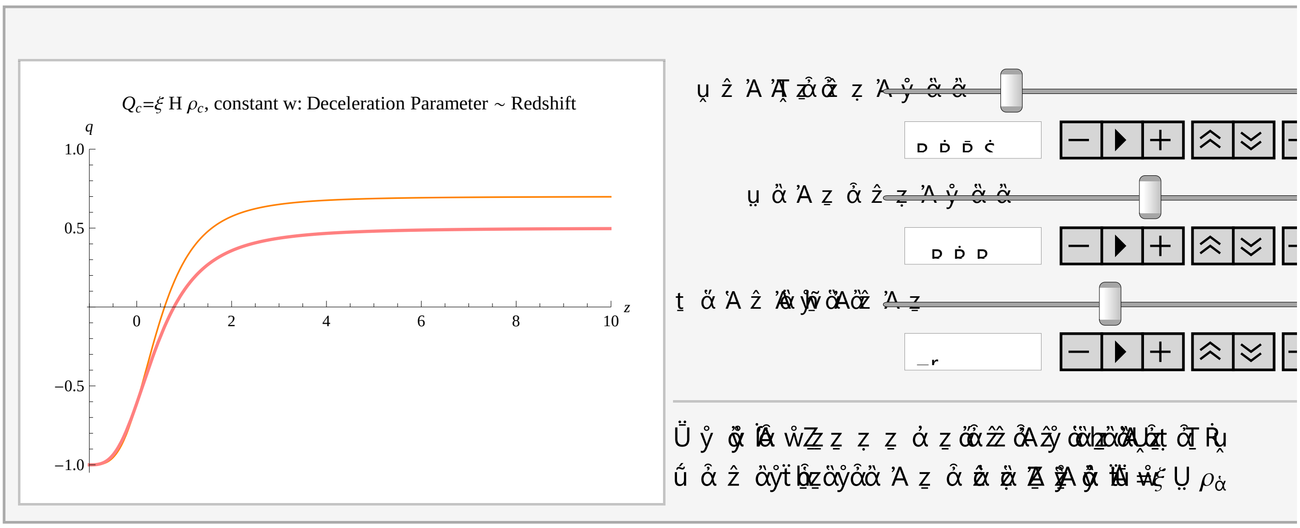
Interaction ξ changes the value of deceleration parameter at $z \to \infty$ limit. EoS changes the the whole shape. Matter fraction determines how fast q varies, but just in a small time scale.

```
varyingICCShowSum
```



A toy to play with is also provied. Slide the bars to view **the effects of different parameters on deceleration parameter**.

```
pldecICCManSum
```



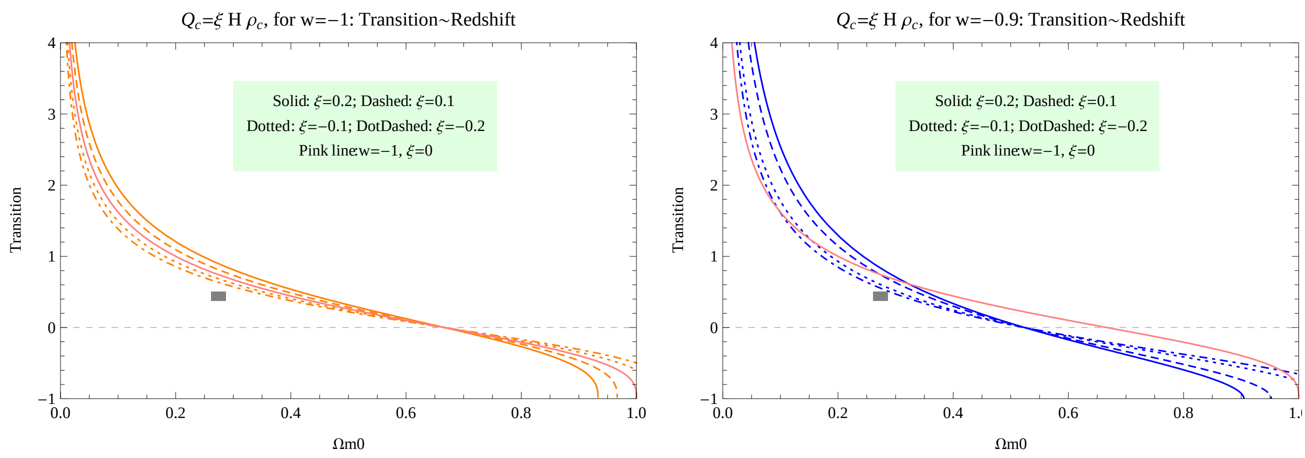
It can be inferred from the expression for Ωm and Ωd that if the transition happens before z=0, increasing couling ξ will bring forward the transition and if the it happens after z=0, increasing coupling ξ will delay the emergence of transition. The following figure shows this result.
Gray rectangle is the region given by Riess (References, Data From, 2).

Orange for w=-1
Blue for w=-0.9

Solid line: ξ=0.2
Dashed line: ξ=0.1
Dotted line: ξ=-0.1
DotDashed line: ξ=-0.2

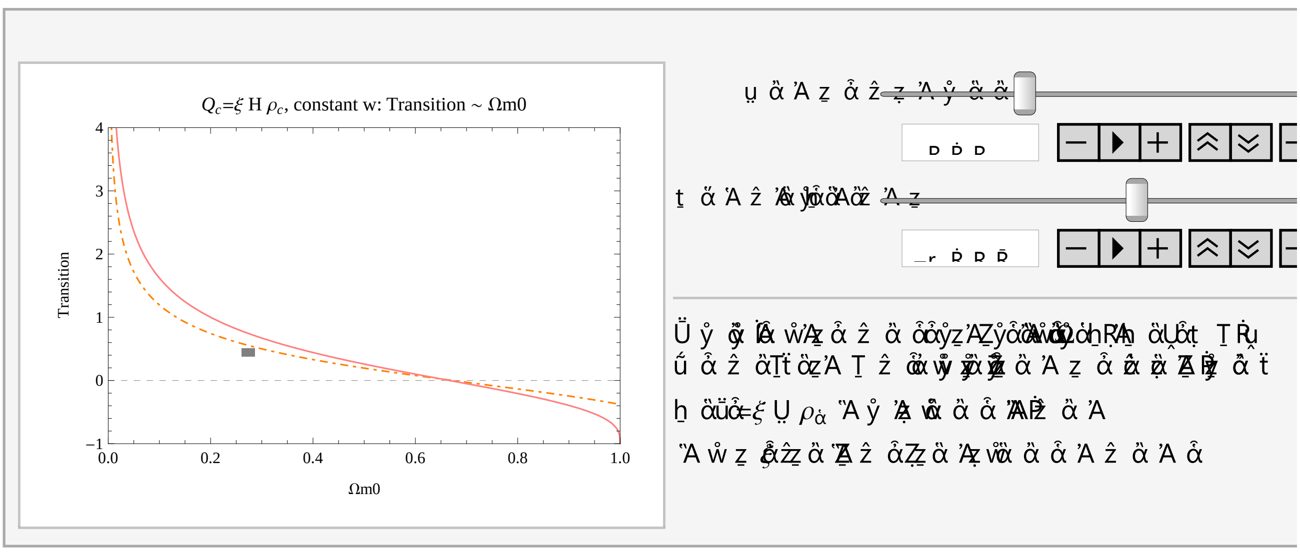
Pink solid line:w=-1, ξ=0

plztrvsΩm0ICCSum



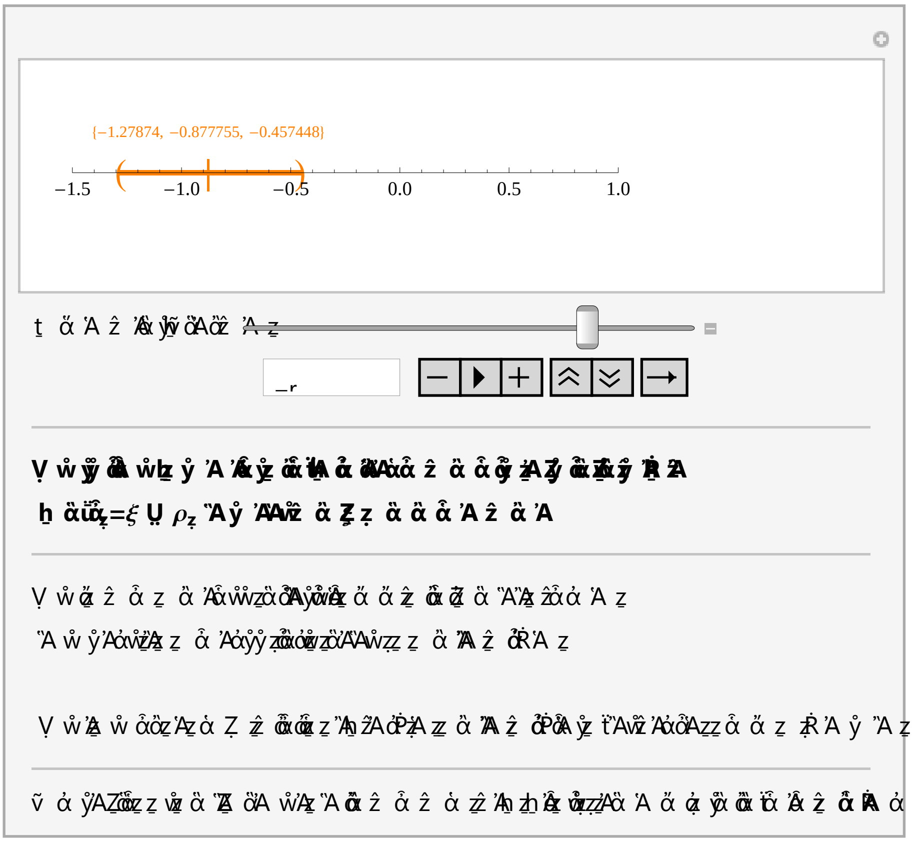
This can also be seen clearly from the following toy. Gray rectangle is the region given by Riess (References, Data From, 2) .

plztrICCManSum



The fitting results of coupling constant ξ can also be dynamic.

fitξICCManSum



For different constant EoS, the fitting results using Ωm0∈(0.261,0.287) with a center value 0.274 and Transition redshift ∈ (0.376,0.508) with a center value 0.426. When EoS is very small, the line cross zero. But that is not so useful.

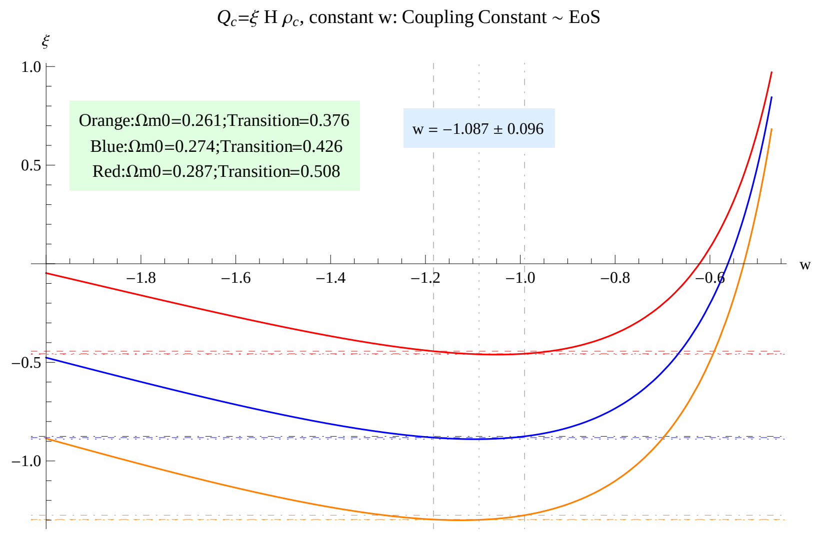
Some data points are derived using w = -1.087 ± 0.096 (from Reference, Data From, 3).

tabξvwExamICC

ξ results for $Q_c = \xi\, H\, \rho_c$ (Fitting data: Data From, 2)			
w	Center	Lower	Upper
−1.183	−0.881565	−1.29687	−0.443589
−1.087	−0.88948	−1.29859	−0.459135
−0.991	−0.875238	−1.27522	−0.456176

A plot showing these data points and the curves of ξ ~ w.

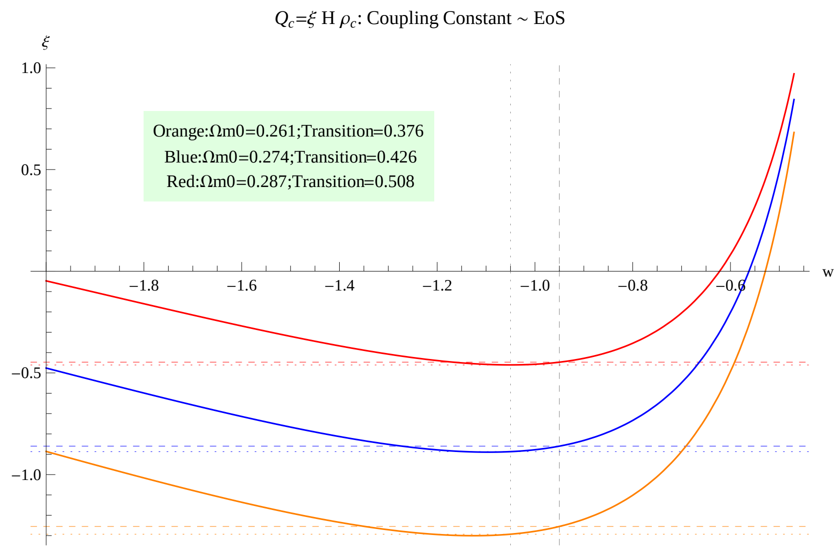
pltξvwExamICC



Or just casually use the following parameters.
(-1 within 5%: (-1.05,-0.95))
w=-1 (-1.279,-0.457) center:-0.878
w=-1.05 (-1.293,-0.461) center:-0.887
w=-0.95 (-1.255,-0.447) center:-0.860

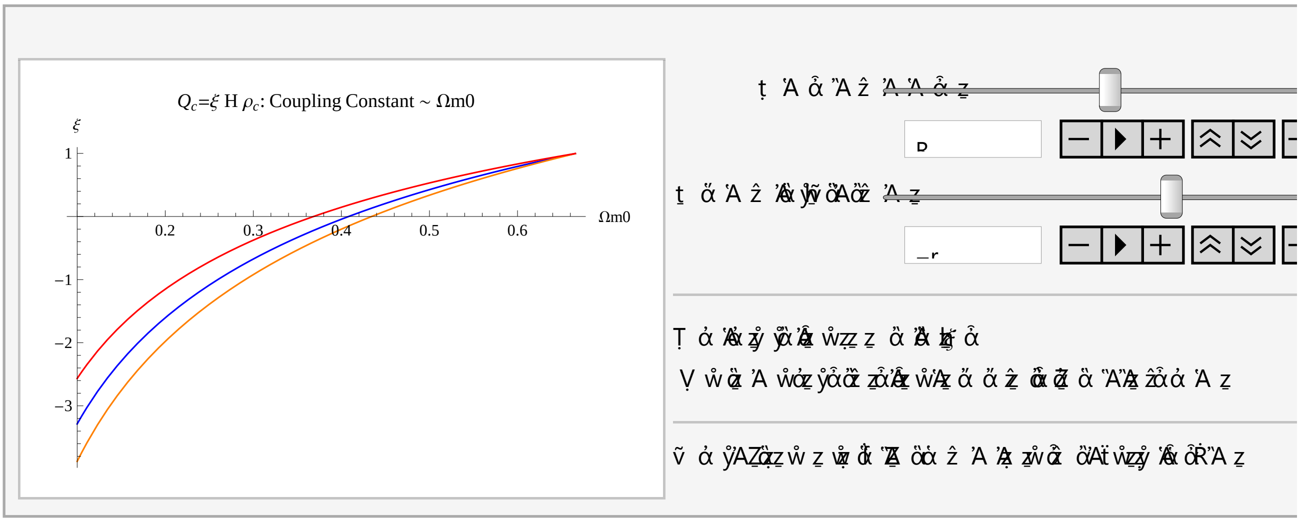
The following graph show how do ξ changes with EoS. The grid lines are the results of $w = -1 \pm 0.05$. Two verticle lines are -1.05 and -0.95 respectively. Horizontal lines are their intersections with the ξ~w lines. EoS does not monotonically change ξ. And the minima of these line occurs at a larger w with an increasing Ωm0.

pltξvwExam1ICC



Now we assume we do not have the observed Ωm0 data, how do this Ωm0 change the result of ξ. In other words, if the observed Ωm0 data float around some value, then how is the fitting result? We also consider the curvature.
In the figure below, it seems that there is a point where three lines converge. This has something to do with the phenomena that

pltξvΩm0ICCManSum



Some data for flat ΛCDM universe. The following data shows how Ωm0 change our results for ξ if we already have transition redshift data {0.426|0.376,0.508}.

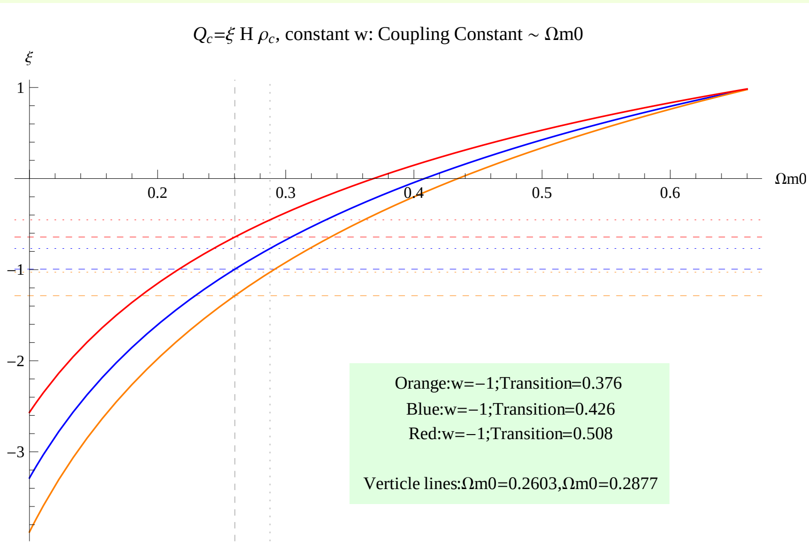
If Ωm0 varies 5 percent from 0.274,

tabξICCSum

For Ωm0∈0.274 (1 ± 0.05) Table of ξ for different Ωm0~Transition combination			
Ωm0⋱Transition	0.426	0.376	0.508
0.2603	−0.994339	−1.28571	−0.641508
0.274	−0.877755	−1.15303	−0.544482
0.2877	−0.767582	−1.02756	−0.452892

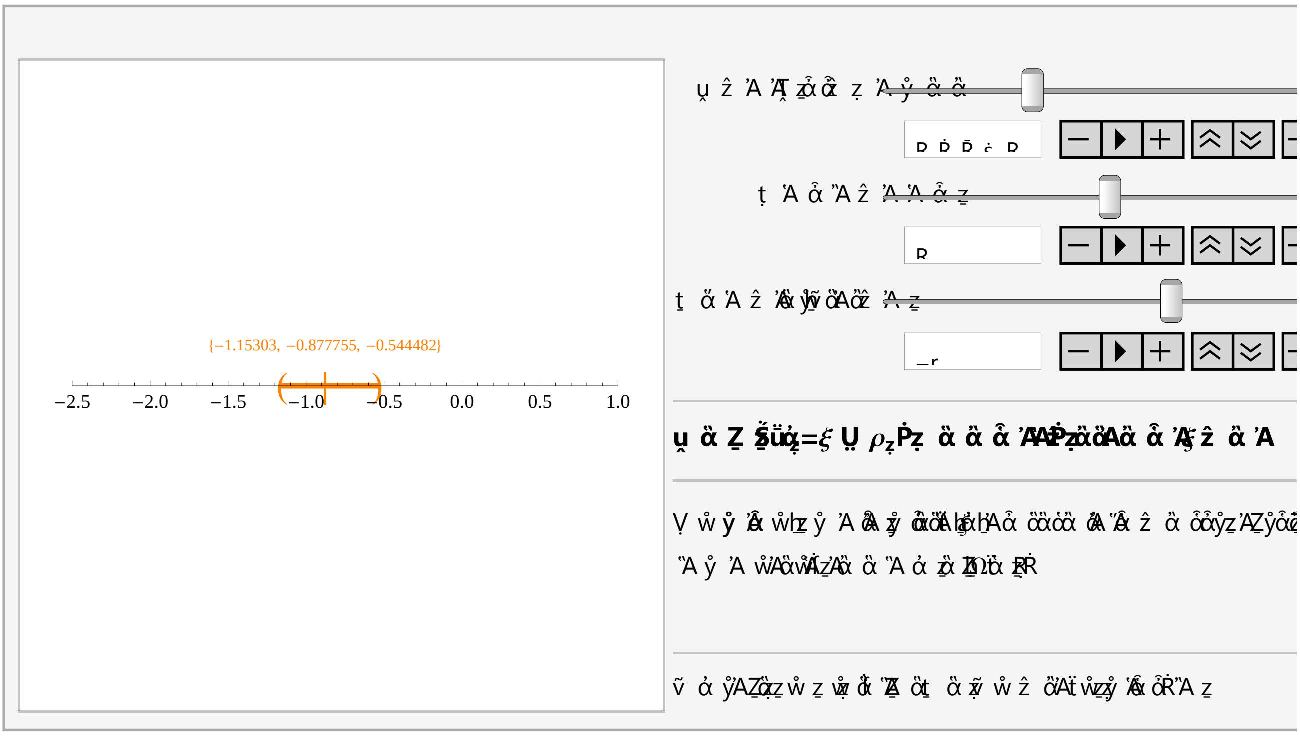
Monotonic line.

pltξvΩm0ICCSum

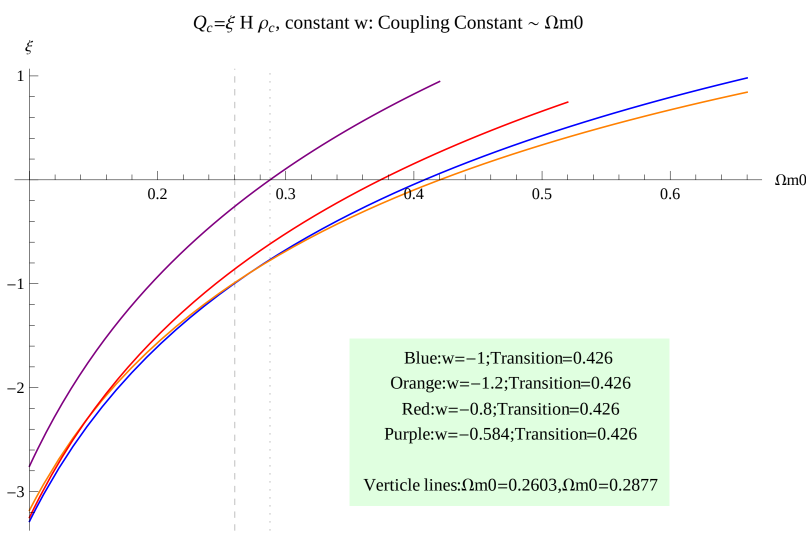


Besides Ωm0, we can also find out the effects of Curvature, EoS. Assuming we have a constrain of Transition redshift (0.376,0.508) with a center at 0.426.

fitξ2ICCManSum



pltξvΩm0ICCSum2



tabξvΩm0ICCSum21
tabξvΩm0ICCSum22

Imaginary part of ξ when transition redshift is 0.426		
⋱	w=−0.3334	w=−0.3333
Ωm0=0.2603	0	3.7462×10^{-14}

EoS value when ξ=0	
⋱	Transition 0.426
Ωm0=0.2877	−0.58406

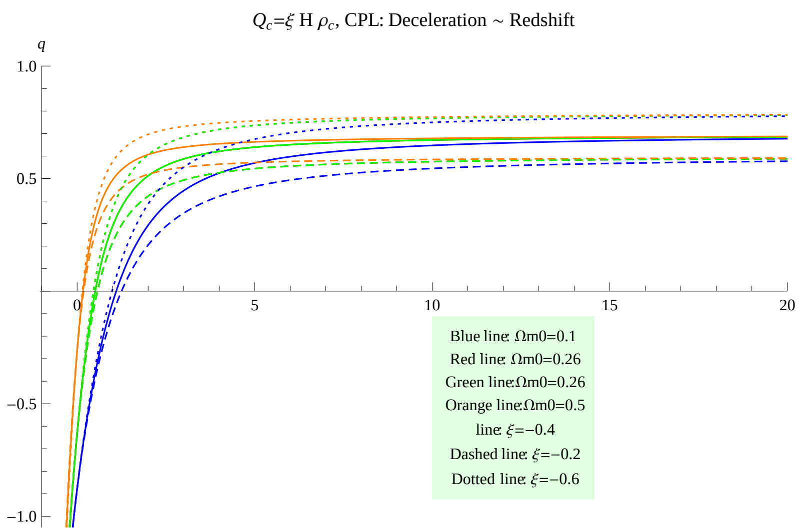
So we give the result that w ∈ (-0.58406, -0.3334) if we constrain Ωm0 = 0.2603 and transition redshift 0.426.

Interacting model $Q_c = \xi H \rho_c$ with constant ξ and CPL parameterized EoS $w = w0 + w1\frac{z}{1+z}$.

For a flat universe, choose the parameters {w0=-1.02,w1=0.6}, the region for interation cosntant ξ should be (-1.04,-0.21) with a center at -0.64, i.e., $-0.64^{+0.42}_{-0.40}$,derived from the (transition redshift, Ωm0) plane, while a result of (-1.01, -0.23) with a center at -0.63, i.e., $-0.63^{+0.40}_{-0.38}$, derived from (transition redshift, $\frac{\Omega m0}{\Omega d0}$) plane.

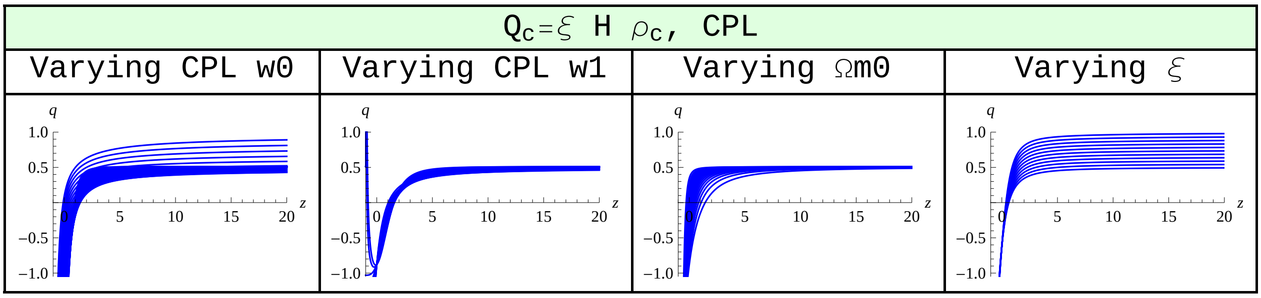
Deceleration parameter is shown below. Behaves similiar to the constant ξ constant w situation.

pldecICCPLShowSum



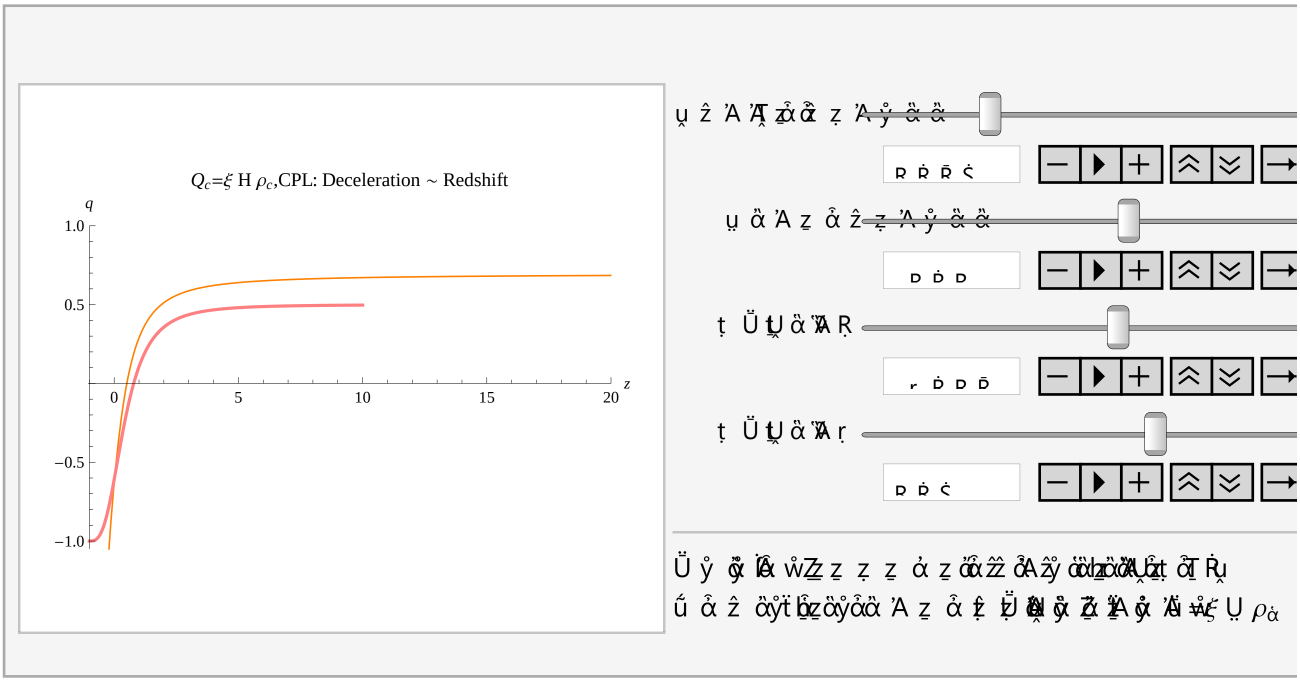
The following plots shows the effect of different parameters. Each plot shows how do the deceleration parameter vs redshift line change under uniformly distributed w0,w1,Ωm0 or ξ.
w0 moves the line up or down, but not monotonously;
w1 changes the late time behavior;
Ωm0 changes the slope;
ξ has moves the line up or down;

varyingICCPLShowSum



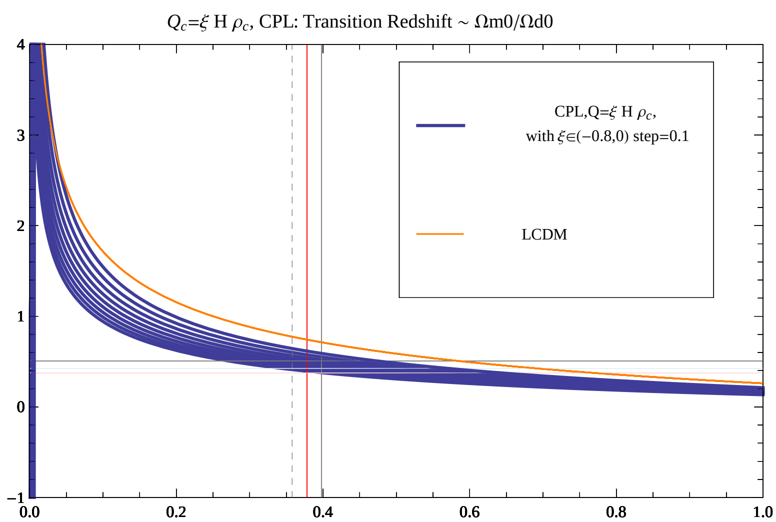
A toy to play with the q~z plot. When ξ=0, w0=-1, w1=0, the curve reduced to LCDM curve.

`pldecICCPLManSum`



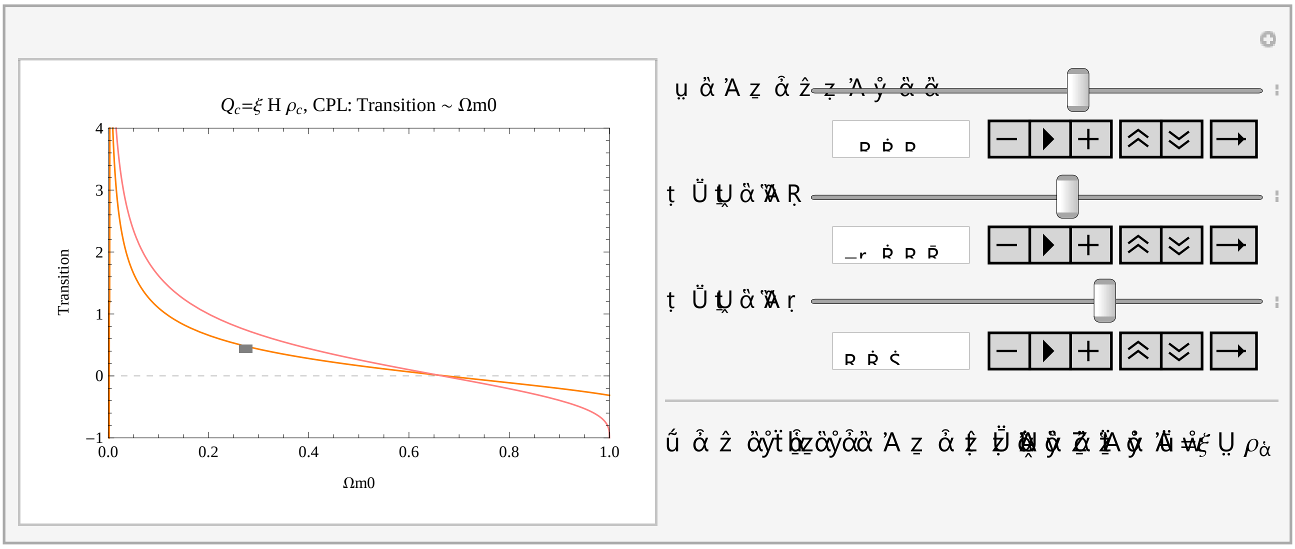
A plot shows how bad it is to use transition redshift to constrain interacting model. This is a CPL parameterized example. For $\xi \in$(-0.8,0), the line just stays near the allowed region constrained by Riess's results (References, Data From, 2).

`pltransrICCPLDense`



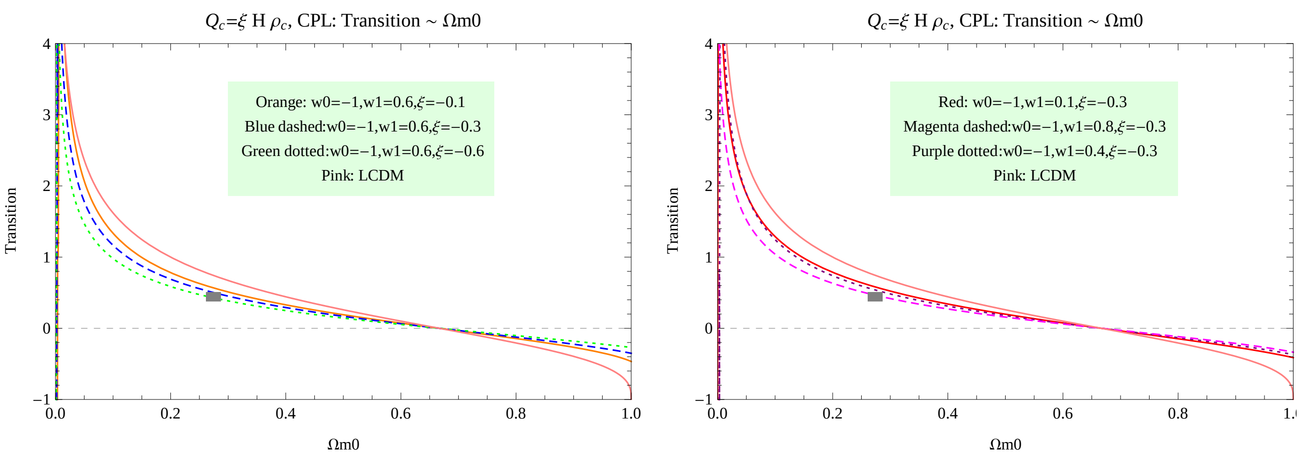
From the manipulate below, we find for w0=-1, there is a point on this transition ~ Ωm0 curve do not change with coupling constant ξ and w1. (Well, what's the use of that...)

```
plztrICCPLManSum
```



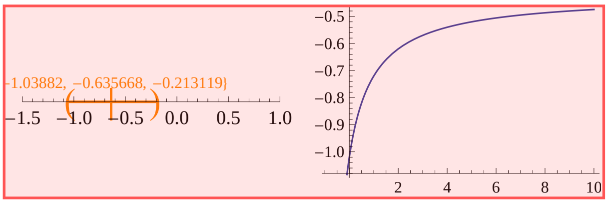
An explicit proof of this statement.

```
plztrExamICCPLSum
```



A manipulate of the fitting results.

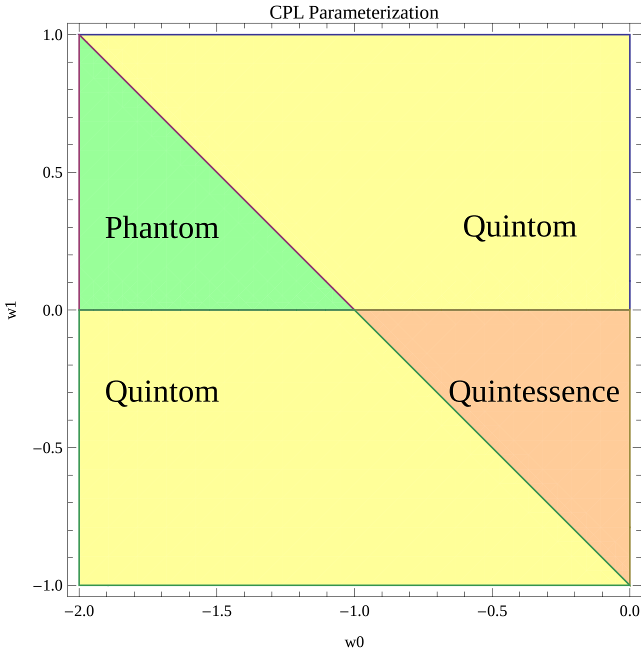
```
fitξICCPLManSum
```


{-1.03882, −0.635668, −0.213119}
−1.5 −1.0 −0.5 0.0 0.5 1.0
−0.5
−0.6
−0.7
−0.8
−0.9
−1.0
2 4 6 8 10

Category

For different w0 and w1 in its EoS equation,

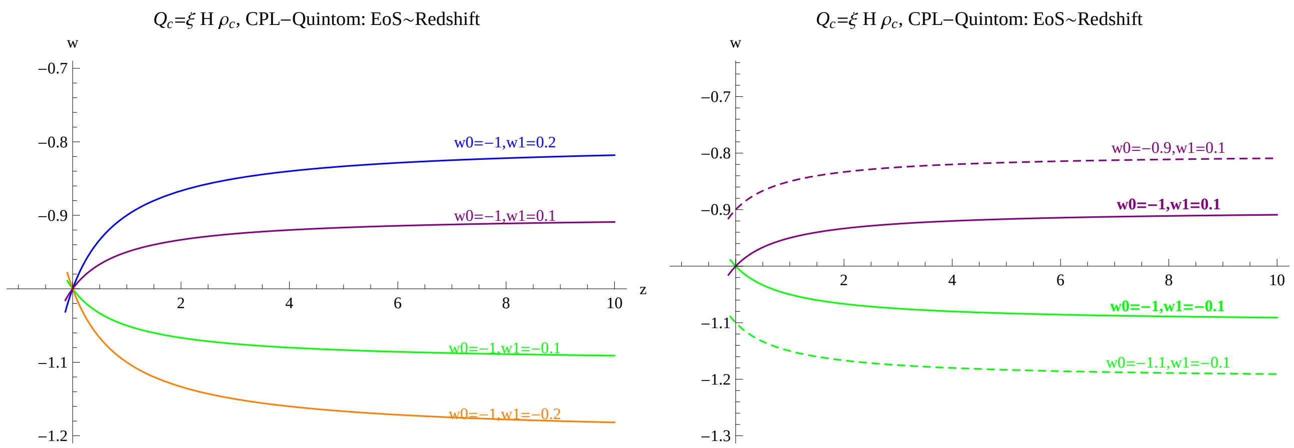
```
plEoSPhaseICCPL
```



Quintom

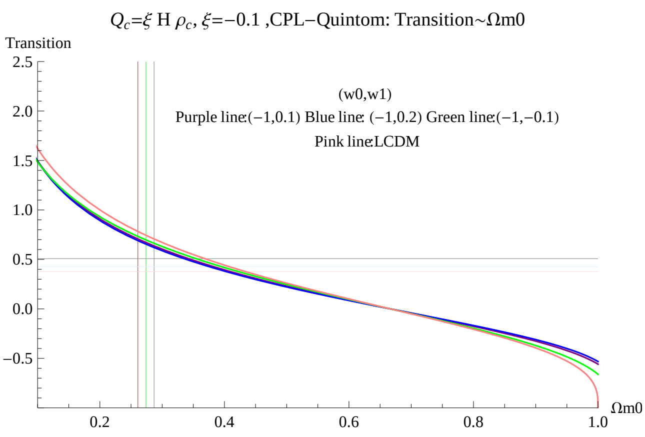
Color illustrations for the following two figures.
“Purple line:(-1,0.1)_ Blue line: (-1,0.2) Green line:(-1,-0.1)\n_ Pink line:LCDM”
“Purple Dashed:(-0.9,0.1) Green Dashed:(-1.1,0.1)”

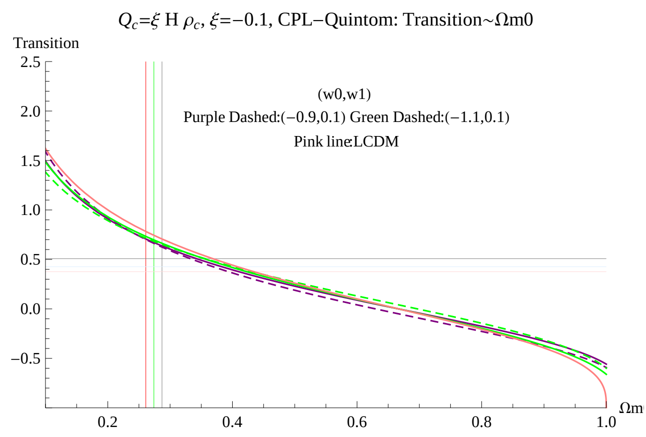
```
plEoSICCPLQuintomSum
```



Plots of Transition redshift vs Ωm0.
Legends are shown on the plots. Hard to distingush from each other.

plztrICCPLQuintomSum



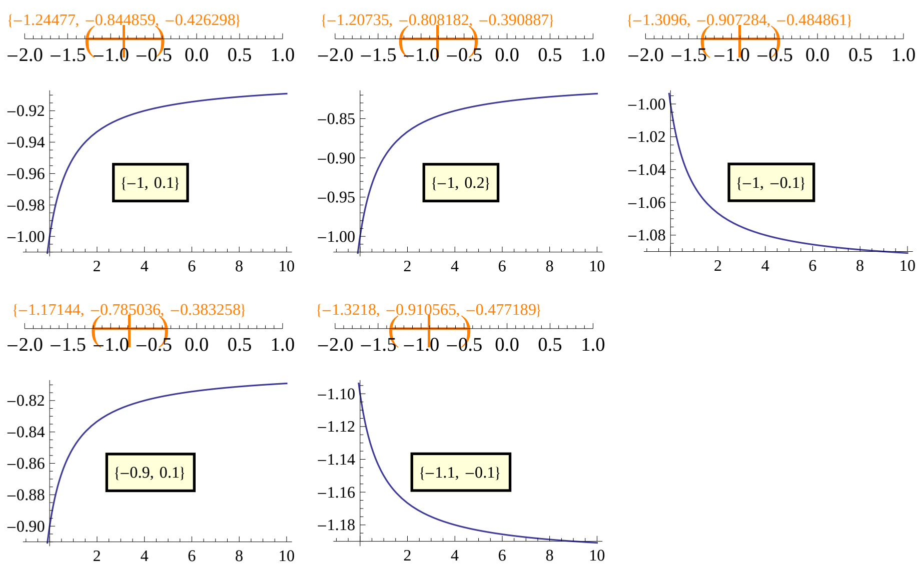


For **different EoS**, the fitting results are different. The following table and plots show how do w0 and w1 change the results.

tabξvwExamICCPLQuintom

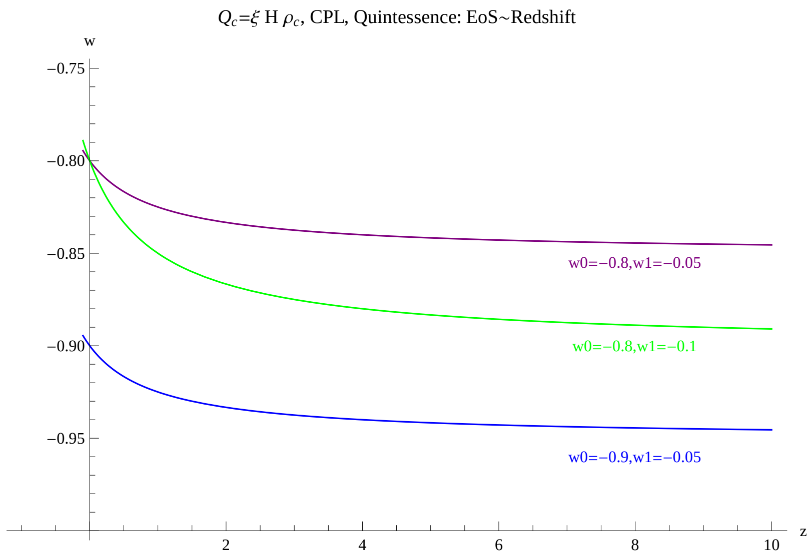
ξ results for $Q_c = \xi\ H\ \rho_d$, CPL,Quintom.			
{w0,w1}	Center	Lower	Upper
{−1, −0.1}	−0.907284	−1.3096	−0.484861
{−1, 0}	−0.877755	−1.27874	−0.457448
{−1, 0.1}	−0.844859	−1.24477	−0.426298
{−0.9, 0.1}	−0.785036	−1.17144	−0.383258
{−1.1, −0.1}	−0.910565	−1.3218	−0.477189

pltfitξICCPLQuintomSum



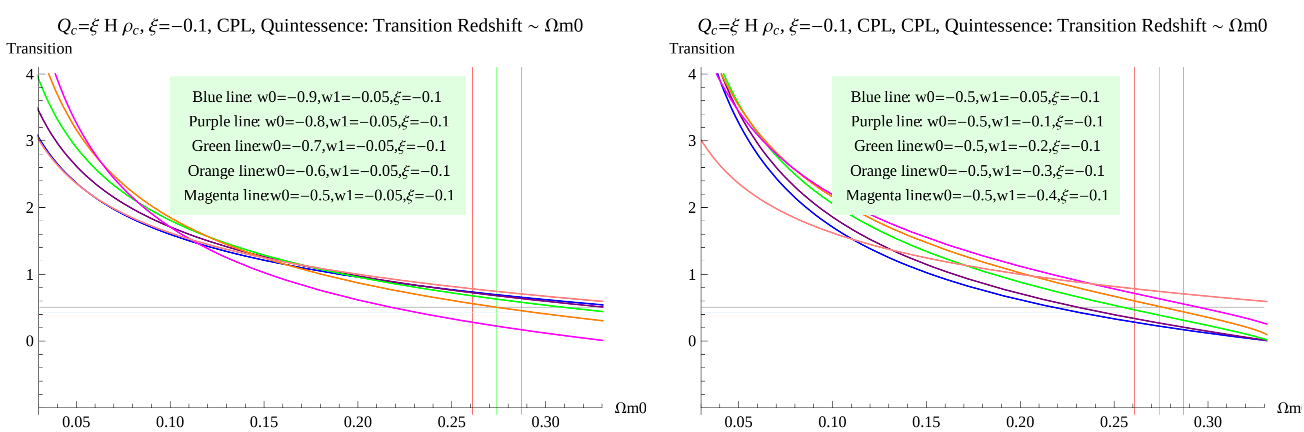
Quintessence

plEoSICCPLQuintessenceSum



Different from constant w results,

plztrICCPLQuintessenceSum

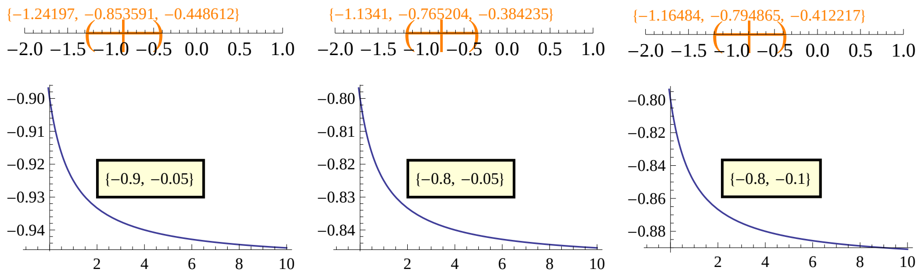


Some ξ fitting results are shown below. This shows how do w0 and w1 change ξ results.

tabξvwExamICCPLQuintessence

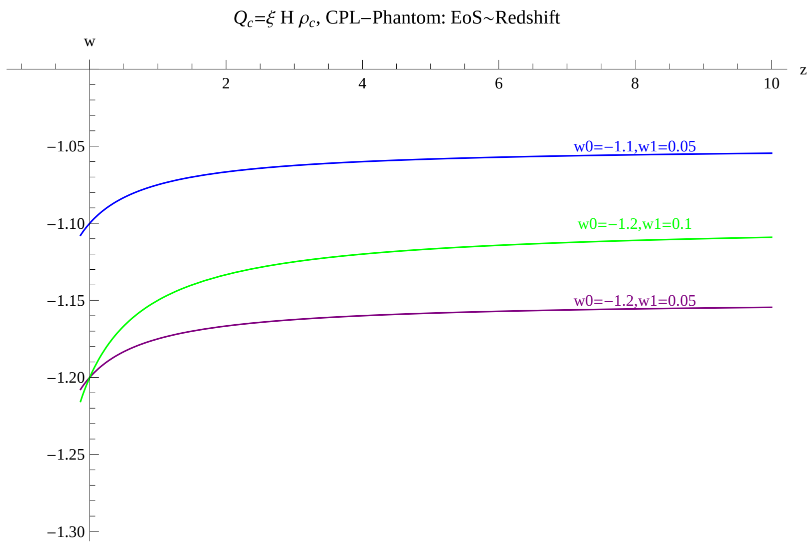
ξ results for Q_c=ξ H ρ_d, CPL,Quintessence.			
{w0,w1}	Center	Lower	Upper
{−0.9, −0.05}	−0.853591	−1.24197	−0.448612
{−0.8, −0.05}	−0.765204	−1.1341	−0.384235
{−0.8, −0.1}	−0.794865	−1.16484	−0.412217

pltfitξICCPLQuintessenceSum

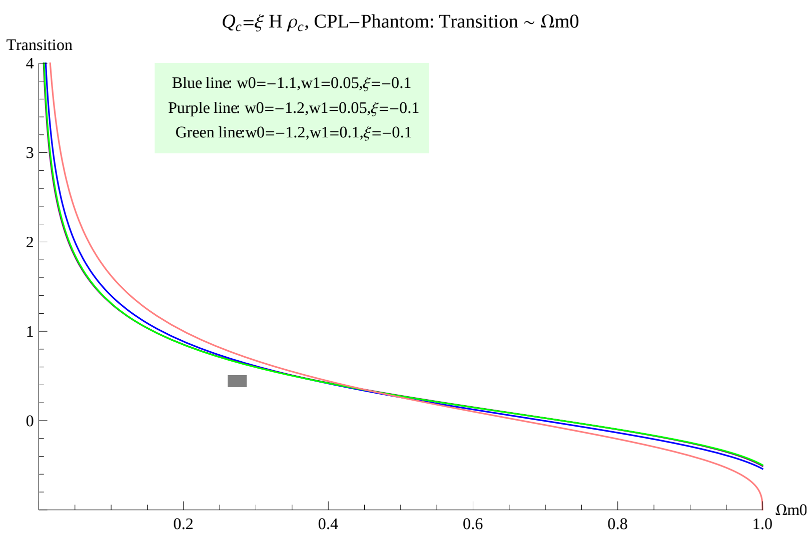


Phantom

plEoSICCPLPhantomSum



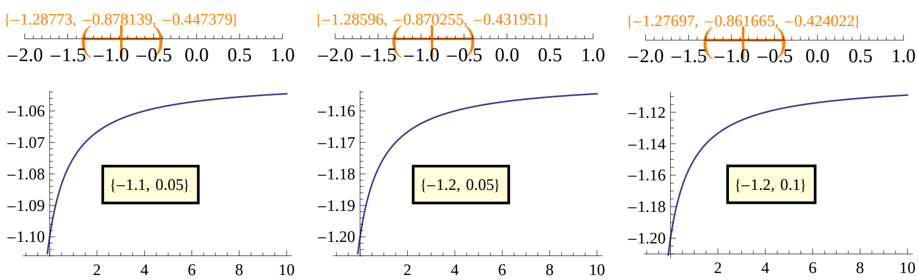
plztrICCPLPhantomSum



tabξvwExamICCPLPhantom

ξ results for Q_c=ξ H ρ_d, CPL,Phantom.			
{w0,w1}	Center	Lower	Upper
{-1.1, 0.05}	-0.878139	-1.28773	-0.447379
{-1.2, 0.05}	-0.870255	-1.28596	-0.431951
{-1.2, 0.1}	-0.861665	-1.27697	-0.424022

pltfitξICCPLPhantomSum

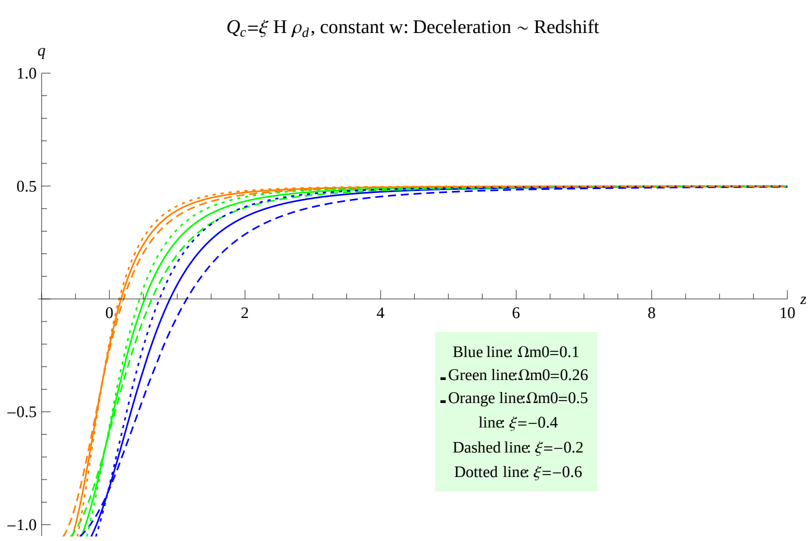


■ Interacting model $Q_c = \xi H \rho_d$ with constant ξ and constant EoS w.

Derived from (transition redshift, Ωm0) plane, the allowed region for coupling constant ξ is (-1.06,-0.42) with a center at -0.76, i.e., $-0.76^{+0.34}_{-0.30}$, taken the case that the universe is flat, and choose the EoS parameter {w=-1}.
Derived from the (transition redshift, $\frac{\Omega m0}{\Omega d0}$) plane, the allowed region of coupling constant ξ is (-1.07,-0.41) with a center at -0.76, i.e., $-0.76^{+0.35}_{-0.31}$.

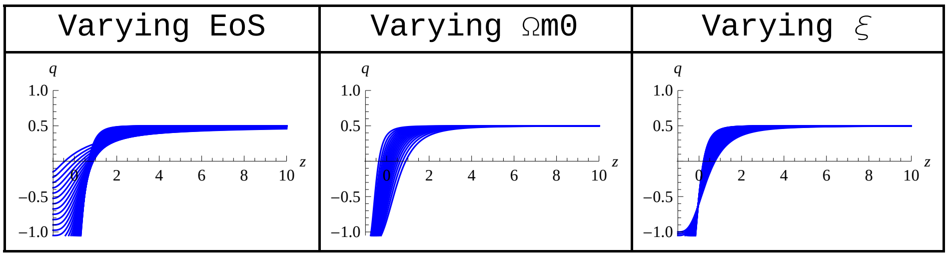
The plots of deceleration parameter are shown below. At the limit z->Infinity, the deceleration parameter ALL goes to $\frac{1}{2}$.
Theoretically, this limit is $\frac{1}{2}$ which is not related to any parameters, with $3\,w+\xi<0$.

pldecI2CCShowSum



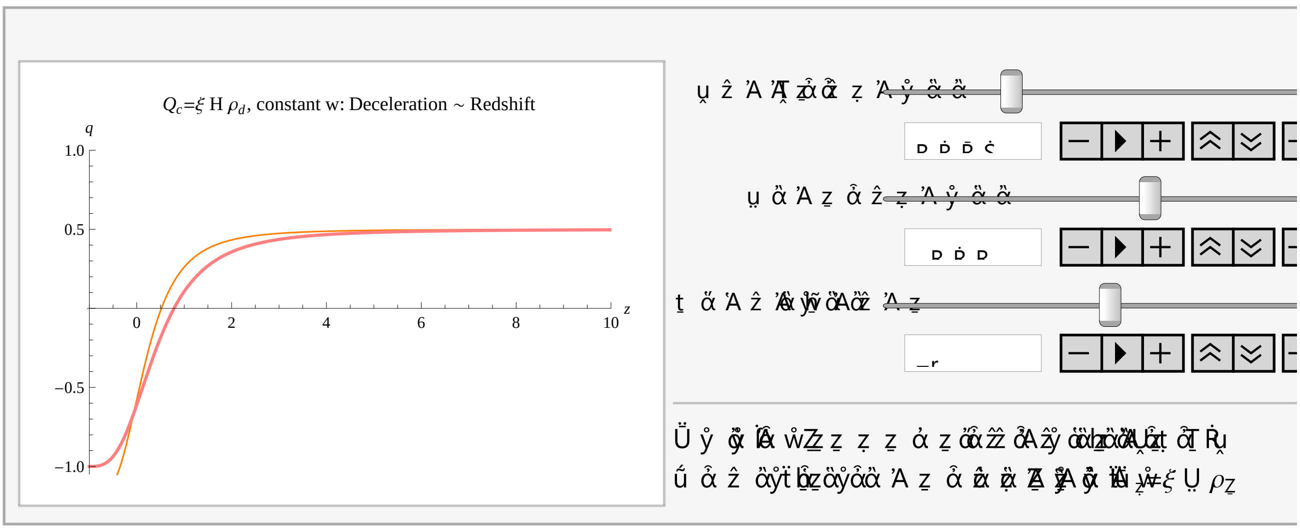
To check the effect of different parameters, another plot is shown.

```
varyingI2CCShowSum
```



A toy to play with deceleration vs z curve is also provied

```
pldecI2CCManSum
```



If the transition happens before z=0, increasing couling ξ will delay the transition.
If the it happens after z=0, increasing coupling ξ will bring forward the emergence of transition.

The following figure shows this result.

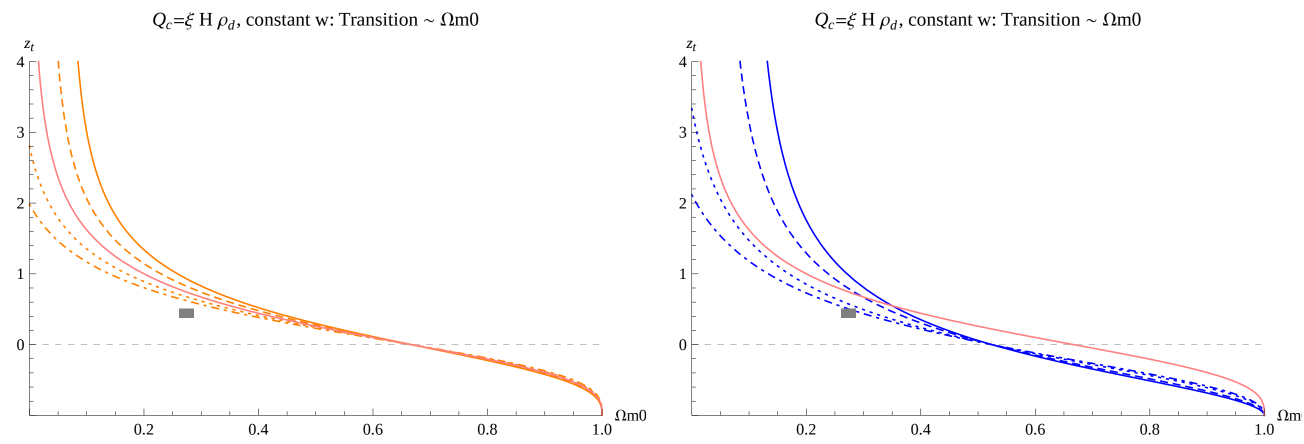
Gray rectangle is the region given by Riess.

Orange for w=-1
Blue for w=-0.9

line: ξ=0.2
Dashed: ξ=0.1
Dotted: ξ=-0.1
DotDashed: ξ=-0.2

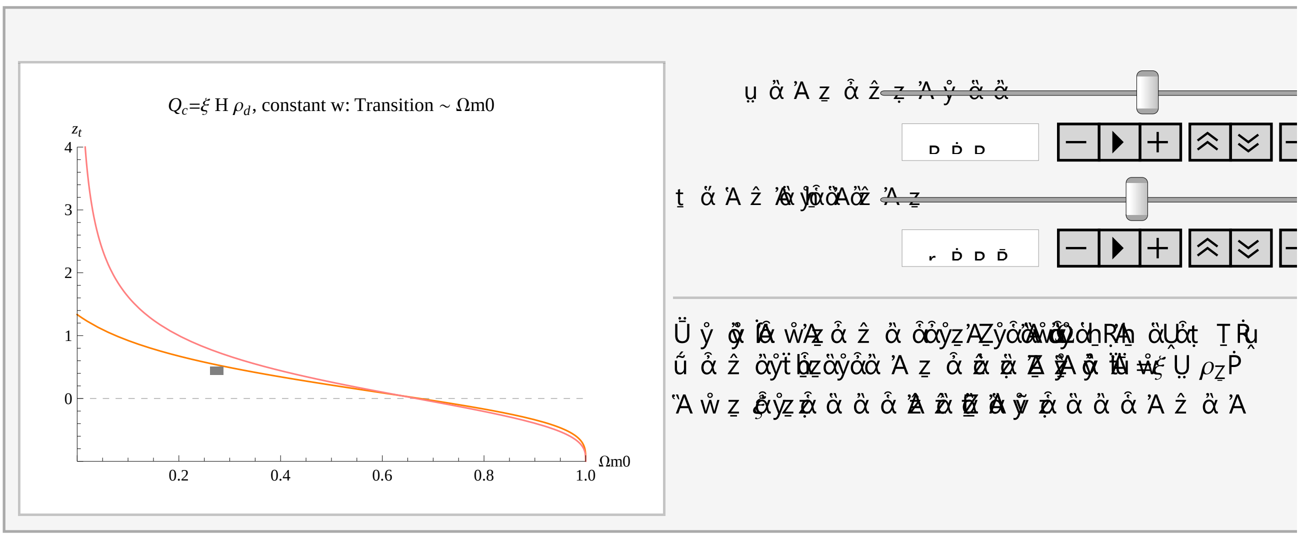
Pink line:w=1, ξ=0

```
plztrvsΩm0I2CCSum
```



A toy of transition redshift. Gray rectangle is the allowed region of Ωm0~Transition redshift

```
plztrI2CCManSum
```



The fitting results of coupling constant ξ is

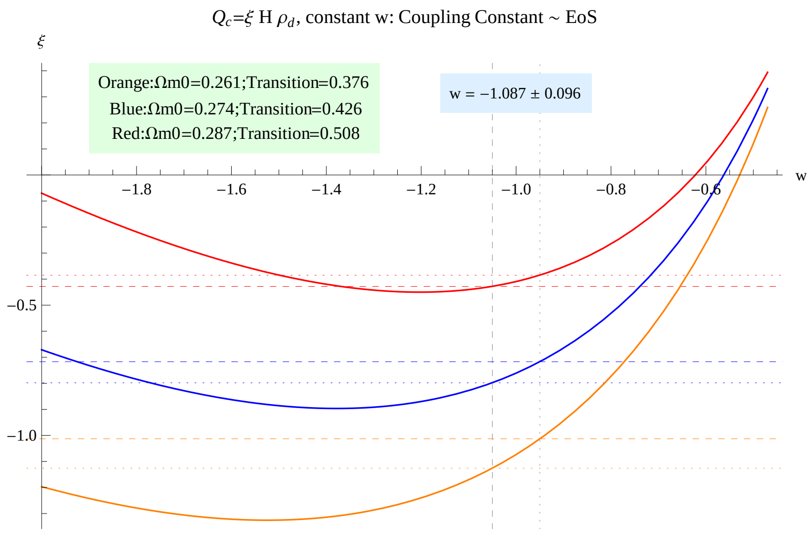
```
fitξI2CCManSum
```

For different constant EoS, the fitting results using Ωm0∈(0.261,0.287) with a center value 0.274 and Transition redshift ∈ (0.376,0.508) with a center value 0.426. When EoS is very small, the line might cross zero. But that is not so useful.

Some data: (-1 within 5%: (-1.05,-0.95))
w=-1 (-1.074,-0.409) center:-0.761
w=-1.05 (-1.126,-0.428) center:-0.798
w=-0.95 (-1.013,-0.385) center:-0.717

The following graph show how do ξ changes with EoS. The grid lines are the results of $w = -1 \pm 0.05$. Two verticle lines are -1.05 and -0.95 respectively. Horizontal lines are their intersections with the ξ~w lines.

pltξvwExam1I2CC

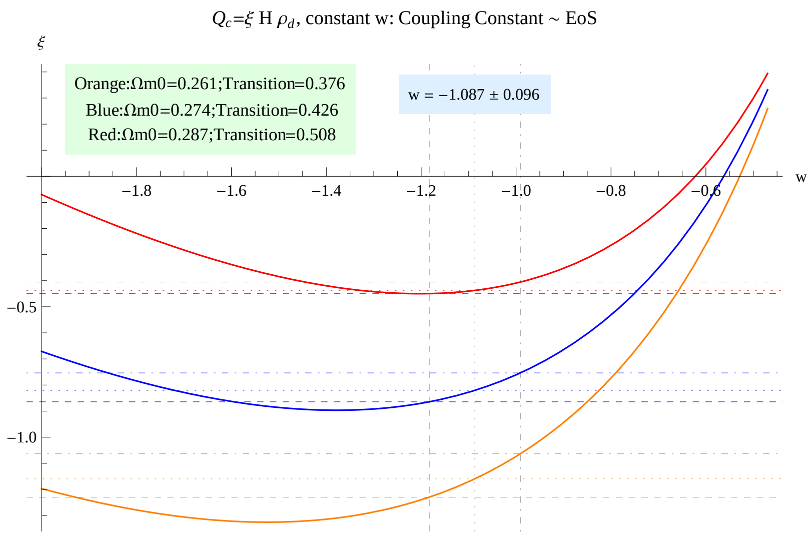


Or we can use some fitting results from WMAP etc. Take the example of w=-1.087±0.096.

tabξvwExamI2CC

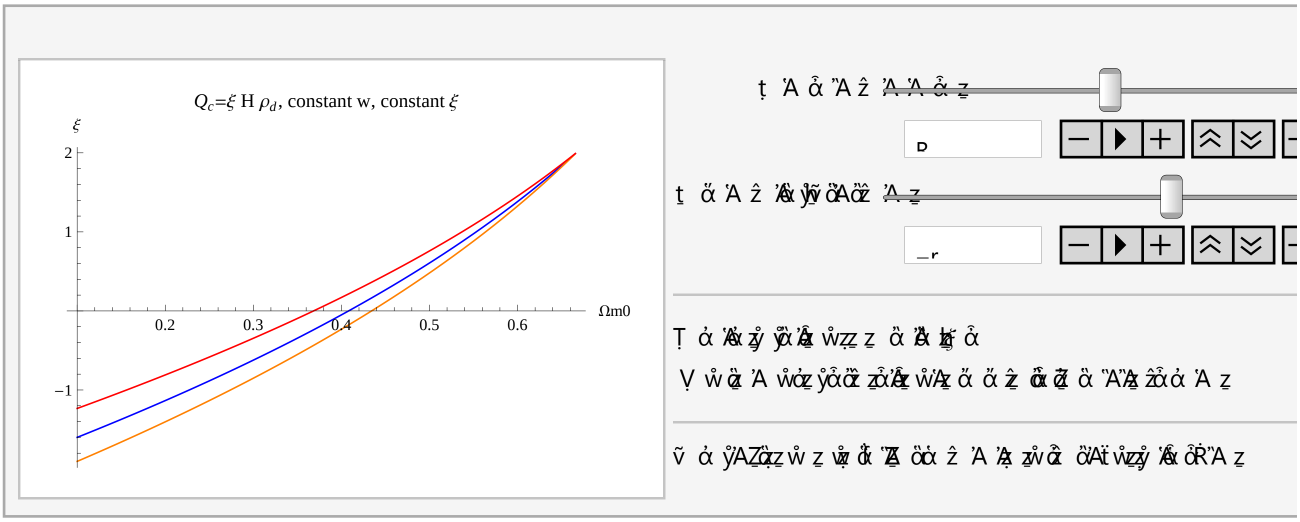
Q_c=ξ H ρ_d,Constant w. (Data used:Data From, 2)			
w	Center	Lower	Upper
−1.183	−0.864289	−1.22984	−0.449552
−1.087	−0.820486	−1.15946	−0.437339
−0.991	−0.753634	−1.06346	−0.405262

pltξvwExamI2CC



Now we assume we do not have the observed Ωm0 data, how do this Ωm0 change the result. In other words, if the observed Ωm0 data float around some value, then how is the fitting result? We also consider the curvature.

pltξvΩm0I2CCManSum

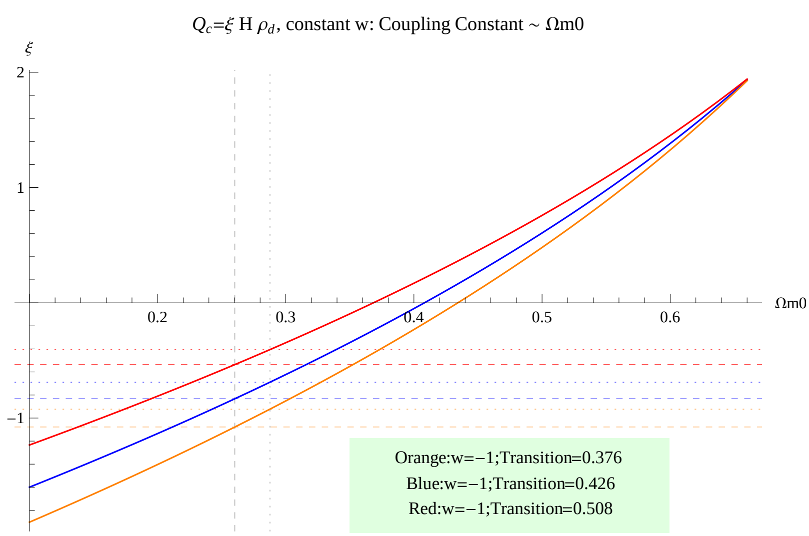


If Ωm0 varies 0.05 percent from 0.274,

tabξI2CCSum

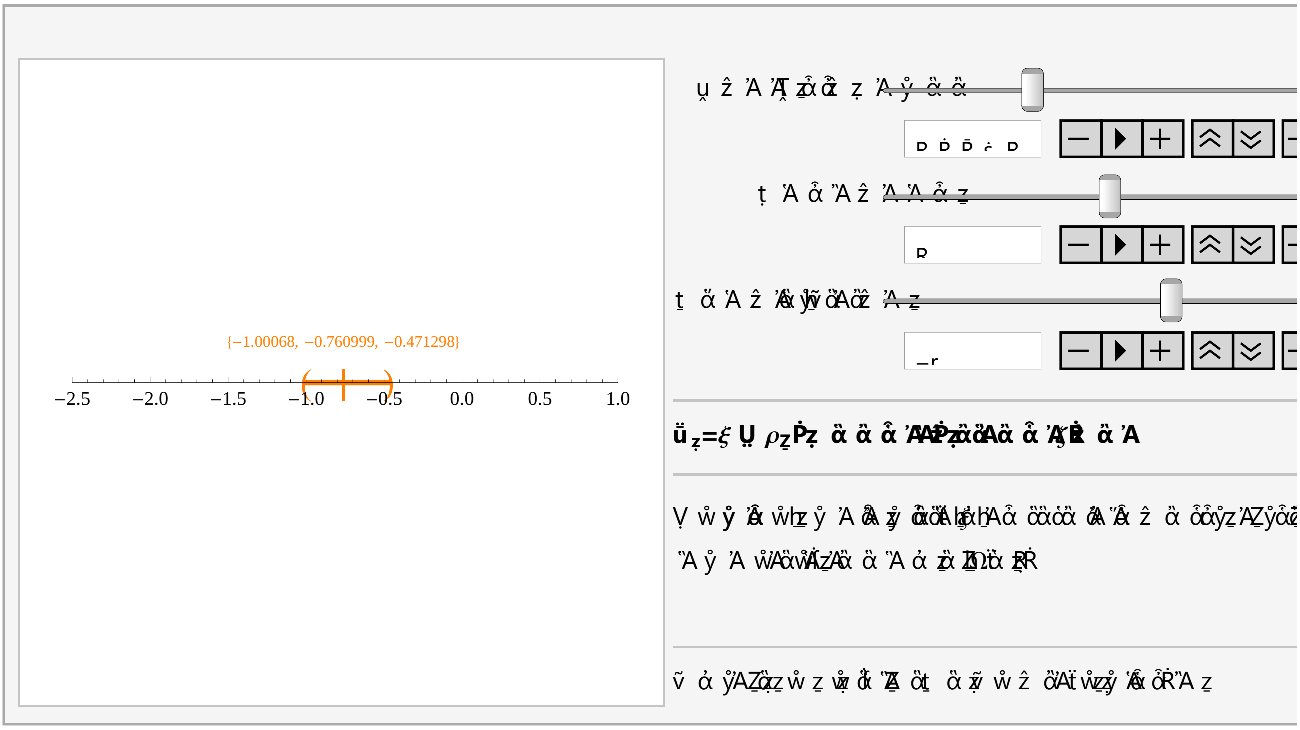
For Ωm0∈0.274 (1 ± 0.05) Table of ξ for different Ωm0~Transition combination			
Ωm0⋱Transition	0.426	0.376	0.508
0.2603	−0.832284	−1.07758	−0.53584
0.274	−0.760999	−1.00068	−0.471298
0.2877	−0.688664	−0.922602	−0.40585

pltξvΩm0I2CCSum



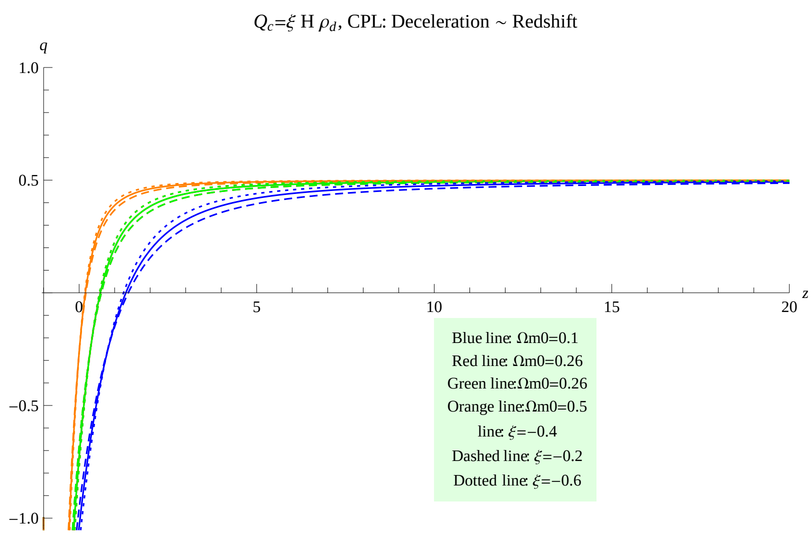
In addition, we can also find out the effects of Curvature, EoS. Assuming we have a constrain of Transition redshift (0.376,0.508) with a center at 0.426.

fitξ2I2CCManSum

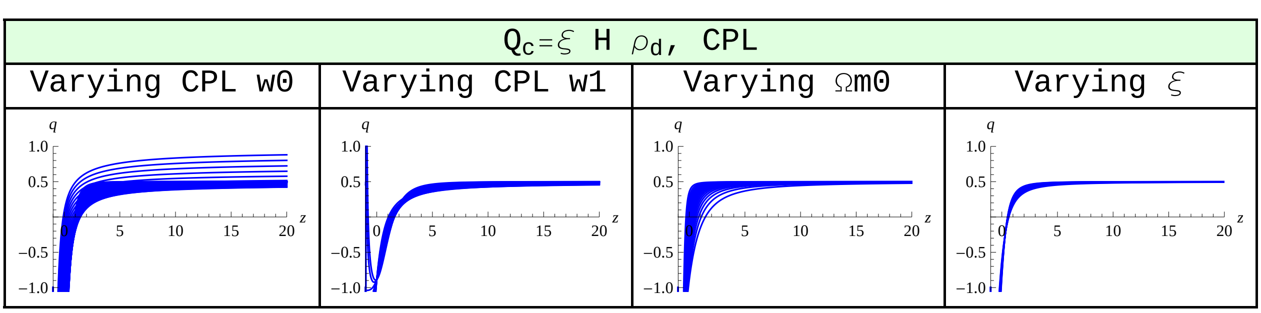


Interacting model $Q_c = \xi H \rho_d$ with constant ξ and CPL parameterization.

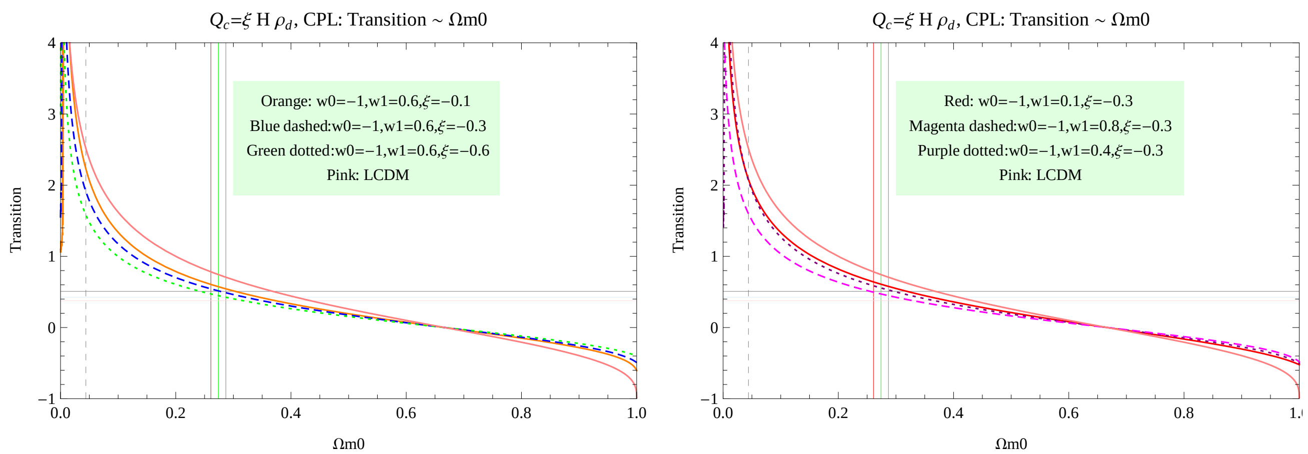
pldecI2CCPLShowSum



varyingI2CCPLShowSum

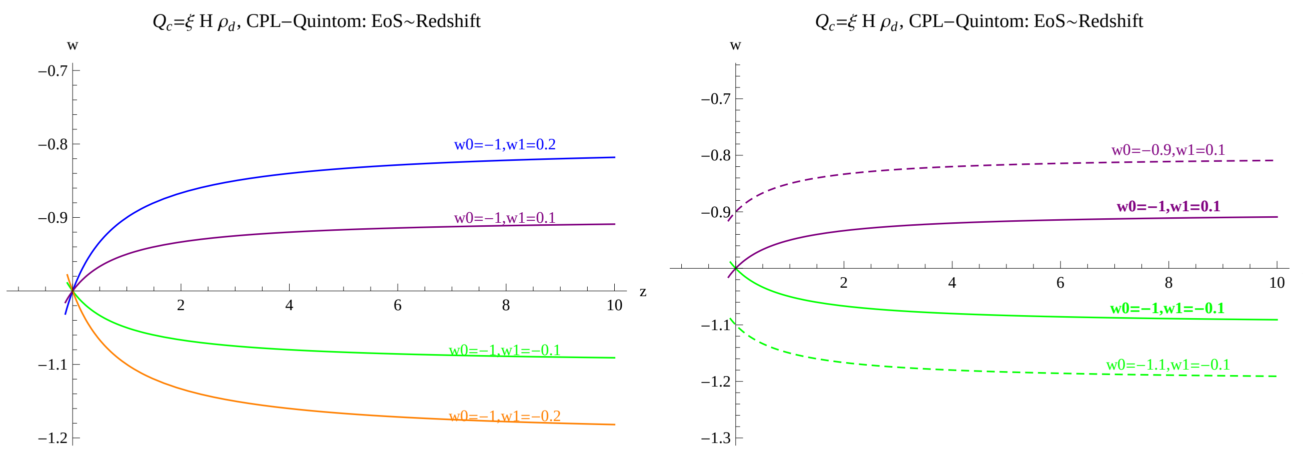


plztrExamI2CCPLSum

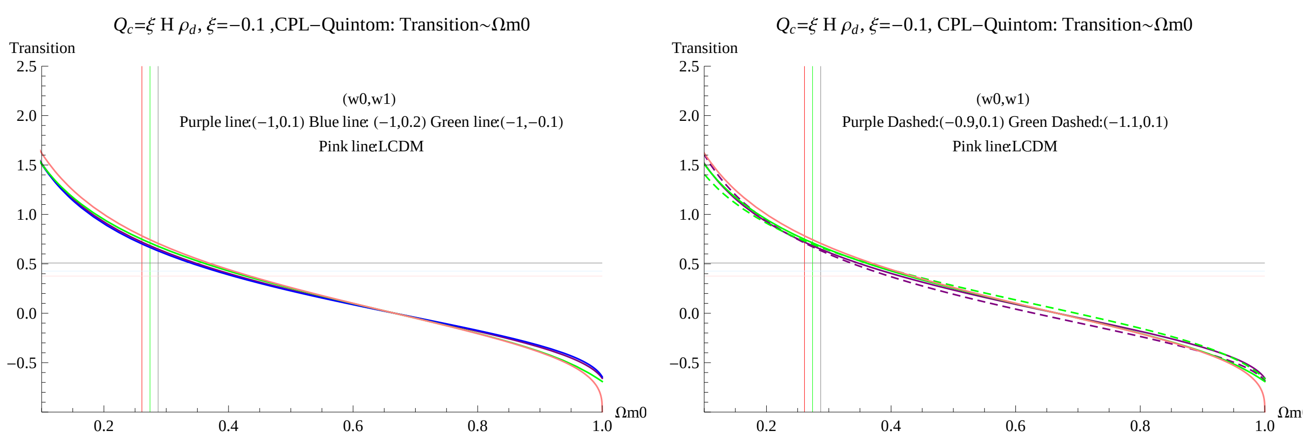


Quintom

plEoSI2CCPLQuintomSum



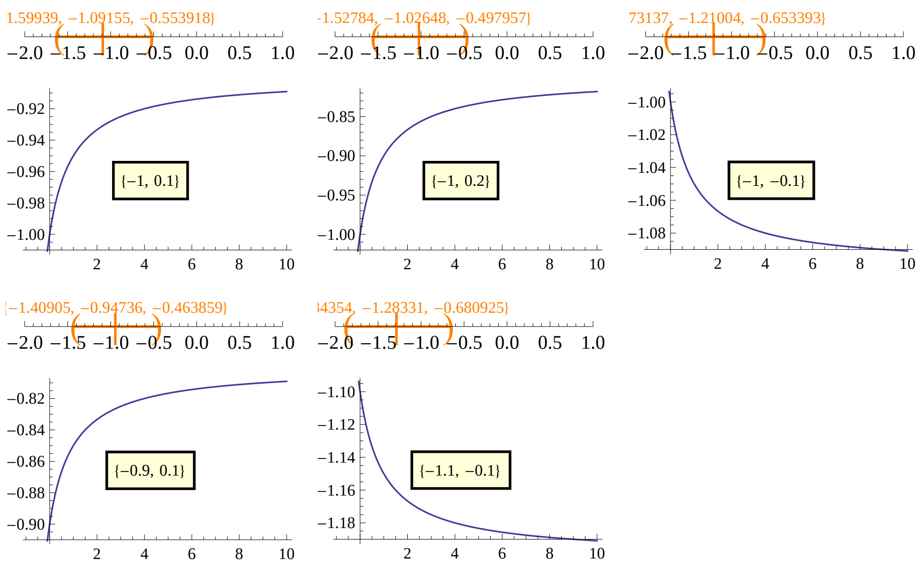
plztrI2CCPLQuintomSum



tabξvwExamI2CCPLQuintom

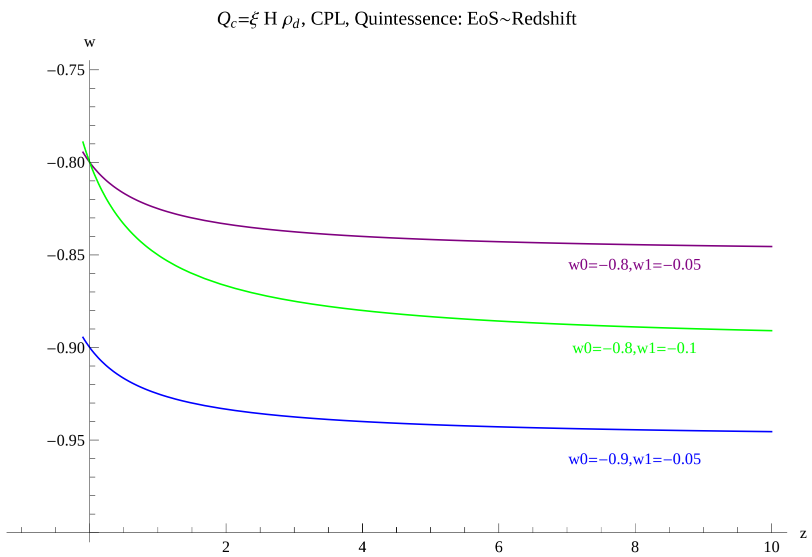
ξ results for $Q_c=\xi\ H\ \rho_d$, CPL,Quintom.			
{w0,w1}	Center	Lower	Upper
{−1, −0.1}	−1.21004	−1.73137	−0.653393
{−1, 0}	−1.15265	−1.66715	−0.605615
{−1, 0.1}	−1.09155	−1.59939	−0.553918
{−0.9, 0.1}	−0.94736	−1.40905	−0.463859
{−1.1, −0.1}	−1.28331	−1.84354	−0.680925

pltfitξI2CCPLQuintomSum

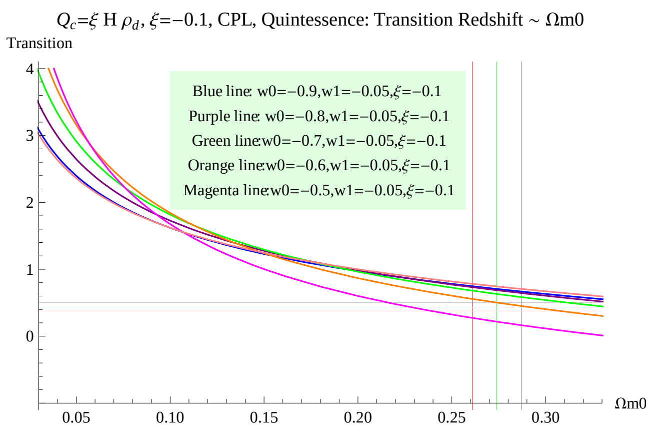


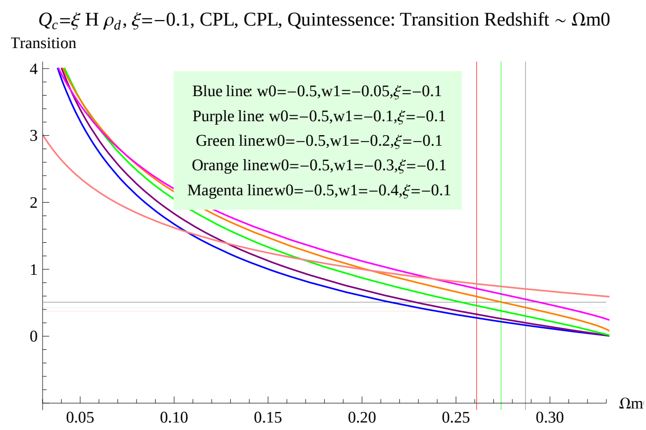
Quintessence

plEoSI2CCPLQuintessenceSum



plztrI2CCPLQuintessenceSum

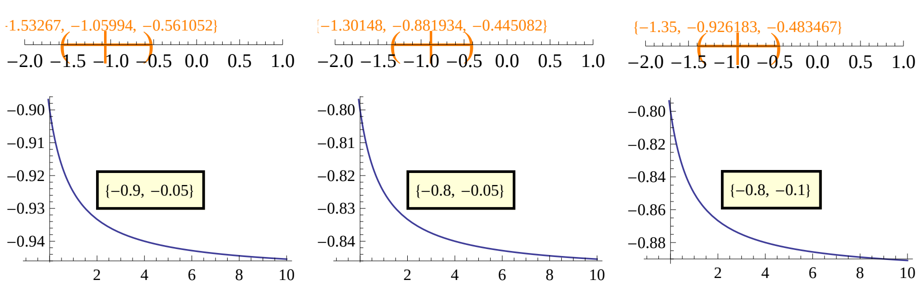




tabξvwExamI2CCPLQuintessence

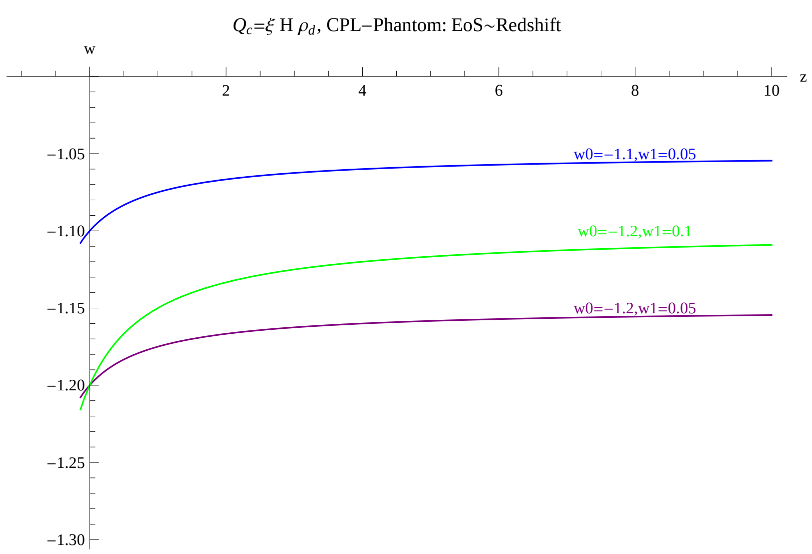
ξ results for Q_c=ξ H ρ_d, CPL,Quintessence.			
{w0,w1}	Center	Lower	Upper
{−0.9, −0.05}	−1.05994	−1.53267	−0.561052
{−0.8, −0.05}	−0.881934	−1.30148	−0.445082
{−0.8, −0.1}	−0.926183	−1.35	−0.483467

pltfitξI2CCPLQuintessenceSum

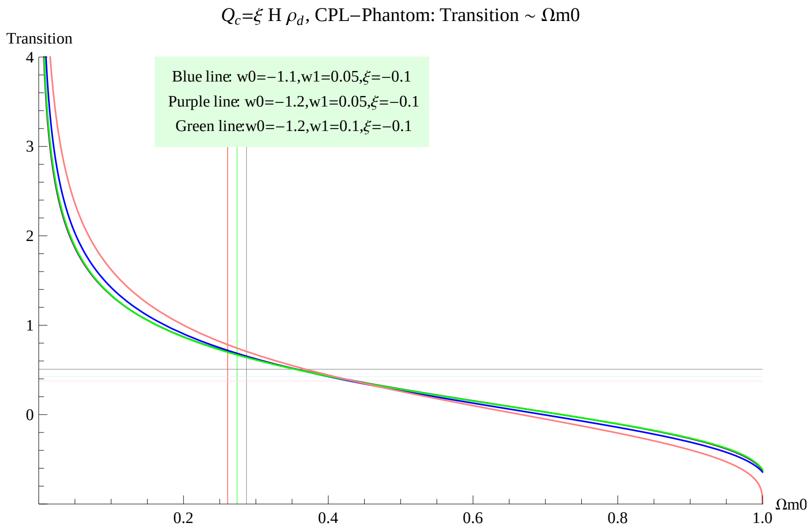


Phantom

plEoSI2CCPLPhantomSum



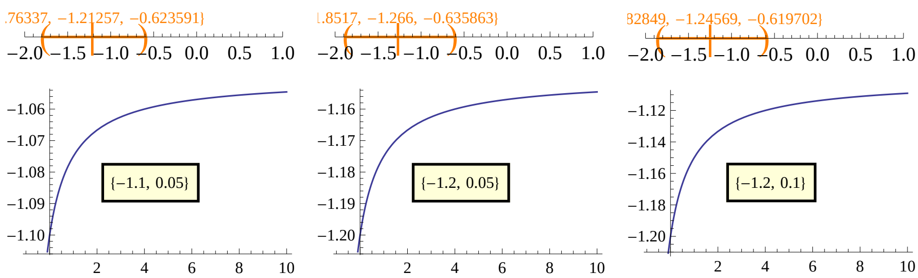
plztrI2CCPLPhantomSum



tabξvwExamI2CCPLPhantom

ξ results for $Q_c=\xi$ H ρ_d, CPL,Phantom.			
{w0,w1}	Center	Lower	Upper
{−1.1, 0.05}	−1.21257	−1.76337	−0.623591
{−1.2, 0.05}	−1.266	−1.8517	−0.635863
{−1.2, 0.1}	−1.24569	−1.82849	−0.619702

pltfitξI2CCPLPhantomSum



References

Data From

1. CPL data

Combining SN1a, BAO 3, WMAP5, H(z)   (From arXiv:0909.0596)

$\Omega m0 = 0.269^{+0.017}_{-0.008}$, $w0 = -0.97^{+0.12}_{-0.07}$, $w1 = 0.03^{+0.26}_{-0.75}$.

2. LCDM

From WMAP: Ωm0=0.265
arXiv:astro-ph/0611572, Riess et al   :
arXiv:1205.4688 : Ωm0=0.247 (+0.013, -0.013) and Transition 0.426 (+0.082, -0.050)

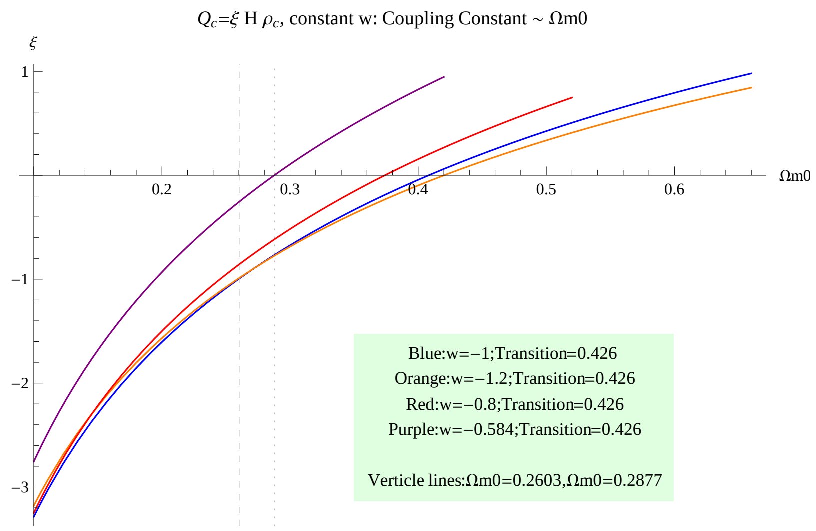
3. Equation of state

From arXiv:1202.0545v1
w = -1.087±0.096.

Supplementary

- **Interacting model $Q_c = \xi H \rho_c$ with constant ξ and constant EoS w.**

pltξvΩm0ICCSum2



tabξvΩm0ICCSum21
tabξvΩm0ICCSum22

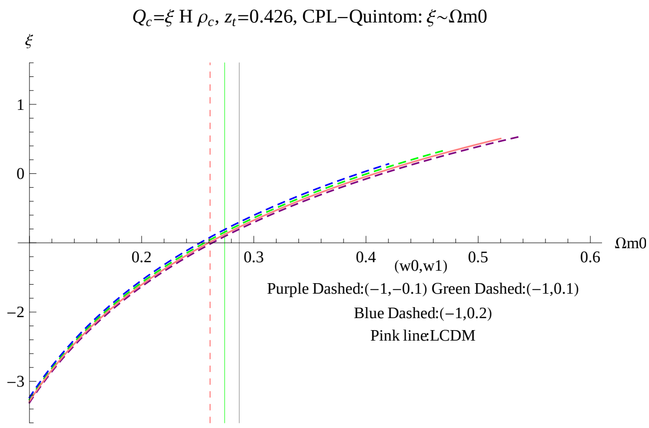
Imaginary part of ξ when transition redshift is 0.426		
⋱	w=−0.3334	w=−0.3333
Ωm0=0.2603	0	3.7462×10^{-14}

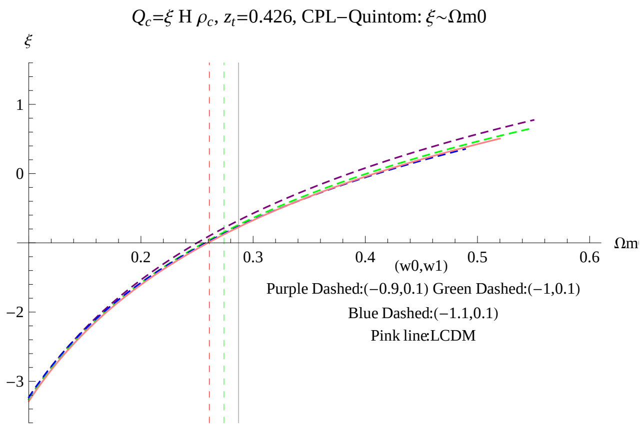
EoS value when ξ=0	
⋱	Transition 0.426
Ωm0=0.2877	−0.58406

So we give the result that w∈(-0.58406, -0.45064) if we constrain Ωm0=0.2603 and transition redshift 0.426.

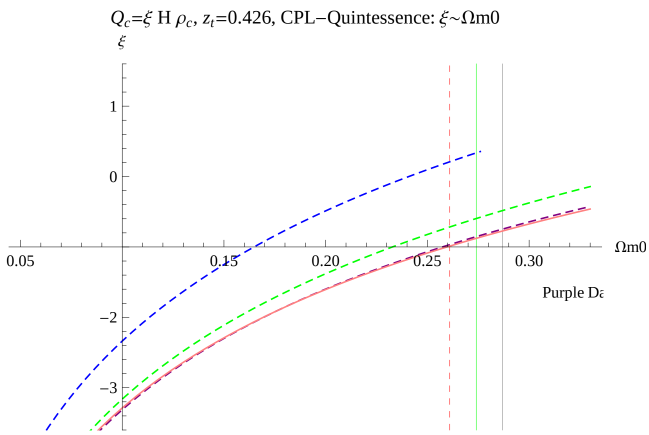
- **Interacting model $Q_c = \xi\, H\, \rho_c$ with constant ξ and CPL parameterized EoS $w = \mathrm{w0} + \mathrm{w1} \frac{z}{1+z}$.**

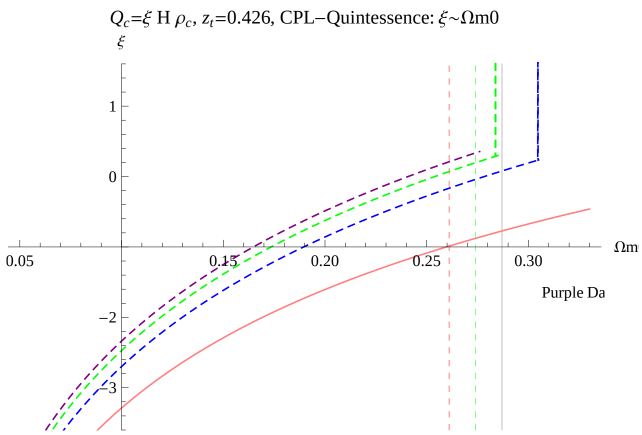
pltξvΩm0ICCPLQuintomSum



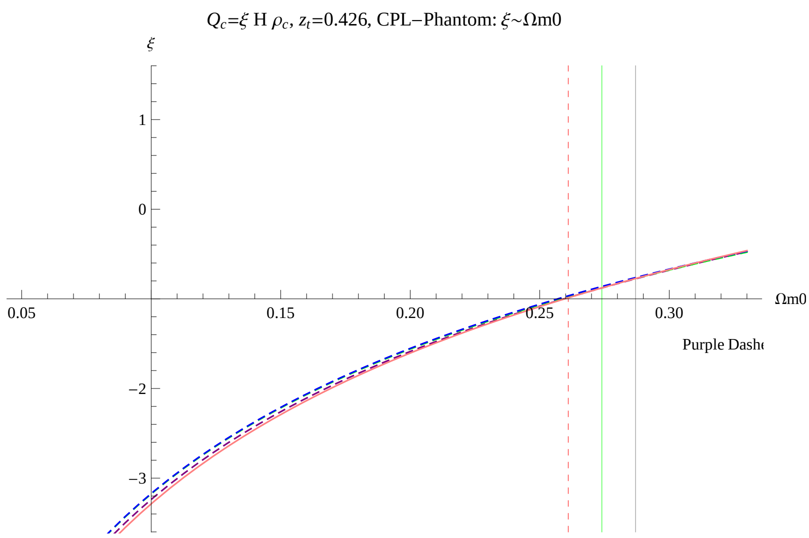


pltξvΩm0ICCPLQuintessenceSum

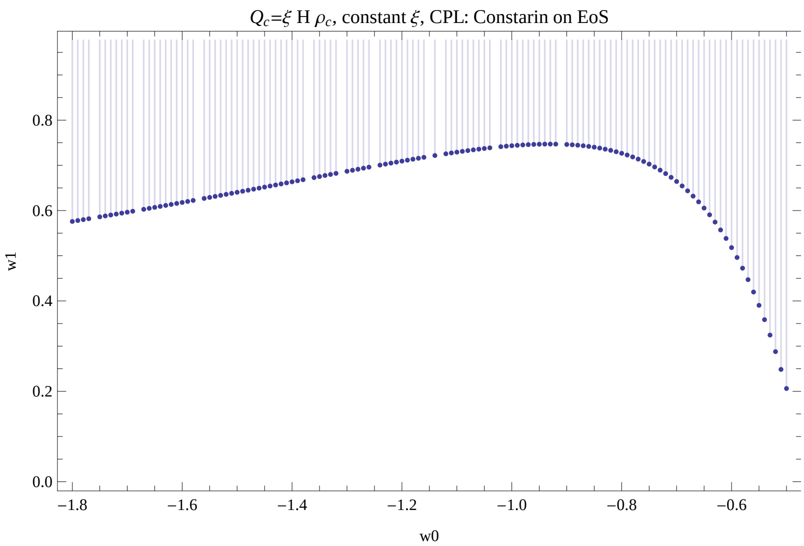




pltξvΩm0ICCPLPhantomSum

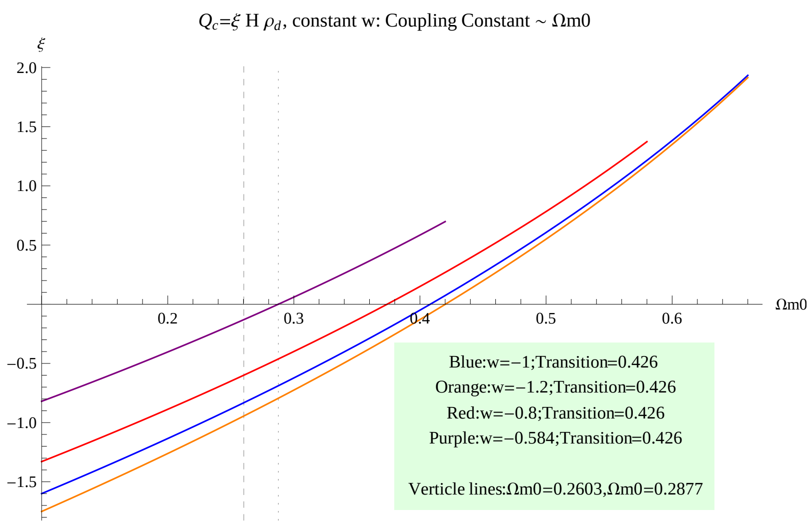


pltw0vw1ConsICCPL



- Interacting model $Q_c = \xi H \rho_d$ with constant ξ and constant EoS w.

pltξvΩm0I2CCSum2



tabξvΩm0I2CCSum21
tabξvΩm0I2CCSum22

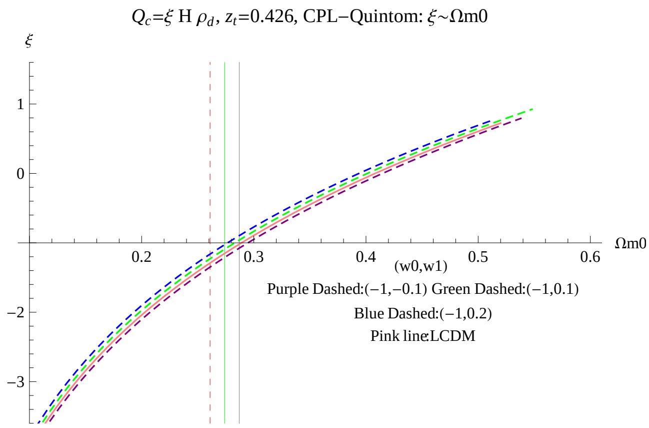
Q_c=ξ H ρ_d, constant w: ξ when transition redshift is 0.426		
⋱	w=−0.4506	w=−0.4507
Ωm0=0.2603	−17.5369	0.351603

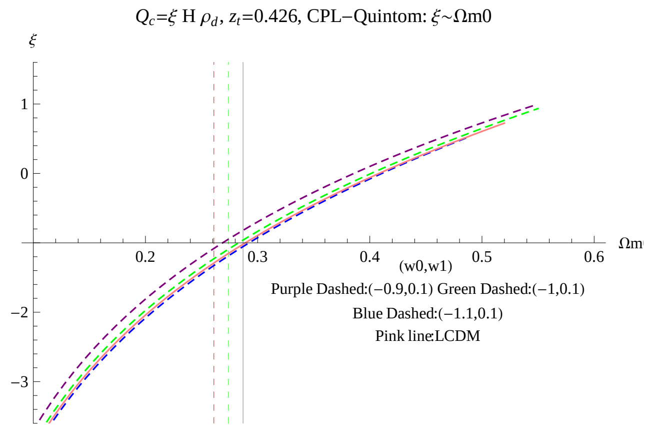
Q_c=ξ H ρ_d, constant w: EoS value when ξ=0	
⋱	Transition 0.426
Ωm0=0.2877	−0.58406

So we give the result that w ∈ (-0.58406, -0.4507) if we constrain Ωm0 = 0.2603 and transition redshift 0.426.

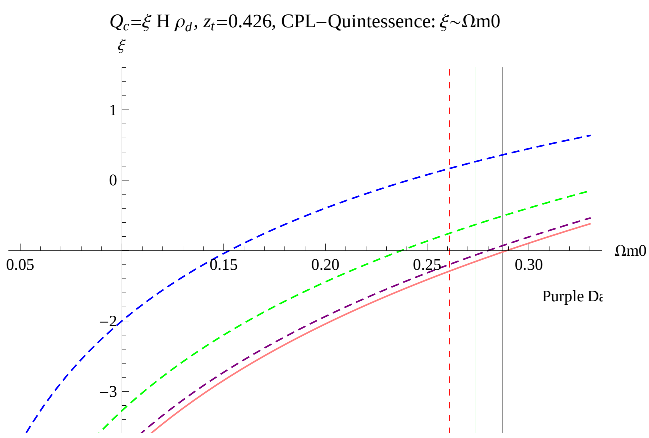
Interacting model $Q_c = \xi H \rho_d$ with constant ξ and CPL parameterization.

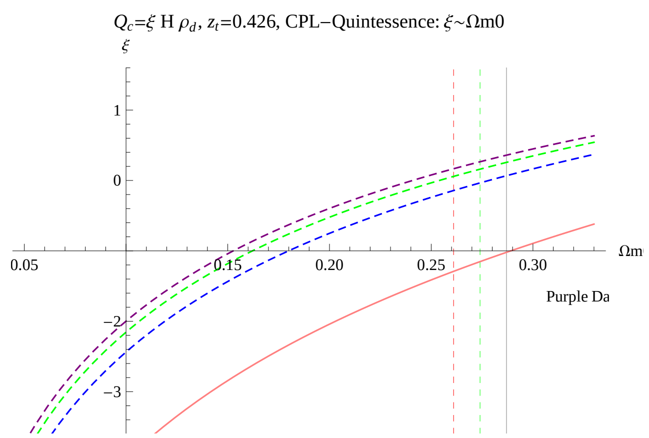
pltξvΩm0I2CCPLQuintomSum



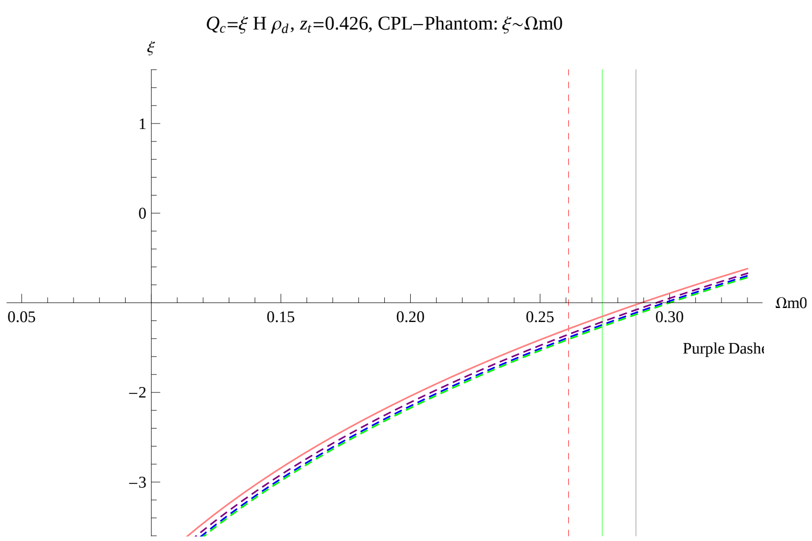


pltξvΩm0I2CCPLQuintessenceSum

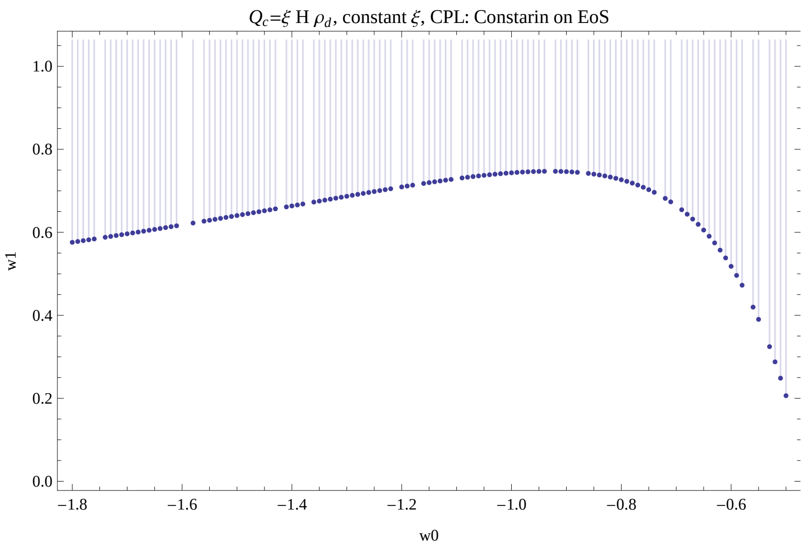




pltξvΩm0I2CCPLPhantomSum



pltw0vw1ConsI2CCPL

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Try quitting other applications and then retry.
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