Actividad 2.2

martes, 16 de febrero de 2021 08:24 p. m.

Herramientas básicas

Fuente: J.E. Hopcroft, J.D. Ullman. Introduction to Automata Theory, Languages, and Computation. Addison-Wesley, (1979), pp. 10-11.

1.1 In the tree of Fig. 1.4,

a) Which vertices are leaves and which are interior vertices?
b) Which vertices are the sons of 5?
c) Which vertex is the father of 5?
d) What is the length of the path from 1 to 9? R = 4
e) Which vertex is the root?

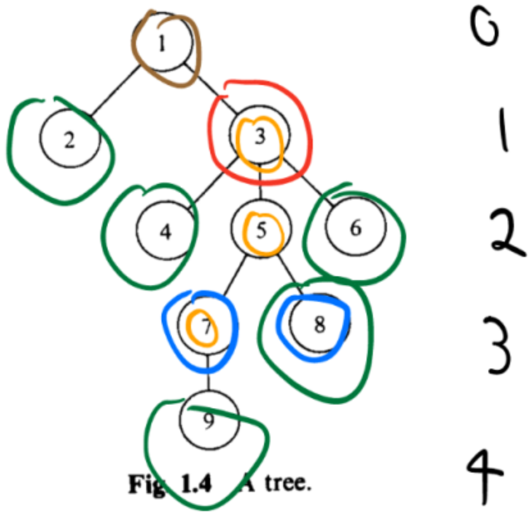

Fig. 1.4 A tree.

a) $\sum_{i=0}^{a} i = \frac{n(n+1)}{2} \longrightarrow$

$n=1$: $1 = \frac{1(1+1)}{2}$, $1=1$

$n=2$: $1+2 = \frac{2(2+1)}{2}$, $3=3$

$n=k$: $\Sigma^k = \frac{k(k+1)}{2} \rightarrow n = k+1$

$$\Sigma^k + k + 1 = \frac{(k+1)((k+1)+1)}{2} \rightarrow \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)((k+1)+1)}{2}$$

$$\frac{(k+1)(k+2)}{2} = (k+1)\left((k+1)+1\right)\left(\frac{1}{2}\right) \rightarrow \frac{(k+1)(k+2)}{2} = (k+1)\left((k+1)+1\right)\left(\frac{1}{2}\right)$$

$$\boxed{\frac{(k+1)(k+2)}{2} = \frac{(k+1)(k+2)}{2}}$$

b) $\sum_{i=0}^{n} i^3 = \left(\sum_{i=0}^{n} i\right)^2$

$$\frac{n^3(n^3+1)}{2} = \left(\frac{n(n+1)}{2}\right)^2$$

$n = k$

$$\frac{(k)^3((k+1)^3+1)}{2} = \left(\frac{k(k+1)}{2}\right)^2$$

$n = k+1$

$$\left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3 = \left(\frac{(k+1)((k+1)+1)}{2}\right)^2$$

$$\frac{x^4+6x^3+13x^2+12x+4}{4} = \frac{x^4+6x^3+13x^2+12x+4}{4}$$