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Herramientas básicas

Fuente: J.E. Hopcroft, J.D. Ullman. Introduction to Automata Theory, Languages, and Computation. Addison-Wesley, (1979), pp. 10-11.

1.1 In the tree of Fig. 1.4,

a) Which vertices are leaves and which are interior vertices?
b) Which vertices are the sons of 5?
c) Which vertex is the father of 5?
d) What is the length of the path from 1 to 9? $\ell = 4$
e) Which vertex is the root?

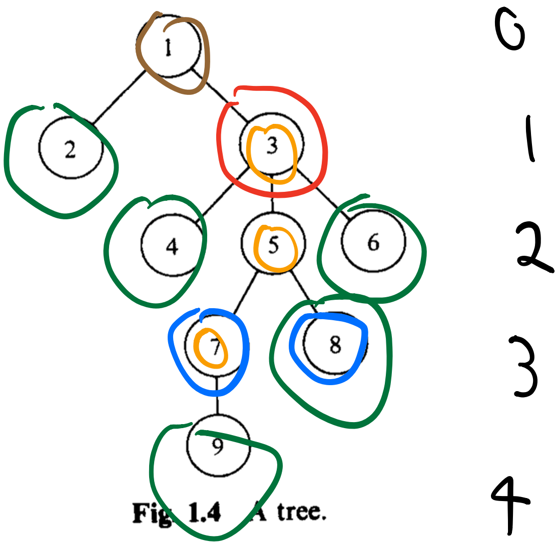

Fig. 1.4 A tree.

1.2 Prove by induction on n that

a) $\displaystyle\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$ b) $\displaystyle\sum_{i=0}^{n} i^3 = \left(\sum_{i=0}^{n} i\right)^2$

Resuelto en la página 3

***S 1.3** A palindrome can be defined as a string that reads the same forward and backward, or by the following definition.

1) ϵ is a palindrome.
2) If a is any symbol, then the string a is a palindrome.
3) If a is any symbol and x is a palindrome, then axa is a palindrome.
4) Nothing is a palindrome unless it follows from (1) through (3).

Prove by induction that the two definitions are equivalent.

*** 1.4** The strings of balanced parentheses can be defined in at least two ways.

1) A string w over alphabet $\{(,)\}$ is balanced if and only if:
   a) w has an equal number of ('s and)'s, and
   b) any prefix of w has at least as many ('s as)'s.
2) a) ϵ is balanced.
   b) If w is a balanced string, then (w) is balanced.
   c) If w and x are balanced strings, then so is wx.
   d) Nothing else is a balanced string.

Prove by induction on the length of a string that definitions (1) and (2) define the same class of strings.

1.7 Find the transitive closure, the reflexive and transitive closure, and the symmetric closure of the relation

$$\{(1, 2), (2, 3), (3, 4), (5, 4)\}.$$

1.8 Prove that any equivalence relation R on a set S partitions S into disjoint equivalence classes.

1.2 Prove by induction on n that

a) $\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$ b) $\sum_{i=0}^{n} i^3 = \left(\sum_{i=0}^{n} i\right)^2$

$n = 1$

$1 = \frac{1(1+1)}{2}$

$1 = 1$

$n = 2$

$1 + 2 = \frac{2(2+1)}{2}$

$n = k$

$\Sigma^{k} = \frac{k(k+1)}{2}$

$n = k+1$

$$\Sigma^{k} + k+1 = \frac{(k+1)\,((k+1)+1)}{2}$$

$$\frac{k(k+1)}{2} + k+1 = \frac{(k+1)\,((k+1)+1)}{2}$$

$$\frac{(k+1)\,(k+2)}{2} = (k+1)\,((k+1)+1)\left(\frac{1}{2}\right)$$

$$\frac{(k+1)\,(k+2)}{2} = (k+1)\,((k+1)+1)\left(\frac{1}{2}\right)$$

$$\frac{(k+1)\,(k+2)}{2} = \frac{(k+1)\,(k+2)}{2}$$

b) $\sum_{i=0}^{n} i^3 = \left(\sum_{i=0}^{n} i\right)^2$

$$\frac{n^3(n^3+1)}{2} = \left(\frac{n(n+1)}{2}\right)^2$$

$n = k$

$$\frac{(k)^3((k+1)^3+1)}{2} = \left(\frac{k(k+1)}{2}\right)^2$$

$n = k+1$

$$\left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3 = \left(\frac{(k+1)((k+1)+1)}{2}\right)^2$$

$$\frac{x^4 + 6x^3 + 13x^2 + 12x + 4}{4} = \frac{x^4 + 6x^3 + 13x^2 + 12x + 4}{4}$$