Transformações de Variáveis Aleatórias

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Tranformações de Variáveis Aleatórias Contínuas

Definições Iniciais

$$Y = g(X)$$

$$X = g^{-1}(Y)$$

$$F_Y(y) = \mathbb{P}\{Y \leq y\}$$

$$\mathbb{P}\{g(X) \leq y\} = \mathbb{P}\{X \leq g^{-1}(Y)\}$$

$$\mathbb{P}\{X \leq g^{-1}(Y)\} = F_X(g^{-1}(Y))$$

$$f_Y(y) = F'_Y(y) = \tfrac{d}{dy}F_X(g^{-1}(y)) = f_X(g^{-1}(y))\tfrac{d}{dy}g^{-1}(y)$$

$$f_Y(y) = F'_Y(y) = \tfrac{d}{dy}F_X(g^{-1}(y))$$

Expressões Finais

$$f_Y(y) = f_X(g^{-1}(y))\frac{d}{dy}g^{-1}(y)$$

$$f_Y(y) = -f_X(g^{-1}(y))\frac{d}{dy}g^{-1}(y)$$

$$f_Y(y) = \left| f_X(g^{-1}(y))\frac{d}{dy}g^{-1}(y) \right|$$

Resoluções $X \sim N(\mu, \sigma^2)$

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$X \sim N(\mu, \sigma^2)$, $Y = \frac{X-\mu}{\sigma}$ com $X \sim N(0,1)$

$Y = g(X) = \frac{X-\mu}{\sigma}$, para $\sigma \neq 0$

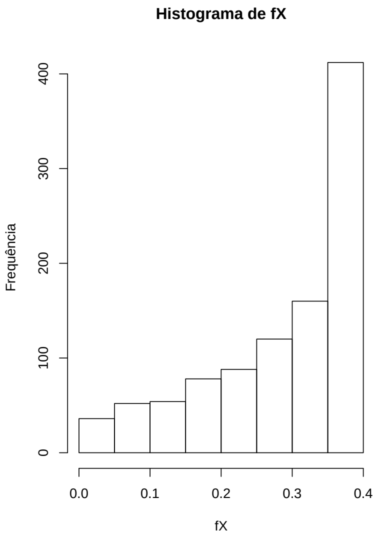
$g^{-1}(Y) = X = Y\sigma + \mu$

$\frac{d}{dy} g^{-1}(y) = \frac{d}{dy}(y\sigma + \mu) = \sigma$

$f_Y(y) = f_X(g^{-1}(y)) \frac{d}{dy} g^{-1}(y) = f_X(y\sigma + \mu)\sigma$

$$f_Y(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(y\sigma+\mu-\mu)^2}{2\sigma^2}} \sigma = \boxed{\frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}}$$

$Y = \frac{X-\mu}{\sigma}$ (Histogramas)



$X \sim N(\mu, \sigma^2)$, $Y = \frac{\sigma}{X-\mu}$ com $X \sim N(0,1)$

$Y = g(X) = \frac{\sigma}{X-\mu}$, para $X \neq \mu$

$g^{-1}(Y) = X = \frac{\sigma}{Y} + \mu$

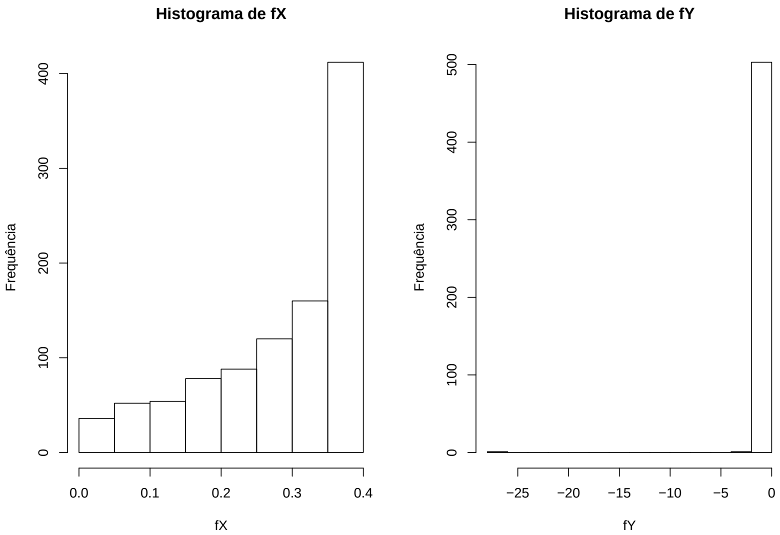
$\frac{d}{dy} g^{-1}(y) = \frac{d}{dy}(\frac{\sigma}{y} + \mu) = -\sigma y^{-2}$

$f_Y(y) = f_X(g^{-1}(y)) \frac{d}{dy} g^{-1}(y) = f_X(\frac{\sigma}{y} + \mu)(-\sigma y^{-2})$

$f_Y(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\frac{\sigma}{y}+\mu-\mu)^2}{2\sigma^2}}(-\sigma y^{-2}) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{1}{y})^2}(-\sigma y^{-2})$

$f_Y(y) = \left| -\frac{1}{y^2\sqrt{2\pi}} e^{-\frac{1}{2y^2}} \right|$

$Y = \frac{\sigma}{X-\mu}$ (Histogramas)



$$X \sim N(\mu, \sigma^2),\ Y = (\tfrac{X-\mu}{\sigma})^3 \text{ com } X \sim N(0,1)$$

$$Y = g(X) = (\tfrac{X-\mu}{\sigma})^3 \text{ para } \sigma \neq 0$$

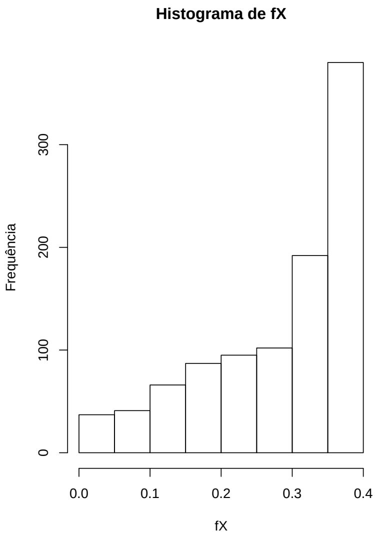
$$g^{-1}(Y) = X = \sigma Y^{\frac{1}{3}} + \mu$$

$$\tfrac{d}{dy} g^{-1}(y) = \tfrac{d}{dy}(\sigma y^{\frac{1}{3}} + \mu) = \tfrac{1}{3}\sigma y^{-\frac{2}{3}}$$

$$f_Y(y) = f_X(g^{-1}(y))\tfrac{d}{dy} g^{-1}(y) = f_X(\sigma y^{\frac{1}{3}} + \mu)(\tfrac{1}{3}\sigma y^{-\frac{2}{3}})$$

$$f_Y(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\sigma y^{\frac{1}{3}} + \mu - \mu)^2}{2\sigma^2}} (\tfrac{1}{3}\sigma y^{-\frac{2}{3}}) = \left| \; y^{-\frac{2}{3}} \frac{1}{3\sqrt{2\pi}} e^{-\frac{1}{2} y^{\frac{2}{3}}} \quad \right|$$

$Y = (\frac{X-\mu}{\sigma})^3$ (Histogramas)



$X \sim \varepsilon(1)$, transformar em $Y = X^p$, para $p \neq 0$

$$f_X(x) = \lambda e^{-\lambda x}$$

$$Y = g(X) = X^p$$

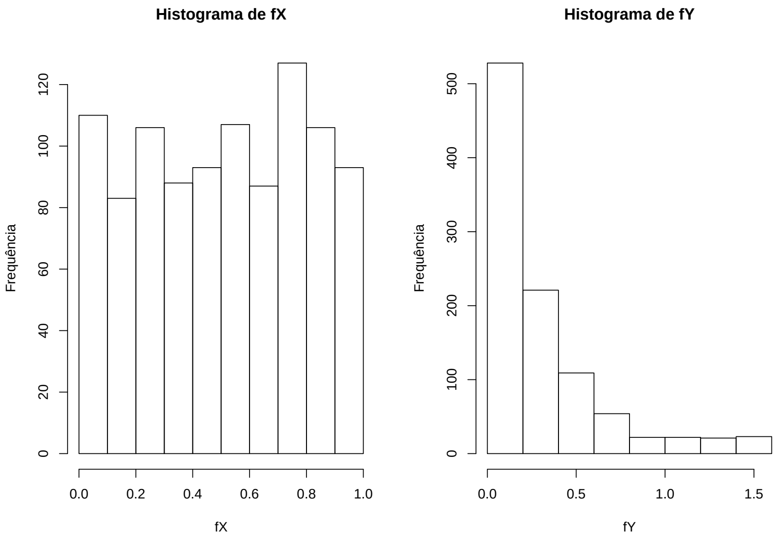
$$g^{-1}(Y) = X = Y^{\frac{1}{p}}$$

$$\frac{d}{dy} g^{-1}(y) = \frac{d}{dy} y^{\frac{1}{p}} = \frac{1}{p} y^{\frac{1-p}{p}}$$

$$f_Y(y) = -f_X(g^{-1}(y)) \frac{d}{dy} g^{-1}(y) = -f_X(y^{\frac{1}{p}}) \frac{1}{p} y^{\frac{1-p}{p}}$$

$$f_Y(y) = -(\lambda e^{-\lambda y^{\frac{1}{p}}}) \frac{1}{p} y^{\frac{1-p}{p}}$$

$Y = X^p$ (Histogramas)



$X \sim U(0,1)$, transformar em $Y = aX + b$

$$f_X(x) = \frac{1}{b-a}\mathbb{I}_{[a,b]}(x)$$

$$Y = g(X) = aX + b$$

$$g^{-1}(Y) = X = \frac{a}{Y} - b$$

$$\frac{d}{dy}g^{-1}(y) = \frac{d}{dy}(\frac{a}{y} - b) = -ay^{-2}$$

$$f_Y(y) = f_X(g^{-1}(y))\frac{d}{dy}g^{-1}(y) = f_X(\frac{a}{y} - b)(-ay^{-2})$$

$$f_Y(y) = \frac{1}{b-a}(-ay^{-2}) = -\frac{a}{y^2(b-a)}$$

$Y = aX + b$(Histogramas)