Portfolio Hedging with Futures - Analysis Walkthrough

BU623 Derivatives | Wilfrid Laurier University | TQM Hedge Fund Case Study

Executive Summary

TQM Hedge Fund manages **$1.035 billion** across four regional equity portfolios. This analysis recommends the **Firm-Wide 3-Factor Hedge**:

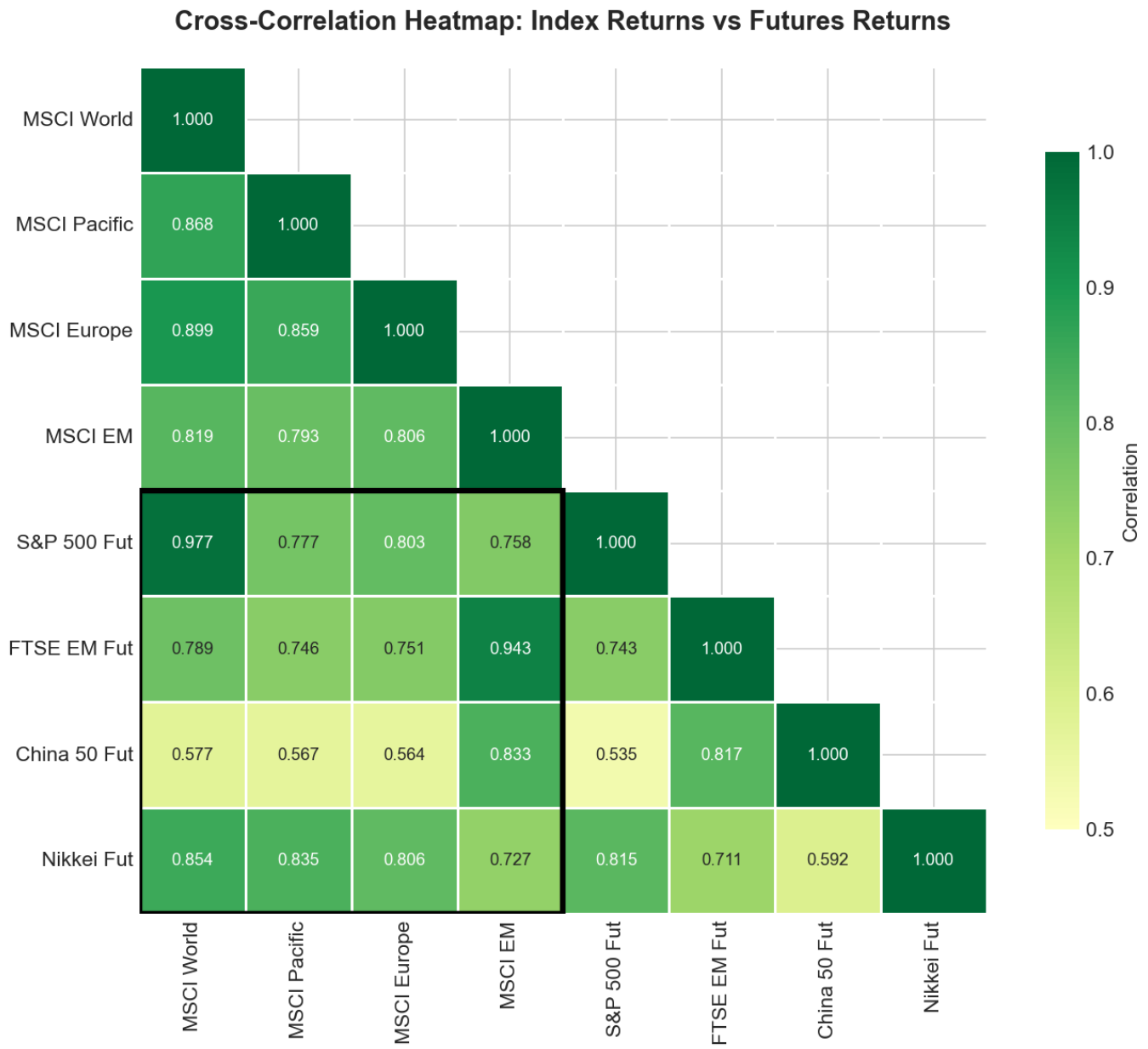
- **Adj R^2 = 92.65%** (single regression, directly interpretable)
- **3 futures only** (lowest management cost)
- **Contracts**: 2,256 S&P 500 + 4,640 FTSE EM + 1,644 Nikkei (short)

1. Portfolio Overview

Portfolio	Index	Value ($)	Weight
MSCI World	Global DM	500,000,000	48.3%
MSCI EM	Emerging Mkts	200,000,000	19.3%
MSCI Europe	European DM	175,000,000	16.9%
MSCI Pacific	Asia-Pacific	160,000,000	15.5%
Total		**1,035,000,000**	100%

2. Correlation Analysis

The correlation heatmap shows the relationship between index returns and futures returns:



Key Observations:

- MSCI World ↔ S&P 500: **0.977** (very high - direct hedge)
- MSCI EM ↔ FTSE EM: **0.943** (very high - direct hedge)
- MSCI Europe ↔ S&P 500: **0.803** (moderate - cross-hedge)
- MSCI Pacific ↔ Nikkei: **0.835** (high - direct hedge)

The black box highlights the key cross-correlations between futures (rows) and indices (columns).

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3. Contract Calculation Methodology

Formula

The optimal number of futures contracts is:

$N^* = h^* \times (V_s / V^f)$

Where:

- **N*** = Optimal number of contracts
- **h*** = Hedge ratio (beta from regression)
- **V_s** = Value of spot position (portfolio value)
- **V^f** = Value of one futures contract = Futures Price × Multiplier

Contract Specifications

Future	Multiplier	Price	Contract Value (V^f)
S&P 500	50	$4,202.50	$210,125
FTSE EM	100	$668.30	$66,830
China 50	2	$20,487.50	$40,975
Nikkei	5	$29,020.00	$145,100

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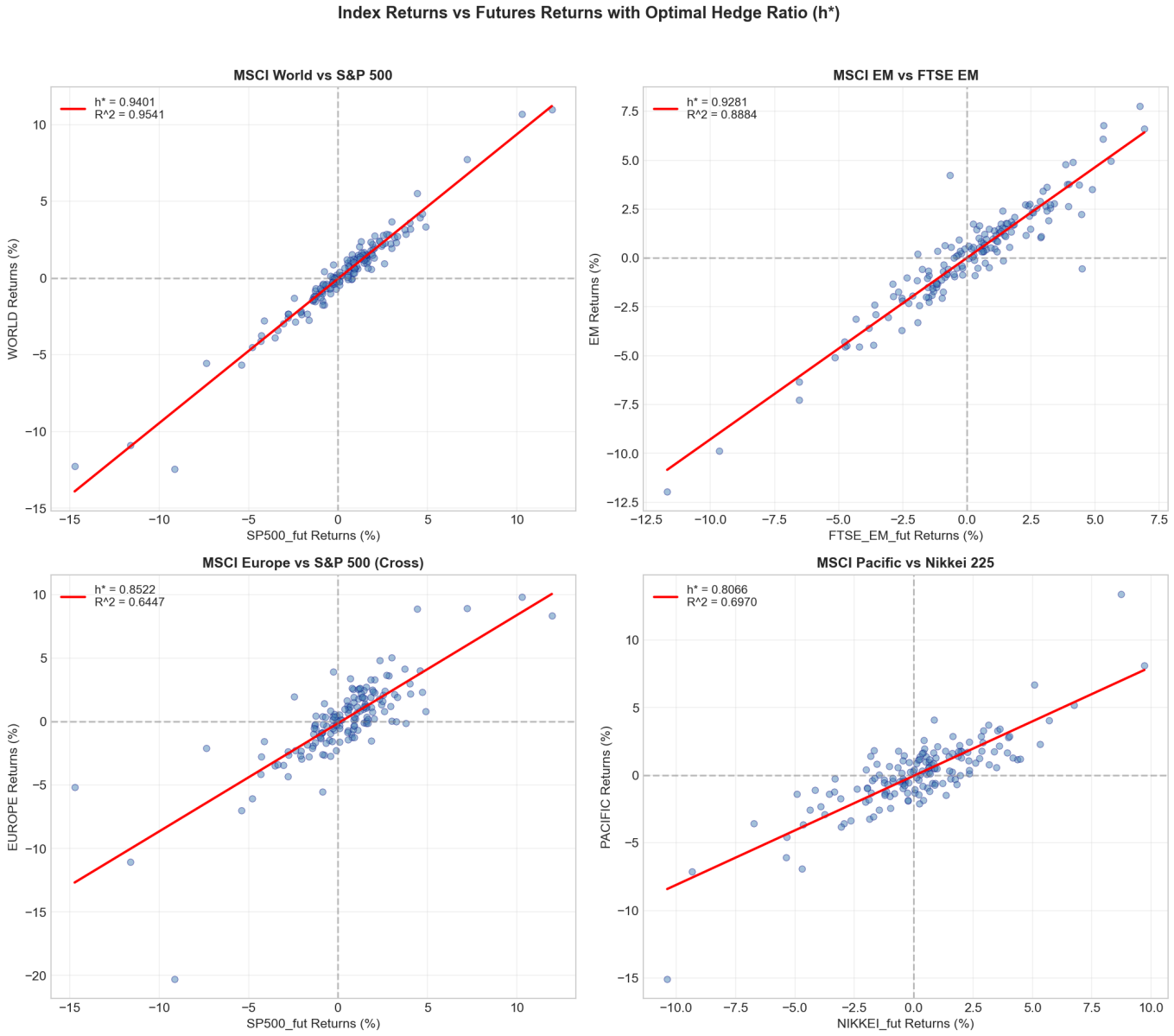
4. Single-Future Hedging

Scatter Plots with Regression Lines



Regression Summary

Portfolio	Futures	h* (β)	R^2	t-stat	Contracts	Value ($M)
MSCI World	S&P 500	0.9401	95.41%	56.79	2,237	470
MSCI EM	FTSE EM	0.9281	88.84%	35.12	2,777	186
MSCI Europe	S&P 500	0.8522	64.47%	16.77	710	149
MSCI Pacific	Nikkei	0.8066	69.70%	18.88	889	129
TOTAL					**6,613**	**$934M**

5. 4-Factor Regression (Feature Selection)

4-Factor Regression Results (* = p < 0.05)
Used to identify significant futures for multi-future hedging

Portfolio	Adj R^2	SP500 β	SP500 t	FTSE_EM β	FTSE_EM t	CHINA50 β	CHINA50 t	NIKKEI β	NIKKEI t
MSCI World	0.9689	0.7417*	28.03	0.1511*	4.64	-0.0417	-1.68	0.1517*	5.74
MSCI EM	0.9096	0.1360*	3.16	0.6265*	11.84	0.2034*	5.05	0.0242	0.56
MSCI Europe	0.7368	0.2999*	3.53	0.3931*	3.76	-0.1068	-1.34	0.4408*	5.19
MSCI Pacific	0.7491	0.1194	1.69	0.3471*	3.99	-0.1070	-1.62	0.5239*	7.42

5.1 MSCI World - Model Summary

Future	β (h*)	t-stat	p-value	Significant?
SP500	0.7417	28.03	<0.001	✓ Yes
FTSE_EM	0.1511	4.64	<0.001	✓ Yes
CHINA50	-0.0417	-1.68	0.094	No
NIKKEI	0.1517	5.74	<0.001	✓ Yes

Adj R^2 = 96.89% | Significant: SP500, FTSE_EM, NIKKEI

5.2 MSCI EM - Model Summary

Future	β (h*)	t-stat	p-value	Significant?
SP500	0.1360	3.16	0.002	✓ Yes
FTSE_EM	0.6265	11.84	<0.001	✓ Yes

Future	β (h*)	t-stat	p-value	Significant?
CHINA50	0.2034	5.05	<0.001	✓ Yes
NIKKEI	0.0242	0.56	0.574	No

Adj R^2 = 90.96% | Significant: SP500, FTSE_EM, CHINA50

5.3 MSCI Europe - Model Summary

Future	β (h*)	t-stat	p-value	Significant?
SP500	0.3000	3.53	<0.001	✓ Yes
FTSE_EM	0.3931	3.76	<0.001	✓ Yes
CHINA50	-0.1068	-1.34	0.181	No
NIKKEI	0.4408	5.19	<0.001	✓ Yes

Adj R^2 = 73.68% | Significant: SP500, FTSE_EM, NIKKEI

5.4 MSCI Pacific - Model Summary

Future	β (h*)	t-stat	p-value	Significant?
SP500	0.1194	1.69	0.093	No
FTSE_EM	0.3471	3.99	<0.001	✓ Yes
CHINA50	-0.1070	-1.62	0.108	No
NIKKEI	0.5239	7.42	<0.001	✓ Yes

Adj R^2 = 74.91% | Significant: FTSE_EM, NIKKEI

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6. Multi-Future Decision Analysis

Based on the 4-factor results, we evaluate whether adding futures improves R^2 enough to justify added complexity and contracts.

6.1 MSCI World Decision

Model	Adj R^2	Futures	h* Values	Contracts	Value
Single	95.41%	SP500	0.9401	2,237	$470M
3-Factor	96.89%	SP500+FTSE_EM+NIKKEI	0.74/0.15/0.15	1,765+1,130+523	$740M

Decision: Use Single Hedge (S&P 500)

- +1.48pp R^2 improvement not worth +$270M contract value
- ✓ Simpler, lower margin

6.2 MSCI EM Decision

Model	Adj R^2	Futures	h* Values	Contracts	Value
Single	88.84%	FTSE_EM	0.9281	2,777	$186M
3-Factor	90.96%	SP500+FTSE_EM+CHINA50	0.14/0.63/0.20	133+1,876+993	$109M

Decision: Use Single Hedge (FTSE EM)

- +2.12pp R^2 improvement
- 3-factor actually reduces contract value but adds complexity
- ✓ Adequate R^2 with simpler execution

6.3 MSCI Europe Decision

Model	Adj R^2	Futures	h* Values	Contracts	Value
Single	64.47%	SP500	0.8522	710	$149M
2-Factor	~72%	SP500+NIKKEI	0.46/0.51	384+617	$170M
3-Factor	73.68%	SP500+FTSE_EM+NIKKEI	0.30/0.39/0.44	250+1,029+531	$181M

Decision: Use 2-Factor (S&P 500 + Nikkei)

- Single hedge has high basis risk (64.47% R^2)
- 2-factor improves to ~72% with manageable contract increase
- 3-factor adds FTSE_EM for only ~1.5pp more - diminishing returns

6.4 MSCI Pacific Decision

Model	Adj R^2	Futures	h* Values	Contracts	Value
Single	69.70%	NIKKEI	0.8066	889	$129M
2-Factor	74.91%	FTSE_EM+NIKKEI	0.30/0.60	715+657	$143M

Decision: Use 2-Factor (FTSE_EM + Nikkei)

- +5.2pp R^2 improvement is meaningful
- Contract value increase modest ($14M)
- Captures both EM and Japan exposure

6.5 Multi-Future Summary

Portfolio	Model	Futures	Contracts	Value
MSCI World	Single	SP500	2,237	$470M
MSCI EM	Single	FTSE_EM	2,777	$186M
MSCI Europe	2-Factor	SP500 + NIKKEI	384 + 617	$170M

Portfolio	Model	Futures	Contracts	Value
MSCI Pacific	2-Factor	FTSE_EM + NIKKEI	715 + 657	$143M
TOTAL		**6 futures**	**7,387**	**$969M**

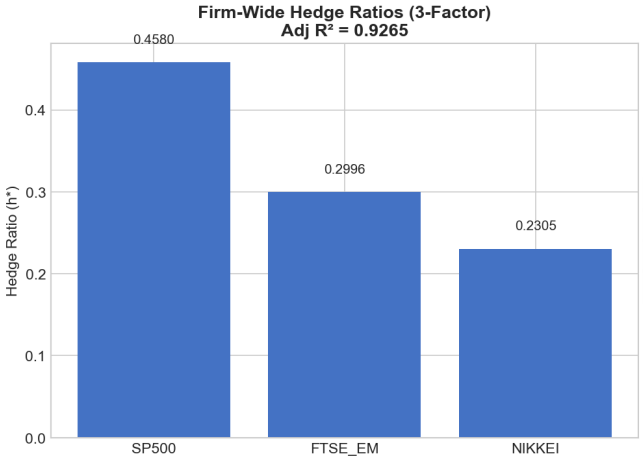
7. Firm-Wide Portfolio Hedge

7.1 Why Firm-Wide?

Instead of hedging each portfolio separately (6 futures), we hedge the entire $1.035B as one unit:

- **Diversification benefit**: Portfolio correlations reduce overall risk
- **Fewer futures (3)**: Lower management cost
- **Single R^2 metric**: Directly interpretable

7.2 3-Factor Model (Excludes Insignificant China50)



Firm-Wide Contracts (3-Factor)
Total Portfolio: $1035M

Future	h*	Contracts	Value
SP500	0.4580	2256	$474.0M
FTSE_EM	0.2996	4640	$310.1M
NIKKEI	0.2305	1644	$238.5M

Future	β (h*)	t-stat	Contracts	Value
SP500	0.4580	***	2,256	$474M
FTSE_EM	0.2996	***	4,640	$310M
NIKKEI	0.2305	***	1,644	$239M
TOTAL			**8,540**	**$1,023M**

Adj R^2 = 92.65%

Contract Calculation

```
N*(SP500)   = 0.4580 × (1,035,000,000 / 210,125) = 2,256 contracts
N*(FTSE_EM) = 0.2996 × (1,035,000,000 / 66,830)  = 4,640 contracts
N*(NIKKEI)  = 0.2305 × (1,035,000,000 / 145,100) = 1,644 contracts
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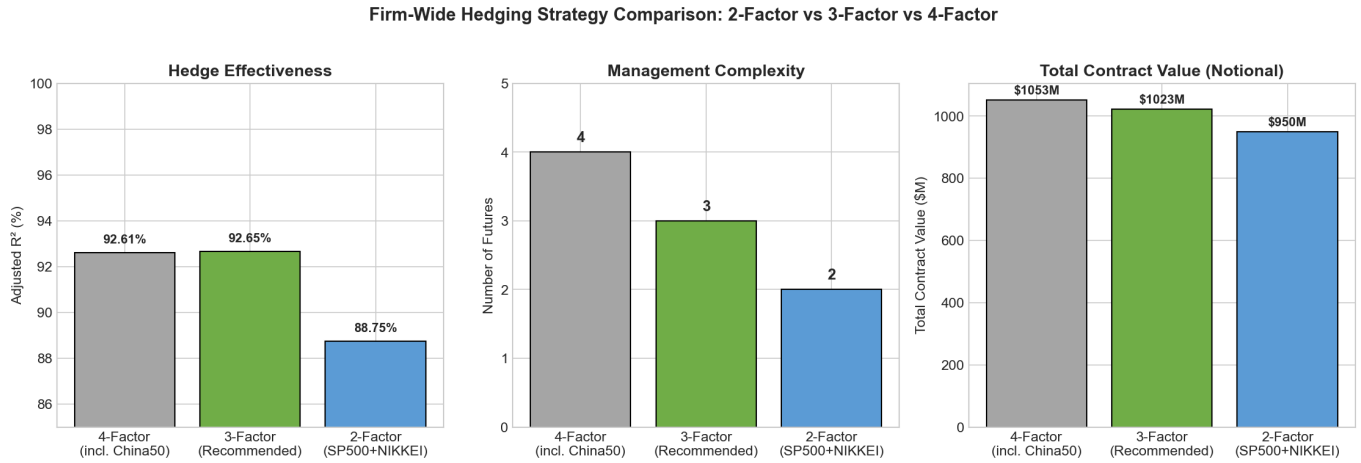
7.3 2-Factor Model (S&P 500 + Nikkei Only)

For comparison, we also evaluate a simpler 2-factor model using only S&P 500 and Nikkei futures:



Future	β (h*)	t-stat	Contracts	Value
SP500	0.5937	***	2,924	$614M
NIKKEI	0.3244	***	2,314	$336M
TOTAL			**5,238**	**$950M**

Adj R^2 = 88.75%

Trade-off Analysis:

- 2-Factor loses ~4pp Adj R^2 but requires only 2 futures vs 3
- 2-Factor contract value is ~$73M lower
- 2-Factor excludes emerging market exposure (FTSE_EM)

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8. Strategy Comparison

Strategy	# Futures	Contract Value	Notes
Single-Future (4 portfolios)	4	$934M	Simple, 1 future per portfolio
Multi-Future (Handpicked)	6	$969M	World/EM: single; Europe/Pacific: 2-factor
Firm-Wide 4-Factor	4	$1053M	Adj R^2=92.61%, includes insig. China50
Firm-Wide 3-Factor	3	$1023M	Adj R^2=92.65%, RECOMMENDED
Firm-Wide 2-Factor (SP500+NIKKEI)	2	$950M	Adj R^2=88.75%, Simplest

Strategy	# Futures	Contracts	Value	Notes
Single-Future	4	6,613	$934M	1 per portfolio

Strategy	# Futures	Contracts	Value	Notes
Multi-Future	6	7,387	$969M	Handpicked
Firm-Wide 4F	4	~10,377	$1,053M	Includes China50
Firm-Wide 3F	**3**	**8,540**	**$1,023M**	**Adj R^2=92.65%**
Firm-Wide 2F (SP500+NIKKEI)	2	5,238	$950M	Adj R^2=88.75%

> **Key Insight**: Firm-Wide 3-Factor achieves the highest Adj R^2 (92.65%) with only 3 futures. The 2-Factor (SP500+NIKKEI) offers simplicity at a modest R^2 cost.

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9. Conclusion & Recommendation

Recommended: **Firm-Wide 3-Factor Hedge**

Metric	Value
Adj R^2	**92.65%**
# Futures	**3**
Contracts	8,540
Contract Value	$1,023M

Final Position (Short)

Future	h*	Contracts	Value
S&P 500	0.4580	2,256	$474M
FTSE EM	0.2996	4,640	$310M
Nikkei	0.2305	1,644	$239M
TOTAL		**8,540**	**$1,023M**

Why Firm-Wide 3-Factor is Optimal:

1. **Highest Adj R^2 (92.65%)** - Best hedge effectiveness
2. **Fewest Futures (3)** - Minimal management complexity
3. **Lower Margin** - Fewer positions = less capital
4. **Diversification** - Benefits from portfolio correlations
5. **No Irrational Positions** - Excludes insignificant China50

Appendix: Files Generated

File	Description
`01_correlation_heatmap.png`	Index-futures correlation matrix

File	Description
02_scatter_plots.png	Single-future regression plots
05_four_factor_table.png	4-factor regression results
07_firmwide_summary.png	Firm-wide hedge summary
09_strategy_comparison.png	Strategy comparison chart
all_contracts.csv	All contract calculations

How to Reproduce

```
python hedging_analysis.py
```