

Question 1:

1	2	3	4	5	6
7	8	9	10	11	17
12	13	14	15	16	17

a) Three elements of  $N \in X$ :

$1 \in X$  - base case

$4 \in X$  – as  $n+3 \in X$ , where 1 is n

$2 \in X$  – as  $n*2 \in X$ , where 1 is n

b) Three elements of  $N \notin X$ :

$3 \notin X$  as it is 1 and it is not  $n+3$  or  $n*x$  for any  $n \in X$

$6 \notin X$  .....

$9 \notin X$  .....

c)  $X \in N/3n$

$x = \{1,2,4,5,7,8,10,11,13,14,16\}$

Question 2:

a) • the 2nd player has the winning strategy if the piles contain an equal number of coins:

If player one needs to move from an equal piles, he must change to a position in which the two piles have an unequal number of chips, and then player two can return to a position with an equal number of chips (perhaps the terminal position).

• the 1st player has the winning strategy if the piles contain an unequal number of coins:

The first player will make the piles equal in his first move therefore he has the winning condition since he can do this after every player two's turn.

b) When opponent finally moves so that there is exactly one pile of size greater than one, reduce that pile to zero or one, whichever leaves an odd number of piles of size one remaining.

Question 3:

b)

(i) "The computer consists of three parts: a CPU, a memory unit, and a bus for communication with the environment."

(ii) "The emergency button can be pushed; this will halt the computer, which will then not do anything further."

The second statement (ii) can be expressed in modal logic M:

$[\text{button pushed}][\text{cp halt}] \neg \langle - \rangle \text{true}$

The first statement (i) cannot be formalized in modal logic M because it has no actions. It is just an enumeration or a description of something.

Question 4:

a) Actions:

iszero , increment, decrement

States:

$C_2$  ;  $C_2^1$  ;  $C_2^2$

Transitions:

$C_2 \xrightarrow{\text{iszero}} C_2$

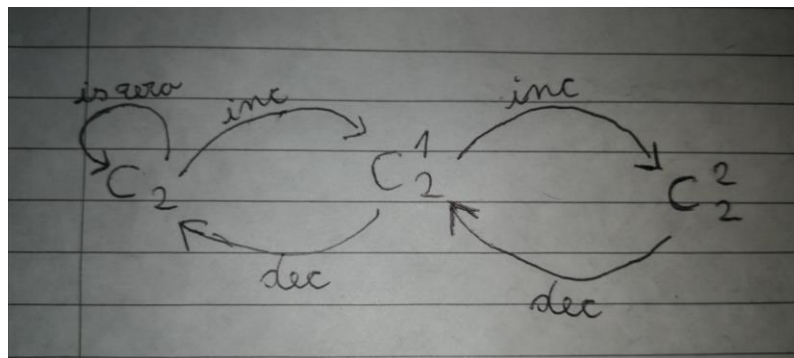
$C_2 \xrightarrow{\text{inc}} C_2^1$

$C_2^1 \xrightarrow{\text{inc}} C_2^2$

$C_2^2 \xrightarrow{\text{dec}} C_2^1$

$C_2^1 \xrightarrow{\text{dec}} C_2$

b)



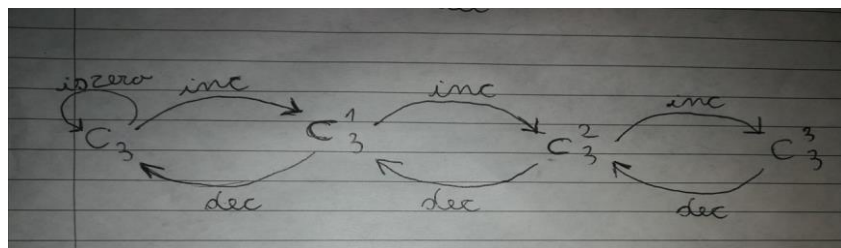
c)

$C_2 \models \langle \text{is zero} \rangle \langle \text{inc} \rangle ([C_2] \text{ true} \wedge [C_2^1] \text{ true})$

$C_2^1 \models \langle \text{inc} \rangle \langle \text{dec} \rangle ([C_2^2] \text{ true} \wedge [C_2] \text{ true})$

$C_2^2 \models [\text{dec}] \langle C_2^1 \rangle \text{ true}$

d)



Definition:

$C_3 \stackrel{\text{def}}{=} \text{iszero}.C_3 + \text{inc}.C_3^1$

$C_3^1 \stackrel{\text{def}}{=} \text{inc}.C_3^2 + \text{dec}.C_3$

$C_3^2 \stackrel{\text{def}}{=} \text{inc}.C_3^3 + \text{dec}.C_3^1$

$C_3^3 \stackrel{\text{def}}{=} \text{dec}.C_3^2$