On MV-algebras with convexity operators

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joint work with Tommaso Flaminio

The notions of *convexity* plays a central rôle in logic and mathematics. Starting from a seminal idea of Brown [**1**], we propose an axiomatic approach to convex combinations in the realm of MV-algebras [**2**]. More in detail, we will expand the language of MV-algebras by an uncountable family of binary operations $cc_\alpha(\cdot,\cdot)$ (one for every $\alpha \in [0,1]$) axiomatized so to capture the basic properties of convex combinations in $[0,1]$. The so resulting algebras are called *convex* MV-algebras (or CMV-algebras for short).

CMV-algebras form a variety. Our first result shows that CMV-algebras are termwise equivalent to *Riesz* MV-algebras [**3**] and, consequently, the variety of CMV-algebras is generated by the standard CMV-algebra, that is the standard MV-algebra where the operators cc_α are interpreted in the usual way: for each $x, y, \alpha \in [0,1]$, $cc_\alpha(x,y)$ is $\alpha x + (1-\alpha)y$.

States of MV-algebras [**4**] are analogous to finitely additive probabilities on boolean algebras and, for every MV-algebra $\mathbf{A}$, its states form a subset of $[0,1]^A$ which coincide with the topological closure of the convex hull of the MV-homomorphisms of $\mathbf{A}$ in the standard MV-algebra $[0,1]_{MV}$. Thanks to this characterization of the states space, we will show that each state of a finitely dimensional MV-algebra $[0,1]^X$ (with X finite) has a faithful representation in the free CMV-algebra $|X|$-generated.

References

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