This exercise simulates the optimal control of an electric train in both closed and open loop. In the first part we will be looking at some problems of optimal control.

Part One: Preparation (Optional)

1. For the system:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ b \end{bmatrix} u$$

where b = 1, find $-1 \le u(t) \le 1$ that solves the minimum time problem with $x(0) = [-1\ 0]^T$ and $x(T) = [0\ 0]^T$. Then, find the corresponding control law u = γ(x) that solves the same problem. Simulate the problems of closed and open loop. Compare the responses of the systems of closed and open loop when b = 1.02

2. For the system:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} x + \begin{bmatrix} 4 \\ 5 \end{bmatrix} u$$

With initial conditions $x_1(0) = 0$, $x_2(0) = -1$, find u(t) that minimizes:

$$J = 5(x_1(10) - 10)^2 + 8x_2^2(10) + \int_0^{10} u^2 dt .$$

3. For the system:

$$\dot{x} = \begin{bmatrix} 0 & \cos \omega t \\ \sin \omega t & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Find controller u = f(x,t) that minimizes:

$$J = x(T)^T Q_f x(T) + \int_0^T x^T Q x + R u^2 dt$$

Where $Q_f = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}, Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ R = 1 \ , T = 100$ and $\omega = 2\pi$

Find function f. Then, simulate the closed loop system. Solve the same problem when $\omega = 0$ and $T = \infty$

Part Two: Optimal Control of an Electric Train

Description

In this exercise, we are dealing with the minimum energy control of an electric train. We assume that the train is driven by a DC motor with constant excitation and controlled drum current. The engine exerts torque on the wheels via a drive system. We also believe that there is a possibility of regenerative braking (ie the electric motor can "brake" the train, gaining energy).

Modeling

The electrical equations (permanent state) of the machine are:

$$V_a = RI_a + \omega(M_{af} I_f)$$

where V_a is the drum voltage, R is the resistance of the drum winding, ω is the angular velocity of the machine and M_{af}, If can be considered constant for the specific problem (stator-rotor induction and stator current respectively). The torque of the machine is given by:

$$T = (M_{af} I_f) I_a .$$

The train exerts friction in proportion to speed and air resistance in proportion to the square of velocity. Based on the above, reducing all the moments of inertia and forces, on the axis of a wheel we can write the model of the train in the form of state equations:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -k_1 x_2 - k_2 x_2^2 + k_3 u$$

where x_1 is the position of the train, x_2 is the speed and u is the drum current of the engine.

Optimal Control

4. For the cost criterion:

$$J = c_1 (x_1(T) - x_{1f})^2 + c_2 x_2^2(T) + \int_0^T k_4 x_2 u + Ru^2 dt ,$$

να γραφούν οι αναγκαίες συνθήκες για την ελαχιστοποίησή του και να διατυπωθεί το αντίστοιχο πρόβλημα συνοριακών τιμών υπό την προϋπόθεση ότι $I_{min} \leq u \leq I_{max}$. Αρχικά θεωρούμε ότι $x_1(0)= x_2(0)= 0$.

5. Solve numerically using Matlab the above boundary value problem and simulate the system. The values of the constants are:

$k_1 = 0{,}5$, $k_2 = 0{,}1$, $k_3 = 1$, $c_1 = c_2 = 1000$, $x_{1f} = 10$, $R = 0{,}3$, $k_4 = 10$ $I_{min} = -2$, $I_{max} = 2$ και $T = 10$

6. Simulate the system with the input that resulted in the previous query but having initial values at x_1 and x_2 slightly different from before.