**UNIVERSITATEA POLITEHNICA TIMIS¸OARA**

**ALGEBRA LINIAR˘ A S¸I GEOMETRIE˘**

**DIFERENT¸IALA˘**

Exerci¸tii

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Cuprins

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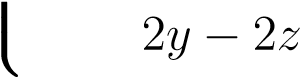
CAPITOLUL 1

RECAPITULARE

### 1.1 Sisteme liniare

Sa˘ se rezolve urma˘toarele sisteme liniare:

 *x* − *y* + 2*z* = −3

 2*x* + 3*y* − *z* = 9  −*x* + *y* − 2*z* = 1 a) b) *x* + *y* = 2

3*x* + 2*y* + *z* = 6

 *x* + 4*y* − 3*z* = 12; = 3;

 

3*x* − 6*y* − 3*z* = 5 *x* + *y* + *z* = 5

 −*x* + 2*y* + *z* = 0  *x* + *z* = 4

c) d)

*x* − 2*y* − *z* = 0 2*x* + *y* + 2*z* = 10

 2*x* − 4*y* − 2*z* = 1;  *y* = 2*.*

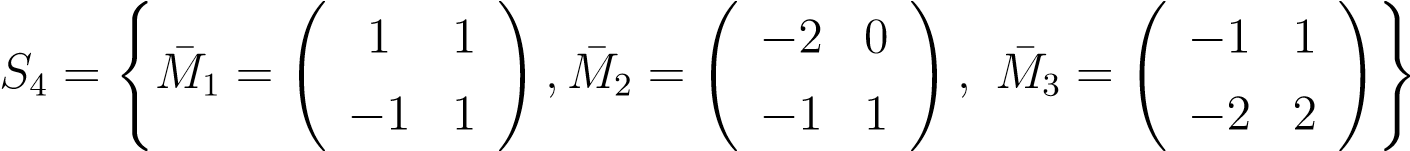
CAPITOLUL 2

SPAT¸II VECTORIALE

**2.1 Dependen¸t˘a ¸si independen¸t˘a liniar˘a. Sistem de generatori. Baze.**

1. Sa˘ se arate ca˘ urma˘toarele sisteme de vectori sunt liniar dependente ¸si s˘a se scrie rela¸tia de dependen¸t˘a liniara˘ dintre ei.

* + 1. *S*1 = {*v*¯1 = (2*,*−1)*,v*¯2 = (4*,*−2)*,v*¯3 = (−6*,*3)*,v*¯4 = (10*,*−5)} ⊂ R2;
    2. *S*2 = {*v*¯1 = (1*,*−1*,*0)*,v*¯2 = (2*,*2*,*1)*,v*¯3 = (5*,*0*,*0)*,v*¯4 = (1*,*1*,*1)} ⊂ R3;
    3. *S*3 = {*p*¯1 = *X*3 + *X*2 + *X* + 1*,p*¯2 = *X*3 − 2*X*2 − *X, p*¯3 = 2*X*3 − *X*2 + 1} ⊂ R3[*X*];

d)

⊂ M2(R);

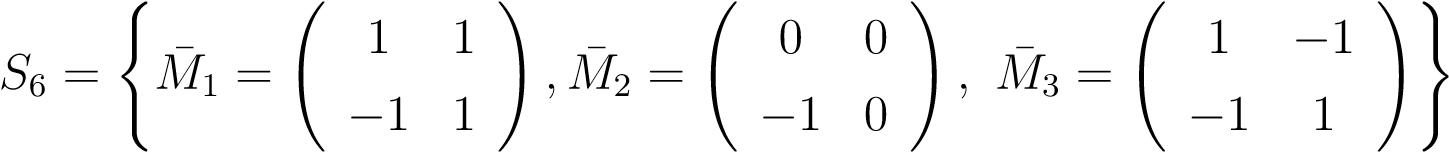
2. Sa˘ se studieze liniar dependen¸ta urma˘toarelor sisteme de vectori ¸siˆın caz de dependen¸ta˘ liniara˘ sa˘ se scrie rela¸tia de dependen¸t˘a. Care din ele sunt sisteme de generatori pentru spa¸tiile vectoriale date? Care sunt baze?

a)*S*1 = {*v*¯1 = (−1*,*−3)*, v*¯2 = (1*,*2)*, v*¯3 = (1*,*1)} ⊂ R2;

* 1. *S*2 = {*v*¯1 = (1*,*2*,*1)*, v*¯2 = (−2*,*3*,*5)*, v*¯3 = (2*,*5*,*3)*, v*¯4 = (3*,*−1*,*−4)}

⊂ R3;

* 1. *S*3 = {*v*¯1 = (2*,*3*,*1*,*−1)*, v*¯2 = (7*,*9*,*5*,*2)*, v*¯3 = (3*,*4*,*3*,*−1)} ⊂ R4;
  2. *S*4 = {*p*¯1 = *X*3 + *X*2 + *X* + 1*, p*¯2 = *X*3 + *X*2 + *X*, *p*¯3 = *X*3 + *X*2} ⊂ R3[*X*].
  3. *S*5 = {*p*¯1 = 2*X*2 + *X* + 1*, p*¯2 = *X*2 + 2*X* + 1*, p*¯3 = *X*2 + *X* + 2*, p*¯4 = 2*X*2 + 2*X* + 2} ⊂ R2[*X*];

f)

⊂ M2(R);

      

 ¯ =  −31 *, M*¯2 =  −21 *, M*¯3 =  −11 

g) *S*7 = *M*1

  3   1   1 

⊂ M3*,*1(R).

### 2.2 Schimb˘ari de baze

1. Fie sistemele de vectori *B*1 = {*v*¯1 = (−1*,*1*,*3)*, v*¯2 = (0*,*1*,*−1)*, v*¯3 = (1*,*−2*,*0)} ⊂ R3

¸si

*B*2 = {*v*¯1 = (2*,*−1*,*3)*, v*¯2 = (1*,*3*,*2)*, v*¯3 = (1*,*−4*,*2)} ⊂ R3.

* 1. S˘a se arate ca˘ *B*1 ¸si *B*2 sunt baze ale lui R3
  2. S˘a se determine matricea de trecere de la baza *B*1 la baza.
  3. S˘a se g˘aseasca˘ coordonatele vectorului ¯*v* = (−2*,*1*,*3) relativ la bazele *B*1 ¸si *B*2.

1. Fie sistemele de vectori
   1. *B*1 = {*u*¯1 = (1*,*3)*, u*¯2 = (2*,*1)} ⊂ R2 ¸si *B*2 = {*v*¯1 = (2*,*−1)*, v*¯2 = (−1*,*1)} ⊂ R2;
   2. *B*1 = {*r*¯1 = *X*2 +*X* −2*, r*¯2 = 7*X* −1*, r*¯3 = −2*X*2 +*X* +1} ⊂ R2[*X*] ¸si *B*2 = {*p*¯1 = *X*2 + 3*X* − 1*, p*¯2 = 2*X* + 1*, p*¯3 = −2*X*2 − *X*} ⊂ R2[*X*] ;

      

  0  ¯ 1 1 

* 1. *B*1 = *N*¯1 =  1 *, N*2 =  0 *, N*¯3 =  1  ⊂ M3*,*1(R) ¸si

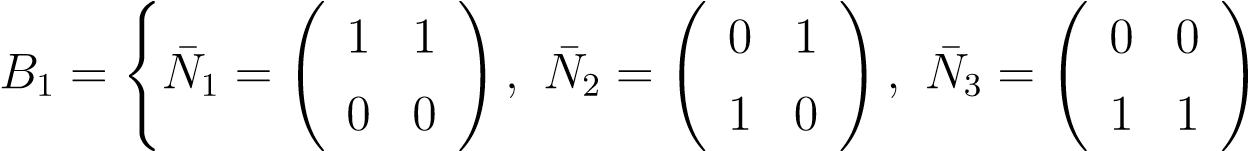
  1   1   0 

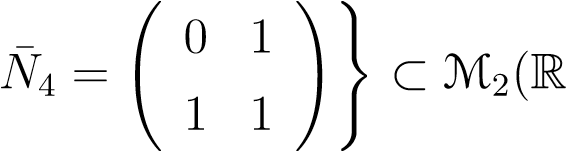
      

  0  ¯ 1 3 

*B*2 = *M*¯1 =  1 *, M*2 =  1 *, M*¯3 =  2  ⊂ M3*,*1(R);

  0   1   1 

* 1.  *,*

) ¸si

( !

|  |  |  |
| --- | --- | --- |
| 2 1  *B*2 = *M*¯1 =  2 0 | *, M*¯2 = | 1. 1 2. 1 |

! !

1 2

*, M*¯3 = ,

1 2

!)

1 1

*M*¯4 = ⊂ M2(R).

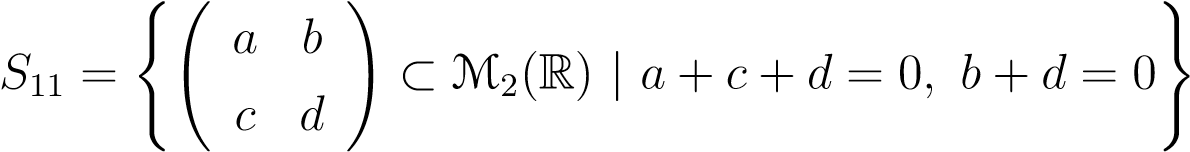
1 1

Sa˘ se arate pentru fiecare caz c˘a *B*1 ¸si trecere de la baza .

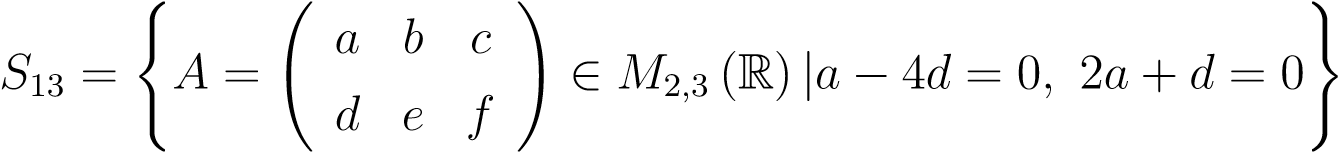
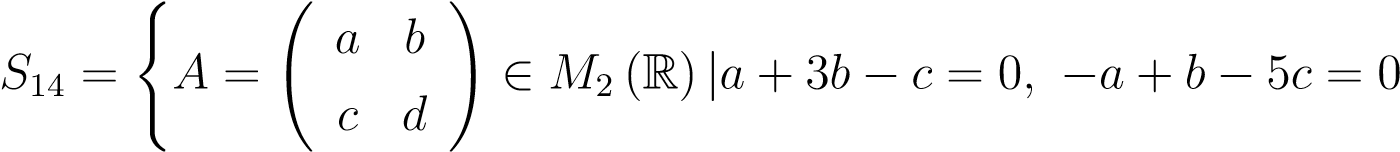
*B*2 sunt baze ¸si sa˘ se determine matricele de

### 2.3 Subspa¸tii vectoriale

5. Sa˘ se studieze daca˘ urm˘atoarele mul¸timi sunt subspa¸tii vectoriale. Pentru subspa¸tiile vectoriale ga˘site sa˘ se ga˘seasc˘a caˆte o baza˘:

1. *S*1 = {(*x,y*) ∈ R2| *x* + 1 = 0};
2. *S*2 = {(*x,y*) ∈ R2| *y* = 0};
3. *S*3 = {(*x,y*) ∈ R2| *x*2 + 4*y* = 0};
4. *S*4 = {(*x,y,z*) ∈ R3 | 3*x* − *y* + 2*z* = 0};
5. *S*5 = {(*x,y,z*) ∈ R3 | *x* − 2*y* + 3*z* = 0*,* 3*x* − *y* − *z* = 0};
6. *S*6 = {(*x,y,z*) ∈ R3| *x* = *y* − *z,y* = *x* + *z,z* = *y* − *x*};
7. *S*7 = {(*x,y,z*) ∈ R3|*x* + *y* − 2*ez* = 0};
8. *S*8 = {*aX*2 + *bX* + *c* ∈ R2[*X*] | *a* = *c*};
9. *S*9 = {*aX*2 + *bX* + *c* ∈ R2[*X*]| − *a* + *b* + 2*c* = 0*,a* + *b* − *c* = 0};
10. *S*10 = {*aX*2 + *bX* + *c* ∈ R2[*X*]| *a* − *b* − *c* = 0*,a* + *b* = 0};
11. ;

|  |  |
| --- | --- |
|      *a*  l) *S*12 = *A* =  *c*    *e* |   *b*  *d*  ∈ *M*3*,*2 (R)|*a* + *e* = 0*, d* − *f* = 0*,*    *f* |

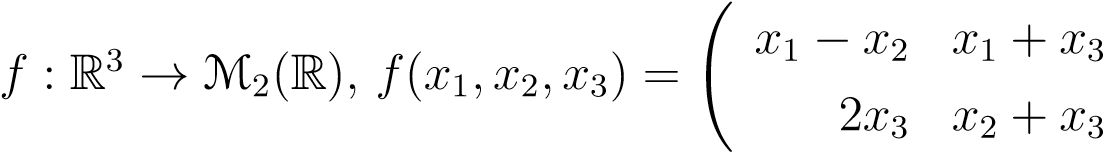
1. + 2*c* = 0*,* 3*b* − *c* = 0};
2. ;
3. *,*
4. + *d* − 3 = 0}.

CAPITOLUL 3

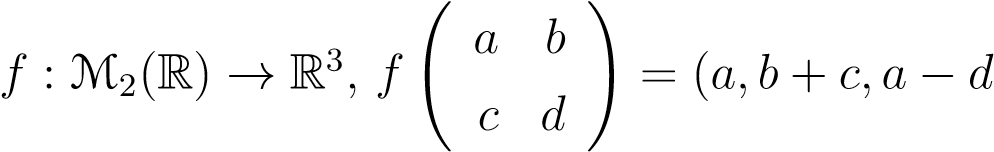
APLICAT¸II LINIARE

1. Fie

a) *f* : R3 → R1[*X*], *f*(*a,b,c*) = (*a* + *b*)*X* + *c*;

!

b);

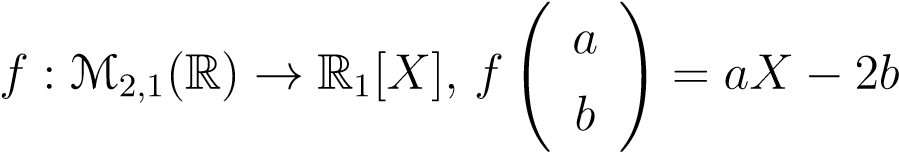
* 1. );
  2. *f* : R2 → R3 *f*(*x,y*) = (2*x* + *y,*−2*x* − *y,*4*x* + 2*y*);
  3. *f* : R2[*X*] → R3, *f*(*aX*2 + *bX* + *c*) = (2*a* − *b,c,*3*a*).
  4. S˘a se determine matricele aplica¸tiilor liniare ˆın perechea de baze canonice;
  5. S˘a se determine Ker(*f*) ¸si dimensiunea lui Ker(*f*). Este *f* injectiva˘? Justifica¸ti ra˘spunsul.
  6. Sa˘ se determine Im (*f*) ¸si dimensiunea lui Im(*f*). Este *f* surjectiva˘? Dar bijectiva˘? Justifica¸ti ra˘spunsul.

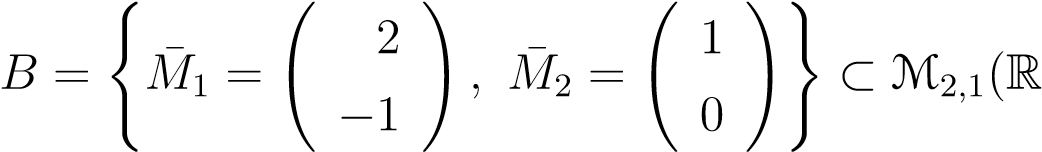
2. Sa˘ se determine pentru urma˘toarele aplica¸tii liniare nucleul, imaginea ¸si caˆte o baza˘ pentru fiecare, precum ¸si matricele ˆın perechile de baze indicate:

a) *f* : R3 → R2, *f*(*x*1*,x*2*,x*3) = (2*x*1 + *x*2 − *x*3*,*3*x*1 + 2*x*3) ˆın perechea de baze

*B* = {*v*¯1 = (1*,*1*,*0)*, v*¯2 = (0*,*1*,*1)*, v*¯3 = (1*,*0*,*1)} ⊂ R3 ¸si

*B*0 = {*u*¯1 = (1*,*−1)*, u*¯2 = (1*,*1)} ⊂ R2;

7

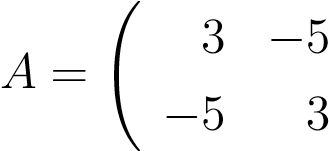
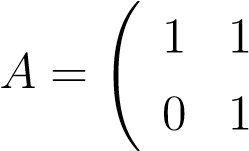
b)ˆın perechea de baze

) ¸si

*B*0 = {*p*¯1 = 2*X* + 1*, p*¯2 = −*X*} ⊂ R1[*X*];

c) *f* : R2[*X*] → R3, *f*(*aX*2 +*bX* +*c*) = (3*a*−2*b*−2*c,*−*a*+4*b*+2*c,a*−2*b*) ˆın perechea de baze

*B* = {*p*¯1 = 2*X* + 1*, p*¯2 = −*X*2 − 1*, p*¯3 = *X*2 + *X* + 1} ⊂ R2[*X*] ¸si *Bc* baza canonic˘a din R3;

1. *f* : R3 → R2[*X*]*, f*(*a,b,c*) = (*a*−*b*+*c*)*X*2+(−2*a*+*b*)*X*−*a*+*b*−*c*ˆın perechea de baze *Bc* baza canonica˘ din R3 ¸si *B* = {*p*¯1 = *X* −1*, p*¯2 = −*X*2+2*X, p*¯3 = *X*2−3} ⊂ R2[*X*].
2. Fie operatorul liniar *f* : R2 → R2, *f*(*x*1*,x*2) = (7*x*1*,*2*x*1 + 7*x*2). S˘a se determine matricea [*f*]*Bc* asociata˘ lui *f* relativ la baza canonica˘, valorile proprii ¸si subspa¸tiile proprii corespunza˘toare. Este [*f*]*Bc* diagonalizabila˘?
3. Sa˘ se studieze dac˘a urma˘toarele matrice sunt diagonalizabile. ˆIn caz afirmativ s˘a se aduca˘ la forma diagonal˘a ¸si sa˘ se precizeze baza ˆın care are aceasta˘ forma˘:

!!

a); b);

c) *A* = [*f*]*Bc*, unde

2 2

*f* : R → R *, f*(*x*1*,x*2) = (9*x*1 − 3*x*2*,*−3*x*1 + *x*2);

   

3 0 −1 1 0 3

d) *A* =   −2 1 1  ; e) *A* =  2 1 2 ;

   

3 −1 −1 3 0 1 f) *A* = [*f*]*Bc*, unde *f* : R2[*X*] → R2[*X*],

*f*(*aX*2 + *bX* + *c*) = (2*a* − *b* + 2*c*)*X*2 + (5*a* − 3*b* + 3*c*)*X* − *a* − 2*c*;

   

0 1 1 −2 −2 −3

g) *A* =  1 0 1 ; h) *A* =  2 3 6 ;

   

2 2 1 −1 −2 −4

   

0 0 0 1 −2 0

i) *A* =  0 −1 1 ; j) *A* =  −2 8 2 ;

   

0 0 −1 0 2 1

   

−1 0 −3 0 1 0

   

k) *A* =  3 2 3 ; l) *A* =  −4 4 0 .

   

−3 0 −1 −2 1 2

ˆIn cazulˆın care *A* este inversabil˘a, s˘a se calculeze folosind teorema lui Hamilton-Cayley *A*−1.

CAPITOLUL 4

FORME BILINIARE. FORME PATRATICE˘

1. Se da˘ aplica¸tia *ϕ* : R2 × R2 → R definita˘ prin

*ϕ*[(*x*1*,x*2)*,*(*y*1*,y*2)] = *x*1*y*1 + 3*x*1*y*2 − *x*2*y*1 − 2*x*2*y*2*.*

Sa˘ se scrie matricea lui *ϕ* ˆın baza canonica˘ din R2. Este *ϕ* simetrica˘?

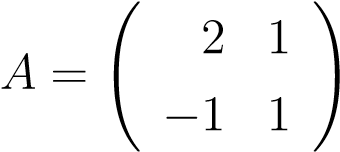
1. Fie forma biliniara˘ *ϕ* : R3 × R3 → R care are in baza canonic˘a matricea *A* =

|  |  |  |
| --- | --- | --- |
|   2    −1    0 | 1  1  1 |   0  2 . Se cere:    −2 |

* 1. S˘a se determine expresia analitica˘ a lui *ϕ*;
  2. S˘a se calculeze *ϕ*[(1*,*−1*,*0)*,*(2*,*1*,*−1)];
  3. S˘a se determine matricea lui *ϕ* ˆın baza

*B* = {*v*¯1 = (1*,*−1*,*2)*, v*¯2 = (−1*,*0*,*2)*, v*¯3 = (3*,*−1*,*1)} ⊂ R3*.*

1. Se da˘ forma biliniara˘ *ϕ* : R1[*X*] × R1[*X*] → R ce are ˆın baza canonica˘ din R1[*X*] ,

matricea .

* 1. S˘a se determine expresia analitica˘ a lui *ϕ*;
  2. Fie baza *B* = {*p*¯1 = *X* − 2*, p*¯2 = 2*X* − 1} ⊂ R1[*X*] ; Sa˘ se scrie matricea lui *ϕ* ˆın

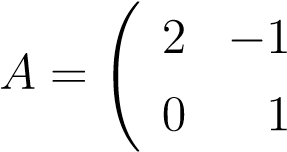
baza *B*.

1. Fie forma biliniara˘ *ϕ* : R1[*X*] × R1[*X*] → R,

*ϕ*(*aX* + *b,cX* + *d*) = *ad* + *bc*

Sa˘ se scrie matricea lui *ϕ* ˆın baza canonica˘ din R1[*X*], respectiv ˆın baza *B* = {*p*¯1 = 2*X* − 3*, p*¯2 = −*X* + 1}. Este *ϕ* o form˘a simetrica˘?

1. Se d˘a forma biliniara˘ *ϕ* : R2 × R2 → R ce are ˆın baza canonica˘ din R2 , matricea

!

;

* 1. S˘a se determine expresia analitica˘ a lui *ϕ*.
  2. Fie baza *B* = {*v*¯1 = (1*,*1)*, v*¯2 = (−1*,*1)} ⊂ R2. S˘a se determine [*ϕ*]*B*.

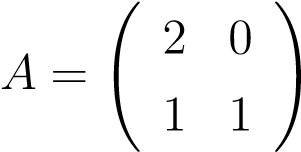
1. Se da˘ forma biliniara˘ *ϕ* : R3 × R3 → R definita˘ prin

*ϕ*[(*x*1*,x*2*,x*3)*,*(*y*1*,y*2*,y*3)] = *x*1*y*1 + *x*2*y*2 + 4*x*3*y*3 + *x*1*y*2 + *x*2*y*1 + 2*x*1*y*3 + 2*x*3*y*1 + 2*x*2*y*3 + 2*x*3*y*2*.*

Sa˘ se determine matricea lui *ϕ* ˆın baza

*B* = {*v*¯1 = (1*,*1*,*0)*, v*¯2 = (1*,*0*,*1)*, v*¯3 = (0*,*1*,*1)} ⊂ R3

¸si sa˘ se calculeze *ϕ*[(3*,*−1*,*1)*,*(1*,*−1*,*2)].

1. Fie forma biliniara˘ *ϕ* : R2 × R2 → R ce are ˆın baza canonica˘ matricea.

Sa˘ se determine expresia analitic˘a a formei pa˘tratice asociate.

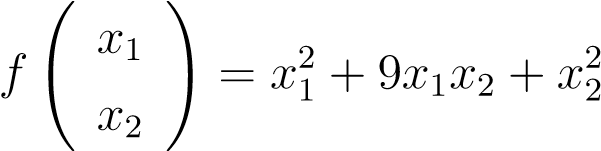
1. Sa˘ se determine expresia analitica˘ a polarei formei pa˘tratice

11

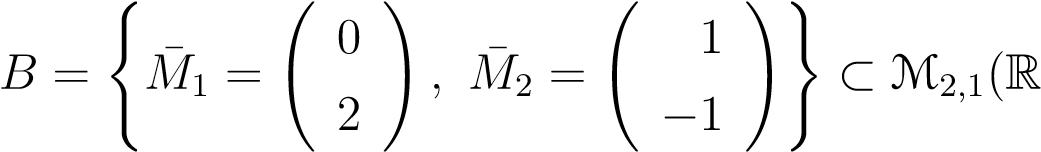
*f* : R3 → R,

*.*

1. Fie forma p˘atratic˘a *f* : M2*,*1(R) → R,

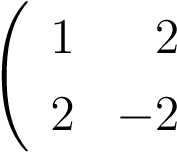
*.*

* 1. S˘a se determine polara lui *f*;
  2. S˘a se scrie matricea asociata˘ lui *f* ˆın baza canonica˘ din M2*,*1(R), respectiv ˆın baza

);

* 1. S˘a se reduc˘a *f* la forma canonic˘a folosind metoda valorilor proprii ¸si sa˘ se precizeze o baz˘a relativ la care *f* are aceast˘a forma˘.

1. Fie forma patratic˘a *f* : R2 → R ce are ˆın baza canonic˘a din R2 matricea *A* =

!

. Sa˘ se determine forma canonica˘ a lui *f* ¸si sa˘ se precizeze o baza˘ rela-

tiv la care *f* are aceast˘a forma˘.

1. Se da˘ forma biliniar˘a simetric˘a *ϕ* : R3×R3 → R definita˘ prin *ϕ*[(*x*1*,x*2*,x*3)*,*(*y*1*,y*2*,y*3)] =

4*x*1*y*1 − 5*x*2*y*2 − 5*x*3*y*3 + *x*1*y*2 + *x*2*y*1 − 3*x*1*y*3 − 3*x*3*y*1. Sa˘ se determine forma pa˘tratic˘a *f* asociata˘ lui *ϕ*, sa˘ se reduca˘ *f* la forma canonica˘ ¸si s˘a se precizeze o baz˘a relativ la care *f* are aceast˘a forma˘.

1. Fie forma p˘atratic˘a *f* : R3 → R definita˘ prin:
   1. ;
   2. ;
   3. .

ˆIn fiecare caz, sa˘ se reduc˘a *f* la forma canonica˘ ¸si sa˘ se precizeze o baza˘ relativ la care *f* are aceast˘a forma˘;

1. Fie forma p˘atratic˘a *f* : R3 → R care are in baza canonica˘ matricea:

 

5 3 0

* 1. *A* =  3 2 −3 ;

 

0 −3 5

 

3 0 −4

* 1. *A* =  0 7 0 .

 

−4 0 3

Sa˘ se determine expresia analitica˘ a lui *f*, precum ¸si forma canonica˘ asociata˘ lui *f*; s˘a se precizeze o baza˘ relativ la care *f* are aceast˘a forma˘.

CAPITOLUL 5

## SPAT¸II VECTORIALE EUCLIDIENE

1. Sa˘ se determine valorile pentru *α* ∈ R astfel ˆıncaˆt vectorii

*u*¯ = (3*,*1*,α*)*, v*¯ = (−6*,*5*,α*)

sa˘ fie ortogonali.

1. Se dau vectorii ¯*u* = ¯*i* + 2¯*j* + *k*¯ ¸si ¯*v* = −2¯*i* + ¯*j* + 2*k*¯. S˘a se calculeze:
   1. Lungimile vectorilor ¯*u* ¸si ¯*v*;
   2. Unghiul dintre vectorii ¯*u* ¸si ¯*v*;
   3. S˘a se determine versorii vectorilor ¯*u* ¸si ¯*v*.
2. Se dau vectorii ¯*u* = (1*,*−2*,*3) ¸si ¯*v* = (0*,*3*,*2). S˘a se calculeze:
   1. unghiul dintre cei doi vectori;
   2. ¯*u* × *v*¯.

−→

1. Se dau punctele *A*(2*,*2*,*1) ¸si *B*(4*,*1*,*3). Sa˘ se determine lungimea vectorului *AB*;
2. Fie punctele *A*(4*,*−2*,*2), *B*(3*,*1*,*1), *C*(4*,*2*,*0) ¸si *D*(0*,*0*,*9). S˘a se determine lunigimea ˆın˘al¸timii din *D* a tetraedrului *ABCD*.
3. Fie punctele *A*(1*,*2*,*−1), *B*(1*,*0*,*3), *C*(2*,*1*,*2) ¸si *D*(2*,*3*,*4). Sa˘ se calculeze:
4. Aria triunghiului *ABC*;
5. Lungimea medianei din *A* a triunghiului *ABC*, perimetrul triunghiului *ABC* ¸si ma˘sura unghiului *ABC*;
6. Lungimea ˆına˘l¸timii duse din *D* ¸si volumul tetraedrului *ABCD*.
7. Sa˘ se verifice ca˘ sistemul de vectori

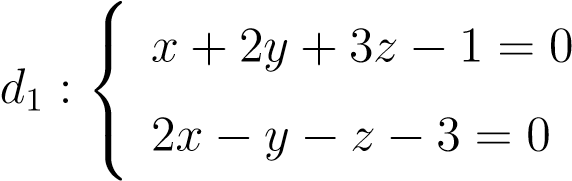
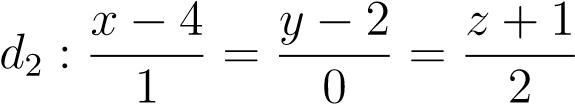
*B* = {*v*¯1 = (−1*,*−1*,*1)*, v*¯2 = (1*,*−2*,*−1)*, v*¯3 = (1*,*0*,*1)}

este un sistem ortogonal de vectori. Sa˘ se ortonormeze baza *B* folosind procedeul de ortonormare Gramm-Schmidt.

1. Utilizaˆnd procedeul lui Gramm-Schmidt, sa˘ se ortonormeze bazele:
   1. *B* = {*v*¯1 = (1*,*1)*, v*¯2 = (2*,*3)};
   2. *B* = {*v*¯1 = (1*,*0*,*1)*, v*¯2 = (0*,*−1*,*2)*, v*¯3 = (1*,*0*,*0)};
   3. *B* = {*v*¯1 = (2*,*1*,*2)*, v*¯2 = (3*,*3*,*0)*, v*¯3 = (1*,*−1*,*−5)};
   4. *B* = {*v*¯1 = (1*,*−2*,*0)*, v*¯2 = (0*,*−2*,*1)*, v*¯3 = (2*,*1*,*2)}.

## 5.1 Dreapta ¸si planul ˆın spa¸tiu

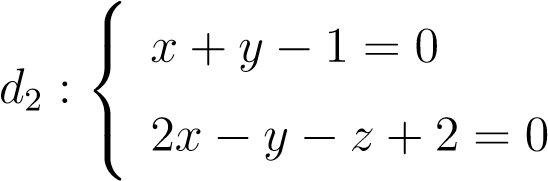
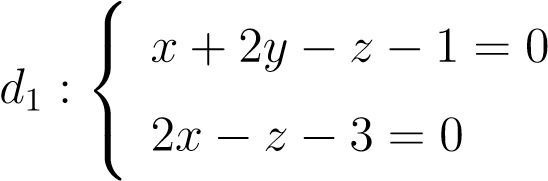
1. Se dau punctele *A*(3*,*−1*,*3), *B*(2*,*1*,*−1) ¸si dreptele

 ¸si  *.*

Sa˘ se determine ecua¸tiile carteziene ale dreptelor:

* 1. *AB*;
  2. *d*3 ce con¸tine punctul *A* ¸si este paralela˘ cu dreapta *d*1;
  3. *d*4 ce con¸tine punctul *B* ¸si este perpendicular˘a pe planul *π*, unde *π* este determinat de direc¸tiile lui *d*1 ¸si *d*2 ¸si trece prin punctul *A*.

1. Se dau dreptele

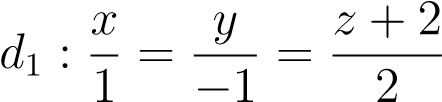
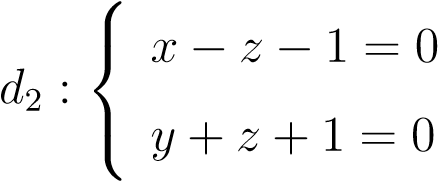
¸si

*.*

Sa˘ se determine:

* 1. ecua¸tiile carteziene ale dreptei *d*1;
  2. ecua¸tiile parametrice ale dreptei *d*2;
  3. unghiul dintre direc¸tiile celor doua˘ drepte.
  4. S˘a se stabileasc˘a daca˘ cele doua˘ drepte se intersecteaza˘.

1. a) Sa˘ se scrie ecua¸tia dreptei *d* ce trece prin punctul de intersec¸tie al dreptelor

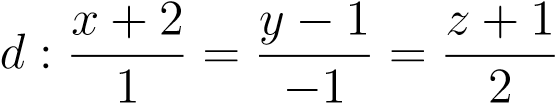
¸si

*.*

¸si este perpendicular˘a pe planul *π* : 2*x* + *y* − 3*z* − 3 = 0;

b) S˘a se calculeze distan¸ta de la punctul *A*(1*,*0*,*−1) la dreapta *d* ¸si la planul *π*.

1. Se dau punctele *A*(3*,*−1*,*0), *B*(−2*,*1*,*−1), *C*(1*,*1*,*−1) ¸si dreapta

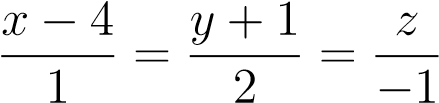
 *.*

Sa˘ se determine ecua¸tia planului care con¸tine:

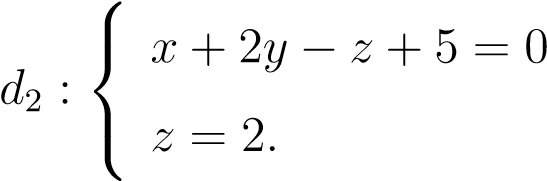
1. punctele *A*, *B* ¸si *C*;
2. punctul *A* ¸si este perpendicular pe dreapta *d*;
3. dreptele *d* ¸si *AB*.
4. a) S˘a se g˘aseasca˘ coordonatele simetricului punctului *A*(1*,*−1*,*1) fa¸ta˘ de planul *π*1 : *x* − *y* + 2*z* + 2 = 0;
   1. S˘a se ga˘seasca˘ coordonatele simetricului punctului *B*(−1*,*−1*,*2) fa¸t˘a de planul *π*2 : *x* + *y* + 3*z* + 7 = 0;
   2. Sa˘ se ga˘seasca˘ coordonatele simetricului punctului *C*(−1*,*2*,*6) fa¸ta˘ de planul *π*3 :

2*x* + *y* − *z* = 0.

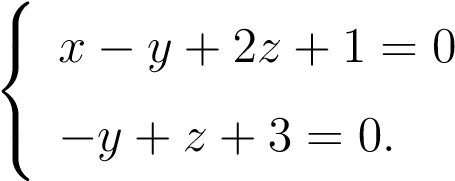
1. a) Sa˘ se ga˘seasca˘ coordonatele simetricului punctului *A*(0*,*−1*,*2) fa¸ta˘ de dreapta *d*1 :

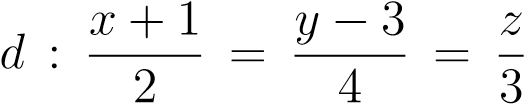


* 1. S˘a se ga˘seasc˘a coordonatele simetricului punctului *B*(−1*,*−1*,*0) fa¸t˘a de dreapta

 ;

* 1. Sa˘ se ga˘seasca˘ coordonatele simetricului punctului *C*(1*,*1*,*1) fa¸t˘a de dreapta *d*3 :

 .

1. a) S˘a se studieze dac˘a dreapta  este paralela˘ cu planul *π* :

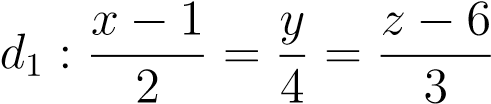
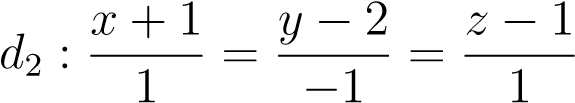
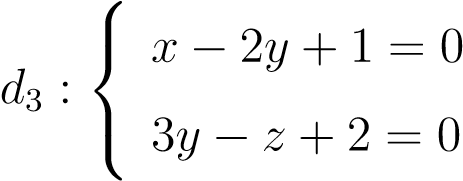
3*x* − 3*y* + 2*z* − 5 = 0;

* 1. S˘a se calculeze distan¸ta de la punctul *A*(1*,*−1*,*0) la dreapta *d* ¸si la planul *π*.

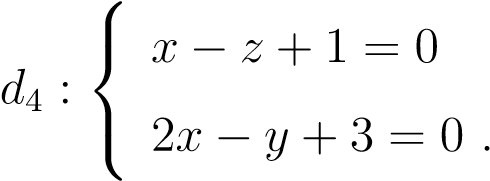
1. Fie punctele *A*(1*,*−1*,*2), *B*(5*,*1*,*−2), *M*0(1*,*−2*,*4) ¸si planul *π* : −*x* + 2*y* − 2*z* + 1 = 0.

Sa˘ se determine distan¸ta de la punctul:

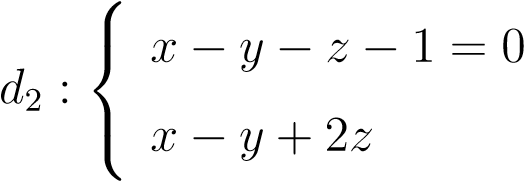
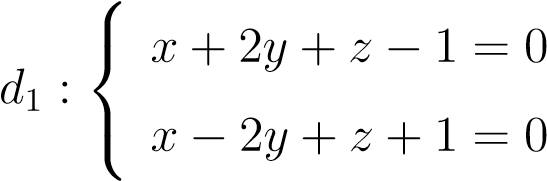
* 1. *A* la *B*;
  2. *M*0 la planul *π*;
  3. *M*0 la dreapta *AB*.

1. Sa˘ se calculeze distan¸ta dintre *M*(1*,*0*,*−2) ¸si:
   1. planul *π*1 : 3*x* − *y* − 2*z* + 1 = 0
   2. planul *π*2 : 6*x* − 2*y* − 4*z* − 3 = 0;
   3. dreapta
   4. dreapta ;
   5. dreapta

*.*

* 1. dreapta

1. Sa˘ se determine unghiul dintre:
   1. planele *π*1 : *x* + 2*y* − 2*z* − 1 = 0 ¸si *π*2 : *x* + *y* + 1 = 0;
   2. dreptele

¸si

+ 1 = 0 ;

* 1. dreapta *d*1 ¸si planul *π*2.

CAPITOLUL 6

GEOMETRIA DIFERENT¸IALA A˘ CURBELOR S¸I SUPRAFET¸ELOR DIN E3

### 6.1 Elemente de geometrie diferen¸tial˘a a curbelor

1. a) Sa˘ se scrie ecua¸tia tangentei la curba

 3 *x*(*t*) = *t*



C : *y*(*t*) = *t*2

 *z*(*t*) = 1

care trece prin punctul *M*0(−1*,*1*,*1);

b) Sa˘ se g˘aseasca˘ puncteleˆın care planul rectificant este paralel cu planul *π* : 2*x*−3*y*+

5 = 0.

1. Sa˘ se scrie ecua¸tia tangentei ¸si a planului normal la curba:



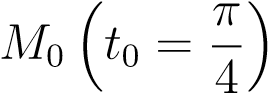
 *x*(*t*) = *t*3 − 2*t,*

* 1. C : *y*(*t*) = 4*t* + 2*,* ˆın punctul *M*0(*t*0 = 1);

 *z*(*t*) = −*t*2 + *t*



 *x*(*t*) = *a*cos2 *t,*

* 1. C : *y*(*t*) = *a*sin*t*cos*t,* ˆın punctul;

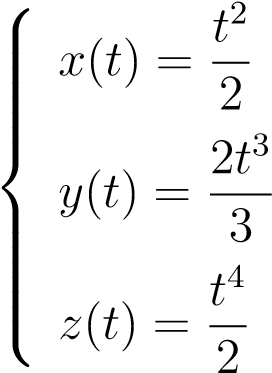
 *z*(*t*) = *a*sin*t*

( *y* = *x*2*,*

* 1. C : ˆın punctul *M*0(2*,*4*,*6).

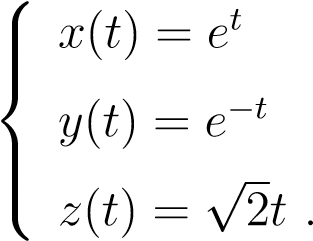
*z* = 3*x*

1. Sa˘ se determine ecua¸tiile dreptelor ¸si planelor triedrului lui Fr´enet ˆın punctul *M*0(*t*0 = 1) al curbei

C :

*.*

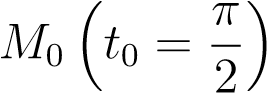
1. Sa˘ se determine ecua¸tiile dreptelor ¸si planelor triedrului lui Fr´enetˆın punctul *M*0(1*,*1*,*0) al curbei

C :

1. Sa˘ se ga˘sesca˘ punctele ˆın care binormala la curba

C : *α*(*t*) = (*t*3 − 1)¯*i* + (*t*2 + 1)¯*j* + *tk, t*¯ ∈ R

este perpendicular˘a pe planul *π* : *x* − 3*y* + 3*z* − 11 = 0.

1. a) Sa˘ se determine versorii triedrului lui Fr´enet ˆın punctul  al curbei



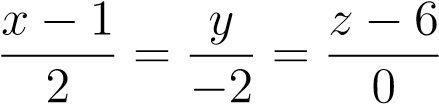
*x*(*t*) = 1 − cos*t*

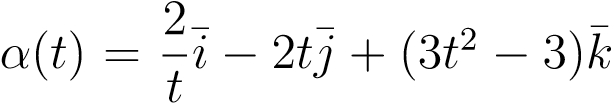
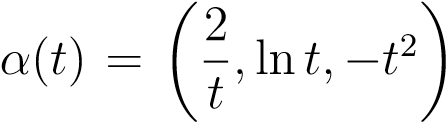


C : *y*(*t*) = sin*t , t* ∈ [0*,π*]*.*

 *z*(*t*) = *t*

b) Sa˘ se ga˘sesca˘ punctele ˆın care planul rectificant este perpendicular pe dreapta *d* :

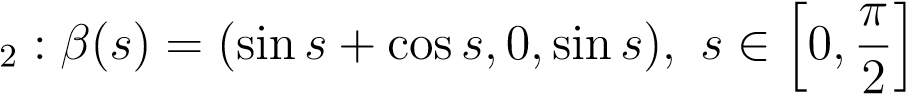
*.*

1. Sa˘ se determine punctele de pe curba C :ˆın care planul normal la curba˘ este perpendicular pe planul *π* : 6*y* + *z* + 5 = 0.
2. Sa˘ se determine punctele de pe curba C : ˆın care binormala la

curba˘ este paralela˘ cu planul *π* : *x* − *y* + 8*z* − 1 = 0.

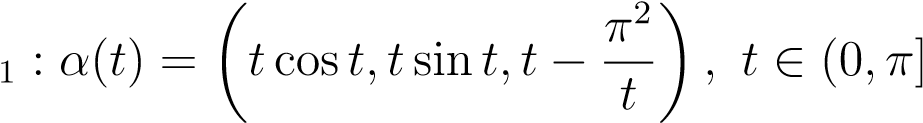
1. Sa˘ se determine unghiul curbelor a)

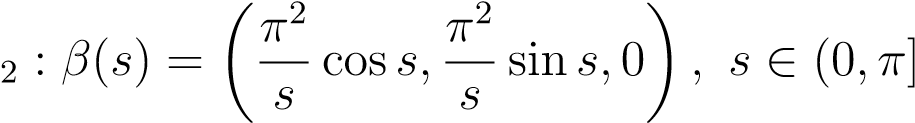
C1 : *α*(*t*) = (1 + cos*t,*1 − sin*t,*1)*, t* ∈ [0*,π*]

C *.* b)

C1 : *α*(*t*) = (1 − sin*t,*2*,*1 + cos*t*)*, t* ∈ [0*,π*]

C2 : *β*(*s*) = (*s* − 1*,*2*s,s*)*, s* ∈ R*.* c)

C

C*.*

1. Sa˘ se calculeze lungimea arcelor de curba˘ *AB*



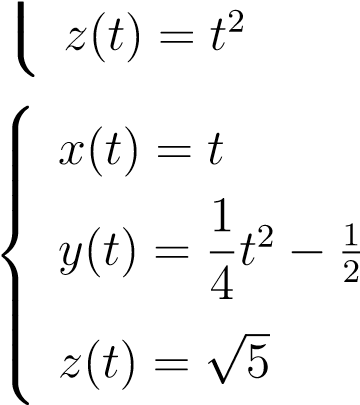
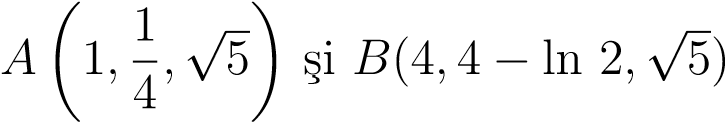
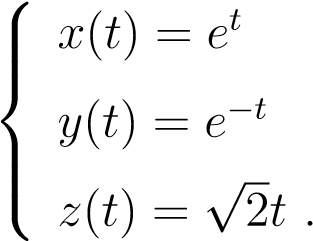
 *x*(*t*) = cos3(*t*)

* 1. C : *y*(*t*) = sin3(*t*) unde *A*(1*,*0*,*−1) ¸si *B*(0*,*1*,*−1)

 *z*(*t*) = −1



 *x*(*t*) = 2*t*

* 1. C : *y*(*t*) = ln t unde *A*(2*,*0*,*1) ¸si *B*(2*e,*1*,e*2)
  2. C : ln t unde
  3. C : unde

1. Sa˘ se calculeze curbura curbei

 2 + 2*t x*(*t*) = *t*



C : *y*(*t*) = 3*t* − 5

 *z*(*t*) = 3

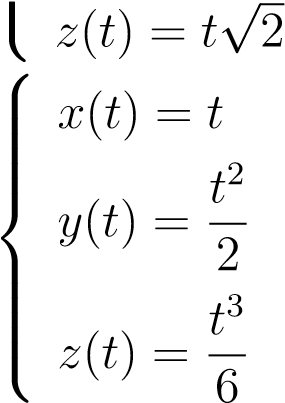
ˆın punctul

1. Sa˘ se calculeze curbura ¸si torsiunea curbelor

 *t x*(*t*) = *e*



* 1. C : *y*(*t*) = *e*−*t*



* 1. C :

ˆın punctul *M*0(*t*0 = 1);

### 6.2 Elemente de geometria diferen¸tial˘a a suprafe¸telor

1. a) Sa˘ se scrie ecua¸tiile planului tangent ¸si ale normalei ˆın punctul *M*0 (*u*0 = 2*,v*0 = 0) la curba:



 *x* = *uev*

*S* : *y* = *ue*−*v*

 *z* = *uv.*

* 1. S˘a se g˘aseasca˘ punctele ˆın care planul tangent este paralel cu axa *Ox*.

1. a) Sa˘ se scrie ecua¸tiile planului tangent ¸si ale normaleiˆın punctul *M*0 (3*,*5*,*7) la curba:



 *x* = 2*u* − *v*

*S* : *y* = *u*2 + *v*2

 *z* = *u*3 − *v*3*.*

* 1. S˘a se g˘aseasca˘ punctele ˆın care normala este perpendiculara˘ pe planul *π* : 3*y* +2*z* − 1 = 0.

1. Sa˘ se determine unghiul ¸si elementul de arc al curbelor de coordonate *C*1 : *u* = 2 ¸si *C*2 : *v* = 0 ale suprafe¸telor: a)



 *x* = *u* + sin*v*

*S* : *y* = *u* − sin*v*

 *z* = *v*

b)



 *x* = *u*cos*v*

*S* : *y* = *u*sin*v*

 *z* = *u*

c)





*S* :



1. Fie suprafa¸ta de ecua¸tie

*x* = *u* + *v y* = *u* − *v . z* = *uv*

(*S*) : *z* = *x*3 + *y*3*.*

Sa˘ se determine coeficien¸tii primei forme fundamentale, ecua¸tia planului tangent ¸si a normalei ˆın punctul *M*(1*,*2*,*9)*.*

1. a) Sa˘ se studieze dac˘a curba *u* = *v* situata˘ pe suprafa¸ta

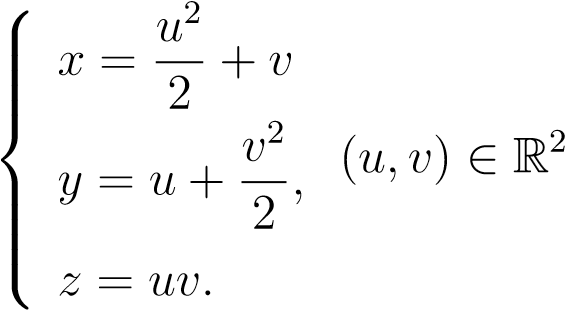
*r*¯(*u,v*) = (*u*2 + *v, u*2 − *v, uv*)*,* (*u,v*) ∈ R2

este o curba˘ plan˘a;

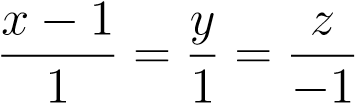
b) Sa˘ se ga˘seasca˘ punctele ˆın care normala este perpendiculara˘ pe planul *π* : *x* + *y* −

4*z* − 5 = 0*.*

1. Se considera˘ suprafa¸ta de ecua¸tii

*.*

* 1. S˘a se scrie elementul de arie al suprafe¸tei, sa˘ se arate ca˘ curba *u* = *v* este o curb˘a plana˘ ¸si sa˘ se calculeze curbura sa;
  2. Sa˘ se ga˘seasca˘ punctele ˆın care planul tangent este perpendicular pe dreapta *d* :

.

1. Se considera˘ suprafa¸ta de ecua¸tii



 *x* = *u*2 + 2*u* + 1 2

*S* : *y* = *u*2 − 2*u* − 1 (*u,v*) ∈ R *.*

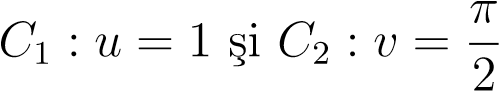
 *z* = *v,*

* 1. S˘a se determine curba C de pe suprafa¸ta *S* pentru *v* = −1 ¸si elementul de arc al curbei;
  2. S˘a se determine metrica ¸si matricea primei forme fundamentale;
  3. Sa˘ se ga˘seasca˘ puncteleˆın care normala este perpendiculara˘ pe planul *π* : 2*x*−*y*+7 =

0.

1. Se considera˘ suprafa¸ta de ecua¸tii

|  |  |
| --- | --- |
|  |  |
|  *x* = *u* cos*v y* = *u* sin*v*   *z* = 2 sin(2*v*)*,* | (*u,v*) ∈ R × [0*,π*]*.* |

* 1. Sa˘ se g˘aseasc˘a metrica asociata˘ suprafe¸tei ¸si ecua¸tia planului tangent la suprafa¸ta˘ˆın punctul *M*0(*u*0 = 1*,v*0 = 0)*.*
  2. S˘a se calculeze elementul de arc ¸si unghiul curbelor.
  3. Sa˘ se g˘aseasca˘ puncteleˆın care planul tangent este paralel cu planul *π* : *y*+*z*−3 = 0. 9. Se considera˘ suprafa¸ta de ecua¸tii

|  |  |
| --- | --- |
|  |  |
|  *x* = *u* cos*v y* = *u* sin*v*   *z* = *u* + *v,* | (*u,v*) ∈ R × (0*,*2*π*)*.* |

Se cere unghiul dintre curbele de coordonate *C*1 : *u* = 0 ¸si *C*2 : *v* = 0 ¸si elementul lor de arc al celor dou˘a curbe.

1. Sa˘ se determine punctele suprafe¸tei:

*r*(*u,v*) = *u* sin*vi* + *u* cos*vj* + (*u* − *v*)*k,* (*u,v*) ∈ R2

ˆın care normala este perpendiculara˘ pe planul (*π*) : *x* + *y* = 0*.*

1. Sa˘ se determine punctele suprafe¸tei:

*r*(*u,v*) = (*u* + *v*)*i* + (*u* − *v*)*j* + (*uv*)*k,* (*u,v*) ∈ R2

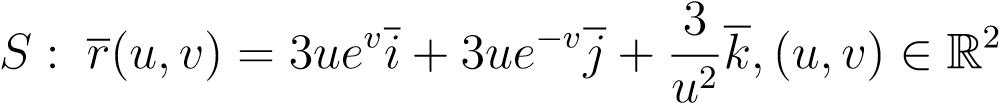
ˆın care planul tangent este paralel cu planul (*π*) : *x* + *y* − 2*z* = 0*.*

1. Care este unghiul dintre planul tangent la suprafa¸ta:

*S* : *r*(*u,v*) = (*u* + *v*)*i* + (*u* − *v*)*j* − (*uv*)*k,* (*u,v*) ∈ R2*,*

ˆın punctul *M*(*u*0 = 1*,v*0 = 0) ¸si dreapta (*d*) : *x* = *y* = *z*?

1. Fie suprafa¸ta

*.*

* 1. S˘a se determine unghiul curbelor de coordonate *C*1 : *v*=ln 2 ¸si *C*2 : *u* = −3
  2. Sa˘ se g˘aseasca˘ punctele suprafe¸tei ˆın care planul tangent este paralel cu planul deecua¸tie *x* − *y* + *z* = 2*.*
  3. Elementul de arie al suprafe¸tei.

1. Fie suprafa¸ta

*S* : *r*(*u,v*) = *u* cos*vi* + *u* sin*vj* + 2*uk,*(*u,v*) ∈ R × (0*,*2*π*)*.*

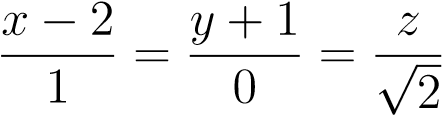
* 1. Sa˘ se scrie ecua¸tia planului determinat de normala la suprafa¸t˘aˆın punctul *P*(0*,*1*,*2).
  2. S˘a se determine unghiul curbelor C ¸si C*.*
  3. Coeficien¸tii primei forme fundamentale ale suprafe¸tei.

1. Fie suprafa¸ta

*S* : ¯*r*(*u,v*) = (sin(*u* + *v*)*,*cos(*u* + *v*)*,*2*u*)*,*(*u,v*) ∈ (0*,*2*π*) × (0*,*2*π*)*.*

Se cere:

* 1. Calcula¸ti curbura ¸si torsiunea curbei C :  situata˘ pe suprafa¸ta *S* ˆın punctul *A*(*u*0 = 0)*.*
  2. Calcula¸ti unghiul dintre tangenta la curba C ˆın punctul *A*(*u*0 = 0) ¸si dreapta

*.*

* 1. Elementul de arie al suprafe¸tei.

1. Fie suprafa¸ta

*S* : ¯*r*(*u,v*) = (*eu*+*v,e*−*u*+*v,e*2*u*)*,*(*u,v*) ∈ R2*.*

Se cere:

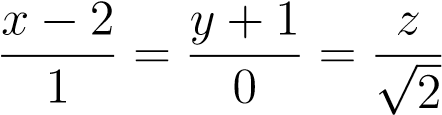
* 1. Ecua¸tia planului tangent ¸si a normalei la suprafa¸t˘a ˆın punctul *M*(*u*0 = 0*,v*0 = 0)*.*
  2. Ecua¸tia binormalei ¸si a planului osculator ˆın punctul *A*(*u*0 = 0) la curba C : *v* = 0 situata˘ pe suprafa¸ta *S.*
  3. Calcula¸ti distan¸ta de la *M*(2*,*3*,*5) la planul osculator determinat la punctul b).

1. Fie suprafa¸ta

*S* : ¯*r*(*u,v*) = (sin*u* cos*v,*sin*u* sin*v,*cos*u*)*,*(*u,v*) ∈ (0*,*2*π*) × (0*,*2*π*)*.*

Se cere:

* 1. Calcula¸ti curbura ¸si torsiunea curbei C :  situata˘ pe suprafa¸ta *S* ˆın punctul *A*(*u*0 = 0)*.*
  2. Calcula¸ti unghiul dintre tangenta la curba C ˆın punctul *A*(*u*0 = 0) ¸si dreapta

*.*

1. Fie suprafa¸ta

*S* : ¯*r*(*u,v*) = (*eu* cos*v,e*−*u* sin*v,e*2*u*)*,*(*u,v*) ∈ R × (0*,*2*π*)*.*

Se cere:

* 1. Ecua¸tia planului tangent ¸si a normalei la suprafa¸t˘a ˆın punctul *.*
  2. Ecua¸tia binormalei ¸si a planului osculator ˆın punctul *A*(*u*0 = 0) la curba C :

situata˘ pe suprafa¸ta *S.*

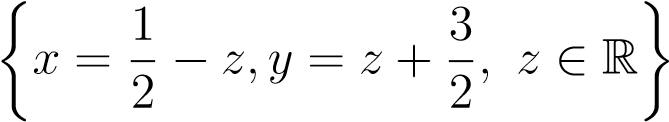
CAPITOLUL 7

RASPUNSURI˘

Ra˘spunsuri verificate de As.dr.mat. Ene Remus Daniel ¸si As.drd.mat. Pa¸sca Ma˘d˘alina Sofia

### CAPITOLUL 1

#### Sisteme liniare

a) Sol={*x* = −*z,y* = 3 + *z, z* ∈ R} b) Sol=

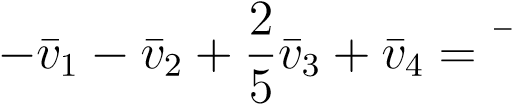
c) Sistem incompatibil d) Sistem incompatibil

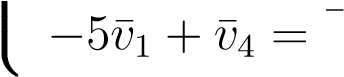
### CAPITOLUL 2 SPAT¸II VECTORIALE

**Dependen¸t˘a ¸si independen¸t˘a liniar˘a. Sistem de generatori. Baze.**

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 −2¯*v*1 + ¯*v*2 = ¯0

1. a) RD: 3¯*v*1 + ¯*v*3 = ¯0 b) RD: 0;

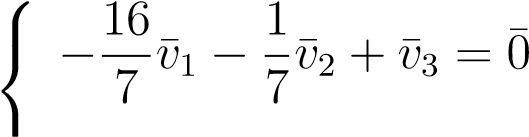
0;

c) RD: −*p*¯1 − *p*¯2 + ¯*p*3 = ¯0; d) RD: −*M*¯1 − *M*¯2 + *M*¯3 = ¯0;

1. a) Sistem liniar dependent, sistem de generatori, nu e baza˘,

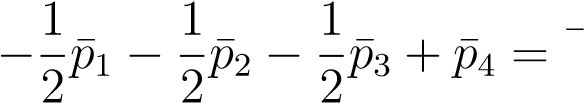
RD: −*v*¯1 − 2¯*v*2 + ¯*v*3 = ¯0;

1. Sistem liniar dependent, nu este sistem de generatori, nu e baza˘,

RD: 

 −*v*¯1 + ¯*v*2 + ¯*v*4 = ¯0;

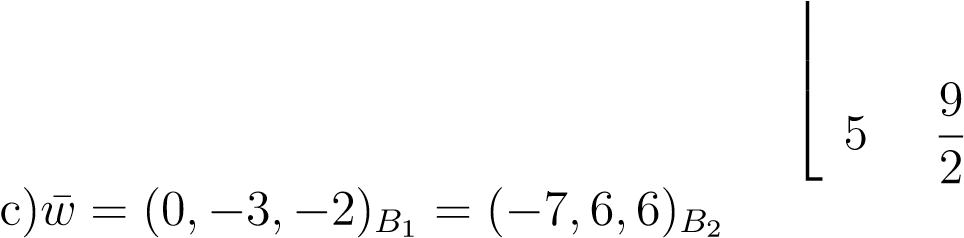
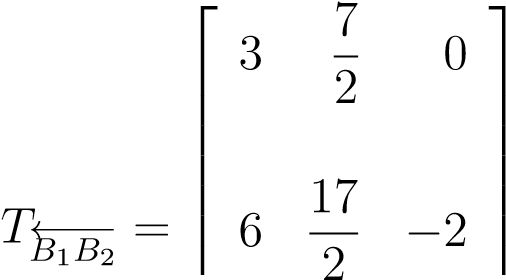
1. Sistem liniar independent, nu este sistem de generatori, nu e baza˘
2. Sistem liniar independent, nu este sistem de generatori, nu e baza˘
3. Sistem liniar dependent, sistem de generatori, nu e baz˘a,

RD: 0;

1. Sistem liniar independent, nu este sistem de generatori, nu e baza˘
2. Sistem liniar dependent, nu este sistem de generatori, nu e baza˘,

RD:−*M*¯1 + 2*M*¯2 + *M*¯3 = ¯0;

#### Schimb˘ari de baze

(3) a) *B*1 ¸si *B*2 sunt baze b)

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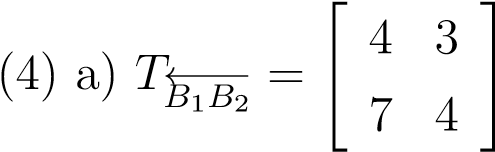


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1

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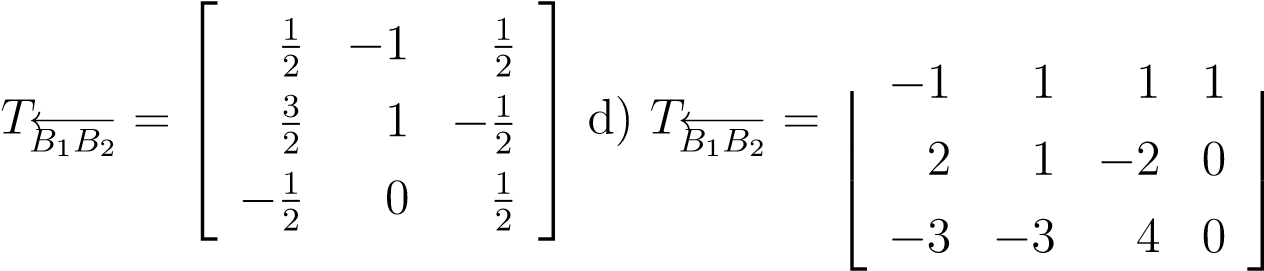
 1 2 0

b) *T*←−−−*B*1*B*2 =  −1 1 1 

 

1. 1 1

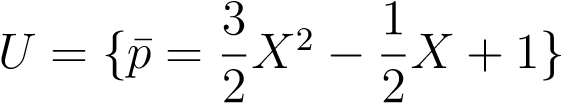
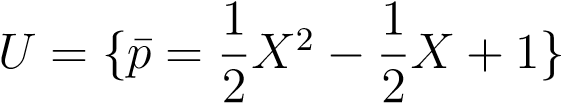
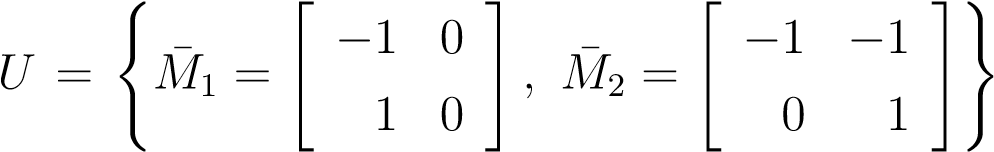
 

1. 1 −1 0

  c)

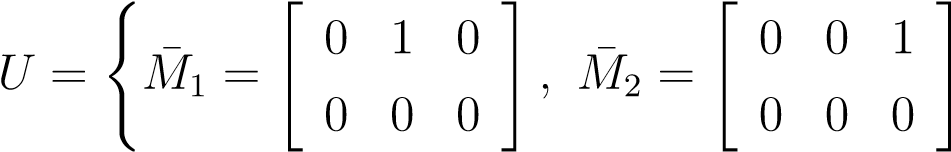
#### Subspa¸tii vectoriale

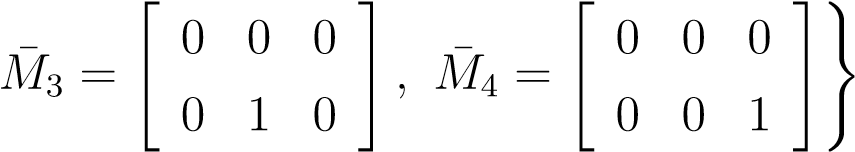
(5) a) *S*1 nu este subspa¸tiu vectorial

1. *S*2 este subspa¸tiu vectorial ¸si *U* = {*u*¯ = (1*,*0)} este o baza˘ a sa
2. *S*3 nu este subspa¸tiu vectorial
3. *S*4 este subspa¸tiu vectorial ¸si *U* = {*u*¯1 = (1*,*3*,*0)*, u*¯2 = (0*,*2*,*1)} este o baza˘ a sa
4. *S*5 este subspa¸tiu vectorial ¸si *U* = {*u*¯ = (1*,*2*,*1)} este o baza˘ a sa
5. *S*6 este subspa¸tiu vectorial ¸si *U* = {*u*¯1 = (1*,*1*,*0)*, u*¯2 = (−1*,*0*,*1)} este o baza˘ a sa
6. *S*7 nu este subspa¸tiu vectorial
7. *S*8 este subspa¸tiu vectorial ¸si *U* = {*p*¯1 = *X*2 + 1*, p*¯2 = *X*} este o baza˘ a sa
8. *S*9 este subspa¸tiu vectorial ¸si este o baza˘ a sa
9. *S*10 este subspa¸tiu vectorial ¸si  este o baza˘ a sa
10. *S*11 este subspa¸tiu vectorial ¸si este o

baza˘ a sa

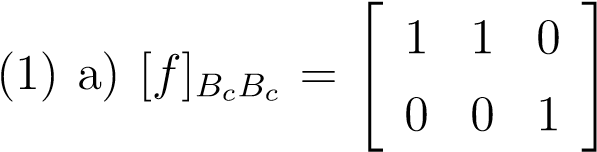
|  |  |  |
| --- | --- | --- |
|      −1 l) *S*12 este subspa¸tiu vectorial ¸si *U* = *M*¯1 =  0    1  a sa |    0 0  0 *, M*¯2 =  0     0 0 |    1.     este o baza˘   1.      1 |

1. *S*13 este subspa¸tiu vectorial ¸si  *,*

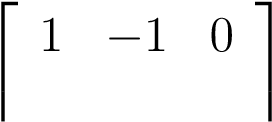
 este o baza˘ a sa

1. *S*14 nu este subspa¸tiu vectorial

### CAPITOLUL 3 APLICAT¸II LINIARE

, Ker(*f*)=*L*({(−1*,*1*,*0)}), dim(Ker(*f*))=1, *f* nu este injectiv˘a,

Im(*f*)=*L*({*X,X,*1}), dim(Im(*f*))=2, *f* este surjectiv˘a.

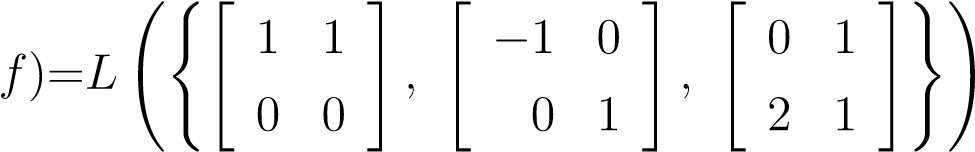
1 0 1

1. [*f*]*BcBc* =  , Ker(*f*)=*L*({(0*,*0*,*0)}), dim(Ker(*f*))=0, *f* este injectiva˘,

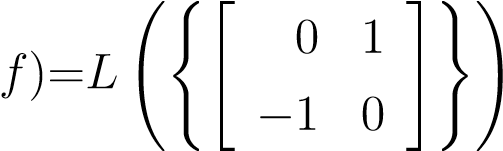
 0 0 2 

 

* 1. 1 1

Im(, dim(Im(*f*))=3, *f* nu este surjectiv˘a.

 

* 1. 0 0 0

1. [*f*]*BcBc* =  0 1 1 0 , Ker(, dim(Ker(*f*))=1, *f* nu este

 

* 1. 0 0 −1

injectiva˘, Im(*f*)=*L*({(1*,*0*,*1)*,*(0*,*1*,*0)*,*(0*,*1*,*0)*,*(0*,*0*,*−1)}), dim(Im(*f*))=3, *f* este surjectiv˘a.

 

* 1. 1

1. [*f*]*BcBc* =  −2 −1 , Ker(*f*)=*L*({(1*,*−2)}), dim(Ker(*f*))=1, *f* nu este injectiva˘,

 

4 2

Im(*f*)=*L*({(2*,*−2*,*4)*,* (1*,*−1*,*2)}), dim(Im(*f*))=1, *f* nu este surjectiv˘a.

 

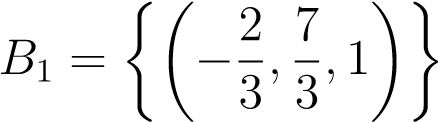
* 1. −1 0

1. [*f*]*BcBc* =  0 0 1 , Ker(*f*)=*L*({0}), dim(Ker(*f*))=0, *f* este injectiv˘a,

 

* 1. 0 0

Im(*f*)=*L*({(2*,*0*,*3)*,* (−1*,*0*,*0)*,* (0*,*1*,*0)}), dim(Im(*f*))=3, *f* este surjectiv˘a, deci *f* este bijectiva˘.

(2) a) Ker(*f*)=*L*(*B*1), unde ) este o baza˘ pentru Ker(*f*),

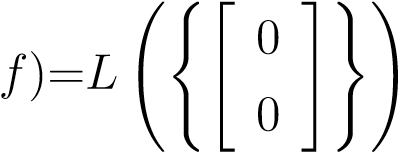
Im(*f*)=*L*({(2*,*3)*,* (1*,*0)*,* (−1*,*2)})*, B*2 = {(2*,*3)*,* (1*,*0)} este o baza˘ pentru Im(*f*),

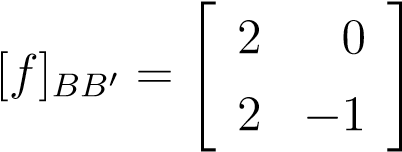
" #

0 −1 −2

[*f*]*BB*0 =

3 1 3

1. Ker(, Im(*f*)=*L*(*B*2)*, B*2 = {*X,*−2} este o baz˘a pentru Im(*f*),



1. Ker(*f*)=*L*({0}), Im(*f*)=*L*(*B*2), unde *B*2 = {(3*,*−1*,*1)*,*(−2*,*4*,*−2)*,*(−2*,*2*,*0)} este o

 

−6 −1 −1

baza˘ pentru Im(*f*), [*f*]*BBc* =  10 −1 5 

 

−4 −1 −1

1. Ker(*f*)=*L*(*B*1), unde *B*1 = {(1*,*2*,*1)} este o baza˘ pentru Ker(*f*),

Im(*f*)=*L*({*X*2 − 2*X* − 1*,* −*X*2 + *X* + 1*, X*2 − 1})*, B*2 = {*X*2 − 2*X* − 1*,* −*X*2 + *X* + 1}

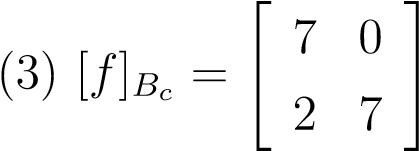
 

−2 −1 4

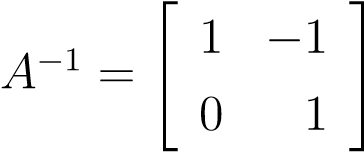
este o baza˘ pentru Im(*f*), [*f*]*BcB* =  0 1 −2 

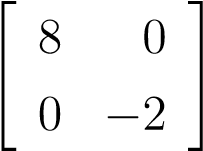
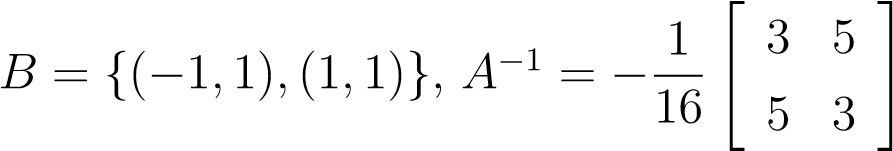
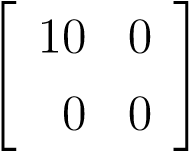
 

1 0 −1

, valoare proprie {7}, *Sλ*=7 = *L*({(0*,*1)}), [*f*]*Bc* nu este diagonaliza-

bila˘.

(4) a) Nu este diagonalizabil˘a,

1.  ˆın baza
2.  ˆın baza *B* = {(−3*,*1)*,*(1*,*3)}, nu este inversabil˘a

 

0 1 1

1. Nu este diagonalizabila˘, *A*−1 =  1 0 −1 

 

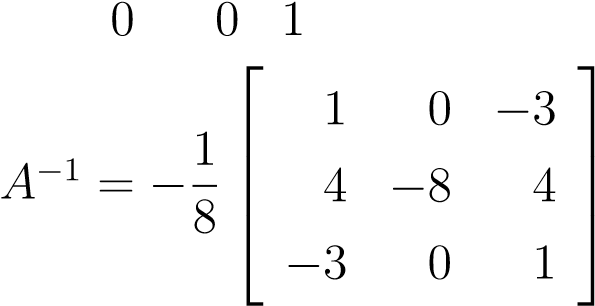
−1 3 3

 

4 0 0

1.  0 −2 0  ˆın baza *B* = {(3*,*4*,*3)*,*(−1*,*0*,*1)*,*(0*,*1*,*0)},

 



 

−6 2 −3

1. Nu este diagonalizabila˘, *A*−1 =  −7 2 −4 

 

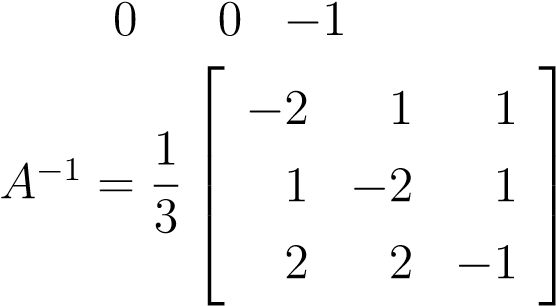
3 −1 1

 

3 0 0

1.  0 −1 0  ˆın baza *B* = {(1*,*1*,*2)*,*(−1*,*0*,*1)*,*(−1*,*1*,*0)},

 



 

0 2 3

1. Nu este diagonalizabila˘, *A*−1 =  −2 −5 −6 

 

1 2 2

1. Nu este diagonalizabila˘ , nu este inversabila˘

 

9 0 0

1.  0 1 0  ˆın baza *B* = {(−1*,*4*,*1)*,*(1*,*0*,*1)*,*(−2*,*−1*,*2)}, nu este inversabil˘a

 

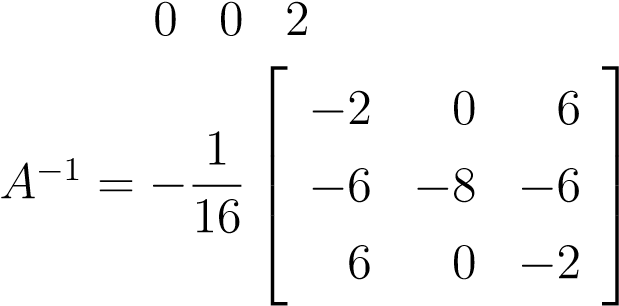
0 0 0

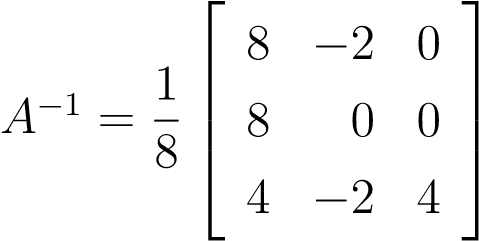
 

−4 0 0

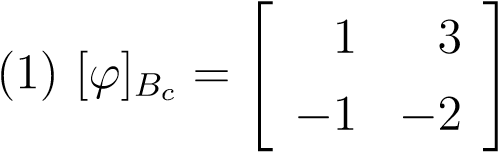
1.  0 2 0  ˆın baza *B* = {(1*,*−1*,*1)*,*(−1*,*0*,*1)*,*(0*,*1*,*0)},

 



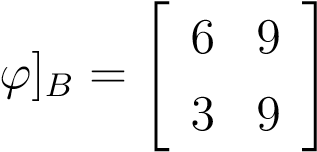
1. Nu este diagonalizabila˘

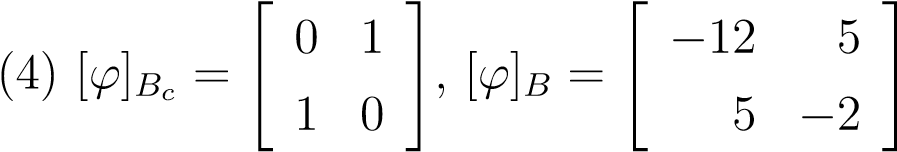
### CAPITOLUL 4 FORME BILINIARE. FORME PATRATICE˘

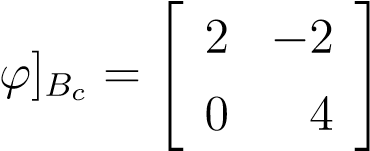
, nu este simetrica˘

1. a) *ϕ*[(*x*1*,x*2*,x*3)*,*(*y*1*,y*2*,y*3)] = 2*x*1*y*1 − *x*2*y*1 + *x*1*y*2 + *x*2*y*2 + *x*3*y*2 +2*x*2*y*3 − 2*x*3*y*3

|  |  |  |  |
| --- | --- | --- | --- |
| b) *ϕ*[(1*,*−1*,*0)*,*(2*,*1*,*−1)] = 8 |   −11 c) [*ϕ*]*B* =  −11    −4 | −15 −6  −15 |   1  −11     14 |

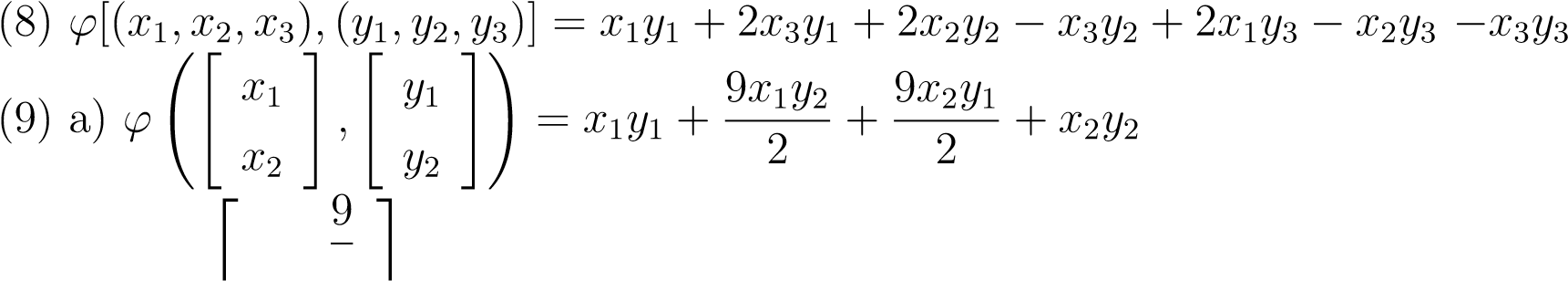
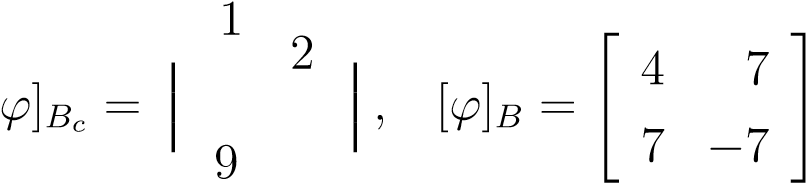
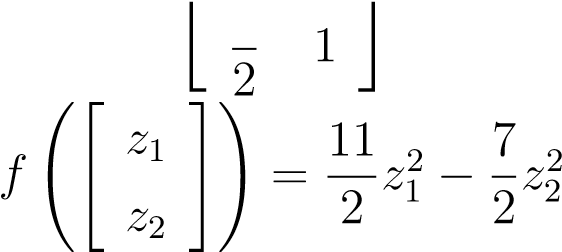
1. a) *ϕ*[*aX* + *b,cX* + *d*] = 2*ac* − *bc* + *ad* + *bd* b) [

, este simetrica˘

(5) a) *ϕ*[(*x*1*,x*2)*,*(*y*1*,y*2)] = *x*1*y*1 − *x*1*y*2 + *x*2*y*2, b) [

|  |  |  |
| --- | --- | --- |
|   4  (6) [*ϕ*]*B* =  6    6 | 6  9  9 |   6  9 *, ϕ*[(3*,*−1*,*1)*,*(1*,*−1*,*2)] = 16    9 |

(7) Nu i se poate asocia o forma˘ p˘atratica˘

1. [
2.  ˆın baza *B* = {(1*,*1)*,*(−1*,*1)}

 ˆın baza *B* = {(−1*,*2)*,*(2*,*1)}

,

 ˆın baza *B* = {(1*,*−1*,*3)*,*(0*,*3*,*1)*,*(−10*,*−1*,*3)}

 ˆın baza *B* = {(1*,*1*,*0)*,*(0*,*0*,*1)*,*(−1*,*1*,*0)}

1.  ˆın baza *B* = {(−1*,*0*,*1)*,*(0*,*1*,*0)*,*(1*,*0*,*1)}
2.  ˆın baza *B* = {(−1*,*−2*,*1)*,*(−1*,*1*,*1)*,*(1*,*0*,*1)}

,

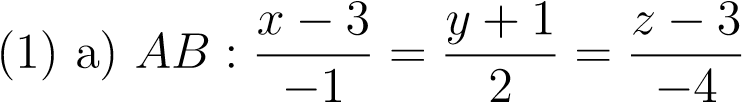
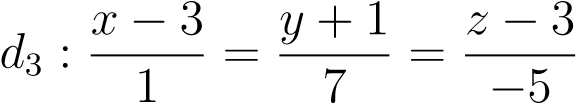
 ˆın baza *B* = {(−1*,*−1*,*1)*,*(1*,*0*,*1)*,*(−1*,*2*,*1)}

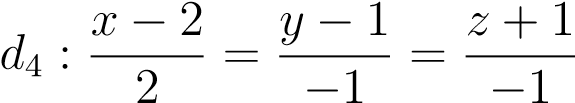
b),

 ˆın baza *B* = {(−1*,*0*,*1)*,*(0*,*1*,*0)*,*(1*,*0*,*1)}

### CAPITOLUL 5

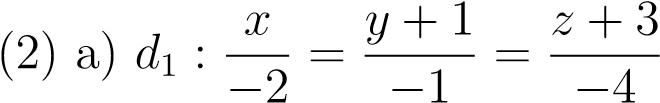
#### Dreapta ¸si planul ˆın spa¸tiu

 b)

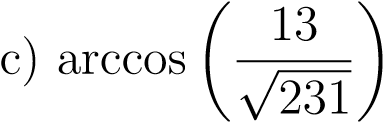
c)

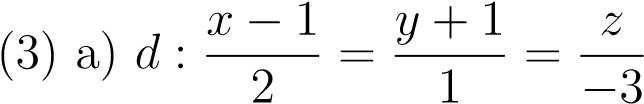
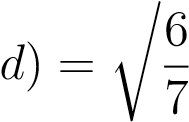
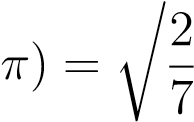


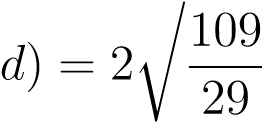
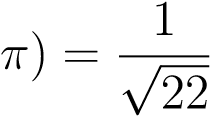
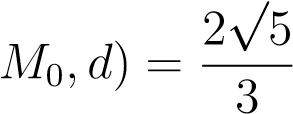
 *x* = −*t*

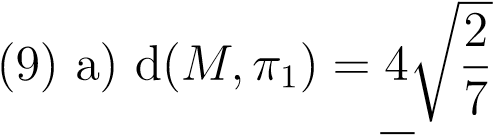
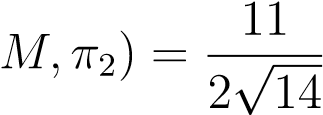
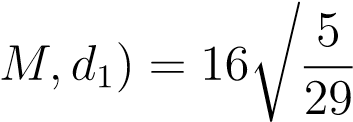
 b) *d*2 : *y* = *t* + 1

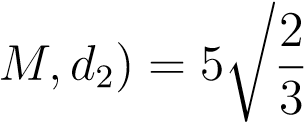
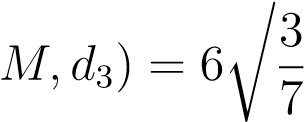
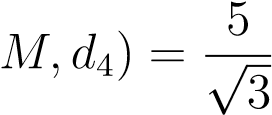
 *z* = −3*t* + 1*.*

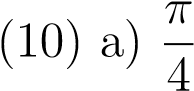
 d) nu se intersecteaz˘a.

 b) d(A,, d(A,

1. a) *π*1 : *y* + 2*z* + 1 = 0 b) *π*2 : *x* − *y* + 2*z* − 4 = 0 c) *π* : *x* + 3*y* + *z* = 0.
2. a) *A*1(−1*,*1*,*1) b) *B*1(−3*,*−3*,*−4) c) *C*1(3*,*4*,*4).
3. a) *A*1(6*,*−5*,*0) b) *B*1(−1*,*−1*,*4) c) *C*1(3*,*5*,*−1).
4. a) da b) d(A,, d(A,
5. a) d(A,B)=6 b) d(*M*0*,π*) = 4 c) d(.

 b) d( c) d(

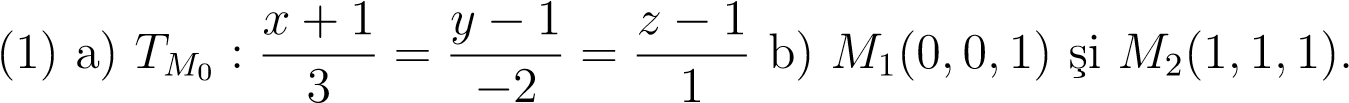
d) d( e) d( f) d(.

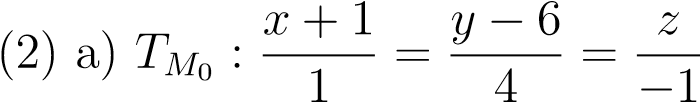
 b)  c) .

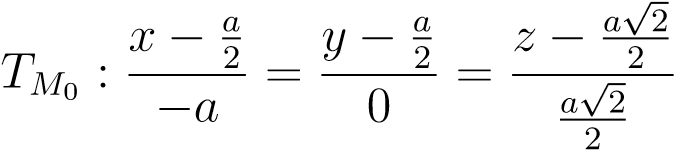
**CAPITOLUL 6 GEOMETRIA DIFERENT¸IALA A CURBELOR S¸I SU-˘**

### PRAFET¸ELOR DIN E3

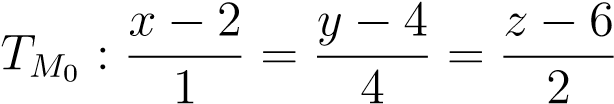
#### Elemente de geometria diferen¸tial˘a a curbelor

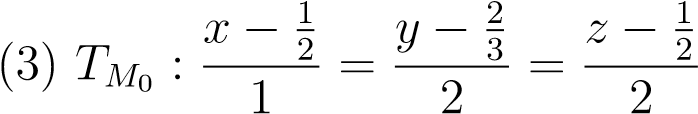
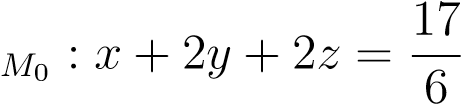


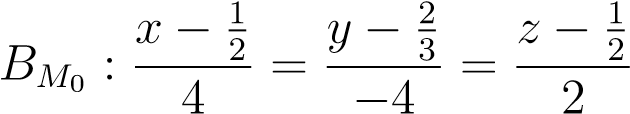
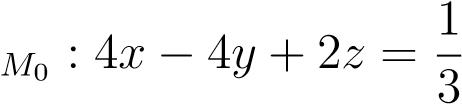
, pl norm*M*0 : *x* + 4*y* − *z* = 23.

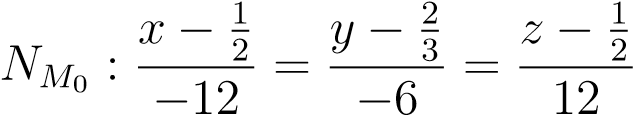
√

b), pl norm*M*0 : 2*x* − *z* = 0.

c)3, pl norm*M*0 : *x* + 4*y* + 3*z* = 36.

, pl norm

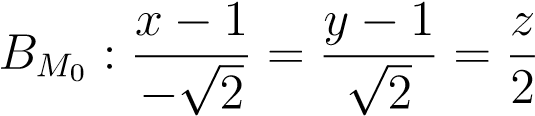
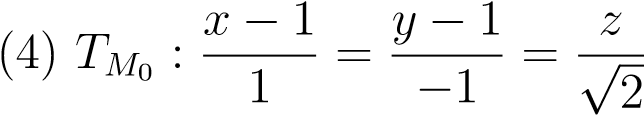
, pl osculator

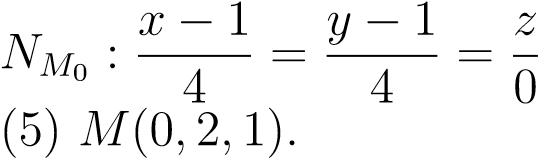
, pl rectificant*M*0 : 6*x* + 3*y* − 6*z* = 2.

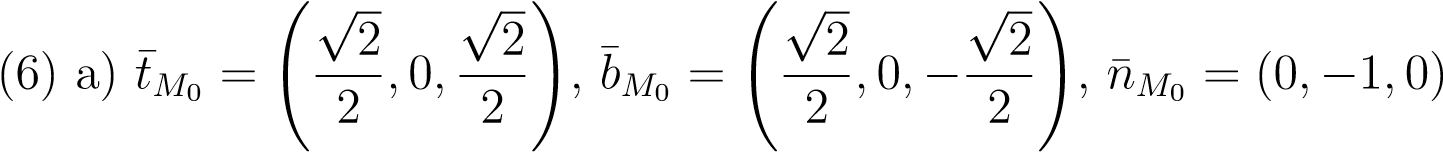
√

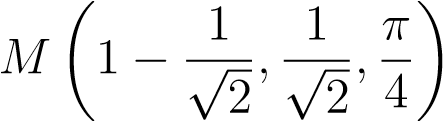
, pl norm*M*0 : *x* − *y* + 2*z* = 0

√

 , pl osculator*M*0 : *x* − *y* − 2*z* = 0

, pl rectificant*M*0 : *x* + *y* = 2.



b).

1. *M*(1*,*−4*,*9).
2. *M*(1*,*ln2*,*−4).