SpotFi: Decimeter Level Localization Using WiFi

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Wonseok

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Overview

- Background
  - We need such a localization system that is:
    - Easy to install (Deployable)
    - Work on any WiFi devices (Universal)
    - With a good enough accuracy (Accurate)
  - …which existing methods failed to achieve
    - ArrayTrack → 6-8 antennas required
    - LTEye → rotatable antennas required
    - Ubicarse → additional sensors required

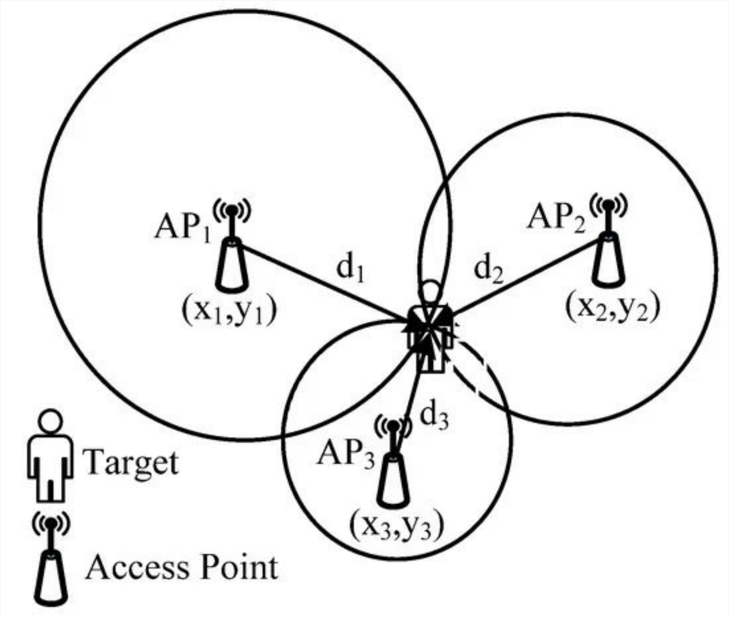
- Goal
  - A more accurate, ubiquitous WiFi localization system

Approaches

- Triangulation Localization
  - By RSSI (Received Signal Strength Indicator)
    - Estimate distance d from at least 3 APs

$$d = d_0 \cdot 10^{\frac{P_t - \mathrm{RSSI}}{10n}}$$

  - By AoA (Angle of Arrival)
    - Intersect lines at AoAs
  - By timestamps
    - Estimate distance d from ToF



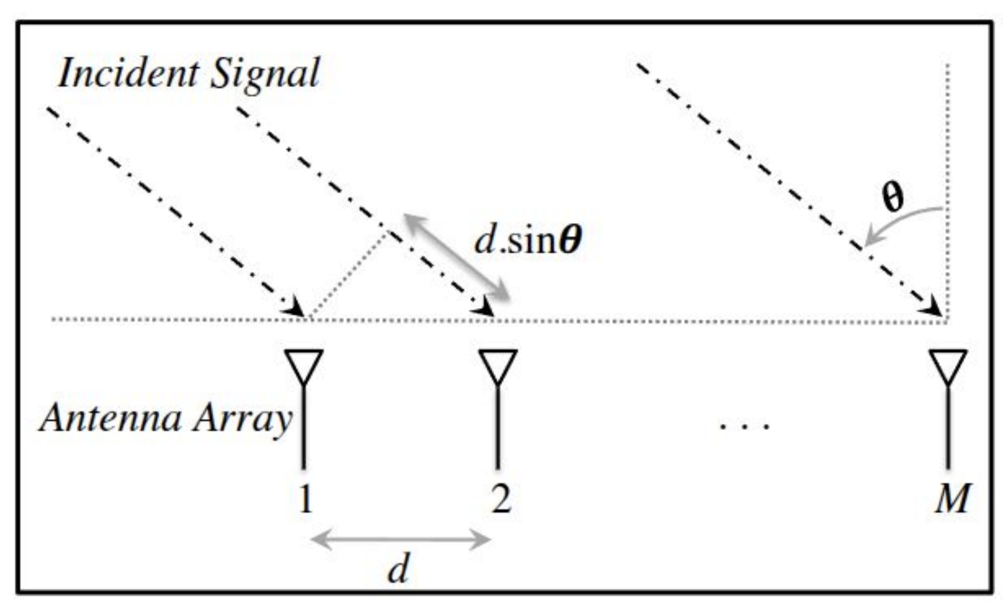
- Fingerprinting Localization
  - Build a RSSI vector map(fingerprint) at known locations
  - Return the location with the most similar fingerprint

⇢ Median accuracy of 0.6m

MUltiple SIgnal Classification

- AoA and CSI
  - *m*-th antenna' phase shift $\Phi^m(\theta_k)$ is given as
    - θ_k : *k*-th propagation path' angle of arrival
    - λ : Wavelength

$$\boxed{\Phi^m(\theta_k) = e^{-j2\pi\frac{d(m-1)\sin(\theta_k)}{\lambda}}}$$



  - A received signal *x* decomposes into *L* propagation paths
    - γ_k: The first antenna's channel attenuation

$$\boxed{x_m = \sum_{k}^{L} \gamma_k \Phi^m(\theta_k)} \longrightarrow \quad x_m = CSI\ !!$$

⇢ **AoA**(θ_k) lies in CSI matrix

CSI Matrix

- CSI Matrix Representation
  - 1. Vectorize received signal *x* into all *M* antennas (1 x M case)

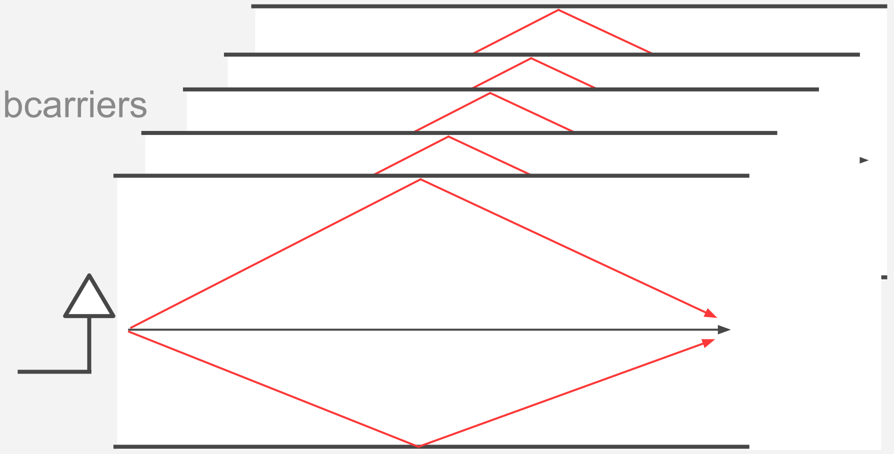
$$\begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_M \end{bmatrix} = \begin{bmatrix} \Phi^1(\theta_1) & \cdots & \Phi^1(\theta_L) \\ \Phi^2(\theta_1) & & \Phi^2(\theta_L) \\ \cdots & & \\ \Phi^M(\theta_1) & & \Phi^M(\theta_L) \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \cdots \\ \gamma_L \end{bmatrix} \Rightarrow \vec{x} = A\vec{\gamma}$$

    – x_m: Received signal at *m*-th antenna
    – *A* : Steering matrix (vectors)

  - 2. Expand it to *N* subcarriers

$$[\vec{x}_1 \quad \cdots \quad \vec{x}_N] = A\,[\vec{\gamma}_1 \quad \cdots \quad \vec{\gamma}_N] \Rightarrow X_{(M\times N)} = A_{(M\times L)}\Gamma_{(L\times N)}$$

    – Steering vectors *A* doesn't change
    – is a CSI matrix for single Tx antenna



Key Idea

- Signal Space, Noise Space

$$\textit{the eigenvectors of } XX^H \textit{ corresponding to the eigenvalue zero,} \\ \textit{are orthogonal to the steering vectors in } A$$

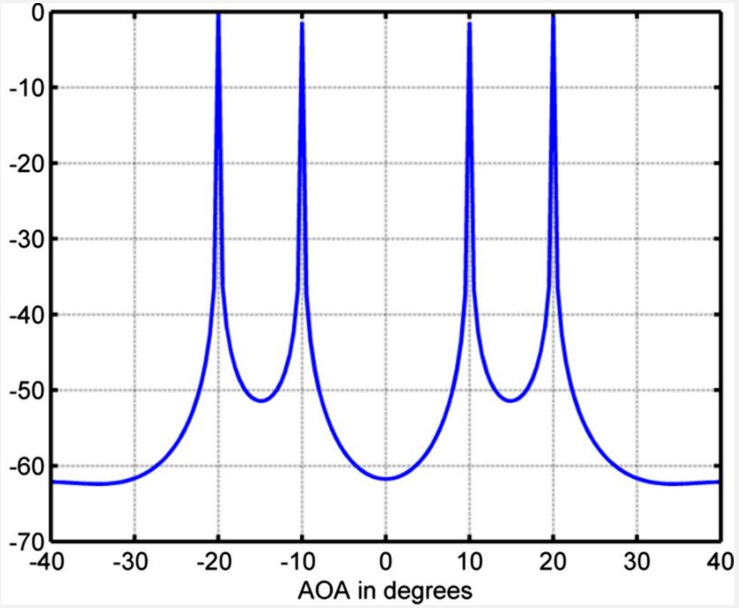
$$u \in N(XX^H) \quad \Rightarrow \quad u \perp R(A)$$

  - 1. Find noise eigenvectors U_n from XX^H's eigen-decomposition
  - 2. Form a steering vector $\vec{a}(\theta)$ with candidate AoA θ
  - 3. Find peaks at **MUSIC Spectrum**

  ⇢ Peaks are the dominant paths' AoA

- MUSIC Spectrum

$$P_{MUSIC}(\theta) = \frac{1}{||U_n^H \vec{a}(\theta)}$$



Limitations

$$X_{(M \times N)} = A_{(M \times L)} \Gamma_{(L \times N)}$$

L: number of propagation paths
M: number of antennas
N: number of subcarriers

- 1. XX^H must have the rank of L
  - $M > L$ from A: **More Rx antennas** than the number of propagation paths
  - $N > L$ from Γ: **More subcarriers** than the number of propagation paths

- 2. MUSIC alone cannot find LoS (direct path)
  - It only measures **orthogonality** between steering vector θ and noise subspace

⇢ Cannot tell which is the direct path among multipaths

Frequency Shift

- Phase Shift Across Frequency
  - Time of Flight(ToF) introduces phase shift across subcarriers
    - Phase shift at *n*-th subcarrier $\Omega^n(\tau_k)$ at a fixed antenna:

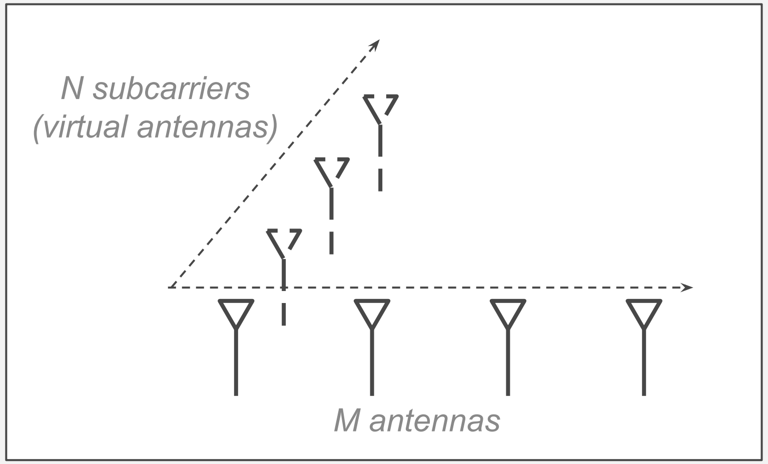
$$\boxed{\Omega^n(\tau_k) = e^{-j2\pi f_\delta (n-1)\tau_k}}$$

f_δ : Frequency spacing between two subcarriers
τ_k : ToF of *k*-th propagation path

⇢ We can extend steering vector $\vec{a}(\theta_k)$ to $\vec{a}(\theta_k, \tau_k)$
- By jointly estimating phase shift with antennas and subcarriers

- Virtual Sensor Array
  - Subcarriers act as virtual spaces
    - Virtually extending *M* antennas to *MN* antennas
  - New steering vector $\vec{a}(\theta_k, \tau_k) \in C^{MN\times 1}$

$$\vec{a}(\theta,\tau) = [\overbrace{1,\ldots,\Omega_\tau^{N-1}}^{\text{antenna 1}}, \underbrace{\Phi_\theta,\ldots,\Omega_\tau^{N-1}\Phi_\theta}_{\text{antenna 2}},\ldots,\overbrace{\Phi_\theta^{M-1},\ldots,\Omega_\tau^{N-1}\Phi_\theta^{M-1}}^{\text{antenna M}}]^\top$$



CSI Smoothing (1)

- Insufficient Rank

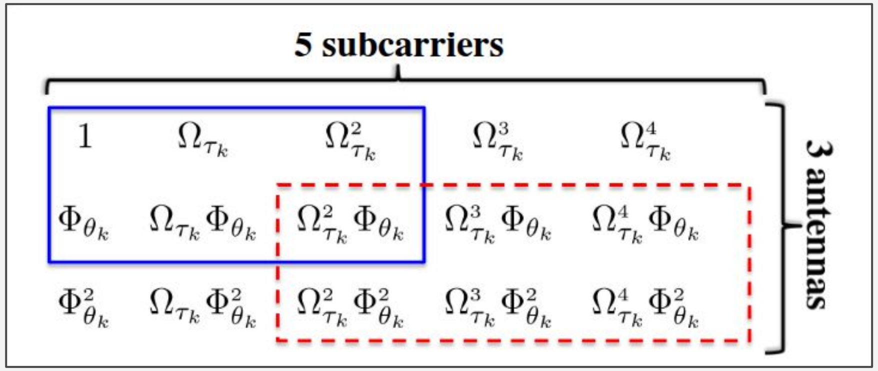
$$\begin{bmatrix} CSI_{1,1} \\ CSI_{1,2} \\ \cdots \\ CSI_{M,N} \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \Omega_{\tau_1}^2 & \Omega_{\tau_2}^2 & & \Omega_{\tau_L}^2 \\ \cdots & & & \\ \Phi_{\theta_1}^M \Omega_{\tau_1}^N & \Phi_{\theta_2}^M \Omega_{\tau_2}^N & & \Phi_{\theta_L}^M \Omega_{\tau_L}^N \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \cdots \\ \alpha_L \end{bmatrix} \longrightarrow \boxed{X_{(MN\times 1)} = A_{(MN\times L)}\Gamma_{(L\times 1)}}$$

  - $rank(XX^H) \leq min(rank(A), rank(\Gamma)) = 1$
    - Only a single (AoA, ToF) pair will be found

⇢ We need a way around to expand $\Gamma_{(L\times 1)}$

CSI Smoothing (2)

- Shifting Trick



$\xrightarrow{L = 2}$

$$\underbrace{\begin{bmatrix} csi_{1,1} & csi_{2,3} \\ csi_{1,2} & csi_{2,4} \\ csi_{1,3} & csi_{2,5} \\ csi_{2,1} & csi_{3,3} \\ csi_{2,2} & csi_{3,4} \\ csi_{2,3} & csi_{3,5} \end{bmatrix}}_{\text{CSI values at the two subarrays}} = \underbrace{\begin{bmatrix} 1 & 1 \\ \Omega_{\tau_1} & \Omega_{\tau_2} \\ \Omega^2_{\tau_1} & \Omega^2_{\tau_2} \\ \Phi_{\theta_1} & \Phi_{\theta_2} \\ \Omega_{\tau_1}\Phi_{\theta_1} & \Omega_{\tau_2}\Phi_{\theta_2} \\ \Omega^2_{\tau_1}\Phi_{\theta_1} & \Omega^2_{\tau_2}\Phi_{\theta_2} \end{bmatrix}}_{\text{Same steering matrix}} \underbrace{\begin{bmatrix} \alpha_1 & \alpha_1\Omega^2_{\tau_1}\Phi_{\theta_1} \\ \alpha_2 & \alpha_2\Omega^2_{\tau_2}\Phi_{\theta_2} \end{bmatrix}}_{\text{Independent vectors of gains}}$$

- Rectangular subarrays can be written as linear combination
  - The subarrays are linearly scaled to each other
    - $\Omega^2_{\tau_k}$ to the column direction (→)
    - Φ_{θ_k} to the row direction (↓)
- Choose the rectangular block $R_{W \times H}$ that ensures $rank(XX^H) \leq L$
  - *(Size of R) > L*
  - *(Total shifts of R)* > L

$$\boxed{X'_{(size(R) \times shift(R))} = A'_{(size(R) \times L)} \Gamma'_{(L \times shift(R))}}$$

Super Resolution MUSIC Algorithm

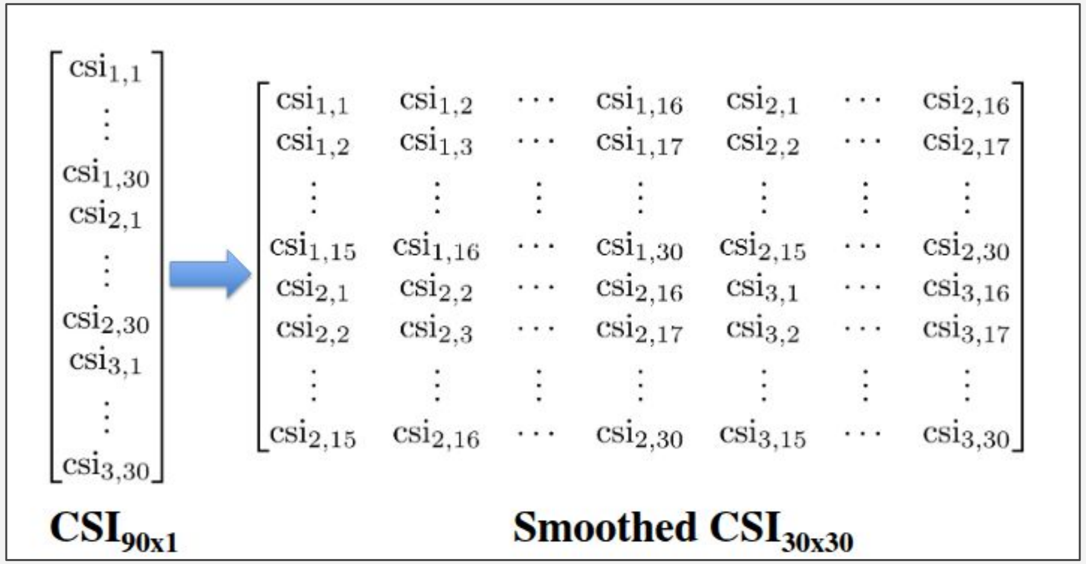
- Smoothed CSI Matrix

$$\boxed{X'_{(size(R)\times shift(R))} = A'_{(size(R)\times L)}\Gamma'_{(L\times shift(R))}}$$

ex) 3 antennas, 30 subcarriers

  - Shift block $R_{2 \times 15}$
  - size(R) = 30 > L
  - shift(R) = 30 > L

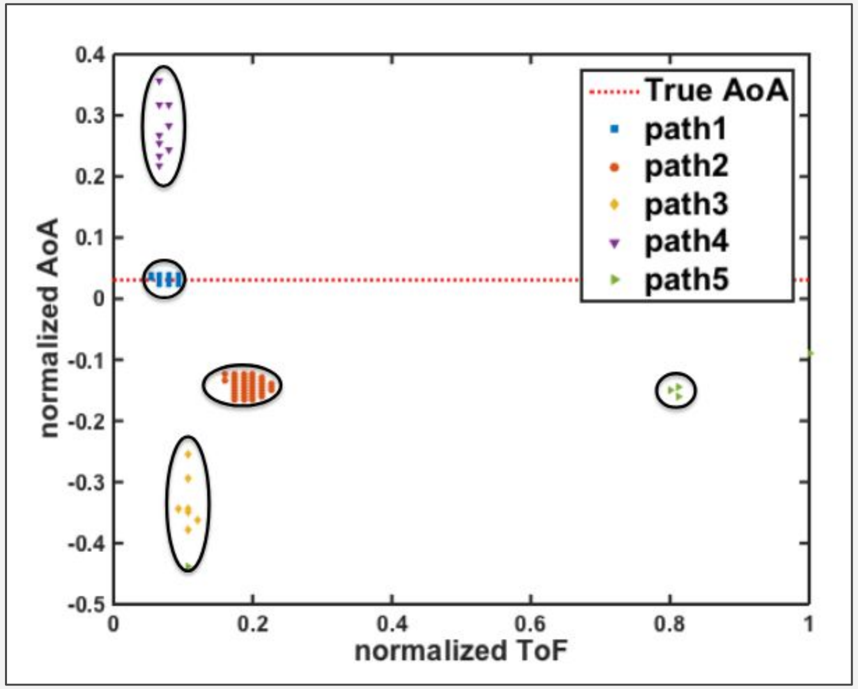
⇢ Smoothed CSI matrix $X'_{30 \times 30}$



- MUSIC Algorithm
  - 1. Find noise vector from $X'X'^H$'s eigen-decomposition
  - 2. Form a steering vector $\vec{a}(\theta,\tau)$ with candidate AoA and ToF
  - 3. Find peaks at **Super Resolution** MUSIC Spectrum $P'_{MUSIC}(\theta,\tau)$

θ τ

Direct Path Likelihood



- Key Idea
  - Use the consistency of AoA-ToF estimates across packets
    - Small variances, small ToF
- AoA-ToF Cluster
  - The same path but different packets will be clustered together
    - The diameter will be a function of variations
- Likelihood Score (Weight)

$$\boxed{\text{likelihood}_k = \exp\left(w_C C_k - w_\theta \sigma_{\theta_k} - w_\tau \sigma_{\tau_k} - w_s \bar{\tau}_k\right)}$$

C_k : Number of points in cluster k

σ_{τ_k}, σ_{θ_k} : Variances of ToF, AoA

$\hat{\tau}_k$: Mean of ToF

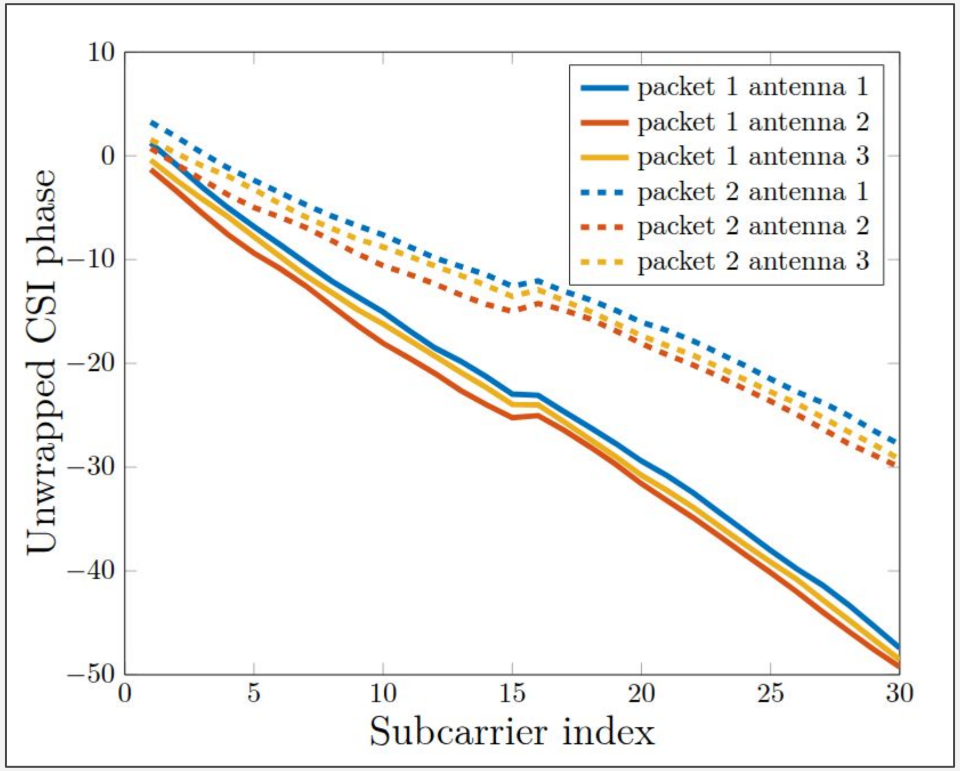
⇢ Highest likelihood = most stable + shortest → likely **direct path**

Modified CSI Phase (1)

- Problem
  - The receiver introduces a random STO (Sampling Time Offset)
    - STO: The receiver samples slightly after the arrival of signal
      - Due to timing unsynchronization

$$\tau^{\text{measured}} = \tau^{\text{true}} + \tau^{\text{STO}}$$

⇢ Inconsistent ToF across packets



- Solution
  - Eliminate the variance due to changing STO
    - ToF sanitization algorithm

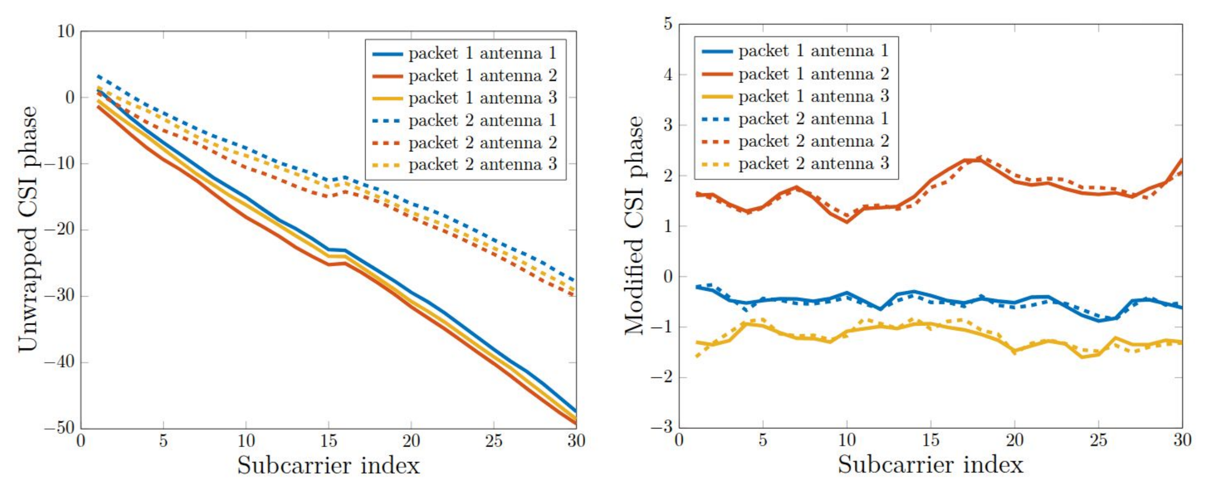
Modified CSI Phase (2)

- Solution
  - ToF sanitization algorithm
    - Obtain the linear trend of unwrapped CSI phase

$$\hat{\tau}_s = \arg\min_{\rho} \sum_{m,n=1}^{M,N} \left(\psi(m,n) + 2\pi f_\delta (n-1)\rho + \beta\right)^2$$

  - Modified CSI Phase
    - Subtract the linear trend from the CSI phase

$$\hat{\psi}_i(m,n) = \psi_i(m,n) + 2\pi f_\delta (n-1)\hat{\tau}_{s,i}$$



(a) Unwrapped CSI phase (b) Modified CSI phase

→ ToF sanitization on CSI matrix + super resolution

$\tau_{s,i}$: STO for i-th packet

$\psi_i(m,n)$: CSI phase response for *i*-th packet at *n*-th subcarrier of *m*-th antenna

Key Idea

- Data
  - Localization
    - AoA (Angle of Arrival)
    - RSSI (Received Signal Strength Indicator)
  - Likelihood Score
    - ToF (Time of Flight)
      - Not used in localization due to delay (STO*)

$$\boxed{d = d_0 \cdot 10^{\frac{P_t - \mathrm{RSSI}}{10n}}}$$

- Objective Function
  - Find the location that minimized the following cost function

$$\boxed{\sum_{i=1}^{R} l_i \left[(\bar{p}_i - p_i)^2 + (\bar{\theta}_i - \theta_i)^2 \right]}$$

R : # of APs

l_i : Likelihood score

$\hat{p}_i, \hat{\theta}_i$: Predicted RSSI, AoA of i-th AP at target location (x,y)

p_i, θ_i : Measured RSSI, AoA of *i*-th AP

*__STO__: Sampling Time Offset

Algorithm

Data: CSI and RSSI measurements

Result: Location of the target (x, y)

for each AP **do**

    for each packet **do**

        (ToF Sanitization) Remove linear fit of CSI phase response

        (CSI Smoothing) Obtain smoothed CSI matrix X

        (Super Resolution) Obtain AoA-ToF pairs using MUSIC algorithm

    end

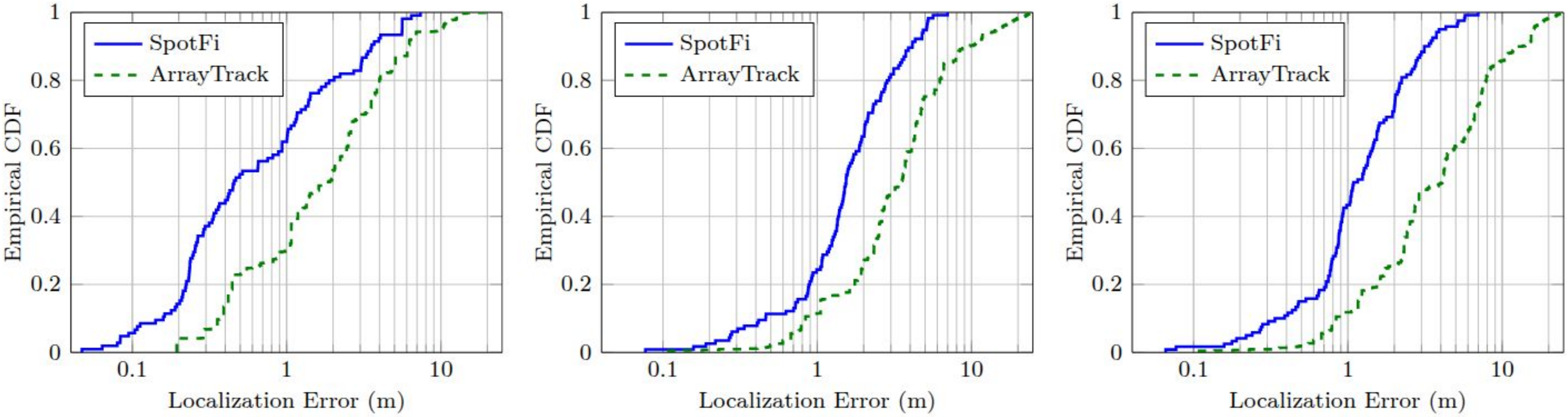
    (AoA-ToF Cluster) Cluster AoA-ToF from multiple packets

    (Direct Path Likelihood) Compute likelihood score and pick the highest one

end

(Localization) Minimize the objective function with predicted location (x, y)

Experiments



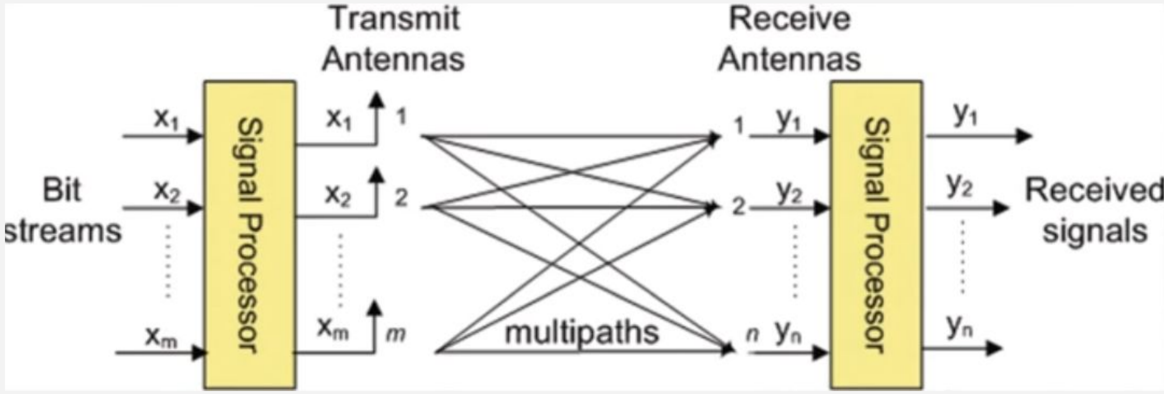
(a) Indoor office deployment (b) High NLoS deployment (c) Corridors

RSSI, CSI

- Received Signal Strength Indicator (RSSI)
  - How strong a signal is when it reaches a receiver
    - P_t : Transmitter's power in dBm
      - Predefined default transmit power (varies by protocols)
    - $L_p(d)$: Path loss approximation at distance *d*

    ex) -20 dBm → roughly 0.01mW at receiver

$$\boxed{\mathrm{RSSI} = P_t - L_p(d)}$$



- Channel State Information (CSI)
  - A matrix H of channel coefficients h_{ij}
  - h_{ij} is affected by both propagation path and frequency

Signal Space, Noise Space

- Without Noise (ideal case)

$$\boxed{XX^H = A\Gamma\Gamma^H A^H \quad \rightarrow \text{rank } L}$$

  - *L* non-zero eigenvalues
  - *M-L* zero eigenvalues
  - *A* spans '**Signal Space'** only, so does XX^H

- With Noise

$$\boxed{XX^H = A\Gamma\Gamma^H A^H + \sigma^2 I}$$

  - All eigenvalues are shifted by σ^2
  - Small eigenvalues of σ^2 are purely by noise

⇢ Thus, null space is said to be '**Noise Space**'

$$X = A\Gamma + N$$

$$\downarrow$$

$$XX^H = (A\Gamma + N)(A\Gamma + N)^H$$

$$\downarrow$$

$$XX^H = A\Gamma\Gamma^H A^H + A\Gamma N^H + N\Gamma^H A^H + NN^H$$

$$\because E(NN^H) = \sigma^2 I \qquad \downarrow \qquad \because E(A\Gamma N^H) = E(N\Gamma^H A^H) = 0$$

$$XX^H = A\Gamma\Gamma^H A^H + \sigma^2 I$$