802.11 with Multiple Antennas for Dummies

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Wonseok

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Wireless Channels

- Limited Capacity
  - The max performance is determined by Shannon's Limit
    - **C**: Capacity (bit/sec/Hz)
    - **B**: Bandwidth (20 Mhz in 802.11a/g)
    - **S/N**: Signal to Noise Ratio in power
  - Doubling of signal power yields in + 1 bit/sec/Hz

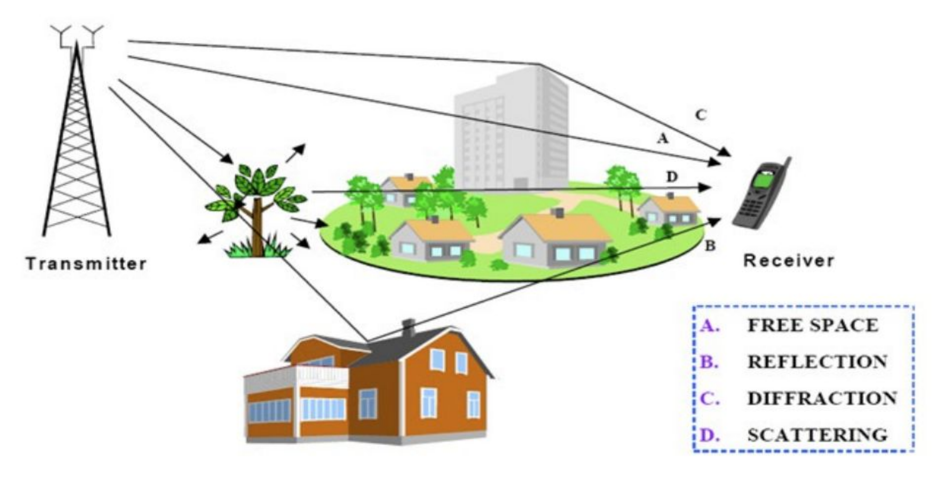
$$C = B \log_2 (1 + \frac{S}{N})$$

⇢ The key for speed had lied in **SNR**

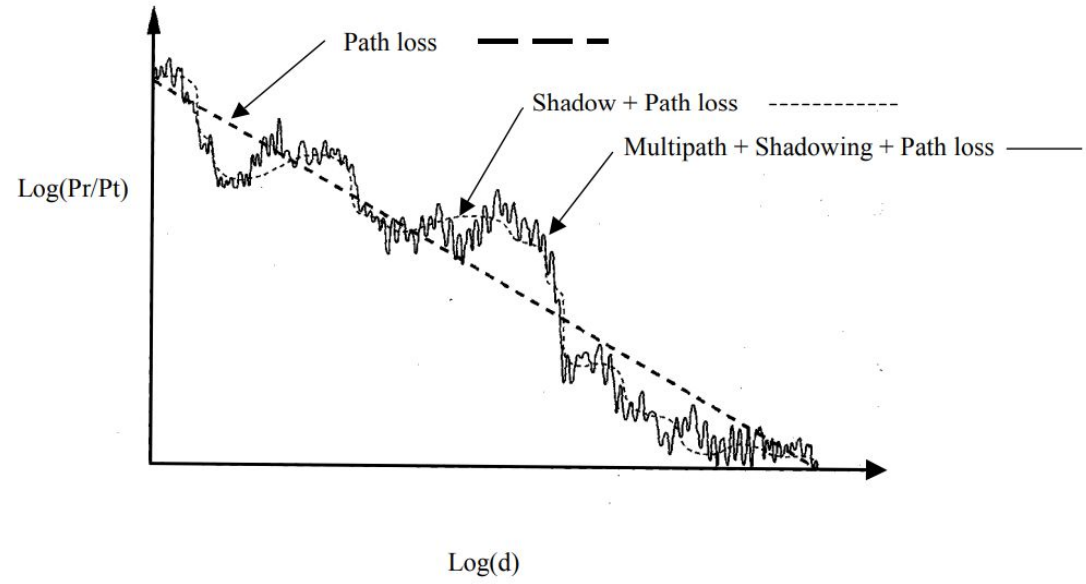
- Attenuation (Weakening)
  - Slow fading
    - Gradual fluctuations of received power over long periods of time
    - *ex) path loss, shadowing*
  - Fast fading
    - Large swings in received power over relatively short periods
    - *ex) multi-path, doppler effect*

Attenuation

- Multi-Path
  - Multi-path effects depend on the phases of signals
    - Some unlucky frequencies in 20 MHz may be wiped out
      - ⇢ Frequency Selective
  - Independent for locations separated by half a wavelength
    - Half a wavelength = 180 degrees of shift = 6cm for 2.4Ghz
    - Ref. Clarke's fading model

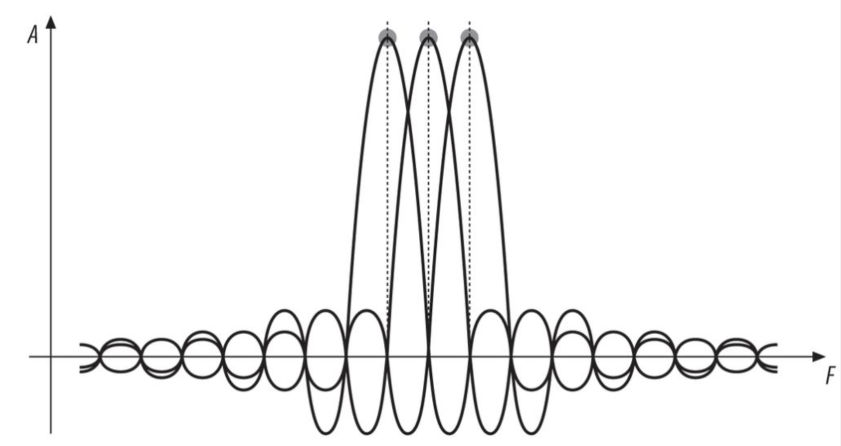


- Attenuated Signal
  - Slow fading + fast fading
  - Signal fluctuates in effect of multipath and doppler

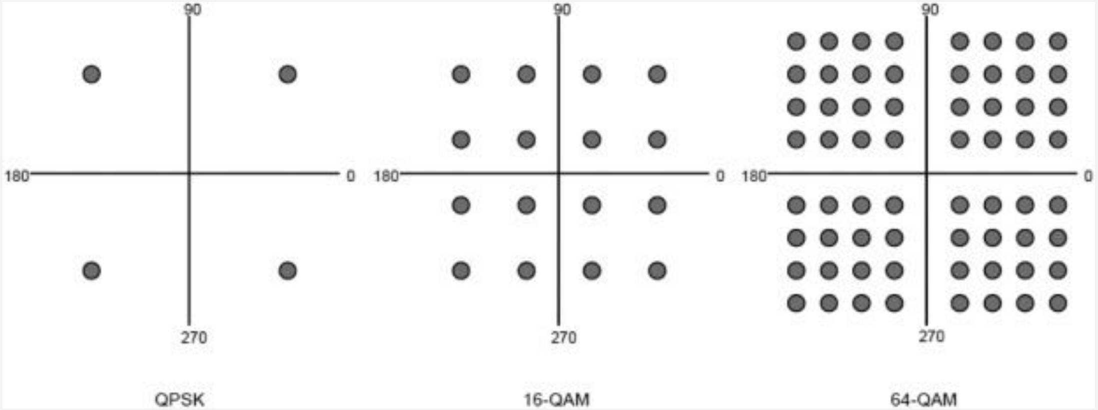


OFDM

- Orthogonal Frequency Division Modulation
  - 20 Mhz into 64 subcarriers
    - 48 subcarriers for data transfer
    - 4 subcarriers for pilot tones
    - 12 subcarriers for guard (unused)
  - Subcarriers are orthogonally divided in frequency domain
    - All other subcarriers do not contribute to the carrier's waveform (amplitude)
  - OFDM benefits from frequency, time and spatial *diversity*
    - *Diversity*: spreading of information with redundancy

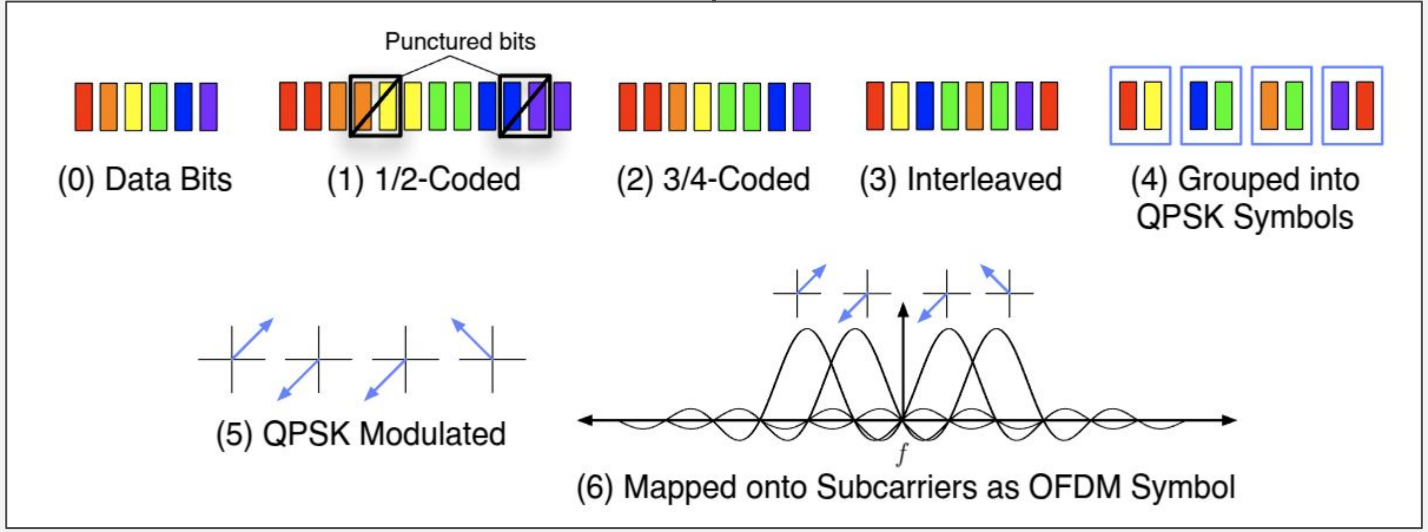


- Quadrature Amplitude Modulation (QAM)
  - M-ary bit modulation scheme in IQ* plane
    - *ex) 16-QAM(4 bits), 32-QAM(5 bits), ...*
  - Each symbol is carried to a subcarrier



Diversity Benefits of OFDM

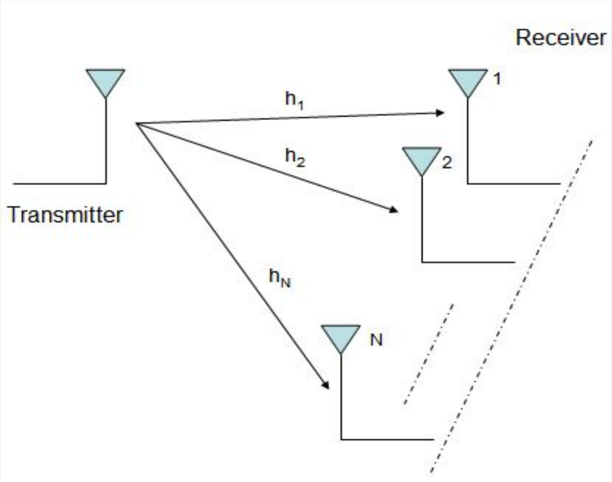
- Frequency diversity
  - Different frequencies, different fading
    ⇢ *Frequency selective*
  - Spread data bits into multiple subcarriers
    – **Coding scheme**: adds redundant bits by coding rate R
    – **Interleaving**: spread the redundancy across subcarriers
  ⇢ Less likely dropping a frame out of deep fade



- Time diversity
  - Slower symbols in parallel
    – 312.5 kHz per symbol
      - $T_{SYM} = \frac{1}{312.5k} = 3.2\mu s$
  - Instead of fast symbols on a wideband
  ⇢ Less likely to be distorted by quick change

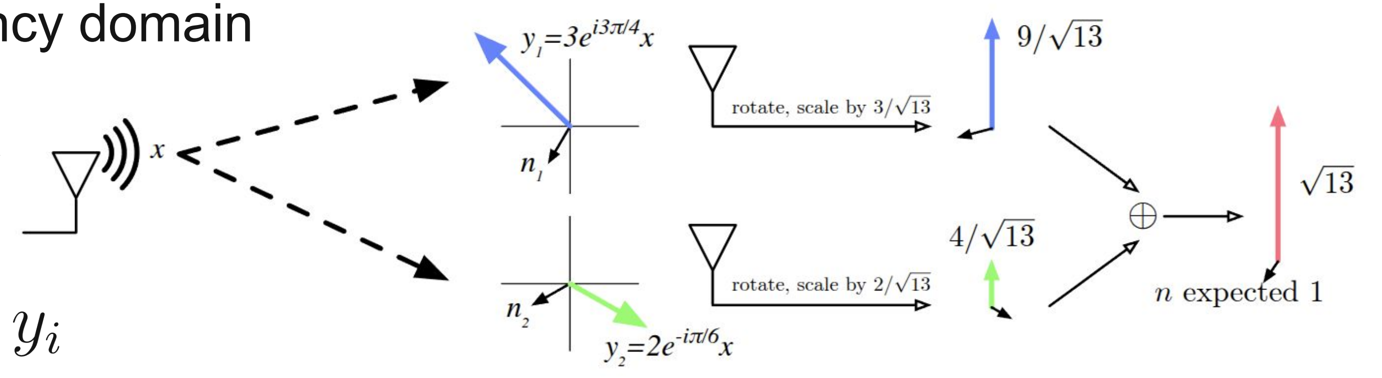
Spatial Diversity - Receiver

- Selection Combining (SEL)
  - Select the antenna with the strongest SNR*
    - all the other antennas are ignored
  - Standard method of 802.11a/g APs



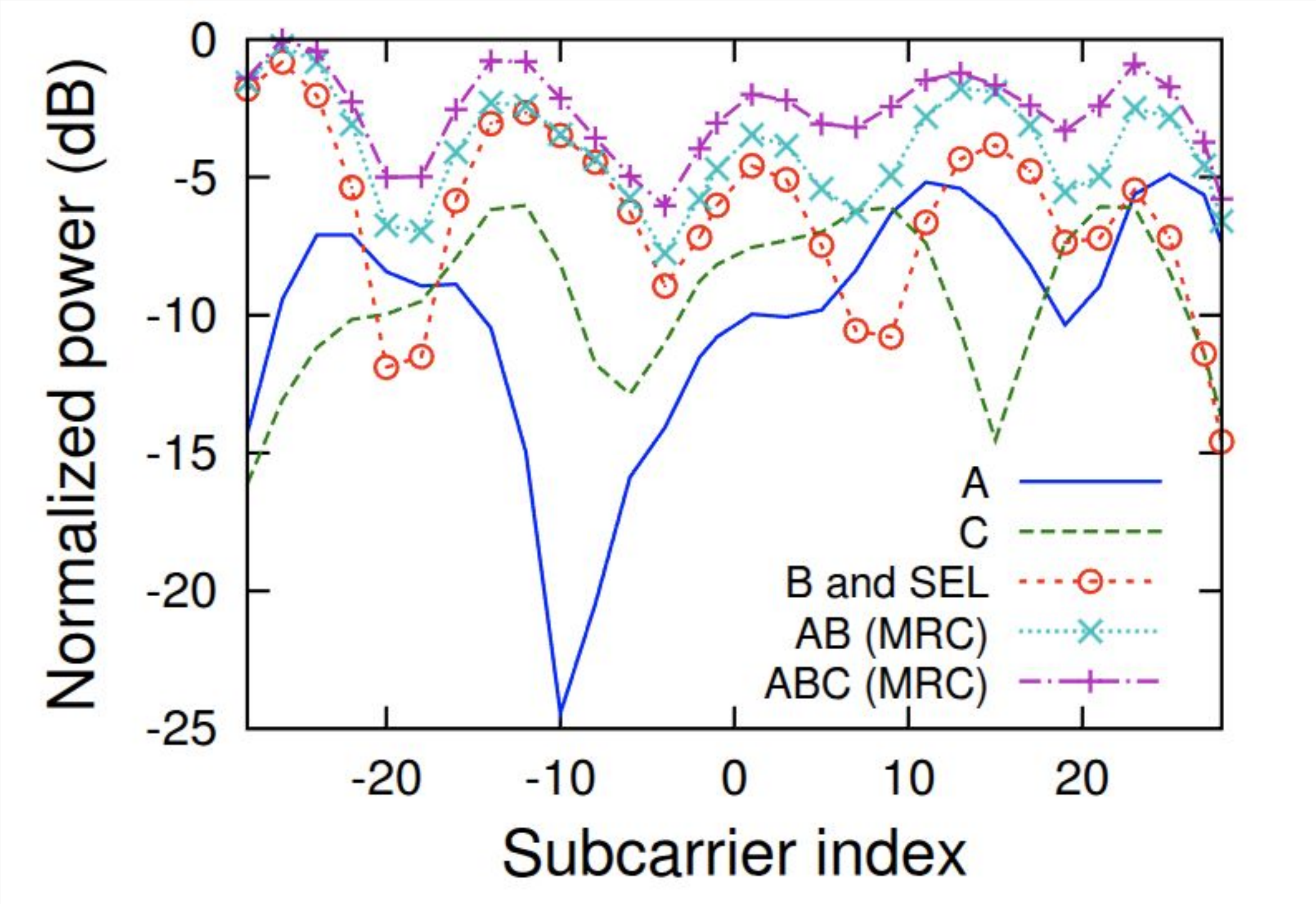
- Maximum Ratio Combining (MRC)
  - 1. Estimate the channel gains H in frequency domain
    - By computing from preamble's training field
  - 2. Rotate and scale the received signals Y
    - By applying the complex conjugate of h
  - 3. Sum up the aligned channel responses y_i

⇢ MRC is known to be optimal to maximize SIMO capacity



*SNR: Signal to Noise Ratio

Comparison: SEL and MRC

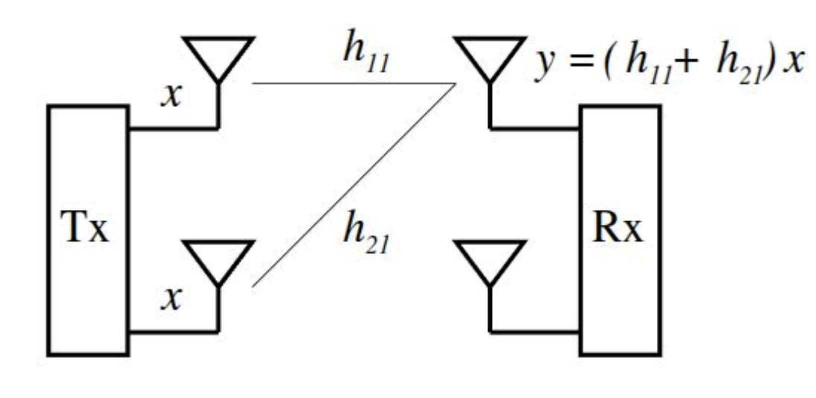


- Antenna A has the worst fluctuations
  - The signal variation of 20 dB
- Antenna B is chosen in SEL
  - The signal variation of 15 dB
- Weighted sum of signals in MRC
  - The signal variation of 5 dB

Spatial Diversity - Transmitter

- Selection Combining (SEL)
  - Select the antenna with the strongest SNR*

- Maximum Ratio Combining (MRC)
  - Channel state feedback
    - Transmitter must know channel's state beforehand
      - Alternatively, transmitter can learn the channel gains when it receives a packet
  - Precoding
    - Phase adjustment (based on phase shift of channel)
    - Amplitude weighting (based on SNR)
  - Beamforming
    - The signals combine constructively at the receiver's antenna



*SNR: Signal to Noise Ratio

Direct-Mapped MIMO

- Direct-Mapped
  - Transmitter is blind to its channel → directly sends data stream without precoding

- Zero Forcing (ZF)
  - Eliminate interference H to zero
  - H is not always invertible in practice
    - $|H| \to 0, \quad \text{then} \quad H^{-1} \to \infty$
      - ⇢ **Noise amplification** problem

$$\vec{y} = H\vec{x} + \vec{n} \quad \longrightarrow \quad H^{-1}\vec{y} = \vec{x} + \underbrace{H^{-1}\vec{n}}_{\text{noise}}$$

- Minimum Mean Squared Error (MMSE)
  - Find H that minimizes MSE between $\vec{x}$ and $W\vec{y}$
    - Minimizing the total error of interference and noise together
  - MMSE suppresses noise amplification problem

$$\vec{y} = H\vec{x} + \vec{n} \quad \longrightarrow \quad \vec{x} \approx W\vec{y}$$

$$argmin_W \mathrm{MSE} = \mathbb{E}\left[\|\mathbf{x} - \mathbf{Wy}\|^2\right]$$

$$\downarrow$$

$$W = \left(H^H H + \sigma_n^2 \mathbf{I}\right)^{-1} H^H$$

Precoded MIMO

- Challenge of Direct-mapped
  - Paths are assumed to be uncorrelated → not often
    - LoS(Line of Sight) makes H correlated
      - Noise amplification problem

- Solution
  - Use Singular Value Decomposition for precoding
    - U : unitary matrix (receiver-side rotation)
    - S : diagonal matrix (scaling factor)
    - V : unitary matrix (transmitter-side rotation)

      ⇢ Use V for precoding

$$\boxed{H = USV^H}$$

Precoded MIMO (2)

- Operation

Transmitter: $$\hat{x} = Vx$$

↓

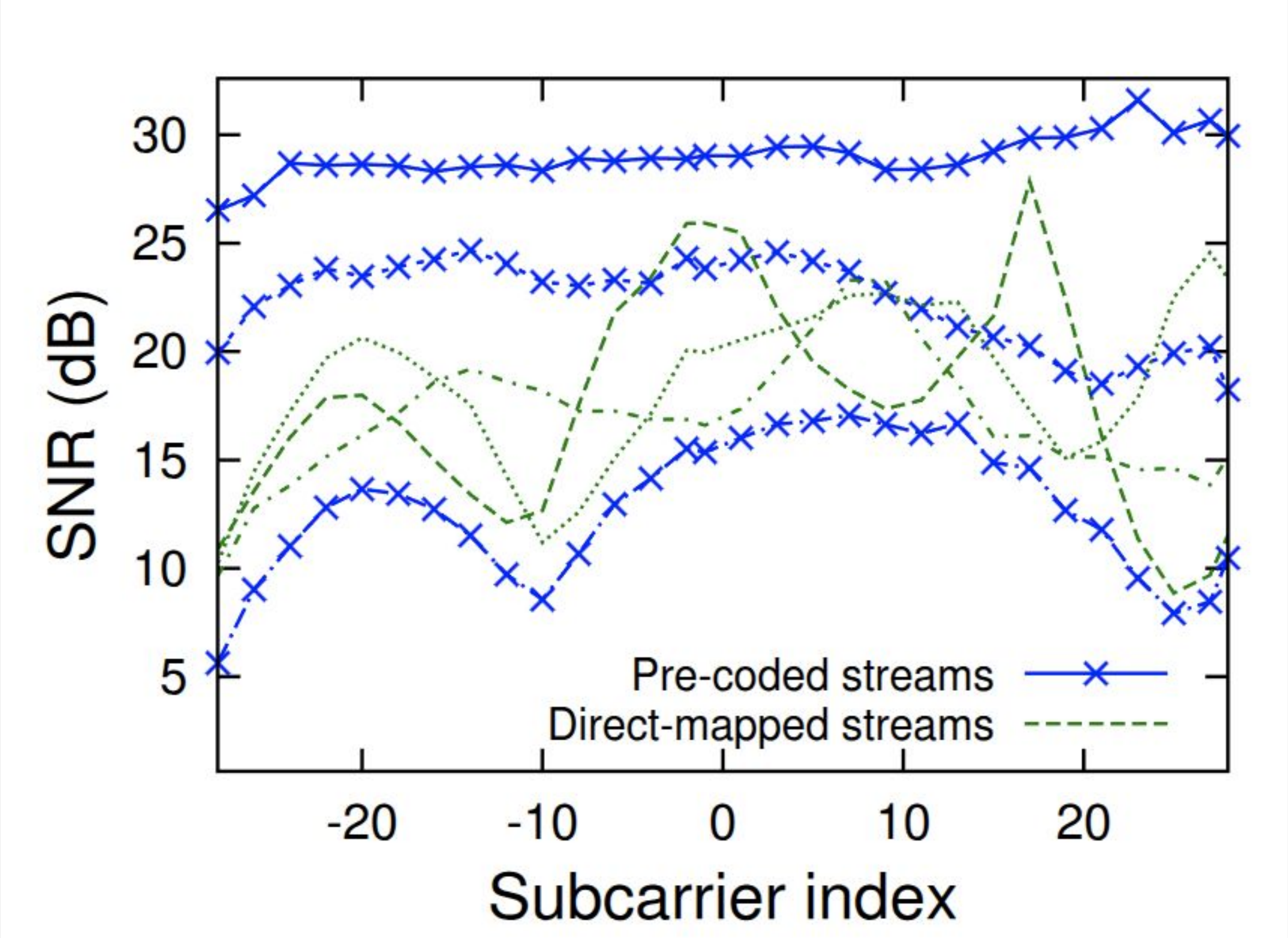
Channel: $$y = H\hat{x} + n = (US)x + n$$

↓

Receiver: $$\hat{y} = U^H y = Sx + U^H n$$

- No noise amplification
  - U is unitary matrix → only rotates the noise

Comparison: Direct-mapped and Precoded



Doppler Effect

- Frequency/Phase Shift
  - v : speed of the receiver (m/s)
  - λ : wavelength of the signal (m)
  - θ : angle to direction of motion

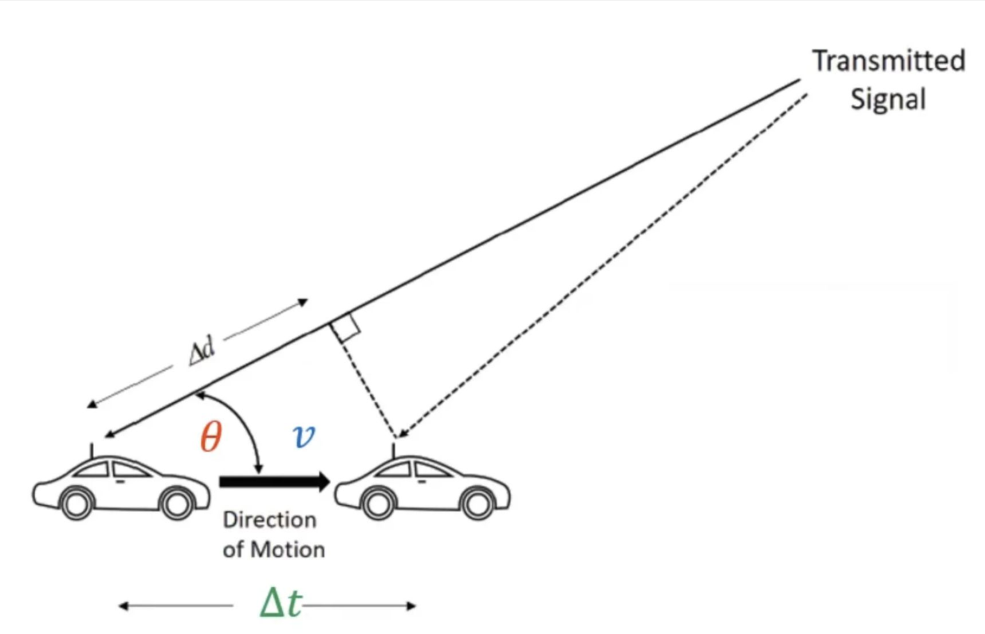
$$\boxed{f_D = \frac{v}{\lambda} cos\theta}$$

$$(\because \frac{\Delta\phi}{\Delta t} = w = 2\pi f_D,\ \Delta\phi = \frac{2\pi}{\lambda}\Delta d)$$

- Example on OFDM
  - 64 QAM at 2.4 GHz band
    - $\lambda = 0.125(m)$
    - $v = 100km/h \approx 30m/s$
    - $T_{SYM} = 4\mu s$

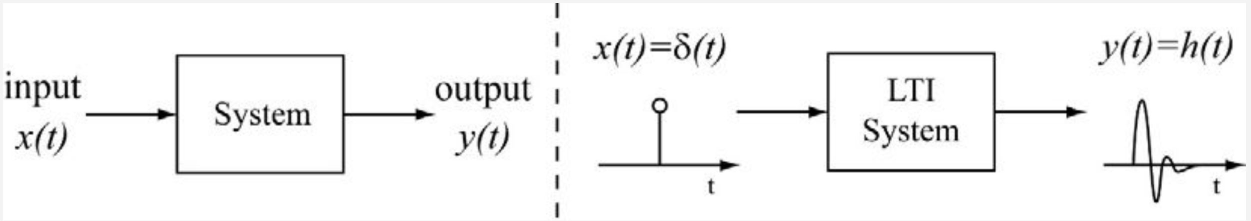
      $\dashrightarrow f_D = 240Hz$

      $\Delta\phi = 2\pi f_D T_{\mathrm{sym}} \approx 0.006\,\mathrm{rad}\,(\approx 0.34°)$
  - if 100 OFDM symbols, then <u>34 degrees</u> of shift



LTI System

- Linear Time Invariant (LTI)
  - Linear relationship between input and output
    - Additivity and homogeneity(scaling)
  - The output is shifted as well as the input
    - $x(t-t_0) \xrightarrow{system} h(t-t_0)$
  - The system corresponds to a channel in wireless communication



- Impulse Response h(x)
  - System's response when the input is Dirac Delta $\delta(t)$
  - Impulse response completely defines LTI's behavior
    - By convolving the continuous input with h(x)
  - 'Channel gain' is the impulse response in frequency domain

$$\delta(t-\tau) \rightarrow h(t-\tau)$$

Homogeneity

$$x_i\delta(t-t_i) \rightarrow x_i h(t-t_i)$$

Additivity

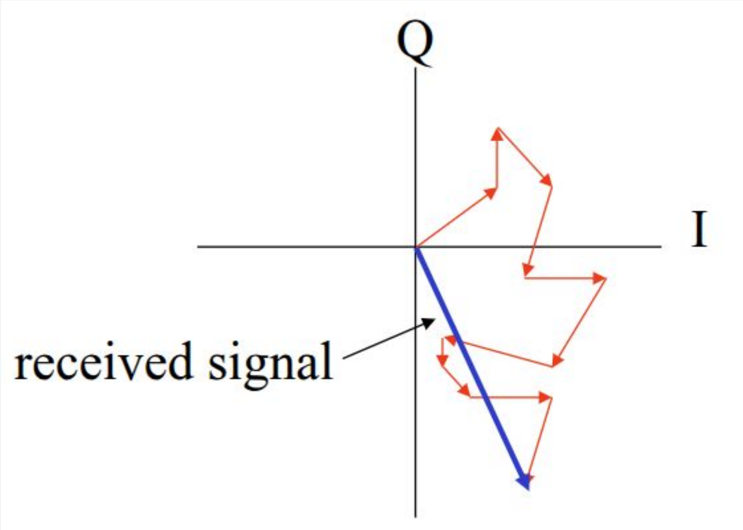
$$x(t) = \sum x_i\delta(t-t_i) \rightarrow \sum x_i h(t-t_i)$$

$\Delta t \rightarrow 0$

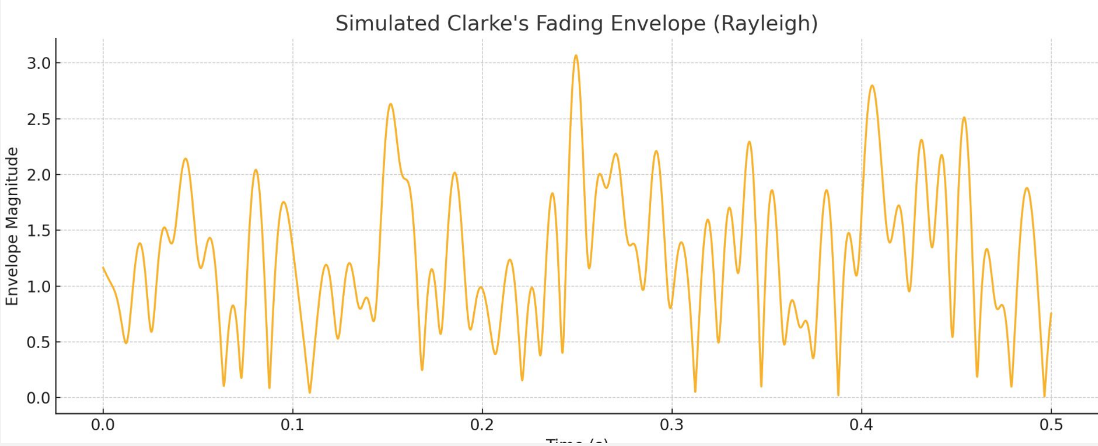
$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau \rightarrow y(t) = x(t) * h(t)$$

Multipath Fading Models

- Rayleigh's Fading
  - No dominant line-of-sight path
  - Many multipath components
  - Amplitudes follow Rayleigh Distribution
    - $R = \sqrt{Z_1^2 + Z_2^2},\ Z \sim N(0,1)$
  - Phases follow uniform distribution (random)



- Clarke's Model
  - Theoretically based on Rayleigh's fading
  - The model assumes receiver is moving
    - Doppler effect is added to Rayleigh Distribution
  - The model gives fading waveform over time

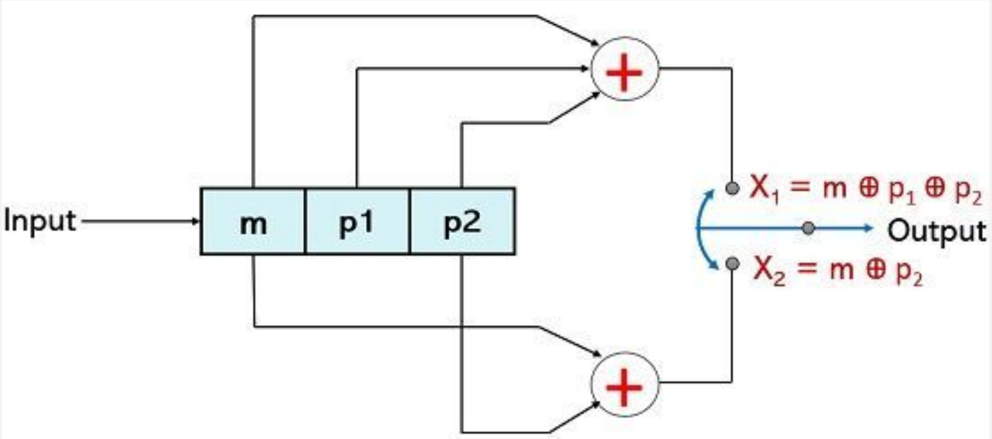


Convolutional Code

- Forward Error Correction (FEC)
  - Let receiver detects and corrects error bits

    ↔ Backward Error Correction (BEC)
    - Receiver only detects, then requests re-transmission

- XOR encoder with shifting
  - Constraint length **K**
    – The encoder remembers K bits at a time (K=3 for m, p1, p2)
  - Coding rate **R**
    – The number of output functions (R=2 for X_1, X_2)
  - Shift, multiply and add : convolution-like



Input	Register	Output (X_1X_2)
1	100	11
0	010	10
1	101	00

ex) 101 -> 00 10 11, K=3, R=1/2