1) Prove ~~the~~ DeMorgan's theorem

a) $\overline{(A+B)} = \overline{A} \cdot \overline{B}$

Soln: Lets assume $A=1, B=0$

$\overline{A+B} = \overline{1+0} = \overline{1} = 0$

and $\overline{A} \cdot \overline{B} = \overline{1} \cdot \overline{0} = 0 \cdot 1 = 0$

Therefore $\overline{A+B} = \overline{A} \cdot \overline{B}$

B) $\overline{A \cdot B} = \overline{A} + \overline{B}$

Soln: Lets assume $A=1, B=0$

$\overline{A \cdot B} = \overline{1 \cdot 0} = \overline{0} = 1$

and $\overline{A} + \overline{B} = \overline{1} + \overline{0} = 0+1 = 1$

Therefore $\overline{A \cdot B} = \overline{A} + \overline{B}$

2) Simplify the following expressions using DeMorgan's

a) $\overline{\overline{A}B\overline{C}}$ Breaking longest bar into two places: Between 1st and 2nd terms and between 2nd and 3rd terms.

$\overline{\overline{A}} + \overline{B} + \overline{\overline{C}}$

Then applying the identity $\overline{\overline{x}} = x$ to $\overline{\overline{A}}$ and $\overline{\overline{C}}$

$\therefore A + \overline{B} + C$

Applying DeMorgan's theorem

$A\overline{B}C$

2D) $\overline{\bar{A} + \bar{B}C}$

Breaking longest bar

$\bar{\bar{A}}\ \overline{\bar{B}C}$

Breaking the shorter bar:

$\bar{\bar{A}}(\bar{\bar{B}} + \bar{C})$

Applying $\bar{\bar{X}} = X$ to $\bar{\bar{A}}$ and $\bar{\bar{B}}$

$\bar{\bar{A}}(\bar{\bar{B}} + \bar{C}) = A \cdot (B + \bar{C})$

Using distributive law

$(A \cdot B) + (A \cdot \bar{C})$

2E) $\overline{\bar{A}\ \bar{B}}$

Breaking longest bar

$\bar{\bar{A}} + \bar{\bar{B}}$

Applying $\bar{\bar{X}} = X$ to $\bar{\bar{A}}$ and $\bar{\bar{B}}$

$A + B$

2F) $\overline{A + \bar{B}}$

Breaking longest bar

$\bar{A}\ \bar{\bar{B}}$

Applying $\bar{\bar{X}} = X$ to $\bar{\bar{B}}$

AB

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b) $\overline{(M+\overline{N})(\overline{M}+N)}$

Breaking the longest bar:

$\overline{(M+\overline{N})}+\overline{(\overline{M}+N)}$

Breaking the two shorter bars:

$(\overline{M}\,\overline{\overline{N}})+(\overline{\overline{M}}\,\overline{N})$

Applying the identity $\overline{\overline{x}}=x$ to $\overline{\overline{N}}$ and $\overline{\overline{M}}$

i.e. $\overline{M}N+M\overline{N}$

c) $\overline{A\overline{(B+\overline{C})}D}$

Breaking the longest bar

$\overline{A\overline{(B+\overline{C})}}+\overline{D}$

Breaking the 2nd longest bar

$\overline{A}+\overline{\overline{(B+\overline{C})}}+\overline{D}$

Breaking the last longest bar

$\overline{A}+(\overline{\overline{B}}\,\overline{\overline{\overline{C}}})+\overline{D}$

applying $\overline{\overline{x}}=x$ to $\overline{\overline{B}}$ and $\overline{\overline{\overline{C}}}$

Therefore:

$\overline{A}+(\overline{\overline{B}}\,\overline{\overline{\overline{C}}})+\overline{D}=\overline{A}+(B\overline{C})+\overline{D}$

Applying DeMorgan's theorem

$\overline{A}+(B\overline{C})+\overline{D}$ using distributive law on $\overline{A}$ and $B\overline{C}$

$(\overline{A}+B)\cdot(\overline{A}+\overline{C})+\overline{D}$

3A) $X = (M+N)(\bar{M}+P)(\bar{N}+\bar{P})$

Using the consensus law: $(x+y)(\bar{x}+z)(y+z) = (x+y)(\bar{x}+z)$

$\therefore X = (M+N)(\bar{M}+P)$

B) $Y = (\bar{A}B\bar{C} + AB\bar{C} + B\bar{C}D)$

Using associative law $a+b+c = a+(b+c) = (a+b)+c$

$a = \bar{A}B\bar{C} \quad b = AB\bar{C} \quad c = B\bar{C}D$

$\therefore Y = a+(b+c)$

$Y = \bar{A}B\bar{C} + (AB\bar{C} + B\bar{C}D)$

3c) $Z = \bar{A}(A+B) + (B+AA)(A+\bar{B})$

Applying Idempotent law on AA: $AA = A$

$Z = \bar{A}(A+B) + (B+A)(A+\bar{B})$

$\bar{A}\cdot(A+B) = \bar{A}A + (\bar{A}+B) =$

And $\bar{x}\cdot x = 0$ (Compliment law)

$\bar{A}A + (\bar{A}+B) = 0 + (\bar{A}+B) = \bar{A}+B$

$\therefore Z = (\bar{A}+B) + (B+A)(A+\bar{B})$

let $a = (\bar{A}+B) \quad \bar{a} = (A+\bar{B}) \quad b = (B+A)$

$\therefore Z = a + \bar{a}b$

Applying redundancy law $(x\bar{y}+y = x+y)$:

$Z = a+b = (\bar{A}+B)+(B+A)$

$\therefore Z = (\bar{A}+B)+(B+A)$